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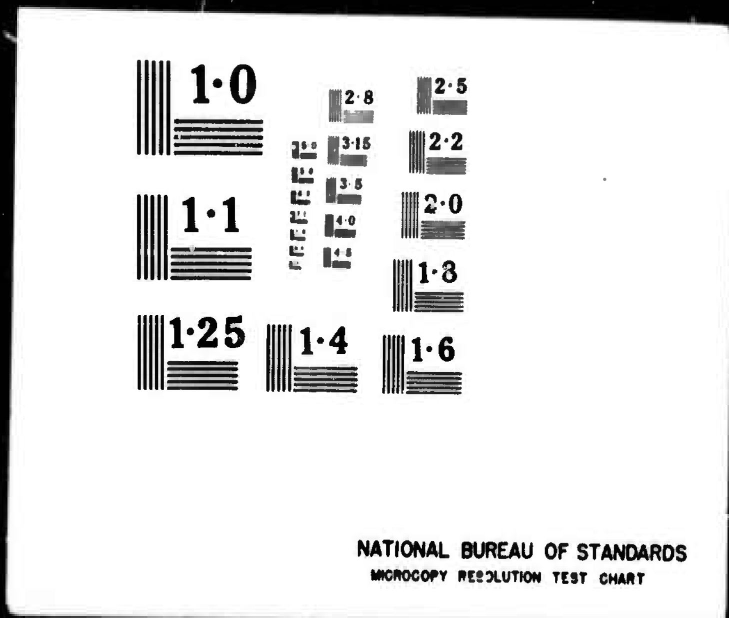
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ELECTRONICS RESEARCH LABORATORIES

632 WEST 125th STREET

NEW YORK, NEW YORK 10027

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MULTICHANNEL ELECTRO-OPTICAL RECORDER FOR THE SYNTHETIC SPECTRUM RADAR

TECHNICAL REPORT T-5/321

By

M. Arm
M. King

Prepared for

Director
Advanced Research Projects Agency
Washington, D.C. 20301

and

Electronics Division
Directorate of Engineering Sciences
Air Force Office of Scientific Research
Office of Aerospace Research
U.S. Air Force
Arlington, Virginia 22209

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ABSTRACT

The Synthetic Spectrum Radar requires an on-line recorder capable of producing a phase-aligned photographic record of its 100 IF output signals. We describe herein an electro-optical configuration capable of producing such a photographic recording. A theoretical analysis of the recorder is provided and leads to expressions relating the light amplitude transmittance of the record to the electrical input signal. Alternate modes of operation of the recorder are discussed.

AUTHORIZATION

The research described in this report was performed at the Electronics Research Laboratories of Columbia University. This report was prepared by M. Arm and M. King.

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Submitted by:

L. Lambert
Assistant Director

Approved by:

For L. H. O'Neill
Professor of Electrical Engineering
Director

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I. INTRODUCTION

This report describes an ingenious application of the techniques of electro-optical signal processing to the problem of on-line recording of pulsed carrier signals which arises in the Synthetic Spectrum Radar. In this pulsed coherent radar, it is desired to record on line 100 separate intermediate-frequency output signals on photographic film, referring the phases of all recorded signals to the phase of the output of a previously designated reference channel. Further information about the signal processing requirements of the Synthetic Spectrum Radar may be found in a companion report.[1]

Section II of this report presents a discussion of a one-channel recorder for a continuous sinusoidal signal. The third section then advances the complexity of the single-channel recorder by assuming that the signal to be recorded is a pulsed carrier signal. After the discussion in Section IV, which considers the effects of photographic film properties on the recording process, all the basic details of one channel of the final 100-channel recorder will have been presented. Section V describes the 100-channel recorder for the Synthetic Spectrum Radar. This final configuration is achieved by spatially multiplexing 100 of the single channel pulsed-signal recorders described in the earlier sections of the report.

We have also included an appendix which describes two possible modifications of the electro-optical configuration described in the main sections of the report. The merits and demerits of the alternate configurations, which involve a

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[1] For numbered references, see Section VI.

change in the mode of operation of the spatial light modulator, are compared with the mode discussed in the main body of the report. It is concluded that for the purposes of this study, one of the alternate modes is basically equivalent and the other inferior to the one in the main body of the report.

II. SIMPLIFIED SINGLE CHANNEL RECORDER

Consider the electro-optical system sketched in Fig. 1. The spatial light modulator is illuminated from the left by plane polarized light (having wavelength λ_L, frequency f_L, and uniform intensity I_o) which is propagating at right angles to the direction of the acoustic wave. An electric signal $v(t)$ drives the acoustic light modulator so that a disturbance $\psi(x,t)$ travels in the positive x direction at speed s. In the absence of the spatial filter in plane 2, lenses 1 and 2 serve to image the light leaving plane 1 onto plane 3. Plane 2 is the back focal plane of lens 1, and it is in this plane that spatial filtering will be performed in order to enable us to make useful signal recordings.

Suppose that $v(t)$ is a cosine wave of temporal frequency $f_o (= \omega_o/2\pi)$ and of phase ϕ such that we can characterize the acoustic disturbance, which is simply proportional to a traveling-wave version of $v(t)$, by the traveling phase grating:

$$\psi(x,t) = \psi_m \cos[\omega_o(t - \frac{x}{s}) + \phi]. \tag{1}$$

$\psi(x,t)$ is interpreted as the phase perturbation accorded to a ray of light traveling in a straight line through the spatial light modulator at position x and time t.

If ψ_m, the peak phase deviation, is not large, the effect of the moving phase grating upon the light leaving the modulator is well represented by assuming that three plane waves of light are produced in the acousto-optical

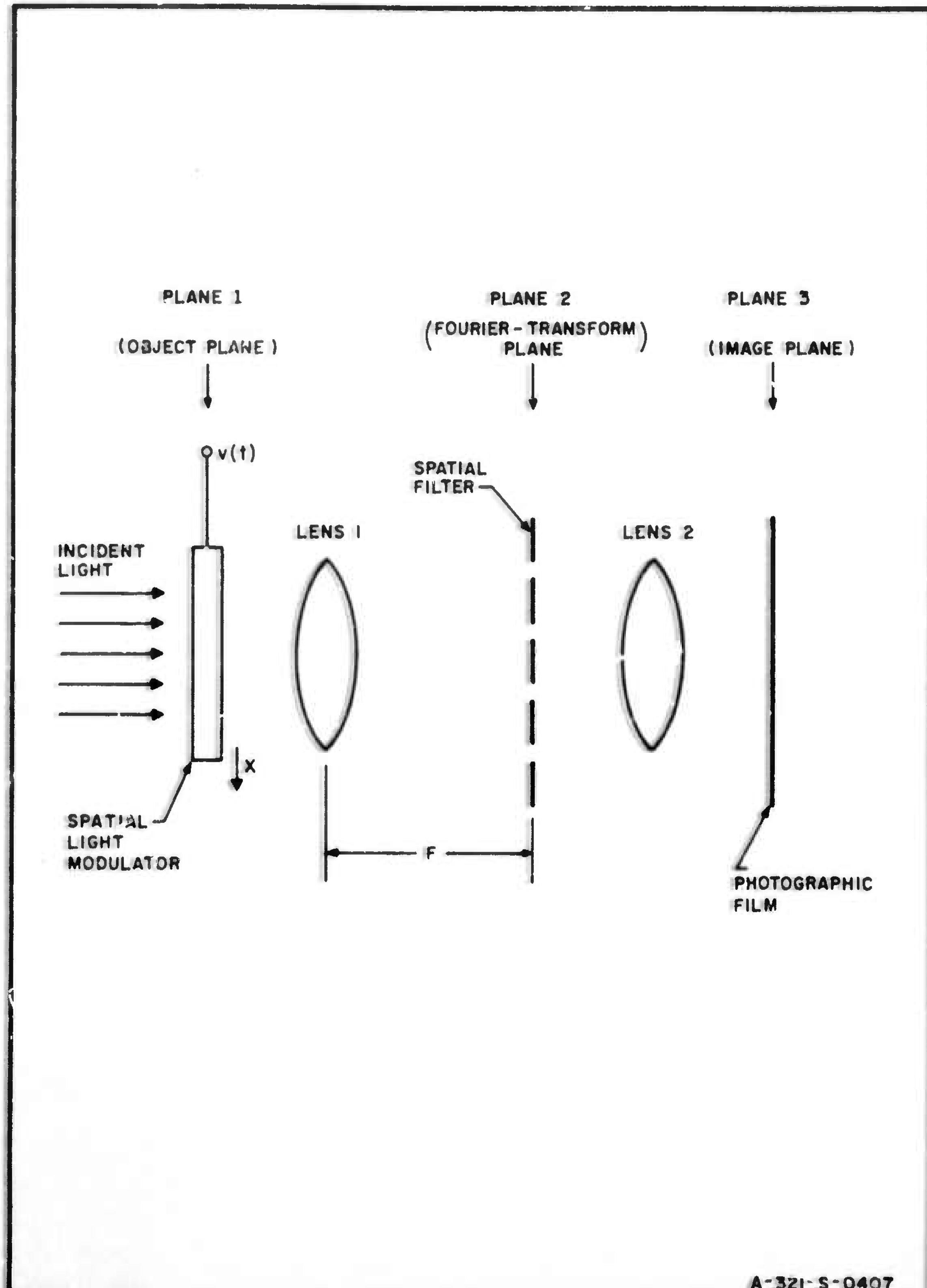

FIG. 1 SIMPLIFIED SINGLE-CHANNEL RECORDER

interaction.[2] These plane waves are, of course, the 0, +1, and -1 orders associated with the acoustic grating. In addition, for $\psi_m < 0.3$, we may associate with the 0 order the amplitude $I_o^{1/2}$ and with each of the first orders the amplitude $I_o^{1/2}\psi_m/2$. Furthermore, the theory of Raman and Nath[3] allows us to express this distribution of light amplitude in the form

$$E_1(x,t) = I_o^{1/2}\left(1 + j\,\frac{\psi_m}{2}\,e^{-j[\omega_o(t-\frac{x}{s})+\phi]} + j\,\frac{\psi_m}{2}\,e^{j[\omega_o(t-\frac{x}{s})+\phi]}\right) \tag{2}$$

$E_1(x,t)$ is interpreted as the total light amplitude in plane 1.

If no spatial filtering is executed in plane 2, and assuming unity magnification, the image field in plane 3, will be a perfect duplication of $E_1(x,t)$. However, it fits our purpose to remove one of the grating orders, say the last term in Eq. (2), by means of a simple optical stop in plane 2. When this is done, the field in plane 3 becomes

$$E(x',t) = I_o^{1/2}\left[1 + j\,\frac{\psi_m}{2}\,e^{-j[\omega_o(t-\frac{x}{s})+\phi])}\right]. \tag{3}$$

Computing the intensity in plane 3, we find that

$$I(x',t) = I_o(1 + \psi_m \sin[\omega_o(t-\frac{x'}{s}) + \phi]), \tag{4}$$

where we have ignored the intensity component proportional to ψ_m^2. Observe that the intensity in plane 3 consists of a constant component and a component which is an image of the traveling perturbation $\psi(x,t)$, shifted in phase by 90 deg. Since our goal is to produce a static image (so it can be photographed), we must find means to convert the moving image of Eq. (4) to a stationary one.

The grating image is made stationary by letting the incident intensity I_o become a sinusoidal function of time. In particular, we let the intensity be given by

$$I_o \longrightarrow I_o(t) = \frac{I_o}{2}\,[1 + b\sin(\omega_o t + \phi_o)]. \qquad (5)$$

Note that the time-varying intensity component has the same carrier frequency as the input signal $v(t)$, but a different phase. In Eq. (6), $b \leq 1$ so that the intensity never becomes negative. Substituting the time-varying intensity of Eq. (5) into Eq. (4), we find that the intensity in plane 3 may be written in the form:

$$I(x',t) = \frac{I_o}{2}\Bigg[1 + b\sin(\omega_o t + \phi_o) + \psi_m \sin\left(\omega_o\left[t - \frac{x'}{s}\right] + \phi\right)$$
$$+ \frac{b\psi_m}{2}\left\{\cos\left(-\omega_o \frac{x}{s} + [\phi - \phi_o]\right) - \cos\left(\omega_o\left[2t - \frac{x'}{s}\right] + \phi_o + \phi\right)\right\}\Bigg]. \qquad (6)$$

Observe that $I(x',t)$ is composed of five separate terms inside the brackets, only two of which have no sinusoidal time dependence.

In order to compute the effect of $I(x',t)$, we must evaluate $\int_o^T I(x',t)dt$, which is defined as the exposure of the photographic film in joules/m^2. T is the exposure time, and if it includes many cycles of the terms in $I(x',t)$ which depend on time sinusoidally, only those terms which are independent of time will contribute significantly to the integral. Assuming this to be the case, we find that the exposure is given by

$$\mathcal{E}(x') = \frac{I_o T}{2}\left(1 + \frac{b\psi_m}{2}\cos[\omega_o \frac{x'}{s} - (\phi - \phi_o)]\right);$$

$$T \gg \frac{2\pi}{\omega_o} . \qquad (7)$$

Observe that a stationary image is formed by the processor, and that the phase of the recorded sinusoidal signal is equal to the difference in phase between $v(t)$ and the sinusoidal component of I_o as given in Eq. (5). In addition, notice that the image of the grating rides on a bias which is independent of the input signal $v(t)$, and that the amplitude of the sine-wave image is proportional to the amplitude, ψ_m, of the input signal $v(t)$.

III. COMPLETE SINGLE-CHANNEL RECORDER FOR PULSED-CARRIER INPUT SIGNAL

Figure 2 is a sketch of the pulsed signal recorder discussed in this section. Note that a temporal light modulator has been placed in the light path to the left of the spatial light modulator, and that a moving strip of photographic film has been placed in the image plane.

We now assume that the input $v(t)$ to the spatial light modulator is a pulsed carrier signal such that the traveling phase perturbation is

$$\psi(x,t) = \sum_{k=1}^{\infty} \psi_k \, \mathrm{rect}\left(\frac{t-kT_o}{T_1}\right) \cos[\omega_o(t-\frac{x}{s}) + \phi_k] ; \qquad (8)$$

$$\mathrm{rect}\left(\frac{t}{T_1}\right) = \begin{cases} 1 & |t| \leq T_1/2 \\ 0 & |t| > T_1/2 \end{cases} .$$

We see that $\psi(x,t)$ is expressed as a series of rectangular pulses of amplitude ψ_k, phase ϕ_k, duration T_1, and period T_o.

In addition, let the voltage input to the temporal light modulator be such that the light intensity incident on the acoustic light modulator is the following function of time:

$$I_o(t) = \frac{I_o}{2} \sum_{k=1}^{\infty} \mathrm{rect}\left(\frac{t-kT_o}{T_2}\right) [1 + b \sin(\omega_o t + \theta_k)] . \qquad (9)$$

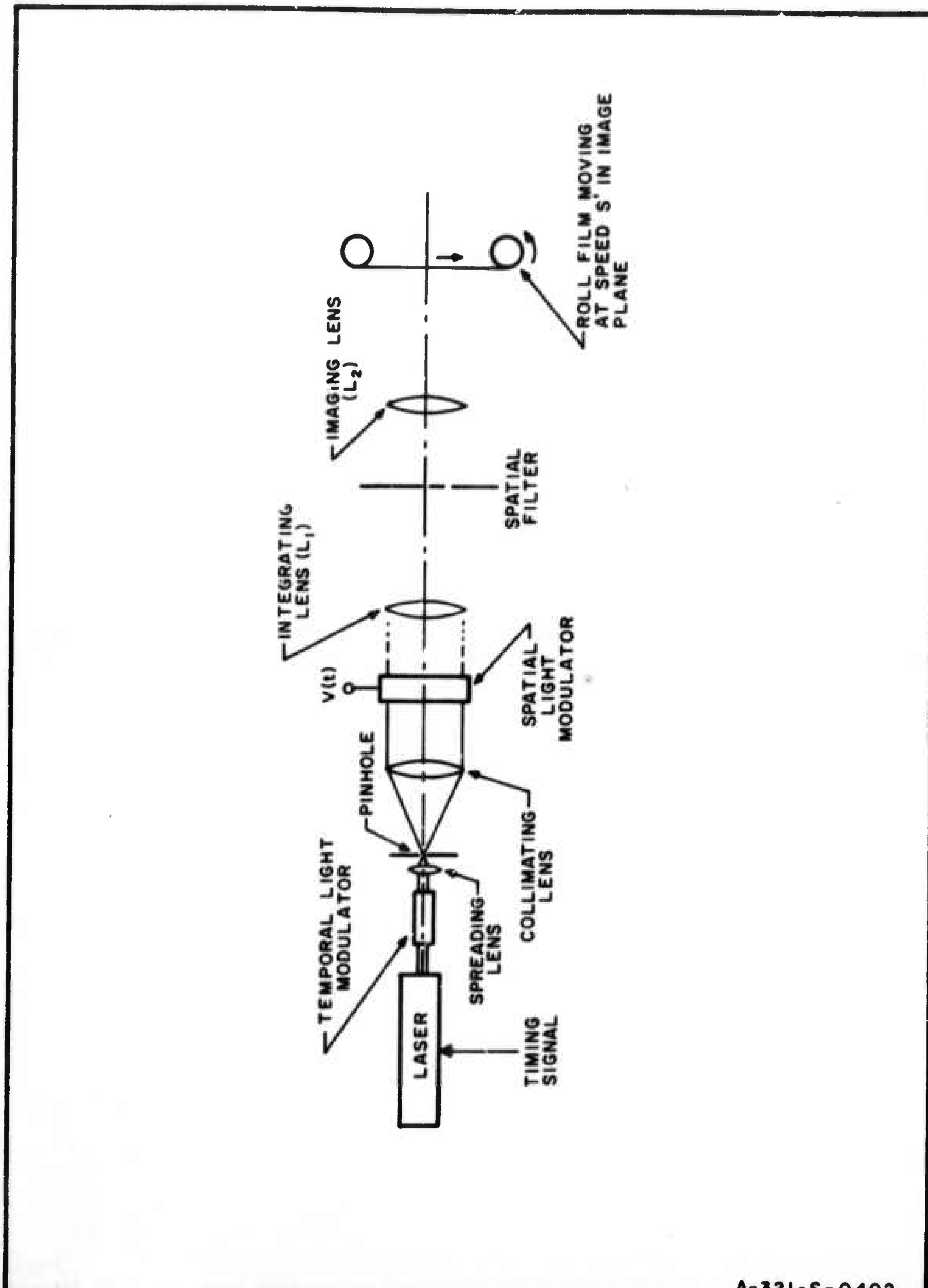

FIG. 2 SINGLE-CHANNEL RECORDER FOR PULSED SIGNALS

The duration of each pulse of light, as well as the exposure time of the film, is equal to T_2. We require T_2 to be small enough so that the light is on only when the aperture of the spatial modulator is filled with the corresponding pulse of $\psi(x,t)$. If the length of the aperture is P meters, and the sonic velocity is s m/sec, then it is required that T_1 be less than P/s sec, and that T_2 satisfy the inequality:

$$T_2 \leq \frac{P}{s} - T_1 \qquad (10a)$$

Recall in connection with Eq. (7) that we pointed out the exposure time T_2 must include many cycles of ω_o. That is,

$$T_2 \gg \frac{2\pi}{\omega_o} . \qquad (10b)$$

It is clear from the work of the previous section that if the film is moved in the interval between pulses of v(t), it is possible to record images of successive pulses side by side on the strip. However, if the repetition rate of the pulses is signficant, and if successive pulses must be recorded coherently, it will be difficult to produce an accurate and precise film motion. It is felt that movement of the film at constant speed is a better technique for the Synthetic Spectrum Radar. Thus, the remainder of the discussion will assume that a constant-speed film transport mechanism will be used.

Let the film be moving in the x direction at speed s'. In addition let us assume that an aperture-limiting slit has been placed in front of the film plane, so that only n cycles of the grating image are allowed to reach the photographic film. If successive pulses are to be recorded side by side on the film, the speed s must be large enough to move the film a distance equal to or greater than the slit width in the basic pulse period. Since the slit width is n sonic wavelengths (for unity magnification between object and image planes), it follows that s' must obey the inequality

$$s' \geq \frac{ns}{f_o T_o} . \qquad (11a)$$

A requirement which limits the maximum value of s' may be obtained by noting that for the image not to be blurred, it is necessary that the film move only a small fraction of a recorded wavelength during the exposure time. Allowing the film to move no more than ℓ sonic wavelengths during exposure time T_2, where $\ell \ll 1$, we find that s' must also obey the inequality

$$s' \leq \frac{\ell s}{f_o T_2} . \qquad (11b)$$

Combining Eqs. (11a) and (b), we find that the film speed s' must be within the limits given by

$$\frac{ns}{f_o T_o} \leq s' \leq \frac{\ell s}{f_o T_2} . \qquad (11c)$$

It is also interesting to note that Eq. (11c) leads to a constraint on the relative values of the exposure time T_2 and the period T_o of $v(t)$. This relationship is

$$\frac{T_2}{T_o} \leq \frac{\ell}{n} . \tag{11d}$$

Thus it is seen that the duty cycle of the temporal light modulator must be less than the ratio of the allowable image smearing to the image length. Alternately, it may be stated that the exposure time T_2 must be no more than a small fraction ℓ/n of the period of the signal $v(t)$.

Assuming that these constraints have been obeyed, it follows from the results of the previous section that the exposure produced on the film strip by $\psi(x,t)$ as given in Eq. (8) and $I_o(t)$ as given in Eq. (9) may be expressed as

$$\mathcal{E}(x') = \frac{I_o T_2}{2} \sum_{k=1}^{\infty} \operatorname{rect}\left(\frac{x'-ks'T_o}{ns/f_o}\right) \cdot$$

$$\left(1 + \frac{b\psi_k}{2} \cos\left[\frac{\omega_o}{s}(x' - ks'T_o) - (\phi_k - \theta_k)\right]\right). \tag{12}$$

It is seen that the film is exposed to a series of grating images, and that the phase of the kth image is equal to the phase difference between the corresponding pulses of $\psi(x,t)$ and $I_o(t)$ plus the indexed constant $\omega_o ks'T_o/s$. It is easy to show that this indexed constant produces a constant calibration factor when the film record is subsequently used for coherent integration as in the Synthetic Spectrum Radar. Also, if the recording-system parameters are selected so that $\omega_o s'T_o/s$ equals an integer times 2π, this phase factor may be ignored altogether. Note in addition that the exposure is proportional to T_2, the exposure time, and to I_o, the average intensity of incident light during each pulse of light. We also point out the presence of a constant DC component of exposure in each image, and the fact that the amplitude of the grating component is proportional to ψ_k, the amplitude of the corresponding component of $\psi(x,t)$.

IV. RESPONSE OF PHOTOGRAPHIC FILM

So far, we have only computed the exposure accorded to a strip of photographic film, and have not considered the response of the film. This section presents a discussion of pertinent film characteristics.

It is assumed that the film record will be read out in a coherent optical system, as is the case for the Synthetic Spectrum Radar. In this situation we are interested in the linearity of the film's transfer characteristic. A typical photographic negative film has a transfer characteristic similar to the one sketched in Fig. 3. This graph shows the dependence of amplitude transmittance on exposure for a negative film after development. We observe that as exposure increases from zero, the transmittance starts from a maximum value Q_{max} and falls off slowly at first, then more rapidly. It is near the center of this region of strong dependence on exposure that the film response is closest to being linear. As exposure increases further, the transmittance falls off more and more slowly, approaching zero.

It is apparent from Fig. 3 that the requirements of a linear transfer characteristic lead us to bias each exposure at $\mathcal{E}_o$, in the center of the linear region of the curve. This way the variations in exposure will be converted linearly to variations of the transmittance about Q_o. Note from Eq. (12) that each grating image rides on a constant exposure bias equal to $I_oT_2/2$. Adjustment of I_o and T_2 provides a straightforward means for producing the most linear record of each grating image.

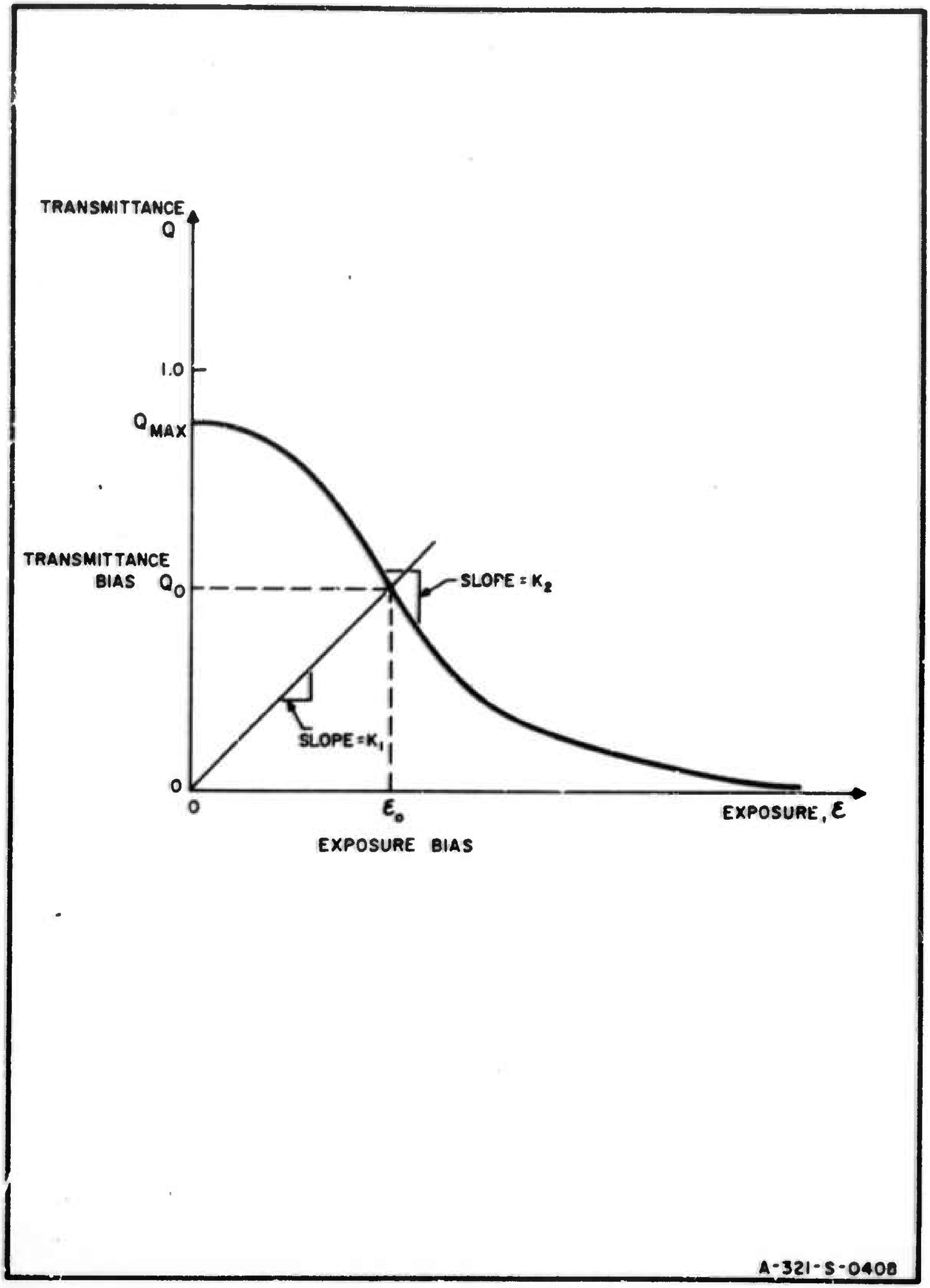

FIG. 3 PHOTOGRAPHIC FILM CHARACTERISTIC

If the best bias point of the film is such that $Q_o/\mathcal{E}_o$ is equal to K_1 and the incremental slope at the bias point is equal to $-K_2$ (as shown in Fig. 3), then the amplitude transmittance of the photographic film when subjected to the exposure of Eq. (12) is

$$Q(x') = \frac{I_o T_2 K_1}{2} \sum_{k=1}^{\infty} \text{rect}\left(\frac{x' - ks'T_o}{ns/f_o}\right) \cdot$$

$$\left[1 - \frac{bK_2\psi_k}{2K_1} \cos\left[\frac{\omega_o}{s}(x' - ks'T_o) - (\phi_k - \theta_k)\right]\right] \cdot \qquad (13)$$

Note that in the gaps between adjacent pulse images on the film the exposure is zero and the transmittance is Q_{max}. It is implicit in Eq. (13) that ψ_k is not large enough to necessitate the inclusion of nonlinear terms in the film response.

We conclude from Eq. (13) that if the film is exposed in its linear region to the light in the image plane of the recorder shown in Fig. 2, then a linear photographic record of each phase-referenced pulse image is produced. In the next section we will show how this technique may be applied to the Synthetic Spectrum Radar.

By means of summarizing Eq. (13), we point out that every pulse image rides on a transmittance bias given by

$$Q_o = \frac{I_o T_2 K_1}{2} \qquad (14a)$$

and the amplitude of the transmittance for the kth grating image is

$$Q_k(x') = Q_o \frac{bK_2}{2K_1} \psi_k , \qquad (14b)$$

where

$$\frac{bK_2}{2K_1} \psi_k < 1.$$

It is possible to modify slightly the mode of the spatial light modulator and while maintaining the same type of signal-recording property that was shown for the system drawn in Fig. 2. The properties of the other two possible modes are discussed and compared with the above configuration in the Appendix to this report. It is shown there that for the purposes of this report one of these configurations is inferior to the one discussed above, while the other may be considered equivalent to the configuration of Fig. 2.

V. RECORDER FOR SYNTHETIC SPECTRUM RADAR

In the case of the Synthetic Spectrum Radar, we desire to record on film all the IF output signals, referencing their phases to the zero channel IF output. The form of the electrical output from the mth IF channel is

$$R_m(t) = \sum_{k=1}^{\infty} A_{mk} E(t - kT_o - T_{mk}) \cos(\omega_o t + \phi_{mk}), \qquad (15)$$

where the envelope $E(t)$ varies very slowly with respect to the carrier ω_o. The electro-optical recorder system for performing this task is drawn schematically in Fig. 4.

We see that the system is illuminated by a laser which is gated on and off in synchronism with each radar return pulse. The laser beam then passes through a temporal light modulator, which is driven by the signal in the reference or zeroth channel of the radar IF. The purpose of the signal conditioner is to provide a constant-amplitude version of the reference signal to the temporal light modulator. Spatial filtering and collimation of the laser beam are then performed by the spreading lens, pinhole, and collimating lens. After this collimated light passes through a rectangular aperture, it is incident upon a 100-channel spatial light modulator. The mth channel of the spatial light modulator is driven by $R_m(t)$, producing the disturbance $\psi_m(x,t)$. The signals $\psi_m(x,t)$ are spatially multiplexed, that is, they travel side by side in the x direction without interfering with each other. Light intensity $I_o(t)$ is

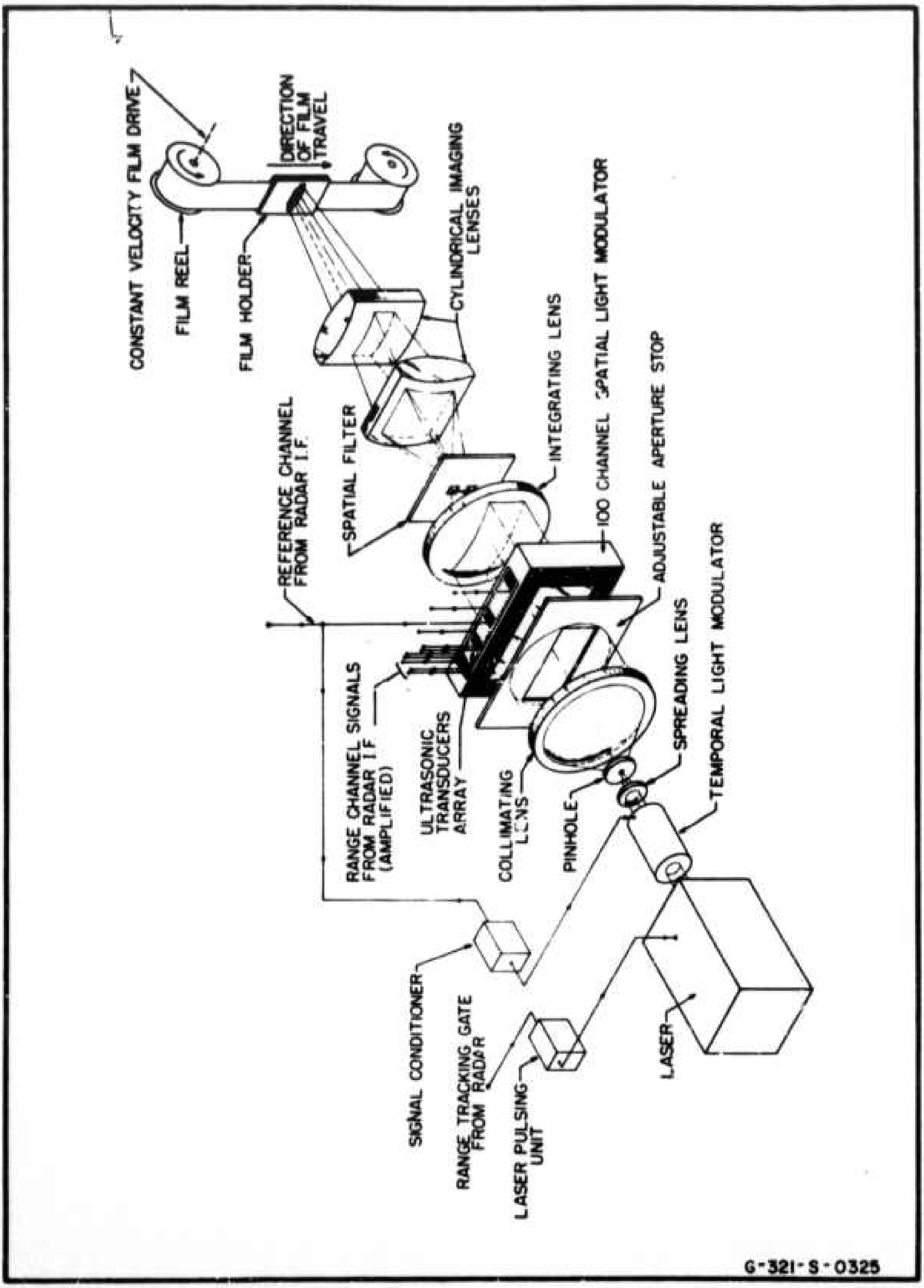

FIG. 4 ELECTRO-OPTICAL RECORDER FOR THE SYNTHETIC SPECTRUM RADAR

incident on the spatial light modulator. The light transmitted by the spatial light modulator is spatially filtered in the focal plane of the integrating lens so that only the zero order and one first order are allowed to pass to the cylindrical imaging lenses. The cylindrical imaging lenses enable the photographic image to be contracted in width so it fits on a standard-size strip of film, while leaving the scale of the orthogonal dimension free to be adjusted independently. The film is driven at constant speed through the film holder.

In the light of the discussions in the preceding sections, we express the optical phase perturbation in the mth channel of the spatial light modulator as

$$\psi_m(x,t) = \sum_{k=1}^{\infty} \psi_{mk} E\left(t - \frac{x}{s} - kT_o - T_{mk}\right) \cos\left(\omega_o\left[t - \frac{x}{s}\right] + \phi_{mk}\right). \tag{15}$$

We now assume that the signal driving the temporal light modulator, which is derived from $R_o(t)$, is amplitude-stabilized by the signal conditioner, and that each radar return pulse turns on the laser to a constant intensity for time T_2. In this case, we find that the light intensity incident on the spatial light modulator may be expressed as

$$I_o(t) = \frac{I_o}{2} \sum_{k=1}^{\infty} \mathrm{rect}\left(\frac{t - kT_o - T_{mk}}{T_2}\right)\left(1 + b \sin[\omega_o t + \phi_{ok}]\right). \tag{16}$$

If the envelope $E(t)$ associated with pulses of $\psi_m(x,t)$ varies slowly over the aperture of the spatial light modulator, and if in addition this envelope varies little during the exposure time T_2, then for the purposes of the signal recorder we may assume that $\psi_{mk}E(t-\frac{x}{s}-kT_o-T_{mk})$ is simply equal to a constant ψ_{mk} during the exposure time. This condition is satisfied in the Synthetic Spectrum Radar. Under these assumptions, it is seen that the object-image relationships in each channel of this recorder are identical to those derived for the single-channel recorder in Sec. IV except for the possible scale-change due to the cylindrical imaging lenses. Applying the results of Sec. III and the discussion of film response in Sec. IV, we can show that the amplitude transmittance in the mth channel of the developed film strip is simply given by

$$Q_m(x') = \frac{I_oT_2K_1}{2} \sum_{k=1}^{\infty} \text{rect}\left[\frac{x'-ks'(T_o+T_{mk})}{ns/f_o}\right] \cdot$$

$$\left(1-\frac{bK_2\psi_{mk}}{2K_1}\cos\left[\frac{\omega_o}{s}(x'-ks'T_o)-(\phi_{mk}-\phi_{ok})\right]\right) \qquad (17)$$

in the exposed areas of the film and by the constant Q_{max} in the gaps between the exposed areas. Note that except for the phase factor $\omega_o ks'T_o/s$, each pulse in each channel is recorded linearly and has been phase-referenced with respect to the phase of the zeroth-channel signal. However, it is possible to adjust s' so that the factor $\omega_o s'T_o/s$ is equal to an integer times 2π, in which case the phase $\omega_o ks'T_o/s$ becomes another integer times 2π and may be disregarded.

There is a second, more involved reason for disregarding the phase $\omega_o ks'T_o/s$ in the case of the Synthetic Spectrum Radar; the next signal-processing step in the Synthetic

Spectrum Radar is coherent integration. In this situation the stepped phase increments of $\omega_o s' T_o / s$ will simply appear as a constant frequency displacement when coherently integrated. (In fact, since the phase steps $\omega_o s' T_o / s$ rad in the interpulse period T_o, we see that the constant apparent frequency shift is $\omega_o s' / s$ rad/sec). Clearly, this constant frequency shift can be calibrated out of the results of coherent integration. Thus, we see that the electro-optical recorder described in this report is able to perform a significant and difficult portion of the signal processing for the Synthetic Spectrum Radar.

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VII. APPENDIX

ALTERNATE MODES OF OPERATION

It was shown in the main body of this report that the desired recording properties are obtainable for the Synthetic Spectrum Radar when the laser light is normally incident on the spatial light modulator, and when the zero order and one of the first orders of the resulting acoustic grating diffraction pattern are allowed to pass through the spatial filter. We will show in this section that two alternative schemes are of interest; (1) light is normally incident on the spatial modulator and only the two first orders are passed by the spatial filter, and (2) light is incident on the spatial modulator at the Bragg angle of the acoustic grating and the zero and first orders are passed by the spatial filter. We will call the mode of operation described at the beginning of this paragraph the N_{01} mode, while the two following modes will be called the N_{11} mode and the B_{01} mode, respectively. It will be made apparent that for our purposes the B_{01} mode is equivalent to the N_{01} mode, while the N_{11} mode is inferior to the other two, because of its severely limited dynamic range and because it halves the unambiguous Doppler range when used for the Synthetic Spectrum Radar.

We will now find the relationship between film transmittance and input signals for the N_{11} mode. Paralleling the discussion of Sec. II, for a continuous wave input signal and in connection with Fig. 1, we express the total electric field arriving in plane 3 of Fig. 1 when the two first orders are passed by the spatial filter as

$$E(x',t) = jI_o^{\frac{1}{2}} \frac{\psi_m}{2} \left(e^{j[\omega_o(t - \frac{x'}{s}) + \phi]} + e^{-j[\omega_o(t - \frac{x'}{s}) + \phi]} \right) \tag{A-1}$$

$$= jI_o^{\frac{1}{2}} \psi_m \cos [\omega_o(t - \frac{x'}{s}) + \phi_o] .$$

The intensity associated with $E(x',t)$ is

$$I(x',t) = \frac{I_o \psi_m^2}{2} \left[1 + \cos 2[\omega_o(t - \frac{x'}{s}) + \phi \right] . \tag{A-2}$$

We observe that stationary and traveling components of intensity are present, and that at any given point x', the intensity varies at circular frequency $2\omega_o$. Thus, I_o must be amplitude modulated at circular frequency $2\omega_o$ in order to produce a stationary sinusoidal pattern. That is,

$$I_o \rightarrow I_o(t) = \frac{I_o}{2} [1 + b \cos 2(\omega_o t + \phi_o)] . \tag{A-3}$$

Substituting in Eq. (A-2), it is found that

$$I(x',t) = \frac{I_o \psi_m^2}{4} \left\{ 1 + b \cos 2(\omega_o t + \phi_o) + \cos 2[\omega_o(t - \frac{x'}{s}) + \phi] \right.$$

$$\left. + \frac{b}{2} \left(\cos 2[-\omega_o \frac{x'}{s} + (\phi - \phi_o)] + \cos 2[\omega_o(2t \ \frac{x'}{s}) + \phi + \phi_o] \right) \right\} . \tag{A-4}$$

If this intensity is integrated by the photographic film for time $T_2 \gg \frac{2\pi}{\omega_o}$, it follows that the exposure is given by

$$\mathcal{E}(x') = \frac{I_o \psi_m^2 T_2}{4} \left(1 + \frac{b}{2} \cos 2[\frac{\omega_o x'}{s} - (\phi - \phi_o)]\right) \quad . \qquad \text{(A-5)}$$

This equation is analogous to Eq. 7 for the N_{01} mode, and represents the exposure accorded the photographic film in the N_{11} mode for a continuous-wave input to the acoustic light modulator. Note that the DC exposure term in the N_{11} mode is dependent on input signal amplitude ψ_m, while the ratio of the peak of the sinusoidal component to the DC component is equal to a constant, b/2. We also observe that the amplitudes of both the DC and sinusoidal components of the exposure are proportional to ψ_m^2. Finally, note that the phase of the sinusoidal component of exposure is equal to twice the phase difference $(\phi - \phi_o)$.

By following the approach used in Secs. III-VI, we can show in a rather straightforward manner that if the recorder for the Synthetic Spectrum Radar is operated in the N_{11} mode, the transmittance of the mth channel of the photographic film record that is produced will be

$$Q_m(x') = \frac{I_o T_2 K_1}{4} \sum_{k=1}^{\infty} \psi_{mk}^2 \operatorname{rect}\left[\frac{x' - ks'(T_o + T_{mk})}{ns/f_o}\right] \bullet \qquad \text{(A-6)}$$

$$\left(1 - \frac{bK_2}{2K_1} \cos[2 \frac{\omega_o}{s}(x' - ks'T_o) - (\phi_{mk} - \phi_{ok})]\right),$$

where all symbols have the same meaning as they did in the main body of this report. This transmittance is similar to the transmittance obtained for the N_{01} mode, as given by Eq. 17. However, note that in this case each recorded grating rides on a variable transmittance bias given by

$$Q_{omk} = \frac{I_o T_2 K_1}{4} \psi^2_{mk} \quad (N_{11}) , \tag{A-7a}$$

while we see from Eq. 17 that in the N_{01} mode the transmittance bias is a constant given by

$$Q_o = \frac{I_o T_2 K_1}{2} \quad (N_{01}) . \tag{A-7b}$$

However, note from the film characteristic in Fig. 3 that if the bias shifts significantly, the incremental slope K_2 will also change, resulting in a decidedly nonlinear record of the grating amplitude ψ^2_{mk}. This tendency to become nonlinear limits the dynamic range of the N_{11} mode. Notice, by comparison, that the constant bias of the N_{01} mode allows much larger variations in ψ_{mk} before nonlinear effects may be expected.

Another disadvantage of the N_{11} mode may be seen by the fact that the phase of the recorded pulse is equal to $2(\phi_{mk} - \phi_{ok})$; in order to avoid Doppler ambiguity in the coherent integration that will be performed on the film record for the Synthetic Spectrum Radar, we require that the magnitude of the pulse-to-pulse phase shift in any channel be less than π; this condition may be written as

$$\left| 2(\phi_{mk+1} - \phi_{ok+1}) - 2(\phi_{mk} - \phi_{ok}) \right| < \pi \quad . \tag{A-8a}$$
$$(N_{11})$$

In contrast, for the N_{01} system, the phase of the recorded pulse is equal to $(\phi_{mk} - \phi_{ok})$, so that the phase ambiguity condition may be written:

$$\left| (\phi_{mk+1} - \phi_{ok+1}) - (\phi_{mk} - \phi_{ok}) \right| < \pi \; . \tag{A-8b}$$
$$(N_{01})$$

Upon comparison of Eqs. (A-8a) and (A-8b) we conclude that the photographic record obtained in the N_{01} mode has twice the allowable pulse-to-pulse phase shift as the N_{11} mode. This means that the unambiguous Doppler range is twice as great in the N_{01} mode as it is in the N_{11} mode. Thus, we conclude that for our purposes the N_{11} mode is inferior to the N_{01} mode of the electro-optical signal recorder for the Synthetic Spectrum Radar because of the limitations on dynamic range and unambiguous Doppler range.

We now turn out attention to the Bragg mode (B_{01}). In this mode, the spatial light modulator is tilted so that the light beam is incident on the acoustic beam at the Bragg angle of the acoustic grating. If the acoustic wavelength is λ_s and the optical wavelength in the spatial light modulator is λ_L, then the Bragg angle is approximately equal to $\lambda_L/2\lambda_s$. The Bragg mode is of interest because Gill[4] has shown that under certain conditions* the acoustic grating produces only a zero order and <u>one</u> first order; all other grating orders are negligible. He shows that the electromagnetic wave emerging from the spatial light modulator, when the phase perturbation is given by

$$\psi(x,t) = \psi_m \cos\left[\omega_o\left(t - \frac{x}{s}\right) + \phi\right] \tag{A-9}$$

is simply

$$E_1(x,t) = I_o^{\frac{1}{2}}\left[\cos\frac{\psi_m}{2} + \sin\frac{\psi_m}{2}\, e^{j\left[\omega_o\left(t - \frac{x}{s}\right) + \phi\right]}\right]. \tag{A-10}$$

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* We will not discuss these conditions here, except to say that they can be met easily and do not hamper the functioning of the recording system.

The first term in the brackets of Eq. (A-10a) is the zero order light, and note that it has amplitude $I_o^{\frac{1}{2}} \cos \frac{\psi_m}{2}$, while the second term is the first order light, and has amplitude $I_o^{\frac{1}{2}} \sin \frac{\psi_m}{2}$.

We now assume that the Bragg-mode spatial light modulator is used in the recorder of Fig. 1, and that Eq. (A-10a) expresses the light amplitude that is emergent from this modulator. Under the additional assumption that ψ_m is small, we may express $E_1(x,t)$ as

$$E_1(x,t) = I_o^{\frac{1}{2}} \left(1 + \frac{\psi_m}{2} e^{j[\omega_o(t-\frac{x}{s}) + \phi]} \right) . \quad \text{(A-10b)}$$

If the spatial filter is set up to pass the two grating orders, it is obvious that the electric field in the image plane is the same as $E_1(x,t)$; that is, the field in the image plane is

$$E(x',t) = I_o^{\frac{1}{2}} \left(1 + \frac{\psi_m}{2} e^{j[\omega_o(t-\frac{x'}{s} + \phi]} \right) . \quad \text{(A-11)}$$

Computing the intensity in the image plane we find

$$I(x',t) = I_o(1 + \psi_m \cos [\omega_o(t - \tfrac{x}{s}) \cdot \phi]) \quad . \quad \text{(A-12)}$$

This expression is analogous to Eq. (4) for the N_{01} mode. If we let the light intensity I_o be a function of time as given by

$$I_o \rightarrow I_o(t) = \frac{I_o}{2} [1 + b \cos (\omega_o t + \phi_o)] \quad , \quad \text{(A-13)}$$

the intensity in the image plane becomes

$$I(x',t) = \frac{I_o}{2}\left\{1 + \psi_m \cos\left[\omega_o\left(t - \frac{x'}{s}\right) + \phi\right] + b\cos(\omega_o t + \phi_o) + \frac{b\psi_m}{2}\left(\cos\left[-\frac{\omega_o x'}{s} + (\phi - \phi_o)\right] + \cos\left[\omega_o\left(2t - \frac{x'}{s}\right) + \phi + \phi_o\right]\right)\right\} \tag{A-14}$$

Proceeding as we did in Sec. II, we now assume that the film integrates $I(x',t)$ for time $T_2 \ll 2\pi/\omega_o$, so that the film exposure is

$$\mathcal{E}(x') = \frac{I_o T_2}{2}\left(1 + \frac{b\psi_m}{2}\cos\left[\frac{\omega_o x'}{s} - (\phi - \phi_o)\right]\right) . \tag{A-15}$$

Comparison with Eq. 7 shows that this is identical to the exposure obtained in the N_{01} mode. It is apparent that since the Bragg mode is functionally identical to the N_{01} mode for a continuous-wave input, it will also be functionally identical when used in the 100-channel recorder for the Synthetic Spectrum Radar sketched in Fig. 4.

In summary, we point out that this Appendix has described and compared the electro-optical recorder when operated in the N_{01} mode, the N_{11} mode, and the Bragg mode; it has been shown that when applied to the requirements of the Synthetic Spectrum Radar, the N_{01} and Bragg modes are satisfactory and equivalent in function, while the N_{11} mode is inferior to the other two because of its limited dynamic range and unambiguous Doppler range.

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13. ABSTRACT

The Synthetic Spectrum Radar requires an on-line recorder capable of producing a phase-aligned photographic record of its 100 IF output signals. We describe herein an electro-optical configuration capable of producing such a photographic recording. A theoretical analysis of the recorder is provided and leads to expressions relating the light amplitude transmittance of the record to the electrical input signal. Alternate modes of operation of the recorder are discussed.

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