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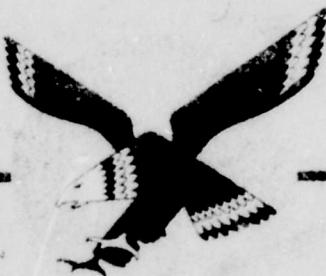
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REPORT ZP-7-022 TN

DATE 2-14-56

MODEL XN-65

TITLE

ANALYSIS OF MISSILE
TANK GEOMETRIES

ZP-7-022 TN



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Model 7 - Preliminary
Design and Systems
GROUP Analysis

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APPROVED BY Krafft A. Ellerick

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MODEL 7
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FOREWORD

This is a technical note pertaining to missile tank geometry.
It provides a list of formulas that may be referred to in reports
and other technical papers where their development is incidental.

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SUMMARY

Mathematical relations are presented pertaining to various missile tank and bulkhead configurations. Although some of these relations are available in handbooks, they are included in this technical note together with a number of newly derived formulas. In this manner a comprehensive summary of equations is obtained, permitting rapid computation of volumes, surfaces and other geometric characteristics of a great variety of shapes.

The purpose of this technical note is to eliminate the need for repeating the derivations of frequently used equations in preliminary design and layout of missiles.

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INTRODUCTION

Reports concerning missile tank geometry have been compiled and it is expected that studies of this sort will continue. The studies indicate the necessity for a list of reference formulas applying to tank geometry that may be drawn from and referred to, eliminating the need of developing them within a report or the repetition of their development in related reports. Although it is felt proper to make calculations within a report, geometric formula development is incidental at the time of writing and should be available elsewhere.

This technical note is presented for the purpose of fulfilling this requirement and effort has been made to include a broad range of mathematical expressions to meet the needs of missile tank designers.

These formulas presented consist of mathematical expressions for tank and bulkhead volumes, their surface areas and other pertinent geometric information that is not readily available in standard handbooks. Formulas for partially filled tank shapes are included along with those of partially filled bulkheads.

In general, this report is compiled for use with studies pertaining to the SM-65 vehicle, however, it includes geometries for other shape tanks and bulkheads that may be used in future development studies.

No attempt is made by this report to qualify the choice of any particular tank or bulkhead.

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INTRODUCTION (cont'd.)

The first part following this introduction contains a comprehensive list of formulas together with reference numbers which may be referred to in technical studies. Each of these formulas refer to its origination whether "classic" or whether derived within this report. If it is derived within this report, its location is referred to by page number.

The second part is devoted to the derivation of the uncommon formulas referred to in the first section. They are presented in a form that gives the designer an understanding of the fundamental derivation and allows him to change the formula's original parameters without time consuming effort.

The formulas contained in this report have been checked. However, errors may exist and their appearance should be brought to the authors attention for correction.

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PART I

SUMMARY OF EQUATIONS

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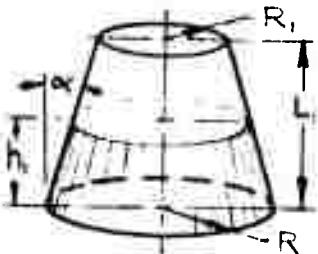
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V_T IS TOTAL VOLUME
 S_T IS TOTAL SURFACE AREA

CONE FRUSTUM TANK REF. PAGES 6 & 7



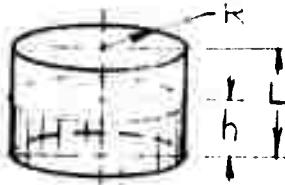
$$(1) V_T = \frac{\pi}{3} \tan \alpha (R^3 - R_1^3)$$

$$(2) S_T = \frac{\pi L (R + R_1)}{\cos \alpha} *$$

$$= \pi (R + R_1) \sqrt{L^2 + (R - R_1)^2}$$

$$(3) V_{(OF\ h_i)} = \pi h_i (R^2 - Rh_i \tan \alpha + \frac{1}{3} h_i^2 \tan^2 \alpha)$$

CYLINDRICAL TANK (CLASSIC FORMULAS)

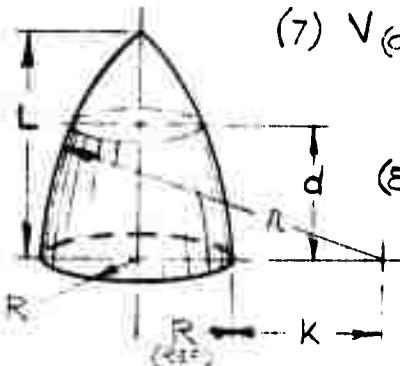


$$(4) V_T = \pi R^2 L$$

$$(5) V_{(OF\ h)} = \pi R^2 h$$

$$(6) S_T = 2 \pi R L *$$

OGIVE TANK (CIRCULAR) REF. PAGE 8



$$(7) V_{(OF\ d)} = \pi [k^2 d - k \{ d \sqrt{r^2 - d^2} + r^2 \sin^{-1}(\frac{d}{r}) \} + r^2 d - \frac{1}{3} d^3]$$

$$(8) S_{(OF\ d)} = 2 \pi r [d - k \sin^{-1}(\frac{d}{r})] *$$

*NOTE. - TANK ENDS ARE NOT INCLUDED IN SURF. AREAS

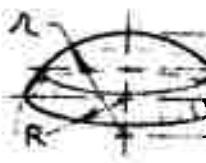
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SPHERICAL BULKHEAD REF. PAGES 9, 10 & 11



$$(9) V_t = \frac{1}{3} \pi H^2 (3R - H)$$
$$(10) = \frac{1}{6} \pi H (H^2 + 3R^2)$$

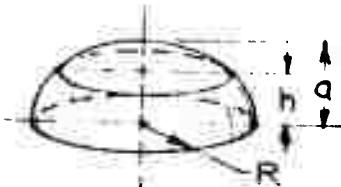
$$(11) R = \frac{H}{2} + \frac{R^2}{2H}$$

$$(12) S_t = 2\pi R H *$$

use eqn 9 twice ~~(13) $V_{(OF\ H)} = \pi d [(\lambda^2 - h^2) - d(\frac{1}{3} d + h)]$~~
~~(13) in ERROR.~~

$$h = R - H \quad \text{OR} \quad h = \sqrt{R^2 - R^2}$$

ELLIPTICAL BULKHEAD REF. PAGES 12, 13 & 14
(IN CYLINDRICAL TANK)



$$(14) V_t = \frac{2}{3} \pi R^2 a$$

$$(15) V_{(OF\ H)} = \pi h \left(\frac{R}{a}\right)^2 \left(a^2 - \frac{1}{3} h^2\right)$$

$$(16) S_t = \pi \left\{ a \sqrt{C_1 a^2 + R^2} + \frac{R^2}{C_1} \left[\log(a \sqrt{C_1} + \sqrt{C_1 a^2 + R^2}) - \log R \right] \right\} *$$

$$C_1 = K_1^4 - K_1^2 \quad \text{WHERE} \quad K_1 = R/a$$

LOG IS OF BASE "e"

(17) RADIUS OF CURVATURE (ρ) DIST. X ABOVE
BASE PLANE

$$\rho = \frac{[R^2 + K_1^2 X^2 (K_1^2 - 1)]^{1/2}}{K_1^2 R^2}$$

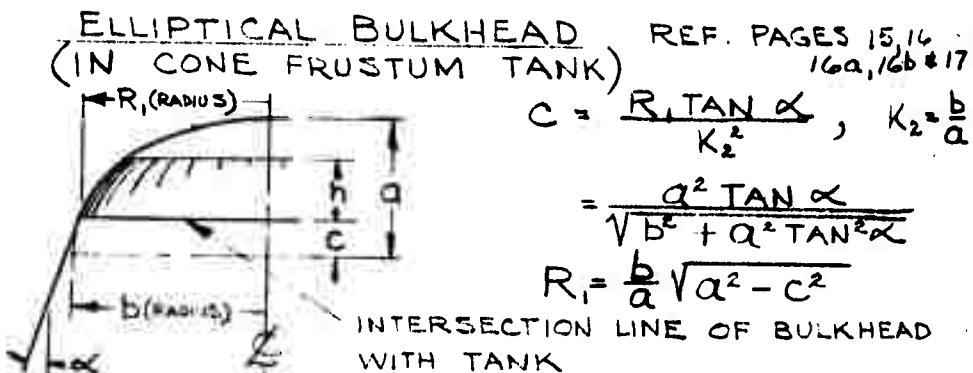
* NOTE :- BULK'D BASE IS NOT INCLUDED IN SURF. AREA

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α = ANGLE OF CONE FRUSTUM TANK

(18) V_T (TOTAL VOL. OF BULK'D FROM BASE [TANK'S INTERSECTION] TO APEX)

$$= \pi \left(\frac{b}{a} \right)^2 \left[a^2(a - c) + \frac{1}{3}(c^3 - a^3) \right]$$

(19) V (VOLUME FROM BASE TO h'')

$$= \pi \left(\frac{b}{a} \right)^2 \left[a^2(h - c) + \frac{1}{3}(c^3 - h^3) \right]$$

(20) S (SURFACE AREA OF BULK'D FROM BASE [TANK'S INTERSECTION] TO APEX)

$$S = \pi \left[a \sqrt{c_1 a^2 + b^2} + \frac{b^2}{\sqrt{c_2}} \log(a \sqrt{c_2} + \sqrt{c_2 a^2 + b^2}) \right. \\ \left. - c \sqrt{c_1 c^2 + b^2} - \frac{b^2}{\sqrt{c_2}} \log(c \sqrt{c_2} + \sqrt{c_2 c^2 + b^2}) \right]$$

a = BULK'D MINOR SEMI-AXIS (HEIGHT)

b = " MAJOR " "

c = DIST BETWEEN MAJOR AXIS AND BULK'D INTERSECTION WITH TANK = $R_1 \tan \alpha / K_2$

$c_1 = K_2^4 - K_2^2$ WHERE $K_2 = b/a$

LOG IS BASE "e"

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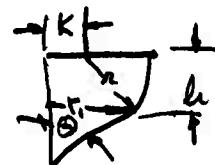
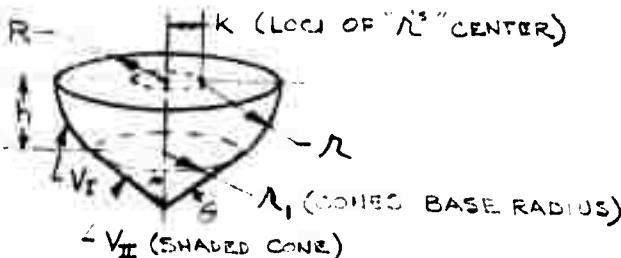
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TORICONICAL BULKHEAD

REF. PAGES 18 & 19



$$(21) V_T = V_I + V_{II}$$

$$V_I = \pi \left\{ h(r^2 + K^2) + K \left[h\sqrt{r^2 - K^2} + r^2 \sin^{-1}\left(\frac{h}{r}\right) \right] - \frac{h^3}{3} \right\}$$

$$V_{II} = \frac{1}{3} \pi r_1^2 \cot \theta$$

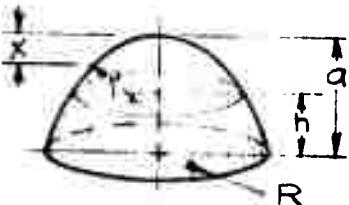
$$(22) S = S_I + S_{II} \quad \left(\sin^{-1} \frac{h}{r} = \theta \text{ RAD.} \right)$$

$$S_I = 2\pi r [K \sin^{-1}\left(\frac{h}{r}\right) + h] \quad (\text{TORUS})$$

$$S_{II} = \frac{\pi r_1^2}{\sin \theta} \quad (\text{CONE})$$

PARABOLIC BULKHEAD

REF. PAGES 20 & 21



$$(23) V_T = \frac{1}{2} \pi R^2 a$$

$$(24) V_{(OF h)} = \frac{1}{2} \pi R^2 h [2 - \frac{h}{a}]$$

$$(25) S_I = \frac{1}{6} \pi Ra \left[\left(4 + \frac{R^2}{a^2}\right)^{\frac{3}{2}} - \left(\frac{R}{a}\right)^3 \right]$$

$$(27) \text{ RADII'S OF CURVATURE } (\rho)$$

$$\rho = \frac{1}{2Ra} (4ax + R^2)^{\frac{3}{2}}$$

X = VERTICAL DIST FROM VERTEX TO
WHERE ρ IS DESIRED (ON THE SURFACE)

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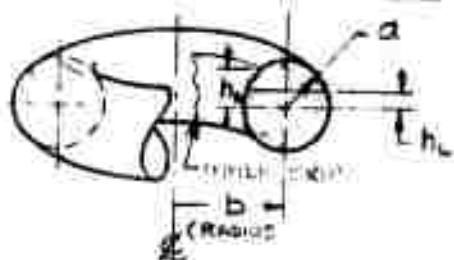
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TORUS TANK

REF PAGES 22 THRU 26



$$(28) \quad V_T = 2\pi^2 b a^2 = 19.739 b a^2$$

$$(29) \quad \text{PARTIALLY FILLED TORUS TO } h_L \text{ (NOTE: } h_L \text{ MAY BE + OR -)} \quad \text{VOL. OF LIQUID} =$$

$$V_L = 2\pi a^2 b \left\{ K \sqrt{1-K^2} + \sin^{-1} K + \frac{\pi}{2} \right\}$$
$$K = \frac{h_L}{a}$$

$$(30) \quad S_T = 4\pi^2 a b$$

(31) OUTER SKIN SURFACE AREA OF $2 h_s$
TOTAL WIDTH AND SYMMETRICAL ABOUT
THE HORIZONTAL CENTERLINE.

$$S_O = 4\pi a \left[b \sin^{-1} \left(\frac{h_s}{a} \right) + h_s \right]$$

(32) INNER SKIN ($2 h_s$ WIDE)

$$S_I = 4\pi a \left[b \sin^{-1} \left(\frac{h_s}{a} \right) - h_s \right]$$

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PART II
DERIVATIONS

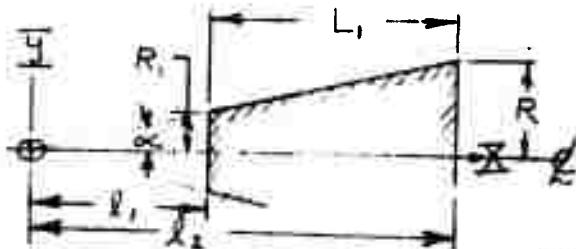
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CONE FRUSTUM TANK



$$L_1 = l_2 - l_1 \\ = \frac{R - R_1}{\tan \alpha}$$

$$y = x \tan \alpha \quad l_1 = \frac{R_1}{\tan \alpha}, \quad l_2 = \frac{R}{\tan \alpha}$$

VOLUME (ENCLOSED FROM R_1 TO R)

$$\begin{aligned} V_T &= \pi \int_{l_1}^{l_2} y^2 dx = \pi \int_{\frac{R_1}{\tan \alpha}}^{\frac{R}{\tan \alpha}} x^2 \tan^2 \alpha dx \\ &= \pi \tan^2 \alpha \left[\frac{x^3}{3} \right]_{\frac{R_1}{\tan \alpha}}^{\frac{R}{\tan \alpha}} \\ &= \frac{1}{3} \pi \tan^2 \alpha \left(\frac{R^3}{\tan^3 \alpha} - \frac{R_1^3}{\tan^3 \alpha} \right) \\ &= \underline{\underline{\frac{\pi}{3} \tan \alpha (R^3 - R_1^3)}} \quad OR = \underline{\underline{\frac{\pi L_1}{3} (R^2 + RR_1 + R_1^2)}} \end{aligned}$$

SURFACE AREA

$$\begin{aligned} S &= \left(\frac{R_1 + R}{2} \right) \cdot 2\pi \cdot \frac{L_1}{\cos \alpha} \\ &= \underline{\underline{\frac{\pi L_1 (R_1 + R)}{\cos \alpha}}} \quad OR = \pi(R + R_1) \sqrt{L_1^2 + (R - R_1)^2} \end{aligned}$$

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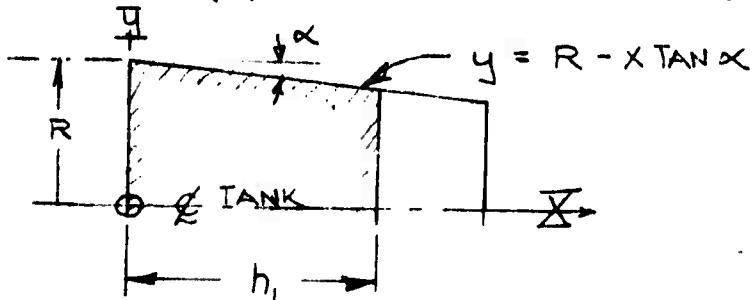
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CONE FRUSTUM TANK (CONT'D)

PARTIALLY FILLED TANK IN TERMS OF BASE RADIUS (R), CONE ANGLE (α) AND HEIGHT OF LIQUID (h) ABOVE TANK'S BASE.



VOLUME 0 TO h ,

$$\begin{aligned} V &= \pi \int_0^h y^2 dx \\ &= \pi \int_0^h (R - x \tan \alpha)^2 dx \\ &= \pi \int_0^h (R^2 - 2Rx \tan \alpha + x^2 \tan^2 \alpha) dx \\ &= \pi \left[R^2 x - Rx^2 \tan \alpha + \frac{1}{3} x^3 \tan^2 \alpha \right]_0^h \\ &= \pi h \left[R^2 - Rh \tan \alpha + \frac{1}{3} h^2 \tan^2 \alpha \right] \end{aligned}$$

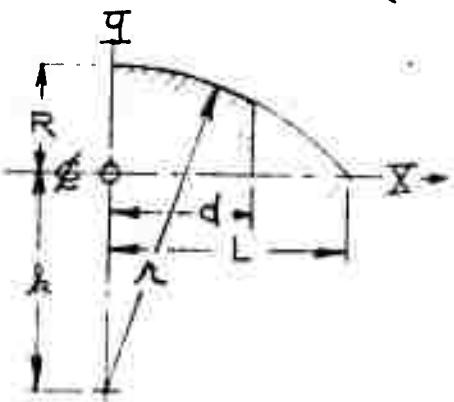
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OGIVE TANK (CIRCULAR) (RADIUS NOSE)



$$x^2 + (y + h)^2 = r^2$$

$$y = -h + \sqrt{r^2 - x^2}$$

$$\frac{dy}{dx} = -x(r^2 - x^2)^{-\frac{1}{2}}$$

VOLUME (ENCLOSED TO "d")

$$\begin{aligned} V &= \pi \int_0^d y^2 dx = \pi \int_0^d (-h + \sqrt{r^2 - x^2})^2 dx \\ &= \pi \int_0^d (h^2 - 2h\sqrt{r^2 - x^2} + r^2 - x^2) dx \\ &= \pi \left[h^2 x - 2h \cdot \frac{1}{2} \left\{ x\sqrt{r^2 - x^2} + r^2 \sin^{-1}\left(\frac{x}{r}\right) \right\} \right. \\ &\quad \left. + r^2 x - \frac{1}{3} x^3 \right]_0^d \\ &= \pi \left[h^2 d - h \left\{ d\sqrt{r^2 - d^2} + r^2 \sin^{-1}\left(\frac{d}{r}\right) \right\} \right. \\ &\quad \left. + r^2 d - \frac{1}{3} d^3 \right] \end{aligned}$$

SURFACE AREA (TO d)

$$\begin{aligned} S &= 2\pi \int_0^d (-h + \sqrt{r^2 - x^2}) \left[1 + \frac{x^2}{r^2 - x^2} \right]^{\frac{1}{2}} dx \\ &= 2\pi \int_0^d (-h + \sqrt{r^2 - x^2}) \left[\frac{r}{\sqrt{r^2 - x^2}} \right] dx \\ &\rightarrow 2\pi \int_0^d \left(\frac{-h}{\sqrt{r^2 - x^2}} + r \right) dx = 2\pi r \left[-h \sin^{-1}\left(\frac{x}{r}\right) + x \right]_0^d \\ &= 2\pi r \left[d - h \sin^{-1}\left(\frac{d}{r}\right) \right] \end{aligned}$$

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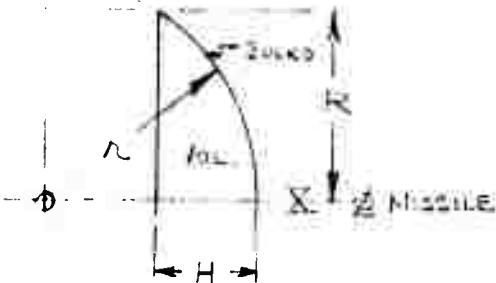
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SPHERICAL BULKHEAD

(THE FOLLOWING APPLIES TO A BULK'D
FOR A CYLINDRICAL OR TAPERED TANK)

ST



R = MISSILE TANK RADIUS
r = BULK'D RADIUS
H = BULK'D HEIGHT

H + R KNOWN
r = ?

$$x^2 + y^2 = r^2 \quad y^2 = r^2 - x^2$$

$$(r-H)^2 + R^2 = r^2 \quad \text{OR} \quad r^2 - 2rH + H^2 + R^2 = r^2$$
$$r = \frac{H}{2} + \frac{R^2}{2H}$$

VOLUME

IN TERMS OF BULK'D HEIGHT (H) & RADIUS (r)

$$\begin{aligned} V &= \pi \int_{r-H}^r y^2 dx = \pi \int_{r-H}^r (r^2 - x^2) dx \\ &= \pi \left(r^2 x - \frac{x^3}{3} \right) \Big|_{r-H}^r \\ &= \pi \left[(r^2 r - \frac{r^3}{3}) - (r^2(r-H) - \frac{(r-H)^3}{3}) \right] \\ &= \pi \left[Hr^2 - \frac{1}{3}r^3 + \frac{(r-H)^3}{3} \right] = \frac{H^2}{3}\pi(3r-H) \end{aligned}$$

IN TERMS OF BULK'D HEIGHT (H) & TANK RAD (R)

$$r = \frac{H}{2} + \frac{R^2}{2H} \quad \text{IN THE ABOVE FORMULA}$$

$$V = \frac{1}{6}\pi H [H^2 + 3R^2]$$

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SPHERICAL BULKHEAD (CONT'D)

SURFACE AREA

$$S = 2\pi \int_{r-H}^r y \, ds \quad ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$\begin{aligned}x^2 + y^2 &= r^2 \\2x \, dx + 2y \, dy &= 0 \quad \frac{dy}{dx} = -\frac{x}{y}\end{aligned}$$

$$S = 2\pi \int_{r-H}^r y \sqrt{1 + \left(-\frac{x}{y}\right)^2} \, dx$$

$$= 2\pi \int_{r-H}^r \sqrt{y^2 + x^2} \, dx$$

$$= 2\pi \int_{r-H}^r \sqrt{r^2 - x^2 + x^2} \, dx$$

$$= 2\pi \int_{r-H}^r r \, dx$$

$$= 2\pi r x \Big|_{r-H}^r$$

$$= 2\pi r [r - (r - H)]$$

$$= \underline{\underline{2\pi r H}}$$

FOR HEMISPHERE $H = r$

$$S = 2\pi r^2 \quad (\text{HEMISPHERE})$$

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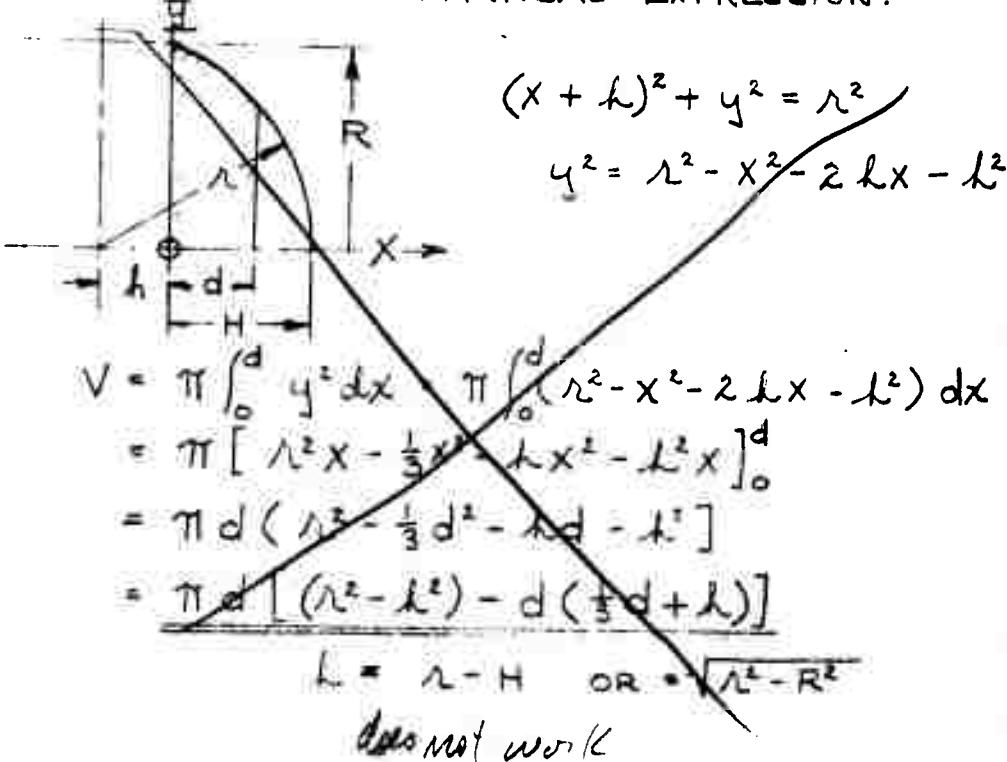
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SPHERICAL BULKHEAD (CONT'D)

VOLUME OF LIQUID IN PARTIALLY FILLED BULK'

IN TERMS OF LIQUID DEPTH (d), BULK'D RADIUS (λ) AND DIST. (h) WHICH IS THE DISTANCE BETWEEN THE BULK'D BASE AND THE CENTER POINT OF THE BULK'D RADIUS.

NOTE: IT IS DESIRABLE TO SHIFT THE X AXIS TO THE BULK'DS BASE FOR REASONS OF SIMPLIFICATION WHEN (h) IS INTRODUCED INTO THE MATHEMATICAL EXPRESSION.



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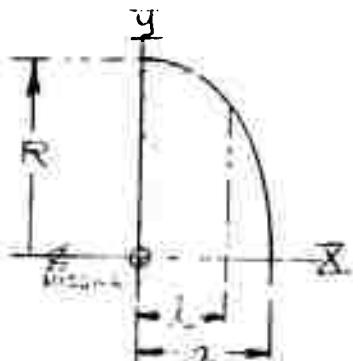
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ELLIPTICAL BULKHEAD

VOLUME (ENCLOSED BY THE BULKHEAD
IN A CYLINDRICAL TANK)



GEN. ELLIP. EQUATION

$$\frac{x^2}{c^2} + \frac{y^2}{R^2} = 1$$

$$y^2 = R^2 - (K_1 x)^2$$

$$K_1 = R/a$$

R = TANK RADIUS

a = BULK'D HEIGHT

VOLUME ENCLOSED TO h.

$$\begin{aligned} V &= \pi \int_0^h y^2 dx = \pi \int_0^h [R^2 - (K_1 x)^2] dx \\ &= \pi \left[R^2 h - \frac{1}{3} K_1^2 h^3 \right] \\ &= \pi h \left(\frac{R}{a} \right)^2 \left(a^2 - \frac{1}{3} h^2 \right) \quad K_1 = R/a \end{aligned}$$

VOLUME ENCLOSED TO a (ENTIRE BULK)

$$V = \pi \left[R^2 h - \frac{1}{3} K_1^2 a^3 \right] \quad (\text{FROM ABOVE})$$

WHEN $h = a$ AND $K_1 = R/a$

$$V = \pi \left[R^2 a - \frac{1}{3} \frac{R^2}{a^2} \cdot a^3 \right]$$

$$= \frac{2}{3} \pi R^2 a$$

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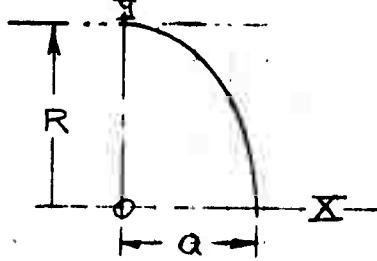
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ELLIPTICAL BULKHEAD (CONT'D)

SURFACE AREA (BULK'D IN CYLINDRICAL TANK)



$$y^2 = R^2 - (K_1 x)^2$$

$$K_1 = R/a$$

$$2y dy = -2K_1^2 x dx$$

$$\frac{dy}{dx} = -\frac{K_1^2 x}{y}$$

$$S = 2\pi \int_0^a y ds$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= 2\pi \int_0^a y \sqrt{1 + \frac{K_1^4 x^2}{y^2}} dx$$

$$= 2\pi \int_0^a \sqrt{y^2 + K_1^4 x^2} dx$$

$$= 2\pi \int_0^a \sqrt{R^2 - K_1^2 x^2 + K_1^4 x^2} dx$$

$$= 2\pi \int_0^a \sqrt{R^2 + (K_1^4 - K_1^2) x^2} dx$$

$$\text{LET } C_1 = K_1^4 - K_1^2$$

$$= 2\pi \int_0^a \sqrt{C_1 x^2 + R^2} dx$$

$$= 2\pi \left[\frac{x}{2} \sqrt{C_1 x^2 + R^2} + \frac{R^2}{2\sqrt{C_1}} \log(x\sqrt{C_1} + \sqrt{C_1 x^2 + R^2}) \right]_0^a$$

$$S = \pi \left\{ a \sqrt{a^2 + R^2} + \frac{R^2}{4C_1} [\log(a\sqrt{C_1} + \sqrt{a^2 + R^2}) - \log R] \right\}$$

a = BULK'D HEIGHT R = TANK RADIUS

$C_1 = K_1^4 - K_1^2$ WHERE $K_1 = R/a$

LOG. IS TO BASE "E"

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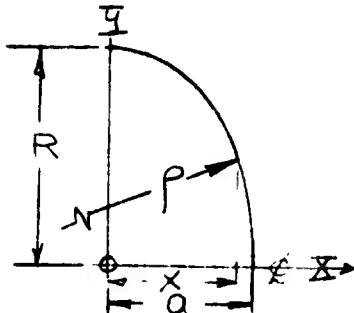
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ELLIPTICAL BULKHEAD (CONT'D)

RADIUS OF CURVATURE (ρ)



$$y^2 = R^2 - (K_1 X)^2$$

$$K_1 = \frac{R}{a}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

$$y = (R^2 - K_1^2 X^2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = -K_1^2 X (R^2 - K_1^2 X^2)^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{(R^2 - K_1^2 X^2)^{\frac{1}{2}} (-K_1^2) - (-K_1^2 X) \frac{1}{2} (R^2 - K_1^2 X^2)^{-\frac{1}{2}} (-2K_1^2 X)}{R^2 - K_1^2 X^2}$$

$$= \frac{(R^2 - K_1^2 X^2)^{-\frac{1}{2}} [(R^2 - K_1^2 X^2)X - K_1^2] - (-K_1^2 X) \frac{1}{2} (-2K_1^2 X)}{R^2 - K_1^2 X^2}$$

$$= \frac{-K_1^2 (R^2 - K_1^2 X^2) - K_1^4 X^2}{(R^2 - K_1^2 X^2)^{\frac{3}{2}}} = \frac{-K_1^2 R^2}{(R^2 - K_1^2 X^2)^{\frac{3}{2}}}$$

$$\rho = \frac{\left[1 + \left\{ -K_1^2 X (R^2 - K_1^2 X^2)^{-\frac{1}{2}} \right\}^2 \right]^{\frac{3}{2}}}{-\frac{K_1^2 R^2}{(R^2 - K_1^2 X^2)^{\frac{3}{2}}}}$$

$$= \frac{(R^2 - K_1^2 X^2)^{\frac{3}{2}} [1 + K_1^4 X^2 (R^2 - K_1^2 X^2)^{-1}]^{\frac{3}{2}}}{-K_1^2 R^2}$$

$$= -\frac{[R^2 + K_1^2 X^2 (K_1^2 - 1)]^{\frac{3}{2}}}{K_1^2 R^2}$$

82.8
31.5

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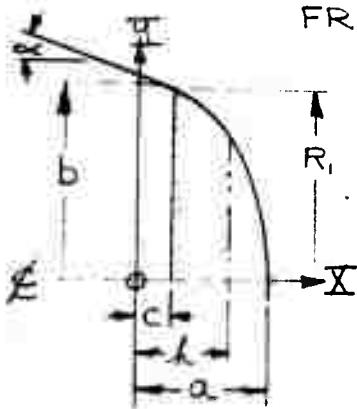
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ELLIPTICAL BULKHEAD (CONT'D)

VOLUME (ENCLOSED BY THE BULKHEAD
AT THE END OF A CONICAL
FRUSTUM TANK)



GEN ELLIPTICAL EQUATION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y^2 = b^2 - (K_2 x)^2$$

$$K_2 = b/a$$

WHERE:

a = MAJOR SEMI-AXIS

b = MINOR SEMI-AXIS

h = PROPELLANT LEVEL

$c = R_{TAN} \times \frac{K_2}{K_2}$ (INTERSECTION OF TANK WITH BULK'D)

VOLUME ENCLOSED FROM "C" TO "h"

$$\begin{aligned} V &= \pi \int_c^h y^2 dx = \pi \int_c^h [b^2 - (K_2 x)^2] dx \\ &= \pi \left[b^2 h - \frac{1}{3} K_2^2 h^3 - b^2 c + \frac{1}{3} K_2^2 c^3 \right] \\ &= \pi \left[b^2(h-c) + \frac{1}{3} K_2^2 (c^3 - h^3) \right] \\ (\text{OR,}) \quad &= \pi \left(\frac{b}{a} \right)^2 \left[a^2(h-c) + \frac{1}{3} (c^3 - h^3) \right] \end{aligned}$$

$$K_2 = b/a$$

VOLUME ENCLOSED FROM "C" TO "a"

$$\begin{aligned} V &= \pi \left[b^2(a-c) + \frac{1}{3} K_2^2 (c^3 - a^3) \right] \\ (\text{OR}) \quad &= \pi \left(\frac{b}{a} \right)^2 \left[a^2(a-c) + \frac{1}{3} (c^3 - a^3) \right] \end{aligned}$$

$$K_2 = b/a$$

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ELLIPTICAL BULKHEAD (CONT'D)

(BULK'D AT END OF CONICAL FRUSTUM TANK CONT'D)

GENERALLY SPEAKING. THE VALUE K_2 (RATIO OF MAJOR TO MINOR AXIS) IS PRE-DETERMINED BY STRUCTURAL CONSIDERATIONS. ALSO THE VALUE OF R_1 (RADIUS OF TANK END) AND α (CONICAL FRUSTUM HALF ANGLE) ARE KNOWN FROM THE TANK GEOMETRY. FROM THESE (K_2 , R_1 , α) THE BULKHEAD GEOMETRY MAY BE SET AS FOLLOWS.

BULKHEAD'S MAJOR AXIS "b"

$$y^2 = b^2 - (K_2 x)^2 \quad (\text{BASIC EQUATION})$$

$$b = \sqrt{y^2 + (K_2 x)^2}$$

$$\text{WHEN } y = R_1, \quad x = c = \frac{R_1 \tan \alpha}{K_2}$$

$$b = \sqrt{R_1^2 + (K_2 \frac{R_1 \tan \alpha}{K_2})^2}$$

$$= \frac{R_1}{K_2} \sqrt{K_2^2 + \tan^2 \alpha}$$

$$\alpha = \frac{b}{K_2}$$

IF THE BULK'DS HEIGHT "H" BEYOND THE TANK END IS KNOWN IN ADDITION TO K_2 , R_1 , α THEN,

$$H = a - c \quad (\text{FROM FIGURE OF PREVIOUS PAGE})$$

$$= a - \frac{R_1 \tan \alpha}{K_2}$$

$$a = H + \frac{R_1 \tan \alpha}{K_2}$$

$$b = K_2 a$$

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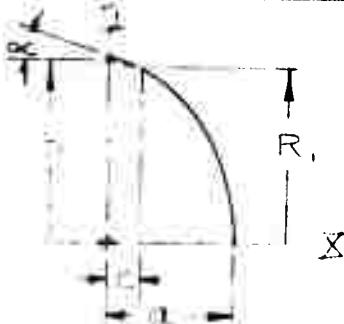
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ELLIPTICAL BULKHEAD (CONT'D)

(BULK'D AT END OF CONICAL FRUSTUM TANK-CONT'D)

VALUE OF "C" AND "R"



C = DISTANCE BETWEEN MAJOR
AXIS AND BULKHEAD
INTERSECTION WITH TANK

R_t = TANK AND BULKHEAD
INTERSECTION RADIUS
(RADIUS AT TOP OF TANK)

a = BULK'D MINOR SEMI AXIS
b = " MAJOR " "

$$y^2 = b^2 - (K_2 x)^2 \quad \text{BULK'D GENERAL FORMULA}$$

$$K_2 = b/a$$

$\frac{dy}{dx}$ = SLOPE AT ANY POINT ON ELLipse

$$2y \frac{dy}{dx} = 0 - 2K_2^2 x \quad (\text{DIFF. OF ABOVE FORMULA})$$
$$\frac{dy}{dx} = -\frac{2K_2^2 x}{2y} = -K_2^2 \frac{x}{y}$$

ALSO $\frac{dy}{dx} = -\tan \alpha$

$$\text{THEN } -\tan \alpha = -K_2^2 \frac{x}{y}$$

$$\text{WHEN } x = C \quad y = R_i$$

$$\tan \alpha = K_2^2 \frac{C}{R_i}$$

$$C = R_i \frac{\tan \alpha}{K_2^2}$$

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ELLIPTICAL BULKHEAD (CONT'D)

(BULK'D AT END OF CONICAL FRUSTUM TANK - CONT'D)

VALUE OF "C" AND "R_i" (CONT'D)

$$\tan \alpha = K_2^2 \frac{x}{y} \quad (\text{FROM PREVIOUS PAGE})$$

$$\text{WHEN } x = C \quad y^2 = b^2 - (K_2 C)^2$$

$$\tan^2 \alpha = K_2^4 \frac{x^2}{y^2}$$

$$\tan^2 \alpha (b^2 - K_2^2 C^2) = K_2^4 C^2$$

$$(K_2^4 + K_2^2 \tan^2 \alpha) C^2 = b^2 \tan^2 \alpha$$

$$C = \frac{b \tan \alpha}{K_2 \sqrt{K_2^2 + \tan^2 \alpha}} = \frac{b \tan \alpha}{\frac{b}{a} \sqrt{\frac{b^2}{a^2} + \tan^2 \alpha}}$$
$$= \frac{a^2 \tan \alpha}{\sqrt{b^2 + a^2 \tan^2 \alpha}}$$

$$\text{WHEN } x = C \quad y = R_i$$

$$\text{THEN FROM } y^2 = b^2 - K_2^2 x^2$$

$$R_i^2 = b^2 - K_2^2 C^2$$

$$R_i = \sqrt{b^2 - \frac{b^2}{a^2} C^2}$$

$$= \frac{b}{a} \sqrt{a^2 - C^2}$$

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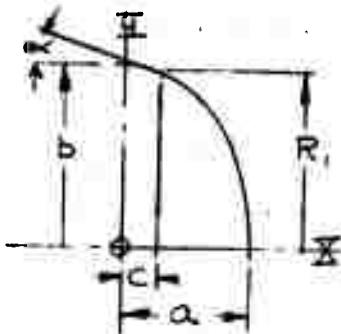
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ELLIPTICAL BULKHEAD (CONT'D)

SURFACE AREA (BULK'D AT SMALL END OF CONICAL FRUSTUM TANK)



$$y^2 = b^2 - (K_2 x)^2$$

$$K_2 = \frac{b}{a}$$

SURFACE AREA OF THE ELLIPTICAL BULK'D FROM LINE OF TANK INTERSECTION IS

$$S = \int_c^a y \, ds$$

FROM WHICH, (SEE DEVEL. ON PREVIOUS PAGE)

$$S = 2\pi \int_c^a \sqrt{C_2 x^2 + b^2} \, dx$$

$$\text{WHERE } C_2 = K_2^4 - K_2^2$$

$$= 2\pi \left[\frac{x}{2} \sqrt{C_2 x^2 + b^2} + \frac{b^2}{2\sqrt{C_2}} \log(x\sqrt{C_2} + \sqrt{C_2 x^2 + b^2}) \right]_c^a$$

$$S = \pi \left[a \sqrt{C_1 a^2 + b^2} + \frac{b^2}{\sqrt{C_2}} \log(a\sqrt{C_2} + \sqrt{C_2 a^2 + b^2}) \right. \\ \left. - c \sqrt{C_1 c^2 + b^2} - \frac{b^2}{\sqrt{C_2}} \log(c\sqrt{C_2} + \sqrt{C_2 c^2 + b^2}) \right]$$

WHERE: a = BULK'D MINOR SEMI AXIS (HEIGHT)

b = " MAJOR "

c = DIST. BETWEEN MAJOR AXIS AND BULK'D INTERSECT WITH TANK = ~~R_{TANK}~~

$C_1 = K_1^4 - K_1^2$ WHERE $K_1 = \frac{b}{a}$

LOG IS TO BASE "e"

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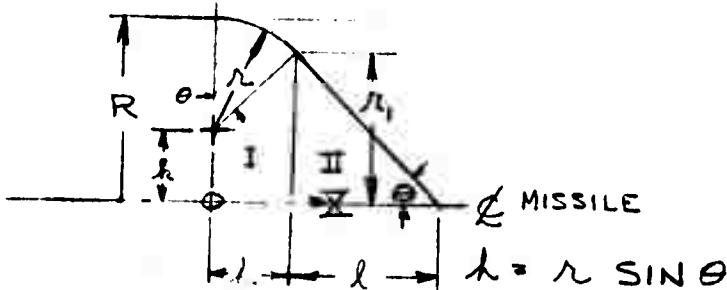
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TORICONICAL BULKHEAD

VOLUME I (ENCLOSED BY BULKHEAD AT
END OF CYLINDRICAL TANK)



VOLUME I

$$\begin{aligned} x^2 + (y - h)^2 &= r^2 \\ y^2 &= h^2 + 2h\sqrt{r^2 - x^2} + r^2 - x^2 \\ V_I &= \pi \int_0^h y^2 dx = \pi \int_0^h (h^2 + 2h\sqrt{r^2 - x^2} + r^2 - x^2) dx \\ &= \pi \left\{ h^2 x + 2h \frac{1}{2} \left[x\sqrt{r^2 - x^2} + r^2 \sin^{-1}\left(\frac{x}{r}\right) \right] + r^2 x - \frac{x^3}{3} \right\} \Big|_0^h \\ &= \pi \left\{ h(l(r^2 + h^2)) + h \left[h\sqrt{r^2 - l^2} + r^2 \sin^{-1}\left(\frac{l}{r}\right) \right] - \frac{l^3}{3} \right\} \end{aligned}$$

VOLUME II

$$\begin{aligned} V &= \frac{1}{3} A_B l \quad (\text{GENERAL CONE FORMULA}) \\ &= \frac{1}{3} \pi r_i^2 \times r_i \cot \theta \\ &= \frac{1}{3} \pi r_i^3 \cot \theta \end{aligned}$$

$$\begin{aligned} r_i &= R - l + h \cot \theta \\ &= h + l \cot \theta \\ \text{OR} &= h + r \cos \theta \end{aligned}$$

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TORICONICAL BULK'D (CONT'D)

SURFACE AREA

SURFACE ENCLOSING VOLUME I

$$S_I = 2\pi \int_0^h y \, ds \quad ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$y = h + \sqrt{r^2 - x^2}$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{r^2 - x^2}} \quad \left(\frac{dy}{dx}\right)^2 = \frac{x^2}{r^2 - x^2}$$

$$S_I = 2\pi \int_0^h (h + \sqrt{r^2 - x^2}) \left[1 + \frac{x^2}{r^2 - x^2} \right]^{\frac{1}{2}} \, dx$$

$$= 2\pi \int_0^h (h + \sqrt{r^2 - x^2}) \left(\frac{r}{\sqrt{r^2 - x^2}} \right) \, dx$$

$$= 2\pi \int_0^h \left(\frac{h r}{r^2 - x^2} + \sqrt{r^2 - x^2} \times \frac{r}{r^2 - x^2} \right) \, dx$$

$$= 2\pi r \int_0^h \left[\frac{h}{r^2 - x^2} + 1 \right] \, dx$$

$$= 2\pi r \left[h \sin^{-1}\left(\frac{x}{r}\right) + x \right]_0^h$$

$$= \underline{2\pi r \left[h \sin^{-1}\left(\frac{h}{r}\right) + h \right]}$$

SURFACE ENCLOSING VOLUME II

$$S_{II} = \frac{\text{AREA OF BASE}}{\sin \theta} = \frac{\pi r^2}{\sin \theta} \quad (\text{CLASSIC})$$

TOTAL SURF. AREA OF TORICONICAL BULK'D IS

$$S_T = S_I + S_{II}$$

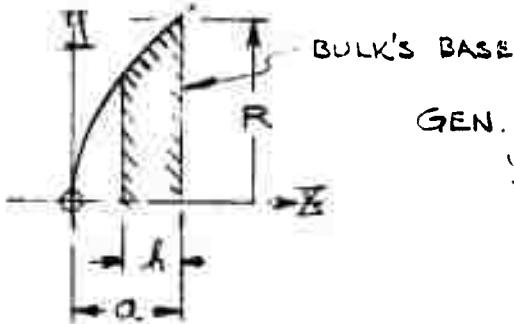
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PARABOLIC BULKHEAD



GEN. PARABOLIC EQUATION

$$y^2 = Ax$$

$$A = \frac{R^2}{a}$$

VOLUME ENCLOSED FROM "h" TO "a" (^{SHADeD AREA})

$$\begin{aligned} V &= \pi \int_{a-h}^h y^2 dx = \pi \int_{a-h}^h Ax dx \\ &= \pi \frac{R^2}{a} \int_{a-h}^h x dx \\ &= \frac{\pi R^2}{2a} x^2 \Big|_{a-h}^h = \frac{\pi R^2}{2a} [a^2 - (a-h)^2] \\ &= \frac{1}{2} \pi R^2 \left[2h - \frac{h^2}{a} \right] = \underline{\frac{1}{2} \pi R^2 h \left(2 - \frac{h}{a} \right)} \end{aligned}$$

WHEN $h = a$ THEN,

$$V_t = \underline{\frac{1}{2} \pi R^2 a}$$

SURFACE AREA OF ENTIRE BULK'D

$$\begin{aligned} S &= 2\pi \int_0^a y ds \quad ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \\ &= 2\pi \int_0^a y \sqrt{1 + \left(\frac{2x}{a}\right)^2} dx = \pi \int_0^a \sqrt{4Ax + A^2} dx \\ &= \pi \left[\frac{2}{3} \cdot \frac{4A}{3} (4Ax + A^2)^{\frac{3}{2}} \right]_0^a = \frac{\pi}{6A} [4Aa + A^2]^{\frac{3}{2}} - A^3 \\ &= \frac{\pi R^3}{6A} \left[\left(4 + \frac{R^2}{a^2}\right)^{\frac{3}{2}} - \frac{R^3}{a^3} \right] = \underline{\frac{\pi Ra}{6} \left[\left(4 + \frac{R^2}{a^2}\right)^{\frac{3}{2}} - \left(\frac{R}{a}\right)^3 \right]} \end{aligned}$$

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PARABOLIC BULK'D CONT'D

RADIUS OF CURVATURE (ρ)

$$\rho = \frac{y^2 = Ax}{\frac{d^2y}{dx^2}} \quad A = \frac{R^2}{a}$$

$$y = A^{\frac{1}{2}}x^{\frac{1}{2}} \quad \frac{dy}{dx} = \frac{1}{2}A^{\frac{1}{2}}x^{-\frac{1}{2}}$$
$$\frac{d^2y}{dx^2} = -\frac{1}{4}A^{\frac{1}{2}}x^{-\frac{3}{2}}$$

$$\begin{aligned}\rho &= \frac{[1 + (\frac{1}{2}A^{\frac{1}{2}}x^{-\frac{1}{2}})^2]^{3/2}}{-\frac{1}{4}A^{\frac{1}{2}}x^{-\frac{3}{2}}} \\ &= \frac{[1 + \frac{1}{4}Ax^{-1}]^{3/2}}{-\frac{1}{4}A^{\frac{1}{2}}x^{-\frac{3}{2}}} \\ &= -\frac{4x^{\frac{3}{2}}(4x+A)^{\frac{3}{2}}}{A^{\frac{1}{2}}} = \frac{-4x^{\frac{3}{2}}(4x+A)^{\frac{3}{2}}}{8x^{\frac{1}{2}}A^{\frac{1}{2}}} \quad *$$

$$A \cdot \frac{R^2}{a} = \frac{1}{2} \frac{(4x + \frac{R^2}{a})^{\frac{3}{2}}}{(\frac{R^2}{a})^{\frac{1}{2}}}$$

$$= \frac{1}{2Ra} (4ax + R^2)^{\frac{3}{2}}$$

* NOTE: THE MINUS SIGN APPEARING
HAS NO SIGNIFICANCE AND IS
THEFORE DROPPED FROM THE
EQUATION.

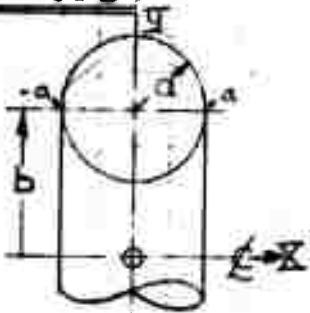
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TORUS



$$x^2 + (y - b)^2 = a^2$$

$$y - b = \pm \sqrt{a^2 - x^2}$$

$$y = b \pm \sqrt{a^2 - x^2}$$

$$\frac{dy}{dx} = \mp x(a^2 - x^2)^{-\frac{1}{2}}$$

$y_u = b + \sqrt{a^2 - x^2}$ IS THE EQUATION FOR THE UPPER SEMICIRCLE
 $y_l = b - \sqrt{a^2 - x^2}$ " " " " LOWER "

SURFACE AREA

$$S = 2\pi \int y \, ds$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

AREA GENERATED BY REVOLVING THE UPPER SEMI-CIRCLE

$$S_u = 2\pi \int_{-a}^a (b + \sqrt{a^2 - x^2}) \left[1 + \frac{x^2}{a^2 - x^2} \right]^{\frac{1}{2}} dx$$

AREA GENERATED BY REVOLVING THE LOWER SEMI-CIRCLE

$$S_l = 2\pi \int_{-a}^a (b - \sqrt{a^2 - x^2}) \left[1 + \frac{x^2}{a^2 - x^2} \right]^{\frac{1}{2}} dx$$

TOTAL SURFACE AREA $S_t = S_u + S_l$

$$\begin{aligned} S_t &= 2\pi \int_{-a}^a y_u \, ds + 2\pi \int_{-a}^a y_l \, ds = 2\pi \int_{-a}^a (y_u + y_l) \, ds \\ &= 2\pi \int_{-a}^a [b + \sqrt{a^2 - x^2} + (b - \sqrt{a^2 - x^2})] \, ds \\ &= 2\pi \int_{-a}^a 2b \, ds \\ &= 4\pi b \int_{-a}^a \left[1 + \frac{x^2}{a^2 - x^2} \right]^{\frac{1}{2}} dx = 4\pi b \int_a^a \left(\frac{a^2}{a^2 - x^2} \right)^{\frac{1}{2}} dx \\ &= 4\pi a b \int_{-a}^a (a^2 - x^2)^{-\frac{1}{2}} dx \end{aligned}$$

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TORUS (CONT'D)

$$\begin{aligned} S_T &= 4\pi ab \left(\sin^{-1} \frac{x}{a} \right) + C \Big|_a^a \\ &= 4\pi ab \left[\sin^{-1}(1) - \sin^{-1}(-1) \right] \\ &= 4\pi ab \left[2 \cdot \frac{\pi}{2} \right] \\ &= \underline{4\pi^2 ab} \end{aligned}$$

VOLUME (ENCLOSED BY TORUS)

$$\begin{aligned} V_T &= \pi \int_{-a}^a (y_U^2 - y_L^2) dx \\ &= \pi \int_{-a}^a [(b + \sqrt{a^2 - x^2})^2 - (b - \sqrt{a^2 - x^2})^2] dx \\ &= \pi \int_{-a}^a [b^2 + 2b\sqrt{a^2 - x^2} + a^2 - x^2 - b^2 + 2b\sqrt{a^2 - x^2}] dx \\ &= 4\pi b \int_{-a}^a \sqrt{a^2 - x^2} dx \\ &= 4\pi b \cdot \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right) \right] \Big|_{-a}^a \\ &= 2\pi b \left[a^2 \sin^{-1}(1) - a^2 \sin^{-1}(-1) \right] \\ &= 2\pi b \left[a^2 \cdot \frac{\pi}{2} - a^2 \cdot (-\frac{\pi}{2}) \right] \\ &= 2\pi b [2a^2 \cdot \frac{\pi}{2}] \\ &= \underline{2\pi^2 b a^2} \end{aligned}$$

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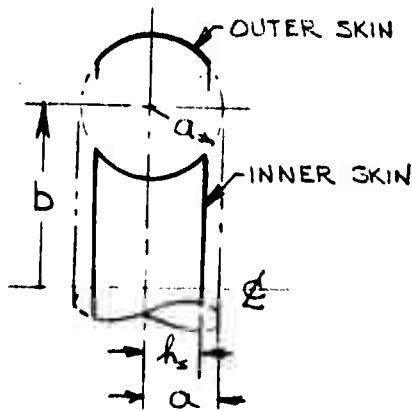
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TORUS (CONT'D)

THE DESIGN OF A TORUS TANK IS USUALLY SUCH THAT HIGH STRESSES ARE DEVELOPED IN THE SKIN OF THE "DO-NUT" HOLE WHILE THE TANK IS UNDER PRESSURE. THIS OFTEN DICTATES HEAVIER GAGE SKINS IN THIS AREA. ALSO LIGHTER SKINS ON THE OUTSIDE OF THE TORUS. THE FOLLOWING FORMULAS ARE DEVELOPED FOR THESE LOCAL AREAS.



$$x^2 + (y - b)^2 = a^2$$

$$\begin{cases} y_0 = b + \sqrt{a^2 - x^2} & (\text{UPPER}) \\ y_1 = b - \sqrt{a^2 - x^2} & (\text{LOWER}) \end{cases}$$

SURFACE AREA OF OUTER SKIN (2 h_o WIDE)

$$\begin{aligned} S_o &= 2 \times 2\pi \int_0^{h_o} y_0 \, ds & ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \\ &= 4\pi \int_0^{h_o} (b + \sqrt{a^2 - x^2}) \left[1 + \frac{x^2}{a^2 - x^2} \right]^{\frac{1}{2}} \, dx \\ &= 4\pi \int_0^{h_o} (b + \sqrt{a^2 - x^2}) \left[\frac{a}{\sqrt{a^2 - x^2}} \right] \, dx \\ &= 4\pi \int_0^{h_o} \left(\frac{ab}{\sqrt{a^2 - x^2}} + \frac{a^2}{\sqrt{a^2 - x^2}} \right) \, dx \\ &= 4\pi a \left[b \sin^{-1}\left(\frac{x}{a}\right) + x \right]_0^{h_o} \\ &= 4\pi a \left(b \sin^{-1}\left(\frac{h_o}{a}\right) + h_o \right) * \end{aligned}$$

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TORUS (CONT'D)

SURFACE AREA OF INNER SKIN (2 h, WIDE)

$$\begin{aligned} S_i &= 2 \times 2\pi \int_0^{h_s} y_i \, ds \\ &= 4\pi \int_0^{h_s} (b - \sqrt{a^2 - x^2}) \left[1 + \frac{x^2}{a^2 - x^2} \right]^{\frac{1}{2}} dx \\ &= 4\pi a \int_0^{h_s} \left(\frac{b}{\sqrt{a^2 - x^2}} - 1 \right) dx \\ &= 4\pi a \left[b \sin^{-1}\left(\frac{x}{a}\right) - x \right]_0^{h_s} \\ &= \underline{4\pi a \left(b \sin^{-1}\left(\frac{h_s}{a}\right) - h_s \right)} * \end{aligned}$$

CHECK FOR ABOVE S_i & S_t EQUATIONS:

$$\begin{aligned} S_t &= S_o + S_i \quad \text{WHEN } h_s = a \\ &= 4\pi a \left(b \sin^{-1}\left(\frac{a}{a}\right) + a \right) + 4\pi a \left(b \sin^{-1}\frac{a}{a} - a \right) \\ &= 4\pi a \left(b \sin^{-1}(1) + a + b \sin^{-1}(1) - a \right) \\ &= 4\pi a \left(2b \cdot \frac{\pi}{2} \right) \\ &= 4\pi^2 a b \quad (\text{THIS CHECKS WITH FORMULA FOR TOTAL AREA OF TORUS}) \end{aligned}$$

* NOTE: $\sin^{-1}\left(\frac{h_s}{a}\right)$ IS DEFINED AS THE ANGLE WHOSE SINE IS $\frac{h_s}{a}$. TO EVALUATE THIS FUNCTION FIND $\frac{h_s}{a}$ VALUE IN A TRIG. SINE TABLE TO OBTAIN IT'S CORRESPONDING ANGLE IN DEGREES. MULTIPLY THE DEGREES BY .01745 TO OBTAIN RADIANS WHICH IS PLUGGED DIRECTLY INTO THE FORMULA IN PLACE OF " $\sin^{-1}\left(\frac{h_s}{a}\right)$ ".

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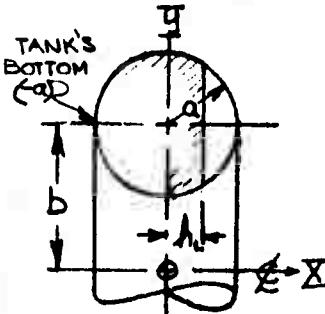
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TORUS (CONT'D)

VOLUME OF PARTIALLY FILLED TORUS TANK



$$x^2 + (y - b)^2 = a^2$$

$$\text{VOLUME OF FULL TANK} \\ = 2\pi^2 b a^2 \quad (\text{SEE PG. 23})$$

$$V' = \pi \int_{-a}^a (y_0^2 - y_1^2) dx \quad (\text{SEE PG.23})$$

$$= 4\pi b^3 \frac{1}{2} \left[x \sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right) \right]_a$$

$$= 2\pi b \left[h\sqrt{a^2 - h^2} + a^2 \sin^{-1}\left(\frac{h}{a}\right) - a^2 \sin^{-1}(-1) \right]$$

$$= 2\pi b \left\{ h\sqrt{a^2 - h^2} + a^2 \left[\sin^{-1}\left(\frac{h}{a}\right) + \frac{\pi}{2} \right] \right\}$$

"*h*" MAY BE (+) OR (-)

IF $\frac{h}{l}$ IS REGARDED AS A PROPORTION OF 'a' THEN $\frac{h}{l} = K a$. SUBSTITUTING THIS IN THE FORMULA ABOVE, THEN,

$$V_L = \pi \alpha^2 b \left\{ K \sqrt{1-K^2} + \sin^{-1} K + \frac{\pi}{2} \right\}$$

$$T = \frac{h}{a}$$