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# AN OPTICAL RAY TRACING PROGRAM

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1 May 1974

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A useful aid in the design of optical systems is a digital computer program for tracing a light beam through an optical system. The optical system is comprised of a large class of reflecting and refracting optical elements which include cylindrical optics. The mathematical technique developed for tracing rays does not require rotational symmetry and is restricted to geometrical optics. RAYTRACE, the ray tracing program, includes graphics among its output options. 95

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AN OPTICAL RAY TRACING PROGRAM

The purpose of this report is to develop a digital computer program for tracing rays through an optical system as a useful aid in the design and development of optical systems.

The author wishes to express his appreciation to Dr. Charles J. Albers for the many helpful discussions during the course of the work.

ROBERT WILLIAMSON II
Captain, USN
Commander

H. S. NIKIRK
By direction

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# Chapter 1

# INTRODUCTION

A useful tool for the designer of an optical system is a technique to determine the effect of deviations from the design positions of both the input beam and the system's optical surfaces. This report develops a technique for tracing a light beam through an optical system for application on a digital computer. The optical system is comprised of a large class of reflecting and refracting surfaces that include cylindrical optics. The ray tracing technique is restricted to geometrical optics and does not require rotational symmetry to specify a quadric surface.

RAYTRACE is the name of the computer program that traces the light beam through the optical system. As a light beam is traced through an optical surface, the number of rays that have failed to pass the surface is given. The rays that are outside a given minimum and maximum radius at the output distance from the surface are considered to be ray failures. After the beam has been traced through the optical system, the beam can be retraced through the system with new input data by entering the data through the console typewriter. The output options include a listing of the output beam parameters and graphics at the output of any particular optical surface. The graphics consist of a cross sectional view and/or knife edge scan of the output beam at the output distance from the optical surface.

A plane surface with the optical property of refraction or reflection is the basic component of an optical system. The relationship between the ray incident to the plane surface and the ray refracted or reflected from the plane surface is developed for a plane surface system. This relationship is referred to as the input-output ray equations. Since most mirrors and lenses that comprise an optical system are quadric surfaces rather than planar, the relationship between a quadric surface and a plane surface must also be established. Definitions of the parameters and symbols used throughout the text are listed in the Glossary, Appendix A.

## MATHEMATICAL BACKGROUND

The coordinate systems defined in this report are right handed orthogonal coordinate systems whose axes are $X_j$, $Y_j$, and $Z_j$. The subscript j designates a particular coordinate system where j = 1, 2, ... n. An alphabetic subscript in conjunction with a numerical subscript designates a particular point in a particular coordinate system. For example, point A in the 1 coordinate system is expressed as $X_{1A}$, $Y_{1A}$, and $Z_{1A}$. Unit vectors, which are parallel to the $X_j$, $Y_j$, and $Z_j$ axes, are expressed by $\hat{X}_j$, $\hat{Y}_j$, and $\hat{Z}_j$. Transformations between the various coordinate systems are determined by symbolic representations or Piograms (ref. (a)).

The direction of a vector

$$\vec{V} = A\hat{X} + B\hat{Y} + C\hat{Z} \tag{1}$$

is defined by the three angles $\alpha$, $\beta$, and $\gamma$ measured from the positive X, Y, and Z axes, respectively, to the vector V. The cosines of $\alpha$, $\beta$, and $\gamma$ are known as the direction cosines of the vector V. The unit vector along the line of action of V is given by

$$\hat{V} = \cos\alpha\ \hat{X} + \cos\beta\ \hat{Y} + \cos\gamma\ \hat{Z}. \tag{2}$$

Since the magnitude of the unit vector is equal to one, obviously

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1. \tag{3}$$

The direction of a vector can be determined by the coordinates of two points ($X_A$, $Y_A$, $Z_A$) and ($X_B$, $Y_B$, $Z_B$). The relationship of the distance between the two points and the direction cosines can be expressed by

$$\frac{X_B - X_A}{\cos\alpha} = \frac{Y_B - Y_A}{\cos\beta} = \frac{Z_B - Z_A}{\cos\gamma} \tag{4}$$

where $\cos\alpha$, $\cos\beta$, and $\cos\gamma$ are not equal to zero. If a denominator is zero then the corresponding numerator is considered to be zero.

## OPTICAL BACKGROUND

Since an optical surface can be either refracting or reflecting, one relationship between the input ray and the output ray is developed to handle both cases. Straight line propagation is assumed. Taking the refraction case first, as shown in Figure 1, the refracting plane surface is designated by the $Z = 0$ plane, and the incident ray has direction

$$\hat{V}_I = \cos \alpha_I \hat{X} + \cos \beta_I \hat{Y} + \cos \gamma_I \hat{Z} \qquad (5)$$

where the subscript I denotes the incoming or incident ray. The refracted ray has direction

$$\hat{V}_R = \cos \alpha_R \hat{X} + \cos \beta_R \hat{Y} + \cos \gamma_R \hat{Z} \qquad (6)$$

where the outgoing or refracted ray is denoted by the subscript R.

From Figure 1 and the dot product, the cosine of the incident angle is given by

$$\cos \tau_I = \hat{Z} \cdot \hat{V}_I = -\cos \gamma_I \qquad (7)$$

and

$$\sin \tau_I = \sqrt{1 - \cos^2 \tau_I} = \sqrt{1 - \cos^2 \gamma_I}. \qquad (8)$$

From Snell's Law

$$\sin \tau_R = \frac{n_I}{n_R} \sin \tau_I \qquad (9)$$

where $n_I$ is the index of refraction of the medium of the incident ray and $n_R$ is the index of refraction of the medium of the refracted ray. The ratio of $n_I$ to $n_R$ is defined by the symbol N. From equations (8) and (9)

$$\cos \tau_R = \sqrt{1 - \sin^2 \tau_R} = \sqrt{1 - N^2 (1 - \cos^2 \gamma_I)}. \qquad (10)$$

Since the direction cosine is measured from the positive axis

$$\cos \tau_R = \cos (180° - \tau_R) = -\cos \tau_R \qquad (11)$$

where $\cos \gamma_R$ is the direction cosine of the refracted ray. Therefore,

$$\cos \gamma_R = -\sqrt{1 - N^2 (1-\cos^2 \gamma_I)}. \qquad (12)$$

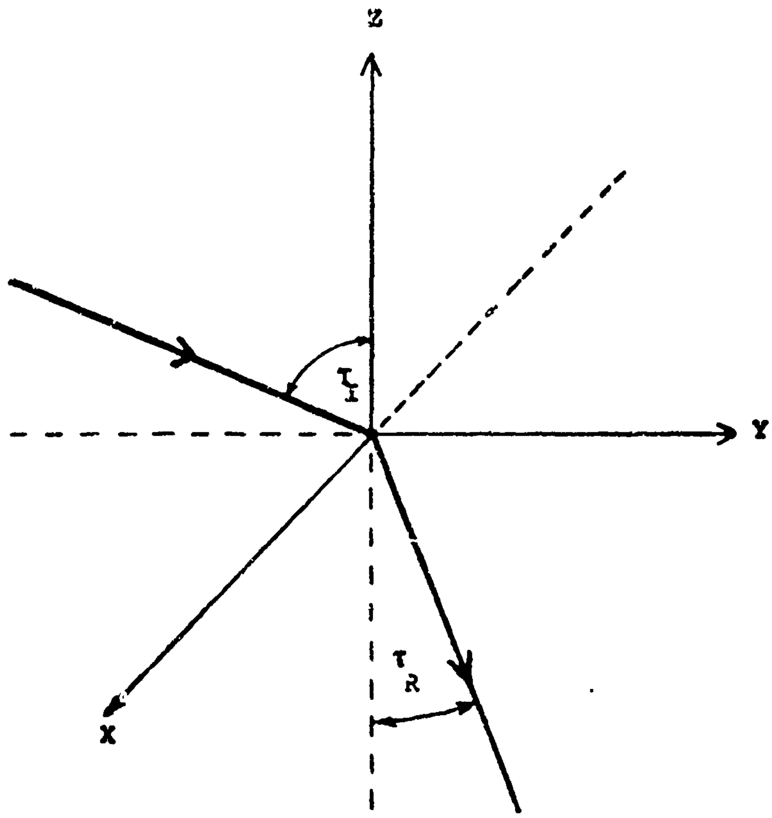



Figure 1 Refracting Surface

Since the component of the ray parallel to the refracting surface does not change direction, it must be true that

$$\frac{\cos \alpha_R}{\cos \beta_R} = \frac{\cos \alpha_I}{\cos \beta_I}. \tag{13}$$

From equation (3)

$$\cos^2 \alpha_R + \cos^2 \beta_R + \cos^2 \gamma_R = 1. \tag{14}$$

Using equations (12) and (14)

$$\cos^2 \alpha_R + \cos^2 \beta_R = 1 - \cos^2 \gamma_R = N^2 \, (1 - \cos^2 \gamma_I).$$

Utilizing equation (13)

$$\cos^2 \beta_R \left(\frac{\cos^2 \alpha_I}{\cos^2 \beta_I}\right) + \cos^2 \beta_R = N^2 \, (1 - \cos^2 \gamma_1)$$

$$\cos^2 \beta_R \left(\frac{\cos^2 \alpha_I + \cos^2 \beta_I}{\cos^2 \beta_I}\right) = N^2 \, (1 - \cos^2 \gamma_I)$$

$$\cos^2 \beta_R \left(\frac{1 - \cos^2 \gamma_I}{\cos^2 \beta_I}\right) = N^2 \, (1 - \cos^2 \gamma_I).$$

Therefore,

$$\cos^2 \beta_R = N^2 \cos^2 \beta_I$$

or

$$\cos \beta_R = N \cos \beta_I. \tag{15}$$

By the same method

$$\cos \alpha_R = N \cos \alpha_I. \tag{16}$$

Therefore, the direction of the refracting ray is

$$\hat{V}_R = \cos \alpha_R \, \hat{X} + \cos \beta_R \, \hat{Y} + \cos \gamma_R \, \hat{Z}$$

where

$$\cos \alpha_R = N \cos \alpha_I \qquad (17a)$$

$$\cos \beta_R = N \cos \beta_I \qquad (17b)$$

$$\cos \gamma_R = -\sqrt{1 - N^2 \ (1-\cos^2 \gamma_I)}. \qquad (17c)$$

For a reflective surface, the angle of the incident ray is equal to the angle of the reflected ray as shown in Figure 2. To obtain the reflected ray direction, the sign of the normal component of the incident ray in equation (5) is reversed. Therefore, the relationship of an incident ray with direction

$$\hat{V}_I = \cos \alpha_I \ \hat{X} + \cos \beta_I \ \hat{Y} + \cos \gamma_I \ \hat{Z}$$

and a refracted or reflected ray with direction

$$\hat{V}_R = \cos \alpha_R \ \hat{X} + \cos \beta_R \ \hat{Y} + \cos \gamma_R \ \hat{Z}$$

is obtained by rewriting equations (17)

$$\cos \alpha_R = N \cos \alpha_R \qquad (18a)$$

$$\cos \beta_R = N \cos \beta_R \qquad (18b)$$

$$\cos \gamma_R = -RP \sqrt{1 - N^2 \ (1-\cos^2 \gamma_I)} \qquad (18c)$$

where

$$RP = \begin{cases} 1 & \text{refraction} \\ -1 & \text{reflection} \end{cases}$$

$$N = \begin{cases} n_I/n_R & \text{refraction} \\ 1 & \text{reflection} \end{cases}.$$

In developing the equations establishing the relationship between the input rays and the output rays, the complement to the angle of incidence and the complement to the angle of refraction or reflection is used. If $\theta_I$ is the complement of the angle of incidence, then $\theta_R$, the complement of the angle of refraction, can be determined from Snell's Law.

$$\sin \ (\pi/2-\theta_R) = N \sin \ (\pi/2-\theta_I)$$

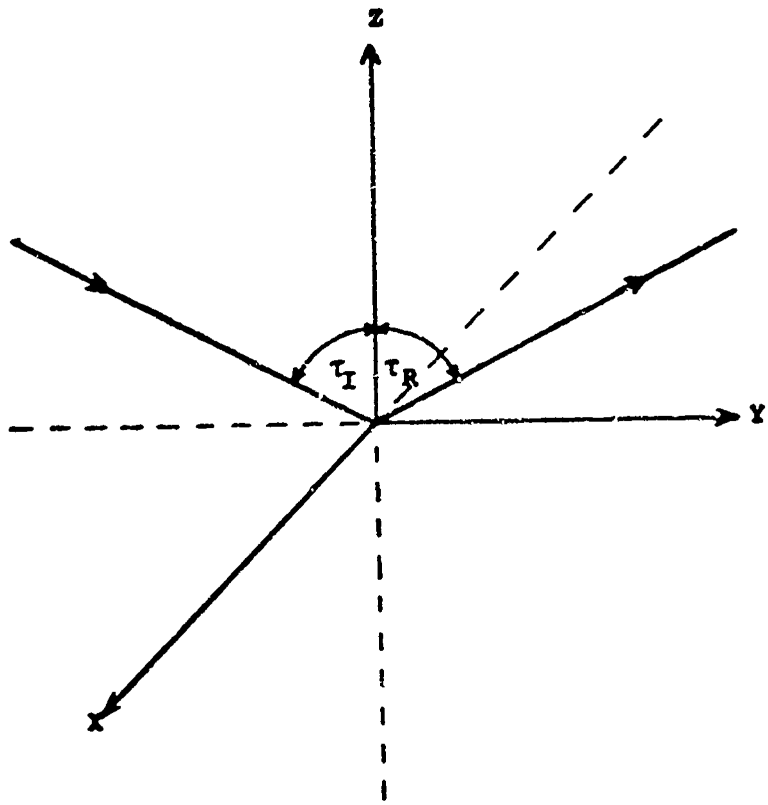



Figure 2 Reflecting Surface

or

$$\cos \theta_R = N \cos \theta_I . \tag{19}$$

Therefore, $\theta_R$ for the output ray can be determined by rewriting equation (19) as follows

$$\theta_R = RP \cos^{-1} (N \cos \theta_I) \tag{20}$$

where

$$|N \cos \theta_I| \leq 1$$

for the refracted ray to exist.

# Chapter 2

# PLANE SURFACE SYSTEM

## REFERENCE COORDINATE SYSTEMS

The plane surface, whether it be reflecting or refracting, is used as the basic element of an optical system in this report. This plane surface along with the associated references comprise the plane surface system. This system consists of a reference input ray, a reference plane, and a reference output ray. The output ray is either a refracted ray or a reflected ray.

Coordinate system 1 is the coordinate system of the plane surface reference. As shown in Figure 3, the plane $Z_1 = 0$ is the refracting or reflecting surface of the reference plane with the origin as the reference point of incidence. The plane of incidence for the reference input ray is the $X_1 = 0$ plane.

Originally, the coordinate system attached to the plane surface is coincident with coordinate system 1. However, the plane surface can move both in translation and rotation with respect to the plane surface reference system. The rotation angles, $\varepsilon$ and $\eta$, are measured with respect to the coordinate system attached to the plane surface. The coordinate system attached to the plane surface is designated coordinate system 3. The rotation angles, $\varepsilon$ and $\eta$, are defined by the Piogram (ref. (b) and (c)) of Figure 4. The coordinate $(X_{1D}, Y_{1D}, Z_{1D})$ describes the translation of the origin of the plane surface from its reference position, the origin of coordinate system 1. The $Z_3 = 0$ plane is the refracting or reflecting surface after the translation and rotation of the plane surface.

The coordinate system of the reference input ray as shown in Figure 3 is coordinate system 2. The origin of this coordinate system is located at a distance $R_I$ from the origin of system 1 along the reference input ray with direction $\hat{z}_2$. The $X_1 = 0$ plane is coincident with the $X_2 = 0$ plane.

Coordinate system 4 is the coordinate system of the reference output ray. The origin of coordinate system 1 and the origin of coordinate system 4 are separated by the distance $R_R$ along the output reference ray with direction $\hat{z}_4$. The $X_1 = 0$ plane is coincident with the $X_4 = 0$ plane.

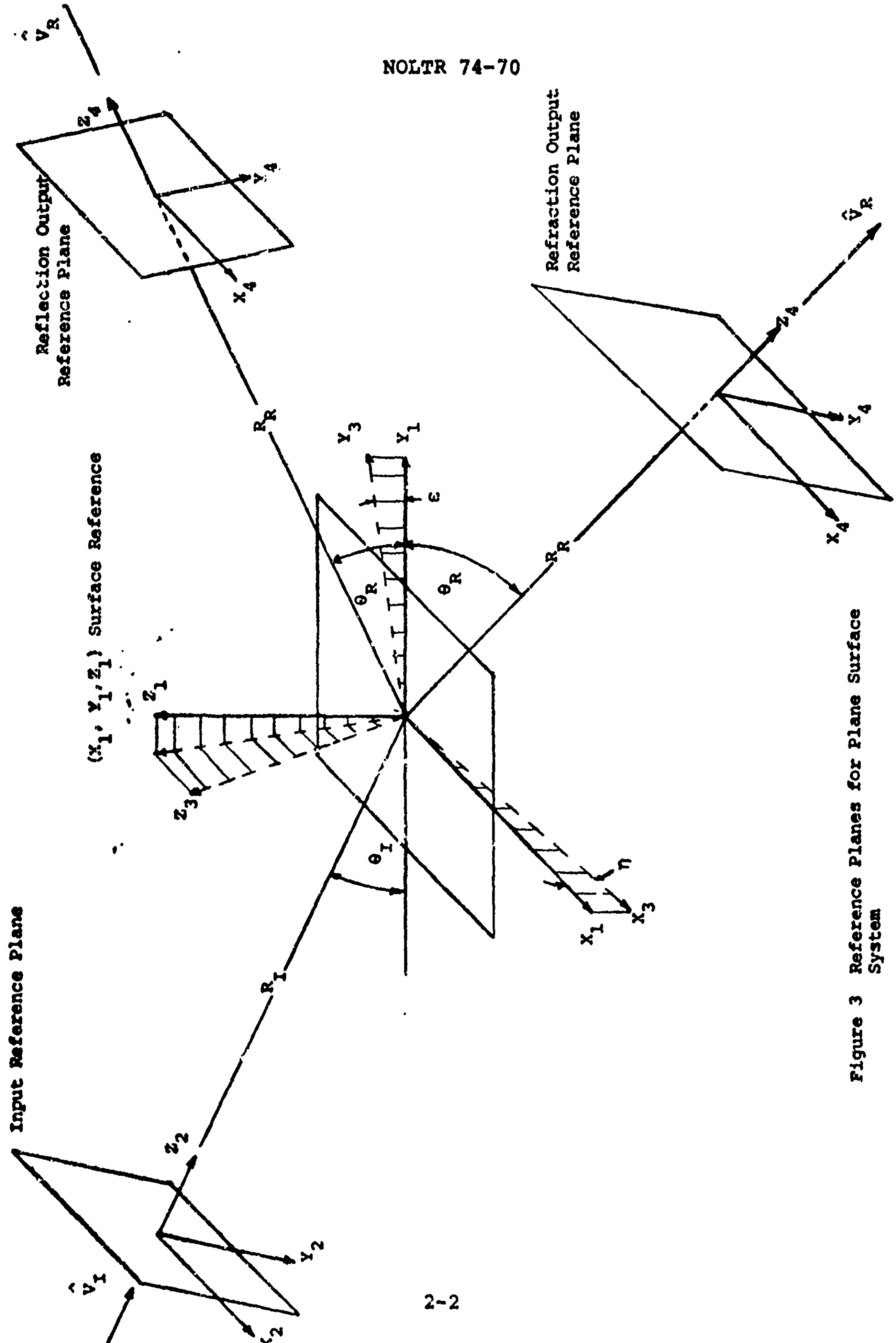



Figure 3 Reference Planes for Plane Surface System

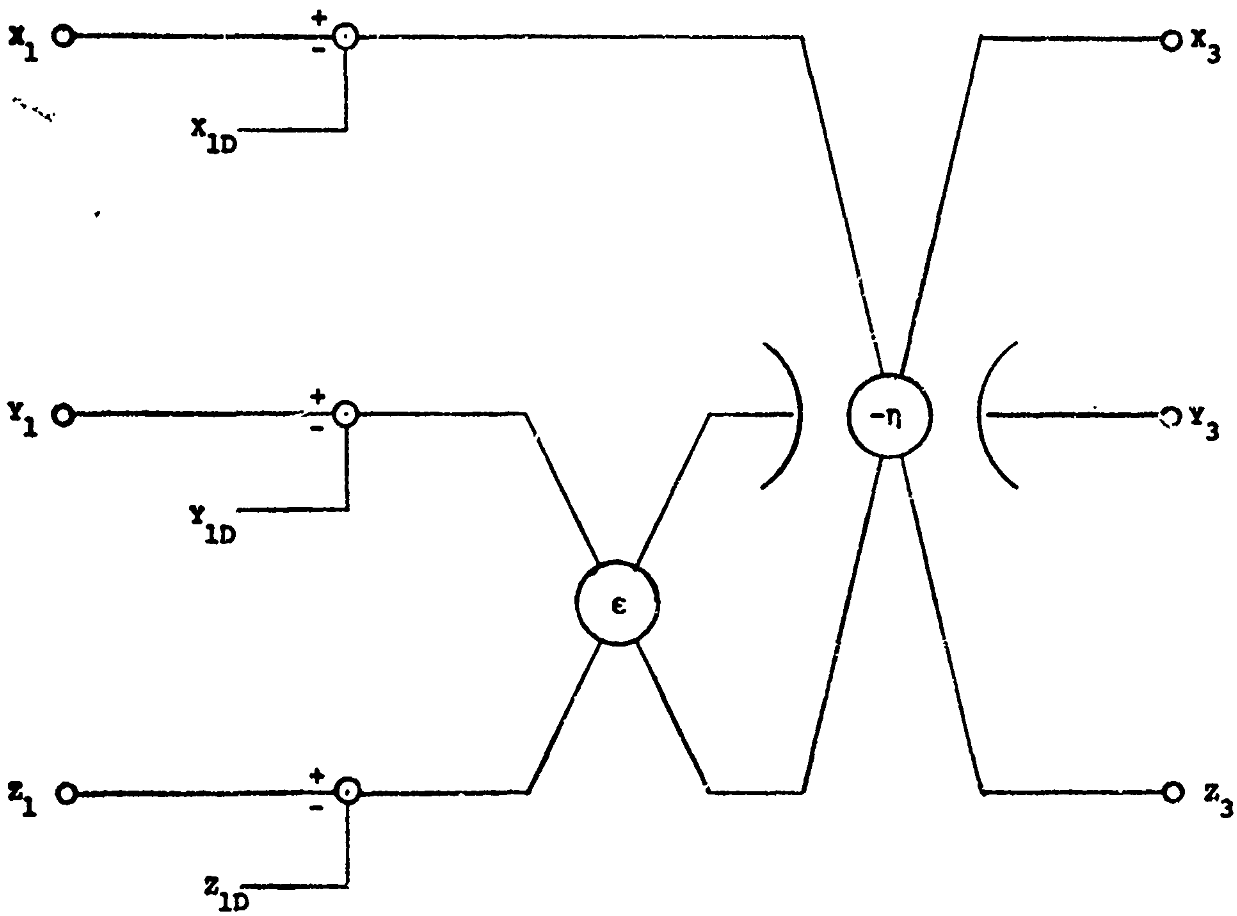



Figure 4 Piogram Relating System 1 to System 3

## Input Ray Deviation

The input ray deviation from the input reference ray with direction $\hat{Z}_2$ can be uniquely described by two linear positions and two angular positions. The linear position is simply the distance from the origin in the $Z_2 = 0$ plane. The coordinates of the input ray deviation from the input reference ray are $(X_{2D}, Y_{2D})$. The angular deviation from the input reference direction is the deviation from the $\hat{Z}_2$ direction. These two angular deviations from the input reference direction are $\emptyset_2$ and $\psi_2$. As shown in Figure 5 $\emptyset_2$ is the angle that the projection of the input ray on $Y_2 = 0$ plane makes with the $\hat{Z}_2$ direction. $\psi_2$ is the angle that the projection of the input ray on the $X_2 = 0$ plane makes with the $\hat{Z}_2$ direction.

The direction cosines of the input ray can be determined from the input ray's deviation angles and equation (3). From Figure 5

$$X = V \cos \alpha_2 \tag{21}$$

$$Y = V \cos \beta_2$$

$$Z = V \cos \gamma_2$$

and

$$\tan \emptyset_2 = X/Z \tag{22}$$

$$\tan \psi_2 = Y/Z.$$

Therefore, by substitution of equation (21) in equation (22)

$$\tan \emptyset_2 = \cos \alpha_2 / \cos \gamma_2,$$

$$\tan \psi_2 = \cos \beta_2 / \cos \gamma_2. \tag{23}$$

Equation (3) can be written as

$$\cos^2 \alpha_2 + \cos^2 \beta_2 = \sin^2 \gamma_2$$

or

$$\frac{\cos^2 \alpha_2}{\cos^2 \gamma_2} + \frac{\cos^2 \beta_2}{\cos^2 \gamma_2} = \frac{\sin^2 \gamma_2}{\cos^2 \gamma_2}. \tag{24}$$

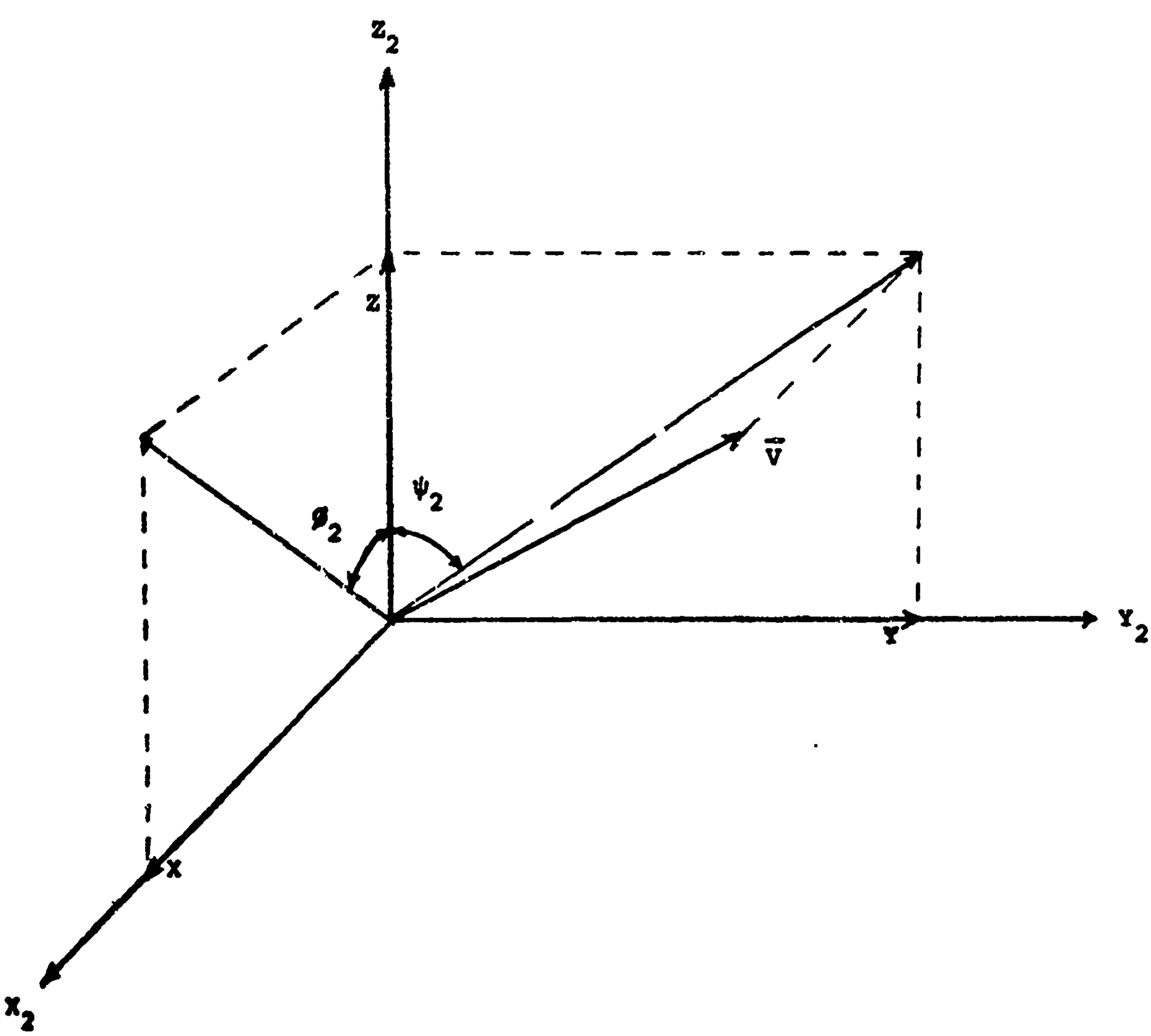



Figure 5 Angles Defining Input Ray

Using equation (23) in equation (24)

$$\tan^2 \phi_2 + \tan^2 \psi_2 = \tan^2 \gamma_2. \tag{25}$$

Hence, equations (23) and (25) can be used to determine the direction cosines of the input ray. The same considerations apply to the output ray.

Input-Output Ray Equations

The linear deviations of the input ray from the input reference ray are $X_{2D}$ and $Y_{2D}$. The input ray direction is

$$\hat{V}_I = \cos \alpha_2 \hat{X}_2 + \cos \beta_2 \hat{Y}_2 + \cos \gamma_2 \hat{Z}_2 \tag{26}$$

with angular deviations of $\phi_2$ and $\psi_2$. The plane surface translations are $X_{1D}$, $Y_{1D}$, and $Z_{1D}$ with angular rotations of $\varepsilon$ and $\eta$.

The Piogram of Figure 6 relates the input reference system (coordinate system 2) to the coordinate system attached to the plane surface (coordinate system 3). Since unit vectors in rectangular coordinate systems are independent of position, the Piogram of Figure 6 is used to determine the relationships among the unit vectors by setting $R_I = X_{1D} = Y_{1D} = Z_{1D} = 0$. Then the input ray direction with respect to coordinate system 3 is determined from these relationships.

The transformation of vectors from system 2 to system 3 is given by

$$T_I = \begin{bmatrix} \cos \eta & \cos (\theta_I+\varepsilon) \sin \eta & \sin (\theta_I+\varepsilon) \sin \eta \\ 0 & -\sin (\theta_I+\varepsilon) & \cos (\theta_I+\varepsilon) \\ \sin \eta & -\cos (\theta_I+\varepsilon) \cos \eta & -\sin (\theta_I+\varepsilon) \cos \eta \end{bmatrix} \tag{27}$$

Using equation (27), the direction of the input ray with respect to system 3 is

$$\hat{V}_I = \cos \alpha_3 \hat{X}_3 + \cos \beta_3 \hat{Y}_3 + \cos \gamma_3 \hat{Z}_3 \tag{28}$$

where

$$\begin{bmatrix} \cos \alpha_3 \\ \cos \beta_3 \\ \cos \gamma_3 \end{bmatrix} = T_I \begin{bmatrix} \cos \alpha_2 \\ \cos \beta_2 \\ \cos \gamma_2 \end{bmatrix}. \tag{29}$$

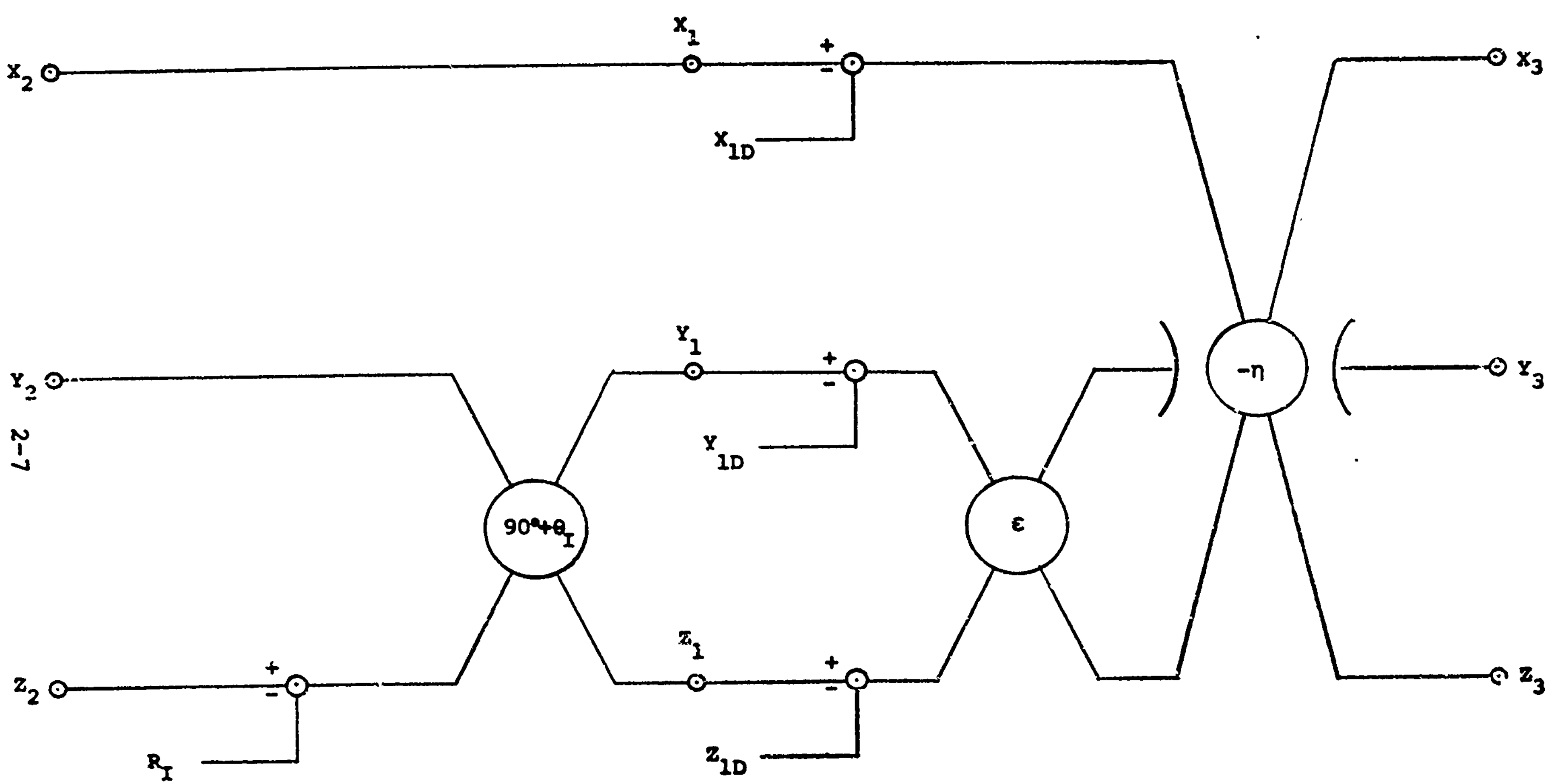



Figure 6 Piogram Relating Input Reference to Plane Surface

The coordinate of the point where the input ray intersects the input reference plane in terms of system 3 coordinates is denoted by the subscript Q. The coordinate relationship can also be determined from the Piogram in Figure 6 with the equation given as follows:

$$\begin{bmatrix} X_{3Q} \\ Y_{3Q} \\ Z_{3Q} \end{bmatrix} = T_I \begin{bmatrix} X_{2D} \\ Y_{2D} \\ -R_I \end{bmatrix} - \begin{bmatrix} \cos\eta & \sin\varepsilon \sin\eta & -\cos\varepsilon \sin\eta \\ 0 & \cos & \sin\varepsilon \\ \sin\eta & -\sin\varepsilon \cos\eta & \cos\varepsilon \cos\eta \end{bmatrix} \begin{bmatrix} X_{1D} \\ Y_{1D} \\ Z_{1D} \end{bmatrix} \qquad (30)$$

Using equation (4) the equations for the point of incidence on the plane surface in system 3 can be determined. The point of incidence is denoted by the subscript P and the plane of incidence is given by the plane $Z_3 = 0$. Therefore, the point of incidence on the plane surface is

$$X_{3P} = X_{3Q} - Z_{3Q} \frac{\cos \alpha_3}{\cos \gamma_3} \qquad (31a)$$

$$Y_{3P} = Y_{3Q} - Z_{3Q} \frac{\cos \beta_3}{\cos \gamma_3} \qquad (31b)$$

$$Z_{3P} = 0$$

In order to determine the direction of the output ray equation (18) is used. In addition, equation (20) is used to determine the complement of the angle of refraction or reflection. The subscript R denotes the output ray and the prime (') refers to the output ray direction. The direction of the output ray with respect to system 3 is

$$\hat{V}_R = \cos \alpha'_3 \hat{X}_3 + \cos \beta'_3 \hat{Y}_3 + \cos \gamma'_3 \hat{Z}_3 \qquad (32)$$

where

$$\cos \alpha'_3 = N \cos \alpha_3$$

$$\cos \beta'_3 = N \cos \beta_3$$

$$\cos \gamma'_3 = -RP \sqrt{1 - N^2 (1-\cos^2 \gamma_3)}$$

and

$$\theta_R = RP \cos^{-1} (N \cos \theta_I)$$

In Figure 7 the Piogram shows the relationship between the coordinate system attached to the plane surface (system 3) and the output reference plane (system 4). The transformation of vectors from system 3 to system 4 is

$$T_R = \begin{bmatrix} \cos\eta & 0 & \sin\eta \\ \sin\eta\cos(\theta_R+\varepsilon) & -\sin(\theta_R+\varepsilon) & -\cos\eta\cos(\theta_R+\varepsilon) \\ \sin\eta\sin(\theta_R+\varepsilon) & \cos(\theta_R+\varepsilon) & -\cos\eta\sin(\theta_R+\varepsilon) \end{bmatrix}. \quad (33)$$

The direction of the output ray with respect to system 4 is

$$\hat{V}_R = \cos\alpha_4\,\hat{X}_4 + \cos\beta_4\,\hat{Y}_4 + \cos\gamma_4\,\hat{Z}_4 \quad (34)$$

where

$$\begin{bmatrix} \cos\alpha_4 \\ \cos\beta_4 \\ \cos\gamma_4 \end{bmatrix} = T_R \begin{bmatrix} \cos\alpha'_3 \\ \cos\beta'_3 \\ \cos\gamma'_3 \end{bmatrix}.$$

The coordinates of the point where the output ray intersects the plane surface in terms of system 4 coordinates are given by

$$\begin{bmatrix} X_{4Q} \\ Y_{4Q} \\ Z_{4Q} \end{bmatrix} = T_R \begin{bmatrix} X_{3P} \\ Y_{3P} \\ Z_{3P} \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & \sin\theta_R & \cos\theta_R \\ 0 & -\cos\theta_R & \sin\theta_R \end{bmatrix} \begin{bmatrix} X_{1D} \\ Y_{1D} \\ Z_{1D} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ R_R \end{bmatrix} \quad (35)$$

The point of incidence of the output ray on the output reference plane can be determined by using equation (4) and using the fact that the plane of incidence is given by the plane $Z_4 = 0$. Therefore, the point of incidence of the output ray on the output reference plane is

$$X_{4D} = X_{4Q} - Z_{4Q}\frac{\cos\alpha_4}{\cos\gamma_4} \quad (36a)$$

$$Y_{4D} = Y_{4Q} - Z_{4Q}\frac{\cos\beta_4}{\cos\gamma_4} \quad (36b)$$

$$Z_{4D} = 0 \quad (36c)$$

The output ray can then be uniquely described by two linear positions and two angular positions with respect to the output reference plane using equations (23) and (36). The linear and angular positions are

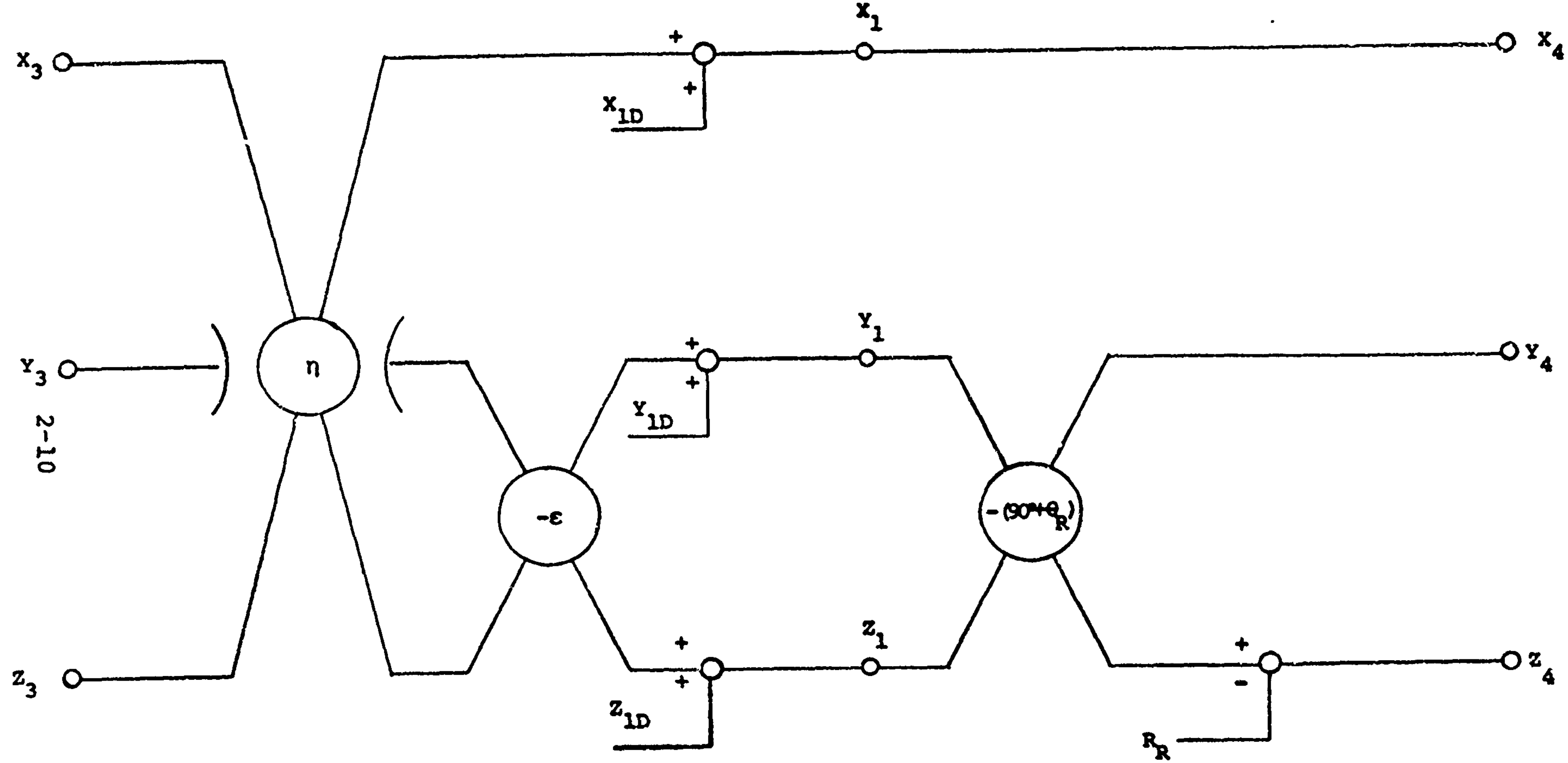



Figure 7 Piogram Relating Plane Surface to Output Reference

$$X_{4D} = X_{4Q} - Z_{4Q} \frac{\cos \alpha_4}{\cos \gamma_4} \tag{37a}$$

$$Y_{4D} = Y_{4Q} - Z_{4Q} \frac{\cos \beta_4}{\cos \gamma_4} \tag{37b}$$

$$\phi_4 = \tan^{-1} (\cos \alpha_4/\cos \gamma_4) \tag{37c}$$

$$\psi_4 = \tan^{-1} (\cos \beta_4/\cos \gamma_4). \tag{37d}$$

These output ray parameters, $X_{4D}$, $Y_{4D}$, $\phi_4$, and $\psi_4$ are used as input parameters for the optical surface that follows.

INTERFACE BETWEEN TWO PLANE SURFACES

When two plane surfaces are in series, the output coordinate system of the first surface may not be coincident with the input coordinate system of the second surface as shown in Figure 8. If the two coordinate systems are not coincident an interface between the two systems is required. This interface from the output coordinate system of the first surface to the input coordinate system of the second surface can be accomplished by a rotation about the reference ray or the $Z_4$ axis as shown in Figure 8. A positive rotation is determined by the righthand rule about the reference ray direction. The magnitude of the rotation is the angle, $\lambda$, between the positive X axis of the output plane of the first surface and the positive X axis of the input of the second surface. The Piogram in Figure 8 shows the relationship between the output ray parameters of the first surface with the input reference plane of the second surface.

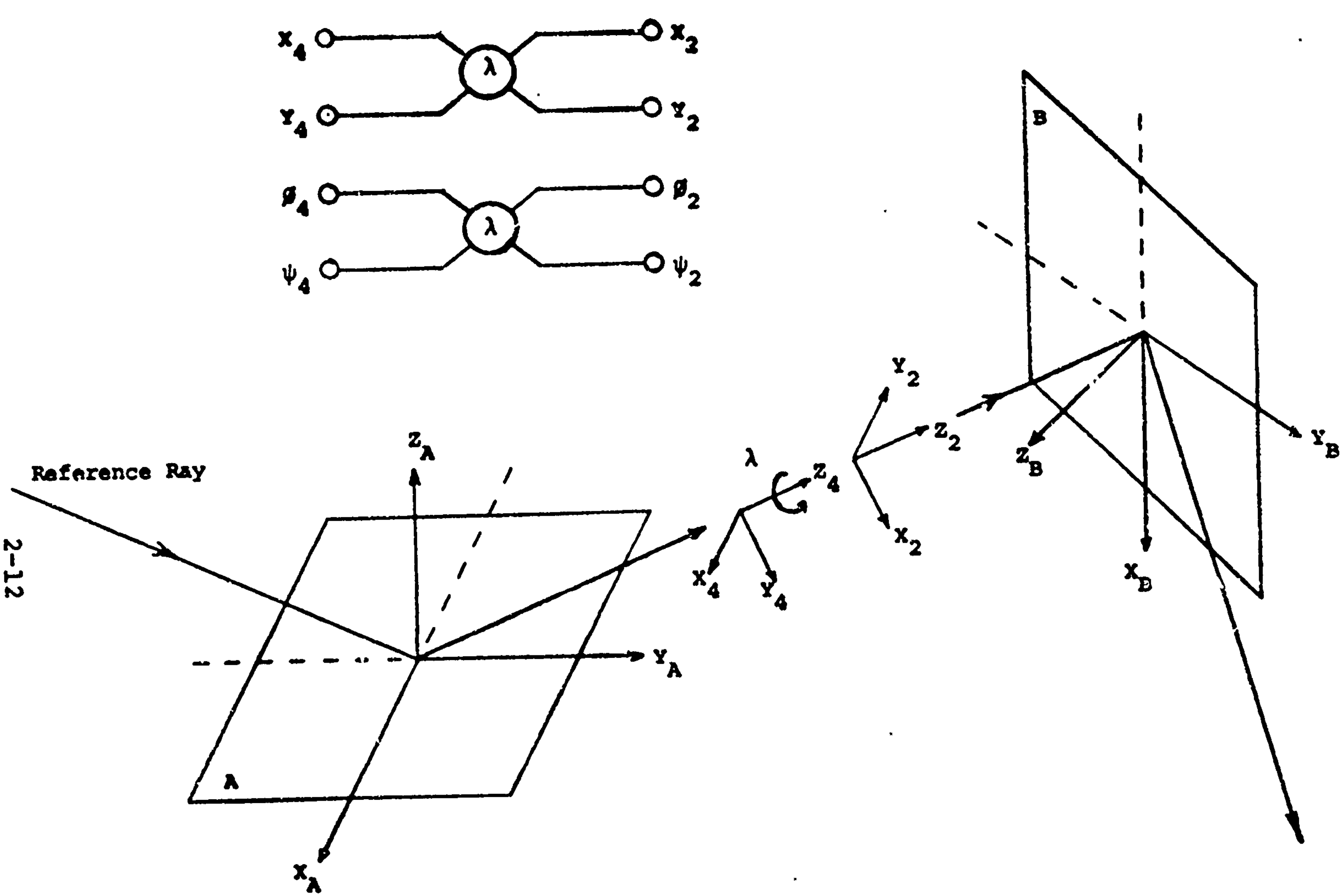



Figure 8 Interface of Two Successive Plane Surfaces with Piogram

# Chapter 3

## QUADRIC SURFACES

The elements that comprise an optical system may have quadric surfaces rather than plane surfaces. Therefore, if a plane surface is considered to be the basic subsystem of an optical system, then the relationship between a quadric surface and the plane surface system must be developed. The basic equation of a quadric surface can be written as

$$AX^2 + BY^2 + CZ^2 + DZ = E \tag{38}$$

where the Z-axis is the axis of symmetry. The center of symmetry may translate in any direction with respect to a reference. However, here misalignment of the axes is not allowed. The translation of the center of symmetry in the XY plane is known as decentration. Focusing is the translation of the center of symmetry along the Z-axis. Therefore, equation (38) becomes

$$A(X-C_x)^2 + B(Y-C_y)^2 + C(Z - (C_z+F))^2 + D(Z - (C_z+F)) = E \tag{39}$$

where $C_x$ and $C_y$ are the decentration coordinates, $C_z$ is the Z coordinate for the center of symmetry, and F is the focusing parameter.

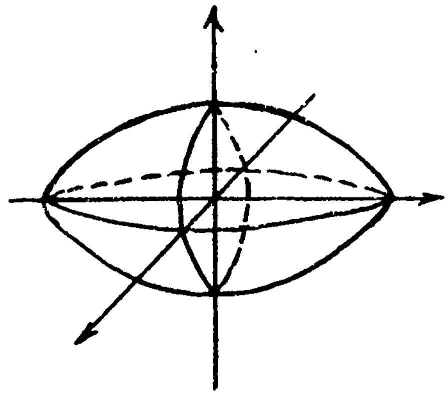
The types of quadric surfaces generally considered for an optical system are an ellipsoid, a hyperboloid of two sheets, and an elliptic paraboloid as shown in Figure 9. The equations for these surfaces are

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} + \frac{Z^2}{c^2} = 1 \text{ (ellipsoid)} \tag{40a}$$

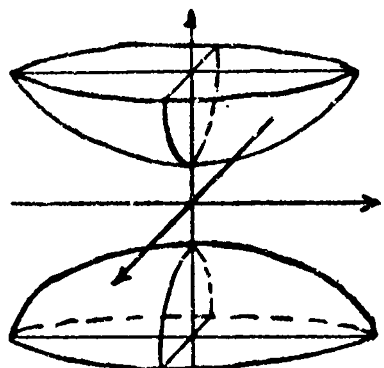
$$-\frac{X^2}{a^2} - \frac{Y^2}{b^2} + \frac{Z^2}{c^2} = 1 \text{ (hyperboloid of two sheets)} \tag{40b}$$

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} \pm \frac{Z}{c} = 0 \text{ (elliptic paraboloid)} \tag{40c}$$

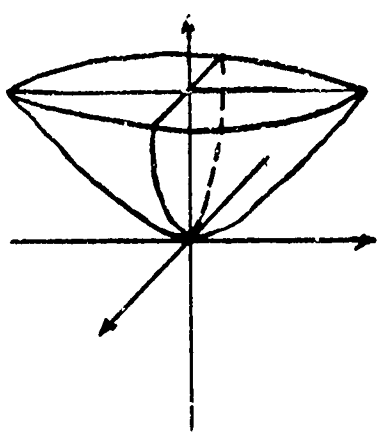
with the Z-axis as the axis of symmetry. Spheres and cylinders are simply special cases of these quadric surfaces. The constants a, b, and c are the semi-axes lengths for an ellipsoid. If a = b = c,



a) Ellipsoid



b) Hyperboloid of Two Sheets



c) Elliptic Paraboloid

Figure 9 Quadric Surfaces

a sphere is the result. If $a = \infty$, the result is an elliptic cylinder with elements parallel to the X-axis. For a hyperboloid of two sheets, the constant c is the vertex of the surface on the Z-axis. If $f_x$ is the focus of the hyperbola in the Y = 0 plane and $f_y$ is the focus in the X = 0 plane then

$$a^2 = f_x^2 - c^2$$

and

$$b^2 = f_y^2 - c^2.$$

In the case of an elliptic paraboloid the constant c is the location of an ellipse along the Z axis with the semi-axes lengths of a and b. Table 1 lists the constants that are substituted into equation (39) to obtain the desired quadric surface.

A relationship between a quadric surface and a plane surface system can be developed by solving for the linear translation and angular rotation of the plane surface from the reference position to a point tangent to the input ray incident on the quadric surface. The translation and rotation parameters are equivalent to the coordinate $(X_{1D}, Y_{1D}, Z_{1D})$ and the angles, $\varepsilon$ and $\eta$, as outlined in Chapter 2. Figure 10 shows a quadric surface with the reference coordinate system (system 1) at the vertex of the surface and an input reference system (system 2). These two reference systems are separated by the distance R. The center of symmetry for the quadric surface is the $Z = C_z$ plane.

Referring to Figure 10, the direction of the input ray is

$$\hat{V}_I = \cos \alpha_2 \hat{X}_2 + \cos \beta_2 \hat{Y}_2 + \cos \gamma_2 \hat{Z}_2 \tag{41}$$

with a deviation coordinate of $(X_{2D}, Y_{2D})$. From equation (29) and setting $\varepsilon = \eta = 0$ and $\theta_I = \pi/2$, the direction cosines and the deviation coordinate with respect to system 1 are

$$\begin{aligned} \cos \alpha_1 &= \cos \alpha_2 \\ \cos \beta_1 &= -\cos \beta_2 \\ \cos \gamma_1 &= -\cos \gamma_2 \end{aligned} \tag{42}$$

and

$$\begin{aligned} X_{1Q} &= X_{2D} \\ Y_{1Q} &= -Y_{2D} \end{aligned} \tag{43}$$

Table 1

CONSTANTS FOR VARIOUS QUADRIC SURFACES

| Parameters / Quadric | A | B | C | D | E | Comments |
|---|---|---|---|---|---|---|
| Ellipsoid | $1/a^2$ | $1/b^2$ | $1/c^2$ | 0 | 1 | a, b, c = semi-axes length |
| Hyperboloid of Two Sheets | $-1/a^2$ | $-1/b^2$ | $1/c^2$ | 0 | 1 | c = vertex of surface on z-axis |
| Elliptic Paraboloid | $1/a^2$ | $1/b^2$ | 0 | $\frac{1}{c}$ | 0 | c = Z location of ellipse corresponding to semi-axes length a and b |

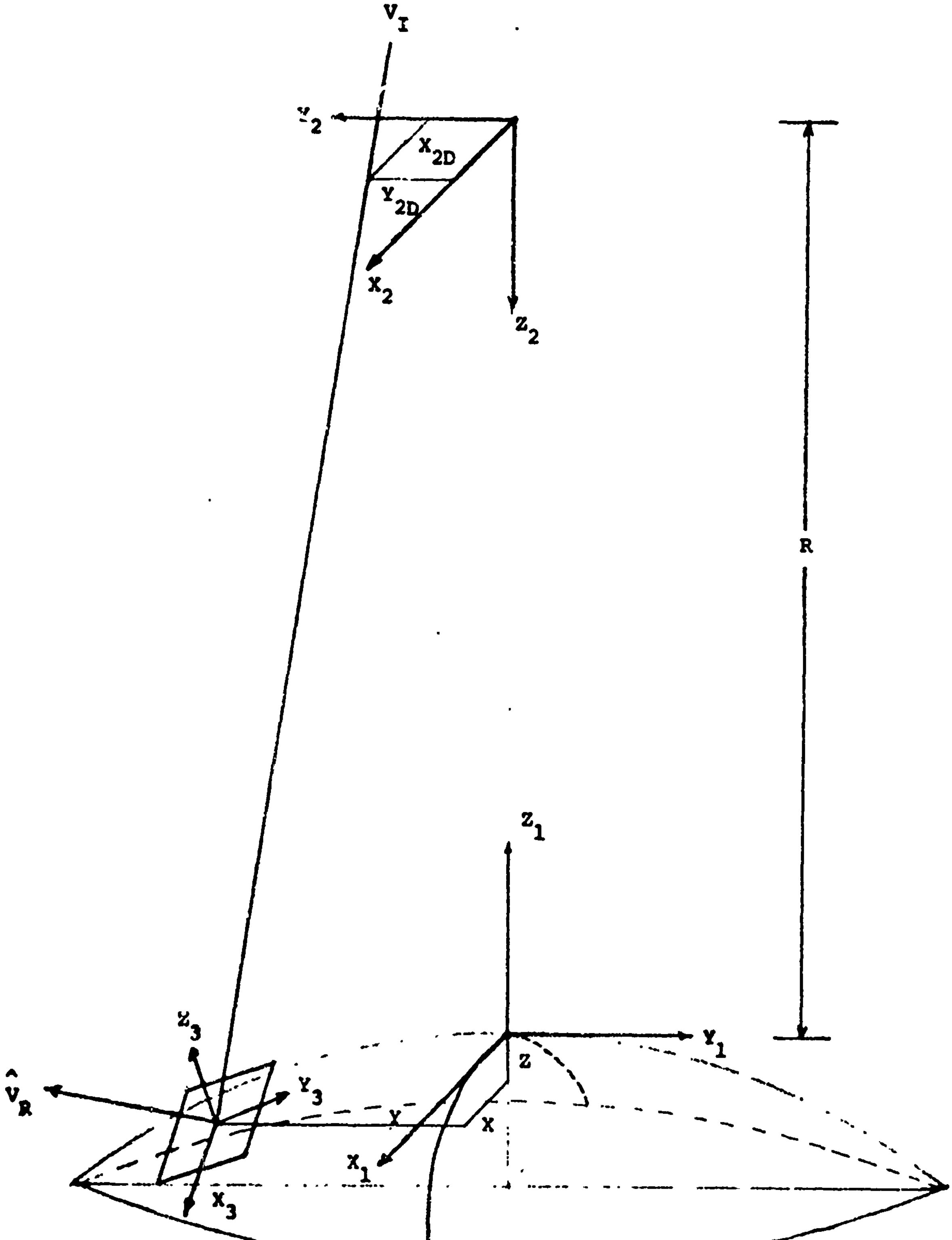



Figure 10  Reference Systems for Quadric Surface and Input Ray

The point of incidence is determined from equation (4) and is

$$X = X_{1Q} + (Z-R) \cos \alpha_1 / \cos \gamma_1 \qquad (44)$$

$$Y = Y_{1Q} + (Z-R) \cos \beta_1 / \cos \gamma_1$$

where (X, Y, Z) is the incident coordinate of the input ray on the quadric surface. By substituting equation (44) into equation (39), the result is

$$A\ (G_x + Z \tan \phi - R \tan \phi)^2 + B\ (G_y + Z \tan \psi - R \tan \psi)^2 \qquad (45)$$

$$+ C\ (Z - G_z)^2 + D\ (Z - G_z) = E$$

where

$$G_x = X_{1Q} - C_x$$

$$G_y = Y_{1Q} - C_y$$

$$G_z = C_z + F$$

$$\tan \phi = \cos \alpha_1 / \cos \gamma_1$$

$$\tan \psi = \cos \beta_1 / \cos \gamma_1 .$$

By combining terms, equation (45) is rewritten as

$$\mathcal{A} Z^2 + \mathcal{B} Z + \mathcal{C} = 0 \qquad (46)$$

where

$$\mathcal{A} = A \tan^2 \phi + B^2 \tan^2 \psi + C$$

$$\mathcal{B} = 2A\ (G_x - R \tan \phi) \tan \phi + 2B\ (G_y - R \tan \psi) \tan \psi - 2CG_z + D$$

$$\mathcal{C} = A\ (G_x - R \tan \phi)^2 + B\ (G_y - R \tan \psi)^2 + CG_z^2 - DG_z - E$$

with two conditions that

$$\mathcal{B}^2 - 4\mathcal{A} \geq 0 \qquad (47a)$$

$$|\mathcal{A}| + |\mathcal{B}| \neq 0 \qquad (47b)$$

If either of these conditions are not met, then the input ray does not intersect the quadric surface.

From equation (46), the z-coordinate of incidence is

$$Z = \begin{cases} \dfrac{-\mathcal{B} + S\sqrt{\mathcal{B}^2 - 4\mathcal{A}\mathcal{C}}}{2\mathcal{A}}, & \mathcal{A} \neq 0 \text{ and } \mathcal{B}^2 - 4\mathcal{A}\mathcal{C} \geq 0 \\ -\mathcal{C}/\mathcal{B}, & \mathcal{A} = 0 \text{ and } \mathcal{B} \neq 0 \end{cases} \qquad (48)$$

where S equals ±1. S is the parameter that determines whether the quadric surface is concave or convex. In order to solve for the correct sign of S, let $\phi = \psi = F = C_x = C_y = X_{1Q} = Z_{1Q} = 0$. Substituting into equation (48) the result is

$$Z = \begin{cases} \dfrac{2CG_z - D + S\sqrt{D^2 + 4\,EC}}{2C} & D^2 + 4\,EC \geq 0 \text{ and } C \neq 0 \\ G_z + \dfrac{E}{D} & C = 0 \text{ and } D \neq 0 \end{cases} \qquad (49)$$

Using Table 1 and equation (49) the result for an ellipsoid is

$$Z = G_z + Sc.$$

Setting the center of symmetry to zero, Z must be positive for a convex surface and negative for a concave surface. Therefore, S = +1 for a convex surface and S = -1 for a concave surface. For a hyperboloid of two sheets the equation is

$$Z = G_z + Sc$$

By setting the center of symmetry to zero the surface is concave for S = +1 and convex for S = -1. The resulting equation for the Z-coordinate of the incident ray on the surface of an elliptic paraboloid is

$$Z = G_z$$

This equation provides no information on the concavity or convexity of the surface. This information can come directly from equation (40c). If $c < 0$, the surface is concave to the input ray and if $c > 0$ the surface is convex to the input ray. A summation of the preceding results on the concavity or convexity of a quadric surface is given in Table 2.

Substituting the result of equation (48) into equation (44) yields the X and Y coordinate of the point of incidence of the input ray on the quadric surface. Now, the normal to the surface is

$$\bar{N} = \bar{\nabla}\,[A(X-C_x)^2 + B(Y-C_y)^2 + C(Z-G_z)^2 + D(Z-G_z) - E] \qquad (50)$$

Table 2

DETERMINATION OF CONCAVITY AND CONVEXITY OF A QUADRIC SURFACE

| | Parameter | Concave | Convex |
|---|---|---|---|
| Sphere | S | -1 | 1 |
| Ellipsoid | S | -1 | 1 |
| Hyperboloid of Two Sheets | S | 1 | -1 |
| Elliptic Paraboloid | c | <0 | >0 |

and the equation for the unit normal is

$$\hat{N} = \frac{1}{N} [\{2A(X-C_x)\} \hat{X} + \{2B(Y-C_y)\} Y + \{2C(Z-G_z) + D\} \hat{Z}] \quad (51)$$

where

$$N = \sqrt{4A^2 (X-C_x)^2 + 4B^2 (Y-C_y)^2 + [2C (Z-G_z) + D]^2}. \quad (52)$$

From the Piogram of Figure 4, the normal to the plane surface is

$$\hat{Z}_3 = \sin \eta \hat{X}_1 - \sin \varepsilon \cos \eta \hat{Y}_1 + \cos \varepsilon \cos \eta \hat{Z}_1 . \quad (53)$$

Since $\hat{N}$ and $\hat{Z}_3$ are both normal vectors to the plane surface, the respective components of the unit vectors are equivalent. Therefore,

$$\sin \eta = 2A (X-C_x)/N$$

or

$$\eta = \sin^{-1} (2A (X-C_x)/N). \quad (54)$$

Also,

$$\sin \varepsilon \cos \eta = -2B (Y-C_y)$$

$$\cos \varepsilon \cos \eta = 2C (Z-G_z) + D$$

and the above two equations yield

$$\tan \varepsilon = \frac{-2B (Y-C_y)}{2C (Z-G_z) + D}$$

or

$$\varepsilon = \tan^{-1} \left( \frac{-2B (Y-C_y)}{2C (Z-G_z) + D} \right) \quad (55)$$

By translating and rotating the plane surface from the reference plane to the incident point of an input ray on a quadric surface, the plane surface system can be used to trace a ray from its input plane to the output plane for a variety of optical element shapes. The relationship between the plane surface system and a quadric surface is summarized as follows:

$$X_{1D} = X_{1Q} + (Z_{1D}-R_I) \cos \alpha_1/\cos \gamma_1 \quad (56a)$$

$$Y_{1D} = Y_{1Q} + (Z_{1D}-R_I) \cos \beta_1/\cos \gamma_1 \quad (56b)$$

$$z_{1D} = \begin{cases} \dfrac{-\mathcal{B} + s\sqrt{\mathcal{B}^2 - 4\mathcal{A}\mathcal{C}}}{2}, & \mathcal{A} \neq 0 \text{ and } \mathcal{B}^2 - 4\mathcal{A}\mathcal{C} \geq 0 \\ -\mathcal{C}/\mathcal{B}, & \mathcal{A} = 0 \text{ and } \mathcal{B} \neq 0 \end{cases} \qquad (56c)$$

$$\eta = \sin^{-1} (2A\ (X-C_x)/N) \qquad (56d)$$

$$\varepsilon = \tan^{-1} \left( \frac{-2B\ (Y-C_y)}{2C\ (Z-G_z) + D} \right) \qquad (56e)$$

# Chapter 4

# RAYTRACE PROGRAM

## DATA VARIABLES

The ray tracing program, RAYTRACE, has been prepared for running on the CDC 3200 computer using a Gould electro-static printer for the graphics output. Appendix B describes and lists each routine in the RAYTRACE program along with a flow chart of the program. The input data necessary for tracing a beam through an optical surface are stored in the DV array. Each time a beam is to be traced through an optical surface, the DV array is initialized to zero except for the run number, the magnitude of the DV array, and the number of elements in the optical system. A list of the DV array variables is shown in Table 3.

Most of the variables in the DV array are self explanatory. The variables from DV(1) to DV(21) are the parameters required to describe the optical surface for the simulation of a beam passing through that surface. These parameters have been described in the previous chapters. DV(22) describes the optical shape of the surface. If the variable is equal to one the shape is convex. For a concave surface the variable is set to minus-one. It is important to note that all angular input data must be radians. In addition, the units for the linear data must agree.

Only one input parameter DV(23) is needed for the graphics output of a cross sectional view and a knife edge scan at the output reference plane of an optical surface. This parameter is the radius of the cross section one wishes to observe at the output distance from the surface and must be greater than the beam radius in order to plot the data. For reasonable scales of the axes, this radius should be some even multiple of four. DV(24) and DV(25) provide boundary conditions for the beam at the output reference plane. These parameters are the maximum and minimum radius, respectively.

## PROGRAM CONTROLS

The user has continuing control over the program through the use of the console typewriter and sense switches. After a beam has passed through an optical surface, the run number, the surface number, and the number of ray failures are printed on the typewriter

Table 3

DV ARRAY VARIABLES

| | | | |
|---|---|---|---|
| DV(1) | $\theta_I$ | DV(16) | D |
| DV(2) | $R_I$ | DV(17) | E |
| DV(3) | $R_R$ | DV(18) | $C_x$ |
| DV(4) | $\lambda$ | DV(19) | $C_y$ |
| DV(5) | $\varepsilon$ | DV(20) | $C_z$ |
| DV(6) | $\eta$ | DV(21) | F |
| DV(7) | $X_{1D}$ | DV(22) | Optical Shape |
| DV(8) | $Y_{1D}$ | DV(23) | Radius of Cross Sectional Area |
| DV(9) | $Z_{1D}$ | DV(24) | $R_{max}$ |
| DV(10) | RP | DV(25) | $R_{min}$ |
| DV(11) | N | DV(26) | Number of Present Optical Element |
| DV(12) | S | DV(27) | Run Number |
| DV(13) | A | DV(28) | Magnitude of DV Array |
| DV(14) | B | DV(29) | Number of Elements in Optical System |
| DV(15) | C | DV(30)-DV(50) | Blank |

in order to help the user decide on what course of action to take. If any graphics output is desired, sense switch (2) is set. If any graphs are plotted, the graph can be replotted immediately with a different dimension or scale by an appropriate input through the typewriter. Setting sense switch (3) will list the output linear and angular positions of each ray in the beam. The beam can be retraced through the same optical system by setting sense switch (4) and inputting the new parameters through the typewriter for the appropriate optical surface. Initially, the input beam to the optical system has been generated and stored in a data file on disc. After the beam has been traced through the system it may be desirable to transfer this beam to the input beam data file. This can be accomplished by setting sense switch (5). As one becomes familiar with the program, some of the messages can be deleted by setting sense switch (6).

## DATA STRUCTURE

### a. Input

The data for each optical system are read from the card reader and stored in a data file. The data for each optical system consist of heading information and data for each successive surface in the system. As the beam is traced through a surface the data for that surface is transferred from the data file to the DV array. Figure 11 shows a typical setup for an input data deck. If the same input beam is used or if sense switch (5) is used to transfer a resultant output beam to the input beam data file, the input data deck can consist of more than one optical system.

The heading cards for an optical system are placed in front of the system's surface data. This information is read and stored in memory as Hollerith constants from column one through column seventy-one. As each card is read, column seventy-two is tested for a zero, a one, or a two. A zero means that more heading information will follow. A one indicates that all heading information has been read into memory. The data card with a one in column seventy-two must separate the heading cards of the system from its surface data. A two in column seventy-two signifies the end of the computer run and this card must be the last card of the input data deck.

After the heading information has been read as indicated by the one card, the data of the first optical surface of the system are read and stored in the DV array. The format of this data is 4(I4, E14.7) for each card. The location of each variable in the DV array is denoted by the I4 format and the value of that variable by the E14.7 format. The magnitude of the DV array is fifty. As each data card is read, the location of each variable is compared with fifty. When a comparison matches, all of the input data for that particular surface have been read and stored in memory. Therefore, the last data card of the input data for an optical surface should be an integer fifty in I4 format. The

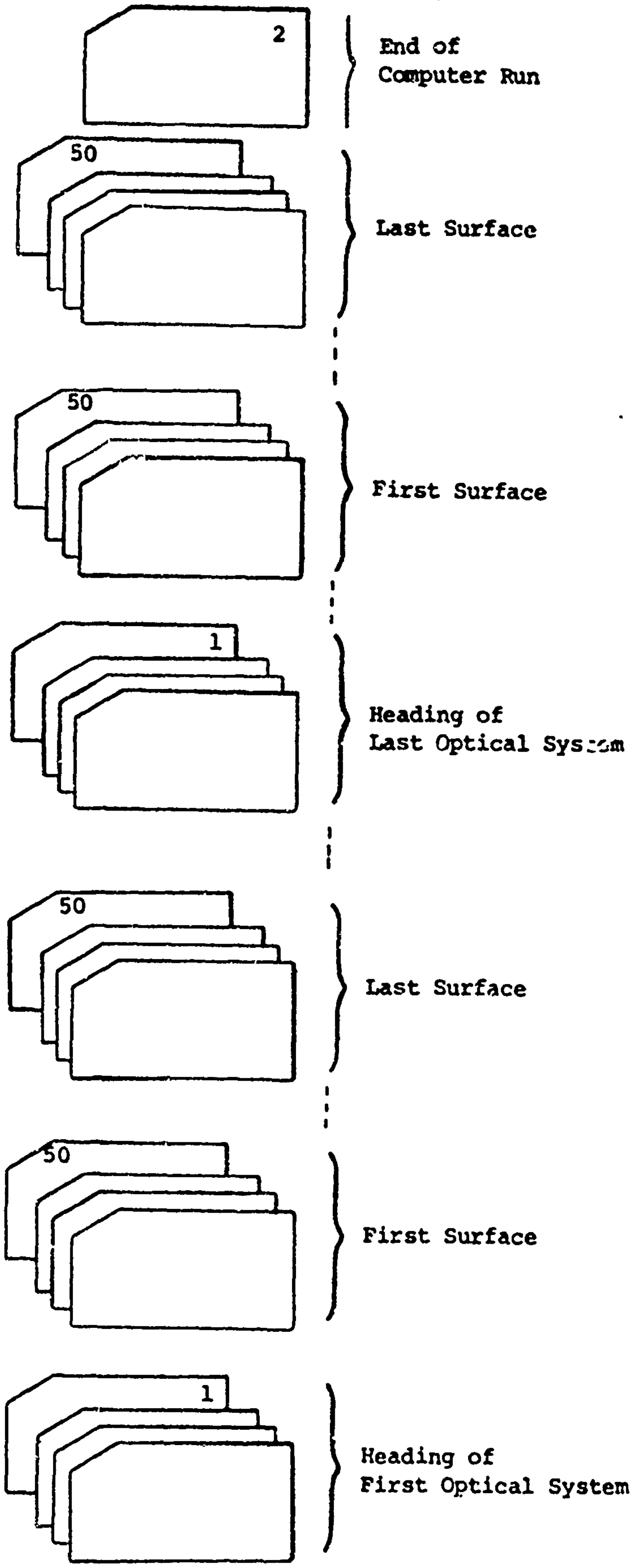



Figure 11 Data Deck Setup

data for each surface of an optical system are stacked in the same order as they occur in the system. If a beam is to be traced through another optical system, the data deck is set up as outlined and stacked behind the previous system as shown in Figure 11.

b. Output

The first piece of information written on the line printer is the heading information for an optical system. Then a listing of the data stored in the DV array for the current surface is printed. After the listing of the DV array, two rows of integer numbers are printed. These numbers represent the number of rays stored in each data block on the disc before and after the rays have passed through the optical surface. Then the total number of rays that have failed to pass through the current optical surface is listed. If sense switch (3) is set the output beam data for the current optical surface are listed for each ray. The format is (I5, 1H), 4E20.7) for each ray. This information is the coordinate of the ray in the output plane ($X_{4D}$, $Y_{4D}$) and the angular deviation ($\phi_4$, $\psi_4$) of the ray from the output reference ray.

SAMPLE PROBLEMS

a. Ellipsoidal Mirror

Several problems with known results are presented to test the RAYTRACE program and to show its uses. A concave ellipsoidal mirror, as shown in Figure 12, is chosen as the first sample problem because of one of its properties. If a ray originates at one focus point of an ellipsoidal mirror the ray will reflect from the mirror and pass through the other focus point. The parameters of the quadric surface are a = 6, b = 6, and c = 10. If the reference system of the quadric surface is placed at the vertex, the center of symmetry is at Z = 10. The input and output reference planes are at the focus points Z = 2 and Z = 18, respectively. The input reference ray is coincident with the Z-axis. With this sample problem in mind, a beam from a point source was generated and stored in the input beam data file. With the point source at the focus point Z = 2, the beam should be focused at the other focus point after tracing the beam through the mirror. Figure 13 lists the input data deck used for this sample problem. The two runs have the output reference planes at two different distances from the surface. Figure 14 shows the cross sectional view of the beam focused at the focus point of Z = 18. When the output reference plane is moved to Z = 2, Figure 15 shows the beam pattern at this distance from the mirror. A knife edge scan of the beam at the distance Z = 2 is shown in Figure 16.

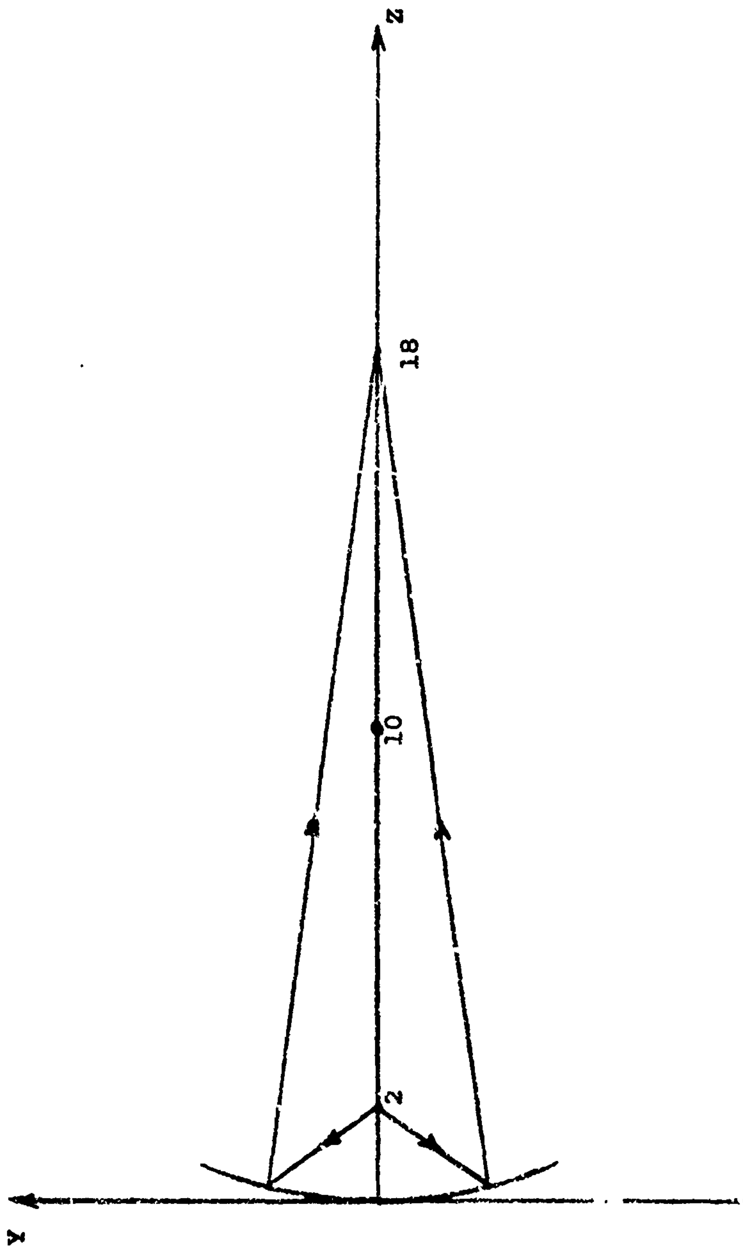



Figure 12 Concave Ellipsoidal Mirror Plane View

```
      RUN NO. 00001     SURFACES = 1 THRU 1      DATE =  8 NOV 1973    0
      TEST RAYTRACE PROGRAM                                             0
      CONCAVE ELLIPSOIDAL MIRROR                                        0
      A=6 , B=6 , C=10 , CZ=10 , RI=2 , RR=18
                                                                        1
  1    15707963+00   2   20000000+00   3   18000000+01   4   00000000+00
  5    00000000+00   6   00000000+00   7   00000000+00   8   00000000+00
  9    00000000+00  10  -10000000+00  11   10000000+00  12  -10000000+00
 13    27777778-02  14   27777778-02  15   10000000-02  16   00000000+00
 17    10000000+00  18   00000000+00  19   00000000+00  20   10000000+01
 21    00000000+00  22  -10000000+00  23   20000000+00  24   17500000+00
 25    00000000+00  26   10000000+00  27   10000000+00  28   30000000+01
 29    10000000+00
 50
      RUN NO. 00002     SURFACES = 1 THRU 1      DATE =  8 NOV 1973    0
      PLOT BEAM PATTERN AND KNIFE EDGE SCAN                             0
      CONCAVE ELLIPSOIDAL MIRROR                                        0
      A=6 , B=6 , C=10 , CZ=10 , RI=2 , RR=18
                                                                        1
  1    15707963+00   2   20000000+00   3   20000000+00   4   00000000+00
  5    00000000+00   6   00000000+00   7   00000000+00   8   00000000+00
  9    00000000+00  10  -10000000+00  11   10000000+00  12  -10000000+00
 13    27777778-02  14   27777778-02  15   10000000-02  16   00000000+00
 17    10000000+00  18   00000000+00  19   00000000+00  20   10000000+01
 21    00000000+00  22  -10000000+00  23   20000000+00  24   17500000+00
 25    00000000+00  26   10000000+00  27   20000000+00  28   30000000+01
 29    10000000+00
 50
                                                                        2
```

Figure 13  Input Data for Ellipsoidal Mirror

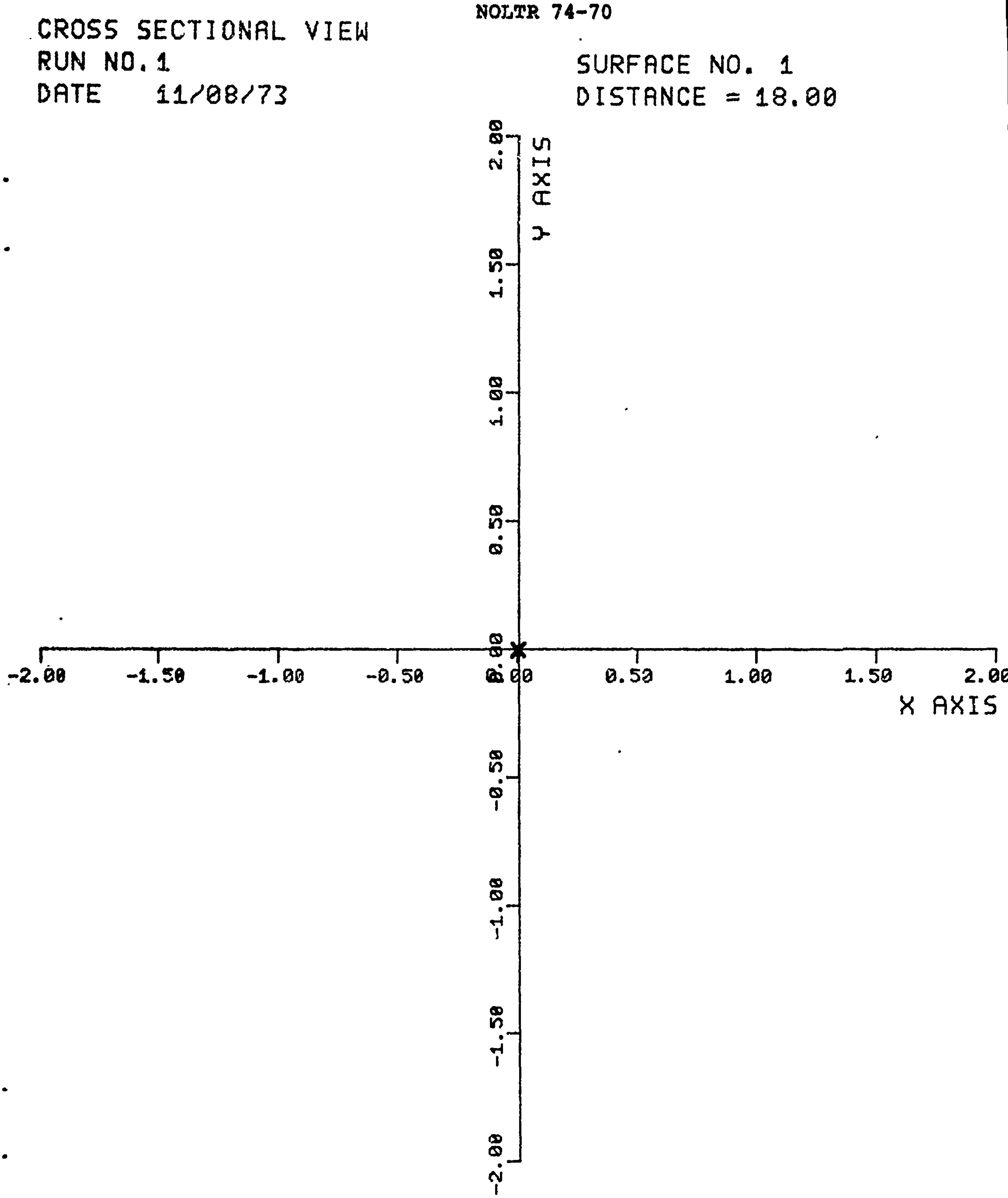



Figure 14 Cross Section of Beam at Focus Point

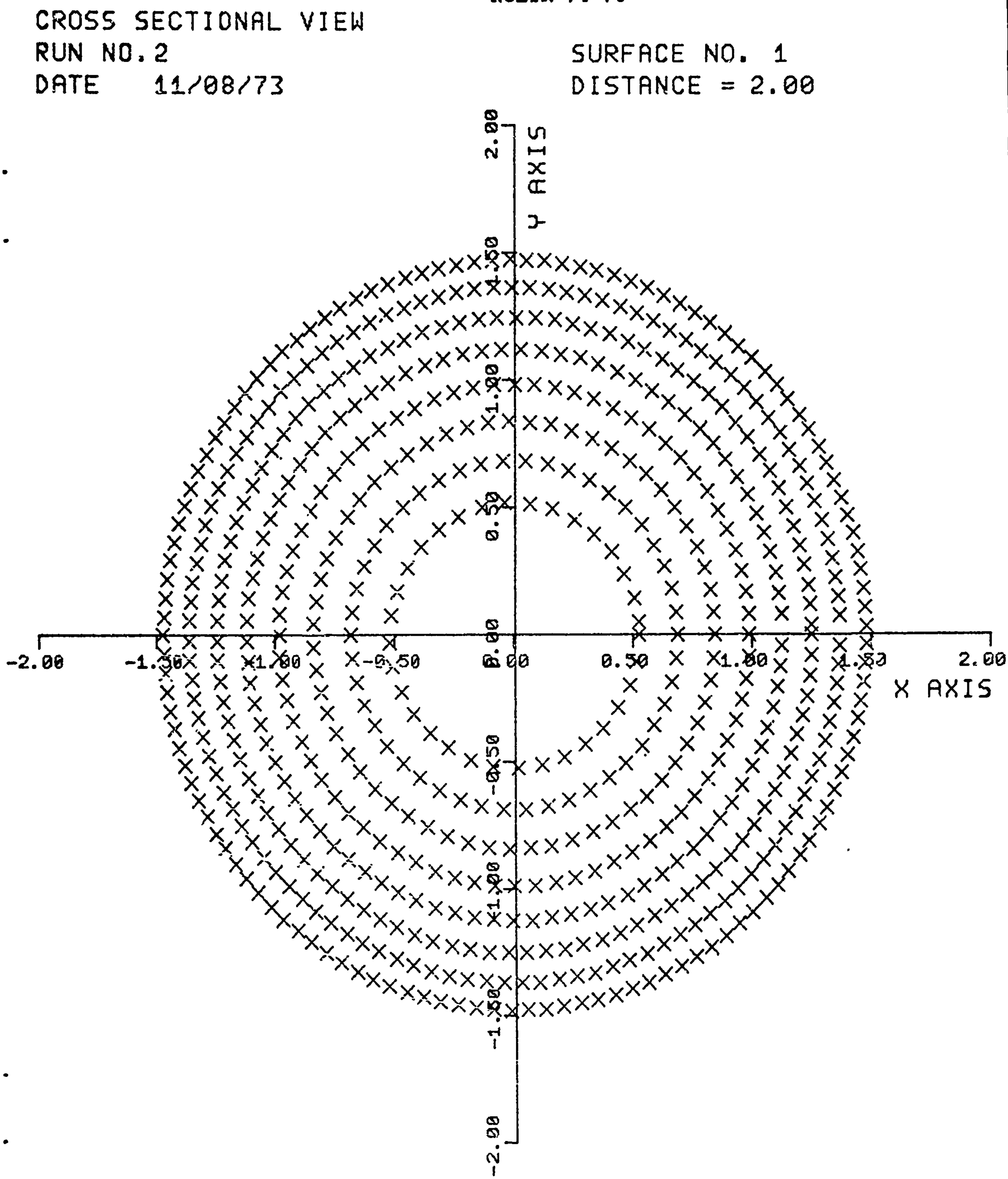



Figure 15 Cross Section of Beam at Z = 2

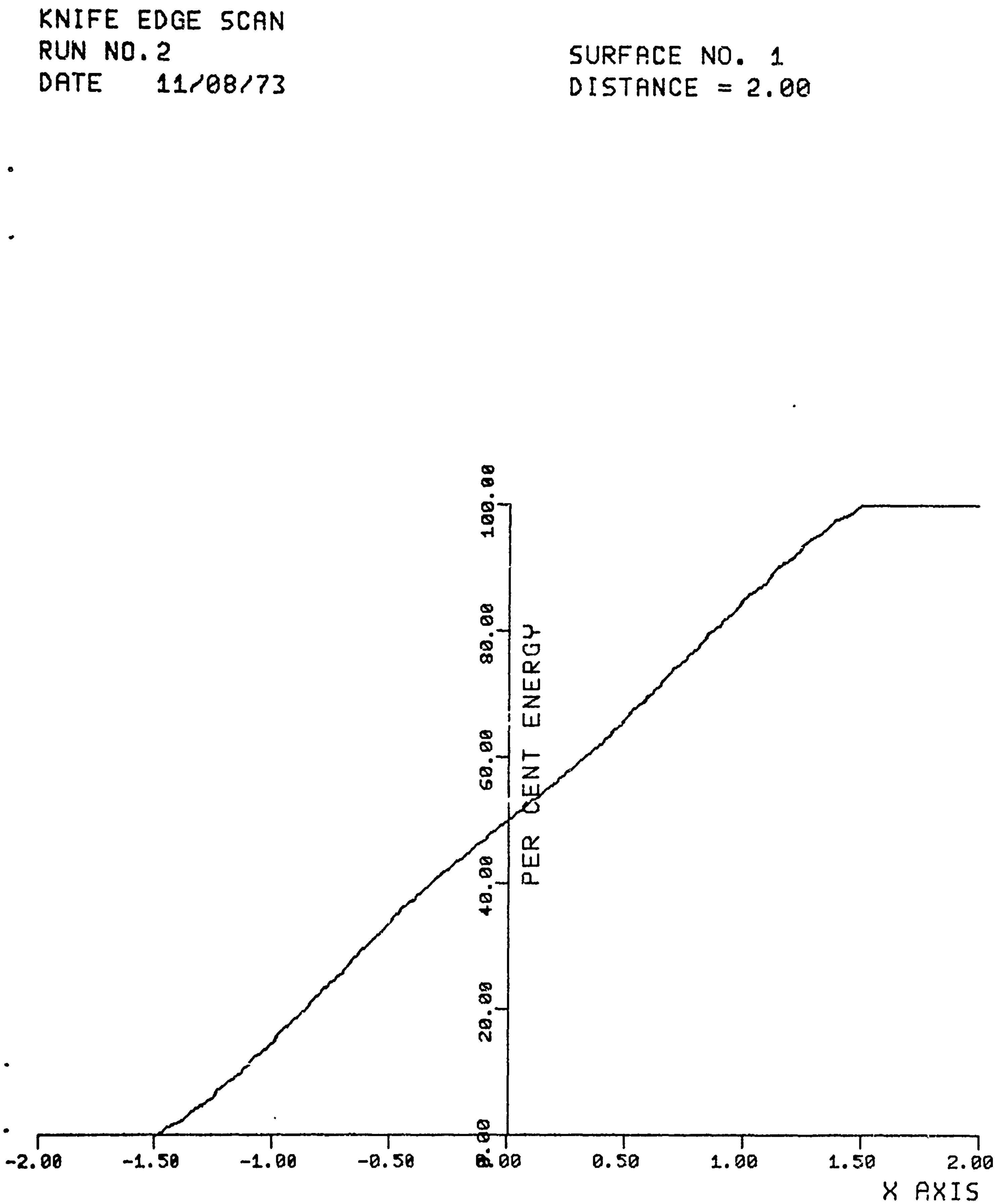



Figure 16 Knife Edge Scan of Beam at Z = 2

b. Elliptic Cylinder

Since the RAYTRACE program is not limited to just quadric surfaces, an elliptic cylinder can be formed from the previous example by setting either a = 0 or b = 0. By setting a = 0 the axis of the cylinder is parallel to the X-axis. If b = 0, then the axis of the cylinder is parallel to the Y-axis. Using the same parameters of the previous problem the focus points are Z = 2 and Z = 18. By placing a point source at the focus point Z = 2 the reflected beam should be focused along a line parallel to the axis of the cylinder at Z = 18 as shown in Figure 17. The input data are listed in Figure 18 for two cylindrical mirrors of different cylindrical axes. Figures 19 and 20 show that the cross section of the reflected beam in the plane of the focus point Z = 18 is a line parallel to the axis of the cylindrical mirror.

c. Telescope

This sample problem will show the use of the focusing parameter or DV(21). This problem is depicted by a telescope which consists of a convex spherical mirror and a concave ellipsoidal mirror as shown in Figure 21. The reference coordinate systems of the two surfaces are placed at the vertices and separated by a distance of 124.2. The spherical mirror is surface one with the center of symmetry at Z = -73.6. Surface two is the ellipsoidal mirror with the quadric constants a = b = 675.13995 and c = 1415.5712. The center of symmetry is at Z = 1415.5712. It should be noted that the dimensions for a particular surface are measured from its respective reference coordinate system. A source of parallel rays is considered to be somewhere between the two mirrors. With the separation distance of 124.2 the beam will focus at infinity after reflecting off the two surfaces. By increasing the distance between the mirrors, the beam will converge at some distance less than infinity. To focus the beam at the far field distances of 70,000 and 100,000 the center of symmetry is moved by -.396 and -.263, respectively. This in effect increases the distance between the mirrors. The input data for focusing the telescope at these two distances are listed in Figure 22. Figures 23 and 24 show that the telescope can focus the beam at the far field distances by the use of the focusing parameter.

d. Telescope with Plane Mirror

Figure 25 shows a telescope of the previous problem with the addition of a plane mirror. The orientation of the various coordinate systems needed to trace a beam through the system is shown for each surface. A dot represents the X-axis out of the page and a cross represents the X-axis into the page. It should be pointed out that the output of surface one is not oriented with the input of surface two even though the planes of incidence are coincident. Therefore, a rotation of $\pi$ radians is necessary to orient the two coordinate systems.

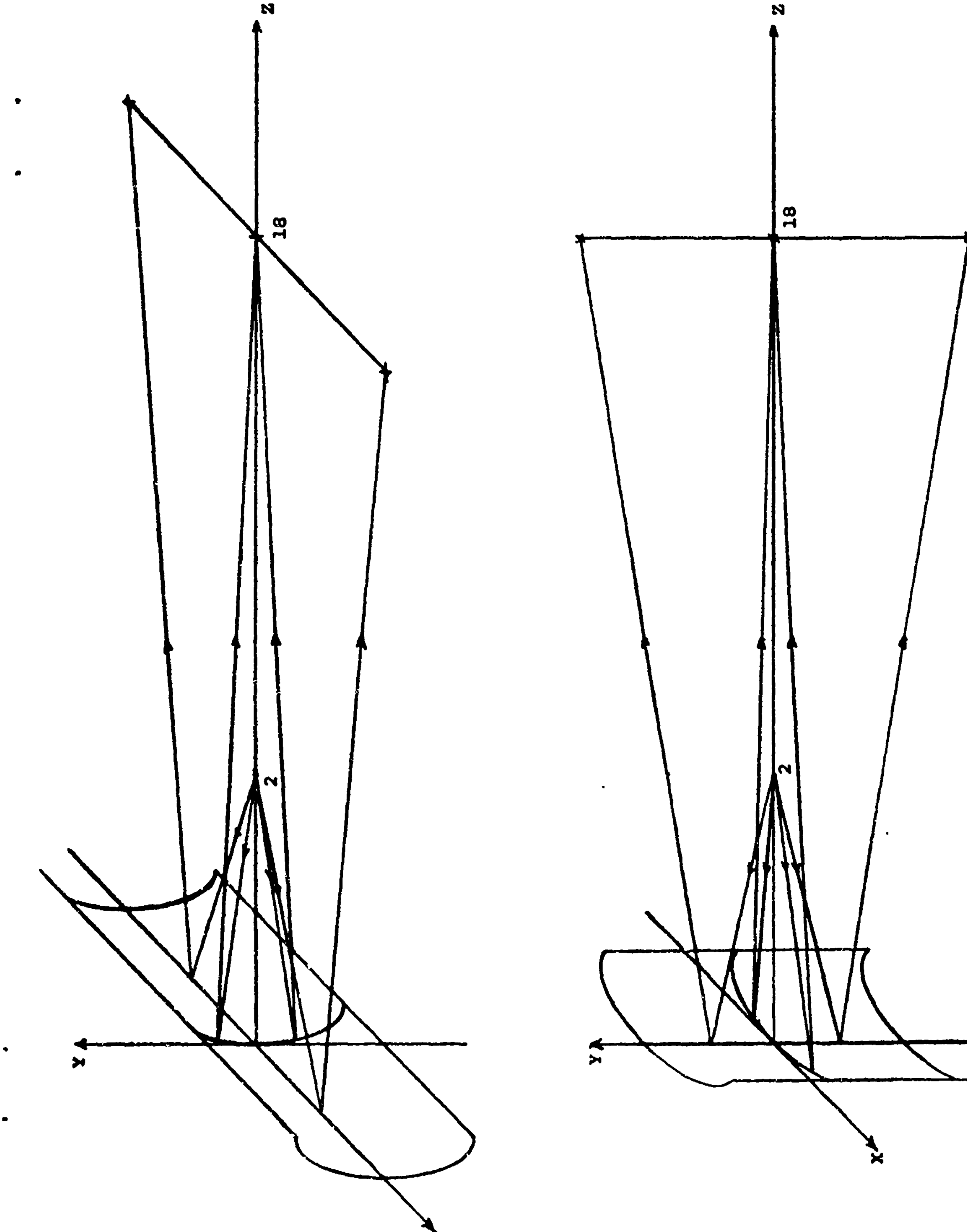



Figure 17 Elliptic Cylinders

```
    RUN NO. 00003     SURFACES = 1 THRU 1      DATE =  7 DEC 1973    0
    CONCAVE ELLIPTICAL CYLINDER                                      0
    A=0 , B=6 , C=10, CZ=10 , RI=2 , RR=18                           0
                                                                     1
 1   15707963+00   2   20000000+00   3   18000000+01   4   00000000+00
 5   00000000+00   6   00000000+00   7   00000000+00   8   00000000+00
 9   00000000+00  10  -10000000+00  11   10000000+00  12  -10000000+00
13   00000000+00  14   27777778-02  15   10000000-02  16   00000000+00
17   10000000+00  18   00000000+00  19   00000000+00  20   10000000+01
21   00000000+00  22  -10000000+00  23   20000000+01  24   20000000+01
25   00000000+00  26   10000000+00  27   30000000+00  28   30000000+01
29   10000000+00
50
    RUN NO. 00004     SURFACES = 1 THRU 1      DATE =  7 DEC 1973    0
    CONCAVE ELLIPTICAL CYLINDER                                      0
    A=6 , B=0 , C=10, CZ=10 , RI=2 , RR=18                           0
                                                                     1
 1   15707963+00   2   20000000+00   3   18000000+01   4   00000000+00
 5   00000000+00   6   00000000+00   7   00000000+00   8   00000000+00
 9   00000000+00  10  -10000000+00  11   10000000+00  12  -10000000+00
13   27777778-02  14   00000000+00  15   10000000-02  16   00000000+00
17   10000000+00  18   00000000+00  19   00000000+00  20   10000000+01
21   00000000+00  22  -10000000+00  23   20000000+01  24   20000000+01
25   00000000+00  26   10000000+00  27   40000000+00  28   30000000+01
29   10000000+00
50
                                                                     2
```

Figure 18 Input Data for Cylindrical Mirror

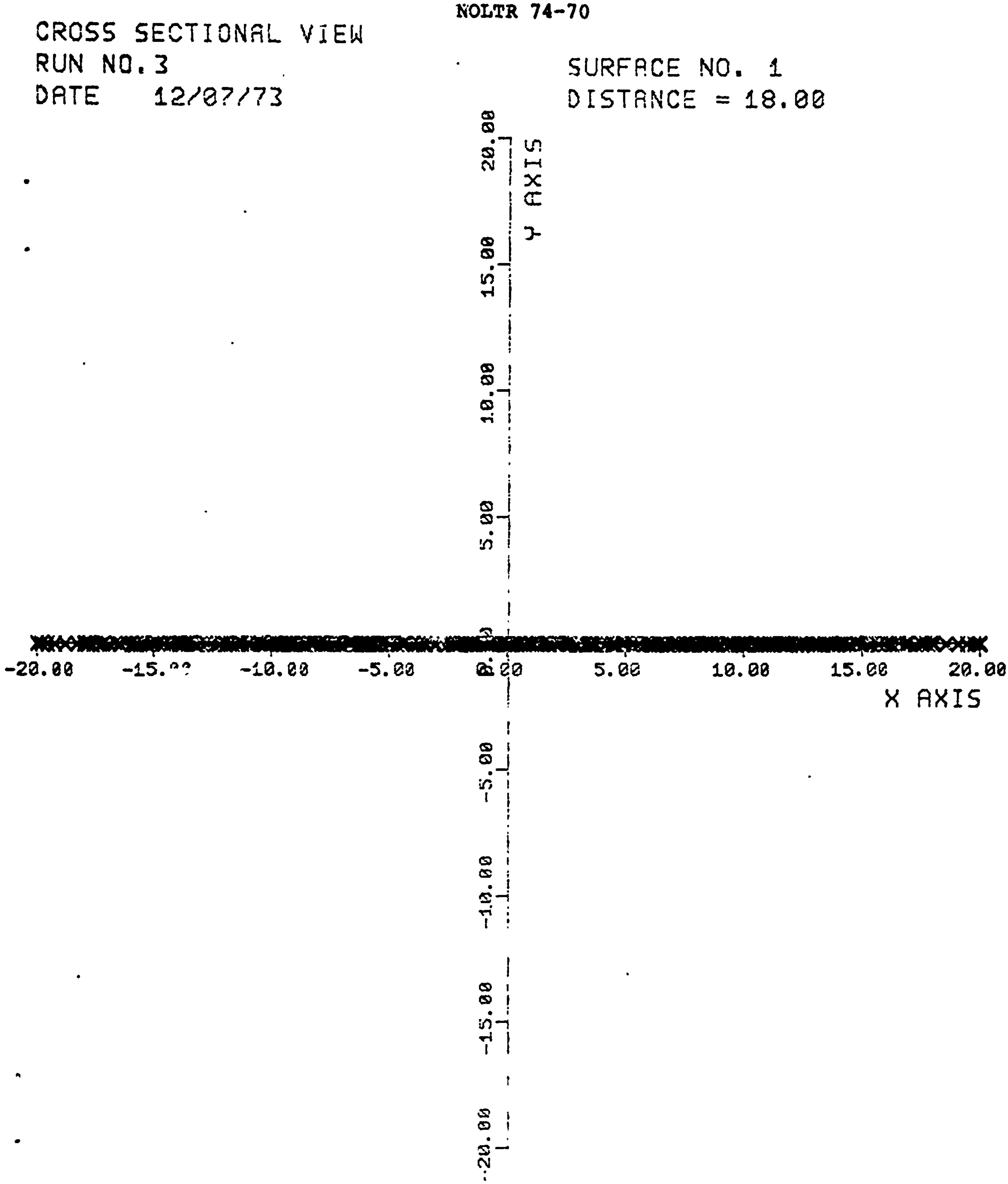



**Figure 19 Reflected Beam from Cylindrical Mirror Parallel to X-Axis**

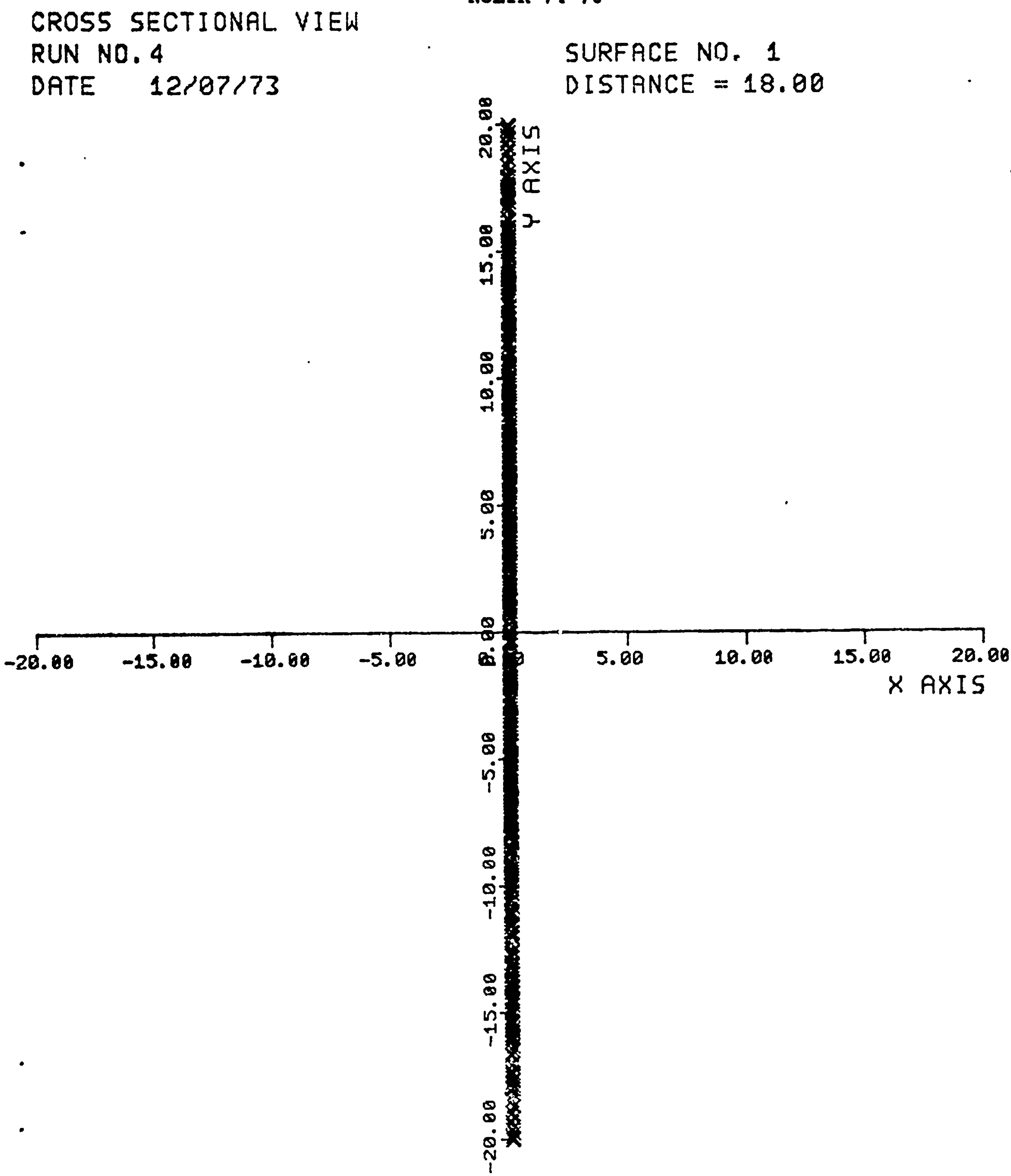



Figure 20 Reflected Beam from Cylindrical Mirror Parallel to Y-Axis

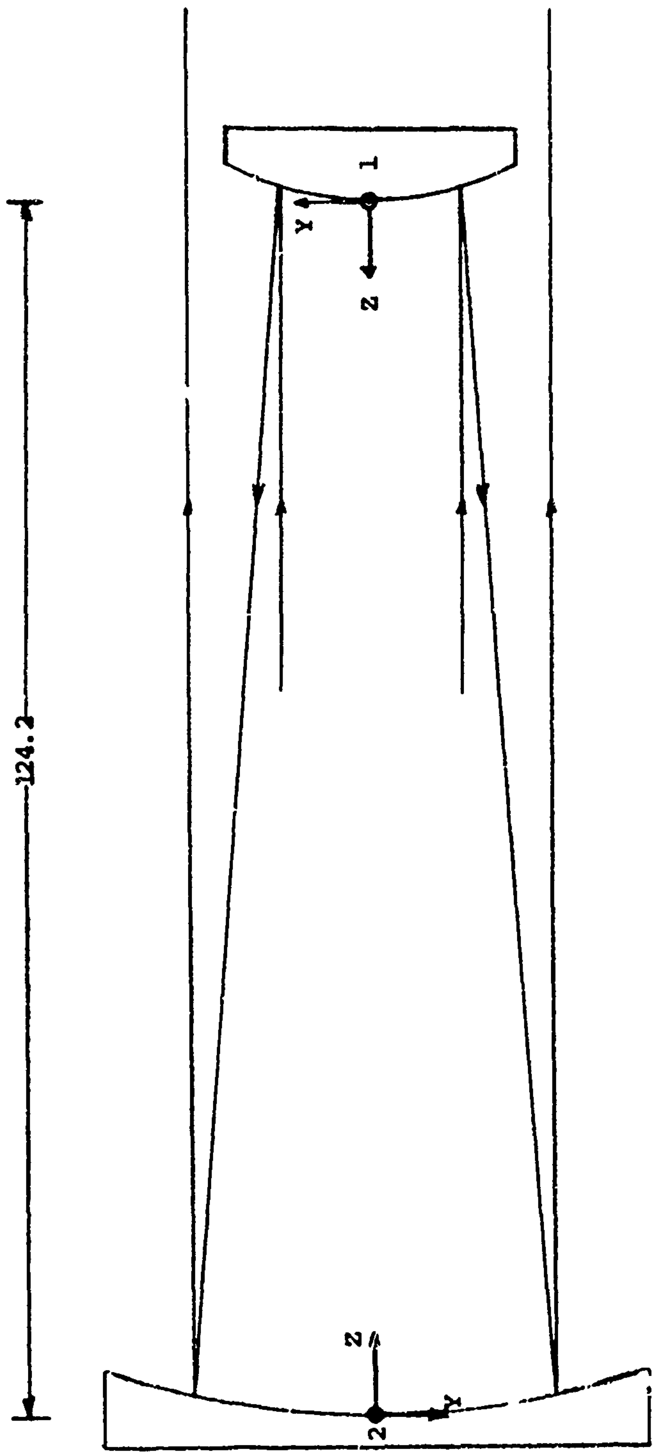



Figure 21 Telescope

```
      RUN NO. 00005     SURFACES = 1 THRU 2      DATE = 12 DEC 1973
      INPUT BEAM : RMAX = 8.0                                          0
                   RMIN = 3.2                                          0
      OUTPUT AT 70,000                                                 0
      SURFACE 1 : F = -0.376                                           0
                                                                       1
 1   15707963+00    2   11000000+02    3   12000000+02    4   00000000+00
 5   00000000+00    6   00000000+00    7   00000000+00    8   00000000+00
 9   00000000+00   10  -10000000+00   11   10000000+00   12   10000000+00
13   10000000+00   14   10000000+00   15   10000000+00   16   00000000+00
17   54169600+03   18   00000000+00   19   00000000+00   20  -73600000+01
21  -37600000-01   22   10000000+00   23   40000000+01   24   40000000+01
25   00000000+00   26   10000000+00   27   50000000+00   28   40000000+01
29   20000000+00
50
 1   15707963+00    2   41990000+00    3   70000000+04    4   00000000+00
 5   00000000+00    6   00000000+00    7   00000000+00    8   00000000+00
 9   00000000+00   10  -10000000+00   11   10000000+00   12  -10000000+00
13   21938775-06   14   21938775-06   15   49904138-07   16   00000000+00
17   10000000+00   18   00000000+00   19   00000000+00   20   14155712+03
21   00000000+00   22  -10000000+00   23   10000000+01   24   40000000+01
25   00000000+00   26   20000000+00
50
      RUN NO. 00006     SURFACES = 1 THRU 2      DATE = 12 DEC 1973
      INPUT BEAM : RMAX = 8.0                                          0
                   RMIN = 3.2                                          0
      OUTPUT AT 100,000                                                0
      SURFACE 1 : F = -0.263                                           0
                                                                       1
 1   15707963+00    2   11000000+02    3   12000000+02    4   00000000+00
 5   00000000+00    6   00000000+00    7   00000000+00    8   00000000+00
 9   00000000+00   10  -10000000+00   11   10000000+00   12   10000000+00
13   10000000+00   14   10000000+00   15   10000000+00   16   00000000+00
17   54169600+03   18   00000000+00   19   00000000+00   20  -73600000+01
21  -26390000-01   22   10000000+00   23   40000000+01   24   40000000+01
25   00000000+00   26   10000000+00   27   60000000+00   28   40000000+01
29   20000000+00
50
 1   15707963+00    2   41990000+00    3   10000000+05    4   00000000+00
 5   00000000+00    6   00000000+00    7   00000000+00    8   00000000+00
 9   00000000+00   10  -10000000+00   11   10000000+00   12  -10000000+00
13   21938775-06   14   21938775-06   15   49904138-07   16   00000000+00
17   10000000+00   18   00000000+00   19   00000000+00   20   14155712+03
21   00000000+00   22  -10000000+00   23   10000000+01   24   40000000+01
25   00000000+00   26   20000000+00
50
                                                                       2
```

Figure 22 Input Data for Telescope

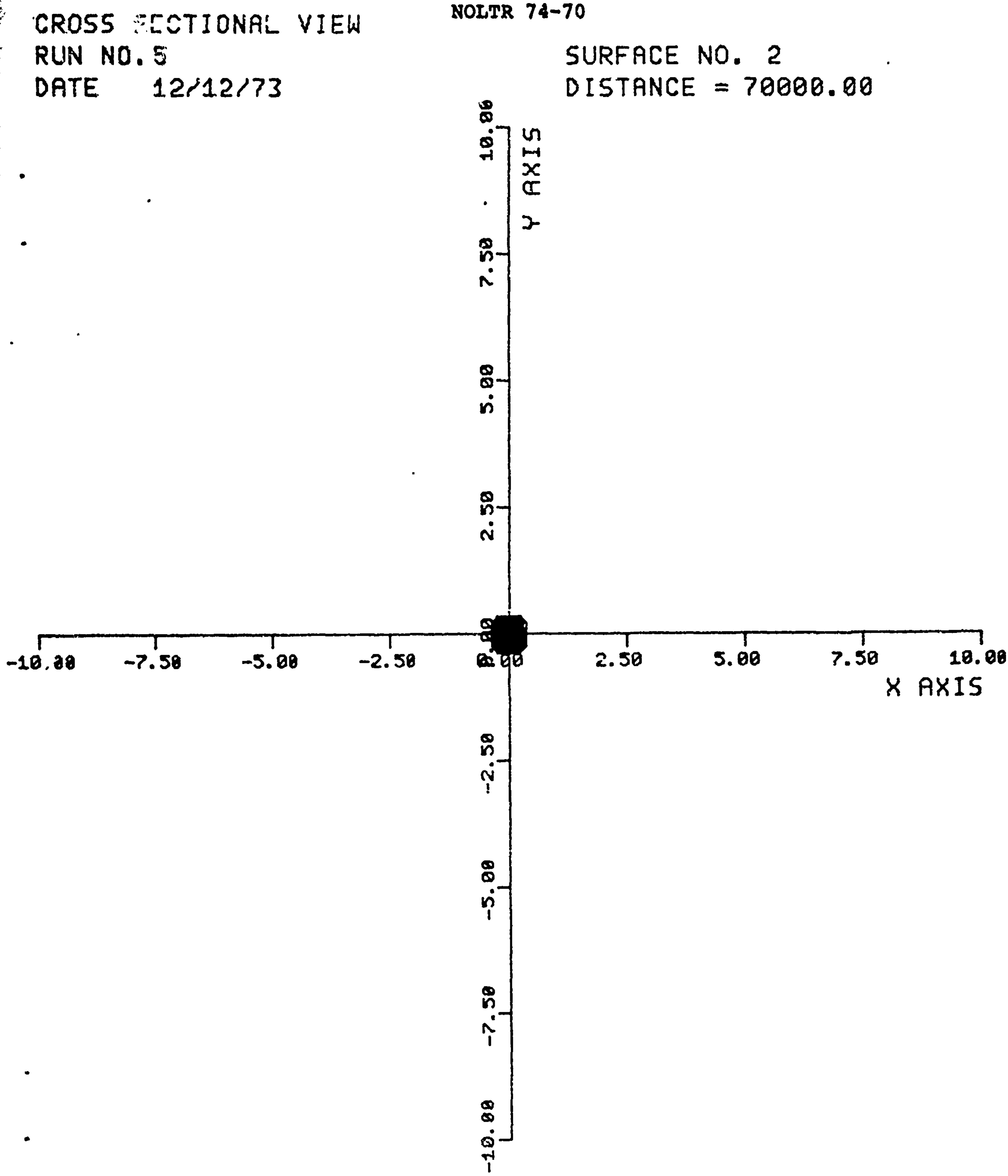



Figure 23 Beam at Z = 70,000

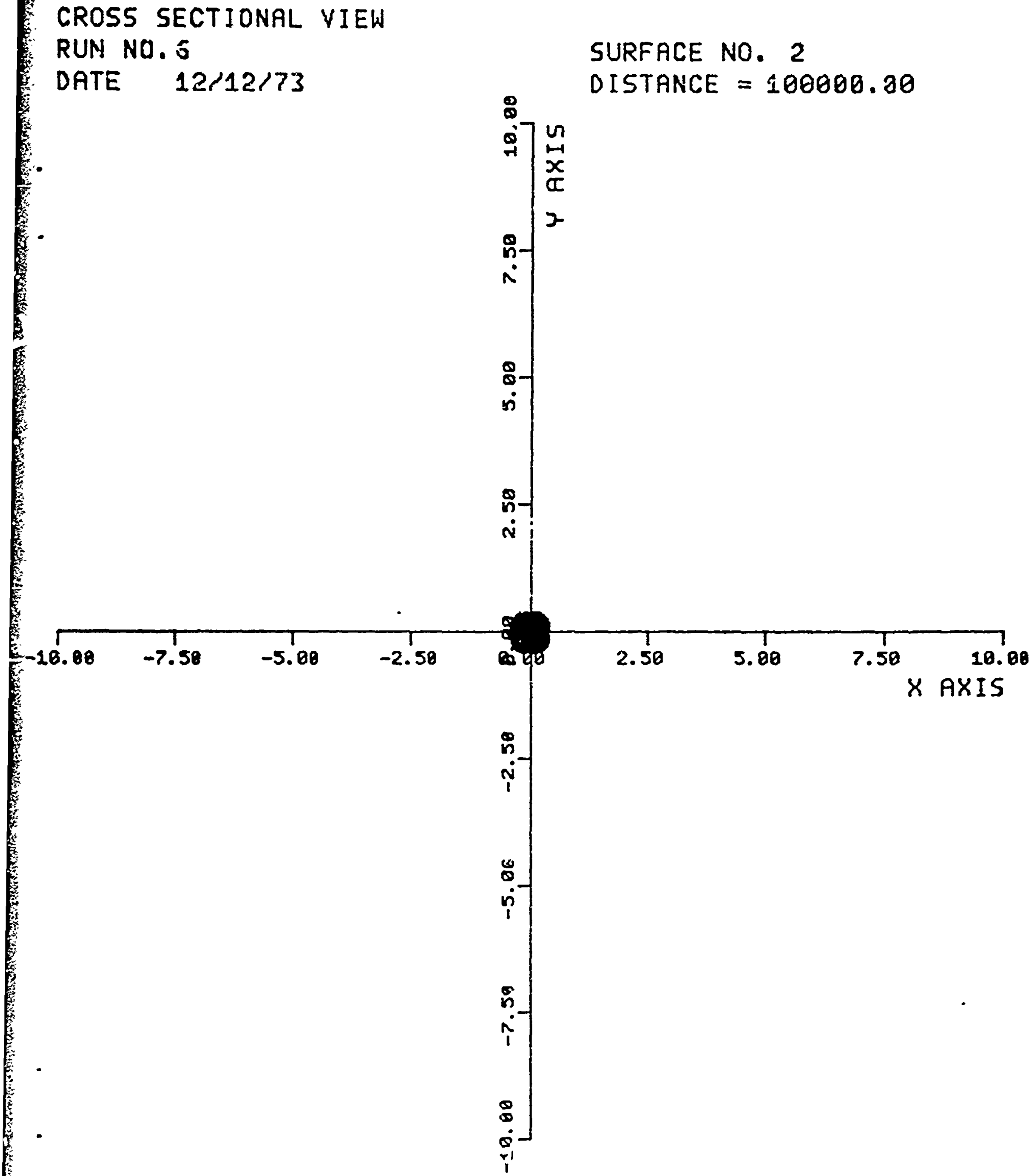



Figure 24 Beam at Z = 100,000

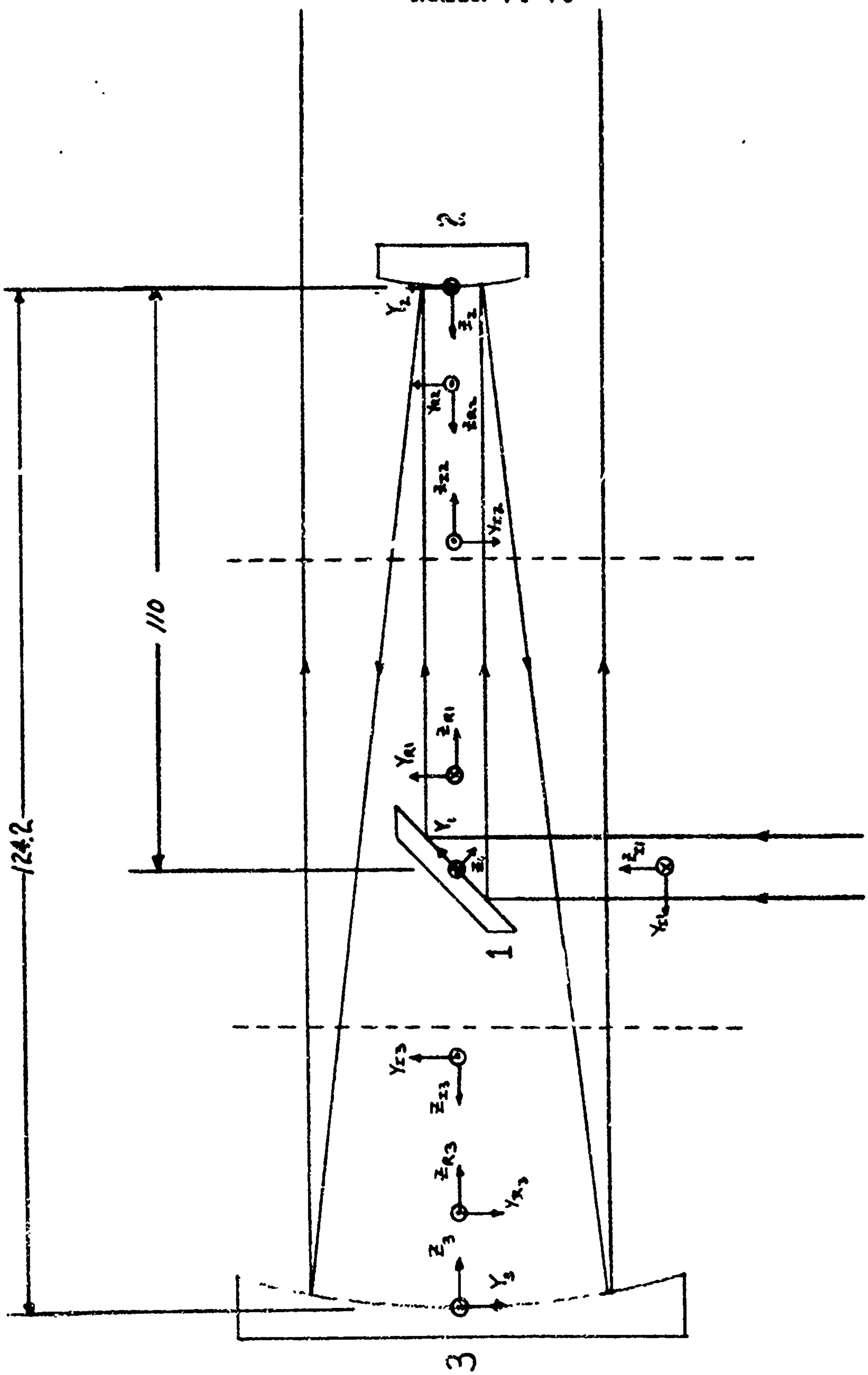



Figure 25 Coordinate Systems for Telescope with Plane Mirror

The previous example has shown that the beam can be focused at some distance in the far field. If the plane mirror has some angular deviation from its reference coordinate system, the focused beam in the far field will move about the origin of the output reference plane. Figure 26 shows a listing of input data where $\epsilon = \eta = 6$ milliradians for the plane mirror. Figure 27 shows how far the focused beam has translated from the origin.

e. Two Thick Lenses

So far the examples have been concerned with mirrors, the RAYTRACE program can handle the refraction case also. Two thick lenses are used to demonstrate this capability as shown in Figure 28. A point source is placed at the focal point of the first lens. After the beam passes the first lens the rays should become parallel. On passing the second lens the rays should focus into a point at the second focal point of the system. The listing of the input data and parameters of the system are given in Figure 29. Figure 30 shows the beam focused at the second focal point of the system after it has been traced through the two lenses.

```
     RUN NO. 00007     SURFACES = 1 THRU 3      DATE = 17 DEC 1973    0
     TELESCOPE WITH PLANE MIRROR                                      0
     INPUT BEAM : RMAX = 8.0                                          0
                  RMIN = 3.2                                          0
     OUTPUT AT  70.000                                                0
     SURFACE 1 : EPSILON = ETA = 6 MILLIRADIANS                       0
     SURFACE 2 : F = -0.376                                           0
                                                                      1
 1   78539816-01   2   20000000+01   3   10000000+02   4   31415927+00
 5   60000000-03   6   60000000-03   7   00000000+00   8   00000000+00
 9   00000000+00  10  -10000000+00  11   10090000+03  12   00000000+00
13   00000000+00  14   00000000+00  15   00000000+00  16   00000000+00
17   00000000+00  18   00000000+00  19   00000000+00  20   00000000+00
21   00000000+00  22   00000000+00  23   16000000+01  24   16000000+01
25   00000000+00  26   10000000+00  27   70000000+00  28   30000000+01
29   30000000+00
30
 1   15707963+00   2   10016000+01   3   12000000+02   4   00000000+00
 5   00000000+00   6   00000000+00   7   00000000+00   8   00000000+00
 9   00000000+00  10  -10000000+00  11   10000000+00  12   10000000+00
13   10000000+00  14   10000000+00  15   10000000+00  16   00000000+00
17   5416[illegible]00+03  18   00000000+00  19   00000000+00  20  -73600000+01
21  -[illegible]7600000-01  22   10000000+00  23   40000000+01  24   40000000+01
25   60000000+00  26   20000000+00
30
 1   15707963+00   2   41990000+00   3   70000000+04   4   00000000+00
 5   00000000+00   6   00000000+00   7   00000000+00   8   00000000+00
 9   00000000+00  10  -10000000+00  11   10000000+00  12  -10000000+00
13   21938775-06  14   21938775-06  15   49904138-07  16   00000000+00
17   10000000+00  18   00000000+00  19   00000000+00  20   1[illegible]155712+03
21   00000000+00  22  -10000000+00  23   40000000+02  24   40000000+02
25   00000000+00  26   30000000+00
30
                                                                      2
```

Figure 26  Input Data for Telescope with Plane Mirror

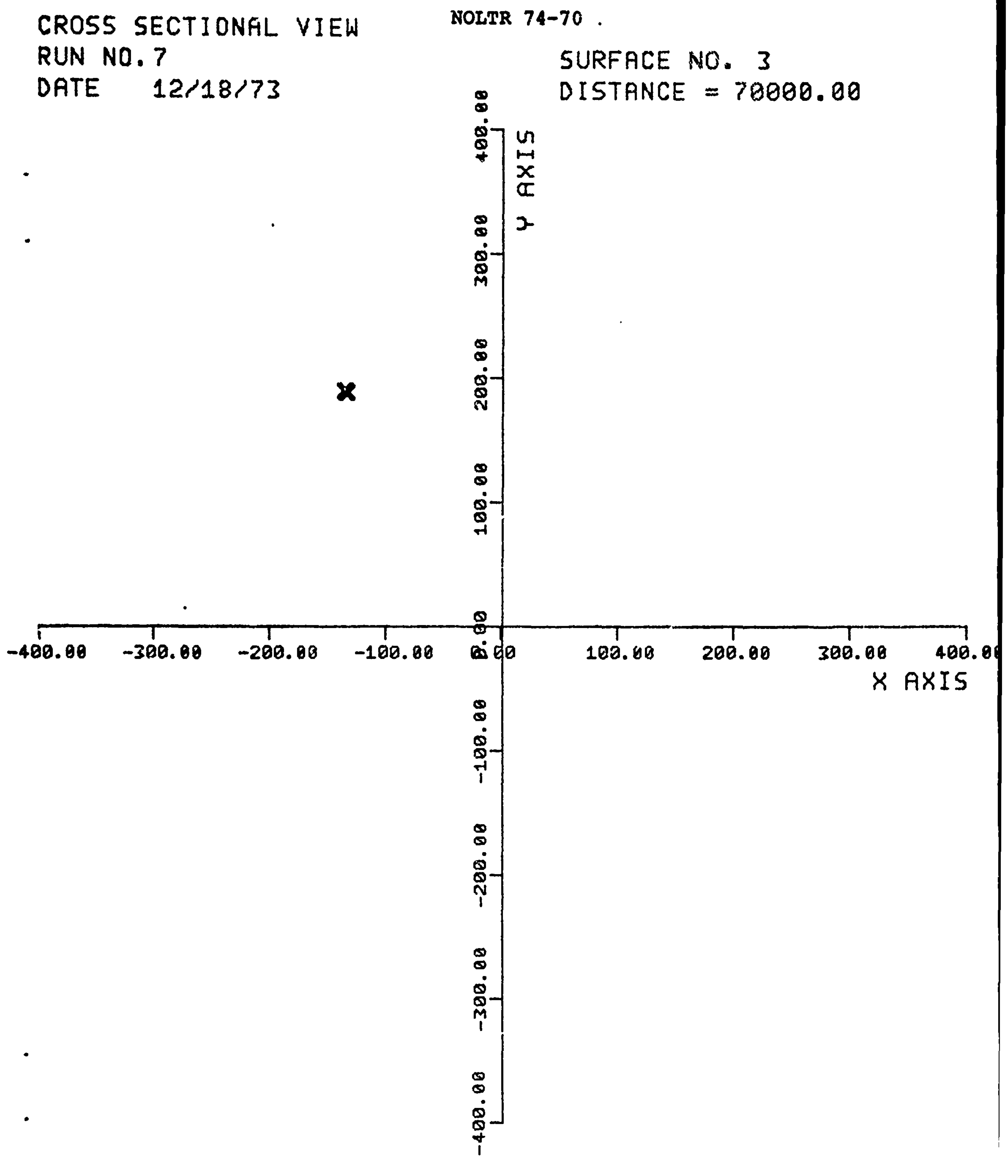



Figure 27 Translation of Focused Beam Due Angular Deviation of Plane Mirror

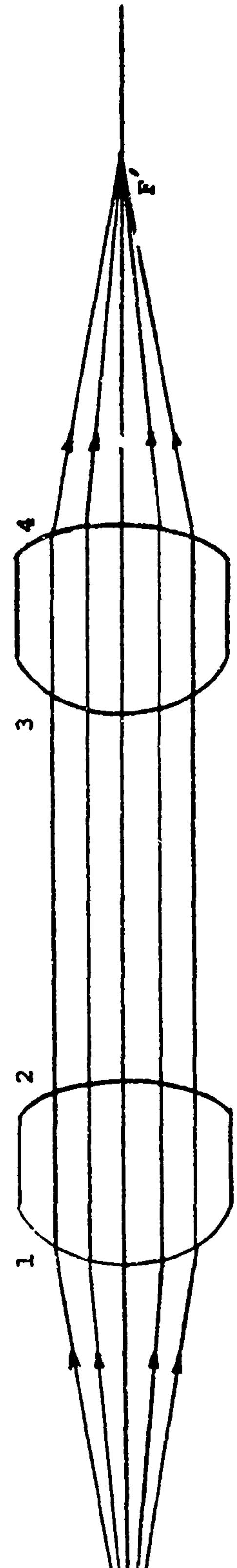



Figure 28 Two Thick Lenses

```
      RUN NO. 00008     SURFACES = 1 THRJ 4      DATE = 21 DEC 1973    0
      TWO THICK LENSES                                                 0
      INPUT BEAM IS POINT SOURCE AT FIRST FOCAL POINT OF SYSTEM        0
      OUTPUT BEAM IS AT SECOND FOCAL POINT OF SYSTEM                   0
                                                                       1
 1    15707963+00   2   35454545+00   3   10000000+00   4   00000000+00
 5    00000000+00   6   00000000+00   7   00000000+00   8   00000000+00
 9    00000000+00  10   10000000+00  11   66666667-01  12   10000000+00
13    10000000+00  14   10000000+00  15   10000000+00  16   00000000+00
17    90000000+00  18   00000000+00  19   00000000+00  20  -30000000+00
21    00000000+00  22   10000000+00  23   40000000+00  24   40000000+00
25    00000000+00  26   10000000+00  27   80000000+00  28   30000000+01
29    40000000+00
50
 1    15707963+00   2   10000000+00   3   31818182+00   4   00000000+00
 5    00000000+00   6   00000000+00   7   00000000+00   8   00000000+00
 9    00000000+00  10   10000000+00  11   15000000+00  12  -10000000+00
13    10000000+00  14   10000000+00  15   10000000+00  16   00000000+00
17    25000000+01  18   00000000+00  19   00000000+00  20   50000000+00
21    00000000+00  22  -10000000+00  23   40000000+00  24   40000000+00
25    00000000+00  26   20000000+00
50
 1    15707963+00   2   35454545+00   3   10000000+00   4   00000000+00
 5    00000000+00   6   00000000+00   7   00000000+00   8   00000000+00
 9    00000000+00  10   10000000+00  11   66666667-01  12   10000000+00
13    10000000+00  14   10000000+00  15   10000000+00  16   00000000+00
17    90000000+00  18   00000000+00  19   00000000+00  20  -30000000+00
21    00000000+00  22   10000000+00  23   40000000+00  24   40000000+00
25    00000000+00  26   30000000+00
50
 1    15707963+00   2   10000000+00   3   31818182+00   4   00000000+00
 5    00000000+00   6   00000000+00   7   00000000+00   8   00000000+00
 9    00000000+00  10   10000000+00  11   15000000+00  12  -10000000+00
13    10000000+00  14   10000000+00  15   10000000+00  16   00000000+00
17    25000000+01  18   00000000+00  19   00000000+00  20   50000000+00
21    00000000+00  22  -10000000+00  23   40000000+00  24   40000000+00
25    00000000+00  26   40000000+00
50
                                                                       2
```

Figure 29  Input Data for Two Thick Lenses

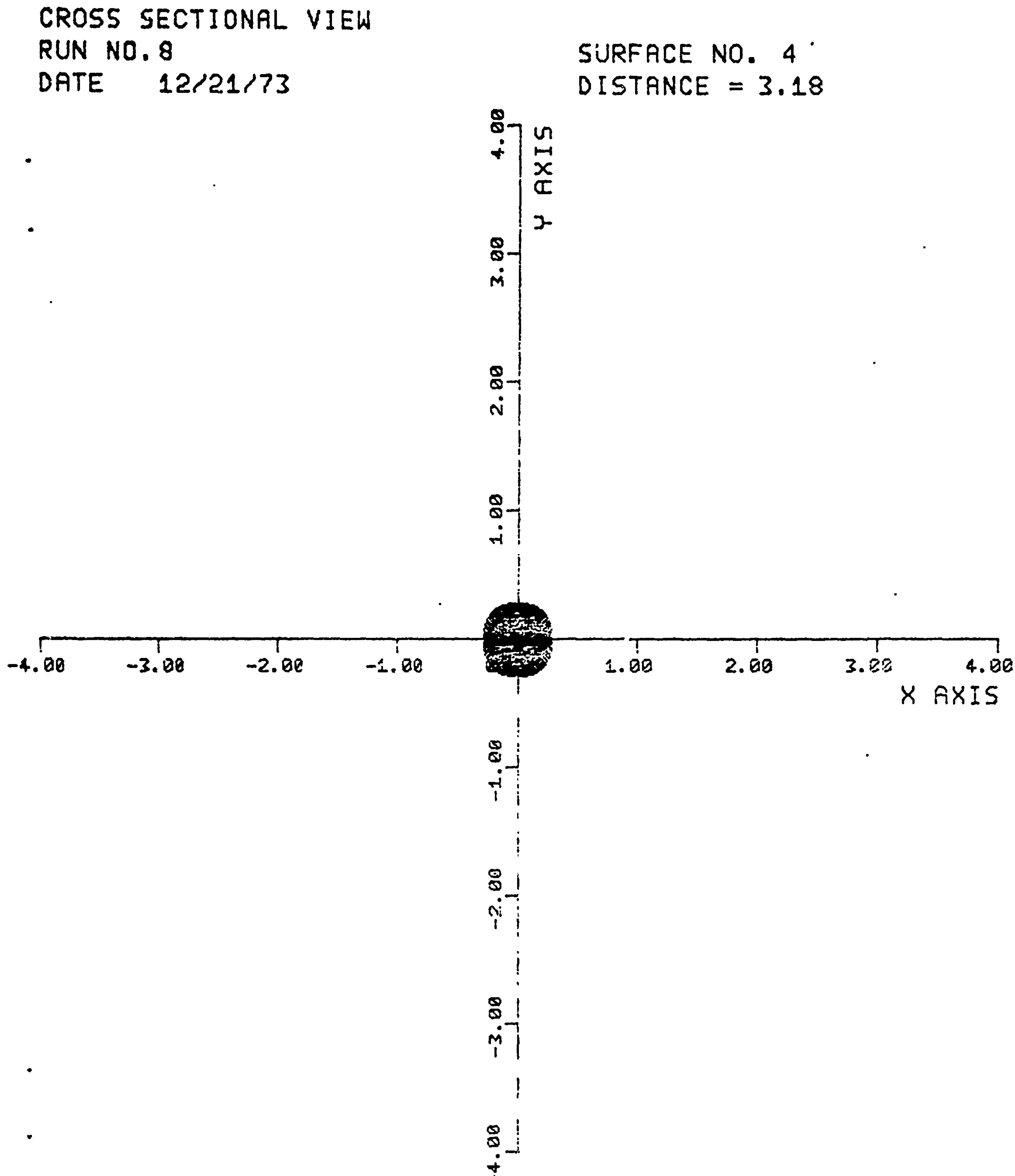



Figure 30 Beam at Second Focal Point

# Appendix A

## GLOSSARY

A - Parameter in the basic equation of a quadric surface

a - Constant used in quadric equations

$\mathcal{A}$ - Constant formed by the combination of terms and used in a quadratic equation for solving the Z coordinate of the incident ray on a quadric surface

B - Parameter in the basic equation of a quadric surface

b - Constant used in quadric equations

$\mathcal{B}$ - Constant formed by the combination of terms and used in a quadratic equation for solving the Z coordinate of the incident ray on a quadric surface

C - Parameter in the basic equation of a quadric surface

$C_x$, $C_y$, $C_z$ - Center of symmetry of a quadric surface

c - Constant used in quadric equations

$\mathcal{C}$ - Constant formed by the combination of terms and used in a quadratic equation for solving the Z coordinate of the incident ray on a quadric surface

D - Parameter in the basic equation of a quadric surface

E - Parameter in the basic equation of a quadric surface

F - Parameter which focuses a quadric surface along the Z-axis of symmetry

$G_x$, $G_y$, $G_z$ - Parameters formed by combination of terms in the basic equation of a quadric surface

N - Ratio of $n_I/n_R$ for a refractive surface and equal to one for a reflective surface

$n_I$ - Index of refraction of the medium of the incident ray

$n_R$ - Index of refraction of the medium of the refracted ray

RP - A constant equal to one,whose sign determines the direction of the output ray from an optical surface (+1 refractive surface, -1 reflective surface)

$R_I$ - Distance from the origin of the input reference plane to the origin of the plane surface reference

$R_R$ - Distance from the origin of the plane surface reference to the origin of the output reference plane

S - A constant equal to one, whose sign determines the concavity or convexity of a quadric surface

$T_I$ - Matrix representing the transformation of vectors from the input reference plane to the plane surface

$T_R$ - Matrix representing the transformation of vectors from the plane surface to the output reference plane

$V_I$ - Incident ray

$V_R$ - Refracted or reflected ray

$X_j$, $Y_j$, $Z_j$ - A right handed orthogonal coordinate system where $j = 1, 2, \ldots n$

$X_1$, $Y_1$, $Z_1$ - Plane surface reference coordinate system

$X_2$, $Y_2$, $Z_2$ - Input reference coordinate system

$X_3$, $Y_3$, $Z_3$ - Coordinate system attached to the plane surface

$X_4$, $Y_4$, $Z_4$ - Output reference coordinate system

$X_{jD}$, $Y_{jD}$, $Z_{jD}$ - Translation of a coordinate system from its reference position

$X_{jQ}$, $Y_{jQ}$, $Z_{jQ}$ - Coordinate point in one coordinate system in terms of another coordinate system

$X_{jP}$, $Y_{jP}$, $Z_{jP}$ - Incident point of ray on $Z = 0$ plane

$\alpha_j$, $\beta_j$, $\gamma_j$ - Angles that define the direction of a ray with respect to the X, Y, and Z axes and their cosines are known as the direction cosines of the ray

$\epsilon$ - Angle of rotation of the plane surface about the $X_1$ axis

$\eta$ - Angle of rotation of the plane surface about the $Y_1$ axis

$\theta_I$ - Angle of the input reference ray makes with the negative $Y_1$ axis in the plane of incidence

$\theta_R$ - Angle the output reference ray makes with the positive $Y_1$ axis in the plane of incidence

$\lambda$ - Angle between the positive $X_4$ axis of an output reference plane and the positive $X_2$ axis of the following input reference plane

$\tau_I$ - Angle of incidence

$\tau_R$ - Angle of refraction or reflection

$\phi_2$ - Angular deviation of the input ray from the positive $Z_2$ axis in the $Y_2 = 0$ plane

$\phi_4$ - Angular deviation of the output ray from the positive $Z_4$ axis in the $Y_4 = 0$ plane

$\psi_2$ - Angular deviation of the input ray from the positive $Z_2$ axis in the $X_2 = 0$ plane

$\psi_4$ - Angular deviation of the output ray from the positive $Z_4$ axis in the $X_4 = 0$ plane

## Appendix B

## RAYTRACE PROGRAM DESCRIPTION

A brief description of each routine in the RAYTRACE program is given along with a listing. A flow diagram of the program is also presented. The parameter R is a Hollerith constant containing the date.

RAYTRACE - The main routine sets up the input beam data and the surface parameters for each optical system. This routine exercises overall control of the program options.

BCOND (RMAX2, RMIN2, NRY) - After the beam has passed through the optical surface to the output reference place, each ray of the beam is compared with a set of boundary conditions for a circular beam at the output reference plane. RMAX2 is the square of the maximum radius and RMIN2 is the square of the minimum radius. NRY is the number of rays.

BEAMIN - Beam data that has been generated and stored on disc in a data file is transferred to a scratch file. The number of data blocks and the number of rays in each block are stored in the NR array.

CROSSECT (R) - The data necessary for plotting a cross sectional view of a beam at the output reference plane of an optical surface is set up and stored.

DATE (R) - The date is retrieved from a system subroutine and is stored in R as a Hollerith constant in the form MM/DD/YY.

GRAPHS (R) - This subroutine receives the generated data from CROSSECT or SCAN and plots the output on the electrostatic printer.

INCIDENT (NRY) - This subroutine calculates the incident position of the input ray on a quadric surface and the angular rotation of the plane surface system required to place the plane tangent to the incident point. NRY represents the number of rays.

INTRFACE - The output ray parameters are interfaced with the input ray parameters of the following surface when the output reference system is out of phase with the following input reference system.

MATVEC (A, B, C, N) - This subroutine combines six different matrix-vector or matrix-matrix operations in one subroutine. A, B, and C are 3 x 3 matrix arrays stored column wise or vectors depending on N where A, B, and N are inputs and C is the output. If capital letters denote 3 x 3 matrices and lower case letters denote vectors, MATVEC computes:

$$N = 0,\ c = Ab$$

$$N = 1,\ c = A^T b$$

$$N = 2,\ C = AB$$

$$N = 3,\ C = A^T B$$

$$N = 4,\ C = AB^T$$

$$N = 5,\ C = A^T B^T$$

where T represents the transpose of a matrix.

READ - Prior to tracing a beam through an optical system, the surfaces are read and stored. An entry through LIST will print the parameters of the particular surface.

RETRACE - If it is desired to change certain parameters in an optical system and retrace the beam through the system, this subroutine will accept input from the typewriter for changes to each surface and will reload the input beam.

SCAN (R) - The data necessary for plotting a knife edge scan of a beam at the output reference plane of a surface is set up and stored.

SINCOS 1 (A,B) - The sine and cosine functions of angle A in radians are calculated with the output stored in the B array. B(1) contains the sine of A with the cosine of A in B(2).

TRACE (NRY, AZERO) - A number of rays, NRY, are traced from the input reference plane through the plane surface system to the output reference plane. If $\theta_I = \pi/2$, then AZERO equals 0. For all other values of $\theta_I$, AZERO equals 1.

TRANSFER - If it is desired to save the output beam data for later use, this subroutine will transfer the beam data from a scratch file to a saved data file.

WRITE - This subroutine will list the output beam parameters on the line printer.

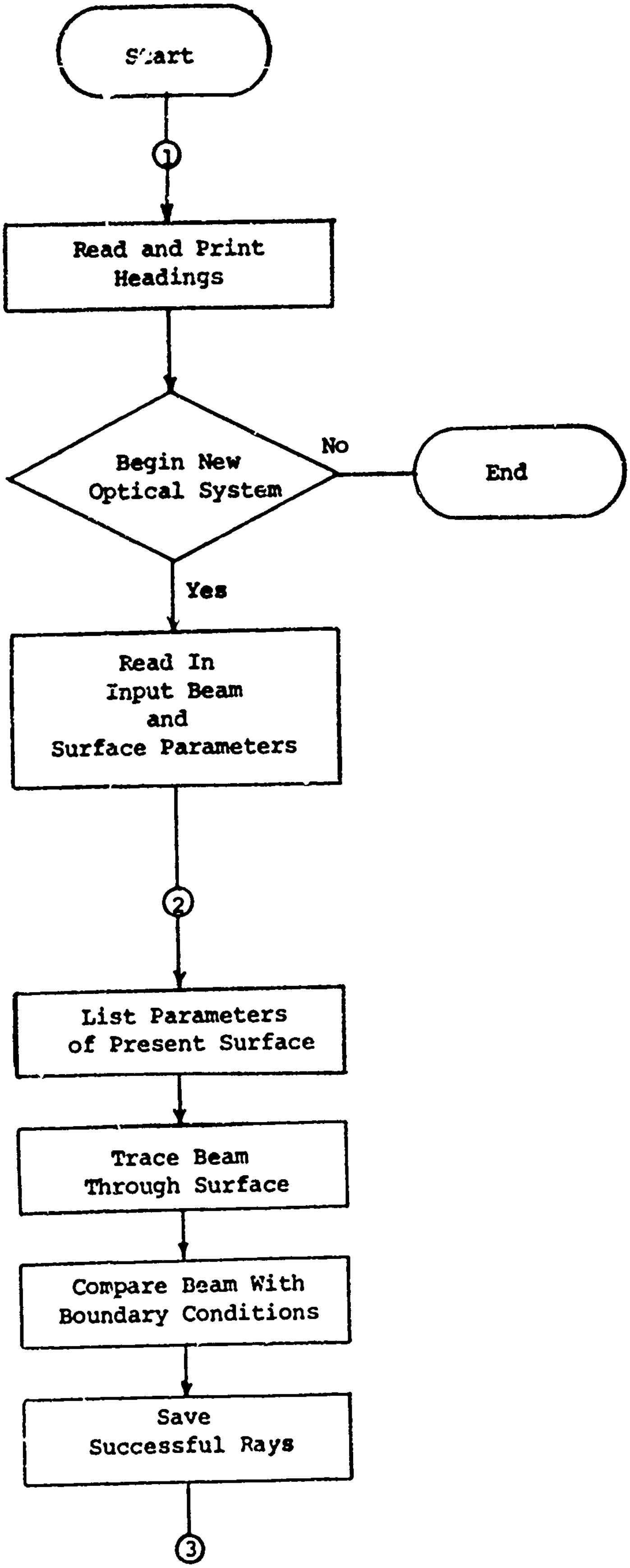



Figure B-1 RAYTRACE Flow Diagram

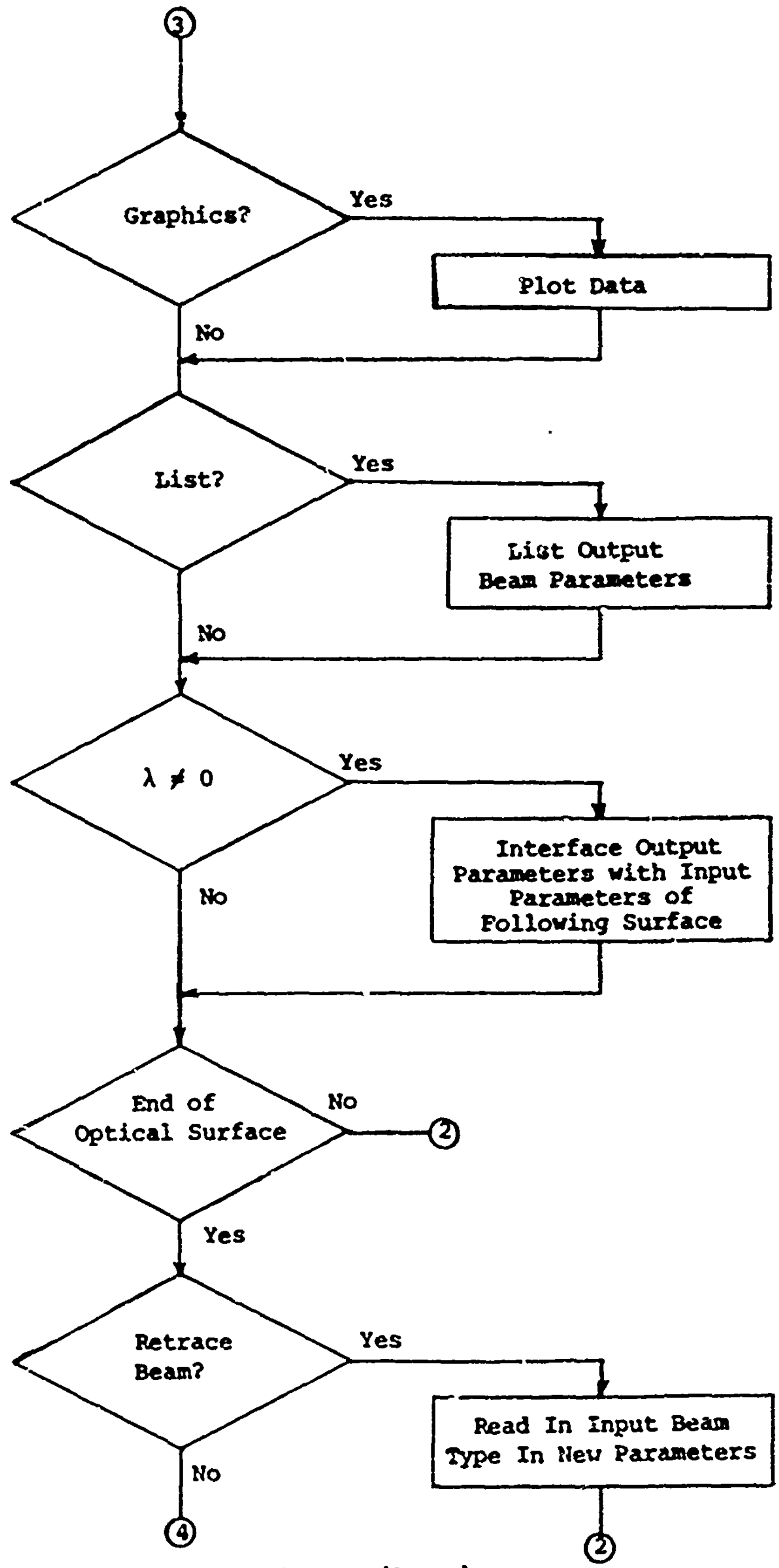



Figure B-1 RAYTRACE Flow Diagram (Cont.)

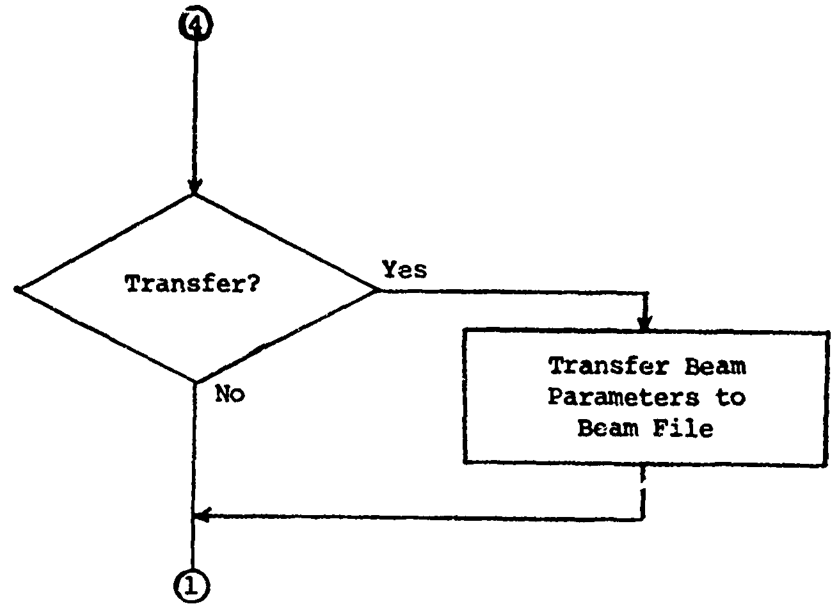



Figure B-1  RAYTRACE Flow Diagram (Cont.)

```
      PROGRAM RAYTRACE
C
C     RAYTRACE IS THE MAIN ROUTINE FOR TRACING RAYS THROUGH
C     AN OPTICAL SYSTEM.
C
C      9 JANUARY 1974 (REVISED)
C
C     DATA FILES ON DISC :
C        BEAM -- THIS FILE CONTAINS A GENERATED BEAM WHERE EACH RAY IS
C                DEFINED BY TWO LINEAR POSITIONS (X,Y) AND TWO ANGULAR
C                POSITIONS (PHI,PSI). THIS FILE HAS A MAXIMUM OF 8 DATA
C                BLOCKS WITH A MAXIMUM OF 250 RAYS IN EACH BLOCK.
C        NRYF -- THIS FILE HAS THE NUMBER OF DATA BLOCKS STORED IN THE
C                BEAM FILE AND THE NUMBER OF RAYS IN EACH BLOCK.
C        RAYF -- SCRATCH FILE USED FOR RAY DATA AS THE BEAM IS TRACED
C                THROUGH THE SYSTEM.
C        XGRF -- THE X POSITION FOR A PARTICULAR GRAPH IS STORED
C                IN THIS FILE.
C        YGRF -- THE Y POSITION FOR A PARTICULAR GRAPH IS STORED
C                IN THIS FILE.
C        STOR -- THE PARAMETERS OF EACH OPTICAL SURFACE ARE STORED
C                IN THIS FILE.
C
C     ARRAYS IN THE RAYTRACE PROGRAM :
C        RAY --  AS INPUT THIS ARRAY CONTAINS THE LINEAR AND ANGULAR
C                POSITIONS OF EACH RAY FROM A DATA BLOCK AT THE INPUT
C                REFERENCE PLANE. AS OUTPUT THIS ARRAY CONTAINS THE RAY
C                POSITIONS AT THE OUTPUT REFERENCE PLANE.
C        DEV --  THIS ARRAY CONTAINS THE ANGULAR (EPSILON,ETA) AND
C                LINEAR (X1D,Y1D,Z1D) DEVIATIONS OF A PLANE SURFACE FROM
C                THE SURFACE REFERENCE PLANE TO THE INCIDENT POINT OF AN
C                INPUT RAY ON A QUADRIC SURFACE
C        IRF --  THIS ARRAY IDENTIFIES RAY FAILURES BY SETTING IRF = 1
C                FOR A FAILED RAY.
C        DV  --  THE PARAMETERS OF A PARTICULAR OPTICAL SURFACE
C                ARE CONTAINED IN THIS ARRAY.
C        NR  --  THE NUMBER OF RAYS IN EACH DATA BLOCK ARE STORED
C                IN THIS ARRAY.
C
C     PROGRAM OPTIONS :
C        SET SENSE SWITCH (2) - GRAPHICS
C        SET SENSE SWITCH (3) - LIST OUTPUT BEAM PARAMETERS
C        SET SENSE SWITCH (4) - RETRACE BEAM THRU SYSTEM WITH NEW INPUT
C        SET SENSE SWITCH (5) - TRANSFER BEAM PARAMETERS TO BEAM FILE
C        SET SENSE SWITCH (6) - DELETE MOST TYPEWRITTEN MESSAGES
C
      COMMON RAY(4,250),DEV(5,250),IRF(250),DV(50),NBLK,NR(24)
    2 FORMAT ( 1H1 )
    4 FORMAT ( 71H
     1                I1)
```

```
    6 FORMAT ( 1H0,41HDO NOT SET SENSE SWITCHES UNTIL REQUESTED )
    8 FORMAT ( 10I8 )
   10 FORMAT ( ///5X25HNUMBER OF RAY FAILURES = ,I4// )
   12 FORMAT ( 1H0,13HRUN NUMBER = ,I5/
     125HOPTICAL SURFACE NUMBER = ,I4/
     225HNUMBER OF RAY FAILURES = ,I4 )
   14 FORMAT ( 1H0,10HGRAPHICS [ / 20HSET SENSE SWITCH (2) )
   15 FORMAT ( 1H0,42HSET REMOTE SWITCH ON ELECTROSTATIC PRINTER /
     132HTYPE 10 FOR CROSS SECTIONAL VIEW/27HTYPE 01 FOR KNIFE EDGE SCAN
     2/23HTYPE 11 FOR BOTH GRAPHS )
   16 FORMAT ( 1H0,24HTYPE IN GRAPHICS CHOICE )
   17 FORMAT ( 2I1 )
   18 FORMAT ( 1H0,27HLIST OUTPUT BEAM PARAMETERS,1H[/
     1  20HSET SENSE SWITCH (3) )
   20 FORMAT ( 1H0,37HTRANSFER BEAM PARAMETERS TO BEAM FILE ,1H[/
     1  20HSET SENSE SWITCH (5) )
   22 FORMAT ( 1H0,41HRETRACE BEAM THRU SYSTEM WITH NEW INPUT [ /
     1  20HSET SENSE SWITCH (4) )
C
C        READ AND WRITE HEADING
C
      CALL DATE (R)
   28 PRINT 2
   30 READ  4,I
      PRINT 4,I
      IF ( I - 1 ) 30,32,60
C
C        READ IN BEAM AND THE OPTICAL SURFACE PARAMETERS.
C
   32 CALL BEAMIN
      CALL READ
   33 IK = 0
C
C        READ IN PRESENT OPTICAL SURFACE PARAMETERS.
C
   34 IK = IK + 1
      CALL FREAD ( 4HSTOR,IK,DV,100 )
   99 IF ( IRSTATF(4HSTOR) .NE. 1 ) GO TO 99
C
C        PRINT PRESENT OPTICAL SURFACE PARAMETERS.
C
      CALL LIST
      AZERO = 0.0
      IF ( DV(1) .NE. 1.5707963 ) AZERO = 1.0
      IF ( SSWTCH (6) .EQ. 2 ) WRITE (59,6 )
      PAUSE
      PRINT 8, ( NR(K),K=1,NBLK )
      NRF = 0
C
C        TRACE EACH BLOCK OF RAYS THROUGH THE OPTICAL SURFACE
```

```
C
      DO 48 K = 1,NBLK
      NRY = NR(K)
      IF ( NR(K) .EQ. 0 ) GO TO 48
      DO 36 I = 1,NRY
   36 IRF(I) = 0
      CALL FREAD ( 4HRAYF,K,RAY,8*NRY )
   38 IF ( IRSTATF(4HRAYF) .NE. 1 ) GO TO 38
      IF ( DV(22) .NE. 0.0 ) CALL INCIDENT ( NRY )
      CALL TRACE ( NRY,AZERO )
C
C        CHECK BOUNDARY CONDITIONS.
C
      RMAX2 = DV(24)*DV(24)
      RMIN2 = DV(25)*DV(25)
      CALL BCOND ( RMAX2,RMIN2,NRY )
C
C        TOTAL RAY FAILURES.
C
      N = 0
      DO 44 I = 1,NRY
      IF ( IRF(I) .EQ. 0 ) GO TO 40
      NRF = NRF + 1
      GO TO 44
   40 N = N + 1
      DO 42 J = 1,4
   42 RAY(J,N) = RAY(J,I)
   44 CONTINUE
      NR(K) = N
      IF ( NR(K) .EQ. 0 ) GO TO 48
C
C        RESTORE RAY PARAMETERS THAT HAVE MET BOUNDARY CONDITIONS.
C
      CALL FWRITE ( 4HRAYF,K,RAY,8*N )
   46 IF ( IWSTATF(4HRAYF) .NE. 1 ) GO TO 46
   48 CONTINUE
      PRINT  8, ( NR(K),K=1,NBLK )
      PRINT 10, NRF
      NRUN = DV(27)
      NPDV = DV(26)
      WRITE (59,12) NRUN,NPDV,NRF
C
C        PROGRAM OPTIONS.
C
      IF ( SSWICHF(6) .EQ. 2 ) WRITE (59,14)
      PAUSE 2
      IF ( SSWTCHF(2) .EQ. 2 ) GO TO 49
      IF ( SSWICHF(6) .EQ. 2 ) WRITE (59,15)
      WRITE (59,16)
      READ (58,17) I1,I2
```

```
      IF ( I1 .EQ. 1 ) CALL CROSSECT ( R )
      IF ( I2 .EQ. 1 ) CALL SCAN ( R )
   49 IF ( SSWTCHF(6) .EQ. 2 ) WRITE (59,18)
      PAUSE 3
      IF ( SSWTCHF(3) .EQ. 1 ) CALL WRITE
      IF ( DV(4) .NE. 0.0 ) CALL INTRFACE
      IF ( DV(26) - DV(29) ) 34,50,60
   50 IF ( SSWTCHF(6) .EQ. 2 ) WRITE (59,22)
      PAUSE 4
      IF ( SSWTCHF(4) .EQ. 2 ) GO TO 52
      CALL RETRACE
      GO TO 33
   52 IF ( SSWTCHF(6) .EQ. 2 ) WRITE(59,20)
      PAUSE 5
      IF ( SSWTCHF(5) .EQ. 1 ) CALL TRANSFER
      GO TO 28
   60 CONTINUE
      END
```

```
      SUBROUTINE BCOND ( RMAX2,RMIN2,NRY )
C
C        31 OCTOBER 1973
C
C        THIS SUBROUTINE WILL TEST EACH RAY AGAINST A SET OF BOUNDARY
C        CONDITIONS (RMAX,RMIN) FOR A CIRCULAR BEAM AT THE
C        OUTPUT REFERENCE PLANE.
C
      COMMON RAY(4,250),DEV(5,250),IRF(250),DV(50),NBLK,NR(24)
      DO 40 J = 1,NRY
      IF ( IRF(J) .EQ. 1 ) GO TO 40
      X2 =  RAY(1,J)*RAY(1,J)
      Y2 =  RAY(2,J)*RAY(2,J)
      R2 =  X2 + Y2
      IF ( R2 .GT. RMAX2 .OR. R2 .LT. RMIN2 ) IRF(J) = 1
   40 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BEAMIN
C
C        29 OCTOBER 1973
C
C        THIS SUBROUTINE WILL READ IN BEAM DATA THAT HAS ALREADY BEEN
C        GENERATED AND STORED ON DISC IN THE BEAM FILE. EACH RAY OF THE
C        BEAM IS DESCRIBED BY TWO LINEAR POSITIONS (X,Y) AND TWO ANGULAR
C        POSITIONS (PHI,PSI). THERE IS A MAXIMUM OF 250 RAYS PER BLOCK.
C
C        IN ORDER THAT THE GENERATED BEAM IS NOT DESTROYED THE BEAM DATA
C        IS TRANSFERED FROM THE BEAM FILE TO THE RAYF FILE.
C
C        THE NRYF FILE CONTAINS THE NUMBER OF BLOCKS STORED IN THE BEAM
C        FILE AND THE NUMBER OF RAYS IN EACH BLOCK.
C
      COMMON RAY(4,250),DEV(5,250),IRF(250),DV(50),NBLK,NR(24)
      CALL FREAD ( 4HNRYF,1,NBLK,25 )
   30 IF ( IRSTATF(4HNRYF) .NE. 1 ) GO TO 30
      DO 40 K = 1,NBLK
      NRY = NR(K)
      CALL FREAD ( 4HBEAM,K,RAY,8*NRY ).
   32 IF ( IRSTATF(4HBEAM) .NE. 1 ) GO TO 32
      CALL FWRITE ( 4HRAYF,K,RAY,8*NRY )
   34 IF ( IWSTATF(4HRAYF) .NE. 1 ) GO TO 34
   40 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CROSSECT ( R )
C
C----- 10 JANUARY 1974 (REVISED)
C
C---- THIS SUBROUTINE STORES ON DISC IN THE XORF FILE AND THE YORF FILE
C---- THE DATA NECESSARY TO PLOT A CROSS SECTIONAL VIEW OF A BEAM
C---- AT THE OUTPUT REFERENCE PLANE OF ANY PARTICULAR SURFACE.
C
      COMMON RAY(4,250),DEV(5,250),IRF(250),DV(50),NBLK,NR(24)
      COMMON/PLTS//IBUF(6000),A(8),P(3),X(8),Y(8),LNE(2),LINX,LABL(5,8)
     1  ,XP(2000),XMAX,XMIN,DX,YMAX,YMIN,DY
      DIMENSION C(250,2)
      EQUIVALENCE (DEV,C)
    2 FORMAT (20HCROSS SECTIONAL VIEW)
    4 FORMAT (20H               Y AXIS)
    6 FORMAT ( 1H@,30HTYPE 1 FOR YES , TYPE 0 FOR NO /
     1  40HREPLOT CROSS SECTION WITH NEW DIMENSIONS )
    8 FORMAT ( I1 )
   10 FORMAT ( 1H@,26HIF MISTAKE ON INPUT TYPE 1 /
     1  42HNEW DIMENSION \ :XXXXXXXX:XX , MISTAKE \ X , 1X,1H@ )
   12 FORMAT ( E12.7,I1 )
   14 FORMAT ( 1H@,26HINPUT MISTAKE RETYPE INPUT )
C
C---- INITIALIZE THE GRAPH PARAMETERS FOR THE CROSS SECTIONAL PLOT.
C
      ENCODE ( 20,2,LABL(1,1) )
      ENCODE ( 20,4,LABL(1,8) )
      P(1)   = 5.00  $  P(2)   = 8.00
      Y(7)   = 4.50  $  Y(8)   = 6.25
      LNE(1) =-2     $  LNE(2) = 3
      XMAX = YMAX =  DV(23)
      XMIN = YMIN = -DV(23)
      DX   = DY   =  DV(23)/4.0
C
C----- FROM EACH DATA BLOCK IN RAYF FILE STORE THE LINEAR POSITIONS
C----- OF EACH RAY IN XORF AND YORF FILE.
C
      DO 30 J = 1,NBLK
      NRY = NR(J)
      IF ( NRY .EQ. 0 ) GO TO 30
      CALL FREAD ( 4HRAYF,J,RAY,8*NRY )
   22 IF ( IRSTATF(4HRAYF) .NE. 1 ) GO TO 22
      DO 24 I = 1,NRY
      DO 24 K = 1,2
   24 C(I,K) = RAY(K,I)
      CALL FWRITE ( 4HXORF ,J,C(1,1),2*NRY )
   26 IF ( IWSTATF(4HXORF) .NE. 1 ) GO TO 26
      CALL FWRITE ( 4HYORF ,J,C(1,2),2*NRY )
   28 IF ( IWSTATF(4HYORF) .NE. 1 ) GO TO 28
   30 CONTINUE
```

```
   32 CALL GRAPHS ( R )
C
C---- OPTION TO REPLOT GRAPH WITH NEW SCALE.
C
      WRITE (59,6 )
      READ  (58,8) I
      IF ( I .EQ. 0 ) GO TO 38
      WRITE (59,10)
   34 READ  (58,12) DIM,MISTAKE
      IF ( MISTAKE .EQ. 0 ) GO TO 36
      WRITE (59,14)
      GO TO 34
   36 XMAX = YMAX =  DIM
      XMIN = YMIN = -DIM
      DX   = DY   =  DIM/4.0
      GO TO 32
   38 RETURN
      END
```

```
      SUBROUTINE DATE(R)
C
C---- THIS SUBROUTINE CALLS THE DATE FROM ACCOUNTS AND STORES IT IN R
C---- AS A HOLLERITH CONSTANT OF THE FORM MM/DD/YY.
C
      MACRO (A)
         EXT        ACCOUNTS
         LDAQ       ACCOUNTS
         SHQ        -6
         SHAQ       -6
         SACH       X+3
         SHA        -6
         SHA        12
         ADAQ       X
         UJP        *+3
X        BCD,C      8,00/00/00
         ENDM
      REAL A
      R = A
      RETURN
      END
```

```
      SUBROUTINE GRAPHS ( R )
C
C----- 24 NOVEMBER 1973 (REVISED)
C
C---- THIS SUBROUTINE WILL PLOT DATA GENERATED OR STORED BY CROSSECT
C---- OR SCAN SUBROUTINES.
C
      COMMON RAY(4,250),DEV(5,250),IRF(250),DV(50),NBLK,NR(24)
      COMMON/PLTS//IBUF(6000),A(8),P(3),X(8),Y(8),INE(2),LINX,LABL(5,8)
     1  ,XP(2000),XMAX,XMIN,DX,YMAX,YMIN,DY
      DIMENSION YP(2000)
      EQUIVALENCE (RAY,YP) , (LABL(26),RR)
      DATA/PLTS/( (LABL(I),I 6,35 ) =
     14HRUN ,4HNO. ,4H    ,4H    ,4H    ,
     14HDATE,4H    ,4H    ,4H    ,4H    ,
     14HSURF,4HACE ,4HNO. ,4H    ,4H    ,
     14HDIST,4HANCE,4H =  ,4H    ,4H    ,
     14H00/0,4H0/00,4H    ,4H    ,4H    ,
     14H    ,4H    ,4H    ,4H  X ,4HAXIS) ,
     1 ( ( X(I),I=1,8 ) = 3(0.50),2(5.00),1.50,5.75,4.75 ) ,
     1 ( ( Y(I),I=1,6 ) = 9.75,2(9.50,9.25),9.25 ) ,
     1 ( ( A(I),I=1,8 ) = 7(0.00),90.0 ) , ( LINX = 4H     )
    2 FORMAT (4H2--0)
      RR = R
      NP = 1
C
C---- READ IN THE LINEAR POSITIONS FOR A PARTICULAR GRAPH
C---- AND STORE IN THE XP AND YP ARRAYS.
C
      DO 26 J = 1,NBLK
      NRY = NR(J)
      IF ( NRY .EQ. 0 ) GO TO 26
      CALL FREAD ( 4HXGRF,J,XP(NP),2+NRY )
   22 IF( IRSTATF(4HXGRF) .NE. 1 ) GO TO 22
      CALL FREAD ( 4HYGRF,J,YP(NP),2+NRY )
   24 IF ( IRSTATF(4HYGRF) .NE. 1 ) GO TO 24
      NP = NRY + NP
      IF ( NP+250 .GE. 1998 ) GO TO 27
   26 CONTINUE
   27 NP = NP - 1
      IF ( NP .EQ. 0 ) GO TO 32
      DO 28 I = 1,NP
      IF ( XP(I) .LT. XMIN ) XP(I) = XMIN
      IF ( XP(I) .GT. XMAX ) XP(I) = XMAX
      IF ( YP(I) .LT. YMIN ) YP(I) = YMIN
      IF ( YP(I) .GT  YMAX ) YP(I) = YMAX
   28 CONTINUE
C
C---- PLOT THE GRAPH.
C
```

```
      CALL GRAPHINT ( IBUF(1),IBUF(6000) )
      K1     = NP + 1  $  K2     = NP + 2
      XP(K1) = XMIN  $  XP(K2) = DX
      YP(K1) = YMIN  $  YP(K2) = DY
      CALL AXISE ( 0.50,P(1),LINX,-4,8.00,0.0,XP(K1),XP(K2) )
      CALL AXISE ( 4.50,1.00,LINX, 4,P(2),90.,YP(K1),YP(K2) )
      XP(K1) = XP(K1) - 0.50*XP(K2)
      YP(K1) = YP(K1) - 1.00*YP(K2)
      CALL LINEE ( XP,YP,NP,1,LNE(1),LNE(2) )
      DO 30 J = 1,8
   30 CALL SYMBOLE ( X(J),Y(J),.14,LABL(1,J),A(J),20 )
      CALL NUMBERE ( 1.50,9.50,.14,DV(27),0.0,-1 )
      CALL NUMBERE ( 6.70,9.50,.14,DV(26),0.0,-1 )
      CALL NUMBERE ( 6.50,9.25,.14,DV(3) ,0.0, 2 )
      WRITE (11,2)
      CALL GRAPHOUT ( 11 )
   32 RETURN
      END
```

```
      SUBROUTINE INCIDENT ( NRY )
C
C----- 24 NOVEMBER 1973 (REVISED)
C
C----- GIVEN AN INPUT RAY DEFINED BY LINEAR AND ANGULAR POSITION
C---- WITH RESPECT TO AN INPUT REFERENCE PLANE. THIS SUBROUTINE WILL
C---- CALCULATE THE INCIDENT POSITION OF THE INPUT RAY ON A QUADRIC
C---- SURFACE AND THE ANGULAR ROTATION OF THE SURFACE REFERENCE PLANE
C---- REQUIRED TO PLACE THE PLANE TANGENT TO THE INCIDENT POINT.
C
      COMMON RAY(4,250), DEV(5,250), IRF(250), DV(50), NBLK, NR(24)
      DO 35 J = 1, NRY
      TANPHI =  TANF ( RAY(3,J) )
      TANPSI =  TANF ( RAY(4,J) )
      TPHI = -TANPHI      $    TPSI =  TANPSI
      DELX =  RAY(1,J)    $    DELY = -RAY(2,J)
      XLAM = DELX-DV(18)
      YLAM = DELY-DV(19)
      ZLAM = DV(20)+DV(21)
      R1 = DV(13)*TPHI       $  R2 = DV(14)*TPSI
      R3 = XLAM-DV(2)*TPHI   $  R4 = YLAM-DV(2)*TPSI
      R5 = DV(15)*ZLAM
C
C----- SOLVE FOR Z INCIDENT POSITION.
C
      A = R1*TPHI + R2*TPSI + DV(15)
      B =  -( 2.0*( R1*R3 + R2*R4 - R5 ) + DV(16) )
      C = DV(13)*R3*R3 + DV(14)*R4*R4 + ZLAM*( R5 DV(16) ) - DV(17)
      Q = B*B - 4.0*A*C
      IF ( Q .LT. 0.0 ) GO TO 30
      IF ( A .EQ. 0.0  .AND.  B .EQ. 0.0 ) GO TO 30
      IF ( A .EQ. 0.0 ) GO TO 20
      DEV(5,J) =  ( B + DV(12)*SQRTF ( Q ) )/( 2.0*A )
      GO TO 25
   20 DEV(5,J) =  C/B
   25 R1       =  DEV(5,J) - DV(2)
C
C----- CALCULATE X AND Y INCIDENT POSITIONS.
C
      DEV(3,J) =  XLAM + R1*TPHI
      DEV(4,J) =  YLAM + R1*TPSI
C
C----- SOLVE FOR ROTATION ANGLES EPSILON AND ETA.
C
      R1       =  2.0*DV(13)*( DEV(3,J) - DV(18) )
      R2       =  2.0*DV(14)*( DEV(4,J) - DV(19) )
      R3       =  2.0*DV(15)*( DEV(5,J) - ZLAM  ) + DV(16)
      R4 = SQRTF ( R1*R1 + R2*R2 + R3*R3 )
      DEV(1,J) =  ATANF ( -R2/R3 )
      DEV(2,J) =  ASINF ( DV(22)*R1/R4 )
```

```
      GO TO 35
   30 IRF(J) = 1
   35 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INTRFACE
C
C---- 24 NOVEMBER 1973 (REVISED)
C
C---- WHEN THE INCIDENT PLANES OF TWO OPTICAL SURFACES ARE NOT PARALLEL
C---- THIS SUBROUTINE INTERFACES THE OUTPUT RAY PARAMETERS WITH THE
C---- INPUT RAY PARAMETERS OF THE FOLLOWING OPTICAL SURFACE.
C
      COMMON RAY(4,250),DEV(5,250),IRF(250),DV(50),NBLK,NR(24)
      DIMENSION A(4),B(2)
C
      CALL SINCOS1 ( DV(4),B )
      DO 28 K = 1,NBLK
      N = NR(K)
      IF ( N .EQ. 0 ) GO TO 28
      CALL FREAD ( 4HRAYF,K,RAY,8*N )
   18 IF ( IRSTATF(4HRAYF) .NE. 1 ) GO TO 18
      DO 24 J = 1,N
      DO 20 I = 1,3,2
      A(I)   =  RAY(I,J)*B(2) + RAY(I+1,J)*B(1)
   20 A(I+1) = -RAY(I,J)*B(1) + RAY(I+1,J)*B(2)
      DO 22 I = 1,4
   22 RAY(I,J) = A(I)
   24 CONTINUE
      CALL FWRITE ( 4HRAYF,K,RAY,8*N )
   26 IF ( IWSTATF(4HRAYF) . NE. 1 ) GO TO 26
   28 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATVEC(A,B,C,N)
      DIMENSION A(9),B(9),C(9),F(9),G(9),H(9)
      IF (N) 10,6,10
   10 GO TO (5,6,5,6,5),N
    6 DO 61 J=1,9
   61 F(J)=A(J)
      GO TO 70
    5 M2=1
      DO 36 K=1,3
      K1=K+6
      DO 36 J=K,K1,3
      F(M2)=A(J)
   36 M2=M2+1
   70 IF (N-1) 71,71,72
   71 M4=1
      DO 73 J=1,3
   73 G(J)=B(J)
      GO TO 80
   72 M4=7
      GO TO (74,74,74,75,75),N
   74 DO 76 J=1,9
   76 G(J)=B(J)
      GO TO 80
   75 M2=1
      DO 66 K=1,3
      K1=K+6
      DO 66 J=K,K1,3
      G(M2)=B(J)
   66 M2=M2+1
   80 M2=1
      DO 30 M1=1,M4,3
      DO 30 K=1,3
      K1=K+6
      M3=M1
      H(M2)=0
      DO 20 J=K,K1,3
      H(M2)=H(M2)+F(J)*G(M3)
   20 M3=M3+1
   30 M2=M2+1
      IF (N-1) 95,95,96
   95 M1=3
      GO TO 90
   96 M1=9
   90 DO 91 J=1,M1
   91 C(J)=H(J)
      RETURN
      END
```

```
      SUBROUTINE READ
C
C - -  7 JANUARY 1974 (REVISED)
C
C - - THIS SUBROUTINE READS IN THE PARAMETERS FOR EACH OPTICAL SURFACE.
C
      COMMON RAY(4,250),DEV(5,250),IRF(250),DV(50),NBLK,NR(24)
      DIMENSION KB(4),VB(4)
C
    2 FORMAT (4(I4,E14.7))
    4 FORMAT (1H0,4X24HOPTICAL SURFACE NUMBER ,I3//(5(I5,1H),E14.7)))
    6 FORMAT ( 1H- )
C
C - - INITIALIZE THE DV ARRAY.
C
      IK = 0
   30 IK = IK + 1
      DO 32 I = 1,26
   32 DV(I) = 0.0
      DO 33 I = 30,50
   33 DV(I) = 0.0
C
C - - READ DATA FOR PARTICULAR OPTICAL SURFACE FROM CARD READER.
C
   34 READ 2, ( KB(I),VB(I),I=1,4 )
      DO 38 I = 1,4
      KK = KB(I)
      IF ( KK ) 36,38,36
   36 DV(KK) = VB(I)
      IF ( KK - 50 ) 38,40,40
   38 CONTINUE
      GO TO 34
C
C -   STORE OPTICAL SURFACE PARAMETERS ON DISC
C
   40 CALL FWRITE ( 4IF,IOR,IK,DV,100 )
   42 IF ( INSTAT(4IF,IOR) .NE. 1 ) GO TO 42
      IF ( DV(26) .LT. DV(29) ) GO TO 30
      RETURN
C
C -   PRINT PARAMETERS OF OPTICAL SURFACE.
C
      ENTRY LIST
      NPDV = DV(26)
      MDV  = DV(28)
      PRINT 4, NPDV,( I,DV(I),I=1,MDV )
      PRINT 6
      RETURN
      END
```

```
      SUBROUTINE RETRACE
C
C      7 JANUARY 1974
C
C     THIS SUBROUTINE WILL ACCEPT INPUT FROM THE TYPEWRITER FOR
C     RETRACING A BEAM THROUGH THE PRESENT OPTICAL SYSTEM.  IN
C     ADDITION THE INPUT BEAM WILL BE RELOADED.
C
      COMMON RAY(4,250),DEV(5,250),IRF(250),DV(50),NBLK,NR(24)
    2 FORMAT ( 1H@,32HCHANGES FOR OPTICAL SURFACE NO.  ,I2/
     115HXX:XXXXXXXX:XXX,12X,1H@ )
    4 FORMAT ( 1H@,14HINPUT FORMAT \/
     133HXX .          (VARIABLE LOCATION) /
     230H:XXXXXXXX:XX  (VARIABLE VALUE) /
     325HX             (1-MISTAKE) )
    8 FORMAT ( I2,E12.7,I1 )
   10 FORMAT ( 1H@,26HINPUT MISTAKE RETYPE INPUT /
     115HXX:XXXXXXXX:XXX,12X,1H@ )
C
C     RELOAD INPUT BEAM.
C
      CALL BEAMIN
      IK = 0
      RUN = 0.0
      IF ( SSWTCHF(6) .EQ. 2 ) WRITE (59,4 )
C
C     INPUT CHANGES THRU TYPEWRITER PER SURFACE.
C
   18 IK = IK + 1
      CALL FREAD ( 4HSTOR,IK,DV,100 )
   20 IF ( IRSTATF(4HSTOR) .NE. 1 ) GO TO 20
      IF ( IK .GE. 1 .AND. RUN .GT. 0.0 ) DV(27) = RUN
      NS = DV(26)
      WRITE (59,2 ) NS
   22 READ  (58,8 ) J,A,MISTAKE
      IF ( MISTAKE .NE  1 ) GO TO 24
      WRITE (59,10)
      GO TO 22
   24 IF ( J .EQ. 50 ) GO TO 26
      IF ( J .EQ. 27 ) RUN = A
      DV(J) = A
      GO TO 22
C
C     RESTORE UPDATED SURFACE PARAMETERS
C
   26 CALL FWRITE ( 4HSTOR,IK,DV,100 )
   28 IF ( IWSTATF(4HSTOR) .NE  1 ) GO TO 28
      IF ( DV(26) .LT  DV(29) ) GO TO 18
      RETURN
      END
```

```
      SUBROUTINE SCAN ( R )
C
C---- 10 JANUARY 1974 (REVISED)
C
C---- THIS SUBROUTINE DEVELOPS THE DATA NECESSARY TO PLOT A KNIFE EDGE
C---- SCAN OF A BEAM AT THE OUTPUT REFERENCE PLANE OF A
C---- PARTICULAR OPTICAL SURFACE.
C
      COMMON RAY(4,250),DEV(5,250),IRF(250),DV(50),NBLK,NR(24)
      COMMON/PLTS//IBUF(6000),A(8),P(3),X(8),Y(8),LNE(2),LINX,LABL(5,8)
     1 ,XP(2000),XMAX,XMIN,DX,YMAX,YMIN,DY
      DIMENSION C(250,2)
      EQUIVALENCE (DEV,C)
    2 FORMAT (20HKNIFE EDGE SCAN      )
    4 FORMAT (20HPER CENT ENERGY      )
    6 FORMAT ( 1H0,30HTYPE 1 FOR YES , TYPE 0 FOR NO /
     1  42HREPLOT KNIFE EDGE SCAN WITH NEW DIMENSIONS )
    8 FORMAT ( I1 )
   10 FORMAT ( 1H0,26HIF MISTAKE ON INPUT TYPE 1 /
     1  42HNEW DIMENSION \ :XXXXXXXX:XX , MISTAKE \ X , 1X,1H@ )
   12 FORMAT ( E12 7,I1 )
   14 FORMAT ( 1H0,26HINPUT MISTAKE RETYPE INPUT )
C
C---- INITIALIZE THE GRAPH PARAMETERS FOR THE KNIFE EDGE SCAN PLOT.
C
      ENCODE ( 20,2,LABL(1,1) )
      ENCODE ( 20,4,LABL(1,8) )
      P(1)   = 1.00  $  P(2)   = 5.00
      Y(7)   = 0.50  $  Y(8)   = 3.00
      LNE(1) = 0     $  LNE(2) = 0
      YMIN =   0.
      YMAX = 100.
      DY   =  20.
      XMAX =  DV(23)
      XMIN = -DV(23)
      DX   =  DV(23)/4 0
C
C---- CALCULATE THE TOTAL NUMBER OF RAYS.
C
      NRY = 0
      DO 22 J = 1,NBLK
   22 NRY = NRY + NR(J)
      YTOTAL = NRY
C
C---- FROM THE SCAN INCREMENT CALCULATE THE NUMBER OF POINTS FOR THE
C---- PLOT AND INITIALIZE THE X AND Y POSITIONS
C
   23 DSCAN = 2 0*XMAX/250.0
      N     = 250
      DO 24 J = 1,N
```

```
      AMULT  = J - 1
      C(J,1) = XMIN + AMULT*DSCAN
   24 C(J,2) = 0.0
C
C--   SCAN THROUGH EACH BLOCK OF RAYS AND TOTAL THE NUMBER OF RAYS
C--   FOR EACH X POSITION.
C
      DO 28 J = 1,NBLK
      NRY = NR(J)
      IF ( NRY .EQ. 0 ) GO TO 28
      CALL FREAD ( 4HRAYF,J,RAY,8*NRY )
   26 IF ( IRSTATF(4HRAYF) .NE. 1 ) GO TO 26
      DO 28 I = 1,N
      DO 28 K = 1,NRY
      IF ( RAY(1,K) .LE. C(I,1) ) C(I,2) = C(I,2) + 1.0
   28 CONTINUE
C
C--   CALCULATE THE PER CENT OF ENERGY AS THE LIGHT BEAM IS SCANNED.
C
      DO 30 J = 1,N
   30 C(J,2) = 100.*C(J,2)/YTOTAL
      CALL FWRITE ( 4HXGRF,1,C(1,1),2*N )
   32 IF ( IWSTATF(4HXGRF) .NE. 1 ) GO TO 32
      CALL FWRITE ( 4HYGRF,1,C(1,2),2*N )
   34 IF ( IWSTATF(4HYGRF) .NE. 1 ) GO TO 34
      IBLK   = NBLK
      INRY   = NR(1)
      NBLK   = 1
      NR(1) = N
      CALL GRAPHS ( R )
      NBLK   = IBLK
      NR(1) = INRY
C
C--   OPTION TO REPLOT GRAPH WITH NEW SCALE
C
      WRITE (59,6 )
      READ  (58,8) I
      IF ( I .EQ. 0 ) GO TO 40
      WRITE (59,10)
   36 READ  (58,12) DIM,MISTAKE
      IF ( MISTAKE .EQ. 0 ) GO TO 38
      WRITE (59,14)
      GO TO 36
   38 XMAX =  DIM
      XMIN =  DIM
      DX   =  DIM/4.0
      GO TO 22
   40 RETURN
      END
```

```
      SUBROUTINE SINCOS1( A,B )
C
C---- THIS SUBROUTINE CALCULATES THE SINE AND COSINE FUNCTION OF
C---- ANGLE A ( IN RADIANS ). THE OUTPUT IS IN THE B ARRAY.
C
      DIMENSION B(2)
      T = A
      B(1) = SIN(T)
      B(2) = COS(T)
      RETURN
      END
```

```
      SUBROUTINE TRACE ( NRY,AZERO )
C
C---- 24 NOVEMBER 1973 (REVISED)
C
C---- THIS SUBROUTINE TRACES A RAY FROM AN INPUT REFERENCE PLANE THROUGH
C---- A REFLECTING OR REFRACTING PLANE SURFACE TO AN OUTPUT REFERENCE
C---- PLANE.
C
C---- THE C ARRAY IS USED FOR INTERMEDIATE CALCULATIONS.
C
C---- ALL ANGLE DATA MUST BE IN RADIANS.
C
      COMMON RAY(4,250),DEV(5,250),IRF(250),DV(50),NBLK,NK(24)
      DIMENSION C(40)
      DO 60 J = 1,NRY
      IF ( IRF(J) .EQ. 1 ) GO TO 60
      IF ( DV(22) .EQ. 0.0 ) GO TO 35
      DO 30 I = 1,5
   30 DV(I+4) =  DEV(I,J)
   35 CONTINUE
      DO 40 I = 1,40
   40 C(I) = 0.0
C
C---- CALCULATE DIRECTION COSINES OF INPUT RAY.
C
      T1 = TANF ( RAY(3,J) )   $   T2 = TANF ( RAY(4,J) )
      C(30) = COS ( ATAN ( SQRT ( T1*T1 + T2*T2 ) ) )
      C(28) = T1*C(30)  $  C(29) = T2*C(30)
      C(31) = RAY(1,J)  $  C(32) = RAY(2,J)  $  C(33) =  -DV(2)
      C(5) = DV(1)
      DO 42 I = 1,2
   42 CALL SINCOS1 ( DV(I+4),C(2*I-1) )
C
C--   I = 1 , CALCULATE POINT OF REFLECTION OR REFRACTION.
C---- I = 2 , CALCULATE POINT OF RAY ON OUTPUT PLANE AND ITS DIRECTION.
C
      DO 50 I=1,2
      CALL SINCOS1 ( C(5)+DV(5),C(6) )
      IF ( DV(5) .EQ. 0.0 ) C(7) = AZERO*C(7)
      C(10) =  C(4)   $  C(13) =  C(3)*C(7)   $  C(16) =  C(3)*C(6)
      C(11) =  0.0    $  C(14) = -C(6)        $  C(17) =  C(7)
      C(12) =  C(3)   $  C(15) = -C(4)*C(7)   $  C(18) = -C(4)*C(6)
      IF ( I.EQ 2 ) GO TO 52
      C(19) =  C(4)   $  C(22) =  C(3)*C(1)   $  C(25) - -C(3)*C(2)
      C(20) =  0.0    $  C(23) =  C(2)        $  C(26) =  C(1)
      C(21) =  C(3)   $  C(24) =  C(4)*C(1)   $  C(27) -  C(4)*C(2)
      GO TO 54
   52 CALL SINCOS1 ( C(5),C(8) )
      C(9) = AZERO*C(9)
      C(19) =  1.0    $  C(22) =  0.0         $  C(25) -  0.0
```

```
      C(20) =  0.0   $  C(23) =  C(8)      $  C(26) =  C(9)
      C(21) =  0.0   $  C(24) = -C(9)      $  C(27) =  C(8)
   54 CALL MATVEC ( C(10),C(28),C(28),I-1 )
      CALL MATVEC ( C(10),C(31),C(31),I-1 )
      CALL MATVEC ( C(19),DV(7) ,C(34),0  )
      DO 56 L = 31,33
   56 C(L) = C(L)-C(L+3)
      IF ( I .EQ. 2 ) C(33) = C(33)-DV(3)
      T1 = C(28)/C(30)
      T2 = C(29)/C(30)
      C(31) = C(31) - T1*C(33)
      C(32) = C(32) - T2*C(33)
      C(33) = 0.0
      IF ( I.EQ.2 ) GO TO 50
      C(28) = DV(11)*C(28)
      C(29) = DV(11)*C(29)
      C(30) = -DV(10)*SQRTF ( 1.0-DV(11)*DV(11)*( 1.0-C(30)*C(30) ) )
      C(5)  =  DV(10)*ACOSF ( DV(11)*COS ( C(5) ) )
   50 CONTINUE
C
C----- OUTPUT RAY RELATIONSHIP.
C
      RAY(1,J) = C(31)
      RAY(2,J) = C(32)
      RAY(3,J) = ATAN ( T1 )
      RAY(4,J) = ATAN ( T2 )
   60 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TRANSFER
C
C     24 NOVEMBER 1973 (REVISED)
C
C     THIS SUBROUTINE WILL TRANSFER THE RAY DATA GENERATED BY THE
C     RAYTRACE PROGRAM FROM THE RAYF FILE TO THE BEAM FILE. THIS WILL
C     ONLY BE NECESSARY IF IT IS DESIRED TO SAVE THE DATA FOR LATER USE.
C
      COMMON RAY(4,250),DEV(5,250),IRF(250),DV(50),NBLK,NR(24)
      CALL FWRITE ( 4HNRYF,1,NBLK,25 )
   52 IF ( IWSTATF(4HNRYF) .NE. 1 ) GO TO 52
      DO 60 K = 1,NBLK
      NRY = NR(K)
      IF ( NRY .EQ. 0 ) GO TO 60
      CALL FREAD ( 4HRAYF,K,RAY,8*NRY )
   54 IF ( IRSTATF(4HRAYF) .NE. 1 ) GO TO 54
      CALL FWRITE ( 4HBEAM,K,RAY,8*NRY )
   56 IF ( IWSTATF(4HBEAM) .NE. 1 ) GO TO 56
   60 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE WRITE
C
C--- -  9 NOVEMBER 1973
C
C---- THIS SUBROUTINE WILL WRITE THE OUTPUT BEAM PARAMETERS.
C
      COMMON RAY(4,250),DEV(5,250),IRF(250),DV(50),NBLK,NR(24)
C
    4 FORMAT ( 1H-,4X11HOUTPUT BEAM,1H: )
    6 FORMAT (I5,1H),4E20.7)
    8 FORMAT ( 1H0,4X15HBLOCK NUMBER = ,I2,5X14HBLOCK SIZE = ,I3// )
      PRINT 4
      DO 26 I = 1,NBLK
      NRY = NR(I)
      IF ( NRY .EQ. 0 ) GO TO 26
      CALL FREAD ( 4HRAYF,I,RAY,8*NRY )
   22 IF ( IRSTATF(4HRAYF) .NE. 1 ) GO TO 22
      PRINT 8, I,NRY
      DO 24 J = 1,NRY
   24 PRINT 6, ( J,( RAY(K,J),K=1,4 ) )
   26 CONTINUE
      RETURN
      END
```

## REFERENCES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