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Low Cycle Fatigue Behavior Under Biaxial Strain Distribution

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Hamilton Standard Division of United Aircraft Corporation

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"LOW CYCLE FATIGUE BEHAVIOR UNDER BIAXIAL STRAIN DISTRIBUTION"

Contract No. DA-31-124-ARO-D-274

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Joseph L. Mattavi

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-		NOMENCLATURE	
	E	conventional strain	
	Ē	equivalent strain	
	σ	conventional stress	
	$\overline{\sigma}$	equivalent stress	
	a,V	exponent expressing plastic stra	in-life dependency
	b	exponent expressing elastic stra	ur-life dependency
	n	number of cycles accumulated a	t a given cyclic strain
		range	
	N	number of cycles to fracture at	a given cyclic strain
		range	
	η	station designation in finite diffe	erence solution
	CeCn	constants	
	h	thickness of station	
	ц	poisson's ratio	
	3	modulus of elasticity	
	0	mass density	
	P	angular velocity	
	; ;	nositive integers	
	Subscripts	POPTATO THOPPER	
	<u>Subscripts</u>	olostio	
	E	Clabuc	
	e	elastic range	

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NOMENCLATURE (Cont.)

f	fracture	
m	mean	
p	plastic	
r	radial	
т	total	
Z	transverse	
ult	ultimate	
θ	circumferential	

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ABSTRACT

This paper presents a linear cumulative damage theory, macroscopically considering elastic and plastic straining, and accounting for biaxiality through the von Mises theory for combined strains. Experimentation on SAE 4340 steel alloy supported this damage hypothesis and confirmed the associated exponential relationship between elastic strains with cyclic life and plastic strains with cyclic life. A limited evaluation of an aluminum casting material, C-355-T61, also supported this exponential dependency. Notching did not significantly alter this low cycle fatigue behavior.

INTRODUCTION

Fracture investigation of structural members exposed to moderate-toheavy loading results in the prominence of one fracture mechanism - fatigue. In view of the multi-variable dependency of this mechanism, preventive design has largely relied upon laboratory testing, supplemented by sound statistical interpretation of the results. This technique is still most often employed today in the high anticipated number of stress cycles spectra of the classic stress-number of cycles fatigue curve. There has evolved, however, a technique for analytically predicting material fatigue behavior in the low-cycle, high-load portion of the fatigue curve. Manson [1, 2] was responsible for establishing the relationship between cycle strain and cycles-to-fracture. This is stated mathematically in the following form.

$$\epsilon_{\rm p} N^{\nu} = {\rm Constant},$$
 (1)

where ϵ_p represents plastic strain range, N indicates cycles to fracture, and ν signifies exponential dependency. Coffin [3] experimentally determined this exponent to have a value of 1/2 and applied the boundary condition that $\epsilon_p = 2 \epsilon_f$ at N = 1/4 where ϵ_f represents strain at fracture or material ductility to obtain C = ϵ_f . Although Coffin's initial work was conducted with copper, Coffin and Tavernelli [4, 5] demonstrated this exponential behavior, applied to an assortment of other metals including steel and aluminum alloys. This work supported the exponent value of 1/2 in the majority of cases. Gross [6] stated that cyclic total strain, rather

INTRODUCTION (Cont.)

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than cyclic plastic strain, is the significant value that yields the same relationship for many different materials when describing low cycle fatigue behavior. Gross's polymaterial power exponent was approximately 1/3.

The test limitations of the cited works are twofold. First, the work was restricted to uniaxial loading, and second, the loading mode was completely reversed. Ives, Kooistra, and Tucker [7] have conducted low cycle biaxial fatigue tests on several pressure vessel steels but have not included effects of mean strain. There have been several papers [8, 9, 10, 11] written on uniaxial low cycle fatigue considering the effects of mean strain. These included several aluminum alloys and a pressure vessel steel. A requisite to properly evaluating the effect of mean strain is the establishment of a cumulative damage theory. Ohji, Miller, and Marin [11] explicitly proposed and experimentally supported a linear damage theory based on strain that is equivalent to Miner's classical theory [12]. Additional uniaxial cumulative damage investigations conducted in the low cycle fatigue regime are contained in references [13, 14, 15, 16]. Miner's theory applied to these studies resulted in fairly satisfactory results. All of the experimental activity conducted with regard to evaluating cumulative damage, however, was of a completely reversed straining nature, and, thus, excluded the contribution of mean strain.

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INTRODUCTION (Cont.)

This present study's purpose was to evaluate low cycle fatigue behavior under a biaxial strain distribution with particular emphasis on mean strain and cumulative damage considerations. Additionally, it was considered highly desirable to select a loading mode that simulated loading of structural members more closely than previous specimen testing. The completely reversed strain cycling cited earlier requires reversal of the load vector. Loading of this nature is generally associated with vibratory modes, which implies high frequency of application or high cycle fatigue. The loading mode utilized in this study is representative of pressure loading and centrifugal loading and is particularly applicable to pressure vessels and rotating members.

THEORETICAL CONSIDERATIONS

Equation (1) can be rewritten as:

$$\epsilon_{\rm p}^{\rm a} {\rm N} = {\rm C}_{\rm p}; {\rm a} = 1/\nu \qquad (2)$$

Although the majority of work has been conducted with emphasis on the plastic strain to life relationship, Manson [17] has shown that this same exponential relationship governs the elastic strain to life dependency, i.e.,

$$\epsilon_{\rm e}^{\rm b}{\rm N} = {\rm C}_{\rm e} \tag{3}$$

where ϵ_e denotes elastic strain range and b the elastic exponent. Smith, Hirshberg, and Manson [18] proposed the following boundary condition based on 11 different alloys, including steel, aluminum, titanium, and

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THEORETICAL CONSIDERATIONS (Cont.)

nickel, to obtain the constant Ce.

$$\epsilon_e = 0.9 \sigma_{ult} / E$$
 when N=10⁵ (4)

which results in

$$C_e = (0.9\sigma_{ult}/E)^b 10^5$$
 (5)

A linear damage theory is proposed to include the elastic case by considering total damage per cycle as being composed of damage attributed to cyclic plastic strain and damage attributed to cyclic elastic strain. Damage per cycle is represented as ϵ_p^a and ϵ_e^b for the plastic and elastic cases, respectively. Total damage is expressed mathematically as

$$D(N_1, N_2) = \epsilon_p^a N_1 + \epsilon_e^b N_2$$
 (6)

where N_1 represents number of plastic cycles, N_2 number of elastic cycles.

Employing Coffin's interpretation of the strain at fracture ϵ_{f} to represent half the strain range of a quarter cycle fatigue excursion and equation (5),

$$C_{p} + C_{e} = (2\epsilon_{f})^{a}/4 + (0.9\sigma_{ult}/E)^{b}10^{5}$$
 (7)

Fracture will be proposed to occur when D $(N_1, N_2) = C_p + C_e$ (8)

or
$$\epsilon_{\rm p}^{\rm R}N_1 + \epsilon_{\rm e}^{\rm b}N_2 = (2\epsilon_{\rm f})^{\rm a}/4 + (0.9\sigma_{\rm ult}/{\rm E})^{\rm b}10^{\rm 5}$$
 (9)

The constant C_e is many orders of magnitude smaller than C_p and can thus be neglected. This can best be illustrated by considering the exponent

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THEORETICAL CONSIDERATIONS (Cont.)

values for SAE 4340 steel reported in reference [18]; a = 1.7, b = 10. Considering an ultimate tensile strength of 150,000 psi and ductility of 20%, $C_p = (2(0.2))^{1.7}/4 = 0.0525$, and $C_e = (0.9 (150000)/30(10^6))^{10} 10^5 = 0.34 \times 10^{-18}$. For similar reasons, if N₁ is within several orders of magnitude comparable to N₂, and ϵ_p is comparable to ϵ_e , the contribution of damage by elastic straining is negligible. This is generally the case when strain cycling plastically, so that equation (9) reduces to

$$\epsilon_{\rm p}^{\rm a} \,{\rm N} = \left(2\epsilon_{\rm f}\right)^{\rm a}/4 \tag{10}$$

There are several techniques proposed to handle the effect of mean strain. Ohji, et al. [11], as a result of a linear damage theory expressed by the following equation,

$$\sum_{i=1}^{k} \epsilon_{p}^{a} N = (2 \epsilon_{f})^{a}/4$$
(11)

proposes equation (10) take the following form as a result of the mean strain $\epsilon_{\rm m}$:

$$\epsilon_{\rm p}^{\rm a} \,\mathrm{N} = \left(2\,\epsilon_{\rm f}\right)^{\rm a}/4 - \left(2\,\epsilon_{\rm m}\right)^{\rm a}/4 \tag{12}$$

Sachs, et al. [10] suggests the equation for mean strain is

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THEORETICAL CONSIDERATIONS (Cont.)

$$\epsilon_{\rm p} N = \epsilon_{\rm f} - \epsilon_{\rm m} \tag{13}$$

This equation assumes a value of a = 2. Re-arranging equation (13) and defining the exponent in the general form, a, results in

$$\boldsymbol{\epsilon}_{\mathrm{p}}^{\mathrm{a}}\mathrm{N} = (2 \ \boldsymbol{\epsilon}_{\mathrm{f}} - 2 \ \boldsymbol{\epsilon}_{\mathrm{m}})^{\mathrm{a}}/4 \tag{14}$$

This expression mathematically implies that damage accumulates nonlinearly as expressed by the following equation

$$\sum_{i=1}^{k} \epsilon_{pN}^{1/a} = (2\epsilon_{f} - 2\epsilon_{m})/4^{1/a}$$
(15)

Manson [19] proposes the same expression as Sachs; however, he supports a linear-cumulative damage theory. Although there is a mathematical conflict indicated, physical interpretation of a shift of the stress-strain curve in a strain direction supports this latter hypothesis. This author subscribes to this hypothesis. Application to equation (5) (with the additional stipulation of no plastic strain cycling, which will be shown later as the case for the particular load mode selected for this study) results in the following expression.

$$\epsilon_{\rm e}^{\rm b}$$
 N = $(2 \epsilon_{\rm f} - 2 \epsilon_{\rm m})^{\rm a}/4$ (16)

For the case of several cyclic strain ranges about the same mean strain

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THEORETICAL CONSIDERATIONS (Cont.)

value, ϵm , this becomes

$$\sum_{i=1}^{k} \epsilon_{e_i}^{b} N_i = (2\epsilon_f - 2\epsilon_m)^{a}/4$$
(17)

Defining Ni as the cyclic life at a cyclic strain range of ϵ_{ei} , equation (17) reduces to Miner's linear cumulative damage expression

$$\sum_{i=1}^{k} \eta_{i} / N_{i} = 1.0$$
(18)

For the case of several different mean strain values, $\epsilon_m(1) \epsilon_m(2) \ldots \epsilon_{mj}$, N_j will be defined as the cyclic life at a cyclic strain range of ϵ_{ei} imposed on a mean strain value of ϵ_{mj} . For this case, fracture will assume to occur when

$$\sum_{i=1}^{K} \sum_{j=1}^{I} \eta_{ij} / N_{ij} = 1.0$$
(19)

Biaxiality effects are accounted for by the effective plastic strain quantity, $\bar{\epsilon}_{p}$, first proposed by Dorn and Thompson [20] based on the von Mises (or Octahedrial Shear Stress) theory.

$$\bar{\epsilon}_{p} = (2)^{1/2} \left((\epsilon_{p_{1}} - \epsilon_{p_{2}})^{2} + (\epsilon_{p_{3}} - \epsilon_{p_{1}})^{2} \right)^{1/2} / 3$$
(20)

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THEORETICAL CONSIDERATIONS (Cont.)

Equivalent stress is defined as

$$\overline{\sigma} = \left((\sigma_1 - \sigma_2)^2 + \sigma_2^2 + \sigma_1^2 \right)^{1/2} / (2)^2; \ \sigma_3 = 0$$
(21)

where $\epsilon_{p...\epsilon_p3}$, and σ_1, σ_2 are principal strains and stresses.

EXPERIMENTAL PHASE

The primary material selected for evaluation was SAE 4340 steel in the Rc 28-32 hardness range. This intermediate hardness was chosen as representing the asympotic value to which hardened SAE 4340 steel strain softens, and annealed SAE 4340 steel strain hardens, as reported in reference [17]. There was some limited additional work conducted with C355, a high precision cast aluminum alloy exhibiting a hardness of 80 BHN. The chemical composition of these alloys is contained in tables 1 and 2. The mechanical properties determined from 0.250-dia., 1.25-inch gage length tensile specimens are contained in tables 3 and 4 for the steel and aluminum alloys, respectively. Although the hardness range specified for the steel alloy was maintained, differences in the mechanical properties (and more critical, stress-strain relationships) warranted classification into three groups which are designated A, B, and C for this report.

The test vehicle employed in this study was a rotating disk which will be referred to as a rotor. Figure 1 is a photograph of the rotor, while figure 2 is a schematic presentation of the rotor. The rotors exhibited a

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EXPERIMENTAL PHASE (Cont.)

6.3-inch OD and tapered hub with a test section thickness of 0.200 inch at a 2.15-inch radius. Notched rotors were also tested and displayed the same configuration with the addition of a 0.030 full radius circumferential notch machined on both sides of the rotor at the test section. The aluminum rotors were identical to the steel rotors in the disk area but contained 60 dummy blades reflected against 18 for the steel rotors.

The rotors were speed cycled in the spin pit facility shown in figure 3. This facility is composed of a pneumatically-driven three-inch OD drive turbine, a six-foot-diameter armour plated containment housing and associated oil mist bearing lubrication system, a vacuum pump for evacuating the pit during testing, and a control console to monitor speed cycling. Photographic equipment for permanent recording of burst modes is also available. Figure 4 is a photograph of one of the steel rotors taken at the instant of burst (55,800 rpm). Strain was related to speed analytically, employing the methodology and analyses developed by Manson [21].

A summary of the pertinent formulation is reproduced in the appendix to provide an understanding of the basic approach. The technique incorporates the experimental determination of stress-to-strain relationships in the plastic range for the subject materials. This was determined from 12 SAE 4340 specimens and six C355 0.250-dia., 1.25-inch gage length,

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EXPERIMENTAL PHASE (Cont.)

specimens that were removed from the hub sections of 18 rotors following fracture. The method presented in the appendix is based on von Mises' theory, which is sometimes referred to as the Deformation Theory, and is valid only if the loading path is such that the ratio of principal strain increments is constant throughout the straining period. This assumption appears to be sound, based on measurements taken by Waldren and Ward [22] on rotating disks at intervals in the loading cycle and the degree of success of Waldren, Percy and Melloc [23] in predicting disk burst speeds utilizing the theory. Care must be exercised, however, in the event the load is reversed, as the original stress-strain path will not be retraced. This is always the case when evaluating fatigue behavior of materials that have been exposed to plastic flow. This is considered in greater detail in the next topic, which presents strain-to-speed response for a rotor.

LOADING MODE STRAIN RESPONSE

Figure 5 is a schematic presentation of the strain response of the test rotor to speed cycling. The stress and strains can be considered equivalent values. It is recognized that the form of the equivalent stress and strain relationships preclude the possibility of negative values; nevertheless, equivalent residual stresses are plotted as compressive, when both radial and tangential components are negative for physical comprehension purposes.

LOADING MODE STRAIN RESPONSE (Cont.)

The plot depicts equivalent, total, plastic and elastic strains, and equivalent stress. Equivalent stress and equivalent plastic strain are direct consequences of the Deformation Theory and are expressed by

$$\overline{\sigma} = (\sigma_{\rm r}^2 + \sigma_{\theta}^2 - \sigma_{\rm r} \sigma_{\theta})^{1/2}$$
(22)

$$\overline{\epsilon}_{p} = 2(\epsilon_{rp}^{2} + \epsilon_{\theta p} + \epsilon_{rp} \epsilon_{\theta p})^{1/2} / (3)^{1/2}$$
(23)

Total equivalent strain is defined through analogy by Manson [17] as

$$\boldsymbol{\epsilon}_{\mathrm{T}} = 2 \left(\boldsymbol{\epsilon}_{\mathrm{T}}^{2} + \boldsymbol{\epsilon}_{\boldsymbol{\theta}}^{2} + \boldsymbol{\epsilon}_{\mathrm{T}}^{2} \boldsymbol{\epsilon}_{\boldsymbol{\theta}}^{2} \right)^{1/2} / (3)^{1/2}$$
(24)

The expression resulting for equivalent elastic strain becomes

$$\overline{\epsilon}_{e} = 2(1+\mu) \,\overline{\sigma}/3E \tag{25}$$

As the rotor is brought from zero speed to a maximum value, a strain excursion along line 0-0'-1 is generated. The respective strain excursion upon returning to zero speed is depicted as line 1-2. Point 2 represents the residual stress and stroin incurred during this first speed cycle. The residual stresses and strains are readily obtained analytically, employing an elastic solution where initial strains are those values existing at point 1. The yield point is noted to have increased to a minimum value equal to that which occurred at maximum speed from the previous loading. This

LOADING MODE STRAIN RESPONSE (Cont.)

behavior is the so-called "Bauschinger Effect" and was supported for the two materials under investigation by examining strain-to-load response when cycling several uniaxial tensile specimens removed from fractured test rotors. Upon re-loading, the strain response is indicated by line 2-3 for the same maximum speed. The cyclic strain range is shown in the figure as ϵ_{e} and is equal to 2 (1 + μ) $\Delta \sigma/3E$ where

$$\Delta \overline{\sigma} = \left((\Delta \sigma_{\mathbf{r}})^{2} + (\Delta \sigma \theta)^{2} - (\Delta \sigma_{\mathbf{r}}) (\Delta \sigma \theta) \right)^{1/2}$$
(26)

It should be emphasized that this model represents the strain response due to speed, i.e., speed is the independent or test input variable and strain the resultant or dependent value. The majority of work done previously has been strain-monitored in that the test specimen was physically forced to cycle over a constant strain interval. This resulted in a varying load input; increasing for strain-hardening materials and decreasing for strain-softening materials. As most structures are low-cycle fatigue-tested through load excursions, it was elected to monitor the testing reported here through centrifugal load or speed. Speed cycling of rotors is seen to reduce to elastically cycling a material that has experienced a reduction in ductility. Equations (16) and (17) proposed to define life under conditions of single and multiple strain ranges are rewritten as follows to include biaxial effects.

$$\overline{\epsilon}_{e}^{b} N = (2 \ \overline{\epsilon}_{f} - 2 \ \overline{\epsilon}_{m})^{a}/4$$
(27)



28)

LOADING MODE STRAIN RESPONSE (Cont.)

$$\sum_{i=1}^{K} \overline{\epsilon}_{e_{i}}^{b} \eta_{i} = (2\overline{\epsilon}_{i} - 2\overline{\epsilon}_{m})^{a}/4 \qquad (2\overline{\epsilon}_{i} - 2\overline{\epsilon}_{m})^{a}/4$$

where $\overline{\epsilon}_{e}$ is designated cyclic elastic equivalent strain and defined by equation (25), and $\overline{\epsilon}_{f}$ and $\overline{\epsilon}_{m}$ are designated total equivalent strain values expressed by equations (23) and (24), respectively.

TEST LOADING MODES

There were three test phases employed in this study that can be classified according to the associated loading mode. Defining load ratio as $R_n = N_{min}/N_{max}$ where N_{min} and N_{max} represent minimum and maximum speeds of a cyclic speed range, the first phase consisted of running in an $R_n = 0$ mode. Physically, this represents stop-start operation of a rotative structure. The second phase consisted of running in an $R_n = R$ mode where 0 < R < 1.0. This represents speed excursions about a normal operating value. The third phase consisted of speed cycling at various successive R_n modes to evaluate cumulative damage effects. Figure 6 is a schematic plot of strain response associated with these loading modes. SAE 4340 material classified into groups A, B, and C in table 3, respectively, were evaluated in the three test phases; C355 was tested in the first phase, R = 0, only.

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TEST RESULTS

Figure 7 contains true stress-true strain diagrams for the two materials and represents the average values obtained from a minimum sample size of three specimens removed from rotors per group. Figures 8 and 9 present the cyclic-elastic strain distribution and plastic strain distribution, respectively, for the polished rotor. Figures 10 and 11 depict the counterparts for the notched rotor. The biaxial strain ratios for the cyclic strains at the test sections of the rotors are 0.42 and 0.32 for the polished and notched rotors, respectively. It should be noted that the ratios for the plastic or mean strains are less than 0.1 at the test section. The mean strains can be considered uniaxial, although biaxial effects were carried along throughout the analytical phases of this investigation.

Figure 12 shows the relationship between equivalent plastic or mean strain with speed and equivalent cyclic elastic strain with speed. These strain values are plotted in dimensionless form, referring to the corresponding values at fracture. The reference for the cyclic strain should be interpreted as the cyclic equivalent strain excursion that would have been realized upon unloading from the speed of fracture if fracture had not occurred.

TEST RESULTS (Cont.)

The test objectives were to ascertain, accounting for biaxiality through von Mises' theory of combined stresses and strains under fatigue loading,

if

- 1. an exponential relationship existed between cyclic life and cyclic strain employing this combined strain theory,
- 2. the exponent of strain-life dependency differed for elastic and plastic straining,
- 3. mean strain could be accounted for through a linear cumulative damage theory, considering the elastic and plastic straining, and if
- 4. notching appreciably affected low cycle fatigue behavior as determined from smooth or polished geometries.

Table 5 contains the results of single cyclic strain range tests for both polished and notched SAE 4340 material. The results for the C355 material are tabulated in table 6.

An examination of equation (28) reveals if mean strain is maintained constant a plot of $\overline{\epsilon}_{e}$, cyclic equivalent elastic strain, versus N (cyclic life) on a log-log coordinate system results in a straight line, the geometric slope having the value 1/b. Conversely, if cyclic strain is constant, a plot of $\epsilon_{f} - \epsilon_{m}$, which will be defined as "available strain" versus N, will also result in a straight line with a geometric slope of 1/a.

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TEST RESULTS (Cont.)

Phase I testing, $R_n = 0$, resulted in essentially constant cyclic strain testing about different mean strain values. Thus, it can be considered to represent the cited converse case. Phase II testing, $R_n = R$, represents the former case when mean strain was maintained constant. As the testing evaluated this behavior at several mean strain values, data of constant cyclic strain nature was also obtained. Figure 13 is a plot of cyclic equivalent strain versus life for polished and notched SAE 4340. The corresponding exponents, "b", are tabulated in table 7. Figure 14 and figure 15 are plots of $\overline{\epsilon}_{f}$ - $\overline{\epsilon}_{m}$ versus life for polished and notched SAE 4340 material, respectively. It should be noted that although the testing associated with the last two figures is considered constant cyclic strain, cyclic strain values $\overline{\epsilon}_e$ deviated from an average value up to 10%. Because small differences in $\overline{\epsilon}_e$ bring about large changes in $\overline{\epsilon}_e^b$, an iteration process based on the experimental value of "b" (previously determined) and equation (27) was conducted to correct the quantity $\overline{\epsilon}_{f}$ - $\overline{\epsilon}_{m}$ to the average cyclic strain value. Figure 16 is a plot of $\overline{\epsilon}_{f}$ - $\overline{\epsilon}_{m}$ versus N for polished and notched aluminum. The values represent uncorrected values, because only the $R_n = O$ loading mode was evaluated for this material. The resulting values of "a" are also contained in table 7.

The average value of "b" obtained for the polished SAE 4340 of 9.15 agrees fairly well with the value of 9.61 obtained by Manson [17] for hardened

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TEST RESULTS (Cont.)

SAE 4340 material. The corresponding value of "a" determined by Manson's work was 1.7 reflected against a value of 3.25 determined from this author's work. The generally accepted value for this quantity based on the results of Coffin and Tavernelli is 2.0; however, for specific material use it is recommended that this exponent be determined experimentally. Notching was seen to result in a somewhat lower value for both "a" and "b". This is consistent with the classical behavior of notched materials in which higher fatigue strengths than the polished configuration are realized in the low cycle fatigue operating regime but with greater strength-to-life gradients. The effect of notching on the aluminum alloy is seen to be more significant than on the steel, although, the average of the two configurations is approximately the same as the corresponding one for steel.

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Table 8 contains the results of the cumulative damage phase of the investigation for both polished and notched rotors. The three loading modes presented schematically in figure 6 were employed in this study. In this third mode, it is noted that the maximum cyclic strain point for the second strain range lies below the transition from elastic to plastic behavior. Considering essentially the same available strain quantity, $\epsilon_{f-} \epsilon_{m}$, to apply to both cyclic strain values as indicated in the figure, results in much greater life than that predicted by a linear damage theory. The implication is that it is not only the strain range that dictates the amount

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TEST RESULTS (Cont.)

accumulated damage, but also the maximum strain associated with each cycle. If this is less than the previous maximum value, damage rate decreases in excess of that indicated by ϵ^{b} . An equivalency that suggested itself was translating the stress-strain curve along the strain axis for this second strain range such that the maximum stress or strain point laid upon it, indicating onset of plastic flow. This would have the effect of increasing the $\epsilon_{f} - \epsilon_{m}$ value by the amount of strain associated with the translation. This technique was employed to obtain the N values shown in the table for this type of loading sequence.

The average $\sum n/N$ for the polished rotors was 0.99 with a low of 0.52 and high of 1.55. The average $\sum n/N$ for the notched rotors was 1.44 with a low of 0.78 and high of 3.06.

Figure 17 is a presentation of a fractographic investigation conducted on several of the test rotors following fracture. Figures 17a and 17b represent the C-355 material. Figures 17c and 17d represent the SAE 4340 alloy. As macroscopic evaluation of a fractured surface operating in the low cycle fatigue regime does not always result in an explicit definition of fracture mode, fractographic techniques were also employed in the posttest investigation. Hamilton Division of United AlfGRAFT CORPORATION

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TEST RESULTS (Cont.)

A two-stage platinum -carbon tape replication technique was employed. The resulting electron fractographs represent a 12, 500-power magnification. Directing attention toward the aluminum, the edge locations are seen to exhibit a relatively flat topography and contain classical fatigue striations. The centers of the fracture surfaces are rather irregular. In fact, they are non-uniformly irregular typical of the varying grain sizes associated with a casting. The fracture mode associated with this appearance is static tensile and shear overload, resulting from a loss in load-supporting area. The life relationships expressed mathematically in this study represent a damage period that includes crack initiation and propagation to the extent that load-carrying capacity is exhausted due to the loss in area. The corresponding figures for steel display the same gross topographic dissimilarities. The center sections, however, exhibit a more uniform "dimpled" appearance indicating a grain structure more typical of forgings. Two sets of markings orthogonal to each other are seen in figure 17d. The horizontal markings in this figure are fatigue striations and are normal to the advancing crack direction. The vertical markings are "river" markings associated with differences in the crystal levels with respect to the crack plane. This fractography work confirms the existence of a fatigue-initiated mode of fracture with the fracture sequence being fatigue crack initiation, fatigue crack propagation, and tensile overload.

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CONCLUSIONS

The following conclusions have been drawn, based on the results of this study primarily considering the SAE 4340 material.

- 1. Biaxial effects can be accounted for by using the von Mises theory of combined stresses and strains.
- 2. A linear damage theory spplied to both elastic and plastic equivalent strains appears reasonable, based on the accumulated damage test results varying both mean strain and cyclic strain.
- 3. An exponential relationship exists between strain and cyclic life. The exponent expressing this dependency was approximately 9.2 for elastic strain and 3.2 for plastic strain.
- Notching does not significantly alter the linear damage theory.
 The associated strain life power coefficients are reduced to 7.8 for elastic strain and 3.1 for plastic strain.
- 5. Limited testing of the cast aluminum alloy C355-T61 indicated that its behavior is similar to that of the tested steel.

The support of this study by U.S. Army Research Office-Durham is gratefully acknowledged.



FIGURE 1. SAE 4340 TEST ROTOR

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FIGURE 2. SCHEMATIC PRESENTATION OF TEST ROTOR



FIGURE 3. TEST FACILITY

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	100	

FIGURE 4. TEST ROTOR AT INSTANT OF BURST N = 55,800 RPM







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FIGURE 9. UNNOTCHED TEST ROTOR TYPICAL PLASTIC STRAIN DISTRIBUTION

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FIGURE 12. EQUIVALENT STRAIN VS SPEED AT TEST SECTION











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EDGE OF FRACTURE SURFACE



CENTER OF FRACTURE SURFACE

FIGURE 17(A). C 355 CYCLES TO FRACTURE - 15



EDGE OF FRACTURE SURFACE



CENTER OF FRACTURE SURFACE

FIGURE 17(B). C 355 CYCLES TO FRACTURE - 147



EDGE OF FRACTURE SURFACE



CENTER OF FRACTURE SURFACE

FIGURE 17(C). SAE 4340 CYCLES TO FRACTURE - 2552



EDGE OF FRACTURE SURFACE



CENTER OF FRACTURE SURFACE

FIGURE 17(D). SAE 4340. CYCLES TO FRACTURE-211

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Stell					TP-67-16	-Т
Table 1	Chemical Composit	ion of SAE 4340 Ste	el Alloy, %			
С	Mn S	i P	S	Cr	Мо	Fe
0.28-0.3	3 0.40-0.60 0.20	-0.35 0.04 Max.	0.04 Max.	0.80-1.1	0.15-0.25	Balance
Table 2	Chemical Composit	on of C355-T61 Al	uminum Allo	<u>y, %</u>		
Cu	Fe Di	Mn Mg	g Zn	Cr	Ti	A1
1.0-1.5	0.2 4.5~5.5	0.3 Max. 0.4-	0.6 0.2 M	ax. 0.08	8 0.30	Balance
Table 3	Mechanical Propert	ies of SAE 4340 Ste	el Alloy			
Group	0.2% Yield Strength,psi	Ultimate Tensil Strength, psi	e Elon, Per	gation cent	Reduction Perc	in Area ent
A	103,000	126,000	2	0	61	
В	114,000	134,000	1	6	53	}
С	132,000	147,000	1	4	45	

Table 4 Mechanical Properties of C355-T61 Aluminum Alloy

0.2% Yield Strength, psi	Ultimate Tensile Strength, psi	Elongation Percent	Reduction in Area Percent
35,000	49,000	8	10

Hamilton Standard A.

Table 5

Summary of SAE 4340 Rotor Test Results

Specimen Number	Surface Condition	Cyclic Speed Range, RPM	Cyclic Strain Range, μ"/"	Mean Strain µ"/"	Number of Cycles
801A (1)	Polished	0-55800	_	297000	1/4
802A	**	0-54000	5360	192000	5
803A		0-52000	4630	116000	739
S04A	88	0-53000	4840	148000	371
805A	**	0-53500	4940	168000	120
S0 6A	11	0-54000	5360	192000	6
807A	**	0-53000	4840	148000	95
808A		0-52000	4630	116000	827
809A		0-51000	4420	87000	1286
S10A	Notched	0-51000		428000	1/4
S11A	17	0-49000	5346	274000	37
S12A	**	0-48000	5079	198000	144
813A	**	0-47000	4810	140000	492
S14A	**	0-46000	4600	114000	714
815A		0-50000	5615	350000	8
S16A	**	0-48000	5079	198000	96
817A	28	0-49000	5346	274000	46
S18A	f 1	0-50000	5615	350000	3

(1) Indicates Group

Hamila	ard A	TP-67-16-T			
Specimen Number	Surface Condition	Cyclic Speed Range, RPM	Cyclic Strain Range µ''/''	Mean Strain µ''/''	Number of Cycles
S19B	Polished	0-56200		264000	1/4
S20B	11	32600-54200	3360	148300	1897
821B	11	31500-55100	3680	192100	39
S22B	11	0-55200	-	200000	1/2
S23B	11	42000-54100	2084	144900	184
S24B	11	44000-53000	1550	102200	10000 (2)
S25B	11	38500-54200	2600	148700	7225
826B	11	20500-53200	4500	147700	40
S27B	11	20500-53200	4200	107300	1230
S28B	11	13800-53200	4620	107700	379
829B	11	19800-54500	4620	163700	79
S30B	11	10000-55000	5280	187300	6
S31B	**	10000-55000	5280	187300	20
S32B	**	20500-54200	4500	147700	34
833B	11	10000-54200	5080	147400	50
S34B	Notched	33000-50000	3110	193400	2552
835B	11	31700-50800	3480	254200	211
S36B	11	30900-51400	3740	301100	48
S37B	88	3050051900	3910	338000	12

(2) Retired

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Specimen Number	Surface Condition	Cyclic Speed Range, RPM	Cyclic Strain Range μ "/"	Mean Strain μ"/"	Number of Cycles
5 38B	11	0-51200	-	288000	1/4
S 39B	11	31950-50350	3320	222300	1164
840B	99	20000-50500	4720	283600	136
841C	Polished	0-57800	-	204000	1/4
842C	Polished	0-57600		193000	1/4
843C		31500-56000	3620	118600	61
844C	11	0-56000	5300	117600	10
845C	99	0-54000	4840	58500	860
S46C	99	0-55000	5070	78300	181
847C		30000-55000	3570	79000	1378
S48C	**	0-55000	5070	78300	35
S 49C	**	0-56000	5300	117600	20
850C	Notched	0-52600	-	293000	1/4
851C	**	0-51000	5660	187800	46
852C	**	0-50000	5530	156400	411
853C	**	25000-51000	4300	188500	367
854C	11	25000-51000	4300	188100	74
855C	**	15000-51000	5170	187800	67
S 56 C	**	0-53300	_	304000	1/4

Hamilton U Standard A.

Table 6

TP-67-16-T

Specimen Number	Surface Condition	Cyclic Speed Range, RPM	Cyclic Strain Range µ"/"	Mean Strain μ"/"	Number o Cycles
L01	Polished	C-50500	3470	7460	90
L02	88	0-52500	3680	13560	2165
L03	11	0-56500	4120	37940	205
L04	11	0-54000	3830	19680	1106
L05	11	0-57100	4190	45100	94
L06	11	0-58600		69700	1/4
L07	11	0-56000	4050	32900	1067
L08	97	0-57500	4260	51470	26
L09	11	0-58000	4320	57840	15
L10	Notched	0-4:9000	4550	81565	1/4
L11	ŦŦ	0-59000	4250	51050	229
L12	11	0-48000	4127	37630	367
L13	**	0-50000	4370	62130	147
L14	**	0-49300	4285	52850	1/4
L15	22	0-47000	4000	28980	349
L16		0-46000	3870	21450	903

Summary of C355 Rotor Test Results

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4370

62130

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0-50000

L17

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Table 7

Exponents of Strain to Life Dependency

Elastic Strain - b Value

Material	Group	Surface Condition	Mean – Strain µ"/"	b	^b ave
SAE 4340	В	polished	147000	9.25	
	В	polished	107000	8.50	
	C	polished	79000	9.70	9.15
SAE 4340	В	notched	215000	7.80	
	С	notched	188000	7.80	7.80
		Plastic Strain -	a Value		
Material	Group	Surface Condition	Cyclic - Strain μ"/"	a	ave
SAE 4340	A	polished	4840	3.00	
	В	polished	4900	2.96	
	С	polished	5070	3.80	3.25
	A	notched	5080	2.50	
	B	notched	3480	3.80	3.10
C355		polished	3970	3.75	
		notched	4160	2.20	

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Table 8

Cumulative Damage Test Results

SAE 4340

Specimen Number	Surface Condition	Cyclic Speed Range – RPM	Cyclic Strain Range - $\mu''/''$	n	N	n/N	S n/N
S57C	polished	0-56000	5301	1	15	0.06	
		0-54200	4983	351	700	0.50	0.56
S58C	11	0-56000	5301	3	15	0.20	
3. de 18. de		0-54200	4983	551	700	0.79	0.99
S59C	11	0-56000	5301	6	15	0.40	
		0-54200	4983	159	700	0.23	0.63
S60C	11	0-56000	5301	1	15	0.06	
		0-54000	4930	324	720	0.46	0.52
S61C	**	0-55000	5072	6	181	0.03	
		0-54000	4840	700	860	0.81	
		0-55000	5072	79	181	0,43	1.27
S62C	2.5	0-56000	5301	6	15	0.40	
		0-55000	5142	51	175	0.29	0.69
S63C	8.8	0-55000	5072	51	175	0.29	
		0-56000	5301	19	15	1.26	1.55
S64C	9.9	0-56000	5301	6	15	0.40	
		20000-56000	4633	35	36	0.97	1.37
865C		0-56000	5301	3	15	0.20	
		30000-56000	3785	64	150	0.43	0.63

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Standa	and a contro	And Table	8 (continued)		TP	-67-16-'	Г
Specimen Number	Surface Condition	Cyclic Speed Range - RPM	Cyclic Strain Range - µ''/''	n	N	n/N	Σ_n/N
866C	99	0-56000	5301	8	15	0.53	
		30000-56000	3785	104	150	0.69	1.22
867C	11	0-55000	5072	101	181	0.56	
		0-54000	4869	820	860	0.95	1.51
			•		Ave.		0.99
S68C	Notched	25000-51000	4300	227	367	0.62	
		0-5 1000	5661	112	46	2.44	3.06
S69C	12	0~51000	5661	2.0	46	0.43	
		25000-51000	4300	227	367	0.62	1.05
S70C	11	0-50000	5529	261	411	0.63	
		0-51000	5661	7	46	0.15	0.78
871C	11	15000-51000	5170	30	67	0.45	
		0-51000	5661	81	46	1.76	2.21
S72C	1 1	0-50000	5529	156	411	0.38	
		0-51000	5661	27	46	0.59	0.97
\$73C	11	25000-51000	4300	100	367	0.27	
		0-51000	5661	33	46	0.72	0.99
S74C		0-51000	5661	20	46	0.43	
		0-50000	5460	261	430	0.61	1.04
					Ave		1 44

APPENDIX

Referring to figure la, when subscripts n and n-1 refer to the nth and (n-1)st station on the rotor, the compatibility and equilibrium equation for each element is

$$\epsilon_{\theta,\eta}^{-r}\eta_{-1}\epsilon_{\theta,\eta-1}^{\prime r}\eta^{-(r}\eta^{-r}\eta_{-1})(\epsilon_{r,\eta}+\epsilon_{r,\eta-1})^{\prime 2r}\eta=0 \qquad (1a)$$

$$\sigma_{\theta,\eta}^{-2r}\eta \sigma_{r,\eta} / r_{\eta}^{-r}\eta - 1 + 2h\eta_{-1} r_{\eta-1} \sigma_{r,\eta}^{-1/h}\eta (r_{\eta}^{-r}\eta_{-1}) + h\eta_{-1} \sigma_{\theta,\eta-1} / h\eta_{-1} r_{\eta-1}^{2} \eta_{-1} / h\eta_{+} r_{\eta}^{2} = 0$$
(2a)

where changes due to deformation are neglected

Strains are related to stresses considering plane stress through the following relationships

$$\epsilon_{\mathbf{r},\eta} = (\sigma_{\mathbf{r},\eta} - \mu_{\eta} \sigma_{\theta,\eta})/E_{\eta} + \overline{\epsilon}_{\mathbf{p},\eta} (\sigma_{\mathbf{r},\eta} - \sigma_{\mathbf{r},\eta}/2)/\overline{\sigma}_{\eta}$$
(3a)

$$\epsilon_{\theta,\overline{\eta}} = (\sigma_{\theta,\eta} - \mu_{\eta} \sigma_{\theta,\eta}) / E_{\eta} + \overline{\epsilon}_{p,\eta} (\sigma_{\theta,\eta} - \sigma_{r,\eta} / 2) / \overline{\sigma}_{\eta}$$
(4a)

where $\bar{\epsilon}_{p}$ and $\bar{\sigma}$ are defined by equations (20) and (21), respectively, in the test and denoted as equivalent strain and equivalent stress. The two additional relationships of the von Mises theory used to obtain equations (3a) and (4a) are

$$\epsilon_{\theta p} + \epsilon_{rp} + \epsilon_{zp} = 0 \tag{5a}$$

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APPENDIX (Cont.)

or volume changes resulting from plastic flow are zero

and

$$(\epsilon_{rp} - \epsilon_{\theta p})/(\sigma_{r} - \sigma_{\theta}) = (\epsilon_{rp} - \epsilon_{zp})/\sigma_{r} = (\epsilon_{\theta p} - \epsilon_{zp})/\sigma_{\theta}$$
 (6a)

or principal plastic shear strains are assumed proportional to principal shear stresses. For this plane stress case, equations (20) and (21) yield

$$\overline{\sigma} = (\sigma_r^2 + \sigma_\theta^2 - \sigma_r^2 \sigma_\theta)^{1/2}$$
(7a)

$$\overline{\epsilon}_{p} = (\epsilon_{rp}^{2} + \epsilon_{\theta p}^{2} + \epsilon_{rp} \epsilon_{\theta p})^{1/2} / (3)^{1/2}$$
(8a)

Equivalent stress is related to equivalent strain by means of a true stresstrue strain curve generated from a uniaxial tensile test of the subject material.

Equations (1a) through (8a) are solved using an iterative technique. The solution of any rotor station, n, in terms of values at the preceding station, n-1, are obtained. Hence, the solution for the entire rotor can be obtained in terms of an assumed quantity at the center. A comparision of the radial stress at the last station with the rim loading, which is readily calculable quantity, indicates whether or not an adjustment in the assumed quantity at the center is warranted.

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APPENDIX (Cont.)

Residual stresses and strains are determined by allowing the rotor to recover elastically from the stress and strain state at maximum speed. Compressive residual stress values must be examined to assure that they do not exceed the compressive yield point of the material. Otherwise, elastic recovery is not justified.



FIGURE 1A. ROTOR ANALYSIS COORDINATE SYSTEM

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