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Effects of Atmosphere Rotation Rate on Orbits and Orbit Determination

APRIL 1967

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Prepared for COMMANDER SPACE SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
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FOREWORD

This report is published by the Aerospace Corporation, El Segundo, California, under Air Force Contract No. AF 04(695)-1001.

This report, which documents research carried out from April 1966 to February 1967, was submitted on 1 May 1967 to G. E. Aichenger (SSYC) for review and approval.

The author expresses his appreciation for the many helpful comments and suggestions by Dr. Charles M. Price and Mr. Richard W. Bruce.

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ABSTRACT

The effect of increased atmosphere rotation rate on low perigee altitude orbits and the effect of underestimation of atmosphere rotation rate on orbit determination are displayed using simulation results. Cross-track changes in satellite position due to increased rotation rate are small. Intrack changes, though larger, are small-error sources in orbit determination and short-term prediction if a seven-parameter fit is used. The angles between satellite inertial velocity and wind vectors that produce zero tangential acceleration and maximum normal acceleration are derived for any wind and for the special case of a circular orbit in a rotating atmosphere. This analysis explains the increased decay rates of some near-polar orbits due to a rotating atmosphere and the inability to predict this effect with frequently used approximations.

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SECTION I

INTRODUCTION AND SUMMARY

The King-Hele (Reference 1) results, displaying high values of atmosphere rotation rate (1.0 to 1.6 times earth rate), prompted a simulation study to determine the effect of a large rotation rate (twice earth rate) on the prediction of satellite position for low perigee altitude orbits. Initially it was believed that large crosstrack errors in polar orbit determination and short-term prediction could result from assuming an atmosphere rotation rate smaller than the true rate.

The Aerospace Corporation orbit determination program, TRACE, was used in the two-part simulation study. In the first part, radial, intrack, and crosstrack differences were obtained by comparing pairs of orbits. Initial conditions were identical for each orbit of a pair, but one was subjected to an atmosphere rotating at twice earth rate, the other to an atmosphere rotating at earth rate. Crosstrack position changes due to a doubled rate proved to be small, even for polar orbits. However, unexpected intrack changes (increased decay rates) occurred for polar orbits.

The second part of the study showed that intrack perturbations due to a doubled atmosphere rotation rate are largely predictable if a seven-parameter fit is used in orbit determination with the ballistic coefficient $W/(C_D A)$ as the seventh parameter. For each orbit, simulated observation data were obtained for the orbit in an atmosphere rotating at twice earth rate. A seven-parameter fit to twelve hours of simulated tracking data was then obtained. For this fit and for the subsequent prediction, the atmosphere was assumed to rotate at earth rate. Using the resulting parameter values, position was then predicted up to twelve hours beyond the fit span. Radial, intrack, and crosstrack differences were obtained by comparing the predicted positions with those of the true orbit. All resulting differences were small.

In the simulations, substantial increases in the decay rates of polar orbits were not expected from commonly used approximations. To explain this, the angle between satellite inertial velocity and wind vectors that produces zero tangential acceleration due to wind is derived. It is slightly less than 90 deg; smaller angles decrease decay rates and larger angles increase them. The angle for maximum normal acceleration is also derived; it is slightly larger than 90 deg. Equivalent results are obtained for the special case of circular orbits in a rotating atmosphere. The equations yield the inclination for zero intrack acceleration and for maximum crosstrack acceleration due to atmosphere rotation for circular orbits. The inability to predict these results with the approximate equations in References 2 and 3 is discussed for the circular-orbit, rotating-atmosphere case.

SECTION II

SIMULATION RESULTS

A. EFFECTS OF ATMOSPHERE ROTATION RATE ON ORBITS

Initial interest was in the effects of increased atmosphere rotation rate on polar orbits. When some effects proved to be substantial, interest turned to the implications with respect to orbit determination and short-term prediction techniques; these are discussed in Section II. B. This section presents results for equatorial orbits as well, to indicate the variation of effects with orbit inclination.

For each of the cases studied, the satellite position at 15-minute intervals was obtained for two orbits over a 12-hour period. Initial conditions were identical for both orbits. Force models used were also identical but with one exception: For one orbit the atmosphere rotated at earth rate; for the other it rotated at twice earth rate. Subtracting appropriate satellite position coordinates of the former from those of the latter at corresponding times yielded the radial, intrack, and crosstrack differences. Keplerian element differences were also obtained.

Table I shows the extremes of the departures of the differences from zero after 12 hours (approximately 8 revolutions). These extremes occurred during the eighth revolution in all cases. The minimum and maximum cross-track differences simply indicate the occurrence of substantial extremes in both positive and negative directions (due primarily to a monotonic inclination change). Other differences grew monotonically except for minor variations during each revolution.

The quantities shown are perigee altitude h_p, apogee altitude h_a, inclination i, latitude of perigee δ_p, and semimajor axis a. The ARDC 1959 atmosphere and a spherical earth were used; and $W/(C_D A)$ was taken as 80 lb/ft^2.

Table I. Extreme Differences after Eight Revolutions Due to Doubling Atmosphere Rotation Rate

Case	Orbit: h_p (n mi) / h_a (n mi)	Orbit: i (deg) / δ_p (deg)	Differences: Radial (ft)	Intrack (ft)	Crosstrack (ft)	a (ft)	i (deg)
1	80 / 180	90 / 34	-60	1179	min, -165 / max, 166	-31	-0.00045
2	80 / 180	90 / 0	-70	1416	min, -186 / max, 186	-39	-0.00056
3	70 / 230	90 / 0	-168	3089	min, -391 / max, 393	-81	-0.0011
4	80 / 180	0 / 0	1864	-39932	0	1092	0
5	70 / 230	0 / 0	3857	-73065	0	1924	0

Cases 1 to 3 involve polar orbits. Cases 4 and 5 involve equatorial orbits which, except for inclination, are like those of Cases 2 and 3, respectively. These pairs of cases display the variation of effects with inclination. Case 1, with perigee latitude at 34 degrees but otherwise the same as the polar orbit of Case 2, shows the effect of reduced winds at perigee due to increased perigee latitude.

The possibility of large crosstrack differences was the primary concern initially; however, it is clear from Cases 1 to 3 that crosstrack differences are minor. (Maximum crosstrack acceleration occurs for inclinations slightly greater than 90 deg. See Section IV. C.) It is easily shown that the polar orbit crosstrack differences of Table I, which take place near the poles, very nearly correspond to the inclination changes in the last column; the perturbed orbit is to the right of the comparison orbit at one pole, to the left at the other pole.

The relatively large intrack differences for polar orbits were unexpected. It is not obvious intuitively that increased wind normal to the orbit plane will produce the increased decay rate indicated by the radial, intrack, and semi-major axis differences for Cases 1 to 3. By comparing Case 2 with Case 4 and Case 3 with Case 5, the polar orbit intrack difference is seen to be about 4 percent of the equatorial orbit intrack difference and oppositely directed. (The equatorial orbit difference is the maximum possible difference for posigrade orbits, since the rotating atmosphere wind is greatest at the equator and the wind vector lies in the orbit plane.) That is, there is a significant loss of satellite energy due to increased wind normal to the polar orbit plane.

It is shown in Section IV. B that for circular orbits (horizontal velocity vectors), the inclination for zero intrack acceleration due to atmosphere rotation is slightly less than 90 deg. (Increased decay rates occur for larger inclinations, decreased decay rates for smaller inclinations.) From Eq. (37), this inclination is computed to be approximately 88. 3 deg at the equator for an orbit like that of Case 3 with an atmosphere rotating at twice earth rate. The Case 3 90-deg inclination thus differs by only 1. 7 deg at perigee from that which would produce no change in decay rate due to twice the earth rate atmosphere rotation, indicating sensitivity of the phenomenon to inclination change. (Commonly used approximations in the literature do not predict these effects. See Section V.)

B. EFFECTS OF UNDERESTIMATED ATMOSPHERE ROTATION RATE ON ORBIT DETERMINATION AND PREDICTION

Tracking was simulated for a 12-hour period for three orbits in an atmosphere rotating at twice earth rate. Then, by assuming an atmosphere rotating at earth rate, seven parameter fits of the tracking data were obtained with the ballistic coefficient $W/(C_D A)$ as the seventh parameter. The resulting epoch parameter values were used to generate predicted positions up to 12 hours past the 12-hour tracking interval. True satellite coordinates were then subtracted from corresponding predicted coordinates to obtain radial, intrack, and crosstrack prediction errors.

Results are displayed in Tables II, III, and IV. Epoch parameter values shown in Table II are for right ascension α, geocentric latitude δ, angle between velocity vector and geocentric vertical β, azimuth of velocity vector from true north Az, distance from geocenter r, magnitude of velocity vector V_o, and the ballistic coefficient $W/(C_D A)$.

The three orbits are similar in shape with perigee altitude at about 68 n mi and apogee altitude at about 209 n mi. Orbit 1 is a polar orbit with perigee at 34°N latitude. Orbits 2 and 3 have inclinations of 65 and 30 deg, respectively, with perigee at the equator (where maximum wind occurs).

In the simulations, the ARDC 1959 atmosphere and the standard TRACE geopotential model were used. Tracking data consisted of exact range, azimuth, and elevation at 30-second intervals during periods of visibility for six tracking stations. The numbers of sets of range, azimuth, and elevation were 69, 49, and 57 for Orbits 1, 2 and 3, respectively. In orbit determination, the standard deviations used were 100 feet for range and 0.1 degree for azimuth and elevation.

Table II shows true epoch parameter values for the three orbits and the "corrections" required in orbit determination due to assuming an atmosphere rotating at earth rate. Note the substantial correction in $W/(C_D A)$ for Orbit 3, which experiences the largest component of wind in the direction of satellite motion because of its low inclination. This correction indicates that in the orbit determination inplane forward acceleration (due to unmodeled high wind) is attributed to reduced drag. The largest of the absolute values of observation residuals for the converged seven-parameter fits are shown in Table III.

The prediction capability of the epoch parameter values for these fitted orbits is indicated in Table IV; it shows the minimum and maximum prediction errors. Errors were obtained at 15-minute intervals from epoch plus 12 hours to epoch plus 24 hours.

The orbits of this section are not the same as those of Table I, in which however, it is clear that the intrack component of satellite position can change by many miles over a 24-hour period if the atmosphere rotation rate is doubled.

Table II. Orbit Parameter Corrections Due to Assumption of Incorrect Atmosphere Rotation Rate

Parameter	Orbit 1		Orbit 2		Orbit 3	
	True	Correction	True	Correction	True	Correction
α (deg)	0	-0.00044	0	0.00020	0	0.00082
δ (deg)	34	0.00060	0	0.00017	0	0.00035
β (deg)	90	-0.00001	90	-0.00004	90	-0.00016
Az (deg)	180	-0.00058	25	0.00042	60	0.00083
r (ft)	21327000	83.0	21327000	41.9	21327000	104.9
V_0 (ft/sec)	25960	-0.11	25960	-0.05	25960	-0.13
$\frac{W}{C_D A}$ (lb/ft^2)	80	-0.363	80	3.895	80	9.262

Table III. Maxima of Absolute Values of 'uals Using Exact Observations

	Orbit 1	bit 2	Orbit 3
Range (ft)	87	113	112
Azimuth (deg)	0.0046	0.0110	0.0181
Elevation (deg)	0.0033	0.0134	0.0036

Table IV. Extremes of Radial, Intrack, and Crosstrack Prediction Errors for the Twelve-hour Post-tracking Interval

	Orbit 1		Orbit 2		Orbit 3	
Component	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
Radial (ft)	-64	87	-34	43	-102	79
Intrack (ft)	-1232	-113	-231	21	268	1042
Crosstrack (ft)	-639	630	-1095	1091	-426	425

The Table IV results indicate that if a seven-parameter orbit determination is used and an atmosphere rotating at earth rate is assumed, even a doubled atmosphere rotation rate does not lead to substantial short-term prediction errors.

C. SIMULATION CONCLUSIONS

- Crosstrack errors in orbit determination and short-term prediction due to underestimation of atmosphere rotation rate are small, even for the exaggerated underestimation used in the simulations.

- Intrack effects on most low-altitude orbits (including 90 deg inclination orbits) due to increased atmosphere rotation rate are substantial. However, if a seven-parameter fit is used, these effects are masked and produce only small intrack errors in orbit determination and short-term prediction.

SECTION III

EFFECTS OF A GENERALIZED WIND

The analysis in this section applies to any wind. It is used in the Section IV analysis of intrack and crosstrack effects of a rotating atmosphere on circular orbits, which helps explain the preceding simulation results. The variation of tangential acceleration A_T and normal acceleration A_N due to wind is discussed. An equation is derived for determining whether wind instantaneously increases or decreases satellite velocity. The angle between satellite velocity and wind vectors for maximum normal acceleration is obtained.

Figure 1 shows earth-centered inertial velocities of the wind $\underline{W}$ and the satellite $\underline{V}_o$, satellite velocity relative to the air $\underline{V}$, and angles between these vectors, γ and θ. Accelerations due to $\underline{V}_o$ (the no-wind case) and $\underline{V}$ are directed opposite to these velocities and have the magnitudes

$$A_o = KV_o^2 \tag{1}$$

$$A = KV^2 \tag{2}$$

where K is considered the same for both velocities in this derivation; that is, the drag coefficient is the same for all directions of satellite motion relative to the atmosphere.

A. TANGENTIAL ACCELERATION

A_T is investigated first. It is given by

$$A_T = A \cos \theta = KV^2 \cos \theta \quad . \tag{3}$$

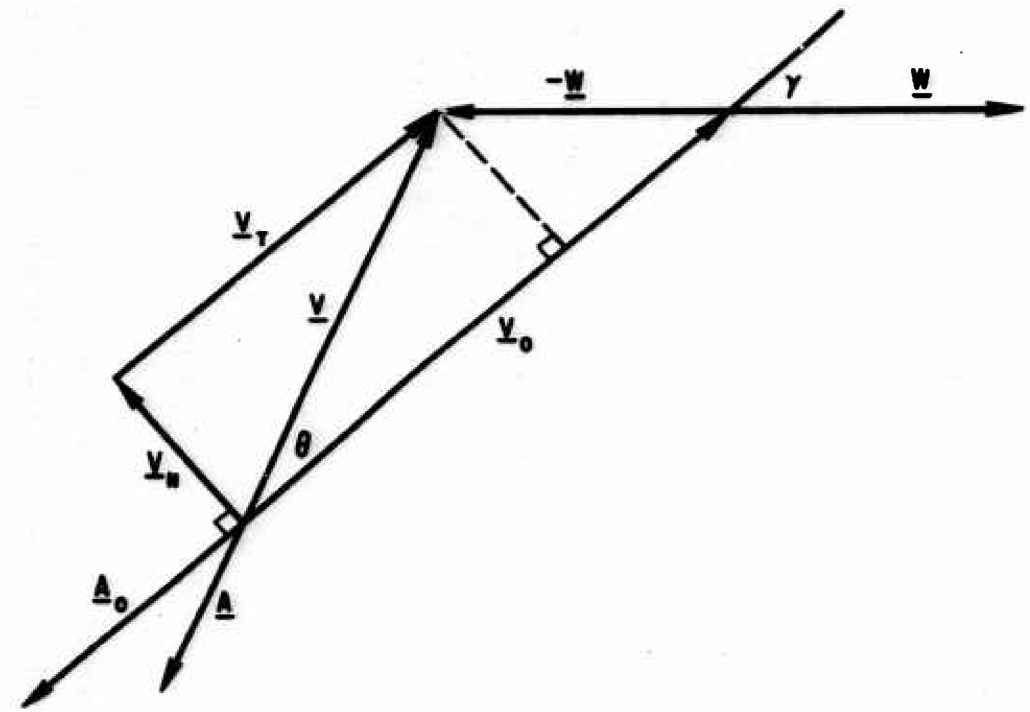

Figure 1. Velocity, Acceleration, and Wind Vectors

From Figure 1, it is evident that

$$V_T = V \cos \theta = V_o - W \cos \gamma \tag{4}$$

$$V^2 = V_o^2 + W^2 - 2V_oW \cos \gamma \quad . \tag{5}$$

From Eqs. (3) and (4),

$$A_T = KVV_T = KV(V_o - W \cos \gamma) \quad . \tag{6}$$

By using V from Eq. (5), this may be written as

$$A_T = K(V_o - W \cos \gamma)\,(V_o^2 + W^2 - 2V_oW \cos \gamma)^{1/2} \quad . \tag{7}$$

The behavior of A_T with increasing γ is readily obtained from Eq. (7). The terms containing $\cos \gamma$ have negative signs. It follows, for fixed V_o and W, that A_T increases monotonically as γ goes from 0 to 180 deg. Further, it is clear from Figure 1 that if $\gamma = 0$ deg

$$A_T = A < A_o \tag{8}$$

implying increased velocity due to wind, and if $\gamma = 180$ deg

$$A_T = A > A_o \tag{9}$$

implying decreased velocity.

The zero of $A_T - A_o$ is now investigated. Introducing $V_o - W \cos\gamma$ into the radical in Eq. (7) and multiplying yields

$$A_T = K\left[V_o^4 + V_o^2W^2 - (4V_o^3W + 2V_oW^3)\cos\gamma + (5V_o^2W^2 + W^4)\cos^2\gamma - 2V_oW^3\cos^3\gamma\right]^{1/2} \quad . \tag{10}$$

A_T is the rate of change of V_o. For no change due to wind,

$$A_T = A_o = KV_o^2 \quad . \tag{11}$$

Now we let γ_o be the value of γ for which Eq. (11) holds. It is clear that Eq. (11) occurs when the sum of the terms after V_o^4 in Eq. (10) is zero. The result is the equation in $\cos\gamma_o$

$$2V_oW^2\cos^3\gamma_o - (5V_o^2W + W^3)\cos^2\gamma_o + (4V_o^3 + 2V_oW^2)\cos\gamma_o - V_o^2W = 0 \quad . \tag{12}$$

From Eqs. (8) and (9) and the discussion following Eq. (7), it follows that only one value of γ between 0 and 180 deg makes $A_T = A_o$. Further, the alternation of signs for the terms of Eq. (12) implies that $\cos\gamma_o \geq 0$. Consequently, $0 \leq \gamma_o \leq 90$ deg. For example, if $V_o = 25,000$ ft/sec and $W = 500$ ft/sec, $\gamma_o = 89.6$ deg; that is, if $\gamma < 89.6$ deg, $A_T < A_o$, implying increased velocity due to wind; if $\gamma > 89.6$ deg, $A_T > A_o$, implying decreased velocity.

Because W is small compared with V_o, γ_o is always near 90 deg, so that $\cos\gamma_o \cong 0$. Also, for any realistic value of W, the absolute value of the coefficient of $\cos\gamma_o$ in Eq. (12) is much larger than are those of $\cos^3\gamma_o$ and

$\cos^2\gamma_o$ because of the term $4V_o^3$. A good approximation for γ_o can therefore be obtained from

$$(4V_o^3 + 2V_oW^2)\cos\gamma_o \cong V_o^2W \quad . \tag{13}$$

This is done by neglecting $2V_oW^2$, which is small compared with $4V_o^3$, and by using

$$\cos\gamma_o = \sin(\pi/2 - \gamma_o) \cong \pi/2 - \gamma_o \quad . \tag{14}$$

This yields

$$\gamma_o \cong \frac{\pi}{2} - \frac{W}{4V_o} \tag{15}$$

which is accurate to within two minutes of arc for the example in the preceding paragraph.

It follows from both Eqs. (12) and (15) that $\gamma_o \leq 90$ deg, and from Eq. (15) that γ_o decreases nearly linearly with increasing W for realistic values of W. Again, if $\gamma < \gamma_o$, satellite velocity increases compared with no wind; if $\gamma > \gamma_o$, satellite velocity decreases. Note that γ_o is independent of K and hence of atmospheric density and ballistic coefficient. Equations (12) and (15) provide a criterion for the instantaneous sign of the change in satellite velocity due to wind, whatever the model.

B. NORMAL ACCELERATION

A_N is given by

$$A_N = KV^2 \sin\theta = KVV_N = KVW\sin\gamma \quad . \tag{16}$$

If Eq. (5) is used in Eq. (16),

$$A_N = KW \sin\gamma\,(V_o^2 + W^2 - 2V_oW\cos\gamma)^{1/2} \quad . \qquad (17)$$

As γ goes from 0 to 180 deg, the radical V increases monotonically. If the properties of $\sin\gamma$ are used, it is then clear that A_N varies from zero at $\gamma = 0$ deg to a maximum, and then to zero at $\gamma = 180$ deg.

Let γ_m be the value of γ for which A_N is a maximum. By rewriting Eq. (17) we obtain

$$A_N = KW\left[(V_o^2 + W^2 - 2V_oW\cos\gamma)(1 - \cos^2\gamma)\right]^{1/2} \quad . \qquad (18)$$

The maximum occurs when the derivative of the radicand is zero. After simplification, this condition is expressed as

$$3V_oW\cos^2\gamma_m - (V_o^2 + W^2)\cos\gamma_m - V_oW = 0 \quad . \qquad (19)$$

The only meaningful solution is

$$\cos\gamma_m = \frac{1}{6W}\left[V_o + \frac{W^2}{V_o} - \left(V_o^2 + 14W^2 + \frac{W^4}{V_o^2}\right)^{1/2}\right] \quad . \qquad (20)$$

By inspection it is seen that $\cos\gamma_m \leq 0$, so that

$$\gamma_m \geq 90 \text{ deg} \quad . \qquad (21)$$

For example, with $V_o = 25{,}000$ ft/sec and $W = 500$ ft/sec, $\gamma_m = 91.2$ deg. Note that γ_m, like γ_o, is independent of atmospheric density and ballistic coefficient.

SECTION IV

ROTATING ATMOSPHERE EFFECTS

Consider a circular orbit about a spherical earth with a spherically symmetric atmosphere rotating at a rate ω_a about the earth's axis. In the following, the magnitudes of the intrack and crosstrack components of velocity relative to the atmosphere and of acceleration due to the atmosphere are investigated. Implications concerning noncircular orbits are discussed.

A. VELOCITY AND ACCELERATION VARIATIONS

Figure 2 shows inclination i and, at P, the satellite latitude δ, and the angle between satellite velocity and wind vectors γ. In the spherical right triangle PQR,

$$\sin\left(\frac{\pi}{2} - \gamma\right) = \cos\gamma = \frac{\cos i}{\cos\delta} \quad . \tag{22}$$

Figure 3 shows the vectors occurring at P in Figure 2. Figure 3 is the same as Figure 1, except that tangential and normal components are in this case intrack (subscript I) and crosstrack (subscript C) components. If r is the satellite's distance from the geocenter,

$$W = r\omega_a \cos\delta \quad . \tag{23}$$

If Eqs. (22) and (23) are used in Eq. (4),

$$V_I = V\cos\theta = V_o - r\omega_a \cos i \quad . \tag{24}$$

Also, from Figure 3,

$$V_C = V\sin\theta = W\sin\gamma \quad . \tag{25}$$

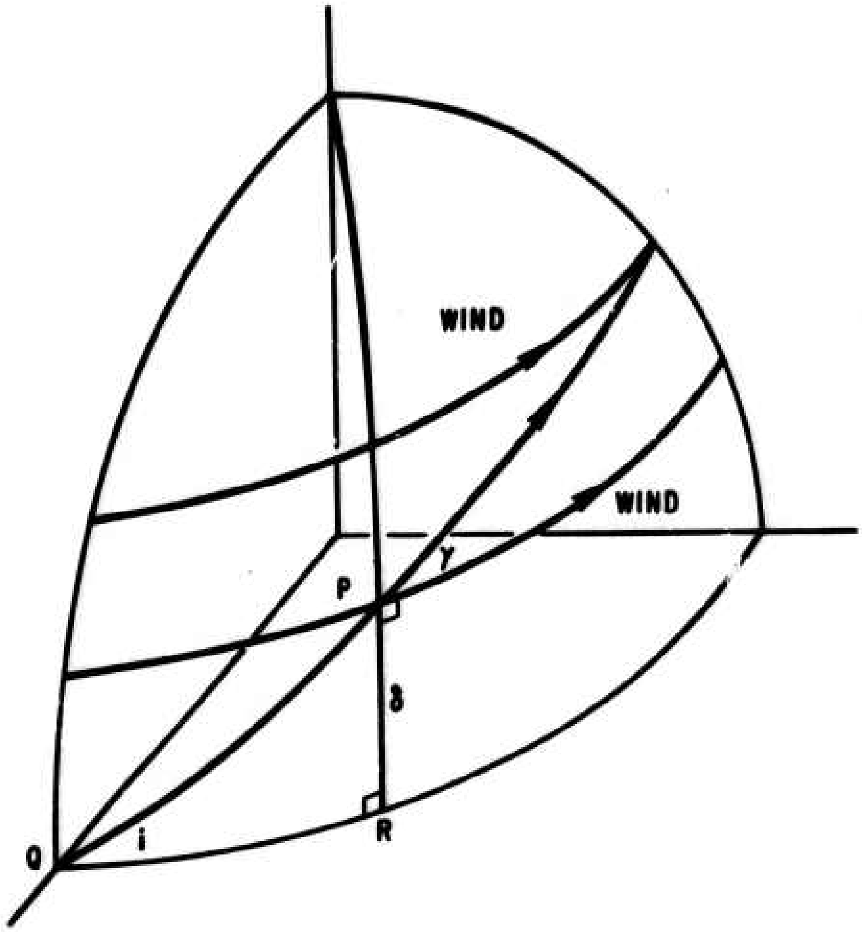

Figure 2. Satellite Orbit and Rotating Atmosphere

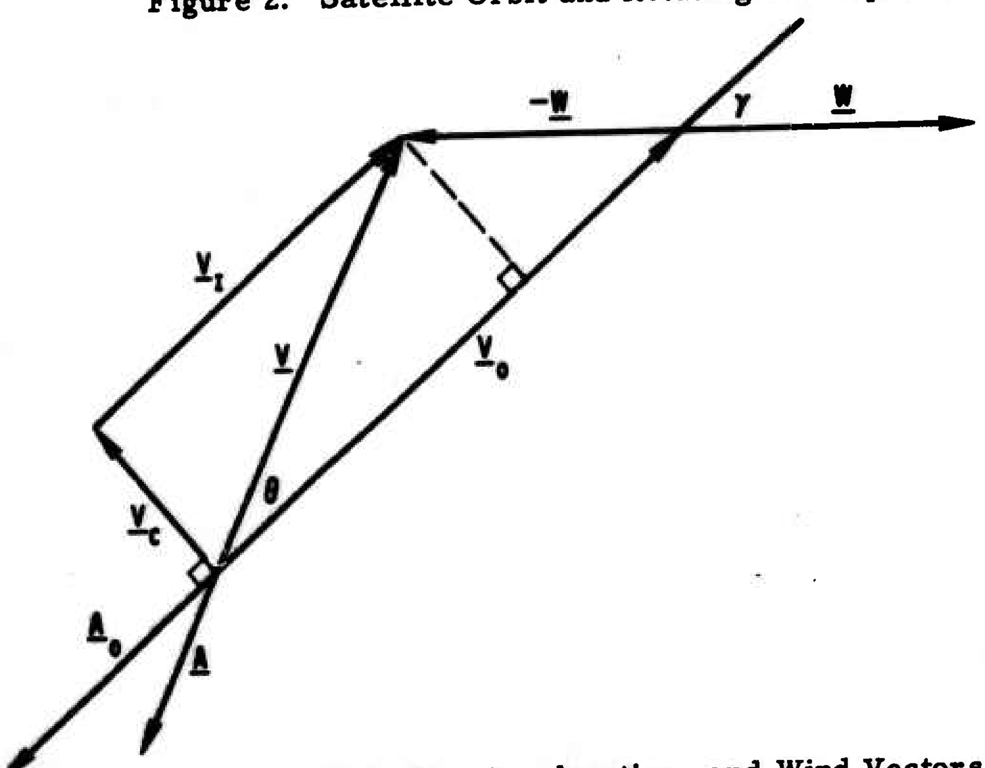

Figure 3. Velocity, Acceleration, and Wind Vectors

By substituting W from Eq. (23) and sin γ from Eq. (22) in Eq. (25) and simplifying,

$$V_C = V \sin\theta = r\omega_a(\cos^2\delta - \cos^2 i)^{1/2} \quad . \tag{26}$$

Note from Eq. (24) that V_I is the same at all points in the orbit for a given inclination; that is, (for 0 deg < i < 90 deg), the tendency to decrease V_I, produced by reduction of γ as the satellite moves, say, northward from the equator, is exactly countered by the reduction in wind with increasing latitude. For retrograde orbits, the tendency to increase V_I produced by increase of γ as the satellite moves northward is also countered by the reduction in wind. In contrast, from Eq. (26), V_C has a maximum at the equator ($\delta = 0$, $\gamma = i$) and is zero at maximum latitude ($|\delta| = i$ or 180 deg - i, $\gamma = 0$ or 180 deg).

Intrack and crosstrack acceleration magnitudes are

$$A_I = KV^2 \cos\theta \tag{27}$$

$$A_C = KV^2 \sin\theta \quad . \tag{28}$$

Although V_I is constant throughout the orbit, A_I is only approximately constant. This can be seen by using Eq. (24) in Eq. (27) to obtain

$$A_I = \frac{KV_I^2}{\cos\theta} = \frac{K}{\cos\theta}\,(V_o - r\omega_a \cos i)^2 \quad . \tag{29}$$

Thus A_I varies inversely as cos θ for any specified inclination. But cos $\theta \cong 1$ for realistic W and V_o so that the variation of A_I is small. In fact,

$$A_I \cong K(V_o - r\omega_a \cos i)^2 \tag{30}$$

is a fair approximation. Note, however, that for i = 90 deg, Eq. (30) is $A_I \cong KV_o^2$, indicating no wind effect; the intrack differences for Cases 1, 2, and 3 in Table I are an indication of the error in Eq. (30).

The variations of A_I and A_C with latitude and inclination are now considered. Using Eqs. (24) and (26) in Eqs. (27) and (28) yields

$$A_I = KVV_I = KV(V_o - r\omega_a \cos i) \tag{31}$$

$$A_C = KVV_C = KVr\omega_a(\cos^2\delta - \cos^2 i)^{1/2} \quad . \tag{32}$$

By using Eqs. (22) and (23), Eq. (5) can be written as

$$V = (V_o^2 + r^2\omega_a^2\cos^2\delta - 2V_o r\omega_a \cos i)^{1/2} \quad . \tag{33}$$

For a specified inclination, only cos δ varies in Eq. (33), so that V is a maximum at the equator and a minimum when the satellite is at maximum latitude (north or south). (This can also be concluded from the variations of V_I and V_C discussed previously.) If a similar argument is used for the radicand in Eq. (32), it follows from Eqs. (31), (32), and (33) that

(a) For any inclination, A_I decreases from a maximum at the equator to a minimum at the greatest distance from the equator ($|\delta| = i$ or 180 deg - i). Again, from Eq. (29), the variation is seen to be small.

(b) For any inclination, A_C decreases from a maximum at the equator to zero at the greatest distance from the equator.

Next consider the effect on A_I and A_C of variation of inclination, holding δ fixed. For $\delta \neq 0$, it is clear that the possible range of values of i is $|\delta| \leq i \leq 180 \text{ deg} - |\delta|$. For any such subset of $0 \leq i \leq 180$ deg, the terms involving cos i in Eqs. (31) and (33) increase monotonically as i increases. The term $-\cos^2 i$ in Eq. (32) increases to zero at i = 90 deg, then decreases thereafter as i increases. Again, from Eqs. (31), (32), and (33), it follows that

(c) For any latitude, A_I increases monotonically as i increases through its possible range.

(d) For any latitude, A_C increases from zero (at $i = |\delta|$) to a maximum and then decreases to zero (at $i = 180$ deg $- |\delta|$) as i increases through its possible range.

B. ZERO INTRACK ACCELERATION

The preceding conclusions (c) and (d) are closely related to those of Section III. First, it is apparent from Figures 2 and 3 that $A_I < A_o$ for $i = 0$ deg and that $A_I > A_o$ for $i = 180$ deg. These conditions indicate, respectively, for the circular orbit and rotating atmosphere, decreased decay rate and increased decay rate due to wind. The condition $A_T = A_o$ (no change in satellite speed due to wind) was investigated in Section III. A. The approximation Eq. (13) is applied here by using Eqs. (22) and (23) with i_o as the inclination that gives $A_I = A_o$. Again if $2V_oW^2$ is neglected, this yields

$$\frac{\cos i_o}{\cos \delta} \cong \frac{W}{4V_o} = \frac{r\omega_a \cos \delta}{4V_o} \tag{34}$$

or

$$\cos i_o \cong \frac{r\omega_a \cos^2 \delta}{4V_o} \quad . \tag{35}$$

The right-hand member is again small so that

$$\cos i_o = \sin(\pi/2 - i_o) \cong \frac{\pi}{2} - i_o \quad . \tag{36}$$

Substituting in Eq. (35) gives

$$i_o \cong \frac{\pi}{2} - \frac{r\omega_a \cos^2 \delta}{4V_o} \quad . \tag{37}$$

Thus, $i_o \leq 90$ deg and i_o depends on δ; that is, i_o increases as latitude increases. Note that there are circular orbits with inclinations slightly below 90 deg that experience increased decay rate at the equator and decreased decay rate near the poles.

C. MAXIMUM CROSSTRACK ACCELERATION

Simililarly, if Eqs. (22) and (23) are used in Eq. (20), i_m, the inclination for maximum A_C, then is given by

$$\cos i_m = \frac{1}{6r\omega_a}\left[V_o + \frac{W^2}{V_o} - \left(V_o^2 + 14W^2 + \frac{W^4}{V_o^2}\right)^{1/2}\right] \quad . \tag{38}$$

Clearly $\cos i_m \leq 0$ implying $i_m \geq 90$ deg. Like i_o, i_m depends on δ, since $W = r\omega_a \cos \delta$. Since $i \cong \gamma$ when $\gamma \cong 90$ deg, and if the results of the example following Eq. (21) are used, it is clear that i_m is close to 90 deg. Further, inspection of Eq. (38) reveals that i_m approaches 90 deg as $|\delta|$ approaches 90 deg, since W approaches zero.

D. COMPARISON WITH SIMULATION RESULTS

The qualitative statements that follow relate the preceding analytical results to the results in Section II. A. These statements are made with the following observations in mind: First, the differences in Table I are in the same direction as, but smaller in magnitude than, those that would have been obtained by comparing twice earth rotation rate and no atmosphere rotation situations. Second, the simulation results are for low-eccentricity orbits; however, radial velocity components are small, particularly in the vicinity of perigee where most of the atmosphere rotation effects are experienced. The simulated effects are taken as resembling the circular orbit effects of the analysis.

- The crosstrack differences for the polar orbits of Cases 1 to 3 in Table I are slightly less than maximum; maximum crosstrack differences would occur for orbital inclinations slightly greater than 90 deg (see Section IV. C).

- As indicated in Section II. A, the 90 deg inclinations are at most only slightly greater than the instantaneous inclination at any point in each orbit that would produce zero intrack acceleration. This is shown in Section IV. B. Again, the intrack differences obtained for these 90-deg-inclination orbits show the sensitivity to inclination change of intrack acceleration due to atmosphere rotation.

- Intrack differences for the polar orbits of Cases 1 to 3 in Table I are an indication of the error in the approximate Eq. (30), which predicts no intrack acceleration for polar orbits due to atmosphere rotation. Eq. (29) does predict this effect. This is discussed further in Section V.

- From paragraph (a) following Eq. (33), it is to be expected that a change in argument of perigee will produce a change in intrack acceleration due to atmosphere rotation. This is illustrated by the intrack differences for Cases 1 and 2 in Table I.

SECTION V

NOTES ON THE LITERATURE

An increased decay rate of polar orbits due to a rotating atmosphere is not expected from frequently used approximations in the literature. This led to the analyses in Sections III and IV. The inclination separating decreased and increased decay rate situations is given approximately by Eq. (37). Although this inclination is close to 90 deg (and less than 90 deg except at the poles), the intrack effect on a 90 deg orbit is substantial (see Table I). Further, for some orbits with inclinations slightly less than 90 deg, a rotating atmosphere increases the decay rate. Thus, any generalization implying that all posigrade orbits have reduced decay rates due to atmosphere rotation is misleading. Discussion here of two references explains the inability of their approximate equations to yield these conclusions.

In Reference 2, an equation is used that can be written for a circular orbit as

$$V = V_o - r\omega_a \cos i + \frac{r^2\omega_a^2}{2V_o}(\sin^2 i - \sin^2 \delta) + \text{smaller terms} \tag{39}$$

using the notation of Section IV. All terms after $-r\omega_a \cos i$ are then neglected. (Note that as i approaches 90 deg the term involving $\sin^2 i$ becomes larger than that involving cos i.) The resulting approximation is

$$V \cong V_o - r\omega_a \cos i \quad . \tag{40}$$

When applied to a circular orbit, Eq. (2.10) in Reference 3 can be rewritten as

$$V^2 = V_o^2\left(1 - \frac{r\omega_a}{V_o}\cos i\right)^2 + r^2\omega_a^2(\cos^2\delta - \cos^2 i) \quad . \tag{41}$$

Reference 3 then neglects $r^2\omega_a^2(\cos^2\delta - \cos^2 i)$, which also yields Eq. (40).

By comparing Eq. (40) with (24), the approximations in both references are seen to be expressions for V_I; that is, the right member of Eq. (40) requires division by cos θ to make the equation exact. For a polar orbit, Eq. (40) yields

$$A_I \cong KV_o^2 \cos\theta \quad . \tag{42}$$

The exact value from Eq. (29) for a polar orbit is

$$A_I = \frac{KV_o^2}{\cos\theta} \quad , \tag{43}$$

and for no wind the exact value is

$$A_I = KV_o^2 \quad . \tag{44}$$

Thus, Eq. (42) differs by a factor of $\cos^2\theta$ from the exact value and is even smaller than the exact value for no wind; it implies a decreased rather than an increased orbit decay rate due to wind for a polar orbit.

Comparing Eqs. (42), (43), and (44) leads to an estimate of the intrack error that results if Eq. (42) is used instead of Eq. (43) for polar orbits. Subtracting Eq. (44) from Eq. (42) yields the change in A_I due to wind, using the approximation, as

$$KV_o^2 \cos\theta - KV_o^2 = KV_o^2(\cos\theta - 1) \quad . \tag{45}$$

Subtracting Eq. (44) from Eq. (43) yields the exact change in A_I due to wind, as

$$\frac{KV_o^2}{\cos\theta} - KV_o^2 = KV_o^2\left(\frac{1}{\cos\theta} - 1\right) \quad . \tag{46}$$

The ratio of Eqs. (45) and (46) reduces to $-\cos\theta$, which is approximately -1. That is, for polar orbits, the acceleration due to wind, if Eq. (42) is used, has about the same magnitude as the true value, but is oppositely directed; or, the error in Eq. (42) is almost twice the exact intrack acceleration due to wind for polar orbits.

A quantitative implication for polar orbits is obtained if Table I is considered to display approximately the intrack differences between orbits subjected to an atmosphere rotating at earth rate and those subjected to a nonrotating atmosphere. The conclusion of the preceding paragraph implies, to first order, that intrack differences using Eq. (42) for intrack acceleration would be equal to the intrack differences for Cases 1, 2, and 3 (polar orbits) of Table I, but oppositely directed.

Further, it is again noted that the Case 2 and 3 differences are approximately 4 percent of the Case 4 and 5 differences, respectively. It follows that the errors in predicting intrack differences due to wind using Eq. (42) for Cases 2 and 3 would be approximately 8 percent of the intrack differences due to wind for Cases 4 and 5. Cases 4 and 5 are selected in this comparison because these equatorial orbits display the maximum intrack differences due to wind for all similar posigrade orbits. When compared with these maximum differences, the errors of Eqs. (40) and (42) in predicting intrack differences due to wind for polar orbits are seen to be substantial.

The error in V^2, if Eq. (40) is used, is obtained by subtracting values of V^2 from Eqs. (40) and (41), the latter being exact. The error is

$$\Delta(V^2) = r^2\omega_a^2(\cos^2\delta - \cos^2 i) \quad . \tag{47}$$

That is, the accuracy of intrack acceleration $A_I = KV^2\cos\theta$ in using Eq. (40) is latitude- and inclination-dependent. By inspection, it is seen that $\Delta(V^2) \geq 0$; the true relative velocity is generally greater than the approximate relative velocity. Also, $\Delta(V^2) = 0$ only if $|\delta| = i$ or 180 deg - i. The maximum value of $\Delta(V^2)$, $r^2\omega_a^2$, occurs when $\delta = 0$ deg and i = 90 deg. Since the variation of cos θ is small, the polar orbit intrack acceleration errors discussed are very nearly maximum. Further, the error does not change rapidly as i departs from 90 deg (with perigee remaining near the equator).

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13. ABSTRACT

The effect of increased atmosphere rotation rate on low perigee altitude orbits and the effect of underestimation of atmosphere rotation rate on orbit determination are displayed using simulation results. Crosstrack changes in satellite position due to increased rotation rate are small. Intrack changes, though larger, are small-error sources in orbit determination and short-term prediction if a seven-parameter fit is used. The angles between satellite inertial velocity and wind vectors that produce zero tangential acceleration and maximum normal acceleration are derived for any wind and for the special case of a circular orbit in a rotating atmosphere. This analysis explains the increased decay rates of some near-polar orbits due to a rotating atmosphere and the inability to predict this effect with frequently used approximations.

14\. KEY WORDS

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Abstract (Continued)