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AUTHORITY

SAMSO ltr, 24 Jan 1972

AIR FORCE REPORT NO.
SD-TR-65-154

AEROSPACE REPORT NO.
TR-669(6540)-7

Digital Computer Program for Error Analysis of Inertial Navigation Systems

AUGUST 1966

Prepared by W. A. FEESS
Guidance and Control Subdivision
and S. BLUMENSTEIN
Computation and Data Processing Center
Electronics Division

Prepared for COMMANDER SPACE SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
Los Angeles, California

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FOREWORD

This report is published by the Aerospace Corporation, El Segundo, California, under Air Force Contract No. AF 04(695)-669.

The requirements for a generalised digital computer program for the error analysis of inertial guidance systems applicable to space missions was established in December 1963. Although under continuing development, the program has been used for system design and analysis studies since June 1964.

This report contains the first complete description of the program and replaces the partial and preliminary ones that have been issued. Its information should be sufficient for most users of the program. This program replaces the one described by R. A. Moore and D. F. Meronek in "A Digital Computer Program for a Generalised Inertial Guidance System Error Analysis" (Reference 1) used previously at Aerospace Corporation. It provides the basic tool for future inertial navigation system error analyses. It was submitted on 24 August 1966 to Captain Ronald J. Starbuck, SSTRT, for review and approval.

Information in this report is embargoed under the U.S. Export Control Act of 1949, administered by the Department of Commerce. This report may be released by departments or agencies of the U.S. Government to departments or agencies of foreign governments with which the United States has defense treaty commitments. Private individuals or firms must comply with Department of Commerce export control regulations.

Approved

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Electronics Division

T. R. Parkin, Director
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Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Captain Ronald J. Starbuck
Project Officer
Space Systems Division
Air Force System Command

ABSTRACT

The theory and assumptions used in developing equations for the error analysis of a general class of inertial navigation systems are described. The computer program developed for their solution is described from a user's point of view. Its application includes the synthesis and/or analysis of inertial navigation systems used in ballistic missile or terrestrial space missions. The program is designed to allow studies of both pure inertial and aided inertial navigation systems, the latter being the process of updating navigation data via data from external sensors.

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SECTION 1

INTRODUCTION

This report describes, from a user's point of view, a computer program developed for the error analysis of a general class of inertial navigation systems. It is generally understood that these systems will be used in connection with either ballistic missile or space mission applications.

Section 2 describes the classes of inertial navigation systems considered, develops the equations necessary to perform an error analysis, enumerates the assumptions made in their derivations, and describes the methods and equations used for data presentation. The equations in Section 2 form the bases for the computer program, which was developed to solve them.

Section 3 deals with the operational aspects of using the computer program to perform error analyses. The input data requirements and procedures are discussed and the resulting output data and formats described. The logical order of the computations resulted in the development of two independent programs: The first, called ERAN, solves the equations presented in Section 2.3; the second, called ØUTP, solves those presented in Section 2.4. These were programmed for the IBM 7090/7094 to be run under the control of the IBM Basic Monitor (IBSYS) Programming System with the assumption that core is set to zero before loading of the programs. There is a certain amount of intentional redundancy in the material presented in Sections 2 and 3. This was done so that once one is familiar with the material presented in Section 2, it will only be necessary to refer to Section 3 for program operations.

Section 4 presents three sample test cases illustrating the input data procedures and the formats of output data, and demonstrating some of the flexibility and capabilities of the program.

Appendices A through C present material augmenting the main body of the report. Appendix A consists of Standard Input Data Forms, Appendix B contains ERAN and ØUTP Input Data for Sample Cases, and Appendix C gives the Output Listings for Sample Cases.

Appendix D is devoted to the subject of updating or correcting navigation data through the use of external data sources utilizing various sensor configurations. Algorithms are derived for three possible schemes of data processing. Although equations have not been programmed, the logical structure of the computer program is designed so that this feature can be incorporated without major revisions.

In Appendix E the method is discussed of treating the effects of aerodynamic drag in orbit when the accelerometers are disconnected from the navigation computer.

Appendix F contains all the figures called out in the report and Appendix G the program definitions and constants.

SECTION 2

EQUATION DEVELOPMENT

2.1 INTRODUCTION

This section defines the classes of inertial navigation systems considered and develops the equations necessary to perform an error analysis of a given configuration.

Section 2.3 relates the derivation of the differential equations of navigation error to a broad class of system errors. The solution of these equations results in the linear transformations (sensitivities) of navigator errors into errors of navigation data. The classes of errors include those of sensor anomalies, initial conditions, terminal control, and, for certain operational procedures, the effect of orbital drag uncertainties.

Section 2.4 describes the equations used for processing these sensitivities into individual navigation vector errors for each error source, and those which statistically sum all vector errors presented as a covariance matrix. Various coordinate systems and processing methods are described for data presentation.

2.2 NAVIGATION SYSTEM CONFIGURATIONS

The inertial navigation system configurations considered are schematically presented in Figure 1. The equations developed for error analysis are applicable to torqued or inertially oriented gimballed-platform systems, and to a certain class of strapped-down systems. The essential sensors used by the navigation system are three accelerometers, which sense the magnitude of the applied external accelerations, and three gyros,* which sense angular dynamics so that the direction of the applied acceleration can be derived. The constraints of accelerometer mounting are such that the three sensing (input) axes cannot be coplanar, but can be nonorthogonal. Gyros are assumed to be mounted in a triad so that their sensing (input) axes are orthogonal. The method of deriving accelerometer orientation from gyro signals is assumed to be one of the following three:

a. Gimballed Platforms. In this configuration - the most conventional - the platform is initially aligned to some auxiliary references. For initial alignment on the ground, the gravity vector, which is sensed by pendulums or the accelerometers, is used for vertical reference; azimuth is referenced to either optical sensors or a gyro compass. For in-orbit alignment, stellar or horizon sensors are used. The gyros measure any deviation of the platform from the initial alignment and their signals are used to torque the platform in such a way that they become null, thus maintaining the initial reference. In some cases, the platform is torqued either to reduce the total gimbal-angle travel, or to maintain prelaunch (earth) rates. To achieve this, the gyros are torqued at prescribed rates, which the platform follows. In this case, the accelerometer transformation matrix is a function of time and computed from the commanded rates. Mathematically, this is identical to configuration (b).

b. Strapped-down/Caged Gyros. In this configuration, the platform is mounted directly (possibly by a shock mount) to the airframe. The gyros sense angular deviations from the initial reference, but are torqued to null their

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*Two single-degree-of-freedom gyros can be oriented to represent one two-degree-of-freedom gyro.

signals. These torquing signals are a measure of the rate-of-change of the platform orientation and are used to compute the transformation matrix required for the accelerometers. The algorithm used is based on the matrix differential equation of direction cosines.

c. Strapped-down/Free Gyros. Until the advent of the electrostatic gyro (ESG), lighweight wide-angle free gyros were not sufficiently accurate to be considered for this application. Free (two-degree-of-freedom) gyros are used in platform configurations but are restricted to small-angle deviations. With its high degree of accuracy, the ESG acts as a potential attitude reference sensor. In this configuration, the angles the gyro case (thus, the accelerometer) makes with respect to the spin axis of the gyro are read out and used to compute the transformation matrix.

The sensor data is processed by a computer to derive position and velocity data. It is assumed that the navigation system computer has perfect algorithms for gravity, and that its word length and integration schemes are such as to produce negligible errors. Most accelerometer outputs are in the form of pulse rates proportional to acceleration, thus there are additional complications in deriving inertial velocity when the platform is not inertially oriented. The algorithms used in these cases become the subject of a separate analysis, which requires detailed knowledge of the hardware characteristics. In general, a special-purpose computer (e.g., a Digital Differential Analyzer) would be required to buffer the processing of accelerometer and gyro data into a suitable form for processing by a general-purpose computer. Generally, this form is sensed velocity data, which is then corrected for the effects of gravity from which the trajectory position and velocity data are determined. In this analysis it is assumed that the error in these computations is small enough to be negligible, or convertible to equivalent sensor errors; therefore, the errors in indicated position and velocity are functions of sensor anomalies only.

Provisions are included for analyses of aided inertial navigation systems in which external sensors are used for measuring position, velocity, and/or platform orientation. The measurements are processed by various techniques and applied as corrections to the navigation system data. These corrections are discussed in Appendix D under Resets. The computer program in its present form does not include the coding of these equations.

2.3 ERROR SENSITIVITY EQUATION DEVELOPMENT

2.3.1 Coordinate Systems and Transformation Matrices

The basic coordinate system used for computing navigation errors is an earth-centered inertial (ECI) system, in which the Z axis is along the earth's polar axis, and the X and Y axes lie in the earth's equatorial plane, forming a right-hand orthogonal axis system. Generally, the convention is that the X axis passes through the Greenwich meridian at time zero.

The notation used for coordinate transformation matrices is the symbol M with two-lettered subscripts to identify the respective coordinate systems; e.g., M_{EK} identifies the transformation matrix, which is used to transform vectors to the ECI coordinate system from the K coordinate system. Conversely, M_{KE} transforms vectors to the K system from the ECI system. The necessary coordinate systems and the transformation matrices are described in the following paragraphs.

2.3.1.1 Platform Coordinate System

The platform axes are designated P_1, P_2, and P_3 and form a right-hand orthogonal coordinate system. The initial transformation matrix is developed by considering the platform axes to be initially aligned with the ECI axes and by applying ordered rotations of ϕ_P about P_3, a negative λ_P about P_2, and a negative ψ_P about P_1. Thus, the matrix that transforms vectors in ECI coordinates to vectors in platform coordinates is

$$M_{PE} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi_P & -S\psi_P \\ 0 & S\psi_P & C\psi_P \end{bmatrix} \begin{bmatrix} C\lambda_P & 0 & S\lambda_P \\ 0 & 1 & 0 \\ -S\lambda_P & 0 & C\lambda_P \end{bmatrix} \begin{bmatrix} C\phi_P & S\phi_P & 0 \\ -S\phi_P & C\phi_P & 0 \\ 0 & 0 & 1 \end{bmatrix} (t = t_o)$$

Figure 2 illustrates the usual initial platform orientation, where ϕ_P is the longitude, λ_P is the geocentric latitude (positive North latitude), and ψ_P is the azimuth (positive is conventional azimuth from North).

Since platform coordinates are orthogonal

$$M_{EP} = M_{PE}^T$$

where T denotes the transpose. M_{PE} can be a function of time (for strapped-down or torqued platforms) and its calculation is discussed in Section 2.3.2.

2.3.1.2 Gyro Coordinate System

It is necessary to assign a coordinate system to each gyro so that the gyro errors can be determined. The gyro axes for each component are right-hand orthogonals and designated G_{i1}, G_{i2}, and G_{i3} (i = number 1, 2, or 3 gyro), where G_{i1} is the sensing (input) axis of the i^{th} gyro. It is assumed that G_{11}, G_{21}, and G_{31} also form a right-hand orthogonal axis system.

As a result of this assumption, the development of the gyro coordinate systems is simplified. Since, for any one configuration, the gyro axes are assumed to be fixed with respect to the platform axes, the gyro coordinate systems are developed with respect to the platform coordinate system. Figure 3 illustrates the initial gyro alignments with respect to the platform axes. The method of specifying gyro orientations is by specification of an axis (1, 2, or 3) and an argument (angles) of successive rotations. Each rotation operates on the gyros as a triad; i.e., all three gyros are being rotated and thus are maintaining their axis orientation with respect to each other during the rotations. The axis of rotation referred to above is that of the number one gyro. Thus, the matrix that transforms vectors in platform coordinates to gyro coordinates (gyro input axes) is

$$M_{GP} = T_{ij} \cdot \cdot \cdot \cdot \cdot \cdot T_{i2} T_{i1}$$

where

i = the axis of rotation (i = 1, 2, or 3)

j = the j^{th} rotation (j = 1, 2, . . . up to 5)

and

$$T_{1j} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta_j & S\theta_j \\ 0 & -S\theta_j & C\theta_j \end{bmatrix}$$ transformation for a rotation about G_{11} axis

$$T_{2j} = \begin{bmatrix} C\theta_j & 0 & -S\theta_j \\ 0 & 1 & 0 \\ S\theta_j & 0 & C\theta_j \end{bmatrix}$$ transformation for a rotation about G_{12} axis

$$T_{3j} = \begin{bmatrix} C\theta_j & S\theta_j & 0 \\ -S\theta_j & C\theta_j & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ transformation for a rotation about G_{13} axis

where θ_j is the angle of the j^{th} rotation and a positive angle is in the sense of a right-hand rotation. Upon completion of this set of rotations, there remains an additional degree of rotational freedom of each gyro about its input axis. By considering this degree of freedom, the vector components along the other two axes of the gyro (axes 2 and 3) are determined, utilizing the matrices

$$M_{G2} = \begin{bmatrix} 0 & C\psi_1 & S\psi_1 \\ S\psi_2 & 0 & C\psi_2 \\ C\psi_3 & S\psi_3 & 0 \end{bmatrix}$$ transforms a vector in gyro coordinates to vector components along the 2 axis of each gyro

$$M_{G3} = \begin{bmatrix} 0 & -S\psi_1 & C\psi_1 \\ C\psi_2 & 0 & -S\psi_2 \\ -S\psi_3 & C\psi_3 & 0 \end{bmatrix}$$

transforms a vector in gyro coordinates to vector components along the 3 axis of each component

where ψ_i is the angle of rotation for the i^{th} gyro and a positive angle is in the sense of a right-hand rotation.

It is convenient to describe the gyro axes in the model of a single-degree-of-freedom gyro, where the G_{i1} is the input reference axis, G_{i2} is the output axis, and G_{i3} is the spin reference axis. Then a rotation of ψ_i = 90 deg of i^{th} gyro will provide an orientation of two gyros with orthogonal input axes and parallel spin axes, a model of a two-degree-of-freedom gyro. Since the gyro (input axes) coordinate system is orthogonal, the following transformations are derived

$$M_{PG} = M_{GP}^T$$

$$M_{GE} = M_{GP}M_{PE}$$

$$M_{EG} = [M_{GP}M_{PE}]^T = M_{EP}M_{PG}$$

2.3.1.3 Accelerometer Coordinate System

The following two options are used for specifying the alignment of accelerometers.

FIRST OPTION

First is the specification of an orthogonal triad and the method of specification is identical with that described for the gyro components; that is, an axis and argument of successive rotations are specified to align the accelerometer input axes. Then an additional degree of freedom about each accelerometer's input axis is specified by an angle β_j. The accelerometer axes for each component are right-hand orthogonal and are designated A_{i1}, A_{i2}, and A_{i3} (i = number 1, 2, or 3 accelerometer), where A_{i1} is considered the input axis. Figure 4 illustrates the initial orientation of the accelerometer axes with respect to the platform axes. The following matrices apply for accelerometers when this option is used.

$$M_{AP} = T_{ij} \cdot \cdot \cdot \cdot \cdot T_{i1} \qquad (j = 1, 2, \ldots \text{ up to } 5)$$

$$M_{A2} = \begin{bmatrix} 0 & C\beta_1 & S\beta_1 \\ C\beta_2 & 0 & -S\beta_2 \\ C\beta_3 & S\beta_3 & 0 \end{bmatrix}$$

transforms a vector in accelerometer coordinates to vector components along the 2 axes of each accelerometer

$$M_{A3} = \begin{bmatrix} 0 & -S\beta_1 & C\beta_1 \\ C\beta_1 & 0 & -S\beta_2 \\ -S\beta_3 & C\beta_3 & 0 \end{bmatrix}$$

transforms a vector in accelerometer coordinates to vector components along the 3 axes of each accelerometer

$$M_{PA} = M_{AP}^T$$

$$M_{AE} = M_{AP} M_{PE}$$

$$M_{EA} = [M_{AP} M_{PE}]^T = M_{EP} M_{PA}$$

SECOND OPTION

The second option allows for a nonorthogonal accelerometer configuration. Here the method of specification is the same (i.e., specification of an axis and an argument); however, each accelerometer is specified independently. The initial orientation of each accelerometer is the same with the accelerometer's input Axis A_{i1} aligned with P_1, A_{i2} with P_2, and A_{i3} with P_3. Therefore, a matrix is developed for each accelerometer k, as follows:

$$M_{APk} = T_{ijk} \cdot \cdot \cdot \cdot \; T_{i2k} T_{i1k} \quad (k = 1, 2, \text{and } 3)$$

From these three matrices, the matrices M_{AP}, M_{A2}, and M_{A3} are formed as

$$M_{AP} = \begin{bmatrix} M_{11} \\ M_{12} \\ M_{13} \end{bmatrix}$$

$$M_{A2} = \begin{bmatrix} M_{21} \\ M_{22} \\ M_{23} \end{bmatrix} M_{PA}$$

$$M_{A3} = \begin{bmatrix} M_{31} \\ M_{32} \\ M_{33} \end{bmatrix} M_{PA}$$

where M_{ik} is the i^{th} row of M_{APk} (k = 1, 2, 3)

Since for this option the accelerometers are nonorthogonal, M_{PA} is developed from the inverse rather than the transpose, and the following relationships result

$$M_{PA} = M_{AP}^{-1}$$

$$M_{AE} = M_{AP} M_{PE}$$

$$M_{EA} = [M_{AP} M_{PE}]^{-1} = M_{EP} M_{PA}$$

2.3.1.4 Initial Condition Coordinate System

These coordinate systems are used to transform errors specified in a convenient coordinate system into errors in ECI coordinates. The initial platform errors are specified in the platform coordinate system and transformed into ECI coordinates by M_{EP}. Two options are provided for specifying initial position and velocity errors. In the first, the errors are assumed to be referenced to platform axes; thus M_{EP} is used. In the second, the errors are assumed to be referenced to the geocentric vertical and directed in azimuth by a specified angle (ψ_I) (see Figure 5). The transformation matrices for position and velocity errors are then

OPTION 1

$$M_{EI} = M_{EP}$$

OPTION 2

$$M_{EI} = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\lambda & 0 & -S\lambda \\ 0 & 1 & 0 \\ S\lambda & 0 & C\lambda \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi_I & S\psi_I \\ 0 & -S\psi_I & C\psi_I \end{bmatrix}$$

where

$$S\phi = \frac{Y}{D} \qquad S\lambda = \frac{Z}{R}$$

$$C\phi = \frac{X}{D} \qquad C\lambda = \frac{D}{R}$$

$$D = \sqrt{X^2 + Y^2} \qquad R = \sqrt{D^2 + Z^2}$$

and

X, Y, Z = initial ($t = t_o$) ECI position coordinates

ϕ and λ = the initial longitude and geocentric latitude, respectively

ψ_I = the azimuth orientation of the coordinate system.

2.3.1.5 Terminal-condition Coordinate System

These coordinate systems are used when it is desired to propagate the effects of terminal-control errors generated by other systems for which there can be no further corrections (e.g., guidance steering and thrust tailoff errors at thrust termination of a ballistic missile). The transformation for terminal-position errors is

$$M_{ET1} = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\lambda & 0 & -S\lambda \\ 0 & 1 & 0 \\ S\lambda & 0 & C\lambda \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi & S\psi \\ 0 & -S\psi & C\psi \end{bmatrix}$$

where

$$S\phi = \frac{Y}{D} \qquad S\lambda = \frac{D}{R} \qquad S\psi = \frac{V_E}{V_H}$$

$$C\phi = \frac{X}{D} \qquad C\lambda = \frac{Z}{R} \qquad C\psi = \frac{V_N}{V_H}$$

$$D = \sqrt{X^2 + Y^2} \qquad R = \sqrt{D^2 + Z^2}$$

$$V_E = \frac{-Y\dot{X} + X\dot{Y}}{D}$$

$$V_N = \frac{-Z(X\dot{X} + Y\dot{Y}) + \dot{Z}D^2}{RD}$$

$$V_H = \sqrt{V_N^2 + V_E^2}$$

and

X, Y, Z = terminal ECI position coordinates

$\dot{X}\ \dot{Y}\ \dot{Z}$ = terminal ECI velocity coordinates

$V_E\ V_N\ V_H$ = east, north, and horizontal velocity vector components

ϕ, λ, ψ = longitude, latitude, and velocity vector azimuth at the termination time

The transformation for terminal velocity errors is

$$M_{ET2} = M_{ET1} \begin{bmatrix} C\gamma & 0 & S\gamma \\ 0 & 1 & 0 \\ -S\gamma & 0 & C\gamma \end{bmatrix}$$

where

$$S\gamma = \frac{X\dot{X} + Y\dot{Y} + Z\dot{Z}}{RV}$$

$$C\gamma = \frac{V_H}{V} \qquad V = (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)^{1/2}$$

and

γ is the flight path angle and equals the angle that the velocity vector makes with the horizontal plane

V is the magnitude of the velocity vector.

These coordinate systems are illustrated in Figure 6.

The other coordinate systems, used for output data and reset, are described in Section 2.4.1 and Appendix D.

2.3.2 Navigation Kinematics

For purposes of error analyses, it is of interest to derive the sensitivities of navigation and platform orientation errors as a function of the anomalies of system parameters (which are random variables). These sensitivities are then used with the statistics of the system parameters to develop the statistical characteristics of the navigation data. For this purpose, the differential equations are linearized so that all the advantages of linear analysis can be utilized. The following sections develop this procedure.

2.3.2.1 Equations of Motion

The differential equation, expressed in ECI coordinates, that describes the vehicle equations of motion for an assumed point mass system is

$$\ddot{\overline{X}} = -\frac{\mu}{R^3}[\overline{X} + \overline{G}] + \ddot{\overline{X}}_s$$

where

μ = gravitational constant

R = magnitude-of-position vector

$\overline{G}$ = vector of higher-order gravity terms

$\ddot{\overline{X}}$ = acceleration vector of the vehicle

$\ddot{\overline{X}}_s$ = sensed acceleration vector due to all external forces (thrust, drag, etc.)

The navigation system's computer mechanizes and solves this equation with the appropriate initial conditions of position and velocity ($\overline{X}_o$ and $\dot{\overline{X}}_o$).* The sensed acceleration components are measured by the accelerometers and transformed into ECI coordinates resulting in

$$\ddot{\overline{X}}_{sm} = [M_{EP}M_{PA}]_1 \overline{A}_{oc}$$

where

$\ddot{\overline{X}}_{sm}$ = the sensed acceleration measured and transformed into ECI coordinates

$[M_{EP}M_{PA}]_1$ = the transformation between accelerometer and ECI coordinates, which the computer uses

$\overline{A}_{oc}$ = accelerometer measurements corrected for calibrated anomalies

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*The choice of coordinate systems made here was for convenience. The results are invariant with the computational coordinate system chosen, provided that the assumption of perfect computer algorithms is valid.

The corrected accelerometer measurements are the result of input accelerations and functionally related correction terms. Thus, the accelerometer equation becomes

$$\overline{A}_{oc} = \overline{A}_s + f(\overline{A})$$

$$= [M_{AP}M_{PE}]_2 \ddot{\overline{X}}_s + f(\overline{A})$$

where

$\overline{A}_s$ = uncorrected accelerometer measurements

$f(\overline{A})$ = the equation for the accelerometer anomalies, which includes corrections for biases, scale factor, nonlinearity, etc., based on instrument calibrations. (This function is discussed in Section 2. 3. 3, Error Sources)

$[M_{AP}M_{PE}]_2$ = the transformation between actual accelerometer axes and ECI coordinates, a function of platform errors, accelerometer alignments, nonlinearities, etc.

Therefore, the equation that is solved by the navigation computer is

$$\ddot{\overline{X}} = -\frac{\mu}{R^3}[\overline{X} + \overline{G}] + [M_{EP}M_{PA}]_1\{[M_{AP}M_{PE}]_2\ddot{\overline{X}}_s + f(\overline{A})\}$$

Generally, it is not required that a distinction be made between $[M_{EP}M_{PA}]_1$ and $[M_{AP}M_{PE}]_2$, except in cases where a given error source affects both computer and platform transformations, e. g., initial position errors.

2. 3. 2. 2 Constraints and Assumptions

Before proceeding to the linearization of the navigation system equations, the following constraints are imposed.

a. Nominal Trajectory Reference

The parameters that affect the sensed acceleration vector (e. g., thrust, weight, wind, control system, etc.) are statistically independent of the navigation parameters (accelerometers, gyros, etc.). Therefore, to include them in the analysis would result in trajectory deviations constrained by the guidance system, but not necessarily in navigation errors of the measured trajectory. The only way in which the two sets of parameters could be entered into the analysis would be from nonlinearities for which Monte Carlo techniques would be required to develop their effects. Fortunately, these nonlinearities are small and can be assessed independently by performing a guidance error analysis and the navigation system error analysis, separately, using the same reference trajectory. The navigation error analysis can be repeated for the maximum perturbed trajectory developed in the guidance error analysis to determine if further analysis is required.

b. First-Order Partials

If the equations are expanded into a Taylor series, the relative magnitudes of second-order partials can be determined. For example, by taking one component of the gravity expression

$$\ddot{X} = -\frac{\mu}{R^3} X = -\frac{\mu}{X^2} \qquad \text{(assuming } R = X,\ Y = Z = 0\text{)}$$

and expanding it to

$$\Delta\ddot{X} = \frac{\partial \ddot{X}}{\partial X} \Delta X + \frac{\partial^2 \ddot{X}}{\partial X^2} \frac{(\Delta X)^2}{2!} + \cdots$$

$$= \frac{2\mu}{X^3} \left(1 - \frac{3\Delta X}{2X} + \cdots\right) \Delta X$$

it is seen that the maximum effect of the second-order partial is a function of $\Delta R/R$. Since $\Delta R/R <<< 1$ for any reasonable system, it can be neglected. Many analyses of inertial guidance systems neglect completely the first-order gravity partials* or treat them as constants.**

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*See for example Reference 2, p 304: "Use of Normalized Integrals."

**See for example Reference 3.

With a similar analysis, the second-order gravity terms ($\tilde{G}$) can be eliminated, even though they present linear terms in the equations.

c. **Small-Angle Approximations**

Applying the small-angle approximation to platform and computer errors is justified on the basis of comparing second-order partials. With this approximation, small-angle rotations can be represented as vectors and the following vector matrix relationships can be used.*

(1) Vector Transformations

$$\bar{\phi}_i = M_{ij}\bar{\phi}_j$$

where

$\bar{\phi}_i$ = angular errors expressed in the i coordinate system due to angular errors in the j coordinate system $\bar{\phi}_j$

M_{ij} = the transformation matrix to the i coordinate system from the j coordinate system.

For example

$$\bar{\phi}_i = \bar{\phi}_E = \begin{bmatrix} \phi_x \\ \phi_y \\ \phi_z \end{bmatrix} = M_{EP}\bar{\phi}_P = M_{EP}\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$$

(2) Matrix Differentials

$$[\delta M_{ij}]_i = [\Phi_i]M_{ij} = \begin{bmatrix} 0 & \phi_3 & -\phi_2 \\ -\phi_3 & 0 & \phi_1 \\ \phi_2 & -\phi_1 & 0 \end{bmatrix} M_{ij}$$

*See Reference 4, pp 251 to 253.

where

$[\delta M_{ij}]_i$ = the change in M_{ij} due to small rotations in the i coordinate system.

$[\Phi_i]$ = a skew symmetric matrix formed from the small angle rotations in the i coordinate system.

For example

$$[\delta M_{PE}]_P = \begin{bmatrix} 0 & \phi_3 & -\phi_2 \\ -\phi_3 & 0 & \phi_1 \\ \phi_2 & -\phi_1 & 0 \end{bmatrix} M_{PE} = [\Phi_P] M_{PE}$$

when ϕ_i are small rotations in platform coordinates.

Also

$$[\delta M_{EP}]_P = [\delta M_{PE}]_P^T = -M_{EP}[\Phi_P]$$

If the small rotations are expressed as rates times time ($\dot{\phi}\delta t = \omega\delta t$), then in the limit as $\delta t \to 0$, the matrix differential equation is

$$\frac{[\delta M_{EP}]_P}{\delta t} = \dot{M}_{EP} = -M_{EP} \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix} = -M_{EP}[\Omega_P]$$

and

$$\dot{M}_{PE} = [\Omega_P] M_{PE}$$

(3) <u>Matrix Transformations</u>

$$[\Phi_j] = M_{ji}[\Phi_i]M_{ij}$$

where

$[\Phi_j]$ = skew symmetric matrix expressed in the j coordinate system due to rotations in the i coordinate system.

For example

$$[\Phi_E] = M_{EP}[\Phi_P]M_{PE}$$

that is

$$\begin{bmatrix} 0 & \phi_z & -\phi_y \\ -\phi_z & 0 & \phi_x \\ \phi_y & -\phi_x & 0 \end{bmatrix} = M_{EP} \begin{bmatrix} 0 & \phi_3 & -\phi_2 \\ -\phi_3 & 0 & \phi_1 \\ \phi_2 & -\phi_1 & 0 \end{bmatrix} M_{PE}$$

This can be shown by using vector matrix relationships as follows: Let vectors in the P coordinate system be related as

$$\overline{Y}_P = \overline{\phi}_P \times \overline{A}_P \qquad = -\overline{A}_P \times \overline{\phi}_P$$

$$= \begin{bmatrix} 0 & A_3 & -A_2 \\ -A_3 & 0 & A_1 \\ A_2 & -A_1 & 0 \end{bmatrix} \overline{\phi}_P \qquad = - \begin{bmatrix} 0 & \phi_3 & -\phi_2 \\ -\phi_3 & 0 & \phi_1 \\ \phi_2 & -\phi_1 & 0 \end{bmatrix} \overline{A}_P$$

$$= - [\Phi_P] \overline{A}_P$$

Transforming the above vectors into E coordinates results in

$$\overline{Y}_E = M_{EP} \overline{Y}_P = -M_{EP} [\Phi_P] \overline{A}_P$$

$$\overline{\phi}_E = M_{EP} \overline{\phi}_P$$

$$\overline{A}_E = M_{EP} \overline{A}_P$$

The vectors $\overline{Y}$, $\overline{\phi}$, and $\overline{A}$ are invariant under an orthogonal transformation; therefore

$$\overline{Y}_E = \overline{\phi}_E \times \overline{A}_E = -[\Phi_E] \overline{A}_E \qquad = -[\Phi_E] M_{EP} \overline{A}_P$$

Equating the two above expressions for $\overline{Y}_E$ results in

$$[\Phi_E] M_{EP} = M_{EP} [\Phi_P]$$

or

$$[\Phi_E] = M_{EP} [\Phi_P] M_{PE}$$

2.3.2.3 Linearization of the Differential Equation

The linearization procedure is established by taking the partial derivatives with respect to each error source. The notation used is: $\delta = \partial/\partial\epsilon_i$ = parti. ' derivative with respect to the i^{th} error source.

Neglecting the higher-order gravity term, we find that the navigation system equation to be linearized is

$$\ddot{\overline{X}} = -\frac{\mu}{R^3}\overline{X} + [M_{EP}M_{PA}]_1[M_{AP}M_{PE}]_2\ddot{\overline{X}}_s + [M_{EA}]\mathcal{f}(\overline{A})$$

When the following assumptions are applied

a. μ is a known constant

b. $\delta\ddot{\overline{X}}_s = 0$, the nominal trajectory reference

c. $\delta M_{EA}\,\mathcal{f}(\overline{A}) \doteq 0$, products of small perturbations are zero (see Section 2.3.3.2 for $\mathcal{f}(\overline{A})$)

the linearized equation becomes

$$\delta\ddot{\overline{X}} = -\frac{\mu}{R^3}\delta\overline{X} + \frac{3\mu}{R^4}\overline{X}\delta R + \{M_{EA}\delta M_{AE2} + \delta M_{EA1}M_{AE}\}\ddot{\overline{X}}_s + M_{EA}\delta\,\mathcal{f}(\overline{A})$$

The following terms in the above expression are expanded as

a. $R = \sqrt{X^2 + Y^2 + Z^2}$

$\therefore \quad \delta R = X^T\delta\overline{X}/R$

b. $\delta M_{AE2} = \delta[M_{AP}M_{PE}]_2 = \delta M_{AP}M_{PE} + M_{AP}\delta M_{PE}$

Since δM_{AP} is in error due to a misalignment of the accelerometer true input axis from the one calibrated, it is treated as an accelerometer error source. Consequently, $\delta M_{AP} = 0$ for these equations. The matrix δM_{PE} results from platform angular misalignments. Therefore

$$\delta M_{AE2} = M_{AP}[\delta M_{PE}]_P = M_{AP}[\Phi_P]M_{PE}$$

where

$[\Phi_P]$ = the skew symmetric matrix made up of platform angular errors expressed in platform coordinates

c. $$\delta M_{EA1} = \delta[M_{EP}M_{PA}]_1 = \delta M_{EP}M_{PA} + M_{EP}\delta M_{PA}$$

Since M_{PA} is calculated from M_{AP}, there is no error in M_{PA}, that is, $\delta M_{PA} = 0$. Here δM_{EP} results from computer errors. Therefore

$$\delta M_{EA1} = [\delta M_{EP}]_P M_{PA} = -M_{EP}[\Theta_P]M_{PA}$$

where

$[\Theta_P]$ = the skew symmetric matrix made up of computer errors expressed in platform coordinates.

By using these relationships, the equation reduces to

$$\delta\ddot{\overline{X}} = \frac{\mu}{R^3}\left[\frac{3\overline{X}\overline{X}^T}{R^2} - I\right]\delta\overline{X} + \left\{M_{EA}M_{AP}[\Phi_P]M_{PE} - M_{EP}[\Theta_P]M_{PA}M_{AE}\right\}\ddot{\overline{X}}_s + M_{EA}\,\delta f(\overline{A})$$

$$= M_G\delta\overline{X} + M_{EP}\{\Phi_P - \Theta_P\}M_{PE}\ddot{\overline{X}}_s + M_{EA}\delta f(\overline{A})$$

where

$$M_G = \frac{\mu}{R^3}\left[\frac{3\bar{X}\bar{X}^T}{R^2} - I\right]$$

$$= \frac{\mu}{R^3}\begin{bmatrix} \frac{3X^2}{R^2} - 1 & \frac{3XY}{R^2} & \frac{3XZ}{R^2} \\ & \frac{3Y^2}{R^2} - 1 & \frac{3YZ}{R^2} \\ \text{(Symmetric)} & & \frac{3Z^2}{R^2} - 1 \end{bmatrix}$$

Using the properties discussed above, these equations are further reduced to

$$\delta\ddot{\bar{X}} = M_G \delta\bar{X} + M_A \bar{\psi} + M_{EA} \delta \mathcal{f} (\bar{A})$$

where

$$M_A = \begin{bmatrix} 0 & -\ddot{Z}_s & \ddot{Y}_s \\ \ddot{Z}_s & 0 & -\ddot{X}_s \\ -\ddot{Y}_s & \ddot{X}_s & 0 \end{bmatrix}$$

$\ddot{X}_s$, $\ddot{Y}_s$, $\ddot{Z}_s$ = components of $\ddot{\bar{X}}_s$

$\bar{\psi} = \bar{\phi} - \bar{\theta}$ = vector expressed in ECI coordinates, which is the difference between computer axes and platform axes,* due to the various error sources (see Section 2.3.3)

$\bar{\phi}$ = the vector expressed in ECI coordinates of platform errors

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*This notation is the same as the one given in Reference 2, p 161.

$\bar{\theta}$ = the vector expressed in ECI coordinates of computer errors

$\delta f(\bar{A})$ = the vector expressed in accelerometer coordinates of the accelerometer errors.

These then constitute the basic differential equations (variational equations) for computing navigation error sensitivities, as a function of the navigation system error sources. These equations are put into a pseudo-state vector matrix form as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \end{bmatrix} = \left[\begin{array}{c|c|c} 0 & I & 0 \\ \hline M_G & 0 & M_A \\ \hline 0 & 0 & 0 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{F}_A \\ \bar{F}_\psi \end{bmatrix}$$

$$\dot{\bar{x}}_i = M_x \bar{x}_i + \bar{F}_i$$

where

$x_1\ x_2\ x_3$ are XYZ position partials due to i^{th} error source

$x_4\ x_5\ x_6$ are $\dot{X}\dot{Y}\dot{Z}$ velocity partials due to i^{th} error source

$x_7\ x_8\ x_9$ are ψ_x, ψ_y, ψ_z orientation partials due to i^{th} error source (also see Section 2.3.3)

$F_A = M_{EA} \frac{\partial f(\bar{A})}{\partial \epsilon_i}$ is the forcing function of accelerometer errors due to i^{th} error source

$F_\psi = \frac{\partial \dot{\psi}}{\partial \epsilon_i}$ is the forcing function of orientation rate errors due to i^{th} error source

$\tilde{x}_i$ is the state vector representing the navigation error sensitivity due to i^{th} error source.

2.3.3 Error Sources

The error sources of the unaided inertial guidance system fall into the general categories of initial conditions, accelerometer, gyro, and platform errors. In addition, terminal errors are included to be able to account for guidance and control errors.

The general notation used for the identification of error sources is a seven-character symbol. Not all are explicitly used when it is convenient to drop certain characters without loss of generality, or when an error source is explicitly defined otherwise. Each character is defined in Table 1.

Table G-1 (in Appendix G) gives the symbol, description, and units for each error-source type presently considered*. Additions to this list are easily accommodated when a certain component does not fit the model error considered here. For purposes of describing error-source models, the phase index is dropped and the error source is represented as a unit vector to derive sensitivities, that is

$$EK\ell m; \quad EK\ell 1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad EK\ell 2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad EK\ell 3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

*NOTE: One of the ℓ indices is dropped for initial- and terminal-type error sources, since $\ell = 3$ for these categories.

Table 1. General Notation Used for Identification of Error Sources

Character	Symbol	Description
1	E	Identifies it as an error source symbol
2	K	Categorical index where K equals: I - initial condition error P - platform error G - gyro error A - accelerometer error T - terminal condition error
3 and 4	ℓ	Numerical ordering of the error-source types within each category ℓ = 00, 01, 02, ... 99
5	m	Identifies a component or axis number m = 1, 2, or 3
6 and 7	n	Phase index used to identify at what time the error source was activated (see Section 2.3.4) n = 01, . . . 12 (For Example, EA102-11 identifies the No.2 accelerometer Type 10 error-source active during the 11th phase of the error analysis.)

2.3.3.1 Initial Condition Errors

As indicated in Section 2.3.1, there are two options for specifying the initial position and velocity errors. For Option 1, the errors cannot be explicitly defined but generally would be the same as for Option 2 (with a change in azimuth direction). In the definitions for initial-condition errors given in Table G-1, it is assumed that Option 2 is being used and that the platform is aligned with respect to vertical. In a launch from an earth site ($t_0 \leq 0$), any uncertainty of the launch location will result in initial-condition errors of position and velocity, and in the transformation matrix M_{EP} of the computer. These are obtained as follows.

2.3.3.1.1 Initial Position Errors

The position error sensitivities transform into ECI coordinates as

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = M_{EI} \begin{bmatrix} EI1m \end{bmatrix} \qquad (m = 1, 2, 3)$$

Initial velocity for navigation is computed, where $t_o \leq 0$, as follows

$$\bar{V}_o = \bar{\omega} \times \bar{R}_o = (\omega_e \bar{Z}_u) \times (X_c \bar{X}_u + Y_o \bar{Y}_u + Z_o \bar{Z}_u)$$

$$= -\omega_e Y_o \bar{X}_u + \omega_e X_o \bar{Y}_u = \dot{X}_o \bar{X}_u + \dot{Y}_o \bar{Y}_u$$

where

$\bar{X}_u, \bar{Y}_u, \bar{Z}_u$ = ECI unit vectors

ω_e = earth rotation rate

Therefore, initial position errors result in the velocity errors

$$\delta\dot{X}_o = -\omega_e \delta Y_o = -\omega_e x_2 = x_4$$

$$\delta\dot{Y}_o = \omega_e \delta X_o = \omega_e x_1 = x_5$$

or

$$\begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix} = \omega_e \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} M_{EI} \left[EI1m \right] \qquad (t_o \leq 0)$$

The transformation matrix used by the computer is developed from the initial position data, therefore errors in position result in errors in M_{EP} (M_{EP} = M_{PE} see Section 2.3.1). A downrange error results in a negative rotation about the 2-axis of M_{PE} and a cross-range error in a positive rotation about the 3-axis. There is no error due to altitude errors. The angular errors are proportional to $1/R_o$; therefore, the computer error in launch coordinates is

$$\bar{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \frac{1}{R_o} \begin{bmatrix} 0 \\ -EI13 \\ EI12 \end{bmatrix}.$$

This is transformed into the ECI coordinates

$$\begin{bmatrix} x_7 \\ x_8 \\ x_9 \end{bmatrix} = \bar{\psi} = -M_{EI}\bar{\theta} = \frac{1}{R_o} M_{EI} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \left[EI1m \right] \qquad (t_o \leq 0)$$

When $t_o > 0$, there is no explicit coupling introduced into velocity and orientation elements of the state vector for position errors and the elements $x_4 \ldots x_9 = 0$.

2.3.3.1.2 Initial Velocity Errors

Initial velocity errors are transformed directly into ECI coordinates as

$$\begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix} = M_{EI} \begin{bmatrix} EI2m \end{bmatrix} \qquad (m = 1, 2, 3)$$

$$x_1 x_2 x_3 x_7 x_8 x_9 = 0$$

2.3.3.1.3 Initial Platform-Orientation Errors

The platform errors ($\bar{\phi}$) are initial rotations about the platform axes and are transformed into ECI coordinates as

$$\begin{bmatrix} x_7 \\ x_8 \\ x_9 \end{bmatrix} = \bar{\Psi} = M_{EP}\bar{\phi} = M_{EP} \begin{bmatrix} EI3m \end{bmatrix} \qquad (m = 1, 2, 3)$$

$$x_1 \ldots x_6 = 0$$

2.3.3.2 Accelerometer Error Sources

In the design and manufacture of accelerometer components, every attempt is made to achieve an output that is a function only of input acceleration along one of its axes. Unfortunately, this is never achievable due to inherent design characteristics and manufacturing tolerances.* The general equation

*See Reference 2 for a discussion of design characteristics, etc.

for an accelerometer's output at a given time can be written as

$$A_o = K_0 + K_1A_1 + K_2A_1^2 + K_3A_1^3 + K_4A_2 + K_5A_3 + K_6A_1A_2 + K_7A_1A_3$$
$$+ K_8\left(A_2^2 + A_3^2\right)^{1/2} + K_9A_1\left(A_2^2 + A_3^2\right)^{1/2} + K_{10}A_2^2 + K_{11}A_3^2$$
$$+ K_{12}A_2A_3 + R$$

where

A_1 = the sensed acceleration along the defined (theoretical) input axis

$A_2\ A_3$ = sensed accelerations normal to A_1

K_ℓ = coefficients that may be functions of time and are the result of design characteristics, manufacturing tolerances, or environmental effects (vibration, temperature, etc.)

R = a remainder term, which includes all higher-order terms assumed to be sufficiently small to neglect.

One notable remainder term, not treated here, is a dynamic response to acceleration transients. Of course, any one component design does not have all the terms presented above; and only those that are significant are included in a given error analysis. The purpose of component calibrations is to measure these coefficients and thereby compensate for their effects. It is assumed that the compensation* is achieved by the following corrections to the accelerometer output

$$A_{oc} = A_o - \left(K_{0c} + K_{2c}A_o^2 + \ldots + K_{5c}A_{o3} + \ldots K_{12c}A_{o2}A_{o3}\right)$$

*Compensations could also be achieved by biasing the target conditions in the guidance equations, etc.

where

A_{oc} = the corrected output of an accelerometer, where all constants and accelerometer outputs have been scaled, based on the calibrated scale factor K_{1c} of each accelerometer

$K_{\ell c}$ = the correction coefficients derived from instrument calibration or based on inherent design characteristics. (Not all are determinable from the above methods.)

A_{oi} = (i = 2, 3) = the accelerations normal to the accelerometer's input axis, derived from the outputs of all three accelerometers as follows

$$\overline{A}_{o2} = M_{A2}\overline{A}_o$$

$$\overline{A}_{o3} = M_{A3}\overline{A}_o$$

where

$\overline{A}_o$ = the vector of accelerometer outputs

$\overline{A}_{o2}$ = a vector composed of the acceleration components along the 2-axis of each accelerometer

$\overline{A}_{o3}$ = a vector composed of the acceleration components along the 3-axis of each accelerometer

M_{A2} and M_{A3} are defined in Section 2.3.1. Since the $K_{\ell c}$ coefficients are sufficiently small with respect to K_{1c} (which implies linear design), their products are negligible and the equation reduces to

$$A_{oc} = (K_0 - K_{0c}) + K_1 A_1 + (K_2 - K_{2c})A_1^2 + \dots (K_{12} - K_{12c})A_2 A_3$$

Taking the partial derivatives of this equation results in

$$\delta A_{oc} = \delta K_0 + \delta K_1 A_1 + \delta K_2 A_1^2 + \ldots . \delta K_{12} A_2 A_3$$

where $\delta A_i = 0$ (nominal trajectory).

Combining the equations of each accelerometer and placing them into the sensitivity form, the final vector matrix form is

$$\delta f(\overline{A}) = \delta \overline{A}_{oc} = [I] \{EA00m\} + \begin{bmatrix} A_{A11} & 0 & 0 \\ 0 & A_{A21} & 0 \\ 0 & 0 & A_{A31} \end{bmatrix} \{EA01m\}$$

$$+ \ldots + \begin{bmatrix} A_{A12}A_{A13} & 0 & 0 \\ 0 & A_{A22}A_{A23} & 0 \\ 0 & 0 & A_{A32}A_{A33} \end{bmatrix} \{EA12m\}$$

$$m = (1, 2, 3)$$

where

$$\overline{A}_{A1} = \begin{bmatrix} A_{A11} \\ A_{A21} \\ A_{A31} \end{bmatrix} = M_{AE} \ddot{\overline{X}}_s$$ acceleration components along input axes

$$\begin{bmatrix} A_{A12} \\ A_{A22} \\ A_{A32} \end{bmatrix} = M_{A2}\bar{A}_{A1}$$

acceleration components along 2-axis

$$\begin{bmatrix} A_{A13} \\ A_{A23} \\ A_{A33} \end{bmatrix} = M_{A3}\bar{A}_{A1}$$

acceleration components along 3-axis

2.3.3.3 Gyro Error Sources

For the purpose of describing the error-source model for gyro components, it is convenient to discuss the model in terms of a single-degree-of-freedom integrating gyro. The general equation for gyro rates can be written

$$\dot{\phi} = C_0 + C_1A_1 + C_2A_3 + C_3A_1A_3 + C_4\omega_3 + C_5\omega_2 + C_6\omega_1 + C_7A_2A_3$$
$$+ C_8A_2 + C_9A_1^2 + C_{10}A_3^2 + C_{11}A_1A_2 + R$$

where

$A_1\ A_2\ A_3$ = sensed accelerations along the input, output, and spin reference axes

$\omega_1\ \omega_2\ \omega_3$ = rates about the input, output, and spin reference axes

C_ℓ = coefficients, which may be functions of time and are the result of design characteristics, manufacturing tolerances, or environmental effects (vibration, temperature, etc.)

R = a remainder term, which includes all higher-order terms assumed to be sufficiently small to neglect.

One notable term is dynamic response to transient inputs. It is assumed that the platform and/or gyro servo loops are designed with sufficient bandpass and static gain to make the effects of transients or sustained rotational dynamic inputs negligible.

As in the case of accelerometers, not all coefficients are applicable to a given design; only those indicative of the particular components are considered in any one analysis. Also, component calibration measures some of these coefficients and the compensation for their effects is assumed to be included in the navigation system equations. The compensation method assumed is the calculation of compensating torquing signals to the gyros.* Thus, the compensated gyro rate equation is

$$\dot{\phi}_c = \dot{\phi} - (C_{0c} + C_{1c}A_{o1} + \dots + C_{4c}\omega_{o3} + \dots + C_{11c}A_{o1}A_{o2})$$

where

$\bar{A}_{o1} = M_{GP}M_{PA}\bar{A}_o$ = acceleration along gyro input axes

$\bar{A}_{o2} = M_{G2}\bar{A}_{o1}$ = acceleration along gyro 2-axes

$\bar{A}_{o3} = M_{G3}\bar{A}_{o1}$ = acceleration along gyro 3-axes

$\bar{\omega}_{o1} = M_{GP}\bar{\omega}_{oP}$ = rates about gyro input axes

$\bar{\omega}_{o2} = M_{G2}\bar{\omega}_{o1}$ = rates about gyro 2-axes

$\bar{\omega}_{o3} = M_{G3}\bar{\omega}_{o1}$ = rates about gyro 3-axes

$\bar{A}_o$ is as defined in Section 2.3.3.2

$\bar{\omega}_{oP}$ = measured or computed vector rates of platform axes

M_{Gi} is as defined in Section 2.3.1

*Use of the compensation of the transformation matrix is more generally in practice, which is equivalent.

By following the same approach as for the accelerometers, the final equations for gyro error sensitivities in vector matrix form become

$$\dot{\Psi} = F_{\psi} = M_{EG}\dot{\phi}_c$$

where

$$\dot{\phi}_c = [I]\{EG00m\} + \begin{bmatrix} A_{G11} & 0 & 0 \\ 0 & A_{G21} & 0 \\ 0 & 0 & A_{G31} \end{bmatrix} \{EG01m\}$$

$$+ \ldots \begin{bmatrix} \omega_{11} & 0 & 0 \\ 0 & \omega_{21} & 0 \\ 0 & 0 & \omega_{31} \end{bmatrix} \{EG06m\}$$

$$+ \ldots + \begin{bmatrix} A_{G11}A_{G12} & 0 & 0 \\ 0 & A_{G21}A_{G22} & 0 \\ 0 & 0 & A_{G31}A_{G32} \end{bmatrix} \{EG11m\}$$

$$(m = 1, 2, 3)$$

where

$$\bar{A}_{G1} = \begin{bmatrix} A_{G11} \\ A_{G21} \\ A_{G31} \end{bmatrix} = M_{GE}\ddot{X}_s$$ acceleration components along gyro input axes

$$\begin{bmatrix} A_{G12} \\ A_{G22} \\ A_{G32} \end{bmatrix} = M_{G2}\overline{A}_{G1}$$

acceleration components along gyro 2-axis

$$\begin{bmatrix} A_{G13} \\ A_{G23} \\ A_{G33} \end{bmatrix} = M_{G3}\overline{A}_{G1}$$

acceleration components along gyro 3-axis

$$\overline{\omega}_1 = \begin{bmatrix} \omega_{11} \\ \omega_{21} \\ \omega_{31} \end{bmatrix} = M_{GP} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

rates about gyro input axes

$$\begin{bmatrix} \omega_{12} \\ \omega_{22} \\ \omega_{32} \end{bmatrix} = M_{G2}\overline{\omega}_1$$

rate components about gyro 2-axis

$$\begin{bmatrix} \omega_{13} \\ \omega_{23} \\ \omega_{33} \end{bmatrix} = M_{G3}\overline{\omega}_1$$

rate components about gyro 3-axis

2.3.3.4 Platform Errors

In addition to the initial condition errors discussed in Section 2.3.3.1, acceleration-sensitive errors arise due to the structural deformation of the gimbals under acceleration loads, and to static servo response due to platform mass unbalances. The general equation for platform acceleration-sensitive errors can be written in the vector matrix form

$$\bar{\phi}_P = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = \begin{bmatrix} A_{P2} & & 0 \\ & A_{P3} & \\ 0 & & A_{P1} \end{bmatrix} \{EP01m\} + \begin{bmatrix} A_{P3} & & 0 \\ & A_{P1} & \\ 0 & & A_{P2} \end{bmatrix} \{EP02m\}$$

$$+ \begin{bmatrix} A_{P2}A_{P3} & & 0 \\ & A_{P1}A_{P3} & \\ 0 & & A_{P1}A_{P2} \end{bmatrix} \{EP03m\} \quad (m = 1, 2, 3)$$

$$\bar{\psi} = \begin{bmatrix} X_7 \\ X_8 \\ X_9 \end{bmatrix} = M_{EP}\bar{\phi}_P \qquad (\bar{F}_\psi = 0)$$

where

$$\bar{A}_P = \begin{bmatrix} A_{P1} \\ A_{P2} \\ A_{P3} \end{bmatrix} = M_{PE}\ddot{\bar{X}}_s$$

acceleration components along platform axes

2.3.3.5 Terminal Condition Errors

The terminal condition errors transform into the following ECI coordinates by utilizing the transformation matrices developed in Section 2.3.1.

Position Errors

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = M_{ET1} \{ET1m\} \quad (m = 1, 2, 3)$$

$$x_4 \ldots x_9 = 0$$

Velocity Errors

$$\begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix} = M_{ET2} \{ET2m\} \quad (m = 1, 2, 3)$$

$$x_1 x_2 x_3 x_7 x_8 x_9 = 0$$

2.3.4 Transition Matrix

Since the majority of error sources are acceleration-sensitive, their forcing functions (excluding accelerometer bias and gyro bias drift) are zero during free-flight sequences; therefore, it is more efficient computationally to propagate the sensitivity vectors across free flight, by using the transition matrix rather than by solving each error-source sensitivity independently. Additionally, when an error-source type has time-dependent statistical characteristics, it becomes both convenient and efficient to subdivide the total trajectory time into phases and treat each error source of this type as an independent error source reinitialized ($\bar{x}_i(t) = 0$) at each phase time. Error source types that were active during previous phases are called inactive vectors, while errors sources that are active during the present phase are termed active vectors. For any one type of error source, all its inactive vectors are updated to the present time by using the transition matrix(es) and combined statistically (see Section 2.4.3) to derive the total effect on the navigation data statistical characteristics. Thus, the following procedure is used to update or propagate vector sensitivities

$$\bar{x}_i(t) = \Phi(t, \tau)\bar{x}_i(\tau)$$

where $\Phi(t, \tau)$ is the transition matrix obtained from the solution of the homogeneous differential equations

$$\dot{\Phi}(t, \tau) = M(t)\Phi(t, \tau), \qquad \Phi(\tau, \tau) = I$$

where

$$M(t) = \begin{bmatrix} 0 & I & 0 \\ M_G(t) & 0 & M_A(t) \\ 0 & 0 & 0 \end{bmatrix}$$

M_G and M_A are defined in Section 2.3.2.

The solution of this equation is achieved by solving the homogeneous differential equations (in ECI coordinates) for each initial condition. Since during free flight $M_A(t) = 0$, the solution requires only six independent solutions; during powered flight nine independent solutions are required. In addition, the following property of the transition matrix(es) is used

$$\Phi(t_i t_k) = \Phi(t_i t_j)\Phi(t_j t_k) \qquad t_i \geq t_j \geq t_k$$

2.3.5 Trajectory Data and Free-flight Equations of Motion

To make an error analysis, the following data is required for the differential equations and the error-source equations:

t	time
$X\ Y\ Z$	nominal position components in ECI coordinate system
$\dot{X}\ \dot{Y}\ \dot{Z}$	nominal velocity components in ECI coordinates
$\ddot{X}_s\ \ddot{Y}_s\ \ddot{Z}_s$	nominal sensed acceleration components in ECI
$\omega_1\ \omega_2\ \omega_3$	nominal platform (body) rates in platform (body) coordinates
$M_{PE}(t)$	direction cosines of platform (body) axes with respect to ECI axes

This data is generated by a trajectory program for a particular vehicle configuration and mission requirement and is written on a magnetic tape, which constitutes a basic input for the error analysis. The last two categories (ω_{Pi} and M_{PE}) are required only for analyzing a strapped-down configuration. For a torqued-platform configuration, the rates (ω_i) are input as a table of rates vs time and $M_{PE}(t)$ is calculated from

$$\dot{M}_{PE} = [\Omega_P]\ M_{PE}$$

where these matrices are as defined in Sections 2.3.1 and 2.3.2.

When error propagation must be evaluated beyond the time for which there is data from the trajectory tape, the program has the capability to generate the required data [$M_G = f(X\ Y\ Z\ t)$] for calculating the transition matrix and

drag error sensitivity. The data is generated from the free-flight equations of motion, based on a simplified oblate earth model (see Reference 5), as follows

$$\begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} = \begin{matrix} \epsilon_1 \dfrac{X}{R^3} \\ \epsilon_1 \dfrac{Y}{R^3} \\ \epsilon_2 \dfrac{Z}{R^3} \end{matrix}$$

where

$$\epsilon_1 = -\frac{GM}{R^2}\left[R^2 + JA^2\left(1 - \frac{5Z^2}{R^2}\right) - \frac{HA^3 Z}{R^2}\left(-3 + \frac{7Z^2}{R^2}\right) + \frac{DA^4}{R^2}\left(\frac{9Z^4}{R^4} - \frac{6Z^2}{R^2} + \frac{3}{7}\right)\right]$$

$$\epsilon_2 = -\frac{GM}{R^2}\left[2JA^2 - \frac{HA^3}{R^2} Z\left(-3 + \frac{3R^2}{5Z^2}\right) - \frac{DA^4}{R^2}\left(\frac{4Z^2}{R^2} - \frac{12}{7}\right)\right] + \epsilon_1$$

where GM, J, A, H, and D are nominal constants, defined in Table G-2 (in Appendix G).

The initial conditions for these equations are obtained from the trajectory tape at termination of the tape data,* or they can be input. The three criteria for terminating the equations of motion are

a. Specified time (t_T)

---

*An option is available to terminate (abort) the trajectory tape data at a time prior to the end of the tape.

b. Specified range angle (θ_T) from termination of the trajectory tape data, i.e., when $\theta = \theta_T$

where

$$\theta = \cos^{-1} \frac{X_T X + Y_T Y + Z_T Z}{R_T R}$$

$X_T Y_T Z_T$ and R_T are the values at the tape termination time

Note: $0 < \theta_T < \pi$

c. Specified altitude h_T. There are two criteria for this termination: The trajectory can be terminated when $h = h_T$ and the slope $(\dot{h})$ is positive, or when the slope is negative.

2.4 DATA PROCESSING EQUATIONS

2.4.1 Basic Coordinate Systems

For the purpose of output data presentation, two coordinate systems are available: the ECI coordinate system and the LH (Local Horizontal) system. The latter is sometimes referred to as the orbit plane, and/or the Radial/Tangential/Normal (RTN) coordinate system. The transformation between ECI coordinates and local coordinates is developed from the nominal trajectory position and velocity vectors. In local coordinates, X is defined as down range, i.e., as directed along the projection of the inertial velocity vector onto the plane normal to the geocentric radius vector; Y is vertical; and Z is cross range, forming a right-hand coordinate system. The local coordinate system is an inertial system at time t, defined by the nominal conditions only, and is used in the transformation of position, velocity, and orientation sensitivity vectors as well as those of transition matrices and covariance matrices. The matrix is defined as

$$M_{LE} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi & -S\psi \\ 0 & S\psi & C\psi \end{bmatrix} \begin{bmatrix} C\lambda & 0 & S\lambda \\ 0 & 1 & 0 \\ -S\lambda & 0 & C\lambda \end{bmatrix} \begin{bmatrix} C\phi & S\phi & 0 \\ -S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{EL} = M_{LE}^{T}$$

where

$$S\phi = \frac{Y}{D} \qquad S\lambda = \frac{Z}{R} \qquad S\psi = \frac{V_E}{V_H}$$

$$C\phi = \frac{X}{D} \qquad C\lambda = \frac{D}{R} \qquad C\psi = \frac{V_N}{VH}$$

$$D = \sqrt{X^2 + Y^2} \qquad R = \sqrt{D^2 + Z^2}$$

$$V_E = \frac{-Y\dot{X} + X\dot{Y}}{D} \qquad V_N = \frac{-Z(X\dot{X} + Y\dot{Y}) + \dot{Z}D^2}{RD}$$

$$V_H = \sqrt{V_E^2 + V_N^2}$$

and X Y Z $\dot{X}$ $\dot{Y}$ $\dot{Z}$ are ECI components of the nominal position and velocity vectors at time t.

NOTE: λ, ψ are left-hand rotations.

It is also convenient to form the matrix $\underline{\mathbf{M}_{LE}}$ as

$$\underline{\mathbf{M}_{LE}} = \begin{bmatrix} M_{LE} & 0 & 0 \\ 0 & M_{LE} & 0 \\ 0 & 0 & M_{LE} \end{bmatrix}$$

$$\underline{\mathbf{M}_{EL}} = \underline{\mathbf{M}_{LE}^T}$$

2.4.2 **Vector Errors**

Vector errors are derived by scaling the sensitivity vectors by an appropriate constant. The constant used in this operation is given the symbol σ_i. (i ranges from 1 to n, which is the total number of error sources, i.e., all active and inactive vectors. See Section 2.3.4.) The units of σ_i for each type of error source are given in Table G-1. Table G-3 gives the conversion factors K_i used by the program for scaling σ_i into the units of the sensitivity vectors. The operation, therefore, is

$$\Delta\bar{X}_i = \sigma_i K_i \bar{x}_i \triangleq \sigma_i \bar{x}_i$$

where the last expression is used throughout, implying the first.

$\Delta\bar{X}_i$ is the error vector in ECI coordinates. The usual implication of σ is that it represents the standard deviation of the particular error source. However, it could also represent the mean value of an error source or be a constant that produces sensitivity vector output in any desired units. When it is desired to output the vector in local horizontal coordinates, the following transformation is made

$$\Delta\bar{X}_i \Big|_{LH} = \underline{M_{LE}}\Delta\bar{X}_i$$

2.4.3 Covariance Matrix

The basic equation for navigation error is

$$\Delta\bar{X} = \sum_{i=1}^{n} \bar{x}_i \epsilon_i$$

In this equation, ϵ_i is the magnitude of the i^{th} error source. Each error source is considered a random variable, the means, variances, and correlations of which are assumed to be known. The accuracy of any navigation system performance is measured in terms of the probability that $\overline{\Delta X}$ (or some function of $\Delta\bar{X}$ - see Section 2.4.5) is within certain specified values. To maximize this probability, it is necessary to compensate for the effects of the means of the error sources. This can be done by the methods discussed in Section 2.3.3, or by offsetting the guidance constants so that the effects are negated at some specified time point. For the latter method, the σ can be input to represent the mean value of each error source and the resulting vectors summed, that is

$$E(\Delta\bar{X}) = \sum_{i=1}^{n} \bar{x}_i E(\epsilon_i)$$

where

$$E(\epsilon_i) = \sigma_i \quad \text{representing } E(\epsilon_i)$$

E = the expectation operator

This calculation is not made in the program, although the magnitude of the mean (expected) value of the navigation vector can be obtained from the covariance matrix output described below, if each error source is correlated with the other by unity. Alternately, the error vectors can be summed by utilizing desk calculators. When it is assumed that the mean values have been compensated, the second statistical moments of navigation data are determined in (covariance) matrix form by

$$E(\Delta \bar{X} \Delta \bar{X}^T) = E(\bar{x}_1\epsilon_1 + \bar{x}_2\epsilon_2 + \cdots)(\bar{x}_1^T\epsilon_1 + \bar{x}_2^T\epsilon_2 + \cdots)$$

$$\sum_{ECI} = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j \rho_{ij} \bar{x}_i \bar{x}_j^T$$

where

ρ_{ij} = the correlation coefficient between the i^{th} and j^{th} error source. (In this way, both time and cross correlation of error sources are handled.)

$\sum_{ECI}$ = a 9 × 9 matrix in which the diagonal elements represent the variances of the navigation data and the off-diagonal elements are the covariances.

Since this is a symmetrical matrix, the correlation coefficients of the navigation data are calculated and presented in the lower half of the matrix. The correlation coefficients are obtained from

$$\rho_{mn} = \frac{E(\Delta X_m \Delta X_n)}{\sigma_m \sigma_n} \qquad m \neq n$$

where $E(\Delta X_m \Delta X_n)$ is the mnth elements of the covariance matrix and σ_m, σ_n are the m and n standard deviations of the m and n coordinates calculated from the square root of the respective diagonal terms. The σ's of the navigation data are also presented in the output.

When vector errors are presented in LH coordinates, the covariance matrix is also presented in this coordinate system. It is computed by

$$\sum_{LH} = \underline{M}_{LE} \sum_{ECI} \underline{M}_{EL}$$

All of the above operations presuppose the condition of linearity, which is usually satisfied, but should not be assumed always to be true. (This is mentioned merely as a precaution about the underlying assumptions of the program.)

2.4.4 Transition Matrix and Trajectory Variables

As stated in Section 2.3.4, the transition matrices are used to propagate not only sensitivity vectors across free flight, but inactive vectors for data processing as well. In addition, when free-flight time histories are desired, the transition matrix plays a paramount role. For various analytical studies, it is desired to obtain the transition matrix, and so its output is made available. Like the previous outputs, it is available in either the ECI or LH coordinate systems. Since it is computed in ECI coordinates, the operation required to present it in local coordinates is easily obtained from

$$\Delta\overline{X}_i(t) = \Phi(t, \tau)\Delta\overline{X}_i(\tau)$$

$$\Delta X_i(t)\Big|_{LH} = \underline{M}_{LE}(t)\Delta\overline{X}_i(t) =$$

$$= \underline{M}_{LE}(t)\Phi(t, \tau)\underline{M}_{EL}(\tau)\Delta\overline{X}_i(\tau)\Big|_{LH}$$

$$\therefore \quad \Phi_{LH}(t, \tau) = \underline{M}_{LE}(t)\Phi(t, \tau)\underline{M}_{EL}(\tau)$$

In addition to the transition matrix, it is convenient and sometimes necessary to know the reference trajectory conditions for which the transition matrix is valid. These are presented at both times (t and τ) and in both ECI and local reference coordinates, the latter being defined and computed from ECI coordinates as

LAT (deg) Geocentric latitude

$$= \sin^{-1}\frac{Z}{R} \qquad -\frac{\pi}{2} \leq \text{LAT} \leq \frac{\pi}{2}$$

LONG (deg) Longitude measured positively east from Greenwich

$$= \tan^{-1}\frac{Y}{X} + \phi_L - \omega_e t \qquad 0 \leq \text{LONG} < 2\pi$$

where ϕ_L is used to reference the ECI coordinates to Greenwich when the trajectory reference is not.

ALT (n mi) Altitude in nautical miles above the surface of an oblate earth

$$= \frac{R - R_e}{6076.1033}$$

where

$$R_e = \frac{A(1 - e)}{(1 + (e^2 - 2e)\cos^2\lambda)^{1/2}}$$

See Section 2.3.5 for definitions.

VEL (ft/sec) Inertial velocity magnitude

$$= (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)^{1/2}$$

FPA (deg) Flight path angle, defined as the angle the inertial velocity vector makes with the local geocentric horizontal

$$= \sin^{-1}\frac{X\dot{X} + Y\dot{Y} + Z\dot{Z}}{RV} \qquad -\frac{\pi}{2} \leq \text{FPA} \leq \frac{\pi}{2}$$

AZ (deg) Azimuth of inertial velocity vector measured clockwise from north.

$$= \tan^{-1} \frac{V_E}{V_N} \qquad 0 \leq AZ \leq 2\pi$$

See Section 2.4.1 for definition of V_E and V_N.

2.4.5 Mission Evaluation

This is an optional output of the program and operates on the final or end conditions of a particular error analysis. It is called upon when a specified mission termination criterion is to be evaluated. The parameters necessary to evaluate mission success can occur in a variety of categories, only a few of which are considered here.

2.4.5.1 Fixed Altitude

This criterion is used primarily for re-entry evaluation and presents the coordinates of down-range (MD) and cross-range (MC) miss at a fixed altitude with respect to earth fixed-target coordinates; in addition, the time dispersion (MT) is presented. The orientation of the down-range miss coordinate is along the projection of the relative velocity vector onto the plane normal to the target geocentric radius vector. The method of computing these quantities is

$$\overline{V}_R = \overline{V} - \overline{V}_T$$

$$= (\dot{X} + \omega_e Y)\overline{X}_u + (\dot{Y} - \omega_e X)\overline{Y}_u + \dot{Z}\,\overline{Z}_u$$

$$= \dot{X}_R \overline{X}_u + \dot{Y}_R \overline{Y}_u + \dot{Z}_R \overline{Z}_u$$

where: $\overline{V}_T$ = target velocity in ECI coordinates

$\overline{V}_R$ = relative velocity in ECI coordinates

$\overline{X}_u \overline{Y}_u \overline{Z}_u$ = ECI unit vectors

$\dot{X}\ \dot{Y}\ \dot{Z}\ X\ Y\ Z$ = components of the nominal position and velocity vectors at the target

$\dot{X}_R\ \dot{Y}_R\ \dot{Z}_R$ = the components of $\overline{V}_R$ in ECI coordinates

The relative velocity vector is transformed into the coordinate system at the target by

$$\begin{bmatrix} \dot{X}_{RR} \\ \dot{Y}_{RR} \\ \dot{Z}_{RR} \end{bmatrix} = M_{REI} \begin{bmatrix} \dot{X}_{R} \\ \dot{Y}_{R} \\ \dot{Z}_{R} \end{bmatrix}$$

where

$$M_{REI} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi & -S\psi \\ 0 & S\psi & C\psi \end{bmatrix} \begin{bmatrix} C\lambda & 0 & S\lambda \\ 0 & 1 & 0 \\ -S\lambda & 0 & C\lambda \end{bmatrix} \begin{bmatrix} C\phi & S\phi & 0 \\ -S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S\phi = \frac{Y}{D} \qquad S\lambda = \frac{Z}{R} \qquad S\psi = \frac{V_{RE}}{V_{RH}}$$

$$C\phi = \frac{X}{D} \qquad C\lambda = \frac{D}{R} \qquad C\psi = \frac{V_{RN}}{V_{RH}}$$

$$D = \sqrt{X^2 + Y^2} \qquad V_{RE} = \frac{-Y\dot{X}_R + X\dot{Y}_R}{D}$$

$$V_{RN} = \frac{-Z(\dot{X}_R X + Y\dot{Y}_R) + \dot{Z}_R D^2}{RD}$$

$$V_{RH} = \sqrt{V_{RE}^2 + V_{RN}^2}$$

and $X\ Y\ Z\ \dot{X}_R\ \dot{Y}_R\ \dot{Z}_R$ are ECI components of the nominal target coordinates and relative velocity vector at time t.

Thus, the relative velocity components in the target coordinates are: $\dot{X}_{RR}$ the altitude rate, $\dot{Y}_{RR}$ the velocity component in the defined cross-range direction ($\approx$ zero), and $\dot{Z}_{RR}$ in the defined down-range direction. When $V_{RE} = V_{RN} = 0$, vertical re-entry, ψ is undefined. In this case, ψ is set to zero so that down-range displacements would be directed north and cross-range east.

Based on the above definitions for a coordinate system, a position error vector can be transformed into this coordinate system to determine the altitude, cross-range and down-range dispersions at the nominal time by

$$\begin{bmatrix} \Delta X_T \\ \Delta Y_T \\ \Delta Z_T \end{bmatrix} = M_{RE1} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$$

where ΔX_T ΔY_T ΔZ_T are altitude, cross-range, and down-range dispersions at the nominal time and ΔX ΔY ΔZ are ECI dispersions for a given error source.

To derive the first-order dispersions for the condition of zero altitude error ($\Delta X_T = 0$), the following constraint equation is used

$$\Delta X_T(t + \Delta t) = 0 = \Delta X_T(t) + \dot{X}_{RR}(t)\Delta t$$

from which the time dispersion is calculated as

$$\Delta t = M_T = -\frac{\Delta X_T}{\dot{X}_{RR}}$$

Using this relation, the cross-range and down-range misses at time $t + \Delta t$ are obtained by

$$\Delta Y_T(t + \Delta t) = M_C = \Delta Y_T(t) + \dot{Y}_{RR}(t)\Delta t$$

$$= \Delta Y_T$$

$$\Delta Z_T(t + \Delta t) = M_D = \Delta Z_T(t) + \dot{Z}_{RR}(t)\Delta t$$

$$= \Delta Z_T - \frac{\dot{Z}_{RR}}{\dot{X}_{RR}} \Delta X_T$$

Thus

$$\begin{bmatrix} M_T \\ M_C \\ M_D \end{bmatrix} = \begin{bmatrix} -\frac{1}{\dot{X}_{RR}} & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{\dot{Z}_{RR}}{\dot{X}_{RR}} & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta X_T \\ \Delta Y_T \\ \Delta Z_T \end{bmatrix}$$

$$\begin{bmatrix} M_T \\ M_C \\ M_D \end{bmatrix} = M_{TR1} M_{REl} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$$

The position vector error for each error source is thus transformed into its dispersion parameters at the target. The covariance matrix of these parameters is developed by partitioning the ECI covariance matrix to include only the position variances and covariances from which the following covariance matrix is determined

$$\sum_{ME1} = M_{TR1} M_{RE1} \sum_{ECI_P} \left(M_{TR1} M_{RE1}\right)^T$$

where

$\sum_{ME1}$ = the covariance matrix for mission evaluation Option 1

$\sum_{ECI_P}$ = the partitioned covariance matrix in ECI coordinates

The nominal trajectory conditions are also listed with this output and include

LAT, LONG, ALT — As described in Section 2.2.4

VEL/R (ft/sec) — Magnitude of relative velocity vector

$$= \sqrt{\dot{X}_R^2 + \dot{Y}_R^2 + \dot{Z}_R^2}$$

FPA/R (deg) — Relative flight path angle, defined as the angle the relative velocity vector makes with the local horizontal

$$= \sin^{-1} \frac{X\dot{X}_R + Y\dot{Y}_R + Z\dot{Z}_R}{RV_R} \qquad -\frac{\pi}{2} \leq FPA/R \leq \frac{\pi}{2}$$

AZ/R (deg) — Azimuth of the relative velocity vector, measured clockwise from north

$$= \tan^{-1} \frac{V_{RE}}{V_{RN}} \qquad 0 \leq AZ/R < 2\pi$$

2.4.5.2 Fixed-range Angle

This criterion is similar to the fixed-altitude case, except the relative range coordinate (ΔZ_T) is fixed and the altitude, cross-range, and time dispersions are determined. The constraint equation is

$$\Delta Z_T(t + \Delta t) = 0 = \Delta Z_T(t) + \dot{Z}_{RR}(t)\Delta t$$

from which the time, cross-range, and altitude dispersions are

$$\Delta t = M_T = -\frac{\Delta Z_T}{\dot{Z}_{RR}}$$

$$\Delta Y_T(t + \Delta t) = M_C = \Delta Y_T$$

$$\Delta X_T(t + \Delta t) = M_V = \Delta X_T(t) + \dot{X}_{RR}(t)\Delta t$$

$$= \Delta X_T - \frac{\dot{X}_{RR}}{\dot{Z}_{RR}} \Delta Z_T$$

Thus

$$\begin{bmatrix} M_T \\ M_C \\ M_V \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1}{\dot{Z}_{RR}} \\ 0 & 1 & 0 \\ 1 & 0 & -\frac{\dot{X}_{RR}}{\dot{Z}_{RR}} \end{bmatrix} \begin{bmatrix} \Delta X_T \\ \Delta Y_T \\ \Delta Z_T \end{bmatrix} = M_{TR2} M_{RE1} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$$

The vector error outputs and covariance are handled in the same way as in the fixed-altitude case.

2.4.5.3 Generalized Linear Transformation

This option is used when linear transformations between ECI position and velocity errors, and some other parameters are known, e.g., the midcourse maneuver velocity sensitivities, as a function of injection errors for a space probe. A matrix is developed from (up to 10) input matrices, and then used to transform injection errors into the desired parameter errors. Each error-source vector is transformed and a covariance matrix is calculated from the partitioned ECI covariance matrix, computed as follows.

The matrix is formed from

$$M = [M_1][M_2] \cdots [M_n] \qquad n \leq 10$$

The vector errors are transformed by

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} = [M] \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta \dot{X} \\ \Delta \dot{Y} \\ \Delta \dot{Z} \end{bmatrix}$$

The covariance matrix is calculated as

$$\sum_{ME3} = M \sum_{\substack{ECI \\ PV}} M^T$$

2.4.6 Platform Reference Attitude

When running a torqued platform or strapped-down case, reference platform orientation is automatically presented as a function of time. Listed in the output are the first two rows of the platform matrix (M_{PE}) and three angles defined and computed as

THETA (θ in deg) Pitch: the angle that the platform 1-axis makes with the local horizontal plane

$$= \sin^{-1}\left(\frac{X(1X)}{R} + \frac{Y(1Y)}{R} + \frac{Z(1Z)}{R}\right) \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

where (1X), (1Y), (1Z) are the elements of the first row of M_{PE} representing the direction cosines of P_1 with respect to the ECI coordinates axes

PSI (ψ in deg) Heading or Yaw: the angle that the projection of the platform 1-axis onto the horizontal plane makes with north, positive clockwise

$$= \tan^{-1}\frac{1E}{1N} \qquad 0 \leq \psi < 2\pi$$

where $1E = \dfrac{-Y(1X) + Y(1Y)}{D}$

$$1N = \frac{-Z[X(1X) + Y(1Y)] + D^2(1Z)}{RD}$$

PHI (ϕ in deg) Roll: the angle that the platform 2-axis makes with the local horizontal plane

$$\phi = -\sin^{-1}\left(\frac{X(2X)}{R} + \frac{Y(2Y)}{R} + \frac{Z(2Z)}{R}\right) \quad -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$$

where (2X), (2Y), (2Z) are the elements of the second row of M_{PE} representing the direction cosines of P_2 with respect to the ECI coordinates axes.

SECTION 3

COMPUTER PROGRAM INPUT/OUTPUT

3.1 INTRODUCTION

This section is intended as a guide to users of the error analysis program in preparing the input data. Brief descriptions of the program capabilities and the procedures for data input are presented, and the available output data formats are described. There are standard forms for preparing the necessary data input, and they are shown in Appendix A.

The program actually consists of two separate programs: The first, called ERAN, computes error sensitivities based on given trajectory data, navigation system instrument configuration, and a model of the component error sources. Trajectory data is supplied via a tape generated by a trajectory program(s) (N-STAGE, TRIP, MVS, etc.). The instrument configuration is supplied as the input data of component orientations. The error models are supplied on an error-source schedule sheet that specifies the error sources to be considered for this configuration and when they are active. Based on this information, the ERAN program computes sensitivity coefficients, and outputs these onto an ERAN tape.

The second program, called ØUTP, basically processes the ERAN tape into the prescribed formats. Inputs to this program are the logical controls, i.e., the desired frequency of output, coordinate systems, formats, etc., and the specification of standard deviations of the error sources, along with any correlation coefficients between error sources. The ERAN tape can be processed many times without rerunning the ERAN Program.

3.2 ERAN INPUT DATA

3.2.1 Trajectory Tape

Prior to running the error analysis program, a trajectory tape must be prepared that contains the following information relative to the nominal mission:

t	Time
$X, Y, Z,$	Nominal position coordinates in the ECI system
R	Magnitude of position (radius) vector
$\dot{X}\ \dot{Y}\ \dot{Z}$	Nominal velocity coordinates in the ECI system
$\ddot{X}_s \ddot{Y}_s \ddot{Z}_s$	Nominal sensed acceleration coordinates in the ECI system
$\omega_1\ \omega_2\ \omega_3$	Rates of platform (body) axis in platform (body) coordinates (optional)
1X 1Y 1Z	Direction cosines of the platform (body)
2X 2Y 2Z	axes with respect to the ECI
3X 3Y 3Z	coordinate system (optional)

Each file on the trajectory tape contains the information for one ERAN case and consists of the following logical records:

1st Record

Word 1 = 1B27
Word 2 = N
Word 3 = 0 (N = size of a data record i.e., 11 or 23)

Continuous Data Point Record

Word 1 = 1B35	Word 5 = Z
Word 2 = t	Word 6 = R
Word 3 = X	Word 7 = $\ddot{X}_s$
Word 4 = Y	Word 8 = $\ddot{Y}_s$

Word 9 = $\ddot{Z}_s$	Word 17 = 1Y
Word 10 = $\dot{X}$	Word 18 = 1Z
Word 11 = $\dot{Y}$	Word 19 = 2X
Word 12 = $\dot{Z}$	Word 20 = 2Y
Word 13 = ω_1	Word 21 = 2Z
Word 14 = ω_2	Word 22 = 3X
Word 15 = ω_3	Word 23 = 3Y
Word 16 = 1X	Word 24 = 3Z

(Words 13 to 24 are omitted if N = 11)

Left Side of a Trajectory Discontinuity

Word 1 = 3B35

Words 2 through 24 are the same for all data records

Right Side of a Trajectory Discontinuity

Word 1 = 5B35

Words 2 through 24 are the same for all data records

Last Record of a Trajectory

Word 1 = 1B29

EØF

All Aerospace Corporation trajectory programs are mechanized to prepare a trajectory tape in the proper format for input to ERAN. The tape writing intervals should be no greater than 4 seconds during powered-flight phases and 32 seconds during coast or free-flight phases. Higher tape densities would cause no problem, but lower densities would tend to degrade the accuracy of the integrations. The integration step size used for ERAN is not necessarily the same as that of the input tape interval, but is controlled by input data. The ERAN Program interpolates between trajectory data points to obtain proper values for integration. The integration routine used by ERAN is based on a fourth-order Runge-Kutta method.

3.2.2 Error Sources

The Error Source Schedule (see Table A-1 in Appendix A) is used to identify which of the available error sources are to be run, and for which time periods (phases) they are to be considered. Table G-1 defines the symbol and units for each error source in the order it appears on the Error Source Schedule. Initial and terminal error sources are listed individually, while component (accelerometers and gyros) and platform errors are listed as error types. Thus, when a component or platform-error type is considered, sensitivities for all three components or axes are automatically and independently run. Each source or type of error can be considered in one or all of 12 possible independent phases of the trajectory. This phase capability is provided to accommodate time-correlated errors and/or independent error-source magnitude changes.* It becomes increasingly more important as the mission time duration increases.

Sections 3.2.2.1 through 3.2.2.5 present a summary of the error-source types and/or error-model equations (see Section 2.3.3).

3.2.2.1 Initial Condition Errors

EI11 → EI13	Initial Position Errors
EI21 → EI23	Initial Velocity Errors
EI31 → EI33	Initial Platform Alignment Errors

If the initial time of the trajectory is equal to or less than zero, the program assumes a launch from an earth fixed pad and automatically calculates

*Phase logic will be used to identify and control "Reset," a feature that updates navigation data as a function of external measurements (see Appendix D).

an initial velocity and vertical alignment error consistent with the initial position error.

3.2.2.2 Accelerometer Errors

$$\Delta A = EA00 + EA01(A_1) + EA02(A_1^2) + EA03(A_1^3)$$
$$+ EA04(A_2) + EA05(A_3) + EA06(A_1A_2)$$
$$+ EA07(A_1A_3) + EA08(A_2^2 + A_3^2)^{1/2}$$
$$+ EA09(A_1)\ (A_2^2 + A_3^2)^{1/2} + EA10(A_2^2)$$
$$+ EA11(A_3^2) + EA12(A_2A_3)$$

where

ΔA = the i^{th} accelerometer error (i = 1, 2, 3)

$EA\ell$ = (ℓ = 00, 01, ...), as defined in Table G-1

A_1 = input axis acceleration of i^{th} accelerometer

A_2, A_3 = acceleration components normal to input axis

3.2.2.3 Gyro Errors

$$\dot{\Phi} = EG00 + EG01(A_1) + EG02(A_3) + EG03(A_1A_3)$$
$$+ EG04(\omega_3) + EG05(\omega_2) + EG06(\omega_1)$$
$$+ EG07(A_2A_3) + EG08(A_2) + EG09(A_1^2)$$
$$+ EG10(A_3^2) + EG11(A_1A_2)$$

where

$\dot{\phi}$ = the i^{th} gyro drift error (i = 1, 2, 3)

$EG\ell$ = (ℓ = 00, 01, ...), as defined in Table G-1

A_j = acceleration along j^{th} axis of i^{th} gyro

ω_j = rate about j^{th} axis of i^{th} gyro

j = 1 input axis j = 2 output axis j = 3 spin reference axis	typical identification

3.2.2.4 Platform Errors

$$\phi_i = EP01i(A_j) + EP02i(A_k) + EP03i(A_j A_k)$$

where

ϕ_i = platform angular error about i^{th} axis (i = 1, 2, 3)

A_j, A_k = platform acceleration components normal to i^{th} axis

$EP\ell$ = (ℓ = 00, 01, ...), as defined in Table G-1

3.2.2.5 Terminal Errors

ET11 → ET13	Terminal Position Errors
ET21 → ET23	Terminal Velocity Errors

Note: These errors can be considered only on the last phase of the error analysis run and can be applied only at the trajectory tape abort time (see Section 3.2.3) or at the end of the trajectory tape.

The procedure for filling out the error-source schedule is as follows:

a. Determine which error sources are to be considered.

b. Establish if any of these error sources are to have non-unity autocorrelation functions; i.e., establish whether multiphase logic is required.

c. In the first column, insert an "X" in each row element that describes the error source to be considered.

d. In the second (and following) columns, insert an "X" in the row elements corresponding to the error sources that require phase logic control for that particular phase* (see example).

The error sources that have been called for by inserting X's in the appropriate squares in Column 1 will be initialized and become active at the start of the case, TSUBO (see Section 3.2.3.5). Insertions of X's into the appropriate squares in Column 2 will cause those error sources to be reinitialized at the start of the second phase (i.e., when the time of the simulation is equal to the value entered into TGØP (see Section 3.2.3.5). The sensitivity vectors from the first phase for those error sources will then become inactive. These inactive vectors will be updated at the desired output times by the transition matrix and be combined statistically with the active vectors to derive the total effect on the navigation data statistical characteristics (see Section 2.3.4). Similarly, X's in the third column will cause those error sources to be reinitialized when the time of the simulation is equal to the value entered into TGØP + 1, etc. In this manner, up to 12 independent sensitivity vectors can be created for each error source on the schedule sheet.

This completes the input on the Error Source Schedule sheet. It essentially indicates to ERAN which error source sensitivities to calculate and whether any of these types require phase logic control.

It will be necessary in setting up the input data for ØUTP to assign sigma values (the standard deviations of each error source) and correlation

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*Error sources that change standard deviations or have time-varying correlation coefficients.

coefficients, when applicable, between error sources. The identification of a given sigma value with a sensitivity vector is accomplished by mentally assigning a number to each error source. The numerical ordering of error sources is determined by starting in the first column of the Error Source Schedule and counting down, then going to the second column, and so forth. Note that three independent error sources are associated with each "X" in an element of a component or platform error.

3.2.3 Orientation and Control Data

The data sheet used to set up the platform and component orientations and to input the necessary program control data and constants is shown as Table A-2 in Appendix A. Control data and constants have preassigned values; therefore, only those numbers that deviate from them need be input. The symbols used are summarized in Table G-2, along with their preassigned numerical values and units.

3.2.3.1 Initial Platform Alignment

There are two options for specifying the initial platform orientation (see Figure 2). The first assumes that the platform 1-axis (P_1) is aligned along the geocentric vertical. The azimuth orientation is specified through an input of PSIP (ψ_p, a left-hand rotation in platform coordinates). With no input in PSIP, the platform would be aligned so that the 3-axis (P_3) would be north and the 2-axis (P_2) east. With a positive ψ_p, the P_3 axis would be rotated toward the east

The second option allows for a more general initial platform orientation; here, three angles are specified for aligning the platform. The initial alignment is such that the P_1, P_2, and P_3 axes are along the ECI X, Y, and Z axes, respectively. PHIP (ϕ_p) rotates the platform positively about its 3-axis; LAMP (λ_p) rotates the platform negatively about its 2-axis; and PSIP (ψ_p) rotates it negatively about its 1-axis, in that order. The option is used when it is desired to align to geodetic or astronomic latitude as a vertical reference for ground alignment, or when platforms are assumed to be aligned in orbit with some stellar instruments.

3.2.3.2 Initial Conditions

There are two options for initial condition specifications (see Figure 5). If no input to PSII (ψ_I) is given, the program assumes the initial conditions to be referenced to platform axes; thus, down range is along the P_3 axis, cross range along the P_2 axis, and altitude along the P_1 axis. If an input of PSII (ψ_I) is specified, the program assumes that vertical or altitude errors are along the geocentric vertical, and down-range errors are referenced ψ_I° from north. Cross-range errors are therefore ψ_I + 90° from north.

3.2.3.3 Gyro Orientation

The initial orientation of the gyros and their axes is illustrated in Figure 3. Gyro alignment is made by specification of an axis of rotation (1, 2, or 3) and an argument (angle) of rotation. The program allows up to five independent rotations in any order desired. Each rotation operates on the gyros as a triad; i.e., all three gyros are being rotated and thus maintain their axis orientation with respect to each other during these rotations. The axes of rotation referred to above are those of the No. 1 gyro. Upon completion of this set of rotations, there remains an additional degree of rotational freedom of each gyro about its input axis, specified by PSI_i (ψ_i) (i = No. 1, 2, or 3 gyro).

3.2.3.4 Accelerometer Orientation

There are two options available for the alignment of accelerometers (see Figure 4). The first is the specification of an orthogonal triad and the method is identical with that described for the gyro components; i.e., an axis and argument are specified for aligning the accelerometer input axes. Then an additional degree of freedom about each accelerometer's input axis is specified by $BETA_i$ (β_i) (i = No. 1, 2, or 3 accelerometer).

The second option allows for nonorthogonal accelerometer configurations. Here the method of specification (of an axis and an argument) is the same; however, each accelerometer is specified independently. The initial orientation of each accelerometer is the same with its 1-, 2-, and 3-axes along those of platforms P_1, P_2, and P_3, respectively. The data sheet for nonorthogonal accelerometers is shown as Table A-3 in Appendix A. To exercise this option, a non-zero entry must be made in data location field ACCEL 10; conversely, a zero entry negates the option.

3.2.3.5 ERAN Control Data

ØUT — This entry controls the ERAN tape writing frequency. If no entry is made, the tape writing frequency for this case will be two records per trajectory discontinuity and two records per ERAN phase discontinuity. If a non-zero entry is made, a single record will be written at each multiple of 100 sec for powered flight, and of 1000 sec for free flight, unless otherwise specified in PPF and PFF.

PPF — This entry controls the tape writing interval during powered flight for values other than the standard 100 sec, when ØUT ≠ 0.

PFF — Similarly this entry controls the tape writing intervals during free flight for values other than the standard 1000 sec.

TSUBO — This is initial time and can be any time greater than or equal to the first time point on the trajectory tape (file). If the first time point (t) on the tape is not zero (t = 0), then the desired starting time must be entered.

TSUBA — This is abort time and can be any time less than the last time point on the tape (file). If an entry is omitted, the tape will be processed from TSUBO to the end of the trajectory tape (file).

TRAJ — When the trajectory tape contains more than one trajectory, this entry identifies the trajectory (file) to be processed (e.g., if the entry is N, ERAN will process the N^{th} file on the tape). If omitted, the next trajectory will be processed. For the first ERAN case this would be File 1.

When consecutive files are being processed on a trajectory tape, starting with File 1, the program will run most efficiently when no entry is made to TRAJ. When an N entry is made, the program will process the N file for that case and all subsequent cases until TRAJ is altered by input.

ENDC — This entry controls the use of the equations of motion in ERAN. These equations model an oblate atmosphere-free earth, used to propagate errors beyond abort time TSUBA. The pair of entries (ENDC and 1) control the termination of the propagation. The options for inputting to ENDC are as follows:

a.	No Entry	terminate at abort time
b.	TIME	terminate at that value of time (sec), which is specified in the next entry (>TSUBA)
c.	THETA	terminate at the value of range angle (deg) beyond the termination of the trajectory tape, which is specified in the next entry ($0 < \theta < 180$)
d.	ALTP	terminate at that value of altitude (ft) with a position slope, which is specified in the next entry
e.	ALTM	same as (d) above, with negative slope.

1 — This identifies the location field where the numerical value of the termination control is to be input.

MAXT — This entry controls the maximum running time of the equations of motion. It is preset to 36,000 sec; i.e., when t = 36,000 the run will be aborted unless a greater value is entered.

DTNP — This entry controls the ERAN integration step size during powered flight. No entry will cause the program to integrate at its nominal 4-sec integration step. *

DTNF — This entry controls the ERAN integration step size during free flight. No entry will cause the program to integrate at its nominal 32-sec integration step. *

BMT — This is the flag used to tdentify a case where the platform(body) axes will not be inertially oriented during the run. A non-zero entry will cause the program to seek one of the options described below.

BRTAB — This flag is used to identify the option to be used for obtaining platform turning rates and platform direction cosines. A non-zero entry will cause the program to determine platform rates from a table of input rates. From this data the direction cosines are derived by integrating the matrix differential equation of direction cosines. No entry will cause the program to read this data (rates and direction cosines) from the trajectory tape.

---

*Note: The program integration routine converges on each multiple of the tape writing interval when ØUT ≠ 0. Therefore, when the value of PPF is less than DTNP, the former would be the integration step size used in powered flight. Similarly, when the value of PFF is less than DTNF, the former is used for the free flight integration step size.

TØØP	This entry and the ten that follow it are used to identify the time to terminate a phase. No entry is needed to terminate the last phase; consequently, for cases in which there is only one phase, no entry is required.

3.2.3.6 **Earth Model Constants**

ØMEGE	rotation rate of the earth
A	equatorial radius of the earth
GM	gravity constant used in the equations of motion
e	ellipticity of the earth
J	constant in the earth's potential function
H	constant in the earth's potential function
D	constant in the earth's potential function
MU	gravity constant (equals GM) used in the variational equations.

The numerical values of these constants are given in Table G-2.

3.2.3.7 ERAN Case Control Data

Since multiple cases from one trajectory tape can be run sequentially in using the ERAN program, two cards are necessary to instruct the program as follows:

END	The preceeding cards contain all the data necessary to run this case.
ENDJØB	This is the last case processed by ERAN. Since it is preprinted on the standard form, it must be crossed out for all cases except the last.

3.2.4 Tabular Input

3.2.4.1 Turning- rate Table

The standard form for platform turning rates, which the program uses if the BRTAB flag (see Section 3.2.3) is non-zero, is shown as Table A-4 in Appendix A. The definitions of symbols and the method to be used to input data are as follows:

ORDER	refers to the order of data interpolation to be used by the program to establish rates between data inputs. A 1 entry will

cause the program to use linear interpolation, a 2 quadratic, etc. The interpolation routine used is a k^{th} order Lagrangian

N	identifies the total number of time points in the table that follows
t_1	first time point of table ($t_1 \leq$ TSUBO)
.	.
.	.
.	.
.	.
.	.
t_N	last (N^{th}) time point of table ($t_N \geq$ TSUBA)
ω_{11}	rate about platform (body) 1-axis at time t_1
.	.
.	.
.	.
ω_{1N}	rate about platform (body) 1-axis at time t_N
ω_{21}	rate about platform (body) 2-axis at time t_1
.	.
.	.
.	.
ω_{2N}	rate about platform (body) 2-axis at time t_N
ω_{31}	rate about platform (body) 3-axis at time t_1
.	.
.	.
.	.
ω_{3N}	rate about platform (body) 3-axis at time t_N

Note: If the table contains many zeros in sequence, they can be entered by writing a Z in the prefix field and the number of zeros to be generated in the value field. As an example, the table for a case of 20 time points, zero rates about the 1- and 3-axis, and the first two rates about the 2-axis, also zero, would look like Table 2.

Table 2. Sample for a Case of 20 Time Points

PRE	LOC	Value	Remarks
I		20	
		value of t_1	
		.	
		.	
		.	
		value of t_{20}	
Z		22	20 zero rates about 1-axis and 2 zero rates about 2-axis
		value of $\omega_2(t_3)$	
		.	
		.	
		.	
		value of $\omega_2(t_{20})$	
Z		20	20 zero rates about 3-axis

Also note that the reverse side of the standard form can be used to continue the rate table.

3.2.4.2 Equation of Motion Initialization

The program is mechanized so that the equations of motion can be initialized independent of a trajectory tape input. This feature is used when it is desired to obtain transition matrices or to use one of the mission evaluation options to derive miss coefficients. The format* for this data is given as Table A-5 in Appendix A.

*A printed standard form is not available.

3.2.5 Multiple Cases

As mentioned previously, ERAN has the capability to run multiple cases from a single trajectory tape. The data used for the first run is retained for the second (and subsequent) runs; thus, only data that requires changes from the preceding runs needs be input. When it is desired to eliminate the effects of an orientation option used in a previous case, it is necessary to input a minus zero in an appropriate location. The three options and the methods of cancelation are as follows:

a.	Platform Orientation Option 2	input minus 0 in PHIP
b.	Initial Condition Orientation Option 2	input minus 0 in PSII
c.	Nonorthogonal Accelerometer	input minus 0 in ACCEL 10, which is the input location for the first axis of rotation of the No. 2 accelerometer component

This last operation will negate the logic that was set up by the previous non-orthogonal case and will therefore interpret the data for the No. 1 component as that required for a triad.

Extreme caution should be exercised when attempting to change the Error Source Schedule for an operation where there are more than 6 phases. Some knowledge of the input routine is necessary to present the intrinsic problem.

The D option (i. e., D in the prefix field of the input word), used to input the Error Source Schedule, causes two words to be stored in the computer, with the last 6 characters being stored in the location immediately following the location of the first 6. When there are no entries in Columns 7 through 12, however, only the first 6 characters are stored and the second location remains unaltered. It is then apparent that an X, entered beyond Column 6 for a previous case, cannot be eliminated without entering at least one X into some other column beyond the 6th for the case in question. Cancellation of the error source for all phases can be achieved by entering zeros into the appropriate locations for the sigma value (see Sections 3.3.3.1 and 3.4). Phase logic, for an error source during phases 7 through 11, can be eliminated when there are less than 12 phases to the case by entering an X in Column 12. Should that be the only entry on the line, the error source would be eliminated for the entire case.

3.3 OUTPUT DATA

The output data processor program (ØUTP) takes the data generated by ERAN and produces output data at the required times, with the prescribed transformations and the proper format. The options available for the above data follow.

3.3.1 Output (Print) Times

Option 0 Output the data only at the terminal condition of the case.

Option 1 Output at the phase discontinuities plus the terminal conditions.

Option 2 Output data called for by Options 0 and 1 and at all trajectory discontinuities where the sensed acceleration goes from non-zero to zero or from zero to non-zero, including the initial time.

Option 3 Process every time point on ERAN tape as determined by the tape density control.

3.3.2 Output Coordinate Systems

ECI Presents data in an Earth Centered Inertial System, where the Z-axis is the earth's polar axis and the X- and Y-axes are in the equatorial plane. Generally, the convention is that the X-axis passes through the Greenwich meridian at time zero, and Y completes a right-hand system; however, these coordinates are determined by the particular trajectory program used to generate the input tape.

LH Presents data in a local horizontal coordinate system, which is inertial and developed from the nominal trajectory position and velocity vectors. X is down range, i.e., directed along the projection of the inertial velocity vector onto the plane normal to the radius vector; Y is vertical, i.e., along the geocentric radius vector; and Z is cross range, forming a right-hand coordinate system.

EVALU Presents additional data at the terminal condition only with a prescribed transformation.* Presently there are the following three options for this output:

EVALU1: Presents the down-range (M_D), cross-range (M_C), and timing (M_T) errors (misses) at a fixed altitude

*A special format is used for these transformations (see Section 3.3.3.4).

EVALU2: Presents the cross-range (M_C), vertical (M_V), and timing (M_T) errors (misses) at a fixed range

EVALU3: General – represents transformation developed from input matrices.

3.3.3 Output Data Formats

The five present formats for output data will be described in Sections 3.3.3.1 through 3.3.3.5. Examples of the formats are presented with the test cases in Appendix C.

3.3.3.1 Vector Errors

In order to present the vector errors at a particular time, ØUTP first updates all inactive vector sensitivities (previous phase sensitivity vectors) by pre-multiplying them by an appropriate transition matrix. Next it scales the vector sensitivities by the proper sigma level (input data of the sigma of a particular error source). This results in a vector error in the ECI coordinate system. Finally, it performs a coordinate transformation, when required. This data is presented in a standard format where

Line 1 gives the run date and job identification (see Section 3.4)

Line 2 is the case identification

Line 3 is the nominal time and coordinate system identification

Line 4 is column headings where DPX, DPY, DPZ are delta-position coordinates to the nearest ft

DVX, DVY, DVZ are delta-velocity coordinates to the nearest 0.01 ft/sec

DOX, DOY, DOZ are delta-platform orientation coordinates to the nearest 0.1 $\widehat{sec}$

Line 5-up is vector error, where the left column identifies the vector as follows:

XXXX	XX
Four characters are used to identify the error-source type, and the component or axis it represents. The identification generally follows the symbols given in Table G-1 with the following changes:	Two characters are used to identify the phase in which this particular error source was initiated (01 to 12)

Initial Condition	E is replaced by O
Accelerometers and gyros	E is dropped and the error type is followed by 1, 2, or 3, indicating which component it represents
Platform	E is dropped and the error type is followed by 1, 2, or 3 indicating which platform axis it represents
Terminal Conditions	E is replaced by T, and T is replaced by O

Note that the order in which the error vectors are presented is that given in Section 3.2.2.

3.3.3.2 Covariance Matrix

The covariance matrix is formed from the expression

$$\sum_{ECI} = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij}\sigma_i\sigma_j\bar{x}_i\bar{x}_j^T$$

where

$\sum_{ECI}$ = covariance matrix in ECI coordinates (9 × 9 matrix)

n = total number of error sources

ρ_{ij} = correlation coefficient between the i^{th} and j^{th} error source. When i=j, ρ=1; when i≠j, ρ=O unless input otherwise

$\sigma_i; \sigma_j$ = standard deviations of the i^{th} and j^{th} error sources

$\bar{x}_i$ = sensitivity vector of i^{th} error source

$\bar{x}_j^T$ = transpose of j^{th} sensitivity vector

To obtain the covariance matrix in the local horizontal coordinate system, the following operation is performed

$$\sum_{LH} = \underline{M_{LE}} \sum_{ECI} \underline{M_{LE}}^T$$

where

$\sum_{LH}$ = covariance matrix in local horizontal coordinates

$\underline{M_{LE}}$ = matrix (9 × 9) which transforms a sensitivity vector from ECI to local coordinates

$\underline{M_{LE}}^T$ = transpose of $\underline{M_{LE}} = \underline{M_{EL}}$

The presentation of this data at a particular time is in a standard format, where

Line 1 = case identification

Line 2 = nominal time and coordinate system identification

Line 3 identifies it as the covariance matrix

Line 4 = column heading (same definition as vector errors)

Lines 5 to 13 = covariance matrix in the following format:

diagonal presents variances of navigation errors in floating point

upper elements present covariances of navigation errors in floating point

lower elements present correlation coefficients in fixed point.

Line 14 = standard deviations (sigmas) of the navigation errors formed from the square root of the variances

Line 15 = trajectory variables in ECI coordinates

Line 16 = trajectory variables in earth reference coordinate system,

where:

LAT = geocentric latitude (deg)

LONG = longitude from Greenwich meridian (deg)

ALT = altitude (n mi) above the surface of an oblate earth (n mi)

VEL = inertial velocity (ft/sec)

FPA = flight path angle defined as the angle the inertial velocity vector makes with the local geocentric horizontal (deg)

AZ = azimuth of the inertial velocity vector, measured clockwise from north.

3.3.3.3 Transition Matrix

The transition matrix is used in the ERAN Program to propagate sensitivity vectors across the free-flight sections of a trajectory, rather than integrating each set of error-source equations.* It is used by ØUTP when presenting a time history of vector errors during free flight in the same manner, i.e., to propagate the sensitivity vectors. ØUTP also uses the transition matrix to update an inactive vector (one generated in a previous phase), when running multiphase cases.

The transition matrix is generated in the usual manner – by solving the homogeneous differential equations (in ECI coordinates) for each initial condition error. To obtain the transition matrix in the local horizontal coordinate system, the following operation is performed:

$$\Phi_{LH}(t, \tau) = \underline{M_{LE}}(t)\Phi_{ECI}(t, \tau)\underline{M_{LE}}^T(\tau)$$

*In the present formulation of the program, it is assumed that the accelerometers are disconnected at termination of a powered phase.

where

$\Phi(t, \tau)_{LH}$ = transition matrix in local horizontal coordinates

$\Phi(t, \tau)_{ECI}$ = transition matrix in ECI coordinates

$\underline{M_{LE}}(t)$ = matrix at time t, which transforms a sensitivity vector from ECI to local coordinates

$\underline{M_{LE}}^T(\tau)$ = transpose of $\underline{M_{LE}}$ at time τ

There is an option to control the output of the transition matrix (see Section 3.4). When called for, the matrix will be presented at each discontinuity (phase or trajectory tape).* In addition, if a time history (print Option 3) is called for, the transition matrix will be presented at the same times as the vector errors and covariance matrix. The presentation of the transition matrix is in a standard format, where

Line 1 = case identification

Line 2 = identifies it as the transition matrix and the applicable time arguments (t, τ)

Line 3 = the coordinate system identification

Line 4 = format identification

Line 5 = column headings

Line 6-14 = transition matrix

Line 15 = trajectory variables in ECI coordinates at time τ

Line 16 = trajectory variables in ECI coordinates at time t

Line 17 = trajectory variables in earth reference coordinate system at time τ

Line 18 = trajectory variables in earth reference coordinate system at time t

3.3.3.4 Mission Evaluation

As indicated in Section 3.3.2, there are presently three options available for presenting data at the end of the case that can be used for evaluating the

*Except for free flight, a transition matrix is not computed in Phase 1.

success of the mission. The data presented when one of these options is called for consists of vector errors and a covariance matrix. The format for each option is as follows:

EVALU1 — Presents data at the reference altitude instead of the reference time

Vector Errors

Line 1 — case identification

Line 2 — nominal time and criterion (ALT)

Line 3 — column headings, where:

MT = timing error to the nearest 0.001 sec

MC = cross-range miss to the nearest ft

MD = down-range miss to the nearest ft

Line 4-up — vector errors, where the left column identifies (described in Section 3.3.1) the vectors

Covariance Matrix

Line 1 — identifies it as a covariance matrix

Line 2 — column heading

Line 3-5 — covariance matrix output (same format as described in Section 3.3.2)

Line 6 — sigma values

Line 7 — headings for nominal trajectory conditions at termination

Line 8 — trajectory conditions where:

LAT, LONG, ALT are as defined in Section 3.3.3.2

VEL/R = magnitude of relative* velocity vector (ft/sec)

FPA/R = magnitude of relative flight path angle (deg)

AZ/R = azimuth of relative velocity vector (deg)

*Velocity vector with respect to rotating earth.

EVALU2 Presents data at the reference range angle instead of at the reference time. The data format is the same as for EVALU1 except as noted below.

Line 2 criterion (ALT replaced by RANGE)

Line 3 MD is replaced by MV (vertical miss to the nearest ft)

EVALU3 This option is used for special cases in which the linear transformation between ECI position and velocity errors and some arbitrary parameters are known; e.g., the midcourse maneuver velocity components as a function of injection errors for a space probe, some orbit elements, etc.

Vector Errors

Line 1 case identification

Line 2 nominal time and identification of EVALUATION OPTION 3

Line 3 column heading (1, 2 .... 6)

Line 4-up vector errors (floating point) where the left column identifies (as described in Section 3.3.3.1) the vector

Covariance Matrix

Line 1 covariance matrix identification

Line 2 column headings (1, 2 .... 6)

Line 3-8 covariance matrix (same format as described in Section 3.3.3.2)

Line 9 sigma values

3.3.3.5 Platform Reference Attitude Time History

When a torqued platform or strapped-down case is being run, a time history of attitudes and direction cosines are given at the end of the case. The times are the same as those called for by the print option. The format for presentation of this data is as follows:

Line 1 identification of type of output

Line 2 column headings, where

THETA (Θ) = angle (deg) platform 1-axis makes with the local horizontal plane

PSI (ψ) = angle (deg) the projection that the platform 1-axis onto the horizontal plane makes with north

PHI (φ) = angle (deg) the platform 2-axis makes with the local horizontal plane

For a strapped-down case these angles would be missile pitch, yaw, and roll angles, respectively.

1X, 1Y, 1Z = direction cosines of the platform 1-axis, with respect to the ECI coordinate system

2X, 2Y, 2Z = direction cosines of the platform 2-axis, with respect to the ECI coordinate system

Line 3-up time history of above data

3.4 ØUTP INPUT DATA

The standard form for input data to ØUTP is presented as Table A-6 in Appendix A. The definitions and procedure for filling out this sheet are as follows:

HJØB1 and HJØB2	Up to 60 characters (30 each), which will be printed as the first line of vector error output
HCASE1 and HCASE2	Up to 60 characters (30 each), which will be printed as the second line of vector errors and first line of all other outputs
ØPT	Print time option as discussed in Section 3.3.1. No input will produce Option O.
SYSTM	Controls output coordinate system. No input results in LH coordinate system. +ECI results in both ECI and LH output. -ECI outputs only in ECI coordinates.
CASE	The number of the case (N) to be processed, N being the N^{th} trajectory processed by ERAN (but not necessarily the N^{th} trajectory on trajectory tape). When outputting consecutive ERAN cases, starting with Case 1, the program will operate most effectively if no entry is made to CASE. When an N entry is made, the program will process the N^{th} case until CASE is altered by input.
SØPT	A non-zero entry will result in storing the sigma values for the next (and subsequent) runs; whereas a zero entry will result in setting all sigma entries to zero after the present case has been completed.
PHIL	Correction term for longitude output. To be used when the trajectory ECI system is not referenced to Greenwich.
EVALU	Identifies the mission evaluation option (if any) to be output.
FØPT	Non-zero entry will result in transition matrix(es) output.
SIGMA	In the LOC. field, the vector number is input and the sigma value for that vector is put in the value field.* Only when a SIGMA value changes is it required to input a new value; otherwise, it will assume the sigma

*An entry of zero will cause that vector to be eliminated from the case.

value of the previous vector error. As an example, if unit sensitivites are desired, then a 1 in the LOC. field and a 1 in the VALUE field will result in output with unit scaling of all error sources.* However, when the SØPT option is used for multiple cases, all desired changes to the sigma table must be entered explicity. In the above example, all desired changes from their assigned unit values would have to be entered; e.g., a change of the sigma value for the first error source would only alter that value, all others retaining their unit values.

RHØ — These are correlation coefficients. Two entries are required to input a correlation coefficient: The first identifies the error sources (by vector number) that are correlated, and the next gives the value of the correlation coefficients. All correlation coefficient data is retained in storage; therefore, when running multiple cases, care must be exercised to not get unwanted correlation into the covariance matrix calculations. A double-zero in the field for assigning vector numbers of a correlation coefficient will result in eliminating the effects (if any) of that previously stored correlation coefficient, as well as of all those that followed on the input sheet.

END — Same control as discussed for ERAN

ENDJØB — Same control as discussed for ERAN

ØUTP has the same logic of data storage (except as noted in the SØPT option) for multiple cases as ERAN. Therefore, only changes to data need be entered for runs following the first. To negate the SYSTM option, six zeros must be entered (i.e., when LH output alone is desired after some other option on the previous case has been chosen).

*Note units of error sources in Table G-1.

When the EVALU3 option is used, the format* for the input data sheet is presented as Table A-7 in Appendix A. The linear transformation matrix [M] used in this option is formed from products of input matrices by

$$[M] = [M_1][M_2][M_3] \cdots [M_n]$$

$$n \leq 10$$

where $[M_i]$ is a (6×6) matrix input.

*A preprint standard form is not available.

SECTION 4

SAMPLE CASES

Three test cases were designed to demonstrate the procedures for filling out input data sheets when exercising the various program options available, and to present data in all of the output formats. The data sheets used to set up the test cases are presented in Appendix B and the output listings from these runs in Appendix C.

The trajectory used for the test cases was one that had been designed to place a payload into a 24-hour synchronous equatorial orbit. Following are the major trajectory sequences:

0 - 464	Powered flight from launch to booster burnout
464 - 477	Separation sequence (coast)
477 - 498	Inject into 100-mile parking orbit
498 - 1380	Coast to first equatorial crossing
1380 - 1685	Inject into transfer orbit
1685 - 20177	Coast in transfer orbit to apogee of 19,300 n mi
20177 - 20288	Inject into synchronous equatorial orbit

The salient features of the test cases and the pertinent input/output options used to obtain these features are now described.

4.1 TEST CASE 1

This is the evaluation of the uncertainty of instantaneous impact prediction (IIP) for premature thrust termination, by using the inertial navigator data and assuming a vacuum re-entry. The same option could be used for evaluating ballistic missile accuracy or guided re-entry vehicle accuracy at a fixed altitude.

The configuration of the inertial navigator was one in which the platform 1-axis was aligned with the geodetic vertical and the 3-axis was north.* The input axes of the gyros and accelerometers were aligned along the platform axes. The error sources considered were initial position (3), initial platform alignment (3), accelerometer bias (3), accelerometer scale factor (3) and gyro bias drift (3). To evaluate the impact accuracy of a thrust termination at 400 sec, the trajectory tape was aborted at 400 sec and the equations of motion were used to integrate during free flight; they were terminated when the altitude went through zero on a negative slope. Although the trajectory tape had only one file, it was necessary to enter a 1 in TRAJ in order to obtain multiple processing of the trajectory. The data sheets used for this run are shown as B-1, Error Source Schedule and B-2, Orientation and Control Data in Appendix B. Since two more cases were to be run by ERAN before being processed by ØUTP, the ENDJØB O card was scratched out in B-2.

The processing of this data was controlled by ØUTP and the data sheet used is shown as B-9. Since it was desired to process the ERAN tape in sequential order, it was not necessary to enter anything in CASE. If processing in a different order, or reprocessing any particular case, had been wanted, it could have been done by using the CASE control. It was required to obtain output in ECI coordinates at the powered flight termination and at

*Since the trajectory was run on a spherical earth model, the geodetic and geocentric latitudes are equal.

impact, and to evaluate the impact errors by using mission evaluation Option 1. As it was desired to save the SIGMA data for the next case, the SØPT option was called for. The 1σ errors for this case are shown in Table 3.

Table 3. One-Sigma Errors for Test Case 1

Vector Number(s)	Error Source	Sigma Value
1, 2, 3	initial position	500 ft (three axes)
4, 5, 6	initial platform orientation	30 $\widehat{\text{sec}}$ (three axes)
7, 8, 9	accelerometer bias	10^{-4}g (3 components)
10, 11, 12	accelerometer scale factor	10^{-4}g/g (3 components)
13, 14, 25	gyro bias drift	0.1 deg/hr (3 components)

The bias and scale-factor error sources of each accelerometer were correlated with a correlation coefficient of 0.5, i.e., the number one accelerometer bias error (7) was correlated with its scale-factor error (10), etc. Since additional cases were to be processed by ØUTP, the ENDJØB O card was scratched, completing the input for this case.

4.2 TEST CASE 2

In this case, an evaluation is made of the altitude, cross-range, and time errors at a fixed range after completion of one orbit. The configuration of the inertial navigator was the same as in Test Case 1, but it was arrived at in a different way. The platform 1-axis was aligned with the vertical as before, but the 3-axis was aligned east. To retain the same gyro orientation with respect to the trajectory, it was necessary to rotate the gyro cluster 90° about its 1-axis. The accelerometer alignment was controlled by using the nonorthogonal accelerometer option (see B-4). The error sources considered were the same as in Case 1, with the addition of terminal errors (applied at the abort time). The trajectory tape was aborted at 500 sec to insert the terminal condition errors (thrust tailoff, guidance equations, etc.) and the equations of motion used for one orbit (approximately 5500 sec). It was desired to have a time history output; therefore, ØUT was made non-zero, PPF was set at 400 and PFF at 2000. The data sheets used to make this run are shown as B-3 through B-5 in Appendix B. It was also necessary to scratch the ENDJØB card.

The data sheet for output processing of this case is shown as B-10. Using Option 3, time history, results in additional output at 400 sec during powered flight and every 2000 sec during orbit. It was desired to have only the LH coordinate system output; therefore, six zeros (000000) were entered in SYSTM to negate the logic from the Case 1 option. The mission evaluation Option 2 was used and the SØPT option cancelled. Since the SIGMA's for the first 15 error sources were held over from Case 1, only the terminal condition error-source sigma values were required. Since no changes in the correlation coefficients from Case 1 were desired, entries in RHØ were not necessary. The ENDJØB card was scratched, completing the input data required for this case.

4.3 TEST CASE 3

This was an evaluation of the errors at injection into the final orbit. The gyro drift was assumed to have an exponential autocorrelation function with a time constant of 2 hr. The accelerometer bias during the trajectory's final powered sequence had a standard deviation three times larger than, and uncorrelated with, that of the initial trajectory sequences. The platform 1-axis was aligned with the geocentric vertical and the 3-axis was north. The initial condition position errors were referenced to platform axes. The platform was torqued about its 3-axis (approximately missile pitch axis) to maintain a small (<20°) angle between the missile and the platform axis throughout the trajectory. The accelerometer and gyro alignment with respect to platform axes was the same as in Case 1. The error sources were the same, with the addition of a gyro-torquing scale-factor error.

To achieve the evaluation described above, the trajectory was divided into 3 phases, with the first terminating in 2 hr, the second in 4 hr, and the third phase at the end of the trajectory (5.63 hr). This allowed an approximation of the effect of the gyro-error autocorrelation function. (More phases would more nearly approach the true effect.)

The changes in the Error Source Schedule were in EAOO, EGOO, EGO6, and the terminal condition errors (see B-6 in Appendix B). The body turning rates are given in B-7, and the changes in the control data (B-8) were the following:

PSIP	= 0, aligns platform 3-axis north ($\psi = 0$)
PHIP	= -0, changes platform alignment option back to 1 and aligns it with respect to geocentric vertical
PSII	= -0, changes IC option back to 1 and causes initial position errors to be along platform axes
I GYRO	= 0, references gyros to platform axes
I ACCEL	= 0, references accelerometers to platform axes

I 10	**= 0, changes accelerometer option back to 1, i.e., orthogonal orientation**
TSUBA	**= chosen to be a time greater than (or could have been equal to) the end of the trajectory tape**
ENDC	**= 0, eliminates use of the equations of motion**
ØUT	**= 0, eliminates use of the intermediate tape writing intervals**
BMT	**= 1, indicates a non-inertial platform case**
BRTAB	**= 1, indicates rates are supplied by an input table**
TGØP	**= 7200, indicates time to end the first phase**
1	**= 14400, indicates time to end the second phase**

The final phase is ended by the termination control.

The data sheet for the output processing of this case is given in B-11. The print option was 2 (phase and trajectory discontinuities) and EVALU set at zero to eliminate its output. Although the SIGMA's were the same for the first 15, they had to be re-entered because the SØPT option was zeroed out in the previous case. The vector errors for this case are shown in Table 4.

Table 4. Vector Errors for Test Case 3

Vector Numbers	Error Source	Sigma Value
1, 2, 3	initial position	500 ft (3 axes)
4, 5, 6	initial platform orientation	30 sec (3 axes)
7, 8, 9 (phase 1)	accelerometer bias	10^{-4}g (3 components)
10, 11, 12	accelerometer scale factor	10^{-4}g/g (3 components)
13, 14, 15 (phase 1)	gyro drift	0.1 deg/hr (3 components)
16, 17	gyro torquer scale factor	0 (Nos. 1 and 2 gyros)
18	gyro torquer scale factor	10^{-4} (No. 3 gyro)
19, 20, 21 (phase 2)	gyro drift	0.1 deg/hr
22, 23, 24 (phase 3)	accelerometer bias	3×10^{-4}g (3 components)
25, 26, 27 (phase 3)	gyro drift	0.1 deg/hr
28, 29, 30	terminal velocity errors	0.1 ft/sec

The correlation of accelerometer bias and scale factor during Phase 1 was assumed to be the same as in Cases 1 and 2, but it had to be re-entered because additional correlation coefficients were being entered. The time correlations of gyro errors were calculated as

$$\rho_{ij} = \exp - \frac{(t_i - t_j)}{7200}$$

where ρ_{ij} is the appropriate correlation coefficient, and

$$(t_i - t_j)^* = 14400 - 7200 = 7200$$
$$20288 - 7200 = 13088$$
$$20288 - 14400 = 5888$$

This completes the description of the input data required for this case.

Printouts listing the cards used for ERAN data and ØUTP data are given for each run. Along with the output listings, they are included in Appendix C.

---

*Finer or coarser time intervals could have been chosen to approximate the autocorrelation function $\exp - \frac{t}{T}$, with more or less phases required for the approximation.

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APPENDIX A

STANDARD INPUT DATA FORMS

CONTENTS

Table A-1. ERAN Error Source Schedule

GENERAL INERTIAL ERRØR ANALYSIS PRØGRAM

X-6 7090 INPUT DATA

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

Originator ______________ CCC ______ JO ________

PROGRAMMER ________ KEYPUNCHED ________ VERIFIED ________ DATE ________ PAGE ____ OF ____

H	ERAN-ERRØR SØURCE SCHEDULE	

PUNCH ONLY X's IN THIS FIELD

	LOC.		LOC
D	EI11	D	EG00
D	EI12	D	EG01
D	EI13	D	EG02
		D	EG03
D	EI21	D	EG04
D	EI22	D	EG05
D	EI23	D	EG06
		D	EG07
D	EI31	D	EG08
D	EI32	D	EG09
D	EI33	D	EG10
		D	EG11
D	EA00	D	
D	EA01	D	
D	EA02	D	
D	EA03	D	
D	EA04	D	
D	EA05	D	
D	EA06	D	
D	EA07	D	
D	EA08	D	
D	EA09	D	
D	EA10	D	
D	EA11	D	
D	EA12	D	EP01
D		D	EP02
D		D	EP03
D		D	
D		D	
D		D	
D		D	
D		D	
D		D	ET11
D		D	ET12
D		D	ET13
D		D	
D		D	ET21
D		D	ET22
D		D	ET23
D		D	

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Table A-2. ERAN Orientation and Control Data

X-1 7090 INPUT DATA

FCC12A-81 (Rev. 11/5/64)

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ____ OF ____

1	7	73
H	ERAN - PLATFØRM GYRØ AND ACCELERØMETER	
H	ALIGNMENTS (ØRTHØGØNAL ØPTIØN ØNLY),	
H	AND CASE CØNTRØL DATA	

SYMBOL	I	LOC.	VALUE	EXP.	SYMBOL	I	LOC.	VALUE	EXP.
Ψ_p		PSIP					TSUBO		
Φ_p		PHIP					TSUBA		
λ_p		LAMP				I	TRAJ		
Ψ_i		PSII							
AXIS	I	GYRØ				A	ENDC		
ANGLE		1					1		
AXIS	I	2					MAXT		
ANGLE		3							
AXIS	I	4					DTNP		
ANGLE		5					DTNF		
AXIS	I	6					BRT		
ANGLE		7					BRTAB		
AXIS	I	8					TGØP		
ANGLE		9					1		
							2		
							3		
Ψ_1		PSI1					4		
Ψ_2		PSI2					5		
Ψ_3		PSI3					6		
							7		
AXIS	I	ACCEL					8		
ANGLE		1					9		
AXIS	I	2					10		
ANGLE		3					11		
AXIS	I	4					ØMEGE		
ANGLE		5					A		
AXIS	I	6					GM		
ANGLE		7							
AXIS	I	8					J		
ANGLE		9					H		
							D		
							MU		
$\beta 1$		BETA1				E	ND	0	
$\beta 2$		BETA2							
$\beta 3$		BETA3							
		ØUT				E	NDIØB	0	
		PPF							
		PFF							

AEROSPACE FORM 2421 REV 6-62

Table A-3. ERAN Nonorthogonal Accelerometer Orientation

X-3 7090 INPUT DATA

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ______ OF ______

H	ERAN - ACCELERØMETER ALIGNMENTS (NONORTHOGONAL)	
H	ALL ANGLES INPUT IN DEGREES	

SYMBOL	F	LOC.	VALUE	EXP.	
					Specification of Accelerometer Triad Orientation (Option 1) or No. 1 Accelerometer of Nonorthogonal Configuration (Option 2)
	I	ACCEL			Axis of 1st rotation
		1			Angle " " "
	I	2			Axis " 2nd "
		3			Angle " " "
	I	4			Axis " 3rd "
		5			Angle " " "
	I	6			Axis " 4th "
		7			Angle " " "
	I	8			Axis " 5th "
		9			Angle " " "
					Specification of Orientation of No. 2 Accelerometer for Option 2
	I	10			Axis of 1st rotation
		11			Angle " " "
	I	12			Axis " 2nd "
		13			Angle " " "
	I	14			Axis " 3rd "
		15			Angle " " "
	I	16			Axis " 4th "
		17			Angle " " "
	I	18			Axis " 5th "
		19			Angle " "
					Specification of Orientation of No. 3 Accelerometer for Option 2
	I	20			Axis of 1st rotation
		21			Angle " " "
	I	22			Axis " 2nd "
		23			Angle " " "
	I	24			Axis " 3rd "
		25			Angle " " "
	I	26			Axis " 4th "
		27			Angle " " "
	I	28			Axis " 5th "
		29			Angle " " "

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Table A-4. ERAN Turning Rates in Platform Coordinates

X-3 7090 INPUT DATA

F00128A-07 12/7/64

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ___ OF ___

1	7	73
H	TURNING RATES IN PLATFØRM (BØDY) CØØRDINATES	
H	ALL RATES IN DEG/SEC	

SYMBOL	P	LOC.	VALUE	EXP.	
ORDER	I	RATES			Order of interpolation K = K^{th} order Lagrangian interpolation 1 = linear interpolation
	X	1	3		
	Y	1	1		
	A		ØMEG1		
	A		ØMEG2		
	A		ØMEG3		
	A		TIME		
N	I				No. of time points (N) The data points must be input in the following manner:*

t_1
⋮
t_N
ω_{11}
⋮
ω_{1N}
ω_{21}
⋮
ω_{2N}
ω_{31}
⋮
ω_{3N}

where: ω_{1i} is the body roll rate for t_i
ω_{2i} " " " pitch " " "
ω_{3i} " " " yaw " " "

* If the roll and/or yaw rates are all zero, it is not necessary to enter N zeros for the roll and/or yaw rates. Simply enter a Z in the prefix field and the number "N" in the value field to generate N zeros.

Table A-5. ERAN Equatic . of Motion Initialization

X-3 7090 INPUT DATA

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ____ OF ____

1	7	73
H	EQUATIØNS OF MØTIØN INITIALIZATIØN	

SYMBOL	PRE	LOC.	VALUE	EXP.	
		EM	1		
		TIME			Sec
X		1			Ft
Y		2			"
Z		3			"
R		4			"
$\dot{X}$		8			Ft/sec
$\dot{Y}$		9			"
$\dot{Z}$		10			"

AEROSPACE FORM ... REV ...

Note: 1) This data is not retained in storage, therefore for multiple cases it must be reinitialized.

2) If a trajectory case is to follow, an EM 0 must be included in its control data.

Table A-6. ØUTP Case Control and Data Input Form

7090 INPUT DATA

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ____ OF ____

1	7	73
RJØ01		
RJØ02		
HCASE1		
HCASE2		

SYMBOL	T/E	LOC.	VALUE	EXP.
	I	[illegible]		
	D	[illegible]M		
	I	CAS[illegible]		
		SOFT		
		PHI[illegible]		
	I	EVA[illegible]		
		RUPT		
		[illegible]		

SYMBOL	T/E	LOC.	VALUE	EXP.
[illegible]	A	RHØ	,,	
VAL[illegible]				
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	[illegible]	ID	0	
		[illegible]	0	

AEROSPACE FORM [illegible]

Table A-7. ØUTP EVALU 3 Input Data Form

X-3 7090 INPUT DATA

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ____ OF ____

EVALU 3 INPUT DATA FØRM

SYMBOL	RE	LOC.	VALUE	EXP.
	I	EVALM		
		1		
		2		
		3		
		.		
		.		
		.		
		.		

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Number of matrices input (n) $n \le 10$

$$M = M_1 M_2 M_3 \ldots M_n$$

The elements of these matrices are identified as follows

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 7 & 8 & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & 36 \end{bmatrix} \begin{bmatrix} 37 & 38 & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & 72 \end{bmatrix}$$

(1) (2)

$$\cdots \begin{bmatrix} [36(n-1)+1] & \cdot & \cdot & \cdot \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & 36n \end{bmatrix}$$

(n)

Note: Zero elements need not be entered unless zeroi[illegible] out an element of a previous case.

GENERAL INERTIAL ERRØR ANALYSIS PRØGRAM

X-6 7090 INPUT DATA

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

Originator ________ CCC ______ JO ______

PROGRAMMER ________ KEYPUNCHED ________ VERIFIED ________ DATE ________ PAGE 1 OF 11

H ERAN-ERRØR SØURCE SCHEDULE

	LOC	PUNCH ONLY X's IN THIS FIELD
D	EI11	X
D	EI12	X
D	EI13	X
D	EI21	
D	EI22	
D	EI23	
D	EI31	X
D	EI32	X
D	EI33	X
D	EA00	X
D	EA01	X
D	EA02	
D	EA03	
D	EA04	
D	EA05	
D	EA06	
D	EA07	
D	EA08	
D	EA09	
D	EA10	
D	EA11	
D	EA12	
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		

	LOC	PUNCH ONLY X's IN THIS FIELD
D	EG00	X
D	EG01	
D	EG02	
D	EG03	
D	EG04	
D	EG05	
D	EG06	
D	EG07	
D	EG08	
D	EG09	
D	EG10	
D	EG11	
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D	EP01	
D	EP02	
D	EP03	
D		
D		
D		
D		
D		
D	ET11	
D	ET12	
D	ET13	
D		
D	ET21	
D	ET22	
D	ET23	
D		

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B-1. Error Source Schedule for Test Case 1

X-1 7090 INPUT DATA

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE 2 OF 11

1	7	73
H	KRAN - PLATFØRM GYRØ AND ACCELERØMETER	
H	ALIGNMENTS (ØRTHØGØNAL ØPTIØN ØNLY).	
H	AND CASE CØNTRØL DATA	

SYMBOL		LOC.	VALUE	EXP.
Ψ_p		PSIP		
Φ_p		PHIP	-80.578831	
λ_p		LAMP	28.555	
Ψ_i		PSII	90	
AXIS	I	GYRØ		
ANGLE		1		
AXIS	I	2		
ANGLE		3		
AXIS	I	4		
ANGLE		5		
AXIS	I	6		
ANGLE		7		
AXIS	I	8		
ANGLE		9		
$\Psi 1$		PSI1		
$\Psi 2$		PSI2		
$\Psi 3$		PSI3		
AXIS	I	ACCEL		
ANGLE		1		
AXIS	I	2		
ANGLE		3		
AXIS	I	4		
ANGLE		5		
AXIS	I	6		
ANGLE		7		
AXIS	I	8		
ANGLE		9		
β_1		BETA1		
β_2		BETA2		
β_3		BETA3		
		ØUT		
		PFF		
		PFF		

SYMBOL		LOC.	VALUE	EXP.
		TSUBO		
		TSUBA	400	
	I	TRAJ	1	
	A	ENDC	ALTM	
		1	0	
		MAXT		
		DTNP		
		DTNF		
		BMT		
		BRTAB		
		TGØP		
		1		
		2		
		3		
		4		
		5		
		6		
		7		
		8		
		9		
		10		
		11		
		ØMEGE		
		A		
		GM		
		J		
		H		
		D		
		MU		
	E	ND	0	

AEROSPACE FORM 2423 REV 6-62

B-2. Orientation and Control Data for Test Case 1

GENERAL INERTIAL ERRØR ANALYSIS PRØGRAM

X-6 7090 INPUT DATA

Originator ______ CCC ____ JO ______

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE 3 OF 11

1	7	73
H	ERAN-ERRØR SØURCE SCHEDULE	

	LOC	PUNCH ONLY X's IN THIS FIELD
D	EI11	
D	EI12	
D	EI13	
D	EI21	
D	EI22	
D	EI23	
D	EI31	
D	EI32	
D	EI33	
D	EA00	
D	EA01	
D	EA02	
D	EA03	
D	EA04	
D	EA05	
D	EA06	
D	EA07	
D	EA08	
D	EA09	
D	EA10	
D	EA11	
D	EA12	
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		

	LOC	PUNCH ONLY X's IN THIS FIELD
D	EG00	
D	EG01	
D	EG02	
D	EG03	
D	EG04	
D	EG05	
D	EG06	
D	EG07	
D	EG08	
D	EG09	
D	EG10	
D	EG11	
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D	EF01	
D	EF02	
D	EF03	
D		
D		
D		
D		
D		
D	ET11	
D	ET12	
D	ET13	
D		
D	ET21	X
D	ET22	X
D	ET23	X
D		

11/20/64 P0012A-01

B-3. Error Source Schedule for Test Case 2

X-3 **7090 INPUT DATA**

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE 4 OF 11

H	XRAN - ACCELEROMETER ALIGNMENTS	
H	ALL ANGLES INPUT IN DEGREES	

SYMBOL		LOC.	VALUE	EXP.	
					Specification of Accelerometer Triad Orientation (Option 1) or No. 1 Accelerometer of Nonorthogonal Configuratio (Option 2)
	I	ACCEL	1		Axis of 1st rotation
		1	90		Angl " " "
	I	2			Axis " 2nd "
		3			Angle " " "
	I	4			Axis " 3rd "
		5			Angle " " "
	I	6			Axis " 4th "
		7			Angle " " "
	I	8			Axis " 5th "
		9			Angle " " "
					Specification of Orientation of No. 2 Accelerometer for Option 2
	I	10	2		Axis of 1st rotation
		11	-90		Angle " " "
	I	12	1		Axis " 2nd "
		13	180		Angle " " "
	I	14			Axis " 3rd "
		15			Angle " " "
	I	16			Axis " 4th "
		17			Angle " " "
	I	18			Axis " 5th "
		19			Angle " "
					Specification of Orientation of No. 3 Accelerometer for Option 2
	I	20	3		Axis of 1st rotation
		21	-90		Angle " " "
	I	22			Axis " 2nd "
		23			Angle " " "
	I	24			Axis " 3rd "
		25			Angle " " "
	I	26			Axis " 4th
		27			Angle "
	I	28			Axis 5th "
		29			Angle " "

AEROSPACE FORM 2511 REV 6-62

7/6/64 PG012A-03

B-4. Nonorthogonal Accelerometer Orientation for Test Case 2

X-1 **7090 INPUT DATA**
PG0..A-31 (Rev. 11/5/64)

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ________ KEYPUNCHED ________ VERIFIED ________ DATE ________ PAGE 5 OF 11

H	ERAN - PLATFØRM GYRØ AND ACCELERØMETER
H	ALIGNMENTS (ØRTHØGØNAL ØPTIØN ØNLY).
H	AND CASE CØNTRØL DATA

SYMBOL	T	LOC.	VALUE	EXP.
Ψ_p		PSIP	90	
Φ_p		PHIP		
λ_p		LAMP		
Ψ_i		PSII		
AXIS	I	GYRØ	1	
ANGLE		1	90	
AXIS	I	2		
ANGLE		3		
AXIS	I	4		
ANGLE		5		
AXIS	I	6		
ANGLE		7		
AXIS	I	8		
ANGLE		9		
Ψ_1		PSI1		
Ψ_2		PSI2		
Ψ_3		PSI3		
AXIS	I	ACCEL		
ANGLE		1		
AXIS	I	2		
ANGLE		3		
AXIS	I	4		
ANGLE		5		
AXIS	I	6		
ANGLE		7		
AXIS	I	8		
ANGLE		9		
β_1		BETA1		
β_2		BETA2		
β_3		BETA3		
		ØUT	1	
		PFF	400	
		PFF	2000	

SYMBOL	T	LOC.	VALUE	EXP.
		TSUBO		
		TSUBA	500	
	I	TRAJ		
	I	RK		
	A	ENDC	TIME	
		1	6000	
		MAXT		
		DTMP		
		DTNP		
		BMT		
		BRTAB		
		TGØP		
		1		
		2		
		3		
		4		
		5		
		6		
		7		
		8		
		9		
		10		
		11		
		ØMEGE		
		A		
		GM		
		J		
		H		
		D		
		MU		
	E	ND	0	

AEROSPACE FORM 2423 REV 6-63

B-5. Orientation and Control Data for Test Case 2

GENERAL INERTIAL ERRØR ANALYSIS PRØGRAM

X-6 7090 INPUT DATA

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

Originator ______ CCC ____ JO ____

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE 6 OF 11

H	ERAB-ERRØR SØURCE SCHEDULE

PUNCH ONLY X's IN THIS FIELD

	LOC	
D	EI11	
D	EI12	
D	EI13	
D	EI21	
D	EI22	
D	EI23	
D	EI31	
D	EI32	
D	EI33	
D	EA00	\ (col. 1), \ (col. 3)
D	EA01	
D	EA02	
D	EA03	
D	EA04	
D	EA05	
D	EA06	
D	EA07	
D	EA08	
D	EA09	
D	EA10	
D	EA11	
D	EA12	
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		

PUNCH ONLY X's IN THIS FIELD

	LOC	
D	EG00	X X X (cols. 1–3)
D	EG01	
D	EG02	
D	EG03	
D	EG04	
D	EG05	
D	EG06	X (col. 1)
D	EG07	
D	EG08	
D	EG09	
D	EG10	
D	EG11	
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D		
D	EP01	
D	EP02	
D	EP03	
D		
D		
D		
D		
D		
D	ET11	
D	ET12	
D	ET13	
D		
D	ET21	X (col. 4)
D	ET22	X (col. 4)
D	ET23	X (col. 4)
D		

11/20/64 PG012A-01

B-6. Error Source Schedule for Test Case 3

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE 7 OF 11

H	BØDY TURNING RATES IN PLATFØRM CØØRDINATES
H	ALL RATES IN DEG/SEC

SYMBOL		LOC.	VALUE	EXP.
ORDER	I	RATES	1	
	X	1	3	
	Y	1	1	
	A		ØMEG 1	
	A		ØMEG 2	
	A		ØMEG 3	
	A		TIME	
	I		6	
			0	
			140	
			560	
			1700	
			19700	
			30000	
	Z		12	
			0.85	
			0.15	
			0.05	
			0.02	
			0	
			0	

Order of Lagrangian interpolation

No. of time points N

The data points must be input in the following manner:*

t_1
⋮
t_N
ω_{11}
⋮
ω_{1N}
ω_{21}
⋮
ω_{2N}
ω_{31}
⋮
ω_{3N}

where: ω_{1i} is the body roll rate for t_i

ω_{2i} " " " pitch " " "

ω_{3i} " " " yaw " " "

* If the roll and/or yaw rates are all zero, it is not necessary to enter N zeros for the roll and/or yaw rates. Simply enter a Z in the prefix field and the number "N" in the value field to generate N zeros.

AEROSPACE FORM 281 REV 6-61

B-7. Turning Rates in Platform Coordinates for Test Case 3

X-1 7090 INPUT DATA
F0012A-81 (Rev. 11/5/64)

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ____ KEYPUNCHED ____ VERIFIED ____ DATE ____ PAGE 8 OF 11

H	FRAN - PLATFØRM GYRØ AND ACCELERØMETER
H	ALIGNMENTS (ØRTHØGØNAL ØPTIØN ØNLY),
H	AND CASE CØNTRØL DATA

SYMBOL		LOC.	VALUE	EXP.
ψ_p		PSIP	0	
ϕ_p		PHIP	-0	
λ_p		LAMP		
ψ_1		PSI1	-0	
AXIS	I	GYRØ	0	
ANGLE		1		
AXIS	I	2		
ANGLE		3		
AXIS	I	4		
ANGLE		5		
AXIS	I	6		
ANGLE		7		
AXIS	I	8		
ANGLE		9		
ψ_1		PSI1		
ψ_2		PSI2		
ψ_3		PSI3		
AXIS	I	ACCEL	0	
ANGLE		1		
AXIS	I	2		
ANGLE		3		
AXIS	I	4		
ANGLE		5		
AXIS	I	6		
ANGLE		7		
AXIS	I	8		
ANGLE		9		
	I	10	-0	
β_1		BETA1		
β_2		BETA2		
β_3		BETA3		
		ØUT	0	
		PPF		
		PPF		

SYMBOL		LOC.	VALUE	EXP.
		TSUBO		
		TSUBA	10	4
	I	TRAJ		
	I	RK		
	A	ENDC	0	
		1		
		MAXT		
		DTNP		
		DTNF		
		BMT	1	
		BRTAE	1	
		TGØP	1200	
		1	14400	
		2		
		3		
		4		
		5		
		6		
		7		
		8		
		9		
		10		
		11		
		ØMEGE		
		A		
		GM		
		J		
		H		
		D		
		MU		
	B	ND	0	
	B	NDJØB	0	

B-8. Orientation and Control Data for Test Case 3

X-1 **7090 INPUT DATA**
PG012A-S2 (Rev. 11/18/64)

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ____ KEYPUNCHED ____ VERIFIED ____ DATE ____ PAGE 9 OF 11

HJØB1	JØHN (1) DØE (3) BLDG (1) N (1) C4 (3) RM (1) 2
HJØB2	TEST (1) CASE (1) 1
HCASE1	IIP (1) DISPERSIØNS (1) EVALU (1) 1
HCASE2	TRAJECTØRY (1) TAPE (1) 962

SYMBOL		LOC.	VALUE	EXP.
	I	ØPT	2	
	D	SYSTM	+ ECI	
	I	CASE		
		SØPT	1	
		PHIL		
	I	EVALU	1	
		FØPT		
		SIGMA	0	
		1	500	
		4	30	
		7	1	-4
		13	0.1	

SYMBOL		LOC.	VALUE	EXP.
1,,1	A	RHØ	7,,10	
VALUE			0.5	
	A		8,,11	
			0.5	
	A		9,,12	
			0.5	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
			,,	
	E	ND	0	

AEROSPACE FORM 2423 REV 4-62

B-9. Case Control and Data Input Form for Test Case 1

X-1 **7090 INPUT DATA**
PC012A-62 (Rev. 11/18/64)

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE 10 OF 11

1	7	73
HJOB1		
HJOB2	TEST ① CASE ① 2	
HCASE1	ORBIT ① DISPERSIONS ① EVALU ① 2	
HCASE2		

SYMBOL	M	LOC.	VALUE	EXP.
	I	ØPT	3	
	D	SYSTM	000000	
	I	CASE		
		SØPT	6	
		PHIL		
	I	EVALU	2	
		FØPT	1	
		SIGMA	0	
		16	5	
		18	1	

SYMBOL	M	LOC.	VALUE	EXP.
1...1	A	PHØ	,,	
VALUE				
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	E	ND	0	
	~~E~~	~~NDOB~~	~~0~~	

AEROSPACE FORM 2425 REV 6-62

B-10. Case Control and Data Input Form For Test Case 2

X-1 **7090 INPUT DATA**

FC012A-32 (Rev. 11/18/64)

AEROSPACE CORPORATION
COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE 11 OF 11

1	7	73
HJØB1		
HJØB2	TEST ① CASE ① 3	
HCASE1	PHASE ① LØGIC ① SYN ① EQ ① MISSIØN	
HCASE2		

SYMBOL		LOC.	VALUE	EXP.
	I	ØPT	2	
	D	SYSTM		
	I	CASE		
		SØPT		
		PHIL		
	I	EVALU	0	
		PØPT		
		SIGMA	0	
		1	500	
		4	30	
		7	1	-4
		13	0.1	
		16	0	
		18	1	-4
		19	0.1	
		22	3	-4
		25	0.1	

SYMBOL		LOC.	VALUE	EXP.
1,,1	A	RHØ	7,,10	
VALUE			0.5	
	A		8,,11	
			0.5	
	A		9,,12	
			0.5	
	A		13,,19	
			0.67	
	A		13,,25	
			0.484	
	A		19,,25	
			0.72	
	A		14,,20	
			0.67	
	A		14,,26	
			0.484	
	A		20,,26	
			0.72	
	A		15,,21	
			0.67	
	A		15,,27	
			0.484	
	A		21,,27	
			0.72	
	A		,,	
	A		,,	
	A		,,	
	A		,,	
	E	ND	0	
	F	NDJØB	0	

AEROSPACE FORM 2429 REV 6-63

B-11. Case Control and Data Input Form for Test Case 3

APPENDIX C

OUTPUT LISTINGS FOR SAMPLE CASES

```
CARD NO  1    H      ERAN-ERROR SOURCE SCHEDULE
CARD NO  2    DEI11 X           DEI12 X           DEI13 X
CARD NO  3    DEI31 X           DEI32 X           DEI33 X
CARD NO  4    DEA00 X           DEA01 X
CARD NO  5    DEG00 X
CARD NO  6    H      ERAN - PLATFORM GYRO AND ACCELEROMETER
CARD NO  7    H      ALIGNMENTS (ORTHOGONAL OPTION ONLY),
CARD NO  8    H      AND CASE CONTROL DATA
CARD NO  9     PHIP -80.578331   LAMP 28.555       PSII 90
CARD NO 10     TSUBA400         ITRAJ 1
CARD NO 11    AENDC ALTM          1    0
CARD NO 12    END    0
CARD NO 13    DET21  X          DET22  X          DET23  X
CARD NO 14    IACCEL1             1    90         I10   2           11   -90
CARD NO 15    I12    1            13   180        I20   3           21   -90
CARD NO 16    H      ERAN - PLATFORM GYRO AND ACCELEROMETER
CARD NO 17    H      ALIGNMENTS (ORTHOGONAL OPTION ONLY),
CARD NO 18    H      AND CASE CONTROL DATA
CARD NO 19     PSIP 90
CARD NO 20    IGYRO 1             1    90
CARD NO 21     TSUBA500
CARD NO 22    &ENDC TIME          1    6000        OUT  1
CARD NO 23     PPF  400          PFF  2000
CARD NO 24    END    0
CARD NO 25    DEA00 X X
CARD NO 26    DEG00 XXX         DEG06 X
CARD NO 27    DET21    X        DET22    X        DET23    X
CARD NO 28    H      BODY TURNING RATES IN PLATFORM COORDINATES
CARD NO 29     RATES            I0     1          X1    3           Y1    1
CARD NO 30    A      OMEG1      A      OMEG2      A     OMEG3       A     TIME
CARD NO 31    I      6                 0                140               500
CARD NO 32           1700              19700            30000       Z     12
CARD NO 33           .85               .15              .08               .02
CARD NO 34           0                 0
CARD NO 35    H      ERAN - PLATFORM GYRO AND ACCELEROMETER
CARD NO 36    H      ALIGNMENTS (ORTHOGONAL OPTION ONLY),
CARD NO 37    H      AND CASE CONTROL DATA
CARD NO 38     PSIP 0            PHIP -0           PSII -0
CARD NO 39    IGYRO 0
CARD NO 40    IACCEL 0          I10    0
```

C-1. ERAN Input Data

```
CARD NO  41       TSUBA10         9
CARD NO  42      AENDC 0              OUT  0
CARD NO  43       BMT  1              BRTAB1
CARD NO  44       TGOP 720C           1     14400
CARD NO  45      END    0
CARD NO  46      ENDJOB0
```

C-1. ERAN Input Data (Concluded)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 1

IIP DISPERSIONS EVAL 1. TRAJECTORY TAPE 562

TIME= 0. COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	0.	500.	-0.	0.03	-0.00	-0.	-0.0	0.	-0.0
0112-01	0.	-0.	500.	0.02	-0.00	-0.00	-4.9	-0.0	0.0
0113-01	500.	-0.	-0.	-0.00	-0.03	-0.02	0.0	0.0	4.9
0131-01	0.	0.	-0.	0.	0.	-0.	0.0	30.0	0.0
0132-01	0.	0.	-0.	0.	0.	-0.	30.0	-0.0	-0.0
0133-01	0.	0.	-0.	0.	0.	-0.	-0.0	0.0	-30.0

C-2. Test Case 1

IIP DISPERSIONS EVAL 1.   TRAJECTORY TAPE 562
COORDINATE SYSTEM LH

TIME= 0.

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0.2500E 06	-0.1305E-02	[illegible]E-02	-0.8941E-07	-0.1601E 02	-0.8714E 01	0.2630E-05	-0.1526E-04	0.2467E 04
0.0000	0.2500E 06	-0.9766E-03	0.1601E 02	0.5960E-07	0.4470E-07	0.9367E-06	-0.3422E-05	-0.5521E-05
-0.0000	-0.0000	0.2500E 06	0.8714E 01	0.2980E-07	0.2980E-07	-0.2467E 04	-0.3287E-05	-0.6692E-05
-0.0000	0.8784	0.4780	0.1329E-02	0.	0.9095E-12	-0.8599E-01	-0.3544E-09	-0.4955E-09
-1.0000	0.0000	0.0000	0.	0.1026E-02	0.5581E-03	0.7586E-10	-0.9313E-09	-0.1580E 00
-1.0000	0.0000	0.0000	0.0000	1.0000	0.3037E-03	-0.1693E-10	0.	-0.8599E-01
0.0000	0.0000	-0.1623	-0.0776	0.0000	-0.0000	0.9243E 03	0.2044E-06	-0.3756E-06
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	0.	0.0000	0.9000E 03	-0.1144E-04
0.1623	-0.0000	-0.0000	-0.0000	-0.1623	-0.1623	-0.0000	-0.0000	0.9243E 03

SIGMA

500.0000	500.0000	500.0000	0.0365	0.0320	0.0174	30.4030	30.0000	30.4030

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
0.	3005559.	-18112597.	9991646.	1320.79	219.17	0.
	LAT	LONG	ALT	VEL	FPA	AZ
	28.555	-80.578	-1.10	1338.85	-0.000	90.000

C-2. Test Case 1 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 1

IIP DISPERSIONS EVAL 1. TRAJECTORY TAPE 562

TIME= 0. COORDINATE SYSTEM ECI

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	72.	-433.	239.	0.03	0.01	0.	-0.0	0.0	0.0
0112-01	39.	-236.	-439.	0.02	0.00	0.	-4.9	-0.8	-0.0
0113-01	493.	82.	0.	-0.04	0.04	0.	0.4	-2.3	-4.3
0131-01	0.	0.	0.	0.	0.	0.	4.3	-25.0	14.3
0132-01	0.	0.	0.	0.	0.	0.	29.6	4.9	0.
0133-01	0.	0.	0.	0.	0.	0.	-2.3	14.1	26.4

C-2. Test Case 1 (Continued)

IIP DISPERSIONS EVAL 1. TRAJECTORY TAPE 562

TIME= 0. COORDINATE SYSTEM ECI

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DDX	DDY	DDZ
0.2500E 06	0.2441E-02	0.2685E-03	-0.1490E-06	0.1823E 02	0.	0.7629E-05	-0.1179E 04	-0.2138E 04
0.0000	0.2500E 06	-0.4053E-03	-0.1823E 02	0.1490E-06	0.	0.1179E 04	-0.1335E-04	-0.3547E 03
0.0000	-0.0000	0.2500E 06	0.1795E-07	0.2750E-07	0.	0.2138E 04	0.3547E 03	-0.1257E-05
-0.0000	-1.0000	0.0000	0.1329E-02	-0.1091E-10	0.	-0.8599E-01	0.1048E-08	0.2587E-01
1.0000	0.0000	0.0000	-0.0000	0.1329E-02	0.	0.5821E-09	-0.8599E-01	-0.1559E 00
0.	0.	0.	0.	0.	0.	0.	0.	0.
0.0000	0.0776	0.1407	-0.0776	0.0000	0.	0.9238E 03	0.3033E 01	-0.1673E 01
-0.0783	-0.0000	0.0236	0.0000	-0.0783	0.	0.0033	0.9061E 03	0.1008E 02
-0.1410	-0.0234	-0.0000	0.0234	-0.1410	0.	-0.0018	0.0111	0.9188E 03

SIGMA

500.0000	500.0000	500.0000	0.0365	0.0365	0.	30.3947	30.1009	30.3114

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
0.	3005559.	-18112597.	9991646.	1320.79	219.17	0.
	LAT	LONG	ALT	VEL	FPA	AZ
	28.555	-80.578	-1.10	1338.85	-0.000	90.000

C-2. Test Case 1 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 1

IIP DISPERSIONS EVAL 1. TRAJECTORY TAPE 562

TIME= 400.000 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-101.	617.	-0.	-0.03	0.64	-0.00	-0.0	-0.0	-0.0
0112-01	18.	1.	505.	0.02	-0.01	-0.08	-4.8	-0.9	0.1
0113-01	514.	0.	-17.	0.01	-0.49	-0.01	0.1	0.0	4.9
0131-01	-2.	-0.	519.	-0.00	-0.00	2.78	-5.7	29.5	-0.0
0132-01	-11.	12.	-387.	-0.04	0.06	-1.22	29.4	5.7	-0.6
0133-01	-481.	458.	8.	-1.75	2.74	0.03	-0.6	-0.1	-30.0
A001-01	-50.	263.	-0.	-0.25	1.36	-0.00	-0.	0.	-0.
A002-01	248.	49.	-5.	1.21	0.25	-0.03	-0.	0.	-0.
A003-01	-5.	-1.	-252.	-0.03	-0.01	-1.24	-0.	0.	-0.
A011-01	-53.	282.	-0.	-0.18	1.01	-0.00	-0.	0.	-0.
A012-01	350.	69.	-8.	1.88	0.39	-0.04	-0.	0.	-0.
A013-01	0.	0.	9.	0.00	0.00	0.04	-0.	0.	-0.
G001-01	-1.	-0.	283.	0.00	0.00	2.03	-7.6	39.3	-0.0
G002-01	-4.	6.	-110.	-0.02	0.04	-0.40	39.3	7.6	-0.9
G003-01	-162.	266.	2.	-0.79	2.03	0.01	-0.8	-0.2	-40.0

C-2. Test Case 1 (Continued)

IIP DISPERSIONS EVAL 1.   TRAJECTORY TAPE 562

TIME=   400.000   COORDINATE SYSTEM   LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0.8107E 06	-0.3140E 06	-0.5684E 04	0.2411E 04	-0.1858E 04	-0.3865E 02	-0.3253E 02	-0.9187E 02	0.2346E 05
-0.3689	0.8936E 06	-0.1591E 04	-0.9245E 03	0.3212E 04	-0.9533E 01	0.8524E 02	-0.7200E 00	-0.2437E 05
-0.0063	-0.0018	0.8277E 06	-0.1763E 02	-0.2355E 01	0.2796E 04	-0.2326E 05	0.2289E 05	-0.3288E 02
0.8046	-0.2939	-0.0058	0.1108E 02	-0.5670E 01	-0.1340E 00	-0.7776E-01	-0.6147E-01	0.8404E 02
-0.5028	0.8281	-0.0006	-0.4152	0.1683E 02	-0.2700E-01	0.8864E-01	0.2540E-03	-0.1657E 03
-0.0111	-0.0026	0.7934	-0.0104	-0.0017	0.1500E 02	-0.8278E 02	0.1517E 03	-0.7511E-01
-0.0007	0.0018	-0.5089	-0.0005	0.0004	-0.4254	0.2523E 04	0.4543E 01	-0.5433E-03
-0.0020	-0.0000	0.5031	-0.0004	0.0000	0.7830	0.0018	0.2501E 04	0.2831E-02
0.5185	-0.5131	-0.0007	0.5026	-0.8039	-0.0004	-0.0000	0.0000	0.2524E 04

SIGMA

900.3628	945.3012	909.7613	3.3283	4.1029	3.8735	50.2341	50.0088	50.2428

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
400.000	7071660.	-17659617.	10008745.	19573.37	6120.72	-2034.31
	LAT	LONG	ALT	VEL	FPA	AZ
	27.751	-69.848	96.25	20608.70	1.289	97.089

C-2.   Test Case 1 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 1

IIP DISPERSIONS EVAL 1. TRAJECTORY TAPE 562

TIME= 400.000 COORDINATE SYSTEM ECI

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	108.	-538.	298.	0.18	-0.53	0.30	-0.0	0.0	0.0
0112-01	46.	-235.	-444.	0.01	0.05	0.07	-4.9	-0.8	-0.0
0113-01	483.	170.	-41.	-0.15	0.41	-0.22	0.4	-2.3	-4.3
0131-01	28.	-247.	-456.	0.16	-1.32	-2.44	4.3	-26.0	14.3
0132-01	-28.	171.	346.	-0.08	0.52	1.11	29.6	4.9	0.
0133-01	-302.	-532.	258.	-0.74	-2.81	1.44	-2.3	14.1	26.4
A001-01	40.	-232.	128.	0.22	-1.20	0.66	0.	0.	0.
A002-01	249.	40.	1.	1.23	0.19	0.01	0.	0.	0.
A003-01	-20.	119.	222.	-0.10	0.58	1.09	0.	0.	0.
A011-01	43.	-248.	137.	0.17	-0.89	0.49	0.	0.	0.
A012-01	352.	57.	1.	1.89	0.29	0.01	0.	0.	0.
A013-01	1.	-4.	-8.	0.00	-0.02	-0.04	0.	0.	0.
G001-01	16.	-135.	-249.	0.42	-0.96	-1.78	5.8	-34.7	19.1
G002-01	-8.	46.	100.	-0.02	0.15	0.38	39.5	6.5	0.
G003-01	-65.	-271.	139.	-0.08	-1.92	1.02	-3.1	16.9	35.1

C-2. Test Case 1 (Continued)

TIME= 400.000 IIP DISPERSIONS EVAL 1. COORDINATE SYSTEM LCI TRAJECTORY TAPE 562

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DDX	DDY	DDZ
0.6243E 06	0.1905E 06	-0.1095E 06	0.1633E 04	0.1134E 04	-0.6439E 03	-0.4827E 02	-0.8116E 04	-0.1166E 05
0.2371	0.1034E 07	-0.9846E 05	0.3043E 03	0.3763E 04	-0.4983E 03	0.8318E 04	-0.6158E 03	-0.3037E 05
-0.1483	-0.1036	0.8739E 06	-0.2205E 03	-0.4451E 03	0.3022E 04	0.1189E 05	0.2956E 05	0.5979E 03
0.7227	0.1046	-0.0824	0.8181E 01	0.2203E 01	-0.1481E 01	-0.2574E 00	-0.2032E 02	-0.1715E 02
0.3310	0.8535	-0.1098	0.1776	0.1881E 02	-0.1890E 01	0.2257E 02	-0.5700E 01	-0.1607E 03
-0.2042	-0.1228	0.8101	-0.1298	-0.1092	0.1593E 02	0.1982E 02	0.1733E 03	0.5805E 01
-0.0012	0.1629	0.2532	-0.0010	0.1036	0.0989	0.2524E 04	0.3033E 01	-0.1673E 01
-0.2052	-0.0121	0.6316	-0.1419	-0.0263	0.8677	0.0012	0.2506E 04	0.1008E 02
-0.2940	-0.5951	0.0127	-0.1195	-0.3303	0.0290	-0.0007	0.0040	0.2519E 04

SIGMA

790.0997	1016.7356	934.8287	2.8603	4.3367	3.9908	50.2378	50.0606	50.1875

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
400.000	7071660.	-17659617.	10008745.	19573.37	6120.72	-2034.31
	LAT	LONG	ALT	VEL	FPA	AZ
	27.751	-69.848	96.25	20608.70	1.289	97.089

C-2. Test Case 1 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 1

IIP DISPERSIONS EVAL 1. TRAJECTORY TAPE 562

TIME= 780.710 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-432.	905.	-0.	-0.27	1.41	-0.00	-0.0	-0.0	-0.0
0112-01	21.	7.	423.	0.01	0.00	-0.34	-4.2	-2.6	0.1
0113-01	499.	0.	-21.	-0.09	-0.51	-0.00	0.1	0.1	4.9
0131-01	-2.	-2.	1488.	-0.00	-0.00	2.21	-16.0	25.4	0.0
0132-01	-36.	28.	-797.	-0.05	0.07	-0.89	25.4	16.0	-0.7
0133-01	-1563.	1146.	18.	-2.23	2.91	0.02	-0.6	-0.3	-30.0
A001-01	-423.	767.	0.	-0.70	1.76	0.00	-0.	0.	-0.
A002-01	565.	412.	-15.	0.78	0.86	-0.02	-0.	0.	-0.
A003-01	-12.	-9.	-682.	-0.02	-0.02	-0.97	-0.	0.	-0.
A011-01	-359.	663.	0.	-0.51	1.41	0.00	-0.	0.	-0.
A012-01	849.	616.	-22.	1.22	1.31	-0.03	-0.	0.	-0.
A013-01	0.	0.	24.	0.00	0.00	0.03	-0.	0.	-0.
G001-01	1.	0.	1001.	0.00	0.00	1.67	-21.3	33.8	0.0
G002-01	-17.	20.	-247.	-0.03	0.05	-0.30	33.8	21.3	-0.9
G003-01	-796.	907.	6.	-1.33	2.29	0.01	-0.7	-0.5	-40.0

C-2. Test Case 1 (Continued)

IIP DISPERSIONS EVAL 1. TRAJECTORY TAPE 562

TIME= 780.710 COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DO!
0.5494E 07	-0.2639E 07	-0.3490E 05	0.7468E 04	-0.6768E 04	-0.5777E 02	-0.5436E 02	-0.8157E 02	0.8122E 05
-0.4894	0.5293E 07	-0.2464E 05	-0.3740E 04	0.1179E 05	-0.4233E 02	0.5945E 02	-0.2585E 01	-0.7065E 05
-0.0070	-0.0050	0.4540E 07	-0.3905E 02	-0.4349E 02	0.6239E 04	-0.7548E 05	0.5251E 05	-0.5899E 02
0.9635	-0.4916	-0.0055	0.1094E 02	-0.9454E 01	-0.6547E-01	-0.6920E-01	0.4304E-01	0.1195E 03
-0.5545	0.9845	-0.0039	-0.5490	0.2711E 02	-0.7763E-01	0.2777E-01	0.1912E-01	-0.1816E 03
-0.0080	-0.0059	0.9457	-0.0064	-0.0048	0.9586E 01	-0.1023E 03	0.9278E 02	-0.5414E-01
-0.0005	0.0005	-0.7060	-0.0004	0.0001	-0.6584	0.2517E 04	0.1098E 02	0.1656E-02
-0.0007	-0.0000	0.4922	0.0003	0.0001	0.5985	0.0044	0.2507E 04	-0.2632E-02
0.6897	-0.6112	-0.0006	0.7193	-0.6940	-0.0003	0.0000	-0.0000	0.2524E 04

SIGMA

2343.9505	2300.6896	2130.7428	3.3070	5.2069	3.0961	50.1740	50.0691	50.2428

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
780.710	13544742.	-13609586.	8237621.	13745.86	14903.67	-7178.12
	LAT	LONG	ALT	VEL	FPA	AZ
	23.220	-48.399	-3.49	21507.98	-9.708	107.160

C-2. Test Case 1 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 1

IIP DISPERSIONS EVAL 1. TRAJECTORY TAPE 562

TIME= 780.710 COORDINATE SYSTEM ECI

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	259.	-844.	474.	0.71	-1.08	0.63	-0.0	0.0	0.0
0112-01	44.	-193.	-374.	-0.01	0.17	0.30	-4.9	-0.8	-0.0
0113-01	378.	305.	-117.	-0.39	0.28	-0.17	0.4	-2.3	-4.3
0131-01	82.	-707.	-1307.	0.12	-1.05	-1.94	4.3	-26.0	14.3
0132-01	-54.	339.	720.	-0.04	0.35	0.82	29.6	4.9	0.
0133-01	-443.	-1679.	860.	0.20	-3.22	1.73	-2.3	14.1	26.4
A001-01	176.	-749.	417.	0.61	-1.56	0.88	0.	0.	0.
A002-01	696.	73.	22.	1.14	-0.09	0.15	0.	0.	0.
A003-01	-54.	322.	598.	-0.08	0.46	0.85	0.	0.	0.
A011-01	157.	-645.	359.	0.52	-1.22	0.69	0.	0.	0.
A012-01	1042.	112.	32.	1.77	-0.12	0.22	0.	0.	0.
A013-01	2.	-11.	-21.	0.00	-0.02	-0.03	0.	0.	0.
G001-01	57.	-475.	-879.	0.10	-0.79	-1.46	5.8	-34.7	19.1
G002-01	-14.	94.	229.	-0.01	0.10	0.29	39.5	6.5	0.
G003-01	-16.	-1064.	568.	0.48	-2.28	1.26	-3.1	18.9	35.1

C-2. Test Case 1 (Continued)

TIME= 780.710 IIP DISPERSIONS EVAL 1. COORDINATE SYSTEM ECI TRAJECTORY TAPE 562

COVARIANCE-MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0.2803E 07	0.4236E 06	-0.2928E 06	0.4102E 04	0.1539E 03	-0.1398E 03	-0.4624E 03	-0.1196E 05	-0.1163E 05
0.0941	0.7232E 07	-0.1424E 07	-0.2512E 04	0.1308E 05	-0.3854E 04	0.1630E 05	-0.7251E 04	-0.1022E 06
-0.0760	-0.2302	0.5292E 07	0.1287E 04	-0.3695E 04	0.8322E 04	0.1765E 05	0.9293E 05	0.7597E 04
0.8448	-0.3220	0.1929	0.8411E 01	-0.5696E 01	0.3120E 01	-0.2515E 01	0.5912E 01	0.2734[illegible] 02
0.0185	0.9769	-0.3228	-0.3946	0.2477E 02	-0.8596E 01	0.1898E 02	-0.3238E 02	-0.196[illegible] 03
-0.0220	-0.3769	0.9516	0.2830	-0.4543	0.1445E 02	0.9507E 01	0.1556E 03	0.347[illegible] 02
-0.0055	0.1206	0.1527	-0.0173	0.0759	0.0498	0.2524E 04	0.3033E 01	-0.1673E 01
-0.1427	-0.0539	0.8069	0.0407	-0.1300	0.8176	0.0012	0.2506E 04	0.1008E 02
-0.1383	-0.7571	0.0658	0.1878	-0.7866	0.1823	-0.0007	0.0040	0.2519E 04

SIGMA

1674.3517	2689.2147	2300.4373	2.9002	4.9768	3.8018	50.2378	50.0606	50.1875

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
780.710	13544742.	-13609586.	8237621.	13745.86	14903.67	-7178.12
	LAT	LONG	ALT	VEL	FPA	AZ
	23.220	-48.399	-3.49	21507.98	-[illegible].708	107.160

C-2. Test Case 1 (Continued)

IIP DISPERSIONS EVAL 1.   TRAJECTORY TAPE 562

TIME=   780.710   CRITERION   ALT

	MT	MC	MD
0I11-01	0.249	9.	4523.
0I12-01	0.002	422.	70.
0I13-01	0.000	-31.	499.
0I31-01	-0.000	1488.	20.
0I32-01	0.008	-796.	102.
0I33-01	0.316	50.	47[illegible]2.
A001-01	0.211	9.	3777.
A002-01	0.114	-27.	2823.
A003-01	-0.002	-681.	-76.
A011-01	0.183	8.	3274.
A012-01	0.170	-40.	4221.
A013-01	0.000	24.	3.
G001-01	0.000	1000.	22.
G002-01	0.005	-247.	85.
G003-01	0.250	22.	4170.

C-2. Test Case 1 (Continued)

COVARIANCE MATRIX

	MT	MC	MD
MT	0.402382E 00	0.833867E 01	0.726621E 04
MC	0.00617	0.454193E 07	0.110958E 06
MD	0.98446	0.00447	0.135389E 09

SIGMAS

0.634	2131.	11636.

NOMINAL TERMINAL CONDITIONS

LAT	LONG	ALT	VEL/R	FPA/R	AZ/R
23.220	-48.399	-3.49	20194.793	-10.346	108.351

C-2. Test Case 1 (Concluded)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 2

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 0. COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	0.	500.	-0.	0.03	-0.00	-0.	-0.0	0.	-0.0
0112-01	0.	-0.	500.	0.02	-0.00	-0.00	-4.9	-0.0	0.0
0113-01	500.	-0.	-0.	-0.00	-0.03	-0.02	0.0	0.0	4.9
0131-01	0.	0.	-0.	0.	0.	-0.	0.0	30.0	0.0
0132-01	0.	0.	-0.	0.	0.	-0.	0.0	-0.0	30.0
0133-01	0.	0.	-0.	0.	0.	-0.	30.0	-0.0	-0.0

C-3. Test Case 2

ORBIT DISPERSIONS EVALU 2   TRAJECTORY TAPE 562

TIME=   0.   COORDINATE SYSTEM   LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
.2500E 06	-0.1305E-02	-0.1027E-02	-0.8941E-07	-0.1601E 02	-0.8714E 01	0.2630E-05	-0.1526E-04	0.2467E 04
-0.0000	0.2500E 06	-0.9766E-03	0.1601E 02	0.5960E-07	0.4470E-07	0.9367E-06	-0.3422E-05	-0.5521E-05
-0.0000	-0.0000	0.2500E 06	0.8714E 01	0.2980E-07	0.2980E-07	-0.2467E 04	-0.3287E-05	-0.6692E-05
-0.0000	0.8784	0.4780	0.1329E-02	0.	0.9095E-12	-0.8599E-01	-0.3544E-09	-0.4935E-09
-1.0000	0.0000	0.0000	0.	0.1026E-02	0.5581E-03	0.7586E-10	-0.9313E-09	-0.1580E 00
-1.0000	0.0000	0.0000	0.0000	1.0000	0.3037E-03	-0.1693E-10	0.	-0.8599E-01
0.0000	0.0000	-0.1623	-0.0776	0.0000	-0.0000	0.9243E 03	0.5962E-06	-0.1096E-05
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	0.	0.0000	0.9000E 03	-0.1144E-04
0.1623	-0.0000	-0.0000	-0.0000	-0.1623	-0.1623	-0.0000	-0.0000	0.9243E 03

SIGMA

500.0000	500.0000	500.0000	0.0365	0.0320	0.0174	30.4030	30.0000	30.4030

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
0.	3005559.	-18112597.	9991646.	1320.79	219.17	0.
	LAT	LONG	ALT	VEL	FPA	AZ
	28.555	-80.578	-1.10	1338.85	-0.000	90.000

C-3. Test Case 2 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 2

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 400.000 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-101.	617.	-0.	-0.03	0.64	-0.00	-0.0	-0.0	-0.0
0112-01	18.	1.	505.	0.02	-0.01	-0.08	-4.8	-0.9	0.1
0113-01	514.	0.	-17.	0.01	-0.49	-0.01	0.1	0.0	4.9
0131-01	-2.	-0.	519.	-0.00	-0.00	2.78	-5.7	29.5	-0.0
0132-01	481.	-458.	-8.	1.75	-2.74	-0.03	0.6	0.1	30.0
0133-01	-11.	12.	-387.	-0.04	0.06	-1.22	29.4	5.7	-0.6
A001-01	-30.	263.	-0.	-0.25	1.36	-0.00	-0.	0.	-0.
A002-01	248.	49.	-5.	1.21	0.25	-0.03	-0.	0.	-0.
A003-01	-5.	-1.	-252.	-0.03	-0.01	-1.24	-0.	0.	-0.
A011-01	-53.	282.	-0.	-0.18	1.01	-0.00	-0.	0.	-0.
A012-01	350.	69.	-8.	1.88	0.39	-0.04	-0.	0.	-0.
A013-01	0.	0.	9.	0.00	0.00	0.04	-0.	0.	-0.
G001-01	-1.	-0.	283.	0.00	0.00	2.03	-7.6	39.3	-0.0
G002-01	-4.	6.	-110.	-0.02	0.04	-0.40	39.3	7.6	-0.9
G003-01	-162.	266.	2.	-0.79	2.03	0.01	-0.8	-0.2	-40.0

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 400.000 COORDINATE SYSYEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DDX	DDY	DDZ
0.8[illegible]07E 06	-0.3140E 06	-0.5684E 04	0.2411E 04	-0.1858E 04	-0.3865E 02	-0.3253E 02	-0.9187E 02	0.2340E 05
-0.3689	0.8936E 06	-0.1591E 04	-0.9245E 03	0.3212E 04	-0.9533E 01	0.8924E 02	-0.7201E 00	-0.2437E 05
-0.0069	-0.0018	0.827[illegible]E 06	-0.1763E 02	-0.2355E 01	0.2796E 04	-0.2326E 05	0.2289E 05	-0.3208E 02
0.8046	-0.2939	-0.0058	0.1108E 02	-0.5670E 01	-0.1340E 00	-0.7776E-01	-0.6147E-01	0.84[illegible]E 02
-0.5028	0.8281	-0.0006	-0.4152	0.1683E 02	-0.2700E-01	0.8864E-01	0.2556E-03	-0.1657E 03
-0.0111	-0.0026	0.7934	-0.0104	-0.0017	0.1500E 02	-0.8278E 02	0.1517E 03	-0.7511E-01
-0.0007	0.0018	-0.5089	-0.0005	0.0004	-0.4254	0.2523E 04	0.4543E 01	-0.5569E-03
-0.0020	-0.0000	0.5031	-0.0004	0.0000	0.7830	0.0018	0.2501E 04	0.3823E-02
0.5185	-0.5131	-0.0007	0.5026	-0.8039	-0.0004	-0.0000	0.0000	0.2524E 04

SIGMA

900.3628	945.3012	909.7613	3.3283	4.1029	3.8735	50.2341	50.0088	50.2428

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
400.000	7071660.	-17659617.	10008745.	19573.37	6120.72	-2034.31
	LAT	LONG	ALT	VEL	FPA	AZ
	27.751	-69.348	96.25	20608.70	1.289	97.089

C-3. Test Case 2 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 2

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 464.664 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-148.	653.	-1.	-0.07	0.75	-0.00	-0.0	-0.0	-0.0
0112-01	17.	2.	497.	0.01	-0.01	-0.15	-4.8	-1.3	0.1
0113-01	514.	0.	-17.	0.00	-0.60	-0.01	0.1	0.0	4.9
0131-01	-3.	-1.	717.	-0.00	0.00	3.38	-7.7	29.0	-0.0
0132-01	635.	-616.	-9.	1.90	-3.38	-0.02	0.6	0.2	30.0
0133-01	-13.	15.	-461.	-0.04	0.07	-1.03	29.0	7.7	-0.6
A001-01	-91.	354.	-0.	-0.39	1.60	-0.00	-0.	0.	-0.
A002-01	327.	90.	-7.	1.37	0.40	-0.03	-0.	0.	-0.
A003-01	-6.	-2.	-338.	-0.03	-0.01	-1.42	-0.	0.	-0.
A011-01	-87.	341.	-0.	-0.21	0.95	-0.00	-0.	0.	-0.
A012-01	476.	130.	-10.	2.24	0.64	-0.05	-0.	0.	-0.
A013-01	0.	0.	12.	0.00	0.00	0.05	-0.	0.	-0.
G001-01	-1.	-0.	441.	0.01	0.00	2.95	-11.9	44.9	-0.0
G002-01	-5.	9.	-130.	-0.02	0.05	-0.17	44.9	11.9	-0.9
G003-01	-241.	412.	2.	-0.94	3.01	0.00	-0.9	-0.3	-46.5

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 464.664 COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DDX	DDY	DDZ
0.1261E 07	-0.5463E 06	-0.1003E 05	0.3729E 04	-0.3001E 04	-0.5629E 02	-0.3793E 02	-0.1510E 03	0.3280E 05
-0.4151	0.1374E 07	-0.4066E 04	-0.1312E 04	0.5319E 04	-0.1835E 02	0.1475E 03	-0.9755E 00	-0.3761E 05
-0.0078	-0.0030	0.1295E 07	-0.2711E 02	-0.5953E 01	0.4605E 04	-0.3233E 05	0.3487E 05	-0.3690E 02
0.8652	-0.2916	-0.0052	0.1474E 02	-0.7598E 01	-0.1547E 00	-0.8109E-01	0.2053E 00	0.1005E 03
-0.5121	0.8694	-0.0010	-0.3793	0.2724E 02	-0.4491E-01	-0.1219E 00	0.2707E-02	-0.2441E 03
-0.0104	-0.0033	0.8409	-0.0084	-0.0018	0.2316E 02	-0.9783E 02	0.2205E 03	-0.6797E-01
-0.0006	0.0023	-0.5117	-0.0004	-0.0004	-0.3662	0.3082E 04	0.6040E 01	-0.3426E-02
-0.0024	-0.0000	0.5538	0.0010	0.0000	0.8283	0.0020	0.3061E 04	0.1276E-01
0.5262	-0.5777	-0.0006	0.4716	-0.8424	-0.0003	-0.0000	0.0000	0.3083E 04

SIGMA

1122.7529	1172.2344	1138.1323	3.8387	5.2189	4.8121	55.5146	55.3239	55.5290

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
464.664	8444418.	-17166351.	9832988.	23153.78	9347.12	-3491.33
	LAT	LONG	ALT	VEL	FPA	AZ
	27.202	-65.748	98.58	25212.21	0.077	98.998

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2   TRAJECTORY TAPE [illegible]

TRANSITION MATRIX.   PARTIALS AT T=   477.664 WITH RESPECT TO T=   464.664

COORDINATE SYSTEM   LH

PARTIALS OF X,Y,Z,XDOT,YDOT,ZDOT,PHIX,PHIY,PHIZ WITH RESPECT TO

X	Y	Z	XDOT	YDOT	ZDOT	PHIX	PHIY	PHIZ
0.9998E 00	-0.1524E-01	-0.2049E-07	0.1300E 02	-0.1981E 00	-0.2682E-06	-0.	0.	-0.
0.1524E-01	0.1000E 01	0.	0.1981E 00	0.1300E 02	-0.1192E-06	-0.	0.	-0.
0.8382E-08	-0.1490E-07	0.9999E 00	0.1192E-06	-0.2384E-06	0.1300E 02	-0.	0.	-0.
-0.1839E-04	-0.1401E-06	0.5400E-12	0.9998E 00	-0.1524E-01	-0.2421E-07	-0.	0.	-0.
0.1401E-06	0.3677E-04	0.6821E-12	0.1524E-01	0.1000E 01	0.3725E-08	-0.	0.	-0.
-0.3979E-12	0.3979E-12	-0.1839E-04	0.6519E-08	-0.1118E-07	0.9999E 00	-0.	0.	-0.
-0.	-0.	-0.	-0.	-0.	-0.	0.9999E 00	-0.1524E-01	-0.2235E-07
0.	0.	0.	0.	0.	0.	0.1524E-01	0.9999E 00	0.3725E-08
-0.	-0.	-0.	-0.	-0.	-0.	0.6519E-08	-0.1118E-07	0.1000E 01

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
464.664	8444418.	-17166351.	9832988.	23153.78	9347.12	-3491.38
477.664	8744396.	-17042791.	9786427.	22995.75	9661.63	-3671.75
	LAT	LONG	ALT	VEL	FPA	AZ
464.664	27.202	-65.748	98.58	25212.21	0.077	98.996
477.664	27.063	-64.834	98.62	25211.77	0.052	99.479

C-3.   **Test Case 2 (Continued)**

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 2

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 477.664 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-159.	661.	-1.	-0.08	0.77	-0.00	-0.0	-0.0	-0.0
0112-01	18.	2.	495.	0.01	-0.01	-0.16	-4.7	-1.3	0.1
0113-01	514.	0.	-17.	0.00	-0.60	-0.01	0.1	0.0	4.9
0131-01	-3.	-1.	760.	-0.00	0.00	3.37	-8.1	28.9	-0.0
0132-01	670.	-650.	-9.	1.94	-3.36	-0.02	0.6	0.2	30.0
0133-01	-14.	16.	-474.	-0.04	0.07	-1.02	28.9	8.1	-0.6
A001-01	-102.	373.	-0.	-0.41	1.61	-0.00	-0.	0.	-0.
A002-01	342.	100.	-7.	1.36	0.42	-0.03	-0.	0.	-0.
A003-01	-7.	-2.	-357.	-0.03	-0.01	-1.41	-0.	0.	-0.
A011-01	-95.	352.	-0.	-0.23	0.96	-0.00	-0.	0.	-0.
A012-01	502.	146.	-10.	2.22	0.68	-0.05	-0.	0.	-0.
A013-01	0.	0.	13.	0.00	0.00	0.05	-0.	0.	-0.
G001-01	-1.	-0.	479.	0.01	0.00	2.94	-13.0	46.0	-0.0
G002-01	-5.	10.	-132.	-0.02	0.05	-0.17	46.0	13.0	-0.9
G003-01	-260.	447.	2.	-0.98	3.01	0.00	-0.9	-0.3	-47.8

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 477.664 COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DQY	DOZ
0.1378E 07	-0.6059E 06	-0.1107E 05	0.3965E 04	-0.3147E 04	-0.5780E 02	-0.3896E 02	-0.1532E 03	0.3506E 05
-0.4215	0.1499E 07	-0.4559E 04	-0.1426E 04	0.5659E 04	-0.1974E 02	0.1468E 03	-0.9672E 00	-0.4085E 05
-0.0079	-0.0031	0.1419E 07	-0.2881E 02	-0.7141E 01	0.4880E 04	-0.3450E 05	0.3777E 05	-0.3777E 02
0.8770	-0.3025	-0.0063	0.1483E 02	-0.7783E 01	-0.1524E 00	-0.8186E-01	0.2150E 00	0.1049E 03
-0.5121	0.8828	-0.0011	-0.3861	0.2741E 02	-0.4783E-01	-0.1246E 00	0.4110E-02	-0.2479E 03
-0.0103	-0.0034	0.8547	-0.0083	-0.0019	0.2298E 02	-0.1018E 03	0.2219E 03	-0.6725E-01
-0.0006	0.0021	-0.5118	-0.0004	-0.0004	-0.3752	0.3204E 04	0.6359E 01	-0.3613E-02
-0.0023	-0.0000	0.5620	0.0010	0.0000	0.8205	0.0020	0.3183E 04	0.1268E-01
0.5275	-0.5893	-0.0006	0.4811	-0.8364	-0.0002	-0.0000	0.0000	0.3206E 04

SIGMA

1173.9390	1224.4565	1191.0800	3.8508	5.2351	4.7939	56.6055	56.4218	56.6213

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
477.664	8744396.	-17042791.	9786427.	22995.75	9661.63	-3671.75
	LAT	LONG	ALT	VEL	FPA	AZ
	27.063	-64.834	98.62	25211.77	0.052	99.439

C-3. Test Case 2 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 2

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 498.375 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-177.	673.	-1.	-0.09	0.81	-0.00	-0.0	-0.0	-0.0
0112-01	18.	3.	491.	0.01	-0.01	-0.18	-4.7	-1.5	0.1
0113-01	514.	0.	-18.	-0.00	-0.61	-0.31	0.1	0.0	4.9
0131-01	-4.	-1.	830.	-0.00	0.00	3.39	-8.8	28.7	-0.0
0132-01	727.	-703.	-9.	2.00	-3.42	-0.02	0.5	0.2	30.0
0133-01	-15.	17.	-495.	-0.04	0.07	-0.99	28.7	8.8	-0.6
A001-01	-120.	405.	-0.	-0.47	1.68	-0.00	-0.	0.	-0.
A002-01	369.	118.	-7.	1.40	0.48	-0.03	-0.	0.	-0.
A003-01	-7.	-3.	-387.	-0.03	-0.01	-1.47	-0.	0.	-0.
A011-01	-109.	370.	-0.	-0.24	0.97	-0.00	-0.	0.	-0.
A012-01	545.	174.	-11.	2.22	0.76	-0.05	-0.	0.	-0.
A013-01	0.	0.	13.	0.00	0.00	0.05	-0.	0.	-0.
G001-01	-1.	-0.	541.	0.01	0.00	3.01	-14.7	47.6	-0.0
G002-01	-6.	11.	-135.	-0.02	0.05	-0.13	47.6	14.7	-1.0
G003-01	-292.	503.	2.	-1.04	3.10	0.00	-0.9	-0.3	-49.8

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2   TRAJECTORY TAPE 562

TIME=   498.375   COORDINATE SYSTEM   LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0.1584E 07	-0.7086E 06	-0.1281E 05	0.4416E 04	-0.3451E 04	-0.6133E 02	-0.4075E 02	-0.1544E 03	0.3894E 05
-0.4299	0.1715E 07	-0.5557E 04	-0.1603E 04	0.6331E 04	-0.2258E 02	0.1519E 03	-0.8923E 00	-0.4618E 05
-0.0080	-0.0033	0.1632E 07	-0.3163E 02	-0.9228E 01	0.5413E 04	-0.3822E 05	0.4249E 05	-0.3903E 02
0.8916	-0.3110	-0.0063	0.1549E 02	-0.8176E 01	-0.1520E 00	-0.8377E-01	0.2580E 00	0.1119E 03
-0.5100	0.8992	-0.0013	-0.3863	0.2891E 02	-0.5384E-01	-0.1525E 00	0.6534E-02	-0.2601E 03
-0.0100	-0.0035	0.8710	-0.0079	-0.0021	0.2367E 02	-0.1081E 03	0.2299E 03	-0.6551E-01
-0.0006	0.0020	-0.5127	-0.0004	-0.0005	-0.3807	0.3406E 04	0.6858E 01	-0.4227E-02
-0.0021	-0.0000	0.5717	0.0011	0.0000	0.8122	0.0020	0.3386E 04	0.1360E-01
0.5301	-0.6040	-0.0005	0.4870	-0.8285	-0.0002	-0.0000	0.0000	0.3408E 04

SIGMA

1258.4632	1309.7279	1277.4120	3.9358	5.3768	4.8647	58.3610	58.1884	58.3791

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
498.375	9221363.	-16835937.	9706828.	23061.90	10315.87	-4016.04
	LAT	LONG	ALT	VEL	FPA	AZ
	26.824	-63.372	98.62	25581.19	0.000	100.132

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TRANSITION MATRIX. PARTIALS AT T= 500.000 WITH RESPECT TO T= 498.375

COORDINATE SYSTEM LH

PARTIALS OF X,Y,Z,XDOT,YDOT,ZDOT,PHIX,PHIY,PHIZ WITH RESPECT TO

X	Y	Z	XDOT	YDOT	ZDOT	PHIX	PHIY	PHIZ
0.1000E 01	-0.1932E-02	-0.4008E-05	0.1625E 01	-0.3139E-02	-0.6514E-05	-0.	0.	-0.
0.1932E-02	0.1000E 01	-0.7451E-08	0.3139E-02	0.1625E 01	-0.2235E-07	-0.	0.	-0.
0.	-0.1118E-07	0.1000E 01	-0.1863E-08	-0.1490E-07	0.1625E 01	-0.	0.	-0.
-0.2298E-05	-0.2220E-08	0.9184E-11	0.1000E 01	-0.1932E-02	-0.4008E-05	-0.	0.	-0.
0.2220E-08	0.4596E-05	-0.4263E-13	0.1932E-02	0.1000E 01	-0.7451E-08	-0.	0.	-0.
-0.3553E-14	-0.1421E-13	-0.2298E-05	0.	-0.1118E-07	0.1000E 01	-0.	0.	-0.
-0.	-0.	-0.	-0.	-0.	-0.	0.1000E 01	-0.1932E-02	-0.4010E-05
0.	0.	0.	0.	0.	0.	0.1932E-02	0.1000E 01	-0.7451E-08
-0.	-0.	-0.	-0.	-0.	-0.	-0.1863E-08	-0.3725E-08	0.1000E 01

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
498.375	9221363.	-16835937.	9706828.	23061.90	10315.87	-401[illegible].04
500.000	9258709.	-16818952.	9700173.	23040.41	10354.42	-4038.29
	LAT	LONG	ALT	VEL	FPA	AZ
498.375	26.824	-63.372	98.62	25581.19	0.000	100.132
500.000	26.805	-63.257	98.62	25580.89	0.000	100.187

C-3. Test Case 2 (Continued)

11/11/65 JOHN DOE BLDG N 6, RM 2 TEST CASE 2

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 500.000 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-178.	674.	-1.	-0.09	0.81	-0.00	-0.0	-0.0	-0.0
0112-01	18.	3.	491.	0.01	-0.01	-0.18	-4.7	-1.5	0.1
0113-01	514.	0.	-18.	-0.00	-0.61	-0.01	0.1	0.0	4.9
0131-01	-4.	-1.	836.	-0.00	0.00	3.39	-8.9	28.6	-0.0
0132-01	732.	-707.	-9.	2.01	-3.42	-0.02	0.5	0.2	30.0
0133-01	-15.	17.	-496.	-0.04	0.07	-0.99	28.6	8.9	-0.6
A001-01	-122.	407.	-0.	-0.47	1.69	-0.00	-0.	0.	-0.
A002-01	371.	120.	-8.	1.40	0.49	-0.03	-0.	0.	-0.
A003-01	-7.	-3.	-389.	-0.03	-0.01	-1.47	-0.	0.	-0.
A011-01	-110.	371.	-0.	-0.24	0.97	-0.00	-0.	0.	-0.
A012-01	548.	176.	-11.	2.22	0.76	-0.05	-0.	0.	-0.
A013-01	0.	0.	14.	0.00	0.00	0.05	-0.	0.	-0.
G001-01	-1.	0.	546.	0.01	0.00	3.00	-14.8	47.7	-0.0
G002-01	-6.	11.	-135.	-0.02	0.05	-0.13	47.7	14.8	-1.0
G003-01	-295.	508.	2.	-1.05	3.10	0.00	-0.9	-0.3	-50.0
T021-02	-0.	0.	-0.	-0.00	5.00	-0.00	-0.	0.	-0.
T022-02	-0.	0.	-0.	-0.00	-0.00	5.00	-0.	0.	-0.
T023-02	-0.	0.	-0.	1.00	0.00	-0.00	-0.	0.	-0.

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU Z TRAJECTORY TAPE 562

TIME= 500.000 COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0.1601E 07	-0.7171E 06	-0.1296E 05	0.4447E 04	-0.3471E 04	-0.6153E 02	-0.4089E 02	-0.1543E 03	0.3926E 05
-0.4305	0.1733E 07	-0.5634E 04	-0.1619E 04	0.6377E 04	-0.2277E 02	0.1519E 03	-0.8831E 00	-0.4661E 05
-0.0080	-0.0033	0.1649E 07	-0.3185E 02	-0.9402E 01	0.5447E 04	-0.3853E 05	0.4287E 05	-0.3914E 02
0.8653	-0.3026	-0.0061	0.1650E 02	-0.8201E 01	-0.1519E 00	-0.8592E-01	0.2583E 00	0.1125E 03
-0.3736	0.6596	-0.0010	-0.2749	0.5393E 02	-0.5422E-01	-0.1518E 00	0.6753E-02	-0.2606E 03
-0.0070	-0.0025	0.6052	-0.0054	-0.0011	0.4864E 02	-0.1086E 03	0.2300E 03	-0.6541E-01
-0.0006	0.0020	-0.5128	-0.0004	-0.0004	-0.2662	0.3422E 04	0.6997E 01	-0.1794E-01
-0.0021	-0.0000	0.5723	0.0011	0.0000	0.5655	0.0020	0.3402E 04	0.1360E-01
0.5303	-0.6031	-0.0005	0.4732	-0.6063	-0.0002	-0.0000	0.0000	0.3424E 04

SIGMA

1265.2606	1316.5268	1284.2856	4.0623	7.3440	6.9741	58.4996	58.3272	58.5172

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
500.000	9258709.	-16818952.	9700173.	23040.41	10354.42	-4038.29
	LAT	LONG	ALT	VEL	FPA	AZ
	26.805	-63.257	98.62	25580.89	0.000	100.187

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TRANSITION MATRIX. PARTIALS AT T= 2000.000 WITH RESPECT TO T= 500.000

COORDINATE SYSTEM LH

PARTIALS OF X,Y,Z,XDOT,YDOT,ZDOT,PHIX,PHIY,PHIZ WITH RESPECT TO

X	Y	Z	XDOT	YDOT	ZDOT	PHIX	PHIY	PHIZ
-0.1425E 01	-0.3399E 01	-0.4809E-03	-0.1216E 04	-0.2038E 04	-0.4763E 00	-0.	0.	-0.
0.9765E 00	0.2208E 01	0.2916E-03	0.2037E 04	0.8195E 03	0.1286E 01	-0.	0.	-0.
0.4566E-05	-0.2909E-03	-0.2118E 00	-0.7537E 00	-0.4475E-02	0.8213E 03	-0.	0.	-0.
-0.1162E-02	-0.1442E-02	-0.5311E-06	-0.1426E 01	-0.9768E 00	-0.7727E-03	-0.	0.	-0.
0.1443E-02	0.5205E-02	0.7905E-07	0.3401E 01	0.2211E 01	0.1937E-02	-0.	0.	-0.
0.7139E-06	-0.5050E-06	-0.1163E-02	-0.8331E-05	-0.1780E-03	-0.2130E 00	-0.	0.	-0.
-0.	-0.	-0.	-0.	-0.	-0.	-0.2130E 00	-0.9770E 00	-0.9991E-05
-0.	-0.	-0.	-0.	-0.	-0.	0.9770E 00	-0.2130E 00	0.7740E-03
-0.	-0.	-0.	-0.	-0.	-0.	-0.7624E-03	0.1551E-03	0.1000E 01

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
500.000	9258709.	-16818952.	9700173.	23040.41	10354.42	-4038.29
2000.000	16941277.	12070075.	-5393493.	-15703.07	17345.02	-10417.33
	LAT	LONG	ALT	VEL	FPA	AZ
500.000	26.805	-63.257	98.62	25580.89	0.000	100.187
2000.000	-14.536	27.112	93.46	25611.65	-0.051	114.861

C-3. Test Case 2 (Continued)

11/11/65 JOHN DOE FLDG N 64 RM 2 TEST CASE 2

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 2000.000 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	UOX	UOY	OOZ
0111-01	-3580.	1790.	-1.	-1.43	4.73	0.00	0.0	-0.0	-0.0
0112-01	-38.	45.	-253.	-0.04	0.07	-0.53	2.4	-4.3	0.1
0113-01	514.	1.	-5.	-0.00	-0.61	0.02	-0.0	C.1	4.9
0131-01	7.	-6.	2638.	0.01	-0.01	-1.69	-26.1	-14.8	-0.0
0132-01	5895.	434.	-13.	0.65	-3.37	0.02	-0.3	0.5	30.0
0133-01	-133.	-1.	-706.	-0.02	0.09	0.79	-14.8	26.1	-0.6
A001-01	-4074.	1204.	-0.	-1.42	4.07	-0.00	-0.	-0.	-0.
A002-01	-3623.	3870.	-23.	-3.07	6.98	0.01	-0.	-0.	-0.
A003-01	73.	-77.	-1123.	0.06	-0.14	0.76	-0.	-0.	-0.
A011-01	-2779.	1007.	-0.	-1.00	3.08	-0.00	-0.	-0.	-0.
A012-01	-5631.	6070.	-36.	-4.80	10.94	0.02	-0.	-0.	-0.
A013-01	-2.	3.	36.	-0.00	0.00	-0.03	-0.	-0.	-0.
G001-01	-15.	18.	2351.	-0.01	0.03	-1.27	-43.5	-24.7	-0.0
G002-01	-117.	25.	-81.	-0.04	0.11	0.19	-24.7	43.5	-1.0
G003-01	-6346.	1240.	1.	-1.92	5.51	0.00	0.5	-0.9	-56.0
T021-02	-10189.	4098.	-0.	-4.88	11.06	-0.00	-0.	-0.	-0.
T022-02	-2.	6.	4106.	-0.00	0.01	-1.06	-0.	-0.	-0.
T023-02	-1216.	2037.	-1.	-1.43	3.40	-0.00	-0.	-0.	-0.

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2   TRAJECTORY TAPE 562

TIME=   2000.000   COORDINATE SYSTEM   LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0.2943E 09	-0.1375E 09	0.3454E 06	0.1407E 06	-0.3524E 06	-0.1694E 03	0.2206E 03	0.4607E 03	0.4967E 06
-0.7825	0.1049E 09	-0.3315E 06	-0.9124E 05	0.2138E 06	0.1847E 03	-0.6409E 03	-0.3333E 03	-0.4900E 05
0.0036	-0.0058	0.3099E 08	0.2738E 03	-0.6118E 03	-0.1306E 05	-0.1585E 06	-0.1175E 06	-0.2562E 02
0.8968	-0.9739	0.0054	0.8370E 02	-0.1990E 03	-0.1469E 00	0.3430E 00	0.3715E 00	0.1157E 03
-0.9358	0.9511	-0.0050	-0.9910	0.4818E 03	0.3271E 00	-0.1067E 01	-0.6358E 00	-0.3797E 03
-0.0037	0.0068	-0.8782	-0.0060	0.0056	0.7133E 01	0.8219E 02	0.8742E 02	0.8932E-01
0.0002	-0.0011	-0.4878	0.0006	-0.0008	0.3267	0.3406E 04	-0.1045E 02	-0.4471E-02
0.0005	-0.0006	-0.3610	0.0007	-0.0005	0.5598	-0.[illegible]31	0.3418E 04	-0.[illegible]533E-02
0.4948	-0.0818	-0.0001	0.2162	-0.2956	0.0006	-0.0006	-0.0000	0.3424E 04

SIGMA

17155.0320	10240.8098	5566.7546	9.1488	21.9503	2.6708	58.3603	58.4672	58.5179

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
2000.000	16941277.	12070075.	-5393493.	-15703.07	17345.02	-10417.33
	LAT	LONG	ALT	VEL	FPA	AZ
	-14.536	27.112	93.46	25611.65	-0.051	114.861

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2   TRAJECTORY TAPE 562

TRANSITION MATRIX.   PARTIALS AT T=   4000.000 WITH RESPECT TO T=   500.000

COORDINATE SYSTEM   LH

PARTIALS OF X,Y,Z,XDOT,YDOT,ZDOT,PHIX,PHIY,PHIZ WITH RESPECT TO

X	Y	Z	XDOT	YDOT	ZDOT	PHIX	PHIY	PHIZ
-0.2018E 01	-0.1414E 02	-0.1143E-02	-0.1334E 05	-0.2518E 04	-0.8324E 01	-0.	0.	-0.
-0.8651E 00	0.2464E 01	0.2508E-02	0.2512E 04	-0.7365E 03	0.3692E 01	-0.	0.	-0.
-0.1951E-02	-0.1130E-01	-0.5165E 00	-0.1102E 02	-0.2527E 01	-0.7192E 03	-0.	0.	-0.
0.1023E-02	-0.1786E-02	-0.2566E-05	-0.2025E 01	0.8659E 00	-0.1760E-02	-0.	0.	-0.
0.1766E-02	0.1580E-01	0.3426E-05	0.1416E 02	0.2477E 01	0.1246E-01	-0.	0.	-0.
0.5962E-05	0.1636E-04	0.1018E-02	0.1203E-01	0.7470E-02	-0.5186E 00	-0.	0.	-0.
0.	0.	0.	0.	0.	0.	-0.5154E 00	0.8570E 00	-0.1067E-02
-0.	-0.	-0.	-0.	-0.	-0.	-0.8570E 00	-0.5154E 00	-0.2460E-02
-0.	-0.	-0.	-0.	-0.	-0.	-0.2662E-02	-0.3537E-03	0.1000E 01

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
500.000	9258709.	-16818952.	9700173.	23040.41	10354.42	-4038.29
4000.000	-21354558.	1230478.	-2040570.	-2463.44	-22494.16	12005.66
	LAT	LONG	ALT	VEL	FPA	AZ
500.000	26.805	-63.257	98.62	25580.89	0.000	100.187
4000.000	-5.449	159.990	92.50	25616.23	0.045	61.909

C-3. Test Case 2 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 2

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 4000.000 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-9968.	980.	-7.	-0.49	11.02	0.01	0.0	0.0	-0.0
0112-01	-237.	30.	-123.	-0.02	0.25	0.59	1.2	4.8	0.1
0113-01	505.	5.	17.	-0.00	-0.60	-0.01	-0.0	-0.1	4.9
0131-01	25.	8.	-2870.	-0.00	-0.01	-0.91	29.1	-7.1	-0.0
0132-01	-9623.	5182.	10.	-5.01	10.06	-0.01	-0.2	-0.6	30.0
0133-01	147.	-101.	967.	0.10	-0.16	0.01	-7.1	-29.1	-0.7
A001-01	-3482.	-1313.	-3.	1.56	3.73	0.01	0.	-0.	-0.
A002-01	-22302.	3126.	6.	-2.24	23.54	0.03	0.	-0.	-0.
A003-01	445.	-66.	1257.	0.05	-0.48	0.36	0.	-0.	-0.
A011-01	-4199.	-317.	-3.	0.56	4.60	0.01	0.	-0.	-0.
A012-01	-35134.	4975.	9.	-3.59	37.08	0.05	0.	-0.	-0.
A013-01	-14.	2.	-41.	-0.00	0.02	-0.01	0.	-0.	-0.
G001-01	-112.	26.	-2442.	-0.02	0.13	-1.00	48.6	-11.9	-0.0
G002-01	-32.	-55.	166.	0.06	0.03	-0.07	-11.9	-48.6	-1.1
G003-01	-422.	-3403.	-3.	3.59	0.35	0.02	0.3	1.1	-50.0
T021-02	-12589.	-3682.	-13.	4.33	12.39	0.04	0.	-0.	-0.
T022-02	-42.	18.	-3596.	-0.01	0.06	-2.59	0.	-0.	-0.
T023-02	-13344.	2512.	-11.	-2.02	14.16	0.01	0.	-0.	-0.

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 4000.000 COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DDX	DDY	DDZ
0.3089E 10	-0.3911E 09	0.7660E 06	0.2677E 06	-0.3255E 07	-0.4221E 04	-0.4303E 04	0.2955E 04	-0.2651E 06
-0.6661	0.1116E 09	-0.1740E 06	-0.9861E 05	0.4163E 06	0.1917E 03	0.1235E 04	-0.1535E 04	0.3257E 06
0.0025	-0.0030	0.2964E 08	0.8421E 02	-0.9506E 03	0.1474E 05	-0.2113E 06	0.1270E 05	-0.2585E 03
0.5057	-0.9802	0.0016	0.9069E 02	-0.2863E 03	-0.4111E-01	-0.6755E 00	0.1412E 01	-0.3300E 03
-0.9998	0.6728	-0.0030	-0.5132	0.3432E 04	0.4357E 01	0.5570E 01	-0.3372E 01	0.2812E 03
-0.0252	0.0060	0.9003	-0.0014	0.0247	0.9046E 01	-0.7365E 02	0.2434E 02	-0.1147E 01
-0.0013	0.0020	-0.6654	-0.0012	0.0016	-0.4199	0.3401E 04	0.5625E 01	0.3143E-02
0.0009	-0.0025	0.0399	0.0025	-0.0010	0.1383	0.0016	0.3423E 04	-0.7744E-03
-0.0815	0.5269	-0.0008	-0.5922	0.0820	-0.0065	0.0000	-0.0000	0.3424E 04

SIGMA

55578.4883	10563.8370	5444.5479	9.5231	58.5827	3.0077	58.3213	58.5061	58.5179

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
4000.000	-21354558.	1230478.	-2040570.	-2463.44	-22494.16	12005.66
	LAT	LONG	ALT	VEL	FPA	AZ
	-5.449	159.990	92.50	25616.23	0.045	61.909

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2   TRAJECTORY TAPE 562

TRANSITION MATRIX.   PARTIALS AT T=   6000.000 WITH RESPECT TO T=   500.000

COORDINATE SYSTEM   LH

PARTIALS OF X,Y,Z,XDOT,YDOT,ZDOT,PHIX,PHIY,PHIZ WITH RESPECT TO

X	Y	Z	XDOT	YDOT	ZDOT	PHIX	PHIY	PHIZ
0.9906E 00	-0.1873E 02	-0.1159E-01	-0.1534E 05	0.4892E 02	-0.2986E 02	-0.	0.	-0.
0.2475E 00	0.9073E 00	0.2744E-02	-0.5094E 02	0.1976E 03	0.6360E 01	-0.	0.	-0.
0.2971E-02	0.3951E-01	0.9649E 00	0.3317E 02	0.5488E 01	0.2201E 03	-0.	0.	-0.
-0.3019E-03	0.4272E-04	-0.1705E-05	0.1004E 01	-0.2476E 00	-0.3478E-02	-0.	0.	-0.
-0.3146E-04	0.2256E-01	0.1413E-04	0.1873E 02	0.9061E 00	0.3559E-01	-0.	0.	-0.
-0.2621E-05	-0.3000E-04	-0.3117E-03	-0.2358E-01	-0.4515E-02	0.9653E 00	-0.	0.	-0.
-0.	-0.	-0.	-0.	-0.	-0.	0.9642E 00	-0.2650E 00	0.3641E-02
0.	0.	0.	0.	0.	0.	0.2650E 00	0.9642E 00	0.2365E-02
-0.	-0.	-0.	-0.	-0.	-0.	-0.4142E-02	-0.1315E-02	0.1000E 01

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
500.000	9258709.	-16818952.	9700173.	23040.41	10354.42	-4038.29
6000.000	14064065.	-13934491.	8408797.	19303.54	15244.45	-7034.90
	LAT	LONG	ALT	VEL	FPA	AZ
500.000	26.805	-63.257	98.62	25580.89	0.000	100.187
6000.000	23.012	-69.803	97.92	25583.39	-0.010	107.378

C-3. Test Case 2 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 2

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 6000.000 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-11325.	733.	27.	-0.21	14.19	-0.02	-0.0	-0.0	-0.0
0112-01	-240.	5.	434.	0.01	0.31	-0.33	-4.2	-2.7	0.1
0113-01	483.	6.	-21.	-0.00	-0.57	-0.00	0.1	0.1	4.9
0131-01	-57.	23.	1553.	-0.02	0.06	3.01	-16.2	25.3	-0.0
0132-01	-16952.	-1239.	9.	2.61	18.49	-0.03	0.6	0.4	30.0
0133-01	311.	20.	-697.	-0.05	-0.33	-0.80	25.3	16.2	-0.7
A001-01	-457.	696.	9.	-0.83	1.91	-0.01	-0.	0.	-0.
A002-01	-23275.	225.	41.	1.18	29.30	-0.07	-0.	0.	-0.
A003-01	498.	-15.	-699.	-0.02	-0.62	-1.29	-0.	0.	-0.
A011-01	-3265.	513.	11.	-0.44	4.67	-0.01	-0.	0.	-0.
A012-01	-36764.	333.	66.	1.88	46.23	-0.11	-0.	0.	-0.
A013-01	-16.	0.	23.	0.00	0.02	0.04	-0.	0.	-0.
G001-01	-188.	21.	1188.	-0.01	0.23	2.73	-27.0	42.1	-0.0
G002-01	81.	19.	-160.	-0.03	-0.06	-0.09	42.1	26.9	-1.2
G003-01	6394.	1053.	4.	-1.71	-5.33	-0.00	-1.0	-0.7	-50.0
T021-02	245.	988.	27.	-1.24	4.53	-0.02	-0.	0.	-0.
T022-02	-149.	32.	1101.	-0.02	0.18	4.83	-0.	0.	-0.
T023-02	-15339.	-51.	33.	1.00	18.73	-0.03	-0.	0.	-0.

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 6000.000 COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0.3454E 10	-0.8258E 07	-0.7059E 07	-0.2052E 06	-0.4274E 07	0.6960E 04	0.2045E 04	-0.1247E 05	-0.8260E 06
-0.0599	0.5504E 07	0.1910E 06	-0.6335E 04	0.2329E 05	0.1870E 03	-0.1402E 04	0.1079E 04	-0.8980E 05
-0.0482	0.0327	0.6220E 07	0.2459E 03	0.8926E 04	0.1452E 05	-0.8328E 05	0.7254E 05	0.6553E 03
-0.7677	-0.5936	0.0217	0.2069E 02	0.2377E 03	-0.5476E 00	0.1147E 01	-0.5094E-01	0.1636E 03
-0.9962	0.1360	0.0490	0.7159	0.5328E 04	-0.8939E 01	-0.3235E 01	0.1438E 02	0.8185E 03
0.0182	0.0123	0.8961	-0.0185	-0.0188	0.4222E 02	-0.1448E 03	0.1766E 03	-0.3459E 00
0.0006	-0.0102	-0.5712	0.0043	-0.0008	-0.3812	0.3417E 04	0.1105E 02	-0.8049E-02
-0.0036	0.0079	0.4983	-0.0002	0.0034	0.4657	0.0032	0.3407E 04	0.1257E-01
-0.2402	-0.6541	0.0045	0.6145	0.1916	-0.0009	-0.0000	0.0000	0.3424E 04

SIGMA								
58769.9321	2346.0510	2493.9410	4.5486	72.9947	6.4980	58.4574	58.3702	58.5179

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
6000.000	14064065.	-13934491.	8408797.	19303.54	15244.45	-7034.90
	LAT	LONG	ALT	VEL	FPA	AZ
	23.012	-69.803	97.92	25583.39	-0.010	107.378

C-3. Test Case 2 (Continued)

ORBIT DISPERSIONS EVALU 2 TRAJECTORY TAPE 562

TIME= 6000.000 CRITERION RANGE

	MT	MC	MV
0111-01	0.468	228.	731.
0112-01	0.010	439.	5.
0113-01	-0.020	-30.	7.
0131-01	0.001	1554.	23.
0132-01	0.700	311.	-1242.
0133-01	-0.012	-702.	20.
A001-01	0.019	17.	696.
A002-01	0.961	456.	221.
A003-01	-0.020	-708.	-15.
A011-01	0.135	69.	513.
A012-01	1.518	721.	326.
A013-01	0.001	24.	0.
G001-01	0.007	1191.	21.
G002-01	-0.003	-161.	19.
G003-01	-0.264	-110.	1054.
T021-02	-0.010	23.	988.
T022-02	0.005	1103.	32.
T023-02	0.633	306.	-54.

C-3. Test Case 2 (Continued)

COVARIANCE MATRIX

	MT	MC	MV
MT	0.589077E 01	0.282758E 04	0.315615E 03
MC	0.42357	0.756497E 07	0.325949E 06
MV	0.05544	0.05053	0.550114E 07

SIGMAS

2.427	2750.	2345.

NOMINAL TERMINAL CONDITIONS

LAT	LONG	ALT	VEL/R	FPA/R	AZ/R
23.012	-69.803	97.92	24209.426	-0.010	108.399

C-3. Test Case 2 (Concluded)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 3

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TIME= 0.

COORDINATE SYSTEM LM

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	0.	500.	-0.	0.03	-0.00	-0.	0.0	0.	-0.0
0112-01	500.	-0.	-0.	0.	-0.03	-0.02	-0.0	-0.	4.9
0113-01	0.	0.	-500.	-0.02	0.	-0.00	4.9	0.0	0.0
0131-01	0.	0.	-0.	0.	0.	-0.	0.0	30.0	-0.0
0132-01	0.	0.	-0.	0.	0.	-0.	30.0	0.	-0.
0133-01	0.	0.	-0.	0.	0.	-0.	0.0	0.	-30.0

C-4. Test Case 3

PHASE LOGIC SYN EQ MISSION   TRAJECTORY TAPE 562

TIME=   0.   COORDINATE SYSTEM   LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0.2500E 06	-0.4230E-03	-0.3642E-03	-0.2980E-07	-0.1601E 02	-0.8714E 01	-0.3092E-05	-0.1526E-04	0.2467E 04
-0.0000	0.2500E 06	-0.9766E-03	0.1601E 02	0.5960E-07	0.4470E-07	-0.1474E-04	-0.1583E-05	-0.6254E-05
-0.0000	-0.0000	0.2500E 06	0.8714E 01	-0.	0.7451E-08	-0.2467E 04	-0.4625E-05	-0.4189E-06
-0.0000	0.8784	0.4760	0.1329E-02	-0.1819E-11	0.	-0.8599E-01	-0.1318E-09	-0.9046E-09
-1.0000	0.0000	-0.	-0.0000	0.1026E-02	0.5581E-03	0.3087E-09	-0.9313E-09	-0.1580E 00
-1.0000	0.0000	0.0000	0.	1.0000	0.3037E-03	0.9949E-10	0.4657E-09	-0.8599E-01
-0.0000	-0.0000	-0.1623	-0.0776	0.0000	0.0000	0.9243E 03	-0.1703E-05	-0.1329E-05
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	0.0000	-0.0000	0.9000E 03	-0.7329E-05
0.1623	-0.0000	-0.0000	-0.0000	-0.1623	-0.1623	-0.0000	-0.0000	0.9243E 03

SIGMA

500.0000	500.0000	500.0000	0.0365	0.0320	0.0174	30.4030	30.0000	30.4030

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
0.	3005559.	-18112597.	9991646.	1320.79	219.17	0.
	LAT	LONG	ALT	VEL	FPA	AZ
	28.555	-80.578	-1.10	1338.85	-0.000	90.000

C-4. Test Case 3 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 3

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TIME= 464.664 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-148.	653.	-1.	-0.07	0.75	-0.00	-0.0	-0.0	-0.0
0112-01	514.	0.	-17.	0.00	-0.60	-0.01	0.1	0.0	4.9
0113-01	-17.	-2.	-497.	-0.01	0.01	0.15	4.5	1.3	-0.1
0131-01	-3.	-1.	717.	-0.00	0.00	3.38	-7.7	29.0	-0.0
0132-01	-13.	15.	-461.	-0.04	0.07	-1.03	29.0	7.7	-0.6
0133-01	-635.	616.	9.	-1.90	3.38	0.02	-0.6	-0.2	-30.0
A001-01	223.	219.	-5.	1.14	0.71	-0.02	-0.	0.	-0.
A002-01	[illegible]	-2.	-338.	-0.03	-0.01	-1.42	-0.	0.	-0.
A003-01	-203.	243.	2.	[illegible]	1.30	0.00	-0.	0.	-0.
A011-01	431.	424.	-10.	2.15	1.37	-0.05	-0.	0.	-0.
A012-01	0.	0.	12.	0.00	0.00	0.05	-0.	0.	-0.
A013-01	-42.	48.	1.	-0.12	0.22	0.00	-0.	0.	-0.
G001-[illegible]	-4.	6.	155.	-0.01	0.04	1.11	37.2	19.2	-0.8
G002-01	-3.	5.	420.	-0.02	0.02	-2.59	19.2	-37.2	-0.2
G003-01	-241.	412.	2.	-0.94	3.01	0.00	-0.9	-0.3	-46.5
G063-01	-408.	566.	5.	-1.41	3.46	0.01	-0.7	-0.2	-39.0

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN EQ MISSION   TRAJECTORY TAPE 562

TIME= 464.664   COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0.1298E 07	-0.5535E 05	-0.1037E 05	0.3829E 04	-0.3973E 04	-0.4805E 02	0.2654E 03	0.6356E 02	0.4873E 05
-0.3738	0.1689E 07	-0.5511E 04	-0.9387E 03	0.7194E 04	-0.3914E 02	-0.3401E 03	-0.2912E 03	-0.5972E 05
-0.0080	-0.0037	0.1284E 07	-0.3054E 03	0.5294E 01	0.4545E 04	-0.2355E 05	0.3519E 05	-0.3810E 03
0.8608	-0.1850	-0.0069	0.1524E 02	-0.9783E 01	-0.1410E 00	0.1005E 01	0.8325E 00	0.1555E 03
-0.5599	0.8890	0.0008	-0.4024	0.3878E 02	-0.7253E-01	-0.3043E 01	-0.1498E 01	-0.3791E 03
-0.0089	-0.0064	0.8472	-0.0076	-0.0025	0.2241E 02	-0.6374E 02	0.2080E 03	-0.9603E 00
0.0045	-0.0051	-0.4019	0.0050	-0.0094	-0.2604	0.2674E 04	0.6239E 01	0.3606E 02
0.0011	-0.0044	0.6028	0.0041	-0.0047	0.8529	0.0023	0.2653E 04	0.1068E 02
0.6302	-0.6770	-0.0050	0.5868	-0.8970	-0.0030	0.0103	0.0031	0.4607E 04

SIGMA

1139.2668	1299.4974	1133.2448	3.9043	6.2274	4.7338	51.7152	51.5045	67.8756

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
464.664	8444418.	-17166351.	9832988.	23153.78	9347.12	-3491.38
	LAT	LONG	ALT	VEL	FPA	AZ
	27.202	-65.748	98.58	25212.21	0.077	98.998

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TRANSITION MATRIX. PARTIALS AT T= 477.664 WITH RESPECT TO T= 464.664

COORDINATE SYSTEM LH

PARTIALS OF X,Y,Z,XDOT,YDOT,ZDOT,PHIX,PHIY,PHIZ WITH RESPECT TO

X	Y	Z	XDOT	YDOT	ZDOT	PHIX	PHIY	PHIZ
0.9998E 00	-0.1524E-01	-0.2049E-07	0.1300E 02	-0.1981E 00	-0.2682E-06	-0.	0.	-0.
0.1524E-01	0.1000E 01	0.	0.1981E 00	0.1300E 02	-0.1192E-06	-0.	0.	-0.
0.8382E-08	-0.1490E-07	0.9999E 00	0.1192E-06	-0.2384E-06	0.1300E 02	-0.	0.	-0.
-0.1839E-04	-0.1401E-06	0.5400E-12	0.9998E 00	-0.1524E-01	-0.2421E-07	-0.	0.	-0.
0.1401E-06	0.3677E-04	0.6821E-12	0.1524E-01	0.1000E 01	0.3725E-08	-0.	0.	-0.
-0.3979E-12	0.3979E-12	-0.1839E-04	0.6519E-08	-0.1118E-07	0.9999E 00	-0.	0.	-0.
-0.	-0.	-0.	-0.	-0.	-0.	0.9999E 00	-0.1524E-01	-0.2235E-07
0.	0.	0.	0.	0.	0.	0.1524E-01	0.9999E 00	0.3725E-08
-0.	-0.	-0.	-0.	-0.	-0.	0.6519E-08	-0.1118E-07	0.1000E 01

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
464.664	8444418.	-17166351.	9832988.	23153.78	9347.12	-3491.38
477.664	8744396.	-17042791.	9786427.	22995.75	9661.63	-3671.75
	LAT	LONG	ALT	VEL	FPA	AZ
464.664	27.202	-65.748	98.58	25212.21	0.077	98.998
477.664	27.063	-64.834	98.62	25211.77	0.052	99.439

C-4. Test Case 3 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 3

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TIME= 477.664 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-15?.	661.	-1.	-0.08	0.77	-0.00	-0.0	-0.0	-0.0
0112-01	514.	0.	-17.	0.00	-0.60	-0.01	0.1	0.0	4.9
0113-01	-18.	-2.	-495.	-0.01	0.01	0.16	4.7	1.3	-0.1
0131-01	-3.	-1.	760.	-0.00	0.00	3.37	-8.1	28.9	-0.0
0132-01	-14.	16.	-474.	-0.04	0.07	-1.02	28.9	8.1	-0.6
0133-01	-670.	650.	9.	-1.94	3.38	0.02	-0.6	-0.2	-30.0
A001-01	235.	232.	-6.	1.12	0.74	-0.02	-0.	0.	-0.
A002-01	-7.	-2.	-357.	-0.03	-0.01	-1.41	-0.	0.	-0.
A003-01	-214.	256.	3.	-0.60	1.30	0.00	-0.	0.	-0.
A011-01	452.	448.	-11.	2.12	1.42	-0.05	-0.	0.	-0.
A012-01	0.	0.	13.	0.00	0.00	0.05	-0.	0.	-0.
A013-01	-45.	50.	1.	-0.12	0.22	0.00	-0.	0.	-0.
G001-01	-4.	7.	169.	-0.01	0.04	1.10	38.2	19.7	-0.8
G002-01	-3.	5.	-454.	-0.02	0.02	-2.59	19.7	-38.2	-0.1
G003-01	-260.	447.	2.	-0.98	3.01	0.00	-0.9	-0.3	-47.8
G063-01	-436.	605.	5.	-1.45	3.46	0.01	-0.7	-0.2	-39.4

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN EQ MISSION   TRAJECTORY TAPE 562

TIME= 477.664   COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DUX	DUY	DUZ
0.1419E 07	-0.6261E 06	-0.1132E 05	0.4081E 04	-0.4177E 04	-0.4906E 02	0.2870E 03	0.9043E 02	0.5224E 05
-0.3850	0.1864E 07	-0.6137E 04	-0.1110E 04	0.7687E 04	-0.4074E 02	-0.3773E 03	-0.3274E 03	-0.6470E 05
-0.0080	-0.0038	0.1406E C7	-0.3223E 02	0.3634E 01	0.4811E 04	-0.2470E 05	0.3809E 05	-0.4063E 03
0.8730	-0.2072	-0.0069	0.1540E 02	-0.1011E 02	-0.1384E 00	0.1053E 01	0.9029E 00	0.1622E 03
-0.5613	C.9013	0.0015	-0.4122	0.3902E 02	-0.7621E-01	-0.3063E 01	-0.1604E 01	-0.3843E 03
-0.0087	-0.0063	0.8604	-0.0075	-0.0026	0.2224E 02	-0.6483E 02	0.2096E 03	-0.1008E 01
0.0046	-0.0053	-0.3959	0.0051	-0.0093	-0.2613	0.2769E 04	0.6574E 01	0.3699E 02
0.0014	-0.0046	0.6128	0.0044	-0.0049	0.8479	0.0024	0.2747E 04	0.1157E 02
0.6356	-0.6868	-0.0050	0.5990	-0.8915	-0.0031	0.0102	0.0032	0.4761E 04

SIGMA

1191.0938	1365.3504	1185.6961	3.9244	6.2469	4.7156	52.6194	52.4159	69.0002

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
477.664	8744396.	-17042791.	9786427.	22995.75	9661.63	-3671.75
	LAT	LONG	ALT	VEL	FPA	AZ
	27.063	-64.834	98.62	25211.77	0.052	99.439

C-4. Test Case 3 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 3

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TIME= 498.375 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-177.	673.	-1.	-0.09	0.81	-0.00	-0.0	-0.0	-0.0
0112-01	514.	0.	-18.	-0.00	-0.61	-0.01	0.1	0.0	4.9
0113-01	-18.	-3.	-491.	-0.01	0.01	0.18	4.7	1.5	-0.1
0131-01	-4.	-1.	830.	-0.00	0.00	3.39	-8.8	28.7	-0.0
0132-01	-15.	17.	-495.	-0.04	0.07	-0.99	28.7	8.8	-0.6
0133-01	-727.	703.	9.	-2.00	3.42	0.02	-0.5	-0.2	-30.0
A001-01	252.	253.	-6.	1.16	0.78	-0.03	-0.	0.	-0.
A002-01	-7.	-3.	-387.	-0.03	-0.01	-1.47	-0.	0.	-0.
A003-01	-233.	279.	3.	-0.62	1.37	0.00	-0.	0.	-0.
A011-01	484.	490.	-12.	2.11	1.50	-0.05	-0.	0.	-0.
A012-01	0.	0.	13.	0.00	0.00	0.05	-0.	0.	-0.
A013-01	-48.	53.	1.	-0.13	0.23	0.00	-0.	0.	-0.
G001-01	-5.	8.	193.	-0.01	0.04	1.14	39.8	20.5	-0.9
G002-01	-3.	6.	-508.	-0.02	0.02	-2.64	20.5	-39.8	-0.1
G003-01	-292.	503.	2.	-1.04	3.10	0.00	-0.9	-0.3	-49.8
G053-01	-482.	666.	5.	-1.52	3.53	0.01	-0.7	-0.3	-40.1

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN EQ MISSION   TRAJECTORY TAPE 562

TIME= 498.375   COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0.1631E 07	-0.7548E 06	-0.1288E 05	0.4549E 04	-0.4647E 04	-0.5078E 02	0.3185E 03	0.1327E 03	0.5824E 05
-0.4013	0.2168E 07	-0.7372E 04	-0.1341E 04	0.8636E 04	-0.4525E 02	-0.4260E 03	-0.3902E 03	-0.7286E 05
-0.0079	-0.0039	0.1616E 07	-0.3494E 02	0.1742E 01	0.5329E 04	-0.2660E 05	0.4284E 05	-0.4447E 03
0.8871	-0.2268	-0.0068	0.1612E 02	-0.1080E 02	-0.1351E 00	0.1110E 01	0.1042E 01	0.1729E 03
-0.5679	0.9153	0.0002	-0.4200	0.4105E 02	-0.8578E-01	-0.3151E 01	-0.1800E 01	-0.4014E 03
-0.0083	-0.0064	0.8763	-0.0070	-0.0028	0.2288E 02	-0.6640E 02	0.2171E 03	-0.1044E 01
0.0046	-0.0054	-0.3871	0.0051	-0.0091	-0.2567	0.2924E 04	0.7097E 01	0.3815E 02
0.0019	-0.0049	0.6255	0.0048	-0.0052	0.8422	0.0024	0.2903E 04	0.1307E 02
0.6441	-0.6989	-0.0049	0.6081	-0.8851	-0.0031	0.0100	0.0034	0.5012E 04

SIGMA

1277.1241	1472.5261	1271.1400	4.0152	6.4069	4.7838	54.0718	53.8798	70.7951

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
498.375	9221363.	-16835937.	9706828.	23061.90	10315.87	-4016.04
	LAT	LONG	ALT	VEL	FPA	AZ
	26.824	-63.372	98.62	25581.19	0.000	100.132

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TRANSITION MATRIX. PARTIALS AT T= 1380.724 WITH RESPECT TO T= 498.375

COORDINATE SYSTEM LH

PARTIALS OF X,Y,Z,XDOT,YDOT,ZDOT,PHIX,PHIY,PHIZ WITH RESPECT TO

X	Y	Z	XDOT	YDOT	ZDOT	PHIX	PHIY	PHIZ
-0.3676E-02	-0.1414E 01	-0.5215E-07	0.2694E 03	-0.8440E 03	-0.7866E-04	-0.	0.	-0.
0.8671E 00	0.1502E 01	0.1863E-07	0.8440E 03	0.7291E 03	0.2861E-04	-0.	0.	-0.
-0.1490E-07	-0.1304E-06	0.4982E 00	0.9537E-06	-0.8392E-04	0.7291E 03	-0.	0.	-0.
-0.1031E-02	-0.5958E-03	0.7458E-10	-0.3676E-02	-0.8671E 00	-0.5960E-07	-0.	0.	-0.
0.5968E-03	0.2712E-02	0.2910E-10	0.1414E 01	0.1502E 01	0.5588E-07	-0.	0.	-0.
-0.3638E-10	-0.2183E-10	-0.1031E-02	-0.1211E-07	-0.6519E-07	0.4982E 00	-0.	0.	-0.
-0.	-0.	-0.	-0.	-0.	-0.	0.4982E 00	-0.8671E 00	-0.8941E-07
0.	0.	0.	0.	0.	0.	0.8671E 00	0.4982E 00	0.2701E-07
-0.	-0.	-0.	-0.	-0.	-0.	0.7451E-08	-0.8941E-07	0.1000E 01

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
498.375	9221363.	-16835937.	9706828.	23061.90	10315.87	-4016.04
1380.724	21408416.	-865589.	1907425.	1979.81	22499.63	-12009.99
	LAT	LONG	ALT	VEL	FPA	AZ
498.375	26.824	-63.372	98.62	25581.19	0.000	100.132
1380.724	5.087	-8.084	96.36	25581.10	0.000	118.122

C-4. Test Case 3 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 3

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TIME= 1380.724 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-1659.	1370.	-1.	-0.92	2.31	0.00	-0.0	-0.0	-0.0
0112-01	514.	-0.	-16.	-0.00	-0.61	0.01	0.0	0.1	4.9
0113-01	-5.	-26.	-113.	0.01	-0.03	0.60	1.1	4.8	-0.1
0131-01	-1.	-6.	2887.	0.00	-0.01	0.83	-29.3	6.6	-0.0
0132-01	-95.	31.	-968.	-0.06	0.09	0.02	6.6	29.3	-0.6
0133-01	-4418.	1232.	17.	-2.63	3.78	-0.00	-0.1	-0.6	-30.0
A001-01	-702.	2147.	-22.	-1.09	3.65	-0.01	-0.	0.	-0.
A002-01	5.	-40.	-1263.	0.02	-0.06	-0.33	-0.	0.	-0.
A003-01	-1715.	694.	4.	-1.11	1.80	-0.00	-0.	0.	-0.
A011-01	-1392.	4025.	-40.	-2.10	6.85	-0.01	-0.	0.	-0.
A012-01	-0.	1.	41.	-0.00	0.00	0.01	-0.	0.	-0.
A013-01	-299.	96.	1.	-0.18	0.28	-0.00	-0.	0.	-0.
G001-01	-51.	27.	926.	-0.04	0.06	0.37	78.0	83.5	-1.9
G002-01	-30.	6.	-2178.	-0.02	0.02	-0.79	83.6	-78.0	1.2
G003-01	-3605.	1882.	2.	-2.68	4.37	-0.00	-0.5	-2.6	-138.0
G063-01	-4330.	1872.	8.	-2.96	4.67	-0.00	-0.2	-1.1	-58.5

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN EQ MISSION   TRAJECTORY TAPE 562

TIME= 1380.724   COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DUX	DUY	DUZ
0.6124E 08	-0.3403E 08	0.7283E 05	0.4307E 05	-0.7837E 05	0.5033E 02	-0.3811E 04	0.1212E 05	0.8860E 06
-0.6837	0.4045E 08	-0.2355E 06	-0.3121E 05	0.7701E 05	-0.9171E 02	0.1511E 04	-0.5151E 04	-0.4063E 06
0.0023	-0.0091	0.1644E 08	0.1206E 03	-0.3819E 03	0.4798E 04	-0.2008E 06	0.2373E 06	-0.5043E 04
0.9577	-0.8540	0.0052	0.3303E 02	-0.6632E 02	0.6003E-01	-0.2519E 01	0.8583E 01	0.6218E 03
-0.8112	0.9807	-0.0076	-0.9346	0.1524E 03	-0.1588E 00	0.3739E 01	-0.1267E 02	-0.9930E 03
0.0046	-0.0104	0.8535	0.0075	-0.0093	0.1923E 01	-0.6108E 02	0.1014E 03	-0.1268E 0[illegible]
-0.0041	0.0020	-0.4192	-0.0037	0.0026	-0.3728	0.1396E 05	0.5891E 01	0.3469E 02
0.0131	-0.0068	0.4950	0.0126	-0.0087	0.6184	0.0004	0.1399E 05	0.1787E 03
0.7400	-0.4175	-0.0081	0.7073	-0.5257	-0.0060	0.0019	0.0099	0.2341E 05

SIGMA

7825.5453	6360.4144	4054.2928	5.7470	12.3467	1.3866	118.1592	118.2660	152.9893

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
1380.724	21408417.	-865589.	1907424.	1979.81	22499.63	-12009.99
	LAT	LONG	ALT	VEL	FPA	AZ
	5.087	-8.084	96.36	25581.10	0.000	118.122

C-4. Test Case 3 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 3

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TIME= 1685.897 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-2597.	1483.	5.	-1.35	3.59	0.00	0.0	-0.0	-0.0
0112-01	521.	-0.	-12.	0.07	-0.80	0.02	-0.0	0.1	4.9
0113-01	14.	-35.	98.	0.03	-0.04	0.80	-0.9	4.8	-0.1
0131-01	11.	-8.	2963.	0.01	-0.02	-0.39	-29.5	-5.6	0.1
0132-01	-124.	16.	-747.	-0.06	0.11	1.57	-5.6	29.[illegible]	-0.5
0133-01	-5520.	472.	25.	-2.29	4.75	-0.03	0.1	-0.6	-30.0
A001-01	-2128.	2965.	-13.	-1.43	5.54	0.00	-0.	-0.	-0.
A002-01	31.	-60.	-1429.	0.05	-0.11	-0.74	-0.	-0.	-0.
A003-01	-2347.	574.	6.	-1.41	2.82	-0.02	-0.	-0.	-0.
A011-01	-4227.	5448.	-34.	-3.63	9.93	0.01	-0.	-0.	-0.
A012-01	-1.	2.	42.	-0.00	0.00	-0.00	-0.	-0.	-0.
A013-01	-372.	26.	2.	-0.09	0.14	0.00	-0.	-0.	-0.
G001-01	-78.	36.	1464.	-0.09	0.17	3.84	66.1	119.9	-2.2
G002-01	-37.	-4.	-2672.	0.[illegible]	0.00	-2.82	119.9	-66.1	1.5
G003-01	-5361.	2246.	-2.	-4.03	10.00	-0.11	0.3	-3.3	-168.6
G063-01	-5959.	1546.	14.	-3.28	7.29	-0.04	0.1	-1.2	-61.6

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN E) MISSION TRAJECTORY TAPE 562

TIME= 1685.897 COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0.1397E 09	-0.7061E 08	0.8490E 05	0.8631F 05	-0.2213E 06	0.6226E 03	-0.1199E 05	0.1732E 05	0.1439E 07
-0.7421	0.6463E 08	-0.1966E 06	-0.5135E 05	0.1454E 06	-0.8878E 02	0.2944E 04	-0.4513E 04	-0.4881E 04
0.0016	-0.0054	0.2062E 08	-0.65[illegible] 02	-0.1352E 03	0.1194E 05	-0.3069E 06	0.3140E 06	-0.7907E 04
0.9709	-0.84[illegible]	-0.0019	0.5[illegible] 02	-0.1489E 03	0.1582E 00	-0.5871E 01	0.4985E 01	0.9501E 03
-0.9353	0.9018	-0.0015	-0.9890	0.4008E 03	-0.7565E 00	0.1610E 02	-0.2349E 02	-0.2434E 04
0.0102	-0.0021	0.5109	0.0041	-0.0073	0.2650E 02	-0.8208E 02	0.6994E 03	0.9136E 01
-0.0072	0.0026	-0.4821	-0.0056	0.0057	-0.1137	0.1965E 05	-0.4959E 01	-0.2538E 02
0.0104	-0.0040	0.4930	0.0047	-0.0084	0.9684	-0.0003	0.1968E 05	0.2603E 03
0.6689	-0.3330	-0.0096	0.6938	-0.6678	0.0097	-0.0010	0.0102	0.3314E 05

SIGMA

11817.7649	8051.9474	4540.8724	7.5227	20.0191	5.1483	140.1878	140.2863	132.0343

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
1685.897	20557451.	6830437.	-2315187.	-7978.25	28282.52	-15889.56
	LAT	LONG	ALT	VEL	FPA	AZ
	-6.100	11.336	141.70	33407.06	5.800	118.073

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN EQ MISSION   TRAJECTORY TAPE 562

TRANSITION MATRIX.   PARTIALS AT T= 7200.000 WITH RESPECT TO T= 1685.897

COORDINATE SYSTEM LH

PARTIALS OF X,Y,Z,XDOT,YDOT,ZDOT,PHIX,PHIY,PHIZ WITH RESPECT TO

X	Y	Z	XDOT	YDOT	ZDOT	PHIX	PHIY	PHIZ
-0.4321E 01	-0.9992E 01	-0.5960E-07	-0.8308E 04	-0.6153E 04	0.2655E-02	-0.	-0.	-0.
0.7864E 00	0.7820E 01	-0.7749E-06	0.9465E 04	0.1376E 04	-0.2045E-02	-0.	-0.	-0.
0.4470E-07	0.7711E-06	-0.2703E 01	0.4730E-03	0.1043E-02	0.2019E 04	-0.	-0.	-0.
-0.5697E-03	-0.1442E-02	-0.4366E-10	-0.1562E 01	-0.7863E 00	0.4396E-06	-0.	-0.	-0.
0.3678E-03	0.2973E-02	-0.2510E-09	0.3134E 01	0.7811E 00	-0.7451E-06	-0.	-0.	-0.
-0.8549E-10	-0.1005E-09	-0.5697E-03	-0.5984E-07	0.3667E-07	0.5561E-01	-0.	-0.	-0.
0.	0.	0.	0.	0.	0.	-0.6178E 00	-0.7863E 00	0.3949E-06
-0.	-0.	-0.	-0.	-0.	-0.	0.7863E 00	-0.6178E 00	0.1863E-07
-0.	-0.	-0.	-0.	-0.	-0.	0.2198E-06	0.3101E-06	0.1000E 01

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
1685.897	20557451.	6830437.	-2315187.	-7970.25	28282.52	-15889.56
7200.000	-71678571.	38640352.	-25823653.	-12155.85	-2318.76	435.44
	LAT	LONG	ALT	VEL	FPA	AZ
1685.897	-6.100	11.336	141.70	33407.06	5.200	118.073
7200.000	-17.595	121.590	10616.61	12382.68	46.749	66.957

C-4. Test Case 3 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 3

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TIME= 7200.000 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
C111-01	-14478.	1713.	-8.	-1.37	2.03	-0.00	0.0	0.0	-0.0
0112-01	2047.	-2.	79.	0.22	-0.20	0.01	-0.1	-0.1	4.9
0113-01	321.	-64.	1341.	0.03	-0.05	-0.01	-3.2	-3.7	-0.1
0131-01	103.	-13.	-8797.	0.01	-0.01	-1.71	22.6	-19.7	0.1
0132-01	196.	-381.	5193.	0.05	-0.10	0.51	-19.7	-22.6	-0.6
C133-01	8913.	-15780.	-122.	2.30	-4.09	-0.02	0.4	0.4	-30.0
A001-01	-42645.	15602.	53.	-5.19	7.88	0.01	0.	-0.	-0.
A002-01	785.	-173.	2361.	0.09	-0.11	0.77	0.	-0.	-0.
A003-01	-1231.	-6814.	-53.	0.49	-1.37	-0.00	0.	-0.	-0.
A011-01	-67109.	18598.	108.	-7.59	11.02	0.02	0.	-0.	-0.
A012-01	-22.	5.	-115.	-0.00	0.00	-0.02	0.	-0.	-0.
A013-01	1260.	-761.	-1.	0.21	-0.24	-0.00	0.	-0.	-0.
GC01-01	-329.	-395.	3794.	-0.00	-0.07	-0.62	-204.7	465.1	3.4
G002-01	76.	78.	1535.	0.03	0.02	1.37	465.1	204.6	9.3
G003-01	-32908.	-9753.	-222.	-2.47	0.60	-0.00	10.1	9.7	-719.9
G063-01	-7298.	-13629.	-128.	0.56	-2.19	-0.01	1.3	1.3	-95.2

C-4. Test Case 3 (Continued)

TIME= 7200.000 PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DUX	DUY	DUZ
0.1062E 11	-0.2575E 10	-0.4943E 07	0.1184E 07	-0.1644E 07	-0.1610E 04	-0.2389E 06	-0.4691E 06	0.2413E 08
-0.6529	0.1465E 10	0.6670E 07	-0.3561E 06	0.5900E 06	0.1313E 04	0.1059E 04	-0.2770E 06	0.8795E 07
-0.0042	0.0154	0.1283E 09	-0.9567E 03	0.2076E 04	0.1920E 05	-0.3708E 06	0.2127E 07	0.1998E 06
0.9788	-0.7925	-0.0072	0.1378E 03	-0.1988E 03	-0.2464E 00	-0.2214E 02	-0.2379E 02	0.1655E 04
-0.9235	0.8920	0.0106	-0.9800	0.2987E 03	0.4856E 00	0.2704E 02	-0.2428E 02	-0.1026E 03
-0.0064	0.0140	0.6907	-0.0086	0.0115	0.6018E 01	0.7134E 03	0.1287E 02	0.1522E 02
-0.0046	0.0001	-0.0643	-0.0037	0.0031	0.5712	0.2592E 06	0.6282E 02	-0.3779E 04
-0.0089	-0.0142	0.3689	-0.0040	-0.0028	0.0103	0.0002	0.2592E 06	-0.3615E 04
0.3222	0.3162	0.0243	0.1939	-0.0082	0.0085	-0.0102	-0.0098	0.5283E 06

SIGMA

103033.5176	38273.9966	11328.3386	11.7403	17.2822	2.4532	509.1027	509.1016	726.8398

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
7200.000	-71678571.	38640352.	-25823653.	-12155.85	-2318.76	435.44
	LAT	LONG	ALT	VEL	FPA	AZ
	-17.595	121.590	10616.61	12382.68	46.749	66.987

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN EQ MISSION   TRAJECTORY TAPE 562

TRANSITION MATRIX.   PARTIALS AT T= 14400.000 WITH RESPECT TO T= 7200.000

COORDINATE SYSTEM LH

PARTIALS OF X,Y,Z,XDOT,YDOT,ZDOT,PHIX,PHIY,PHIZ WITH RESPECT TO

X	Y	Z	XDOT	YDOT	ZDOT	PHIX	PHIY	PHIZ
0.5623E 00	-0.5980E 00	-0.2235E-07	0.5819E 04	-0.3256E 04	-0.2136E-03	0.	-0.	-0.
0.4333E 00	0.1650E 01	-0.2608E-07	0.3293E 04	0.7863E 04	-0.3510E-03	0.	-0.	-0.
0.4657E-07	0.7823E-07	0.6610E 00	0.3967E-03	0.3357E-03	0.6543E 04	0.	-0.	-0.
-0.7464E-04	-0.2245E-04	0.1137E-11	0.6752E 00	-0.4333E 00	-0.1863E-07	0.	-0.	-0.
0.1497E-04	0.1891E-03	0.3638E-11	0.5431E 00	0.1395E 01	-0.5588E-07	0.	-0.	-0.
-0.3183E-11	0.6366E-11	-0.7464E-04	0.5215E-07	0.4[illegible]57E-07	0.7740E 00	0.	-0.	-0.
0.	0.	0.	0.	0.	0.	0.9013E 00	-0.4333E 00	-0.2608E-07
-0.	-0.	-0.	-0.	-0.	-0.	0.4333E 00	0.9013E 00	-0.5122E-07
-0.	-0.	-0.	-0.	-0.	-0.	0.5960E-07	0.3725E-07	0.1000E 01

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
7200.000	-71678571.	38640352.	-25823653.	-12155.85	-2318.76	435.44
14400.000	-126919830.	10368633.	-14220141.	-4058.41	-4678.79	2264.49

	LAT	LONG	ALT	VEL	FPA	AZ
7200.000	-17.595	121.590	10616.61	12382.68	46.749	66.987
14400.000	-6.372	115.165	17644.42	6594.67	30.934	61.983

C-4. Test Case 3 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 3

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TIME= 14400.000 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DDX	DDY	DDZ
0111-01	-23752.	7956.	-21.	-0.76	2.19	-0.00	0.0	0.0	-0.0
0112-01	3073.	-5.	106.	0.08	-0.14	0.00	-0.0	-0.1	4.9
0113-01	562.	-224.	812.	0.02	-0.05	-0.11	-1.3	-4.8	-0.1
0131-01	170.	-45.	-17004.	0.01	-0.01	-0.67	28.9	-6.0	0.1
0132-01	972.	-1136.	6788.	0.07	-0.18	0.01	-8.0	-28.9	-0.6
0133-01	41171.	-46755.	-184.	3.02	-7.31	-0.00	0.2	0.5	-30.0
A001-01	-89132.	52124.	107.	-4.08	10.48	0.00	0.	-0.	-0.
A002-01	1422.	-551.	6616.	0.05	-0.13	0.42	0.	-0.	-0.
A003-01	10723.	-20930.	-63.	1.17	-2.95	0.00	0.	-0.	-0.
A011-01	-128880.	63298.	200.	-5.31	13.77	0.01	0.	-0.	-0.
A012-01	-39.	15.	-233.	-0.00	0.00	-0.01	0.	-0.	-0.
A013-01	3152.	-1890.	-8.	0.17	-0.34	-0.00	0.	-0.	-0.
G001-01	270.	-1349.	-1553.	0.06	-0.18	-0.76	-386.0	330.5	3.4
G002-01	-50.	334.	9952.	-0.01	0.05	0.94	330.5	385.9	9.3
G003-01	-28980.	-33758.	-179.	0.75	-2.84	0.01	4.9	13.1	-719.9
G063-01	14427.	-41013.	-153.	2.18	-5.43	0.00	0.8	2.2	-[illegible]20.9
G001-02	0.	-0.	-0.	0.	-0.	-0.	520.3	428.5	11.3
G002-02	0.	-0.	-0.	0.	-0.	-0.	428.6	-520.3	[illegible]
G003-02	0.	-0.	-0.	0.	-0.	-0.	4.9	13.1	-719.9

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN EQ MISSION   TRAJECTORY TAPE 562

TIME= 14400.000   COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0.3952E 11	-0.2099E 11	-0.4341E 08	0.1716E 07	-0.4386E 07	-0.2216E 04	-0.2621E 06	-0.4472E 06	0.3188E 08
-0.8458	0.1558E 11	0.4385E 08	-0.1106E 07	0.2855E 07	0.1819E 04	-0.5590E 05	-0.1640E 07	0.4693E 08
-0.0100	0.0160	0.4798E 09	-0.2748E 04	0.7378E 04	0.2459E 05	0.5656E 07	-0.6568E 06	0.2665E 06
0.9468	-0.971[illegible]	-0.0138	0.8309E 02	-0.2134E 03	-0.122[illegible]E 00	-0.2747E 01	0.5868E 02	-0.1253E 04
-0.9414	0.9760	0.0144	-0.9990	0.5494E 03	0.3281E 00	0.7080E 01	-0.1808E 03	0.4288E 04
-0.0077	0.0100	0.7743	-0.0093	0.0097	0.2102E 01	0.5916E 03	-0.4305E 03	-0.1928E 02
-0.0017	-0.0006	0.3242	-0.0004	0.00[illegible]4	[illegible].5123	0.6343E 06	0.1445E 03	-0.7600E 04
-0.0028	-0.0165	-0.0376	0.0081	-0.0097	-[illegible].3728	0.0002	0.6346E 06	-0.2023E 05
0.1213	0.2845	0.0092	-0.1040	0.1384	-0.0101	-0.0072	-0.0192	0.1747E 07

SIGMA

198798.2383	124814.354[illegible]	21904.0132	9.1152	23.4388	1[illegible].4499	795.4267	796.6382	1321.5800

TRAJECTORY VARI[illegible]

TIME	X	Y	Z	XDOT	YDOT	ZDOT
14400.000	1269[illegible]9830.	10368633.	-14220141.	-4058.41	-4678.79	2264.49
	LAT	LONG	ALT	VEL	FPA	AZ
	-6.372	115.165	17644.42	6594.67	[illegible]0.934	61.983

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN EQ MISSION   TRAJECTORY TAPE 562

TRANSITION MATRIX.   PARTIALS AT T= 20177.531 WITH RESPECT TO T= 14400.000

COORDINATE SYSTEM   LH

PARTIALS OF X,Y,Z,XDOT,YDOT,ZDOT,PHIX,PHIY,PHIZ WITH RESPECT TO

X	Y	Z	XDOT	YDOT	ZDOT	PHIX	PHIY	PHIZ
0.8750E 00	-0.2525E 00	-0.3353E-07	0.5441E 04	-0.1345E 04	-0.1831E-03	0.	-0.	-0.
0.2291E 00	0.1177E 01	0.7846E-07	0.1345E 04	0.5995E 04	0.4778E-03	0.	-0.	-0.
0.2235E-07	-0.9872E-07	0.9016E 00	0.2441E-03	-0.5112E-03	0.5595E 04	0.	-0.	-0.
-0.3230E-04	-0.4030E-05	0.1137E-11	0.8821E 00	-0.2291E 00	-0.3353E-07	0.	-0.	-0.
0.3738E-05	0.7022E-04	0.2103E-11	0.2508E 00	0.1162E 01	0.9430E-07	0.	-0.	-0.
-0.1137E-11	-0.3979E-12	-0.3230E-04	0.2608E-07	-0.8009E-07	0.9087E 00	0.	-0.	-0.
0.	0.	0.	0.	0.	0.	0.9734E 00	-0.2291E 00	-0.3353E-07
-0.	-0.	-0.	-0.	-0.	-0.	0.2291E 00	0.9734E 00	0.7979E-07
-0.	-0.	-0.	-0.	-0.	-0.	0.3353E-07	-0.8754E-07	0.1000E 01

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
14400.000	-126919830.	10368633.	-14220141.	-4058.41	-4678.79	2264.49
20177.531	-68566476.	-16830542.	-150172.	112.26	-4586.59	2517.13
	LAT	LONG	ALT	VEL	FPA	AZ
14400.000	-6.372	115.165	17644.42	6594.67	30.934	61.983
20177.531	-0.062	-257.306	19294.65	5248.12	1.603	61.325

C-4. Test Case 3 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 3

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TIME= 20177.532 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DDX	DDY	DDZ
0111-01	-29881.	[illegible]007.	-27.	-0.44	2.82	-0.00	0.0	0.0	-0.0
0112-01	3316.	-9.	99.	0.00	-0.13	-0.00	-0.0	-0.1	4.9
0113-01	729.	-433.	123.	0.01	-0.07	-0.13	-0.2	-4.9	-0.1
0131-01	208.	-88.	-19062.	0.00	-0.02	-0.06	30.0	-1.1	0.1
0132-01	1771.	-2074.	6172.	0.08	-0.26	-0.21	-1.1	-30.0	-0.6
0133-01	74066.	-85355.	-183.	3.19	-10.86	0.00	0.1	0.6	-30.0
A001-01	-127453.	98300.	122.	-3.33	14.49	0.00	0.	-0.	-0.
A002-01	1851.	-1043.	8325.	0.03	-0.17	0.17	0.	-0.	-0.
A003-01	25016.	-38301.	-53.	1.45	-4.57	0.00	0.	-0.	-0.
A011-01	-176129.	120394.	221.	-3.93	18.63	0.00	0.	-0.	-0.
A012-01	-51.	29.	-266.	-0.00	0.00	-0.00	0.	-0.	-0.
A013-01	4604.	-3335.	-11.	0.13	-0.48	-0.00	0.	-0.	-0.
G001-01	1156.	-2510.	-5672.	0.09	-0.28	-0.64	-451.4	233.3	3.4
G002-01	-265.	642.	14246.	-0.02	0.07	0.54	233.3	451.3	9.3
G003-01	-8940.	-62402.	-90.	2.38	-5.59	0.02	1.8	13.9	-719.9
G063-01	42127.	-74638.	-130.	2.86	-8.60	0.01	0.3	2.4	-126.5
G001-02	0.	-0.	-0.	0.	-0.	-0.	408.3	536.2	11.3
G002-02	0.	-0.	-0.	0.	-0.	-0.	536.4	-408.3	-6.5
G003-02	0.	-0.	-0.	0.	-0.	-0.	1.8	13.9	-719.9
G001-03	0.	-0.	-0.	0.	-0.	-0.	564.8	111.9	3.6
G002-03	0.	-0.	-0.	0.	-0.	-0.	111.9	-564.7	-10.6
G003-03	0.	-0.	-0.	0.	-0.	-0.	1.4	11.1	-577.6

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN EQ MISSION   TRAJECTORY TAPE 562

TIME= 20177.532   COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DUX	DUY	DUZ
0.7873E 11	-0.6056E 11	-0.8625E 08	0.2051E 07	-0.8927E 07	-0.7969E 03	-0.7117E 05	0.6063E 06	0.5726E 07
-0.9236	0.5462E 11	0.9334E 08	-0.1897E 07	0.7613E 07	0.4659E 03	-0.8449E 05	-0.3633E 07	0.1044E 09
-0.0116	0.0151	0.7039E 09	-0.3269E 04	0.1237E 05	0.1242E 05	0.8095E 07	-0.5196E 07	0.7531E 05
0.8984	-0.9778	-0.0151	0.6620E 02	-0.2522E 03	-0.1467E-01	0.4172E 01	0.1366E 03	-0.3988E 04
-0.9674	0.9905	0.0142	-0.9798	0.1032E 04	0.7815E-01	-0.[illegible]816E 01	-0.3700E 03	0.9695E 04
-0.0032	0.0022	0.5256	-0.0020	0.0027	0.7928E 00	0.2832E 03	-0.4599E 03	-0.3489E 02
-0.0002	-0.0003	0.2829	0.0005	-0.0001	0.2950	0.1163E 07	0.9286E 02	-0.4771E 04
0.0020	-0.0144	-0.1816	0.0156	-0.0104	-0.4788	0.0001	0.1164E 07	-0.3701E 05
0.0116	0.2545	0.0016	-0.2772	0.1679	-0.0223	-0.0025	-0.0195	0.3083E 07

SIGMA

280585.7891	233703.2031	26530.8992	8.1365	32.8889	0.8904	1078.3660	1078.7023	1755.8879

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
20177.532	-68566478.	-16830543.	-150172.	412.26	-4586.59	2517.13
	LAT	LONG	ALT	VEL	FPA	AZ
	-0.062	-257.306	1[illegible]2[illegible]4.65	5248.12	1.603	61.325

C-4. Test Case 3 (Continued)

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TRANSITION MATRIX. PARTIALS AT T= 20288.784 WITH RESPECT TO T= 20177.532

COORDINATE SYSTEM LH

PARTIALS OF X,Y,Z,XDOT,YDOT,ZDOT,PHIX,PHIY,PHIZ WITH RESPECT TO

X	Y	Z	XDOT	YDOT	ZDOT	PHIX	PHIY	PHIZ
0.8773E 00	-0.5696E-02	0.4798E 00	0.9761E 02	-0.6337E 00	0.5338E 02	0.5703E 03	-0.1261E 06	-0.2539E 04
0.5532E-02	0.1000E 01	0.1755E-02	0.6155E 00	0.1113E 03	0.1953E 00	0.2425E 06	-0.1024E 04	-0.1808E 06
-0.4798E 00	0.1114E-02	0.8773E 00	-0.5338E 02	0.1240E 00	0.9761E 02	0.3839E 04	0.2750E 06	0.1750E 04
-0.5209E-06	-0.2339E-08	-0.2849E-06	0.8773E 00	-0.5696E-02	0.4798E 00	0.3100E 02	-0.2517E 04	-0.8656E 02
0.1216E-08	0.1187E-05	-0.5084E-10	0.5532E-02	0.1000E 01	0.1755E-02	0.4843E 04	-0.2046E 02	-0.3610E 04
0.2849E-06	0.3361E-10	-0.5209E-06	-0.4798E 00	0.1114E-02	0.6773E 00	0.1125E 03	0.5491E 04	0.5456E 02
0.	0.	0.	0.	0.	0.	0.8774E 00	-0.5696E-02	0.4798E 00
0.	0.	0.	0.	0.	0.	0.5532E-02	0.1000E 01	0.1755E-02
-0.	-0.	-0.	-0.	-0.	-0.	-0.4798E 00	0.1114E-02	0.8774E 00

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
20177.532	-68566478.	-16830543.	-150172.	412.26	-4586.59	2517.13
20288.784	-68522446.	-17612547.	3822.	1286.42	-10011.03	0.30
	LAT	LONG	ALT	VEL	FPA	AZ
20177.532	-0.062	-257.306	19294.55	5248.12	1.603	61.325
20288.784	0.002	-257.445	19296.30	10093.40	0.001	89.398

C-4. Test Case 3 (Continued)

11/11/65 JOHN DOE BLDG N 64 RM 2 TEST CASE 3

PHASE LOGIC SYN EQ MISSION TRAJECTORY TAPE 562

TIME= 20288.784 COORDINATE SYSTEM LH

	DPX	DPY	DPZ	DVX	DVY	DVZ	DOX	DOY	DOZ
0111-01	-26364.	16156.	14355.	-0.39	2.84	0.20	-0.0	0.0	-0.0
0112-01	2757.	-9.	-1505.	-0.00	-0.21	-0.00	2.4	-0.1	4.3
0113-01	699.	-437.	-262.	0.01	-0.07	-0.25	-0.2	-4.9	-0.0
0131-01	-8965.	-87.	-16830.	-0.00	0.69	-0.05	26.3	-1.0	-14.3
0132-01	4542.	-2084.	4498.	0.33	-0.28	-1.02	-1.1	-30.0	-0.0
0133-01	65696.	-96130.	-35764.	2.83	-10.42	-1.51	-14.3	0.5	-26.4
A001-01	-112653.	99210.	61548.	-2.94	14.59	1.56	0.	0.	-0.
A002-01	5636.	-1038.	6429.	0.11	-0.17	0.13	0.	0.	-0.
A003-01	22284.	-38672.	-12169.	1.29	-4.60	-0.69	0.	0.	-0.
A011-01	-155481.	121499.	85036.	-3.15	18.80	1.68	0.	0.	-0.
A012-01	-162.	29.	-188.	0.23	-0.00	0.42	0.	0.	-0.
A013-01	4066.	-3365.	-2230.	0.12	-0.52	-0.07	0.	0.	-0.
G001-01	-1861.	-3080.	-5299.	-3.14	10.97	5.36	-395.8	230.8	219.8
G002-01	6350.	937.	13286.	-5.24	5.35	12.62	206.6	452.6	-103.3
G003-01	-7293.	-62442.	4027.	2.27	5.99	-0.96	-343.9	12.6	-632.4
G063-01	37607.	-75253.	-20561.	2.57	-6.45	-1.33	-60.4	2.2	-111.1
G001-02	-327.	468.	723.	-6.49	9.34	14.50	360.6	538.5	-185.4
G002-02	251.	638.	-534.	5.07	12.75	-10.58	469.8	-405.3	-263.5
G003-02	0.	633.	12.	0.13	12.64	0.18	-343.9	12.6	-632.4
A001-03	52.	7.	-28.	0.94	0.13	-0.51	0.	0.	-0.
A002-03	-27.	1.	-52.	-0.51	0.02	-0.94	0.	0.	-0.
A003-03	-6.	59.	4.	-0.10	1.07	0.08	0.	0.	-0.
G001-03	-67.	665.	160.	-1.29	13.33	3.31	506.3	116.3	-273.0
G002-03	348.	144.	-756.	6.99	2.89	-15.14	77.4	-575.2	-64.4
G003-03	0.	512.	10.	0.11	10.25	0.15	-281.3	10.3	-517.3
T021-04	0.	0.	-0.	-0.00	0.10	-0.	0.	0.	-0.
T022-04	0.	0.	-0.	0.00	0.00	0.10	0.	0.	-0.
T023-04	0.	0.	-0.	0.10	0.00	0.00	0.	0.	-0.

C-4. **Test Case 3 (Continued)**

PHASE LOGIC SYN FO MISSION   TRAJECTORY TAPE 562

TIME= 20288.784   COORDINATE SYSTEM LH

COVARIANCE MATRIX

DPX	DPY	DPZ	DVX	DVY	DVZ	DUX	DUY	DUZ
0.6165E 11	-0.5408E 11	-0.3331E 11	0.1570E 07	-0.7740E 07	-0.8655E 06	0.5341E 07	-0.2678E 07	0.1747E 07
-0.9248	0.5547E 11	0.2958E 11	-0.1634E 07	0.5767E 07	0.8104E 06	0.5056E 08	-0.3481E 07	0.8988E 08
-0.9758	0.9136	0.1890E 11	-0.7702E 06	0.4561E 07	0.3741E 06	0.5156E 07	-0.3176E 07	-0.5549E 07
0.4166	-0.4570	-0.3787	0.2303E 03	-0.1385E 03	-0.4048E 03	-0.1800E 04	-0.1444E 05	-0.4045E 04
-0.4587	0.5219	0.6835	-0.1340	0.2357E 04	0.1011E 03	0.2835E 04	0.1730E 03	-0.5248E 05
-0.1244	0.1199	0.0349	-0.9296	0.0726	0.8232E 03	0.1491E 04	0.3083E 05	0.1095E 04
0.0169	0.1690	0.0295	-0.0932	0.0459	0.0408	0.1621E 07	-0.1622E 05	0.8127E 06
-0.0099	-0.0136	-0.0226	-0.8760	0.0033	0.9834	-0.0117	0.1179E 07	-0.2983E 05
0.0043	0.2334	-0.0247	-0.1630	-0.6612	0.0233	0.3905	-0.0168	0.2673E 07

SIGMA

248300.5996	235518.0918	137476.6055	15.1766	48.5454	28.6911	1273.0711	1085.9502	1635.0835

TRAJECTORY VARIABLES.

TIME	X	Y	Z	XDOT	YDOT	ZDOT
20288.784	-68522446.	-17612647.	3822.	1286.42	-10011.09	0.30
	LAT	LONG	ALT	VEL	FPA	AZ
	0.002	-257.445	19296.30	10093.40	0.001	89.998

C-4. Test Case 3 (Continued)

	ATTITUDES			DIRECTION COSINES					
TIME	THETA	PSI	PHI	1X	1Y	1Z	2X	2Y	2Z
0.	89.9815	0.	-0.	0.143786	-0.866510	0.478002	0.986510	0.163693	0.
464.664	-3.5815	97.3481	86.4044	0.890277	0.429553	-0.151299	-0.448647	0.770143	-0.453426
477.664	-3.8215	98.3971	86.1639	0.881395	0.444430	-0.160077	-0.465854	0.761655	-0.450401
498.375	-4.1259	99.1109	85.8585	0.867143	0.466897	-0.173408	-0.491874	0.748094	-0.445439
1380.724	4.7986	117.8193	85.0793	0.160065	0.875572	-0.455799	-0.384000	0.104929	-0.143991
1685.897	19.6488	117.7855	70.3174	0.010306	0.881396	-0.472266	-0.996881	-0.027902	-0.073828
7200.000	54.3153	68.8626	35.6738	-0.995668	-0.081356	-0.045022	0.050229	-0.878076	0.475877
14400.000	8.7985	62.5356	81.1369	-0.273320	-0.857448	0.435979	0.958735	-0.205959	0.195979
20177.532	6.4381	61.5917	83.4671	-0.005332	-0.881245	0.472629	0.996920	0.032299	0.071471
20288.784	6.7555	61.5891	83.1695	-0.005332	-0.881245	0.472629	0.996920	0.032299	0.071471

C-4. Test Case 3 (Concluded)

```
CARD NO   1    HJOB1 JOHN DOE   BLDG N 64   RM 2
CARD NO   2    HJOB2 TEST CASE 1
CARD NO   3    HCASE1IIP DISPERSIONS EVAL 1.
CARD NO   4    HCASE2TRAJECTORY TAPE 562
CARD NO   5    IOPT  2          DSYSTM+ECI
CARD NO   6     SIGMAO             1   500         4    30
CARD NO   7     7    1         -4 13   .1
CARD NO   8    ISOPT 1          IEVALU1
CARD NO   9    ARHO  7,,10             .5        A     8,,11             .5
CARD NO  10    A     9,,12             .5
CARD NO  11    END   0
CARD NO  12    HJOB2 TEST CASE 2
CARD NO  13    HCASE1ORBIT DISPERSIONS EVALU 2
CARD NO  14    IOPT  3          DSYSTM000000
CARD NO  15     SIGMAO            16   5          18    1
CARD NO  16    ISOPT 0          IEVALU2         IFOPT 1
CARD NO  17    END   0
CARD NO  18    HJOB2 TEST CASE 3
CARD NO  19    HCASE1PHASE LOGIC SYN EQ MISSION
CARD NO  20    IOPT  2
CARD NO  21     SIGMAO             1   500         4    30
CARD NO  22     7    1         -4 13   .1         16    0         18    1       -4
CARD NO  23     19   .1           22   3       -4 25    .1
CARD NO  24    IEVALUC
CARD NO  25    ARHO  7,,10             0.5       A     8,,11             0.5
CARD NO  26    A     9,,12             0.5       A     13,,19            0.67
CARD NO  27    A     13,,25            0.484     A     19,,25            0.72
CARD NO  28    A     14,,20            0.67      A     14,,26            0.484
CARD NO  29    A     20,,26            0.72      A     15,,21            0.67
CARD NO  30    A     15,,27            0.484     A     21,,27            0.72
CARD NO  31    END   0
CARD NO  32    ENDJOB0
```

C-5. ØUTP Input Data

APPENDIX D

RESETS

CONTENTS

APPENDIX D

RESETS

D.1 INTRODUCTION

"Resets" is the term applied to correction schemes for updating inertial system navigation data as a result of measurements made by sensors other than the inertial elements (gyros and accelerometers). These measurements can be made by ground equipment from which processed navigation data is transmitted to the navigation computer; or airborne sensor data can be used by the computer to update the navigation data. Many sensor configurations and data processing schemes could be used, but only a few are discussed here. It is assumed for this analysis that only a limited number of corrections would be made, although many measurements might be used and pre-filtered to obtain the measured value used for reset.

D.2 MEASUREMENTS CONSIDERED

The measurements considered are components of position, velocity, and/or platform orientation as follows:

a. Position Measurements

(1) altitude

(2) slant range to a ground station

(3) position vector in the ECI or orbit plane (see Section 2.4.1) coordinate system as derived by a ground station and transmitted to the airborne computer

b. Velocity Measurements

(1) altitude rate

(2) slant range rate to a ground station

(3) velocity vector (ECI or orbit plane coordinates) as derived by a ground station

c. Angular Measurements

(1) angular measurements with respect to inertial space, using a stellar sensor

(2) angular measurements with respect to the local vertical (assumed geocentric), using a horizon sensor

(3) platform error vector as derived from multiple stellar sensor or horizon sensor measurements.

In developing the appropriate reset equations, it was desired to retain the common notation of Y for measurement vectors and X for state vectors. For this reason, there is some ambiguity between this notation and that of coordinates of the position and velocity vectors (e.g., see Section D.6). It is hoped that this ambiguity will not cause confusion in interpretation.

Three reset methods are considered that use the same form for correcting the state vector. The concept for deriving the form of the reset equations is obtained from the following considerations. A measurement is functionally related to the navigation data and is corrupted by some noise. This relationship is shown by

$$Y_i = F_i(X) + N_i$$

where

Y_i	is i^{th} measurement
$F_i(X)$	is the functional relationship (see Section D.6 for explicit forms)
X	is the true state vector (9 elements)
N_i	is the error (noise plus bias) of the i^{th} measurement (a random variable)

Likewise a vector of measurements can be constructed as

$$Y = F(X) + N$$

The measurement state vector can be estimated from the navigation system data as

$$\hat{Y} = F(\hat{X})$$

where $\hat{X}$ is the navigation system estimate of the state vector X and is in error by ΔX, that is

$$\hat{X} = X + \Delta X$$

where ΔX is a random variable, which is the sum of the effects up to time (t) of all the error sources (P in number), that is

$$\Delta X = \sum_{i=1}^{P} \frac{\partial X_i}{\partial \epsilon_i} \epsilon_i = \sum_{i=1}^{P} \delta X_i \epsilon_i$$

Then the difference between $\hat{Y}$ and Y can be calculated and used to estimate ΔX and/or ϵ_i. That is

$$\Delta Y = \hat{Y} - Y$$
$$= F(X + \Delta X) - F(X) - N$$

$F(X + \Delta X) - F(X)$ can be expanded into a Taylor Series so that

$$\Delta Y = \frac{\partial F}{\partial X} \Delta X + \cdot \cdot \cdot \cdot \cdot - N$$

where $\partial F/\partial X$ is the matrix formed by taking the partial derivatives of $F(X)$ with respect to X for each measurement. All higher-order terms are assumed to be zero.

Again, for purposes of error analysis, it is desired to obtain the sensitivities of ΔY to each error source. Therefore

$$y = Mx - u$$

where the notation has been adopted that

y = sensitivity (vector for multiple measurements) of the measurement state with respect to the i^{th} error source $\partial \Delta Y / \partial \epsilon_i$

$M = \partial F/\partial X$ is an $m \times n$ matrix in which m equals the number of functionally independent measurements at time t and $n = 9$

x = navigation sensitivity state vector $\partial X/\partial \epsilon_i$

u = unit vector (one for each measurement), as described in Section 2.3.3. (An explicit symbol EKlm is not given here.)

The Reset Equation is developed by letting

$$\hat{X}_R \triangleq \hat{X} - K\Delta Y \text{ (or) } \Delta X_R = \Delta X - K\Delta Y$$

where $\hat{X}_R$ is the estimated state vector after the measurement correction(s) and K is an n x m matrix developed in the sequel.

Taking the partial derivatives of this expression with respect to each error source, it is seen that the state vector sensitivity for each error source is changed when a measurement is made, that is

$$\delta\Delta X_R = \delta\Delta X - K\delta\Delta Y$$

or

$$\begin{aligned} x_R &= x - Ky \\ &= x - KMx + Ku \\ &= (I - KM)\, x + Ku \end{aligned}$$

NOTE: The notation and form of this equation are the same as for Kalman filtering (Reference 6), except that x and u are sensitivity (w. r. s. t. random variable) vectors, rather than vectors of random variables.

It remains to establish the matrix K in the above equation. Three methods are considered here, which are termed: replacement reset, deterministic reset, and linear statistical reset.

The development of each of these methods will now be described.

D.3 REPLACEMENT RESET

In using this technique, it is assumed that the measurement error is much smaller than the state vector error; thus, the measurement difference(s) (ΔY) is assumed to be dependent on the navigation state vector error only. The constraint of the correction is such as to make $\Delta Y_R = 0$, and therefore

$$\Delta Y_R = \hat{Y}_R - Y = 0 = M\Delta X_R - N$$

Taking the partial derivatives of this expression results in

$$y_R = Mx_R - u$$
$$= (I - MK)(Mx - u) \triangleq 0$$

In order for this condition to be satisfied for all x, the first term must equal zero. Therefore

$$MK = I$$

post multipling by $[MM^T]$

$$MK[MM^T] = MM^T$$

from which K is determined as

$$K = M^T[MM^T]^{-1}$$

K is sometimes referred to as the pseudo-inverse of M.

Thus, the reset sensitivity state vectors for all error sources active up to time t, based on this constraint, are

$$x_r = [I - KM]x$$

Additionally, new sensitivity state vectors are generated to account for each measurement error by

$$x_m = Ku$$

which are essentially initial condition errors at time t. x_r and x_m represent the total set of sensitivity vectors x, which are subsequently operated upon as independent sensitivity vectors. Thus, any operations including additional resets at time t would include x_m in the set of state vectors.

A few remarks about $[MM^T]^{-1}$ are in order here. For most measurement types considered (e.g., an individual position measurement, a stellar sensor sighting or a platform reset, or a position and/or velocity correction from the ground), $[MM^T]^{-1} = I$.

For individual velocity measurements and nonorthogonal measurements (e.g., simultaneous altitude and slant range measurements), $[MM^T]^{-1} \neq I$.

Functionally dependent measurements (such as altitude and slant range when the slant range vector is along the radius vector, and overdetermined measurements when two stellar sightings give four measurements of platform angle errors), result in singular inverses. The pseudo-inverse could be invoked for the altitude and slant range example, which would in effect result in the average of the two measurements. The case of the two stellar sightings should be reformulated by partitioning the state vector into the orientation elements only, from which the least squares solution ($[M^TM]^{-1}M^T$) can be used.

The question of sequential (at time t) reset vs simultaneous reset is of interest. The M matrix is formed from row vectors of partials. If the dot product of these vectors is zero, and the cross product is one, then sequential or simultaneous reset is equivalent. However, if both these conditions are not satisfied, the order of sequential reset is important.

An alternate and generally equivalent approach is one in which a transformation matrix is developed, which transforms the state vector in ECI coordinates into a state vector in the measurement coordinate system. The elements that are measured are set to zero, and the resultant state vector is transformed back by the inverse of the transformation matrix. This method suffers in that simultaneous multiple measurements usually cannot be included in the reset technique; therefore, multiple measurements would have to be handled sequentially and sometimes only in a specific order.

D.4 DETERMINISTIC RESET

In this method it is assumed that the measurement differences are solely dependent on an equal number of error sources. A typical example would be the use of a stellar tracker to derive the launch position errors of a mobile missile system, and thus compensate for the error. The technique developed here is not restricted to measurement types or the error sources to be considered. The concept is developed as follows. Let

$$\Delta X_D = D\epsilon_D$$

where

ΔX_D = the error in X due to the error source vector ϵ_D

D = an n x m matrix formed from the sensitivity vectors of ϵ_D

ϵ_D = an m vector of error sources to be determined from the measurement(s)

In using this technique, it is assumed that the measurement error is much smaller than the effect of the state vector error, and that the state vector error due to all other error sources is much smaller than the state vector error due to ϵ_D. Therefore, the measurement differences (ΔY) are assumed to be dependent on the error vector only, so that

$$\Delta Y_D = M\Delta X_D = MD\epsilon_D$$

$\therefore$

$$\epsilon_D = [MD]^{-1}\Delta Y_D$$

However, ϵ_D can only be estimated from ΔY (the quantity derived from the measurements), so that the estimated value of ϵ_D, $\hat{\epsilon}_D$ is

$$\hat{\epsilon}_D = [MD]^{-1}\Delta Y$$

The state vector estimate is corrected by using the estimate of ϵ_D, as follows

$$\hat{X}_R = \hat{X} - D\hat{\epsilon}_D = \hat{X} - D[MD]^{-1}\Delta Y$$

Thus, it is seen that for this correction scheme

$$K = D[MD]^{-1}$$

and

$$x_r = (I - KM)x$$

The sensitivity vector corrections follow the same procedure as for the replacement reset; each sensitivity vector is corrected at time t, and new sensitivity vectors are added to account for the measurement errors. The constraints on $[MD]^{-1}$ are that the measurements be functionally independent and that the error sources to be determined do not have equal sensitivities.

It is apparent from the state vector reset equation that the explicit determination of $\hat{\epsilon}_D$ is not required for corrections of the navigation data. Also, it can be shown that the reset sensitivity state vector for each element of the ϵ_D vector is set to zero at the reset time, provided that the sensitivity matrix D, used in the reset equations, is formed from the sensitivity vectors of the error analysis discussed in detail in the next paragraph. Thus, the navigation errors resulting from ϵ_D vector errors are zero at reset time, regardless of the magnitude of the ϵ_D vector (assuming the elements of ϵ_D are not large enough to invalidate the linearity assumption). If the ϵ_D vector were composed of initial condition elements only, the effects would be negated and new error sources formed by the measurement error vector. On the other hand, if the ϵ_D vector had elements representing forcing functions (such as accelerometers, drag, etc.), it might be desirable to use the estimate of ϵ_D to compensate those parameters, thereby reducing the variance of navigation errors following the reset time. For this procedure, it is necessary to determine the variance of the estimate error, which can be calculated, when desired, from the relationship

$$\hat{\epsilon}_D = [MD]^{-1} \Delta Y$$

$$= [MD]^{-1} \{M\Delta X_D + M\Delta X_{P-D} - N\}$$

$$= \epsilon_D + [MD]^{-1} M \sum_{i=1}^{P-D} \delta X_i \epsilon_i - [MD]^{-1} \sum_{j=1}^{M} \epsilon_j$$

where

ϵ_i are all error sources excluding the set ϵ_D

ϵ_j are the measurement errors

Thus, the estimate error is defined as

$$\Delta\epsilon_D = \hat{\epsilon}_D - \epsilon_D$$

and, assuming that the measurement errors are independent of the other error sources, the variance covariance of the estimate error can be calculated as

$$E(\Delta\epsilon_D \Delta\epsilon_D^T) = [MD]^{-1} \left\{ M \sum M^T + Q \right\} ([MD]^{-1})^T$$

where

> Σ is the covariance matrix of navigation state vector errors excluding ϵ_D parameters at reset time (before the measurement is included)
>
> Q is the covariance matrix of measurement errors at reset time

In the application of this method, the sensitivity matrix D would be obtained in one of the three following ways:

a. Input based on a nominal trajectory

b. Computed by using the normalized integral approach (which excludes the effects of gravity feedback in navigation equations)

c. Computed in a manner similar to the equations of this program.

Approach (a) should be adequate for most applications for the same reason that a nominal trajectory suffices for error analysis. Approach (b) more closely approximates the true sensitivities for a given trajectory and is not a difficult computation for an airborne computer. An analysis of either of the first two methods requires the generation of the D matrix from alternate runs, which would then be treated as input data. Method (c) is self-contained, but more complex for airborne computations.

D.5 LINEAR STATISTICAL RESET

This is equivalent to the Kalman filtering technique (Reference 6), but it is developed differently. Certain assumptions are made to facilitate the development of the problem at hand, but as the technique presented is a general one, assumptions need not be made. The one basic restriction made, on both the Kalman filtering technique and the method here, is that of the best linear estimate of the random variables in the sense of minimizing the mean square error of the estimate. The development uses the conditional expectation function directly. For the scalar case it is well known that

$$\hat{E}(X|Y) = \rho_{XY} \frac{\sigma_X}{\sigma_Y} Y$$

(The notation $\hat{E}$ is adopted to distinguish it from the normal operator E in the conditional expectation function, because of the restriction of best linear estimate. However, if the stochastic processes are Gaussian, the operators are equivalent. This same notation is used in Reference 6.)

$$\hat{E}(X|Y) = \frac{E(XY)\sigma_X}{\sigma_X \sigma_Y^2} Y = E(XY)[E(Y^2)]^{-1} Y$$

For X and Y vectors, it can be shown that the same form holds as

$$\hat{E}(X|Y) = E(XY^T)[E(YY^T)]^{-1} Y = KY$$

where the elements of K in this expression (an n x m matrix) are termed regression coefficients (Reference 7, Chapter 23, p 302).

For the problem at hand, it is desired to estimate ΔX, the error in the state vector estimate, given the measurement difference ΔY, or

$$\Delta\hat{X} = \hat{E}(\Delta X|\Delta Y) = E(\Delta X \Delta Y^T)[E(\Delta Y \Delta Y^T)]^{-1}\Delta Y$$

and based on this estimate, the state vector estimate would be corrected as follows

$$\hat{X}_R = \hat{X} - \Delta\hat{X} = \hat{X} - K\Delta Y$$

where

$$K = E(\Delta X \Delta Y^T)[E(\Delta Y \Delta Y^T)]^{-1}$$

As defined previously

$$\Delta Y = M\Delta X - N \quad , \quad \Delta Y^T = \Delta X^T M^T - N^T$$

and therefore

$$K = E(\Delta X \Delta X^T M^T - \Delta X N^T)[M\,E(\Delta X \Delta X^T)M^T - M\,E(\Delta X N^T) - E(N\Delta X^T)M^T + E(NN^T)]^{-1}$$

Assuming that the inertial navigation system parameters are independent of measurement errors, this reduces to

$$K = \left\{\sum M^T - D_n R_n(t,\tau)\right\}\left[M\sum M^T - M\,D_n R_n(t,\tau) - R_n^T(t,\tau)D_n^T M^T + R_n(t,t)\right]^{-1}$$

where

> Σ is the covariance matrix of navigation errors at reset time
>
> $R_n(t, \tau)$ is a matrix of time correlation functions (including cross-correlation terms) for the measurement errors
>
> D_n is a matrix that propagates the effects of measurements made at a previous time (or at the same time but processed sequentially)
>
> $R_n(t, t)$ represents the covariance matrix of measurement errors at time t.

If it is assumed that the measurements are independent (and time and time-cross-correlation functions are zero), the gain expression can be further reduced to

$$K = \Sigma M^T \left[M \Sigma M^T + Q \right]^{-1}$$

The effects of the previous assumptions can be determined, however, since the calculation of Σ in the program includes the effects of time and time-cross-correlated errors independent of the assumptions for obtaining K. The simplified gain computation represents a more realistic one for airborne use. Finally, the reset state vector sensitivity is

$$x_r = x - Ky$$

$$= [I - KM]x$$

which is in the same form as the previous methods. Additionally, a new vector is formed for each measurement, and the complete set of sensitivity vectors is handled in the same manner as Methods 1 and 2 (in Sections D-3 and D-4).

This reset technique could be extended to include the estimation of the error sources as follows

$$\hat{E}(\epsilon|\Delta Y) = E(\epsilon \Delta Y^T)[E(\Delta Y \Delta Y^T)]^{-1} \Delta Y$$

where ϵ = the error vector to be estimated, and

$$\Delta Y = M\Delta X - N$$
$$= MD\epsilon - N = H\epsilon - N$$

where D = the sensitivity matrix of all error sources.

Therefore

$$\hat{\epsilon} = \hat{E}(\epsilon|\Delta Y) = E[\epsilon\ \epsilon^T H^T - \epsilon N^T][E(\Delta Y \Delta Y^T)]^{-1} \Delta Y$$

Making the same assumptions as before, that $E(\epsilon N^T) = 0$,

$$\hat{\epsilon} = \sum_\epsilon H^T \left[M \sum M^T + Q\right]^{-1} \Delta Y$$

where

$\sum_\epsilon = E(\epsilon\ \epsilon^T)$ = covariance matrix of all error sources

$\sum$ can also be written as $D \sum_\epsilon D^T$

By using the above relationships, $\hat{\epsilon}$ can also be written

$$\hat{\epsilon} = [D^T D]^{-1} D^T K \Delta y = K_\epsilon \Delta y$$

provided that $[D^T D]^{-1}$ exists. If it is singular, then the pseudo-inverse, which always exists, can be developed. However, it is not required, since the original formula does not present this problem.

For purposes of an error analysis, it is necessary to derive the variance of the estimate error ($\Delta\epsilon = \hat{\epsilon} - \epsilon$), which is

$$E(\Delta\epsilon\Delta\epsilon^T) = E\{(\hat{\epsilon} - \epsilon)(\hat{\epsilon}^T - \epsilon^T)\}$$

and reduces to

$$\Sigma_{\epsilon R} = (I - K_\epsilon H)\Sigma_\epsilon$$

where $\Sigma_{\epsilon R}$ is the covariance matrix of the reset error source parameters.

There are several approaches that can be used to compensate the navigation data, given the estimate of the error vector; the most straightforward being to reset the sensor error equation.

It is seen that, in theory, the navigation data and error source parameters can be corrected, given one or more measurements. Thus, from the standpoint of minimum navigation error in the sense of the least-mean-squared-error under the constraint of a linear estimate, an optimum use of the data would incorporate both navigation data and error-source reset.

Practically, the task described would overburden any airborne computer, and in particular the computation of the D matrix. Although the estimate of $\hat{\epsilon}$ could be partitioned so that only a selected few would need to be estimated, all sensitivities are required (or should be) for the calculation of Σ. For that reason it is assumed that calculation of the sensitivities (D matrix elements) would be based on the nominal trajectory; therefore, it would be precomputed and inserted in the airborne computer for flight data processing.

In the special case of free flight (orbit navigation), most forcing functions reduce to zero; and the problem can be reformulated so that Σ can be updated by using the transition matrix in conjunction with those forcing functions acting in orbit (drag, gravity model constants, gyro drift, control system impulses, etc.). The capability to perform this last analysis, that is, orbit reset or orbit navigation, is presently under development in a separate program. A report describing it will be published by J. Meditch of Aerospace Corporation.

D. 6 M MATRIX GENERATION

The M matrix is assumed to be computed by the airborne computer and based on the functional relationships of the measured quantities (Y)* and the navigation data ($\bar{X}$, $\dot{\bar{X}}$, M_{EP}). Each measurement represents a row of the M matrix and is developed as follows for the measurements considered.

D. 6. 1 Altitude

$$Y = h = R - R_e = \sqrt{X^2 + Y^2 + Z^2} - R_e$$

$$\delta Y = \frac{\partial Y}{\partial \bar{X}} \delta X = \frac{\bar{X}^T}{R} \delta \bar{X}$$

and

$$m = \begin{bmatrix} \frac{X}{R} & \frac{Y}{R} & \frac{Z}{R} & 0\ 0\ 0 & 0\ 0\ 0 \end{bmatrix}$$

where it has been assumed that $\partial R_e / \partial X = 0$

D. 6. 2 Slant Range to a Ground Station

The slant range vector is defined as

$$\begin{aligned} \bar{S} &= \bar{R}_G - \bar{R} \\ &= (X_G - X)\bar{X}_U + (Y_G - Y)\bar{Y}_U + (Z_G - Z)\bar{Z}_U \\ &= X_S\bar{X}_U + Y_S\bar{Y}_U + Z_S\bar{Z}_U \end{aligned}$$

*See comment on notation at end of Section D-2.

where

$$X_G = R_e \cos\lambda_G \cos\phi_G$$

$$Y_G = R_e \cos\lambda_G \sin\phi_G$$

$$Z_G = R_e \sin\lambda_G$$

$$R_e = \frac{A(1 - e)}{\sqrt{1 + (e^2 - 2e)\cos^2\lambda_G}}$$

and the terms in this equation are as defined in Section 2. 3. 3.

$$Y = S = \sqrt{X_S^2 + Y_S^2 + Z_S^2}$$

$$\delta Y = \frac{\partial Y}{\partial X}\delta X = -\frac{X_S^T}{S}\delta\bar{X}$$

$$m = \left[-\frac{X_S}{S} \; -\frac{Y_S}{S} \; -\frac{Z_S}{S} \;\; 0\;0\;0 \;\; 0\;0\;0\right]$$

It is assumed that station locations are perfectly known, and also that timing errors are zero.

D. 6. 3 Position Vector

D. 6. 3. 1 ECI Coordinates

$$m = \left[I_{(3\times3)} 0_{(3\times3)} 0_{(3\times3)}\right]$$

D. 6. 3. 2 Local Coordinates

$$m = \left[M_{LE} \, 0_{(3\times3)} \, 0_{(3\times3)} \right]$$

where M_{LE} is defined as in Section 2. 4. 1

D. 6. 4 Altitude Rate

$$Y = \dot{h} = \frac{dR}{dt} = \dot{R} = \frac{\overline{X}^T}{R} \dot{\overline{X}} = \frac{X\dot{X} + Y\dot{Y} + Z\dot{Z}}{R}$$

$$\delta\dot{h} = \frac{\overline{X}^T \delta\dot{\overline{X}}}{R} + \frac{\dot{\overline{X}}^T \delta\overline{X}}{R} - \frac{\dot{R}\delta R}{R}$$

$$= \frac{1}{R^2} \left[(R\dot{\overline{X}}^T - \dot{R}\overline{X}^T)\delta\overline{X} + R\overline{X}^T \delta\dot{\overline{X}} \right]$$

from which

$$m = \left[\frac{R\dot{X} - X\dot{R}}{R^2} \;\; \frac{R\dot{Y} - Y\dot{R}}{R^2} \;\; \frac{R\dot{Z} - Z\dot{R}}{R^2} \;\; \frac{X}{R} \;\; \frac{Y}{R} \;\; \frac{Z}{R} \;\; 0 \; 0 \; 0 \right]$$

D. 6. 5 Slant Range Rate

$$Y = \frac{dS}{dt} = \frac{\overline{X}_S^T \dot{\overline{X}}_S}{S} = \frac{X_S\dot{X}_S + Y_S\dot{Y}_S + Z_S\dot{Z}_S}{S}$$

where

$$\dot{\overline{X}}_S = \dot{X}_S\overline{X}_U + \dot{Y}_S\overline{Y}_U + \dot{Z}_S\overline{Z}_U$$

$$= (\dot{X}_G - \dot{X})\overline{X}_U + (\dot{Y}_G - \dot{Y})\overline{Y}_U + (\dot{Z}_G - \dot{Z})\overline{Z}_U$$

$$= (-\omega_e Y_G - \dot{X})\overline{X}_U + (\omega_e X_G - \dot{Y})\overline{Y}_U - \dot{Z}\overline{Z}_U$$

$$\delta\dot{S} = -\frac{1}{S^2}\left[\left(S\dot{\overline{X}}_S^T - \dot{S}\overline{X}_S^T\right)\delta\overline{X} + S\overline{X}_S^T\delta\dot{\overline{X}}\right]$$

and the form of m is the same as for altitude rate with the substitutions of $\overline{X} = \overline{X}_S$, $\dot{\overline{X}} = \dot{\overline{X}}_S$, $R = S$, and $\dot{R} = \dot{S}$. Station location and timing errors are assumed to be zero.

D. 6. 6 Velocity Vector

D. 6. 6. 1 ECI Coordinates

$$m = \left[0_{(3\times3)} I_{(3\times3)} 0_{(3\times3)}\right]$$

D. 6. 6. 2 Local Coordinates

$$m = \left[0_{(3\times3)} M_{LE} 0_{(3\times3)}\right]$$

D. 6. 7 Stellar Sensor Measurement

It is assumed that the sensor is mounted on the platform and can be slewed in azimuth about the platform 1-axis, and in elevation about the tracker 2-axis to sight on a prescribed star. Figure D-1 shows the tracker axes system with respect to the platform axes. The transformation between a vector in platform

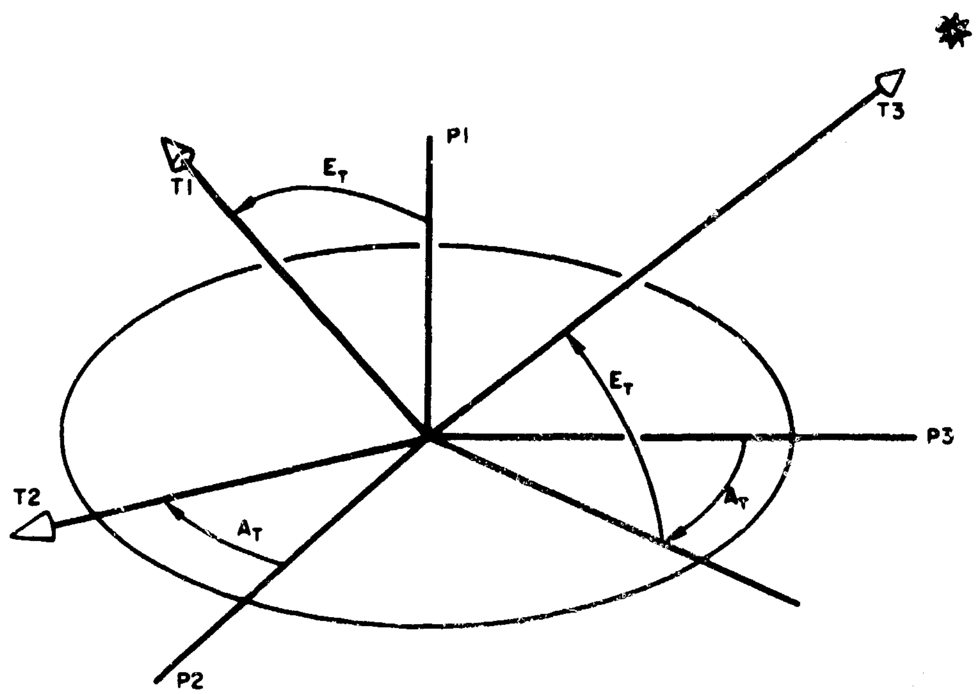

Figure D-1. Stellar Sensor (Tracker) - Coordinate System coordinates and tracker coordinates is

$$M_{TP} = \begin{bmatrix} CE_T & 0 & -SE_T \\ 0 & 1 & 0 \\ SE_T & 0 & CE_T \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & CA_T & -SA_T \\ 0 & SA_T & CA_T \end{bmatrix}$$

where

A_T is the azimuth of the star expressed in nominal platform coordinates

E_T is the elevation of the star expressed in nominal platform coordinates

The tracker is capable of measuring coordinates (small angles) only about its 2-axis (a coordinate along the T1 axis), and about its 1-axis (a coordinate along the T2 axis). Alternately, it can measure the changes in A_T and E_T, which make the above coordinates zero. Mathematically either measurement type is equivalent. Since the platform rotation errors are assumed to be small, they can be treated as vectors, so that

$$\begin{bmatrix} \phi_{1T} \\ \phi_{2T} \end{bmatrix} = \begin{bmatrix} CE_T & -SE_T SA_T & -SE_T CA_T \\ 0 & CA_T & -SA_T \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$$

where

ϕ_{1T} is a measurement along the sensor's 2-axis

ϕ_{2T} is a measurement along the sensor's 1-axis

Therefore, for a single stellar fix with a stellar sensor, two angles can be used to correct the effect of platform errors. In this case

$$m = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & CE_T & -SE_T SA_T & -SE_T CA_T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & CA_T & -SA_T \end{bmatrix}$$

For the case of two independent stellar sightings (two different stars, preferably 90° apart and in the platform's $2_p 3_p$ plane), the platform orientation errors are overdetermined. In this case, least squares or other techniques could be used to process the four measurements so that the platform error angles were determined explicitly. This is implied in the last measurement category (Section D. 6. 9), Platform Error Vector.

D. 6. 8 Horizon Sensor Measurements

A horizon sensor primarily measures small-angle deviations of its mount with respect to local vertical; the measurements can also be processed to indicate altitude. Conventionally, it is mounted on the vehicle frame and used as a reference for the vehicle control system, while maintaining small-angle deviations of the vehicle axes with respect to local vertical.

In conjunction with platform gimbal-angle readout (direction cosines in the case of strapped-down inertial systems) and the navigation system position data, the angles measured by the sensor can be used to correct the platform angular errors, navigation system position errors, or both. For purposes of a general analysis of the use of the horizon sensor, the local horizontal coordinate system will be used. It is assumed that the sensor measures pitch (rotations about the Z-axis of the local horizontal system) and roll (rotations about the X-axis). Transformed into this system, the state vector error is

$$\overline{\Delta X}_L = \underline{M_{LE}}\,\overline{\Delta X}$$

where $\underline{M_{LE}}$ is as described in Section 2. 4. 1.

Under these assumptions, the sensor measures

$$\Delta\theta = -\frac{\Delta X}{R} + \phi_Z - n_\theta \quad \text{pitch measurement}$$

$$\Delta\phi = \frac{\Delta Z}{R} + \phi_X - n_\phi \quad \text{roll measurement}$$

where

ΔX = range error in local coordinates

ΔZ = cross-range error in local coordinates

$\phi_{Z'}$ = platform error about Z in local coordinates

ϕ_X = platform error about X in local coordinates

n_θ and n_ϕ = pitch and roll sensor errors.

Thus

$$m = \begin{bmatrix} -\frac{1}{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{1}{R} & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \underline{M_{LE}}$$

D. 6. 9 Platform Error Vector

In this case, it is assumed that the platform error angles are derived from multiple stellar sensor measurements and expressed in platform coordinates. Therefore

$$m = \left[0_{(3\times3)} \, 0_{(3\times3)} \, M_{PE} \right]$$

The M matrix for any given reset is then constructed, based on any one or more of the m matrices presented above. The combinations, however, must be limited so that elements of the state vector are not overdetermined when the Method 1-type reset is used.

D.7 RESET EQUATION SUMMARY

This section summarizes the pertinent equations presented for reset. Those marked with an asterisk could be conveniently mechanized in the error analysis program.

	$\hat{X} = X + \Delta X$	navigation system estimate of the state vector
	$\Delta Y = \hat{Y} - Y = M\Delta X - N$	navigation system processed measurement difference
	$\hat{X}_R = \hat{X} - K\Delta Y$	navigation system reset estimate of the state vector given the measurement(s) Y
	$\Delta X_R = \Delta X - K\Delta Y$	state vector error following reset
*	$x_r = [I - KM]x$	reset error-source sensitivity vector(s) for all error sources active prior to current reset time
*	$x_m = Ku$	added sensitivity vector(s) to account for the reset measurement error
*	$M = \frac{\partial F}{\partial X}$	measurement sensitivity matrix (see Section D.6)

K is determined by one of the following 3 methods:

a. Replacement

* $K = M^T[MM^T]^{-1}$

b. Deterministic

* $K = D[MD]^{-1}$

where D is formed from the prescribed state vector sensitivities or is input.

$\hat{\epsilon}_D = [MD]^{-1}\Delta Y$	navigation system estimate of the prescribed error sources, given the measurement(s) Y.
$\Delta\epsilon_D = \hat{\epsilon}_D - \epsilon_D$	error of the estimated error sources

$$\Sigma_{\epsilon D} = [MD]^{-1}\left\{M \Sigma M^T + Q\right\}([MD]^{-1})^T$$

covariance matrix of the estimated error sources

where

Σ is the covariance matrix of the navigation state vector errors due to all error sources excluding ϵ_D parameters at reset time (before the measurement is included)

Q is the covariance matrix of measurement error(s) at reset time

c. Linear Statistical

$$* \quad K = \Sigma M^T \left[M \Sigma M^T + Q\right]^{-1}$$

where

Σ is the covariance matrix of the navigation state vector error due to all error sources at reset time

Q is the covariance matrix of measurement error(s) at reset time

and Σ and Q are input.

$$\hat{\epsilon} = \Sigma_{\epsilon} [MD]^T \left[M \Sigma M^T + Q\right]^{-1} \Delta Y$$

navigation system estimate of the error sources, given the measurement Y. ϵ could be partitioned so that only a selected few need be explicitly derived, e.g., $\hat{\epsilon}_D$ as in Method 2

where

D is the sensitivity matrix of all (or partitioned) error sources

Σ_{ϵ} is the covariance matrix of all (or partitioned) error sources

$$\Sigma_{\epsilon R} = [I - K_{\epsilon} MD] \Sigma_{\epsilon}$$

where

$\Sigma_{\epsilon R}$ = the covariance matrix of error sources after reset

$$K_{\epsilon} = \Sigma_{\epsilon} [MD]^T \left[M \Sigma M^T + Q \right]^{-1}$$

APPENDIX E

DRAG ERRORS

APPENDIX E

DRAG ERRORS

In this appendix is discussed the proposed method for estimating the effects of atmospheric drag errors, when the accelerometers are disconnected during orbital flight phases.

If there were no forces experienced by the vehicle during coast periods (parking orbits, transfer orbits, etc.), the most advantageous way to operate the navigation system would be to disconnect the accelerometers during these periods so that the accelerometer bias error would not be integrated. For low-altitude orbits (in the region of 100 n mi), aerodynamic drag force is not negligible, but is generally of the same order of magnitude as accelerometer bias. Consequently, it must be decided either to measure drag via the accelerometer, or to predict it by an empirical formula. The decision would be contingent on which method would result in the least error, i.e., on the uncertainty of accelerometer bias vs the uncertainty of drag calculations. To fully answer this question, a detailed knowledge of the configuration and flight time (function of time of day, month, and year) is necessary. For purposes of the error analysis, these characteristics are generalized so as to assess the relative importance of drag and thereby determine if more detail is required. Therefore, the configuration's ballistic coefficient and the parameters of the atmospheric density model are treated as random variables, with assumed means and standard deviations.

When utilizing a drag model for calculating the sensed acceleration, the equations of motion become

$$\ddot{\bar{X}} = - \frac{\mu}{R^3} \bar{x} + \bar{A}_D$$

where

$\bar{A}_D$ = vector of drag accelerations

$$= - \frac{\left(\frac{C_D S}{m} q\right)}{V} \dot{\bar{X}}$$ (assuming that drag acts along the negative inertial velocity vector)

where

C_D = drag coefficient

S = reference area (ft^2)

m = system mass (slugs)

q = dynamic pressure (lb/ft^2)

$= 1/2\, \rho V^2$

where

ρ = atmospheric density ($slug/ft^3$)

V = magnitude of inertial velocity vector (assuming magnitude of inertial velocity equals magnitude of relative velocity)

$$\therefore \quad \bar{A}_D = - \left(\frac{g_o}{2B} \rho V\right) \dot{\bar{X}} = - A_D \dot{\bar{X}}$$

where

$B = \frac{W}{C_D S}$ = ballistic coefficient

g_o = reference gravity constant (= 32.174)

W = vehicle weight (lb)

The linearized differential equations are

$$\delta\ddot{\bar{X}} = M_G\delta\bar{x} - \delta A_D\dot{\bar{X}} - A_D\delta\dot{\bar{X}}$$

where M_G is as defined in Section 2.3.2

and

$$\delta A_D = A_D\left(\frac{\delta V}{V} + \frac{\delta\rho}{\rho} - \frac{\delta B}{B}\right)$$

The first term in δA_D is derived as follows

$$V = (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)^{1/2}$$

$$\delta V = \frac{\dot{X}}{V}\,\delta\dot{X} + \frac{\dot{Y}}{V}\,\delta\dot{Y} + \frac{\dot{Z}}{V}\,\delta\dot{Z} = \frac{\dot{\bar{X}}^T\delta\dot{\bar{X}}}{V}$$

The second term can be approximated at any given altitude as

$$\rho(h) = \rho(h_o) + \left.\frac{\partial\rho}{\partial h}\right|_{h_o}(h - h_o) = K_1(h_o) + K_2(h_o)h$$

where h_o is the reference altitude, thus $\delta\rho = \delta K_1 + \delta K_2 h + K_2\delta h$.

In general, density variations are sufficiently homogeneous in a region so that $\delta K_2 = \partial\rho/\partial h|_{h_o} \doteq 0$; and δK_1 can be expressed as a percentage of $\rho(h_o)$; i.e., $\delta K_1 = (\delta\rho(h_o)/\rho(h_o))\rho(h_o)$

$$\therefore \qquad \frac{\delta\rho}{\rho} = \frac{\delta\rho(h_o)}{\rho(h_o)} + \frac{1}{\rho}\frac{\partial\rho}{\partial h}\,\delta h$$

Now h is obtained from

$$h = R - R_e$$

where

$$R_e = \frac{A(1 - e)}{(1 + (e^2 - 2e)\cos^2\lambda)^{1/2}}$$

$$\cos^2\lambda = \frac{X^2 + Y^2}{R^2} \quad \text{(geocentric latitude)}$$

$$e = \frac{1}{298.3} \quad \text{(ellipticity)}$$

$$A = \text{equatorial radius}$$

$$\therefore \qquad \delta h = \delta R \qquad (\delta R_e \approx 0)$$

$$= \frac{\overline{X}^T \delta\overline{X}}{R}$$

Thus

$$\frac{\delta\rho}{\rho} = \frac{\delta\rho(h_o)}{\rho(h_o)} + \frac{1}{\rho}\frac{\partial\rho}{\partial h}\left(\frac{\overline{X}^T \delta\overline{X}}{R}\right) .$$

The third term is simply

$$B = \frac{W}{C_D S}$$

$$\delta B = -B\,\frac{\delta C_D}{C_D} \qquad (\delta W = \delta S = 0)$$

Combining these results into the linearized differential equations results in

$$\delta\ddot{\bar{X}} = M_G\delta\bar{X} - A_D\left(\frac{\dot{\bar{X}}^T\delta\dot{\bar{X}}}{V^2} + \frac{1}{\rho}\frac{\partial\rho}{\partial h}\left(\frac{\bar{X}^T\delta\bar{X}}{R}\right) + \frac{\delta\rho(h_o)}{\rho(h_o)} + \frac{\delta C_D}{C_D}\right)\dot{\bar{X}} - A_D\delta\dot{\bar{X}}$$

$$= [M_G + M_{\dot{x}x}]\delta\bar{X} + M_{\dot{x}\dot{x}}\delta\dot{\bar{X}} + \bar{F}_A$$

where

$$M_{\dot{x}x} = -\frac{1}{\rho}\frac{\partial\rho}{\partial h}A_D V\left[\frac{\dot{\bar{X}}\bar{X}^T}{RV}\right]$$

$$M_{\dot{x}\dot{x}} = -A_D\left[\frac{\dot{\bar{X}}\dot{\bar{X}}^T}{V^2} + I\right]$$

$$\bar{F}_A = -(EQOO)A_D\dot{\bar{X}}$$

In this particular case, EQOO is a scalar and equals 1 when deriving the sensitivity of aerodynamic forces (i.e., due to the rss value of $\delta\rho(h_o)/\rho(h_o) + \delta C_D/C_D$).

When considering relatively short time durations (less than 2 orbits) in a drag environment (a 100-mile orbit), these expressions can be further simplified as

a. the coefficient of M_G is $\mu/R^3 \doteq 1.4 \times 10^{-6}$ and

$$M_{\dot{x}x} = \frac{1}{\rho}\frac{\partial\rho}{\partial h} A_D V \doteq \frac{8.10^{-8}}{B}$$

for B in the range $10 < B < 100$

$$M_G >> M_{\dot{x}x}$$

$$\therefore \qquad M_{\dot{x}x} \doteq 0$$

b. the coefficient of $M_{\dot{x}\dot{x}} = A_D \doteq 4 \times 10^{-7}/B < 4 \times 10^{-8}$ and

$$\delta\bar{x} >> \delta\dot{\bar{x}}$$

so that $M_G\ \delta\bar{x} >>> M_{\dot{x}\dot{x}}\ \delta\dot{\bar{x}}$

$$\therefore \qquad M_{\dot{x}\dot{x}} \doteq 0$$

The validity of these assumptions can always be ascertained by utilizing Reference 9, which includes these effects as well as the first earth oblateness term (J_2). The intent here is only to assess the relative importance of atmospheric effects, so that when the results indicate that more accuracy is required, Reference 9 will be used for detailed studies. Thus, the equations for drag uncertainty reduce to the same differential equations as those for the other error sources, with the forcing function of

$$\bar{F}_A = -\ (EQOO) A_D \dot{\bar{X}}$$

where

$$A_D = \left(\frac{g_o}{2B}\rho V\right)$$

$$B = \frac{W}{C_D S} \text{ (an input constant)}$$

$$\log \rho = f(h) + K_B \text{ (an input table plus a scaling constant)}$$

$$\rho = e^{2.30258 \log \rho}$$

It remains to establish methods for specifying the mean values of B and ρ, and some estimate of their deviations. With the assumptions that the region of interest is in orbital velocities at greater than 80 miles, and the configurations of interest are upper stages with payloads attached, the drag coefficient for zero angle of attack and molecular flow can be approximated by

$$C_D = C_{DF} + 0.256 \frac{L}{D}$$

where C_{DF} is the drag coefficient for the shape of the payload section. Typical values are

flat plate $C_{DF} = 2.11$

20° cone $C_{DF} = 2.04$

15° cone $C_{DF} = 2.03$

L = the length of the cylindrical section (excluding cone sections)

D = the diameter of the cylindrical section

Using this formulation of drag coefficient produces approximately a 20-percent uncertainty in its magnitude, provided the assumptions of molecular flow and zero angle of attack are maintained.

The upper atmospheric density is affected by many parameters, by far the most by solar activity. In Reference 10 are discussed the various factors that influence the density and they are summarized as follows:

a. Diurnal (Day-Night Effect). The density varies with the time of day, having its minimum at night and maximum at approximately 2:00 P.M. local standard time. The effect is a function of the angle between the earth sun line and the radius vector and, therefore, depends on lattitude as well as longitude. The effect is small (15 percent) at low altitude (100 n mi) and increases with altitude to more than 100 percent at 200 n mi (utilizing Eq. (10) in the Jacchia formula, given in Reference 11).

b. Solar Activity (11-Year Cycle). There is still a rather large discrepancy in models for this effect, as evidenced by the curves presented in Reference 10: Figure 1 for the Jacchia 1960 model and Figure 2 for the Paetzold 1962 model. However, there is reasonably good agreement in the low-altitude region. The variation between the average densities during active and quiet periods is a factor of 3 and related to the decimetric flux (specifically, the 10.7-cm radiation). Jacchia's model relates density directly, resulting in a factor of 3 for all altitudes. Paetzold's model results in factors greater than 10 at 200 n mi that generally increase with altitude. In addition to the 11-year cycle, there are 27-day cycles and semiannual and annual cycles, which result in approximately 25-percent variations at 100 n mi and also increase with altitude.

c. Magnetic Storms. These are generally unpredictable, but their effects are relatively short-lived, lasting for only a few days. The effects are proportional to the storm's intensity and can vary as much as 40-percent at 100 n mi and much more at higher altitudes.

Based on the above, and on the premise that 100 n mi is the principal altitude for parking orbits, etc., the nominal (mean) atmospheric density model was conservatively chosen as the 1959 ARDC (see Table E-1). It can be

Table E-1. Nominal Atmospheric Density - ARDC 1959 Model

Altitude (n mi)	Altitude (ft)	Log Density ($slug/ft^3$)	Density ($slug/ft^3$)
79^+	480×10^3	-11.4264	4.116×10^{-12}
100^+	608×10^3	-11.9914	1.020×10^{-12}
140^-	850×10^3	-12.6960	2.014×10^{-13}
181^+	1.1×10^6	-13.3055	4.949×10^{-14}
140^+	1.46×10^5	-14.0444	9.028×10^{-15}
296^+	1.8×10^6	-14.6315	2.336×10^{-15}

scaled up or down to include the average solar activity effects.* Combining all the effects of density and drag coefficient uncertainties, so as to estimate a standard deviation for purposes of assessing the relative importance of aerodynamic effects, a conservative value of 0.2 (standard deviation of EQOO) is recommended.

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*Other density models can be easily input, if required, for which Figure E- presents curves.

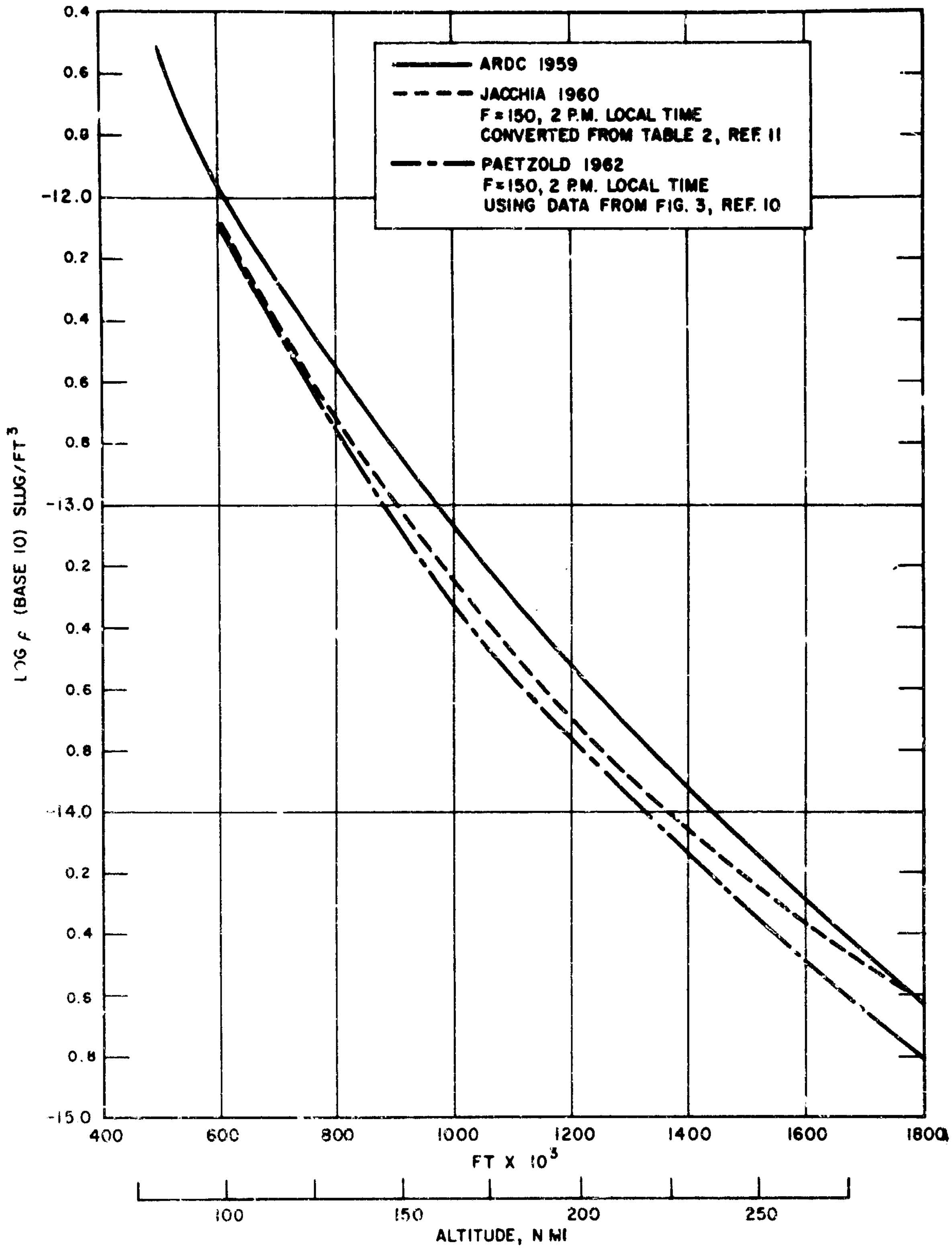

Figure E-1. Log Density vs Altitude Curves

APPENDIX F

FIGURES

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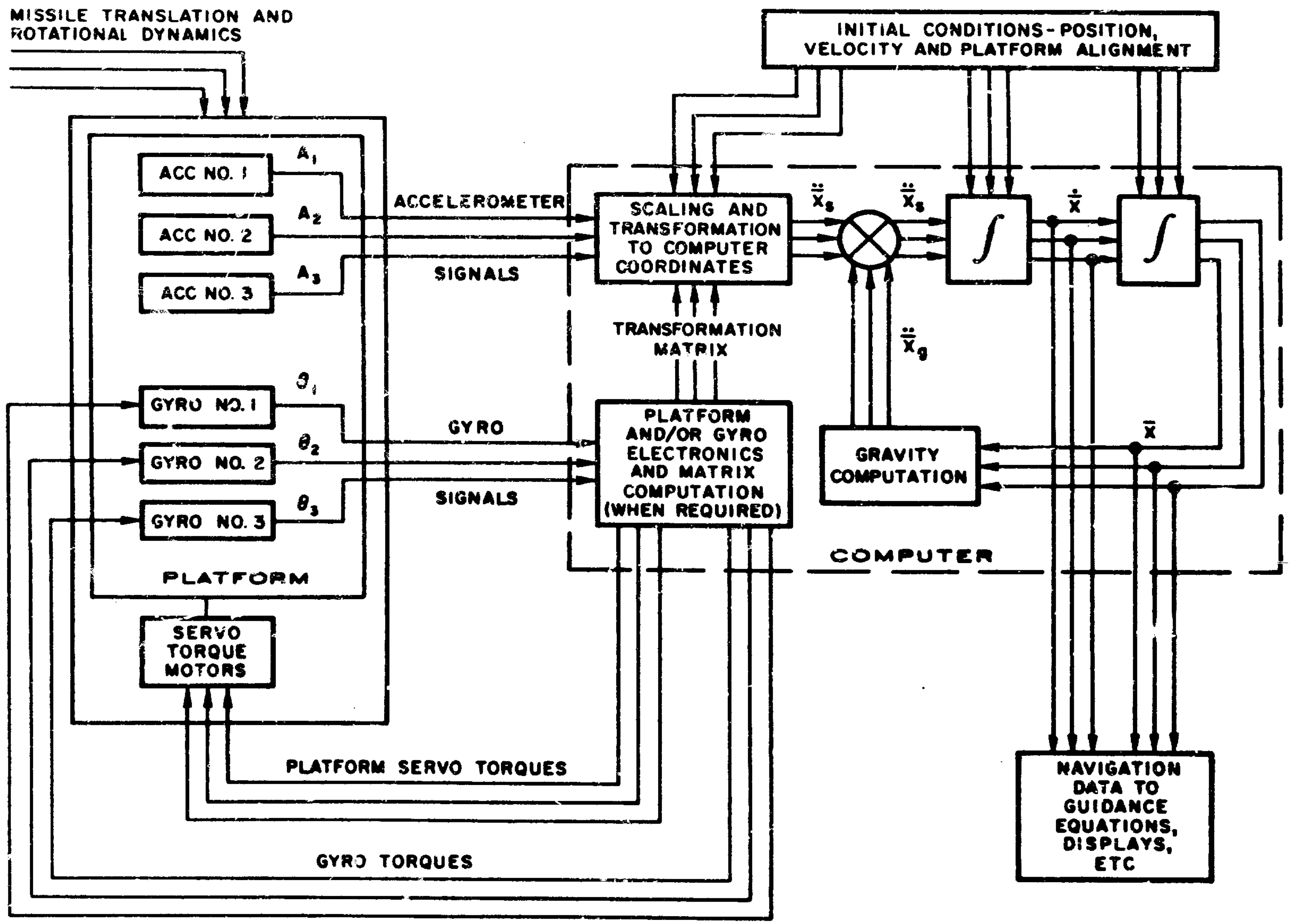


Figure 1. Schematic of Navigation System Configuration

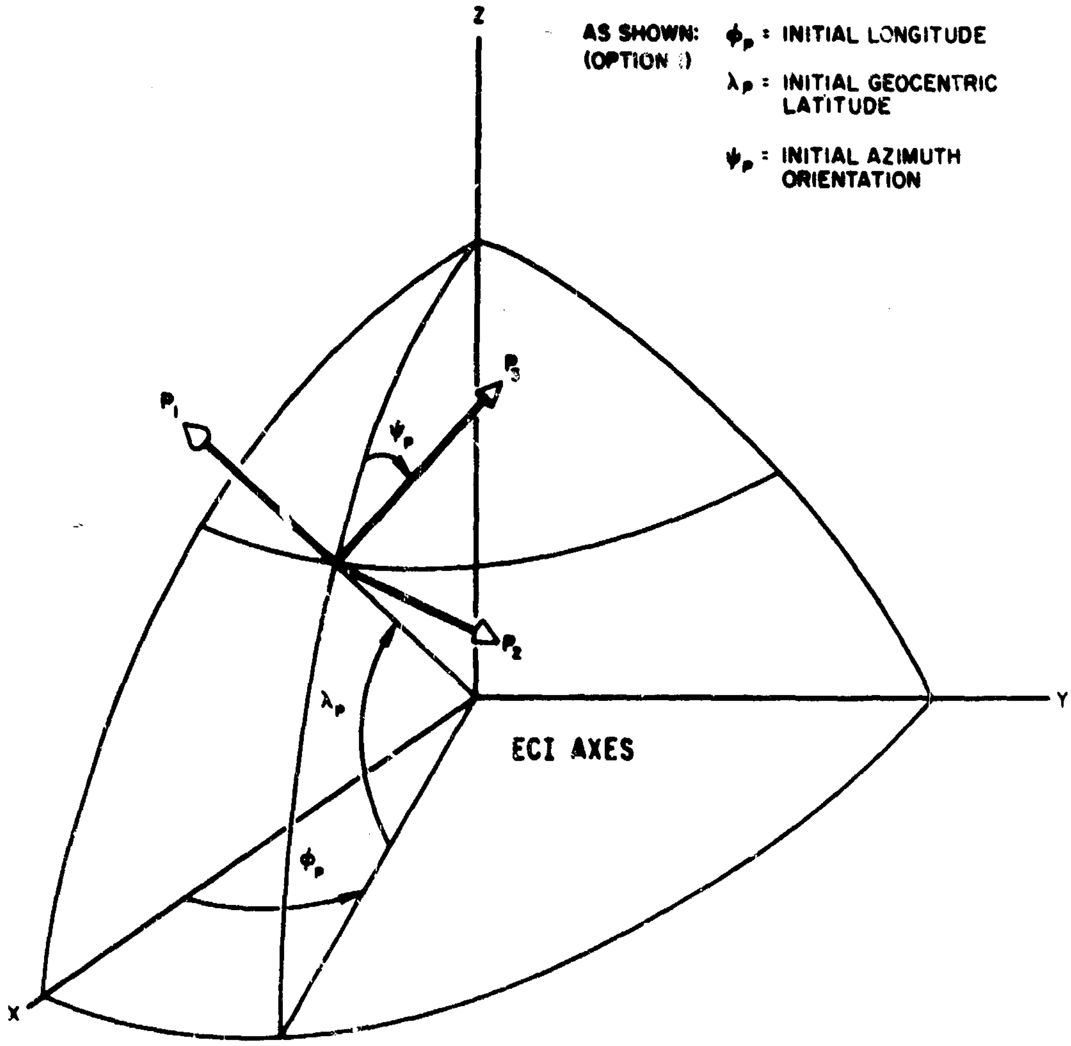


Figure 2. Initial Platform Orientation

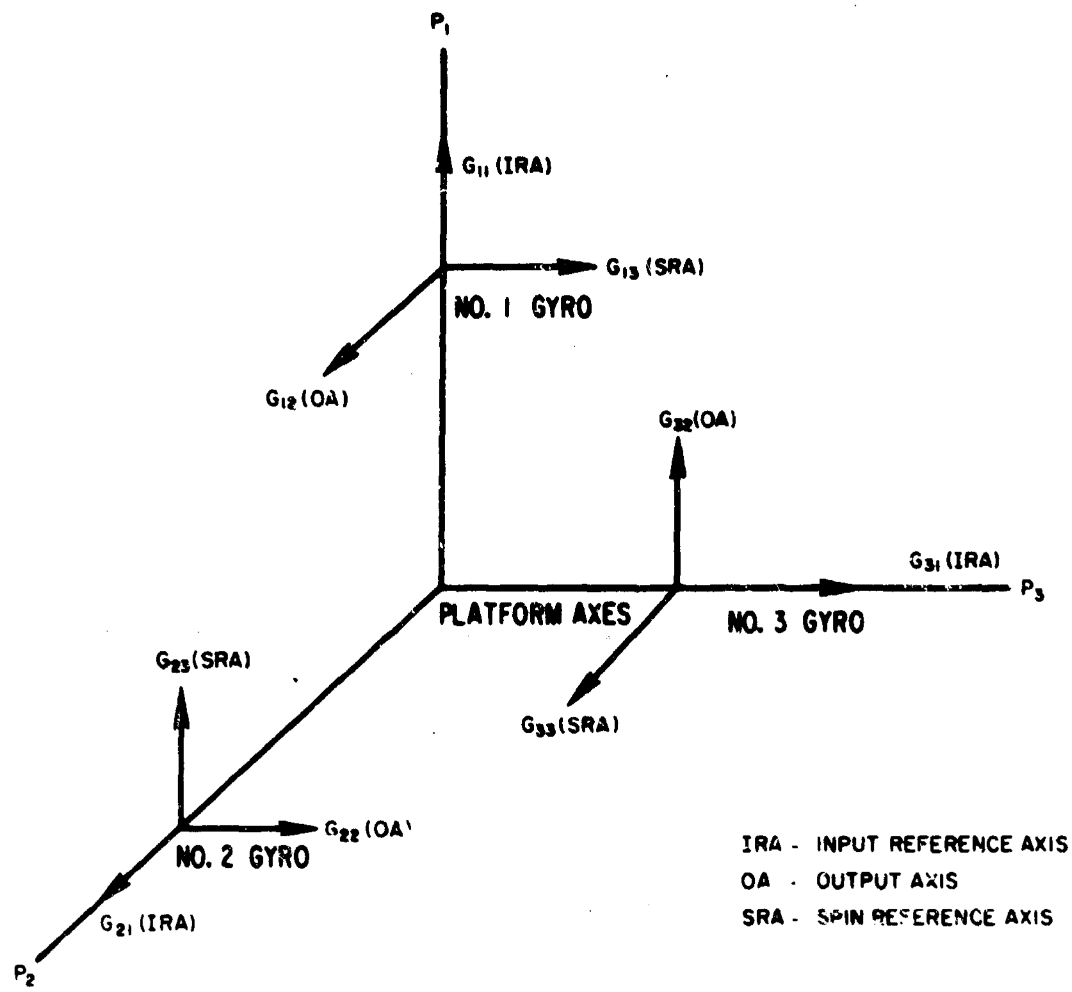


Figure 3. Initial Gyro Orientation

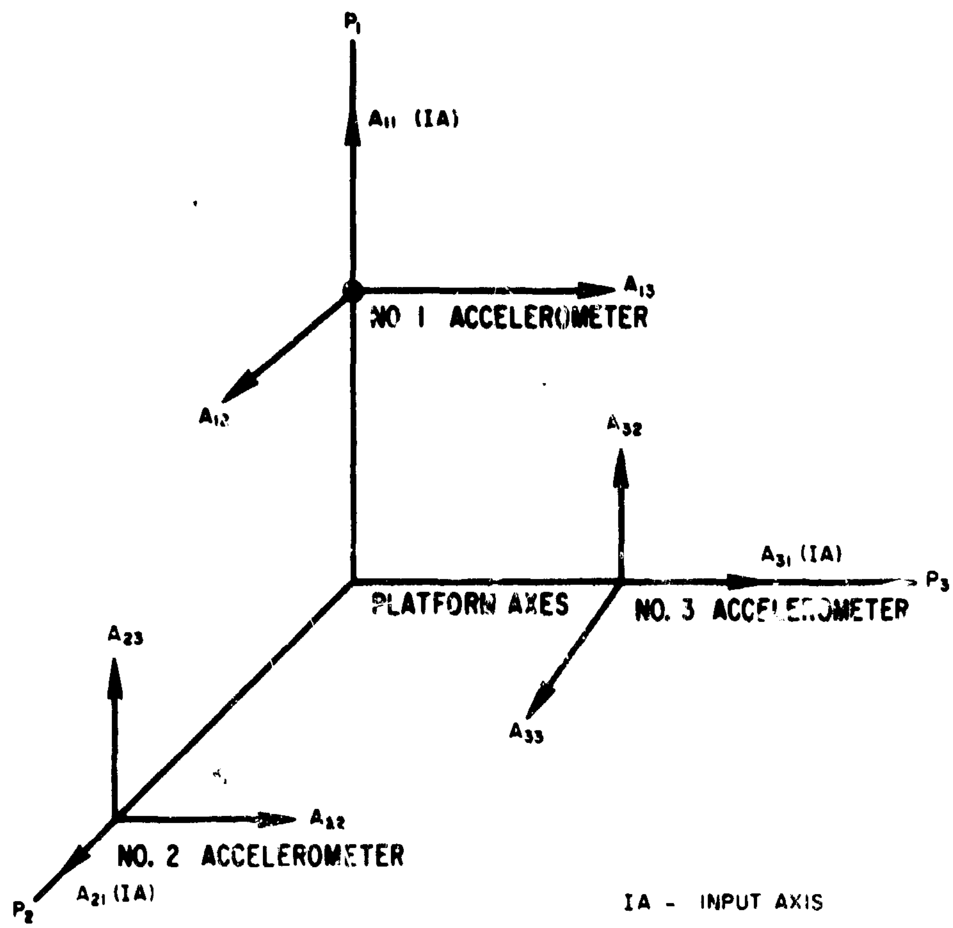
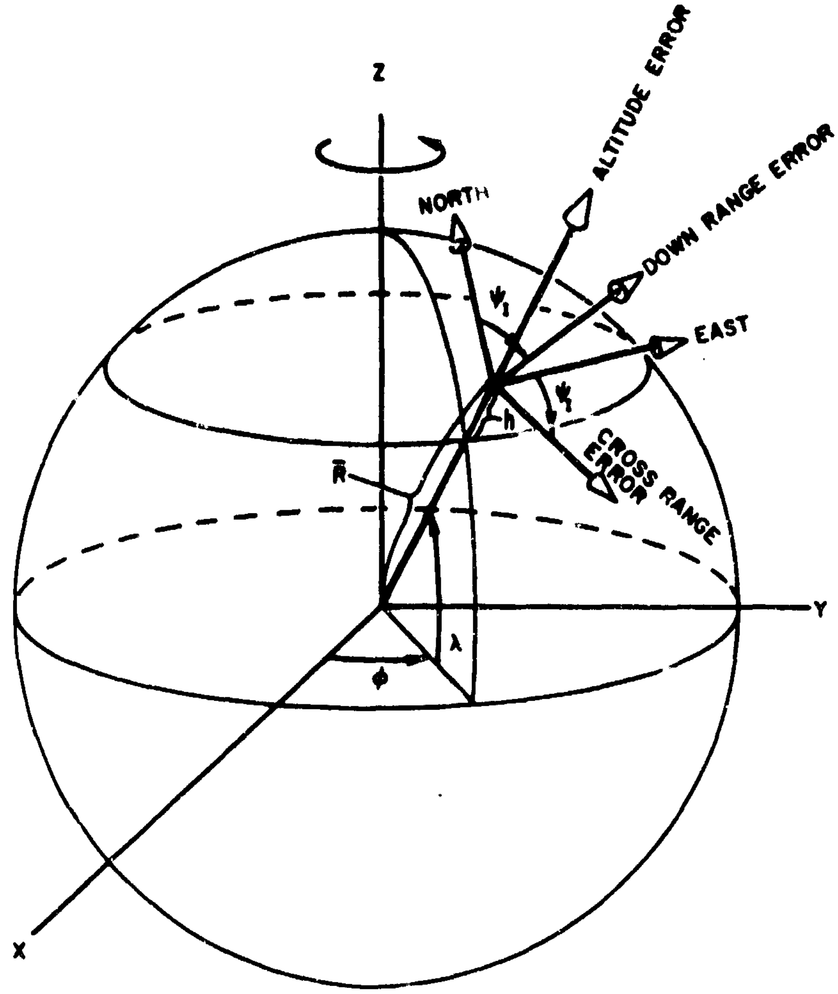


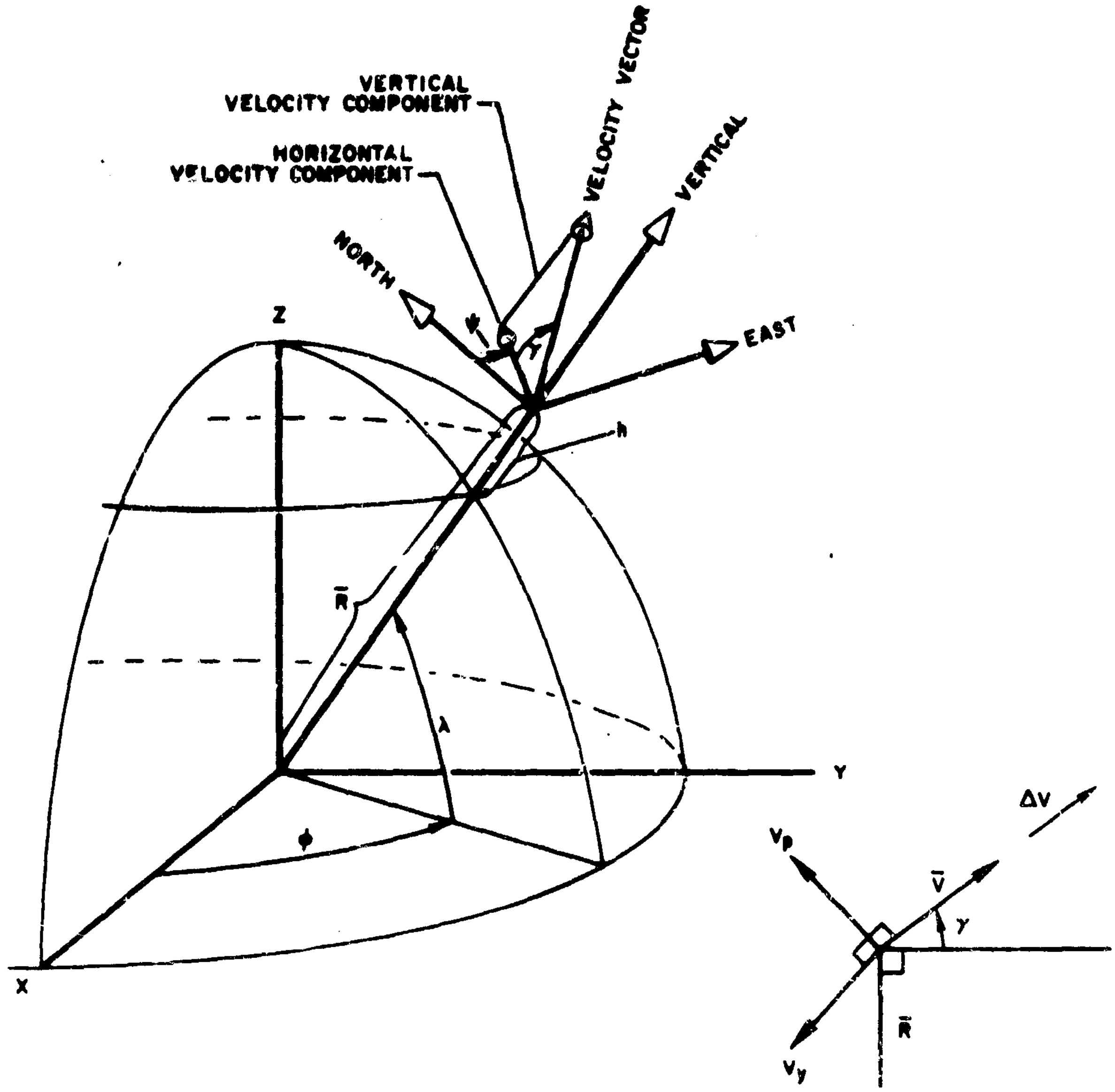
Figure 4. Initial Accelerometer Orientation – Orthogonal Configuration



AS SHOWN, INITIAL CONDITION ORIENTATION OPTION 2

- ϕ INITIAL LONGITUDE
- λ INITIAL GEOCENTRIC LATITUDE
- $\bar{R}$ INITIAL RADIUS VECTOR
- h INITIAL ALTITUDE
- ψ_I REFERENCE AZIMUTH FOR INITIAL CONDITION DOWN-RANGE AND CROSS-RANGE ERRORS MEASURED POSITIVE FROM NORTH TOWARDS EAST

Figure 5. Coordinate System for Initial Condition Errors



DOWN RANGE IS DEFINED ALONG THE PROJECTION OF VELOCITY VECTOR ONTO HORIZONTAL PLANE

V_p IS PITCH COMPONENT OF VELOCITY ERROR

V_y IS YAW COMPONENT OF VELOCITY ERROR

ΔV IS VELOCITY MAGNITUDE ERROR

Figure 6 Coordinate System for Terminal Condition Errors

APPENDIX G

PROGRAM DEFINITIONS AND CONSTANTS

CONTENTS

Table G-1. Error Sources

Symbol	Description	Units
	Initial Condition*	
EI11	Initial altitude error	ft
EI12	Initial cross-range error	ft
EI13	Initial downrange error	ft
EI21	Initial altitude rate error	ft/sec
EI22	Initial cross-range rate error	ft/sec
EI23	Initial downrange rate error	ft/sec
EI31	Initial platform error about 1 (azimuth) axis	$\widehat{sec}$
EI32	Initial platform error about 2 (level) axis	$\widehat{sec}$
EI33	Initial platform error about 3 (level) axis	$\widehat{sec}$
	Accelerometers	
EA00	Accelerometer(s) bias	g
EA01	Accelerometer(s) scale factor	g/g
EA02	Accelerometer(s) second-order nonlinearity	g/g^2
EA03	Accelerometer(s) third-order nonlinearity	g/g^3
EA04	Accelerometer(s) cross-axis sensitivity (misalignment, etc)	g/g(rad)
EA05	Accelerometer(s) cross-axis sensitivity (misalignment, etc)	g/g(rad)
EA06	Accelerometer(s) cross-coupling sensitivity	g/g^2

*These definitions are consistent with initial condition Option 2 and platform orientation Option 1.

Table G-1. Error Sources (Continued)

Symbol	Description	Units
	Accelerometers (Cont'd)	
EA07	Accelerometer(s) cross-coupling sensitivity	g/g^2
EA08	Accelerometer(s) sensitivity to normal acceleration	g/g
EA09	Accelerometer(s) cross-coupling sensitivity to normal acceleration	g/g^2
EA10	Accelerometer(s) cross-axis squared sensitivity	g/g^2
EA11	Accelerometer(s) cross-axis squared sensitivity	g/g^2
EA12	Accelerometer(s) cross-axis [illegible]duct sensitivity	g/g^2
	Gyros*	
EG00	Gyro bias	deg/hr
EG01	Sensitivity due to acceleration along input axis	deg/hr/g
EG02	Sensitivity due to acceleration along spin axis	deg/hr/g
EG03	Sensitivity due to accleration along input and spin axes	$deg/hr/g^2$
EG04	Misalignment about gyro output axis	$\widehat{sec}$
EG05	Misalignment about gyro spin axis	$\widehat{sec}$
EG06	Torquer scale factor error	---
EG07	Sensitivity due to acceleration along output and spin axes	$deg/hr/g^2$
EG08	Sensitivity due to acceleration along output axis	deg/hr/g
EG09	Sensitivity due to acceleration squared along input axis	$deg/hr/g^2$

*Descriptive of single-degree-of-freedom gyro.

Table G-1. Error Sources (Concluded)

Symbol	Description	Units
	Gyros* (Cont'd)	
EG10	Sensitivity due to acceleration squared along spin axis	$deg/hr/g^2$
EG11	Sensitivity due to acceleration along input and output axes	$deg/hr/g^2$
	Platform	
EP01	Rotation about platform i axis due to acceleration along j axis	$\widehat{sec}/g$
EP02	Rotation about platform i axis due to acceleration along k axis	$\widehat{sec}/g$
EP03	Rotation about platform i axis due to acceleration along j and k axes	$\widehat{sec}/g^2$
	Terminal Conditions	
ET11	Terminal altitude error	ft
ET12	Terminal cross-range error	ft
ET13	Terminal downrange error	ft
ET21	Terminal pitch component of velocity error	ft/sec
ET22	Terminal yaw component of velocity error	ft/sec
ET23	Terminal magnitude of velocity error	ft/sec

*Descriptive of single-degree-of-freedom gyro.

Table G-2. Orientation and Control Data

Symbol	Description	Nominal Value	Units
$\psi_p \rightarrow$ PSIP	Euler angles used for initial platform	0	deg
$\phi_p \rightarrow$ PHIP	orientation. Generally ϕ_p is longitude, λ_p	0	deg
$\lambda_p \rightarrow$ LAMP	is latitude and ψ_p is platform azimuth	0	deg
$\psi_I \rightarrow$ PSII	Azimuth reference for initial position and velocity errors (option 2)	0	deg
$\psi_1 \rightarrow$ PSI1	Rotation of No. 1 gyro about its input axis	0	deg
$\psi_2 \rightarrow$ PSI2	Rotation of No. 2 gyro about its input axis	0	deg
$\psi_3 \rightarrow$ PSI3	Rotation of No. 3 gyro about its input axis	0	deg
$\beta_1 \rightarrow$ BETA 1	Rotation of No. 1 accelerometer about its input axis	0	deg
$\beta_2 \rightarrow$ BETA 2	Rotation of No. 2 accelerometer about its input axis	0	deg
$\beta_3 \rightarrow$ BETA 3	Rotation of No. 3 accelerometer about its input axis	0	deg
ØUT	Option for control of ERAN tape writing density	0	-
PPF	Power flight tape writing density (ØUT $\neq$ 0)	100	sec
PFF	Free flight tape writing density (ØUT $\neq$ 0)	1000	sec
TSUEO	Initial time point to read from trajectory tape	0	sec
TSUBA	Abort time for reading trajectory tape	∞	sec

Table G-2. Orientation and Control Data (Continued)

Symbol	Description	Nominal Value	Units
TRAJ	Trajectory or file number to be processed	None*	-
ENDC	Location for the equation of motion termination criterion	-	-
1	Location for the value of terminal control	0	**
DTNP	Powered flight integration step size	4	sec
DTNF	Free flight integration step size	32	sec
BMT	Flag to indicate non-inertial platform	0	-
BRTAB	Flag to indicate reading the rate table to determine platform orientation	0	-
TGØP	Time to end first phase	∞	sec
1	. NOTE: The last phase is always terminated by the termination criterion.	∞	.
2	.	∞	.
.	.	.	.
.	.	.	.
.	.	.	.
N	Time to end the N+1 phase (N+1 = 1, 2 . . . 12)	∞	sec

*No entry results in taking files in sequence.

**TIME (sec), THETA (deg), or ALTP and ALTM (ft)

Table G-2. Orientation and Control Data (Concluded)

Symbol	Description	Nominal Value	Units
ØMEGE	Earth rotation rate	7.2921152×10^{-5}	rad/sec
A	Earth equatorial radius	2.0925696×10^{7}	ft
GM	Gravity constant (used in equations of motion)	1.4076452×10^{16}	ft^3/sec^2
J	Earth potential function constant	1.6234633×10^{-3}	-
H	Earth potential function constant	0	-
D	Earth potential function constant	8.849057×10^{-6}	-
MU	Equals GM (used in variational equations)	1.4076452×10^{16}	ft^3/sec^2
END	Indicates end of ERAN data input for this case	-	-
ENDJØB	Indicates end of job, i.e., there are no more ERAN cases to be run	-	-

Table G-3. Program Constants (Conversion Factors)

From	To	Conversion	From	To	Conversion
$\widehat{sec}$	rad	$\underline{0.48481368 \times 10^{-5}}$	rad	$\widehat{sec}$	2.062648×10^{5}
deg/hr	$\widehat{sec}$/sec	1			
deg/hr	rad/sec	$\underline{0.48481368 \times 10^{-5}}$	rad/sec	deg/hr	2.062648×10^{5}
deg/hr	MERU	66.66667	MERU	deg/hr	0.015
deg/hr/g	rad/sec/ft/sec^2	$\underline{0.15068493 \times 10^{-6}}$	rad/sec/ft/sec^2	deg/hr/g	6.1636364×10^{6}
deg/hr/g^2	rad/sec/(ft/sec^2)^2	$\underline{0.46834379 \times 10^{-8}}$	rad/sec/(ft/sec^2)^2	deg/hr/g^2	2.135184×10^{8}
g	ft/sec^2	$\underline{0.32174 \times 10^{2}}$	ft/sec^2	g	3.1080997×10^{-2}
$\widehat{sec}$	g/g(rad)	$0.48481368 \times 10^{-5}$			
g/g^2	1/ft/sec^2	$\underline{0.31080997 \times 10^{-1}}$	1/ft/sec^2	g/g^2	3.2174×10^{1}
g/g^3	1/(ft/sec^2)^2	$\underline{0.96602838 \times 10^{-3}}$	1/(ft/sec^2)^2	g/g^3	1.0351663×10^{3}
$\widehat{sec}$/g	rad/ft/sec^2	$\underline{0.15068493 \times 10^{-6}}$	rad/ft/sec^2	$\widehat{sec}$/g	6.1636364×10^{6}
$\widehat{sec}$/g^2	rad/(ft/sec^2)^2	$\underline{0.46834379 \times 10^{-8}}$	rad/(ft/sec^2)^2	$\widehat{sec}$/g^2	2.135184×10^{8}
ft	n mi	$0.16457916 \times 10^{-3}$	n mi	ft	$\underline{6.0761033 \times 10^{3}}$

NOTE: Underlined numbers only are program constants.

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Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Aerospace Corporation El Segundo, California	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

DIGITAL COMPUTER PROGRAM FOR ERROR ANALYSIS OF INERTIAL NAVIGATION SYSTEMS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

5. AUTHOR(S) (Last name, first name, initial)

Feess, W. A. and Blumenstein, S

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
August 1966	255	10

8a. CONTRACT OR GRANT NO. AF 04(695)-669 b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) TR-669(6540)-7 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) SSD-TR-66-154

10. AVAILABILITY/LIMITATION NOTICES

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Space Systems Division Air Force Systems Command Los Angeles, California

13. ABSTRACT

The theory and assumptions used in developing equations for the error analysis of a general class of inertial navigation systems are described. The computer program developed for their solution is described from a user's point of view. Its application includes the synthesis and/or analysis of inertial navigation systems used in ballistic missile or terrestrial space missions. The program is designed to allow studies of both pure inertial and aided inertial navigation systems, the latter being the process of updating navigation data via data from external sensors.

UNCLASSIFIED

Security Classification

KEY WORDS

Digital Computer Program

Error Analysis

Inertial Navigation

Guidance

Accelerometers

Resets

Abstract (Continued)

UNCLASSIFIED

Security Classification