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MISSILE ALLOCATION MODELS FOR THE ATTACK OF DEFENDED TARGETS

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Block 20 - ABSTRACT (Cont.)

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Missile Allocation Models for the

Attack of Defended Targets

by

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I. EXECUTIVE SUMMARY

The purpose of this thesis is to analyze a ballistic missile attack against defended targets with the purpose of determining effective targeting tactics for the offense.

Basically we consider a target complex containing a known number of targets. The complex is defended by a known number of defensive missiles divided into two sites, each having an automatic radar-controlled defensive system.

The attacking missiles can be directed at the defensive missile sites or at the targets themselves. If an attacking missile is aimed at the defense site and it penetrates the defensive system, it destroys that entire guarding system. The attack is assumed to be sequential. The offense first commits some number of its attackers to the defensive sites, then it attacks the targets. The defense is assumed to be one-on-one. Both the offensive and defensive missiles are imperfect, each working with some known probability.

The offensive problem is to determine how its fixed force of attackers should be allocated between the two defense sites and the targets in the main complex. We assume that the offense receives no information about the success or failure of its weapons in the course of the attack. The measure of effectiveness used is to maximize the expected number of targets destroyed.

The report describes the computations involved in the allocation models and solves some sample problems for illustration. It also considers the generalization to more than two defense sites.

II. INTRODUCTION

A. GENERAL PROBLEM

This thesis considers the ballistic missile attack of a complex of targets surrounded by defense missiles. In order to gain the highest damage to the targets, it is desired to allocate the attackers optimally between the defense sites and the targets themselves. In the following cases that will be developed, we assume ground to air defender missiles in the models. The attackers might be launched from a submarine underwater, or they might come from the surface of the ocean or from any station on the ground. We assume we know the probability that a missile will hit a distant target. We also assume the attacker has good intelligence information and knows the number of defenders, their placement and the probability of their successfully destroying an attacking missile. Thus we know the hit probability of an attacker and also the installation of the defense sites of the enemy country. The mission is to destroy the targets which are defended by these anti-missile systems. The defense sites themselves are of no value as targets to the attacker except that destroying them permits the offense to reach the desired targets.

The first model presented here is for the attack of a target complex with two defense sites around it using a fixed force of imperfect missiles. The attackers will be directed first at the two defense sites then at the targets themselves. The imperfect defenders are used against the attackers on a one-for-one basis as long as defenders remain. If any attacker penetrates a defense site, all the defenders at that site are destroyed. The problem addressed is the offensive problem of determining how many attackers to send to each defense site and to the target complex.

We discuss next some generalizations to n defense sites, two types of defensive missiles and two defense sites and in another model two types of offensive missiles and two defense sites.

B. MEASURE OF EFFECTIVENESS

This section discusses the choice of the measure of effectiveness used in the allocation model. The ballistic missile attack is analyzed from the offensive point of view. The objective of the analysis is to determine tactics which will permit the offense to use his forces more effectively. The term "more effective use of resources" must be translated into terms which can be used unambiguously to guide the offense in its weapon deployment.

Aside from the deterrent effect, the purpose of the offensive system is to destroy targets. It would be desirable in an actual attack to destroy, if possible, the most valuable set of targets, but then we have the problem of determining or assigning target values. No general agreement can be reached regarding the values to be assigned; and even if agreement could be reached, any values assigned could not reflect interaction between targets. For example, the value of an industrial target depends very much on the continued existence of a power plant to run it.

It is assumed that if some targets have a value which is obviously large compared to most of the others, these targets will be given special consideration in targeting. The majority of the targets, however, are assumed to be of roughly comparable value and the criterion used in this report is to maximize the expected number of targets destroyed. It is assumed here that the targets do not vary in value with time.

For planning purposes on a larger scale it is possible that a more versatile measure of effectiveness would be desired, but for examination of alternative tactics the criterion of maximizing the expected number of targets destroyed serves as a useful means of comparing alternatives.

C. MATHEMATICAL MODEL

Suppose we let

t = the number of the targets in the complex,

a_i = the number of offensive missiles assigned to the defensive site i,

t_i = the number of defensive missiles in each defense site i,

$E(t,a_i)$ = the expected number of targets destroyed.

We can formulate the problem as follows:

$$\text{Max } E(t,a_i)$$

$$\text{subject to } a_i \leq t_i,\ i=1,2,\ldots,n$$
$$a_i \geq 0.$$

III. BASIC PROBLEM DESCRIPTION

A. DESCRIPTION

The basic problem addressed in this report is the problem of allocating a fixed force of imperfect offensive missiles to a fixed set of targets. All of the targets are in the complex. The number of targets is known to the offense. There are two defense sites for the complex. Each defense site is defending the targets in the complex using a known number of imperfect defensive missiles each of which can be used against any missile approaching any target in the complex.

It is assumed that the number of defensive missiles in each defense site is the same. The attack has two basic phases, first the attack on the defense sites, then the attack on the actual targets. The attacks on the defense sites can be assumed to proceed simultaneously if the offense has two launcher systems available. The attacking missiles can be directed to either the targets themselves or to the defensive missile launching complex, probably the control radars. It is also assumed that if an offensive missile which is aimed at one defensive complex penetrates that defense and hits its target, the entire force of defensive missiles in that complex is rendered useless. If an offensive missile is destroyed or misses its intended target it does no damage at all.

The attack can be thought of as sequential, the offensive first directing some number of attackers to both defense sites and then the remainder to the targets themselves. We assume that the offense has no damage assessment capability; that is, he cannot tell which missiles, if any, have successfully penetrated to their targets. We also assume in the report that the defense does not have the capability of attack evaluation; that is, he cannot determine in flight the impact point of an incoming missile accurately enough that he dares to let it pass undefended with the knowledge that it will impact harmlessly. Even if the defense can determine the impact point he is assumed here to be unable to correlate that information in real time with the continued existence or previous death of targets near the impact point. Thus we assume that as long as the defense has missiles available he will not let offensive missiles proceed undefended. We assume that the defense is one-on-one.

The model is an offense-last-move model and assumes that the offense knows both the number of targets and the number of defensive missiles in each complex. Thus the offense will never allocate more attackers to the defensive systems than the number of defensive missiles minus one because the supply of defensive missiles serving as targets will be exhausted at that point.

Since the offensive has no damage assessment capability, the best process for him to follow is to spread as evenly as possible over the targets those re-entry vehicles which are allocated to targets.

The basic problem then is to determine the optimal allocation of the fixed forces of offenders to the defensive systems and the target complexes to maximize the expected number of targets destroyed. See Figure 1. We assume that t is sufficiently large so that each attacker can shoot at a live target.

B. ANALYSIS

For two defense sites protecting the targets in the complex, we use the following notation in the model.

Let

t = the number of targets in the complex,

t_1 = the number of defenders in defense site number one,

t_2 = the number of defenders in defense site number two,

A = the total number of attackers,

a = the number of attackers assigned to targets in the main complex,

a_1 = the number of attackers assigned to defense site number one,

a_2 = the number of attackers assigned to defense site number two,

p_a = probability that an attacking missile kills its target when no defender is used,

p_d = probability that a defensive missile which is assigned to an attacker kills that attacker,

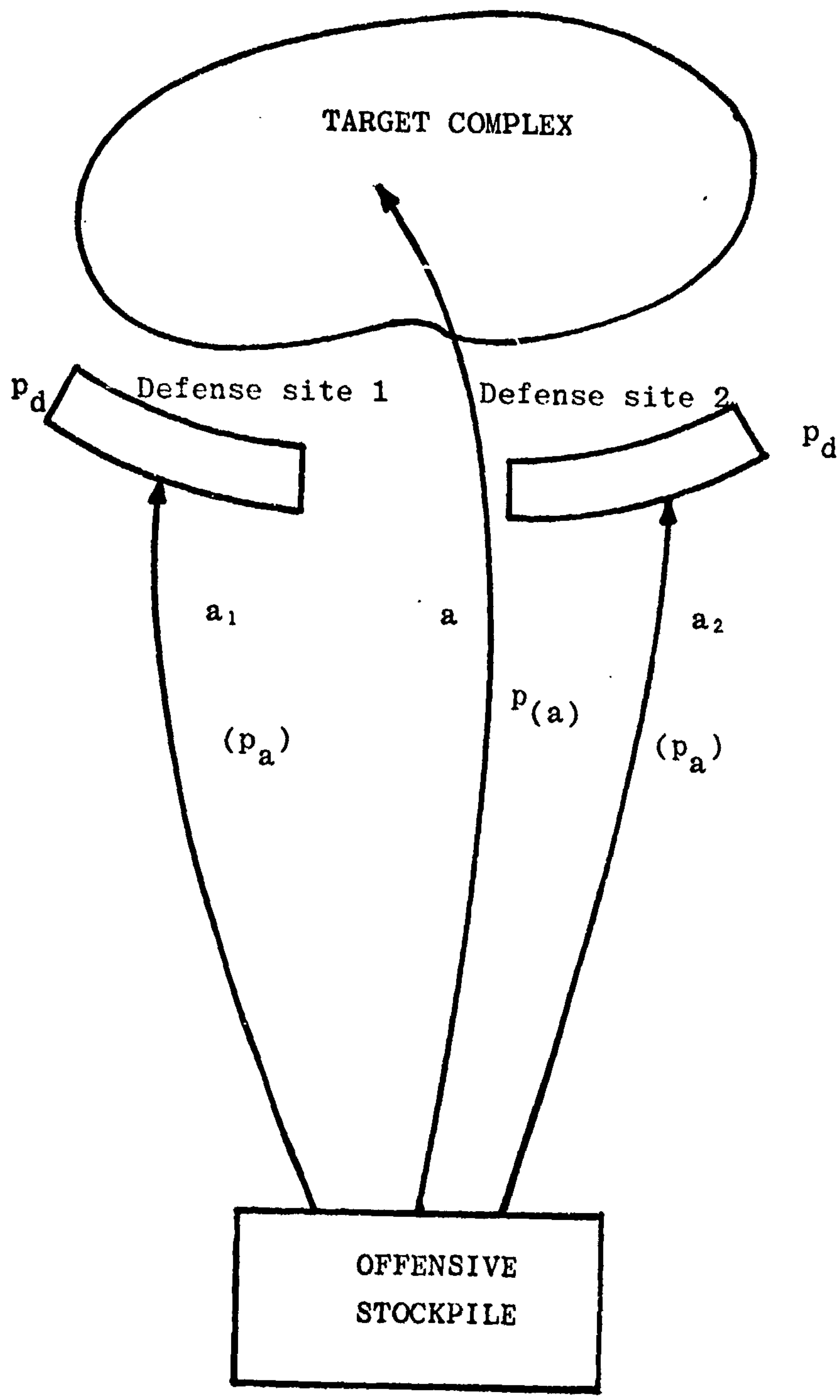


FIGURE 1

The Allocation Model

$p_k = p_a(1-p_d)$, the probability that an attacking missile kills its target when a defender is used.

Suppose the number of defenders in sites one and two are equal. Then we always assign a_1 equal to a_2.

Let

P_i = probability that exactly i defenders remain available for use after the first $a_1+a_2 = 2a_1$ attackers complete the attack on the defense sites.

We have that

$$P_i = \begin{cases} 2(1-p_k)^{t_1-i} \cdot \{1-(1-p_k)^{t_1-i}\} & \text{if } i = t_1-a_1 \\ (1-p_k)^{2t_1-i} & \text{if } i = 2(t_1-a_1) \\ 0 \quad \text{otherwise.} & \end{cases}$$

Number the attackers beginning with the first one which is assigned to a target and let

P_r' = probability that attacker r kills a target.

We have

$P_r' = p_a \cdot$ (probability that target is not defended)
$\quad + p_k \cdot$ (probability that target is defended).

Let

R_r = probability that the r^{th} target is defended,
= probability that r or more defenders remain available for use after the initial attack on the defense sites.

We have

$$R_r = \sum_{i=r}^{2(t_1-a_1)} p_i \;,$$

so that

$$\begin{aligned} p_r' &= p_a \cdot (1-R_r) + p_k \cdot R_r \\ &= p_a - p_a \cdot R_r + p_k \cdot R_r \\ &= p_a - (p_a - p_k) R_r \\ &= p_a - p_a p_d R_r \\ &= p_a - p_a p_d \left[\sum_{i=r}^{2(t_1-a_1)} p_i \right] . \end{aligned}$$

The expected total number of targets killed in the complex will be written as $E(t,a_1)$.

We have

$$\begin{aligned} E(t,a_1) &= \sum_{r=1}^{A-2a_1} p_r' \\ &= \sum_{r=1}^{A-2a_1} \left\{ p_a - p_a p_d \left[\sum_{i=r}^{2(t_1-a_1)} p_i \right] \right\} . \end{aligned}$$

We let

$P^1_{(a)}$ = probability that exactly one of the defense sites survives,

and

$P^2_{(a)}$ = probability that both survive.

Then

$$P^1_{(a)} = 2(1-p_k)^{a_1}\{1-(1-p_k)^{a_1}\}$$

and

$$P^2_{(a)} = (1-p_k)^{2a_1} .$$

Then we have

$$E(t,a_1) = (A-2a_1)p_a - p_a p_d(t_1-a_1)[P^1_{(a)} + 2P^2_{(a)}]$$

Table I helps to illustrate the derivation of this form of $E(t,a_1)$.

Analytical efforts to maximize $E(t,a_1)$ over a_1 have not been successful, but for fixed values of p_a, p_d, A and t_1 it is easy to compute $E(t,a_1)$ for all $a_1 \leq t_1$. The results can be plotted and the best values of a determined. This has been done for a few sample cases and the results are shown in Figures 2 and 3.

Suppose $A = 30$

$t_1 = t_2 = 10$

in the case $a_1 = a_2 = 6$

i / r				(t_1-a_1)				$2(t_1-a_1)$		
	1	2	3	4	5	6	7	8	9	10
1	0	0	0	P^1	0	0	0	P^2	0	0
2	0	0	0	P^1	0	0	0	P^2	0	0
3	0	0	0	P^1	0	0	0	P^2	0	0
4	0	0	0	P^1	0	0	0	P^2	0	0
5	0	0	0	0	0	0	0	P^2	0	0
6	0	0	0	0	0	0	0	P^2	0	0
7	0	0	0	0	0	0	0	P^2	0	0
8	0	0	0	0	0	0	0	P^2	0	0
9	0	0	0	0	0	0	0	0	0	0
.	.									.
.	.									.
.	.									.
18	0	.	.	.	.	.	.	.	.	0

Table I

Illustration of R_r computation for two defense sites.

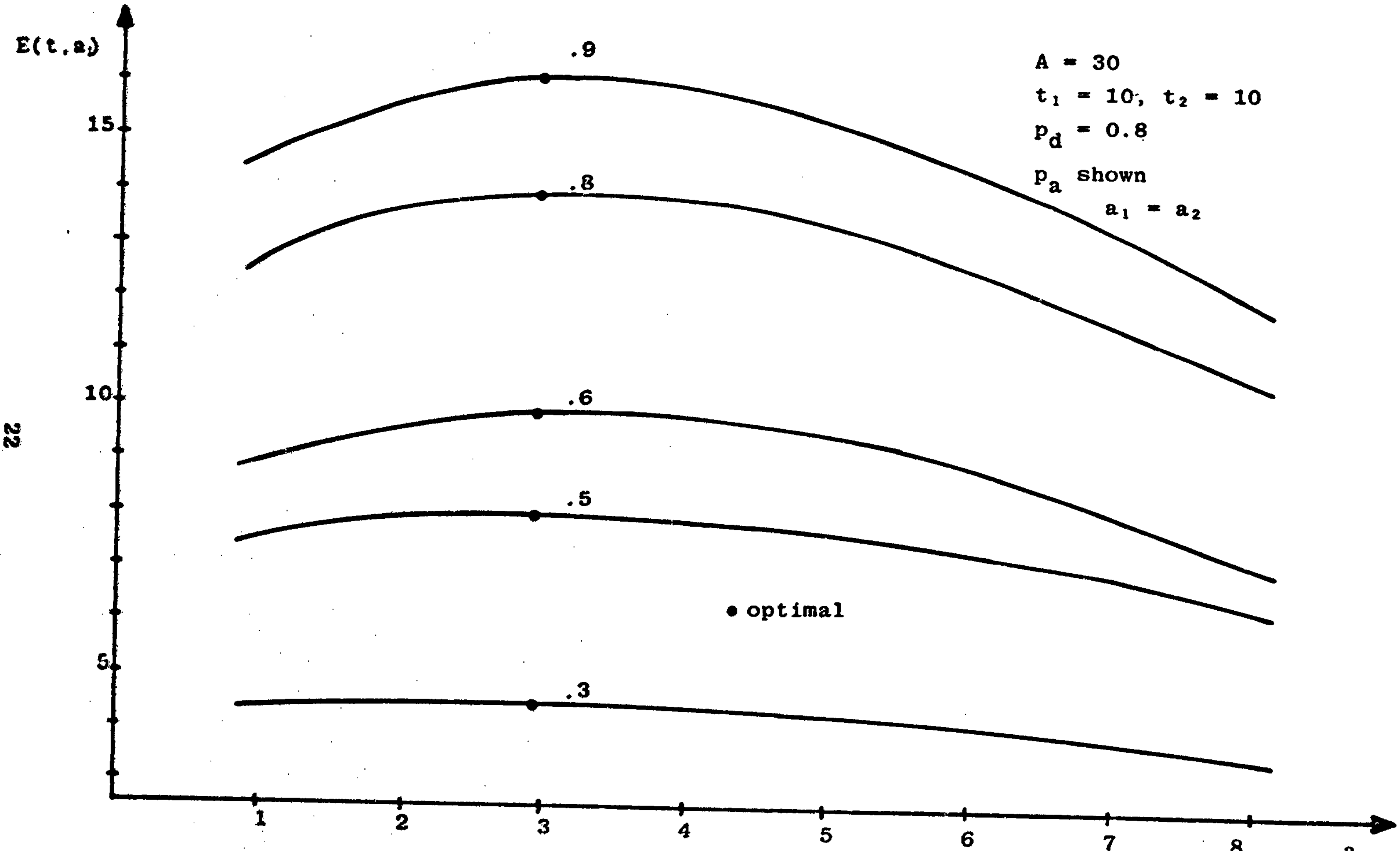


FIGURE 2
Illustration of different hit probabilities of attackers

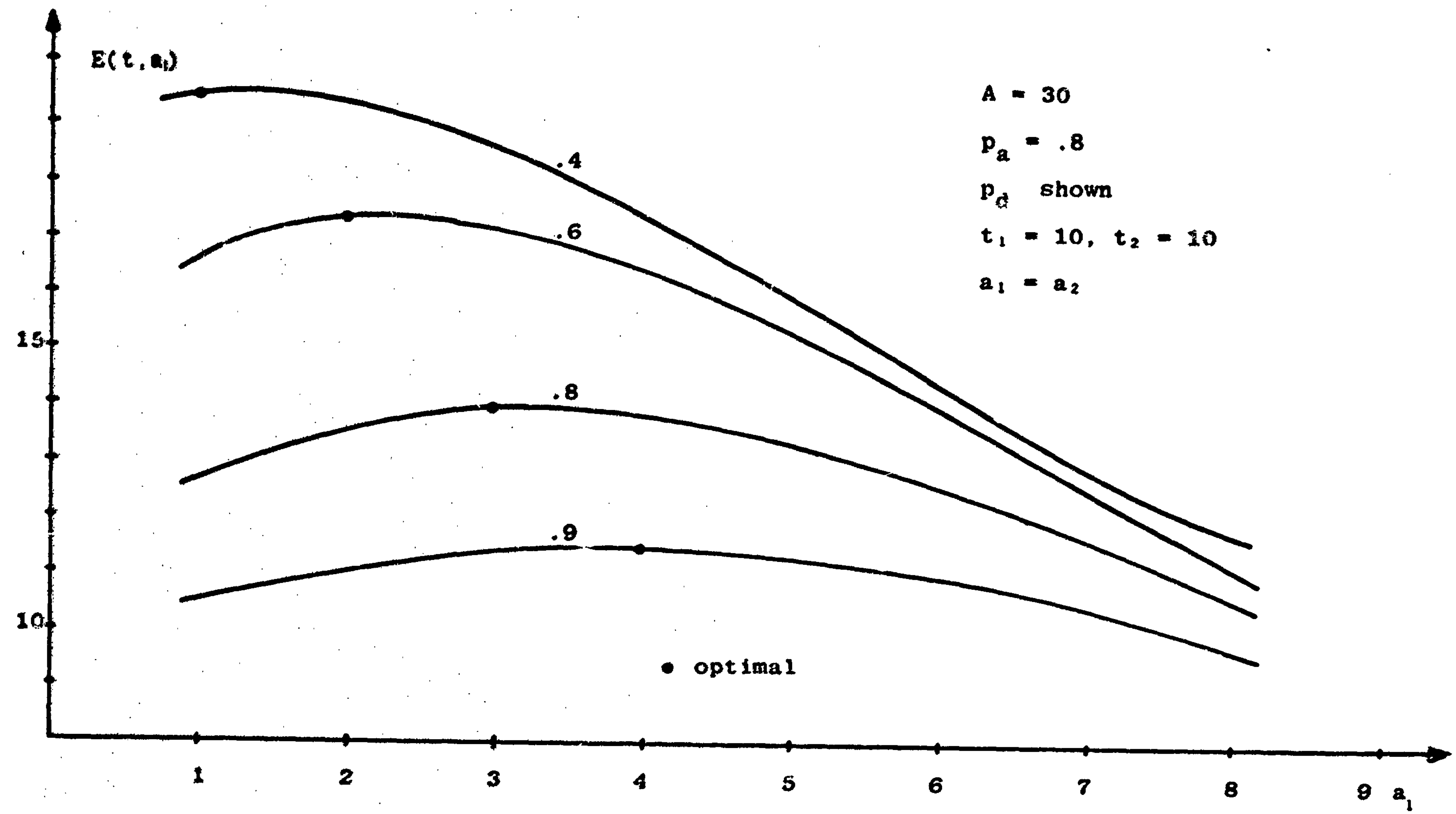


FIGURE 3

Illustration of different probabilities of defenses

IV. GENERALIZATIONS

A. SEVERAL DEFENSE SITES

We consider now a more general case of this model. Suppose the targets in the main complex are very important. The defense might spread out a large number of defensive missiles into several groups around the complex with a separate control system for each site. The basic one-on-one defense is still assumed. The offense must decide the optimal number of attackers to assign to each of the defense sites.

It is assumed that the number of defensive missiles in each defense site is the same. In general the attacker missiles could come from any base or carrier and might be different kinds of missiles launched from different ranges to the targets. Thus they could have different probabilities of hitting the targets. But in the case here we will assume that all of the offensive missiles have the same hit probability.

We will consider the special cases of different types of missile, tactics, and different numbers of defensive missiles in each defense site later.

The number of targets destroyed is still employed as the measure of effectiveness. We continue the assumption that all defensive missiles have the same probability of defense against an attacking missile. The defensive systems are illustrated in Figure 4.

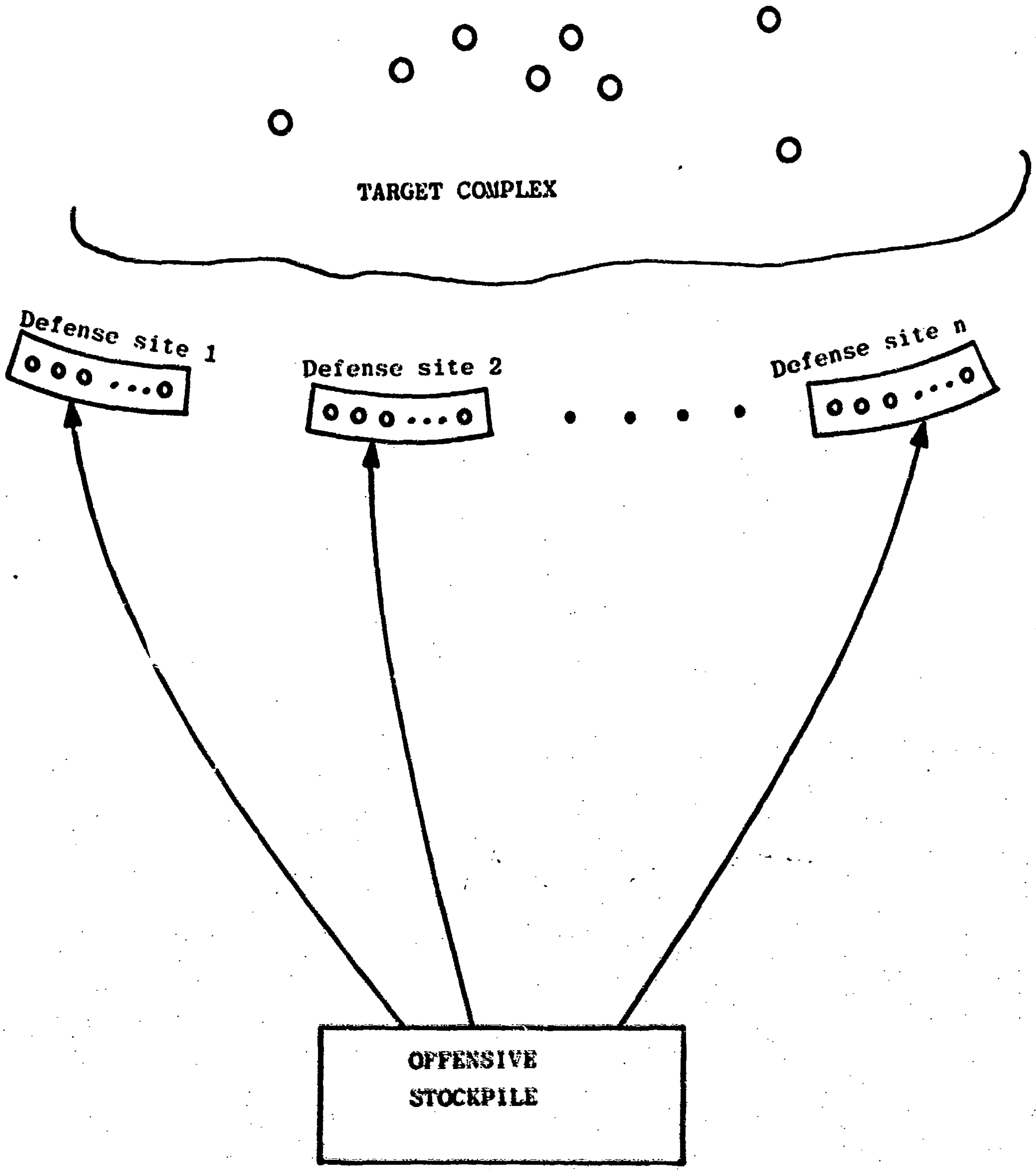


FIGURE 4

The allocation model for n guarding systems

It is assumed that the number of targets is greater than the number of attackers and that the offense has no damage assessment capability as before.

Now, we will look more closely at some special cases. Suppose we have n defense sites and each of them independent of the others. We will develop the ideas from the one defense site, then two, and so on until we obtain the model for n defense sites.

The complication resulting from increasing the number of defense sites comes primarily from the difficulty in expressing the probability that any offensive missile will receive a defender. The expected number of targets destroyed depends of course on the number of defenders which remain available after the initial attack on the defense sites is completed.

Consider the following cases:

i) For the case of one defense site we have

$$E(t,a_1) = (A-a_1)p_a - p_a p_d(t_1-a_1)(1-p_k)^{a_1} \text{ . [Ref. 1]}$$

ii) For two defense sites we have

$$E(t,a_1) = (A-2a_1)p_a-p_a p_d(t_1-a_1)[P^1_{(a)} + 2P^2_{(a)}]$$

or

$$E(t,a_1) = (A-2a_1)p_a-p_a p_d(t_1-a_1)[2(1-p_k)^{a_1}\{1-(1-p_k)^{a_1}\}$$
$$+2(1-p_k)^{a_1}] \text{ .}$$

iii) The situation is somewhat more complicated with three defense sites. What follows is essentially a listing of the possible outcomes of the initial attack on the defense sites.

1) For the case in which two systems have been destroyed (one survives) we have

$$P^1_{(a)} = 3[1-(1-p_k)^{a_1}]^2 \cdot (1-p_k)^{a_1}, \quad i=t_1-a_1.$$

2) If one site has been destroyed (two survive) we have

$$P^2_{(a)} = 3[1-(1-p_k)^{a_1}] \cdot (1-p_k)^{2a_1}, \quad i=2(t_1-a_1).$$

3) If none has been destroyed (three survive) we have

$$P^3_{(a)} = (1-p_k)^{3a_1}, \quad i=3(t_1-a_1).$$

For illustration suppose

$A = 40$

$t_1 = t_2 = t_3 = 10.$

Now, consider the case where $a_1=8$. Table II illustrates this case.

r \ i	1	2	3	4	5	6	7	8	9	10
		(t_1-a_1)		$2(t_1-a_1)$		$3(t_1-a_1)$				
1	0	P^1	0	P^2	0	P^3	0	0	0	0
2	0	P^1	0	P^2	0	P^3	0	0	0	0
3	0	0	0	P^2	0	P^3	0	0	0	0
4	0	0	0	P^2	0	P^3	0	0	0	0
5	0	0	0	0	0	P^3	0	0	0	0
6	0	0	0	0	0	P^3	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
.	.									.
.	.									.
.	.									.
16	0	.	.	.	.	.	.	.	.	0

Table II

Illustration of R_r computation for three defense sites.

We see that

$$E(t,a_1) = (A-3a_1)p_a-p_ap_d(t_1-a_1)[P^1_{(a)}+2P^2_{(a)}+3P^3_{(a)}] .$$

Furthermore, we see that

$$P^1_{(a)} = 3[1-(1-p_k)^{a_1}]^2 (1-p_k)^{a_1}$$

$$P^2_{(a)} = 3[1-(1-p_k)^{a_1}][(1-p_k)^{a_1}]^2$$

$$P^3_{(a)} = [(1-p_k)^{a_1}]^3 .$$

Now, let

$$f = [1-(1-p_k)^{a_1}]$$

$$t = [(1-p_k)^{a_1}] \quad \text{and} \quad f + t = 1,$$

then

$$P^1_{(a)} = 3f^2t$$

$$P^2_{(a)} = 3ft^2$$

$$P^3_{(a)} = t^3 ,$$

which can be written in the "Binomial" form as

$$P^1_{(a)} = \binom{3}{1} f^2t$$

$$P^2_{(a)} = \binom{3}{2} ft^2$$

$$P^3_{(a)} = \binom{3}{3} t^3 .$$

Now we can consider $P^1_{(a)}, P^2_{(a)},\ldots,P^n_{(a)}$ in the general form.

That is

$$P^1_{(a)} = \binom{n}{1} f^{n-1} \cdot t$$

$$P^2_{(a)} = \binom{n}{2} f^{n-2} \cdot t^2$$

$$P^3_{(a)} = \binom{n}{3} f^{n-3} \cdot t^3$$

$$\vdots$$

$$P^i_{(a)} = \binom{n}{i} f^{n-i} \cdot t^i$$

$$\vdots$$

$$P^n_{(a)} = \binom{n}{n} t^n \ .$$

The expected number of targets killed in the case of n defense sites can easily be constructed.

We get

$$E(t,a_1) = (A-na_1)p_a - p_a p_d (t_1 - a_1)[P^1 + 2P^2 + \ldots + iP^i + \ldots + nP^n],$$

where $P^i_{(a)}$ is written as P^i , $i = 1,\ldots,n$.

Consider

$$R_r = (P^1 + 2P^2 + 3P^3 + \ldots + iP^i + \ldots nP^n)$$

which comes from

$$\binom{n}{1} f^{n-1} \cdot t + 2\binom{n}{2} f^{n-2} \cdot t^2 + \ldots + i\binom{n}{i} f^{n-i}\, t^i + \ldots + n\binom{n}{n} t^n,$$

where the probability that j defense sites survive is $P^j = \binom{n}{j} f^{n-j} \cdot t^j$. We can see that this is the expected number of surviving defense sites out of n.

We can write

$$E(t,a_1) = (A-na_1)p_a-p_ap_d(t_1-a_1)\left[\sum_{j=1}^{n} jP^j\right], \; j=1,2,\ldots,n,$$

or

$$E(t,a_1) = (A-na_1)p_a-p_ap_d(t_1-a_1)E(j).$$

A few samples of three defensive systems are presented in Figure 5. We assume that the attackers are all identical and all of the defenders in any guarding complex are the same. The quantities A, a_1, a_2, a_3, t_1, t_2, p_a and p_d are shown on the figure.

If we assume $a_1=a_2=a_3=\ldots=a_n$, it is not hard to determine the optimal number of attackers to each defense site. We will see in the next section the effect of changing p_a, a_1 and a_2, p_d for each defense site.

B. TWO TYPES OF DEFENSIVE MISSILES AND TWO DEFENSE SITES

Consider the case that the defense sites have two different kinds of weapons, and their abilities to destroy the attackers are different.

Suppose the first defense site uses the defenders of Type 1, and each has the probability p_x of intercepting one attacker. The second site uses the defenders of Type 2, and each has the probability p_y of intercepting an attacker. See Figure 6.

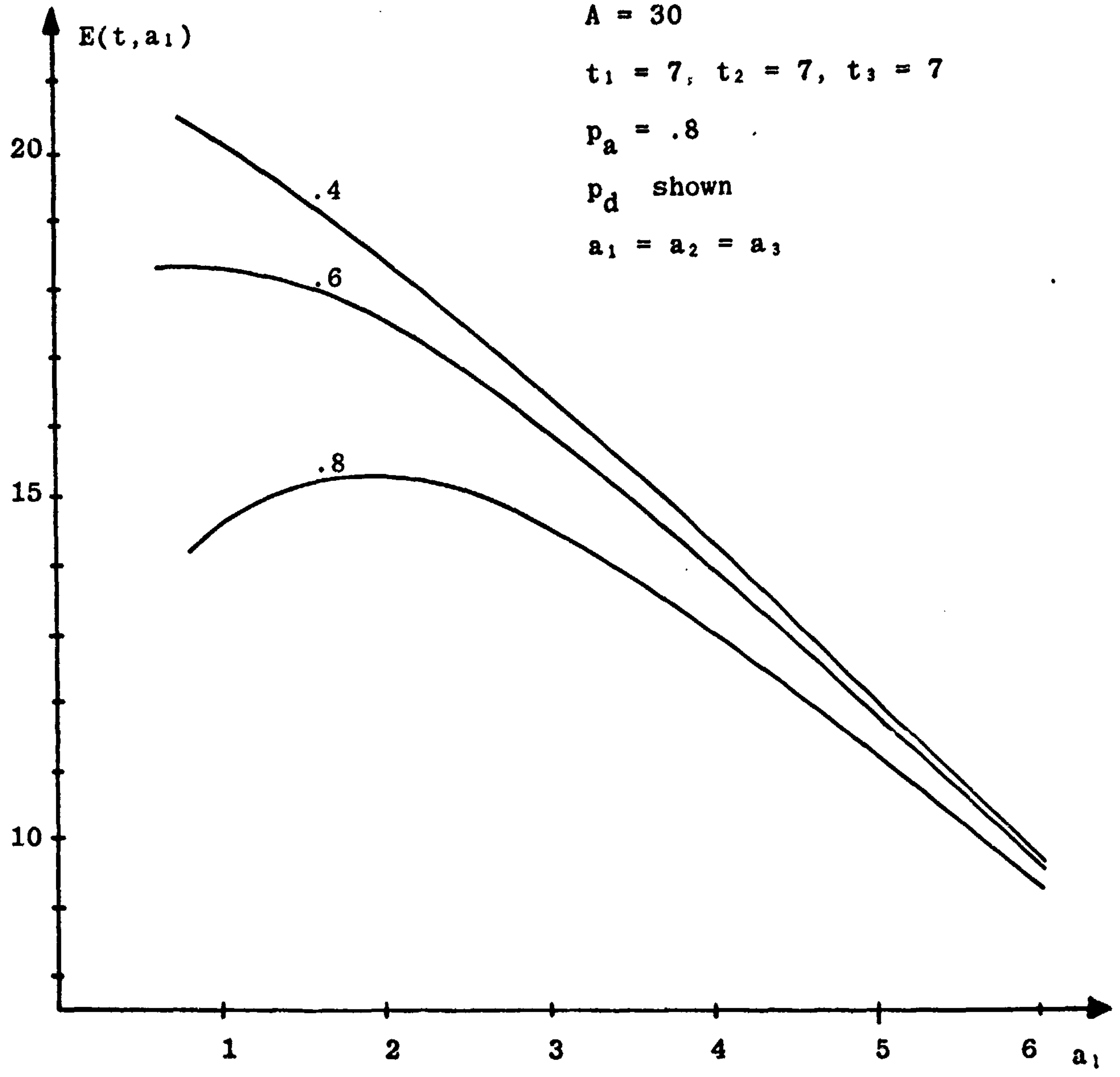


FIGURE 5

Optimal solutions for three defense sites showing different probabilities of defenses

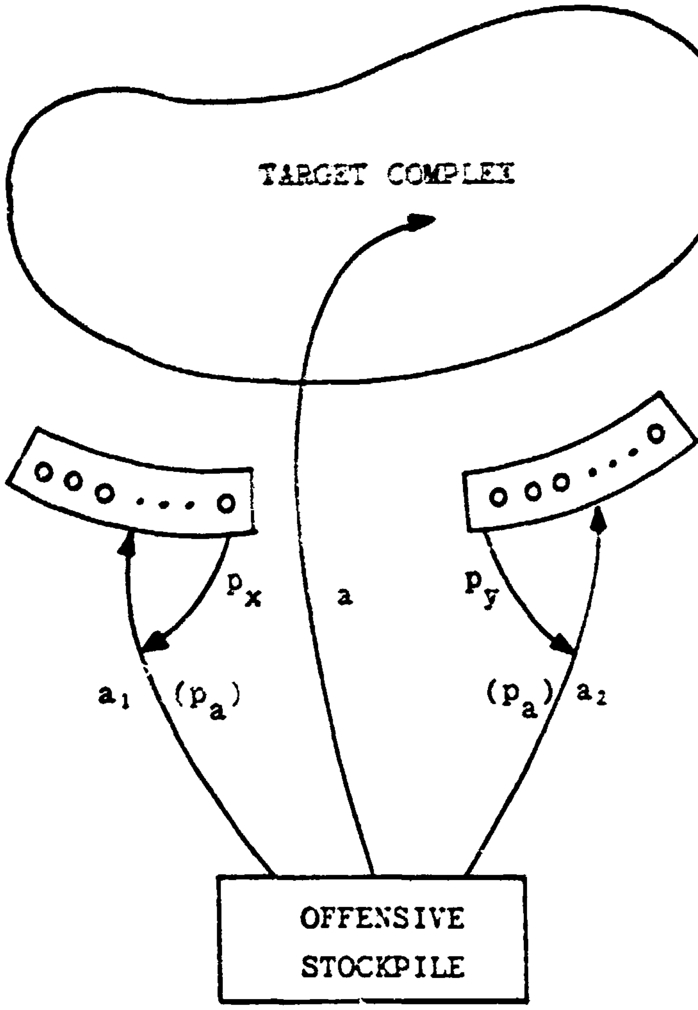


FIGURE 6
Two types of defensive missiles from two defense sites

Let

a_1 = number of attackers assigned to first defense site,

a_2 = number of attackers assigned to second defense site,

a = number of attackers assigned to the targets in the complex,

A = total number of attackers available, a_1+a_2+a.

Let

P_i = probability that i defenders remain in both defense sites together,

$P^{1'}$= probability that only the first defense site still survives after the first attack, (a_1+a_2 attackers).

p^{12} = probability that only the second defense site still survives after the first attack.

p^{2} = probability that both defense sites still survive.

Suppose the numbers a_1, a_2 are not necessarily the same, and the numbers t_1, t_2 are not necessarily the same.

The probability that i defenders remain available is

$$P_i = \begin{cases} p^{11} = [1-(1-p_{ky})^{a_2}](1-p_{kx})^{a_1}, & i=t_1-a_1 \\ p^{12} = [1-(1-p_{kx})^{a_1}](1-p_{ky})^{a_2}, & i=t_2-a_2 \\ p^{2} = (1-p_{kx})^{a_1}(1-p_{ky})^{a_2} & , i=(t_1-a_1)+(t_2-a_2) \\ 0 & , \text{otherwise}, \end{cases}$$

where $p_{kx} = p_a(1-p_x)$,

$p_{ky} = p_a(1-p_y)$.

Now, let

R_r = probability that the r^{th} attacker will be defended.

Then

$$R_r = \sum_{i=r}^{(t_1-a_1)+(t_2-a_2)} P_i \; .$$

The expected total number of targets killed in the complex will be written as $E(t,a_1,a_2)$.

We have that

$$E(t,a_1,a_2) = \sum_{r=1}^{A-(a_1+a_2)} [p_k \cdot R_r + p_a \cdot (1-R_r)]$$

$$= \sum_{r=1}^{A-(a_1+a_2)} \left[p_k \left\{ \sum_{i=r}^{(t_1-a_1)+(t_2-a_2)} P_i \right\} + p_a \left\{ 1 - \left[\sum_{i=r}^{(t_1-a_1)+(t_2-a_2)} P_i \right] \right\} \right]$$

$$= [A-(a_1+a_2)]p_a - p_a p_x (t_1-a_1)(P^{11}+P^2)$$

$$-p_a p_y (t_2-a_2)(P^{12}+P^2).$$

Some examples to illustrate the model are presented in Figures 7 and 8.

C. TWO TYPES OF OFFENSIVE MISSILES AND TWO DEFENSE SITES

1. Tactic 1

We will consider another case. Suppose we have two types of attackers. The one which is assigned to the first defense site has the probability p_b of successfully attacking the target and the other has probability p_c. Other assumptions are the same as before. See Figure 9 below.

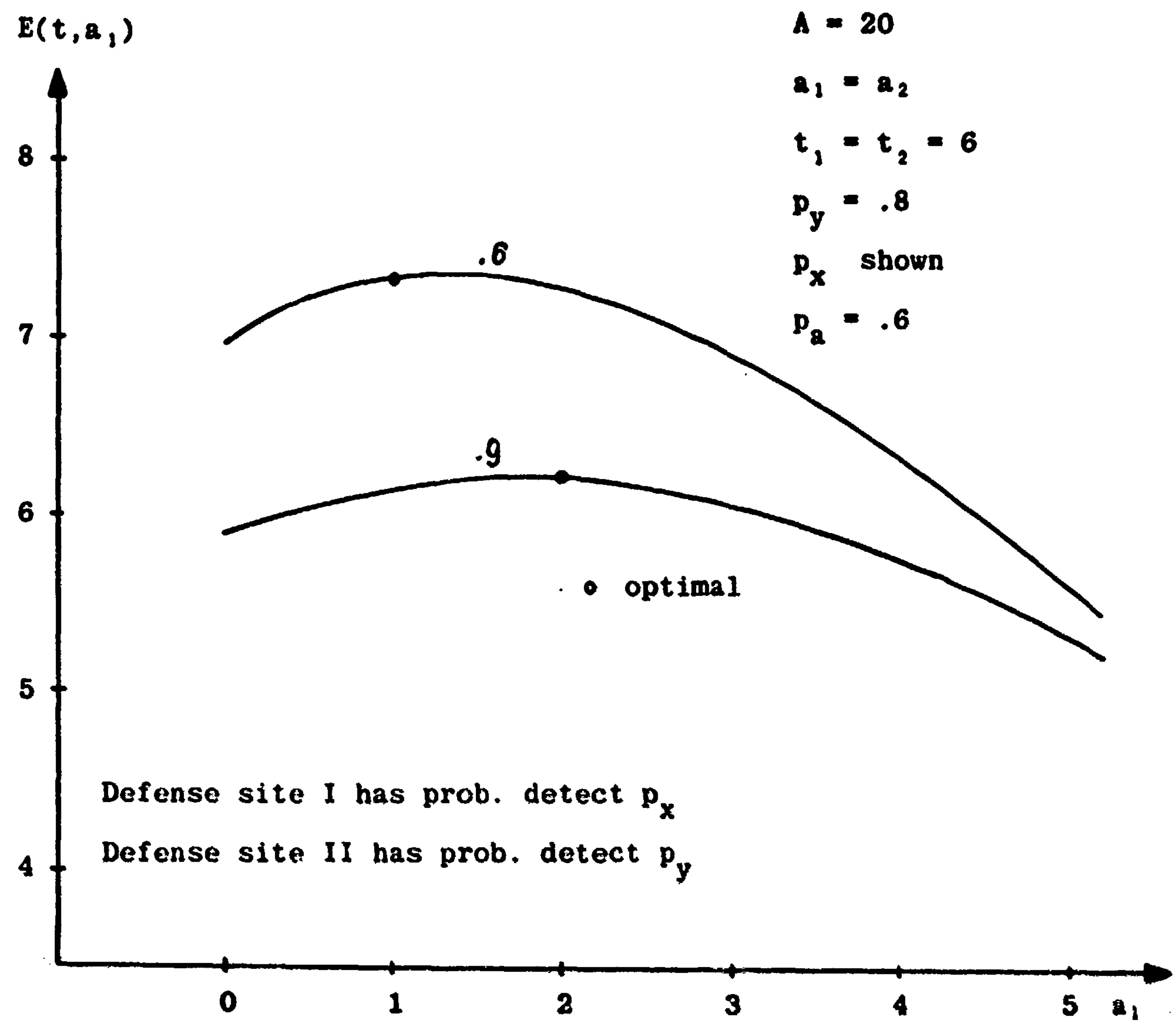


FIGURE 7

Two types of Defenders

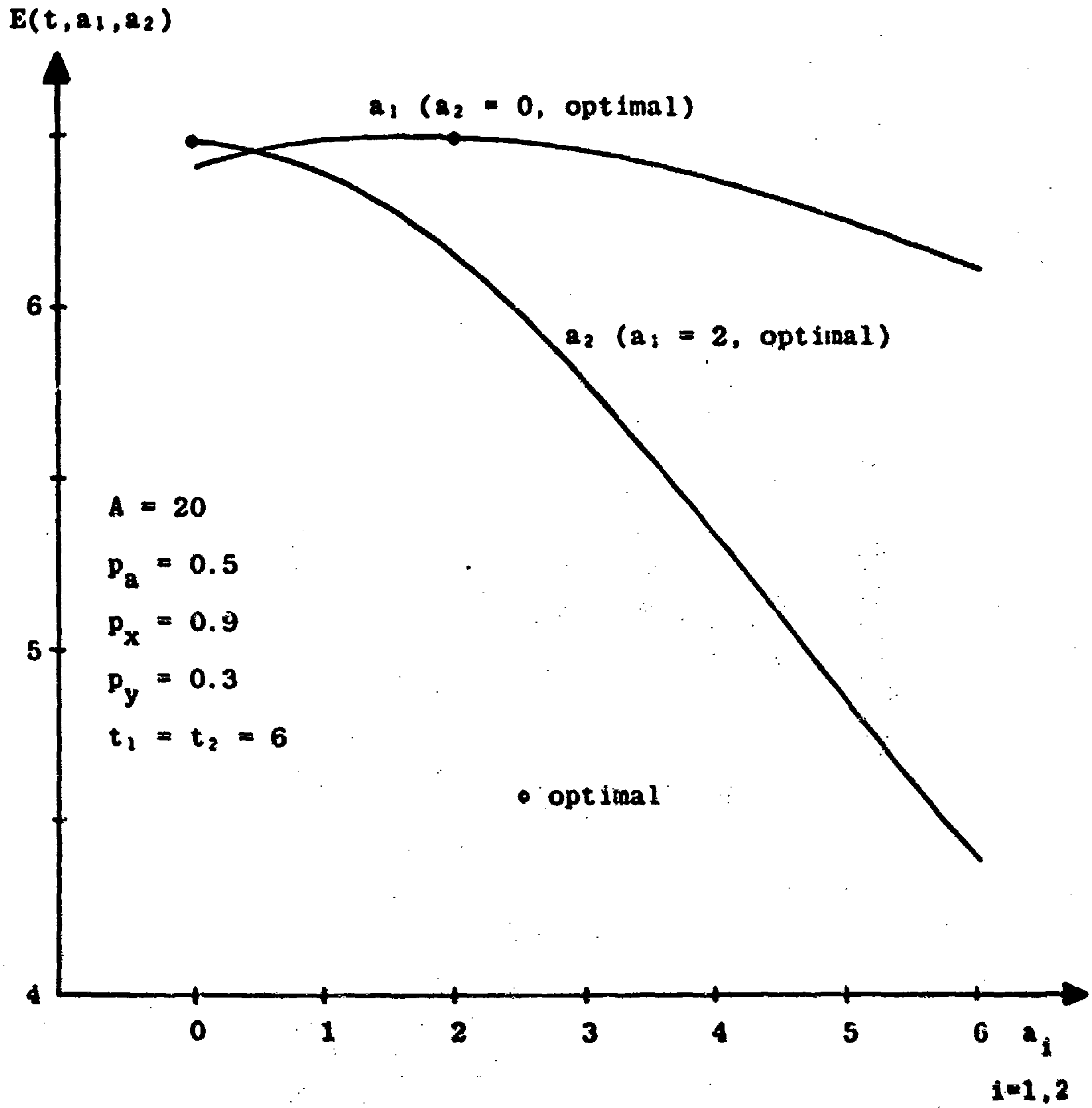


FIGURE 8

Defense Site 1 has prob. detect p_x

Defense Site 2 has prob. detect p_y

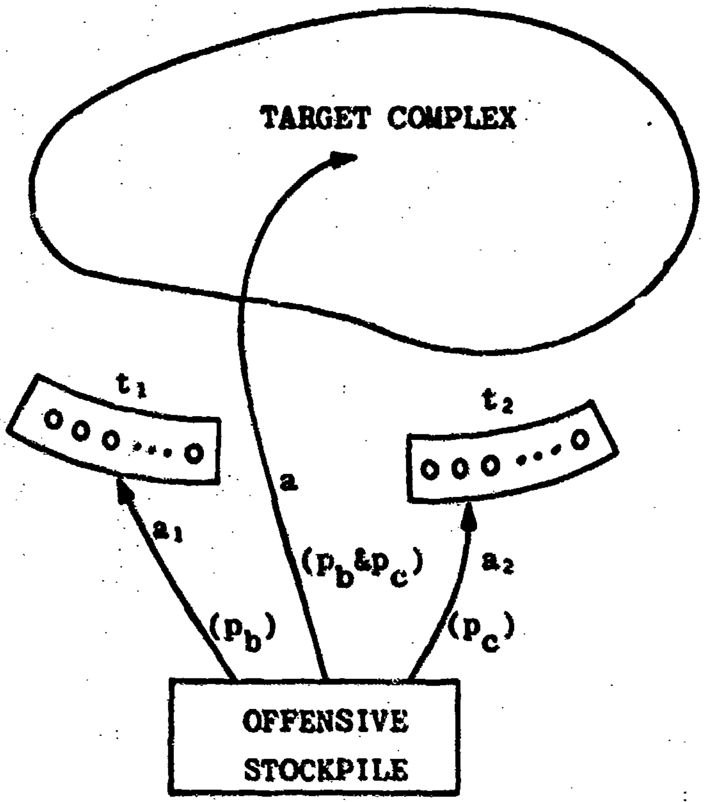


FIGURE 9

Two types of attackers assigned one type to one defense site

Define a_1, a_2, a and A as in part B. The numbers a_1 and a_2 are not necessarily equal, nor are t_1 and t_2.

Let

A_1 = total number of Type 1 attackers,

and

A_2 = total number of Type 2 attackers.

Then $A = A_1+A_2$, total number of attackers, and P_i, P^{i1}, P^{i2}, P^i are the same as case B.

In this case the kill probability p_k depends on the weapon type since there are two types of attackers. The quantity p_k is the probability that an attacker kills its target when a defender is used.

Let

$p_{kb} = p_k = p_b(1-p_d)$, if the first type of attackers is used, and

$p_{kc} = p_k = p_c(1-p_d)$, if the second type of attackers is used.

The probability that exactly i defenders survive the initial attack on the defensive sites is

$$P_i = \begin{cases} P^{11} = [1-(1-p_{kc})^{a_2}](1-p_{kb})^{a_1}, & i=t_1-a_1, \\ P^{12} = [1-(1-p_{kb})^{a_1}](1-p_{kc})^{a_2}, & i=t_2-a_2, \\ P^2 = (1-p_{kb})^{a_1}(1-p_{kc})^{a_2} & , i=(t_1-a_1)+(t_2-a_2), \\ 0 & , \text{otherwise.} \end{cases}$$

The expected number of targets destroyed is

$$E(t,a_1,a_2) = \sum_{r=1}^{(A_1-a_1)+(A_2-a_2)} [p_k \cdot R_r + p_a(1-R_r)]$$

$$\text{where } R_r = \sum_{i=r}^{(t_1-a_1)+(t_2-a_2)} P_i \,.$$

Thus

$$E(t,a_1,a_2) = (A_1-a_1)p_b + (A_2-a_2)p_c - p_b p_d(t_1-a_1)(P^{11}+P^2)$$
$$-p_c p_d(t_2-a_2)(P^{12}+P^2) \;.$$

A few examples are presented in the graph in Figures 10, 11 and 12. Figure 10, illustrates different numbers of missiles in the defense sites 1 and 2 and the different values of p_b and p_c which lead to different optimal numbers of

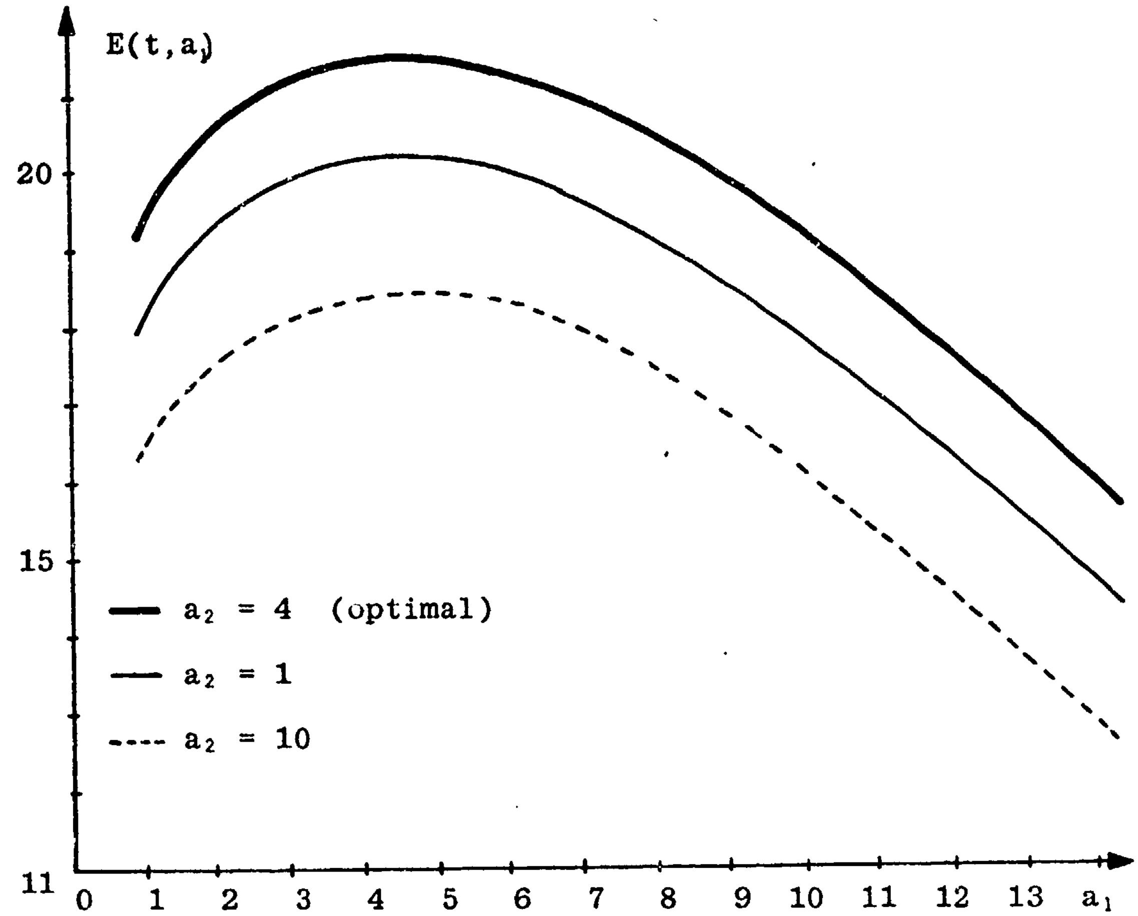


FIGURE 10

Two types of attacking missiles

$A = 42$, $t_1 = 12$, $t_2 = 15$, $p_b = 0.9$, $p_c = 0.7$, $p_d = .08$

a_2 is shown, $a_1 = 5$ is optimal

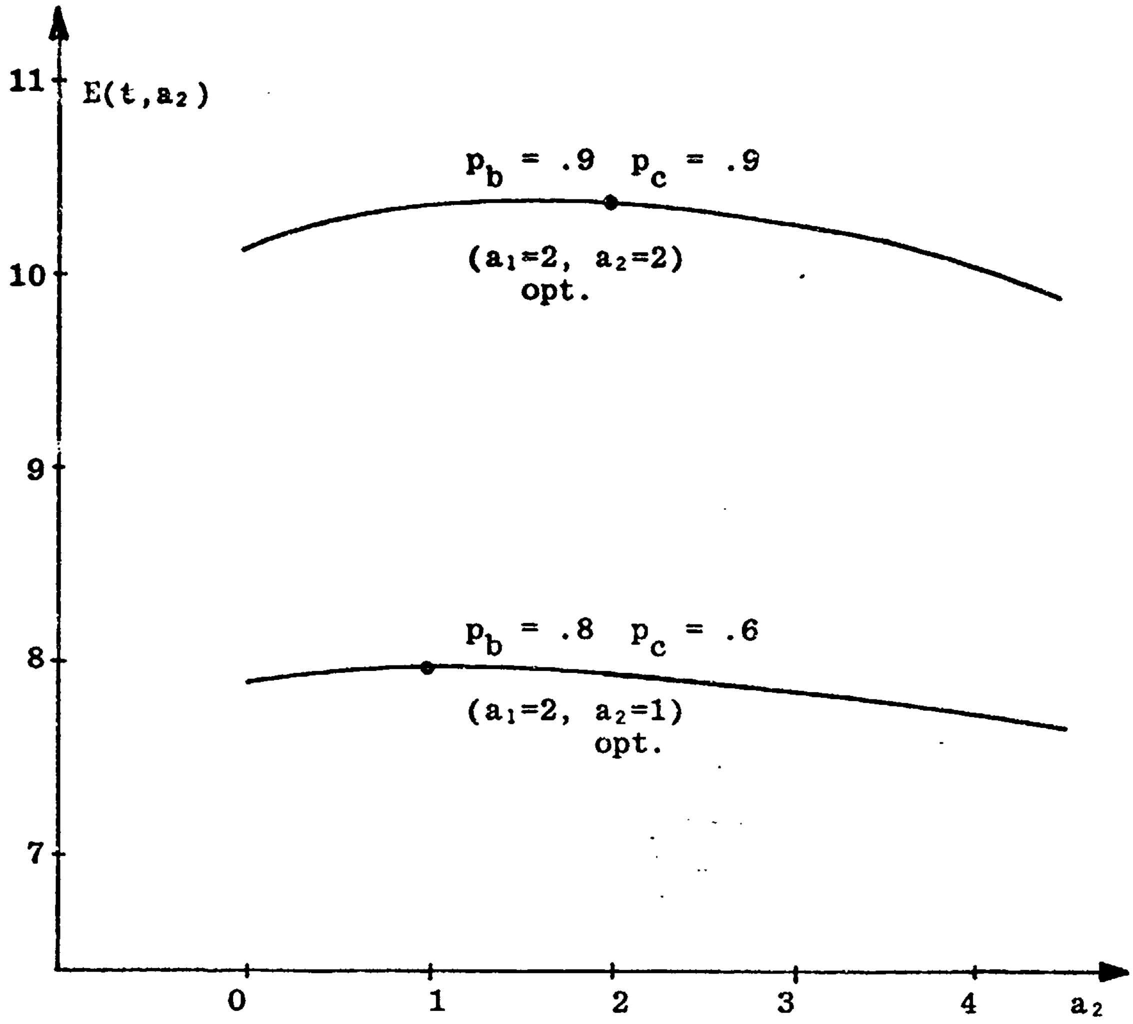


FIGURE 11

Two types of attackers with kill probabilities p_b, p_c resp.

$A_1 = A_2 = 10$ $\qquad$ $a_1 = 2$ (optimal)

$t_1 = t_2 = 5$ $\qquad$ p_b, p_c shown

$p_d = .9$

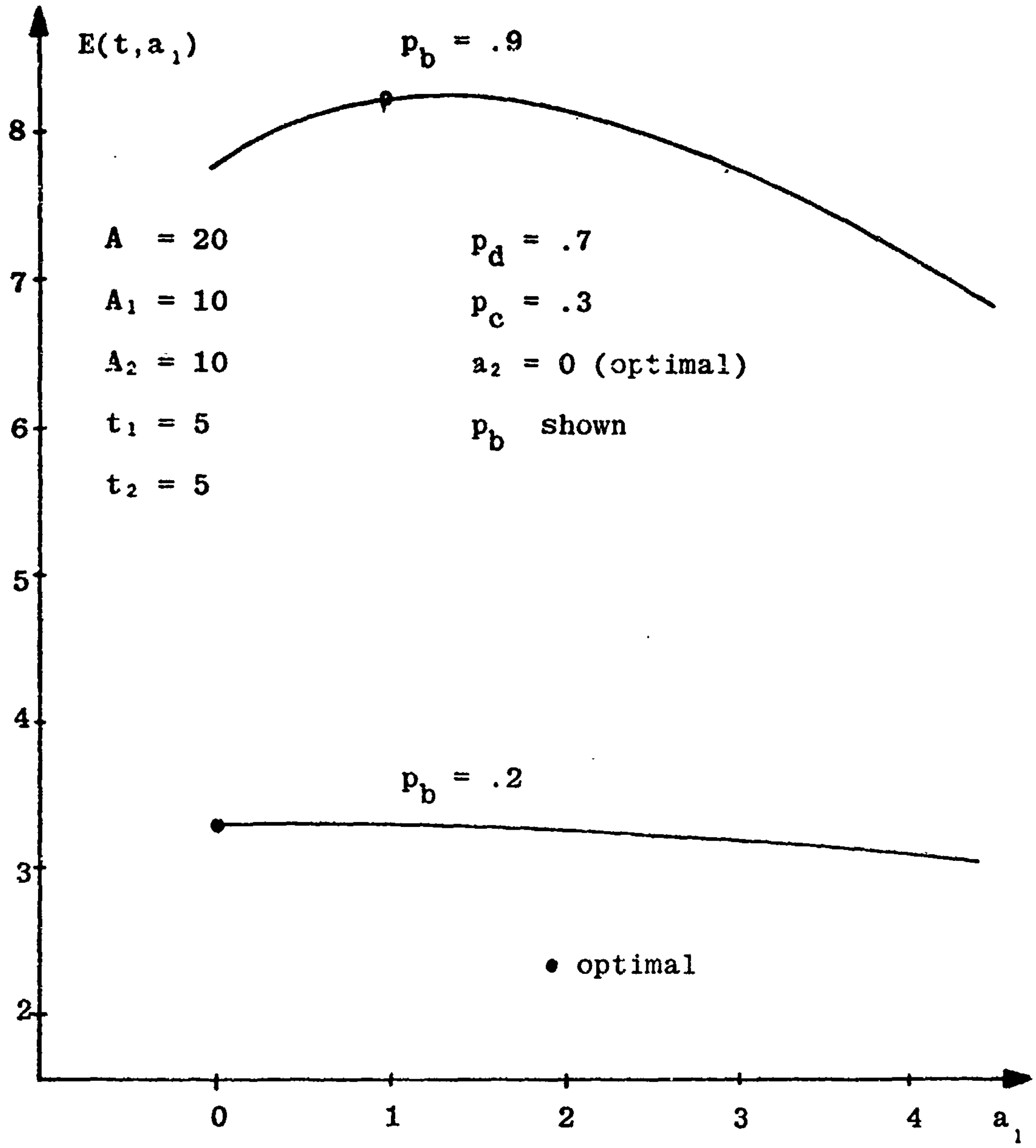


FIGURE 12

p_b = prob. kill target when no defender is used of Type 1

p_c = prob. kill target when no defender is used of Type 2 attackers. Figures 11 and 12 show that the high p_b or p_c values will give the optimal allocations near the middle of $a_2(a_1)$ axis, but for low p_b or p_c the figures show that the optimal allocation will move the number of attackers assigned to the defense sites to a very low number.

2. Tactic 2

As in case 1 suppose we have two types of offensive missiles and the defense still has two defense sites but only one type of defender. Suppose we change our tactics and instead of assigning one type to each defense site we let one type attack both defense sites and the other attack the targets in the main complex. If optimal we also attack targets with some missiles of the first type.

We will call the two types of attackers Type 1 and Type 2. Type 1 and Type 2 have probabilities of hitting a distant target p_a and p_b respectively when no defender is used. See Figure 13.

Let

$p_{k1} = p_a(1-p_d)$ be the probability that a Type 1 attacker kills its target when a defender is used,

and let

$p_{k2} = p_b(1-p_d)$ be the probability that a Type 2 attacker kills its target when a defender is used.

It is assumed that Type 2 (p_b) is always assigned to the targets. Then we have

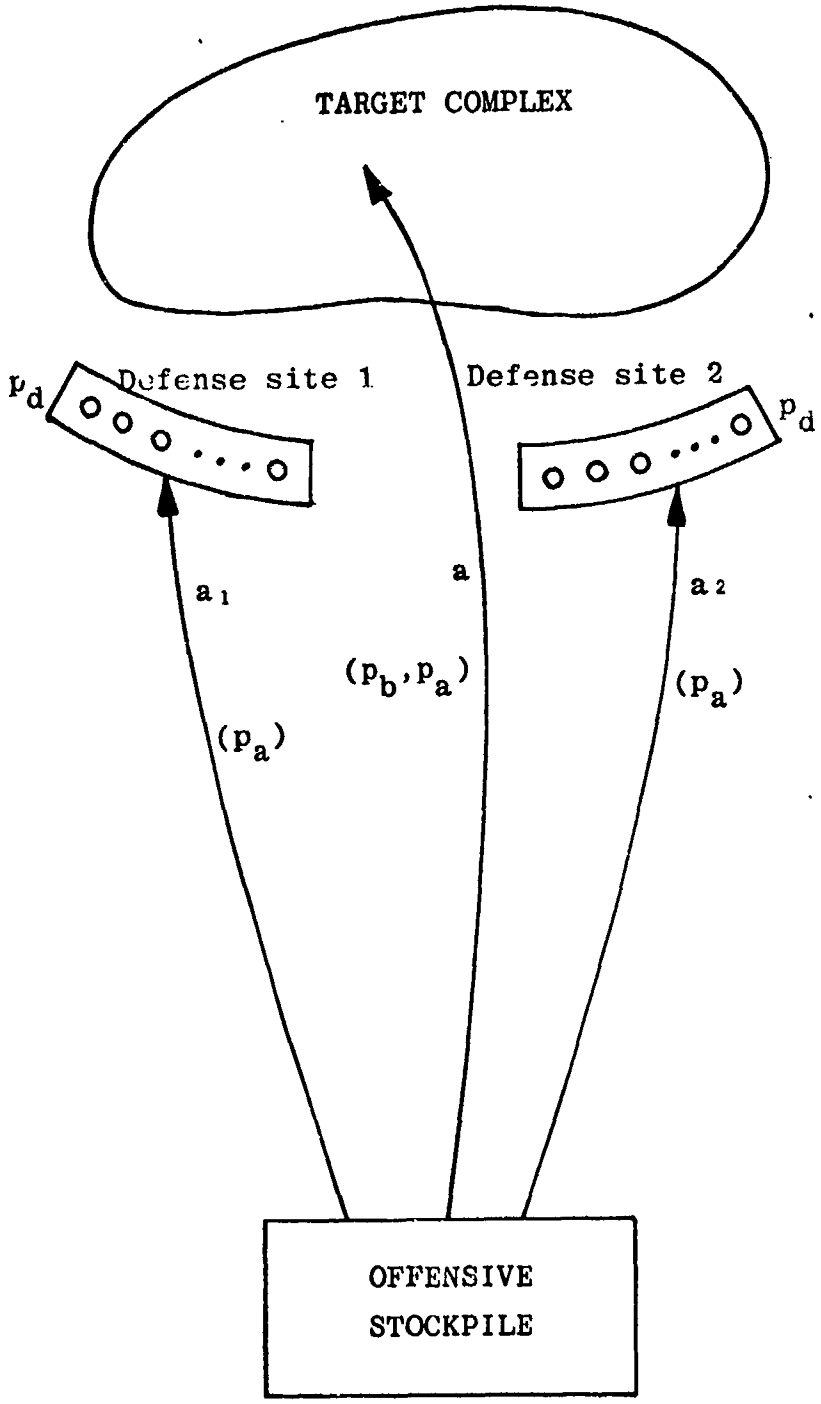


FIGURE 13

Two types of attackers, one type primarily for defense sites and another type exclusively for the targets in the main complex.

$$P_i = \begin{cases} P^{11} = [1-(1-p_{k1})^{a_2}](1-p_{k1})^{a_1}, & i=t_1-a_1, \\ P^{12} = [1-(1-p_{k1})^{a_1}](1-p_{k1})^{a_2}, & i=t_2-a_2, \\ P^{2} = (1-p_{k1})^{a_1}(1-p_{k1})^{a_2} & , i=(t_1-a_1)+(t_2-a_2), \\ 0 \quad \text{otherwise,} & \end{cases}$$

where t_1, t_2 are the numbers of defenders in defense sites 1 and 2 respectively.

Let P'_r = probability that attacker r kills a target. We have

$$P'_r = \begin{cases} p_a(1-R_r) + p_{k1}\cdot R_r, & r=1,2,\ldots,A_1-(a_1+a_2), \\ p_b & , r=A_1-(a_1+a_2)+1,\ldots,A_1-(a_1+a_2)+A_2, \end{cases}$$

where the attackers are numbered beginning with the first Type 1 missile which is assigned to a target in the complex. The Type 2 missiles are all assigned to the targets in the complex and are numbered consecutively following the Type 1 missiles.

As before we can write

$$E(t,a_1,a_2) = \sum_{r=1}^{A_1-(a_1+a_2)} [p_{k1}\cdot R_r+p_a(1-R_r)] + \sum_{r=A_1-(a_1+a_2)+1}^{A_1-(a_1+a_2)+A_2} p_b$$

$$= \sum_{r=1}^{A_1-(a_1+a_2)} [p_{k1}\cdot R_r+p_a(1-R_r)] + A_2p_b$$

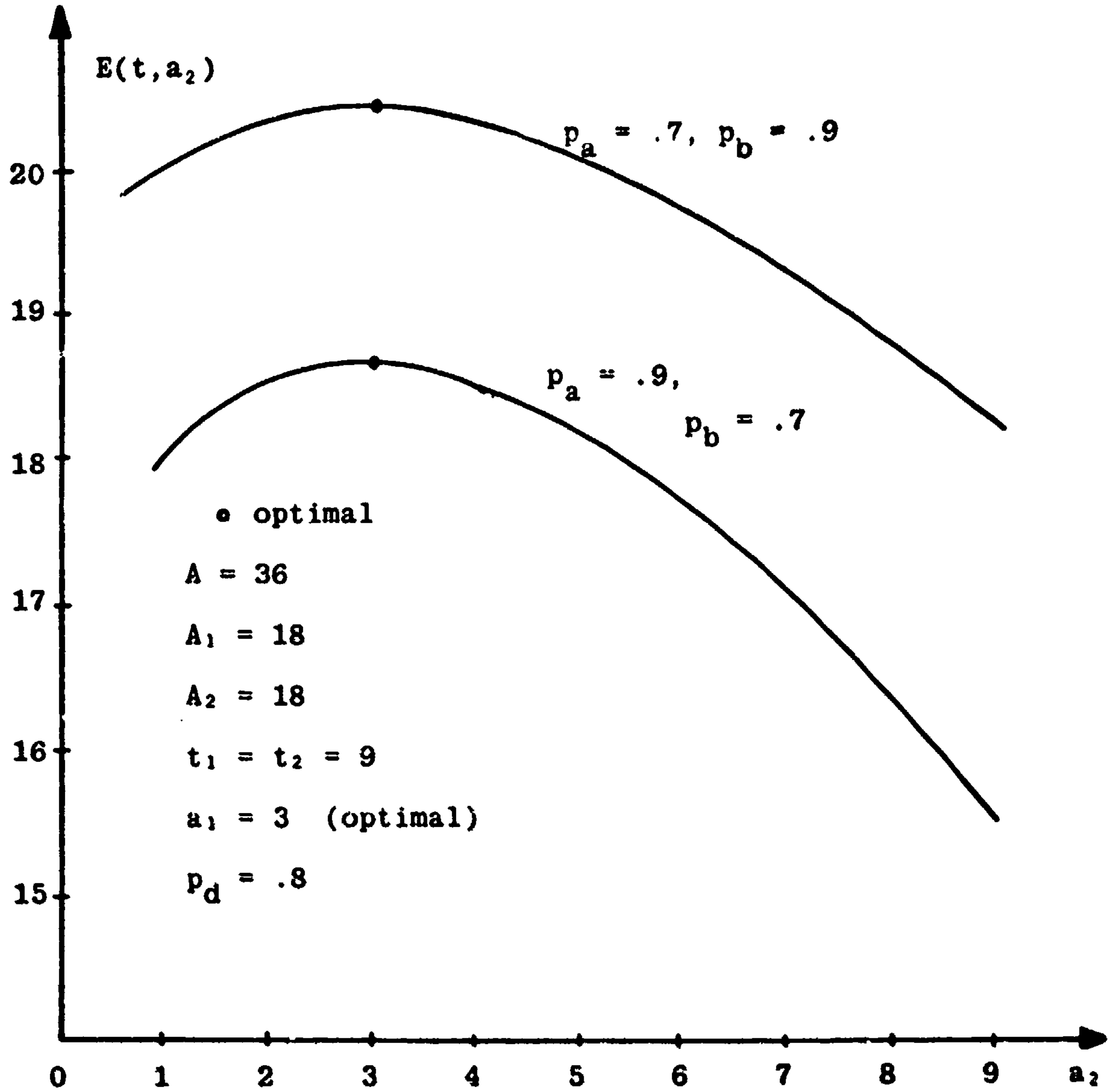
$$= A_2p_b+[A_1-(a_1+a_2)]p_a-p_ap_d(t_1-a_1)(P^{11}+P^2)$$

$$-p_ap_d(t_2-a_2)(P^{12}+P^2)$$

$$= A_2p_b+[A_1-(a_1+a_2)]p_a-p_ap_d[(t_1+t_2)-(a_1+a_2)]\cdot$$

$$(P^{11}+P^{12}+P^2).$$

By fixing the quantities A_1, A_2, p_a, p_b, t_1 and t_2, we can determine the optimal numbers of a_1 and a_2 to assign to the defense sites. A few examples are shown by Figures 14 and 15 in the following pages.



Two types of attackers: Type p_b assigned exclusively to targets in the complex; Type p_a assigned primarily to guarding systems, then to targets.

FIGURE 14

Two types of Attackers

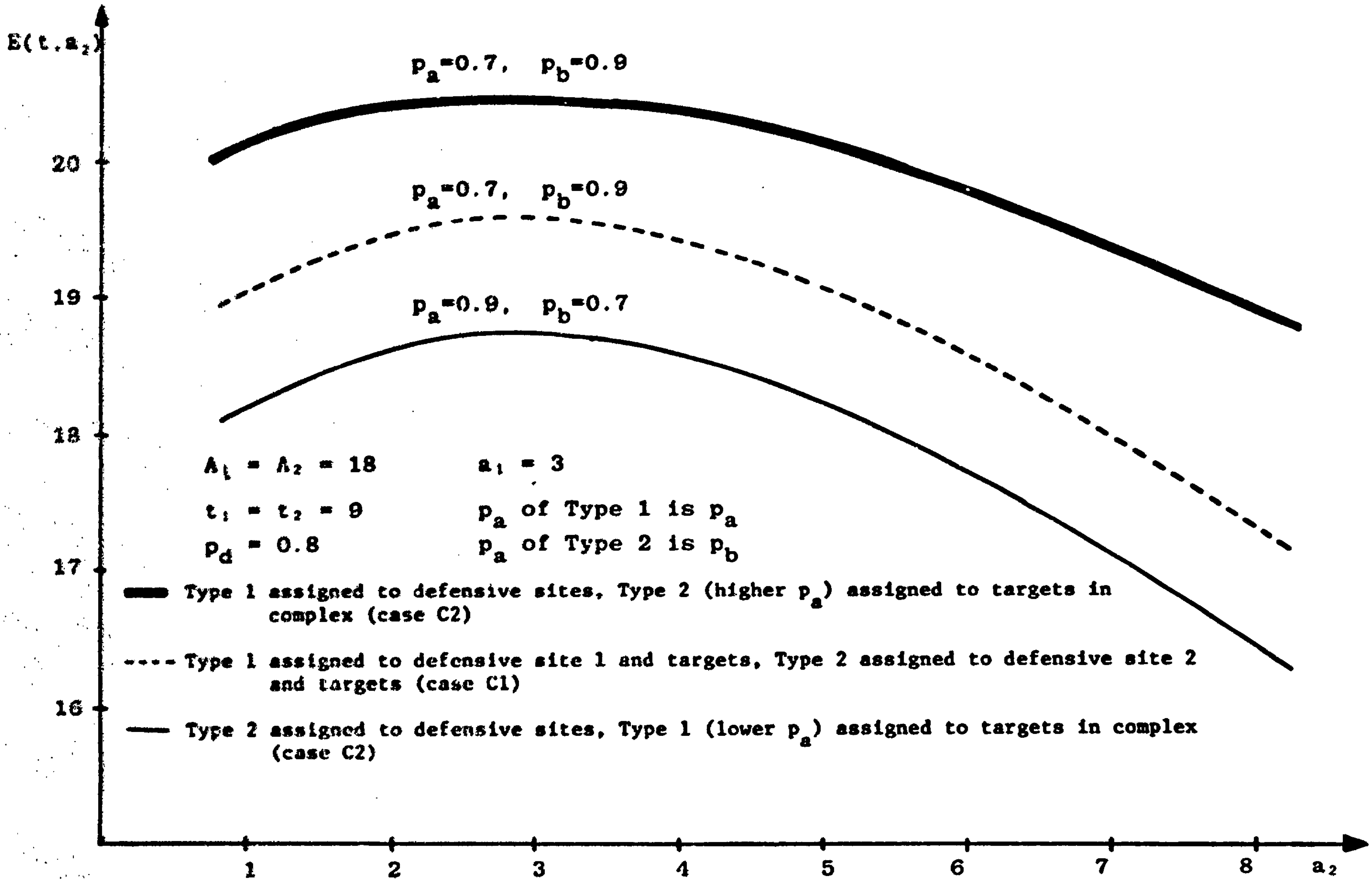


FIGURE 15
Two types of Attackers, A_1, A_2

V. DISCUSSION

Figures 14 and 15 show the results of allocating Type 1 and Type 2 attackers with the probabilities p_a and p_b respectively in various ways to the defense sites and targets. We have computed the maximum number of targets destroyed with fixed numbers of A, A_1, A_2, p_d and a_1. If Type 1 (p_a) is assigned to the defense sites and has a lower probability of hit than Type 2 (p_b) which is assigned to the targets in the complex we obtain a higher expected number of targets destroyed than in the reverse case.

In Case C1 each type of attacker is aimed first at one defense site then at the targets in the complex. Figure 15 shows the expected number of targets destroyed using the same input data as given in Case C2. Case C1 does not give the highest expected number destroyed. The lower p_a assigned to the defense sites and higher p_b assigned to the targets in the main complex gives the best result.

The allocation of identical attackers in Cases A and B of section IV does not seem as interesting as in the Case C1 and C2 where there are different types of attackers. Case A where the number of defense sites is generalized can be used to compute the optimal number of attackers. Likewise the different types of defensive missiles in Case B can be dealt with in the model to obtain the optimal allocation.

Models of different kinds of situations have been developed. The model that gives all different numbers A_1, A_1,...,

A_m, a_1, $a_2, \ldots,$ a_n, p_a, $p_b, \ldots,$ p_d, p_x or p_y is easy to construct and the optimal allocations a_1, $a_2, \ldots,$ a_n can be obtained from it. In these cases the necessary calculations can easily be done using the computer.

It can be seen from the results that if p_a (probability of an attacker hitting the target when no defender is used) is very low such as p_a = .2, .3, we might not need any attackers aimed at the defense sites. Ignoring these defense sites and allocating all of the attackers to the main targets in the complex will give the highest expected number of targets destroyed. In the case that p_a and p_d (probability of a defensive missile intercepting an incoming attacker) are reasonably high, such as p_a and p_d = .7, .8, .9, we should assign some number of attackers to the defense sites first and the rest of them to the targets to obtain the best result.

Other generalizations can be considered, such as changing the structure of the defensive complex to consist of several launcher group controls. We can consider the case that the defenders are not rendered useless until two attackers penetrate. This would be relevant for the case where each launcher group has two control radars, either of which can control the interceptors.

Generalizations could also be made by changing the assumption about the knowledge available to the offense and defense. If the defense has attack evaluation capability, his performance will be improved; or if the offense has damage assessment capability he can increase the expected number of targets destroyed.

A very interesting and apparently difficult extension is to assume that some of the targets have a value which diminishes with time. If some of the targets are offensive missile launchers there is no benefit in attacking the launcher after the missile is gone. Another case which is important is the difference in value of targets. Some targets might be assembly plants for the enemy force, some could be supply depots. Problems with different target values cannot be adequately analyzed in terms of number of targets destroyed. These kinds of generalizations will be left for further research.

LIST OF REFERENCES

1. Naval Postgraduate School Technical Report NPS55SK73081A, An Allocation Model for Attacking Defended Target Complexes with Imperfect Attackers, by G. T. Howard, August 1973.