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THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRICAL ENGINEERING
Radiation Laboratory

Scattering from Small Obstacles on an Infinite Conducting Plane

Technical Report No. 3

MARTIN A. PLONUS

December 1966

Contract AF 04(694)-834
Lieutenant J. Wheatley, Contract Monitor

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Deputy for Ballistic Missile Re-Entry Systems
Norton Air Force Base, California 92409

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FOREWORD

This report was prepared by the Radiation Laboratory of the Department of Electrical Engineering of The University of Michigan under the direction of Dr. Raymond F. Goodrich, Principal Investigator and Burton A. Harrison, Contract Manager. The work was performed under Contract AF 04(694)-834, "Investigation of Re-entry Vehicle Surface Fields (SURF)". The work was administered under the direction of the Air Force Ballistic Systems Division, Norton Air Force Base, California 92409, by Lieutenant J. Wheatley, BSYDF and was monitored by Mr. H. J. Katzman of the Aerospace Corporation.

The publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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ABSTRACT

The scattered field from small (with respect to wavelength) obstacles which are mounted on an infinite, perfectly conducting plane is determined. The obstacles considered are a monopole, half-sphere, half-cylinder, half-loop and a slot. Since the reradiated field due to these obstacles can be identified with the radiation of a combination of electric and magnetic dipoles, the solution is presented as scattering dipole moments. These moments are induced in the obstacle by the incident wave and depend on the direction and polarization of the incident energy.

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I
INTRODUCTION

Large scattering bodies often have protrusions or small obstacles mounted on them. It would be desirable to know the scattering behavior of these obstacles. For example, conical or spherical objects occasionally have slots cut into their surface which serve as telemetry transmitting slots. When such an object is illuminated by a radar, the slots will affect the back scattered signal. In this report, the effects of obstacles will be calculated by considering their size to be small with respect to wavelength. This has several advantages as far as the analysis is concerned. First, it permits the simpler methods of Rayleigh scattering. Secondly, it allows us to use the scattering results of a small obstacle mounted on an infinite, perfectly conducting plane, when we are considering scattering by a small obstacle which is now mounted on a large cone, sphere or some other large "mother" body. As long as the radius of curvature at the point of such a body where the obstacle is placed is large with respect to wavelength, the region around the obstacle can be approximated by an infinite plane and the results for the "obstacle on an infinite plane" can be used.

The work reported here can be divided essentially into two parts. In the first part we are primarily interested in deriving the scattering behavior of some obstacles when these are placed on an infinite plane. In this respect we would like to consider a monopole, a half-sphere, a half-cylinder and a half-loop protruding from a plane, in addition to a slot cut into the plane. The analysis for the scattering of the slot, since it is of a more difficult nature constitutes the second part.

The method of analysis is as follows. Since we are considering obstacles whose size is small with respect to wavelength, the scattering from these obstacles can always be identified with dipole radiation terms. This is, in general, a small obstacle will reradiate the energy intercepted from an incident wave. This reradiation can be identified as the field of a combination of radiating electric and magnetic dipoles. The radiating electric dipoles are normal to the plane, whereas the radiating magnetic dipoles are flush with the plane. The strength of the dipole moments is related to the direction and polarization of the incident wave. These induced dipole moments will be called the scattering dipole moments. Of course once these moments are determined the solution to the obstacle scattering problem is solved. In this report we will determine the scattering moments of a monopole, half-sphere, half-cylinder, half-loop and a slot.

II
MONOPOLE ABOVE AN INFINITE, PERFECTLY CONDUCTING PLANE

The scattered field of a monopole in the region above a perfectly conducting plane can be obtained by using image theory. In Fig. 2-1 a plane wave is incident at angle θ_i. The boundary condition $\hat{n}\times\underline{E}=0$ on the plane can be reproduced (when the conducting plane is removed) by an image wave coming from below at angle $\pi-\theta_i$.

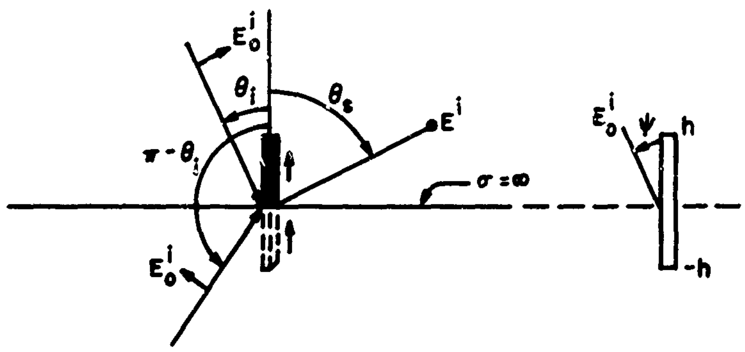


FIG. 2-1

The image wave in the region above the plane is then identified as the reflected wave. The field above the plane can be obtained by placing an image monopole symmetrically below the plane and calculating the field of a dipole of twice the monopole length in free space, i.e.

$$E^s(\theta_s) = E_i(\theta_i)R(\theta_i,\theta_s) + E^i(\pi-\theta_i)R(\pi-\theta_i,\theta_s) \tag{2.1}$$

where $R(\theta_i,\theta_s)$ is the bistatic reflection coefficient of a dipole which transforms an incident field at θ_i to a scattered field at θ_s.

When a cylindrical wire is located along the z-axis and a plane wave is incident at an arbitrary angle θ_i the tangential component of the incident electric field on the surface of the cylinder is

$$E_z^i = E_o \cos\psi \sin\theta \, e^{-jkz\cos\theta + j\omega t} \tag{2.2}$$

where E_o^i makes an angle ψ with the plane containing the incidence direction and the axis of the wire. Introducing the value of E_z^i in the expression for the scattered field vector potential will yield the usual integro-differential equation. This equation can be solved for by variational or iterative procedures. The current distribution depends on k and θ, but its dependence on ψ is simply that of $\cos\psi$. In general the current along a dipole with end points at $z = \pm h$ that is used as a transmitting antenna is well approximated by a sinusoidal distribution. However, the same dipole used as a receiving antenna has a current distribution that varies with the direction of the incident wave and is usually different from sinusoidal unless it is of resonant length. At oblique incidence the induced current does not have symmetry with respect to the center of the wire. When a plane wave is used to excite a wire there is a phase shift along the antenna as the incident wave passes. Only for broadside incidence where the E vector is parallel to the wire are all points of the wire excited in equiphase. For broadside incidence the current is symmetric, i.e. $I_z(d) = I_z(-d)$. For arbitrary incidence a symmetric as well as an antisymmetric current $I_z(d) = -I_z(d)$ is excited in the antenna. Since these induced currents radiate, the scattered far-zone fields produced by them (King, 1956) are

$$E_\theta = \frac{j\omega\mu_o}{4\pi} \sin\theta_s \frac{e^{-j\beta R_o}}{R_o} \int_{-h}^{h} \left[I^s(z') + I^a(z') \right] e^{j\beta z' \cos\theta_s} dz \qquad (2.3)$$

The current must go to zero at the end points: $I(\pm h) = 0$. This also implies that the symmetric or antisymmetric part alone must vanish at the ends of the wire. For example, on a cylinder with resonant length $(h \approx \lambda/4)$ the symmetric component of the induced current dominates, whereas for cylinders with antiresonant length $(h \approx \lambda/2)$ the antisymmetrical current dominates. The antisymmetric current is also characterized by a zero at the center of the wire, whereas the symmetric component has a maximum there.

For wires small with respect to wavelength the incident wave cannot produce enough phase shift as it travels past the wire to excite an antisymmetrical component of current. Hence for small wires the current induced by a plane wave at arbitrary angles of incidence is predominantly symmetric. We therefore conclude that for dipoles not much longer than $2h = \lambda/2$ the symmetric current excited by an incident field from any direction predominates. In general this current is well approximated by a triangular distribution for short antennas, a sinusoidal distribution for resonant antennas, and a constant distribution for long antennas. The dependence on the incident wave is simply $E_0^i \cos\psi \sin\theta$, which has been experimentally verified (King, 1956; Chen and Liepa, 1964) for receiving and transmitting antennas. Such studies also indicate that for antennas such as the short dipole and the half-wave dipole the receiving and transmitting currents of the antennas are nearly identical.

2.1 Resonant and Short Dipoles

Assuming a symmetric sinusoidal current

$$I = I_m \begin{cases} \sin\left[k(h-z)\right] & z > 0 \\ \sin\left[k(h+z)\right] & z < 0 \end{cases} \tag{2.4}$$

is induced in the wire and that the antisymmetric component is zero (we will confine ourselves to wire length $h < \lambda/4$), (2.3) becomes, after integration

$$E_\theta = \frac{j\eta I_m}{2\pi r} e^{-jkr} \left[\frac{\cos(kh\cos\theta_s) - \cos kh}{\sin\theta_s}\right] \tag{2.5}$$

where $\eta = \sqrt{\mu/\epsilon} = 120\pi\,\Omega$ for free space. Total $\underline{E}$ and $\underline{H}$ at long distances from the antenna are at right angles to each other and the direction of propagation, in time phase, and related by η, i.e. $E_\theta = \eta H_\phi$.

Assuming a triangular current distribution

$$I = I_m \begin{cases} k(h-z) & z > 0 \\ k(h+z) & z < 0 \end{cases} \tag{2.6}$$

we obtain

$$E_\theta = \frac{j\eta I_m}{2\pi r} e^{-jkr} \tan\theta_s \sec\theta_s \left[1 - \cos(kh\cos\theta_s)\right] . \tag{2.7}$$

In the limit of $kh \ll 1$, (2.5) and (2.7) yield

$$E_\theta = \frac{j\eta I_m}{4\pi r} e^{-jkr} (kh)^2 \sin\theta_s . \tag{2.8}$$

In the last expression one should note that the assumed current is $I = I_m kh(1 - |z|h)$.

If more accurate (but still approximate) solutions to the induced currents are desired, the methods of Chen and Liepa (1964), King (1956), Tai (1952) Harrison and Heinz (1963), Harrison (1962), Vainstein (1959) and Ufimtsev (1962) can be used.

Now we must relate the magnitude of the current I_m to the incident field. This will be done by applying the law of conservation of energy as follows. The total energy radiated by the induced currents in the antenna can be computed by integrating over a closed surface which coincides with the antenna surface or alternately, choosing a surface of a large sphere of radius R. Equating these two expressions a solution for I_m is obtained.

The tangential component of the incident field is given by (2.2). Since the total tangential field is composed of the incident and the induced field, and since this field must be zero on the surface of the wire we have

$$E_{total} = E^i + E_{ind} = 0$$

which gives for the induced field

$$E_{ind} = -E^i \tag{2.9}$$

The energy radiated from the antenna is

$$W = \frac{1}{2}\,\mathrm{Re}\int_{-h}^{h} \underline{E}\cdot I^{*} dz$$

$$= -\frac{1}{2}\,\mathrm{Re}\int_{-h}^{h} E_o \sin\theta_i \cos\psi\, e^{jkz\cos\theta_i} I_m \sin k(h-|z|)\, dz$$

$$= -\frac{1}{2} E_o \sin\theta_i \cos\psi I_m \int_{-h}^{h} \cos(kz\cos\theta_i)\sin k(h-|z|)\, dz \tag{2.10}$$

for a sinusoidal antenna current. Performing the integration we obtain

$$W = E_o \cos\psi I_m \frac{\cos(kh\cos\theta_i) - \cos kh}{k\sin\theta_i} \quad . \tag{2.11}$$

Recalculating the power radiated by the Poynting method we have

$$W = \oiint \underline{P}_r \bullet \underline{dA} = \oiint \frac{1}{2}\left|E_\theta\right|\left|H_\phi\right| dA = \frac{\pi}{\eta}\int_0^{\pi} \left|E_\theta\right|^2 r^2 \sin\theta\, d\theta \quad . \tag{2.12}$$

Using (2.5) for E_θ, (2.12) becomes

$$W = \frac{\eta I_m^2}{4\pi}\int_0^{\pi} \frac{\left[\cos(kh\cos\theta) - \cos kh\right]^2}{\sin\theta} d\theta \quad . \tag{2.13}$$

Equating this to (2.11) we can solve for I_m

$$I_m = \frac{4\pi E_o \cos\psi(\cos kh\cos\theta_i - \cos kh)}{\eta k\sin\theta_i \int_0^{\pi} \frac{\left[\cos(kh\cos\theta) - \cos kh\right]^2}{\sin\theta} d\theta} \quad . \tag{2.14}$$

The integration when performed yields

$$C = \int_0^{\pi} \ldots = .5772\ldots + \ln 2kh - Ci(2kh) + \frac{1}{2}\sin 2kh\left[Si(4kh) - 2Si(2kh)\right] +$$

$$+ \frac{1}{2}\cos 2kh\left[.5772 + \ln kh + Ci(4kh) - 2Ci(2kh)\right] \qquad (2.15)$$

where the sine and cosine integrals

$$Si(x) = \int_0^{x} \frac{\sin x}{x}\, dx$$

$$Ci(x) = -\int_x^{\infty} \frac{\cos x}{x}\, dx$$

are tabulated.

The scattered field is then

$$E_\theta^s = j2E_o \cos\psi \, \frac{\cos(kh\cos\theta_i)\cos kh}{\sin\theta_i} \cdot \frac{\cos(kh\cos\theta_s) - \cos kh}{\sin\theta_s} \, \frac{e^{-jkr}}{Ckr} \ . \qquad (2.16)$$

2.2 Dipoles Short with Respect to Wavelength

For small kh, (2.16) becomes

$$\lim_{kh \to 0} E_\theta^s = j3E_o \cos\psi \sin\theta_i \sin\theta_s \frac{e^{-jkr}}{2kr} \ . \qquad (2.17)$$

It is apparent that this does not give the correct kh dependence in the limit. The trouble arises from the following: in the emf approach to the power calculation (2.10), it was assumed that the induced current is in phase with the forcing incident field. This is a valid assumption for resonant wire length. However, short dipoles are capacitive, i.e. the induced current leads the voltage by 90°. This can be easily seen from the near field expression for a Hertzian dipole which is

$$E_{\theta} = \frac{-j\eta I d\ell \sin\theta}{4\pi k r^{3}} e^{-jkr} \quad . \tag{2.18}$$

Let us then assume a triangular current which for short dipoles has the form

$$I = I_o \left(1 - \frac{|z|}{h}\right) \exp\left\{j\left[\frac{\pi}{2} - (kh)^n\right]\right\} \tag{2.19}$$

where n is an integer yet to be determined. This current has the correct behavior, namely when $kh \to 0$, $I = jI_o\left(1 - \frac{|z|}{h}\right)$, i.e. current leads the voltage by $\pi/2$. Using this current in the emf calculation for power (2.10) we obtain

$$W = \frac{1}{2} \mathrm{Re} \int_{-h}^{h} E_o \sin\theta_i \cos\psi \, e^{jkz\cos\theta_i} I_o^{*} \left(1 - \frac{|z|}{h}\right) \exp\left\{-j\left[\frac{\pi}{2} - (kh)^n\right]\right\} dz$$

$$= \frac{1}{2} E_o \sin\theta_i \cos\psi I_o^{*} \int_{-h}^{h} \cos(kz\cos\theta_i) \left(1 - \frac{|z|}{h}\right) (kh)^n dz$$

$$= E_o \sin\theta_i \cos\psi I_o^{*} \left[1 - \cos(kh\cos\theta_i)\right] \frac{(kh)^n}{k^2 h \cos^2\theta_i}$$

$$= \frac{1}{2} E_o \sin\theta_i \cos\psi I_o^{*} h(kh)^n \quad . \tag{2.20}$$

The radiated far field from a short wire with an assumed triangular current distribution as in (2.19) is from (2.8)

$$E_\theta = \frac{j\eta kh \sin\theta_s}{4\pi r} I_o \exp\left\{j\left[\frac{\pi}{2} - (kh)^n - kr\right]\right\} . \tag{2.21}$$

The radiated power using the Poyntings method (2.12) is then

$$W = \frac{\pi}{\eta}\int_0^\pi \left|E_\theta\right|^2 r^2 \sin\theta_s d\theta_s = \frac{\eta k^2 h^2}{12\pi} I_o I_o^* \tag{2.22}$$

Equating (2.22) and (2.20) we obtain for the current

$$I_o = \frac{6\pi E_o \sin\theta_i \cos\psi (kh)^n}{\eta k^2 h} \tag{2.23}$$

Therefore, the scattered field using (2.21) is

$$E_\theta^s = j3E_o \cos\psi \sin\theta_i \sin\theta_s (kh)^n \frac{\exp\left\{j\left[\frac{\pi}{2} - (kh)^n - kr\right]\right\}}{2kr} \tag{2.24}$$

In order to determine n we need another condition. To avoid a length discussion, let us use an expression for the back scattering cross section for small kh as given by Van Vleck, Bloch and Hamermesh (1947) and Mack and Reiffen (1964):

$$\sigma = \frac{\lambda^2 (kh)^6 \cos^4\psi \sin^4\theta}{9\pi \left[\log 4h/a_o - 1\right]^2} . \tag{2.25}$$

This expression is valid for $kh < .3$ and $ka_o \ll 1$, where a_o is the radius of the wire. It becomes more accurate as the ratio h/a_o gets larger. In any case this ratio should be chosen such that $2h/a_o > 100$. Using (2.25), n is established as $n = 3$, and the scattered field as

$$E_\theta^s = -\frac{E_o k^3 h^3 \cos\psi \sin\theta_i \sin\theta_s}{\log 4h/a_o - 1} \frac{e^{-jkr - j(kh)^3}}{kr} . \tag{2.26}$$

The $(kh)^3$ term in the phase can be ignored as being small.

That the kh dependence is correct can also be verified from recent results for the radiating antenna (King and Wu, 1965). For $kh \ll 1$ the current is found to be

$$I = \frac{j2\pi V_o^e}{\eta\psi(0)} kh \left(1 - \frac{|z|}{h}\right) \tag{2.27}$$

where $\psi(0)$ is a constant and V_o^e is the applied voltage $E_z = -V_o^e \delta(z)$ at the center of the antenna. It is here again seen that I and V are out of phase by 90^o. For the scattering problem, V_o^e becomes the induced voltage due to the incident field as given by (2.2), i.e.

$$V_o^e = \int_{-h}^{h} E_o \cos\psi \sin\theta_i \, e^{-jkz\cos\theta_i} dz = E_o \cos\psi \sin\theta_i 2h \, . \tag{2.28}$$

Using (2.8), the scattered field then becomes

$$E_\theta^s = - \frac{E_o k^3 h^3 \cos\psi \sin\theta_i \sin\theta_s}{\psi(0)} \frac{e^{-jkr}}{kr} \tag{2.29}$$

which agrees with (2.26) above.

2.3 Monopole over Perfectly Conducting Ground

The scattered field of a monopole of length h above a perfectly conducting plane can then be written using (2.1) and (2.16) as

$$E_\theta^s = 2E_\theta^s \quad \text{of (2.16)} \, . \tag{2.30}$$

The total electric field above the infinite plane is then given by the incident field, the image field and the scattered field. The image field is interpreted as the principal reflected wave from the plane in the absence of any obstacle. For example, when the E-vector is in the plane of incidence ($\psi = 0$) which is taken as the zx plane, the total field for a short monopole is given by

$$\underline{E} = E_o(\cos\theta_i \hat{i} + \sin\theta_i \hat{k})\, e^{jk(x\sin\theta_i - z\cos\theta_i)} + E_o(-\cos\theta_i \hat{i} + \sin\theta_i \hat{k}) \cdot$$

$$\cdot\, e^{jk(x\sin\theta_i + z\cos\theta_i)} - \frac{2E_o k^3 h^3 \sin\theta_i \sin\theta_s}{\log 4h/a_o - 1} \frac{e^{-jkr}}{kr} \quad . \qquad (2.31)$$

It can also be concluded that antisymmetric currents are not excited in a monopole above ground. The induced currents are always symmetric, i.e. $I(z) = I(-z)$. This comes about since the total tangential field at the wire due to the incident and image waves is an even function of z, i.e.

$$(E^i + E^{im})\hat{k} = \hat{k}\, 2E_o \sin\theta_i \cos(kz\cos\theta_i)\, e^{jkx\sin\theta_i} \quad . \qquad (2.32)$$

Since this tangential E field is the total forcing function for small dipoles we conclude that the induced currents are symmetric.

III
SCATTERING FROM A HALF-LOOP ON AN INFINITE PERFECTLY CONDUCTING PLANE

Scattering from a half-loop on a conducting plane can be obtained by considering scattering from a full loop with an incident and an image wave as the forcing functions. Let us therefore first derive expressions for the bistatic scattered field from a full loop in free space.

3.1 Scattering from a Loop in Free Space

Weston (1957) has obtained rigorous solutions for the loop in toroidal coordinates. In the far field, when r is large, the scattered field is given by

$$E_\theta^s = B\cos\theta \sum_{M=1}^{\infty} \frac{(-1)^M}{A(M)} \Big[J_{M+1}(ka\sin\theta) + J_{M-1}(ka\sin\theta)\Big] F_M(\theta_o, \phi) \tag{3.1}$$

where

$$F_M(\theta_o, \phi) = \cos\theta_o \sin\psi_o \cos M\phi \Big[J_{M+1}(ka\sin\theta_o) + J_{M-1}(ka\sin\theta_o)\Big] -$$
$$- \cos\psi_o \sin M\phi \Big[J_{M+1}(ka\sin\theta_o) - J_{M-1}(ka\sin\theta_o)\Big]$$

and

$$E_\phi^s = B\left\{\sum_{M=1}^{\infty} \frac{(-1)^M}{A(M)} \Big[J_{M+1}(ka\sin\theta) - J_{M-1}(ka\sin\theta)\Big] G_M(\theta_o, \phi) + \right.$$
$$\left. + 2\,\frac{\cos\psi_o}{A(0)}\, J_1(ka\sin\theta)\, J_1(ka\sin\theta_o)\right\} \tag{3.2}$$

where

$$G_M(\theta_o, \phi) = \cos\theta_o \sin\psi_o \sin M\phi \Big[J_{M+1}(ka\sin\theta_o) + J_{M-1}(ka\sin\theta_o)\Big] +$$
$$+ \cos\psi_o \cos M\phi \Big[J_{M+1}(ka\sin\theta_o) - J_{M-1}(ka\sin\theta_o)\Big] ,$$

$$B = E_{\shortparallel}(ka)^3 \pi \frac{e^{ikr}}{kr} ,$$

ψ_o = polarization angle measured from normal to plane of incidence,

$$A(M) = \left[(ka)^2 - M^2\right] B(M) + C(M) ,$$

$$B(M) = 2\left[\ln(8S_o) + \psi(1/2) - \psi(M + 1/2)\right] + 2\pi \int_0^{ka} \left[E_{2M}(2z) + iJ_{2M}(2z)\right] dz ,$$

$$C(M) = \pi (ka)^2 \left[E_{2M-1}(2ka) - E_{2M+1}(2ka) + iJ_{2M-1}(2ka) - iJ_{2M+1}(2ka)\right] ,$$

$E_{2M}(z)$ are the Weber functions (Erdelyi et al, 1953),

$J_N(z)$ are the Bessel functions of the first kind,

$\psi(z)$ is the logarithmic derivative of the gamma function,

$S_o = a/a_o$; a, a_o are the radii of the loop, wire, respectively

Without loss of generality, incidence is confined to the xz plane at an angle θ_o with respect to the z-axis. The geometry is shown in Fig. 3-1.

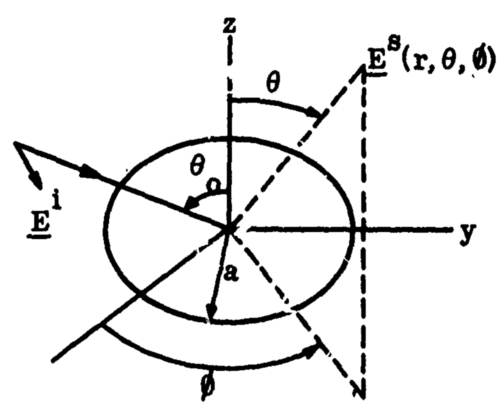


FIG. 3-1: A WIRE LOOP IN THE xy PLANE.

These expressions for the scattered field will simplify when small loops are considered, i.e. when ka < 1. In the following derivations we will retain second order terms of ka, and let terms of order $(ka)^3$ and higher be zero. The above functions, appropriate for small diameter loops are then

$$B(M) = \chi_M + \frac{-\pi(-1)^M (ka)^2}{\Gamma(3/2+M)\Gamma(3/2-M)} + \frac{i2\pi(ka)^{2M-1}}{(2M-1)(2M)!} \tag{3.3}$$

$$C(M) = \pi(ka)^2 \left[\frac{1}{\pi(M^2-1/4)} \left(1 + \frac{3(ka)^2}{M^2-9/4}\right) + \frac{i(ka)^{2M-1}}{(2M-1)!}\left(1 - \frac{(ka)^2}{2M(2M+1)}\right)\right] \tag{3.4}$$

$$A(M) = M^2\chi_M - \frac{M^2 i2\pi(ka)^{2M-1}}{(2M-1)(2M)!} + (ka)^2 \left[\chi_M + \frac{1}{M^2-1/4} + \right.$$

$$\left. + \frac{M^2\pi(-1)^M}{\Gamma(3/2+M)\Gamma(3/2-M)}\right] \tag{3.5}$$

$$F_M = \left(\frac{ka\sin\theta_o}{2}\right)^{M-1} \frac{1}{(M-1)!}\left[A_M^+ + \frac{(\sin\theta_o\, ka/2)^2}{M(M+1)} A_M^-\right] \tag{3.6}$$

$$G_M = \frac{(\sin\theta_o\, ka/2)^{M-1}}{(M-1)!}\left[a_M^- + a_M^+ - \frac{(\sin\theta_o\, ka/2)^2}{M(M+1)}\right] \tag{3.7}$$

$$F_{2M\pm1}(2ka) = \frac{\pm 1}{\pi(1/2 \pm M)}\left[1 - \frac{(ka)^2}{(1/2 \mp M)(3/2 \pm M)}\right] \tag{3.8}$$

$$J_{M+1}(ka\sin\theta_o) \pm J_{M-1}(ka\sin\theta_o) = \frac{(\sin\theta_o\, ka/2)^{M-1}}{(M-1)!}\left[\pm 1 + \frac{(ka\sin\theta_o)^2}{4M(M+1)}\right] \tag{3.9}$$

where

$$\chi_M = 2\left[\ln 8S_o + \psi(1/2) - \psi(M+1/2)\right]$$

$$A_M^{\pm} = \cos\theta_o \sin\psi_o \cos M\phi \pm \cos\psi_o \sin M\phi$$

$$a_M^{\pm} = \cos\theta_o \sin\psi_o \sin M\phi \mp \cos\psi_o \cos M\phi$$

$$\psi(1/2) = -1.96351\ldots$$

Substituting the above results in (3.1) and (3.2) we obtain for the scattered field from a small loop

$$E_\theta^s = B\cos\theta \frac{A_1^+}{\chi_1}\left\{1 - ka\frac{i\pi}{\chi_1} + (ka)^2\left[1 - \left(\frac{\pi}{\chi_1}\right)^2 + \frac{\sin^2\theta}{8} + \frac{A_1^- \sin^2\theta_o}{8A_1^+} - \frac{A_2^+\chi_1}{16A_1^+\chi_1}\sin\theta_o\sin\theta\right]\right\} \qquad (3.10)$$

and

$$E_\phi^s = B\frac{a_1^-}{\chi_1}\left\{S\frac{\chi_1}{a_1^-} - 1 + (ka)\frac{i\pi}{\chi_1} + (kd)^2\left[-1 + \left(\frac{\pi}{\chi_1}\right)^2 + \frac{\sin^2\theta}{8} - \frac{a_1^+\sin^2\theta_o}{8a_1^-} + \frac{a_2^-\chi_1}{16\chi_2 a_1^-}\sin\theta_o\sin\theta\right]\right\} \qquad (3.11)$$

where

$$S = \frac{\cos\psi_o \sin\theta \sin\theta_o}{2\chi_1}$$

These expressions are valid for arbitrary polarizations of the incident field. When the $\underline{E}$ vector is in the plane of incidence, $\psi_o = \pi/2$, and the incident electric field is $\underline{E}^i = \hat{i}_\theta E_o$. For polarization perpendicular to the plane of incidence, $\psi_o = 0$ and $\underline{E}^i = \hat{i}_\phi E_o$. The plane of incidence is the plane formed by the incidence direction and the normal to the loop $\hat{i}_z$.

The above expressions for the scattered fields are moderately complicated. The additional accuracy which the $(ka)^2$ terms give is usually not needed when loops small with respect to a wavelength are considered. Henceforth we will use terms only up to ka, which will give results of sufficient accuracy when $ka \ll 1$. The scattered fields can then be written as

$$E_\theta^s = \frac{B}{\chi_1}\left(1 - ka\frac{i\pi}{\chi_1}\right)(\cos\theta_o \sin\psi_o \cos\theta\cos\phi + \cos\psi_o \cos\theta\sin\phi) \tag{3.12}$$

$$E_\phi^s = \frac{B}{\chi_1}\left(1 - ka\frac{i\pi}{\chi_1}\right)(-\cos\theta_o \sin\psi_o \sin\phi + \cos\psi_o \cos\phi) + \frac{B}{2\chi_1}\cos\psi_o \sin\theta_o \sin\theta \tag{3.13}$$

A further examination of the above two results reveals that the scattered field from a loop is composed of two electric dipole terms and one magnetic dipole term. The scattered electric field can then be given as the sum of the fields of an x and y oriented electric dipole plus a z oriented magnetic dipole, i.e.

$$\underline{E} = \underline{E}^{ed}_{\hat{i}_x} + \underline{E}^{ed}_{\hat{i}_y} + \underline{E}^{md}_{\hat{i}_z} \tag{3.14}$$

where

$$\underline{E}^{ed}_{\hat{i}_x} = c_1(\hat{i}_r x \hat{i}_x) x \hat{i}_r \frac{e^{ikR}}{R}$$

$$= \frac{B}{\chi_1}\left(1 - ka\frac{i\pi}{\chi_1}\right)\cos\theta_o \sin\psi_o\left[\hat{i}_\theta\cos\phi - \hat{i}_\phi\sin\phi\right] \tag{3.15}$$

$$\underline{E}^{ed}_{\hat{i}_y} = c_2(\hat{i}_r x \hat{i}_y) x \hat{i}_r \frac{e^{ikR}}{R}$$

$$= \frac{B}{\chi_1}\left(1 - ka\frac{i\pi}{\chi_1}\right) \cos\psi_o \left[\hat{i}_\theta \cos\theta \sin\phi + \hat{i}_\phi \cos\phi\right] \quad (3.16)$$

$$\underline{E}^{md}_{\hat{i}_z} = c_3(\hat{i}_r x \hat{i}_z) \frac{e^{ikR}}{R} = \frac{B}{2\chi_1} \cos\psi_o \sin\theta_o \left[\hat{i}_\phi \sin\theta\right] \quad (3.17)$$

The factors inside the square brackets are the pattern factors of the corresponding dipole and the terms preceding the square brackets are the dipole moment strength. One can also observe that the scattering dipole moment depends on the amplitude, polarization and direction of the incident field as well as the geometry of the loop. It is clear now that a small loop is not a purely magnetic dipole type of scatterer and therefore cannot be used for measuring the magnetic field in the same way that a thin, short, linear wire can be used to measure the electric field (Justice and Rumsey, 1955). As a matter of fact, for polarization in the plane of incidence ($\psi_o = \pi/2$) the scattered field from a loop always looks like that of an x-oriented electric dipole. Similarly, for incidence normal to the plane of the loop, and for arbitrary polarization, the scattering loop acts as an electric dipole with orientation parallel to the incident electric vector. Hence it appears that the scattering loop behaves more like an electric than a magnetic dipole. At first, such behavior appears puzzling, expecially when one recalls that the antenna patterns of small loops are described in terms of equivalent magnetic dipoles. However, the differences can be explained as follows. For the small radiating loop we have a uniform current which is associated with the magnetic dipole. For the scattering loop the currents induced by the incident field are more complicated and in addition to a uniform current have other components which can be identified with electric dipoles.

3.2 Scattering from a Half-Loop

The above results can now be applied to scattering from a half-loop mounted on a conducting plane as shown in Fig. 3-2. The conducting plane is assumed to be

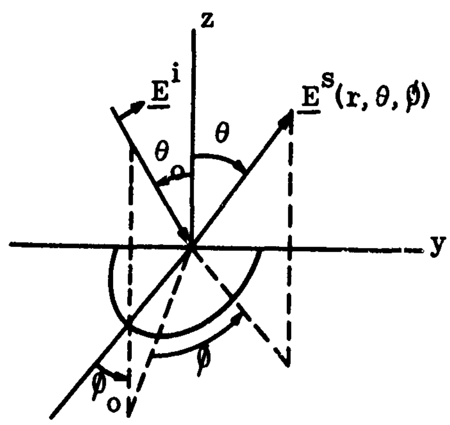


FIG. 3-2: GEOMETRY OF THE HALF-LOOP OF RADIUS d. The Conducting Plane is the zy Plane.

the zy plane. If the incident wave is now confined to the $\phi = 0$ plane a loss of generality would result. The expressions for the full loop can be extended to an arbitrary plane of incidence ϕ_o, simply by replacing every ϕ term by $\phi - \phi_o$. Then, the solution to the half-loop can be constructed from the solution to the full loop by the method of images as follows. The boundary condition that the total tangential electric field be zero on the conducting plane can be obtained by adding an image wave in the $x < 0$ space to the incident field such that

$$\hat{i}_x \times \left[\underline{E}^i(\theta_o, \phi_o, \psi_o) + \underline{E}^{im}(\theta_o, \pi - \phi_o, -\psi_o)\right]\Big|_{x=0} = 0 \quad . \tag{3.18}$$

If the full loop is illuminated by the incident and the image wave, then in the $x > 0$ space the scattered field that is obtained is the scattered field for a half-loop over a conducting plane. The conducting plane coincides with the zy plane. The total scattered field from the half-loop can then be written as

$$\underline{E} = \underline{E}^i(\theta_o, \phi_o, \psi_o) + \underline{E}^r(\theta_o, \pi-\phi_o, -\psi_o) + \underline{E}^s(\theta, \phi; \theta_o, \phi_o, \psi_o) + \underline{E}^s(\theta, \phi; \theta_o, \pi-\phi_o, -\psi_o) \tag{3.19}$$

where $\underline{E}^r$, given by $\underline{E}^{im}$, is the reflected field from the plane, and the remaining two terms are the scattered fields due to the incident and the image wave, respectively.

If the incident wave is given by

$$\underline{E}^i = \underline{E}^i_o \, e^{i\underline{k}\cdot\underline{r} - i\omega t} \tag{3.20}$$

where the incidence direction is given by

$$\hat{k} = -(\sin\theta_o \cos\phi_o \hat{i}_x + \sin\theta_o \hat{i}_y + \cos\theta_o \hat{i}_z) \tag{3.21}$$

and the polarization is

$$\begin{aligned}\hat{E}^i_o &= \hat{i}_\theta \sin\psi_o + \hat{i}_\phi \cos\psi_o \\ &= \hat{i}_x(\cos\theta_o \cos\phi_o \sin\psi_o - \sin\phi_o \cos\psi_o) + \hat{i}_y(\cos\theta_o \sin\phi_o \sin\psi_o + \cos\phi_o \cos\psi_o) - \\ &\quad - \hat{i}_z \sin\theta_o \sin\psi_o\end{aligned} \tag{3.22}$$

where the caps denote unit vectors, the components of the half-loop scattered field can be written as

$$\begin{aligned}&E^s_\theta(\theta, \phi; \theta_o, \phi_o, \psi_o) + E^s_\theta(\theta, \phi; \theta_o, \pi-\phi_o, -\psi_o) = \\ &\quad = \frac{2B}{\chi_1}\left(1 - ka\frac{i\pi}{\chi_1}\right)(\cos\theta_o \sin\psi_o \cos\phi_o - \cos\psi_o \sin\phi_o)\cos\phi\cos\theta\end{aligned} \tag{3.23}$$

and

$$E^s_\phi(\theta,\phi;\theta_o,\phi_o,\psi_o) + E^s_\phi(\theta,\phi;\theta_o,\pi-\phi_o,-\psi_o) =$$

$$= \frac{B}{\chi_1}\left[-2\left(1-ka\frac{i\pi}{\chi_1}\right)(\cos\theta_o\sin\psi_o\cos\phi_o - \cos\psi_o\sin\phi_o)\sin\phi + \cos\psi_o\sin\theta_o\sin\theta\right] \tag{3.24}$$

As in the case of the full loop, the scattered field from a half-loop can be decomposed into dipole fields. Identifying the respective terms, the scattered field from a half-loop on a conducting plane can be written as

$$\underline{E} = \underline{E}^{ed}_{\hat{i}_x} + \underline{E}^{md}_{\hat{i}_z} \tag{3.25}$$

where the scattering electric dipole contribution is

$$\underline{E}^{ed}_{\hat{i}_x} = \frac{2B}{\chi_1}\left(1-ka\frac{i\pi}{\chi_1}\right)(\cos\theta_o\sin\psi_o\cos\phi_o - \cos\psi_o\sin\phi_o)\left[\hat{i}_\theta\cos\theta\cos\phi - \hat{i}_\phi\sin\phi\right] \tag{3.26}$$

and the scattering magnetic dipole contribution is given by

$$\underline{E}^{md}_{\hat{i}_z} = \frac{B}{\chi_1}\cos\psi_o\sin\theta_o\left[\hat{i}_\phi\sin\theta\right] \tag{3.27}$$

When these expressions are compared to the corresponding ones for the full loop it is seen that the presence of the conducting plane doubles the magnetic dipole type of contribution, doubles the electric dipole contribution which is normal to the plane, and cancels the electric dipole field which is parallel to the plane.

3.3 Conclusion

It was shown that the scattered field from a small loop in free space can be identified with three dipole contributions. The equivalent scattering dipole moments which are induced by the incident field are expressed in terms of the loop geometry, the orientation and polarization of the incident wave. The scattered field of the loop shows a wavelength dependence which is $E^s \sim k^2 a^2$. Such a dependence is characteristic of Rayleigh scattering.

Scattering for a half-loop, mounted on a conducting plane, is similar to that for the free space loop. The differences are as follows. The induced magnetic dipole moment is doubled. The normal electric dipole contribution is present in altered form, but the electric dipole term which is parallel to the plane vanishes. These changes, in addition to identifying the image wave with the reflected field from the plane, account for the presence of the conducting plane and the arbitrary direction of incidence.

We have treated scattering from a half-loop and the full loop in terms of the equivalent scattering dipoles. Let us derive explicitly the scattering dipole moments which the incident field induces in the loop. The radiation field of an x oriented dipole with moment μ is given by

$$\underline{E} = \frac{k^2 p}{2\pi\epsilon} \frac{e^{ikR}}{R} \left[\quad \right] \tag{3.28}$$

where the square bracket is that of (3.26). If the dipole is a radiating current element $I\ell$, then we have from the continuity equation that the radiating dipole moment is $p = iI\ell/\omega$, where ℓ is the length of the current element. Equating (3.28) and (3.26) we can solve for the scattering dipole moment which is

$$p = \frac{4\pi^2\epsilon}{\chi_1} E_o^i a^3 \left(1 - ka\frac{i\pi}{\chi_1}\right) (\cos\theta_o \sin\psi_o \cos\phi_o - \cos\psi_o \sin\psi_o). \tag{3.29}$$

The term in the parenthesis can now be identified as the incident wave polarization component which is normal to the conducting plane, i.e.

$$p = \frac{4\pi^2\epsilon}{\chi_1} a^3 \left(1 - ka\frac{i\pi}{\chi_1}\right) \underline{E}_o^i \cdot \hat{n} \tag{3.30}$$

where n is the normal to the plane, in this case $\hat{n} = \hat{i}_x$. The remaining scattering dipoles can be similarly obtained.

IV
PLANE WAVE DIFFRACTION FROM A HALF-SPHERE ON AN INFINITE CONDUCTING PLANE

Scattering from a half-sphere on a conducting plane can be obtained by considering scattering from a sphere with an image wave in addition to the incident wave. Let us therefore first derive general expressions for the bistatic scattered field from a small sphere in free space.

4.1 Scattering from a Sphere

The Mie series is the well-known solution to the scattering problem of a sphere. For the special case of the small sphere, i.e. $ka \ll 1$, the first term only is important. If the illuminating field is a plane wave incident from the negative z-axis:

$$\underline{E}^i = E_o \hat{i}_x e^{i(\omega t - kz)} \tag{4.1}$$

the solution can be written as (Kerr, 1964)

$$\underline{E}^s = \frac{E_o e^{-ikR}}{kR} (ka)^3 \left[\hat{i}_\theta \cos\phi(\cos\theta + \frac{1}{2}) - \hat{i}_\phi \sin\phi(1 + \frac{1}{2}\cos\theta)\right] \tag{4.2}$$

where the caps denote unit vectors. This electric field looks like the field of an electric and magnetic dipole. We can therefore write

$$\underline{E}^s = \underline{E}^{ed}_{\hat{i}_x} + \underline{E}^{md}_{-\hat{i}_y}$$

$$= E'\left[(\hat{i}_r x \hat{i}_x) x\, i_r - \frac{1}{2}(\hat{i}_r x \hat{i}_y)\right] \tag{4.3}$$

where $E' = E_o e^{-ikR}(ka)^3/kR$. That is, the scattered field is that of an x oriented electric dipole and a y oriented magnetic dipole. The incident wave induces in the sphere an electric dipole and a magnetic dipole with moment half that of the electric dipole.

We would like to express the scattered field from a sphere for an incident field that has arbitrary direction and polarization, as for example,

$$\underline{E}^i = E_o \hat{a}_o e^{ik\hat{r}_o \cdot \underline{r} + i\omega t} \tag{4.4}$$

where the incidence direction is

$$\hat{r}_o = \hat{i}_x \sin\theta_o \cos\phi_o + \hat{i}_y \sin\theta_o \sin\phi_o + \hat{i}_z \cos\theta_o$$

and the polarization vector is given by

$$\hat{a}_o = \hat{\theta}_o \sin\psi_o + \hat{\phi}_o \cos\psi_o$$

$$= \hat{i}_x(\cos\theta_o \cos\phi_o \sin\psi_o - \sin\phi_o \cos\psi_\theta) + \hat{i}_y(\cos\theta_o \sin\phi_o \sin\psi_o +$$

$$+ \cos\phi_o \cos\psi_o) - \hat{i}_z \sin\theta_o \sin\psi_o \ .$$

The induced electric dipole will have the same direction as the incident electric vector, whereas the orientation of the induced magnetic dipole is determined by $\hat{r}_o x \hat{a}_o$. This is found by examining (4.2) and (4.3). The scattered field for an arbitrary incident wave can then be written as

$$\underline{E}^s = E' \left[(\hat{i}_r x \hat{a}_o) x \hat{i}_r + \frac{1}{2} \hat{i}_r x (\hat{r}_o x \hat{a}_o)\right]$$

$$= E' \left[\hat{a}_o (1 - \frac{1}{2} \hat{r}_o \cdot \hat{i}_r) + \hat{i}_r \cdot \hat{a}_o (\frac{1}{2} \hat{r}_o - \hat{i}_r)\right] \tag{4.5}$$

where the unprimed quantities denote coordinates of the observation point and the sub-zero coordinates are those of the incident field direction and polarization.

4.2 Scattering from a Half-Sphere

The above results can now be applied to scattering from a half-sphere mounted on a conducting plane. The conducting plane is assumed to be the zy plane. The solution to the half-sphere can be constructed from the solution to the full sphere by the method of images. The boundary condition that the total tangential electric field be zero on the conducting plane can be obtained by adding an image wave in the $x < 0$ space to the incident field such that

$$\hat{i}_x \times \left[\underline{E}^i(\theta_o, \phi_o, \psi_o) + \underline{E}^{im}(\theta_o, \pi - \phi_o, -\psi_o) \right] \Big|_{x=0} = 0 \, . \tag{4.6}$$

When the sphere is illuminated by the incident and the image wave, then in the $x > 0$ space the scattered field that is obtained is the scattered field for a half-sphere on a conducting zy plane. The total scattered field from the half-sphere can then be written as

$$\underline{E} = \underline{E}^i(\theta_o, \phi_o, \psi_o) + \underline{E}^r(\theta_o, \pi - \phi_o, -\psi_o) + \underline{E}^s(\theta, \phi; \theta_o, \phi_o, \psi_o) + \underline{E}^s(\theta, \phi; \pi - \phi_o, -\psi_o) \tag{4.7}$$

where $\underline{E}^r$, given by $\underline{E}^{im}$, is the reflected field from the plane, and the remaining two terms are the scattered fields due to the incident and the image wave, respectively.

Let us examine the scattered part, which is given by the last two terms of (4.7). Adding these two terms will give us an expression which after some algebra can be decomposed into dipole fields. Identifying the respective terms, the scattered field from a half-sphere on a conducting plane can be written as the field of three dipoles

$$\underline{E}^s = \underline{E}^{ed}_{\hat{i}_x} + \underline{E}^{md}_{\hat{i}_y} + \underline{E}^{md}_{\hat{i}_z} \tag{4.8}$$

where the contribution of the scattering electric dipole is

$$\underline{E}^{ed}_{\hat{i}_x} = 2E'(\sin\psi_o \cos\theta_o \cos\phi_o - \cos\psi_o \sin\phi_o) \left[\hat{i}_\theta \cos\theta \cos\phi - \hat{i}_\phi \sin\phi \right] \tag{4.9}$$

and the scattering magnetic dipole contributions are

$$\underline{E}^{md}_{\hat{i}_y} = E'(\sin\psi_o \cos\phi_o - \cos\psi_o \cos\theta_o \sin\phi_o)\left[\hat{i}_\phi \cos\theta \sin\phi - \hat{i}_\theta \cos\phi\right] \qquad (4.10)$$

and

$$\underline{E}^{md}_{\hat{i}_z} = -E' \cos\psi_o \sin\theta_o \left[\hat{i}_\phi \sin\theta\right] \qquad (4.11)$$

The factors inside the square brackets are the pattern factors of an x oriented electric dipole, a y and z oriented magnetic dipole, respectively. The terms preceding the square brackets are the corresponding scattering dipole moment strength. One can observe that the scattering dipole moment depends on the amplitude, polarization and direction of the incident field as well as the geometry of the half-sphere.

Comparing these expressions with the corresponding ones for the full sphere one can see the effect that the introduction of the perfectly conducting plane has. For a sphere in free space, an arbitrary incident wave would in general induce three electric dipoles and three magnetic dipoles, oriented along the x, y, and z axis respectively. The introduction of the condutting plane has then the following effect. It cancels the scattering electric dipoles which are parallel to the plane and doubles the strength of the remaining dipole which is normal to the plane. The magnetic dipole which is normal to the plane is canceled and the strength of the remaining two magnetic scattering dipoles which are oriented parallel to the plane is doubled. These changes, in addition to identifying the image wave with the reflected field from the plane, account for the presence of the conducting plane.

4.3 Conclusion

It was shown that the scattered field from a small half-sphere on a plane can be decomposed into scattering dipole fields which show the characteristic Rayleigh scattering dependence, i.e. $E \sim k^2 a^3$. Let us derive explicitly the scattering dipole moments which the incident field induces in the half-sphere. The radiation field of an x oriented dipole with moment p is given by

$$\underline{E} = \frac{k^2 p}{2\pi\epsilon} \frac{e^{-ikR}}{R} \left[\quad \right] \tag{4.12}$$

where the square bracket is that of (4.9). If the dipole is a radiating current element $I\ell$, then the continuity equation gives us the radiating dipole moment as $p = I\ell / i\omega$, where ℓ is the length of the current element. Equating (4.12) and (4.9) we can solve for the scattering dipole moment which is

$$p = 4\pi\epsilon E_o a^3 (\sin\psi_o \cos\theta_o \cos\phi_o - \cos\psi_o \sin\phi_o) \tag{4.13}$$

The term in the parenthsis can now be identified as the incident wave polarization component which is normal to the conducting plane, i.e.

$$p = 4\pi\epsilon E_o a^3 \hat{a}_o \cdot \hat{n}$$

where $\hat{n}$ is the normal to the plane, in this case $\hat{n} = \hat{i}_x$. The remaining scattering dipoles can be obtained similarly. These results can now be used to obtain the scattered field from a small half-sphere which is mounted on various large bodies like cones, cylinders, etc. As long as the radius of curvature at the point of the body where the half-sphere is placed is large with respect to wavelength, the region around the half-sphere can be approximated by an infinite plane and the above results can then be utilized.

V
SCATTERING BY A HALF-CYLINDER ON A CONDUCTING PLANE

Reflection of plane electromagnetic waves from a perfectly conducting half-cylinder mounted on an infinite conducting plane can be obtained from the solution to the full cylinder and the theorem of images. We will divide this study into two parts. One for horizontally polarized waves with E vector perpendicular to the plane of incidence and the other for vertical polarization or with E vector in the plane of incidence.

5.1 Horizontally Polarized Waves

If we have a plane electromagnetic wave with E vector normal to the plane of incidence impinging upon a conducting half-cylinder as shown in Fig. 5-1, the scat-

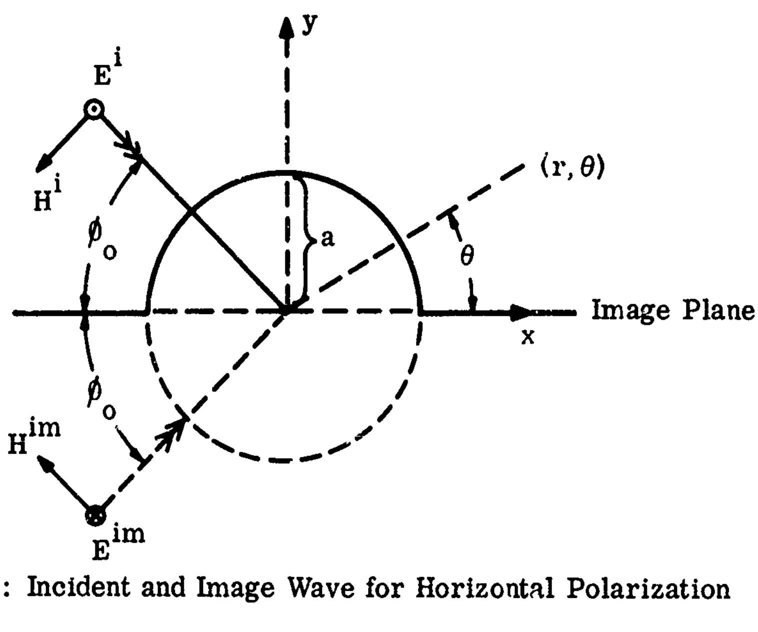


FIG. 5-1: Incident and Image Wave for Horizontal Polarization

tered field from it can be obtained by the addition of an image wave. If the total scattered far field from a full cylinder in the presence of the incident wave is

$$E = E^i + E^s = E_o e^{ikr\cos(\phi_o+\theta) - i\omega t} -$$

$$- E_o \sqrt{\frac{2}{\pi kr}}\, e^{i(kr+\frac{\pi}{4})} \sum_{m=0}^{\infty} \epsilon_m e^{-i\delta_m} \sin\delta_m \cos m(\theta+\phi_o) \tag{5.1}$$

and in the presence of the image wave is

$$E = E^{im} + E^s = -E_o e^{ikr\cos(\phi_o-\theta)} -$$

$$- E_o \sqrt{\frac{2}{\pi kr}}\, e^{i(kr+\frac{\pi}{4})} \sum_{m=0}^{\infty} \epsilon_m e^{-i\delta_m} \sin\delta_m \cos m(\theta-\phi_o) \tag{5.2}$$

then the addition of the above two fields will give the scattered far field from a half-cylinder on a conducting plane. That is

$$E = E_o e^{ikr\cos(\phi_o+\theta)} - E_o e^{ikr\cos(\phi_o-\theta)} +$$

$$+ 4E_o \sqrt{\frac{2}{\pi kr}}\, e^{i(kr+\frac{\pi}{4})} \sum_{m=1}^{\infty} e^{-i\delta_m} \sin\delta_m \sin m\theta \sin m\phi_o \tag{5.3}$$

where the first term is the incident wave, the second term is the reflected wave, the third is the scattered field, and

$$\frac{J_m(ka)}{H_m^{(1)}(ka)} = i e^{-i\delta_m} \sin\delta_m \tag{5.4}$$

5.2 Vertically Polarized Waves

When the E vector is in the plane of incidence as shown in Fig. 5-2, the

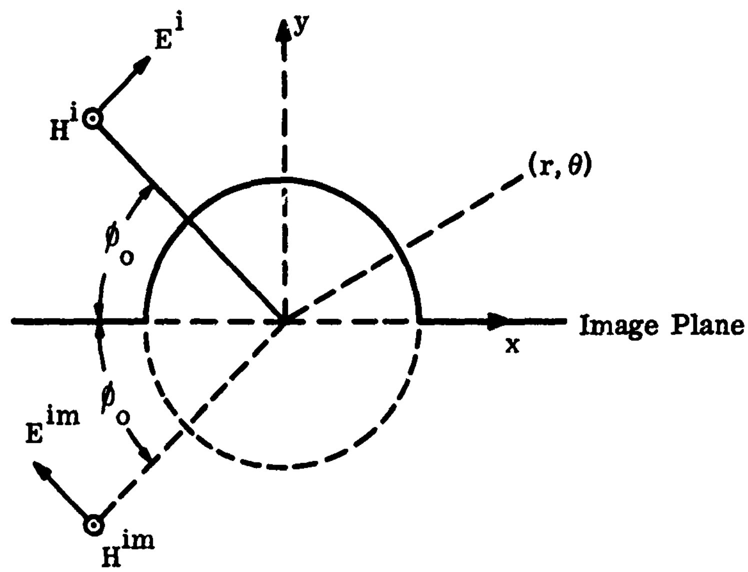


FIG. 5-2: Incident and Image Wave for Vertical Polarization

total scattered far field from a full cylinder in the presence of the incident wave is

$$E = E^i + E^s = E_o e^{ikr\cos(\phi_o + \theta)} -$$

$$-\eta H_o \sqrt{\frac{2}{\pi kr}}\, e^{i(kr + \frac{\pi}{4})} \sum_{m=0}^{\infty} \epsilon_m e^{-i\delta'_m} \sin\delta'_m \cos m(\theta + \phi_o) \qquad (5.5)$$

and in the presence of the image wave is

$$E = E^{im} + E^{s} = E_o e^{ikr\cos(\phi_o - \theta)} -$$

$$-\eta H_o \sqrt{\frac{2}{\pi kr}}\, e^{i(kr+\frac{\pi}{4})} \sum_{m=0}^{\infty} \epsilon_m e^{-i\delta'_m} \sin\delta'_m \cos m(\theta - \phi_o) \qquad (5.6)$$

the solution of the above two fields will give the scattered far field from a half-cylinder on a conducting plane, i.e.

$$E = E_o e^{ikr\cos(\phi_o+\theta)} + E_o e^{ikr\cos(\phi_o-\theta)} -$$

$$-2\eta H_o \sqrt{\frac{2}{\pi kr}}\, e^{i(kr+\frac{\pi}{4})} \sum_{m=0}^{\infty} \epsilon_m e^{-i\delta'_m} \sin\delta'_m \cos m\theta \cos m\phi_o$$

(5.7)

where $\epsilon_m = 1\ (m=0),\ = 2\ (m>0)$ and

$$\frac{J'_m(ka)}{H_m^{(1)'}(ka)} = i e^{-i\delta'_m} \sin\delta'_m \qquad (4.8)$$

A somewhat different approach to solving the same problem was considered by Tai (1948). Extensive scattering patterns for the half-cylinder are also given in this reference.

VI
SCATTERING FROM SMALL SLOTS AND DIPOLES ON A CONDUCTING PLANE

The work presented in this section will be divided into two parts. In the first part we will derive the radiation characteristics of slots in an infinite plane. We will show that small slots radiate like magnetic or electric dipoles which are mounted on a plane. Equivalence relations between magnetic current elements and electric loop currents and between electric current elements and magnetic loop currents will be given. Equivalence relations using duality or Babinet's principle between magnetic (electric) current elements and electric (magnetic) current elements will also be given.

In the second part we will consider the scattering behavior of small slots which are cut in an infinite, perfectly conducting plane. In this case the incident wave will induce dipole moments in the slot which will in turn radiate with the same characteristics as the radiating dipoles of part one. The problem is then to find the scattering dipole moments which are induced by the incident field.

6.1 Radiation from Dipoles on a Conducting Plane

Radiation from electric and magnetic current elements can be obtained from the solution to Maxwell's equations which include magnetic charge and current. Magnetic currents can be used to express discontinuities in the tangential electric field. For example, the tangential field in a slot or an aperture can be looked upon as an equivalent magnetic current sheet, given by $\underline{M} = -\hat{n} \times \underline{E}$. A magnetic dipole which is composed of two oppositely charged magnetic charges separated by an infinitesimal distance can be looked upon as the equivalent of an infinitesimal loop of circulating current, i.e. we will show that $p_m = \mu_o m$ where the magnetic dipole moment is $p_m = q_m \ell$ and the magnetic moment of circulating electric current about an area A is $m = IA$.

If we write Maxwell's equations for time-harmonic fields as

$$\nabla \times \underline{H} = \underline{J} + j\omega\underline{D}$$
$$\nabla \times \underline{E} = \underline{J}_m - j\omega\underline{B} \qquad (6.1)$$
$$\nabla \cdot \underline{D} = \rho$$
$$\nabla \cdot \underline{B} = \rho_m$$

then the solution can be given as

$$\underline{E} = -j\omega\mu\underline{A} - \frac{j\eta}{k}\nabla(\nabla \cdot \underline{A}) - \nabla \times \underline{F} \qquad (6.2)$$

$$\underline{H} = -j\omega\epsilon\underline{F} - \frac{j}{k\eta}\nabla(\nabla \cdot \underline{F}) + \nabla \times \underline{A} \qquad (6.3)$$

where for surface currents the vector potentials are

$$\underline{A} = \iint \frac{\underline{K}\, e^{-jkR}}{4\pi R}\, dS \qquad (6.4)$$

$$\underline{F} = \iint \frac{\underline{M}\, e^{-jkR}}{4\pi R}\, dS \qquad (6.5)$$

and the equivalent surface current densities are related to the field by

$$\underline{M} = -\hat{n} \times \underline{E} \qquad (6.6)$$

$$\underline{K} = \hat{n} \times \underline{H} \qquad (6.7)$$

and $\eta = \sqrt{\mu/\epsilon}$.

The radiated field from a small current element can now be calculated since the integrations in (6.4) and (6.5) can be readily done. Let us express with the aid of the continuity equation the dipole moment p of a current element $I\ell$ as $\underline{p} = \underline{I}\ell/j\omega$. The radiated magnetic field of a current element $I\ell$ is then found to be

$$\underline{H} = \frac{j\omega}{4\pi} e^{j(\omega t - kR)}\left(\frac{jk}{R} + \frac{1}{R^2}\right) \underline{p} \times \hat{R} \qquad (6.8)$$

Similarly, the dipole moment p_m of a small magnetic current element $I_m \ell$ can be expressed with the use of the continuity equation

$$\nabla \cdot J_m + \frac{\partial \rho_m}{\partial t} = 0$$

as $\underline{p}_m = \underline{I}_m \ell / j\omega$. The radiated electric field of a magnetic current element $I_m \ell$ is then

$$\underline{E} = -\frac{j\omega}{4\pi} e^{-jkR}\left(\frac{jk}{R} + \frac{1}{R^2}\right) \underline{p}_m \times \hat{R} \qquad (6.9)$$

6.1.1 Image Theory

When an element radiates above a conducting plane, the total field is obtained by forcing the tangential electric field to be zero at the plane. For the special case of the plane the solution can be obtained much more readily by placing an image source behind the plane and calculating the field due to the two sources with the plane removed. Since a correctly oriented image source will combine with the real source to yield a zero tangential field over a plane which bisects the two sources, the solution so obtained is also the solution to the source above the conducting plane. This is guaranteed by the uniqueness theorem. Figure 6-1 shows current elements and the correctly oriented image elements to produce zero tangential electric field on the dashed line. If a source composed of dipoles is positioned on a conducting plane the effect will be as follows: the conducting plane cancels the electric dipoles tangential to the plane, doubles the electric dipoles normal to the plane, cancels the magnetic dipoles normal to the plane and doubles the magnetic dipoles tangential to the plane.

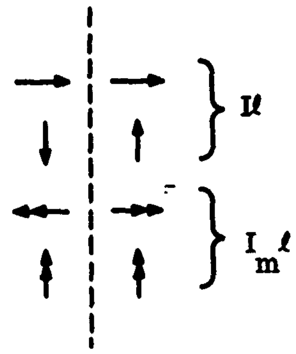


FIG. 6-1: PAIRS OF CURRENT ELEMENTS WHICH PRODUCE ZERO TANGENTIAL ELECTRIC FIELD ALONG THE DASHED LINE.

6.1.2 Radiation from Electric Dipoles on a Conducting Plane

The radiation characteristics of a small current element $I\ell e^{j\omega t}$ protruding normally from a perfectly conducting plane is with the use of (6.8)

$$\underline{H} = \frac{j\omega}{2\pi} e^{-jkR}\left(\frac{jk}{R} + \frac{1}{R^2}\right) \underline{p} \times \hat{R} \quad . \tag{6.10}$$

A z-directed current element at the origin will produce a magnetic field

$$H_\phi = \frac{I\ell}{2\pi} e^{-jkr}\left(\frac{jk}{r} + \frac{1}{r^2}\right) \sin\theta$$

with the conducting plane coinciding with the $z = 0$ plane.

6.1.3 Radiation from Magnetic Dipoles on a Conducting Plane

The electric field of a magnetic current element lying flush on a conducting plane is from (6.9)

$$\underline{E} = -\frac{j\omega}{2\pi} e^{-jkR}\left(\frac{jk}{R} + \frac{1}{R^2}\right) \underline{p}_m \times \hat{R} \tag{6.11}$$

A z-directed element $I_m \ell$ at the origin in a conducting plane will produce an electric field

$$E_\phi = -\frac{I_m \ell}{2\pi} e^{-jkr} \left(\frac{jk}{r} + \frac{1}{r^2} \right) \sin\theta$$

where the conducting plane is any plane containing the z-axis. The magnetic current can be related to a voltage by Maxwell's equations (6.1) as

$$\oint \underline{E} \cdot \underline{d\ell} = -I_m \quad . \tag{6.12}$$

6.2 Radiation from Slots in a Conducting Plane

Radiation from an aperture S_1 in a conducting plane is given by

$$\underline{E}(\underline{r}) = -\frac{1}{2\pi} \iint_{S_1} \underline{M} \times \nabla' \frac{e^{-jkR}}{R} \, dS \tag{6.13}$$

where $R = |\underline{r} - \underline{r}'|$ and the origin is contained in S_1. If the tangential electric field $\underline{M} = \underline{E} \times \hat{n}$ is known in the aperture S_1, the above expression gives the exact radiated field at the observation point $\underline{r}$. Differentiating the Green's function with respect to the source point, we have

$$\nabla' \frac{e^{-jkR}}{R} = jk \left(1 + \frac{1}{jkR} \right) \frac{e^{-jkR}}{R} \hat{R} \quad . \tag{6.14}$$

If we consider fields far from the source in the sense that $r \gg \ell$ (but with no assumption about the size of r relative to λ), we can write

$$|\underline{r} - \underline{r}'| \cong r - \hat{r} \cdot \underline{r}' = r - r' \cos\psi \quad . \tag{6.15}$$

Here ℓ is a characteristic dimension of the aperture S_1. Substituting we have

$$\underline{E}(\underline{r}) = \frac{e^{-jkr}}{2\pi}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\hat{r}\times\iint_{S_1}\underline{M}\, e^{jkr'\cos\psi}\, dS \tag{6.16}$$

where we have retained the approximation (6.15) in the phase, but in the amplitude we have let $R \cong r$. Expression (6.16) can now be readily applied to apertures which are small with respect to wavelength; it is also the correct far field expression for apertures large with respect to wavelength.

6.2.1 <u>Short, Linear Slot</u>

If we have a thin and short slot such that $\Delta \ll \ell \ll \lambda$, where Δ is the thickness and ℓ is the length of the slot, (6.16) can be written as

$$\underline{E} = \frac{e^{-jkr}}{2\pi}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\hat{r}\times\underline{M}\,\ell\Delta \quad . \tag{6.17}$$

If we compare this to (6.11) we find that the short slot behaves like a magnetic dipole with the equivalent dipole moment of the slot as $j\omega p_m = M\ell\Delta = V\ell$. Therefore the equivalent magnetic current is related to the aperture field as

$$\underline{I}_m = \underline{M}\Delta = (\underline{E}\times\hat{n})\Delta = \underline{V}\times\hat{n} \tag{6.18}$$

since $V = E\Delta$. The above expression shows (as did 6.12), that magnetic current is equal to the slot voltage V across the gap, i.e. $I_m = V$. Figure 6-2 shows the notation for a slot

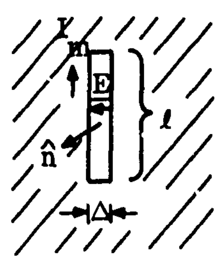


FIG. 6-2: RELATIONSHIP BETWEEN MAGNETIC CURRENT AND SLOT FIELD.

6.2.2 Circular Slot

The integral in (6.16) is now slightly more difficult to evaluate than it was for the linear slot. If we consider a slot as shown in Fig. 6-3, with the radius

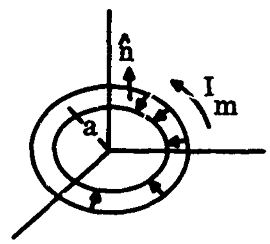


FIG. 6-3: CIRCULAR SLOT OF RADIUS a.

a $\ll \lambda$ and a uniform excitation, the integral in (6.16) can be written with the aid of some vector identities as

$$\iint_{S_1} \underline{M}\, e^{jk\hat{r}\cdot\underline{r}'}\, dS = -jk\hat{r}\times\frac{1}{2}\iint_{S_1}\left[\underline{r}'\times\underline{M}(\underline{r}')\right] dS \qquad (6.19)$$

The last integral can now be defined as the magnetic moment of the magnetic current distribution $\underline{M}$, i.e.

$$\underline{m}_m = \frac{1}{2}\iint_{S_1}\left[\underline{r}'\times\underline{M}\right] dS \qquad (6.20)$$

The magnetic moment is normal to the area of the loop as given by the unit vector $\hat{n}$. The electric field of the magnetic ring current in a conducting sheet then becomes

$$\underline{E} = -jk\frac{e^{-jkr}}{2\pi}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\hat{r}\times(\hat{r}\times\underline{m}_m) \quad . \qquad (6.21)$$

Since in the far field the electric and magnetic fields are related by

$$\underline{E} = -\eta \hat{r} \times \underline{H} \tag{6.22}$$

the magnetic field of the ring current can be written as

$$\underline{H} = j\omega\epsilon \frac{e^{-jkr}}{2\pi} \left(\frac{jk}{r} + \frac{1}{r^2}\right) \hat{r} \times \underline{m}_m \tag{6.23}$$

Therefore at least in the far field the radiation from a circular slot behaves like radiation from a vertical dipole above a conducting plane. Comparing (6.23) to the radiated magnetic field of an electric dipole (6.10) we find that the equivalence relationship is

$$\underline{p} = -\epsilon \underline{m}_m \quad . \tag{6.24}$$

If the magnetic current I_m flows in a closed circuit, whose line element is $\underline{d\ell}$, the magnetic moment of this ring current can be written

$$\underline{m}_m = \frac{I_m}{2} \oint \underline{a} \times \underline{d\ell} \tag{6.25}$$

where $I_m = M\Delta$ and Δ is the width of the slot assumed here as $\Delta \ll a$. Since $\frac{1}{2}(\underline{a} \times \underline{d\ell}) = dA\hat{n}$, where dA is the triangular element of the area inside the ring, (6.25) gives the total area which is bound by the ring. The magnetic moment then has the magnitude

$$m_m = I_m A \tag{6.26}$$

where A is the area regardless of the shape of the circuit and is $A = \pi a^2$ for a circular loop. The equivalence between a current element mounted at right angles on a conducting plane and a magnetic ring current (circular slot) in a conducting plane is

$$I\ell = -j\omega\epsilon I_m A \quad . \tag{6.27}$$

6.3 Radiation from Electric Loop Currents

If a magnetic ring current (actually an infinitesimally thin washer-shaped current with average radius a which flows in a strip between $a+\frac{\Delta}{2}$ and $a-\frac{\Delta}{2}$) could exist in free space, the radiated magnetic field would be, from (6.23)

$$\underline{H} = j\omega\epsilon \frac{e^{-jkr}}{4\pi r}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\hat{r} \times \underline{m}_m \quad . \tag{6.28}$$

It would be only half as strong as the field produced from a magnetic current in a conducting plane. That this must be so can be inferred from the theory of images, that is the conducting plane doubles magnetic current elements that are parallel to it. Therefore it is not necessary to calculate the radiated field from magnetic currents using the vector potential (6.4) and then (6.2) since these are equivalent to (6.13). This becomes more evident when (6.13) is written as

$$\underline{E} = -\frac{1}{2\pi}\nabla \times \iint_{S_1} \underline{M}(\underline{r}')\frac{e^{-jkR}}{R}\, dS \quad . \tag{6.29}$$

Radiation from a loop of electric current can be obtained using the vector potential method. The magnetic field can be expressed with the use of (6.4) and (6.3) as

$$\underline{H} = \nabla \times \iint \frac{\underline{K}\, e^{-jkR}}{4\pi R}\, dS \quad . \tag{6.30}$$

Using (6.14) and (6.15) this can be rewritten for small loop currents as

$$\underline{H}(\underline{r}) = -\frac{e^{-jkr}}{4\pi}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\hat{r} \times \iint_{S_1} \underline{K}\, e^{jk\hat{r}\cdot\underline{r}'} dS \ . \tag{6.31}$$

One can again use the vector identity (6.19) to express the above magnetic field as

$$\underline{H}(\underline{r}) = \frac{jke^{-jkr}}{4\pi}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\hat{r}\,x(\hat{r}\,x\,\underline{m}) \tag{6.32}$$

where $\underline{m}$ is the magnetic moment of the electric current distribution $\underline{K}$, i.e.

$$\underline{m} = \frac{1}{2}\iint_{S_1}\left[\underline{r}'\,x\,\underline{K}(\underline{r}')\right]dS = \frac{1}{2}\int \underline{r}'\,x\,\underline{I}(\underline{r}')\,d\ell \quad . \tag{6.33}$$

In the far field the electric and magnetic fields are related by (6.22), which can be re-expressed as $\eta\underline{H} = r\,x\,\underline{E}$. This permits us to write the electric field of the loop current as

$$\underline{E}(\underline{r}) = jk\mu\,\frac{e^{-jkr}}{4\pi}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\hat{r}\,x\,\underline{m} \quad . \tag{6.34}$$

An examination of the above expression and of (6.9), the electric field of a magnetic dipole element $I_m\ell$ in free space, shows that a magnetic current dipole and electric current loop radiate equally if their moments are related by

$$\underline{p}_m = \mu\underline{m} \quad . \tag{6.35}$$

A magnetic current element and a current loop, positioned as shown in Fig. 6-4

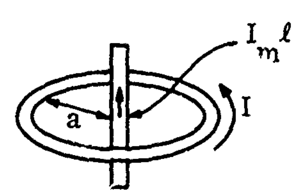


FIG. 6-4: POSITION OF ELECTRIC LOOP CURRENT AND MAGNETIC CURRENT ELEMENT FOR IDENTICAL RADIATION CHARACTERISTICS.

have identical radiation characteristics if (6.35) is satisfied.

If the loop current is confined to a plane, the magnetic moment can again be evaluated with the aid of (6.25) as

$$\underline{m} = IA\hat{n} \qquad (6.36)$$

where $\hat{n}$ is the normal to the plane of the loop, A is the area of the loop which is πa^2 for a circular loop. With this last expression the equivalence relationship (6.35) can be rewritten as

$$\underline{I}_m \ell = j\omega\mu IA\hat{n} \quad . \qquad (6.37)$$

The direction of the electric field from a current loop which is given by $\hat{r} \times \hat{m}$ is normal to any plane containing the axis $\hat{m}$ of the loop. This permits us to introduce a conducting plane along m without disturbing the electric field of the loop (boundary conditions are automatically satisfied). A half loop mounted on a conducting plane, as shown in Fig. 6-5, therefore radiates like a full loop. Therefore

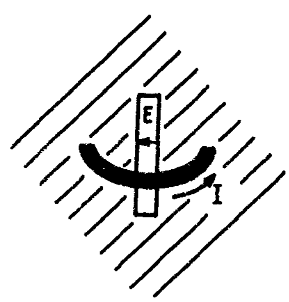


FIG. 6-5: ORIENTATION FOR A SLOT AND A HALF-LOOP ON A CONDUCTING PLANE FOR SIMILAR RADIATION.

comparing the electric fields of a linear slot (6.17) and of a loop (6.34) we find that the relationship for the radiation to be equal is

$$2\underline{I}_m \ell = j\omega\mu\underline{m} = jk\eta IA\hat{n} \qquad (6.38)$$

or

$$2\ell(\underline{V} \times \hat{n}) = j\omega\mu I A \hat{n} \qquad (6.39)$$

We should pause now and examine the expressions that we have identified as magnetic and electric line currents. For example, in the case of the linear and circular slot of width Δ, we have identified the magnetic current with $I_m = M\Delta = (\underline{E} \times n)\Delta$. However, when we talked about a magnetic current element in free space (as for example in 6.19) the wire (or strip) current was related to the surface current density by an integration around the cross section of the wire, i.e. $I_m = \oint M\, d\ell$, which for a flat strip could be written as $I_m \cong M2\Delta$. This however introduces a factor of 2 when compared with the equivalent slot current. To resolve this apparent paradox we only have to look at the vector potential solutions to Maxwell's equations to determine whether the $I_m \cong M2\Delta$ interpretation is correct. The vector potentials (6.4) and (6.5) are solutions to the vector potential wave equations whose inhomogeneous terms are the electric and magnetic currents $\underline{J}$ and $\underline{J}_m$ which for good conductors are crowded on the surface and become the surface currents $\underline{K}$ and $\underline{M}$ respectively. Since these currents flow in a small skin depth layer inside the conducting body we see that, for example, in the case of a thin wire $\oint M\, d\ell = I_m$ gives the total wire current and should be interpreted as such, whether the wire is round, flat, square, etc. The question now arises whether the vector potentials are valid also for points just outside the conducting surface when the interpretation for $\underline{K}$ and $\underline{M}$ must be given as $\hat{n} \times \underline{H}$ and $\underline{E} \times \hat{n}$. This can be answered in the affirmative. Zinke (1941) has shown by a complicated skin-effect analysis that the vector potential just outside the surface of a cylindrical wire is valid and that it is independent of the cross-sectional distribution of the axial current. This permits us then to find an equivalent circular cylinder for an arbitrary cross section. For example (King, 1956, p.20), the equivalent radius r of a thin strip of width Δ is $r = \Delta/4$.

The same comments apply when considering electric currents. For this reason we have written $\oint \underline{K} d\ell = I$ in (6.33).

6.4 Summary of Radiating Dipoles Results

A duality exists between the fields produced by electric and magnetic sources. This can be seen from the expressions (6.8) and (6.9) which give the field from linear electric and magnetic current elements, or (6.34) and (6.28) which are the equivalent expressions for ring currents. From these we can infer the duality relations. These relations show us how to interchange symbols in order to obtain the solution when only electric (magnetic) sources are present from a solution with only magnetic (electric) sources present.

Electric sources only	Magnetic sources only	
$\underline{E}$	$\underline{H}$	
$\underline{H}$	$-\underline{E}$	
$\underline{K}$, I, p, m	$\underline{M}$, I_m, p_m, m_m	(6.40)
$\underline{A}$	$\underline{F}$	
$\mu, \epsilon, \sqrt{\mu/\epsilon}$	$\epsilon, \mu, \sqrt{\epsilon/\mu}$	

The above relations give the equivalence between electric and magnetic currents which have the same physical shape. Another set of equivalence relations can be obtained which relate the fields of linear electric (magnetic) currents to loop magnetic (electric) currents. These are

$$\underline{p}_m = \mu \underline{m} \quad \text{or} \quad \underline{I}_m \ell = V\underline{\ell} = j\omega\mu I A \hat{n} = j\omega \underline{p}_m \tag{6.41}$$

and

$$\underline{p} = -\epsilon \underline{m}_m \quad \text{or} \quad \underline{I}\ell = -j\omega\epsilon I_m A \hat{n} \quad . \tag{6.42}$$

From these we conclude that a linear monopole protruding from a conducting plane radiates identically to a circular slot in a conducting plane if (6.42) is satisfied and that a half loop mounted as shown in Fig. 6-5 and a linear slot radiate equally. Figure 6-6 shows the equivalent radiators.

6.5 Scattering Dipole Moments

Electromagnetic scattering from a small body in free space can be identified with the radiation fields from a combination of electric and magnetic dipoles which have a strength and orientation such that their combined radiation field is identical to that of the scattering body. When a symmetric scattering body is bisected by a conducting plane, the scattered field from the half-body is altered in the following way: the conducting plane cancels the electric dipoles tangential to the plane, doubles the electric dipoles normal to the plane, cancels the magnetic dipoles normal to the plane and doubles the magnetic dipoles tangential to the plane.

6.5.1 Scattering Dipole Moment of a Monopole

The radiated electric far field of an electric dipole can be obtained from (6.8) with the aid of the far field relation $\underline{E} = \eta \underline{H} \times \hat{r}$ as

$$\underline{E} = k^2 \frac{e^{-jkr}}{4\pi\epsilon r} (\hat{r} \times \underline{p}) \times \hat{r} \quad . \tag{6.43}$$

The scattered electric field of a z-oriented monopole of length ℓ was obtained before as (see eq. 2.26)

$$E_\theta^s = -\frac{k^2 \ell^3 E_o^i \cos\psi \sin\theta_i \sin\theta_s}{\log 4\ell/a_o - 1} \frac{e^{-jkr}}{r} \tag{6.44}$$

where $\underline{E}^i \cdot \hat{i}_z = E_o^i \cos\psi \sin\theta_i$ is the tangential component of the incident electric field along ℓ and a_o is the wire radius. An arbitrarily oriented electric monopole will then scatter an electric field

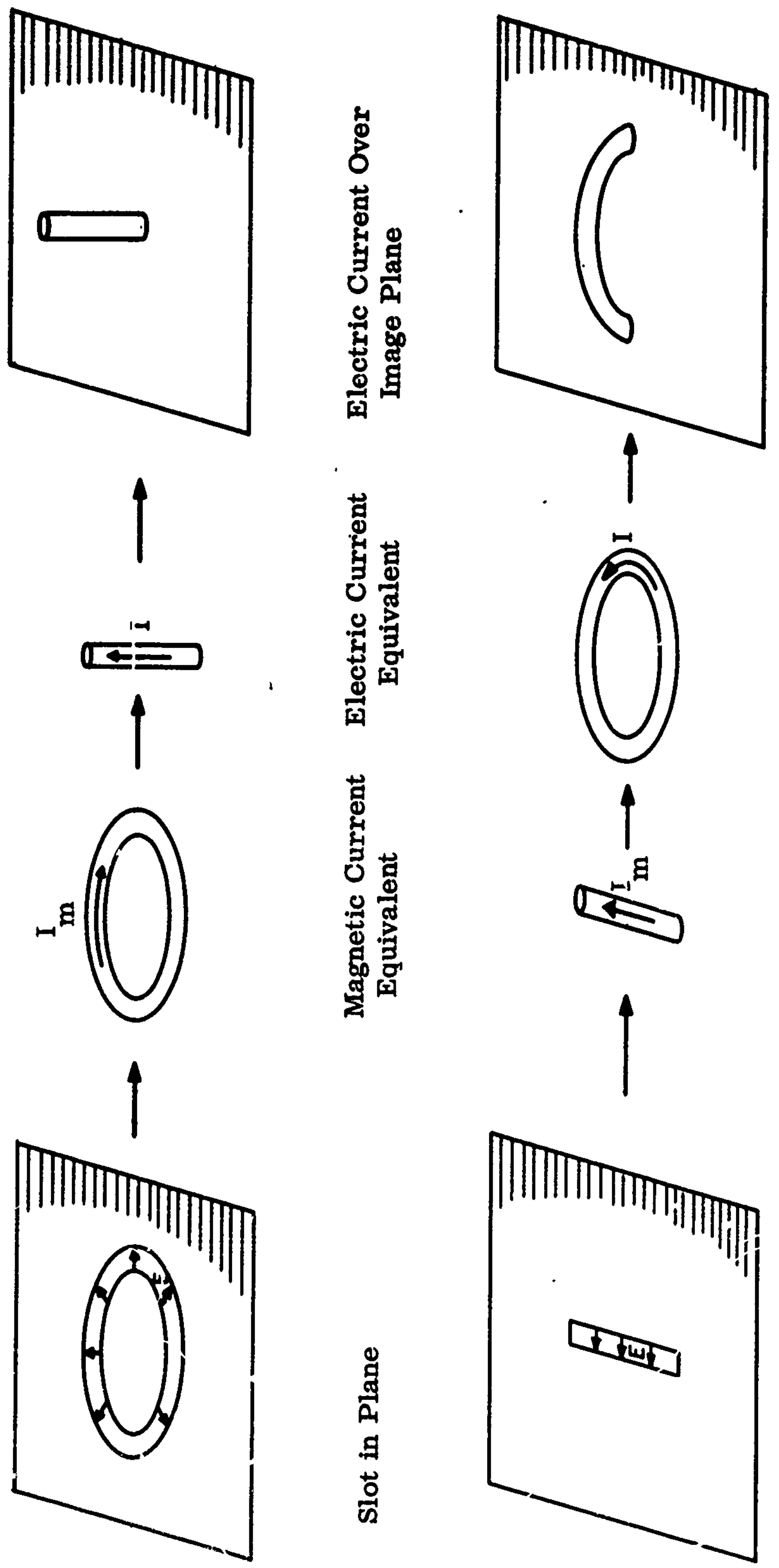


FIG. 6-6: SLOT RADIATORS AND THEIR ELECTRIC CURRENT EQUIVALENTS.

$$\underline{E}^s = -\frac{\underline{E}^i \cdot \hat{\ell}\, k^2 \ell^3}{\log 4\ell/a_o - 1} \frac{e^{-jkr}}{r} (\hat{r} \times \hat{p}) \times \hat{r} \tag{6.45}$$

where the orientation of the element is given by $\hat{\ell} = \hat{p}$. Comparing this expression to (6.43) we can solve for the induced scattering dipole moment, which is

$$\underline{p} = -\frac{4\pi\epsilon\,\ell^3 \underline{E}^i \cdot \hat{\ell}}{\log 4\ell/a_o - 1} \hat{\ell} \tag{6.46}$$

This is the dipole moment of a short piece of wire of length ℓ and cross section radius a_o either in free space or mounted normally on a conducting plane. In the latter case the normal to the plane and the wire direction coincide, i.e. $\hat{n} = \hat{\ell}$.

6.5.2 Scattering Dipole Moments of a Loop and a Half-Loop on a Conducting Plane

In was shown in a previous section that the scattered field from a loop is composed of two electric dipole terms and one magnetic dipole term. The two induced electric dipoles lie in the plane of the loop. One is oriented parallel to the plane of incidence, the other normal to it. Actually, the two electric dipoles were the two components of a single dipole which the electric field $\underline{E}^i$ induces in the loop. This single dipole lies in the plane of the loop, and has the direction which coincides with the projection of $\underline{E}^i$ onto the plane of the loop. The unit vector

$$\hat{p} = (\hat{m} \times \hat{E}^i) \times \hat{m} \tag{6.47}$$

gives the direction of the induced electric dipole moment. This dipole radiates an electric field which is given by

$$\underline{E}^{ed} = \frac{B}{\chi_1}\left(1 + ka\frac{j\pi}{\chi_1}\right)\left\{\hat{r} \times \left[(\hat{m} \times \hat{E}^i) \times \hat{m}\right]\right\} \times \hat{r} \tag{6.48}$$

where $B = E_o^i k^2 a^3 \pi\, e^{-jkr}/r$, $\hat{m}$ is a unit vector along the axis of the loop, $\hat{r}_o$ and

$\hat{r}$ are the direction of incidence and observation point, respectively. Comparing this expression to (6.43), we find that the induced scattering dipole moment is given by

$$\underline{p} = \frac{4\pi^2 \epsilon a^3}{\chi_1}\left(1 + ka\frac{j\pi}{\chi_1}\right)(\hat{m} \times \underline{E}^i) \times \hat{m} \tag{6.49}$$

where $\chi_1 = 2\left[\ln 8S_o - 2\right]$ and S_o is the ratio of loop radius to wire radius.
The incident field also induces a magnetic dipole which is normal to the plane of the loop and reradiates a field given by

$$\underline{E}^{md} = \frac{B}{2\chi_1}\hat{m}\cdot(\hat{r}_o \times \hat{E}^i)\hat{r} \times \hat{m} \quad . \tag{6.50}$$

The induced scattering magnetic moment can be obtained by equating the above expression to (6.34). The moment is then

$$m = -\frac{2\pi^2 a^3}{\chi_1}\hat{m}\cdot(\hat{r}_o \times \hat{\underline{E}}^i)\hat{m} \quad . \tag{6.51}$$

Since $\eta\underline{H} = \hat{r}_o \times \underline{E}$ for a plane wave, (6.51) can be written also as

$$m = -\frac{2\pi^2 a^3}{\chi_1}(\hat{m}\cdot\underline{H}^i)\hat{m} \quad . \tag{6.52}$$

Using (6.41) which relates the magnetic moment to the magnetic dipole moment ($p_m = \mu m$), (6.52) can also be written as

$$\underline{p}_m = -\frac{2\pi^2 a^3 \mu}{\chi_1}(\hat{m}\cdot\underline{H}^i)\hat{m} \tag{6.53}$$

This last expression can be considered as the dual (Babinet's equivalent) of the electric dipole expression (6.48). The induced magnetic moment of the half loop is the same as (6.53) for a full loop except that p_m should be multiplied by a factor of 2 if p_m is to be used in (6.43). (The use of the factor 2 can of course be avoided simply by using (6.53) in conjunction with (6.11).)

When a half loop is mounted on a conducting plane, the electric dipole tangential to the plane cancels, whereas the electric dipole normal to the plane has a scattering dipole moment given by (see 3.30)

$$\underline{p} = \frac{4\pi^2 \epsilon a^3}{\chi_1}\left(1 + ka\frac{j\pi}{\chi_1}\right)(\underline{E}^i \cdot \hat{n})\hat{n} \tag{6.54}$$

where $\hat{n}$ is the normal to the plane. If (6.54) is to be used in (6.43) it should be multiplied by a factor of 2. Of course, if (6.54) is used in the expression for the radiated field of an electric dipole mounted normally on a plane which is given from (6.43) and (6.10) as

$$\underline{E} = \frac{k^2 e^{-jkr}}{2\pi\epsilon r}(\hat{r} \times \underline{p}) \times \hat{r} \tag{6.55}$$

then the use of the additional factor 2 can be avoided.

6.5.3 Scattering from Slots in a Conducting Plane

In this section we will consider scattering from slots or magnetic currents on a conducting plane. In a sense we have solved this problem already when the scattering results for a half loop mounted on a conducting plane were obtained. Let us recall that scattering from a half loop on a plane was equivalent to scattering from an electric dipole normal to the plane and a magnetic dipole tangential to the plane. Since in the radiation problem radiation by a slot and a loop are equivalent, we can reason that the magnetic dipole contribution of the half loop must be also an equivalent slot scattering contribution. However, in the scattering problem, to find the equivalence between the loop radius and the slot length an additional condition besides (6.38) is needed. This added condition can be obtained from circuit theory. In the radiation problem it was shown that the equivalence between a slot and a half loop is given by (6.38), i.e.

$$2V\ell = jk\eta IA \tag{6.56}$$

This is precisely the condition to obtain the electric field radiated by a slot which is given by (6.17) as

$$\underline{E} = \frac{jk\ell V}{2\pi}\frac{e^{-jkr}}{r}\hat{r}\times\hat{\ell} \tag{6.57}$$

from the radiated electric field of a half loop on a plane given by (6.34) as

$$\underline{E} = -k^2\eta IA\frac{e^{-jkr}}{4\pi r}\hat{r}\times\hat{m} \tag{6.58}$$

The orientation of the slot and the axis of the loop are parallel, i.e. $\hat{\ell} = \hat{m}$. In the scattering problem the half loop reradiates as an electric and magnetic dipole. If we assume that the magnetic dipole part of the half loop scattered electric field, which is given by (6.50) as

$$\underline{E} = \frac{k^2a^3\pi e^{-jkr}}{\chi_1 r}\hat{m}\cdot(\hat{r}_o\times\underline{E}^i)\hat{r}\times\hat{m} \tag{6.59}$$

is equivalent to the scattering by a slot, an additional condition which will relate the radius of the loop a to the length of the slot ℓ is needed. Since we are considering slots and loops which are small with respect to wavelength we can use some concepts of circuit theory since that is also based on the assumption that the circuit dimensions are small with respect to wavelength. Ignoring losses, a loop which is a special case of the single turn solenoid is a purely inductive device in which voltage and current are related by

$$V = j\omega LI \tag{6.60}$$

where L is the inductance of the loop. Inductance is defined as flux linkage divided by the current that causes the flux, i.e.

$$L = \frac{\Delta}{I} = \frac{BA}{I} = \frac{\mu H \pi a^2}{I} \quad . \qquad (6.61)$$

The magnetic field at the center of a coil is given by $H = I/2a$ which is a good approximation to the flux linking the loop. The inductance of a small loop then becomes $L = \mu\pi a/2$, and the relationship (6.60) is

$$V = jk\eta\pi aI/2 \qquad (6.62)$$

This expression in connection with (6.56) permits us now to solve for the relationship between the loop radius a and the slot length ℓ which is

$$a = \ell \quad . \qquad (6.63)$$

Therefore scattering from a short slot can be obtained from (6.59) with the substitution $a = \ell$. The scattered field is then

$$\underline{E} = \frac{k^2\ell^3\pi e^{-jkr}}{\chi_1 r} \hat{\ell}\cdot(\hat{r}_o \times \underline{E}^i)\hat{r}\times\hat{\ell} \quad . \qquad (6.64)$$

The factor $\chi_1 = 2(\ln 8S_o - 2)$ can be adopted to a flat strip current since it is known that the equivalent radius of a strip of width Δ is $\Delta/4$. Therefore $S_o = 4a/\Delta$. The above expression for the scattered field from a slot of length ℓ and width Δ can then be written as

$$\underline{E} = \frac{\eta k^2\ell^3\pi e^{-jkr}}{2(\ln 32\ell/\Delta - 2)r} (\hat{\ell}\cdot\underline{H}^i)\hat{r}\times\hat{\ell} \qquad (6.65)$$

where the substitution $\hat{r}_o \times \underline{E}^i = \eta \underline{H}^i$ for a plane wave was made. The accuracy of this expression depends on the accuracy of the derivation for inductance of a loop (6.61). Comparing this expression to the radiated field from a slot (6.17) which can be written as

$$\underline{E} = -\frac{\omega k}{2\pi r} e^{-jkr} p_m \hat{r}\times\hat{p}_m \quad , \qquad (6.66)$$

we obtain for the induced scattering magnetic dipole moment of a slot

$$\underline{p}_m = - \frac{\pi^2 \ell^3 \mu}{(\ln 32\ell/\Delta - 2)} (\hat{\ell} \cdot \underline{H}^i)\hat{\ell} \quad . \qquad (6.67)$$

This expression could also have been obtained more directly, once the relationship (6.63) is known. Substituting a = ℓ into (6.53) which is the scattering magnetic dipole moment of a half loop, (6.67) is obtained.

Perhaps a more direct approach to the scattering of a small slot on a conducting plane is to use the scattering results for a short piece of wire and then apply duality. The induced scattering dipole moment for a thin wire of length ℓ is given by (6.46). The dual of that, with the use of the duality relations (6.40) is

$$\underline{p}_m = - \frac{4\pi\mu\ell^3 \underline{H}^i \cdot \hat{\ell}}{\log 4\ell/a_o - 1} \hat{\ell} \quad . \qquad (6.68)$$

Since the radiated field for a slot is given by (6.17) as

$$\underline{E} = - \frac{\omega k e^{-jkr}}{2\pi r} \hat{r} \times \underline{p}_m \qquad (6.69)$$

we obtain the scattered field of a slot by substituting (6.68) in (6.69) which is then

$$\underline{E} = \frac{2\eta k^2 \ell^3}{\log 16\ell/\Delta - 1} \frac{e^{-jkr}}{r} (\underline{H}^i \cdot \hat{\ell}) \hat{r} \times \hat{\ell} \qquad (6.70)$$

where the substitution for the equivalent radius of a strip $a_o = \Delta/4$ has been made.

Now we are in possession of two expressions for the scattered electric field from a slot. One was derived from the solution of a small half loop on a conducting plane by equating the magnetic dipole portion of the total scattered field to that scattered by a slot. After finding the equivalence between loop radius a and slot length ℓ we were able to obtain the scattered field by a slot which is given by (6.65). The second method to obtain scattering from a slot was the use of duality applied to the scattering results for a dipole. This yielded (6.70). Hopefully these two expressions will agree. After an examination of these equations we find that they will be equivalent if

$$\frac{2}{\log 16\ell/\Delta - 1} = \frac{\pi}{2(\ell n\, 32\ell/\Delta - 2)} \quad . \tag{6.71}$$

A quick check for a slot length-to-width ratio of $\ell/\Delta = 10$ gives for (6.71) the result that 2/4.09 = 3.14/3.77. The source of this small error is probably the equivalence relation between the loop raidus a and slot length ℓ, given by $a = \ell$. For this derivation the inductance of the loop (6.61) was obtained under the assumption that the flux linking the loop is given by the flux at the center of the loop. However, the agreement is substantial, so as not to warrant further refinement.

VII
SUMMARY OF SCATTERING DIPOLE MOMENTS FOR VARIOUS SHAPES

Since we have now obtained the scattered fields for various shapes of interest, which are mounted on a perfectly conducting plane, we will now list their electric and magnetic scattering moments. The normal to the conducting plane is denoted by $\hat{n}$, the incidence direction by $\hat{r}_o$, and the observation direction by $\hat{r}$. The scattering moments are obtained by comparing the scattered dipole fields of the shape of interest to the fields of the corresponding radiating dipoles, and then solving for the induced scattering dipole moments. These scattering dipole moments can now be applied to obtain the scattering from, let us say, a small half-sphere mounted for example on a large cone, sphere, cylinder, etc., provided the simpler problem of radiating dipoles on the large bodies has been solved

For example, if the body on which the dipoles are mounted is an infinite conducting plane, the radiated far field due to the dipoles of electric and magnetic moment p and p_m, respectively, are

$$\underline{E} = \frac{k\omega e^{-jkr}}{2\pi r} \underline{p}_m \times \hat{r} \tag{7.1}$$

$$\underline{H} = \frac{k^2 e^{-jkr}}{2\pi\mu r} \hat{r} \times (\hat{\underline{p}}_m \times \hat{r}) \tag{7.2}$$

for the fields of a magnetic dipole (which is mounted flush on the conducting plane) and

$$\underline{E} = \frac{k^2 e^{-jkr}}{2\pi\epsilon r} (\hat{r} \times \underline{p}) \times \hat{r} \tag{7.3}$$

$$\underline{H} = \frac{k\omega e^{-jkr}}{2\pi r} (\hat{r} \times \underline{p}) \tag{7.4}$$

for the fields of an electric dipole (which is mounted normally on the conducting plane). Knowing the scattering dipole moments for a small object that is mounted on a plane and substituting in the above equations, the scattered field of that object is then determined.

7.1 The Monopole on a Conducting Plane

A monopole of length ℓ mounted normal on a conducting plane has only an electric dipole moment:

$$\underline{p} = -\frac{4\pi\epsilon\ell^3 E^i \cdot \hat{\ell}}{\log 4\ell/a_o - 1}\hat{n} . \quad (7.5)$$

If the monopole is not mounted normally on the plane, the above expression should be multiplied by the factor $(\hat{n}\cdot\hat{\ell})$, since only the normal component contributes to the scattered field.

7.2 The Half-Loop on a Conducting Plane

The half loop has a magnetic and electric dipole moment given by

$$\underline{p}_m = -\frac{\pi^2 a^3 \mu}{\ell n\, 8a/a_o - 2}(\hat{m}\cdot\underline{H}^i)\hat{m} \quad (7.6)$$

$$\underline{p} = \frac{2\pi^2 \epsilon a^3}{\ell n\, 8a/a_o - 2}\left(1 + ka\frac{j\pi}{\ell n\, 8a/a_o - 2}\right)(\underline{E}^i\cdot\hat{n})\hat{n} \quad (7.7)$$

where $\hat{m}$ is the direction of the loop axis, a and a_o are the radii of the loop and wire respectively.

7.3 The Half-Sphere on a Conducting Plane

The half-sphere has both moments

$$\underline{p}_m = -2\pi\mu a^3\left[(\hat{n}\times\underline{H}^i)\times\hat{n}\right] \quad (7.8)$$

$$\underline{p} = 4\pi\epsilon a^3(\underline{E}^i \cdot \hat{n})\hat{n} \tag{7.9}$$

where a is the radius of the sphere.

7.4 A Small Slot on a Conducting Plane

A slot has only a scattering magnetic dipole whose moment is given by

$$\underline{p}_m = -\frac{4\pi\mu\ell^3 \underline{H}^i \cdot \hat{\ell}}{\log 16\ell/\Delta - 1}\hat{\ell} \tag{7.10}$$

where ℓ is the length and direction of the slot and Δ is the slot width, such that $\Delta \ll \ell \ll \lambda$.

REFERENCES

Chen, K.M. and V.V. Liepa (1964) "The Minimization of the Back Scattering of a Cylinder by Central Loading", IEEE Trans. AP-12, 576-582.

Erdelyi, A., W. Magnus, F. Oberhettinger, and F. Tricomi (1953) Higher Transcendental Functions, Vol. II (McGraw-Hill Book Co., Inc., New York).

Harrison, C.W. Jr. (1962) "On the Receiving Characteristics of a Dipole in Proximity to Two Intersecting Conducting Surfaces", IRE Trans. AP-10, 543-547.

Harrison, C.W. Jr. and R.O. Heinz (1963) "On the Radar Cross Section of Rods, Tubes, and Strips of Finite Conductivity", IEEE Trans. AP-11, 459-468.

Jackson, J.D. (1962) Classical Electrodynamics (John Wiley and Sons, Inc., New York).

Justice, R. and V.H. Rumsey (1955) "Measurement of Electric Field Distributions", IRE Trans. AP-3, 177.

Kerr, D.E. (1964) Propagation of Short Radio Waves (Boston Technical Publishers, Inc., Lexington, Mass.).

King, R.W.P. (1956) The Theory of Linear Antennas (Harvard University Press, Cambridge, Mass.).

King, R.W.P. and T.T. Wu (1965) "Currents, Charges, and Near Fields of Cylindrical Antennas", Radio Sci., J. Res. NBS, 69D, 429-446.

Mack, C.L. and B. Reiffen (1964) "RF Characteristics of Thin Dipoles", Proc. IEEE 52, 533-542.

Tai, C.T. (1948) "Reflection of Plane Electromagnetic Waves from a Perfectly Conducting Grounded Half-Cylinder", Harvard University Cruft Laboratory Technical Report No. 40.

Tai, C.T. (1952) "Electromagnetic Backscattering from Cylindrical Wires", J. Appl. Phys. 23, 909-916.

Ufimtsev, P. Ya. (1962) "Diffraction of Plane Electromagnetic Waves by a Thin Cylindrical Conductor", Radioteckhnikai Electronika 7, 241-249.

Vainshtein, L.A. (1959) "Waves of Current in a Thin Cylindrical Conductor", Soviet Phys. - Tech. Phys. 4, 601-615 and 616-626.

Van Vleck, J.H., F. Bloch and M. Hamermesh (1947) "Theory of Radar Reflection from Wires or Thin Metallic Strips", J. Appl. Phys. 18, 274-294.

Weston, V.H. (1957) "Scattering from a Circular Loop", Department of Electrical Engineering, University of Toronto, Report No. 12.

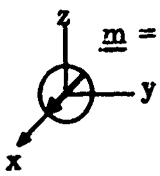
Zinke, O. (1941) "Grundlagen der Strom-und Spannungsverteilung auf Antennen", Arch. Electrotech. 35, p. 67.

APPENDIX

The following six pictures represent radiating dipoles in various basic orientations and their respective fields. Magnetic dipoles of moment m are shown in the first three pictures, followed by three electric dipoles of moment p. In all cases the dipole moments are absorbed in the factor E_o.

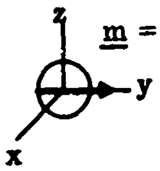
$\underline{m} = m_o\hat{i}_z$

$$\underline{E} = E_o(\hat{r}\times\hat{i}_z) = E_o \sin\theta\,\hat{i}_\phi \frac{e^{-jkR}}{R}$$



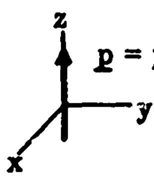
$\underline{m} = m_o\hat{i}_x$

$$\underline{E} = E_o(\hat{r}\times\hat{i}_x) = E_o(\hat{i}_\theta \sin\phi + \hat{i}_\phi \cos\theta\cos\phi)\frac{e^{-jkR}}{R}$$



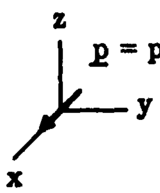
$\underline{m} = m_o\hat{i}_y$

$$\underline{E} = E_o\frac{e^{-jkR}}{R}(\hat{r}\times\hat{i}_y) = E_o(-\hat{i}_\theta\cos\phi + \hat{i}_\phi\cos\theta\sin\phi)\frac{e^{-jkR}}{R}$$



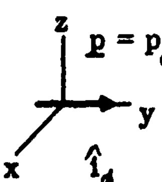
$\underline{p} = p_o\hat{i}_z$

$$\underline{E} = E_o(\hat{r}\times\hat{i}_z)\times\hat{r}\frac{e^{-jkR}}{R} = E_o\hat{i}_\theta\sin\theta\frac{e^{-jkR}}{R}$$



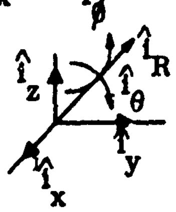
$\underline{p} = p_o\hat{i}_x$

$$\underline{E} = E_o(\hat{r}\times\hat{i}_x)\times\hat{r}\frac{e^{-jkR}}{R} = E_o(-\hat{i}_\phi\sin\phi + \hat{i}_\theta\cos\theta\cos\phi)\frac{e^{-jkR}}{R}$$



$\underline{p} = p_o\hat{i}_y$

$$\underline{E} = E_o(\hat{r}\times\hat{i}_y)\times\hat{r}\frac{e^{-jkR}}{R} = E_o(\hat{i}_\phi\cos\phi + \hat{i}_\theta\cos\theta\sin\phi)\frac{e^{-jkR}}{R}$$



$$\hat{i}_x = \hat{i}_R\sin\theta\cos\phi + \hat{i}_\theta\cos\theta\cos\phi - \hat{i}_\phi\sin\phi$$

$$\hat{i}_y = \hat{i}_R\sin\theta\sin\phi + \hat{i}_\theta\cos\theta\sin\phi + \hat{i}_\phi\cos\phi$$

$$\hat{i}_z = \hat{i}_R\cos\theta - \hat{i}_\theta\sin\theta$$

$$\hat{i}_r = \hat{i}_x\sin\theta\cos\phi + \hat{i}_y\sin\theta\sin\phi + \hat{i}_z\cos\theta$$

$$\hat{i}_\theta = \hat{i}_x\cos\theta\cos\phi + \hat{i}_y\cos\theta\sin\phi - \hat{i}_z\sin\theta$$

$$\hat{i}_\phi = -\hat{i}_x\sin\phi + \hat{i}_y\cos\phi$$

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13 ABSTRACT

The scattered field from small (with respect to wavelength) obstacles which are mounted on an infinite, perfectly conducting plane is determined. The obstacles considered are a monopole, half-sphere, half-cylinder, half-loop and a slot. Since the reradiated field due to these obstacles can be identified with the radiation of a combination of electric and magnetic dipoles, the solution is presented as scattering dipole moments. These moments are induced in the obstacle by the incident wave and depend on the direction and polarization of the incident energy.

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	ROLE	WT	ROLE	WT	ROLE	WT
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