UNCLASSIFIED

AD NUMBER

AD829113

CLASSIFICATION CHANGES

TO: unclassified

FROM: confidential

LIMITATION CHANGES

TO:

Approved for public release, distribution unlimited

FROM:

Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; 27 NOV 1956. Other requests shall be referred to Air Force Space and Missile Systems Organization, Los Angeles, AFB, CA.

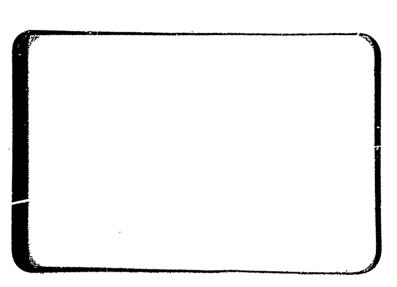
AUTHORITY

14 May 1965, WS107A per document marking; samso, usaf ltr, 28 feb 1972

KONFIDENTIAL/

UNCLASSIFIED







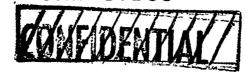


CONVAIR-ASTRONAUTICS
TECHNICAL LIBRARY

UNCLASSIFIED

CONVAIR

A DIVISION OF GENERAL DYNAMICS CORPORATION SAN DIEGO



Best Available Copy

REPORT 24-7-137 DATE 27 November 1956 TIVISION OF GENERAL DYNAMICS CORPORATION SAN DIEGO MODEL __Seven CONVAIR ASTRONAUTICS TITLE A METHOD OF OBTAINING LINEAR AND ENGULAR This document is subject to special export controls and each transmittal to foreign ACCELERATIONS USING LINEAR ACCELEROMETERS governments or foreign nation is may be made only INCLASSIFICATION Changed To:

With proor approval of:
Hq.SAM.D, LA., (a. 90045
Attn: SISD

Authorized Emily Pate 5-14-65 D D 254 / 306 Reclassified By: PREPARED BY_ GROUP Aerophysics REFERENCE CHECKED BY APPROVED B APPROVED BY NO. OF PAGES ___ NO. OF DIAGRAMS___ Chief Development Engineer REVISIONS PAGES AFFECTED DATE BY CHANGE

FORM 1812 A-4

CONVAIR

SAN CE.O.

PAGE 1
RICHBIAN ZA-7-137
MODEL Seven

The state of the s

DATE 27 Nov. 1956



FOREWORD

This study was undertaken to determine the minimum number of linear accelerameters necessary to determine the aerodynamic characteristics of the XSM-65 missiles from flight tests and to obtain the equations for the linear and angular accelerations about the three reference axes in terms of the accelerometer readings.

The final results are dependent on the geometric location of the accelerometers so it is recommended that the locations of the instrumentation accelerometers be checked before using the final equations that are set forth here.



CONVAIR

the second second with the second sec

PAGE 11

R. YORT NO ZA-7-137

MODEL Seven

DATE 27 Nov. 1956

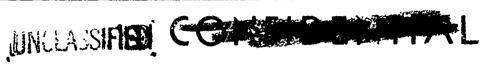


TABLE OF CONTENTS

							Page
POREWORD	•	•	•	•	•	•	i
TABLE OF CONTENTS	•	•	•	•	,	•	ii
LIST OF ILLUSTRATIONS	•	•	•	•	•	•	ii
LIST OF TABLES	•	•	•	•	•	•	iv
SUMMARY	•	•	•	•	•	•	¥
I. Introduction	•	•	•	•	•	•	1
II. Discussion and Analysis	•	•	•	•	•	•	2
REFERENCES	•	•	•	•	•	•	7
APPENDICES	•	•	•	•	•	•	
A. Reducing the Number of Unknowns	•	•	•	•	•	•	8
B. Solving for the Accelerations	•	•	•	•	•	•	11
C. Evaluating the Contribution of Each Instrument.	•	•	•	•	•		14







and the second to the second second to the second of the second s

PAGE 111 REPORT NO 24-7-137 MODEL Seven

DATE 27 Nov. 1956



LIST OF ILLUSTRATIONS

FIGURE									PAG
1.	Relative Accelerometer Locations	•		_		_	_		5



LIST OF TUBLES

CARLE		PAGE
I	Accelerometer and Center of Gravity Locations	6

C O N V A I R

REPORT NO ZA-7-137

MODEL Seven

DATE 27 Nov. 1956

CONFIDENTIAL

SUMMARY

In order to determine the aerodynamic force and moment characteristics of a missile from flight tests, it is necessary to have measurements of the linear and angular accelerations about the three reference axes. Since angular accelerometers having the desired accuracy and capable of withstanding the environment of the XSM-65 missiles are not available, it has become necessary to obtain these measurements through the use of linear accelerometers. This report presents the results of a study to determine the minimum number of accelerometers required to accomplish these measurements and a study to determine the relative contribution of each instrument toward the net measured acceleration.

By analyzing the general equations for the net measured acceleration, it is shown that the contributions of the centripetal accelerations are negligible providing the angular velocities are moderate. This permits a reduction of the required linear acceleration measurements from nine to six.

Then, using the instrument locations current in September 1956, from Reference 1 through _, a numerical analysis was made to determine the relative contribution of each instrument. The results are given in Ecuations 7 to 18.

Equations 7 to 12 are valid for the general case, but Equations 13 to 18 are valid only for the configuration used in this report.

PAGE 1
REPORT NO ZR-7-137
MODEL Seven
DATE 27 Nov. 1956

CONFIDENTIAL

INTRODUCTION

During the design of a missile, estimates are made of the aerodynamic forces and moment characteristics based on theoretical and wind tunnel data. The theoretical values are, of necessity, based on certain assumptions regarding the effects of viscosity and compressibility while the wind tunnel data contains some interference from the system used to support the model. Consequently, it is most desirable to obtain a final check on the accuracy of the predicted characteristics, from flight tests of the full-scale vehicle.

Furthermore, since the KSM-65 is statically unstable, a certain angle of attack limit exists beyond which the gimballed rocket thrust chambers cannot trim the missile. Should this limit inadvertantly be exceeded in the flight test it should be possible to define the limit from the data obtained and thus provide a check on the predicted limit.

In obtaining these aerodynamic characteristics, two primary measurements are the linear and angular accelerations about the three reference axes. Current instrumentation planning includes the use of servo accelerometers to obtain the linear accelerations, but no angular accelerometers that provide the required accuracy and which meet the XSM-65 environmental specification are available in time for the first flight vehicles. Consequently, it is necessary to obtain angular accelerations by using linear accelerometers and a *base-line* technique.

The study reported herein was undertaken to determine the minimum number of linear accelerameters required to define the linear and angular accelerations about the three reference axes. As a supplement, equations were obtained to show these net accelerations in terms of the acceleration measured by each instrument.

PAGE 2
REPORT NO ZA-7-137
MODEL Seven
DATE 27 Nov. 1956

大大的人工人 医克尔特氏 化对对原 医皮肤软件 医克里克氏试验检尿病 化二十二烷基化二十二烷

CONFIDENTIAL

DISCUSSION

The instrument locations current in September 1956 for the ISM-65A S/M4 missile from References 1 through 3 are shown in Figure 1. The geometry is presented in Table I.

As shown in the detailed development of Appendix A, the analysis of the general acceleration of a body requires the measurement of linear accelerations, angular velocity and angular acceleration about each of three orthogonal reference axes. To accomplish this, nine measurements are necessary. However, when the equations for the output of the instruments are written and the maximum expected values of the linear and angular accelerations and angular velocities are substituted from Reference 4, it is evident that for the moderate angular velocities used, the centripetal accelerations are small. Consequently, the number of measurements may be reduced from nine to six leaving only the linear and angular accelerations to be determined. The six equations are:

$$A_{xz} = a_x - \alpha_z (y_x - y_{cg}) + \alpha_y (z_x - z_{cg}) \qquad (2)$$

$$A_{x3} = a_x - \alpha_2(y_3 - y_{cq}) + \alpha_y(z_3 - z_{cq})$$
 (3)

$$A_{ya} = a_y - \alpha_x(X_a - X_{y}) + \alpha_x(Z_a - Z_{y}) \qquad (4)$$

$$A_{ys} = a_y - \alpha_x (x_s - x_{cg}) + \alpha_x (z_s - z_{cg})$$
 (5)

$$A_{26} = a_2 + \alpha_y (x_6 - x_{cg}) + \alpha_y (y_6 - y_{cg})$$
 (6)

PAGE 3
REPORT NO ZA-7-137
MODEL Seven
DATE 27 Nov. 1956

CONFIDENTIAL

Solving for the six unknown accelerations as shown in Appendix B, results in:

$$\alpha_{x} = \frac{A_{y4} - A_{y5} + (X_{4} - X_{5})\alpha_{\xi}}{Z_{4} - Z_{5}} \tag{7}$$

$$\alpha_{y} = \frac{A_{x_{1}}[y_{2}-y_{2}] + A_{x_{2}}[y_{3}-y_{1}] + A_{x_{3}}[y_{1}-y_{2}]}{(y_{2}Z_{x}-y_{1}Z_{3}) + (y_{1}Z_{3}-y_{2}Z_{1}) + (y_{2}Z_{1}-y_{1}Z_{2})}$$
(8)

$$\alpha_{1} = \frac{A_{11}[Z_{1}-Z_{2}] + A_{12}[Z_{3}-Z_{1}] + A_{13}[Z_{1}-Z_{2}]}{(y_{1}Z_{2}-y_{2}Z_{3}) + (y_{1}Z_{3}-y_{3}Z_{1}) + (y_{2}Z_{3}-y_{1}Z_{2})}$$
(9)

$$\alpha_{x} = A_{xi} + \alpha_{z}(y_{i} - y_{ij}) - \alpha_{y}(\bar{z}_{i} - \bar{z}_{ij})$$
 (10)

$$a_y = A_{y+} + \alpha_x (X_4 - X_9) - \alpha_x (Z_4 - Z_9)$$
 (11)

$$a_z = A_{zc} - \alpha_y (x_c - x_{cg}) - \alpha_x (y_c - y_{cg})$$
 (12)

Using the geometry of Figure 1 and Table I, and the center of gravity location from Reference 5 which corresponds to an initial gross weight of 201,254 pounds, the above equations were evaluated. The time chosen, t = 60 seconds, is the time of maximum dynamic pressure and minimum allowable angle of attack. The following equations were obtained in Appendix C:

$$Q_y = .009361 \, A_{XI} - .019490 \, A_{XZ} + .010129 \, A_{XS}$$
 (14)

$$\alpha_{z} = -0.096437 \, A_{K1} + 0.094437 \, A_{K3}$$
 (15)

$$Q_X = .404/94 A_{XV} + .204239 A_{XZ} + .391567 A_{XS}$$
 (16)

CONVAIR

C 1 . MAP

FAGE 4
RETURN NO ZA-7-137
MODEL Seven
DATE 27 Nov. 1956

CONFIDENTIAL

ay =-.099458 Au +.099458 Au +.195761 Ays +.204239 Ays (17)

Q = = .000463 Ax1 -.021829 Ax2 +.021365 Ax8 +.103443 Ay4 -.103443 Ays + A86 (18)

These equations, 13 to 18, are good only for the specified geometry and center of gravity location.

Equations 13 to 18 show the relative contribution of each instrument toward the measurement of the net acceleration about any of the three axes. These equations may be then used to evaluate the effect of an error in any instrument upon the final acceleration measurement accuracy.

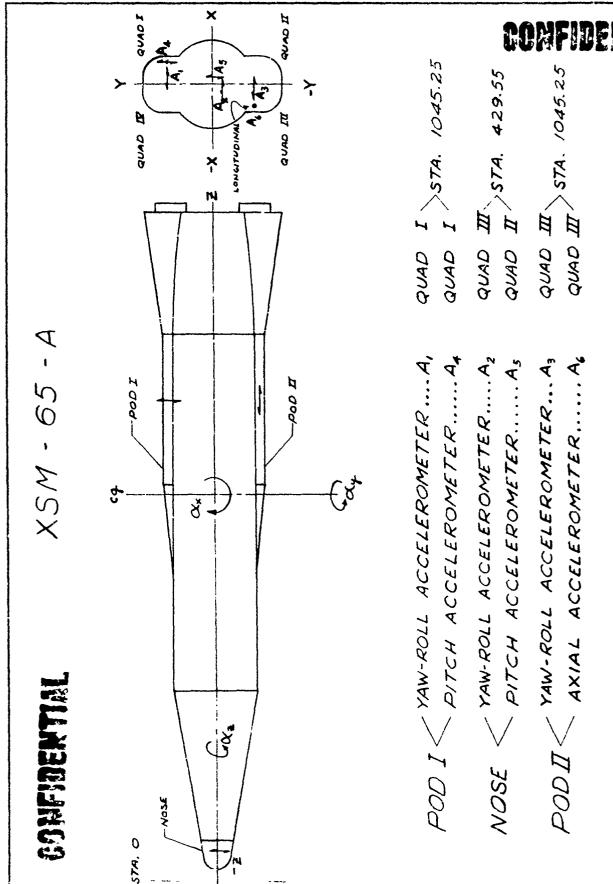
CHECKED BY REVISED BY

BAN DIEGO

MODEL Seven

RELATIVE ACCELEROMETER LOCATIONS

DATE 27 Nov.



CONVAIR
PAGE 6
REPORT NO. ZA-7-137 MODEL Seven DATE 27 Nov. 1956

TABLE I

XSM-65A Accelerometer and

Center of Gravity Locations

Accelerometer	Accelerometer Locations						
	Inches	Inches	Inches				
A1	11.33	63.40	1045.25				
A2	- 0.45	- 2.50	429.55				
A3	-11.33	-63.40	1045.25				
A4	14.28	63.40	1045.25				
A 5	2,00	- 0.30	429.55				
A 6	-14.28	-63.40	1045.25				
C.G. Location at t = 60 sec. G. Wt = 201,254 Lbs.	- 0.84	0 .29	919.50				

FORM 1812 ~A

FA F 7
FEPORT NO ZA-7-137
MODEL Seven
DATE 27 Nov. 1956

CONFIDENTIAL

REFERENCES

- 1. Paul, John, "Traneducer Installation Station 426 Noce," Convair Drawing 7-11798, October 1956.
- 2. Paul, John, "Transducer Installation Station 1045 Quad I," Constir Drawing 7-11.796, October 1956.
- 3. Faul, John, "Transducer Installation Station 1045 Quad III," Convair Drawing 7-11799, October 1956.
- 4. Lukens, D., "Basic Autopilot & Aeroelastic Data for the XSM-65 A and C Series Missiles During the Boost Phase," Convair Technical Note ZU-7-050-TN, December 15, 1955.
- 5. Allenson, J. M., "XSM-65A Mass Distribution Centers of Gravity Moments and Products of Inertia," AW-A-35, September 25, 1956.

PAGE 8
REPORT NO ZA-7-137
MODEL Seven
DATE 27 Nov. 1956

CONFIDENTIAL

APPENDIX A

Reducing the Number of Unknowns

The output of each accelerometer will include components of acceleration resulting from translational acceleration, angular acceleration and angular velocity. The outputs of the various instruments may be written as:

$$A_{xi} = \alpha_{x} - \omega_{z}^{2}(x_{i} - x_{ij}) - \omega_{y}^{2}(x_{i} - x_{ij}) - \alpha_{z}(y_{i} - y_{ij}) + \alpha_{y}(z_{i} - z_{ij})$$
(Ai)

$$A_{X2} = \alpha_{x} - \omega_{z}^{2}(x_{z} - x_{y}) - \omega_{y}^{2}(x_{z} - x_{y}) - \alpha_{z}(y_{z} - y_{y}) + \alpha_{y}(z_{z} - z_{y})$$
(A3)

Ay = ay -
$$\omega_z^2(y_4 - y_9) - \omega_x^2(y_4 - y_9) - \alpha_z(x_4 - x_{cg}) + \alpha_x(z_4 - z_9)$$
 (A4)

These six equations contain nine unknowns, a_X , a_Y , a_Z , w_X , w_Y , w_Z , and C(x,C(y),C(z)) so that in theory three more equations are required for solution. However, by examining the magnitude of each term, three unknowns may be eliminated.

At t = 60 seconds, the time corresponding to maximum dynamic pressure, the instrument outputs were evaluated, using the maximum expected values of the accelerations and angular velocities. The values used are:

$$a_x \sim 16 \text{ ft/sec}^2$$

 $a_y \sim 16 \text{ ft/sec}^2$
 $a_z \sim 2.15 g = 69.166 \text{ ft/sec}^2$
CONFIDENTIAL

manufacture of the second seco

SAN DIEGO

PAGE 9
REPORT NO. ZA-7-137
MODEL Seven
DATE 27 Nov. 1956

CONFIDENTIAL

$$W_X \sim 3$$
 deg/sec = 0.05236 pad/sec

 $W_Y \sim 3$ deg/sec = 0.05236 rad/sec

 $W_Z \sim 3$ deg/sec = 0.05236 rad/sec

 $CX_X \sim 25$ deg/sec = 0.43633 rad/sec

 $CX_X \sim 25$ deg/sec = 0.43633 rad/sec

 $CX_Z \sim 25$ deg/sec = 0.43633 rad/sec

where -

a ~ translational acceleration

 $\omega \sim$ angular velocity

α angular acceleration

Substituting these values and the geometry from Table I into equations A1 to A6 results in:

$$A_{xi} = 16 - 0.003 - 0.003 - 2.295 + 4.572$$
 (A7)

$$A_{x2} = 16 - 0.00009 - 0.00009 + 0.101 - 17.815$$
 (A8)

$$Ax_3 = 16 + 0.002 + 0.002 + 2.316 + 4.572$$
 (A9)

$$A_{44} = 16 - 0.014 - 0.014 - 0.550 + 4.572$$
 (A10)

$$A_{26} = 69.166 - 0.029 - 0.029 - 0.489 - 2.316$$
 (A12)

It is evident in each equation that the contributions of the second and third terms, which are due to the angular velocities, are quite small. Therefore, these terms may be neglected. The six remaining unknowns then are the three translational accelerations and three angular accelerations.

- といい、最後ないないと、なないななないのできたのできゃくないないないないないないないのないないないないできょうか

CONFIDENTIAL

Depending upon the direction of the angular accelerations, when the translational acceleration is at a maximum the instrument outputs may vary between:

Therefore, it is not necessarily apparent from the output of a single instrument whether the translational acceleration has reached its maximum amplitude at any instant of time. This should be kept in mind when analyzing flight records.

PAGE 11
REPORT NO ZA-7-137
MODEL Seven

DATE 27 Nov. 1955

CONFIDENTIAL

APPENDIX B

Solving For the Accelerations

After eliminating the angular velocity terms from Equations A1 to A6 they may be rewritten as:

$$Axi = ax - \alpha_{e}(y_{i} - y_{eg}) + \alpha_{g}(z_{i} - z_{eg})$$
 (B1)

$$A_{x2} = a_x - \alpha_z (y_2 - y_g) + \alpha_y (\bar{z}_2 - \bar{z}_g)$$
 (82)

$$Ax3 = ax - \alpha_2(y_3 - y_{ij}) + \alpha_2(z_3 - z_{ij})$$
 (A3)

$$A_{y*} = \alpha_y - \alpha_z \left(x_4 - x_{cy} \right) + \alpha_x \left(z_4 - z_{cy} \right) \tag{84}$$

$$Ays = \alpha_y - \alpha_z (x_s - x_y) + \alpha_x (z_s - z_y)$$
 (B5)

$$A_{26} = a_{2} + \alpha_{y}(x_{6} - x_{cg}) + \alpha_{x}(y_{6} - y_{cg})$$
 (86)

Forming a determinant from the first three equations leads to:

$$D = \begin{vmatrix} 1 & -(y_1 - y_{eg}) & (Z_1 - Z_{eg}) \\ 1 & -(y_2 - y_{eg}) & (Z_2 - Z_{eg}) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -(y_2 - y_{eg}) & (Z_2 - Z_{eg}) \\ 1 & -(y_3 - y_{eg}) & (Z_3 - Z_{eg}) \end{vmatrix}$$
(87)

$$\mathcal{Q}_{a} = \begin{vmatrix}
A_{xx} & -(y_{1} - y_{2}y) & (Z_{1} - Z_{2}y) \\
A_{xx} & -(y_{2} - y_{2}y) & (Z_{2} - Z_{2}y) \\
A_{xx} & -(y_{3} - y_{2}y) & (Z_{3} - Z_{2}y)
\end{vmatrix}$$
(88)

$$D_{W_2} = \begin{vmatrix} 1 & A_{54} & (2_1-2_9) \\ 1 & A_{55} & (2_3-2_9) \\ 1 & A_{55} & (2_3-2_9) \end{vmatrix}$$
 (89)

$$Dx_y = \begin{vmatrix} 1 & -(y_1 - y_{ig}) & Ax_1 \\ 1 & -(y_2 - y_{ig}) & Ax_2 \\ 1 & -(y_3 - y_{ig}) & Ax_3 \end{vmatrix}$$
 (810)

1

SAN DIEGO

PAGE 12
REPORT NO ZA-7-137
MODEL Seven
DATE 27 Nov. 1956

CONFIDENTIAL

Solving then for Qy, Qz and ax yields

$$\alpha_{y} = \frac{D_{\alpha y}}{D} = \frac{A_{x1} [y_{2} - y_{3}] + A_{x2} [y_{3} - y_{1}] + A_{x3} [y_{1} - y_{2}]}{(y_{3} Z_{2} - y_{2} Z_{3}) + (y_{1} Z_{3} - y_{2} Z_{1}) + (y_{2} Z_{1} - y_{1} Z_{3})}$$
(B11)

$$\alpha_{2} = \frac{D\alpha_{3}}{D} = \frac{A_{x_{1}}[Z_{2} - Z_{3}] + A_{x_{2}}[Z_{3} - Z_{1}] + A_{x_{3}}[Z_{1} - Z_{2}]}{(y_{3}Z_{2} - y_{2}Z_{3}) + (y_{1}Z_{3} - y_{3}Z_{1}) \div (y_{2}Z_{1} - y_{1}Z_{2})}$$
(B12)

$$\alpha_{x} = \frac{D_{x}}{D} = \frac{A_{x_{1}}[(y_{3} z_{2} - y_{1} z_{3}) + y_{y_{1}}(z_{3} - \overline{z_{2}}) + z_{y_{2}}(y_{2} - y_{3})]}{(y_{3} z_{2} - y_{1} z_{3}) + (y_{1} z_{3} - y_{3} z_{1}) + (y_{2} z_{1} - y_{1} z_{2})}$$

$$+\frac{A_{x3}[(y_1Z_1-y_1Z_2)+y_{12}(Z_2-Z_1)+Z_{12}(y_1-y_2)]}{(y_3Z_2-y_1Z_2)+(y_1Z_2-y_3Z_1)+(y_2Z_1-y_1Z_2)}$$
(8/3)

Alternate forms of the solution for ar are

$$\alpha_{x} = A_{xi} + \alpha_{z}(y_{i} - y_{cg}) - \alpha_{y}(z_{i} - z_{cg})$$
 (B14)

$$\alpha_x = A_{xz} + \alpha_z (y_2 - y_{cg}) - \alpha_y (Z_2 - Z_{cg})$$
 (815)

$$Q_x = A_{x3} + Q_2 (y_3 - y_{cg}) - Q_x (Z_3 - Z_{cg})$$
 (816)

PAGE 13 REPORT NO ZA-7-137

MODEL Seven

DATE 27 Nov. 1956



UNCLASSIFIED

and the second of the second o

Now using the solution for Q_z and solving equations (B4) and (B5) for Q_z yields

$$Q_{X} = A_{YA} - A_{YB} + Q_{Z}(X_{A} - X_{S})$$

$$(Z_{A} - Z_{S})$$
(B17)

Then ay may be obtained from either

$$\alpha_y = A_{y*} + \alpha_z (x_4 - x_{cg}) - \alpha_x (Z_4 - Z_{cg})$$
 (B19)

$$Q_{\gamma} = A_{\gamma s} + \alpha_{s} \left(x_{s} - x_{cs} \right) - \alpha_{x} \left(z_{s} - z_{cs} \right) \tag{820}$$

Finally solving B6 for az yields

CONVAIR

UNCLASSIFIED

PAGE 1(14 PT NO ZA-7-137 MODEL Seven DATE 27 Nov 1956



APPENDIX C

Evaluating the Contribution of Each Instrument

Substituting the geometry from Table I into the acceleration equations results in the following:

From (B 12)

$$\alpha_{z} = \left(\frac{-65.70 \text{ Axi} + 0 + 615.70 \text{ Axs}}{78070.76}\right) 12 \tag{C1}$$

$$\alpha_{*} = -.094637 \, A_{*1} + .094637 \, A_{*2}$$
 (C2)

From (B 11)

$$\alpha_y = \left(\frac{60.9 \, A_{x1} - 126.8 \, A_{x2} + 65.9 \, A_{x3}}{78070.76}\right) 12 \tag{C3}$$

$$\alpha_y = .009361 \, A_{E1} - .019490 \, A_{E2} + .010129 \, A_{E3}$$
 (C4)

From (B 17)

$$\alpha_{x} = \left(\frac{A_{ys} - A_{ys} + 12.28(-.094637 A_{x1} + .094637 A_{x3}}{615.70}\right) 12 \quad (C5)$$

Próû (F 13)

$$Q_{X} = \frac{3.555 A_{XI} + 15945 A_{X2} + 30569.9 A_{X3}}{78070.76}$$
 (C7)

$$a_{x} = .404194A_{x1} + .204239A_{x2} + .391567A_{x3}$$
 (C8)

From (B 19)

$$-\frac{15.12}{12}\left(-.094637 \text{ Axi} + .094637 \text{ Axs}\right) - \text{Aye} \tag{C9}$$



FA 12 15 RT NO ZA-7-137 MODEL Seven DATE 27 Nov. 1956

UNCLASSIFIED

COMPANY

The numerical values of the coefficients are good only for the geometry specified in Table I. Since the location of center of gravity will change with time, these coefficients will vary with time. In addition, the location of the instruments may change which would also cause the coefficients to change from the values quoted herein. Consequently before using any of these relations in data reduction, all numerical quantities should be carefully checked.

