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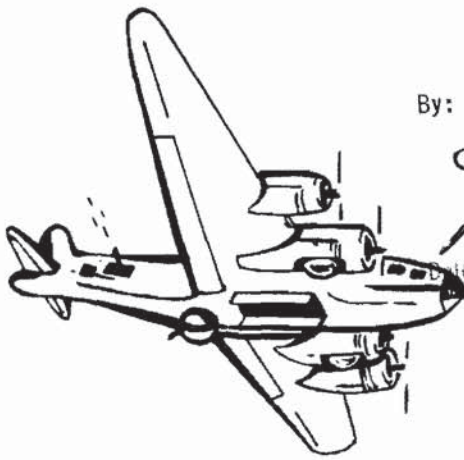

MEMORANDUM
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Lotto Method of Computing Kill Probability

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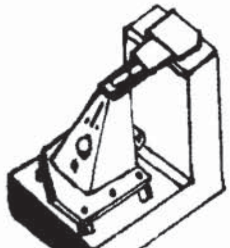

F. G. KING

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MEMORANDUM REPORT NO. 530

December 1950

LOTTO METHOD OF COMPUTING KILL PROBABILITY OF LARGE WARHEADS

F. G. King

Project No. TB3-0138 of the Research and
Development Division, Ordnance Corps

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MEMORANDUM REPORT NO. 530

F. G. King/emj
Aberdeen Proving Ground, Md.
18 December 1950

LOTTO METHOD OF COMPUTING KILL PROBABILITY OF LARGE WARHEADS

ABSTRACT

A method is described for computing the kill probability of large warheads against multiply vulnerable targets, either for a single shot or a number of shots during an engagement. A sample of burst positions is drawn at random from the distribution of guidance errors. This can be done in a number of ways. It has been done so far by drawing a card for each of the three cartesian coordinates of the burst position of each shot. The box of cards is made up to represent guidance errors for particular conditions of engagements. Each burst is positioned with respect to a scale model of the target. The distance and direction of each vital component of the aircraft from the burst is measured. The kill probability for each such component, determined from vulnerability and fragmentation data, is read from a graph for the measured distance and direction. A random number table is used to determine whether a particular component was killed by a particular burst. A score is kept on the number of bursts which killed enough components to bring down the target aircraft. One hundred sample bursts are enough to estimate the probability of kill within the accuracy of the vulnerability data, provided this probability is about .3 or higher. The lotto method can be adapted to estimation of low kill probabilities, but it may be necessary to speed up some of the sampling and summarizing by use of punchcard machines.

Figure 1. Simulator for evaluation of guided missiles.

INTRODUCTION

The problem is to compute the probability that a number of external bursts will kill a large aircraft by throwing fragments into vital components or blasting the structure. If the aircraft can stand the loss of some components, such as one pilot or one engine, so long as others survive, the aircraft is "multiply vulnerable." It is assumed for this problem that kill probabilities on single components have already been estimated from experiments.

The problem was partly formulated as early as World War I by Pearson and Cunningham[1] in England. The solution in terms of abstract mathematical symbols is difficult to carry through in plain numbers, once damage experiments have provided a basis for numerical methods. These Laboratories continued the development of Cunningham's methods,[2] but it was found necessary to make simplifications for the very large volume of computations required in the antiaircraft problem.[3] These simplifications are not considered allowable in the guided missile warhead analysis.

The lotto method proposed in the present report seems very simple when compared with the formulation of the problem in mathematical symbolism. The basic vulnerability data is known for the most part only to order of magnitude and at best only to ten or twenty per cent, so that a simple method is in order. If the basic data were known within one per cent, the lotto method would still be justified as a short-cut to an approximate answer but would be inefficient for getting the answer to one per cent accuracy.

The lotto method has been adopted by BRL for comparison of a number of guided missiles and for study of a family of guided missile warheads.

DESCRIPTION OF THE PROBLEM

The following factors must be considered in computing the kill probability of a bursting shell against a target with duplicated vital components:

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[1] Mathematical Theory of Air Combat and Theory of Air Warfare, L. B. C. Cunningham.

[2] A Method of Computing the Probability of Killing a Multiply Vulnerable Aircraft Target with "N" Rounds of Fragmenting Shell, H. K. Weiss, BRL Memorandum Report No. 495, September 1949 (Confidential).

[3] A Study of a Family of Antiaircraft Weapons, BRL Technical Note No. 119, first issued November 1949.

a. The distance from the target to the nearest point on the path of the missile. This is not known in advance for a particular round. All that is generally given is a two-dimensional probability distribution for the amount and direction of miss.

b. Fuze operation. For an influence fuze, point of burst is dependent upon the path of the missile relative to the target. There is a random element in fuze operation which would make it impossible to predict the exact burst position even if the path of the missile were known in advance.

c. The burst pattern of the warhead. This includes the number of fragments, their distribution by angle and possibly by mass, and their velocity.

d. The orientation of the warhead and its velocity with respect to the air at the time of burst.

e. The orientation of the target and its velocity with respect to the air.

f. The density of the air and the drag coefficient of the fragments.

g. The vulnerable area of each vital component to the fragments at the velocities and angles with which the fragments strike. This includes the shielding of components by armor or other parts of the aircraft.

h. The vulnerability of the structure of the aircraft to the blast wave. This involves the position of the burst, the mass of the casing around the explosive, and the air density.

i. The number of vital components of each type and the combinations which must be killed to destroy the aircraft or prevent accomplishment of its mission. This and the vulnerability of the components depend upon the category of damage chosen.

j. The distances between the components. This determines the probability that two vital components are in the spray of fragments from the same burst. The dispersion of burst positions must be small compared with the dimensions of the aircraft, if a large warhead is to be an effective weapon. A kill on one component may increase or decrease the probability that other components have been killed.

Methods for integrating continuous probability distributions have been developed for taking into account some of these factors and justifying the neglect of others.[1,2,3] These methods are particularly useful for ordinary AA shell. The large number of shell fired in an antiaircraft gun engagement reduces the importance of factor "j" on the preceding page and makes possible simple approximations in the calculation of over-all kill probability

EFFICIENCY OF METHODS USING RANDOM NUMBERS

Mathematical experiments involving the drawing of random numbers to determine the over-all kill probability, such as the lotto method to be described here, are efficient for weapons with high kill probability per shot, as will be seen from the following: The standard error of the kill probability, as determined by mathematical experiments, is given by the theory of the binomial probability distribution as $\sqrt{npq}$ where n is the number of experiments, p is the probability of a kill on the airplane (successful experiment) and q is 1 - p. For example, if the correct but unknown probability is .5, 100 experiments will produce about 50 kills. The standard error $\sqrt{npq}$ in this case is 5. The normal distribution, which is a good approximation to the binomial distribution, gives 2 to 1 odds that there will be between 45 and 55 kills and 20 to 1 odds that there will be between 40 and 60 kills. Such accuracy is better than the accuracy of vulnerability data.

On the other hand, if p is small, as it is for an AA shell, a tremendous number of experiments must be conducted in order to determine the kill probability within reasonable limits. A hundred experiments might produce one kill. One hundred sample engagements of 100 rounds each (10,000 experiments) would be needed to determine an AA engagement kill probability with reasonable accuracy. A scheme is suggested below which might make the lotto method efficient even for AA shell. In this scheme, only the easier part of the mathematical experiment is repeated a large number of times, possibly with the aid of a punchcard machine.

Simulators using electronic and optical equipment are being designed with a built-in randomizing process for "firing" a large number of shots efficiently and in a short time. These simulators employ many or all of

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[1] Cunningham, op. cit.

[2] Weiss, op. cit.

[3] The Survival Probability of a Multiple Component Airplane, A. George Carlton, APL/JHU T681, November 1949 (Confidential).

the basic principles used in the lotto method and should extend their field of application. (The same principles are basic in the "Monte Carlo" method used in mathematical physics.[1])

DETAILED PROCEDURE FOR THE LOTTO METHOD

This is a mathematical experiment in which the events of an engagement between an aircraft and an antiaircraft system are acted out.

a. Each burst is positioned with respect to the target by drawing cards at random from a box representing the distribution of guidance errors at the range in question.

(1) The box can be made up to represent any distribution, theoretical or experimental. For instance, the box could contain a fair collection of missile flight-simulator results. A box of about 1,000 cards has been made up to represent a normal or Gaussian distribution with mean at zero and unit standard error. The cards, of course, make up a discrete distribution. The smallest interval between cards is .01 of the standard error. There are at most four cards representing the same error. At the greater miss distances where a card cannot be given for each possible interval, gaps have been made by withdrawing cards at random. (Beyond three standard errors "small denomination" cards have been made for a total frequency of 100,000. Only one out of a hundred of these are in the main box. When a small denomination card from the main box is used, it goes back into a separate box for small denomination cards, and a card is drawn from this same box as a replacement in the main box.)

(2) For warheads which burst on command from the ground, three cards are drawn at random from the box to represent the error in each of the three cartesian coordinates. The burst is represented in a three-dimensional model in the correct position relative to a scale model of the target aircraft. (The best available models are on a scale of one inch to six feet.) Two stands are used, one for the missile and one for the aircraft. (See Figure 1.) The relative heading of the missile and aircraft is represented on the aircraft stand. The "vertical" miss distance (perpendicular to the trajectory) is set in by sliding a "burst position disk" up or down on the missile stand. Lateral and range errors are set in by moving the missile stand on a table. The aircraft stand is

[1] Journal of American Statistical Association, September 1949.

clamped to the table so as to hold the aircraft at the correct position relative to the center of the error distribution. (The center of the distribution may not be the center of the aircraft.) It has been found more convenient to change the scale markings on the table and the missile stand than to multiply the numbers on the cards by the appropriate standard error of firing in each dimension.

(3) For warheads which are detonated by a fixed-cone fuze, only two cards are drawn at first. After these are drawn to locate the missile trajectory with respect to the target, the trigger position of the fuze is determined by moving the missile stand parallel to the range axis until the fuze cone first touches some part of the aircraft. (The fuze cone is easily generated by rotating the burst-position disk about its diameter representing the missile axis, with the pointer or chain held at a fixed angle, marked on the disk.) Once the "trigger" position is determined, a fuze delay may be added, and a third card is drawn for the fuze error so as to determine the actual burst position. (It may be easier to throw dice to determine fuze delay and fuze error, which together are necessarily positive.) (See note on page 13.)

b. The distance and angle from the burst disk to each vital component is measured.

(1) The disk is rotated about its diameter representing the missile axis, so as to contain the line from disk center to component center. The angle from the longitudinal axis of the warhead to the component is measured on the disk. This angle may be needed merely to determine whether a particular component is in the nose spray, side spray, or no spray at all. In cases where the measurement cannot be made on account of intervening parts of the aircraft, the component is considered to be shielded. In cases of partial shielding, a component is considered to be completely shielded if its center is shielded.

(2) The measurement of distance need not be made with extreme accuracy. The smallest distance which needs to be taken into account is about one foot (one-sixth of an inch on the model). It is permissible to make errors that are smaller than the distances between engines so long as these errors are randomly distributed. For blast damage, the distance to the nearest part of the aircraft structure is measured, and it is determined whether the burst is inside the blast danger volume determined for the particular warhead.

c. The kill probability on each type of vital component is computed for the particular warhead as a function of distance and possibly angle. This computation takes account of fragmentation characteristics and vulnerability data. The altitude of the engagement determines the amount of air slowdown on fragments. The density of fragments per square foot at the component depends upon the limiting angles of the spray which in turn involve missile and target velocity. (As a convenient approximation, the component of target velocity in the direction of the missile velocity may be added to the missile velocity and the other component of target velocity ignored. The exact method of handling target speed, given in the Appendix, involves great labor. A small check sample in a doubtful case will show whether this labor is necessary.)

d. In this mathematical experiment, each vital component is considered to be either killed or not killed by a particular burst. The kill probability, computed as above, turned into yes or no by reading down a random number table. For instance, if the probability that a burst ten feet from an engine will kill the engine is .762 and the next random number read is less than .762, the component is considered killed.

(1) This process is carried out once and for all for each warhead for each type of component. The same yes - no table can be used for different engines exposed to the same burst, since in general the distance between engines on the aircraft is large compared with one foot, and the same random number is unlikely to be read for two engines.

(2) In the case of pilots, two yes - no tables must be constructed from the same probability curve. These may be constructed by using two different columns of the random number table. This procedure is necessary because the distances from the bursts to the pilots are likely to be measured as the same. If the same yes - no table were used for both pilot and co-pilot, they would always be considered to live or to die together.

e. For each burst it is decided whether or not the airplane has been killed, subject to the requirement that a certain number of engines or pilots must be killed. For "A" kills it has been required that both pilots and over half the engines be considered out of action within five minutes. Blast, fire, or direct hit may also kill a target, even though enough engines and pilots survive.

f. A number of sample bursts are drawn as outlined above, say 100 for the smaller errors of guidance and 200 for the larger. The sample size, or at any rate an unbiased sampling procedure, must be decided upon <u>in advance</u>. The kill probability for any given standard error of guidance is estimated by dividing the number of kills on the airplane by the number of trials.

g. To find the cumulative kill probability of a number of missiles with the same accuracy of guidance, it is necessary only to group the elements of the sample.

(1) To get the two-shot probability, the first two bursts out of a sample of 100 are taken as a pair, the third and fourth bursts as another pair, and so on until 50 pairs have been formed. (A vast number of additional pairs can be formed by reusing the bursts in different combinations.) Cumulative damage is of course considered. One pilot can be killed by one burst and the other pilot by the other burst.

(2) The probability of kill on the airplane in an engagement generally involves firing accuracy at a number of different ranges. The damage done by the first burst at one range is added to the damage done by the first burst at each of the other ranges, to get a sample engagement. Thus a hundred bursts at each range can easily be made into a hundred sample engagements. If the bursts are reused in different combinations, such a vast sample of engagements is possible that only a small subsample can ever be summarized. The reliability of results from such a subsample is discussed below.

h. It has been found to be little more work to repeat step "d" a number of times. Step "d" is the second stage in the mathematical experiment where chance is introduced, and it is not necessary that the sample size be the same in this second stage as in the first (step "a"). It is a useful analogy to consider that each time step "d" is repeated, the decision as to whether a given component is killed or not is put up to a different "umpire." Each umpire is just a line or column of the random number table. The random numbers are the successive "moods" of the umpire. Each umpire has one mood for the pilot and one mood for the co-pilot at each burst distance. Results for all the umpires are of course averaged together. The reliability of this average is discussed below.

PRESENT DEVELOPMENTS IN THE USE OF THE LOTTO METHOD

A library system of filing the "firing records" is being worked out. It is planned to have one card for each shot. This card will have the details of the engagement, the distances of the burst from each of the components, and the kill probabilities on the component. The decisions of the umpires in regard to the fate of each component and of the whole airplane should perhaps go on separate cards. It is planned to construct boxes of cards to represent engagements centered at various points in the target's course. Sample engagements can be made up by drawing cards from these boxes. The card system must allow for easy revision of vulnerability data and easy recapitulation of results.

The above requirements suggest a system using IBM cards. This might make it feasible, for instance, to use the lotto method for missiles of low kill probability by machine polling a very large number of umpires.

INFORMATION OBTAINABLE FROM A LIMITED NUMBER OF BURST POINTS

a. Single Shot Kill Probability

It is estimated that, for a study in progress, the use of four umpires instead of one is equivalent to increasing the sample size by thirty per cent. This increase is less than the three hundred per cent that might be expected at first glance, because the four umpires use the same sample burst positions and thus are not independent of each other.

The use of four or more umpires will be a more effective labor saver for missiles of low kill probability per shot. The main labor of sampling is in setting up the burst points and in making geometric measurements. The total number of shots in the sample is the number of burst positions times the number of umpires, but these shots are not spread in space as uniformly as if they were drawn from the given distribution of firing errors shot by shot. Instead the shots are clustered at the points in space which were drawn as burst positions and from which measurements of distance and angle were made.

If only 100 cluster points were used, these might be spaced too thin in three dimensions for estimating the kill probability of small unguided warheads since the target might hardly ever be harmed by the nearest burst drawn in a particular sample of 100 positions. In a spherical normal distribution there is only one chance in three that this nearest burst will be within s/4, where s is the standard error in one dimension. It may be necessary to draw burst points until a kill is obtained and then repeat this procedure until the average length of run is established. (This scheme, in use at Bell Telephone Laboratories in a simulating device, appears especially applicable to duel computations.) Another scheme is to represent the central part of the distribution of firing errors by more cards in the box but to give less weight to cards from the center of the distribution in computing the kill probability. Less weight might be given simply by allowing fewer umpires.

b. Engagement Kill Probability

The reuse of the burst points in making up a large number of sample engagements* destroys the strict statistical independence of the sample engagements in the same way as does the use of a large

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* This procedure is similar to one suggested by an associate, Mr. Ed S. Smith, for a small number of systematically (not randomly), selected burst positions.

number of umpires. Nevertheless, it seems that there will be a saving in labor from use of a sample of non-independent engagements, just as from the use of a sample of decisions by non-independent umpires.

A number of umpires should be used to estimate engagement kill probability from a limited number of burst points. It is ideal but not absolutely necessary to use a different umpire for each engagement.

F. G. King

F. G. King

NOTE: The method of finding the burst position for a cone fuze applies strictly only when the missile trajectory and aircraft flight path are parallel. It was brought out in a conference with personnel of Johns Hopkins Institute for Cooperative Research and the Applied Physics Laboratory that "range axis" must be interpreted to mean "direction of approach of missile and target." The stand must first be placed as if the burst occurred at exactly the range of the target, then brought back along the direction of approach, and then brought forward until the fuze cone touches some part of the aircraft.

APPENDIX I

Exact Method of Accounting for Target Speed in Estimating Kill Probability by the Lotto Method

(Only the main conclusions are given below.)

Consideration was given to the inclusion of some sort of lead-computing linkage on the burst-position disk. However, the most exact method is also the simplest to engineer. A rubber or putty cap can be fitted on the front of each vital component. A spine projects in front of the component in the direction of flight. It is held in position by the putty or rubber. On this spine is marked the position the center of the component will be in at the end of each millisecond after the time of burst. A chain attached to the burst-position disk is also marked to show the position of the fragment at each millisecond. The collision point for each component is then found by moving the chain along the possible future positions of the component until the milliseconds match.

All of the elements of the problem are now completely determined for a particular burst against a particular component. For instance, the remaining velocity of the fragment can be added vectorially to the target speed. From the same vector diagram exact directions of fragment strike can be read. Vulnerability data can be read off or interpolated for the particular angle and velocity of strike. The probability of kill on the component can then be determined and turned into yes or no by use of a random number table.

However, there are some ticklish points to consider in making a determination that pretends to be this exact. The density of fragments per square foot at the future position of the component is not the same as it would be if the component were not moving. There is a "scoop" effect such that the density of fragments is multiplied by the secant of the angle through which the target motion changes the direction of fragment strike. The striking line is better approximated in direction by the line from the burst position to the present position of the component than by the line to the future position. This means that shielding is better approximated by considering the present position of the aircraft. To be really exact, the curve in which a fragment appears to an observer in the aircraft to travel should be sketched. (Positions of observers and fragment are given by the space model at each millisecond. The fragment appears to curve toward the tail as if blown down-wind.)

This procedure just outlined would greatly slow down the kill decisions for the sample burst positions. Many of the effects that are bothered with must be of the second order. For example, the complete solution must allow for target motion which is not along the axis of the missile. However, the effect of a component of target motion perpendicular to the missile axis is opposite for misses on opposite sides of the aircraft. It would presumably take an enormous sample to show the second-order effect remaining. As another example, the complete solution must consider that the fragment is being slowed up by air resistance while the target is not. This becomes important only in cases where the target might kill itself by sweeping up spent fragments.

It should be noticed that the approximate method suggested in the main body of the report for handling target motion projects the target motion onto the trajectory as seen from the ground. It can be shown that the fragment directions and velocities so obtained are good averages for replacing the exact angles and velocities all around the shell. Neglect of the component of target velocity perpendicular to the trajectory restores circular symmetry to the fragment spray.

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