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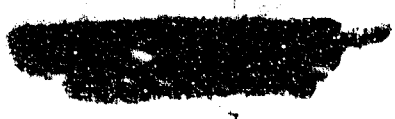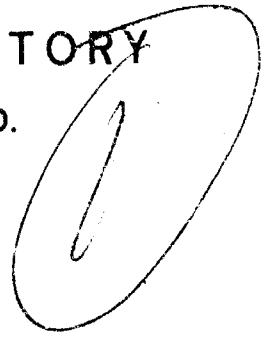
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AUTHORITY

Army Armament Research and Development Center, APG, MD ltr dtd 31 Mar 1977

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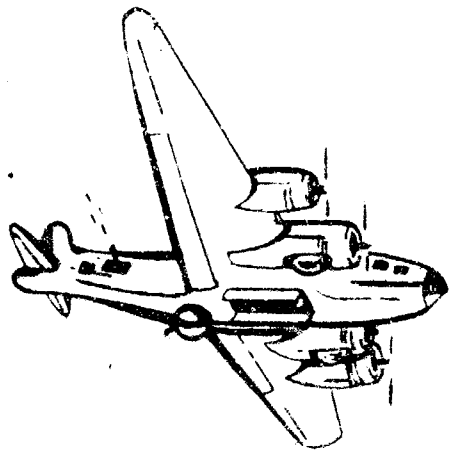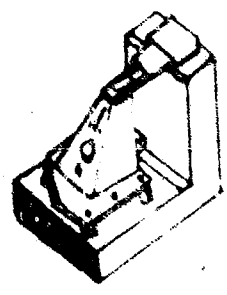
BALLISTIC RESEARCH LABORATORY

ABERDEEN PROVING GROUND, MD.

MEMORANDUM
REPORT NO. 461

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THE LOCATION BY MEANS OF A SINGLE CAMERA, OF A PROJECTILE WITH A KNOWN LINE OF FIRE

BY

A. P. MORSE

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BALLISTIC RESEARCH LABORATORIES
MEMORANDUM REPORT.

A. P. Morse

Morse/emh
Aberdeen Proving Ground, Md.
5 June 1947   10 P.

Exterior Ballistic Laboratory
Problem No. E-189

THE LOCATION BY MEANS OF A SINGLE CAMERA, OF A PROJECTILE WITH A KNOWN LINE OF FIRE.

Abstract   BRL-MR-461

This report gives the details of a reduction procedure for converting the photographic measurements of a rocket in flight into ballistic data pertinent to the rocket trajectory. It opens with a careful description of the geometric problem involved. Transformations between the conventional coordinate systems at the camera are described; by means of these, the formulas for the projectile's position are written down. The methods are essentially vectorial throughout. The report concludes with a suggested computation scheme and an example.

The Location, by Means of a Single Camera, of
A Projectile with a Known Line of Fire

Agree that: the line of fire is the intersection of
the vertical trajectory plane with the horizontal plane
through the camera (that is the center of the camera lens):
the line of sight is the intersection of the latter plane
with the vertical plane through the optical axis; ε is the
angle of inclination of the optical axis; φ so measures the
angle formed by the line of sight and the perpendicular
from the camera to the line of fire that φ is positive in
Fig. I and negative in Fig. II.
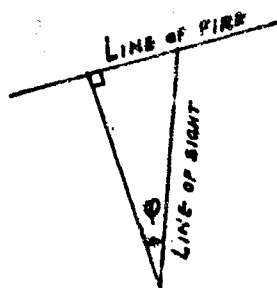
CAMERA
FIG. I
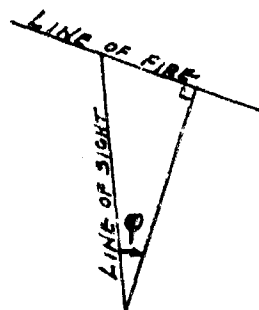
CAMERA
FIG II

Thus φ might be defined as the amount of counterclockwise
(picture the clock face up and directly beneath the camera)
turning necessary to bring the line of sight perpendicular
to the line of fire, it being understood that clockwise
turning is accorded a negative value. Also agree that: D
is the distance from the camera to the point at which the
line of sight intersects the line of fire. ρ is the distance
from the camera to the point $\bar{O}$, at which the optical axis
pierces the trajectory plane.

Imagine that the origin of 3 space is so placed at $\bar{O}$
that: the positive first axis (say k_1) is horizontal and
passes directly above the camera; the second (say k_2) axis
is horizontal, perpendicular to the k_1 axis, and points clock-
wise about the camera (that is to the camera man's right); the
third (say k_3) axis points vertically upward. Let C be the
point at which we find the center of the camera lens and adopt
the general convention whereby $C = (C_1, C_2, C_3)$. Let the ζ_1
axis agree with the k_2 axis and let the η_1 axis so pass
through $\bar{O}$ in an upwardly direction that η_1 is perpendicular to

both the optical and the ξ_1 axis. Let the x* axis pass horizontally through the origin in the trajectory plane and point clockwise about the camera, and let the y* axis agree with the k_3 axis.

It should be noted that a ray through C with standard coordinates (ξ, η) pierces the plane determined by ξ_1 and η_1 in a point whose (ξ_1, η_1) coordinates are $(\rho\xi, \rho\eta)$.

Let $u^{(1)}$, $u^{(2)}$, $v^{(1)}$, $v^{(2)}$ be the unit points on the respective positive ξ_1, η_1, x*, y* axes, and let

$$\tau = v^{(1)} \times v^{(2)}.$$

A check reveals:

$u^{(1)} = (0, 1, 0),$

$u^{(2)} = (\sin \varepsilon, 0, \cos \varepsilon),$

$v^{(1)} = (-\sin \varphi, \cos \varphi, 0),$

$v^{(2)} = (0, 0, 1),$

$\tau = (\cos \varphi, \sin \varphi, 0),$

$C = (\rho\cos \varepsilon, 0, -\rho\sin \varepsilon).$

Since the trajectory plane passes through $v^{(1)}$, $v^{(2)}$, and O, it is evident that the line through O and τ is perpendicular to the trajectory plane. Accordingly the trajectory plane consists of those points R for which

$$R \cdot \tau = 0.$$

Letting P be any point and Q be such a point in the trajectory plane that Q is collinear with C and P, choose λ so that

$$(1 - \lambda)C + \lambda P = Q.$$

Clearly

$$(1 - \lambda)\tau \cdot C + \lambda\tau \cdot P = \tau \cdot Q = 0,$$

$$\lambda = \frac{\tau \cdot C}{\tau \cdot C - \tau \cdot P}$$

$$Q = \frac{(\tau \cdot C)P - (\tau \cdot P)C}{\tau \cdot C - \tau \cdot P}.$$

Suppose now: that P and Q are related as above; that I, the photographic image of Q, has plate measurements (p_1, p_2); and that standard coordinates (ξ, η) of the ray CQ have been introduced in such a way that

$$P = \rho\xi u^{(1)} + \rho\eta u^{(2)}$$

$$\xi = A_{10} + A_{11}p_1 + A_{12}p_2$$

$$\eta = A_{20} + A_{21}p_1 + A_{22}p_2.$$

Note that P lies in the (ξ_1, η_1) plane and that $(\rho\xi, \rho\eta)$ are the coordinates of P with respect to the (ξ_1, η_1) axes.

Let $(x*, y*)$ be the coordinates of Q with respect to the horizontal trajectory $x*$ axis and the vertical $y*$ axis and verify that:

$$x* = v^{(1)} \cdot Q \qquad y* = v^{(2)} \cdot Q;$$

$$x* = \frac{(\tau \cdot C)(P \cdot v^{(1)}) - (\tau \cdot P)(C \cdot v)^{(1)}}{\tau \cdot C - \tau \cdot P},$$

$$y* = \frac{(\tau \cdot C)(P \cdot v^{(2)}) - (\tau \cdot P)(C \cdot v^{(2)})}{\tau \cdot C - \tau \cdot P};$$

$$P \cdot v^{(1)} = \rho\xi(u^{(1)} \cdot v^{(1)}) + \rho\eta u^{(2)} \cdot v^{(1)}$$

$$= \rho\xi \cos\varphi - \rho\eta \sin\varphi \sin\epsilon,$$

$$P \cdot v^{(2)} = \rho\xi(u^{(1)} \cdot v^{(2)}) + \rho\eta(u^{(2)} \cdot v^{(2)})$$

$$= \rho\eta \cos\epsilon,$$

$$\tau \cdot P = \rho\xi(u^{(1)} \cdot \tau) + \rho\eta(u^{(2)} \cdot \tau)$$

$$= \rho\xi \sin\varphi + \rho\eta \cos\varphi \sin\epsilon,$$

$$\tau \cdot C = \rho \cos\varphi \cos\epsilon,$$

$$C \cdot v^{(1)} = -\rho \sin \varphi \cos \varepsilon,$$

$$C \cdot v^{(2)} = -\rho \sin \varepsilon;$$

$$(\tau \cdot C - \tau \cdot P)x^* = (\rho \cos \varphi \cos \varepsilon)\left[\rho \xi \cos \varphi - \rho \eta \sin \varphi \sin \varepsilon\right]$$

$$+ (\rho \sin \varphi \cos \varepsilon)\left[\rho \xi \sin \varphi + \rho \eta \cos \varphi \sin \varepsilon\right]$$

$$= \xi \rho^2 \cos \varepsilon,$$

$$(\tau \cdot C - \tau \cdot P)y^* = (\rho \cos \varphi \cos \varepsilon)\rho \eta \cos \varepsilon$$

$$+ (\rho \sin \varepsilon)\left[\rho \xi \sin \varphi + \rho \eta \cos \varphi \sin \varepsilon\right]$$

$$= \xi \rho^2 \sin \varphi \sin \varepsilon + \eta \rho^2 \cos \varphi;$$

$$(\tau \cdot C - \tau \cdot P) = \rho \cos \varphi \cos \varepsilon - \xi \rho \sin \varphi - \eta \rho \cos \varphi \sin \varepsilon;$$

$$x^* = \frac{\xi \rho \cos \varepsilon}{\cos \varphi \cos \varepsilon - \xi \sin \varphi - \eta \cos \varphi \sin \varepsilon}$$

$$y^* = \frac{\xi \rho \sin \varphi \sin \varepsilon + \eta \rho \cos \varepsilon}{\cos \varphi \cos \varepsilon - \xi \sin \varphi - \eta \cos \varphi \sin \varepsilon}$$

Now let F be the intersection of the horizontal line of sight and the line of fire. Let the x axis lie along the line of fire and point clockwise about the camera. Let the y axis pass through F and point vertically upward, let (x, y) be the coordinates of Q with respect to these axes, and let D be the distance from C to F. It follows that

$$D = \rho \cos \varepsilon, \quad y = y^* + \rho \sin \varepsilon, \quad x = x^*.$$

$$x = \frac{\xi D}{\cos \varphi \cos \epsilon - \xi \sin \varphi - \eta \cos \varphi \sin \epsilon} ,$$

$$y = \frac{D \cos \varphi \sin \epsilon + \eta D \cos \varphi \cos \epsilon}{\cos \varphi \cos \epsilon - \xi \sin \rho - \eta \cos \varphi \sin \epsilon} ;$$

$$x = \frac{\xi D \sec \varphi}{\cos \epsilon - \xi \tan \varphi - \eta \sin \epsilon} ,$$

$$y = \frac{D \sin \epsilon + \eta D \cos \epsilon}{\cos \epsilon - \xi \tan \varphi - \eta \sin \epsilon} ;$$

$$\xi = A_{10} + A_{11} P_1 + A_{12} P_2 ,$$

$$\eta = A_{20} + A_{21} P_1 + A_{22} P_2 .$$

The last two equations, which are fitted as a rule by least squares, formulate the passage from the plate measurements (P_1, P_2) to the standard coordinates (ξ, η).

Some computational suggestions follow.

Agree: that Matrix I is

$$
\begin{Vmatrix}
\cos \epsilon & -\tan \varphi & -\sin \epsilon \\
0 & \sec \varphi & 0 \\
\sin \epsilon & 0 & \cos \epsilon
\end{Vmatrix} ;
$$

that Matrix II is

$$
\begin{Vmatrix}
1 & 0 & 0 \\
A_{10} & A_{11} & A_{12} \\
A_{20} & A_{21} & A_{22}
\end{Vmatrix} ;
$$

that Matrix III is the classical row into column result of multiplying Matrix I into Matrix II.
Let

$$\begin{Vmatrix} R_{00} & R_{01} & R_{02} \\ R_{10} & R_{11} & R_{12} \\ R_{20} & R_{21} & R_{22} \end{Vmatrix}$$

be Matrix III, and let

$$X = \frac{x}{D}, \qquad Y = \frac{y}{D}.$$

Note that

$$x = DX, \qquad y = DY,$$

and that the computation of (X, Y) from (p_1, p_2) may be effected in the following way.

Column	Entry	Heading	Footing	Checks
1	Frame Numbers	Frame number		
2	First Plate Coordinates	Plate Measurement, p_2	$\Sigma 2$	
3	Second Plate Coordinates	Plate Measurement, p_2	$\Sigma 3$	
4	$R_{00}+R_{01}p_1+R_{02}p_2$	D	$\Sigma 4$	$\Sigma 4 = nR_{00}+R_{01}\Sigma 2+R_{02}\Sigma 3$
5	$(R_{10}+R_{11}p_1+R_{12}p_2)/4$	X	$\Sigma 4.5$	$\Sigma 4.4 = nR_{10}+R_{11}\Sigma 2+R_{12}\Sigma 3$
8	$(R_{20}+R_{21}p_1+R_{22}p_2)/4$	Y	$\Sigma 4.8$	$\Sigma 4.8 = nR_{20}+R_{21}\Sigma 2+R_{22}\Sigma 3$

N.B. n = the number of entries in column 1.

N.B. "$\Sigma 2$" means "sum column 2"; "$\Sigma 4.5$" means "sum the products of corresponding entries of column 4 and column 5."

A good check on the matrix multiplication of Matrix I into Matrix II may be based on the inner product - sum formula

$$\left(\sum_{i=0}^{2} r_i \right) \cdot \left(\sum_{j=0}^{2} C_j \right) - \sum_{(i,j)=(0,0)}^{(2,2)} R_{ij} = 0$$

where r_i is the ith row of Matrix I, C_j is the jth column of Matrix II, and R_{ij} is, as agreed, the $(i,j)^{th}$ element of Matrix III.

If the above reduction is used on the present (Spring 1945) data (viz. Take 473) obtained by the Photographic Measurements Section, then the following rules should be followed:

Use 5 decimal trigometric tables and make all entries in Matrix I to 5 decimals, as in

$$\begin{vmatrix} .99347 & .02892 & -.11411 \\ 0 & 1.03020 & 0 \\ .11752 & 0 & .98366 \end{vmatrix}$$

Make entries in Matrix II to 5 decimals in column 1 and to 7 decimals in columns 2 and 3, as in

$$\begin{vmatrix} 1 & 0 & 0 \\ .00251 & .0011949 & .0000129 \\ .00065 & .0000023 & .0011941 \end{vmatrix} \cdot$$

Make each entry in Matrix III to 8 decimals, as in

$$\begin{vmatrix} .99345801 & .00003429 & -.00013589 \\ .00221493 & .00123099 & .00001329 \\ .11815938 & .00000226 & .00117459 \end{vmatrix}$$

Employ the inner product-sum formula to obtain a 7 decimal check. Before using Matrix III round each of its entries to 5 decimals in column 1 and to 7 decimals in columns 2 and 3, as in

$$\begin{Vmatrix} .99346 & .0000343 & -.0001359 \\ .00221 & .0012310 & .0000133 \\ .11816 & .0000023 & .0011746 \end{Vmatrix}$$

Matrix III may now be used in obtaining (X, Y) from (p_1, p_2) as indicated above. In this connection write d, X, and Y to 5 decimals.

Since the standard error to be feared in a determination of either x or y is usually about .15 feet, the rounding errors introduced into X and Y should be negligible as long as D is less than 10,000 feet.

Although the above matrices could be so altered that their use would lead directly to x and y instead of X and Y it does not appear desirable to make this change. Firstly, frame by frame determination of (x,y) is seldom needed. In fact usually only the center value of (x,y) is of interest. Secondly it is clear that any linear function of the x's and y's may be evaluated by applying it to the X's and Y's and multiplying the result by the scale factor D. Thirdly the present matrix arrangements are more uniform and more easily remembered than those met within the proposed alternative.

If, as is frequently the case, ϵ and Matrix II remain constant for a number of rounds it is desirable from some points of view to arrange the computation differently. In Matrix I replace "tan φ" and "sec φ" by "0" to obtain Matrix I_0. Multiply Matrix I_0 into Matrix II to obtain Matrix III_0 and let B_0, B_1, B_2 be the three rows of Matrix III_0. Check that $B_1 = (0, 0, 0)$ recall the distributivety of matrix multiplication and observe

$(R_{00}, R_{01}, R_{02}) = B_0 + (-\tan \varphi)(A_{10}, A_{11}, A_{12})$

$(R_{10}, R_{11}, R_{12}) = B_1 + (\sec \varphi)(A_{10}, A_{11}, A_{12})$

$(R_{20}, R_{21}, R_{22}) = B_2$

These formulas make clear the simplicity with which a calculating machine may be used to obtain any given positional entry in Matrix III for each of a number of rounds. Simple sum checks suggest themselves.

It should be noted, however, that efficient use of this method depends on obtaining Matrix III for each of a number of rounds before Matrix III is used for any one round.

If frame by frame determination of x and y are desired, a new matrix may be so formed that its rows are respectively

$$B_0 + (- \tan \varphi)(A_{10}, A_{11}, A_{12})$$

$$(D \sec \varphi) \quad (A_{10}, A_{11}, A_{12})$$

$$D B_2$$

From this new matrix x and y may be arrived at in the same manner as X and Y were reached via Matrix III.

A. P. Morse
by R.H.K.
A. P. Morse

-10-

AD-805 132

TSDM FORM 88 (13 MAR 47)

Morse, A. P.

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ABSTRACT

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