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LARGE-ARRAY SIGNAL AND NOISE ANALYSIS

Special Scientific Report No. 10

EQUALIZATION STUDIES

Prepared by

Terence W. Harley

Frank H. Binder, Program Manager

TEXAS INSTRUMENTS INCORPORATED
Science Services Division
P.O. Box 5621
Dallas, Texas 75222

Contract No. AF 33(657)-16678

Prepared for

AIR FORCE TECHNICAL APPLICATIONS CENTER
Washington, D.C. 20333

Sponsored by

ADVANCED RESEARCH PROJECTS AGENCY
ARPA Order No. 599
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Principal authors of this report are

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T.W. Harley	Section IV
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SECTION I

SUMMARY

This report reviews the following four tasks pertaining to response equalization problems:

- Evaluating a new technique using large signals for equalizing seismometers
- Examining the concept of designing regional equalization filters for subarray outputs
- Analyzing statistically the coefficients used to equalize the noise data
- Developing the theory to incorporate statistical phase fluctuations in the correlation statistics

Section II discusses the technique using large signals to equalize seismometers. The method is based on finding the minimum-phase filters which equalize the signal power spectra for a set of seismometers. The method is tested by using the nine in-line seismometers (NW-SE arm) of LASA subarray C2 for a large Aleutian Islands event. Four sets of filters are designed (using two gate lengths) to equalize eight channels to the reference channel (seismometer 10). The reference channel is subtracted from the other channels before and after equalization and the error traces compared. The equalized signals are significantly more similar for the first few cycles of the arrival. After the first few cycles, there is little difference between the two sets of signals — probably because of interfering scattered energy. The technique appears to be valid, but a complete evaluation (including an analysis of cost vs improvement) is recommended.

Section III discusses the designing of a set of regional equalization filters for LASA subarray outputs. Average Levinson filters are designed for three subarray outputs, using three events from the Aleutian Islands region. Then, the filters are applied to seven events from this region, and results are compared with those obtained for both amplitude equalization and individual Levinson equalization. Regional equalization filtering is found to be possible — but only if the epicentral region is very small. Thus, regional equalization filtering does not appear to be practical to implement.

Section IV shows that variations in the coefficients used to equalize the noise are caused by a combination of seismometer gain fluctuations and variations in estimating the noise average power (i. e., the zero-lag autocorrelation function estimate). Thus, the anticipated use of the noise equalization coefficients to study the nature of statistical gain fluctuations is not possible.

Section V shows how statistical phase fluctuations can be included in the correlation statistics. This is a generalization of gain fluctuation (a 1-point stochastic filter) to an n-point stochastic filter. If the filter weights are independent, it is shown that phase fluctuation is accounted for in the same manner as gain fluctuation (i. e., scaling the autocorrelation function). The scalar is 1 plus the sum of the variances of the n points.

SECTION II

SEISMOMETER EQUALIZATION USING LARGE SIGNALS

A. INTRODUCTION

The problem of equalizing seismometers occurs frequently in data analysis; for example, seismometer equalization is necessary in computing wavenumber spectral estimates and in designing multichannel filter systems from measured noise data and a theoretical signal model. Techniques for equalizing seismometers using large signals range from simple amplitude equalization to Levinson equalization. The method chosen by the analyst depends on the nature of his problem; each method offers advantages and is subject to limitations.

This section presents a new method using large signals to equalize seismometers. Given a reference channel and a channel to be equalized, this method finds the minimum-phase filter which, when applied to the channel to be equalized, causes it to have the same power spectrum as the reference channel; i.e., if $g_r(t)$ and $g_e(t)$ are the two channels with power spectra $G_r(f)$ and $G_e(f)$, respectively, the desired filter is the minimum-phase filter with power response $G_r(f)/G_e(f)$.

Differences in seismometer responses are due to differences in instrument and amplifier responses and in subsurface structure. Instrument response and impulse response of a layered medium are minimum phase. (Amplifier response may not be.) Additionally, the difference between two minimum-phase systems is minimum phase, so the choice of a minimum-phase filter appears to be a reasonable approximation. Note that the filter which is calculated will equalize the two signals only if the difference between them is actually minimum phase; also, the two signals will not be equalized if the spectral ratio changes outside the design gate.

A program to design and apply these equalization filters uses a new technique for estimating power spectra. Developed by Texas Instruments Incorporated, it reduces end effects and is particularly suited to short-duration signals. Filter design and application are actually accomplished entirely in the time domain by applying to the channel to be equalized its own whitening filter and the inverse of the reference-trace whitening filter.

Figure II-1 shows the results of a test case used to check the method; the third trace is the actual signal arrival, and the first trace is the output of a predetermined minimum-phase filter applied to the signal arrival. Using the first trace as the reference, the third trace is then equalized using the equalization program. Note that a very short gate is used to estimate the power spectra of the two traces. The second trace shows the estimate obtained. Considering that only a very short gate and a 10-point filter are used, a very good estimate of the target trace was obtained.

B. DATA PRESENTATION

Figure II-2 shows the Andreanof Islands event which was used to evaluate the method. The nine in-line seismometers of subarray C2 (Figure II-3) were chosen, with seismometer 10 as the reference trace.

Signal sample rate is 0.05 sec; the finer sampling is preferred to the normal 0.1 sec to improve overall accuracy, especially when aligning the signals. Figure II-4 shows that the signal peaks slightly above 1.0 cps. The signal is amplitude-equalized (using a 900-point gate) and static-corrected to the nearest integer sample using the signal-reference crosscorrelation function (Figure II-5). The bottom nine traces in Figure II-5 are the error traces obtained by subtracting the reference channel (seismometer 10) from the other channels. (The error traces are scaled up by 2, relative to the signal traces.) Note that the error traces become larger as seismometer separation increases. To evaluate the equalization method, the difference traces are compared with those obtained after equalization.

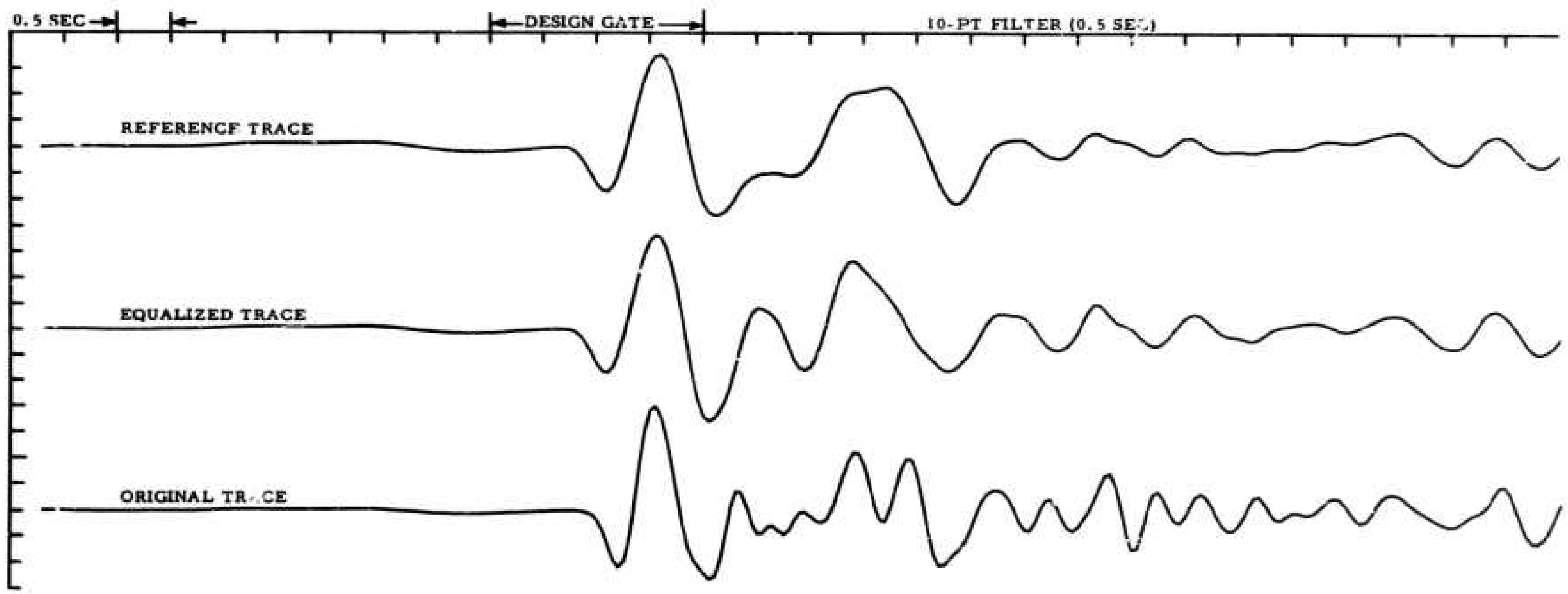
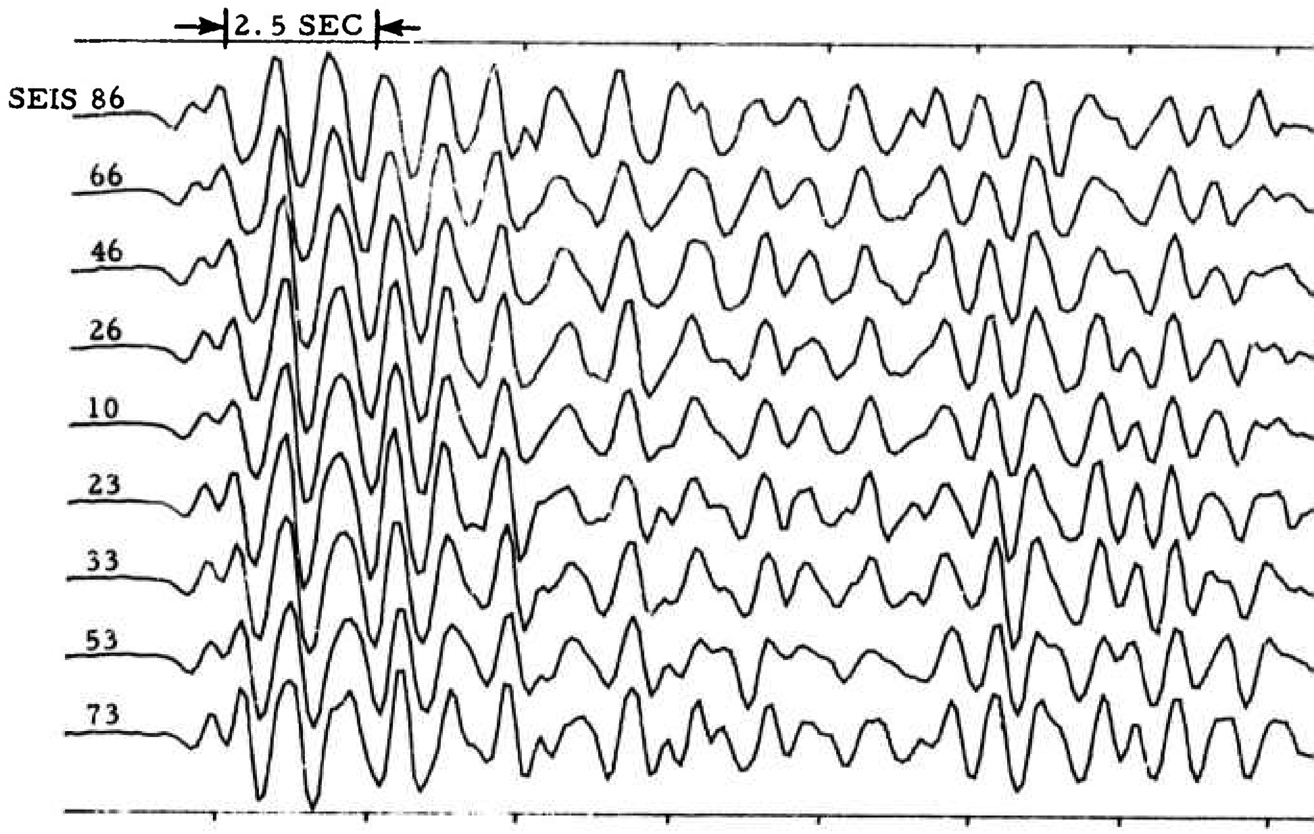


Figure II-1. Results of a Test Case Used to Check Equalization Method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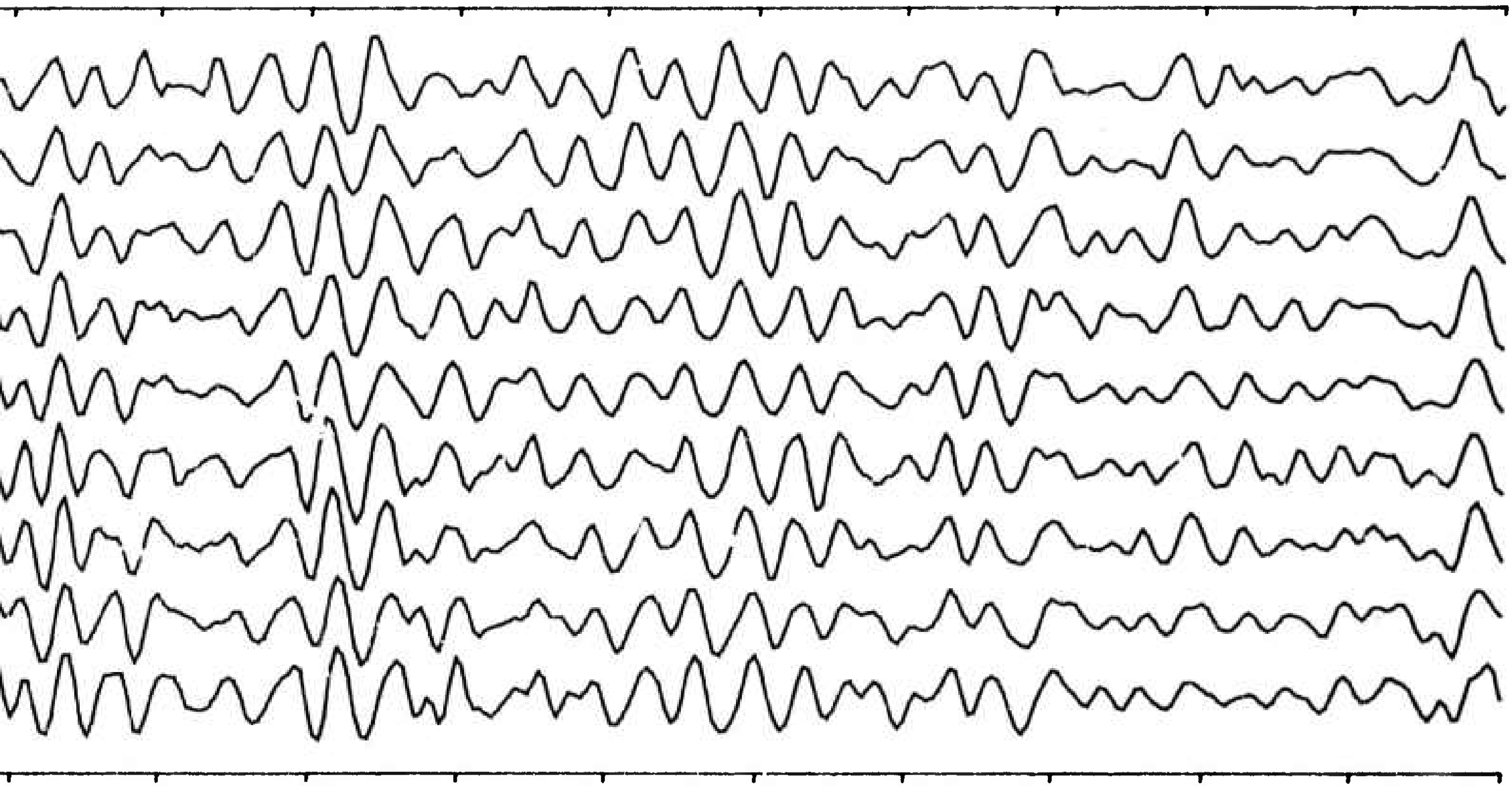

B

Figure II-2. Original Data, Andreanof Islands Event, Subarray C2

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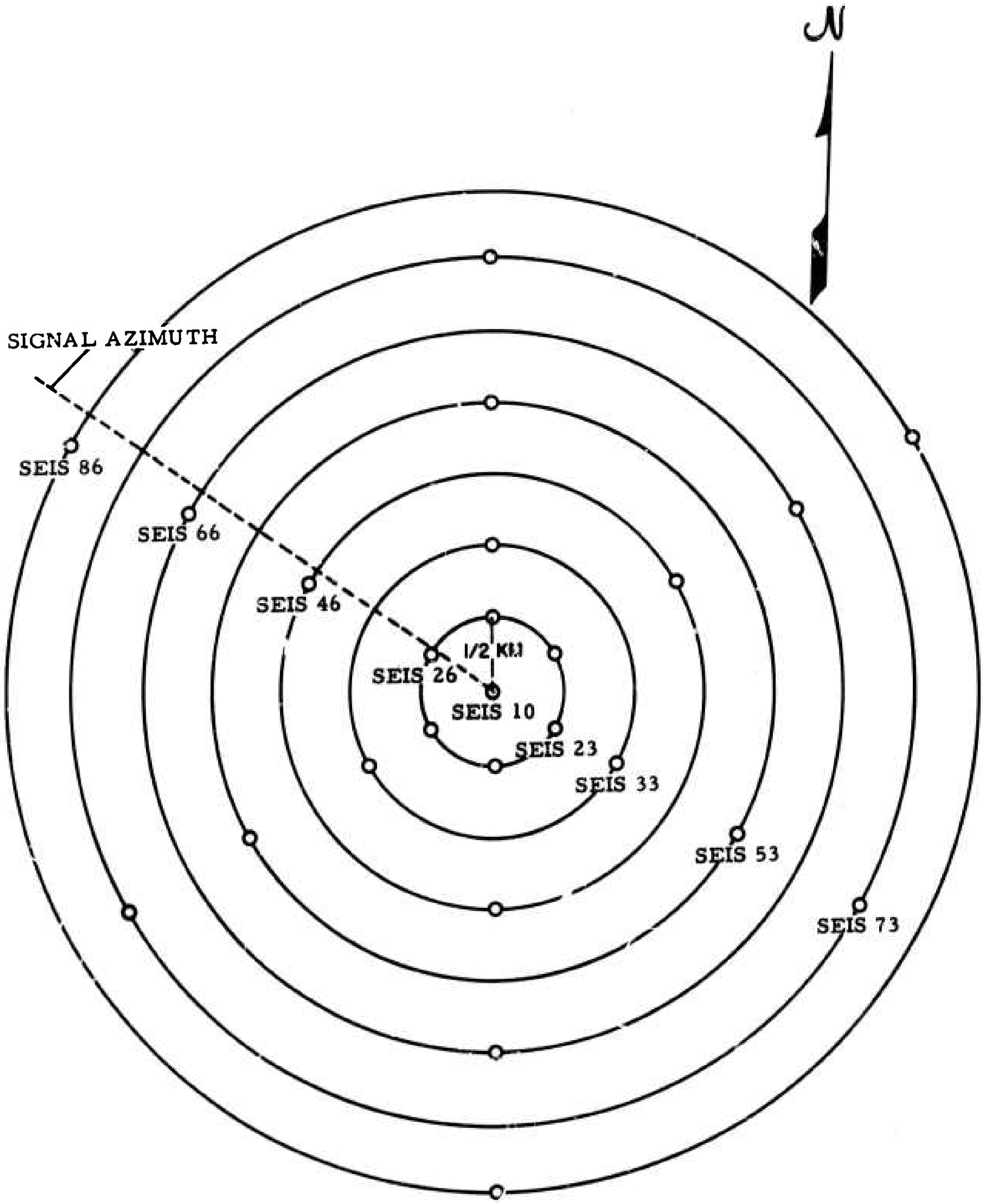


Figure II-3. Subarray C2

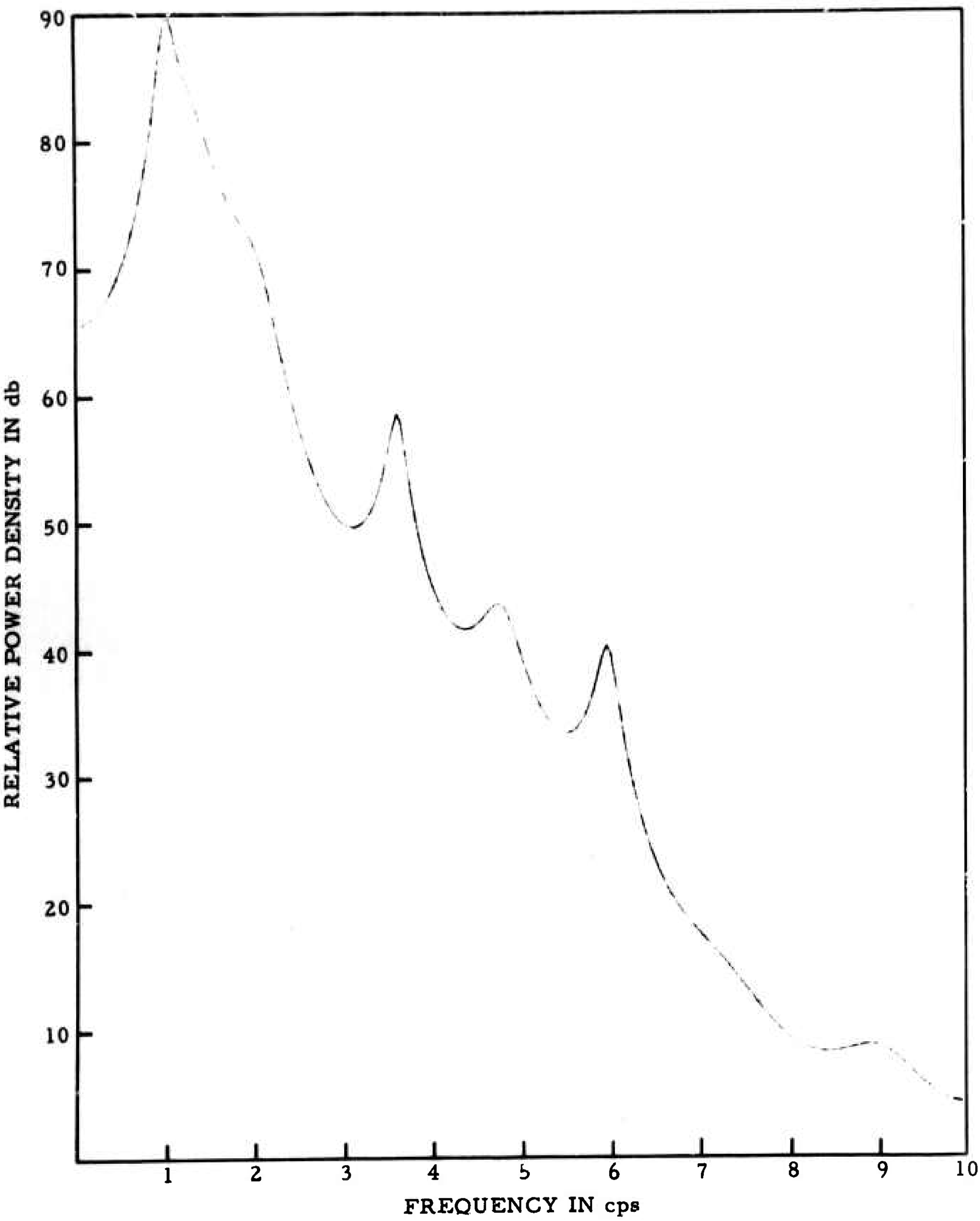
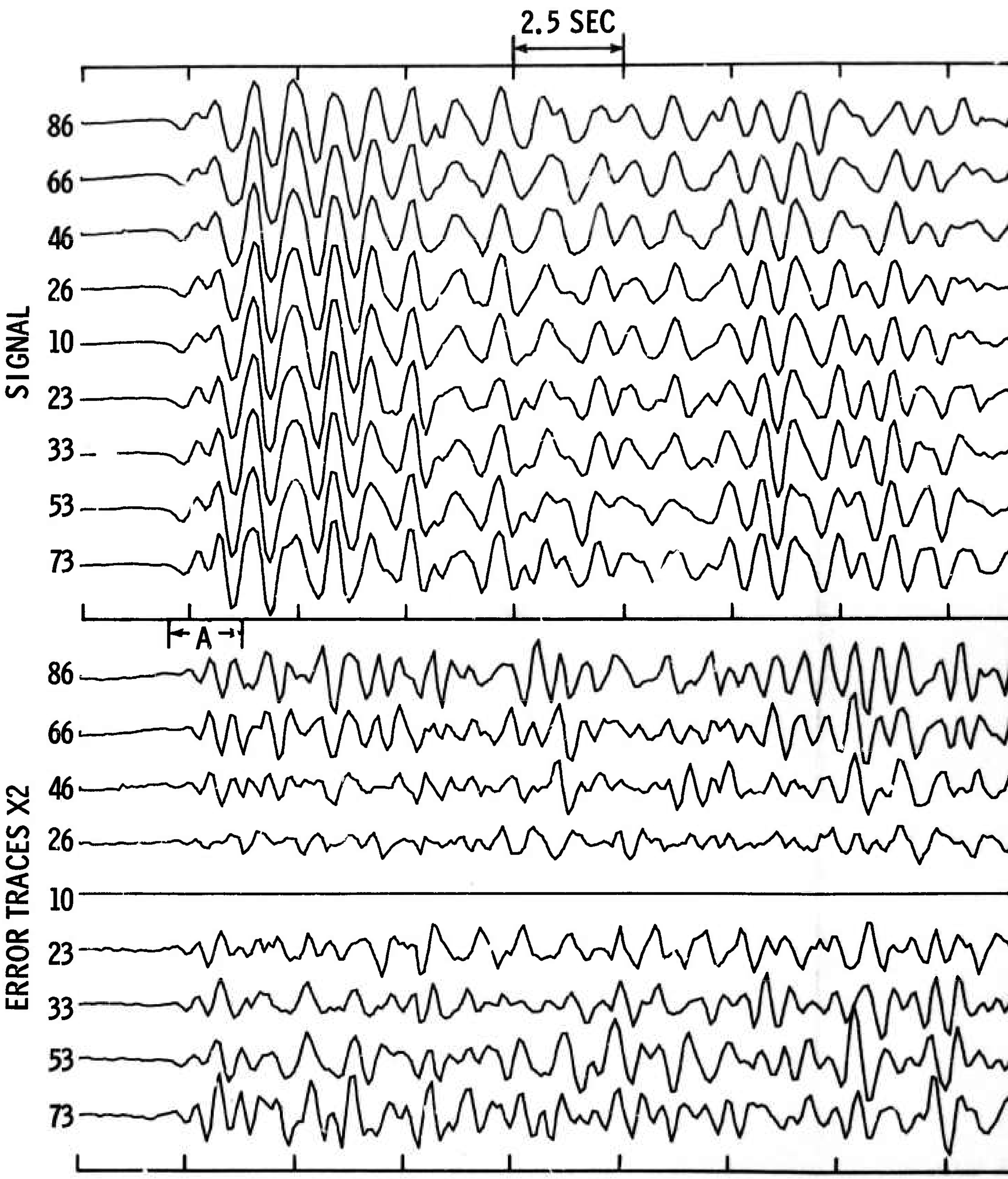


Figure II-4. Power Density Spectrum of Andreanof Islands Event, Seismometer 86



A = REGION WHERE EQUALIZATION
HAS BEEN EFFECTIVE

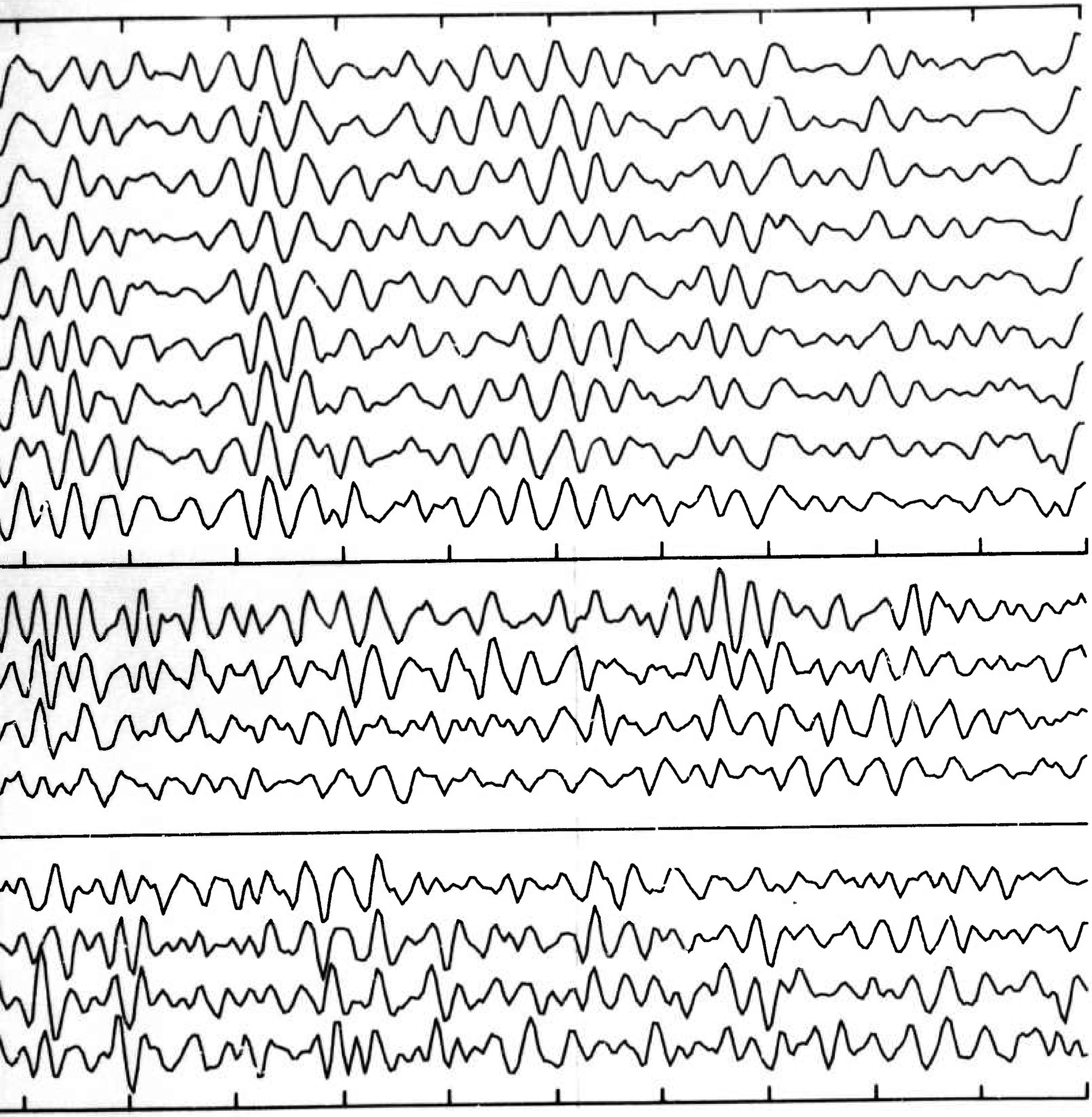


Figure II-5. Amplitude-Equalized, Static-Corrected Andreanof Islands Event

To make valid the comparison between error-trace sets it is necessary to align the signal and reference traces as exactly as possible before subtraction. Assume that we have two unit-amplitude, monochromatic, 1-cps cosine waves, and that one wave is displaced by Δ sec ($\Delta < \Delta t$, the sample rate) relative to the other. To examine the effect of the misalignment on the error trace, we compare the average power in the error trace with that in the original trace. Let

$$X_1(t) = \cos 2\pi t$$

$$X_2(t) = \cos 2\pi (t+\Delta) \qquad \text{since } A = f = 1$$

Let Y (t) be the difference trace:

$$\begin{aligned} Y(t) &= X_1(t) - X_2(t) \\ &= \cos 2\pi t - \cos 2\pi (t+\Delta) \\ &= (\cos 2\pi t)(1 - \cos 2\pi \Delta) \\ &\quad + (\sin 2\pi t)(\sin 2\pi \Delta) \end{aligned}$$

For the original trace,

$$\begin{aligned} \overline{X(t)^2} &= \int_0^1 \cos^2 2\pi t\,dt \qquad \text{since } T = 1 \\ &= \frac{t}{2} + \frac{\sin 4\pi t}{8\pi} \Big|_2^1 \\ &= \frac{1}{2} \end{aligned}$$

For the error trace,

$$\overline{Y(t)^2} = \int_0^1 \Big\{ \cos 2\pi t (1 - \cos 2\pi\Delta) + \sin 2\pi t \sin 2\pi\Delta \Big\}^2 dt$$

$$= (1 - \cos 2\pi\Delta)^2 \int_0^1 \cos^2 2\pi t \, dt$$

$$+ \sin^2 2\pi\Delta \int_0^1 \sin^2 2\pi t \, dt$$

$$+ 2 (1 - \cos 2\pi\Delta) \sin 2\pi\Delta \int_0^1 \cos 2\pi t \sin 2\pi t \, dt$$

$$= (1 - \cos 2\pi\Delta)^2 \left\{ \frac{t}{2} + \frac{\sin 4\pi t}{8\pi} \Big|_0^1 \right\}$$

$$+ \sin^2 2\pi\Delta \left\{ \frac{t}{2} + \frac{\sin 4\pi t}{8\pi} \Big|_0^1 \right\}$$

$$= \frac{1}{2} \left\{ 1 + \cos^2 2\pi\Delta + \sin^2 2\pi\Delta - 2 \cos 2\pi\Delta \right\}$$

$$= \frac{1}{2} [2 - 2 \cos 2\pi\Delta]$$

Thus,

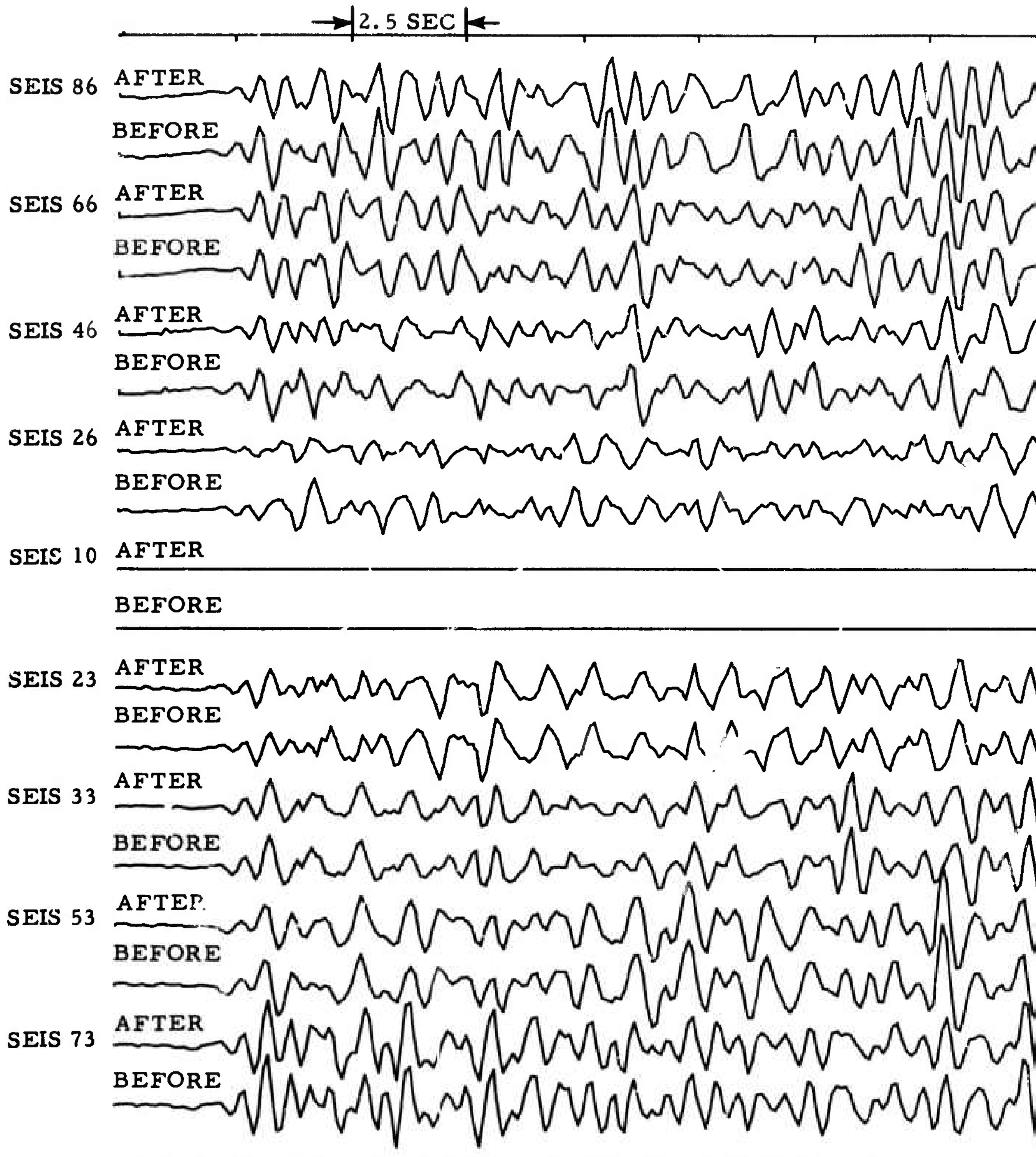
$$\overline{Y(t)^2} \Big/ \overline{X(t)^2} = 2 - 2 \cos 2\pi\Delta$$

Assuming that Δ is one-half block (or, in our case, 0.025 sec), the error-trace average power would be just 16 db less than that of the original trace. The observed error traces are 6 db to 18 db below the signal traces, so a one-half block misalignment error would contribute significantly to the error-trace power.

Since trace alignment is critical, a program has been written in order to interpolate the signal between samples so that the maximum misalignment will be much less than one-half sample. Appendix A outlines the program which is referred to as the microstatics program. Figure II-6 shows that the error traces obtained for the amplitude-equalized data are smaller — sometimes by a significant amount (e.g., seismometers 46, 26, and 53) — after application of microstatics.

The amplitude-equalized microstatic-corrected traces of Figure II-5 are used as the "original" data for comparison with the outputs of the equalization filters designed. To insure a valid comparison, the equalized data are also amplitude-equalized and microstaticly corrected before the error traces are obtained.

Figures II-7 through II-9 show the equalized traces for 10-, 20-, and 30-point filters, respectively. Each figure shows the design gate; note that the same signal segment is used in each case. There appears to be no consistent difference in the performances of the three filters; the filter length that gives the smallest error trace varies from channel to channel. A comparison of the equalized and original error traces shows that equalization has significantly improved the signal similarity over the first few cycles; beyond the first few cycles, there is little difference between the sets of traces. A possible explanation for this behavior is that scattered energy accounts for a large proportion of the difference between the signal traces; i.e., the first few cycles of the signal arrival are free of scattered-energy, so the differences are largely due to the seismometer responses, which the filters correct quite well. However, the differences due to scattered energy later become significant with respect to seismometer response differences; and no overall improvement is obtained. The design gate used to determine the filters includes much more of the signal than just the first few cycles. This implies that the filters are not especially attuned to the first few cycles but indeed use general spectral information and the minimum-phase assumption to achieve equalization.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SEIS 86 AFTER
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SEIS 66 AFTER
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SEIS 46 AFTER
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SEIS 26 AFTER
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SEIS 53 AFTER
BEFORE
SEIS 73 AFTER
BEFORE

A

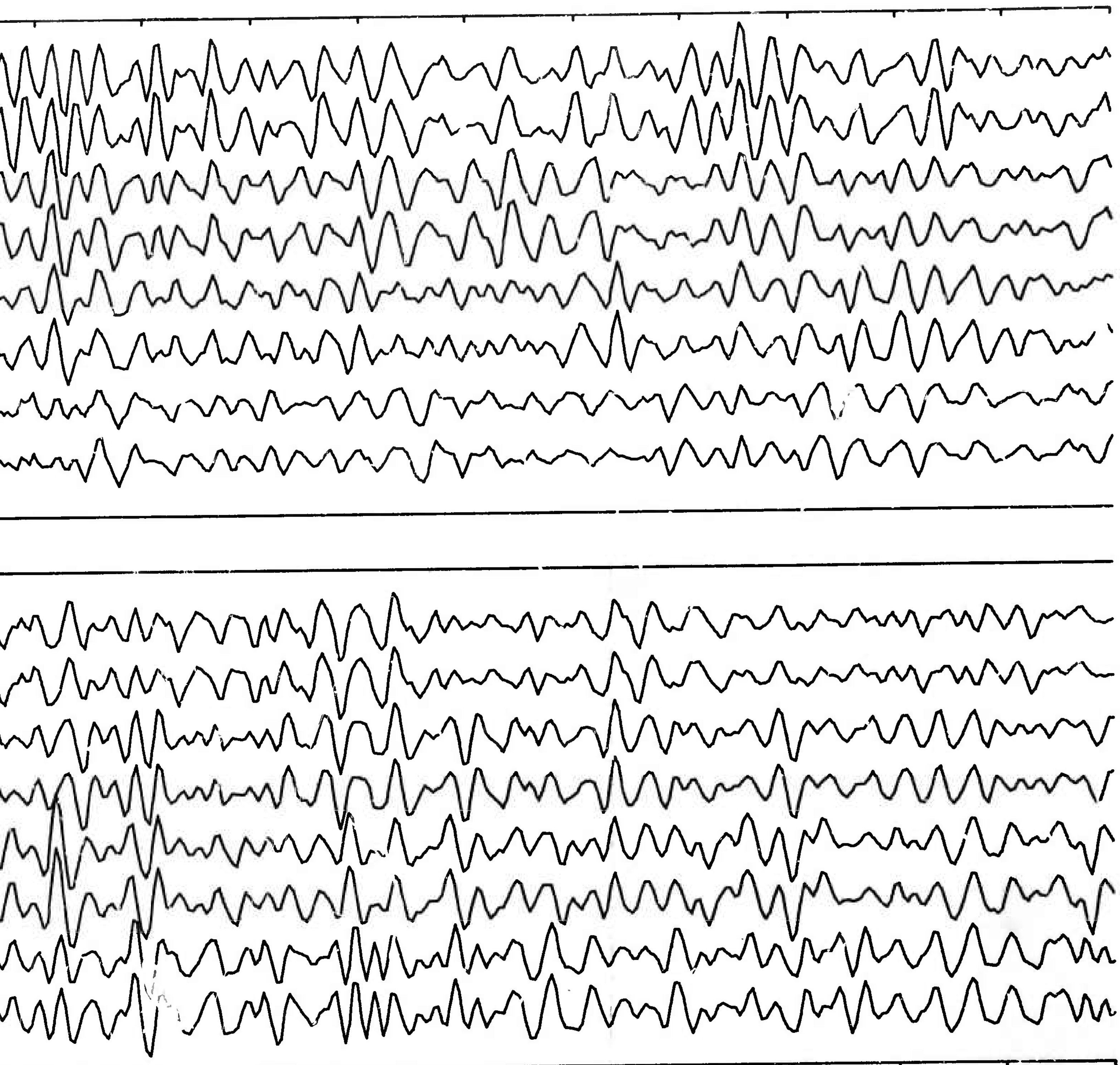

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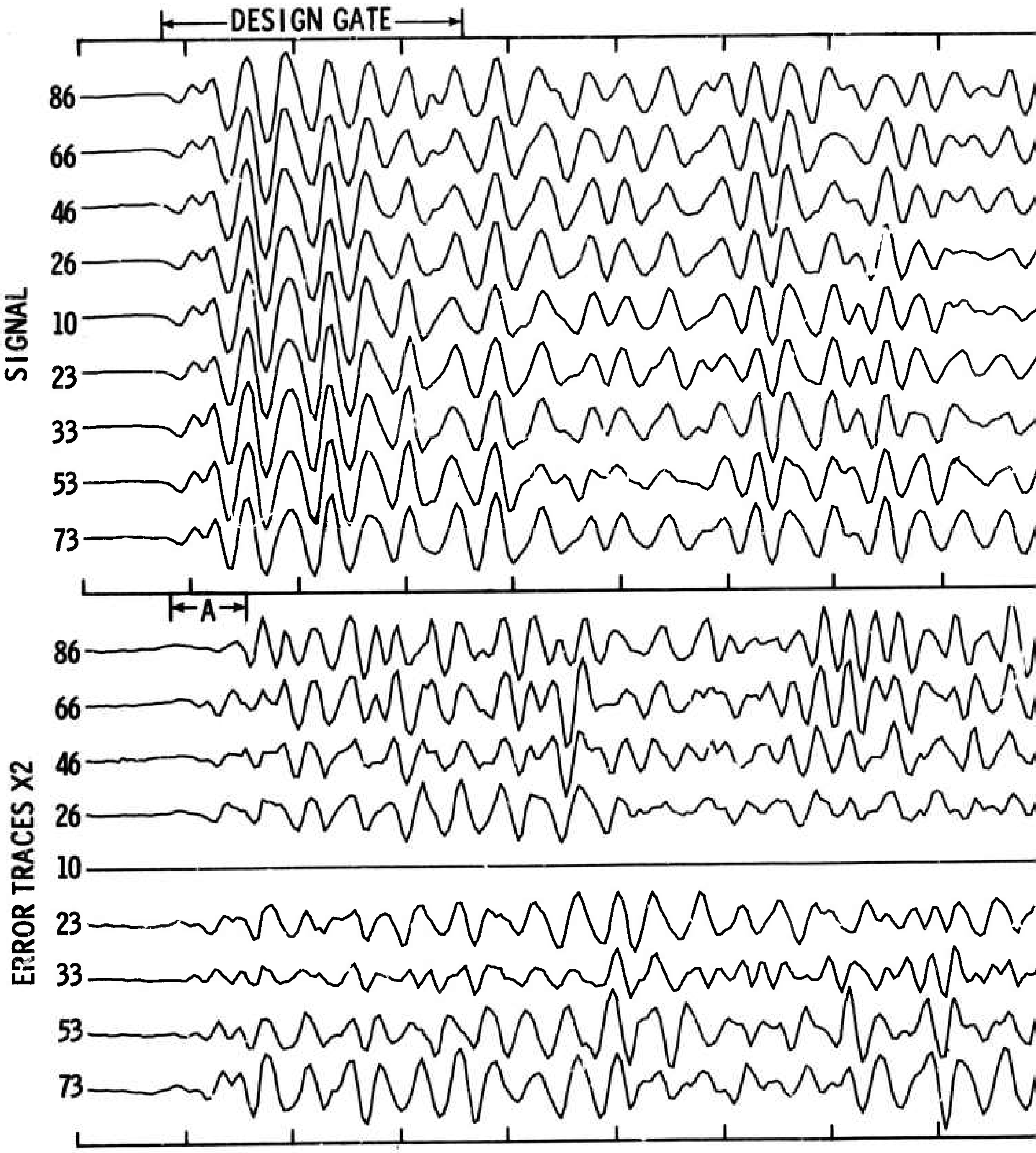
Figure II-6. Comparison of Error Traces for Amplitude-Equalized Data Before and After Microstatics



A = REGION WHERE EQUALIZATION HAS BEEN EFFECTIVE

A

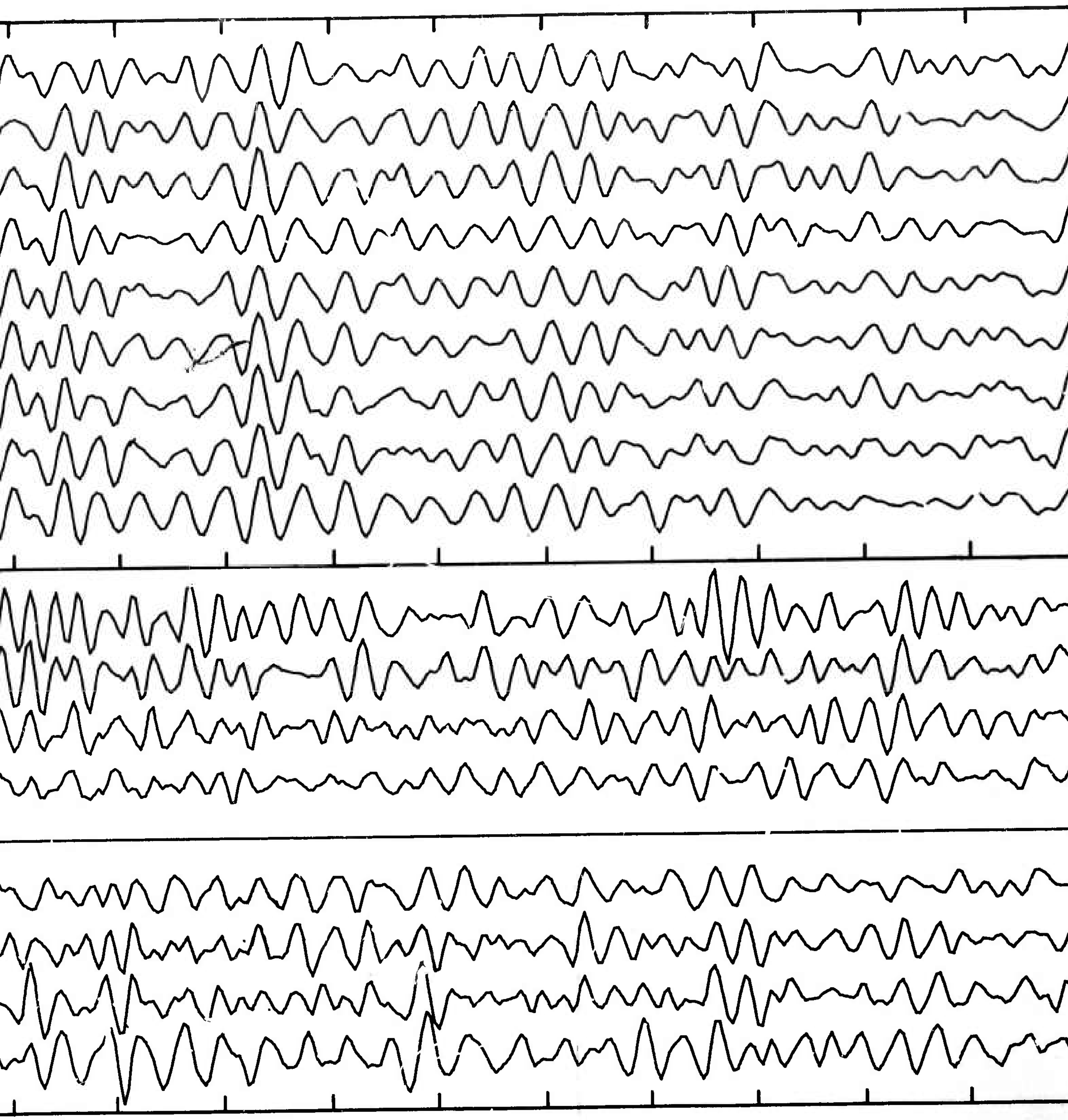
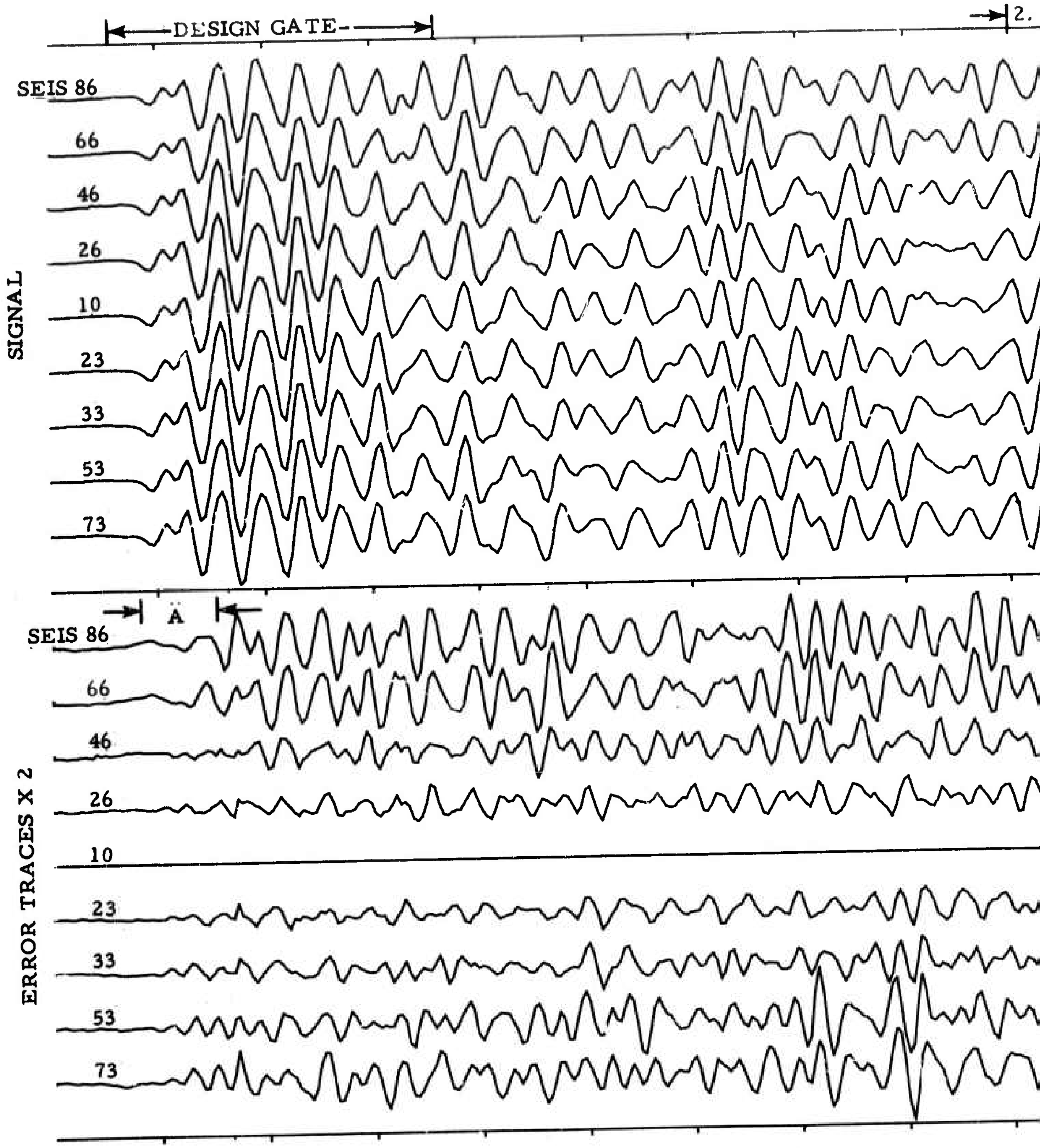


Figure II-7. Equalized Signal Using a 10-Point Filter



A = REGION WHERE EQUALIZATION
HAS BEEN EFFECTIVE

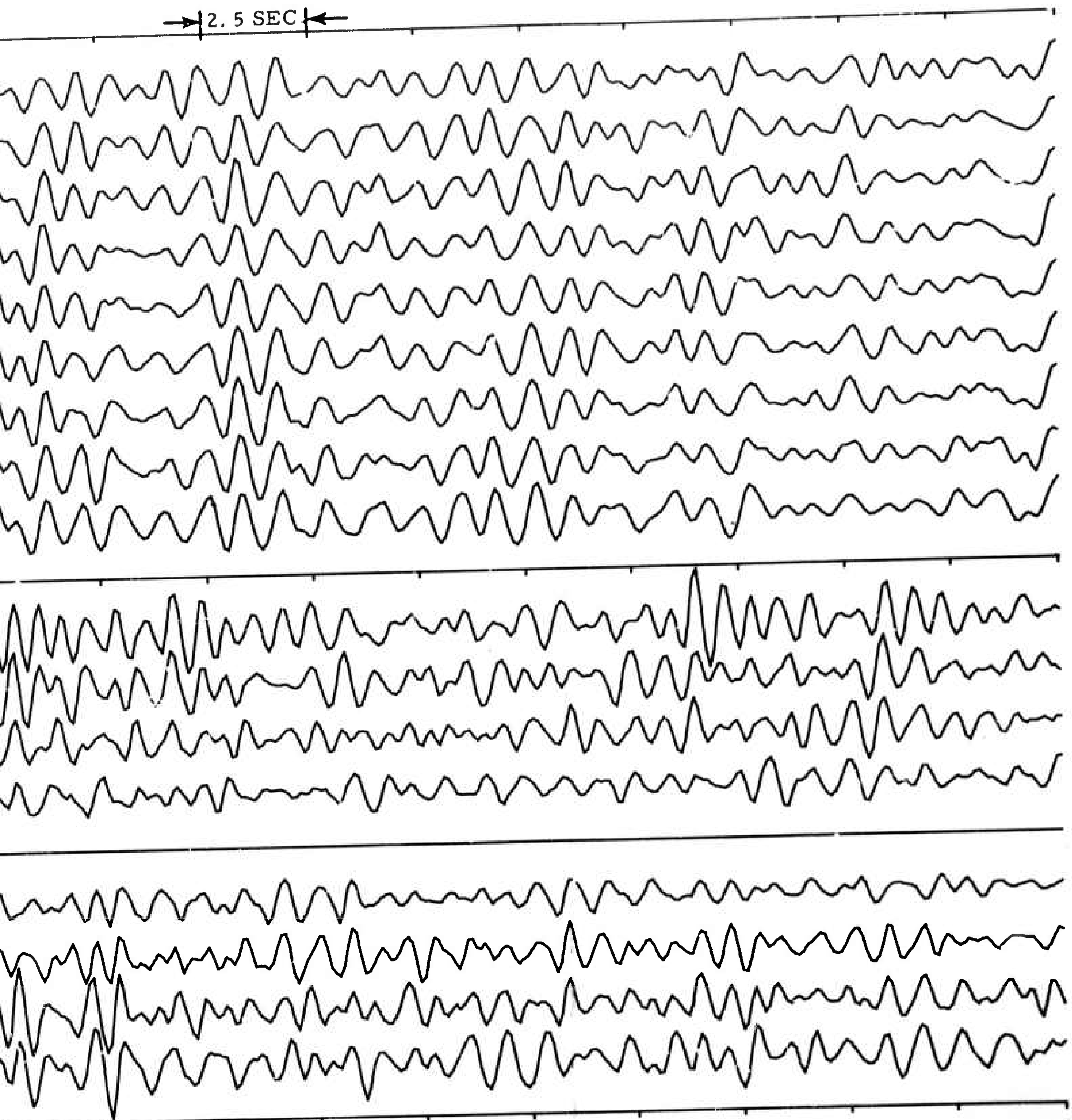
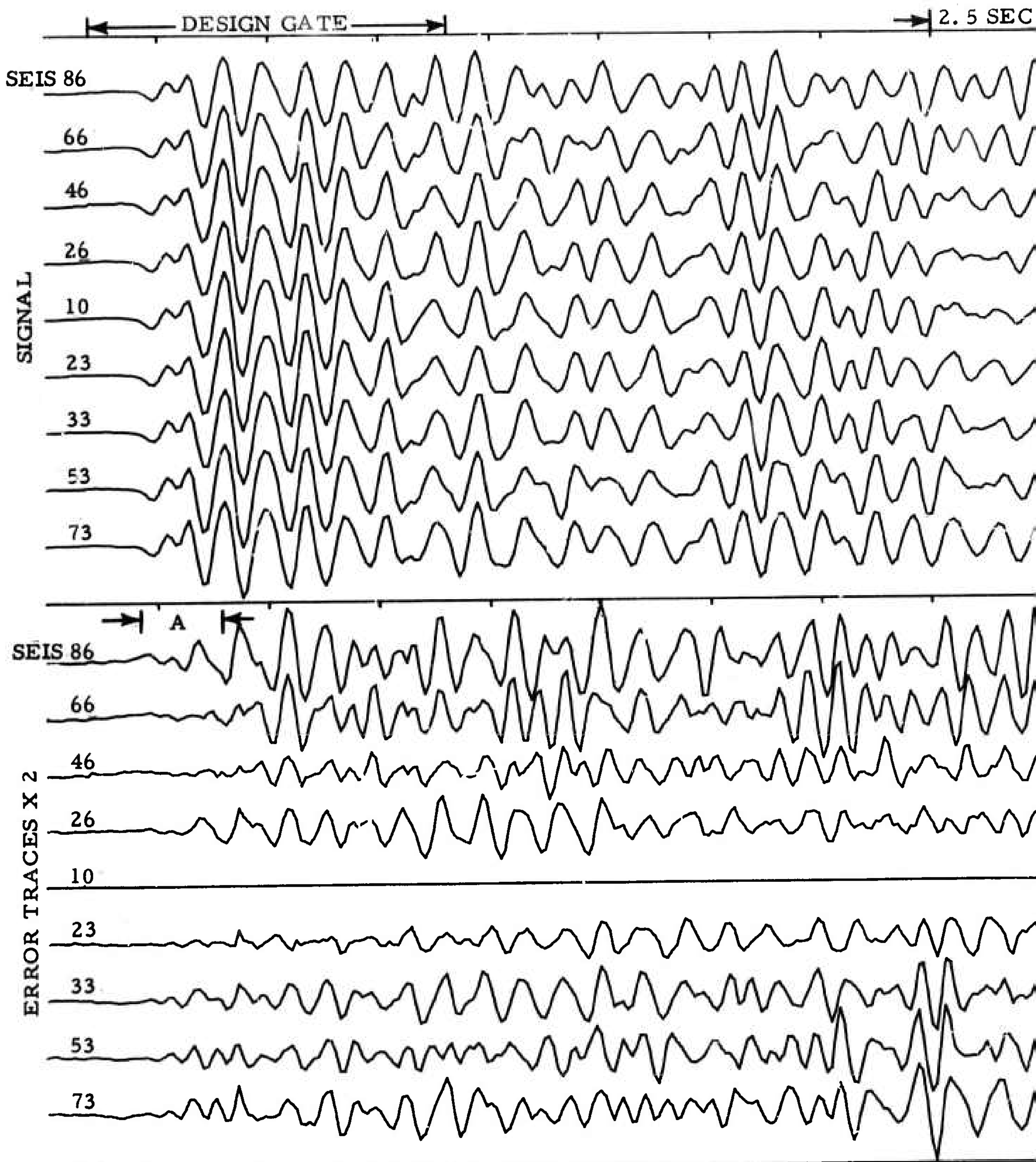


Figure II-8. Equalized Signal Using a 20-Point Filter



A = REGION WHERE EQUALIZATION HAS BEEN EFFECTIVE

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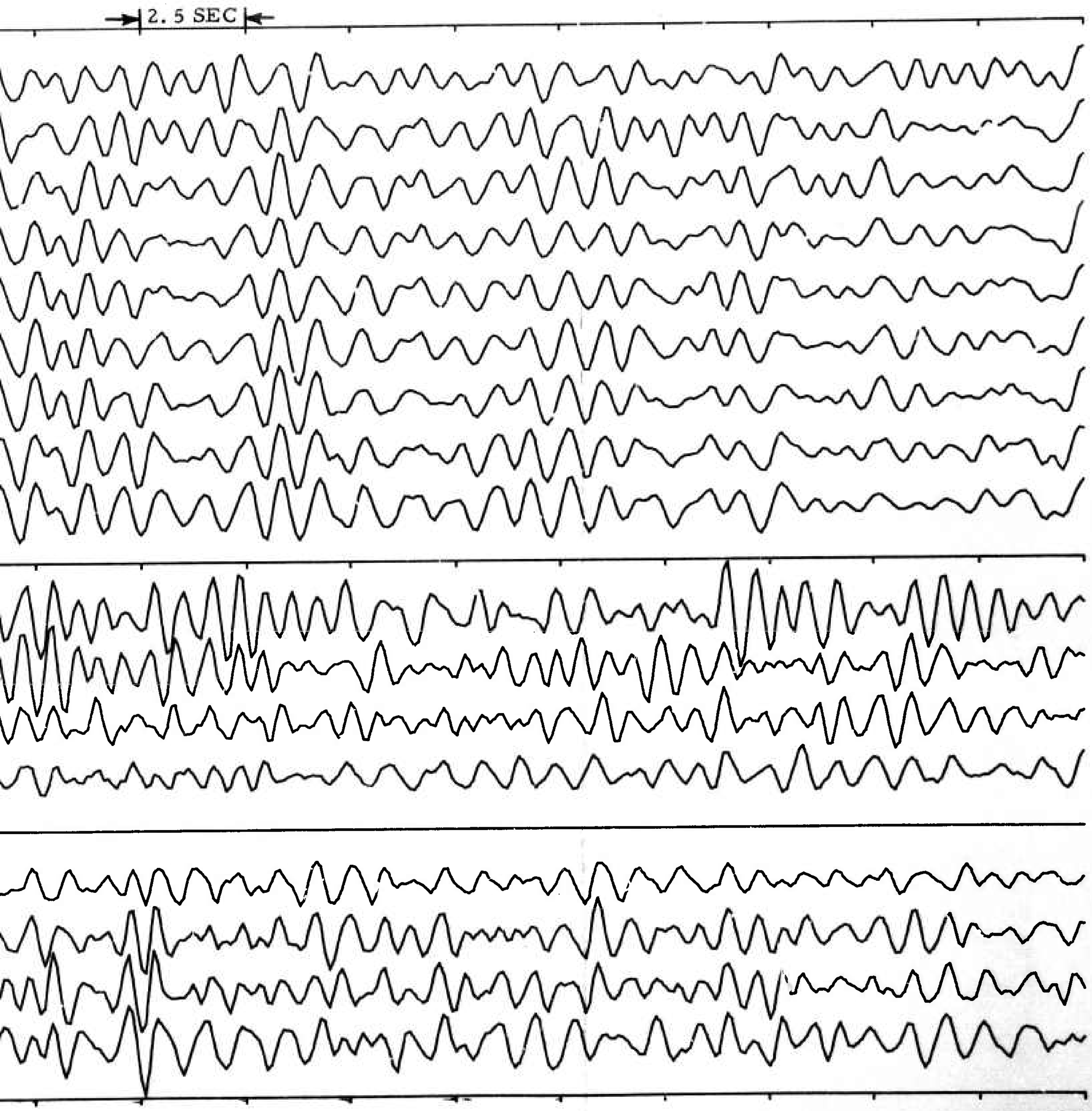


Figure II-9. Equalized Signal Using a 30-Point Filter

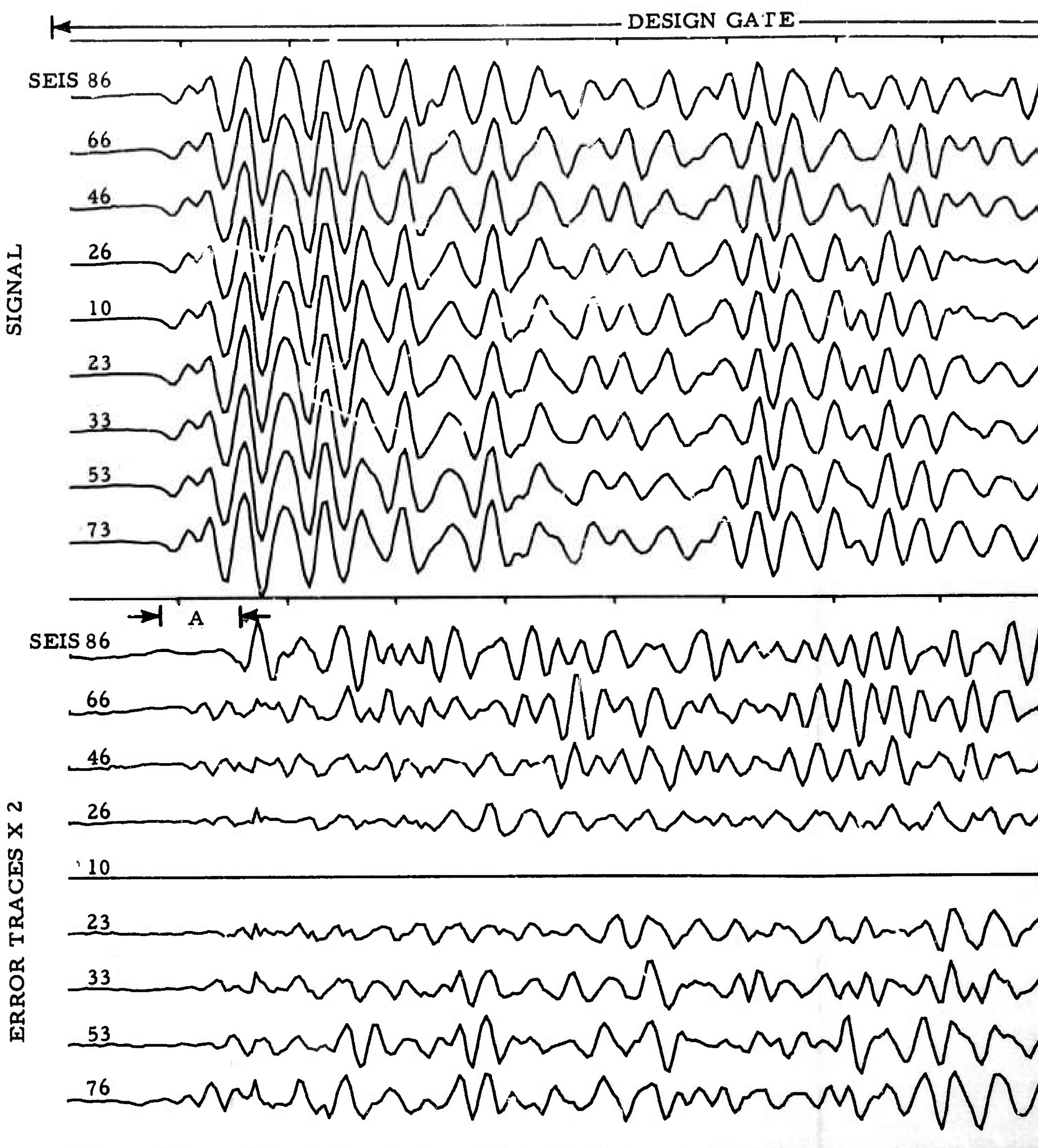
Figure II-10 shows the equalized and error traces for a 50-point filter designed from a 600-point gate. Again equalization is most effective for the first few cycles. Also, the error traces are slightly lower overall than are the original error traces; the reason for this is not known.

Figure II-11 shows the signal-to-reference-trace spectral ratios before and after equalization. As can be seen, the filters equalize the signal spectra quite well. The large positive ratios at high frequency before equalization are due to the lower noise level for the deeper reference trace (seismometer 10). Thus, results might have been better with a different seismometer as the reference.

C. CONCLUSIONS

Equalization significantly improves signal similarity for the first few cycles of the signal arrival. Later in the arrival, no improvement is obtained, possibly due to the effect of scattered energy.

Since the equalization technique works only if the assumption that signal differences are minimum phase is met, it appears that actual differences are approximately minimum phase. Further study, therefore, is recommended to more fully evaluate the method.



A = REGION WHERE EQUALIZATION HAS BEEN EFFECTIVE

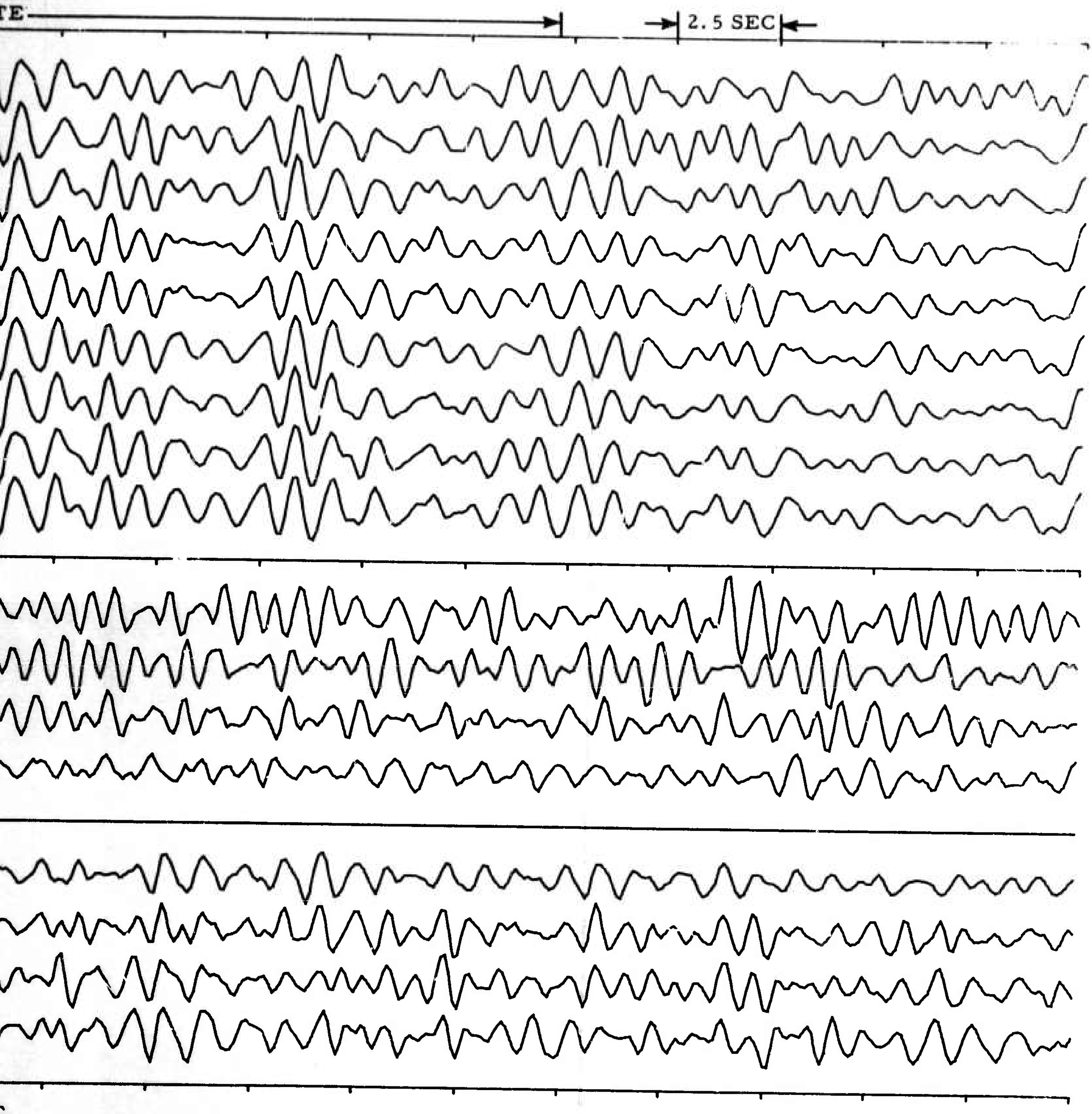
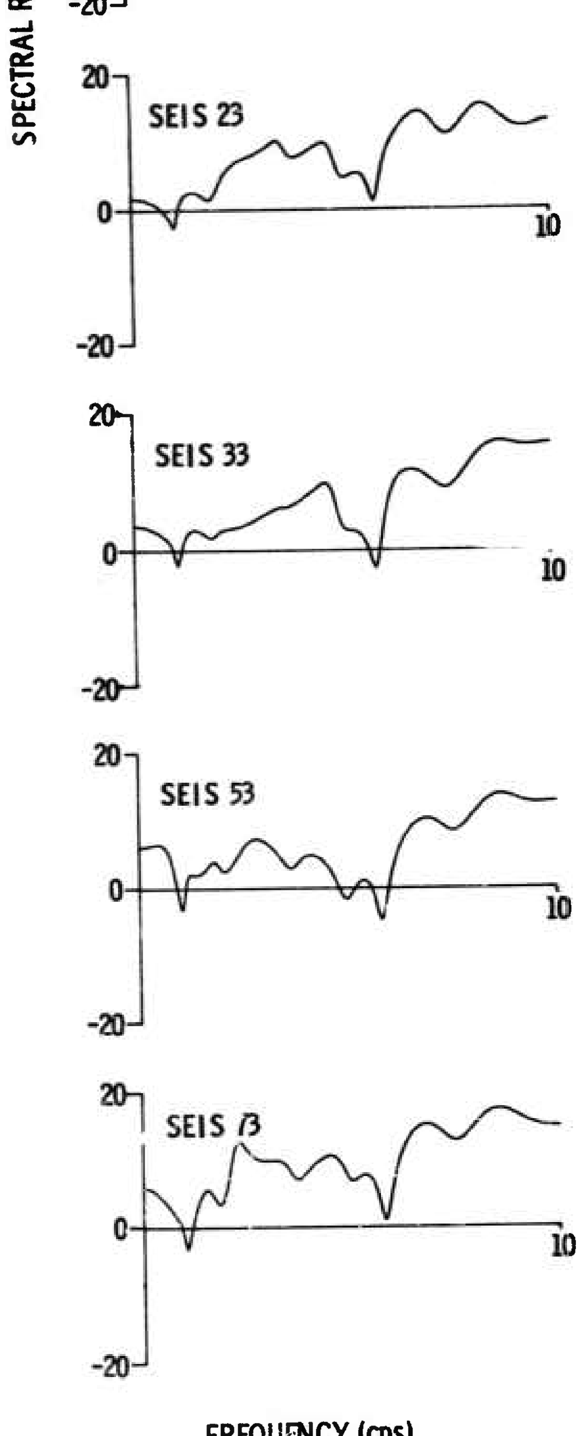


Figure II-10. Equalized Signal Using a 50-Point Filter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10
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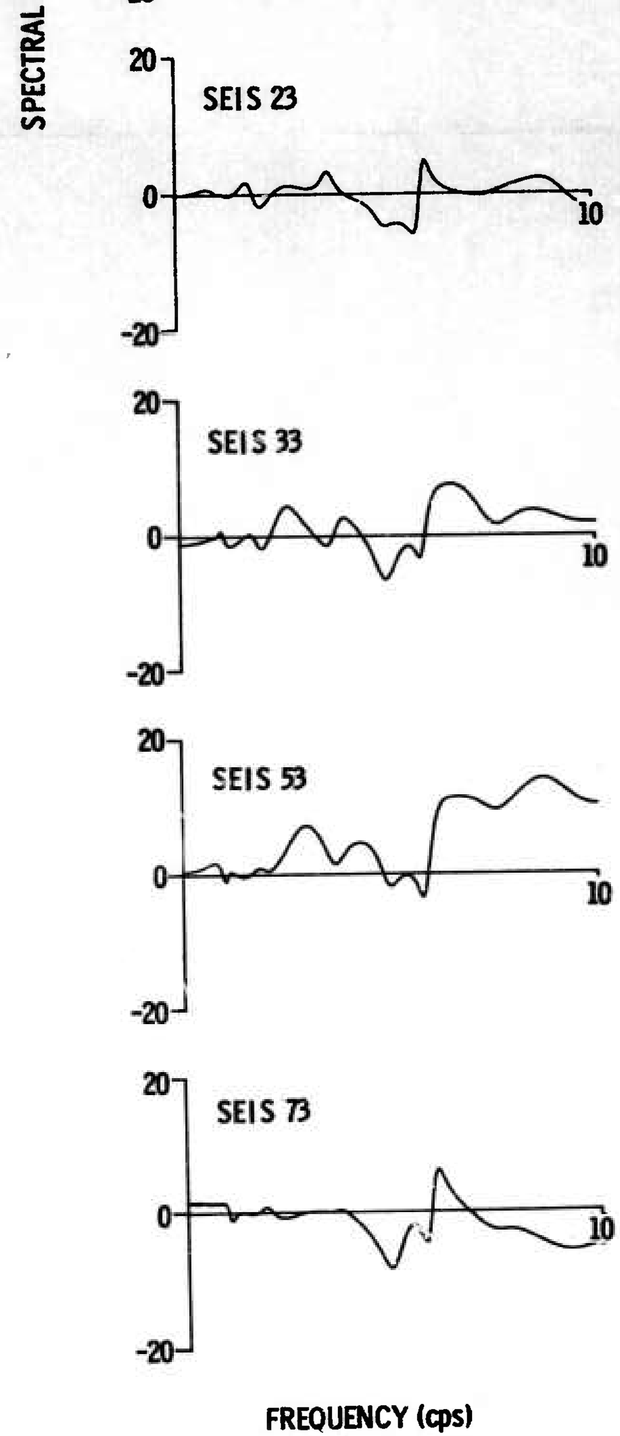
SEIS 33
SEIS 53
SEIS 73
FREQUENCY (cps)


SPECTRAL
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SEIS 33
SEIS 53
SEIS 73
FREQUENCY (cps)

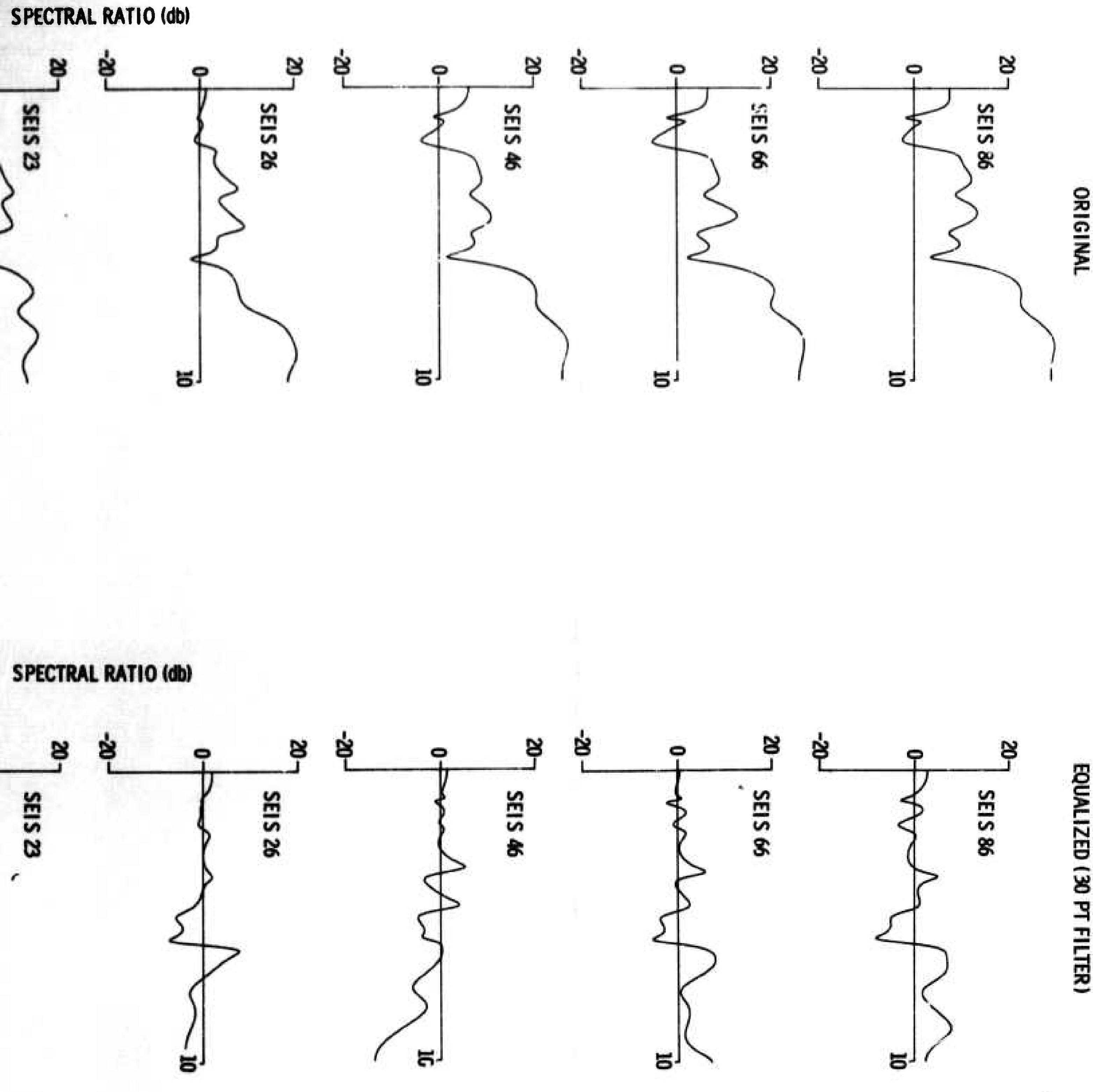

Figure II-11. Signal-to-Reference-Trace Spectral Ratios Before and After Equalization

SECTION III

EVALUATION OF REGIONAL EQUALIZATION FILTERS FOR LASA SUBARRAY OUTPUTS

The purpose of this study is to determine the feasibility of designing for LASA subarray outputs a set of equalization filters which can be applied to all events from a given epicentral region. If regional equalization were feasible, a set of equalization filters for each region could be designed and the correct set automatically applied to every event before large-array processing.

A. METHOD OF ANALYSIS

Table III-1 shows seven events from the Aleutian Islands area which were chosen for the analysis. Subarray outputs from B1, B2, C3, and D2 (Figures III-1 through III-7) were used for each event. These subarrays were chosen because the moveout anomalies were the same for each event, thus allowing the averaging of crosscorrelation data. The events have been resampled to a 0.1-sec rate and bandpass-filtered with a zero-phase 0.8- to 2.8-cps digital filter (Figure III-8). Table III-2 lists the peak-signal-to-rms-noise ratios.

In all cases, events are equalized to the subarray B1 output. The three equalization procedures are as follows.

- Amplitude Equalization

  The square root of the ratio of the energy on the reference trace to that on the trace to be equalized is computed, and the trace to be equalized is scaled by the resulting number.

- Levinson Equalization

  For each event, 21-point Levinson equalization filters are designed using 100-point (10-sec) gates. The filters are then applied to the appropriate traces.

Table III-1

EVENTS USED IN REGIONAL EQUALIZATION FILTER STUDY

Event No.	Region	Coordinates		Date	Origin Time (GMT)	Depth (km)	M_b	Δ (°)	Azimuth (°)
		Lat.	Long.						
13	Andreanof Islands	51.2N	178.8W	11/10/65	03:58:28	33	4.1	46.1	304.0
17	Andreanof Islands	51.4N	179.9W	11/23/65	02:17:49	48	5.6	46.5	304.1
24	Andreanof Islands	51.2N	178.1W	01/05/66	07:01:58	33	5.0	45.9	303.6
33	Fox Islands	51.3N	170.4W	05/04/66	07:09:37	33	3.9	41.2	300.8
35	Aleutian Islands	53.0N	168.0W	06/11/66	11:19:28	33	-	39.2	302.6
36	Aleutian Islands	52.8N	170.3E	06/12/66	06:49:09	33	4.5	51.4	309.5
40	Andreanof Islands	51.4N	179.1W	11/23/65	06:16:26	33	4.3	46.8	304.5

Table III-2

SIGNAL-TO-NOISE RATIOS OF EVENTS STUDIED

	Signal-To-Noise Ratios	
Event No.	LASA	Subarray B1
13	44.8	20.8
17	869.0	455.5
24	201.0	83.5
33	64.8	12.5
35	40.0	16.2
36	68.9	30.0
40	53.8	28.0

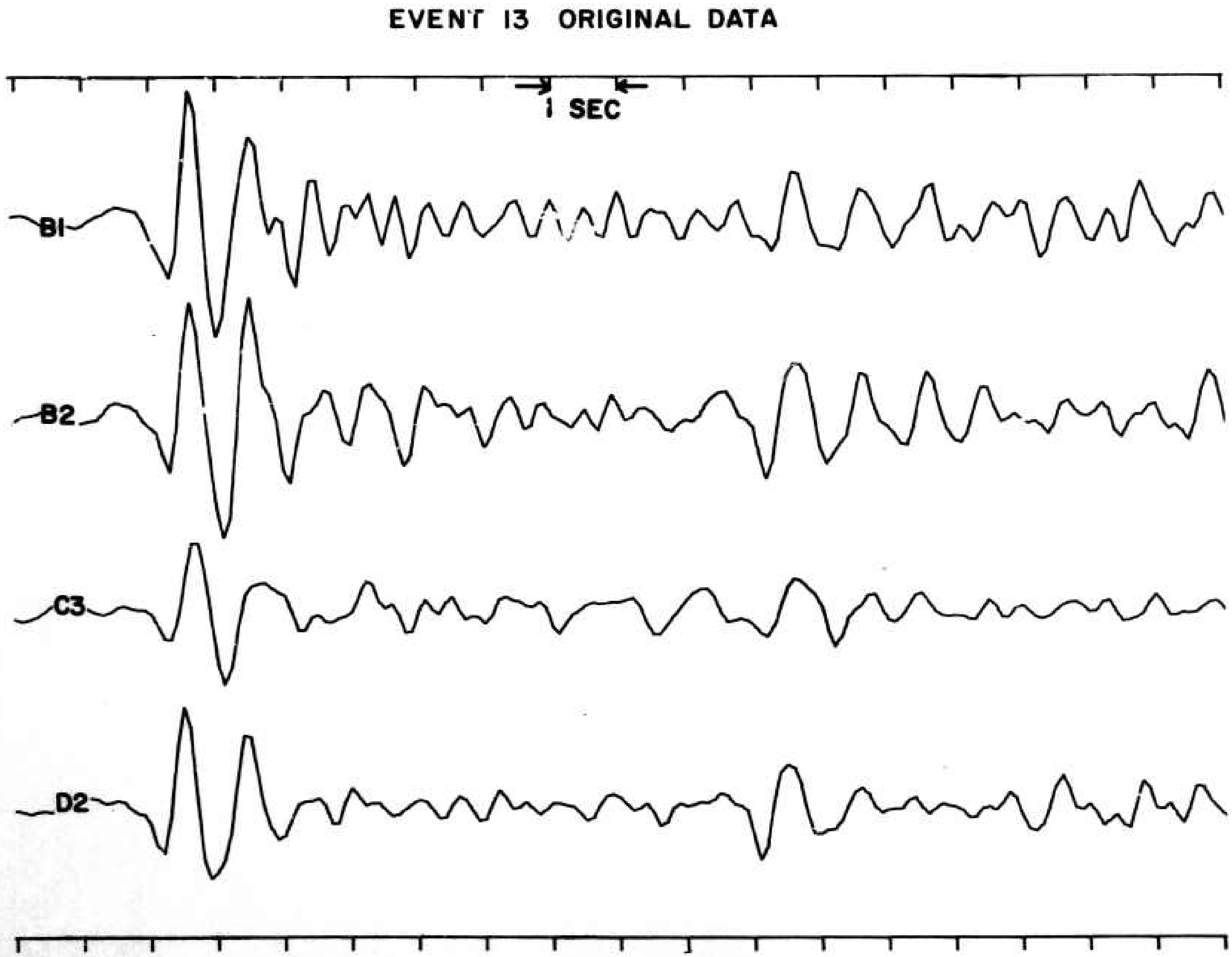

Figure III-1. 11 November 1965 Andreanof Islands Event

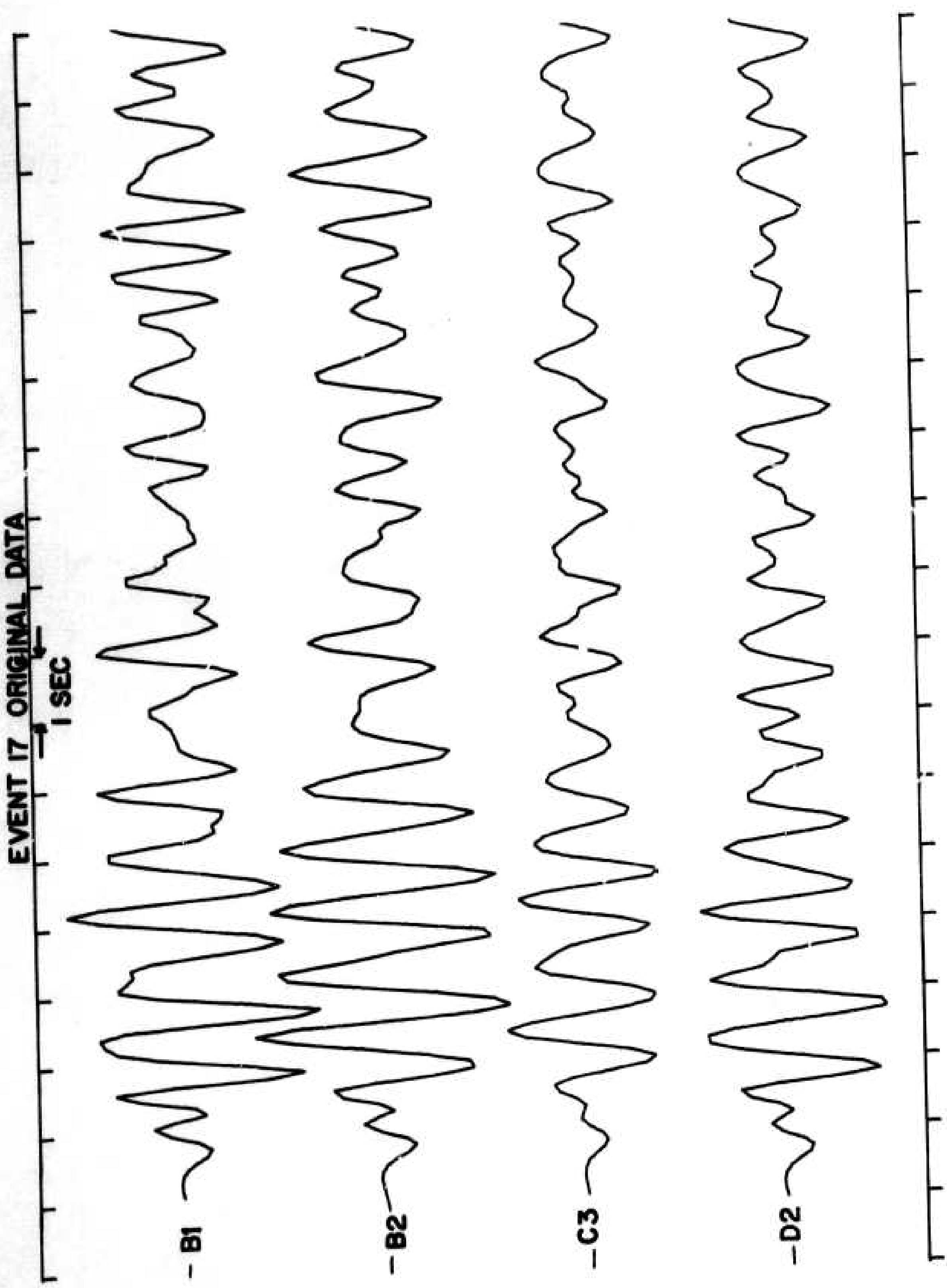

Figure III-2. 23 November 1965 Andreanof Islands Event

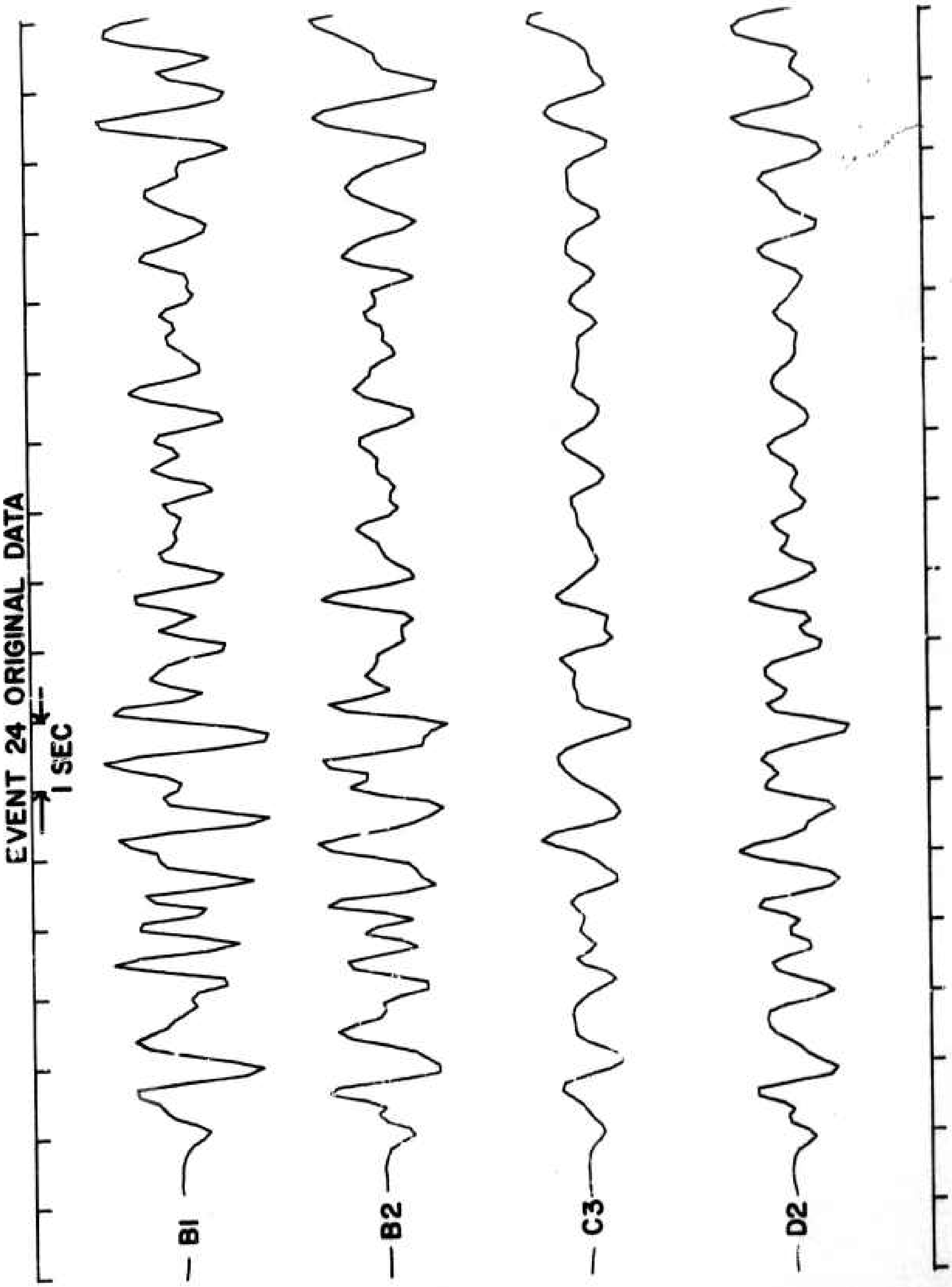

Figure III-3. 5 January 1966 Andreanof Islands Event

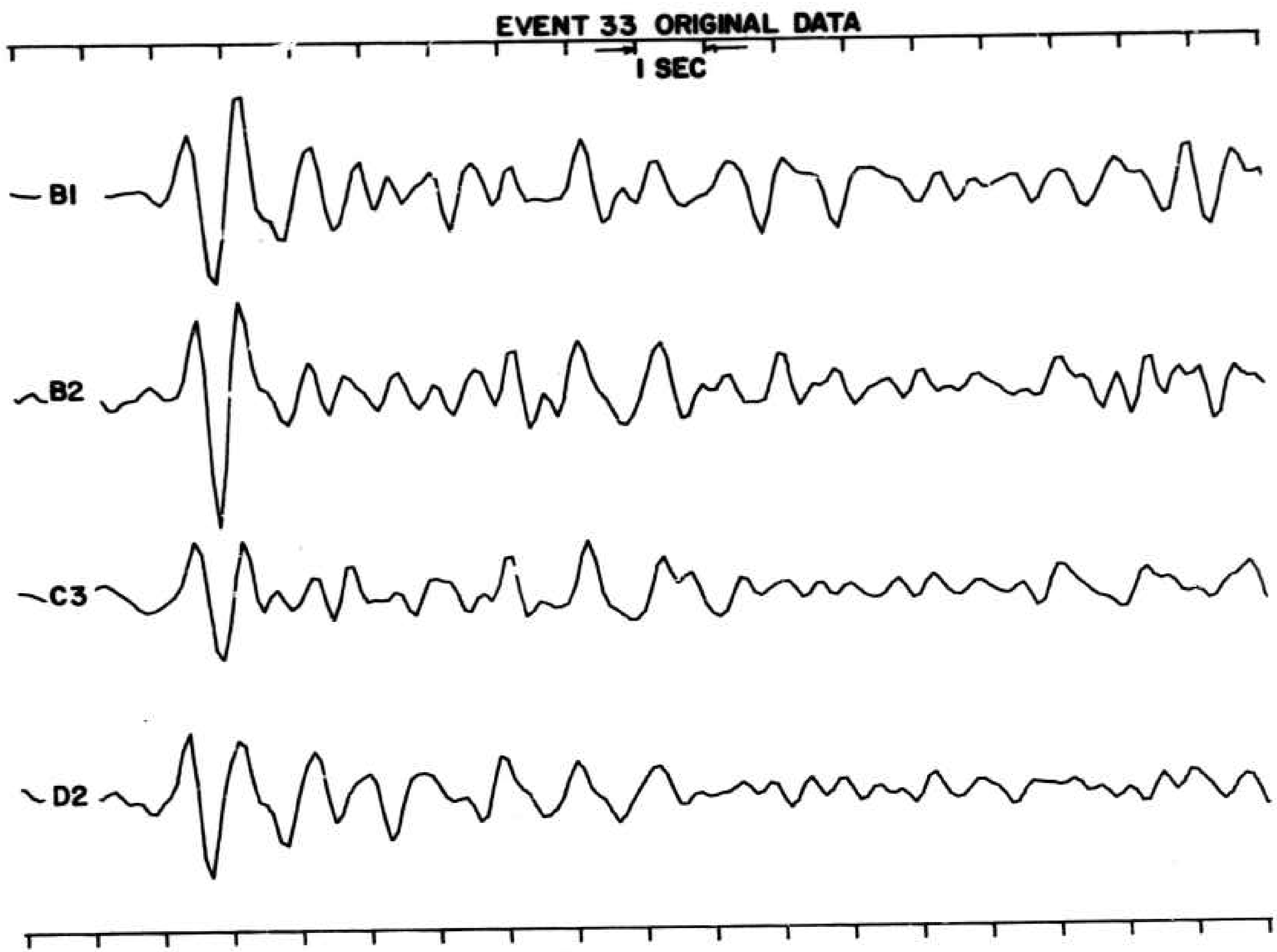

Figure III-4. 4 May 1966 Fox Islands Event

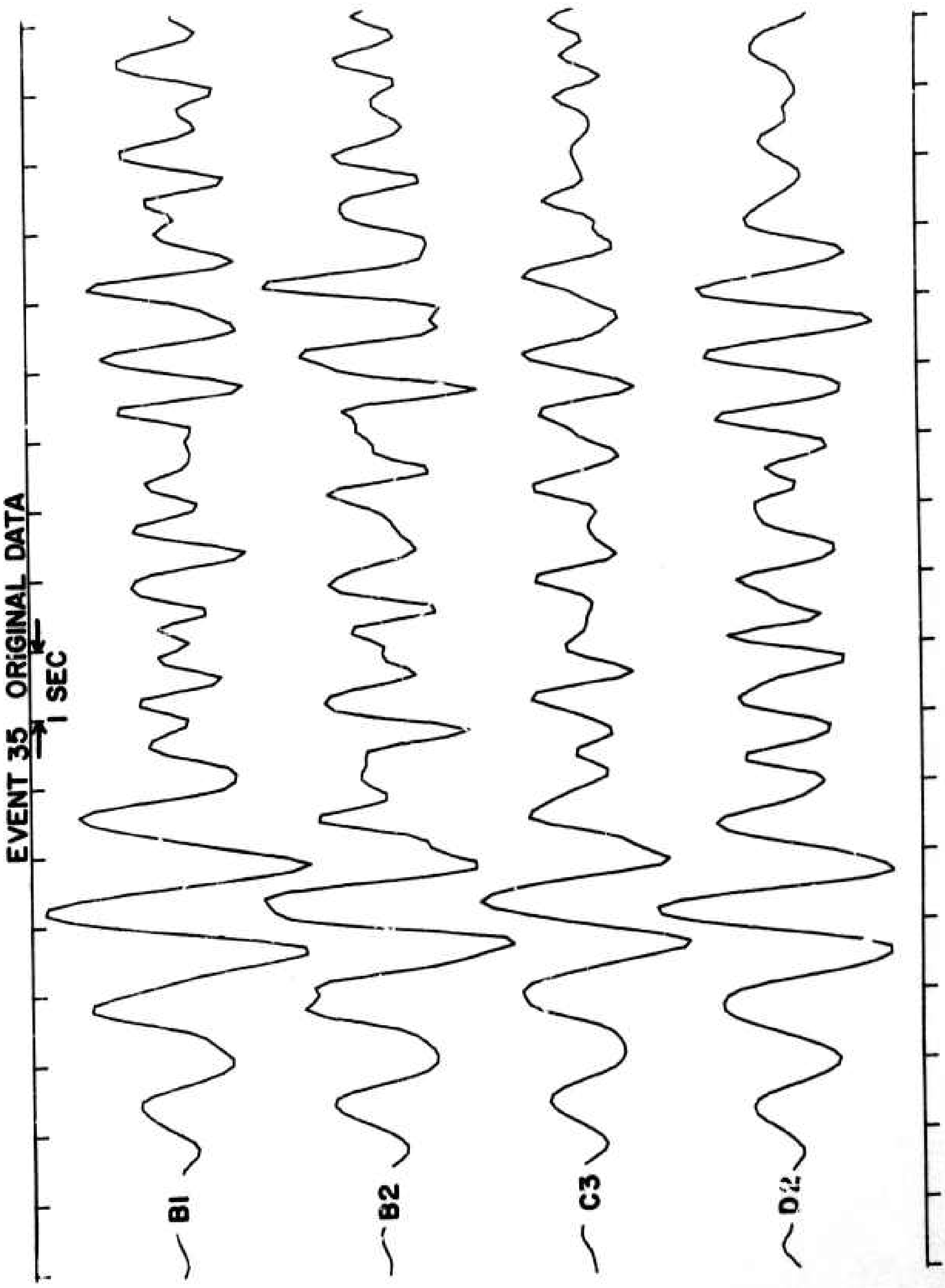

Figure III-5. 11 June 1966 Aleutian Islands Event

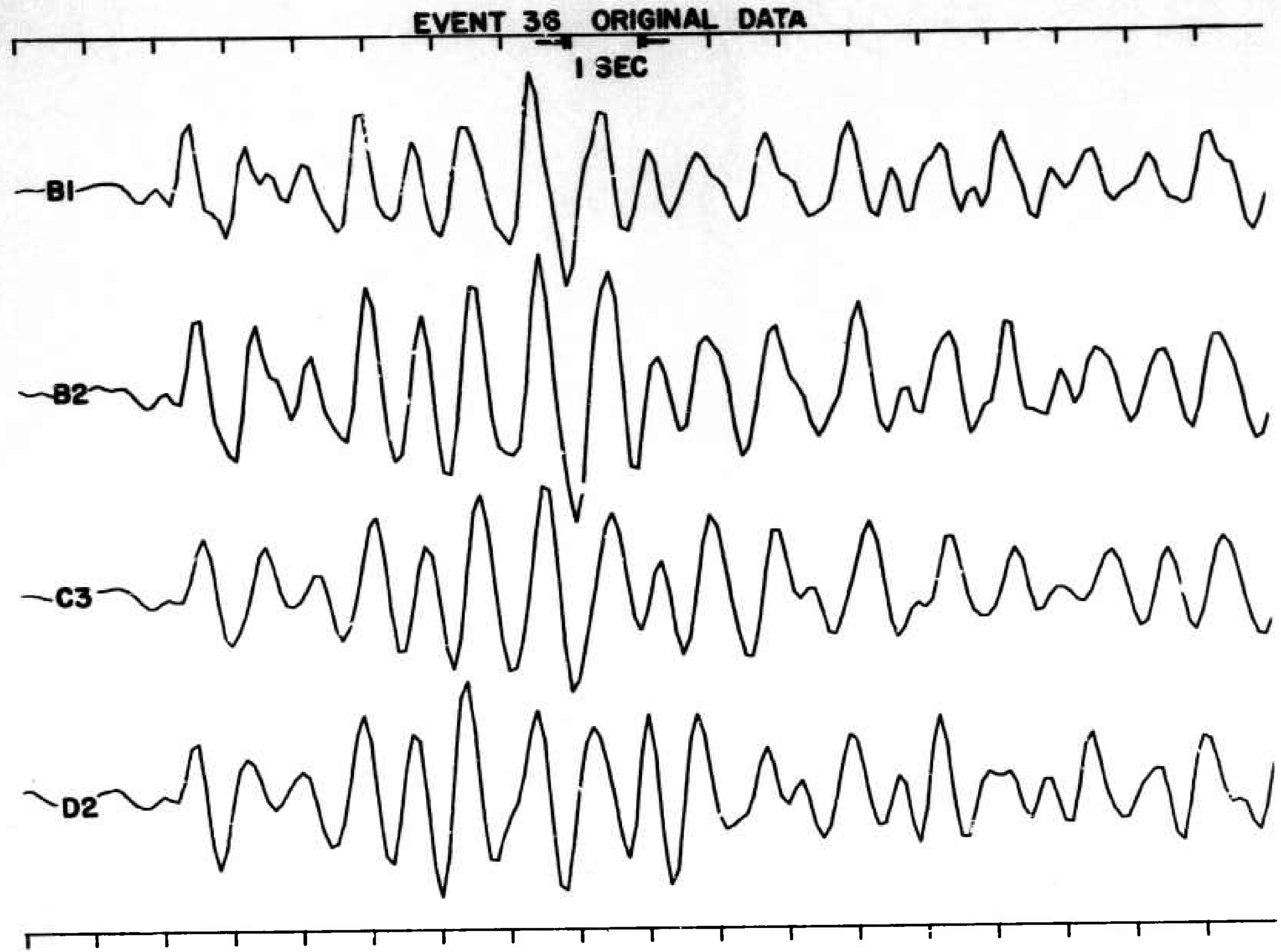

Figure III-6. 12 June 1966 Aleutian Islands Event

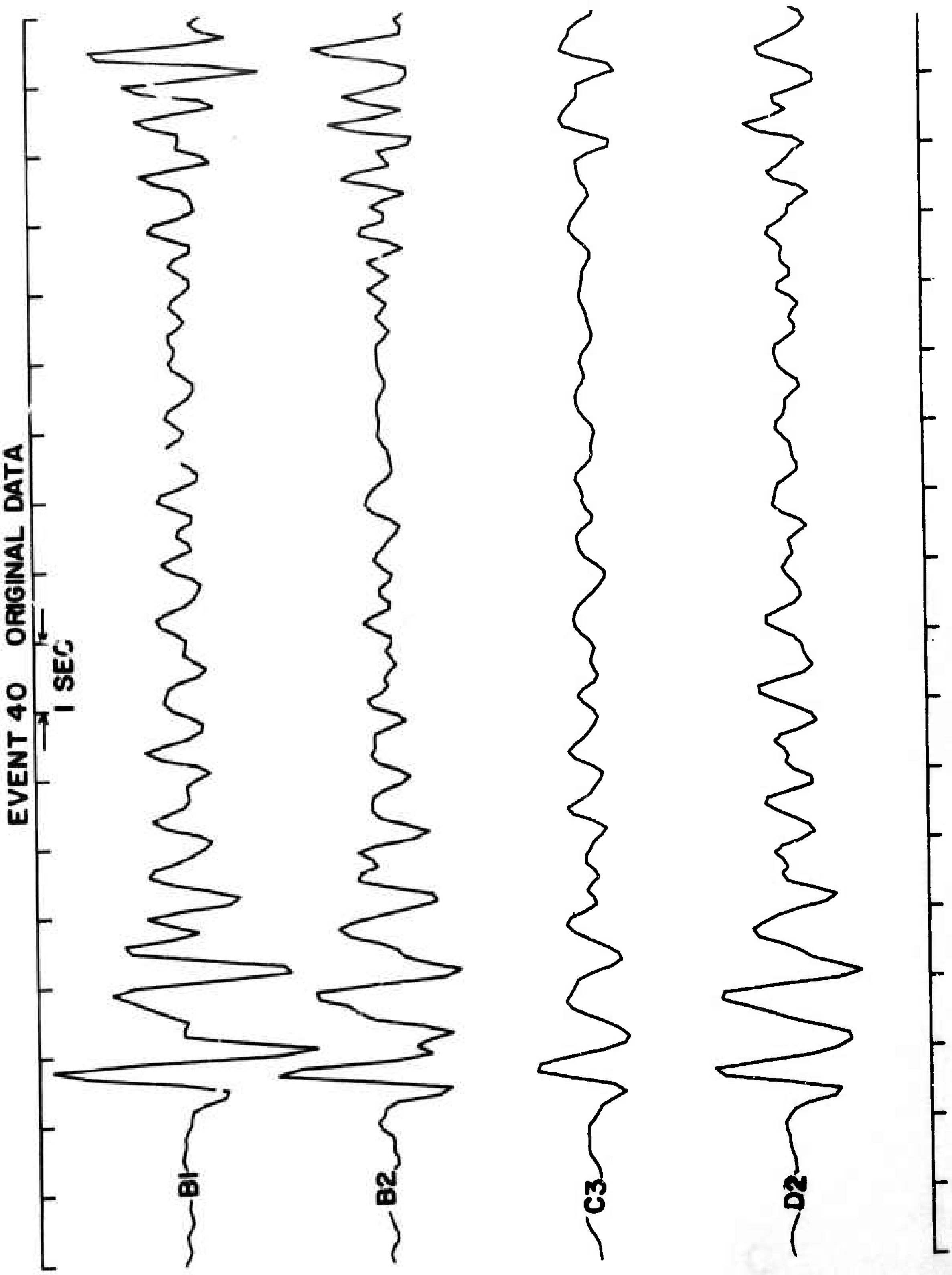

Figure III-7. 23 November 1965 Andreanof Islands Event

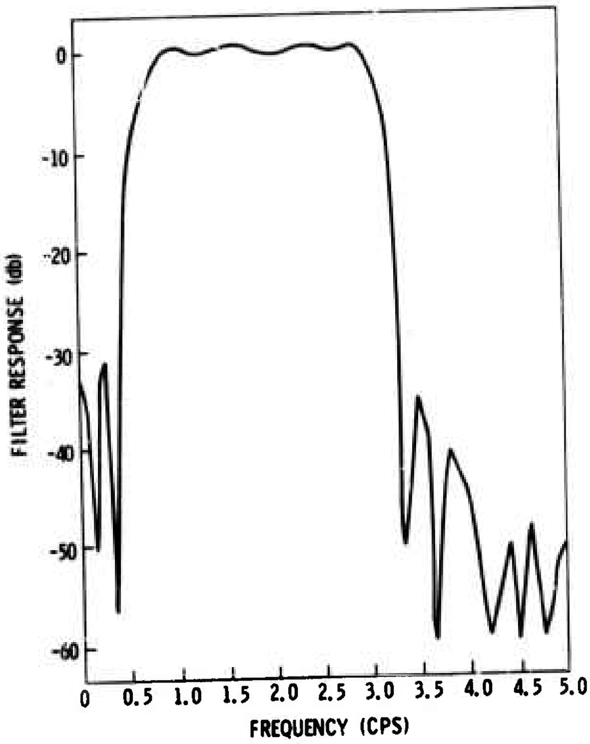

Figure III-8. Amplitude Response of 3.7-sec Zero-Phase Bandpass Filter (100-msec Sample Rate; 0.8- to 2.8-cps Passband)

- Average Levinson Equalization

  Correlation data for the three largest events (17, 24, and 36) are scaled to be approximately the same size; auto- and crosscorrelations are stacked and an "average" Levinson filter designed for each of the B2, C3, and D2 outputs. The average filters are then applied to the traces for all events.

To compare Levinson equalization with amplitude and individual Levinson equalization, the correlation coefficient* between B1 and the other three subarray outputs is computed for each set of equalized traces. Note that scaling does not affect the correlation coefficient, so the correlation coefficients after amplitude equalization are the same as the correlation coefficients for the original data.

*Texas Instruments Incorporated, 1967: Short-Period Signal Waveform at LASA, LASA Spec. Rpt. No. 8, Contract AF 33(657)-16678, 1 Aug.

B. DATA PRESENTATION

Figures III-9 through III-15 show the amplitude-equalized, the Levinson-equalized, and the average Levinson-equalized traces for each event. Table III-3 lists the correlation coefficients obtained after each type of equalization. The events are grouped by epicentral location; the first four are from the Andreanof Islands, and the last three are from other locations in the Aleutians.

A comparison of the raw data (Figures III-1 through III-7) with the amplitude-equalized data (top set of traces in Figures III-9 through III-15) shows that amplitude equalization is necessary for all events. Amplitude equalization would probably be sufficient also for most processing schemes for events 13, 17, 35, and 36, since the correlation coefficients are close to unity.

Levinson equalization improves the signal similarity for all events. All correlation coefficients except those for subarray D2, event 24, and subarray C3, event 40, are greater than 0.9 after Levinson equalization. The increased waveform similarity is most evident for event 40 (Figure III-15).

Average Levinson equalization improves the signal similarity for events 13, 17, 24, and 40, does not appreciably change the similarity for event 35, and decreases the similarity for events 33 and 36. For the four events where similarity is improved, the improvement is less than that obtained by Levinson equalization. Note that these four events have almost identical epicenters (within approximately 100-km distance); the three remaining events have significantly different epicenters (up to 700-km distance). It thus appears that average equalization is possible but that average filters are critically dependent on event location and therefore can be applied only to events from a very small epicentral region. This strong dependence of the filter set on event location appears to make average equalization impractical because of the large number of filter sets that would be required.

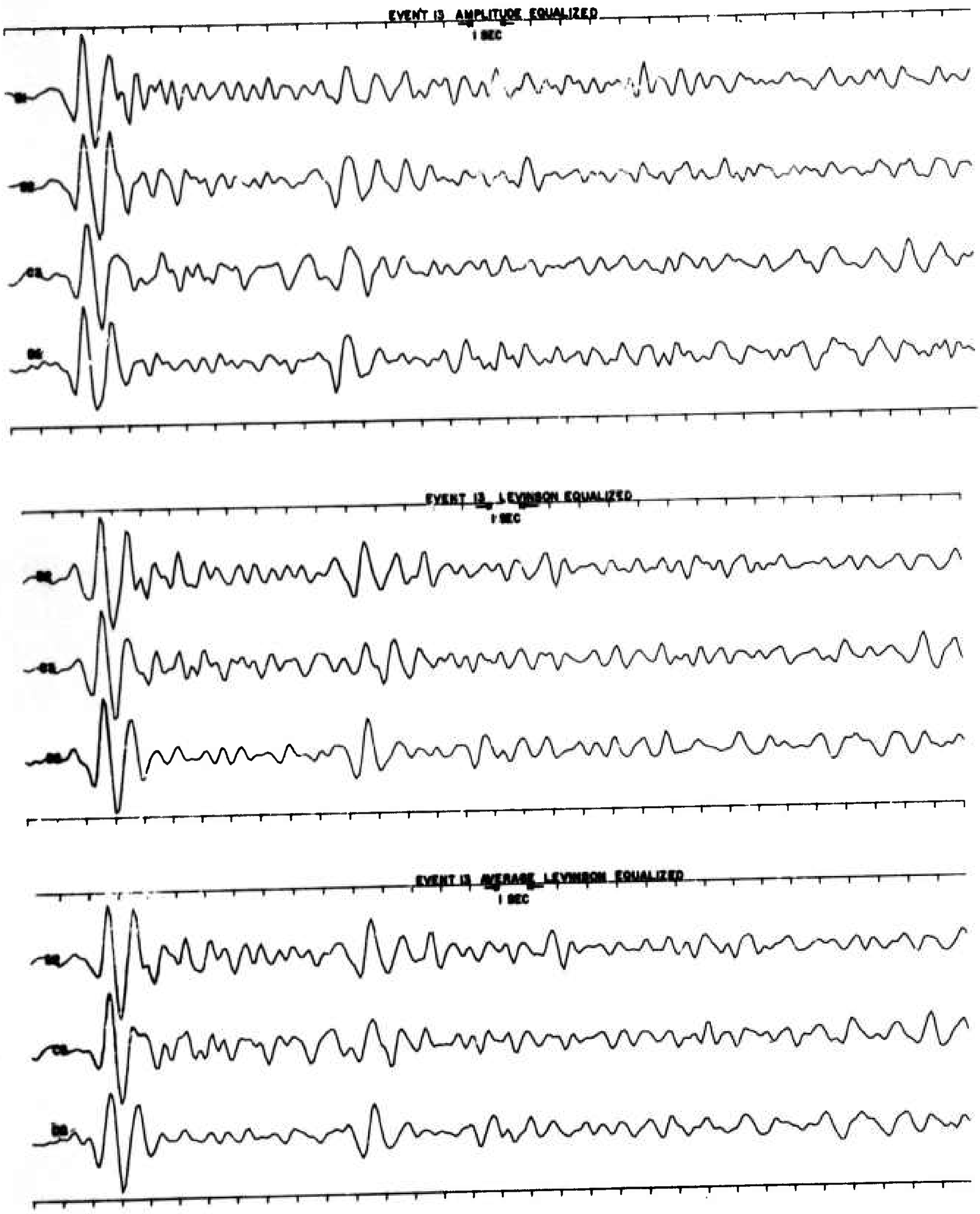

Figure III-9. Equalized Data, Event 13

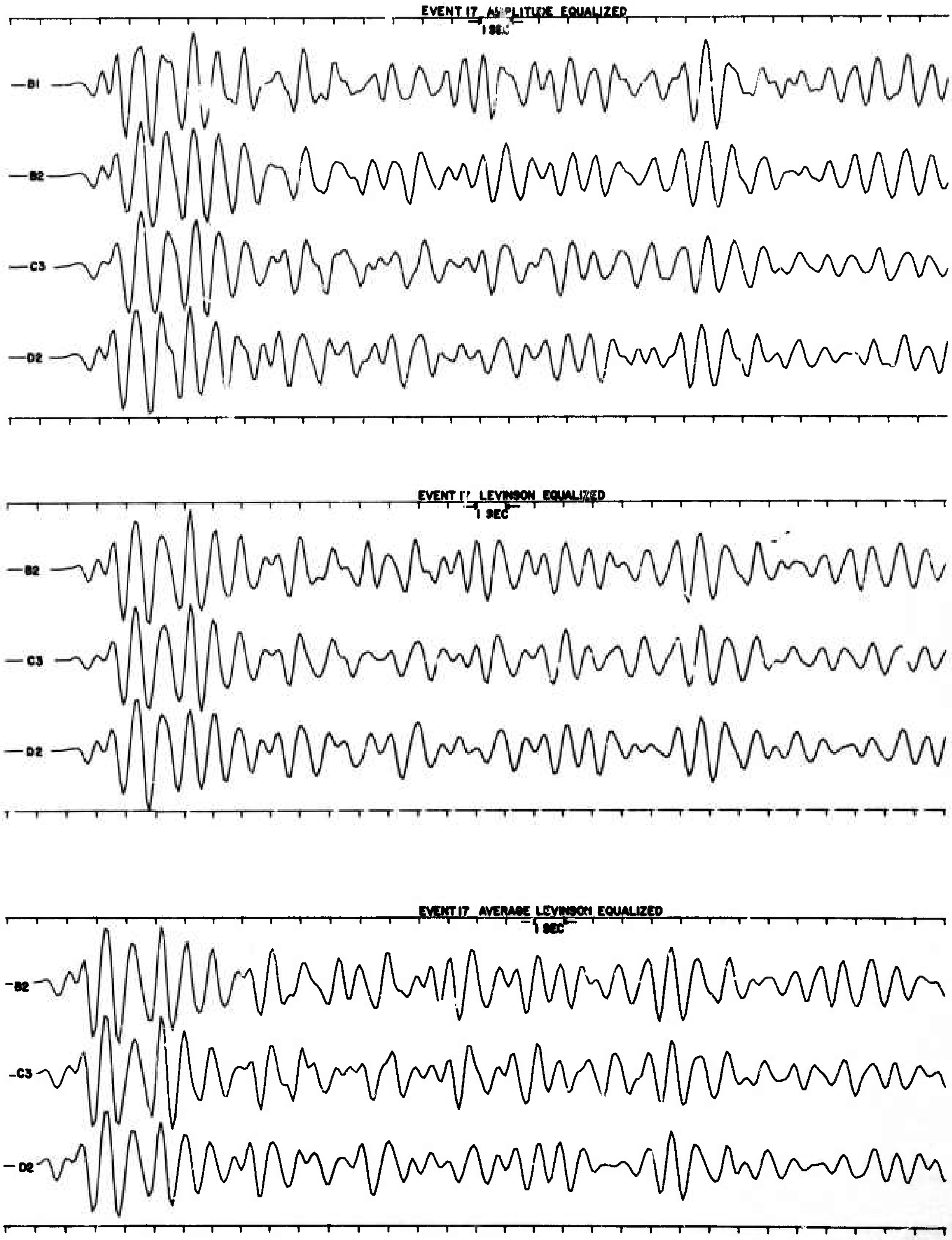

Figure III-10. Equalized Data, Event 17

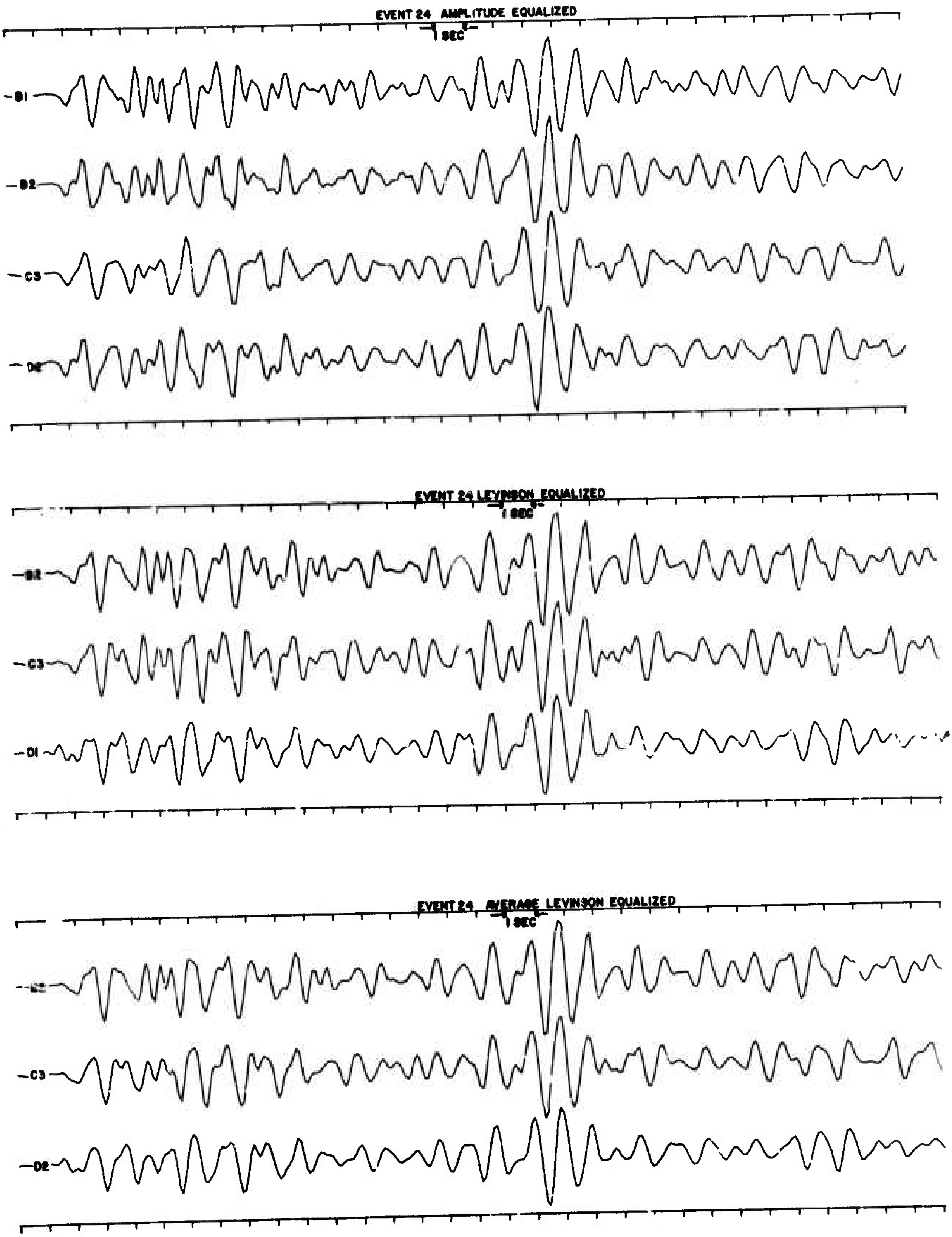

Figure III-11. Equalized Data, Event 24

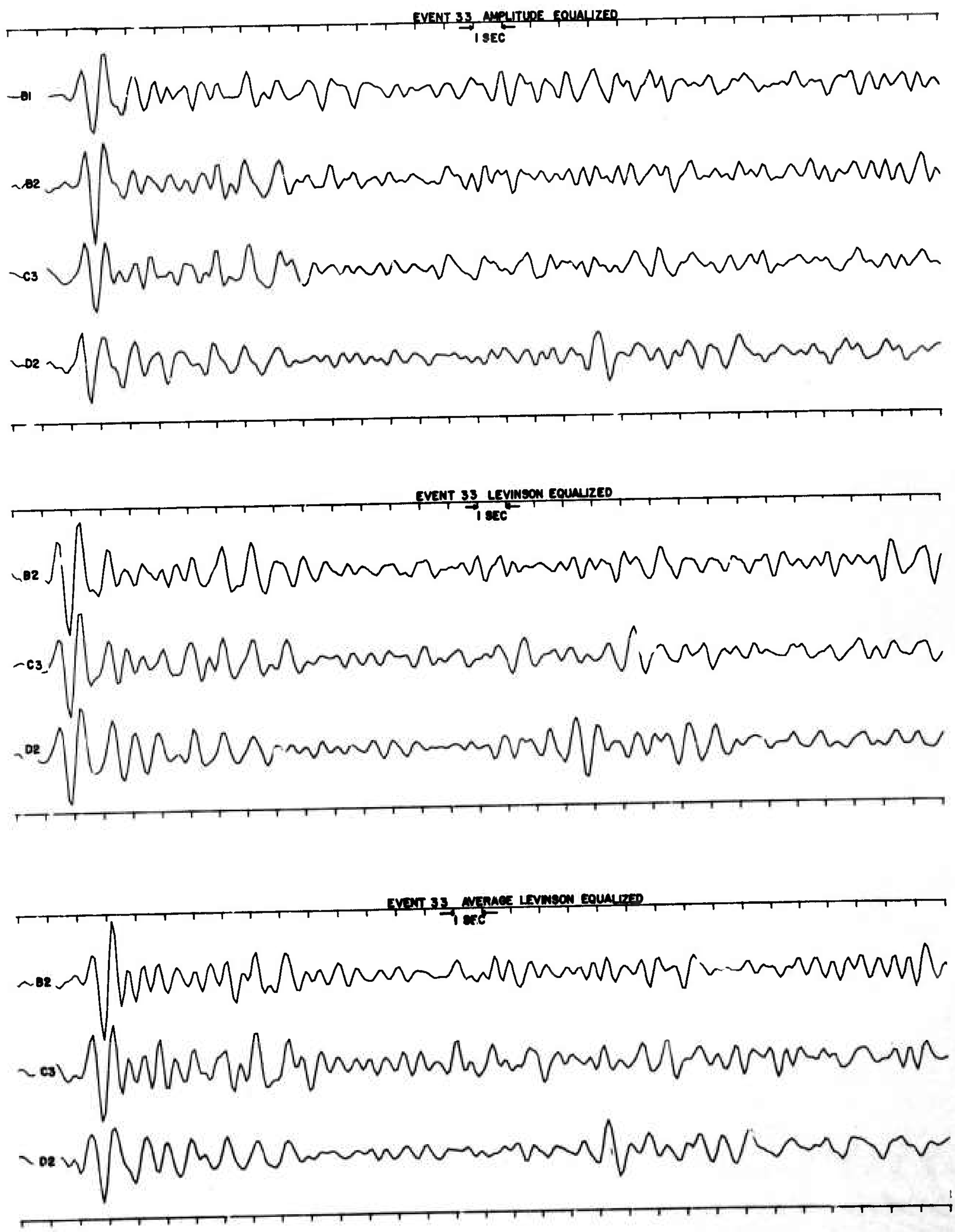

Figure III-12. Equalized Data, Event 33

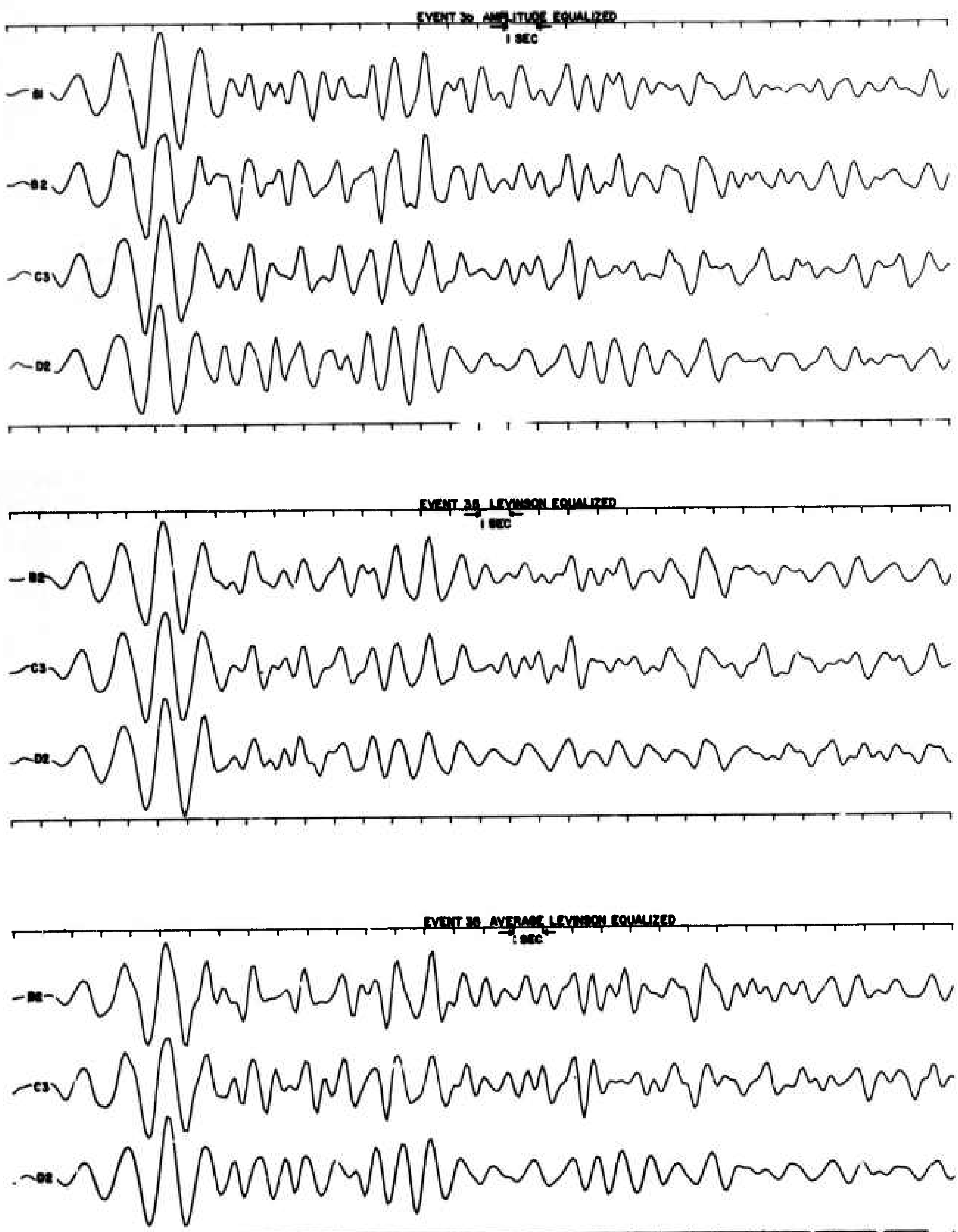

Figure III-13. Equalized Data, Event 35

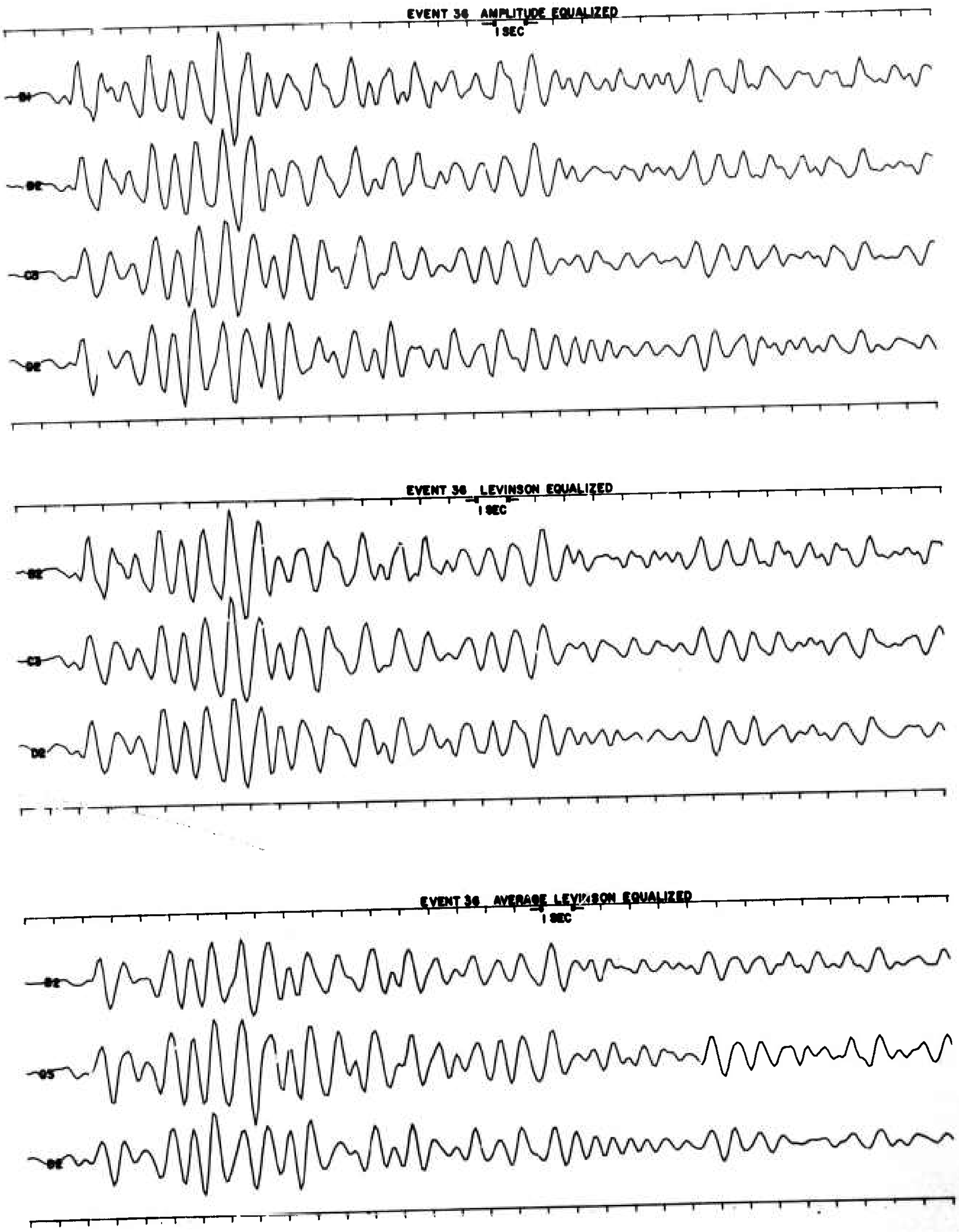

Figure III-14. Equalized Data, Event 36

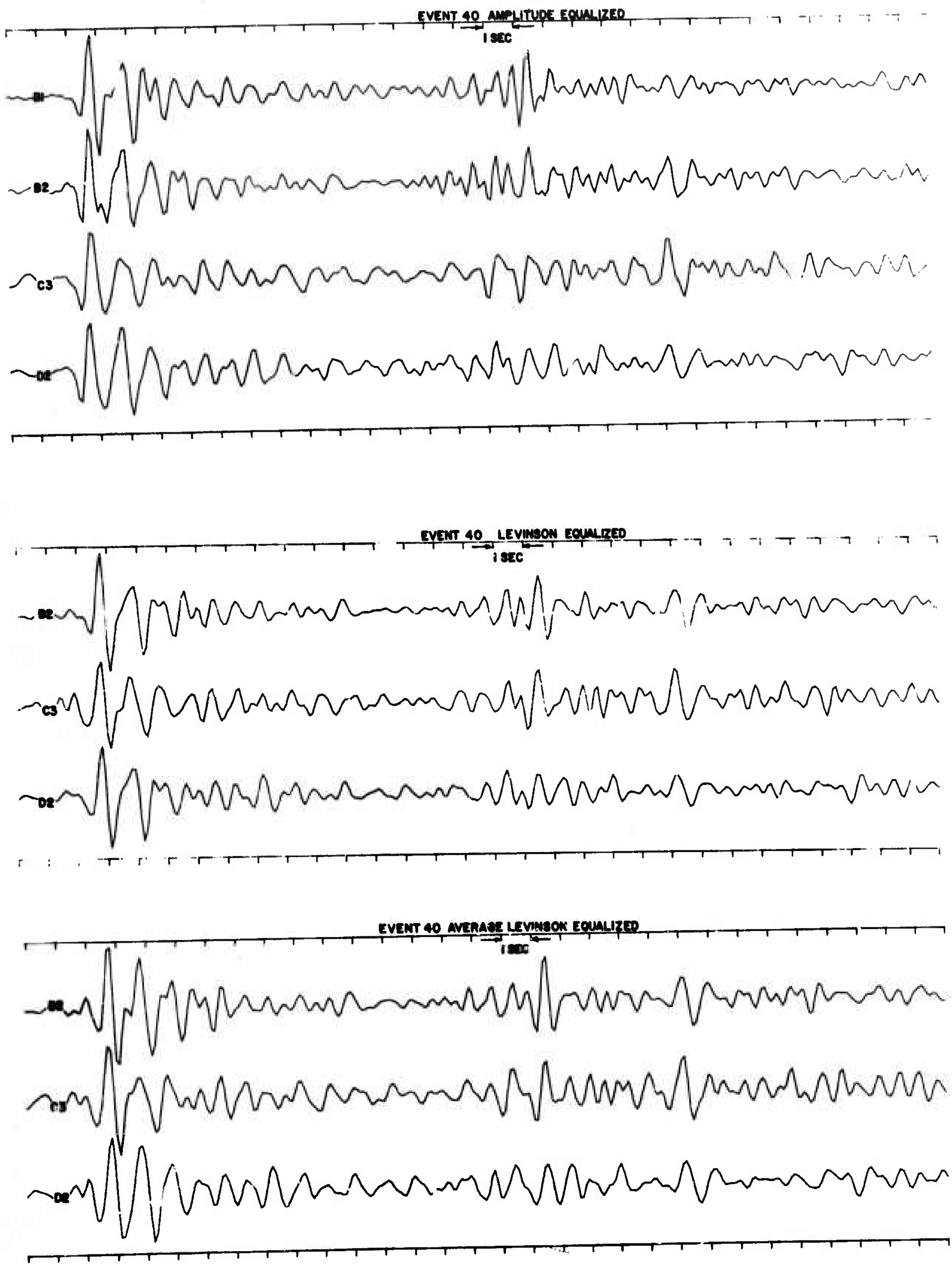

Figure III-15. Equalized Data, Event 40

Table III-3

CORRELATION COEFFICIENTS FOR ORIGINAL AND EQUALIZED DATA

Event No.	Subarray	Δ (°)	Azimuth (°)	Correlation Coefficients		
				Orig & Amp Eq	Lev Eq	Avg Lev Eq
13	B2	46.1	304.0	0.907	0.975	0.964
	C3			0.852	0.971	0.914
	D2			0.930	0.930	0.924
17*	B2	46.5	304.1	0.887	0.975	0.958
	C3			0.920	0.950	0.928
	D2			0.902	0.945	0.931
24*	B2	45.9	303.6	0.754	0.931	0.915
	C3			0.722	0.942	0.884
	D2			0.723	0.803	0.772
40	B2	46.8	304.5	0.756	0.969	0.901
	C3			0.763	0.873	0.846
	D2			0.811	0.948	0.840
33	B2	41.2	300.8	0.851	0.944	0.803
	C3			0.791	0.936	0.751
	D2			0.825	0.917	0.824
35	B2	39.2	302.6	0.930	0.991	0.928
	C3			0.960	0.992	0.975
	D2			0.975	0.991	0.975
36*	B2	51.4	309.4	0.943	0.980	0.871
	C3			0.911	0.949	0.876
	D2			0.849	0.959	0.836

*Used in design of average Levinson filter

Note that event 36 has poorer signal similarity after average equalization, even though it is one of the three (with events 17 and 24) used in the average filter design. Apparently, events 17 and 24 together dominate in the average filter design. It appears that, if event 36 had been omitted from the average filter design, correlation coefficients for events 13, 17, 24, and 40 after average equalization would have been almost as good as those obtained after individual equalization for these events.

C. CONCLUSIONS

As observed previously,* amplitude equalization of LASA subarray outputs would generally be sufficient for most processing schemes.

The design of a set of regional equalization filters appears to be feasible but only if the epicentral region is very small. Consequently, implementation seems impractical except for specific small areas of interest.

*Ibid

SECTION IV

ANALYSIS OF NOISE PREEQUALIZATION COEFFICIENTS

A previous report shows that the noise at a subarray must be equalized prior to multichannel processing in order to obtain consistent rejection at low frequency (below 1.0 cps).* The technique adjusts the rms noise level on each trace to a common value; i.e., the quantity

$$\sigma_i = \left\{ \frac{1}{N} \sum_{j=0}^{N} X_{ji}^2 \right\}^{1/2} \qquad \text{for } i = 1, \ldots, NC \quad j = 1, \ldots, N$$

where

σ_i is rms noise level on i^{th} trace

X_{ji} is j^{th} sample on i^{th} trace

N is number of samples in each trace

NC is number of channels in a subarray

is computed for every trace and the quantity ρ_i is found such that

$$\rho_i = \frac{\sigma_r}{\sigma_i}$$

where

ρ_i is scalar to apply to i^{th} trace

σ_r and σ_i are rms noise levels on reference trace and i^{th} trace, respectively

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*Texas Instruments Incorporated, 1967: Subarray Processing, Large-Array Signal and Noise Analysis, Spec. Rpt. No. 3, Contract AF 33(657)-16678, 16 Oct.

This technique is simple to implement and, because of the peaked noise power spectra, effectively equalizes the data at low frequency. Consequently, it was applied at all subarrays for all noise samples.

The large number of equalization coefficients collected appear to provide the data necessary to undertake a statistical analysis of seismometer-gain fluctuations. Suppose we express the noise on any channel at any time as

$$X_{ji} = K_i X'_{ji}$$

where

K_i is gain of i^{th} seismometer-amplifier system

X'_{ji} is ground motion at i^{th} seismometer at time j

K_i is assumed to be a random variable which is statistically independent of the gain on any other seismometer. From physical considerations, it appears reasonable to assume that K_i is constant over the time interval (0, N); i.e., the seismometer gain is assumed to be a slowly varying function. Then we can express ρ_i as

$$\rho_i = \frac{\sigma_r}{\sigma_i} = \left\{ \frac{\frac{1}{N}\sum_{j=0}^{N}\left(K_r X'_{jr}\right)^2}{\frac{1}{N}\sum_{j=0}^{N}\left(K_i X'_{ji}\right)^2} \right\}^{1/2}$$

$$= \frac{K_r}{K_i} \left\{ \frac{\frac{1}{N}\sum\left(X'_{jr}\right)^2}{\frac{1}{N}\sum\left(X'_{ji}\right)^2} \right\}^{1/2}$$

$$= \frac{K_r}{K_i} \left\{ \frac{\varphi_{rr}(0)}{\varphi_{ii}(0)} \right\}^{1/2}$$

where

$\varphi_{rr}(0)$ and $\varphi_{ii}(0)$ are zero-lag values of autocorrelation function on channels r and i, respectively

or

$$\rho_i = \frac{K_r}{K_i}\,\alpha$$

where

$$\alpha = \left[\varphi_{rr}(0) \big/ \varphi_{ii}(0)\right]^{1/2}$$

If α is nearly constant for each noise sample, for an assumed distrubution of the K's, the distribution of the ρ's can be calculated. (For example, if the K's are uniformly distributed and $\alpha \approx 1$, distribution of the ρ's can be derived as in Appendix B.) It would then be possible to compare the theoretical and observed distribution of the ρ's.

The key to the problem is to determine whether the assumption that α is nearly constant is reasonable; estimating the variance of α is necessary. Because the time series is autocorrelated (i.e., the X_{ji}'s are not independent), the exact distribution of α cannot be determined. However, the variance of α can be estimated by making some simplifying assumptions.

Assume the X_{ji}'s are normally and identically (but not independently) distributed. Then $\varphi_{ii}(0)$ is approximately X^2 distributed with N_e degrees of freedom, where N_e can be estimated from the spectral shape by determining the number of frequencies which contribute significantly to the total power. Because of the peaked noise spectrum, significant power levels are confined to about one-tenth the bandwidth, so $N_e \sim \frac{1}{10}$ N. Usually about 2000 points are used to compute $\varphi_{ii}(0)$ so that $N_e \sim 200$.

Assume $\varphi_{rr}(0)$ and $\varphi_{ii}(0)$ are independent. Since the reference in this case was seismometer 21, this assumption would be approximately true for the seismometers on the outer edges of a subarray.* Then

$$\alpha^2 = \frac{\varphi_{rr}(0)}{\varphi_{ii}(0)}$$

is approximately F-distributed with 200 degrees of freedom for both numerator and denominator. Thus, α is distributed as $F^{1/2}_{200,200}$.

The variance of α is

$$\begin{aligned} \mathrm{Var}(\alpha) &= E(\alpha^2) - E^2(\alpha) \\ &= E\left[F\right] - E^2\left[F\right]^{1/2} \\ &= \frac{N_e}{N_e - 2} - E^2\left[F^{1/2}\right] \end{aligned}$$

It is shown in Appendix C that $E\left[F^{1/2}\right] \sim 1$ when N_e becomes large,

$$\therefore \ \mathrm{Var}(\alpha) = \frac{N_e}{N_e - 2} - 1 = \frac{2}{N_e - 2}$$

and the standard deviation of α is approximately ±10 percent. Consequently, the assumption that α is constant does not appear to be reasonable, and the variations in the equalization coefficients cannot be attributed to seismometer-gain fluctuations only.

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*Texas Instruments Incorporated, 1966: LASA Data Analysis and MCF Support, Final Rpt., Contract AF 19(628)-5167, 31 Aug.

Figure IV-1 shows a plot of the equalization coefficients ρ_i as a function of time obtained for several seismometers. Note that for two noise samples on the same day, considerable difference in ρ_i is observed, implying that α indeed varies considerably from noise sample to noise sample. Variations in ρ_i also occurred for the inner seismometers (where the assumption that the two time series are independent does not hold), which indicates the variance of α is still significant. Figure IV-1 does indicate, however, that the seismometer gains at low frequency are different from channel to channel because the curves are separated by amounts larger than the variations observed.

In summary, it is impossible to interpret variations in the ρ_i's solely in terms of gain fluctuations. Instead, the variations are due to a combination of gain fluctuations and variations in the estimate of α.

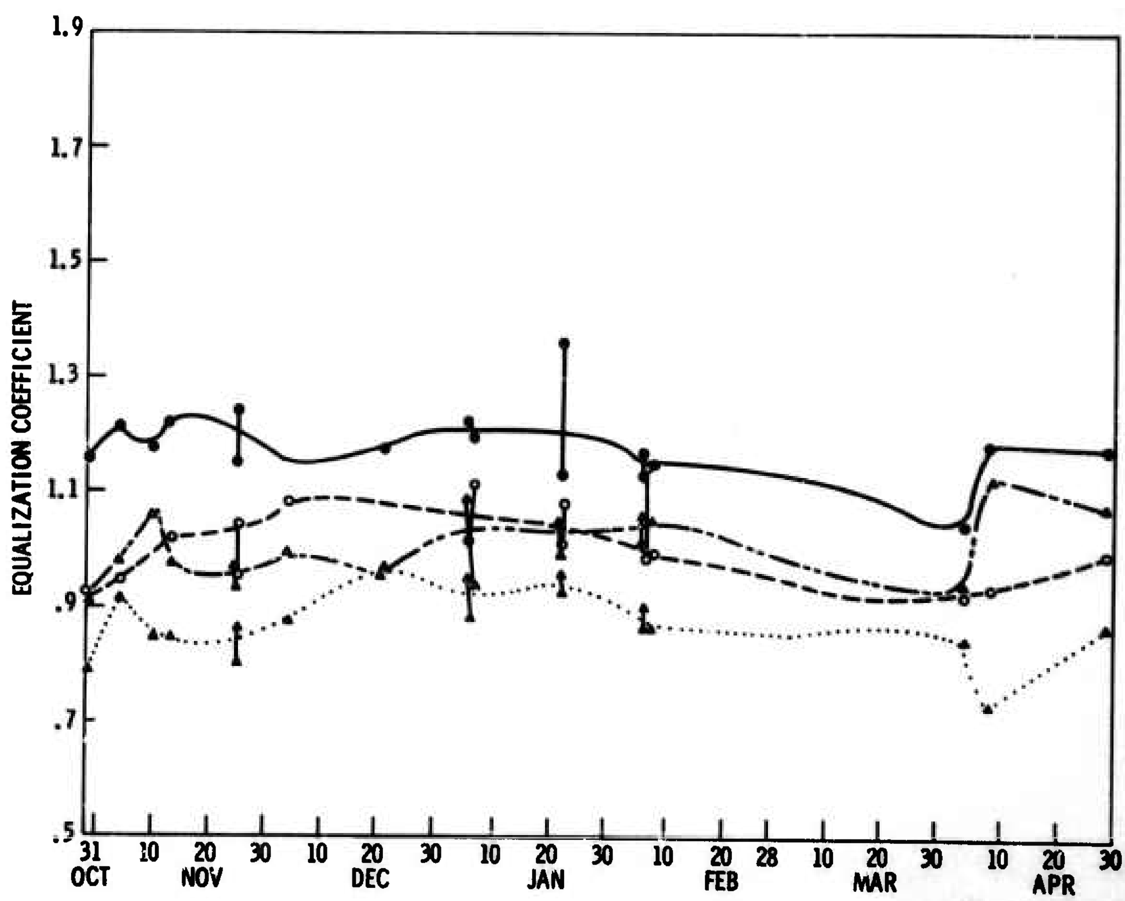

Figure IV-1. Variations in Noise Equalization Coefficients for Seismometers 31, 82, 44, and 75, Subarray B3

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SECTION V

STATISTICAL PHASE FLUCTUATIONS

Considerable work was done in the past on the problem of adding statistical gain fluctuation to the correlation statistics. The purpose was to prevent misdesigning multichannel-filter systems by using gain differences to separate measured noise from an equalized signal model. There has been some interest in the phase inequalities among sensors at the LASA. This section generalizes the method of gain fluctuation to incorporate statistical phase fluctuations into the correlation matrix.

Gain fluctuation was incorporated by adding at each sensor a 1-point stochastic filter having an expected value of 1 and any assigned variance. The variance determined the amount of gain fluctuation added. This idea can be generalized to gain and phase fluctuation by adding a $(2L+1)$-point stochastic filter at each seismometer. These filters have the following properties:

- Expected value of $\delta(t)$
- Diagonal covariance matrix (independent filter weights)
- No correlation between channels

To determine the crosscorrelation between two channels which have stochastic filters with the properties just defined, the outputs of the two channels are

$$f_1'(t) = f_1(t) * h_1(t)$$

$$f_2'(t) = f_2(t) * h_2(t)$$

where

$f_1'(t)$ and $f_2'(t)$ are outputs of channels 1 and 2

$h_1(t)$ and $h_2(t)$ are stochastic filters

$f_1(t)$ and $f_2(t)$ are inputs to channels 1 and 2

$*$ stands for convolution

The crosscorrelation is

$$\Phi_{12}'(m) = \overline{f_1'(t)\, f_2'(t+m)}$$

$$= \overline{\left[\sum_{\sigma=-L}^{+L} f_1(t-\sigma\Delta t)\, h_1(\sigma\Delta t)\right] \sum_{\xi=-L}^{L} f_2(t+m-\xi\Delta t)\, h_2(\xi\Delta t)}$$

Writing out the correlation estimate as a summation, rearranging terms, and taking the expectation inside the summation gives

$$\Phi_{12}'(m) = \sum_n \sum_\xi \sum_\sigma f_1(n\Delta t - \sigma\Delta t)\, f_2(n\Delta t - \xi\Delta t + m)\, \overline{h_1(\sigma\Delta t)\, h_2(\xi\Delta t)} \quad (5\text{-}1)$$

Since $h_1(t)$ and $h_2(t)$ are assumed independent,

$$\overline{h_1(\sigma\Delta t)\, h_2(\xi\Delta t)} = \overline{h_1(\sigma\Delta t)}\ \overline{h_2(\xi\Delta t)} = \delta(\sigma\Delta t)\, \sigma(\xi\Delta t)$$

and the triple summation (Equation 5-1) reduces to

$$\Phi_{12}'(m) = \sum_n f_1(n\Delta t)\, f_2(n\Delta t + m) = \Phi_{12}(m)$$

Thus, the crosscorrelations are unchanged by the addition of independent stochastic filters at each seismometer.

The autocorrelation is

$$\Phi_{11}'(m) = \overline{f_1'(t)\, f_1'(t+m)}$$

and similarly can be reduced to the form

$$\Phi_{11}'(m) = \sum_n \sum_\xi \sum_\sigma f_1(n\Delta t - \xi\Delta t)\, f_1(n\Delta t - \sigma\Delta t + m)\, \overline{h_1(\xi\Delta t)\, h_1(\sigma\Delta t)}$$

Now the filter weights are independent with mean $\delta(t)$; therefore,

$$\overline{h_1(\xi\Delta t)\, h_1(\sigma\Delta t)} = \begin{bmatrix} \lambda_{-L} & \cdot & \cdot & \cdot & 0 \\ & \lambda_{-1} & & & \\ \cdot & & 1+\lambda_0 & & \\ \cdot & & & \lambda_1 & \\ \cdot & & & & \ddots \\ 0 & & & & \lambda_L \end{bmatrix} = \delta_{\sigma\xi}\, \Lambda_\sigma \qquad (5\text{-}2)$$

where

$\delta_{\sigma\xi}$ is Kronecker delta

Λ_σ is σ^{th} diagonal element of the matrix in Equation 5-2

The autocorrelation then reduces to

$$\Phi_{11}'(m) = \sum_n \sum_{\sigma=-L}^{L} f_1(n\Delta t - \sigma\Delta t)\, f_2(n\Delta t - \sigma\Delta t + m)\, \Lambda_\sigma = \sum_{\sigma=-L}^{L} \Lambda_\sigma \sum_n f_1(n\Delta t)\, f_2(n\Delta t + m)$$

which, if end effects are ignored, reduces to

$$\Phi_{11}'(m) = \left[\sum_{\sigma} \Lambda_{\sigma}\right] \Phi_{11}(m)$$

Thus, all lags of the original autocorrelation are scaled by the multiplicative constant

$$\sum_{\sigma=-L}^{L} \Lambda_{\sigma}$$

which is equivalent to 1 plus the sum of the variances assigned to the random sample weights. This result is the exact extension of that obtained for the 1-point stochastic filter, where all lags of the autocorrelation were scaled by 1 plus the variance of the random sample weight.

APPENDIX A

DESCRIPTION OF PROGRAM TO COMPUTE MICROSTATIC CORRECTIONS ON SIGNAL TRACES

This program is written so that the signal traces can be aligned to a fraction of a sample, which is necessary in order to use the difference between the reference and signal traces as a method of evaluating the effect of equalization. The program is divided into three steps.

(1) The traces are statically aligned to the nearest integer sample

(2) The best estimate of the microstatic shift α is determined

(3) The interpolation filter which applies the shift α is designed and applied to the traces

The maximum lag of the signal-reference crosscorrelation is used to align the signals to the nearest integer sample.

To estimate α, treat the crosscorrelation function between the signal and reference traces φ_{sT} as an ordinary time trace. Then find the Wiener filter which estimates the crosscorrelation at $\tau = (n+1/2)\,\Delta t$. That is, solve

$$[\varphi][f] = [c]$$

where

φ is autocorrelation of the crosscorrelation

f is interpolation filter

c is inverse transform of the "spectrum" of the crosscorrelation at $\tau = (n+1/2)\,\Delta t$, $n = 0, \pm 1 \ldots$

It can be shown that φ can be obtained by convolving the autocorrelation of the signal and target traces; i. e.,

$$\varphi = [X(t)\ \varphi\ T(t)]\ \varphi\ [X(t)\ \varphi\ T(t)]$$

$$= [X(t)\ \varphi\ X(t)] * [T(t)\ \varphi\ T(t)]$$

where

- $X(t)$ is signal trace
- $T(t)$ is target trace
- φ is correlation
- $*$ is convolution

The interpolation filter then is applied to φ_{sT}, and the value of the maximum crosscorrelation lag $\varphi\left(\pm\frac{\Delta t}{2}\right)$ is compared with $\varphi(0)$. Assume $\varphi\left(\frac{\Delta t}{2}\right)$ is larger than $\varphi\left(-\frac{\Delta t}{2}\right)$. Then, if the inequality

$$\frac{\left|\varphi(0) - \varphi\left(\frac{1}{2}\Delta t\right)\right|}{\max\left[\varphi(0),\ \varphi\left(\frac{1}{2}\Delta t\right)\right]} < 10^{-7}$$

is satisfied, α is chosen as either 0 or $\Delta t/2$, whichever has the largest crosscorrelation value. Otherwise, the process is repeated until the inequality is satisfied or until 15 iterations are completed (usually six or seven iterations are sufficient).

Having chosen α, we proceed to the third step, which is to compute the interpolation filter for the signal trace. This is achieved by solving

$$[\varphi_{xx}][\Gamma] = [K]$$

where

φ_{xx} is autocorrelation of X(t)

Γ is interpolation filter

K is inverse transform of the spectrum X(t) at $\tau = n\Delta t + \alpha$, $n = 0, \pm 1 \ldots$

This is exactly the same equation as in the second step, except here we are dealing with the signal trace instead of the crosscorrelation between the signal and target traces. The filter Γ then is applied to X(t) to microstatically align it with the reference trace.

APPENDIX B

PROBABILITY DISTRIBUTION OF THE RATIO OF INDEPENDENT, IDENTICALLY DISTRIBUTED, UNIFORM, RANDOM VARIABLES

APPENDIX B

PROBABILITY DISTRIBUTION
OF THE RATIO OF INDEPENDENT,
IDENTICALLY DISTRIBUTED, UNIFORM,
RANDOM VARIABLES

Suppose X and Y are independent, identically distributed, random variables with probability density function (PDF)

$$f(x) = f(y) = \frac{1}{b-a} \qquad \begin{matrix} a < x < b \\ a < y < b \end{matrix}$$

$$= 0 \qquad \text{elsewhere}$$

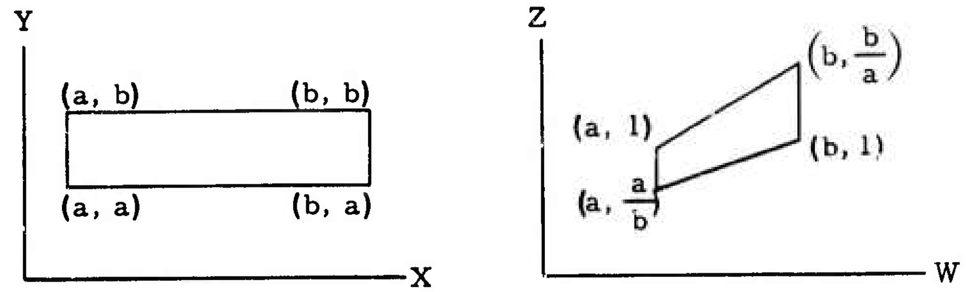
We want to find the PDF of Z = X/Y. Let W = X and Z = X/Y. Solving for X and Y in terms of W and Z gives X = W and Y = W/Z. Then,

$$[J] = \begin{bmatrix} \frac{\partial X}{\partial W} & \frac{\partial X}{\partial Z} \\ \frac{\partial Y}{\partial W} & \frac{\partial Y}{\partial Z} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & -\frac{W}{Z} \end{bmatrix} = \frac{W}{Z^2}$$

Therefore,

$$f(w, z) = \frac{1}{(b-a)^2} = \frac{w}{z^2}$$

The mapping from the XY to WZ plane is as follows.



To find f(z), integrate out w in f(w, z) as follows:

$$f(z) = \frac{1}{(b-a)^2 z^2} \int_w w\,dw$$

$$= \frac{1}{(b-a)^2 z^2} \int_{w=a}^{w=bz} w\,dw \qquad \frac{a}{b} < z < 1$$

$$= \frac{1}{(b-a)^2 z^2} \int_{w=az}^{b} w\,dw \qquad 1 < z < \frac{b}{a}$$

$$\therefore\ f(z) = \frac{1}{2(b-a)^2 z^2} \left[(bz)^2 - a^2\right] \qquad \frac{a}{b} < z < 1$$

$$= \frac{1}{2}\left[\frac{b^2}{(b-a)^2} - \frac{a^2}{(b-a)^2 z^2}\right]$$

Similarly,

$$f(z) = \frac{1}{2}\left[-\frac{b^2}{(b-a)^2 z^2} - \frac{a^2}{(b-a)^2}\right] \qquad 1 < z < \frac{b}{a}$$

Therefore, the PDF of z is

$$f(z) = \frac{1}{2}\left[\frac{b^2}{(b-a)^2} - \frac{a^2}{(b-a)^2 z^2}\right] \qquad \frac{a}{b} < z < 1$$

$$= \frac{1}{2}\left[\frac{b^2}{(b-a)^2 z^2} - \frac{a^2}{(b-a)^2}\right] \qquad 1 < z < \frac{b}{a}$$

$$= 0 \qquad \text{elsewhere}$$

Check to determine that PDF integrates to 1.

$$\int_z f(z)\,dz = \frac{1}{2}\left\{\int_{z=a/b}^{1} \frac{b^2}{(b-a)^2}\,dz - \int_{z=a/b}^{1} \frac{a^2}{(b-a)^2 z^2}\,dz\right.$$

$$\left. + \int_{z=1}^{b/a} \frac{b^2}{(b-a)^2 z^2}\,dz - \int_{z=1}^{b/a} \frac{a^2}{(b-a)^2}\,dz\right\}$$

$$= \frac{1}{2}\left[\frac{b}{b-a} - \frac{a}{b-a} + \frac{b}{b-a} - \frac{a}{b-a}\right]$$

$$= \frac{1}{2}\left[\frac{b-a}{b-a} + \frac{b-a}{b-a}\right] = \frac{1}{2} \cdot 2 = 1$$

Find the mean of Z.

$$E\left[Z\right] = \int_{z} z\,f(z)\,dz$$

$$= \frac{1}{2}\left\{\int_{a/b}^{1}\left[\frac{b^2 z}{(b-a)^2} - \frac{a^2}{(b-a)^2 z}\right]dz\right.$$

$$\left. + \int_{1}^{b/a}\left[\frac{b^2}{(b-a)^2 z} - \frac{a^2 z}{(b-a)^2}\right]dz\right\}$$

$$= \frac{1}{2}\left\{\frac{b^2}{(b-a)^2}\cdot\frac{z^2}{2}\Bigg|_{a/b}^{1} - \frac{a^2}{(b-a)^2}\log z\Bigg|_{a/b}^{1}\right.$$

$$\left. + \frac{b^2}{(b-a)^2}\log z\Bigg|_{1}^{b/a} - \frac{a^2}{(b-a)^2}\frac{z^2}{2}\Bigg|_{1}^{b/a}\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{2} + \frac{a^2}{(b-a)^2}\log\left(\frac{a}{b}\right) + \frac{b^2}{(b-a)^2}\log\left(\frac{b}{a}\right) - \frac{1}{2}\right\}$$

$$= \frac{1}{2(b-a)^2}\left[a^2\log\left(\frac{a}{b}\right) + b^2\log\left(\frac{b}{a}\right)\right]$$

$$= \frac{1}{2(b-a)^2}\left(b^2 - a^2\right)\left[\log b - \log a\right]$$

$$= \frac{(b+a)}{2(b-a)}\log\frac{b}{a}$$

APPENDIX C

EXPECTED VALUE OF THE SQUARE ROOT OF F

APPENDIX C

EXPECTED VALUE OF THE SQUARE ROOT OF F

We want to find the expected value of $F^{1/2}$, where F is F-distributed with n degrees of freedom for both numerator and denominator. For simplicity, assume that n is even (as it was for all computations). The PDF of F is

$$f(F) = \frac{(n-1)!}{\left\{\left(\frac{n}{2}-1\right)!\right\}^2} \frac{F^{\frac{n-2}{2}}}{(1+F)^n}$$

$$E\left[F^{1/2}\right] = \frac{(n-1)!}{\left\{\left(\frac{n}{2}-1\right)!\right\}^2} \int_0^\infty \frac{F^{\frac{n-1}{2}}}{(1+F)^n} dF$$

Let

$$F = \left(\frac{n+1}{n-1}\right) t$$

$$dF = \left(\frac{n+1}{n-1}\right) dt$$

Then,

$$E\left[F^{1/2}\right] = \frac{(n-1)!}{\left\{\left(\frac{n}{2}-1\right)!\right\}^2} \int_0^\infty \frac{\left(\frac{n+1}{n-1} t\right)^{\frac{n-1}{2}}}{\left[1+\frac{n+1}{n-1} t\right]^n} \left(\frac{n+1}{n-1}\right) dt$$

$$= \frac{(n-1)!}{\left\{\left(\frac{n}{2}-1\right)!\right\}^2} \left(\frac{n+1}{n-1}\right)^{\frac{n+1}{2}} \int_0^\infty \frac{t^{\frac{n-1}{2}}}{\left[1+\frac{n+1}{n-1}t\right]^n} dt$$

$$= \frac{\left(\frac{n-1}{2}\right)!\left(\frac{n-3}{2}\right)!}{\left\{\left(\frac{n-2}{2}\right)!\right\}^2} \left\{ \frac{(n-1)!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-3}{2}\right)!} \left(\frac{n+1}{n-1}\right)^{\frac{n+1}{2}} \int_0^\infty \frac{t^{\frac{n-1}{2}}}{\left[1+\frac{n+1}{n-1}t\right]^n} dt \right\}$$

The expression in braces is the integral of an F-distribution with n + 1, n - 1 degrees of freedom over the whole range of the random variable and must be unity.

$$\therefore \; E\left[F^{1/2}\right] = \frac{\left(\frac{n-1}{2}\right)!\left(\frac{n-3}{2}\right)!}{\left\{\left(\frac{n-2}{2}\right)!\right\}^2}$$

Using Stirling's approximation for n!,

$$E\left[F^{1/2}\right] = \frac{e^{-\left(\frac{n-1}{2}\right)}\left(\frac{n-1}{2}\right)^{\frac{n}{2}} e^{-\left(\frac{n-3}{2}\right)}\left(\frac{n-3}{2}\right)^{\frac{n}{2}-1}}{e^{-(n-2)}\left(\frac{n-2}{2}\right)^{2\left(\frac{n}{2}-\frac{1}{2}\right)}} = \frac{\left(\frac{n-1}{2}\right)^{\frac{n}{2}}\left(\frac{n-3}{2}\right)^{\frac{n}{2}-1}}{\left(\frac{n-2}{2}\right)^{n-1}}$$

As n becomes large,

$$E\left[F^{1/2}\right] \approx \frac{\left(\frac{n}{2}\right)^{n-1}}{\left(\frac{n}{2}\right)^{n-1}} \approx 1$$

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13. ABSTRACT

~~This~~ The report reviews the following four tasks pertaining to response equalization problems: evaluating a new technique using large signals for equalizing seismometers; examining the concept of designing regional equalization filters for subarray outputs; analyzing statistically the coefficients used to equalize the noise data; and developing the theory to incorporate statistical phase fluctuations in the correlation statistics.

The technique using large signals to equalize seismometers is based on finding the minimum-phase filters which equalize the signal power spectra for a set of seismometers. Four sets of filters are designed to equalize eight channels to the reference channel. The reference channel is subtracted from the other channels before and after equalization and the error traces compared. The equalized signals are significantly more similar for the first few cycles of the arrival. After the first few cycles, there is little difference between the two sets of signals – probably because of interfering scattered energy.

Average Levinson filters are designed for three subarray outputs, using three events from the Aleutian Islands region. Then the filters are applied to seven events from this region, and results are compared with those for both amplitude equalization and individual Levinson equalization. Regional equalization filtering is found to be possible – but only if the epicentral region is very small.

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