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STAGNATION POINT HEAT
TRANSFER FOR HYPERSONIC FLOW

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A DIVISION OF GENERAL DYNAMICS CORPORATION

CONVAIR SCIENTIFIC RESEARCH LABORATORY

RESEARCH NOTE 1

STAGNATION POINT HEAT TRANSFER FOR HYPERSONIC FLOW

Mary F. Romig

SUMMARY

A design method is given for obtaining stagnation point heat transfer on a spherical nose in hypersonic flow. The heat transfer equation incorporates real gas effects through use of dissociated air properties and a fitted curve for the stagnation point velocity gradient. Comparison with numerical solutions of the compressible dissociated stagnation point heat transfer indicates agreement within $\pm$ 5%. The derived equation for heat transfer is a function only of free stream Mach number, free stream pressure and body nose radius.

SYMBOLS

C = constant in heat transfer equation (Eq. (1))

h = enthalpy

i = dimensionless enthalpy = h/h_B

$i*$ = dimensionless reference enthalpy (Eq. (3))

k = ratio of free stream density to density behind the normal shock

M = Mach number

p = pressure

Pr = Prandtl number = viscosity x specific heat/thermal conductivity

q = compressible stagnation heat transfer rate

Q = the ratio $q/\sqrt{u_o'}$

R_N = radius of the nose

R = gas constant

s = surface distance around nose

T = temperature

u = velocity

u' = velocity gradient = du/ds

Greek

γ = ratio of specific heats

λ = factor in heat transfer equation (Eq. (13))

μ = viscosity

ρ = density

Subscripts

∞ = free stream

o = stagnation

w = wall

B = base (at T_B = 400°R)

i = incompressible

Superscript

* = quantity evaluated at reference enthalpy

ANALYSIS

The equations presented in this note are the result of a semi-empirical approach to the problem of determining stagnation point heat transfer. This approach arose from the fact that no closed exact solution exists for the differential equations determining the compressible flow heat transfer at the stagnation point. It has long been the practice (see e.g., Ref. 1) of aerodynamicists to modify the well-verified incompressible flow boundary layer heat transfer equationsso they could be used for compressible flow.

Consider the incompressible stagnation point heat transfer equation, given by Sibulkin[2]

$$q_i = C \sqrt{\rho \mu} \sqrt{u_e'} \; (h_e - h_w) \tag{1}$$

where C = 0.763 for Pr = 1.

An incompressible flow heat transfer equation can be used to obtain compressible flow heat transfer if it is possible to account for the effects of compressibility on the air property values ($\rho\mu$) and the stagnation point velocity gradient (u_e'). In the discussion following,

these two problems are treated separately since the velocity gradient may
be determined without reference to the boundary layer equations.

Various temperature levels have been suggested at which to evaluate
the properties $\rho\mu$. Among them are the wall temperature, the stream
temperature, or an average temperature. The best approximation to flat
plate compressible heat transfer has been obtained by evaluating $\rho\mu$ at
a "reference temperature", which is a function of local Mach number and
wall-to-stream temperature ratio. A complete discussion of this technique,
beyond the scope of this paper, may be found in Ref. 1.

It has been well established[1] that compressibility effects for flat
plate laminar flow can be introduced through use of such a reference
temperature. It is assumed that this method will apply to stagnation
point flow as well. If it is further assumed that real gas effects are
included through use of a reference enthalpy, rather than a reference
temperature, and the proper air properties, then Eq. (1) may be used
to determine stagnation point heat transfer in hypersonic flow. It will
be seen later that there is some evidence to support these assumptions.

In order to simplify the analysis several further assumptions will
be made. These may be modified without invalidating the following
development.

(1) Newtonian flow equations are used for surface pressure and
   stagnation point velocity gradient

(2) $M_\infty \gg 1$ and $h_o \gg h_w$, i.e., the wall is cool compared to the stream

(3) $Pr = 1$

If assumptions (2) and (3) are changed, the analysis presented here
can be applied to supersonic flow as low as $M_\infty = 3$. The curve-fitted
equations shown later for u_o' and $\rho\mu$ also permit this extension.

The following assumptions are less subject to modification.

(4) Free stream temperature $T_\infty = 400°R = T_B$

(5) The body shape in the stagnation region is spherical.

Although assumption (4) greatly simplifies the final equations it was chosen primarily so that the normal shock solution of Ref. 6 could be utilized for the velocity gradient. It is not known at the present time how large an effect the change of initial temperature would have on the normal shock solution.

On the basis of these assumptions, the compressible stagnation point heat transfer becomes

$$q = 0.763 \sqrt{\rho^*\mu^*} \sqrt{u_s'} \; h_o \tag{2}$$

for hypersonic flow. The dimensionless reference enthalpy, defined in Ref. 1, reduces to

$$i^* = 0.5 \; (i_o + i_w)$$

at the stagnation point. From assumption (2)

$$i^* = 0.5 \; i_o \tag{3}$$

where $i = h/h_B$.

The stagnation enthalpy, determined from free stream conditions, is

$$\frac{h_o}{h_\infty} = 1 + \frac{\gamma_\infty - 1}{2} \; M_\infty^2$$

For $h_\infty = h_B$ (assumption (4) and $M_\infty >> 1$,

$$i_o = \frac{\gamma_\infty - 1}{2} \; M_\infty^2 \; . \tag{4}$$

and, from Eq. (3), for $\gamma_\infty = 1.4$,

$$i* = 0.1 \, \tilde{M}_\infty^2 \tag{5}$$

so that Eq. (2) becomes (for $h_B = 96.8$ BTU/lb)

$$q = 0.763 \times 96.8 \times 0.2 \sqrt{\rho^* \mu^*} \sqrt{u_e^{\prime}} \, M_\infty^2 \tag{6}$$

Examination of Eqs. (5) and (6) shows it would be desirable to express $\rho^* \mu^*$ (or $i*$) and $u_e^{\prime}$ as functions of given (free stream) conditions. It is possible to do this through use of fitted curves and assumptions (1)-(5).

The Air Properties

For moderate supersonic Mach numbers the values of $\rho^* \mu^*$ to be used in Eq. (6) are comparatively well-defined. As the air temperature increases with speed, however, the various constituents become dissociated and complex chemical reactions occur. In this analysis the air will be assumed to be in dissociation equilibrium. The density and viscosity selected for use in this analysis are associated with the most recent complete set of air properties[3]. While the thermal properties (p, T, ρ, h) have subsequently been revised[4] the transport properties have yet to be computed. It is suggested that when new transport properties become available they be incorporated into the analysis in a manner similar to the following procedure.

Fig. 1 shows the variation of $\rho\mu/\rho_B \mu_B$ with enthalpy. The pressure dependency of density is incorporated into the base density by the expression

$$\rho_B = \frac{p(\text{local})}{RT_B}$$

where $T_B = 400°R$. The variation of $\rho\mu/\rho_B \mu_B$ with pressure due to dissociation is slight (within $\pm$ 6% from 0.1-10 atmospheres) so the curve for p = 1

atmosphere is plotted in Fig. 1. A good approximation to the curve of Fig. 1 is given by

$$\rho\mu = \rho_B\mu_B \, (1)^{-.28}$$

or, in this particular case, from Eq. (5),

$$\rho^*\mu^* = 1.9 \, \rho_B\mu_B(M_\infty)^{-.56} \tag{7}$$

Now, the above equation is still a function of local flow properties through the density ρ_B. The local pressure, in this case the stagnation pressure, can be removed through use of the Newtonian flow approximation at hypersonic Mach numbers (Ref. 7, p.265),

$$P_0 = P_\infty + \gamma_\infty \, M_\infty^2 \, P_\infty \approx \gamma_\infty \, M_\infty^2 \, P_\infty$$

so that

$$\rho^*\mu^* = \frac{1.9\gamma_\infty\mu_B}{RT_B} \, P_\infty \, M_\infty^{1.44} \tag{8}$$

The Velocity Gradient

The stagnation point velocity gradient for hypersonic flow on a sphere can be given by the constant-density Newtonian flow approximation developed by Li and Geiger[5]

$$u_e' = \frac{u_\infty}{R_N} \sqrt{k(2 - k)} \tag{9}$$

where k is the ratio of free stream density to density behind the normal shock. Values of $\sqrt{k(2 - k)}$ are given in Fig. 2 for the normal shock solution of Ref. 3, which is based on the recent air properties of Ref. 4. Fig. 2 shows that u_e' can be approximated by

$$u_\bullet^! = \frac{0.8}{R_N} \, u_\infty (M_\infty)^{-.232} = \frac{0.8}{R_N} \, \sqrt{\gamma_\infty R T_\infty} \, M_\infty^{.768} \qquad (10)$$

neglecting the variation in $\sqrt{k(2-k)}$ with free stream pressure. The curve given by Eq. (10) also agrees with the low speed test data of Ref. 5.

Another method of obtaining $u_\bullet^!$ for hypersonic flow is to consider use of an effective γ rather than use of real gas air properties. Lees[7] suggests the formula, which at high Mach numbers reduces to

$$u_\bullet^! \approx \frac{u_\infty}{R_N} \, \sqrt{\frac{\bar{\gamma} - 1}{\bar{\gamma}}} \qquad (11)$$

where $\bar{\gamma}$ is suggested to vary between 1.1 - 1.2 for high temperature flow. This equation is also shown in Fig. 2 for $\bar{\gamma} = 1.1$, 1.2 and 1.4. It can be seen that using a constant value of γ restricts the formula for $u_\bullet^!$ to a given Mach number region. For this reason Eq. (10), which is applicable over a wide range of Mach numbers, is recommended for general use. In specific instances where the Mach number range permits use of a constant γ, Eq. (11) is recommended on the basis of its simplicity.

Stagnation Point Heat Transfer

The use of Eqs. (8) and (10) now allow the heat transfer, Eq. (6), to be expressed solely as a function of free stream Mach number, pressure and nose radius. After the constants are evaluated, with $T_\infty = T_B = 400°R$, then

$$q = 0.0145 \, M_\infty^{3.1} \, \sqrt{\frac{P_\infty}{R_N}} \qquad (12)$$

The restrictions on Eq. (12) are that $M_\infty \gg 1$, the wall is much cooler than the stagnation point air and free stream temperature $T_\infty = 400°R$. These conditions could correspond to hypersonic flight in the isothermal altitudes.

COMPARISON WITH OTHER SOLUTIONS

Unfortunately, we cannot present an experimental comparison for Eq. (12) since existing test data in this flight regime are classified. In most cases of numerical solutions for the compressible stagnation point the determination of $\rho\mu$ and $u'_\bullet$ are left in the equation as parameters to be determined by the reader. Thus, the only verification of Eq. (12) which can be offered is to check the validity of the use of a reference enthalpy, i.e., to determine how well the method approximates the conditions of compressibility and dissociation.

The reference enthalpy method can be tested in this way by inserting the curve-fitted equation for $\rho^*\mu^*$ (Eq. (7)) into the heat transfer equation (Eq. (2)) and comparing with the numerical solution of Mark[8]. This solution of the compressible stagnation point boundary layer equations include use of the dissociated air properties of Ref. 3. In Mark's equation,

$$q = \left\{ \frac{\left(\rho_w\mu_w/\rho_B\mu_B\right)}{\sqrt[4]{\rho_o\mu_o/\rho_B\mu_B}} \quad \frac{\lambda}{Pr_w} \right\} \sqrt{u'_\bullet} \; h_o \sqrt{\rho_B\mu_B}$$

where λ , a function of the shear stress, was determined by numerical integration of the boundary layer equations. Eq. (2) can be written

$$q = \left\{ 1.052 \; M_\infty^{-.28} \right\} \sqrt{u'_\bullet} \; h_o \sqrt{\rho_B\mu_B}$$

The terms in brackets should compare and will indicate the effectiveness of the reference enthalpy in approximating compressible heat transfer. The ratio

$$\frac{q(i^*)}{q(Mark)} = \frac{1.052(M_\infty)^{-.28} Pr_w \sqrt{\rho_o\mu_o/\rho_B\mu_B}}{(\rho_w\mu_w/\rho_B\mu_B) \; \lambda} \tag{13}$$

is plotted in Fig. 3 for the range of flight velocities covered by Ref. 8. The agreement is within $\pm$.5% for the three wall temperature values given by Mark. This tends to substantiate the assumptions made previously that the reference enthalpy method can be used at the stagnation point, and

furthermore that it accounts for real gas effects on the compressible boundary layer.

Another calculation which can be made points out the effects of compressibility and dissociation on the stagnation point boundary layer heat transfer. A plot of such a calculation also offers a simple graphical method of obtaining heat transfer from a standard solution. The function $Q = q/\sqrt{u_o'}$ can be determined from Eqs. (2) and (7) for compressible flow. The incompressible $Q_i = q_i/\sqrt{u_o'}$ is given in Eq. (1), evaluated at a given wall temperature T_w. The ratio

$$\frac{Q}{Q_i} = \frac{\sqrt{1.9}(M_\infty)^{-.28}}{\sqrt{\rho_w\mu_w/\rho_B\mu_B}} \qquad (14)$$

is plotted in Fig. 4 for $T_w = 2000°R$. It indicates the effect of compressibility and dissociation on the boundary layer. Since Q_i is comparatively easy to evaluate, Fig. 4 presents a rapid method of obtaining Q, which with Eq. (10), gives the value of compressible heat transfer.

It can be seen in Fig. 4 that evaluation of the air properties at the wall conditions could overestimate heat transfer by as much as 30% at Mach 20. Since Eq. (1) is sometimes used to obtain engineering estimates the possible error induced should be kept in mind.

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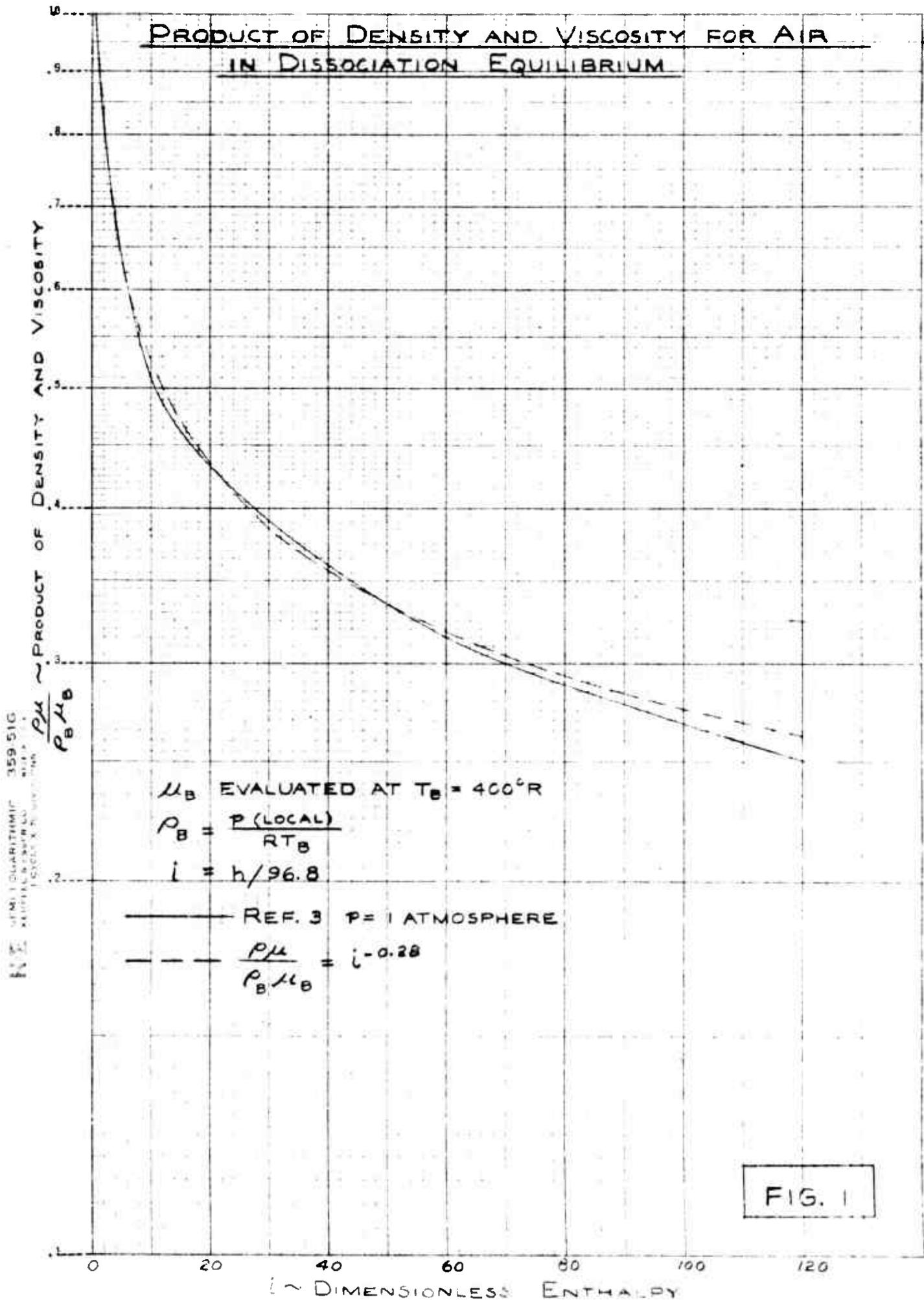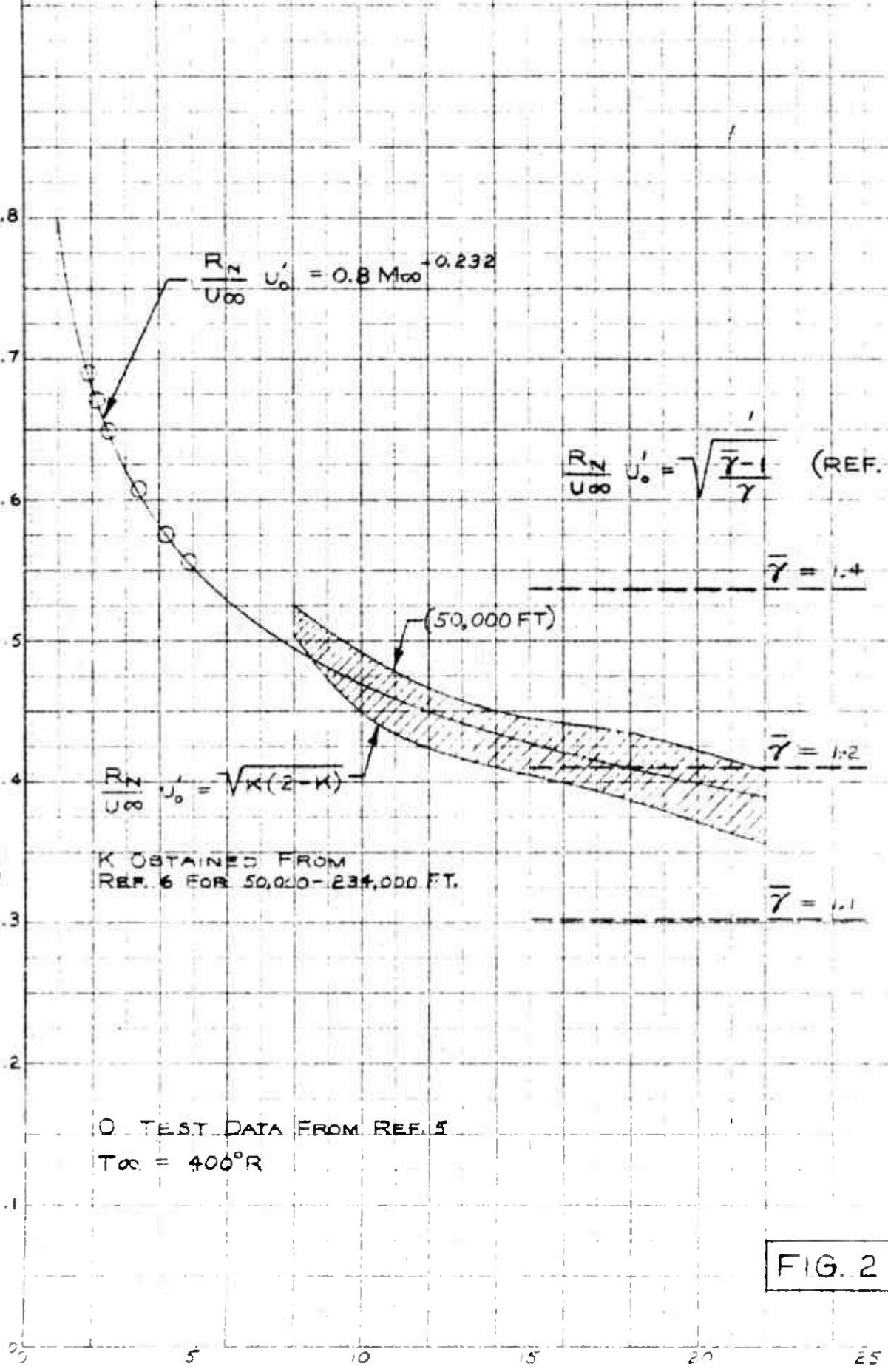PRODUCT OF DENSITY AND VISCOSITY FOR AIR IN DISSOCIATION EQUILIBRIUM

μ_B EVALUATED AT $T_B = 400°R$

$\rho_B = \dfrac{P\,(LOCAL)}{RT_B}$

$i = h/96.8$

—————— REF. 3 $P = 1$ ATMOSPHERE

— — — — $\dfrac{\rho\mu}{\rho_B\mu_B} = i^{-0.28}$

$\dfrac{\rho\mu}{\rho_B\mu_B} \sim$ PRODUCT OF DENSITY AND VISCOSITY

$i \sim$ DIMENSIONLESS ENTHALPY

FIG. 1

STAGNATION POINT VELOCITY GRADIENT

$$\frac{R_N}{U_\infty} U_o' = 0.8 M_\infty^{-0.232}$$

$$\frac{R_N}{U_\infty} U_o' = \sqrt{\frac{\bar{\gamma}-1}{\gamma}} \quad (\text{REF. 7})$$

$\bar{\gamma} = 1.4$

(50,000 FT)

$$\frac{R_N}{U_\infty} U_o' = \sqrt{K(2-K)}$$

$\bar{\gamma} = 1.2$

K OBTAINED FROM
REF. 6 FOR 50,000 - 234,000 FT.

$\bar{\gamma} = 1.1$

O TEST DATA FROM REF. 5
$T_\infty = 400°R$

FIG. 2

$M_\infty \sim$ FREE STREAM MACH NUMBER

COMPARISON OF NUMERICAL AND APPROXIMATE COMPRESSIBLE HEAT TRANSFER SOLUTIONS FOR AIR IN DISSOCIATION EQUILIBRIUM
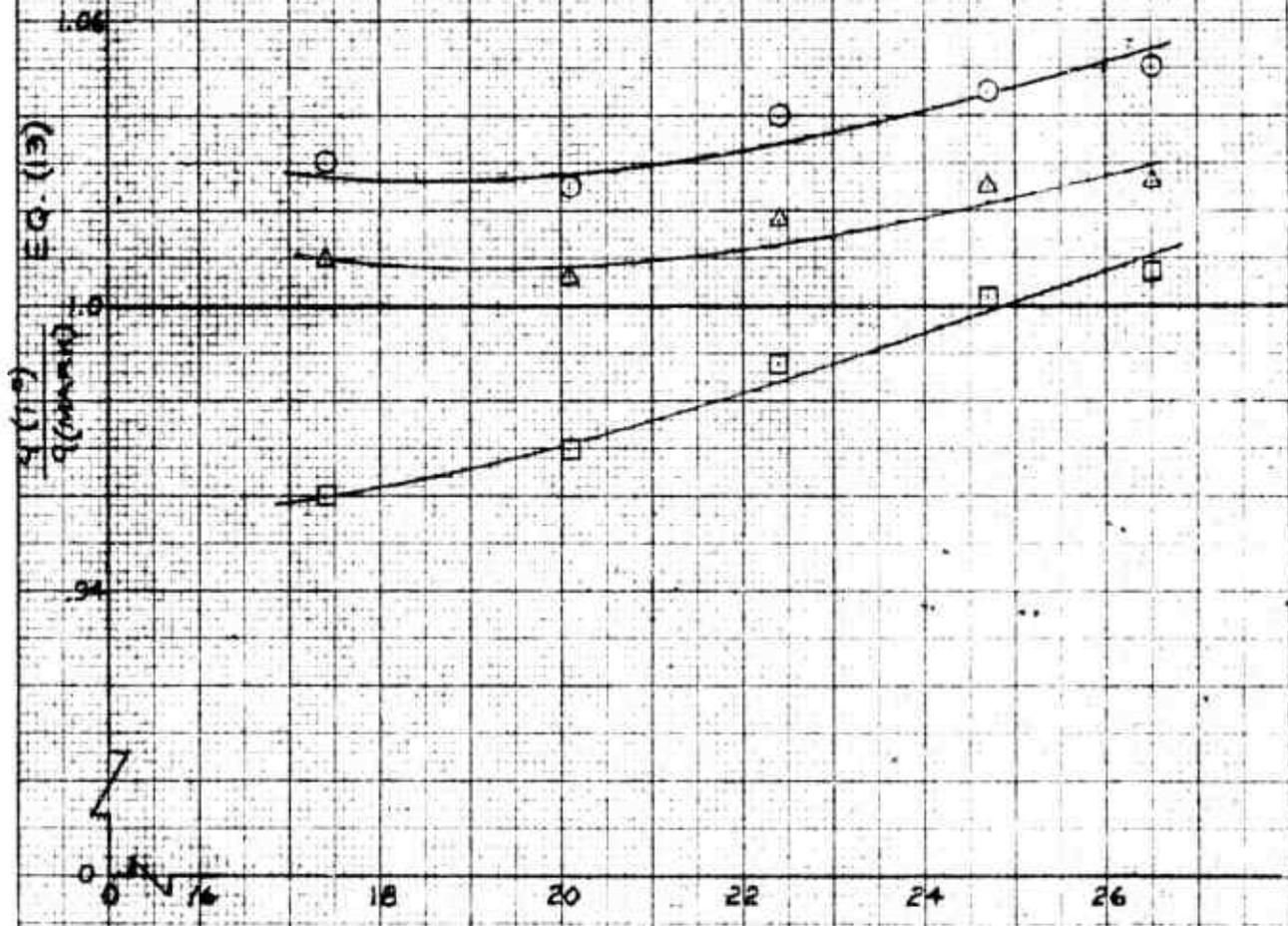
FIG. 3

COMPRESSIBILITY AND REAL GAS EFFECTS ON THE STAGNATION POINT HEAT TRANSFER PARAMETER

Q/Q_i ~ HEAT TRANSFER PARAMETER

M_∞ ~ FREE STREAM MACH NUMBER

Q/Q_i FROM EQ. (14)

FIG. 4