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AN EVALUATION OF THE USE OF HIGH-RESOLUTION WAVENUMBER SPECTRA FOR AMBIENT-NOISE ANALYSIS

Special Report No. 8

ADVANCED ARRAY RESEARCH

SCIENCE SERVICES DIVISION

TEXAS INSTRUMENTS
INCORPORATED
SCIENCE SERVICES DIVISION
13500 NORTH CENTRAL EXPRESSWAY

GLOSSARY OF ACRONYMS

CPO	Cumberland Plateau Observatory
MCF	Multichannel Filter
SNR	Signal-to-Noise Ratio

Air Force Technical Applications Center, Wash, D.C.

VELA T/7701

AN EVALUATION OF THE USE OF HIGH-RESOLUTION WAVENUMBER SPECTRA FOR AMBIENT-NOISE ANALYSIS

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Prepared by

William A. Johnson James A. Bonner

John P. Burg, Program Scientist

George D. Hair, Program Manager

Telephone: 1-214-238-3473

TEXAS INSTRUMENTS INCORPORATED
Science Services Division
P.O. Box 5621
Dallas, Texas 75222

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SECTION I
INTRODUCTION

This report describes the results of an experiment to compare and evaluate several approaches to computing high-resolution wavenumber spectra of seismic noise. The experiment was conducted prior to simultaneous ambient noise-field studies at each array station constituting the network of stations. Frequency-wavenumber spectra are extremely useful for determining the frequency and velocity structure of the noise field existent at an array station and may be used to locate and describe all the components of the ambient noise field. Principal objectives of the ambient noise study were to determine

- The degree to which various network stations see energy from the same noise sources
- The mechanism and characteristics of these noise sources
- The relative proportions of bodywave, surface wave, and random noise present at each station

An understanding of these characteristics is a necessary prerequisite to the development and implementation of network-processing procedures for upgrading seismic event detection, location, and identification.

The results of the network noise analysis are presented in a companion report, Advanced Array Research Special Report No. 6.[1] Included in that report are studies of the spatial distribution of coherent noise at five array stations in the northern hemisphere. A principal tool used in those studies is the frequency-wavenumber spectral estimation technique determined best suited for noise-field analysis as a result of the experiment described in this report.

Since the network array stations were all of small aperture, conventional techniques for computing wavenumber spectra could not achieve the wavenumber resolution required to resolve and accurately locate multiple noise sources, especially bodywave noise sources. The high-resolution approach to wavenumber analysis has previously been successfully demonstrated for transient signals; however, it is also applicable to coherent seismic noise studies. Objectives of this experiment were to establish the validity of the high-resolution wavenumber spectral estimate for wavenumber analysis of coherent seismic noise and to determine those parameters necessary for optimum implementation of the technique for such analysis.

SECTION II
THE EXPERIMENT

A recently developed technique[2] for computing frequency-wavenumber spectra with very high wavenumber resolution has been successfully used to locate event epicenters through the computation of wavenumber spectra from short-period recordings of their P waves. The technique, although equally valid for computing wavenumber spectra for noise fields, had been little used for studying the composition of ambient noise; there was reason to believe that the parameters for optimum implementation of the technique for transient-signal wavenumber location might not be optimum for the case of several rather diffuse and quasi-stationary noise sources.

The reported experiment was designed to compare and evaluate several approaches to computing high-resolution wavenumber spectra of ambient noise. A single noise sample recorded at CPO on a day of intense and well-defined storm activity (16 October 1964) along the eastern seaboard was used for all comparisons.

The CPO station was selected on the basis of extensive prior study of the ambient noise there;* that study had shown the noise to consist of a very characteristic and time-stationary component plus a component directly attributable to storms on or near the North American continent. With strong storms located at Cape Hatteras (tropical storm Isabel) and on the Newfoundland coast, the belief was that the noise field at CPO that day could be easily predicted and that the prediction could be considered in establishing the validity of the various techniques being compared.

The sample chosen was a 6.67-min sample of noise recorded by nine short-period vertical-component seismometers; Z5 was not usable.

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*Contract No. AF 33(657)-14618

The low-resolution conventional technique and several variations of the high-resolution technique were used to compute wavenumber spectra for this noise sample. Figure 1 is a chart of the various techniques and the methods to estimate the auto- and crosspower density spectra used as input.

While many interesting comparisons are possible from such a variety of approaches, those most pertinent to the purposes of this experiment are

- The conventional wavenumber spectrum computed using a set of crosspower spectra derived by Fourier transformation of correlation functions compared with one computed using crosspower spectra derived by direct Fourier transformation of array outputs
- Comparison of high-resolution spectra obtained from the same two sets of crosspower spectra
- Comparison of high-resolution spectra computed with respect to different reference sensors
- Comparison of high-resolution spectra obtained by two methods of averaging spectra computed with respect to different reference sensors
- Averaged high-resolution spectra compared with individual high-resolution and conventional spectra
- Comparison of high-resolution spectra computed using different signal-to-noise ratios (SNR) for design of the multichannel filter which is the basis of the high-resolution technique
- High-resolution spectra computed at frequencies other than 1 Hz from data equalized on the basis of 1-Hz calibrations compared with those computed from frequency-dependent normalized data

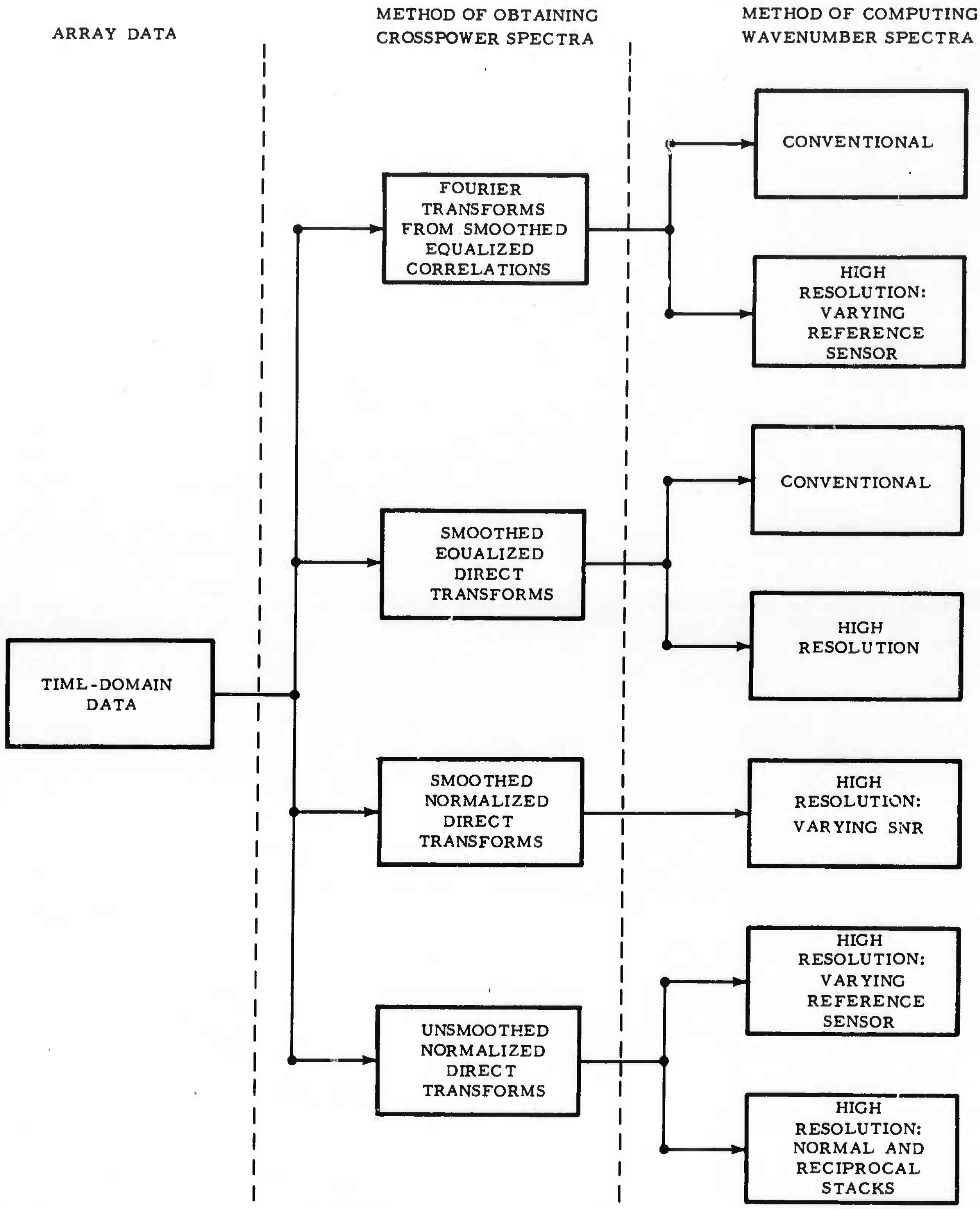

Figure 1. Comparison of Wavenumber Spectral Estimation Techniques

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SECTION III

CONVENTIONAL VS HIGH-RESOLUTION TECHNIQUES

Conventional and high-resolution methods are compared to establish their suitability for noise analysis. Figure 2 (a, b, and c) displays wavenumber spectra computed at 0.3, 0.5, and 1.0 Hz, respectively, from smoothed correlation and smoothed direct-transform crosspower matrices for both techniques.

Correlations are computed for ±5 sec over 6.67 min of data amplitude equalized at 1.0 Hz, Parzen-smoothed, and Fourier-transformed, resulting in a frequency resolution of 0.1 Hz. The data traces also are Fourier-transformed directly over the same data gate and are Danielle-smoothed to yield a 0.1-Hz frequency resolution. High-resolution wavenumber spectra are computed using Z3 as the reference sensor and an SNR of 0.1.

Each spectrum indicates coherent energy from the storm east of the station (low-pressure center 1, Figure 3). At 0.3 Hz, excellent agreement in wavenumber location is seen for all methods, with a marked improvement in wavenumber resolution with the high-resolution technique. Although the storm center is located almost due east of the station, all spectra indicate energy from slightly south of east. Similarly, when using correlation-derived spectra, agreement is good in both azimuth and velocity at 0.5 Hz. The conventional technique using smoothed direct-transform data agrees well with the correlation-derived spectra. The high-resolution technique using smoothed direct-transform data locates the peak slightly farther south, although there is also an indication of energy from the east. The effect is somewhat more pronounced in Figure 2c, in which spectra at 1.0 Hz are shown. Again, the conventional wavenumber spectra agree well, locating the energy source slightly south of east.

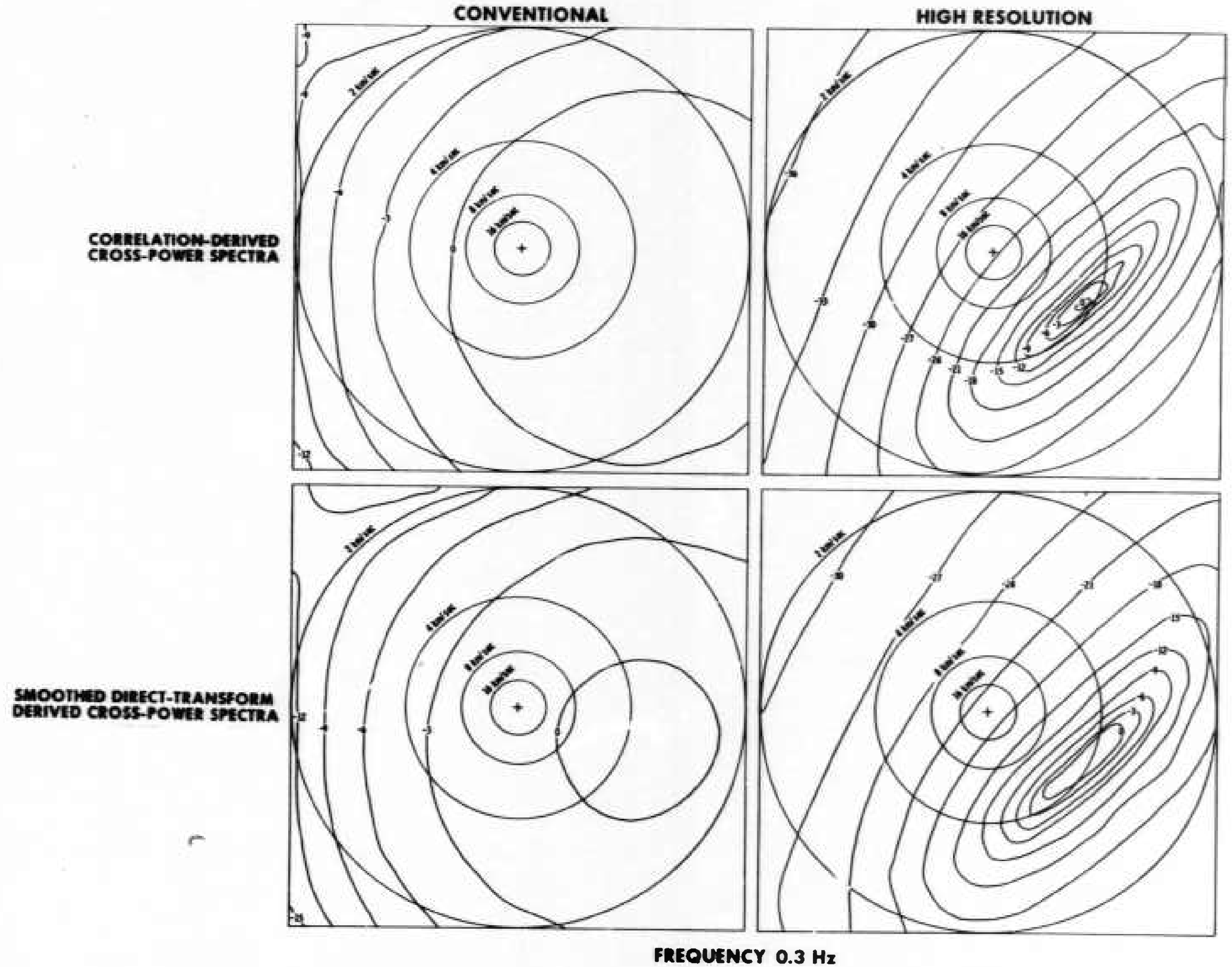

Figure 2a. Comparison between Conventional and High-Resolution Wavenumber Spectra Using Correlation-Derived and Direct-Transform-Derived Crosspower Matrices: F = 0.3 Hz

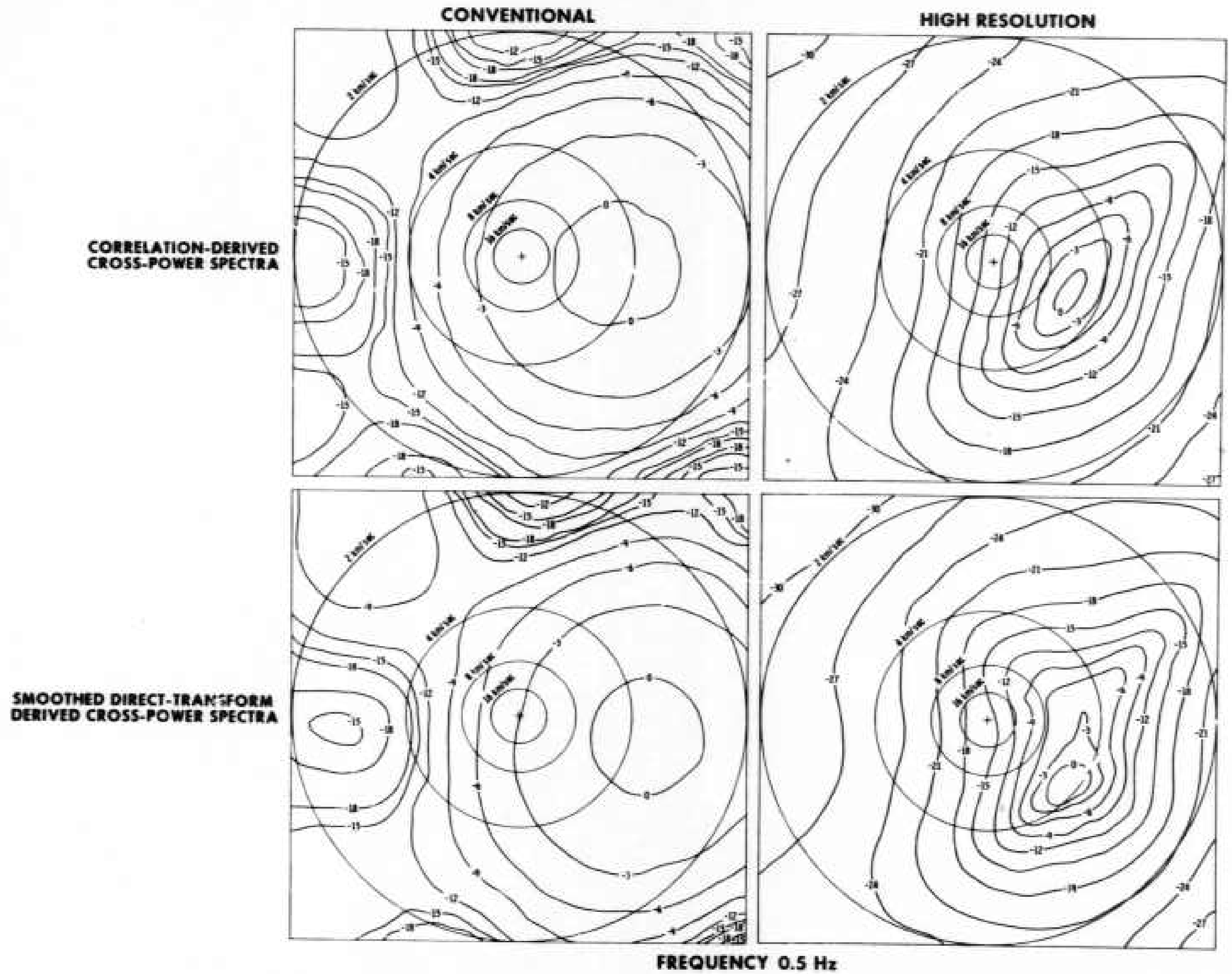

Figure 2b. Comparison between Conventional and High-Resolution Wavenumber Spectra Using Correlation-Derived and Direct-Transform-Derived Crosspower Matrices: F = 0.5 Hz

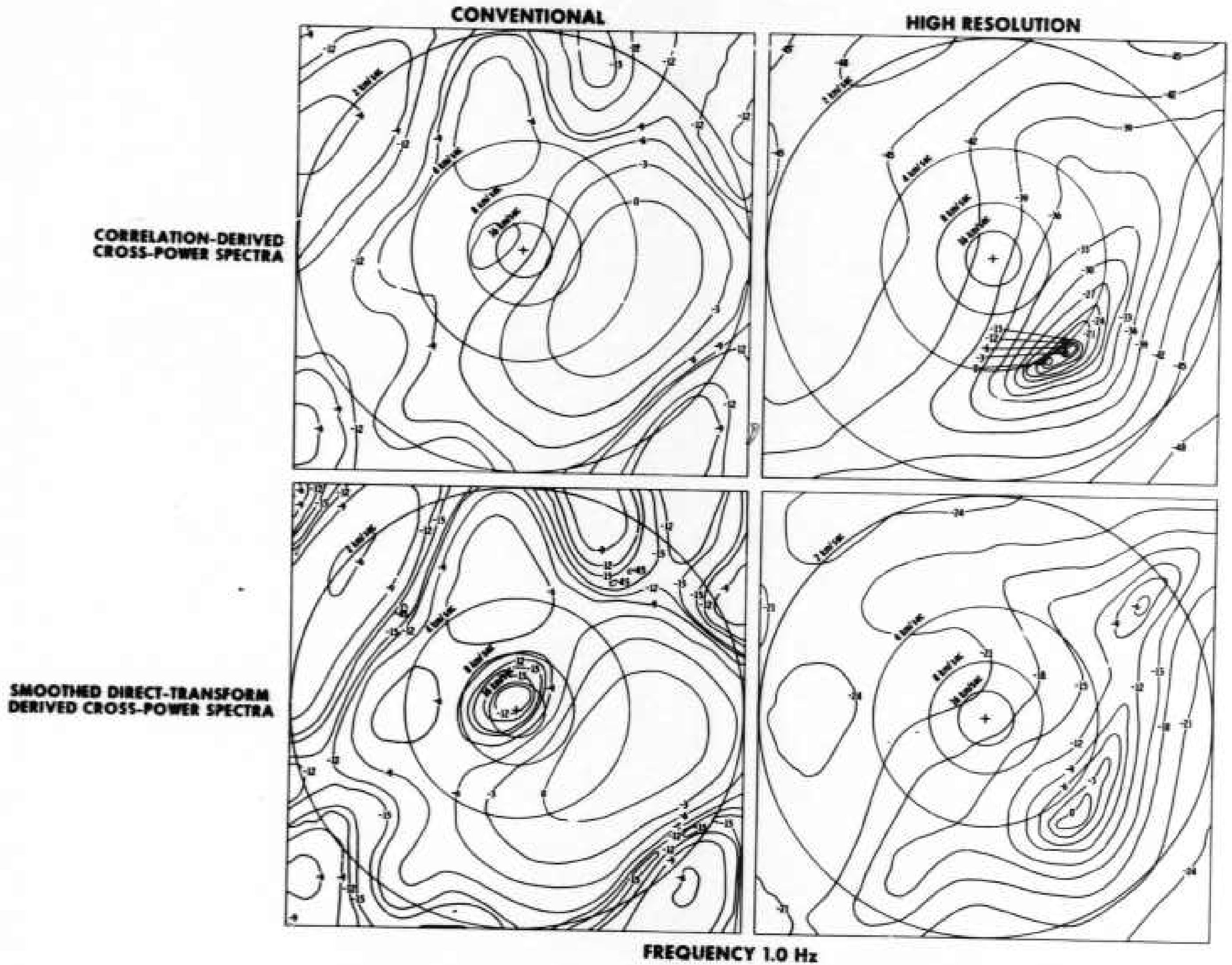

Figure 2c. Comparison between Conventional and High-Resolution Wavenumber Spectra Using Correlation-Derived and Direct-Transform-Derived Crosspower Matrices: F = 1.0 Hz

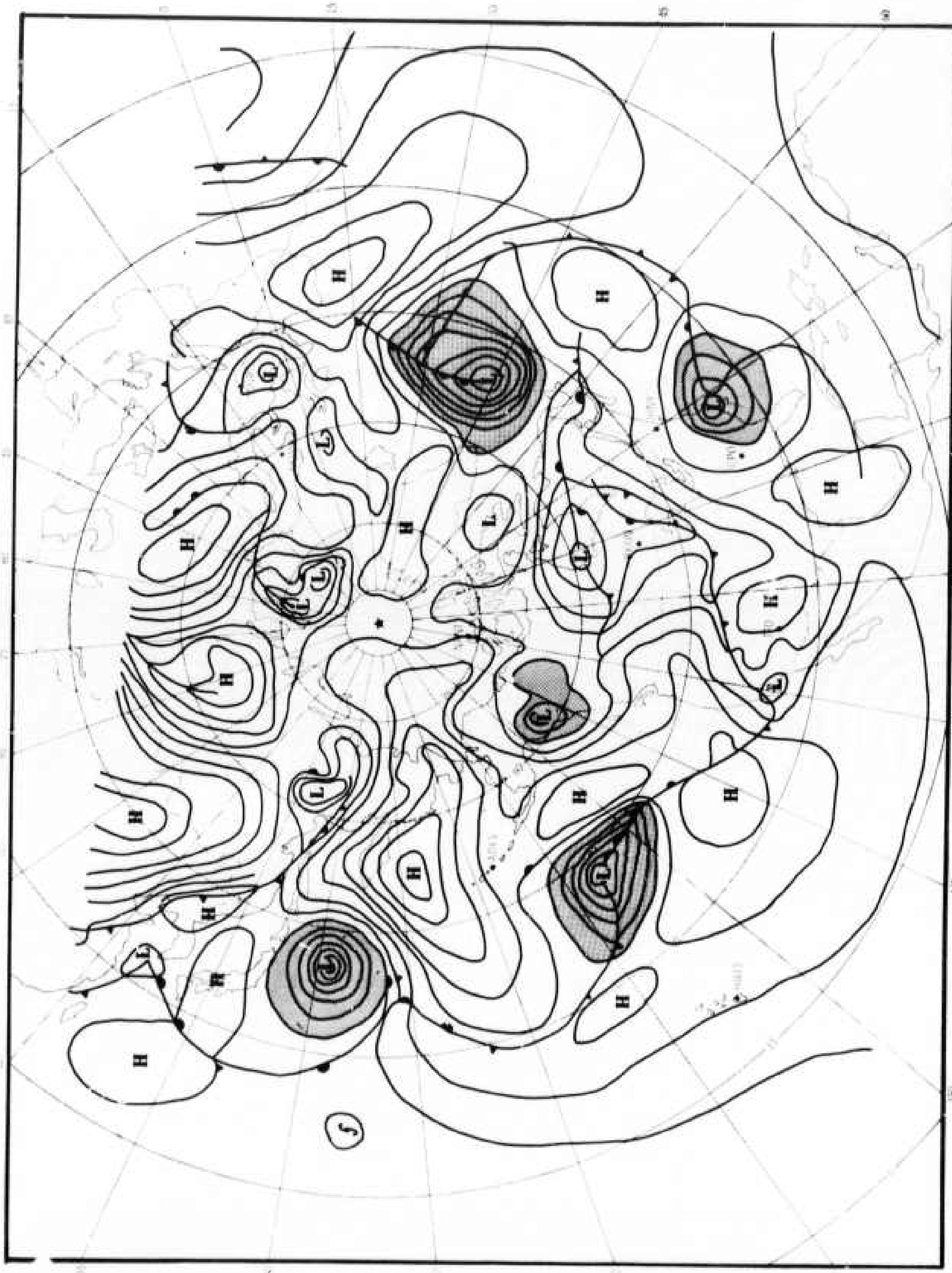

Figure 3. Surface Weather Map, 16 October 1964 (1800 GMT)

The high-resolution spectra tend to locate the peak further south, with indication of additional energy from the northeast. This effect, probably caused by the choice of reference sensor and/or SNR, is explored further in subsection D of Section IV.

Figure 4 shows the CPO array geometry used.

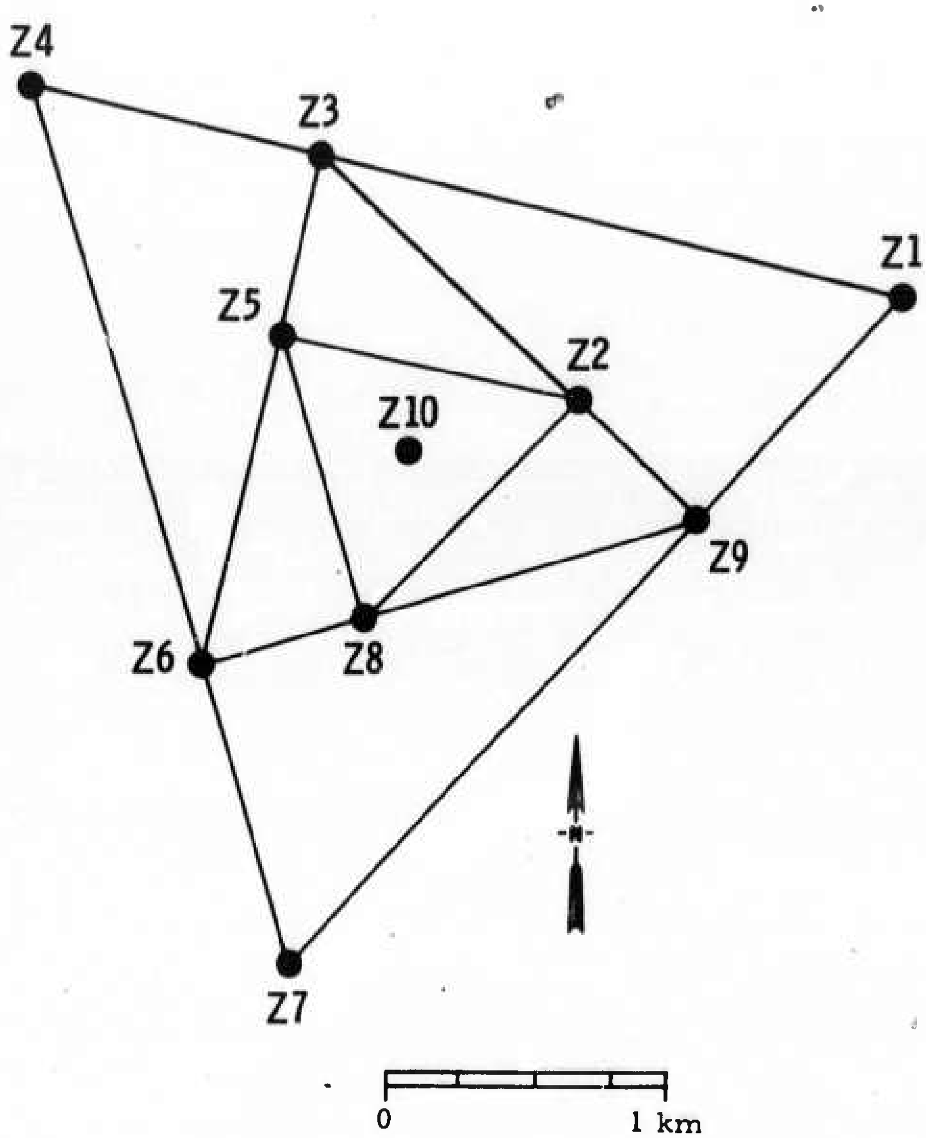

Figure 4. CPO Array Geometry

Direct-transform methods are computationally simpler and yield comparable results; therefore, correlation-derived crosspower spectral density matrices are abandoned in favor of direct-transform-derived crosspower spectral density matrices.

The generally good agreement of conventional and high-resolution wavenumber spectra and of both with known noise sources establishes the validity of the high-resolution approach to wavenumber spectra of ambient noise. The slight spectral variations observed for different high-resolution approaches suggest that more investigation of the proper parameters for high-resolution computation is in order.

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SECTION IV

COMPARISON OF SEVERAL TECHNIQUES FOR COMPUTING HIGH-RESOLUTION WAVENUMBER SPECTRA

The various parameters of the high-resolution technique investigated — all of which have some degree of effect on the wavenumber spectral estimate — are

- The means of compensating for intra-array inequalization
- The SNR used in multichannel-filter (MCF) design
- The method of estimating and smoothing the crosspower-density spectra
- The choice of reference sensor for MCF design

Investigated also are two approaches to the smoothing of the wavenumber spectra to average out the distorting effects of inequalization and nonspace-stationary components in the noise field. The first approach averages the wavenumber power responses of several MCF's, each designed with respect to a different reference sensor. The other approach averages the wavenumber spectra computed with respect to several different reference sensors. Since each high-resolution wavenumber spectrum is simply the reciprocal of the wavenumber power response of an MCF designed to whiten as a function of wavenumber, these two approaches amount to averaging either before or after taking the power-response reciprocal.

A. EQUALIZATION VS NORMALIZATION

One method of compensating for intra-array inequalizations is to multiply each trace by a scalar constant proportional to the amplitude response of the corresponding seismometer as determined by calibration.

This method is simple and easy to implement, but the calibration information is usually only available at one frequency and is only valid over a small frequency range on either side of the calibration frequency; thus, it does not compensate correctly at other frequencies. As an additional disadvantage, such inequalization causes as earth filtering and seismometer-to-earth coupling are not taken into account.

Conversely, another method which requires the normalization of crosspower matrices does not require any a priori information, takes all effects into account, and is performed independently at each frequency. If the time traces are somewhat well amplitude-equalized, these two procedures should yield approximately equivalent results.

Crosspower matrices are normalized according to the equation

$$\Phi'_{ij} = \Phi_{ij} \Big/ \sqrt{\Phi_{ii}^2 \cdot \Phi_{jj}^2}$$

where Φ_{ii} is the autopower density of the i^{th} trace and Φ_{ij} is the measured crosspower density between the i^{th} and j^{th} traces.

High-resolution wavenumber spectra are computed at 0.3 Hz from both normalized and amplitude-equalized (at 1.0 Hz) crosspower matrices of smoothed direct transforms. Z3 is the reference sensor, and the filter-design SNR is 0.1.

Figure 5 displays the wavenumber spectra obtained and indicates that the methods compare favorably, although they are not computed at the equalization frequency. This agreement implies that amplitude equalization differences between channels are small and that either process can adequately compensate for them. Normalization is preferable, however, since the required scale factors are readily available and no manipulation of time traces is required.

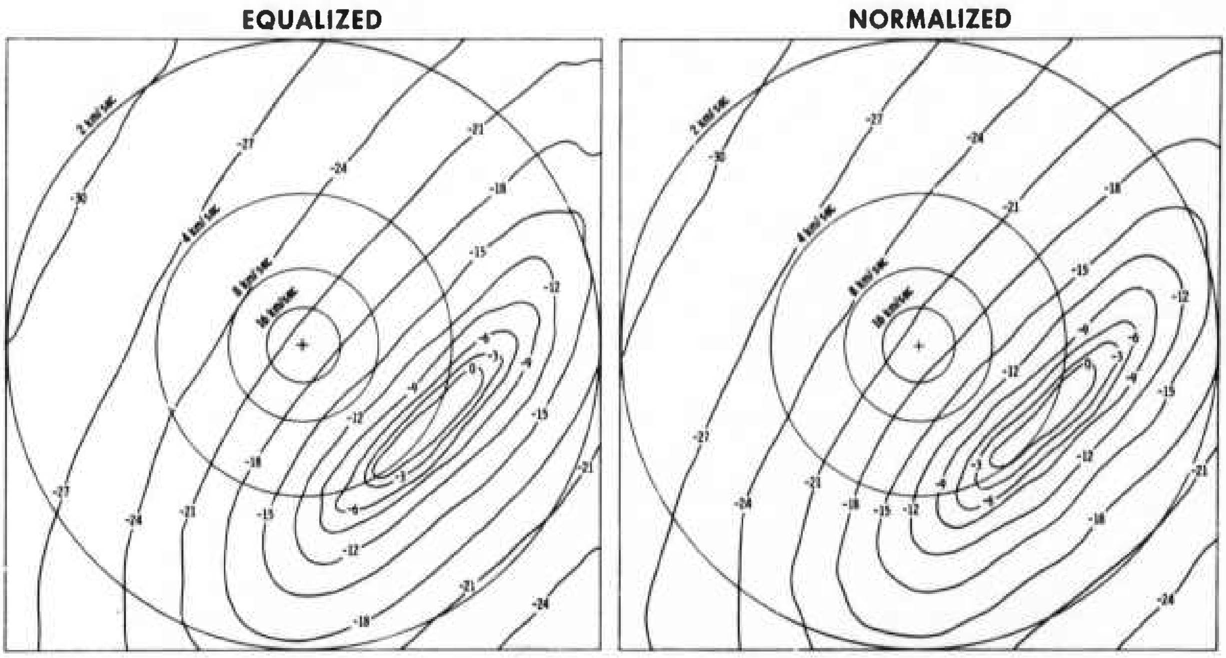

Figure 5. Comparison of High-Resolution Wavenumber Spectra Using Equalized and Normalized Crosspower Matrices of Smoothed Direct Transforms

B. EFFECT OF VARYING SNR

The SNR used in multichannel whitening-filter design affects the dynamic range and, to some extent, the wavenumber resolution of the high-resolution wavenumber spectrum.

An evaluation of the variation in resolution with SNR is conducted to establish a proper SNR for the ambient-noise analysis. High-resolution spectra are computed at 0.5 Hz. Normalized, smoothed, direct transforms of the entire 6.67-min data gate are used. Frequency resolution is 0.1 Hz. The SNR is varied from 0.001 to 0.5, and Z3 is used as the reference sensor. Figure 6 displays the spectra.

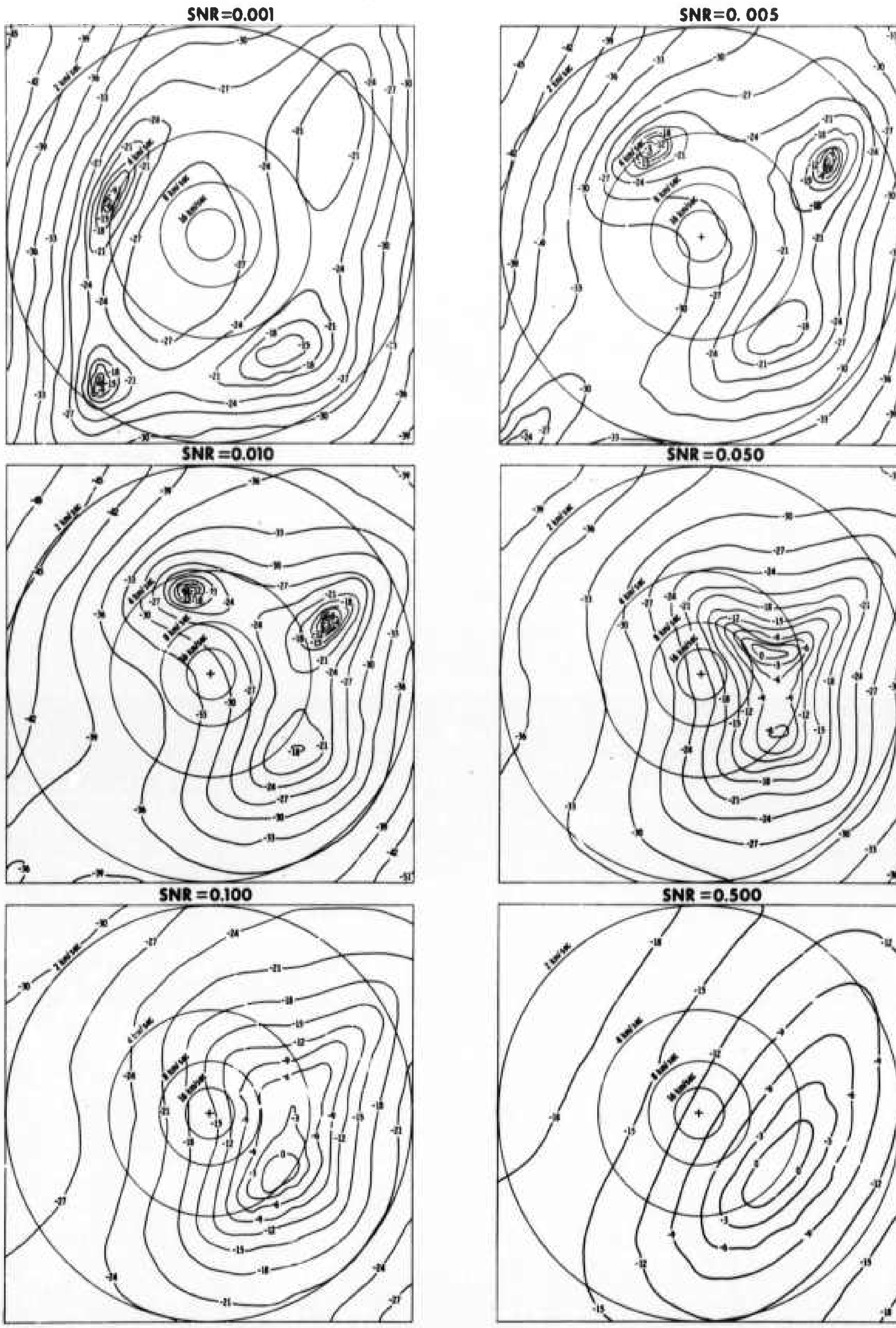

Figure 6. Comparison of High-Resolution Wavenumber Spectra, as a Function of SNR, Using Smoothed Direct Transforms

For this noise sample, coherent energy at 0.5 Hz from the east-southeast had been detected earlier. This coherent peak is observed for signal-to-noise ratios of 0.1 and 0.5. As the SNR decreases, the dominant peak is observed to be resolved into weaker stationary peaks which are apparently valid; however, relative comparison of peak sizes obviously is not possible. At an SNR of 0.001, the coherent energy source in the southeast is poorly located in wavenumber space and the spectrum is believed invalid. The reason for poor performance when very small SNR is used is the almost total lack of a constraint on the MCF wavenumber response at wavenumbers where no coherent energy is present.

From previous studies of transient signals using high-resolution wavenumber spectra,[3] SNR's from 0.1 to 0.01 are found suitable. The preceding evaluation indicates valid spectra for ambient noise using SNR's at or above 0.1. The choice of SNR appears to have more effect when using smoothed direct-transform-derived spectral matrices. Similar experiments using crosspower matrices of unsmoothed direct transforms indicate a relative insensitivity to SNR above 0.01; this apparent sensitivity or insensitivity to SNR is, to a large extent, due to the effects of smoothing when using direct-transform spectra.

C. SMOOTHED VS UNSMOOTHED CROSSPOWER MATRICES

High-resolution wavenumber spectra computed from both smoothed and unsmoothed spectra are compared. Figure 7 displays wavenumber spectra at 1.0 Hz computed using three different spectral matrices obtained by three different approaches to the estimation of crosspower spectra. Crosspower spectra are estimated from Fourier transforms of Parzen-smoothed crosscorrelations and unsmoothed and Danielle-smoothed direct Fourier transforms of the traces. All matrices are computed over the same 6.67-min data gate and normalized. An SNR of 0.01 is used, and Z3 is the reference sensor. Smoothed and unsmoothed spectra have frequency resolutions of 0.1 Hz and 0.0025 Hz, respectively.

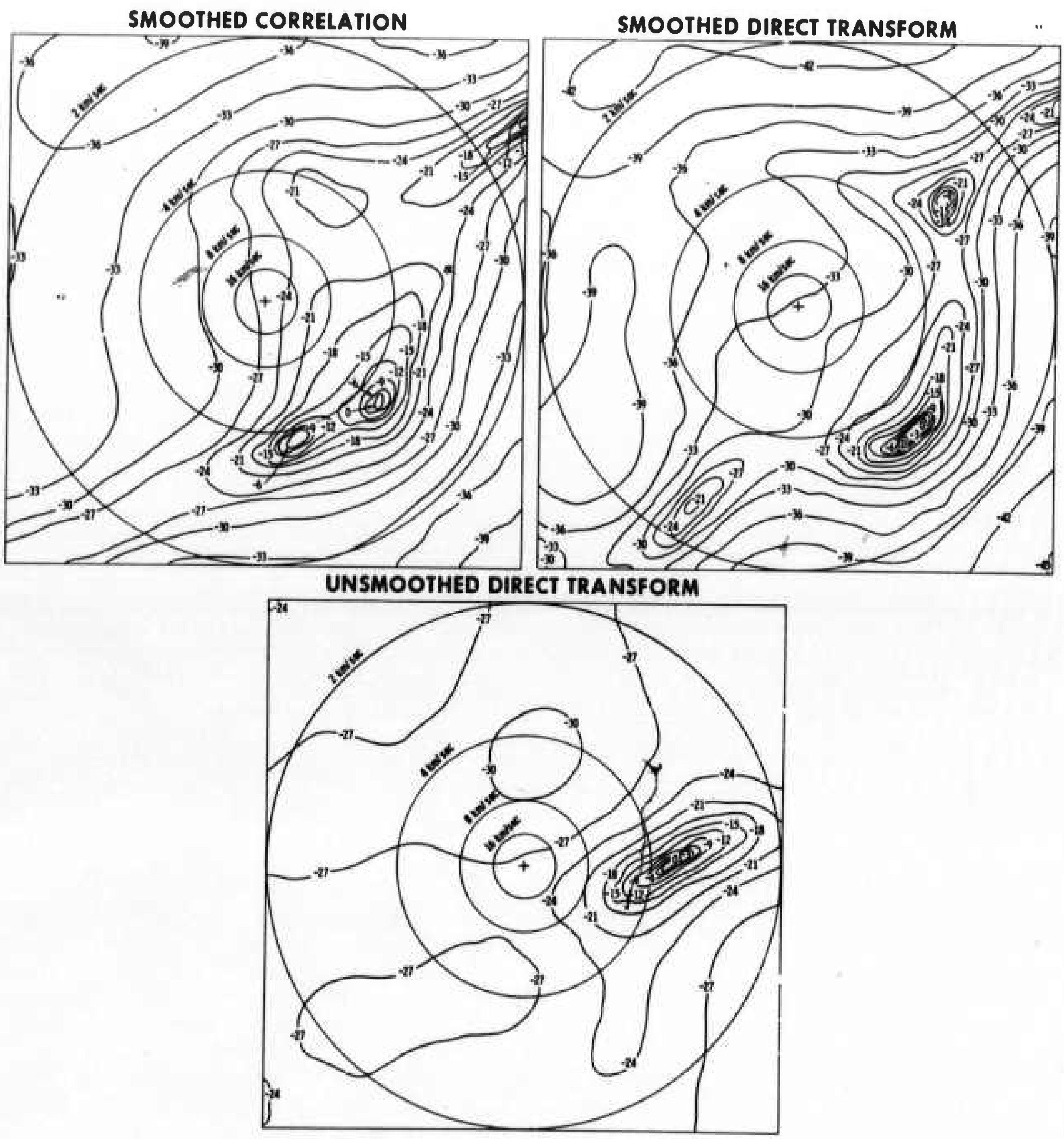

Figure 7. Comparison of High-Resolution Wavenumber Spectra Using Smoothed and Unsmoothed Direct Transforms and Smoothed Correlations

Wavenumber spectra computed from smoothed crosspower matrices are approximately equivalent and have several separated peaks both north and south of east. The spectrum computed from the unsmoothed direct-transform matrix indicates a single peak approximately due east, corresponding to tropical storm Isabel. As previously observed, at an SNR of 0.1, smoothed crosspower wavenumber spectra agree well with those obtained from unsmoothed crosspower.

Wavenumber spectra derived from unsmoothed direct transforms seem to represent a more stable and more reasonable estimate of the coherent noise. Previous work with transient signals shows that crosspower smoothing can virtually wipe out a wavenumber spectral peak due to a coherent transient signal when the crosspower spectra are smoothed over a frequency band that is wide with respect to the rate of change of the crosspower-phase spectrum. (The rate of change can be very high, even for moderate-sized arrays.) If the noise field is viewed as consisting of many overlapping transients, significant information could be lost by smoothing. If the noise field is composed of time-separated transients in a random noise field, smoothing is necessary to produce an interpretable spectrum due to the random noise; however, care must be taken to avoid losing the transient information.

D. EFFECT OF VARIATION IN REFERENCE SENSOR

Previous studies show that significant variations can exist in wavenumber spectra computed using different reference sensors because of intra-array inequalization and nonspace-stationary components in the noise field.[3]

An evaluation of reference-sensor choice is performed using both smoothed and unsmoothed spectra. Figures 8 and 9 display the effects of wavenumber spectra at 1.0 Hz for smoothed and unsmoothed crosspower matrices, respectively. The matrices are normalized, and an SNR or 0.01 is used in the filter design.

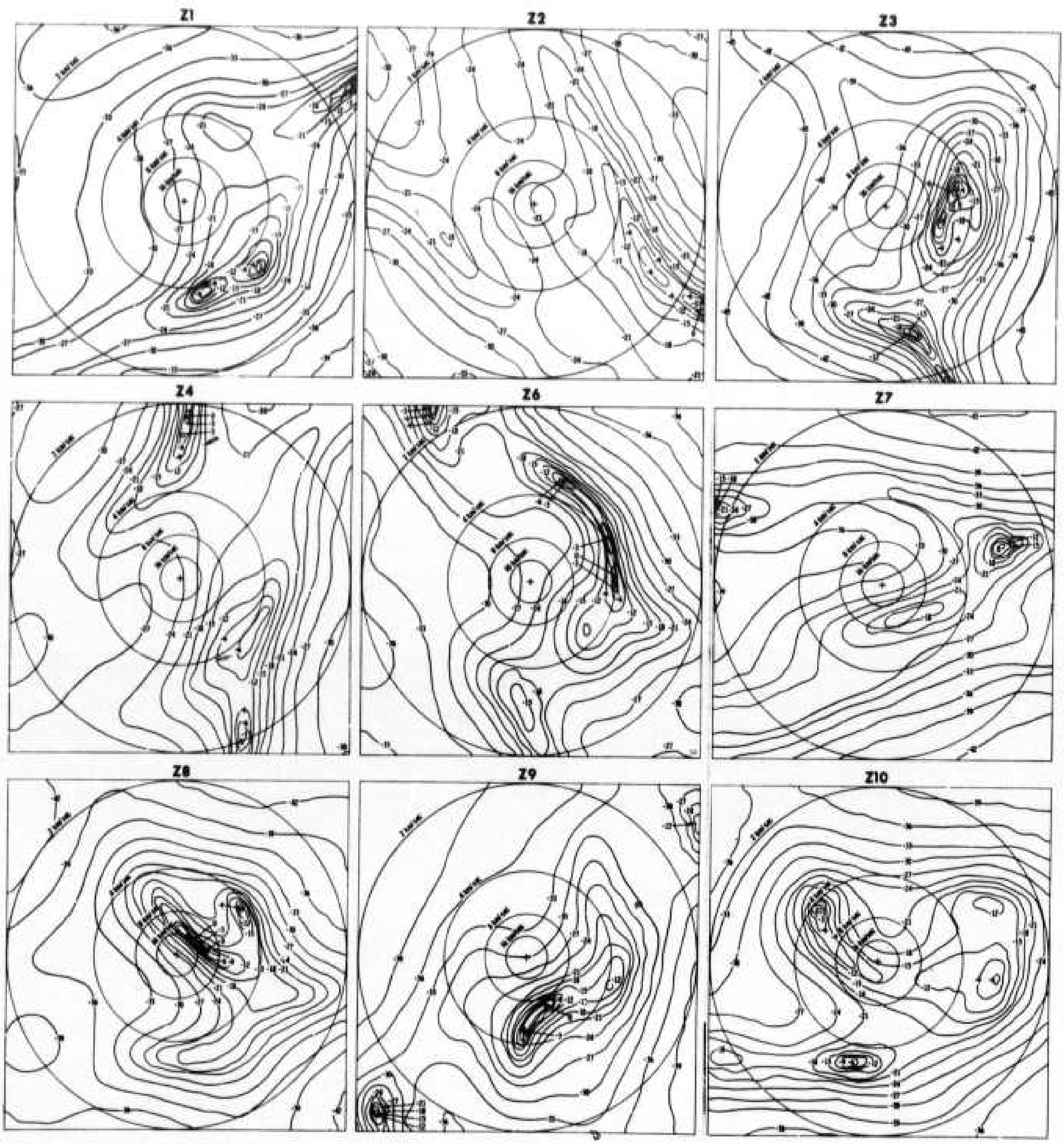

Figure 8. Comparison of High-Resolution Wavenumber Spectra, as a Function of Reference Sensor, Using Smoothed Correlation-Derived Matrices

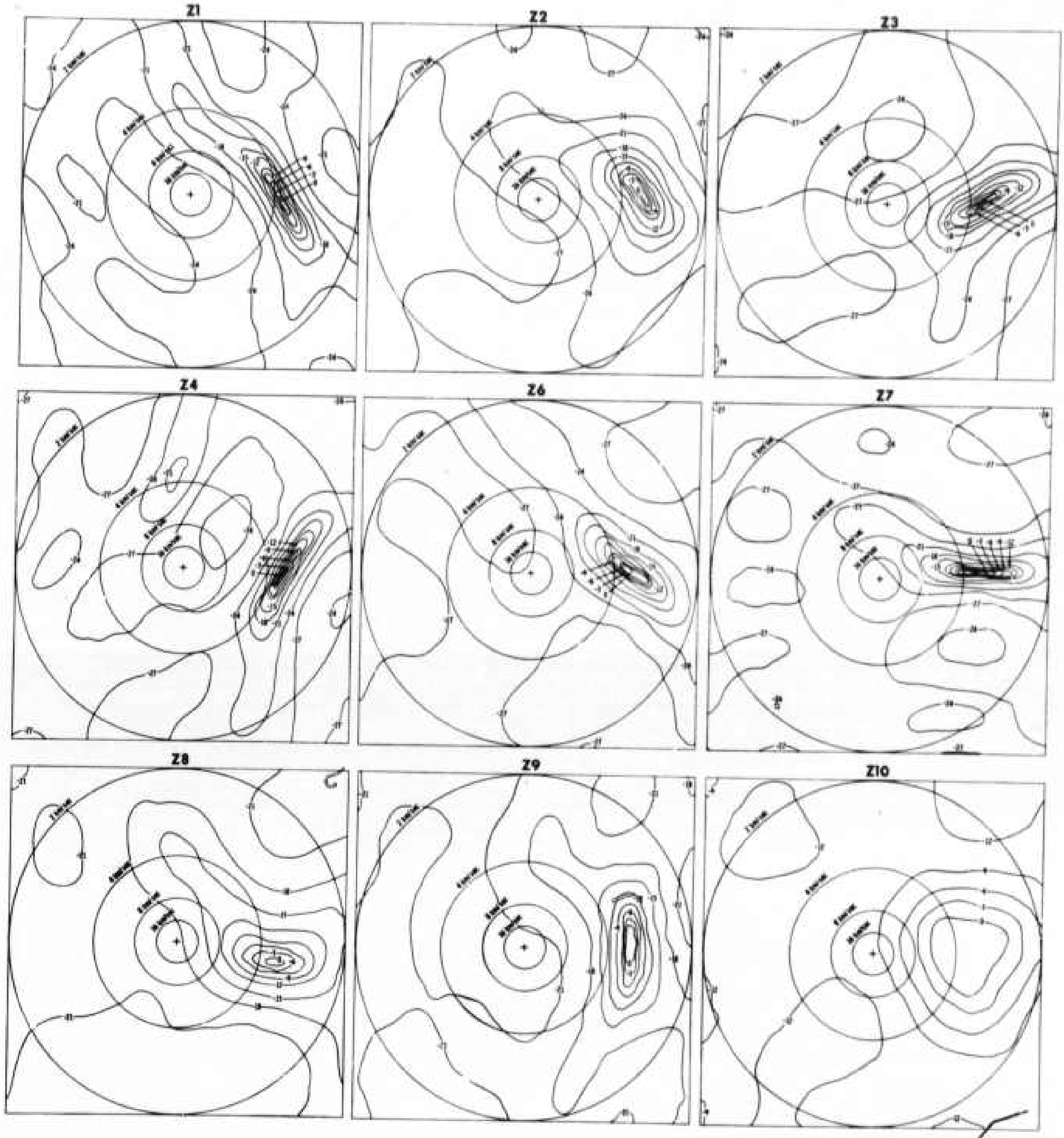

Figure 9. Comparison of High-Resolution Wavenumber Spectra, as a Function of Reference Sensor, Using Unsmoothed Direct Transforms

The smoothed spectra display considerable variation in spatial resolution with choice of reference sensor. The indicated source to the east is consistent with conventional and high-resolution spectra presented earlier. The unsmoothed spectral estimates are apparently more stable and represent the best possible estimate in this case.

E. AVERAGING OF WAVENUMBER SPECTRA

Interpretation and source location are hindered by the elongated peaks and the wandering of peaks sometimes observed in high-resolution spectra computed with respect to a single reference sensor. These peaks can be resolved by computing several spectra using different reference sensors and then stacking these spectra.

Essentially, this operation averages out variations in the noise fields as seen by different reference sensors and the effects due to variations of instrument responses and ground coupling. Spatial resolution is gained by the summing technique accompanied by a reduction in dynamic range (peak amplitude to background level). Peaks become circular rather than elliptical.

Figure 10 presents spectra computed at 1.0 Hz for two stacking methods, i.e., normal and reciprocal. The normal stacking method consists of summing the wavenumber power responses of multichannel whitening filters designed with different reference sensors. The power responses of the filters, it should be noted, are actually the reciprocals of the wavenumber spectral estimate. The reciprocal stacking method uses the same MCF wavenumber power responses but averages the reciprocals of the individual power responses. Computation of each of the spectra is from unsmoothed direct transforms. The crosspower matrices are normalized, and a 0.01 SNR is used in each MCF design.

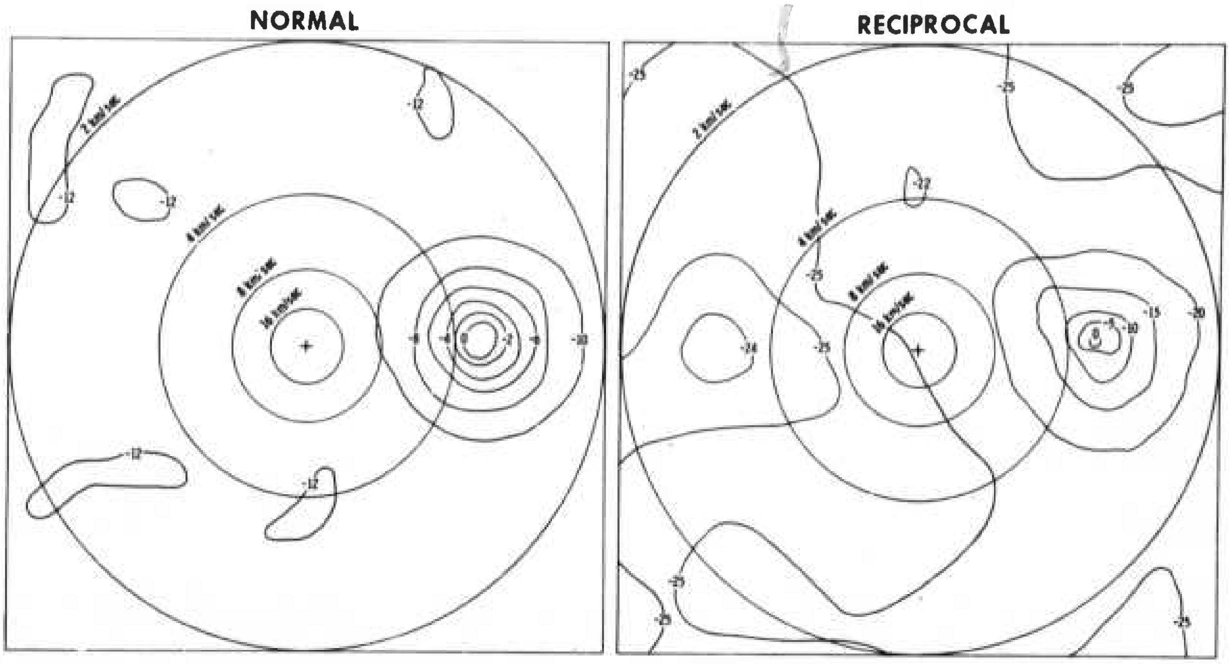

Figure 10. Comparison of Two Averaging Techniques for Smoothing High-Resolution Wavenumber Spectra

Additional discrimination between coherent peaks and overall background level is achieved by the reciprocal stacking procedure because there is a dependence of peak resolution on the relation between reference sensor and source azimuth. The appendix presents a shortcut computational technique for each of the two stacking methods, eliminating the computation of individual filters and filter responses for each reference sensor. The techniques, applicable only to unsmoothed crosspower matrices, represent dramatic reductions in the total computations required.

SECTION V

CONCLUSIONS

The validity of seismic noise analysis using the high-resolution technique is demonstrated. Several implications arise that are pertinent to the network surveillance problem. Apparently, coherent seismic noise (in this case, storm-generated) is sufficiently time-stationary over periods of at least several minutes to permit such analysis. In addition, plane-wave assumptions appear to hold for this energy. Consequently, high-resolution wavenumber analysis should indicate relative stability of the background noise enhancing the ability to detect transient signal arrivals from earthquakes or explosions.

This behavior may be rather significant when assessing the practicality of such systems for seismic event detection and location. Slow variations in the wavenumber spectra caused by movement or changes in intensity of storms might be tracked adaptively and those noise components removed from the spectrum.

Several conclusions are drawn regarding various approaches to computation of the high-resolution wavenumber spectra. Wavenumber spectral estimates made from unsmoothed crosspower spectra appear more stable, less sensitive to filter design SNR, and computationally much simpler than those made from smoothed crosspower spectra.

Summation of several MCF wavenumber power response, where each MCF is designed with respect to a different reference sensor, stabilizes the wavenumber estimate.

Summing of individual spectral estimates (i.e., the reciprocals of the MCF wavenumber responses) yields better resolution of coherent peaks relative to the background level.

SECTION VI
REFERENCES

1. Texas Instruments Incorporated, 1968: Network Studies - Noise Characteristics, Advanced Array Research, Spec. Rpt. No. 6, Contract F33657-67-C-0708-P001, 15 Feb.

2. Texas Instruments Incorporated, 1966: High-Resolution Wavenumber Spectra with Applications for Detection and Location of Weak Signals (U), Study of Statistical Discrimination between Earthquakes and Explosions, Scientific Rpt. No. 1, Contract AF 19(628)-4173, Jun. (S)

3. Texas Instruments Incorporated, 1967: Research on High-Resolution Frequency-Wavenumber Spectra, Large-Array Signal and Noise Analysis, Spec. Scientific Rpt. No. 2, Contract AF 33(657)-16678, 20 Apr.

APPENDIX

DERIVATION OF FORMULAS FOR DIRECT COMPUTATION OF SMOOTHED HIGH-RESOLUTION WAVENUMBER SPECTRA

APPENDIX

DERIVATION OF FORMULAS FOR DIRECT COMPUTATION OF SMOOTHED HIGH-RESOLUTION WAVENUMBER SPECTRA

The high-resolution wavenumber spectra smoothing techniques compared in this report would both be prohibitively complex and expensive if each individual MCF had to be designed and its wavenumber power response computed. Fortunately, when unsmoothed direct-transform-derived cross-power spectra are used, these smoothed wavenumber spectra can be obtained directly at a fraction of the computational effort.

The derivation of the direct-solution formulas for both smoothing techniques uses a theorem for the exact inverse of a special type of matrix. This special type of matrix is the same form as the matrix which must be inverted in obtaining the multichannel whitening filters. The theorem states that, for any column vectors $\underline{B}$ and $\underline{C}$ and nonsingular matrix A for which $A + \underline{B}\underline{C}^T$ is nonsingular,

$$\left(A + \underline{B}\,\underline{C}^T\right)^{-1} = A^{-1} - \frac{A^{-1}\,\underline{B}\,\underline{C}^T A^{-1}}{1 + \underline{C}^T A^{-1}\,\underline{B}}$$

where superscript T denotes transpose.

A. AVERAGE OF WAVENUMBER POWER RESPONSES OF N MULTICHANNEL WHITENING FILTERS

Let $\underline{X}$ be an Nx1 complex column vector with elements composed of the N complex numbers obtained by direct Fourier transformation of the outputs of an N element array. The crosspower spectral matrix is then $\underline{X}\underline{X}^H$ where superscript H denotes conjugate transpose. Sought is a multichannel filter which will whiten, as a function of wavenumber, the space-time field, the crosspower matrix of which is $\underline{X}\underline{X}^H$. The equation to be solved for the complex Nx1 filter-weight vector $\underline{F}_i$ is given by

$$\underline{F}_i^* = \left(sI + \underline{X}\underline{X}^H\right)^{-1} \underline{\Gamma}_i = \frac{1}{s} Z \underline{\Gamma}_i$$

where the asterisk denotes conjugate, s is a real scalar, I is the identity matrix, $\underline{\Gamma}_i$ is the i^{th} column of sI, and i is the reference sensor. The real scalar s is the signal-to-noise ratio (SNR) to be used for filter design.

The wavenumber power response of the filter $\underline{F}_i$ at the wavenumber $\vec{k}$ is given by

$$\underline{V}^T \underline{F}_i \underline{F}_i^H \underline{V}^*$$

where $\underline{V}^T = \left(e^{j2\pi\vec{k}\cdot\vec{x}_1} \quad e^{j2\pi\vec{k}\cdot\vec{x}_2} \quad \ldots\ldots \quad e^{j2\pi\vec{k}\cdot\vec{x}_N}\right)$ and $\vec{x}_1, \vec{x}_2, \ldots\ldots, \vec{x}_N$ are the vector distances from any arbitrary reference point and the N sensors. The sum over all N sensors of the wavenumber power responses of the whitening filters designed with respect to each sensor is

$$\sum_{i=1}^{N} \underline{V}^T \underline{F}_i \underline{F}_i^H \underline{V}^* = \underline{V}^T \left[\sum_{i=1}^{N} \underline{F}_i \underline{F}_i^H\right] \underline{V}^*$$

$$= \underline{V}^T Z^* \left[\sum_{i=1}^{N} \frac{1}{s^2} \underline{\Gamma}_i^* \underline{\Gamma}_i^T\right] Z^T \underline{V}^* = \underline{V}^T Z^* Z^T V^*$$

Using the theorem for exact inverse of $A + \underline{B}\,\underline{C}^T$ and substituting $sI = A$, $\underline{X} = \underline{B}$, and $\underline{X}^H = \underline{C}^T$, the following equation is obtained:

$$Z = s\left(sI + \underline{X}\underline{X}^H\right)^{-1} = s\left[\frac{1}{s} I - \frac{I\underline{X}\underline{X}^H I}{s^2 + s\underline{X}^H I \underline{X}}\right] = I - \frac{\underline{X}\underline{X}^H}{s + \underline{X}^H \underline{X}}$$

Then,

$$\underline{Z}^* \underline{Z}^T = \left[I - \frac{\underline{X}^* \underline{X}^T}{s + \underline{X}^T \underline{X}^*} \right] \left[I - \frac{\underline{X}^* \underline{X}^T}{s + \underline{X}^T \underline{X}^*} \right] = I - \underline{X}^* \underline{X}^T \left[\frac{2s + \underline{X}^H \underline{X}}{(s + \underline{X}^H \underline{X})^2} \right]$$

The sum of N wavenumber power responses is then

$$\sum_{i=1}^{N} \underline{V}^T \underline{F}_i \underline{F}_i^H \underline{V}^* = \underline{V}^T \underline{V}^* - \underline{V}^T \underline{X}^* \underline{X}^T \underline{V}^* \left[\frac{2s + \underline{X}^H \underline{X}}{(s + \underline{X}^H \underline{X})^2} \right]$$

$$= N - C \, \underline{V}^T \underline{X}^* (\underline{V}^T \underline{X}^*)^*$$

Note that the constant C is independent of the wavenumber vector $\vec{k}$ and thus must be evaluated only once. For each wavenumber at which the power-response sum is evaluated, the vector dot product $\underline{V}^T \underline{X}^*$ must be evaluated since $\underline{V}^T$ is a function of wavenumber. Multiplying the complex number $\underline{V}^T \underline{X}^*$ by its conjugate and by C and then subtracting this real number from N, the desired response sum is obtained. Note that this response sum is always greater than 0 (for nonzero s) and less than N. Therefore, its reciprocal, which is the estimate of the wavenumber spectrum, is always positive and finite. Note also the lack of sensitivity to the choice of s.

B. AVERAGE OF HIGH-RESOLUTION WAVENUMBER SPECTRA COMPUTED WITH DIFFERENT REFERENCE SENSORS

If the sum of M of the N possible individual wavenumber spectra is desired, this is given by

$$\sum_{i=1}^{M} \frac{1}{\underline{V}^T \underline{F}_i \underline{F}_i^H \underline{V}^*} \qquad M \leq N$$

which is not as easily evaluated as was the reciprocal of the sum of wavenumber responses. However, some simplification is possible.

Let

$$\underline{V}^T \underline{F}_i \underline{F}_i^H \underline{V}^* = \underline{Y}^T \underline{\Gamma}_i \underline{\Gamma}_i^T \underline{Y}^* = y_i y_i^*$$

where

$$\underline{Y}^T = \left[y_1\ y_2 \cdots y_N\right] = \underline{V}^T \left[I - \frac{\underline{X}^H \underline{X}}{s + \underline{X}^H \underline{X}} \right]$$

Then, the sum of M wavenumber spectra is obtained by evaluating

$$\sum_{i=1}^{M} \frac{1}{y_i y_i^*}$$

Thus, the operations required to evaluate this sum at each wavenumber are as follows: multiply the row vector $\underline{V}^T$ by the M columns of the constant matrix

$$I - \frac{\underline{X}^H \underline{X}}{s + \underline{X}^H \underline{X}}$$

corresponding to the M reference sensors selected; multiply the resulting M complex numbers by their conjugates; and take the reciprocal of each resultant real number and sum.

The required number of complex multiplies and adds is $M(N + 1)$, which is well above the $N + 1$ needed for the first technique but well below the number required to solve for each filter and filter response individually.

The second technique, although requiring more computation, is preferred. When summing power responses over all N multichannel filters as shown in the first technique, the spectral window effects of the conventional technique are regenerated. Examination of the equation for this sum reveals it to be a nonlinear transformation of the conventional wavenumber spectrum given by $\underline{V}^T \underline{X}^* \underline{X}^T \underline{V}^*$. Of course, summing over all N filter responses is not necessary and a direct solution similarly derived is available for the sum of any subset of N filter responses.

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13. ABSTRACT

A principal objective of the network studies is to characterize the worldwide ambient-noise field. An excellent tool for such studies is the frequency-wavenumber spectrum of the ambient noise recorded by an array station.

Since all network array stations were of small aperature, conventional techniques for computing wavenumber spectra could not achieve the wavenumber resolution required to resolve and accurately locate multiple noise sources, especially bodywave noise sources. To establish the procedure to use for computing the spectra, the experiment described in this report was performed prior to the computation of ambient-noise wavenumber spectra for the network studies.

Analysis of ambient seismic noise using the high-resolution technique is shown to be valid. Wavenumber spectral estimates made from unsmoothed crosspower spectra appear more stable, less sensitive to SNR, and computationally much simpler than those made from smoothed crosspower spectra. The summing of several MCF wavenumber power responses, each designed with respect to a different reference sensor, stabilizes the wavenumber estimate. The summing of individual spectral estimates, i.e., the reciprocals of the MCF wavenumber responses, yields better resolution of coherent peaks relative to background noise levels.

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