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SEARCH PROCEDURES IN THE PRESENCE OF
FALSE CONTACTS

James M. Dobbie

Arthur D. Little, Incorporated

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SEARCH PROCEDURES IN THE PRESENCE OF FALSE CONTACTS

by

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ABSTRACT (continued)

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PREFACE

The work described in this report was done as a sub-task of ONR Project NR 364-000: Methods for Analysis of Fleet Tactical Effectiveness and Performance (MAFTEP). The project was sponsored by the Naval Analysis Programs (Code 462) under the direction of Mr. R. J. Miller. Mr. J. R. Simpson was the ONR Project Scientific Officer.

The research was performed by Arthur D. Little, Inc. in Cambridge Massachusetts. The principal investigators were J. M. Dobbie and B. O. Koopman. Contributions also were made by S. Marks and M. F. Stankard.

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CONTENTS

SUMMARY

This final report on Search Procedures in the Presence of
False Contacts is written in the form of an administrative report
since most of the results produced under the contract will appear
in OPERATIONS RESEARCH or in the SIAM Journal on Applied Mathematics.

A review was made of previous work on false target models.
Most of this work is concerned with the effects of false targets
on existing search plans, and procedures to use after contacts
occur. Only a few authors consider the problem of planning the
search when false contacts can be anticipated. The few papers that
are pertinent were reviewed thoroughly.

In formulating search problems in the presence of false
contacts it became evident that certain properties of searches, such
as additivity and compatibility, play a central role in the formula-
tion and solution. These properties were reviewed in the case of no
false contacts and extended to the case of search in the presence
of false contacts. A principle of stability was used to show that the
false target distribution under conditions of least "information" is
the Poisson distribution.

Search problems were formulated for the case in which false
contacts are generated by real stationary objects that are investi-
gated when contacted. It was shown that the formulation and solution
of the standard optimization functionals are contingent on the number
of false targets found, in general, and possibly on the locations
and times of contact as well. The optimization functionals were
found to be difficult to write and more difficult to solve. The
expected time functional was constructed when the number of false
targets is bounded and has a known value or a known distribution.
A procedure was developed to find the optimal distribution of effort,
and illustrated with a simple example.

Several problems involving moving targets without the compli-
cation of false contacts were formulated and solved in preparation
for the study of problems involving both moving targets and false
contacts. First, a two-cell model of search for a moving target
was formulated as a continuous-time Markov process with known constant
transition rates and detection rates. The distribution of searching
effort that maximizes the probability of detection in time T was found.
Also, the distribution that minimizes the expected time to detection
was found.

In the second problem it was assumed that an evader attempts to avoid detection by moving away from the vicinity of the searcher when he becomes aware of the presence of the searcher. It was assumed that the evader's rates of transition to other regions are proportional to the searcher's rate of search in the region in which the evader is located. The distribution of effort that maximizes the probability of detection in a given time was found for the two-cell problem. The corresponding problem for n cells was formulated, and a method was derived for finding the complete set of extremals.

Next, it was assumed that false signals could be generated by two different mechanisms and the four problems obtained by adding false signals to the two problems involving moving targets were studied. It was assumed that false contacts are generated as Poisson processes with known constant rates that are either independent of the target's location or are dependent on the target's presence, and that contacts are investigated until identified correctly as the target or the decision is made that the contact is not the target.

The distribution of the search fraction of the effort was found to maximize the probability of detection in a given time for both false contact assumptions when the target motion is induced by the searcher. A procedure that can be used to obtain the optimal distribution was found for the case in which the transition rates are independent of the searcher's location and the false contact rates are independent of the target's location. The optimization problem has not been solved for the case in which the transition rates are independent of the searcher's location and the false contacts are generated by the target.

A theory of search in the presence of false contacts is necessary for the construction of search plans under these conditions. The theory developed under this contract and by others is judged to be adequate to begin the construction of search plans that explicitly allow for false targets and the possibility of false contacts from other causes. Physical limitations on the searcher's motion, that were ignored in deriving the theoretical results, may prevent the attainment of a distribution that is a good approximation to the optimal distribution. The conversion of the theory into practical plans is an important step that has not received adequate attention.

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1. Introduction

Search theory, search plans, and search doctrine have been developed almost entirely without explicit consideration of false contacts. The effects of the subsequent investigation and disposition of such contacts have been studied in some particular cases. However, little has been done to incorporate false contacts and their effects explicitly into the search plans and search doctrine.

An essential requirement for the construction of search plans is the development of a comprehensive theory of search and subsequent action, in the presence of false contacts. The objectives of this study are to review existing search theory and search plans, and to develop a theory of search and subsequent action in an environment of false contacts as a basis for constructing search plans and procedures that include the possibility of false contacts.

2. The False-Contact Search Problem

False contacts may be generated by real objects that can't be distinguished from the target with certainty, except by a close inspection. For example, in search by active sonar for a submerged submarine some possible generators of false contacts are wrecks on the bottom, sea mounts and bottom irregularities that reflect sound waves, whales, other marine life, and decoys (including bubble clouds produced by the interaction with water of chemicals expelled from the target submarine.)

False contacts also may be generated by anomalies or variations (usually called noise) in the signal response, produced by fluctuations in the performance of the detection equipment and fluctuations in the medium through which the energy passes.

When a contact is made the searcher may decide to take no immediate action, while continuing to observe the contact indication; or, decide to stop searching and investigate the contact; or, decide to proceed with appropriate action, on the assumption that the contact is the target. If no immediate action is taken, the searcher may decide later to investigate the contact or to take some other action. Under combat conditions it may not be feasible to investigate the contact; the searcher can ignore the contact, attempt to maintain contact, or make an attack.

As the search proceeds the searcher may obtain additional information on the generators of false contacts, their types, numbers, and locations. Some of the information will be obtained by direct observation and some of it by deduction. For example, the number and

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locations of contacts, and the times at which the contacts were made, can be observed and recorded. Also, results of investigations, if made, would be known. Inferences can be made about the number and location of false targets not contacted, under some conditions, from the observed results.

The searcher may be able to use some of the additional information in the continuation of the search. In general, the extent to which he can do so will depend on the contingencies that have been anticipated and provided for in the search plans, and the availability of the required information. For example, the searcher may be using search plans that anticipate the finding of real objects that generate false contacts. Each time such an event occurs the search plan may change in a way that requires the estimation of the posterior distribution of residual false targets. The posterior distribution of residual false targets depends on the distribution of effort that the searcher has applied up to that epoch. The applied distribution of effort almost always differs from the planned distribution, and often by large amounts. Hence, to estimate the posterior distribution of residual false targets under realistic assumptions it is necessary to record the actual distribution of search effort. If no provision is made for the collection of this information and for its use in computing the posterior distribution, a different search plan must be used.

The false contact search problem is diverse and complex. False contacts may be generated by real objects and by random fluctuations. Real objects may be stationary or moving, and may or may not be identifiable. Various action options are available to the searcher when contacts occur. Additional information that becomes available as the search proceeds may be used to improve the search plan, provided the search plan includes the relevant contingencies. It is evident that a mathematical abstraction that includes all the possibilities would be very complicated.

3. Some Fundamental Properties of Search

One of the essential considerations in search theory and search plans is the cumulative one: to determine the effect of adding further searches (viz., searching effort) to searches already made but which have not achieved success; and also, to find the optimum distribution of searching effort. Under what conditions can the successive searches be regarded as compatible observations, so that the probabilities of combined events can be calculated directly from the laws of probability applied to successive trials?

Some fundamental properties of searches were examined in references [3] and [8]. The properties include additivity of search efforts, consistency of two or more searches, and compatibility of two or more searches. Although these properties apply to search for a single target without false contacts, some or all may not be applicable in other search problems. Here, we are concerned particularly with the case of false contacts.

The occurrence of a false contact is an event that may make the search that is made after the event incompatible with the search that is made before the event. Under these conditions it may be possible to improve the search plan by making it contingent on these events. This possibility was studied in reference [3]; the results are summarized in Section 5 below.

4. Review of False Target Models

In the process of formulating models of search theory in the presence of false targets we reviewed [1] the few published papers that treat the problem, and some other sources. Most of the work on false targets is concerned with their effects on the expected results with an existing search plan (or with a raid recognition procedure). A few papers are concerned with the problem of what procedures to use in investigating contacts after they occur. Very few authors consider the problem of how to plan the search when false targets are known to be present or false contact can be anticipated.

The first serious attempt to develop a theory of search in the presence of false targets is that of Stone and Stanshine [11]. This theory was continued by Stone, Stanshine and Persinger [12]. They assume that false contacts are generated by real objects that can be marked, when located and identified, so that they will not require investigation should they be contacted again. They write an equation for the expected time to find the target under the assumption that the search plan does not depend on the number of false targets that have been found and eliminated. If search plans that are contingent on the finding of false targets are admitted, the optimal search plan might depend on the number of found false targets, in general.

5. False Contacts Generated by Real Stationary Objects

We assume that exactly one stationary target is known to be in a region R of Euclidean n-space, with known location density function $f(x)$. During the search, false contacts may be generated by other real objects in R, called false targets, which can't be distinguished from the target except by a close inspection. The number of

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false targets in R may be known, or unknown with a known number density function. We assume that the false targets are independently distributed with known location density functions. They may be identically distributed with common location density function $g(x)$, or otherwise. Our assumptions are more general than those of Stone and Stanshine[11], who explicitly introduce only the collective location density function $\delta(x)$. Other properties of the false target population must be inferred from the assumptions on false contacts, by means of which one can deduce that their number density function for false targets is the Poisson function.

If a contact is made, the search will be interrupted, an investigation will be started, and continued until the contact has been identified. If the contact is the target, the search will be stopped. If the contact is a false target, its location will be recorded and the position marked, perhaps with a buoy, so that another investigation will not be made, should it be contacted again. The search will be resumed.

In formulating the search problem and writing the expression for the optimization functional we include the number of found false targets as a contingency. A search plan in our class of search plans may depend on the number and locations of the found false targets, as well as on the elapsed search time and the target density function.

The original formulation [2] of the problem was revised [3]. Equations for the location and number distributions of residual false targets were obtained. In general, the number mass function of the residual false targets depends on the number found and eliminated. Thus, the optimal plan for a class of search plans that may depend on the number of found false targets will differ from the optimal plan for a class of search plans that do not make use of this information, in general. An exception is the Poisson distribution used by Stone and Stanshine [11]. Only for this distribution is the residual number distribution not altered when a false target is found and eliminated.

The functionals for the probability of detection in a given time and the expected time to detection are difficult to construct and optimize when the search plan may depend on the number of found false targets. A method of constructing the expected time functional for finite number distributions has been obtained. A method of optimizing this functional by combining Dynamic Programming with other optimizations has been outlined, and illustrated with a simple example. These results were published [3].

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6. Search for Moving Targets

In preparation for the study of search for moving targets in the presence of false contacts several problems involving search for moving targets with no false contacts were formulated and solved. These problems are described below.

The problem of optimal allocation of search effort to find a moving target is significantly more difficult than the corresponding problem for a stationary target, in general. In some cases the problem can be reduced to that of a stationary target by using transformations, as demonstrated by Stone and Richardson [10] for targets having conditionally deterministic motion. Hellman [7] has shown that diffusion theory can be used to solve the problem when the motion is sufficiently random. Very few moving-target problems that do not have these special properties have been formulated and solved.

We started with a modification of a two-cell problem studied by Pollock [9] in an effort to find a formulation that could be generalized to n cells. Pollock formulates a two-cell Markov model, in which the transition probabilities between looks and the detection probabilities per look are known constants. He finds the search strategy that minimizes the expected number of looks to find the target, and the strategy that maximizes the proability of detection with a given number of looks. The problems are easy to formulate, difficult to solve, the solutions are complicated, and the method of solution does not appear to be applicable to more than two cells.

We replaced Pollock's discrete formulation by a continuous-time Markov model [4]. The modified model is easy to formulate and difficult to solve. The solution is complicated but can be computed from explicit equations without iteration. Generalization to more than two cells, although difficult, appears to be possible. These results will appear in OPERATIONS RESEARCH.

In the second problem [5] we assumed that the target moves from cell to cell in response to the searcher's activities, in an effort to avoid the region in which the search currently is being made. If the target is in cell 1 and the searcher does not look there, the target stays in cell 1. The rate at which the target moves from cell 1 to another cell is assumed to be proportional to the density of searching effort in cell 1 with known constant proportionality factor. Thus, the target may become aware of the presence of the searcher and move to another cell to avoid detection. An example is a submerged submarine that is alerted to the presence of a searcher by the ping of a sonar that is being operated in the active mode. Other examples occur when the target has the capability of passive reception of any energy radiated by the searcher at ranges exceeding the

detection range.

We examined only the problem of maximizing the detection probability in a given time, starting with the two-cell problem and then generalizing to n cells. Although the generalization for this problem is easier than the corresponding generalizations of the problems in [4] and [9], a complete solution of the n-cell problem has not been obtained. However, a method was derived for finding the complete set of extremals, which contains the solution. These results will appear in the SIAM Journal on Applied Mathematics.

7. Search for Moving Targets in the Presence of False Contacts

The two problems described in the previous section were modified to include Poisson-distributed false contacts.

A target moves back and forth between two cells, while false signals are generated by a Poisson process. The stochastic target motion may be independent of the searcher's activities, or it may be induced by the searcher. The false signals may be generated by noise or other mechanisms that are independent of the target, or they may be generated by the target. Contacts are investigated until it is discovered that the contact is indeed the target or until a decision is made that the contact is not the target. The probability of making a wrong decision is assumed to be negligible. The problem is to find the distribution of that portion of the effort that is available for searching to maximize the probability of detection in a given time.

The problem was solved for both false contact assumptions when the target motion is induced by the searcher. A procedure that can be used to obtain the optimal distribution was formed for the case in which the transition rates are independent of the searcher's location and the false contact rates are independent of the target's location. The optimization problem has not been solved for the case in which the transition rates are independent of the searcher's location and the false contacts are generated by the target. The results [6], which have been submitted for publication, are the only known solutions of search optimization problems involving both moving targets and false contacts.

8. Reports and Publications

Results obtained under the contract were submitted in seven documents: references [1] through [6] and reference [8]. An earlier version of [3] was submitted in January 1972 as ADL-73538-2 with the same title.

One paper [3] has been published in OPERATIONS RESEARCH
and a second paper [4] is scheduled for publication in that jorunal.
A third paper [5] has been accepted for publication in the SIAM
Journal on Applied Mathematics. Another paper [6] has been submitted
for publication.

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