AD-787 069

THEORETICAL PRINCIPLES OF THE EXPERIMENTAL DEPLETION OF LIQUID ROCKET ENGINES

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Wright-Patterson Air Force Base, Ohio

29 August 1974

AD 787069

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Foreign Technology Division Air Force Systems Command U. S. Air Force	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

THEORETICAL PRINCIPLES OF THE EXPERIMENTAL DEPLETION OF LIQUID ROCKET ENGINES

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Translation

5. AUTHOR(S) (First name, middle initial, last name)

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6. REPORT DATE 1973	7a. TOTAL NO. OF PAGES ~~379~~ 396	7b. NO. OF REFS 104
8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)	
b. PROJECT NO.	FTD-MT-24-836-74	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.		

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Foreign Technology Division Wright-Patterson AFB, Ohio

13. ABSTRACT

19

DD FORM 1 NOV 65 1473

UNCLASSIFIED
Security Classification

EDITED MACHINE TRANSLATION

FTD-MT-24-836-74 29 August 1974

THEORETICAL PRINCIPLES OF THE EXPERIMENTAL DEPLETION OF LIQUID ROCKET ENGINES

By: V. A. Makhin and N. P. Milenko

English pages: 379

Source: Teoreticheskiye Osnovy Eksperimental'noy Otrabotki Zhrd, 1973, pp. 1-280

Country of Origin: USSR
Requester: FTD/PDTA
This document is a SYSTRAN machine aided translation, post-edited for technical accuracy by: Robert D. Hill

PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ë in Russian, transliterate as yë or ë. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

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GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

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rot	curl
lg	log

GREEK ALPHABET

Alpha	Α	α	ɑ		Nu	Ν	ν	
Beta	Β	β			Xi	Ξ	ξ	
Gamma	Γ	γ			Omicron	Ο	ο	
Delta	Δ	δ			Pi	Π	π	
Epsilon	Ε	ε	ϵ		Rho	Ρ	ρ	ϱ
Zeta	Ζ	ζ			Sigma	Σ	σ	ς
Eta	Η	η			Tau	Τ	τ	
Theta	Θ	θ	ϑ		Upsilon	Τ	υ	
Iota	Ι	ι			Phi	Φ	φ	ϕ
Kappa	Κ	ϰ	κ	ϰ	Chi	Χ	χ	
Lambda	Λ	λ			Psi	Ψ	ψ	
Mu	Μ	μ			Omega	Ω	ω	

Makhin, V. A., Milenko, N. P., Pron', L. V. Theoretical Principles of the Experimental Development of Liquid-Propellant Rocket Engines M., "Machine-Building", 1973, 284 pages.

Examined in the book are theoretical questions of the experimental development of liquid-propellant rocket engines (ZhRD) [ЖРД]. General considerations on the experimental development stages and types of tests, their planning and analysis of the results, the development of programs of experimental works and applied methods of the theory of reliability are set forth. An analysis of emergency situations is given, and models of failures with the use of equations of dynamics of operating conditions in the units of the ZhRD are described. The question of the modeling of the physical processes in the development of elements and systems of the ZhRD is briefly touched upon.

The book is intended for engineers and scientists of the appropriate fields of technology. It can also be useful for instructors, graduate students and students of senior courses of the colleges.

There are 31 tables, 77 figures and a bibliography list of 104 titles.

INTRODUCTION

A feature of the contemporary epoch is the scientific and technical progress which encompassed virtually all spheres of the national economy. It was revealed most considerably in the field of rocket and space technology. In order to present a high level of the perfection of carrier rockets and space vehicles, it suffices to indicate the achievements of recent years (delivery to the moon of the automatic researcher "Lunokhod-II", flights of manned space vehicles around the earth and moon, the prolonged flights of automatic stations to Venus and Mars, etc.). These achievements are largely predetermined by the creation of powerful reliably acting liquid-propellant rocket engines (ZhRD) [ЖРД] and engine installations (DU) [ДУ]. At the same time, there arose new problems, first of all - the problem of the reliability of functioning of the ZhRD, conditioned by considerations of the provision for a reliability of missile-space complexes, the safety control of flights, and so on.

It is indisputable that questions of reliability in one form or another always confronted creators of engines of rockets. But if in the early development period of technology they were limited to qualitative information, then at present the quantitative evaluation of the reliability level is required.

This necessity appears especially sharply in stages of the experimental development of the ZhRD, since the endeavor of the program and volumes of different types of tests and the moment of completion of the development are determined by the achieved level of reliability, which must correspond to requirements for technical assignment for the development of ZhRD.

Despite the fact that the reliability theory was formed into an independent science very recently (in the last two decades), in its development two trends were clearly determined.

The first - the statistical-probability trend, based on the statistical representation of mass phenomena or events, assumes the admission, systematization and processing of information according to test results. In this case the used methods of the probability theory and mathematical statistics do not directly consider the physical features of the investigated processes.

The second trend is based on the physical approach to the problem of reliability. It assumes the detection of failures, the investigation of reasons for their manifestation and elimination by means of the appropriate modifications.

The first trend was developed basically by mathematicians and the second - by specialists engineers and designers - the direct creators of the ZhRD. Both trends, in supplementing one another, serve the same purposes; however, until recently they were the object of vigorous discussions [5, 11, 21, 37, 54, 76, 81, 87, 103].

In order to explain the essence of that which is occurring, let us note that the statistical-probability methods of the evaluation of reliability, which are based on the use of only qualitative information (classified according to the criterion "success-failure"), cannot satisfy the requirements given to the evaluation of the reliability of complex technical devices which are contemporary

ZhRD. (Methods based on the fixing of the time of operation time for ZhRD are not used in the majority of the cases).

This position is explained by the fact that despite the comparatively high informativeness of the tests with which by means of telemeterings dozens and hundreds of dynamic characteristics are recorded, for calculating the reliability only one evaluation - "success" or "failure" is utilized. Therefore, the confirmation of high levels of reliability requires a large volume of tests. It is possible to distinguish three basic reasons which make unacceptable the classical methods of the evaluation of the reliability of the ZhRD based on the qualitative information:

1) the high level of reliability of the ZhRD;

2) the complexity of the design, the high cost of manufacture and testing;

3) the limitedness of periods of development and production.

For these reasons, for reliability tests a small number of articles is isolated, as a result of which it is impossible to determine all the failures, the "weak" places of the design and technology.

The manifestation of design and engineering failures and their subsequent development in the stage of operation are for many reasons extremely undesirable. This is why the acceptance of substantiated solutions is especially important in the final stages of development. Nevertheless, it is necessary to accept the solution on the basis of tests of a limited quantity of articles. This means that the informativeness of the tests must be high.

With the probability methods of the evaluation of reliability based on qualitative information, the result of unit testing is considered as a simple event, the evaluation of the probability of which is based on the assumption about the frequency stability of test results. This assumption conditions the indispensable fulfillment of requirements of the identity of tested copies of articles and the constancy of conditions of the conducting of tests. However, these requirements contradict the research nature of the development of the ZhRD, since they eliminate the possibility of the use for evaluations of the reliability of a considerable part of the information.

Consequently, the statistical-probability methods, based on qualitative information, cannot be (without gross deviations) matched with programs of the experimental development of the ZhRD, and requirements of the technical assignment with respect to reliability are not connected with the real possibilities of their confirmation. The methods considered eliminate the possibility of the determination of a specific type of failure; therefore, in the process of development it is not possible to study the features of the design.

The result of tests in the process of design development is not the only one and is not always the main figure of merit of the engine tested. The greater in this stage the failures will be revealed and the more effective the development will be carried out, the more reliable the engine in the process of the operation will prove to be. In examining the problem concerning the operation of engine, the researcher must not be limited to the fixing of results of the tests, but there must be taken into consideration the results of measurements of parameters which characterize the physical processes, systems and test conditions with which the defects and failures, the state of the material, and so on are recorded.

Only this approach to the evaluation of the operation of the ZhRD in combination with the statistical methods, which consider the full volume of information, represents the convincing characteristic of its reliability.

In recent years the reliability theory began to be developed on the joint of two examined directions - physical and statistical.

The task is to formulate the physical bases of the engineering analysis in mathematical form and obtain a quantitative evaluation of the test results. Posed in this case are questions concerning an increase in the informativeness of the tests, a decrease in the quantity of articles, the early development of "weak" places and failures, and the evaluation of reliability indices and their forecasting for prolonged periods of operation. In all the cases these evaluations must concur with the statistical evaluations obtained in the stage of operation according to the results of a large number of natural tests.

To provide a reliability of the ZhRD the following is necessary.

1. Check the operation of the engine with any possible combinations of external and internal factors, which determine the conditions of its functioning.

2. Determine the conformity of the operating characteristics and parameters of the engine to the assigned requirements.

In the solution to this problem in all stages of development, an important place is assigned to the early detection and prediction of failures. Until recently this was done by methods of empirical search. However, the contemporary requirements given to the development of the ZhRD require the development of scientifically substantiated programs with the substantiation of the different plans of experimental works and volumes of tests.

Theoretical bases for this have basically already been developed, i.e., this is the theory of experiment, mathematical modeling, theory of similitude and dimensionality, and so on. Unfortunately, a description of the application of these methods, with rare exception, is isolated in the literature according to different fields of knowledge.

Since the ZhRD is a complex of complex assemblies and systems which are distinguished by the nature and mechanism of the physical processes, methodologically it would be erroneous to be restricted during the planning of its development to any one of the methods. This purpose corresponds to the implementation of technical diagnostics into the practice of the investigation and search of emergency situations of the ZhRD.

The test theory is closely connected with theories of mathematical, physical and structural modeling.

The complex use of basic positions of these theories and methods must be the basis of the theory of the experimental development of complex technical systems. In connection with the ZhRD this theory would correspond to the contemporary requirements for the provision of the assigned level of reliability.

CHAPTER I

GENERAL CONSIDERATIONS WITH RESPECT TO THE PROVISION FOR THE RELIABILITY OF THE ZhRD

§ 1. TECHNICAL ASSIGNMENT FOR THE DEVELOPMENT OF THE ENGINE

The technical assignment from the side of the customer is the initial basic document which determines the complex of operational requirements, design characteristics and operational indices by which must be satisfied by the engine developed. Set forth in the technical assignment is information which determines the ultimate purpose of the engine, its use and operation, the enumeration of the basic parameters, including the kind of fuel, some design features, and finally the period and cost of the development [62].

The basic parameters include the energy and mass parameters: the specific and gross thrust of the engine, the pressure in the combustion chamber and gas generator, fuel component ratios, flow rates per second, characteristics of the turbopump unit, ranges of control, total operating time, cyclograms of starting and shuting down, overall dimensions, mass, butt dimensions, and so on.

The conditions of application are determined by the ultimate purpose (engine of a ballistic missile, carrier of space vehicles, etc.).

Understood by design features are certain assigned (or proposed by the customer) designed solutions, which consider the contemporary achievements of science or the development of this field of technology. Taking this into account, in a technical specification there can be indicated what engine must be (with an "open" or "closed" circuit), the type of suspension of the combustion chamber, the presence of a turbopump unit TNA [THA - turbopump assembly] or of a pressure feed system of the propellant components, the form of cooling of the combustion chamber; the methods of propellant ignition, the ranges of control of the basic parameters and certain other characteristics, for example, reusability, the interconnection with other systems of the complex, and so on.

The performance properties of the ZhRD are determined by the following factors:

- convenience of installation;

- the possibility of periodic inspections of a technical state;

- conditions of storage and transportation;

- temperature and humidity conditions in operation;

- temperature and pressure of the operating components during storage, servicing and in the prelaunch period;

- the maximum permissible difference in temperatures of the components;

- the maximum time of contact with the components or with their pairs;

- the service life for the varied conditions of operation;

- the levels of vibrations, excess pressure and short-term increase in temperatures;

- impact loads;

- axial and radial accelerations, etc.

For the ZhRD of carrier rockets of space vehicles, the operating requirements should be related to levels of solar and cosmic radiation, the effect of conditions of weightlessness, the cyclic recurrence of the change in operating modes, active and passive phases of the flight, the trajectory correction of the flight, and so on.

In the final development stages and with deliveries of the finished products, the technical specifications are one of the basic documents for the solution to the problem concerning the quality of fulfilment of the order and concerning the degree of its conformity to the established requirements. In this case the technical specifications are the basis of the developed technical and operational documentation. In a number of cases it can be supplemented by individual particular specifications for completion elements and systems, assemblies of automation, instruments, sensors, and so on.

§ 2. TECHNICAL DESIGN

The technical design is developed on the basis of the technical specifications, preliminary design data in the form of the advance design, theoretical and experimental research of the SRI [НИИ - Scientific Research Institute], the design bureaus, and a priori information about product-prototypes.

The technical design must contain:

- a list of specifications;

- substantiation of the accepted design and engineering and circuit solutions:

- application with the hydraulic and thermogasdynamic calculations of operating processes, stress analyses of main assemblies and units;

- the selection of the optimum nozzle configuration;

- working drawings;

- the technological and operational and technical documentation, which includes a description of the design of the article, instructions on tests and practical use.

The basic figures of merit of the implemented technical design for the engine can be considered the following characteristics:

- the level of operational reliability;

- the value of specific thrust;

- the specific mass of the engine (as the ratio of mass of the ZhRD, filled with propellant components, to thrust in a void);

- the cost of development.

All of them, to a sufficient degree, depend on one another. Thus, for instance, attempts to increase the combustion efficiency of the propellant or combustion chamber pressure for the purpose of an increase in the specific thrust, to a certain degree, can lead to a reduction in the reliability. An attempt to insure reliability by an increase in strength of the assemblies makes the mass characteristics worse. In turn, the achieving of this goal by applying more ideal structural materials will lead to an increase in the cost. In no less degree the reliability and cost prove to be interrelated [53]. It is quite obvious that a continuation of the experimental development of the engine for the purpose of increasing the achieved level of reliability leads directly to an increase in its cost. On the other hand, a decrease in the cost because of the volume of test is connected with a lowering in the reliability level.

With all this the indicated requirements depend directly on the ultimate purpose of the engine. The requirements for the minimum cost, maximum specific thrust and low mass, to a certain degree, are general. The value of specific thrust and mass of the engine determine the maximum flying range (or payload). However, the very values of their derivatives in terms of distance depend on the specifications and especially the purpose of rocket stage. Thus, for instance, for an intercontinental ballistic missile the reduction by one percent of specific thrust or an increase of several percent of the mass of the engine of the first (lower) stage for the purpose of increasing the reliability will not lead to any essential decrease in the flying range.

At the same time a change thus in engine performances of the upper stages already plays a significant role. The higher the stage, the greater the value of partial derivatives.

In each specific case the selection of the examined engine characteristics in the technical design must be substantiated by the solution of the particular problems in optimization.

The optimization and normalization of the reliability of engines of lower and upper stages must consider features of the launching structure, operating characteristics of the article, including the safety of its use. Thus, for instance, the reliability of first stage according to these considerations can be higher than the reliability of the upper stages.

§ 3. CLASSIFICATION OF SYSTEMS AND ELEMENTS OF ENGINE INSTALLATIONS

The programs and types of tests must maximally consider the physical and design features of the systems and elements.

Understood by the system is the complex of interacting elements which are found in a functional interconnection and are considered as the simplest links in the structural scheme of the reliability of the article. Some systems can be mounted from the standard units (elements), which, in turn, can be independent or correlated with one another. For the newly developed mechanical systems, such possibilities, as a rule, are limited. However, the use in pneudraulic systems of DU [ДУ - engine installation] of separate standard elements of the type of pressure relay, electropneumatic valves, etc. is not excluded. The specific feature of the considered rocket systems is virtually always cross-correlation between the elements, which creates definite difficulties in their autonomous development.

Tests are considered to be autonomous if separate elements of the system are tested. But in a number of cases when the complex integrated system consists of less complex systems, then tests of the latter are also autonomous. Thus, for instance, the engine installation, which consists of several independent ZhRD, can be considered as an integrated system and the separate ZhRD - as its self-contained units. In this case tests of the ZhRD are autonomous in relation to the DU. The tests themselves of the DU in relation to the ZhRD will be complex. This does not exclude the possibility of conducting complex tests of the DU in the composition of other more complex systems as, for example, a rocket.

From this point of view, by the simplest condition of the division of systems into elements one should examine the structural scheme of the reliability or design concept of the article. They are both interconnected.

Both the systems and elements as a whole can be subdivided into systems and elements of single and repeated action.

The technical devices of single action cannot be included in the operation twice, since the processes which occur in them are irreversible. The number of such devices can include the different pyrotechnic means, explosive diaphragms, sheared stop elements, explosive bolts, and so on. In connection with the DU, the elements of single action are certain assemblies of automation, means of the control of starting and stage separation, and so on.

The basic assemblies (combustion chambers, turbopump units, gas generators and others, including such elements of automation as electropneumatic valves, regulators, throttles, etc.) are devices of repeated action.

There are also the type of combined mechanisms which combine in themselves functions of single and repeated action.

Although the tests of elements of single action are characterized as destructive tests, nevertheless, many parts remain suitable for further use and can be used in reliability tests, tests up to failure, etc.

An example of systems of single action can be the design of a fuel tank with a pressure feed system of the propellant components (Fig. 1.1). The internal volume of the tank is divided by an elastic diaphragm into two hermetically sealed cavities, the right one of which is filled with liquid component, and the left is filled with compressed gas. The operating principle is the following.

With the displacement of propellant component from the right cavity of the tank the diaphragm is deformed and consecutively occupies positions 0, 1, 2, 3, and 4. As the final result it lies on the internal surface of the right hemisphere.

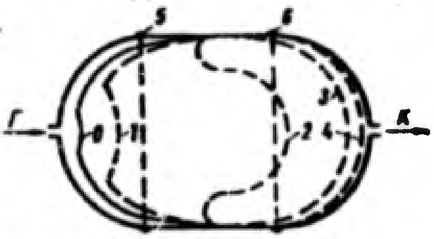


Figure 1.1. Tank with an elastic separator diaphragm:
Г - compressed gas; К - liquid propellant component; 0, 1, 2, 3, 4 - successive positions of the separator diaphragm with the emptying of the tank; 5, 6 - weld seams.

Thus in this system the elastic diaphragm is the element of single action. The hemispheres of the tank can be used several times, since within limits of a change in the operating pressure they retain their efficiency. However, in a number of cases they can be subjected to tests up to their breakdown.

For this purpose after the full displacement of propellant component from the tank, in the gas cavity it is necessary to continue to increase the pressure. Upon reaching the tensile strength, the walls of the tank are destroyed.

In designing the calculation of the diaphragm is conducted in such a way that in the process of its operation up to the moment of total displacement of liquid from the tank its elasticity would be retained because of strain and fluidity of the material.

In the structural scheme of the reliability of the tank, the weakest link, in all probability, will be the diaphragm. Tests for the reliability of the diaphragm will be examined below.

§ 4. GENERAL REQUIREMENTS FOR TESTS AND PROGRAMS OF DEVELOPMENT

In the reliability programs of complex technical systems (and ZhRD related to them), the tests are the most important part, which requires the maximum contribution of economic, material and physical expenditures.

The tests should be considered as the natural continuation of design and research works, which are concluded with the creation of experimental models. At present the level of the accuracy of the design works and theoretical calculations from the viewpoint of the evaluation of the physical functioning of the ZhRD does not make it possible to eliminate testing from the general plan of operations with respect to the provision for reliability. The efficiency of the ZhRD is estimated only on the basis of the test results.

The first stage of the tests is the designer's development of experimental models, which has as its goal the refinement of the specifications and the selection of the regular version of the design. Then there follows the finishing of the regular version of the article and the evaluation of its operational and technical characteristics. The development of the complex technical systems is completed by government tests [57, 53].

The task of the planning of experimental works and development of test programs includes the maximally full development of potential possibilities of the design and the elimination of its "weak" places. In this case all efforts of the experimenters and designers must be directed toward the possibility of an early failure prediction of the developed articles.

The tests are a unique base for the introduction of design modifications with respect to the revealed defects and malfunctions. In this case the earlier they are detected, then in a more favorable position the project will prove to be. The introduction of design modifications in the process of the operation is completely inadmissible according to the many obvious considerations. For this reason the earliest diagnostics of malfunctions and an evaluation of reliability before the completion of the designer's development of the ZhRD become a task of paramount importance. In light of this, the entire importance of the development and acceptance of rational programs which correspond to the given requirements becomes obvious. This task is considered by the specialists as being virtually the most important [62], and, therefore, for its solution there must be enlisted the most qualified colleagues of the KB [КБ - Design Office], SRI, experts and testers, and also design-developers of specific systems.

The experimental program must contain the following basic information:

- the purpose of the test, the object and place of the test work;

- the type of bench equipment, the composition of the measuring and recording equipment;

- factors (parameters) which determine the test conditions;

- laws of the change and levels of the variation of factors;

- the type and site of installation of monitoring sensors on the article, and the methods of their attachment;

- periods of the measurement of parameters in the process of testing;

- methods of the decoding of the recordings of parameters and the processing of experimental data;

- methods of flaw detection of the material after a test, etc.

In the process of the tests a large complex of technical and organizational problems concerning the provision for a high quality of the development is solved. In a number of branches of industry the designer's development of technical systems is carried out not only with special experimental offices but, mainly, at commercial plants under the condition of their sufficient manning by technical and engineering personnel and the most active participation of the experimental and design offices [57].

Sometimes the tests are begun even in the early planning stages when the designer by empiricism fills the absence of the theoretical data necessary for calculation or checks their reliability.

From the viewpoint of the planning of development, the tests are subdivided into several stages [62]. The initial stages are characterized by laboratory and experimental studyings, and the final stages - by a demonstration of the reliability under the assigned operating conditions with the necessary confirmation of the conformity of the operational and technical characteristics of the article to the technical specifications for development. The juridicial completion of the development should be considered as a confirmation of the assigned levels of reliability and outputs characteristic of the ZhRD according to results of all types of tests.

Since the quantity of tests of the articles under actual conditions of operation is very limited as a result of their high cost, the launching failures and emergency results lead to an additional increase in the expenditures; therefore, it is necessary to attain a maximum effectiveness of the ground development. The latter is provided by the simulation of conditions of full-scale tests and an account of the effect of all operational factors which condition the certainty of the characteristics of reliability during the bench development of separate elements and systems.

The solution to the problem is achieved by the development of experimental programs of ground tests, the basic goal of which is the appearance of all possible forms of malfunctions and failures and the elimination of them by means of the modification of design and perfection of the production processes. In the development of aircraft engines this consists of the basic goal of the modification tests [1].

Less substantiated is the statement of problem which places as the purpose only a confirmation of the efficiency of the accepted version of the design, since it proceeds from the condition of the demonstration of reliability of the ZhRD and not from the viewpoint of the determination of its "weak" places. A striving for the achievement of success can be created by a tendentious subjective approach to the evaluation of the test results. In this case sometimes the serious malfunctions and failures can be explained by reasons not characteristic to this design.

With ground development the tendency to the appearance of failures of elements and systems by means of the making rigid of the possible operating modes and test conditions, taking into account the repeated use of the materiel part, contributes to the creation of highly reliable engine installations, in a number of cases even in shorter periods with a less amount of expenditures.

At present abroad ever greater attention is given to the design and development of ZhRD of repeated use for those stages of rockets of space carriers and vehicles which are returned to earth. The guaranteed service life of such engines is no longer limited to single startings and several minutes of operation. Thus, for instance, the oxyhydrogen ZhRD of the Aerojet General firm with a thrust equal to 230 tons ($0.225 \cdot 10^7$ N) is designed for repeated use with the number of switchings on of not less than 500 and the total useful life under operating conditions on the order of 10 hours [92]. In this case the possibility of repeated loadings of the stage with propellant components is provided.

This imposes its features on the methods of development and the types of tests themselves. The number of such features includes the cyclic mode of operation of the engine under terrestrial conditions and conditions of weightlessness; positive and negative axial g-forces (connected with acceleration and braking of the stages upon the return to earth); the admissibility of impact loads which appear at the moment of landing; the cyclic recurrence of a change in the temperature conditions for some design elements in very wide limits (from high temperatures of operating conditions to temperatures of the initial components) and so on.

No less important and complex in this case should be considered the provision for the assigned service time, including the operating time under operating conditions. If for the ZhRD of ballistic missiles these requirements do not cause any special difficulties, then for engines of repeated use they are already basic. Taking into account the noticeable effect of the start-up conditions and the periodicity of switchings on the total technical service life, the programs of service-life tests must also provide such conditions of the engine operations in which the quantity of startings and cyclic recurrence of the change in the modes would be maximum.

The requirements for making the operating modes rigid are general for the bench and flight tests. However, if bench tests can be carried out under different maximum and boundary conditions and modes which exceed those which are assigned, then the flight tests, as a rule, are limited by maximum levels of the basic parameters permissible in technical specifications and records. The making rigid of the engine operating modes during flight tests is more expediently achieved as a result of the combination of limiting values of different factors.

In the process of the ground development by means of the realization of a different form of programs from the materiel part of the experimental models on each assembly, unit and ZhRD, as a whole the maximum of information about reliability must be extracted.

In the rational program not one element of the design which preserved the efficiency must be included in the number of articles not suitable for further use.

In summation, the basic requirements for the optimum program of the carrying out of the firing tests of the engine in the final stages of development can be formulated.

1. The conditions and operating modes, including bench equipment, must be maximally approximated to the actual conditions of operation.

2. The test program must provide for modes which consider not only the nominal but also maximum levels of the variation of external and internal factors and also their most adverse combinations.

3. In the process of the tests there must be conducted a variation in temperature of the propellant components and design, inlet pressures, some internal factors, programmed reversal of controlling elements and control drives of combustion chambers, including permissible deviations in the chemical and mass composition of the propellant components.

4. The measuring system must provide monitoring and recording of virtually all the basic parameters and levels of the significant factors with the necessary accuracy.

5. The program must not allow the conducting of the next testing without a decoding and analysis of results of the foregoing experiments.

6. The volume of the tests must be sufficient for the single-valued evaluation of results of tests and confirmation of the assigned level of reliability.

7. The program must provide for the maximum use of the materiel part.

8. The planning of the tests, the program of operations and the processing of experimental data must be based on methods of mathematical statistics, factor analysis and the theory of reliability.

9. Program must provide means of a further increase in the informativeness of the tests.

The experimental and design tests are not any isolated stage of the creation of a technical system; they pass as a steady source of reliable information through all periods of development, production and operation.

§ 5. REQUIREMENTS FOR BENCH EQUIPMENT

The realization of a complex experimental program requires complex bench equipment and the presence of an ideal system of telemetering control, including metering equipment. The development of the starting and switching off of ZhRD of high-altitude stages requires the creation of such bench-test systems as high-altitude chambers with knock-out plugs, vacuum systems in the form of ejectors, etc. For high-thrust engines these installations must possess large overall dimensions at sufficiently high power or productivity of the power systems. For these purposes, in the USA (at the Arnold center), underground high-altitude chamber whose depth is 76.2 m and diameter is 30 m is built, and it ensures a rarefaction which corresponds to conditions at an altitude of 38 km [95]. The high-altitude chamber was used for the development of the J-2 engine (Rocketdyne) intended for the second stage of the carrier rocket Saturn-5. At the same time the Boeing firm created a vibration table for the testing of the completely assembled rockets and stages [82], and at Edwards A.F.B. (USA) there is a stand for firing tests of stages S-1C of the carrier rocket Saturn-5 [91] with the thrust of the DU equal to 3400 tons ($0.333 \cdot 10^8$ N) during a flight operating time of approximately 150 s.

The purpose of the ground-based test can be formulated as a provision for the required levels of reliability of the ZhRD by the most economically available means, which is reached by the partial replacement of full-scale flight tests by bench tests.

Virtually this means that results of the bench tests can be reliable only with their full conformity to flight conditions. For this static and dynamic similarity or the simulation of conditions of flight tests of articles on a stand must be observed. This problem determines the basic requirement given to bench equipment.

By examining the dynamics of the ZhRD in the pre-operational period, with launching, start, in flight under operating conditions and with switching off it is possible to arrive at the conclusion that each of these systems is characterized by a complex of determined factors. Thus, for instance, the pre-operational period and starting of the engine are characterized by a temperature and humidity mode and ambient pressure, which are determined, basically, by atmospheric conditions or the microclimate of the firing structure.

These factors have maximum and minimum levels assigned in technical specifications and records. Similarly the temperatures of the propellant components and design are established. In turn, the pressure at the engine inlet in the pre-operational period varies in accordance with the boost pressure of the tanks. The order of the passage of instructions and functioning of elements of automation occur depending on the cyclogram of the start.

After the opening of main valves there occurs motion of the propellant components along the internal engine communications, caused by pressure differentials which are determined, mainly, by characteristics of the feed main lines (their length, cross section, local losses to hydraulic resistance, and vibrations) and the boost pressure of the tanks. After the start of the engine axial overloads appear. The latter, in turn, render influence on an increase in the inlet pressures before pumps.

The test stand for conducting firing tests of a ZhRD must have a stand with a thrust-measuring device, a centralized system of the feed of propellant components with fittings and meters of flow rates per second and a control panel with monitoring-measuring equipment and registering equipment.

The basic requirements of the test-stand equipment can be the following:

- the provision for the identity or for a dynamic similarity of the system's characteristics of feed of the engine with propellant components, including the conformity of the inertia, wave and hydraulic characteristics of the feed main lines;

- the provision for the conformity of the laws of a change in the inlet pressures into the engine, pressures in the combustion chamber and gas generator, the time of the approach or entrance of the propellant components to the main units;

- the provision for a change over wide limits of the temperature of the propellant components and design, inlet pressures, control of flow rates per second and thrust;

- for engines of the upper stages the achievement of the degree of the conformity of high-altitude conditions (rarefaction of the surrounding medium, the absence of a convective exchange between the structural parts and the medium, the effect of solar radiation);

- the provision for the identity of the passage of instructions with time;

- the use of the design of mounting points of the engine and its units which corresponds to the regular;

- the convenience and possibility of conducting tests during as short periods as possible;

- the efficiency of the tests;

- the maximum automation and simplification of the test program;

- the presence of systems of noise suppression, neutralization and industrial flows;

- the presence of measuring and recording equipment of one type.

The last requirement is for the elimination of the possibility of the appearance of errors in the comparison and determination of the controllable parameters of the ZhRD because of the diversity of the equipment, the distinction in the characteristics of the sensors and procedures for their conversion.

As a whole the test stands for the conducting of firing tests must provide the possibility of the development of ZhRD of different design, which are similar in thrust and operate on similar propellant components. Test storage tanks of components must provide the test operation of ZhRD during several assigned service lives of operation without refueling.

The fittings of test stands must allow emergency engine cutoff and the cessation of feed of propellant components with several autonomous panels duplicated with each other and located in different rooms connected by means of communication. At the same time it is necessary to have an automatic system of emergency cutoff which reacts to a sharp increase in the temperature in the test bays, to a pressure drop in the feed lines and so on.

As a whole the test stand must satisfy all requirements of safety engineering and have means for fire extinguishing, sanitary service, decontamination equipment, and so on.

The test stands used for the development of the ZhRD are universal, while test stands for the monitoring of the testing of commodity articles are special. In this case they are tuned to definite dynamic characteristics of the specific engine.

The abundance of programs with experimental developing requires different technical equipment and the creation of special test stands. Some of them can be utilized, let us say, only for flow tests of systems and various kinds of hydrotests; others - for the autonomous test of combustion chambers; the third - for autonomous test of ZhRD in ground conditions; the fourth - for the same test but already under conditions of high vacuum or weightlessness; the fifth - for the testing of engine plants which consist of the clusters of self-contained units of ZhRD; and the sixth - for the testing of stages of rockets, and so on.

As a result the same engine can simultaneously undergo tests on several test stands. Thus, for instance, the development of the engine F-1 of the carrier rocket "Saturn-5" was conducted in practice simultaneously approximately on 10 test stands, 3 of which were utilized for the testing of the DU in the composition of the stages.

Depending on the thrust vector angle to the horizon, the test stands can be vertical, inclined and horizontal.

The degree of perfection of test stands is characterized by the material equipment, thrust-weight ratio, protection from possible explosions of the engines and propellant components, the composition of the monitoring-measuring equipment, and the time of preparation and conducting of firing tests, including restarts. The higher the material equipment of the test stands, the more informative the results of the testings and the more effective the ground development.

§ 6. INFORMATIVENESS OF THE TESTS

The information obtained in the process of the tests can be of two forms.

1. Quantitative parametrical information about the physical characteristics of the article and the technical state of its

elements. The primary source of this information consists of the measuring meters for the fixing of the established or rapidly changing processes. They include various kinds of sensors for the measurement of pressure, flow rates per second, temperature, number of revolutions (rotation frequency), levels of liquids, and all possible sensors of the inspection of the passage of different instructions. This information from the test stand or the side of the article is transferred to the ground memory devices using the wire or radio-channel system of telemeterings. With respect to total volume the parametrical information is the most considerable. It suffices to give an example [80] when during the flight test of the carrier rocket "Saturn-5" only from the first stage S-1C the telemetry data from 890 parameters are taken and transmitted to earth by means of 12 channels with interrogation frequency of 12 to 120 Hz. This makes it possible to conduct virtually a continuous recording of all basic parameters of the DU, eliminating the rapidly changing processes and high-frequency oscillations. As required their recording can be conducted during static tests.

This volume of information gives virtually a full representation about the physical processes and technical state of the article during its functioning at any moment of time, including at the moment of the manifestation of a failure, including the prehistory and aftereffect. This is very important in the analysis of the reason for failure and the entire emergency situation. An exception, of course, is the cases when in the place of the manifestation of the failure the inspection measurement was absent or its reliability was low.

However, in the quantitative evaluation of reliability use of the indicated volume of information is difficult.

2. Qualitative and quantitative characteristics in the form of the frequency of operation without breakdown, time of operating time between failures, and so on.

It is completely obvious that the parametrical and quantitative characteristics possess considerably greater accuracy than the qualitative. The latter are reliable only with a large number of tests. By knowing the accuracy of measurement instruments, it is possible to estimate sufficiently accurately the parameters literally according to unit tests.

Thus, if the numerical value of the quantitative factor makes it possible to establish its level sufficiently accurately, then the qualitative factor is characterized only by the evaluation "yes-no", "success-failure", and so on. Nevertheless, for the majority of the systems and elements which do not have telemeterings, the qualitative information is basic form in the statistical evaluation of the reliability indices. Let us examine this question in somewhat more detail.

The reliability indices can be determined only from the test results. Some of them can be estimated according to results of the short-term service-life, inspection, standard, acceptance and other types of tests. But there are characteristics connected with the evaluation of guaranteed service lives and durability which require the conducting of prolonged expensive tests, for example, up to breakdown. The actual level of the reliability of the articles, which considers the effect of all operational factors, can be estimated according to results of the operation under actual conditions. This makes it possible to determine the composition of malfunctions and failures, guaranteed service life, structural unfinished products, deficiencies in the technology of manufacture and operation. All this requires prolonged periods of operation, tests and high material expenditures. In order to determine the indicated characteristics in advance, even at the stage of the experimental development, the conducting of tests which simulate the conditions of a full-scale operation is necessary.

Of great significance also is the well-adjusted system of organization and account of the statistical data. Determined by it are forms of defects and failures and quantitative indices according to characteristics of reliability, and technical measures, plans of experimental operations and a test program with respect to the provision for reliability are developed. An important role in this is played by the very methods of the evaluation of reliability, since they define the composition and volume of information. Its task consists of the collection, classification, generalization and processing of statistical data. The information must enter in proportion to the manifestation of failures or through strictly defined time intervals. The information must objectively and fully reflect all facts of the manifestation of the failure.

The difficulty in the development of a system of the information about the reliability of articles entails the diversity of information about the nature, conditions, reason for the emergence of the failure, the mean time between failures for the given moment of time, and so on. The operation time for the different elements is determined by the nature of their functioning. For elements which operate in the service-life mode, the operation time is determined by the time of operation without breakdown. For the elements which function in the mode of the switching systems, it is characterized by a quantity of cycles up to failure. For articles which operate in modes of multiple switching-on, the accumulated operating time is determined by the number of cycles and operating time in the steady-state conditions.

The characteristic of the operation time is a very important criterion which determines the physical process of operation of the article with which the failure was revealed. An incorrect selection of this criterion, as a rule, leads to the acceptance of erroneous solutions. Value of operation time as the reliability of the articles is determined by design perfection of the articles, the technology of the manufacture and test conditions. This last factor, sometimes when evaluating reliability, is not given the proper attention, which unavoidably leads to miscalculations and errors. Sometimes, in totaling the failures, they are not differentiated depending on test conditions. For example, it is possible to sum up the failures which were revealed in tests of articles, carried out, it would appear, in virtually identical conditions, with the exception of the ambient temperature. Subsequently, after prolonged operation it was found that, namely, the temperature is the decisive factor which determines the limits of operation of the system.

Consequently, in the processing of statistical data the generalization of information about failures caused by different conditions is inadmissible. This is possible only when by results of the tests themselves it is proved that the factors which determined the distinction in the conditions do not lead to a change in the failure rate.

For the merit rating of the test results, usually used are different statistical methods based on the fixing of parameters of the article, defect levels, and so on.

There is a certain interest in this respect in the statistical method, which is based on the summation of failures in the process of the development of the article not allowing for design distinctions in its separate modifications. Noted on figure 1.2 along the axis of the ordinates systematically according to the measure of the storage of experimental data is the failure level m_0, and along the axis of the abscissas - the number of tests N. Obtained as a result is the discrete distribution function of failures depending on the volume of the tests and degree of input into the article of design changes. As a whole this dependence reflects the effectiveness of the development and can be divided into several stages, each of which characterizes a change in the reliability of the article in proportion to its modification with a sufficient number of experiments, which confirm the effectiveness of the accepted measures. On figure 1.2 there are two such stages -

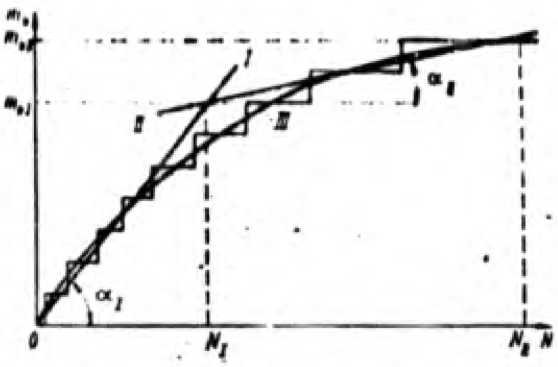


Figure 1.2. Dependence of the failure level on the number of tests in the process of the development.

I and II. Stage I corresponds to segment (N_I-0), and stage II - (N_{II}-N). Each of them has an intensity level of failures defined by the tangent of the corresponding slope angle (α_I or α_{II}):

$$\mathrm{tg}\,\alpha_I = \frac{m_{0I}}{N_I};\ \mathrm{tg}\,\alpha_{II} = \frac{m_{0II} - m_{0I}}{N_{II} - N_I}.$$

At the moment which corresponds to the point of intersection of straight lines I and II, introduced into the design or technology of the manufacture of the article were changes which, from the viewpoint of an engineering solution, can be estimated as being effective. A further analysis of these solutions makes it possible to insure an increase in the reliability. If $\alpha_{II} \geqslant \alpha_I$, then this can be considered as a symptom of the ineffectiveness of the produced modifications or errors and miscalculations, as a result of which the designer and experimenter proved to be on the wrong track.

This method of the processing of statistical data by itself is not new, but its use is practical both for the evaluation of the effectiveness of the modifications and systematization and generalization of the experimental data. When evaluating the effectiveness of the modifications it proceeds from the qualitative information and does not consider the quantitative and parametrical information; and therefore, this method is not operational. Its accuracy depends on a large volume of tests. When using quantitative information the problem of the evaluation of the effectiveness of modifications can be solved with a less quantity of expenditures and during a shorter period. One of the possible means in the solution to the problem with the use of quantitative and parametrical information is the method of the construction of mathematical models of the investigated processes, which will be examined below.

Returning again now to Fig. 1.2, let us focus attention on the fact that the obtained experimental dependence actually symbolizes an "increase" in the reliability of the engine in the process of its development. In this case we have in mind the clear interconnection which exists between the quality of the development and the quantity of the conducted tests. Thus, for instance, characteristic III is in a certain kind of theoretical dependence, which establishes the functional interconnection of the number of failures and quantity of tests of the specific model of the engine, not allowing for knowledge of the general form of the distribution law.

By knowing both these indices, it is not difficult to construct the dependence of the probability of failure-proof operation P on the number of tests N, utilizing the relation

$$P = 1 - \frac{m_0}{N}.$$

This dependence outwardly will not differ from characteristics of Fig. 1.2.

Virtually during the entire period of the development the reliability level is changed and is characterized by a gradual increase in the reliability index. Therefore, in a number of sources [53, 70] the broken line of discrete form is replaced by the exponential curve

$$P = 1 - ae^{-b(N-1)},$$

where a and b are coefficients which characterize the results of the tests.

The entire complexity of these solutions lies in the fact that the coefficients a and b, as a rule, are unknown, and to determine them accurately by a limited quantity of tests is virtually impossible [70]. Moreover, the known methods of the calculation, based on the regression analysis and the method of least squares, are assumed to be the stochastic independence of the observations, and since in our case the observations (or test results) are dependent on each other, then the non-fulfillment of the known assumptions leads to additional errors. A partial solution to the problem can be found in work [53], where it is proposed to divide the entire process of the experimental development into several intervals, each of which is characterized by a relatively stable test program. According to the results of these tests, for each interval the point evaluation of the reliability and the confidence interval corresponding to it is determined.

The informativeness of the tests and interval estimates of reliability in the closest manner are connected with each other. The greater the volume of information with any quantity of tests or the higher their informativeness, the narrower the confidence interval and more precise the evaluation.

The quantitative reliability index is the most universal and objective evaluation criterion of the test results. Therefore, its use for determining the degree of perfection of the development of the engine in different stages of the experimental works is very desirable. At the same time, if the time of test work is limited and small models are accepted, then in the implementation of a separately taken program such evaluations cannot always be conducted. Therefore, in practice in the implementation of each program it is necessary to approach the expansion of the volume of tests, without increasing in this case the sample size of the experimental models. This makes it possible to raise considerably the informativeness of the tests as a result of an increase in their duration, cyclic recurrence of the modes and expansion of the ranges of variation of the factors which determine the test conditions.

On the basis of an analysis of test results according to the output parameters which characterize the physical process, the informativeness of the tests can be increased by an increase in the accuracy of test measurements and their reliability, and also by an increase in the number of points to be checked.

§ 7. PLANNING OF THE DEVELOPMENT

The plan of experimental works is composed by taking into account the staging nature of the development, scales of the test production, productivity of the test bench equipment, assigned periods of development, cost of the tests and other material expenditures. The basic nucleus of this plan is the totality of experimental particular programs, and the output criterion of quality of the plan should be considered the achieved level of reliability of the developed engine.

Taking into account the above named factors, the plan for experimental works as a final result is determined by the volume of all types of tests in accordance with the established stages of development. The development of the optimum plan is a rather complex problem, since many factors not only cannot always be estimated quantitatively but also determined qualitatively. The problem is aggravated still by the fact that the basic element of the plan, the program, differs by the great variety in quantity, types of tests, duration, and volume of works and on expenditures.

From the viewpoint of an analysis of programs of the concluding finishing tests and reliability tests, it should be pointed out that this stage is the most laborious and is considerably more expensive than quality control with serial production. However, it is necessary, since the consequences of erroneous solutions, permissible when selecting a regular design, as a rule, prove to be very difficult. For this reason the output quality control cannot replace the previous stage of the finishing test, which thoroughly determines the quality of the design and accepted technology of production.

Actually the plan for experimental works coordinates the staging nature of the development of the engine depending on the factors enumerated above and determines the periods of realization of the particular programs. As is known, the purpose of these programs in connection with the accepted version of the design is the determination of the tactical-technical characteristics of the article and the determination of "weak" places of design and also the technology of production.

For each model the majority of the tactical-technical characteristics is determined unambiguously in the process of the tests, and their spread for this engine model is more precisely formulated with the processing of the statistical data.

The appearance of the "weak" places is connected with the random appearances of the failures. This indicates the qualitative or quantitative determination of the reliability of the design from test results of the limited selection of articles within limits of the specific program.

Let us assume that the program determines a certain set of conditions of the tests. It is required on the basis of experimental data to determine if the engine under given conditions is efficient.

It is quite obvious that it is possible to answer this question unambiguously, having estimated the reliability quantitatively. But if the reliability levels are assigned high, then the volume of the tests must respectively be great. But since there is an abundance of programs and specific problems in parallel to the considered program, the total volume of the tests increases in proportion to their number.

As a whole the realization of the dozens of programs on a specific engine amounts to hundreds and even thousands of bench tests. In proportion to this the expenditures grow, and we must strive for a decrease in this and, consequently, also an approach to a decrease in the volume of all works.

Taking into account that which was given above, the determination of the volume of experimental works comprises the basic goal of the developed plan.

In examining this question, let us say that in the absolute majority of the cases after successful test work the materiel part of the experimental model retains its efficiency (an exception, of course, must be considered assembly tests up to destruction, etc.).

This explains one very important requirement for the plan: the repeated use of the materiel part in the process of the realization of one program. Virtually this means the successive test work up to failure, as a result of which in the process of the realization of one program the characteristics of the mean time between failures can be determined. In turn, for their effective use the objective distribution laws, taking into account the statistical and physical features of the specific design, are necessary.

The accuracy of the statistical evaluations of test results depends on how the accepted law corresponds to the true distribution of failures. The question concerning the properties and advantages of those or other laws will be examined in detail by us later in Chapter V. In this section let us say that the use of the exponential distribution has a limited application for mechanical systems and that the most reliable model, in the opinion of the specialists, is the distribution of Weibull [Translator's note: spelling not verified] [62].

Thus the development of the plan for experimental works must be composed of the following stages.

1. The study and analysis of the tactical-technical assignment for the development of designed and technical specifications and records.

2. The collection and generalization of data on scales of production, characteristics of the available test equipment (test stands and laboratories), the degree of their conformity to the projected program of works, degree of modernization or the development of new forms of equipment, and so on.

3. An evaluation and analysis of the economic indices of the cost of the tests, expenditures for modernization and development of new forms of bench equipment.

4. Determination of goals and problems of experimental research in each particular program.

5. The development and agreement of programs for all types of tests of elements and systems, including the autonomous development of the assemblies and estimated, finishing and qualification tests in connection with the existing bench equipment [62].

6. Determination of the volume of tests for each of the programs and quantity of the expendable materiel part of the elements and systems, taking into account the optimization of all characteristics of the developed plan.

The particular programs are developed on the basis of factor plans, taking into account the specific conditions. With this plan of experimental works, the "flexible" reaction to test results must be provided, which makes it possible in proper time to reduce or expand the volume and program. A decrease in the volume of tests can be caused by the premature achieving of the goal, and the expansion, for example, - by the introduction into the program of new factors, and so on.

As a whole the plan for experimental works must serve the goals of the provision for the assigned level of reliability with the optimum means and during the assigned periods.

CHAPTER II

EXPERIMENTAL DEVELOPMENT OF THE LIQUID-PROPELLANT ROCKET ENGINE. STAGES AND TYPES OF TESTS

§ 1. GENERAL CHARACTERISTICS OF THE TESTS

The reliability programs of ZhRD [ЖРД - liquid-propellant rocket engine] differ by great variety. The realizations of such programs should be considered as tests which, from the viewpoint of the object of their conducting, are divided into two basic forms: bench and flight tests. However, from the viewpoint of volume both these forms cannot be considered equivalent.

Bench tests of ZhRD and their assemblies include the entire series of problems connected with ground development, the provision for reliability and output quality control of the commodity articles.

Flight tests are, first of all, the complex testing of the efficiency of all rocket systems. Correspondingly, the cost of one complex flight test is many times higher than the cost of bench test of the separately taken autonomous system of the ZhRD. Therefore, the total amount of flight tests consists of altogether only a part or several percent of the total number of bench tests [83, 86].

Since the bench tests are the realization of a large enumeration of programs, they are divided into several stages, conferring a definite ultimate purpose to each stage.

The content of each of the stages of the ground development will be examined in § 2, 3 and 4.

The ZhRD must possess sufficiently high reliability, even when systems of redundancy are present, since its reliability determines the reliability of the rockets, carriers and space vehicles, and the life of the firing structures.

In the course of bench tests, solved virtually completely are questions of the provision of a reliability of design and developed systems under conditions of flight and operation, which, as is known, are the totality of combinations of various kinds of factors, which cause design, technological and operating characteristics of the articles. Their large number and diversity create the need for developing a complex of programs for ground and bench development, which require the simulation of all actual conditions of operation, including flight. Therefore, the creation of unique equipment for the conducting of bench tests, a large quantity of measuring devices and recorders, systems for the servicing and storage of the propellant components, compressed gases, means of protection of service personnel and technology is necessary (see Chapter I).

The degree of the development of the ZhRD and its operational reliability are determined, first of all, by the completeness of the simulation of the actual conditions of operation and rocket flight. This is connected with the need for careful development of the programs, prolonged preparation of the tests, their clear conducting, and the processing and analysis of the obtained information.

Bench tests usually begin from the autonomous development of the assemblies. In this case the efficiency of elements and systems is checked, and their performance characteristics, not allowing for the effect of the adjacent systems of the technical project are determined. After the completion of autonomous development there follow complex tests during which the efficiency and parameters of the engine with the interaction of the assemblies and systems are checked. In this case the degree of the significance of correlation and interaction of assemblies of the ZhRD is estimated. If it is low or does not lead to a disturbance of the operational modes, the effectiveness of the autonomous development will be high. But if the correlation leads to a disturbance of the conditions of normal functioning of the assemblies, the effectiveness of the autonomous development is low. In a number of cases for very dynamic systems, it is equal to zero. This, in turn, requires the conducting of a large quantity of more complex and expensive comprehensive tests. From this viewpoint optimum will be the design solution which ensures the high efficiency of the autonomous tests with the development of the separate elements of the systems.

The developed correlations between elements of the systems virtually cannot be achieved by simulation during autonomous tests.

Taking into account the simulation of the effect of the environment and external factors, the programs of experimental works can differ in the diversity of the technical solutions, the duration of the tests, the levels of variation of the factors, the cyclic velocity of the replacement of systems, and so on. Sometimes a number of specific routines can be realized: reliability tests, a check of the effect on efficiency of a different kind of technological defects, and so on.

The planning of the development requires the conducting of such types of the test which for the specific assignment, in being most informative, taking into account all the expenditures would be as optimum as possible. Taking this into account the test program must involve the analysis of different types of tests with the indication of their advantages and expected results, the evaluation of the optimum periods of the completion of the works and material expenditures. The existing programs most frequently consider the operating reserve and the experiment of the enterprise and not its real possibilities.

Bench tests can be divided into following stages [62]:

I - experimental, estimated tests;

II - tests on the effect of the ambient medium, taking into account the effect of external and internal factors;

III - qualification tests.

§ 2. EXPERIMENTAL, ESTIMATED TESTS

In these tests the efficiency of one or several designed versions of the design of the basic assemblies and the ZhRD as a whole is estimated. The possibilities of the functioning and dynamic interaction are determined, and also a preliminary evaluation of the fundamental characteristics is conducted. In the very initial stage of the estimated tests it is checked to see how this designed scheme or design is realized under the assigned operating conditions. Subsequently, the possibilities placed into project are developed; there occurs improvement from the viewpoint of efficiency, reliability, cost, ease of control, increase in the warranty periods of operation and all the remaining properties and quality coefficients. This development is conducted in the direction of the provision for the assigned operational indices. But usually the designer and customer are not limited to this and carry out tests under more rigorous conditions, and the maximum possibilities of the regular version of the article are determined [62]. The tests are subdivided into:

- hydraulic flow tests and strength tests;

- scavenging by "cold" and "hot" gases;

- short-term repeated and standard tests of elements and systems under laboratory conditions;

- static, dynamic and vibration tests of separate assemblies, subassemblies and articles as a whole;

- monitoring and standard tests of systems on full-scale test stands;

- corrosion, climatic and other forms of tests [62, 57].

Tests of this stage involve the autonomous development of virtually all designed subassemblies and assemblies. Sometimes the exception can be the completion elements and standardized articles when the assigned operating modes and test conditions do not exceed those which are permissible. They include the bearings, gaskets, sealing elements of any types, certain assemblies of automation, sensors, pressure relays, and so on.

Test programs of the basic assemblies are distinguished by a special diversity: the combustion chamber, turbine, fuel pumps and gas generator. The purpose of these tests is also directed at the selection of the optimum characteristics and early diagnostics of the failures. Thus, for instance, the autonomous tests of gas turbines are for selecting the design with maximum efficiency of the calculated carrying capacity of the nozzle cascade. For this aerogasdynamic tests of the flow part are carried out [102], and by these the optimum shape of the blades of the turbine rotor and guide vane of the stator is selected.

Virtually all the structural elements of the assemblies must undergo autonomous strength tests. For closed volumes (casings of pumps and turbines) these tests are static and are accomplished by the method of hydraulic pressure tests. In a number of cases for the definite selections such tests are carried out to failure. Impellers of the pumps and turbine disks undergo dynamic tests on special test stands with increased revolutions. In this case a definite selection of the articles is tested up to failure with the aid of centrifugal forces by the method of a gradual increase in the number of revolutions of the rotor.

An important moment in the developing of programs of autonomous development should be considered the provision for simulation of loads and the achievement of their conformity to full-scale tests of the assemblies under operating conditions. In the process of strength tests the "weak" places of the structure are checked. According to their results the computed values of strength characteristics with the actual are compared, the spreads of the characteristics of strength are established, and solutions of the finishings of the design or a change in the technology of production are accepted.

As a whole the tests are completed by the flaw detection of the materiel part and the analysis of the results.

With the autonomous development of the turbines, just as for the fuel feed pumps, a very important role is played by the methods of physical modeling. For these purposes in a number of cases models of reduced dimensions, which are sometimes made with transparent walls are created. In these cases tests can be conducted in air (for turbines) or on model fluid (for pumps) with the addition of special chemical dyes.

The autonomous development of virtually each new design are preceded by model tests. The purpose of these tests is for the selection of the optimum design and refinement of the rated characteristics.

The autonomous tests of the combustion chamber and gas generator are directed at the selection of the efficient design and as complete a study as possible on the stable combustion zone, ranges of the emergence of low and high frequencies, measurement of amplitudes of oscillations, determination of measures for their liquidation. In this case an evaluation of the region of stable combustion, depending on a change in the different factors such as the combustion chamber pressure, the fuel component ratio and others [16], is conducted.

No less important is the evaluation of modes of cooling of the combustion chamber with different combinations of external and internal factors. In this case the most adverse combination for the chamber should be considered as the minimally permissible flow rate of the cooling component, pressure boosting and correlation coefficient with the minimum pressure differential in the jacketed cavity.

In the process of autonomous tests, more precisely formulated are the basic energy characteristics and their dependences on controlled parameters, external factors (inlet pressures and temperatures) and some internal factors determined by the spread of technological characteristics, for example, pressure differentials on the injection head.

Together with the firing tests, the combustion chamber and its elements undergo hydrotests for the purpose of determining the pressure differentials on lines of the fuel and oxidizer, the evaluation of the uniformity of zones of atomization, the flow rates per second through separate fuel and oxidizer nozzles, angles and quality of the atomization, the distribution of the fuel component ratio over the cross section of the chamber, and so on.

The autonomous finalizing of assemblies by their strength tests is completed. The purpose of the tests is to check for reserves of strength after a repeated static and dynamic load by the pressure of liquid or gas. In this case the question concerning the pressure leak test of cavities and connections, joint welds, and soldered joints is simultaneously solved.

With conducting of the autonomous development of the assemblies, a very important problem should be considered as the maximum approach of the experimental program to the actual conditions of testing in the composition of the article. Since in the autonomous development of the assemblies it is not always possible to achieve this entirely, then the selection of the program must provide for the most critical conditions of its conducting for each specific assembly. Thus, for instance, the automated treatment of the combustion chamber of the engine F-1 of the carrier rocket "Saturn-5" was conducted with the pressure feed system of the propellant components.

The achievement of the identity of conditions and characteristics of the starting, which correspond to the regular ones, is complicated, especially because the normal conditions on this stage of development are still entirely unknown. In such cases it is expedient to derive extreme operating modes which would correspond on the one hand, to very "rigid" gradients of the buildup of pressure and temperatures in the chamber and, on the other hand, to conditions of the providing a protracted starting. In this case the design must satisfy not only the requirements with respect to the provision of strength, but also the stability of the interchamber processes, combustion stability, and so on.

Dynamic and vibration tests are conducted on special test stands. which make it possible to load the assemblies for a long time and change the type of load. Thus, for instance, fuel supply lines, casings of the fittings and pumps and cavities of injector assemblies can be checked for the action of hydraulic impacts. In this case the state of materiel part is recorded, stress and value of pressure at "peak" loads are recorded. The loads themselves can be single, cyclic, and alternating with damping or generatable frequency and amplitude.

Vibration tests are conducted on vibration tables and have as a goal the determination of frequency and form of natural oscillations by the method of the establishing of resonance frequencies. For this, in the plane of vibration and normal to it vibration pickups are installed.

By steady changing of the oscillation frequency the first resonance frequency is located. By the same method all others can be determined. Since the natural vibration frequency depends on the structural esasticity, the analysis of the resonance frequencies and forms makes it possible to establish their physical nature. In accordance with the test results, design finishings can be carried out.

For determining the damping decrement of oscillations, the structure is introduced into the resonance of the appropriate tone, whereupon the oscillator is instantly turned off. In this case the period of attenuation of the oscillations must be minimum.

The experimental and estimated tests are most frequently carried out under conditions of the environment. According to their results theoretical dependences and calculation data, which refer to the system as a whole, are refined. Depending on this, some parameters, the structural solutions of separate elements and subassemblies, while in a number of cases, the fundamental design of the ZhRD, can be corrected. The nature of the experimental tests is search. In the course of their conducting quantitative and qualitative characteristics for different designs and structural solutions are constantly compared. Test procedures, as a rule, are based on an empirical, step-by-step search. The reasonably constructed plan provides for the virtually parallel conducting of test for several versions of the same subassembly or assembly. On the basis of the analysis of the test results the version close to optimum is selected. A large role in this is played by the conclusions of leading specialists, who consider the experience of the enterprise, the SRI and other organizations. In this case it is incorrect to strive for a decrease in the number of initial versions and a decrease in the volume of the tests.

With the completion of the stage of the experimental and estimated works, individual technical solutions with respect to the elements and system as a whole are refined. Cyclograms of starting and shut-off are selected, and the basic parameters, performance characteristics of the elements, the technology of their production, technical specifications and records, methods of reception and input inspection, and so on are determined.

Measures of technical and organization plan are carried out. In the assigned stage, in accordance with formed practice, for example, in the USA, the customer, as a rule, does not participate in the tests. This is explained by the fact that produced in this stage is the selection of the optimum design solutions, but not the evaluation of the technical state of the article and production processes with serial production.

This position is preserved also when the program is financed completely by the customer. After the completion of the stage of experimental-design works the customer can participate in the analysis of the achieved results, since on their basis the solution of the regular version of the design is taken [62].

§ 3. TESTS ON THE EFFECT OF THE ENVIRONMENT TAKING INTO ACCOUNT THE INFLUENCE OF EXTERNAL AND INTERNAL FACTORS

In this stage the design modification close to the regular is tested.

Tests on the effect of the environment in the general plan for the experimental works are distinguished by a variety of programs and types and conditions of the tests. The purpose of the test is the following:

1) determination of the regular design of the ZhRD, its separate subassemblies and assemblies;

2) confirmation of the efficiency of the regular design in accordance with the assigned technical specifications and records and determination of the functional connections between the input

and output characteristics of the ZhRD of both the system and its elements;

3) establishment of operating ranges of a change in the basic parameters for all subassemblies and assemblies;

4) evaluation of the technological process of manufacture of engines with serial production.

The indicated enumeration covers the basic stages of the experimental works and in practice is found in full agreement with the general requirements for test programs on the effect of the environment and various kinds of factors [62]. In accordance with this, the test procedures in each specific case are determined by the technical assignment, are developed by a designer group and agree with the customer. The volume of the tests is determined by the departments of the designer or by the subdivision of reliability. Posed in a number of foreign sources [37, 62] is the question concerning the determination of the volume of tests as functions of the subdivision of reliability - the subdivision of the direct subordination to the leader of the project. The volume of the tests must be found in accordance with the technical specifications for the development of the project and with the requirements for test programs for reliability.

By their nature the tests of this development stage can be the most diverse - from the simplest corrosion tests, conducted in atmospheric conditions, to complex programs when the starting of the ZhRD under conditions of weightlessness or a rarefied medium is checked.

In speaking about tests on the effect of external factors, we have in mind the evaluation of the efficiency of the ZhRD depending on the temperature of the propellant components and structural elements, inlet pressures on the fuel and oxidizer lines, barometric pressure, humidity, and so on.

Tests on the effect of external in internal factors in their volume are the most extensive part of the reliability program. The inclusion into the experiment of those or other factors and their combinations is conditioned by the degree of test readiness of this stage of the evolved design. Depending on the ultimate purpose of the program of experimental works, the test conditions can be natural or those which are simulated. Tests under actual conditions are economically always more expedient than those with the simulation of the environmental conditions and environmental factors. The test programs, as a rule, provide for a check of the efficiency of the design of the article at low and high temperatures. Being guided by economic considerations, it is easy to arrive at the conclusion that tests at low and high values of temperatures of the propellant and design are expediently carried out in the appropriate seasons and, as far as possible, in the appropriate geographical or climatic zones.

In a number of cases the conducting of such tests in the arctic or subtropic latitudes from the viewpoint of economic expenditures, can prove to be more advantageous than tests under conditions of the simulation of the assigned temperature range on a test stand. This is related, first in all, to tests of large-dimension and scale articles. The decisive criterion in this case must be the economic cost estimate of the program of experimental works. It is appropriate to note that by carrying out testing under actual conditions it is not possible to insure the desired values of all factors, especially when the discussion is about the interactions of several factors simultaneously in one program.

Thus the simulation of the effect of external factors with the finalizing of the complex technical systems is an inevitable phenomenon. The simulation of test conditions, as a rule, requires unique equipment: vibration tables, thermostats, cooling installations, altitude chambers, pulsators of pressures, receivers, quick-break switches, vibrators, and so on. It is natural that the cost of this equipment is determined not only by its purpose, but also by the productivity, useful volume and characteristics which are found in direct dependence on the scale and mass of the developed articles and systems.

During the finalizing of the engines a special place is also assigned to their tests for strength and airtightness. Above it has already been noted that such tests are given to separate structural elements with autonomous finalizing. However, from the viewpoint of the system, autonomous tests can prove to be insufficient, since in this case the points of connection of the elements, joint welds, mounts, and so on will not be verified.

Strength tests of closed volumes, as a rule, are carried out by the method of hydraulic pressure tests; however, for checking the strength of open cavities, external suspensions, mountings of assemblies, and means of installation, system tests, as a whole for the influence of a different form of overloads are necessary. Depending on the specific nature of the program and design features, the articles can be subjected to tests for the influence of axial overloads and, in a number of cases, radial overloads. In this case the completely assembled engine as the test object in regular suspension is fastened to a centrifuge [93]. In accordance with program, the closed cavities can be found in a loaded or unloaded state; in other words, these tests can include strength tests.

The most important places of the structure are equipped with telemetric sensors, which measure local stresses, vibrations and other parameters. This form of test of the ZhRD can be carried out on special installations (Fig. 2.1). These tests give valuable information about the strength and rigidity of fastening of the main lines, master instruments and sensors, nozzle, nozzle adapters, automation, and so on. Similar tests were carried out in the USA on the sustainer engine of the "Apollo" spacecraft (Rialto, California) [93].

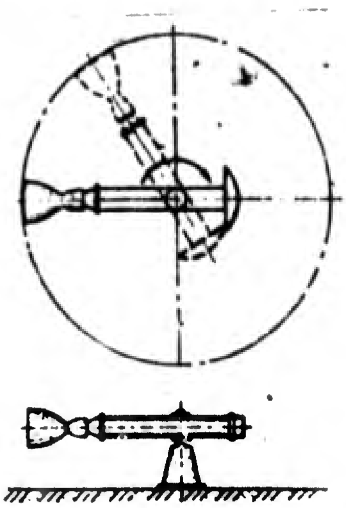


Figure 2.1. Engine testing on a centrifuge.

Fairly complicated in the simulation of test conditions is the provision for weightlessness, determined also by external and internal factors. The inclusion of these factors into the program of the experimental finalizing of the second stage is necessary for the engines of space vehicles and apparatuses and also for engines of the upper stages of the rockets, which enter into the operation under conditions of weightlessness or axial overloads close to zero. It suffices to say that the starting of the ZhRD under normal conditions is distinguished by the dynamicity and complexity of the elapsing processes. The conditions of weightlessness all the more have a noticeable effect on it, which cannot be disregarded.

The absence of the forces of gravity changes the physical state of the quiescent or moving fluid. Under these conditions the drops seemingly tear off from the bulk of the liquid component, and they can chaotically be moved along the gas cavity of the tank and become mixed with gases of the pressurized system. Those, in turn, freely penetrate into the liquid phase, forming gas occlusions and a gas-liquid emulsion. The emulsions are characterized, in the first place, by the instability of the processes and, therefore, cannot insure the stable start-up conditions. In this case this mode is determined not only by the design features and characteristics of the ZhRD, but also by the parameters and characteristics of the feed lines, including the tanks.

Consequently, the experimental programs of the finalizing of the starting of the engine under bench-test conditions must provide for not only the simulation of conditions of weightlessness (first of all, the entire absence of axial overloads), but also the simulation of full-scale feed system. Taking into account the complexity of this question, in a number of cases it proves to be expedient to carry out such tests of the ZhRD in the composition of the DU [ДУ - engine installation].

Simultaneously, let us note that the basic requirement for the provision for a smooth start of the ZhRD under these conditions is the entering into the fuel-feed main lines of "pure" components, i.e., in liquid phases without gas pockets. This requirement is ensured by means of a number of design solutions. One of them can be called the separation of liquid components from the gas in tanks with elastic diaphragms (see Fig. 1.1). In conformable to this requirement in bench-test finalizing, the checking of its fulfillment must be ensured.

The actual recreation of the physical pattern of the similarity of the behavior of propellant components in the presence of weightlessness under terrestrial conditions is a very complex task. Therefore, it is possible to solve by simulating the flight of the DU on the given phase of the trajectory. For this it is possible to use the so-called "towers of weightlessness", which outwardly represent hollow, vertically located altitude-test structures.

Inside this tower there are guides which ensure the free or forced movement of the platform (similar to the compartment of an elevator of a high-rise apartment). On the platform the DU is installed so that the thrust vector from the engine would be directed vertically upward. The DU prepared for testing is installed on the upper tier of the tower, and then at the necessary moment together with platform it can accomplish a "free" drop downward along the guides under the action of gravity. Later, at a certain time, on instructions from the control panel the engine is turned on. The thrust appearing in this case ceases the "free" drop in the platform.

Under the action of the increasing thrust of the engine, its motion downward seemingly is impeded and finally is ceased entirely. The latter is possible if the thrust level of the engine exceeds the mass of the platform with the DU. At the moment of stopping the platform, automatically with the aid of special clamps, is fixed with respect to the guides, and the engine is turned off.

As follows from the description, the starting of the engine in practice completely occurs under conditions of weightlessness, i.e., "free" drop of the DU.

The "towers of weightlessness" are expensive and unique structures. Therefore, in a number of cases they are substituted by manned installations. For example, a container with the DU can be ejected from an aircraft, and after the starting and shut-down of the engine it descends to earth with the aid of parachute systems.

In both cases both the platform with the DU and the container are provided with systems of remote control in the form of autonomous recording mechanisms and radio-channel transmitters. Besides the parameters of DU, also axial overloads during the entire period of the tests, beginning from the moment of the "free" drop up to the engine cutoff, are recorded.

The cycle of tests on effect of external and internal factors is concluded with tests of the engine and its assemblies for reliability and longevity (see Chapter IV).

The various kinds of longevity tests which require the simulation of environmental factors for a long time, which considerably exceeds the operating cycle of the enterprise, are the most laborious. Here appear the interruptions in the conducting of the experiment, which can influence, to a greater or lesser degree, the final result.

Let us assume that the program provides the tests of the ZhRD for the full service life. It is natural that for engines with high guaranteed service life with respect to operating time, the fuel reserve in bench-test capacities can prove to be insufficient. The periodic refueling of tanks is connected with the time curtailment of the tests. And this can lead to the following.

First, the curtailment of the tests will cause the sudden cooling of the structural elements and assemblies subjecte to the fuel action (combustion chamber, nozzle, gas generator, turbine) as a result of the evaporation of fuel residues in the working cavities. A restart, on the contrary, will cause their intensive heating. It is quite obvious that the sudden cooling, just as the subsequent intensive heating of the complex welded or soldered structures, which consist of parts of different thicknesses and made of different metals, leads as a final result to the concentration of internal stresses, the deformation of the elements, the appearance and development of microscopic cracks and defects, i.e., to a change in the quality of the structure as a whole.

In the second place, with the restart, which is very dynamic with respect to the conditions and short in time, sudden failures as a result of the scattering of loads and decrease in mechanical characteristics because of the effect of impact loads are possible. The studies carried out by a number of authors [54] indicate that with the starting of the majority of the ZhRD the intensity of the appearance of failures considerably increases in comparison with the steady-state condition. This is especially manifested in the stages of the designer finalizing of the ZhRD.

Thus in service-life tests of the engines, conducted in several stages, with one of the restarts let us assume that a failure occurred. But this failure, for the same reason, could appear somewhat later under operating conditions. Then it can be explained by completely different reasons, which will lead to the appearance of additional hypotheses subject to checking.

Thirdly, the aggressive propellant components can corrosively affect the heated structural elements in the process of the curtailment of the tests. This affects the overall state of the engine performances and, in summation, the results of the entire program.

This example confirms that the disturbance of the objective conditions of the execution of the program (time curtailment of the tests) affects the final result: the same type of failure can be related to the transitional or steady-state condition or is explained by an entire complex of reasons.

The effectiveness of the tests from the viewpoint of quality and authenticity of the determination of the output characteristics, first of all, depends on the "purity" of the conducting of the experiment. The time curtailment of the tests does not always noticeably show up in the output characteristics, for example, in tests for leakage, shaking, and so on.

No less important a question in the development of the test program on the effect of various kinds of factors is the selection of values of maximum levels.

Let us assume that a program with limiting values of the factors accepted in accordance with the technical assignment is implemented. In the qualitative analysis of the test results it is not always possible to distinguish the effect of the external factors of the probable deviations of separate parameters. Therefore, the limits of the investigated factors must somewhat exceed the given ones. This excess can be not less than 10-15% of the nominal level [62]. The need for this "reserve" according to the basic parameters which determine the test conditions becomes noticeable the nearer the assigned levels are to the boundary operating conditions of the ZhRD.

The expansion of ranges of a change in parameters of the engine with finalizing is produced, mainly, for the purpose of an increase in the informativeness of the tests. This can be shown in the following example.

Let us assume that operating "square" - $p_{\kappa}\times k$ - of a change in the combustion chamber pressure p_{κ} and the fuel component ratio k is investigated (Fig. 2.2). The points designated by asterisks indicate the tests carried out within the assigned limits of a change in the adjusting parameters p_{κ} and k. Simultaneously we are interested in tests with the maximum levels of these parameters, which correspond to the points noted by circles. However, errors of adjustment, amplified by the effect of the uncontrollable random factors, does not always make it possible to achieve the intended goal. Instead of the expected results, others can be obtained. Moreover, measuring errors create an additional scattering. As a result of this, there is practically always the probability of obtaining realizations which do not correspond entirely to the planned programme. Let us assume that in Fig. 2.2 they are designated by circles. In summation, according to these data, taking into account the foregoing realizations, we must judge the efficiency of the engine at limiting values of parameters p_{κ} and k, which is undesirable. Therefore, in order to have reliable data, tests of the foregoing program must be repeated, but this, in turn, cannot be recognized as being rational.

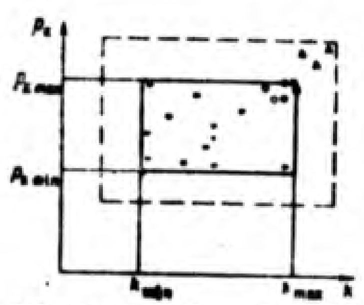


Figure 2.2. Operating "square" of a change in parameters p_{κ} and k.

As a result in order to exclude the random errors and the need for the conducting of repeated tests, especially when not one-two factors but several vary, the researchers proceed along the path of the expansion of ranges of the changes in parameters, expanding them beyond the assigned limits (triangles in Fig. 2.2). Such ranges of variation of factors can be reached by means of the corresponding adjusting of parameters of DU with a certain "guaranteed" reserve for the maximum permissible, boundary and other conditions, and also as a result of the expansion of ranges of the variation of input factors (pressures, temperatures, etc.).

Test work on nonrated, extreme modes and their combinations considerably expands the volume of information even with the same volume of experiments. Consequently, not only the informativeness of the tests but also the authenticity of the produced evaluations are increased.

§ 4. QUALIFICATION TESTS

Qualification tests are distinguished from those which have preceded by the staging nature of their conducting and not by the content of the programs. In accordance with the practice formed in the USA, such tests are characterized as being final [62]. In fact these are tests of a specific sample of the regular articles manufactured in accordance with technical specifications and records and accepted for the participation in flight tests. The program of qualification tests corresponds to tests of stage II, taking into account all the changes introduced into the project in the course of experimental and design works and the stage of tests on the effect of various kinds of factors.

The design of the ZhRD at the beginning of the qualification tests can undergo substantial changes. Therefore, the statistics of tests of a regular design, as a rule, is small and cannot satisfy the requirements given to the confirmation of reliability. It is natural that there appears the need for the repetition of a certain part of the test programs of the foregoing stage, which in turn, must lead to the lengthening of the total periods of finalizing of the ZhRD. This lengthening of the periods is easily anticipated, and therefore it is expedient to previously include it in the plan of the experimental works. Thus, for instance, in the USA during the development of rockets and space objects, for the implementation of programs of qualification tests 5-6 months are needed and in a number of cases more [62].

The final program of the qualification tests, including the ground finalizing, should be considered as the bench tests of the ZhRD in the composition of engine plants and stages of the rocket. Thus, for instance, in the USA all newly developed ZhRD pass firing tests in the composition of the DU with the technological tanks completely reproducing full-scale, including automation and fuel-feed main lines and then the tests in the composition of full-scale rocket stages [83, 86]. In a quantitative respect the volume of these tests is quite great. It suffices to give these examples:

- DU of the first stage of the carrier "Saturn-5" (of the firm Boeing), which consists of five autonomous F-1 ZhRD, underwent not less than 25 firing tests;

- DU of the second stage "Saturn-5" (of the firm North American), which consists of five J-2 ZhRD, - about 60 tests [83, 86].

The same can be said also about other projects. The quantity of firing tests of DU with industrial tanks corresponds approximately to the order named.

Such experiments make it possible to obtain very valuable information about the efficiency of the ZhRD in the composition of the engine plants and also in a complex with other airborne systems and to reveal and estimate the interaction of the engines and systems with relatively low material expenditures in comparison with the flight tests.

The solution to the beginning of the flight tests of the rocket must be accepted after the full completion of the program of qualification tests.

§ 5. FLIGHT TESTS

In the course of experimental-designer works, tests on the effect of the complex of factors and qualification tests, the efficiency of all systems of the DU under the influence of the ambient medium and different combinations of external and internal factors is checked. However, a sufficiently complete simulation of flight conditions during bench tests is impossible to achieve.

For full-scale objects under terrestrial conditions, it is difficult to insure the simulation of vibration, weightlessness, axial overloads, high vacuum, and so on. A no less complex and often unresolved problem is the provision for combinations of the complex of dynamic factors. All this raises the question of the need for the conducting of flight tests under operating conditions. At the same time the flight tests should be considered as their own kind of final complex tests of the totality of the developed ground and airborne systems. According to their results the degree of the conformity of the bench and flight tests is more precisely formulated, as a result of which some programs of bench tests can be corrected in an appropriate manner. In this stage one can speak about the correction of programs of acceptance tests, standard tests, and so on.

However, the flight tests have shortcomings. The basic of them is the high cost, and also their somewhat less informativeness due to the limited quantity of measuring means and telemetering channels, and the limited possibility of the regulation of the physical processes in comparison with laboratory and bench tests.

Flight tests of the ZhRD can be carried out together with the article of this developed project or in the composition of the serial similar articles of earlier developments, which would completely or partially satisfy the requirements for the technical assignment. Such tests, in all probability, are justified in their conducting in stages of the designer finalizing for the purpose of decreasing the total material expenditures. This practice has found wide use, for example, in the USA in the development of missile complexes and space objects [86].

Despite an attempt to simulate test conditions of engines of upper stages under terrestrial conditions, the bench tests do not make it possible to completely reproduce the dynamics of the starting of the ZhRD. Therefore, in a number of cases in the USA the engines of upper stages undergo the initial stage of the flight tests on less expensive carriers. For the purpose of the illustration of this position, it is possible to give examples when the engine J-2, before its setting on the carrier rocket Saturn-5, passed the first flight tests in the composition of carrier's second stage Saturn-1; the engine RL-10, before its setting on the carrier's second stage Saturn-1 passed the first flight tests in the composition of the carrier Atlas-Centaur; engines of the stage Agena, before setting on the Atlas, underwent flight tests in carrier's composition Thor.

Consequently, the less expensive or manufactured guaranteed service life of the carrier and strategic missiles can be used as unique "flight-test stands." The purpose of such tests is for a check of the efficiency of the ZhRD in practice with the full simulation of conditions of a full-scale objective use. Test data make it possible to obtain reliable information about the flight characteristics of the ZhRD, to compare them with test-stand characteristics and finally to finish the latter for the purpose of the achievement of as complete a conformity of the basic parameters during bench and flight tests as possible. All this can be considered as the measure directed toward a reduction in the total material expenditures.

The question concerning "flight test-stands" deserves special attention. With their aid even in the early stages of development testing of the starting of the engine under conditions of weightlessness or at low values of the axial overloads in the rarefied medium, etc. can be accomplished. Virtually such tests make sense in all cases when simulation on the test stand of the actual conditions of tests is economically more expensive, requires special equipment, modernization of the test stand, the creation of new structures, and so on. Even with equal expenditures, tests of a certain selection of engines under these flight conditions can prove to be advisable, since from the viewpoint of the reliability results of the simulation on the test stand of conditions of the occurrence of complex physical processes will virtually always be less precise than during full-scale tests.

At the same time tests on "flight-test stands" cannot completely replace the usual bench tests, at least from the viewpoint of the organization of a check with serial production. Therefore, any flight tests should be combined with bench tests, attaining in this case their complete congruence. We keep in mind that the dynamic characteristics from the basic parameters under bench and flight conditions must be identical, which is easily checked by the usual coincidence of single-type oscillograms.

With the emergence to flight tests of rockets in the first stage the replacement of the upper stages by mock-ups equivalent to full-scale stages in geometric, mass and aerodynamic respects can be assumed. This fact considers the fact that when going to flight tests the probability of obtaining failure from the first stage is sufficiently great. Consequently, the probability with the emergency flight of the first stage to lose the materiel part of the upper stages without any useful information about their quality is great. In the case of the identical technical equipment of the lower and upper stages when proceeding to flight tests, a great risk is possible. Therefore, for example, in the USA flight tests of first stages are carried out, as a rule, with mock-ups of the following stages, and only after the confirmation of a certain acceptable level of reliability are they converted to tests of the upper stages in regular performance.

The flight test programs of articles can considerably differ from each other by flight mission. In this case in connection with the ZhRD from one testing to the next, programs of the provision of thrust momentum because of the variation in operating time and mode of the control of the basic parameters are changed. The remaining factors are determined by the environment and the tuning precision of separate systems and elements, and therefore they can respectively be changed in a random manner. If the quantity of programs is limited, then the processing of results can be conducted by methods of the "passive" experiment. However, for the purpose of the coverage of extreme test conditions, it is expedient to expand the limits of the variation of some factors up to the assigned levels and thereby approximate the flight tests to conditions of the conducting of the "active" experiment. The latter fact prove to be very important from the viewpoint of increasing the quality of the finishing.

Furthermore, an important stage which precedes the flight tests must be the forecasting of test results by means of methods of mathematical modeling [80], the analysis of the previous information, the diagnostics of failures, and the analysis of possible emergency situations.

No less important should be considered the subsequent modeling of physical processes by means of the obtained results for forecasting conditions of normal functioning of the DU with all possible combinations of the factors not encompassing this program.

The flight tests are one of the stages of the proving-ground tests [53], in process of which the interaction of all elements of the missile complex is checked. Proving-ground tests involve transporting and climatic testings, and tests with extreme temperature and humidity modes after prolonged storage, and so on.

As a rule, the articles which underwent the entire complex of ground-based proving-grounds tests undergo flight tests. The information obtained in this case is used in the testing of the conformity of tactical-technical characteristics of the complex and its systems for requirements of the technical assignment.

§ 6. TYPES OF TEST WITH SERIAL PRODUCTION. QUALITY CONTROL AND RELIABILITY

Serial production of the ZhRD is a complex process in which together with the industrial-technological factors the moral-psychological and other factors act. Technical and economical progress and industrial efficiency are the basic figures of merit of the commodity production. Taking this into account, it is indisputable that the quality and reliability of the articles depend not only on the degree of their design perfection, perfection of bench-test finalizing, but in no less measure on the quality and stability of the industrial processes of manufacture, the implementation of the foremost methods of production and control, and process, step-by-step, preventive and acceptance inspection.

Each stage of serial production is characterized by the conducting of the appropriate tests on the basis of physical and statistical methods of the selective and continuous forms of inspection.

Statistical studies [17] showed that with bench and flight tests of the ZhRD in a whole number of cases failures are explained by technological factors. They comprise approximately 30% of the total number of failures. Moreover, the named figure in a certain kind should be considered as being conditional, since in the analysis of the reasons for the emergency state it is not always possible to divide clearly the failures into design, production and operational. This is especially related to cases when reasons are not unambiguously established or are explained as the consequence of several hypotheses.

If the failure is explained only by production reasons and occurred within limits of the allowances established by technical specifications and records, it can be explained only by its inadequacy. Such failures are considered to be caused by technological factors of the first kind. Unlike them there are technological factors of the second kind, caused by departures from the production processes, and in a number of cases they are explained by its rough disturbances. Technological factors of the second kind depend on random causes. The process of their appearance is chaotic.

Technological factors of the first kind are virtually inseparable from the design. Therefore, the problems connected with them of the provision for high levels of reliability depend on the degree of perfection of the designer finishing of the article and technical specifications and records and on the degree of the adjusting nature of the production. If the technological factor of the first kind are inherent in design and documentation, then factors of the second kind characterize only the specific production. It is natural that under these conditions the specific measures which do not allow or lower the probability of the appearance of defective products and thereby ensure the high level of quality can be determined. As a whole this can be reached by the absence of rejects among the commodity articles, on the one hand, and because of the stability of the basic parameters on the other. The latter is most frequently explained by technological factors of the first kind.

Applied in industry is a whole number of physical and statistical methods of quality control of the manufacture of parts and inspection of the airtightness of cavities and connections [50]. The physical methods include ferromagnetic, luminescent, colored defectoscopy, radioscopy by X-rays and gamma-rays, ultrasonic and finally visual inspection of the parts by special optical and lighting meters. The indicated methods make it possible to check the quality of manufacture of the parts without their failure and contribute to the development of the concealed defects connected with the breakdown of the surfaces and internal continuity of the metals. Each method has its features which determine the field of its use. Thus, for instance, the ferromagnetic method can be applied only for quality control of parts manufactured from ferromagnetic alloys. Most universal in this respect can be the method of the X-ray check, with the aid of which the weld quality, shells, continuity of the cast housings, and so on are checked.

The inspection of airtightness of closed volumes can be accomplished by methods of the insertion of vessels, which are located under pressure, into a bath with water, the saponification of the joints; gas leak detectors, and so on.

In the process of serial production the physical methods of non-breaking forms of inspection make it possible to obtain the basic volume of information about the quality of commodity articles. These methods are used on not only on separate elements but also systems. Thus, for instance, in the USA careful radiographic inspection, ultrasonic radiographic inspection, ultrasonic inspection and tests with the use of other physical methods of the pressure leak test of cavities, welds, completeness of the housings of assemblies are given to completely assembled engines [62].

Despite their entire effectiveness, the methods of physical checking are all the same indirect. They do not exclude and cannot replace the standard tests on checking for the efficiency and evaluation of the stability of the basic parameters. As experience shows, the most efficient was and remains complete inspection, and remaining forms of effectiveness only in a different measure can approach it. As follows from a statistical analysis, for the small-scale manufacture, which exists in rocket and space technology, the methods of sampling inspection are always efficient. In the case of a small number of batches, the statistically substantiated plans and volumes of sampling are absent. For this reason in a number of American projects the reliability of commodity supplies of the ZhRD is checked by the methods of current control [80]. Thus, for instance, acceptance tests of the engine F-1 of the Rocketdyne firm are constructed on the basis of repeated firing tests as varieties of current control of all the commodity articles.

The newly manufactured engines pass full-scale service-life tests (160 s) in accordance with the flight program and then firing tests according to a specific routine with several switchings with the total duration from 100 to 300 s. The final stage of the acceptance tests consists of bench tests of clusters of the ZhRD in DU in the composition of stage (5 self-contained units) with a duration of 100-150 s. After this type of standard tests the DU goes to the commodity assembly without the subsequent bulkhead.

This form of continuous acceptance inspection imposes responsibilities on the design and undoubtedly must be taken into account even at stages of design and finishing of the ZhRD. In turn, this led to a review of points of view on the ZhRD both on the system of one-time use, and sometimes and on the rocket stage. If earlier the ZhRD was examined only as a system of single action and one-time use, then at present in the light of new ideas and requirements it becomes a system of multiple switching-on. The ZhRD in its operational capabilities is considerably similar to aircraft turbojet engines.

Contemporary ZhRD allow high technical and guaranteed service life, repeated startings and stops without supplementary installation works [80]. Therefore, the question arises concerning the failure of the automation of single action. It goes without saying that the aforesaid does not have a common nature and is not related to all ZhRD as a whole but must in each specific case correspond to the purpose of the engine. But today this is already indisputable for ZhRD of carrier rockets of space vehicles.

Together with the continuous forms of quality control of the articles characteristic to small-scale production, statistical samplings of inspection have found extensive application at present. In a number of cases plans with variable sample sizes are used [62]. According to this plan the sample size from first batch can be 1:1, i.e. all 100% of the articles are tested. For the subsequent batch the plan 1:4 is utilized, i.e., 25% of the articles are tested, and then 1:10 and so on.

The sample size changes depending on results of the tests and statistical data, which confirm the quality of the commodity production. If it deteriorates, the plan must provide for an immediate rigidification of inspection and transfer to an increased sample size of the articles for standa. .ests. Such selective plans must "flexibly react" to the failures of articles during delivery tests and in the process of their operation.

Statistical methods are constructed on the basis of single and repeated selection. At present statistical methods of quality control and the reliability of the commodity production are basically standardized [63, 68] and are extensively used in large-scale and mass production. These methods of inspection are constructed on the basis of standard tests, which in connection with the ZhRD, correspond in duration and operating mode to the flight program.

Standard tests are given not only to the systems but also their elements. At the same time with serial production all the possible special and technological tests for the purpose of checking the stability of production in terms of programs of wide technological studies are carried out.

CHAPTER III

THE PLANNING OF FINAL DEVELOPMENT AND ANALYSIS OF TEST RESULTS

§ 1. FEATURES OF THE PLANNING OF TESTS OF COMPLEX TECHNICAL SYSTEMS

The planning is preceded by information analysis, and the experiment is carried out for the purpose of checking one of the possible hypotheses. However, in the early stages of the search works it is not always possible to foresee the final result. If the planning of the tests is conducted in the stage of design, it must consider the presence of a priori information in the form of systems - analogs. The planning of tests must be based on reliable information. If the information is barely reliable or is absent, the tests cannot be planned. It is difficult to plan the first experiment, it is desirable to plan the second, and it is already completely necessary to plan the third.

In the conducting of the first test series used more frequently is the test search, owing to which it is as if the designer "feels" the threshold of the efficiency of the system. For the complex technical systems are carried out the autonomous tests of assemblies and subassemblies, and only after their completion are the autonomous tests of the system as a whole begun. The autonomous testing makes it possible to establish the interconnection and functioning of elements of this subassembly. In this case it in no way considers the cross correlation, which under conditions of the functioning of complex dynamic systems can be considerable. Cases are known when the great effect of the correlations between elements of separate systems nullified results of autonomous final development. In similar cases the final development virtually entailed the conducting of the expensive complex tests. Economically this cannot be recognized as being advisable. In view of this for a complex system, the design, which to a certain degree excludes the interaction of elements of the system on each other, should be called rational.

It is expedient to carry out the planning of the experiments in all stages, but its effectiveness is especially obvious for the complex technical systems when there is a complex of significant factors. During the planning the subsequent stage must consider results of the preceding stage, and therefore the timely processing of primary data is important.

The experiment gives information which confirms the calculation and corrects errors in the design. From the test results judgement is made about the correctness of the solutions taken. The experiment confirms one hypothesis, rejects another or bears a new one. However, the intuitive approach to the development of experimental programs can lead to errors. From practice examples are known when from the first unsuccessful testing an acceptable version of the design was rejected and accepted, and as was subsequently found, is inadequate.

The experimental final development of the ZhRD [ЖРД - liquid-propellant rocket engine] and the planning of tests connected with it have as their purpose the search for optimum structural solutions which satisfy the requirements of the technical assignment. The search is conducted in the direction of shortening in the total periods of final development, the provision for quality and reliability, and the minimization of economic expenditures.

Each engine must satisfy the requirements which determine its design, engineering and operating characteristics. For this reason the final development is connected with the functional test of articles with different combinations of external and internal factors.

The external factors include temperatures of the propellant components and structural elements, inlet pressures, dynamic momenta changed in accordance with the test program, and so on.

The internal factors are the parameters conditioned by design and engineering characteristics, features of production, tuning precision, and so on. It is possible to change them by means of an input of special design measures or engineering assemblies.

The final stage of development should be considered as the transition to the serial production with which the reliability of the commodity supplies is provided by the perfection of the design final development, the degree of perfection of the production processes and industrial equipment, and the current intermediate and output inspection.

Thus, the scientific solution to the problem must be conducted in directions of the provision for the reliability of articles in the process of final development, on the one hand, and the inspection of commodity supplies, on the other hand. This, first of all, is the selection of the efficient design of engine which satisfy the assigned requirements with respect to the provision for the operational indices and reliability. The latter must be reached by the optimum planning of final development for the purpose of a comprehensive efficiency of all elements with the different combinations of the external and internal factors. In this case the important questions are the shortening of the quantity of the tests, total periods of final development and economic expenditures under the condition of fulfilling the requirements for the technical assignement. Besides the technical problems connected with the design and final development, there appears a number of difficulties connected with the large volume of experimental works in which the completeness of the coverage of a large quantity of factors which affect the efficiency of the engine is considered.

In the process of the final development when the problem of the providing of design-engineering reliability is solved, the design engine performances can change. In this stage significant factors can be the different technical parameters which must be varied by the developer when selecting an efficient design. The ZhRD refer to a number of those complex systems which do not always allow a successive change absolutely in all factors. They are dynamic and in a number of cases are correlated with each other, as a result of which the varying of one factor can affect the levels of others. This is the basic difficulty in the use of factor plans for the final development of similar technical systems.

As a whole tests of the ZhRD consist of a factor experiment which can be simple or complex. Therefore, the simplest classification which would divide tests into stationary, single-factor and multifactor is necessary. Understood by the factor experiment is that type of tests with which all levels of one factor are found in combinations with levels of the variation of others. The stationary includes tests which are carried out under conditions of normal operation at fixed levels of loads. Test data can be utilized directly when evaluating the reliability. They are the most widely accepted type of tests with serial production.

The single-factor tests include those by which, arbitrarily or in accordance with a planned program, the determined factors are changed; however this condition is observed: within the limits of one program or group of tests one factor is varied. As an example is inspection tests with the adjustment of the engines for different combustion-chamber pressure. In this case, unlike other characteristics, the pressure is considered as the varied factor. The purpose of the tests is for an evaluation of the effect of this factor on the efficiency of the system. The variation can be accomplished both within limits of the assigned boundaries for normal operation and beyond them. The latter case is an example of the so-called "accelerated" tests.

For the complex systems which total the effect of several factors, single-factor experiments are insufficient, since they do not always correspond to conditions of normal operation. As an example it is possible to give the description of the operation of a controllable ZhRD in the interaction of the propellant feed systems, the change in thrust and the fuel component **ratio**. Three of the factors are named here. In reality there are considerably more: they include oscillations of inlet pressures as a result of the instability of the performance of the pressurization systems, a decrease in the hydrostatic liquid columns in the tanks, an increase in the axial overloads and temperatures of propellant components when systems of "hot" supercharging are present, and so on. In these conditions the solution of problem is possible only by means of the implementation of the multifactor experiment in which part of the factors or majority of them are changed simultaneously. These tests are called multifactor. The methods of their setting and planning are urgent problems of the present.

The advantages of multifactor experiments over the single-factor can be formulated in the following way.

1. The multifactor experiments have considerably greater informativeness and, consequently, higher efficiency with which the same the purpose is reached with less expenditures.

2. In multifactor experiments the evaluation of the effect of each varied factor is conducted by the generalization of all results and not by part of them, as in the analysis of data obtained in single-factor experiments.

3. They make it possible to judge the effect and interactions of linear effects on the final results.

4. Unlike the single-factor experiments, the multifactor most fully correspond to the real physical processes.

Since there are errors in the adjustment of the engines to the basic parameters and to various kinds of errors, with a sufficient degree of accuracy it is possible to confirm that for complex systems the stationary and single-factor experiments virtually do not exist. In multifactor experiments when is varied a certain quantity of factors at previously known levels, sometimes important is the fulfillment of the condition of randomization, which increases the accuracy of the calculations because of the elimination of the effect of a number of random causes. Under this condition, as a rule, the random test procedure must be provided. The randomization makes it possible to obtain more uniform and reliable results. The condition of randomization can significantly affect the final conclusion, which is easily shown in the following example.

Let us assume that the problem is posed to compare the actual data on flow-tests of two versions of the design of combustion chambers under bench-test conditions. For this several copies of the articles for each version of the design is distinguished. In the process of hydraulic tests, for reasons independent of the experimenter, a certain drift of the flow rate per second of the fluid tested occurred. If the flow tests were conducted in succession (first articles of one design and then - another), then it could be found that some articles were flow-tested with low flow rate while others - with increased. Correspondingly, inaccurate measurements will be obtained, and incorrect conclusions are made. But if with the flow tests chambers of both versions were alternated randomingly, the measuring errors will be averaged and the average values of results will be nearer to the true values.

Thus, even the incomplete randomization of the tests **makes** it possible in some way to avert an increase in errors. With the observance of the objective condition of randomization, the order in the test work can be established in accordance with the table of random numbers or by means of coin-tossing ("heads"-"tails").

The informativeness of tests of the ZhRD is great, and the field of its use in the objective estimate of the obtained results must be sufficiently large. More frequently this evaluation is qualitative, since there are definite difficulties in the selection of the criteria (parameters) which characterize the output quality of the process. In the stage of the design final development the informative evaluation of the results is, first of all, an explanation of the possible reasons for failures.

For the designer and an experimenter the complex problems are the search for the optimum solutions, schemes, characteristics and conditions of the operation. An important role in this is played by the empirical, step-by-step search. But it by itself is not economically favorable, and that is why it is necessary to use the mathematical statistics, which makes it possible to solve the same problems but with less means.

It is known that any physical phenomenon can be described by means of the analytical expressions which reflect entirely the physical essence of the processes, and this is not always required. Sometimes it suffices to connect the "input" of the system with its "output". Similar solutions are known in mathematical statistics and ever more frequently are being used in practice. The problem which does not examine the intermediate physical processes but only the "input" and "output" of the systems are symbolically called the "black box." For a complex technical system it can be represented in the form of the diagram on Fig. 3.1, where x_1, x_2, x_3, ..., x_n are the input parameters or factors which include the variables ("input" of the system); y is the output parameter ("output" of the system).

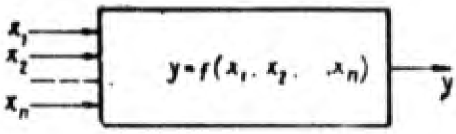


Figure 3.1. Diagram of the "black box" for the deterministic processes.

In the general meaning of the word, "input" can be the deterministic, random, dynamic, and independent variable from the viewpoint of the interaction of the factors finally correlated. The term "output" can be, in turn, one or there can be several of them. In the solution to the problems the most universal can prove to be the methods of cybernetic diagnostics based on the identification of the dynamic processes. In a theoretical plan some objectives are achieved [47, 59, 85, 89], but from the viewpoint of practice their use causes definite difficulties, about which we will speak later.

The finishing works for virtually any technical system represents a series of consecutive experiments conducted for the purpose of the provision of fundamental characteristics, depending on the complex of external and internal factors. Taking this into account, the entire final development must be constructed on the basis of the optimum planning of the experimental works. These goals correspond to the full factor plans with which the number of the tests covers the effect of all factors and their interactions. If we accept the quantity of factors equal to n, and the levels of variation to d, then the number of tests N in the full factor experiment will be equal to d^n. Already from this dependence it is evident that the applicability of such plans is very limited, since for their realization a large volume of experimental works is required. Even with two levels of variation and the quantity of factors more than five, the volume of tests is:

with n=6, N=64;
with n=8, N=256 and so on.

Hence we have the conclusion: depending on the complexity and cost of the experiment, the full factor plan can be applied with the quantity of independent variables $n \leq 5$. With a large number of factors there arises the question concerning the shortening of the tests. For this purpose the fractional plans, steep ascension over the surface of response, and so on will be determined. A definite interest is represented by Д-optimum plans, orthogonal planning, and so on. At present there is a large quantity of literature on the theoretical problems of the design of experiments [2, 47, 67, 68, 72].

The purpose of the planning of the experiments is the determination of the composition of the significant factors, their possible combinations and the effect on the output quality of the investigated process. The methods of planning are varied and for different physical problems cannot be standard. The common indispensable property of all the plans is the construction of a mathematical model on the basis of the processing of experimental data. The experiment itself is the physical realization of the planned program from the viewpoint of the provision for input factors. The output quality of the process is determined by properties of system itself to react to the "input".

In conclusion it should be noted that the planning of the experiments and mathematical modeling do not eliminate empirical search but supplement it.

§ 2. THE USE OF FACTOR PLANS FOR TESTS OF COMPLEX TECHNICAL SYSTEMS

The basic goal of the tests is the obtaining of the necessary information. However, the same result is reached by different means. The required volume of information can be obtained with considerably less quantity of tests than follows from the condition of the adopted full factor experiment. The effect is reached

a) by the rational selection of the number and interval of the variation of factors; b) by a decrease in the quantity of indistinguishable combined effects; c) by the elimination of the repetition of results in the conducting of the experiments; d) by the determination of extreme test conditions; and e) by the forecasting of the results.

The optimum nature of the factor plans consists in the extraction from the experiment of the maximum of information about the investigated processes, on the one hand, and the minimization of the number of tests on the other hand. In this setting the solution to the problem can be the fractional factor plans, i.e., the fractional replicas. Their meaning consists in the replacement of the interactions of the linear effects of the new fictitious variable with the ordinal number (n+1). In a physical sense the fragmentation of the full factor plan into units occurs. Thus, for instance, if there is a plan for type 2^n, then it can be divided into the even number of r units from which in the initial stage one is realized. Correspondingly, the quantity of realizable tests is decreased and in this case will be $N=\frac{1}{r}2^n$. The fractional factor experiment is the mathematical instrument utilized during the planning of the tests.

Let us examine some of its properties. Let us suppose an experiment of the type 2^4 with different variations in factors x_1, x_2, x_3, and x_4 is examined. The matrix of the test is determined:

- by the line of linear terms

$$x_0; x_1; x_2; x_3; x_4,$$

where x_0 is the fictitious variable equal to unity;

- by the double interactions

$$(x_1x_2); (x_1x_3); (x_1x_4); (x_2x_3); (x_2x_4); (x_3x_4),$$

- by the triple interactions

$$(x_1 x_2 x_3);\ (x_1 x_2 x_4);\ (x_1 x_3 x_4);\ (x_2 x_3 x_4)$$

- and by the interaction $(x_1 x_2 x_3 x_4)$.

In accordance with expressior $N=2^4$ the quantity of tests for a full factor plan will be equal to 16. For a decrease in the volume of works, let us divide the full plan into two units or half-replicas. Then each of them is described by the appropriate line:

- unit No. 1

$$x_1;\ x_2;\ x_3;\ x_4;\ (x_1 x_2 x_3);\ (x_1 x_2 x_4);\ (x_1 x_3 x_4);\ (x_2 x_3 x_4);$$

- unit No. 2

$$x_0;\ (x_1 x_2);\ (x_1 x_3);\ (x_1 x_4);\ (x_2 x_3);\ (x_2 x_4);\ (x_3 x_4);\ (x_1 x_2 x_3 x_4).$$

As a whole both units from the viewpoint of the total effect are equivalent to the full factor plan. The simplest principle in the determination of the half-replicas is the recording in the form of lines where factor x_0 and all the even interactions refer to one unit, and all others - to the second unit. Entering into the first unit are factors with the determining contrast (+1) and into the second, (-1).

It is completely obvious that the division of the full factor plan into units is not limited by half-replicas. The fragmentation can be continued by the division into a fourth of a replica and further. In the limit plans wh ch in the quantity of the tests are equal to the number of va ied factors plus one ($N=n+1$) can be created. The feature of fractional factor plans lies in the fact that the units taken can be augmented up to the full plan by the realization of the remaining replicas. In the course of the tests fractional plans can easily be reconstructed from replicas of larger multiplicity into smaller and vice versa. This property makes it possible in an operational way to change the plans even in the course of the tests themselves.

The plans can be reconstructed, expanded or at the proper time be discontinued.

A decrease in the experiments with the fractional factor plans is achieved because of the introduction into the matrix of tests of combined estimates. In this case the effects of the interaction are replaced by new factors, participate in the real process and are estimated on a level with the others. Under the effect of the factor let us agree to understand the reaction of the response of the system to a change in the level of the "input" variation. The presence of the matrix of the tests makes it possible in time to examine the effects of each of the linear terms on output quality. Thus, for instance, for the factor plan of type 2^3 (Table 3.1) the sums on each of the columns which determine the effects of the linear terms and their interactions can be obtained. These are:

- for the linear factors

$$X_1 = +x_0 + x_1 - x_2 + x_1x_2 - x_3 + x_1x_3 - x_2x_3 + x_1x_2x_3;$$
$$X_2 = +x_0 - x_1 + x_2 + x_1x_2 - x_3 - x_1x_3 + x_2x_3 + x_1x_2x_3;$$
$$X_3 = +x_0 - x_1 - x_2 - x_1x_2 + x_3 + x_1x_3 + x_2x_3 + x_1x_2x_3;$$

- for the interactions

$$X_1X_2 = x_0 - x_1 - x_2 + x_1x_2 + x_3 - x_1x_3 - x_2x_3 + x_1x_2x_3;$$
$$X_1X_3 = x_0 - x_1 + x_2 - x_1x_2 - x_3 + x_1x_3 - x_2x_3 + x_1x_2x_3;$$
$$X_2X_3 = x_0 + x_1 - x_2 - x_1x_2 - x_3 - x_1x_3 + x_2x_3 + x_1x_2x_3;$$
$$X_1X_2X_3 = +x_0 + x_1 + x_2 - x_1x_2 + x_3 - x_1x_3 - x_2x_3 + x_1x_2x_3.$$

Similarly the effects of linear factors and their interactions for any plan for type 2^n can be estimated. The consequence of estimates of effects is the variance analysis of the test results, which shows which of the factors and interactions have a decisive effect on the investigated process.

Table 3.1

№	x_0	x_1	x_2	x_3	x_1x_2	x_1x_3	x_2x_3	$x_1x_2x_3$	Variant of tests	Y
	1	p_k	k	φ_k	$p_k k$	$p_k \varphi_k$	$k \varphi_k$	$p_k k \varphi_k$		
1	+	+	—	—	—	—	+	+	X_1	Y_1
2	+	—	+	—	—	+	—	+	X_2	Y_2
3	+	—	—	+	+	—	—	+	X_3	Y_3
4	+	+	+	+	+	+	+	+	$X_1X_2X_3$	Y_4

An analysis of the total effects showed that the division into units of the full factor plan has no meaning with a small quantity of linear terms, since the total effects in the interpretation of results of the tests become indistinguishable, i.e., combined. For the purpose of the investigation of indistinguishable effects, we again will turn to the plan 2^3 (see Table 3.1).

Let us assume that parameters p_H, k, ϕ_H - pressure in the chamber, fuel component ratio and the coefficient which characterizes the combustion efficiency of the fuel, are investigated. Let us divide the full factor plan into two units, from which we realize the replica

$$x_1; x_2; x_3; x_1x_2x_3.$$

In this case the matrix of the tests will take such a form at which the interaction $(x_1x_2x_3)$ is considered as a new independent variable.

Let us estimate further the total effects which correspond to the variants of the tests:

- for the linear factors

$$[p_{\text{к}}] = X_1 - X_2 - X_3 + X_1X_2X_3;\ [k] = -X_1 + X_2 - X_3 + X_1X_2X_3;$$
$$[\varphi_{\text{к}}] = -X_1 - X_2 + X_3 + X_1X_2X_3;$$

- for the interactions

$$[p_{\text{к}}k] = -X_1 - X_2 + X_3 + X_1X_2X_3;\ [p_{\text{к}}\varphi_{\text{к}}] = -X_1 + X_2 - X_3 + X_1X_2X_3;$$
$$[k\varphi_{\text{к}}] = X_1 - X_2 - X_3 + X_1X_2X_3;\ [p_{\text{к}}k\varphi_{\text{к}}] = X_1 + X_2 + X_3 + X_1X_2X_3.$$

The conducted analysis of linear effects and their interactions showed that the effects [k] and $[p_H\phi_H]$; $[k\phi_H]$ and $[p_H]$ and also $[\phi_H]$ and $[p_Hk]$ are indistinguishable. The effects are considered to be indistinguishable if their moduli are equal, i.e., their estimates do not depend on the sign.

It is completely obvious that this type of planning for this quantity of factors will be barely effective, since of the eight effects three are repeated.

The examined means of the analysis and determination of the indistinguishable effects is bulky. Moreover, the degree of its complexity increases in proportion to the increase in the number of factors. But the estimate itself of the indistinguishable effects is important prior to the test conducting for the purpose of the elimination from the program of barely effective experiments. From this estimate the selection of any plan must begin.

When selecting the multiplicity of the replica of the fractional plan, an important moment is the establishment of the generating relationships and determining contrasts without which the most important interacting effects for the considered number of factors cannot be estimated. In such a case when the half-replica of type 2^{r-1} is examined, the determining contrast is equal to the product of the linear effects

$$J = x_1x_2x_3, \ldots, x_n.$$

Obtained respectively can be the equivalent effects

$$x_1 J = x_1^2 x_2 x_3, \ldots, x_n,$$

or $x_1 = x_2 x_3, \ldots, x_n$
and similarly

$$x_2 = x_1 x_3, \ldots, x_n; \; x_3 = x_1 x_2 x_4, \ldots, x_n \text{ etc.}$$

Together with the linear effects their interactions are similarly determined

$$x_1 x_2 = x_3 x_4, \ldots, x_n; \; x_1 x_3 = x_2 x_4, \ldots, x_n; \; x_2 x_3 = x_1 x_4, \ldots, x_n \text{ etc.}$$

In the case of the higher multiplicity of the fractional replica, it is necessary to solve which linear effects should be equated with the interacting effects. As was already mentioned, these are a tribute to the method of D. Finni [68, 69], which makes it possible to reduce the volume of tests in comparison with the full factor plan. It is desirable to solve the problem concerning the equating of the effect on the basis of an analysis of experimental data and the physical essence of the investigated process or by using the available a priori information.

The quantity of generating relationships is equal to the multiplicity of the fractional replica. Thus, for instance, if we accept the fractional plan 2^{n-k}, then the quantity of generating relationships is taken equal to k. That which corresponds to this quantity will be the number of determining contrasts. The latter is always easy to find in each specific case by means of the multiplication of the left and right sides of the generating relationships by the appropriate linear effect. Let us assume that in a certain plan of the type 2^{7-3} one of the generating relationships $x_6 = x_1 x_2 x_4 x_5$ is accepted, then the determining contrast for this relationship can be found by the following means of

$$x_6^2 = x_6 (x_1 x_2 x_4 x_5) \text{ or } 1J = x_1 x_2 x_4 x_5 x_6.$$

Similarly the problem for other generating relationships and their determining contrasts is solved. In summation, after obtaining k values for the contrasts, it is necessary to estimate all the indistinguishable effects of the first and second orders by using the rule discussed above. The interacting effects of the third and higher orders can be disregarded.

On the basis of that given, it is easy to examine the half-replicas, which consider effect of 3, 5 and more factors. In this case it becomes obvious that with an increase in the number of factors the portion of indistinguishable effects and their paired interactions decreases. From this viewpoint it is possible to arrive at the conclusion very important for practice that the advantages of the fractional plans become evident only with a large number of factors when $n \geq 5$.

The meaning and value of the conducted analysis consist in the fact that for a realization there must be accepted the unit in which the quantity of indistinguishable linear factors and their paired interactions is equal to zero. This approach somewhat disturbs the principle of randomization but increases the effectiveness of the tests themselves, which in this case is more important. More accurately, after accepting the unit for realization, we introduce an estimate of its quality, since by itself the selection of the plan still means nothing.

The estimate of quality is the determination of the resolution of the replica. In a fractional factor experiment it is not possible to distinguish the regression coefficients between the linear terms and some of their interactions. The higher the divisibility of the replica, the greater the indistinguishability of the effects and vice versa. The full factor plans do not have the indistinguishable effects. The number of indistinguishable effects is the resolution of the fractional replica.

Further for the specific fractional plan the problem concerning the estimate of the regression coefficients, taking into account the accepted generating relationships, is solved, since it precisely, as a final result, interests the experimenter. For the purpose of the definition of dependences for the regression coefficients of indistinguishable combined effects, let us examine an example for the half-replica of plan 2^4 with the determining contrast

$$J = x_1x_2x_3x_4.$$

The indistinguishable effects for the paired interactions are

$$x_1x_2 = x_3x_4;\ x_1x_3 = x_2x_4;\ x_1x_4 = x_2x_3.$$

For the indicated interactions the selective estimates of regression coefficients will be respectively equal.

For the purpose of practical use it is possible to propose a simpler method of the selection of the fractional factor plan, which is, as a rule, optimum. The method entails the following.

For a fractional factor the quantity of tests must satisfy the condition

$$N = 2^{n-k} > n + 1.$$

Let us assume that in the experiment four linear factors ($n=4$) participate, and then with $k=1$ the quantity of the tests $N=8$. Thus, in the presence of four factors it is expedient to utilize a half-replica of the plan 2^{4-1}.

If the quantity of factors is equal to eight ($n=8$), then the recommended fractional plan is 1/16 - a replica of type 2^{8-4} and so on.

The following stage after the selection and analysis of the fractional factor plan is the compilation of a matrix initial data X and test programs. Let us show the virtually convenient method of programming in an example.

Let us assume that in the plan of type 2^3 for a realization there is accepted the half-replica - x_0, x_1x_2, x_1x_3, x_2x_3. Then the quantity of tests will be equal to four. However, a program is unknown to us. The principle of its construction can be proposed as the following.

Assigned to the matrix of the initial data is the column, "variant of the tests", according to Table 3.2.

Table 3.2

№	X				Variant of tests
	x_0	x_1	x_2	x_3	
1	+	−	−	−	X_0
2	+	+	+	−	X_1X_2
3	+	+	−	+	X_1X_3
4	+	−	+	+	X_2X_3

For the column x_0 the conditional level with the sign (+) is accepted. The first line of real factors is written at levels characterized by a "-" sign. Further for the "variants of the tests" and "effects" the "+" sign is placed at the point of intersection of the line and column which have identical notations (x_1-x_1), (x_2-x_2) and (x_3-x_3). In all the remaining cases the test program is characterized by the "-" sign. On this the first stage of planning is concluded, and further experiments follow.

Subsequently, the relizable plan is corrected by taking into account the test results. For a more effective use of the experimental data, expedient is the construction of mathematical models of the investigated processes whose optimization will make it possible to solve theoretically the problems of processing and subsequently only more precisely formulate the obtained results by carrying out inspection tests.

§ 3. STUDY OF THE MATHEMATICAL MODELS OF COMPLEX PHYSICAL PROCESSES

In the initial stages of the development of complex technical systems, as a rule, the reasons for the failures are unknown, and it is not possible to forecast them. The search for optimum conditions is conducted empirically by the generalization of the experiment of the final development of the analog systems, not excluding the intuitions of the developer. As is known, this leads to a large volume of the experimental works whose decrease is possible on the basis of the use of statistical methods of the modeling of the physical processes. In accordance with this there can be examined the method based on the description of the surface of the response of the "output" of the system to the "input" in a certain vector space. Each of the x_i vectors is characterized by properties of this factor and the surface of the response as a whole by properties of the vector of the output quality Y. In general the function of the response for a certain number n of the factors taken into consideration can be written

$$y=f(x_1, x_2, x_3, \ldots, x_n)=f(x_i).$$

For complex physical models the analytical form of the function is previously unknown. Unknown also are the equations which describe the full dynamics of the investigated processes as a result of their extreme complexity. In a number of cases separate particular dependences for intermediate processes can be known, for example, the effect of the total flow rates per second and fuel component ratios for combustion chamber pressure of the ZhRD and so on. The investigation is conducted virtually with the full (or partial) ignorance of the internal mechanism of the phenomena. This makes it possible for us to be limited to the series expansion:

$$y=\sum_{i=0}^{n} b_i x_i+\sum_{i<j}^{n} b_{ij} x_i x_j+\sum_{i=1}^{n} b_{ii} x_i^2+\ldots, \quad (\,.\,1)$$

where b_i, b_{ij} and b_{ii} are the regression coefficients which determine the degree of the factor and their interactions on the output characteristic y.

The functional dependence $y=f(x)$ represents the system of normal equations which sometimes can be augmented by dependences which describe the intermediate processes or functional connections of the separate parameters with the "output" of the system. In this case the initial model of the "black box" will be supplemented by new characteristic features which make it qualitatively more ideal, since the accuracy is increased. The determination of regression coefficients is the basic difficulty in the construction of the model. By the processing of experimental data there can be reproduced the physical essence of the model, on the basis of which the prevailing factors and their interaction are revealed, the optimum solutions are found, and a program of further studies is planned. As a rule, mathematical models of complex technical systems characterize the determined operating modes.

For the purpose of testing the reliability of the model, there are carried out inspection tests in which the theoretical and calculation data on the estimate of the expected output quality are compared with the experimental. If the expected effect is not achieved, the enumeration of the factors taken into consideration is analyzed, and successive approximations are made. There can be several such approximations. Nevertheless, this does not lead to substantial increase in the volume of the tests. The total effect, as a rule, is significant and becomes more noticeable, the more complex the process itself is. The method of regression is applied for the solution of different problems, but, unfortunately, it is not possible to consider them as ideal. There is a number of limitations of the mathematical and physical plan which narrow the field of its use. The basic of them can be formulated in the following form.

1. Input factors and observed data must be independent of one another.

2. Results of the measurements must follow the law of the normal density of distribution.

3. Dispersions of the estimates must be constant and the selective estimates - uniform.

4. Measuring errors of the input factors are low in comparison with errors in the determination of the output quality.

The measuring system, sensors and feed channels must provide the reliability of the information about the investigated processes without distorting the physical essence. For the absolute majority of the systems, the first three of conditions, as a rule, are fulfilled. The last condition does not always make it possible to apply statistical methods of modeling for the study, for example, of the rapidly changing processes and their parameters. The frequently obtained results in many respects depend on metrology. In this case the degree of their accuracy and reliability is determined not only by errors of measurement but sometimes by the place of location of the feed channels and the accuracy of their manufacture. The importance of this fact can be shown in an example.

Let us assume that the static pressure is measured along the contour of the nozzle exit of a ZhRD. The actual value of this parameter on a specific engine, besides the parameters of the working medium, depends on the cross-sectional area of the nozzle. The measured value of pressure, besides the others, will be determined by the perpendicularity of the axis of the channel to the tangent of the nozzle contour at the assigned point. This limitation is correct for the majority of the models and in each specific case must be considered from the viewpoint of an estimate of the accuracy of the obtained results.

In speaking about the statistical methods of the planning of experiments, it should be noted that at present it is not possible to recommend the optimum plans characteristic to the specific model to the experimenter. In different areas of technology they can be different, and the advantages of some over others are not obvious. The general ones for all programs are:

1) determination of the number of factors and levels of variation;

2) the selection of intervals of variation;

3) the selection of the optimum plan for the conducting of experiments;

4) the composition of the matrix of tests and its realization;

5) the processing of experimental data and the determination of the mathematical model.

In the simplest cases the regression coefficients can be determined by the least-squares method and in more complex cases - with the aid of a digital computer.

Let us examine as an example the plan which corresponds to the problem of determining the maximum value of specific thrust of an engine in the variation simultaneously of two factors - combustion chamber pressure $p_{к}$ and the fuel component ratio k. With the calculated optimum values of their parameters, it is possible conditionally to consider them as being independent. The variation is conducted at two levels on the "square" of $p_{к} \times k$ (Fig. 3.2).

This test program is characterized as a plan for type 2^2. With the full factor experiment the quantity of the tests will

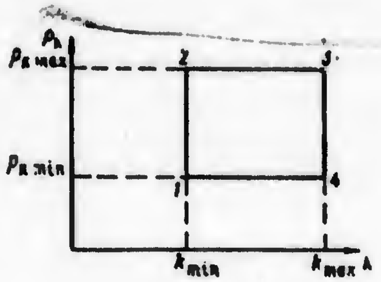


Figure 3.2. Geometric representation of the plan for the experiment ($p_\kappa \times k$).

be equal to four (N=4). The order of the recording of variants of the tests is shown in Table 3.3.

Table 3.3

№	X				Y
	x_0	p_κ	k	$p_\kappa k$	
1	+	−	−	+	y_1
2	+	+	−	−	y_2
3	+	−	+	−	y_3
4	+	+	+	+	y_4

For the given experiment the mathematical model will take the form

$$y = b_0x_0 + b_1p_\kappa + b_2k + b_{12}p_\kappa k + \varepsilon,$$

where ε is the error of the experiment.

With the appearance of the third independent variable, for example, factor ϕ_κ, which characterizes the combustion efficiency of the fuel, the matrix X is correspondingly supplemented by new terms:

$$\varphi_\kappa,\ p_\kappa\varphi_\kappa,\ k\varphi_\kappa,\ p_\kappa k\varphi_\kappa.$$

In summation, we will turn to the full factor experiment of type 2^3. With the introduction of the third factor the quantity of tests will increase two times and correspondingly will be N=8. The full factor experiment can geometrically be represented by the vertexes of a tube.

An experiment of such type satisfies the conditions of the matrix of planning (Table 3.4).

Table 3.4

№	X								Variant of tests	Y
	x_0 1	x_1 $p_к$	x_2 k	x_3 $\varphi_к$	x_1x_2 $p_к k$	x_1x_3 $p_к\varphi_к$	x_2x_3 $k\varphi_к$	$x_1x_2x_3$ $p_к k\varphi_к$		
1	+	−	−	−	+	+	+	−	X_0	y_1
2	+	+	−	−	−	−	+	+	X_1	y_2
3	+	−	+	−	−	+	−	+	X_2	y_3
4	+	−	−	+	+	−	−	+	X_3	y_4
5	+	+	+	−	+	−	−	−	X_1X_2	y_5
6	+	+	−	+	−	+	−	−	X_1X_3	y_6
7	+	−	+	+	−	−	+	−	X_2X_3	y_7
8	+	+	+	+	+	+	+	+	$X_1X_2X_3$	y_8

The mathematical model for the given experiment will take the form

$$y = b_0x_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_2x_1 + \\ + b_5x_1x_3 + b_6x_2x_3 + b_7x_1x_2x_3 + \varepsilon.$$

With the introduction of the fourth factor the quantity of tests will increase to 16, and the geometric model can be represented by vertexes of an octahedral prism.

From that stated above it follows that the full factor experiment does not eliminate the need for the setting of a large number of tests. However, there are methods of their reduction. The latter will be examined below.

The defining moment in the setting of the factor experiment, as we already verified, is the determination of the quantity of independent variables. Nevertheless, in early stages of the final development the quantity of factors taken into consideration can be incomplete. With the input of new independent variables the regression coefficients in the linear equations are changed (with the exception of the orthogonal planning). Therefore, in the processing of results of tests (especially the first series), the directivity and significance of the separate factors should be accepted tentatively, avoiding premature conclusions. After the conducting of a test series of the experiments when the calculation data concur with the actual, the composition of the varied factors can be considered full, and an analysis of their effect is reliable.

A test of the solution of the individual problems connected with the final development of the complex technical systems showed the practical value of statistical methods of the modeling of processes. However, reliable solutions are not always possible, especially when the function of the response in the investigated region has a discontinuity. The possibilities of regressive analysis are limited by the conditions of its applicability.

More universal, despite the apparent complexity, are the methods of identification based on a correlation analysis. The function of response in this case is the equation of dynamics

$$y(t)=\sum_{i=1}^{n}\int_{0}^{\infty}x_i(t-\theta)R_i(\theta)\,d\theta, \qquad (3.2)$$

where R_i is the pulse transient function of the argument θ, which is the interval of variation of the time factor t.

This model takes the form shown in Fig. 3.1, where all factors of the "output" and "input" are the dynamic functions, which consider not only the final results or the value of the factors but also their prehistory on this time interval.

The methods of the description of this mathematical model in principle are known [59], and we indicate only some of them for the purpose of the systematization of the material. If the "inputs" are not correlate, the function of response can be written in the form of the system of integral equations:

$$\left.\begin{aligned} K_{x_1y} &= \int_0^\infty K_{x_1}(\tau-\theta)R_1(\theta)\,d\theta; \\ K_{x_2y} &= \int_0^\infty K_{x_2}(\tau-\theta)R_2(\theta)\,d\theta; \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ K_{x_ny} &= \int_0^\infty K_{x_n}(\tau-\theta)R_n(\theta)\,d\theta, \end{aligned}\right\} \quad (3.3)$$

and here K_{x_i} is the correlation function of the i-th "input", and K_{x_iy} - the correlation function between to the ith "input" and "output", respectively.

In mathematical statistics in the particular case in the presence of limitations, this solution is known as the problem of Weiner and Pelegren [Translator's Note. Spelling not verified]. When the constant values of input factors are present, the function of response will take the form

$$y(t) = \sum_{i=1}^{n} x_j \int_0^\infty x_i(t-\theta)R_i(\theta)\,d\theta. \quad (3.4)$$

The study of complex technical systems is the frequently encountered case for which the input factors are test conditions or design characteristics which are virtually stable within limits of one experiment. A determination of the mathematical model of the process from experimental data, in fact, is reduced to a correlation analysis.

In the processing of statistical data it is sometimes convenient to speak about the discrete values of the input and output factors, depending on the quantity of the tests:

$$x_1(N);\ x_2(N), \ldots, x_i(N), \ldots x_n(N).$$

In this case, if the discussion is about the parameters, then each of the factors is the linear characteristic following the law of the normal density of distribution. Similarly expressed is the realization of the characteristic of "output" - y(N). If in the course of investigation there appears the need for characterizing the process by several output factors with the same "input", the calculation is repeated several times in proportion to the number of "inputs." The solution can prove to be bulky but does not cause any fundamental difficulties. The problem is solved on a computer or on a special computer - synthesizer, used for the solution of integral equations. With a certain approximation the mathematical model of this process, as the surface of response, can be described by an integral equation of the form

$$y(N)=\sum_{i=1}^{n}\int_{0}^{\infty}x_i(N-\Delta)R_i(\Delta)\,d\Delta,$$

where Δ is the interval of the variation of the quantity of tests.

If the input factors are independent, the correlation functions of the connection of the "output" and "input" are defined as

$$K_{x_1y}(N)=\int_{0}^{\infty}K_{x_1}(N-\Delta)R_1(\Delta)\,d\Delta;$$

$$K_{x_2y}(N)=\int_{0}^{\infty}K_{x_2}(N-\Delta)R_2(\Delta)\,d\Delta;$$

. . . .

$$K_{x_ny}(N)=\int_{0}^{\infty}K_{x_n}(N-\Delta)R_n(\Delta)\,d\Delta.$$

With the presence of experimental data the initial expression for the search of the correlation function will take the form

$$K_x(m)\approx\frac{1}{N}\int_{0}^{N}x(m)\,x(m+\xi)\,dm.$$

With calculations of the correlation function the considered quantity of N tests is subdivided into k small groups Δ:

$$N = \Delta k;$$
$$N > (m + \xi) > m > 0; \xi = \Delta\mu,$$

where μ=0, 1, 2, 3 ...

By replacing the integral by the sum sign, under these assumptions made, we will obtain

$$K_x(\mu) \approx \frac{1}{k+1} \sum_{\nu=0}^{k} x_{\mu+\nu} x_\nu.$$

The calculation of the cross-correlation function can be conducted similarly with a sufficient degree of accuracy

$$K_{xy}(\mu) \approx \frac{1}{k+1} \sum_{\nu=0}^{k} y_{\mu+\nu} x_\nu.$$

In the general case when the input factors are mutually correlated, the equations for determining correlation and pulse transient functions can be represented with the aid of spectral densities S(ω), which are defined as the Fourier transform from the correlation function K(N):

$$S_{x_1y} = X_1(j\omega) S_{x_1}(\omega) + \ldots + X_n(j\omega) S_{x_nx_1}(\omega);$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$S_{x_ny} = X_1(j\omega) \cdot S_{x_1x_n}(\omega) + \ldots + X_n(j\omega) S_{x_n}(\omega),$$

where $X_i(j\omega) = \frac{1}{d} \sum_{k=1}^{n} (-1)^{k+i} S_{x_ky}(\omega) d_{ki},$

and d_{k1} - is the corresponding minor of the determinant d:

$$d = \begin{vmatrix} S_{x_1}(\omega) \ldots S_{x_nx_1}(\omega) \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ S_{x_1y_n}(\omega) \ldots S_{x_n}(\omega) \end{vmatrix}.$$

As a whole the presented mathematical apparatus makes it possible to utilize the experimental data in the form of input and output characteristics for the construction of models of complex physical processes, including models of forecasting. Just as in the case of the regressive model, function y(N) must be adequate.

A mathematical model of the form (3.1) can characterize, strictly speaking, only a steady-state process, for example, the engine operation under operating conditions. In the dynamic processes besides the model (3.2) there is not excluded the use of a polynomial of form (3.1), for which each matrix X and vector Y strictly corresponds to a definite argument of time t. In this sense matrices of the initial data and the vectors are represented in the form of a set of separate consecutive discrete values of factors x and characteristic y. If we conditionally accept the quantity of the measured parameters x(t) equal to the number of factors, then the matrix of the tests can be presented in the form of Table 3.5.

Table 3.5

No.	Time	X	Y
		x_1 x_2 x_3 . . . . x_n	
1		x_{11} x_{12} x_{13} . . . . x_{1n}	y_1
2		x_{21} x_{22} x_{23} . . . . x_{2n}	y_2
3		x_{31} x_{32} x_{33} . . . . x_{3n}	y_3
4		x_{41} x_{42} x_{43} . . . . x_{4n}	y_4
. . .	t_i	. . . .	. . . .
. . .		. . . .	. . . .
. . .		. . . .	. . . .
. . .		. . . .	. . . .
N		x_{N1} x_{N2} x_{N3} . . . . x_{Nn}	y_N

Characteristic for this form of recording is the fact that in the process of the processing of tests, from each experiment several measurements of the instantaneous value of this factor x(t), equal to the number of discrete points t, are taken. Previously one should say that this form of matrix X is not optimum. It is completely obvious that the quantity of intervals of the measurement of the instantaneous values of factors x and characteristic y must be as large as possible.

The more dynamic the investigated process, and the higher the gradients of the change in its characteristics, the more accurate the initial data for determining the mathematical model must be determined. The assignment of the initial data for a dynamic model in the form of discrete values of factors is a very complex problem. But its solution is essential, since for the majority of the mechanical systems the functional dependence of a change in the instantaneous values of factors $x_1(t)$; $x_2(t)$; $\ldots x_n(t)$, as a rule, are unknown. However, the form of the telemetering recording of these factors can be known. For this let us examine Fig. 3.3. From an analysis of the telemetering recordings it follows that the accuracy of the modeling in this case will be affected by the number of experiments and the quantity of intervals Δt of the time of measurement of the instantaneous values of parameters x(t), $\Delta t_{ij}=t_i-t_j$ with $i>j$. For this, if we speak about the accuracy of the modeling of steady-state processes, the volume of the tests, was established in accordance with the resolution of the fractional replica. This condition remains valid also in this case when dynamic process is investigated. But, unfortunately not only does this condition determine the accuracy of our model. One should consider the selection of the interval of measurement Δt_{ij} as being no less important. Its effect can be shown in the following example.

In Fig. 3.3 in the construction of the dynamic model six intervals of the measurement of parameters $x_1(t)$ and $x_2(t)$ are taken. In this case the moments of recording of the parameters x_1 and x_2 do not coincide with their extreme levels. Nevertheless, the model characteristic on the first curve (dashed line)

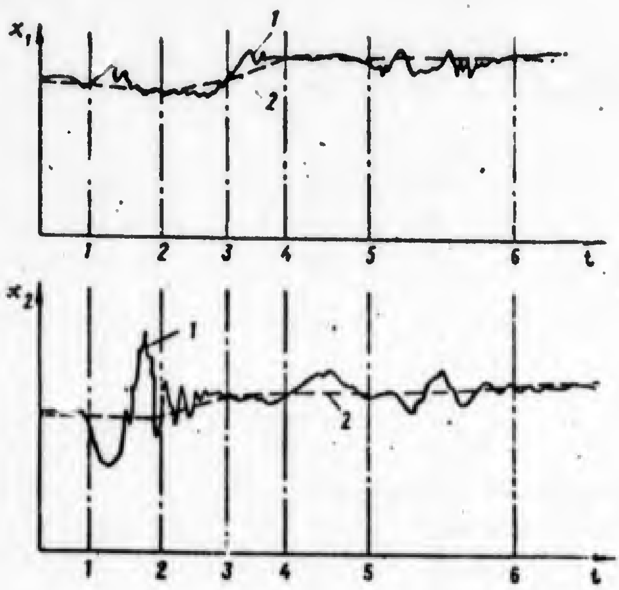


Figure 3.3. Recording of dynamic characteristics of parameters x_1 and x_2 and their models: 1 - oscillographic recording of the process; 2 - model curve.

characterizes to a certain degree the investigated process. On the second curve the model characteristic virtually completely does not correspond to the real process. For the providing of their fullest conformity to the physical process, obviously, the time intervals Δt_{12} and Δ_{45} and Δt_{56} must be additionally divided into smaller sections. To do this, in the processing of the primary information, obtained in the form of telemetering data, all the significant factors should be tied to one time scale. Then it is arbitrary to assign the time intervals of the measurements which most fully considered the physical features of the behavior of the considered parameters. Moreover, from the viewpoint of the selection of the quantity of intervals of the measurement, there are limitations only of the storage capacity of the computer and the time of its operation.

Further, at each established moment of time t_i with respect to all parameters from the telemetering curves the measurements which correspond to the specific experiment are taken and placed into a table of the type of Table 3.6.

Thus, the quantity of tables corresponds to the volume of the experiments. Their data are generalized and placed into matrices of initial data (Table 3.7).

Table 3.6

Time	Results of experiment	X	Y
t_1	$x_{11}\ x_{12}\ \ldots$	x_{1n}	y_{t1}
t_2	$x_{21}\ x_{22}\ \ldots$	x_{2n}	y_{t2}
t_3	$x_{31}\ x_{32}\ \ldots$	x_{3n}	y_{t3}
...	...	...	...
t_T	$x_{T_1}\ x_{T_2}\ \ldots$	x_{Tn}	y_{tT}

Table 3.7

No.	$x(t_i)$	$Y(t_i)$
	$x_1\ x_2\ x_3\ \ldots\ x_n$	
1	$x_{11}\ x_{12}\ x_{13}\ \ldots\ x_{1n}$	$y_1(t_i)$
2	$x_{21}\ x_{22}\ x_{23}\ \ldots\ x_{2n}$	$y_2(t_i)$
3	$x_{31}\ x_{32}\ x_{33}\ \ldots\ x_{3n}$	$y_3(t_i)$
...	...	...
N	$x_{N_1}\ x_{N_2}\ x_{N_3}\ \ldots\ x_{Nn}$	$y_N(t_i)$

The calculation of coefficients of the model of the process is conducted for each point in time t_i, i.e., the quantity of calculations in proportion to the number of time intervals. Further, by totaling the results of the conducted calculations, it is possible to obtain the mathematical model of the dynamic process y(t).

Examined until recently were the deterministic and random processes, which consider only quantitative information. In this case we completely do not touch upon qualitative information. In the study of mathematical models this condition was constantly observed: independently of the levels of variation and the number of introduced factors there was observed strictly an additive effect, by which the equality of the left and right sides of the linear equations at the point values of all variables was necessary. At the same time there is a vast circle of problems which require the account not only of the quantitative but also qualitative information classified according to the "yes-no" or "success-failure" criterion. This condition is frequently encountered in the solution of problems of the theory of games and an analysis of models of reliability. With the input of qualitative data the additivity of the equations of regression is disturbed, which makes the trial-and-error methods of constant coefficients unacceptable. Since the qualitative factors randomly can vary from zero to unity, it is not possible to achieve the equality of the left and right sides of the linear equations in all cases.

Nevertheless, there are mathematical methods which make it possible to make the final effects of such models similar to the additive because of a change in the scale of the measurement of the portion of the qualitative factor with the aid of any transform. In this case it is important that the converted value would change between very wide limits, for example, from infinity to zero, with a change in the very portion of the qualitative factor from zero to unity. For similar purposes in mathematical statistics the transforms into logits and probits are usually used. Used in some works of D. Finni and F. Yeyts [26] the transforms into logits are used with the aid of the expression

$$y = \frac{1}{2} \ln\left(\frac{P}{\Delta}\right),$$

where P is the probability, and Δ characterizes the interval of its measurement.

The form of this expression for a logit is not limited to the example given above. It can depend on the condition of the problem to be solved. Thus, for instance in the analysis of models of reliability, the transform into logits can be conveniently conducted by means of the equality

$$y = -\ln(F\ \Delta).$$

In this case understood by P is the frequency in the form of the probability of trouble-free operation estimated from a group of tests N_i ($P=m_i/N_i$, where m_i is the number of successful tests). The instantaneous value Δ_i in this case is equal to the ratio N_i/N. If for z we accept a quantity of groups in this program, then there must be provided the equality $N=\sum_{i=1}^{z} N_i$.

It is appropriate to note that the logit of D. Finni makes sense when approximately identical intervals or portions of the sampling Δ_i are present. If they are changed in the whole interval from zero to unity, then the expression for the logit does not determine the frequency. Therefore, in the analysis of models of reliability more acceptable is the expression

$$y_i=-\ln(P\,\Delta_i) \text{ or } y_i=-\ln\left(\frac{m_i}{N}\right).$$

At the values of the probability $P=1$, the equation of the logit $y=-\ln P\Delta$ completely corresponds to the expression $y=\ln P/\Delta$, which corresponds to the logit of D. Finni.

In the compilation of the matrix of initial data, the processing of test results somewhat differs from the usual methods. For this purpose all the tests are divided into groups with averaged (or approximately identical) values of the parameters. For each group the output quality is estimated by the experimental frequency approximated by the expression for the transform into the logits.

In the classification of the test results on each group (matrix row), the testing during which the material part remained undamaged is considered to be successful. Simultaneously, if there appears the need for considering a certain change in the design, it is possible to characterize by the input of new factor as an objective quantitative characteristic. For example, distinctions in the design of the injector assemblies of the combustion chambers can be expressed by a change in the combustion efficiency of fuel and the dimensions of the chamber by the value of the reduced length, and so on.

Let us examine now the question concerning the evaluation of the accuracy of modeling. It is natural that the accumulated error depends on errors in the method itself, the composition of the factors, and the volume and quality of information. When using the indicated models, the dispersion of the error in the description of the surface of the response can be estimated by a correlation matrix or the linearization method of the obtained function

$$\sigma_y^2 = \sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i}\right)^2 \sigma_{x_i}^2 + \sum_{i=1}^{n} \left(\frac{\partial^2 y}{\partial b_i}\right)^2 \sigma_{b_i}^2,$$

where $\sigma_{x_i}^2$ is the dispersion of estimates of the independent variables x.

When using methods of correlation functions the accuracy depends, first of all, on such factors as the interval of integration from the quantity of tests N, the step of integration and the number of ordinates of the correlation function to be determined in interval considered $0 \leq m \leq m_{max}$.

From the viewpoint of mathematical statistics, examined in this chapter are the stationary random processes for which known is the distribution law of the parameters and its characteristic in the form of the mathematical expectation and dispersion, which in the process of the testing are unstable. This can be uniquely confirmed by examining the input factors for which the distribution law during normal conditions is normal. The same can be said also about the output quality y. But since the random process of the characteristic change is stationary, it can be sufficiently accurately approximated by the linear combination of harmonic oscillations with the random amplitude and phase

$$y(N) = A \sin(\omega_i N + \varphi_i).$$

The analytical investigation of this function, given in work [59], makes it possible to draw the conclusion that for the calculation of the correlation function with an error of not more than 2% it is necessary to have the relationships $m_{max} \leq 0.1N$ and $\mu_{max} \leq 0.1\Delta$.

Thus the accuracy of the calculation of the correlation function K(N) is determined by the volume of statistical data (or by the interval of observation and the magnitude of the instantaneous value of argument m, which changes in the limit from from 0 to m_{max} and from the scale factor Δ).

The procedure for the determination of correlation functions on universal digital computers and the accuracy of their estimates are examined in specialized literature on computer technology in application to statistical studies [59].

In the analysis of mathematical models it is necessary to keep in mind that the connection between the "output" and "input" can be functional if the defined values of levels of the significant factors x correspond to any defined value y. This connection can be stochastic when at defined values of input factors the "output" randomly with defined probabilities can take different values. Thus the probability model of reliability is the special case of the stochastic model.

The use of probability models in the study of physical processes is based on the general principles of their reduction to the definite type of mathematical dependences. In a number of cases these models cannot absolutely accurately reflect the real processes. For this reason their formalization is connected with some simplifications. At the same time it should be noted that the precise reflection of the physical phenomena is not always required. Nevertheless, any formalization does not eliminate the evaluation of the accuracy which is reached by the input of the accumulated error of modeling ε. Practice shows that the advantages from the formalization of the processes in the planning of the tests, the forecasting of results and in general the modeling considerably exceed those negative consequences which can be observed as a result of an inaccuracy in the methods themselves.

With the physical investigations and in the analysis of qualitative indices, it is necessary to consider that not all factors introduced into the experiment are significant. Therefore, their effectiveness can be the different directivity. Secondary factors should be eliminated, and their effect should be considered by means of the averaging of results when evaluating the error of modeling.

The special case of the stochastic form of the bond is the correlation between the "output" and "input." The random values $\vec{x}$ and y are correlated if the mathematical expectation y_0 depends on the mathematical expectation of the levels of the significant factors x_0.

The most important moment of the construction of the mathematical model undoubtedly is the selection of the form of the bond between the "output" and "input." These bonds can be different. Besides those which were examined above, others can be proposed:

a) the linear model

$$y = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_n x_n, \tag{3.5}$$

which is the special case of the polynomial (3.1);

b) the power model

$$y = a_0 x_1^{a_1} x_2^{a_2} \ldots x_n^{a_n}. \tag{3.6}$$

c) the exponential model

$$y = a_0 a_1^{x_1} \cdot a_2^{x_2} \ldots a_n^{x_n} \tag{3.7}$$

The selection of model is dictated by features of the physical process. Nevertheless, the selection of the form of the bond can be automated on the basis of the sorting of different analytic functions with the aid of a digital computer. This selection involves the calculation of parameters of correlation at different variants of the mathematical interpretation of the process, the analytical comparison of the results and the selection of the best version. As an objective criterion of the selection of the optimum form of the connection there can be used the coefficient of multiple correlation under the condition of the adequacy and satisfactory physical interpretation of the very model of the process. Accepted as the optimum is the model for which the coefficient of multiple correlation has a maximum value.

For this it is necessary to develop the algorithm of sorting which would make it possible in the shortest period to find the optimum form of the bond. The calculating program must provide for the reproduction of the model and modeling of processes with all the known forms of the bond of "input" with "output."

In the process of the final development of the complex technical systems it is not always possible to recreate virtually the pattern of clear planning of the experiments. There appears the mass of particular problems and programs which force us to retreat from the initially developed plans, subject them to correction, introduce new ideas, and so on. Besides this by virtue of different reasons, the experiments carried out at the "non-systematic" levels of variation appear. It is admissible that in this case there can be not the maximum but intermediate values of some parameters.

Thus in the process of the final development of technical systems there appears a complex of statistical data characteristic both for active and passive experiments. Then there arises the question in which measure the obtained experimental data can be used in the development or refinement of the mathematical models of the physical processes.

Investigations show that in the construction of the models there is no distinction in the initial experimental programs of special importance. It is important to insure the condition of the account of the effect of all the significant factors from the viewpoint of the completeness of their coverage and the identity of the operating conditions. We have in mind that the totality of the statistical data which characterize the steady-state conditions of operation can be augmented by results of the passive experiment carried out also in steady-state conditions. It is inadmissible to generalize the test results which refer to transient conditions with data for steady-state conditions and vice versa. In the accomplishing of these conditions virtually any experimental data in an arbitrary order can be included in the matrix of the tests X and the column vector Y for aftertreatment. With this the more the initial data is utilized for these purposes, the larger area they cover, and the less the step of the variation of factors, the more precise the model.

§ 4. RANKING OF THE SIGNIFICANT FACTORS AND THE SELECTION OF THEIR NUMBER. ADEQUACY OF THE MODEL

An increase in the number of factors leads to the complication of the model and solution of the problem of planning of experiments as a whole. Therefore, the inclusion of supplementary factors requires their analysis from the viewpoint of the evaluation of significance, which in turn is determined by the physical structure of the investigated process. In the determination of the initial data for any plan in the form of quantity and rank of the significant factors, it is possible to accept the objective solution that it does not exclude an account and subjective opinion of the researchers.

Utilized in the first case are methods of mathematical statistics which consider the composition of the initial data, and in the second case - the subjective evaluations of separate specialists who introduce supplementary information and therefore deserve attention. Both methods are constructed independently of one another, but they should not mutually exclude each other.

Let us examine the method of the ranking of factors by means of interrogation. This means that before the planning of the experiments each of the specialists must express an opinion about the degree of the significance of the considered factors or about their effect on the output quality of the process. It is natural that this opinion must be based not on intuition and the assumptions of the interrogated person but on his personal experiment and on a priori information known to him. It can appear that this type of interrogation introduces nothing into the calculation except subjective estimates. However, the generalization of somewhat even subjective evaluations of specialists of this field of knowledge at times is much nearer to truth than the procedure deprived of the physical essence. In this case it is expedient to utilize the rank method of the formalization in the selection of the factors. Its essence can be shown in an example.

Let us assume that the engineer who planned the experiment prepared the initial data and program in the form of a matrix of tests. In this case the factors were arranged in the order of their assumed significance, which is determined by the degree of the effect of each of the factors on the investigated process. In summation, the line of the initial data which characterize "input" is obtained:

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7.$$

However, for objectivity there was produced an interrogation with which four specialists independently of each other represented their data. The results of interrogation are given in Table 3.8.

Table 3.8

Specialist	x_1	x_2	x_3	x_4	x_5	x_6	x_7
1	1	3	2	5	4	6	7
2	3	1	2	5	4	7	6
3	2	1	4	3	5	6	7
4	1	4	3	6	5	7	2

The data presented can be subjected to an analysis of variance.

An evaluation of the uniformity or discrepancy in the opinions of the specialists can be conducted with the aid of the F-criterion.

If the obtained value of the criterion does not enter in the zone of the F-distribution, then this means that the opinions of specialists about ranks are contradictory.

The degree of the coordination of results of the interrogation can also be checked by means of the coefficient of concordation [2] [Translator's Note. Term not verified] computed from the formula

$$\omega = \frac{12S}{m^2(n^3 - n)},$$

where S is the sum of the standard deviations of the sum of ranks from the average value; m - the quantity of interrogated specialists.

In mathematical statistics there are many methods of the determination of significance criteria, but they do not always satisfy the experimenters, since some of them do not have a clear connection with the physical interpretation of the problem. In this respect one should consider the methods of analysis of variance more acceptable.

If the conclusion about the mismatch in the opinions of the specialists is obtained, then it is possible to continue the investigation about the ranks. For this purpose let us construct a diagram which strictly gives an answer to the question posed. For this purpose it is possible to utilize the usual scale-number system in which given as the "first" place is the quantity of points equal to the number of factors n. Given as the "second" place is the quantity of points equal to (n-1) and so on. In the construction of the diagram, plotted along the axis of the ordinates is the sum of the obtained points divided by the number of specialists. Then on the axis of abscissas we obtain strictly the order of the significance of the factors. In our case (Table 3.8) this is

$$x_1, x_2, x_3, x_5, x_4, x_7, x_6.$$

A diagram of the ranking is shown in Fig. 3.4.

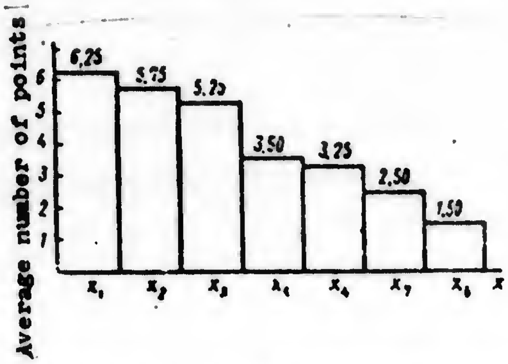


Figure 3.4. Diagram of the ranking of significant factors.

Sometimes the specialists who participate in the interrogation supplement the presented enumeration of factors or eliminate some of them. In this case the problem is solved similarly. In this case the processing and analysis of the interrogation can be conducted by taking into account the condition of randomization or without it. It is appropriate to note that the randomization of the order of the representation of results lowers the error and unconditionally gives a more precise result.

After these comparatively simple operations there can be accepted the solution to the composition of the significant factors, which is an important condition for the substantiation of the test plan

With the machine methods of the processing of the tests for the ranking of the factors, the particular set relationships which characterize the correlation between each input factor and the output quality can be utilized. This type of automated ranking of factors x according to the degree of the "purified" closeness of the connection with the characteristic y can be conducted in the following way.

1. The matrix for the set correlation ratios η_{yx_i}, η_{x_iy} and $\eta_{x_ix_j}$ of the form of Table 3.9 is determined.

Table 3.9

	x_1	x_2	x_3	...	x_n	y
x_1	1	$\eta_{x_1x_2}$	$\eta_{x_1x_3}$	...	$\eta_{x_1x_n}$	η_{x_1y}
x_2	$\eta_{x_2x_1}$	1	$\eta_{x_2x_3}$	...	$\eta_{x_2x_n}$	η_{x_2y}
x_3	$\eta_{x_3x_1}$	$\eta_{x_3x_2}$	1	...	$\eta_{x_3x_n}$	η_{x_3y}
...	...	...	...	...	...	...
x_n	$\eta_{x_nx_1}$	$\eta_{x_nx_2}$	$\eta_{x_nx_3}$	...	1	η_{x_ny}
y	η_{yx_1}	η_{yx_2}	η_{yx_3}	...	η_{yx_n}	1

2. On the basis of this matrix the analytical expressions for y_i and y are found:

$$y_i = b'_0 + b'_1 x'_1 + \ldots + b'_k x'_k;$$
$$y = b_0 + b_1 x_1 + \ldots + b_k x_k + \ldots + b_n x_n,$$

where x'_j are factors on which y_i depend; $1 \leq j \leq k$.

3. The particular correlation ratios are determined from the formula

$$\dot{\eta}_{y_i y} = \frac{b_1' b_1 \sigma_{x_1}^2 + \ldots + b_k' b_k \sigma_{x_k}^2}{\sqrt{(b_1')^2 \sigma_{x_1}^2 + (b_k')^2 \sigma_{x_k}^2} + \sqrt{b_1^2 \sigma_{x_1}^2 + \ldots + b_n^2 \sigma_{x_n}^2}}.$$

4. The ranking of the factors is carried out according to the degree of distribution of the correlation ratio in the interval from zero to unity:

$$0 < \bar{\eta}_{y_i y} < 1.$$

A shortcoming of this method should be considered its complexity and sensitivity to the volume of information.

Both the described methods are constructed virtually independently of one another. However, only the method of expert evaluations, which is based on knowledge of physics of the processes, the experience of experts and a priori information, can objectively introduce concrete factors into the plans. The method of statistical evaluations, in turn, makes it possible to exclude the trivial from them. Consequently, both methods not only do not exclude but mutually supplement each other. In practice both methods establish at which composition of factors the model more precisely corresponds to the experimental data.

Therefore, when selecting the composition and rank of the significant factors, it is expedient to use both methods with the only difference being that the method of interrogation is applied when selecting the composition and rank of the significant factors and the method of correlation ratios - for their subsequent ranking by machine methods in the examination of results of the tests.

At the same time the testing of the significance of coefficients of the model is necessary. In certain cases it has a direct relation to the evaluation of the significance of the factors themselves and their linear effects. This is observed when in the testing of the significance of coefficients of the model some of them proved to be insignificant. The reason for this could prove to be the trivial effect from the factor which by mistake was included in the experiment. As a final result, if the established model is reliable, the insignificant factors must have coefficients close to zero.

Products of coefficients of the model by independent variables and their interactions do not always make it possible to establish the quantitative effect and directivity of each of the factors on the investigated process. In a certain sense the exact solution of this problem gives orthogonal planning. In such case the regression coefficients are calculated from the formulas

$$b_0 = \frac{1}{N}\sum_{j=1}^{N} y_i \quad \text{and} \quad b_i = \frac{1}{N}\sum_{j=1}^{N} y_j x_{ij}.$$

The orthogonal plans provide the independence of the coefficients and their estimates, which is very important for practice. This is especially desirable with the input of the new revealed factors or with elimination from the experiment of trivial ones. The condition of orthogonality takes the form

$$\sum_{1}^{N} x_{iN} x_{jN} = 0 \text{ with } i \neq j;$$

$$\sum_{1}^{N} x_{iN}^2 = 0 \text{ with } i = j.$$

With the accomplishing of the indicated conditions, dispersions of the estimates are minimum and equal to each other.

Determination of the significance of coefficients of the model makes sense before the test work, since the screening of at least one of them leads to a considerable simplification in the experimental program. When several test series are carried out, proving useful can be the method according to which the assumed significance of the factors prior to experiment is judged, and the value and directivity of the effect after the processing of the primary data are checked. If there is no agreement even in the directivity of the effect, then this factor from a further program is excluded as not corresponding to the physical essence of the investigated process. A new mathematical model is constructed on the basis of this premise. The tests are continued. If it seems that the convergence of experimental and theoretical data were not reflected by the solution to the screening of the factors, the solution taken remains in force. But if this led to undesirable consequences, then the factor excluded from the program is again included in the experiment. Operations on the checking of the significance of the factors as the final result can be automated.

The methods of testing of the significance of regression coefficients are well-known [2], and the conducting of them does not represent special complexity.

To evaluate the adequacy of the model, we usually use the variance ratio in the form of Fischer's criterion

$$F = \frac{\sigma_y^2}{\bar{\sigma}_y^2},$$

where

$$\sigma_y^2 = \frac{\sum_{i=1}^{N} (y_i - y_0)^2}{N-1}; \quad \bar{\sigma}_y^2 = \frac{\sum_{i=1}^{N} (y_i - \bar{y}_i)^2}{(N-n)-1}.$$

In this case understood by characteristic y_0 is the mathematical expectation, and by $\bar{y}_1$ - the computed values of the variable y.

An evaluation of the adequacy of the model is necessary for the testing of the conformity of test results of the obtained mathematical model. If model is adequate, further study of the surface of the response and the optimization of its parameters can be carried out. The reasons for the inadequacy of the model can consist also in the unsuccessful selection of intervals of the variation of factors in the direction of their decrease. The model can be inadequate if at the zero value of the parameter of optimization the extreme field immediately proved to be reached.

Thus in the case of the inadequacy of the model, it is required to repeat the experiments with the changed intervals of variation. For complex technical systems this is undesirable. At present the only output which prevents a similar error is, in our opinion, the fictitious realization of the developed plan by a computer with the aid of test results of analog systems. Such a "guarantee" of the adequacy of the model, strictly speaking, is conditional, but virtually this measure will always be justified.

Physically the convergence of estimates of the adequacy of the developed and analog models can be explained by the fact that the development of new systems, as a rule, is conducted in the direction of the modernization and perfection of "previous" plans. Because of this the intervals of the variation of basic physical parameters, which determine the process itself, can be very close, although their basic levels are different.

§ 5. PASSIVE AND ACTIVE EXPERIMENT IN THE FINAL DEVELOPMENT OF SYSTEMS AND ELEMENTS

In a number of cases when the experimental final development of some system is conducted by methods of empirical search, the division of reliability can be found in the position of the "passive" observer who processes the test results but who does not interfere actively with the experiment, in particular, in the sorting of the test programs. Thus it occurs, for example, when selecting the nominal values of some design and technological characteristics of the units and when evaluating the extreme values of the basic parameters, and their allowances and scatters.

An example of a passive experiment can be called the hydraulic flow tests of combustion chambers for the purpose of determining the pressure differentials along the cooling channel, injector assembly, and so on.

A passive experiment is carried out in the absence of sufficiently precise data on the functional connections between the system's elements when the experimenter actually "gropingly" finds one of the possible ways of providing the efficiency of the designed version of the system. In this case it is impossible to estimate which one of the ways of providing the assigned operational indices is optimum. For a solution to this problem, by the methods of empirical search, it is necessary to carry out a large volume of experimental works. It is possible to reduce their quantity if the functional connections of the dynamic characteristics of the system or the mathematical model of the investigated process are known. In this case the experimenter can utilize methods of mathematical statistics and computer technology, including mathematical programming. However, in the passive experiment it is not possible to foresee results beforehand.

With the realization of the active experiment there are possibilities for the control of it. In this case it is checked how accurately test results confirm the theoretical data obtained by the optimization of the mathematical model of the real physical process. This is strictly the basic difference in the methods of the conducting of passive and active experiments.

However, their fundamental evaluation requires a critical approach to the statistical methods of the planning of experiments in general. Before processing the experimental data, the physical process virtually is not considered. The same plan of the full factor experiment, for example, with four independent variables x_1, x_2, x_3, and x_4 and two levels of variation (plan of 2^4 type) can be used in an equal measure both for the ZhRD [ЖРД - liquid-propellant rocket engine] and an electronics device. A certain physical feature of the process can be considered only when selecting the generating relationships of the fractional factor plan. However, this selection in a larger measure is a subjective process rather than an objective one, and therefore even here errors are not eliminated. The same can be said also about the majority of other statistical methods of the planning of the experiments.

If a priori information is absent or not considered, it is not possible to create the reliable plans which reflect the physical essence of the processes. It is possible to speak about the objectivity of the methods only in the presence of experimental data with the aid of which the mathematical model of the process is preliminarily described. Objective plans can be constructed only on the basis of its analysis. In summation it is possible to raise the question concerning the development of the method of the planning of tests for the purpose of studying the complex physical processes, which considers in the initial stage a priori and then a posteriori information. The essence of the method consists in the fact that in the development of concrete plan there is applied information not only about the composition and ranks of the factors (results of expert estimates), but mainly the a priori model of the process

$$y_a = f_1(\vec{x}).$$

Values of y_a are established according to test results of the prototype engine.

A distinctive feature and advantage of this method are the use in the development of a new plan of the mathematical model of the previous plan. This introduces into the mathematical structure of the plan physical features of the investigated process with respect to the concrete system and supplements the volume of the utilized information, which, in the initial stages of the experimental works, can be borrowed from analog systems. The planning of the active experiment symbolizes the transfer from a priori information to a posteriori information under the condition of the modeling and subsequent optimization of the physical processes.

In accordance with this the step-by-step method of planning, based on the study of mathematical models of processes, is proposed. In the first stage of the planning the model is accepted as being a priori and can be assigned in the form of appropriate coefficients (partial derivatives of characteristics of system) for a certain composition of the significant factors, which determine the efficiency of the existing analog model. The planning can be conducted by the method of steep ascending of the response along the surface. Sufficiently fully this method is presented in works [2, 47]. The advantages of this method are the small volume of experiments for the realization of the optimum plans and also its continuity in the development of test programs of complex technical systems from the viewpoint of convenience in the interval variation of the independent variables.

The quantity of tests of the first stage is accepted on the basis of the condition

$$N_1 > n_a + 1,$$

where n_a is the quantity of factors of the a priori model.

On the basis of a mathematical analysis of the models, it can be concluded that the sufficiently precise description of a number of the physical processes can be based on the volume of information which corresponds to the dependence

$N > 6n$.

In this stage the real interval and step of variation should be assign in relative values, which quantitatively correspond to levels of the variation of factors of the a priori model.

The second a posteriori stage of planning is preceded by the realization of the plan of the first stage, the processing of test results and the refinement of the a posteriori model in the form of the regression coefficients and the composition of the factors. In this case it is necessary to remember that the volume of a priori information in the first stage considerably exceeded the a posteriori, as a result of which it is as though the "absorption" of information of the test results of the first stage can occur. If it seems that the results of two forms of information are incompatible with each other or the composition of the factors essentially differs, then the a priori data are excluded in general as reliable.

In the implementation of plans for each stage, the test procedure is established by taking into account the limitations on randomization. The tests are conducted in a sequence determined by means of a table of random numbers or by means of coin tossing (heads-tails).

In principle the proposed step-by-step planning of the final development of the complex technical systems can be conducted also on the basis of fractional factor plans, D-optimum planning and other statistical methods.

A similar type of plan for a certain number of factors n can be schematically represented in the form of Table 3.10. It is completely obvious that the development of this plan cannot be limited to two stages, but further planning does not cause any fundamental difficulties, since it is conducted by the usual methods.

Table 3.10

	Composition of the significant factors							Output quality
A priori model	x_1'	x_2'	x_3'	....	x_i'	....	x_n'	
Coefficient of the model	B_1'	B_2'	B_3'	....	B_i'	....	B_n'	
Main level	a_1	a_2	a_3	....	a_i	....	a_n	
Upper level	+	+	+	....	+	....	+	
Lower level	−	−	−	....	−	....	−	
Interval of variation	Δ_1	Δ_2	Δ_3	....	Δ_i	....	Δ_n	
Step of variation	$\Delta_1 B_1'$	$\Delta_2 B_2'$	$\Delta_3 B_3'$	....	$\Delta_i B_i'$	....	$\Delta_n B_n'$	
Planned tests of stage I:								
4	$a_1+\Delta_1 B_1$	....	....	....	....	....	$a_n+B_n'\Delta_n$	y_4
1	......	....	....	....	....	....	......	....
7	......	....	....	....	....	....	......	....
.	.							.
.	.							.
.	.							.
N_1	$a_1+N_1\Delta_1 B_1'$	....	....	....	....	....	$a_n+N_1\Delta_n B_n'$	y_{N_1}
Coefficient of the refined model	B_1	B_2	B_3	....	B_i	....	B_n	
Refined composition of the factors	x_1	x_2	x_3	....	x_i	....	x_n	
Refinement of the step	$\Delta_1 B_1$	$\Delta_2 B_2$	$\Delta_3 B_3$	....	$\Delta_i B_i$	....	$\Delta_n B_n$	
Planning of tests of stage II:								
3	$a+\Delta_1 B_1$	....	....	....	....	....	....	y_3
8	$a+2\Delta_1 B_1$	....	....	....	....	....	....	y_8
5	$a+3\Delta_1 B_1$	....	....	....	....	....	....	y_5
1	$a+4\Delta_1 B_1$	....	....	....	....	....	....	y_1
6	$a+5\Delta_1 B_1$	....	....	....	....	....	....	y_6
.								.
.								.
.								.
N	$a+N\Delta_1 B_1$	....	....	....	....	....	....	y_N

§ 6. METHODS OF THE SOLUTION OF MATHEMATICAL MODELS OF COMPLEX PROCESSES

Stages of the planning of tests are completed by the realization of the developed programs, as a result of which experimental material for the construction of mathematical models of the investigated processes is accumulated. The initial data for the calculation are conveniently represented in the form of matrices of initial data X and vectors Y:

$$X=\begin{vmatrix} x_{11}x_{21}x_{31}\ldots x_{n1} \\ x_{12}x_{22}x_{32}\ldots x_{n2} \\ \ldots\ldots\ldots\ldots \\ x_{1N}\,x_{2N}\,x_{3N}\ldots x_{nN} \end{vmatrix};\quad Y=\begin{vmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{vmatrix},$$

where each line characterizes the experiment or group of tests conducted virtually in stable conditions.

As the final result all forms of the models (3.1), (3.5), (3.6) and (3.7) can be reduced to linear form by the corresponding transformation of the initial information:

1) the linear model

$$y=b_0+b_1x_1+b_2x_2+\ldots+b_nx_n+\varepsilon;$$

2) the nonlinear model

$$y=b_0+b_1x_1+\ldots+b_nx_n+b_{11}x_1^2+b_{12}x_2^2+b_{1n}x_n^2+b_{21}x_1x_2+\ldots+\varepsilon$$

3) the exponential model

$$\ln y=\ln a_0+a_1\ln x_1+a_2\ln x_2+\ldots+a_n\ln x_n+\varepsilon;$$

4) the exponential model

$$\ln y=\ln a_0+x_1\ln a_1+x_2\ln a_2+\ldots+x_n\ln a_n+\varepsilon.$$

The solution for each form of models can be obtained when the number of terms of the linear model R satisfies in the last resort at least the condition of the providing of the square matrix X, i.e.,

$$N > R + 1.$$

For linear, power and exponential models R=n. The most complex from the viewpoint of realization is the solution for a nonlinear model, since in this case the greatest quantity of tests with just one composition of the factors is required. This is explained by the presence of not only linear terms but of their interactions and exponential indices. Moreover, the higher the power of the polynomial which describes the process, the greater the volume of realizations N. However, investigations show that with modeling of the complex physical processes the use of a polynomial of the second power is sufficient [47]. But if the discussion is about the local zone of the surface of the response or about the investigation of its extremum, as a rule, it is possible to be restricted to a plane. The quantity of terms for the polynomial of the square is shown in Table 3.11.

Table 3.11

n	b_0	b_i	b_{ii}	b_{ij}	s
5	1	5	5	10	21
6	1	6	6	15	28
7	1	7	7	21	36
8	1	8	8	28	45
9	1	9	9	36	57
10	1	10	10	45	66

Here b_i is the quantity of linear terms; b_{ii} - the quantity of quadratic terms; b_{ij} - the quantity of members of interactions; b_0 - the quantity of fictitious variables; n - the quantity of factors.

From practice it follows that the quantity of tests necessary for the solution of the nonlinear model satisfies the condition

$$N \geqslant (5 \div 6)n.$$

In all the remaining cases examined above the following condition is correct:

$$N \geqslant (n+1).$$

Hence a certain lower limit for the volume of tests necessary for the description of the mathematical model of the investigated process can be established.

The processing of the test results is conducted, as a rule, by the method of least squares. With the number of factors greater than 4-5, the calculation of the coefficients should be conducted in a computer. In matrix recording the solution can be presented in the following order.

1. The matrix of the initial data is transformed - X^*.

2. The multiplication of matrices is conducted - XX^*.

3. Matrix 2 is inverted - $(XX^*)^{-1}$.

4. The transformed matrix X^* is multiplied by vector Y - X^*Y.

5. Matrices 3 and 4 are multiplied - $(XX^*)^{-1}(X^*Y)$.

The final expression represents the column vector of coefficients of linear equations.

The statistical analysis of dependences 4 and 5 showed that in the solution to problems on the digital computer some difficulties can be encountered; for example, with matrix inversion the determinant can prove to be close to zero, or with the multiplication of the matrices the large numbers which cannot be written in the cell of the computer are obtained. Therefore, in the development of calculating program it is expedient to provide also the machine method of the selection of the scale of initial information.

In summation, the algorithm of the calculating program can be represented in the following way.

1. Determination of the composition of significant factors, the introduction of the initial data and ranking of independent variables.

2. Solution of the models: linear, nonlinear, power and exponential. Determination of the coefficients of linear equations.

3. Selection of the scale of initial information and the checking of its equipment to one general set (exclusion of experimental data which fell outside $\pm 2\sigma$).

4. Checking of the significance of the regression coefficients and adequacy of the models.

For further investigations that form of the model which in the best measure satisfies the F-criterion of the significance or has a minimum error of the linear approximation is accepted.

§ 7. PREDICTION OF TEST RESULTS

Prediction is an element of dynamic programming. It is utilized to evaluate the possible technical state of systems with the most responsible tests, for example, the day before the putting of a space vehicle into orbit or with bench tests of the stages, and so on. The basic means for the prediction of test results at present should be considered the methods based on the description of mathematical models of complex physical processes by the generalization and processing of experimental data. The mathematical form of the model in this case takes the form

$$y=\sum_{i=0}^{n} b_i x_i+\ldots+\varepsilon,$$

where coefficients b_i are considered as the partial derivatives of the individual characteristics of the DU [ДУ - engine installation].

If the model with the known degree of accuracy (1-ε) corresponds to the investigated process, then on the eve of next responsible testing initial data of the program of planned experimental works can be introduced into it. In accordance with the assigned accuracy an answer - prediction can be obtained. This makes it possible for the designer and experimenter to program the model on a computer and as required to carry out "experiments" on the computer before their full-scale realization and thus foresee the expected result. Depending on the computing program, the prediction can be conducted according to the output dynamic characteristics of the system or by the evaluation of the probability of the providing of its good working order. The methods examined above of the construction of mathematical models of processes by the processing of a priori information serve virtually in an equal measure both the problems of planning and prediction of the test results.

One of the real examples of prediction can be the model "Mark-7A" [80] utilized for the prediction of results of planned flight tests of the DU of the first stage of the carrier rocket "Saturn-5." A description of this model was carried out according to results of bench and flight tests of the DU in the composition of stages S-1C, acceptance tests of F-1 engines in the composition of a cluster on the technological stage and so on. The diagram of the model "Mark-7A" provides for an account of the effect of input factors and their maximum deviations on the real physical process. As a whole the computing program includes the model of the DU, model of a change in the flight mass of the rocket stage in proportion to the consumption of the propellant components from tanks and model of external measurements. Results of the calculation as the final result are utilized for determining the maximum deviations of parameters of the flight trajectory during the finite phase. In proportion to the entrance of a posteriori data, the coefficients of the effect of characteristics of the DU can be more precisely formulated. Furthermore, with the aid of the program of the model "Mark-7A", for different flight situations about 250 parameters of the DU can be calculated.

The computing program considers the effect and instability of the thrust, specific thrust, fuel component ratios, boost pressure, mass of the filled tanks, temperature and specific mass of the propellant components, and so on. Many of the indicated parameters are established by taking into account the specific date of the planned testing and the real adjusting and weight characteristics. For this purpose the telemetering with check-technical acceptance tests of the DU are entirely used.

From the viewpoint of the technological possibilities, the same computing programs and mathematical models can be utilized both for a prediction of results of the tests, the modeling of the functioning of the DU, and for the subsequent analysis of results of the realized experiment.

In summation, a block diagram of the universal mathematical model of the article presented in Fig. 3.5 can be examined. As can be seen from the diagram, the model has two "inputs" (for parameters of the DU and the weight characteristics of the

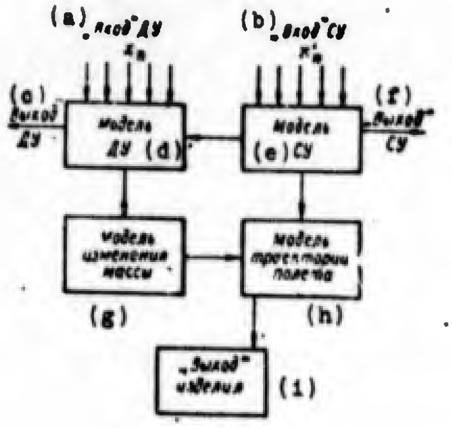


Figure 3.5. Block diagram of the computing program on the basis of the mathematical model of the article.
Key: (a) "Input" of DU; (b) "Input" of SU [translator's Note. Acronym unknown]; (c) Output of DU; (d) Model of DU; (e) Model of SU; (f) "Output" of SU; (g) Model of mass change; (h) Model of flight trajectory; (i) "Output" of article.

stage. The model is not limited to the functional "output" of the DU but makes it possible to determine the degree of the effect of the input parameters of the DU, weight characteristics and flight program of the article on extra-trajectory deviations. As required the problem can be limited only to a determination of the output characteristics of the DU. This model contains elements virtually characteristic for any rocket system. However, in each concrete case the computing program must possess elements and properties characteristic only to this plan. The presented method of prediction is of practical importance, since it makes it possible to combine the mathematical model of the "prediction" with methods of factor analysis. This considerably expands the methodological statement of problems in prediction examined in some works of A. G. Ivakhnenko [25].

The solution to problems of prediction and also in the final development of complex technical systems is important. The specific nature of these problems lies in the fact that the model of the process is being constantly improved from one experimental program to another. And this means that the careful modeling of intermediate versions of the plan and their investigation makes it possible to "experiment" with the aid of computing programs on a computer and not by the conducting of full-scale tests. Before rejecting any structural solution in favor of another, they both must be investigated by means of modeling. In final development the planning and realization of the experiments must be carried out in the quantities necessary for the description of the mathematical model of the physical process and for the conducting of monitoring tests, desirably, extreme combinations of the significant factors. In all the remaining cases one should investigate the process and determine characteristics of the system by means of predicting and mathematical modeling. Only such plans make it possible to reduce optimally the quantity of tests.

As follows from an analysis of the method of steep ascension on the surface of the response and other statistical methods of the planning of tests, the volume of experimental works necessary for a description of mathematical models of the processes is completely realistic. Usually it does not exceed several dozens of tests, but in a number of cases when the number of factors is 7 or 8 the solution to the problem can be reduced to 15-20 tests. In the initial stages of the development of complex technical systems, the prediction can be reduced to the construction of an approximate model. This, in turn, also lowers the requirements for the volume of full-scale tests.

§ 8. FACTOR PLANS WITH THE QUANTITY OF LEVELS OF VARIATION MORE THAN TWO

In the existing literature on mathematical statistics, the plans of type 2^n are sufficiently fully examined. However, there is a wide range of problems connected with the tests and final development of the complex technical systems for which the quantity of levels of variation d can be more than two. Such plans have the conventional designation d^n, where d can be any integer. In our specific case $d>2$.

Plans for type 3^n are of the greatest practical interest. These plans are not any exclusion in the test theory, since they comprise an integral part of programs of the final development for the majority of the technical systems.

The practical conditionality of the considered plans is explained by the complexity of the systems themselves, the assigned range of variation and physical features of the investigated processes. Thus, for instance, for controllable ZhRD there are, as a rule, three basic operating modes - nominal, boosted and throttled. It is natural that these systems for real articles are actually the assigned levels of the variation of the basic parameters. Some of them differ by a special range. In their number, in connection with the ZhRD, one should name the temperatures of structural elements and propellant components. For them the range of variation can comprise of dozens and even hundreds of degrees. This leads to the fact that in the investigation of the entire range of a change in the factors, sometimes the physical process itself is changed. The latter is easily shown in an example.

At the low values of temperatures of the propellant and structural elements, the starting of the engine, as a rule, occurs under conditions of the normal flow of liquid along the main-line channels and through the injectors of the combustion chamber and gas generator. At high temperatures the picture sometimes sharply changes. It is natural that the characteristics of the course of the processes of starting in both cases differ from one another. The maximum levels of the variation of factors are characterized by this phenomenon. Within the considered temperature range one should expect the course of the intermediate processes.

Consequently, if for the complex technical system, which has a broad range of the variation of factors, we are restricted only to the two maximum levels, then their replacement can be accompanied by a change in the physical processes. In that case the latter cannot always be modeled. If this is possible, then it is only by means of the input of new factors which characterize the replacements of the physical constants. It is more expedient to introduce supplementary levels of the variation.

Thus, the use of factor plans of type d^n in the study of complex physical processes is a practical need.

In a certain measure plans of type 3^n have already been investigated [72]. The method of their dividing into units is developed. Plans 4^n are investigated very little, especially from the viewpoint of practical use.

It should be noted that the methods taken in mathematical statistics of the substantiation of the factor plans of type d^n with the number of levels of variation more than two encounter considerable difficulties in implementation, especially when we are speaking about tests of complex technical systems, including the ZhRD. Experience shows that the dividing of plans of type 3^n into units with the number of factors n>5 also requires an excessive volume of experimental works, and the supplementary division of the plans into units sharply lowers the reliability of the obtained results.

A comparison according to the number of experiments of plans of type 3^n and their fractional replicas with plans of type 2^n is presented in Table 3.12.

Table 3.12

n	1	2	3	4	5	6	7
2^n	2	4	8	16	32	64	128
3^n	3	9	27	81	143	729	2187
3^{n-1}	1	3	9	27	81	243	729
3^{n-2}			1	9	27	81	243
2^n+2^n	4	8	16	32	64	128	256
$2^n+2^n+2^n$	6	12	24	48	96	192	384

From the table it is evident that in comparison with plans of type 2^n for the total factor experiment 3^n, the quantity of tests is one order more. The matrix of the tests for plan 3^n is shown in Table 3.13. In this case the three levels of the variation, the upper, lower and nominal, are designated, respectively, by (+), (-) and (0).

Table 3.13

$N=3^3$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
X_1	0	0	0	0	0	0	0	0	0	+	+	+	+	+	+	+	+	+	–	–	–	–	–	–	–	–	–
X_2	0	0	0	+	+	+	–	–	–	0	0	0	+	+	+	–	–	–	0	0	0	+	+	+	–	–	–
X_3	0	+	–	0	+	–	0	+	–	0	+	–	0	+	–	0	+	–	0	+	–	0	+	–	0	+	–

The given matrix causes the full combination of test conditions of this plan. Despite a comparatively small number of varied factors, it reflects its entire complexity from the viewpoint of realization. Therefore, the experimenters and engineers are forced to search for means more acceptable for practice in the solution to similar problems.

Although there is a distinction in the levels of variation of the factors, plans of type 3^n in principle can be compared with plans of type 2^n. For this purpose the following three diagrams are used: a, b and c (Fig. 3.6).

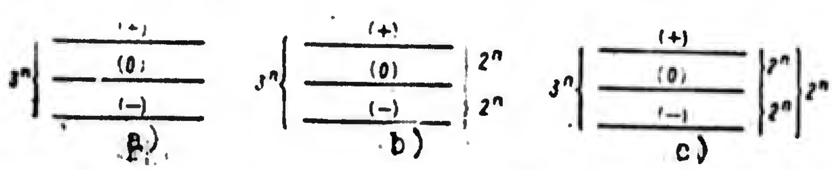


Figure 3.6. Diagrams of the plans for factor experiments of the type 3^n.

Diagram a represents a factor experiment of type 3^n.

Diagram b symbolizes the same factor experiment 3^n, but in this case an attempt is made to encompass three levels of variation with the aid of plans of type 2^n. We call the latter that which was combined with the designation (2^n+2^n), unlike plan 2×2^n, which is the double realization of the plan of type 2.

At the same time diagram c of the combined plan of the type $(2^n+2^n+2^n)$ can be examined.

Matrices of tests of plans of diagrams b and c are represented in the form of tables. Table 3.14 characterizes the plan 2^n with levels of variations (+) and (-); Table 3.15 - the matrix of the same plan with levels (0) and (-); Table 3.16 - the matrix of the plan with levels (+) and (0).

Table 3.14

№	x_0	x_1	x_2	x_3
1	+	−	−	−
2	+	+	−	−
3	+	−	+	−
4	+	−	−	+
5	+	+	−	+
6	+	+	+	−
7	+	−	+	+
8	+	+	+	+

Table 3.15

№	x_0	x_1	x_2	x_3
1	−	−	−	−
2	−	0	−	−
3	−	−	0	−
4	−	−	−	0
5	−	0	−	0
6	−	0	0	−
7	−	−	0	0
8	−	0	0	0

Table 3.16

№	x_0	x_1	x_2	x_3
1	+	+	+	+
2	+	0	+	+
3	+	+	0	+
4	+	+	+	0
5	+	0	+	0
6	+	0	0	+
7	+	+	0	0
8	+	0	0	0

By comparing the combined plans of types (2^n+2^n) $(2^n+2^n+2^n)$ with plan 3^n (see Table 3.12) and also plans (2^3+2^3) $(2^3+2^3+2^3)$ with plan 3^3, it is possible to note the following feature. According to the quantity of the tests, the combined plans are considerably more economical than plans 3^n, but they do not consider the combination of all maximum levels for the assigned number of factors (experiments No. 6, 8, 12, 16, 20 and 22 in Table 3.13).

Let us examine how it is necessary to consider these combinations of the levels of variation. Obviously, this depends on the condition of the problem. For this purpose it is expedient to examine the new variable - the conditional probability of the simultaneous observation of combinations of all maximum levels of variation.

In analyzing the diverse variants of the solution to the particular problem, it is easy to note the following. If the distributions of levels of factors are equiprobable, then the program of tests must consider all the maximum combinations provided by plan 3^n. But by itself this case is a rare phenomenon. More frequent maximum levels follow the normal distribution law. Then the combinations of all maximum levels for n factors are unlikely events, which gives to us the basis for recommending the combined plans (2^n+2^n) $(2^n+2^n+2^n)$ instead of the plans with the quantity of levels of variation of more than two.

It is appropriate also to note that the solution to the practical problems which covers n factors does not stipulate the variation of all variables with an identical quantity of levels. Thus, for instance, if it is required to estimate the starting of the engine in a sufficiently wide temperature range, for example, at four levels of variation, then there is completely no need for assigning the four levels for the other significant factors: inlet pressure, time of the arrival into the operation of automation elements and others. Moreover, the division of the full range of the variation of these factors into four levels (three zones) can lead to the fact that the regression coefficients in linear equations will prove to be insignificant, and the model is inadequate.

Thus, there appears a new problem - the construction of the combined plans with a variable number of levels of variation. If n_1 factors has three levels, and n_2 - a total of two, then the full factor experiment of this plan will be characterized by the number of tests $N=3^{n_1}\cdot 2^{n_2}$, where $n_1+n_2=n$. The quantity of the tests for plans of type $3^{n_1}\cdot 2^{n_2}$ can be judged from Table 3.17.

Table 3.17

n_2	n_1 = 1	2	3	4	5	6	7
0	3	9	27	81	243	729	2187
1	6	18	54	162	486	1458	4374
2	12	36	108	324	972	2916	
3	24	72	216	648	1944		
4	48	144	432	1296			
5	96	288	864				
6	192	576					

The broken line along the diagonal intercects the plans with an identical number of factors ($n_1+n_2=7$).

From an examination of Tables 3.12 and 3.17 it follows that a plan of type $3^{n_1}\cdot 2^{n_2}$ is considerably more economical than plans 3. With this the quantity of tests is less, the less the factors with three levels of variation.

In the limit with $n_2\to 0$ plans of type $3^{n_1}\cdot 2^{n_2}$ approach plans 3^n and, on the contrary, with $n_1\to 0$ they approach plans 2^n.

In the presence of all three factors the plan of type $3^1\cdot 2^2$, unlike plan 3^3, is characterized by 12 experiments instead of 27. The matrix of this plan is given in Table 3.18 (compare it with the matrix of the plan of type 3^3 in Table 3.13).

The determining contrast of this plan will be

$$J = x_1^3(x_2x_3).$$

Table 3.18

$N=3^1\times2^2$	1	2	3	4	5	6	7	8	9	10	11	12
X_1	0	0	0	0	+	+	+	+	−	−	−	−
X_2	+	+	−	−	+	+	−	−	+	+	−	−
X_3	+	−	+	−	+	−	+	−	+	−	+	−

The division of the plan $3^1\times2^2$ into units leads to the fact that even for a half-replica the inner effects are indistinguishable. Therefore, planning of this type as a whole is unsatisfactory. Consequently, the plan $3^1\times2^2$ requires the realization of all experiments (N=12). Similarly, it is possible to examine plans $(3^1\times2^3)$, $(3^1\times2^4)$ and so on.

From the viewpoint of the practical use of plans of type $3^{n_1}\times2^{n_2}$, in a number of cases the combined plans can prove to be advisable also. Diagrams of their realization are shown in Fig. 3.7.

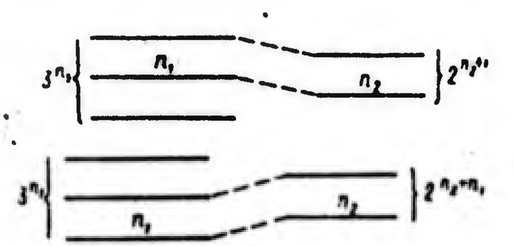


Figure 3.7. Diagrams of the combined plans for factor experiments.

However, one should recognize that the combined plans possess a deficiency as a result of which the maximum combinations of the levels of variation of all the significant factors are eliminated. In a number of cases this is inadmissible, for example, in the implementation of test programs, according to which the operation of the ZhRD at extreme values of the external and internal factors is checked.

The realization of the plans of type $3^{n_1}\times2^{n_2}$ can occur also by another way, which is most promising. In this case each group of factors n_1 and n_2, being independent of each other, is determined by the program which requires for its realization a conditional quantity experiments N_1 and N_2. **Correspondingly,** $N_1=3^{n_1}$ and $N_2=2^{n_2}$, and the total quantity of tests $N=N_1N_2$.

By analogy with plans of type 2^n and 3^n, fractional plans with the multiplicity of replicas k_1 and k_2 can be proposed:

$$N=3^{n_1-k_1}\times 2^{n_2-k_2}.$$

For the purpose of their illustration, let us examine a certain plan of the type $3^1 \cdot 2^{3-1}$ (with one half-replica), which considers the effect of four significant factors: x_1, x_2, x_3, and x_4, one (x_2) of which is varied on three levels ($n_1=1$) and three (x_1, x_3, x_4) on two ($n_2=3$). In this case from the conditional plan of 2^{3-1} the following unit is realized:

$$x_1,\ x_3,\ x_4,\ x_1x_3x_4.$$

In summation, by utilizing ideas of the construction of matrices of the initial data given in § 3 of this chapter , we can pass to the compilation of the test program of the combined plan of the type

$$N=3^1\times 2^{3-1}.$$

As follows from the given relation, the total volume of tests for the realization of this plan consists of the selection N=12. In this case the conditional quantity of the experiments for factors with three and two levels of variation will correspondingly be

$$N_1=3 \text{ and } N_2=4.$$

The test procedure is established by taking into account the limitation on randomization in accordance with the table of random numbers.

Similarly, problems for a large quantity of factors can be solved. In this case the program correspondingly becomes complicated, but the initial premises remain as before.

If the quantity of factors with three levels of variation increases more than three, it is recommended to introduce additionally in Table 3.19 the column "variants of tests of the subplan 3^n." This formalization will make it possible to simplify somewhat the method of the construction of the test program for this combined plan. With the number of factors $n_1>2$ the test program can be based on fractional replicas of a subplan of the type 3^{n_1}. However, in the solution of practical problems it is recommended to limit even the most complex plans according to the number of factors with three and more levels of variation. Otherwise the realization of this plan is impeded not only due to an excessive increase in the volume of the tests, but also from the viewpoint of the evaluation of its effectiveness.

Table 3.19.

№	No. of text	X					Variants of tests of sub-plan 2^n	Y
		x_0	x_1	x_2	x_3	x_4		
1	7	+	+	−	−	−	X_1	y_7
2	11	+	−	−	+	−	X_3	y_{11}
3	9	+	−	−	−	+	X_4	y_9
4	4	+	+	−	+	+	$X_1X_3X_4$	y_4
5	12	+	+	0	−	−	X_1	y_{12}
6	3	+	−	0	+	−	X_3	y_3
7	6	+	−	0	−	+	X_4	y_6
8	8	+	+	0	+	+	$X_1X_3X_4$	y_8
9	5	+	+	+	−	−	X_1	y_5
10	1	+	−	+	+	−	X_3	y_1
11	10	+	−	+	−	+	X_4	y_{10}
12	2	+	+	+	+	+	$X_1X_3X_4$	y_2

The effectiveness of the proposed plans, as it was shown above, lies in the fact that the solution of the same problem is achieved with a less quantity of expenditures in comparison with plans of type 3^n.

All the factor plans in a certain sense should be considered as being discrete, since they consider the strictly fixed levels of the variations in which the transfer from one level to another is accomplished in stages.

In the controllable systems this condition is unacceptable. The experimenter is virtually interested in the entire operating range of a change in the factor from the upper to the lower limit.

By utilizing factor plans of type d^n for the study of the physical processes, we apply beforehand the hypothesis according to which if the system is efficient at the maximum levels of variation a and b, then it is efficient within the range ab. Having composed the model of the considered process, we can conduct mathematical modeling. The results are checked by means of monitoring tests, which are carried out at intermediate values of the ranges of variation of a series of basic factors.

The quantity of monitoring tests must be selected in accordance with the solution of the specific problem and the analysis of the findings. In a number of cases in the study of the physical processes, the volume of the monitoring tests can be 15-20% of the total number of experiments accepted for determining the mathematical model. Virtually this means that if the model which considers the effect of seven significant factors is investigated and described by the polynomial of the second power, then in accordance with that which was given in § 6 the quantity of tests $N \geq 42$. Then the volume of monitoring tests is equal to approximately eight. The total amount of the tests $N_{\Sigma} \geq 42+8=50$.

This solution is sufficient for the investigation of the full range of the change in the factors which determine the efficiency of the considered systems. Moreover, it does not require the division of the assigned range of the variation of the factors into an infinite number of levels. As a rule, it can be limited to three and, in very rare cases, four levels.

§ 9. GENERAL METHODS OF THE VARIANCE ANALYSIS USED IN THE PROCESSING OF EXPERIMENTAL DATA

In the process of the processing of results of the observations, which belong to one or different general sets, several selective average sets, which are characterized by great scatter, can be obtained. The question concerning which selective average sets correspond to the actual deviation is solved by means of the variance analysis. The mathematical model utilized in this case is accepted as being linear.

Let us examine two cases of the variance analysis: single-factor and two-factor.

Single-factor variance analysis. Under conditions of the conducting of a single-factor experiment one factor with fixed levels is varied. The mathematical model of process, taking into account the possible errors, in this case is described by an equation of the form

$$y_{ij}=x_0+bx_i+e_{ij},$$

where y_{ij} is the point value of the output quality under the jth condition and with the ith test; x_0 - true average; x_i - instantaneous value of the factor; ε_{ij} - possible error of the experiment.

Limits of a change in values are i=1, 2, 3, ...n and j=1, 2, 3, ...m.

The reliability of this model is provided within limits of the fulfillment of conditions of the applicability of regressive analysis.

At present the mathematical apparatus for the conducting of a variance analysis is developed quite fully. As one of the convenient procedures the schemes examined in work [72] can be recommended. For the testing of the conformity of test results to the taken hypothesis the F-distribution with (m-1) and (n-m) degrees of freedom is used. In such a case when the value of the F-criterion is lower than values of $F_{(1-\alpha);(m-1);(n-m)}$, the considered hypothesis is accepted, otherwise it is rejected.

Values of the criterion $F_{(1-\alpha);(m-1);(n-m)}$ characterize by themselves the boundary of the critical zone of the F-distribution with (m-1) (n-m) degrees of freedom.

Actually accepted beyond the critical zone is the confidence limit of the F-distribution with the confidence coefficient α. This distribution does not exclude other methods of checks.

The initial data for the conducting of the variance analysis of results of a single-phase experiment are conveniently presented in the form of Table 3.20.

Table 3.20

	Test conditions				
	1	2	. . . .	m	
	x_{11} x_{12} . . . x_{1n}	x_{21} x_{22} . . . x_{2n}	. . . .	x_{m1} x_{m2} . . . x_{mn}	
Sum	D_1	D_2	. . . .	D_m	$D=\sum_{i=1}^{m} D_i$
Number of observations	n_1	n_2	. . . .	n_m	

The variance analysis itself for the randomized single-factor plan is conveniently carried out in accordance with the diagram presented in Table 3.21.

Table 3.21

Source of variation	Number of degrees of freedom	Sum of standard deviations	Mean square	F-criterion
Between test conditions	$m-1$	$S_n = \sum_{i=1}^{n} \frac{D_i^2}{n_i} - \frac{D^2}{N}$	$\frac{S_n}{m-1}$	$\frac{S_n (N-m)}{(m-1) S_\epsilon}$
Error of the experiment	$\sum_{i=1}^{m} (n_i - 1) = N - m$	$S_\epsilon = \sum_{i=1}^{m} \sum_{j=1}^{n_i} y_{ij}^2 - \sum_{i=1}^{m} \frac{D_i^2}{n_i}$	$\frac{S_\epsilon}{N-m}$	
Sum	$\sum_{i=1}^{m} n_i - 1 = N - 1$	$S_\Sigma = \sum_{i=1}^{m} \sum_{j=1}^{n_i} y_{ij}^2 - \frac{1}{N} D^2$		

One should understand the unbiased estimates of dispersions as the sum of the standard deviations and their relation to the number of the degrees of freedom as the mean squares. In this case the difference in the sum of standard deviations $(S_\Sigma - S_n)$ is equal to the unbiased estimate of the variance of error permissible in the experiment:

$$S_\epsilon = S_\Sigma - S_n.$$

If the planning of the single-factor experiment is unit planning, the mathematical model is somewhat modified. There appears the supplementary variable, which considers the interunit effect. Under the condition of the realization of the full randomization of the tests within the units, the variance analysis of this model is similar to the analysis of the two-factor experiment, not allowing for the effect of the interaction of the independent variables. The fundamental equation of the variance analysis is the expression for the sum of standard deviations, which in connection with the two-factor experiment will take the form

$$S_{\Sigma} = S_{b} + S_{n} + S_{e},$$

where S is the sum of the standard deviations between the units.

The variance analysis for the randomized block plans for single-factor experiments is conveniently carried out in accordance with Table 3.22.

Table 3.22

Source of variation	Number of degrees of freedom	Sum of standard deviations	Mean square	F-criterion
Between units	$n-1$	$S_{b} = \sum_{i=1}^{n} \frac{D_i^2}{m} - \frac{D^2}{N}$	$\frac{S_b}{n-1}$	$\frac{S_b (m-1)}{S_e}$
Between test conditions	$m-1$	$S_{n} = \sum_{j=1}^{m} \frac{D_j^2}{n} - \frac{D^2}{N}$	$\frac{S_n}{m-1}$	$\frac{S_n (n-1)}{S_e}$
Error of the experiment	$(n-1)(m-1)$	$S_e = \sum_{i=1}^{n}\sum_{j=1}^{m} y_{ij}^2 - \sum_{i=1}^{n} \frac{D_i^2}{m} - \sum_{j=1}^{m} \frac{D_j^2}{n} + \frac{D^2}{N}$	$\frac{S_e}{(n-1)(m-1)}$	
Sum	$N-1$ $N=mn$	$\sum_{i=1}^{n}\sum_{j=1}^{m} y_{ij}^2 - \frac{D^2}{N}$		

Two-factor variance analysis. In the two-factor experiment two factors at some fixed levels are varied. The mathematical model of the process, taking into account the possible errors, is described by the equation of the form

$$y_{ij} = x_0 + b_1 x_1 + b_2 x_2 + b_3 x_1 x_2 + \varepsilon_{ij},$$

where the product of the independent variables and the appropriate regression coefficients characterize the effects from the effect of both factors and their interaction. The reliability of this model is provided within limits of the applicability of the regressive analysis.

However, together with the effects of interaction, the sums from squares of independent variables are not considered as trivial terms. The diagram of the variance two-factor analysis for the independent variables A and B is shown in Table 3.23. In this case the quantity of levels of factor a is equal to p and of factor b is equal to d (i=1, 2, 3, ..., d; j=1, 2, 3, ..., p; m=1, 2, 3, ..., n).

The single-factor and two-factor variance analyses are based on the method of testing of the hypotheses and can be used for the qualitative estimate of characteristics of the systems from the viewpoint of their conformity to the assigned requirements.

Similarly the investigation for the three-factor and more experiment can be suggested. However, as a rule, this is of no practical importance. In an analysis of the effect on the system of a considerable number of factors, it is expedient to estimate quantitatively their effect on the output characteristics of the system. The procedure is given above. The investigation is conducted by the construction of the mathematical model of the process.

§ 10. APPLICATION OF A VARIANCE ANALYSIS FOR THE SOLUTION TO PARTICULAR PROBLEMS

Let us assume that of the four versions of corrective engines of the space vehicle, it is necessary to select the one with the optimum characteristic which determines the aeronautical engineering qualities. This characteristic can be the thermal stability of the uncooled nozzle exit, which determines the stability of the specific impulse or the momentum itself. A similar problem can be solved under bench conditions, but in this case there appears the need for the modeling of a number of the external factors such as the high degree of the rarefaction of the ambient medium, thermal radiation, and so on.

Table 3.23

Source of variation	Number of degrees of freedom	Sum of standard deviations	Mean square	F-criterion
Factor a	$p-1$	$S_a = \frac{1}{nd}\sum_{j=1}^{p} a_j^2 - \frac{D^2}{N}$	$\frac{S_a}{p-1}$	$\frac{S_a(n-1)pd}{(p-1)S_e}$
Factor b	$d-1$	$S_b = \frac{1}{np}\sum_{i=1}^{d} b_i^2 - \frac{D^2}{N}$	$\frac{S_b}{d-1}$	$\frac{S_b(n-1)pd}{(d-1)S_e}$
Interaction ab	$(p-1)(d-1)$	$S_{ab} = \frac{1}{n}\sum_{i=1}^{d}\sum_{j=1}^{p}\left(\sum_{m=1}^{n} y_{ij}\right)^2 - \frac{D^2}{N} - S_a - S_b$	$\frac{S_{ab}}{(p-1)(d-1)}$	$\frac{S_{ab}(n-1)pd}{(p-1)(d-1)S_e}$
Error	$pd(n-1)$	$S_e = \sum_{i=1}^{d}\sum_{j=1}^{p}\sum_{m=1}^{n} y_{ijm}^2 - \frac{1}{n}\sum_{i=1}^{d}\sum_{j=1}^{p}\left(\sum_{m=1}^{n} y_{ijm}\right)^2$	$\frac{S_e}{pd(n-1)}$	
Sum	$N-1$ $N=pdn$	$S_\Sigma = \sum_{i=1}^{d}\sum_{j=1}^{p}\sum_{m=1}^{n} y_{ijm}^2 - \frac{D^2}{N}$		

With known initial data the planned tests can be characterized as the factor experiment in which from one test to the next is changed, and the specific impulse Y_{ij} and engine type D_j are fixed, where i and j=1, 2, 3... Each engine passes 4 tests. In summation, we have a factor experiment of the type of 4×4 (N=16). By solving the blem concerning the arrangement of the corrective engines on full-scale object, it is possible to examine two variants:

1) on each space vehicle engines of one type are installed;

2) on one vehicle along different channels of the trajectory correction of the flight engines of different types are installed.

It is quite obvious that when using the first variant the features of the specific flight program of each spaceship, the psychological features of the behavior of the cosmonauts, a possible random maneuver, maximum disturbances, etc. will not be taken into account. The plan for this variant can be presented in the form of Table 3.24.

Table 3.24

Number of vehicle	1	2	3	4
Distribution of types of engines	I II III IV	I II III IV	I II III IV	I II III IV

This plan is ineffective, since it does not make it possible to determine the net affect of the varied factor on the output quality. In this case the error in the estimate will be maximum.

The plan for the second variant of the arrangement of the engines (composed by taking into account the condition of

Table 3.25

Number of vehicle	1	2	3	4
Distribution of types of engines	III	I	IV	I
	II	IV	II	IV
	IV	II	III	I
	III	II	I	III

randomization) can be presented in the form of Table 3.25. In this case the following condition was satisfied: one engine type on one article is encountered only once.

Since in the second case the plan is completely randomized, the errors determined by the design features of the carriers and vehicles and by a distinction in the flight program are averaged.

The mathematical model of this plan is expressed by the dependence

$$Y_{ij}=x_0+bD_j+\varepsilon_{ij}.$$

However, the already brief analysis detects errors of this method of planning. Specifically, just as with the mixed plan (variant 1), the randomized plan (variant 2) does not consider the features of the flight of all vehicles. Thus, for instance, from Table 3.25 it is evident that an engine of type III will not consider the features of tests of article No. 2, of type I - article No. 1, and of type II - features of article No. 4.

Tests of articles No. 1, 2 and 4 have a common shortcoming, which entails the fact that the engine types III, II and I are tested twice in each case, while engines I, III and II were not installed on these articles and were not tested at all. And only with flight tests of article No. 3 are engines of all types tested. From all points of view this test is most informative. The test plan, which was similar to that of article No. 3, is a unit plan, and the order of the arrangement of engines of all types on the articles is completely randomized. This type is called the completely randomized unit planning. This type of mathematical model is expressed by the relation for two factors of the experiment

$$Y = x_0 + b_i D_i + b_j D_j + \varepsilon_{ij},$$

where D_i is the factor which determines the design features of the articles and the distinction in their flight programs; D_j - the factor which evaluates the effect of the characteristics of the engine types; b_i, b_j - the corresponding regression coefficients; ε_{ij} - total error of the tests.

In spite of the fact that we called the given experiment two-factor, in fact it is single-factor: altogether only one factor - the engine type, is varied.

For this reason the interaction of factors D_i and D_j is not examined.

Let us allow that tests were carried out in accordance with the proposed plan, and results which in coded form were reduced in Table 3.26 were obtained.

Table 3.26

Number of vehicle	Engine type				Sum
	I	II	III	IV	
1	0	−1	3	4	6
2	5	−1	0	2	6
3	1	2	0	1	4
4	−2	4	−2	0	0
Sum D_i	4	4	1	7	16
Sum $\sum_{i=1}^{4} Y_{ij}^2$	30	22	13	21	$\sum_1^i \sum_1^j Y_{ij}^2 = 86$

For variants of the engine types the sum of the squares of results of the planning is defined as

$$S_т = \sum_{j=1}^{k=4} \frac{D_i^2}{n} - \frac{D^2}{N} = 22 - 16 = 6,$$

and for variants of tests of the engines

$$S_в = \sum_{j=1}^{n=4} \frac{D_j^2}{k} - \frac{D^2}{N} = 20{,}5 - 16 = 4{,}5.$$

The total sum of the squares will be

$$S_з = \sum_{j=1}^{n=4}\sum_{i=1}^{k=4} Y_{ij}^2 - \frac{D^2}{N} = 86 - 16 = 70.$$

The sum of the squares for an estimate of the error experiment will be defined as the difference

$$S_e = S_з - S_в - S_т = 70 - 6 - 4{,}5 = 59{,}5.$$

In conclusion let us find the mean squares:

$$S_{т.ср} = \frac{S_т}{n-1} = \frac{6}{4-1} = 2; \quad S_{в.ср} = \frac{S_в}{k-1} = \frac{4{,}5}{4-1} = 1{,}5;$$

$$S_{eср} = \frac{S_e}{(n-1)(k-1)} = \frac{59{,}5}{(4-1)(4-1)} = 6{,}6.$$

Let us reduce the results of the calculation in Table 3.27.

Table 3.27

Factor to be varied	Number of degrees of freedom	Sum of the squares	Mean square
Number of vehicle	3	6,0	2,0
Engine type	3	4,5	1,5
Error of variance	9	59,5	6,6
Sum	15	70	

On the basis of the adopted variance analysis, the possible hypotheses, for example, about the equality of the average characteristics of all engine types, can be checked.

For this purpose we use the criterion of F-distribution with a risk level of 5%:

$$F_{3,9} = \frac{2{,}0}{6{,}6} = 0{,}303 < 3{,}86.$$

Under the condition of the accomplishing of this inequality, we can confirm that the average values of characteristics of the indicated engine types are approximately identical, and that is why we cannot show a preference for any of them.

As a result of the adopted variance analysis the following conclusion can be made: the average characteristics of the output quality for all engine types are approximately identical.

If together with the unit randomized planning the analysis of the usual randomized plan was conducted, then the same conclusion would be obtained. However, in the case of unit planning of the errors in the evaluation of the variance would prove to be somewhat less. This again indicates the higher effectiveness of the unit randomized planning in comparison with the others.

For the purpose of studying the effect and interaction of the factors on the final effect, let us examine this example.

Let us assume that we want to estimate the effect of the combustion-chamber pressure $p_к$ and the fuel component ratio k on the specific thrust of the engine P_1. Accepting the condition that the considered ZhRD operates at an optimum (or close to this) value of k, we will consider that both parameters are virtually independent. For the purpose of the elimination of the effect of other factors, the tests are carried out under other stable conditions.

The factor experiment was realized for the levels of variation 1 and 2 with the quantity of tests $N=2^2$ (Fig. 3.8). From the figure one can see that with the variation of parameter k within limits of levels 1-2 and constant pressure $p_к$, the specific thrust increases; at a constant value of k and an increase in parameter $p_к$, the effect is repeated. The graphs show that the degree of this effect is determined by the slope angle of the line segment toward the horizontal axis. The slope angle of the straight line to a great degree depends on the levels of variation (1-2) (see Fig. 3.8a). It is quite obvious that in the specific case we cannot say what the degree of this effect is, since on the graph a change in p_1 is determined by the scale factor. An analysis of the dependences also indicates the presence of the interaction between the factors. Thus, for instance, in Fig. 3.8b and c the lines which correspond to different pressures of $p_к$ and to coefficients k, strictly speaking, are not parallel, which can be explained only by presence of the interaction of the factors to be varied.

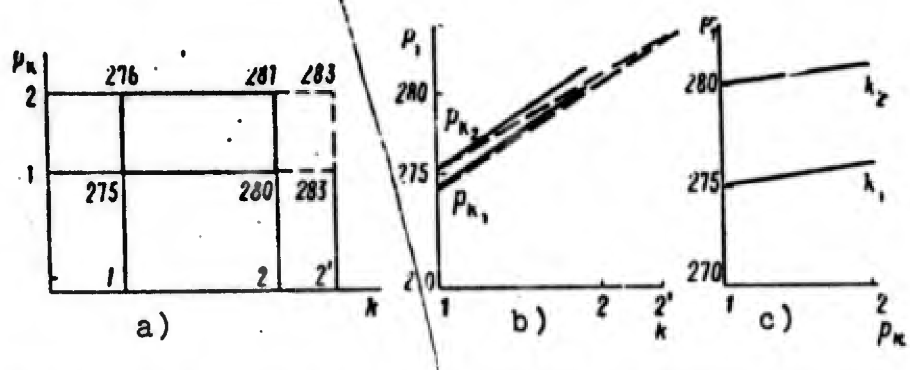


Figure 3.8. Dependences between the parameters $p_к$, k and p_1:

It is definitely impossible to estimate the greater or lesser degree of the effect of parameters $p_к$ or k due to the aforementioned reasons. When this must be done, a statistical check of the significance of the interaction is given. But it can be carried out when the quantity of observations with each combination is more than one.

To evaluate the interaction of the factors, let us somewhat expand the experiment. For this purpose let us introduce the supplementary level of the variation of parameter $p_к$ - its nominal value, besides the two maximum levels (boosting and throttling). More accurately a factor experiment of the type 2×3 with fixed levels of the variation of parameters $p_к$ and k is planned. In the development of the program it is decided to check each combination twice. In summation, the total quantity of the tests will be 2N. Each of the experiments is given an ordinal number. All the tests are carried out under the condition of the observance of the objective method of randomization.

The mathematical model appears in a following manner:

$$y = b_0x_0 + b_ip_к + b_jk + b_{ij}p_кk + \varepsilon_{ij}.$$

In this case the regression coefficients b_i, b_j and error ε_{ij} correspond to levels of the variation of the factors: $p_к$-i=1, 2, 3; k-j=1, 2.

In the presence of three levels of variation for parameter $p_к$ and two for parameter k, this program has 3·2=6 experimental conditions, or there exists 6-1=5 degrees of freedom between the variants of the tests and (2-1)6=6 degrees of freedom within the variants. The total number of the degrees of freedom for 12 tests will be 12-1=11.

In accordance with the considered plan, results of the tests which in coded form are placed in Table 3.28 are obtained. Data for the variance analysis are represented in Table 3.29.

By utilizing the test results, we will obtain the sums of squares which correspond to the effect of the factor $p_к$. Further let us estimate the effect of the main effect for parameter $p_к$ in terms of the sum of the squares:

$$S_{p_к} = \frac{14^2 + 6^2 + 0^2}{6} - \frac{20^2}{12} = 5.4.$$

Table 3.28

k	$p_к$ max	$p_к$ nom	$p_к$ min	$\sum_1^3 D_i$
max	6 4	6 2	8 1	27
min	3 1	0 –2	–4 –5	–7
$\sum_1^2 D_j$	14	6	0	

Table 3.29

Variant of tests	$p_к$ max k_{max}	$p_к$ max k_{min}	$p_к$ nom k_{max}	$p_к$ nom k_{min}	$p_к$ min k_{max}	$p_к$ min k_{min}	D_i
Levels of factors	6 4	3 1	6 2	0 –2	8 1	–4 –5	19 1
Sum	10	4	8	–2	9	–9	$20=\sum_1^2 D_i$
Sum of squares	52	10	40	4	65	41	$212=\sum_{i=1}\sum_{j=1} y_{ij}^2$

The sum of squares of the main effect k will be

$$S_к = \frac{27^2+(-7)^2}{6} - \frac{20^2}{12} = 96{,}7.$$

The sum of the squares of the interaction of parameters $p_н \times k$ is defined as the difference in the sums of squares between the variants of the tests and the sum:

$$(S_{pк} + S_к) = 102{,}1.$$

The obtained data make it possible to complete the variance analysis conducted for the purpose of studying the effect of factors $p_н$ and k on the investigated process.

Results of the calculation are given in Table 3.30.

Table 3.30

Source of variation	Number of degrees of freedom	Sum of squares	Mean square
p_K	2	5,4	2,7
k	1	96,7	96,7
$p_K \times k$	2	37,6	18,8
Errors of variance	6	39,0	6,5
Sum	11	178,7	

In conclusion, by utilizing the F-criterion, let us check the hypothesis on the effect of parameter p_H on specific thrust. The essence of the hypothesis: parameter p_H affects P_1. For the confidence coefficient 0.95 $F_{2.6}=\frac{2.7}{6.5}<4.76$. Since the criterion $F_{2.6}$ is considerably less than critical value of 4.76, the advanced hypothesis is competent within the assigned limits of a change in parameter p_H.

Let us examine the second hypothesis about the effect of parameter k on P_1. Then

$$F_{1,6}=\frac{96,7}{6,5}>5,99.$$

The hypothesis is rejected as being inconsistent.

Similarly the hypothesis about the effect of the interaction of $p_H \times k$ on the output parameter P_1 can be examined.

The examples examined bear a somewhat illustrative nature. Specifically, we solved the problem in which the degree of the significance of factors p_H and k was estimated by means of the variance analysis. This analysis can be applied for the solution of particular problems by a selection of those or other design variants of the developed systems and elements when, as a result of the effect of the complex of factors, the selection of the optimum variant is difficult. However, as a whole the possibilities of a variance analysis are considerably more extensive.

CHAPTER IV

RELIABILITY TESTS

§ 1. GENERAL POSITIONS, PURPOSE AND DESIGNATION

In the course of development works a considerable volume of the statistical data is accumulated, and it can be used for the qualitative and quantitative evaluation of reliability. However, a careful analysis of them shows that for the majority of the systems and assemblies these data cannot be fully accepted. The reason is that they are related to different forms of tests and programs and different design modifications of the same type of subassembly, assembly or element. But mainly, in practice, all the tests conducted in stages of the design final development of the ZhRD [ЖРД - liquid-propellant rocket engine] are characterized by a great variety of programs which by no means always corresponds to the actual conditions of operation. Almost each subsequent test does not repeat that which precedes and differs from one another by a number of design modifications. Sometimes they can be considered as tests of diverse variants of engines with similar values of the basic parameters.

Consequently, from the viewpoint of mathematical statistics, the results of these tests cannot be considered those which belong to one general aggregate, since they are related to different test objects. Therefore, in the final stage of development, from the full volume of statistical data only part of it can be used to evaluate the reliability. The value of the remaining part of the experimental data consists in the formation of correct paths of the empirical search for the regular variant of the design, the determination of limits of efficiency, reserves of strength, stability of the processes, and so on.

In the future one should expect the appearance of optimum methods of the evaluation of reliability, which make it possible to consider the entire volume of information independently of the test programs and levels of the variation of quantitative and qualitative factors.

At present the problem is to expand in an optimum way the volume of information and respectively statistical data accepted for evaluating the reliability of the regular variant of design. Largely this problem can be solved by special tests of limited selections of the articles and their elements, which are called reliability tests. Their optimum nature is based on the property of the ergodicity of the systems and elements, owing to which on a limited number of articles a large quantity of tests is carried out, and in this way the minimum of the material and economic expenditures is reached. In principle the qualitative information, classified according to the criterion "success-failure", is supplemented by quantitative information (by the time between failures, the distribution of statistical characteristics, their mathematical expectation, and the dispersion of estimates). The advisability of reliability tests lies in the fact that with their conducting by means of telemetering data there is considered virtually the entire course of the change in the fundamental characteristics of the articles in the process of functioning and not the single fact of the onset of failure, as when evaluating the reliability of systems by discrete methods. This means of the solution to the problem provides the reduction in the required quantity of articles because of the expansion of the volume of information [21, 87].

In the analysis of results of reliability tests, the failures caused by the rough disturbance of industrial processes of production cannot be considered.

Depending on the purpose, the reliability tests differ by a great variety of programs. Programs are widely known for the purpose of checking for reserves of strength, evaluation of a guaranteed service life, quantity of cycles up to failure, service lives and storage, determination of the "weakest" link, the possible type of failure, and so on.

Reliability tests of experimental models, in a certain measure, make it possible to determine their "weak" places, but as a whole they cannot characterize the real levels of reliability of the regular articles. This is explained, in the first place, by the possible design modifications, which unavoidably must be introduced into the articles with their improvement in the process of the tests, and in the second place, by the distinction in the experimental production from the serial. Therefore, reliability tests must also be given to the regular articles manufactured under conditions of serial production for regular technology. Because of this the reliability tests, from the viewpoint of the evaluation of the reliability of the indices, can be most informative in final stages of development.

Most advisable prove to be the test plans for finishing constructed on the basis of reliability tests. For this purpose the qualification tests of articles (III stage of development) can be planned according to standard programs with the only difference being that the time of their conducting is not limited to the guaranteed service life. The tests are continued up to the detection of primary failure or for the period and quantity of inclusions many times exceeding normal conditions [83, 86]. In a number of cases this is not an exception also for the article of single action. Thus, for instance, the engine F-1 (of the firm Rocketdyne), after the completion of the autonomous final development of the assemblies and units, was tested according to the programs which combine the elements of qualification tests and reliability tests.

Up to the moment of the completion of tests for the testing of the conformity of the engine to technical specifications, there were carried out on the order of 800 firing tests with a total duration of approximately 82,000 seconds [83, 86]. More accurately, in the execution of this program each engine was tested on the average of about 35 times (one switching on is assigned). In this case the average operation time for each model of the engine was 35,700 s (flight operating time ~160 s).

Test programs for articles of single and repeated action are different. Specifically, the latter most frequently undergo cyclic and resource tests [37, 62, 104].

The cyclic tests are determined by operating conditions of the system or element during transient conditions. Consequently, with the test data failures characteristic to these systems can be revealed. In this case the mean statistical probability of the manifestation of failures for a broad class of articles is rather large. Specifically, for ZhRD the portion of failures which were revealed under conditions of starting and shut-off makes up a considerable part of the total number of emergency results [37].

The service-life tests are determined by operating conditions of the system or element on the steady-state modes of operation. In the majority of cases wear and gradual failures are characteristic of this type of tests.

As a rule, reliability tests of one selection of the articles are carried out in the fixed conditions, which correspond to the actual conditions of the operation or to the extreme combinations of significant factors revealed in the process of the final development or operation of the system.

§ 2. ORGANIZATION OF TESTS AND THE PROCEDURE FOR THEIR CONDUCTING

The feature of programs for the reliability test is the wide use of the material part which passed other types of tests (standard, checking, etc.). In this case the accumulated operating time is considered during the processing of the statistical data. In the process of the tests a constant monitoring of the technical state of the articles is conducted. In this case accepted as failures are destructions of material part or the yields of the check parameters as tolerances. The latter can be accompanied by a smooth change in the characteristics for the gradual failures or intermittent - for random failures (Figs. 4.1 and 4.2).

Figure 4.1 distinctly shows the moments of the beginning of deviations of the check parameter from a certain working level (τ_1) and its handling tolerance (τ_2). From the viewpoint of the efficiency of the system, the exposure time of failure should be considered the moment τ_2, but from the viewpoint of the technical state in a number of objective programs accepted as the exposure time of failure is the point in time τ_1, since a change in the characteristic of the parameter, its break, indicates the emergence of a malfunction, and a further period is the development of failure.

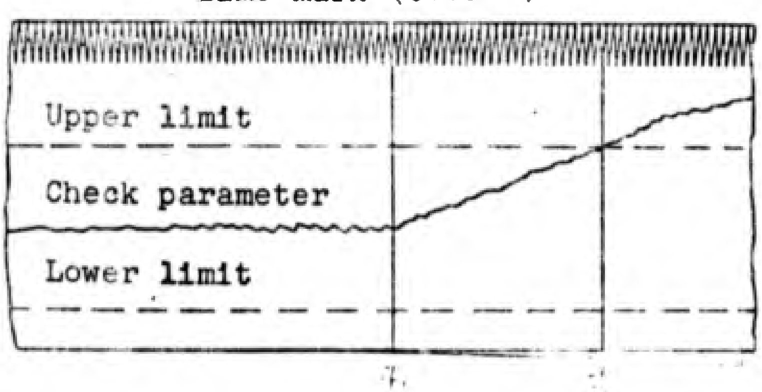


Figure 4.1. Oscillogram of the recording of the check parameter in the period of the manifestation of gradual failure.

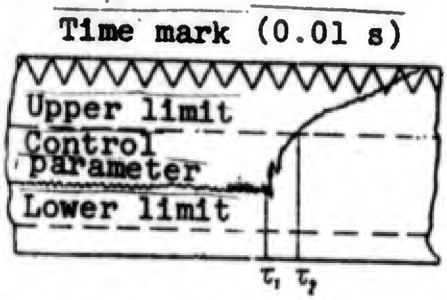


Figure 4.2. Oscillogram of the recording of the check parameter in the period of the manifestation of random failure.

The moment of the onset of a random failure (see Fig. 4.2) is characterized by the abrupt change in the nature of the behavior of the parameter and by the intermittent tolerance limit. The time of the onset of the sudden jump is calculated, as a rule, in millionths of a second.

Above the discussion was about parameters for which during the entire process of the tests constant checking is accomplished. Unfortunately, not all the articles can be organized.

Let us allow reliability tests are conducted on the electropneumatic valve, for which the power stroke of the relief valve is the check parameter. In the process of repeating-alternating switch-ons (cyclic tests), it is sufficiently complicated to measure valve stroke. For realization of this purpose it would be necessary to finish all articles of the selection and introduce design changes, and this is undesirable. Under normal conditions for the measurement of the valve stroke it is necessary to stop the tests, which also is undesirable, since operating conditions are disturbed, test periods are lengthened, the accuracy of results of the investigation is reduced and so on. Sometimes it is necessary to do this. In such cases it is necessary to take measures in order that the error of the experiment would be minimum and periods of the realization of the program be not lengthened very much.

Let us assume that the following condition is accepted: check measurements are accomplished every 1000 cycles. Then it can seem that in the interval between 8000 and 9000 cycles a failure occurred. For the actual number of cycles at which the failure occurred, it is possible to accept the average value of operation time (8500 cycles) or with the responsible program - 8000. But both are undoubtedly inaccurate.

In a number of programs in order to decrease the possible errors, a variable test plan is proposed. To do this first one tests the article and determines the tentatively expected limits in which the failure can occur. Let us assume that it occurred at 8000 cycles. Then, taking into account the running-in period and the possible scatter, an inspection of the course of the relief valve is conducted according to the following plan: 0, 100, 500, 3000, 6000, 7000, 7500 and further every 500 cycles. For the purpose of shortening the periods of the program, tests of an entire sampling (or half) are conducted in parallel on one test stand.

The test procedures are approximately thus. Prior to the beginning of the tests all articles of the established sampling pass through the receiving inspection in accordance with technical specifications for commodity supplies. In order of exception some, especially responsible articles, can undergo additional testings over the established program: for example, testing for vibration stability along three mutually perpendicular axes, the determination of resonance frequencies, allowances for functioning at resonance frequencies, and so on.

As an example let us examine the test procedures of elements of automation, in particular, the pressure relay, as a function of which enters the delivery of the electrical instruction after the achievement of a certain pressure level in the working cavity. Reliability tests are conducted by cycles up to failure. In this case as a result of simulation of the actual conditions of operation for the period of time of each cycle, the following are determined: a) the effect of the action of resonance frequencies; b) cyclic strength by pressure, which varies from zero to the upper limit and then down to zero; c) the stability of the contact pair with the maximum current load.

Through each 500 cycles the articles undergo vibration tests at the assigned levels of vibration overloads and frequencies for the period of time provided by the program at the maximum pressure in the working cavity. Periodically after each cycle of tests the points of the functioning of the contact pair at the maximum temperature of the medium are recorded.

The registration of cases of closing and opening with the direct and return stroke of the rod can be conducted with the aid of the loop oscillograph. The test results are processed and represented in the form of tables and curves. At the same time there is given a brief analysis of the experimental data by which there is established the frequency range of the vibration overloads with which the articles are vibration-proof, the error in the functioning of the articles at resonance frequencies, and so on. Accepted as a failure is the destruction of material part or the output of the check parameter (in the case examined above - the pressure of functioning of the contact pair) as the established limits.

According to the experimental data it is possible to define the limits of the field of the scattering of failures as the ratio of the maximum value of operation time x_{max} to the minimum x_{min}. For a number of mechanical systems it can prove to be sufficiently large $x_{max}/x_{min}=3-8$, especially when the discussion is about extreme values of the field of scattering determined for the general totality of articles [57]. The indicated scattering is determined, mainly, not by features of the physical processes but to greater degree by the stability of properties of the materials, the technology of manufacture, and so on. For this reason for the broad class of articles the indicated limit is basically retained. The presence of a large scattering of failures cannot help reflect the test procedures, especially during article testing of repeated action with a large guarantee period of operation. In this case the tests up to the failure of an entire sampling can be conducted not always due to short periods of the realization of the program according to economic considerations. In connection with this a number of test programs for reliability have a limitation which is characterized by this condition: the tests are carried out up to an accrued operating time of not more x_a. Virtually this means that according to relation to the actual limits of the scattering of failures the assigned magnitude of operating time x_a can be distributed in the following way:

1. $x_{min} < x_a < x_{max}$;
2. $x_{min} < x_{max} < x_a$;
3. $x_a < x_{min} < x_{max}$.

In the first case, with testing up to accrued operating time x_a, a certain defect level (in the extreme case, at least one) is obtained. The remaining articles maintained their efficiency. In mathematical statistics of this type the result is conventionally designated as tests of the truncated sample. Under specific conditions the test results can characterize the scattering of the failures.

In the second case the entire sampling of the articles underwent tests up to failure, and the program proved to be completely fulfilled. According to clear reasons the performance evaluation of scattering in this case is more precise than in the first.

The third case is characterized by the fact that the failures are not detected. From the viewpoint of the expressed goal, the test data should be considered less resultant, since the characteristics of scattering and the very type of failure are unknown. In a number of cases after the verification test of the articles of this sampling for conformity to the technical specifications of testing, it is expedient to continue. When the purpose is to confirm a certain level of reliability $P(x, \alpha)$ with the confidence level α, the level x_a can be determined in the following way.

For reliable tests the lower boundary of the probability of trouble-free operation can be described by means of the exponential distribution [76]:

$$P_H(x, \alpha) = \exp\left[\frac{x \ln(1-\alpha)}{x_0}\right],$$

where the mathematical expectation x_0 corresponds to the accumulated operating time of the articles of selection N, then

$$x_0 = N x_a.$$

After transformation we will obtain

$$x_a = \frac{x \ln(1-\alpha)}{N \ln P_H(x, \alpha)}.$$

In this case one should accept for the instantaneous value of parameter x the assigned operating time or guaranteed service life. Usually it makes sense to assign level x_a in relative values of (x_a/x); then x=1.

Taking this into account,

$$x_a = \frac{\ln(1-\alpha)}{N \ln P_H(1, \alpha)}.$$

The graphic dependence of parameters x_a and N on characteristic $P_H(1, \alpha)$ is shown in Fig. 4.3.

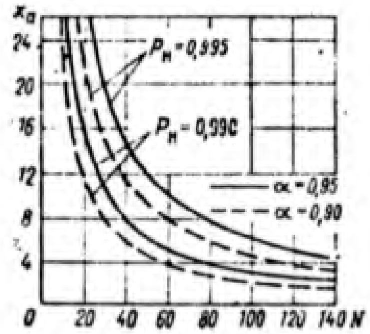


Figure 4.3. Dependence of parameter x_a for reliable tests.

The level which is established for parameter x_a for different systems and elements, as follows from Fig. 4.3, can be the different. Virtually this depends not only on the assigned level of reliability and sample size, but also on the complexity of the article, degree of automation and cost of the tests, the presence of bench equipment, methods of check, and so on. In the USA during the test of full-scale ZhRD the average accrued operating time of the engine exceeds the assigned flight operating time by dozens and hundreds of times. The same order also refers to the quantity of switch-ons - cycles. Thus, for instance, for the engine F-1 on one of the stages of the qualification tests the average accrued operating time was about 220 guaranteed service lives (35,700 s, [illegible] s) [83, 86].

After the completion of test programs for reliability, the experimental data are processed and all articles are made defective. The destroyed parts undergo metallographic analysis. The reasons for failures are thoroughly analyzed for the purpose of conducting further modifications of the design for an increase in the achieved level of reliability.

The procedure for the processing of experimental data will be examined below.

§ 3. ACCELERATED TESTS

Depending on the program in a number of cases the reliability tests can be considered as being accelerated. This is explained by the fact that for some articles the assigned guaranteed operating time is sufficiently high. Thus, for instance, for the corrective systems of space vehicles and vehicles intended for flights to Mars, the guaranteed operating time under conditions of normal operation must consist of years. Naturally, there arises the question as to how to conduct the complete service-life tests of similar systems, even if the testing for flight operating time causes almost insurmountable difficulties. A reasonable solution to this question is the use of procedures of accelerated tests [51, 104] in which there occurs a stiffening of the operating conditions of the article, owing to which a shortening in the total periods of realization of the programs is achieved.

Let us assume that a certain system operates in the assigned cyclic system (Fig. 4.4). Correspondingly the full operating cycle is characterized by the time of activation t_1, the operation on the cruising mode t_2 and shut-down t_3. In a number of cases it is convenient to speak about the sum of the times (t_1+t_3) as about the operating time on transient conditions. It is quite obvious that the gradients of a change in the operating conditions with starting and shut-down depend on the cyclogram of control and physics of the process and to a great degree cannot be changed. Unlike them, operating time on the cruising mode virtually can vary from zero to the full time between failures. If we accept $t_2=0$, then in this extreme case the oscillogram of the operating process will take the form of b or c (see Fig. 4.4).

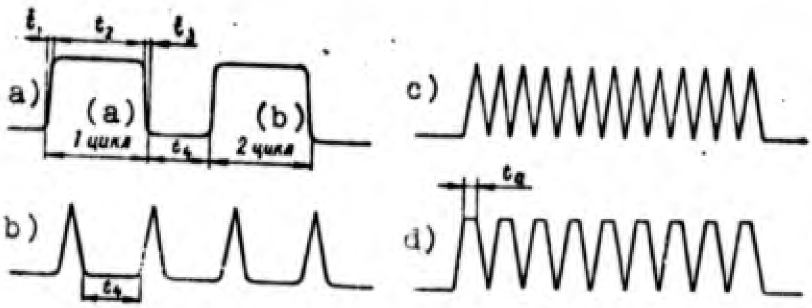


Figure 4.4. Cyclic modes of operation.
Key: (a) 1 cycle; (b) 2 cycles.

Both oscillograms differ by the fact that in the case of b between separate switch-ons there is the interval of "rest" t_4. In the case of c it is absent. Already from this viewpoint mode c is considerably more strained than mode b and all the more than mode a. The tests conducted from oscillogram c are accelerated in relation to the tests under conditions of normal operation.

Both oscillograms b and c are characterized by the output of the check parameter on the assigned mode and then by its sharp decrease (shut-down). Oscillogram c reflects the so-called "machine-gun" operating mode. However, such types of tests are possible only for systems and elements of repeated action, and furthermore their circle is limited. If this kind of program can be used in accelerated tests, for example, of elements of automation of the EPK [ЭПК - electroneumatic valve] type, then they can not always be realized for thermal systems and elements which require a certain delay in t_a in the steady-state mode for the purpose of the stabilization of thermal and dynamic processes. Thus, for instance, for full-scale ZhRD, according to the Aerojet-General Corporation [54], this time consists of not less than 10 s. Taking the aforesaid into account, the curve of the distribution of failure rate $\lambda(t)$ somewhat "delays" in relation to the dynamic characteristic of the increase in the check parameter. In summation, for this class of systems the program of accelerated tests, which is characterized by oscillogram d can be used.

In order to characterize quantitatively the distinction in programs of the type b, c and d, let us introduce characteristic C_r - the cyclic velocity of the replacement of the modes [104] as the number of cycles (switch-ons) per unit time:

$$C_r = \frac{x}{t},$$

where x is the number of cycles which was produced by the system for time t. Under certain conditions for a defined class of systems of repeated action, the cyclic tests can be examined as being accelerated in relation to the service life.

The shortening of the total test periods up to the onset of failures is achieved by means of a "weight increase" in the operating modes. The degree of this "weight increase" is characterized by the coefficient η - the ratio of the failure rates of two types of tests:

$$\eta = \frac{\lambda_1}{\lambda_2}.$$

The stability of the coefficient η, as will be shown below, can be provided for by the exponential distribution of failures when characteristics λ_1 and λ_2 are constant. In all the remaining cases the insuring of the stability of coefficients η is very difficult. This is explained by the fact that for any mechanical system in a certain range of a change in the parameters it is characteristic to retain the form of the distribution of the law of failures, which in this range the physical picture of the process is not changed. Consequently, after accepting the condition of the constancy of the distribution law of failures, it is easy to be convinced of the following.

Let us assume that the system investigated by us is subordinated to the Weibull distribution with a failure rate

$$\lambda(x) = \frac{\beta}{x_0} x^{\beta - 1},$$

where x_0 is the mathematical expectation; β - constant coefficient, distribution index.

Figure 4.5 shows the functional dependence of characteristic λ(x) on the number of cycles x and index β. If with regular tests the failure rate is close to the index of distribution β=4, then because of a change in the mode and test conditions the failure rate will be changed and will be close to the index β=2.

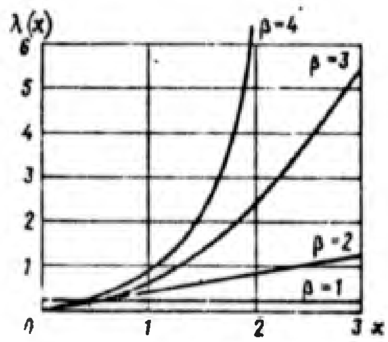


Figure 4.5. Failure rate for the Weibull distribution (x_0=5).

Let us examine the nature of a change in the operating mode of interest to us. It is easy to establish that the coefficient of "weight increase" is an unstable value and is changed in the process of the tests in accordance with the dependence shown in Fig. 4.6.

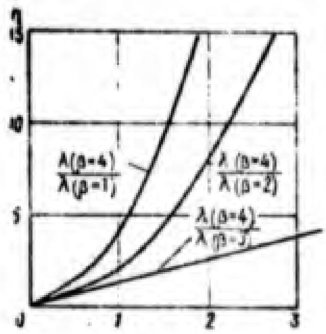


Figure 4.6. Dependence of the coefficient of "weight increase" of mode η on the number of cycles x and on the distribution index β.

Hence there follows the important conclusion that the coefficient of "weight increase" η can be accepted as the characteristic of accelerated tests only under the condition of the exponential distribution of failures. In the majority of the other cases it gives ambiguous solutions, and an attempt to estimate the failure rate for the new type of test by means of this coefficient can lead to serious errors. This is explained by the fact that the ratio of failure rates is a certain functional dependence which cannot be expressed by the constant coefficient, since it is a certain function with new independent variables. The latter characterize parameters of the types of accelerated tests used.

One of the main disadvantages of the method of accelerated tests is the need for experimental research as the means for the evaluation of the reliability of results. Virtually always for all systems it is required to carry out together accelerated tests with full-scale tests. But this can also be justified if, for example, it is necessary by accelerated tests to confirm the reliability of commodity articles with very long guaranteed operating time.

Then on the stage of development work both types of tests are conducted, the experimental dependences between them are established, and subsequently only accelerated tests are conducted, and a conversion to the actual conditions of their conducting is made.

The appropriate experimental studies are being carried out by us and abroad. Thus, for instance, according to results of works of the firm of Arinc Research [Translator's Note. Name Arinc is not verified] (USA) [104], obtained by the processing of statistical data on tests of rocket systems, failure rate during the accelerated tests increases according to the relation

$$\frac{\lambda_1}{\lambda_2} = 1 + C_r f_c$$

where λ_1 and λ_2 are failure rates with the accelerated cyclic and regular service-life tests, respectively; f_c - the coefficient which determines the time of the conducting of service-life tests equivalent to one cycle of the operation.

According to data of the firm Arinc Research, for the vast class of the rocket systems of one-time use, the coefficient f_c is a constant value $f_c = 8$ [104]. In this case the distribution law of failures was not exponential but normal. In the case of the normal distribution, the coefficient of the "weight increase" empirically proved to be equal to a certain function with the variable parameter C_r:

$$\eta = 1 + C_r f_c.$$

In all probability, for other distributions and other types of tests there can exist another form of this function with the new independent variables which characterize the parameters of this type of tests. If there are experimental values of characteristics λ_1 and λ_2 for one form of tests and different systems of one class, then it is not difficult to determine the form of the function which characterizes the coefficient "weight increase" by means of the methods of mathematical statistics, in particular, the method of least squares.

Thus we examined briefly one of the types of the accelerated tests, which is characterized by the acceleration of the rate of the test work, i.e., by the cyclic velocity of the replacement of the modes.

At the same time the types of accelerated tests in which the "weight increase" of the operating mode is accomplished by an increase in levels of the basic parameters can be represented. The difficulties of these tests consist not of the determination of the failure type but the setting of the value of the correlation of the obtained data with the expected characteristics in the actual conditions of operation, since an increase in the level of the parameter which causes the failure can be accompanied by a partial or total variation in the physical processes. In such cases the revealed failures can be in no way characteristic for the articles during their tests under actual conditions.

An example to this is the excessive boosting of the combustion chamber of the ZhRD according to pressure and temperature. The latter, as is known, can lead to the appearance of vapor in channels of coolant passage, the decomposition of the propellant components, to its coking, i.e., to the phenomena which are not characteristic for the standard conditions of operation of the ZhRD. In this case the difficulties of the conducting of accelerated tests consist also in the limited possibilities of the systems themselves. Thus, for instance, the boosting of the combustion chamber can more frequently be limited not to the manifestation of its failures, but to the possibilities of other units: gas generator, TNA [THA - turbopump assembly] and pumps which control the elements and especially their hydrodynamic characteristics. Even when there is the time and means it proves to be sufficiently difficult to establish the degree of correlation of the modes. Test work with the excessive boosting of the basic parameters (up to levels at which there does not occur a change in the physical processes) is useful for determining the efficiency of the basic assemblies of the ZhRD under boundary test conditions. Virtually it is expedient to combine the tests with a certain guaranteed boosting of the basic parameters with cyclic and service-life programs.

The problem of accelerated tests of mechanical systems and their elements as independent field is rather complex and little investigated. Probably its solution must be based both on the experiment and mathematical statistics. Useful in this respect can prove to be the work experience in electronic elements [51].

For the planning of accelerated tests, an account of the effect of separate factors, and also methods of the construction of mathematical models in a practical plan the materials of Chapter III can be used.

§ 4. FEATURES OF RELIABILITY TESTS OF ELEMENTS OF SINGLE ACTION

In the period of functioning the elements of single action are characterized by the irreversibility of the physical processes. For this reason with their reliability tests the cyclic or other types of tests, which are characterized by such levels of parameters as cyclic recurrence or the replacement of the modes cannot be used. The reliability tests must entirely correspond to the physical phenomena which take place under conditions of normal operation. An analysis of these conditions shows that in the absolute majority of the cases the physical processes which determine the efficiency of the elements of single action are accompanied by a change in the strength characteristics. Consequently, they can be referred to the type of elements for which operational mode is defined by the ratio of such parameters as "load" and the "strength". Both parameters follow the law of normal distribution with densities $f_1(x)$ and $f_2(x)$, mathematical exceptations x_0' and x_0'' respectively, and the rms deviations

σ_1 and σ_2. In connection with the probability of the failure-free operation, it is possible to establish dependence P(x) in terms of the Laplace function F_0:

$$P(x) = F_0\left(\frac{x_0 - x}{\sigma}\right), \quad (4.1)$$

where $(x_0-x)/\sigma$ is the quantile of normal distribution

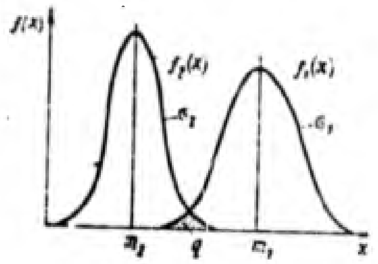


Figure 4.7. Dependence of the relationships of densities of the distribution of "load" and "strength": m_1, σ_1 - parameters of "strength"; m_2, σ_2 - parameters of "load".

From a practical point of view the condition when mathematical expectations x_0' and x_0'' are sufficiently great in comparison with the appropriate rms deviations σ_1 and σ_2 is valid. On Fig. 4.7 the area q characterizes the unreliability of the system and is quantitatively equal to (1-P). In this case the total deviation of the mathematical expectation for this distribution will be

$$x_0 - x = (x_0' - x) - (x_0'' - x) = x_0' - x_0''.$$

The total rms deviation σ will be equal to

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}\,.$$

Then the probability of trouble-free operation of the system, determined by the relationship "load" and "strength", is equal to

$$P = F_0\left(\frac{x_0' - x_0''}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right). \quad (4.2)$$

This dependence properly determines the test program for the reliability of elements of single action, according to which according to tests of a certain sampling of one-type articles mathematical expectations x_0' and x_0'', and also their dispersions σ_1^2 and σ_2^2 are estimated. For the determination of the evaluations of strength, in all probability it will be required to carry out tests up to destruction.

For the purpose of the illustration of this method, let us examine two examples.

Example 1. It is proposed with the aid of special tests to estimate the design and engineering reliability of a spherical tank with an operating pressure p_2:

$$(p_2 - \Delta p) < p_2 < (p_2 + \Delta p),$$

where Δp is the deviation from operating pressure.

Correspondingly, the rms scatter of "load" can be defined as $\sigma_2 = \Delta p / t_\alpha$, where t_α is the quantile for normal distribution with the assigned confidence probability α.

To evaluate the strength, there are usually carried out hydraulic tests up to destruction of a certain sampling of one form of the articles which passed all forms of the current and output check provided by technical specifications and records. In the process of the tests the maximum pressures at which there occurs a loss in strength are recorded. According to the given results parameters x_0' and σ_1 are estimated. Further from formula (4.2) the mean statistical value of probability $\overline{P}$ is determined.

Example 2. It is proposed to conduct reliability tests of the tank (see Fig. 1.1). Reliability tests of the outer covering of the tank, if necessary, can be carried out in the same way as was stated in Example 1. Reliability tests of the elastic diaphragm can be organized in the following way.

A certain sampling of single-type articles undergoes tests. Tests on all the articles are carried out in turn in two stages. In the course of the first stage from the fuel cavity by the maximum operating pressure, the full displacement of the fluid before the fitting of the elastic diaphragm on the internal surface of the hemisphere of the tank is conducted. Then the pressure is discarded, and the testing ceases. After the testing of the first stage the entire sampling of the tanks is headed for the shop where the supporting hemispheres are sheared by a cutter on the perimeter of the weld joint 6.

In the course of tests of the second state there are carried out hydraulic tests of the fuel cavity of the tank up to the destruction of the diaphragm devoid of solid support. With the tests there occurs a further bulge and the thinning of the diaphragm, which concludes with its explosion. The final volume of the fuel cavity is determined by the linear deformation of the diaphragm and can be accepted for an estimate of the reserve of its strength, which, as is known, determines the reliability of this element.

A feature of the test programs for reliability for some articles can be the time constraint of functioning. It entails the fact that the operating element of the system must not only function normally but also operate not later than a certain preset time. In the process of the tests in this case not only the load and strength but also the triggering time of the working element are measured. This parameter is also characterized by the mathematical expectation and rms scatter. If we utilize the previously established dependences for the estimate of the reliability of this type of element, then a solution can be found with the aid of the two-dimensional normal distribution [8].

Reliability tests of elements and systems of single action, just as systems and elements of repeated action, have the following aspects. Initially stated is the purpose of determining the assigned characteristics of reliability and then the type of failure or weakest link. With this the analysis of the test results is not concluded, but examined are promising programs, for example, the possibility of the perfection of the design from the viewpoint of an increase in the achieved level of reliability and economic advisability of the continuation of the works, and so on. The rationally composed programs should, as a minimum, proceed from the condition of the optimization of attainable levels of reliability, periods of final development and economic expenditures.

The primary task of reliability tests is an estimate of the maximum possibilities of the article, its design solutions with the most adverse combinations of external and internal factors under the actual conditions of operation, and also the evaluation of the reserves, which guarantee the efficiency of the article under these conditions. It is expedient to designate the latter sometimes as being more rigid than is provided for by the regular program.

§ 5. RELIABILITY TESTS UNDER UNSTABLE CONDITIONS. PLANNING OF RELIABILITY TESTS

Until now, speaking about test conditions, we considered them as being strictly fixed. In this case for systems and elements of single action the following control parameters were recorded: load and strength, and for systems and elements of the repeated action, as a rule, the accrued operating time in the form of the time of the operation without breakdown or quantity of cycles. In a number of experimental programs the conditions of test work are accepted to be sufficiently rigid, and they correspond to the maximum unlikely combinations of external and internal factors. This approach to the estimate of reliability can sometimes cause substantiated objections. There arises the question, what should be the levels of the parameters which determine the test conditions for the optimum programs? This leads to the thought that it is desirable to have several test programs for reliability which would consider not only the maximum combinations of the significant factors, but also their nominal values. Moreover, for the combinations of maximum levels, there are maximum and minimum values. Their account by itself already requires the realization of at least two programs. All this as a final result will substantially reflect the general periods of the final development of the systems and material expenditures. Let us see if it is not possible to develop one general program which would consider in an optimum way the variation of the significant factors at several levels.

In the planning of such tests their program approaches conditions of the realization of the factor plan. The possible distinction lies in the fact that at the beginning of the reliability tests there is a sufficiently large volume of statistical data on the final adjustment, and there is the possibility of sufficiently fully determining the composition and rank of the significant factors. At the same time the mathematical model of the process is known, or it is possible to determine it. For the output quality of the process in this case the investigated nature of reliability (control parameter, the time of accrued operating time up to failure, number of cycles, "load', "strength") is acquired.

The indicated characteristic is the argument of the function of the distribution of failures by which the tested system or element follows. We assume that the distribution of failures corresponds to the law of Weibull, then by taking into account the knowledge of the real model of accrued operating time the expression for the probability of trouble-free operation can be presented as

$$P(x_i) = \exp\left[-\frac{y^{\beta}(x_i)}{y_0}\right].$$

where $y_0 = \left[\frac{y_{cp}}{\Gamma\left(\frac{1}{\beta}+1\right)}\right]^{\beta}$.

In this case the mathematical expectation y_{cp} is the scale characteristic for the parameters of distribution y_0 and β. The question can be similarly solved for other forms of distributions distinct from the Weibull law. In a number of cases the form of the law can be previously introduced into the calculating program. The analytical interconnection of the results of reliability tests with parameters of the distribution law of failures can be established by means of the plausibility function [76].

For an illustration of this method let us examine the plan for the reliability tests in which the variation six of quantitative factors is considered: x_1, x_2, x_3, x_4, x_5, x_6. The test procedure is established by taking into account limitations on randomization. The tests are carried out in the sequence determined with the aid of the table of random numbers.

The basis of the proposed plan is the method of steep ascending over the surface of the response with which accepted as the initial form of the model of the physical process is the polynomial of the first degree

$$y = b_0 x_0 + \sum_{i=1}^{n} b_i x_i.$$

Hence from this condition there is initially carried out $N \geq (n+1)$ experiments in the program presented in the plan for the reliability test (Table 4.1).

In our case there are 8 experiments. The variation of factors is conducted at two levels: upper and lower. By the processing of the results by the method of least squares the regression coefficients of the mathematical model are determined.

Table 41

Variable factor	x_1	x_2	x_3	x_4	x_5	x_6	Output quality: experiment	Output quality: calculation
Main level	a_1	a_2	a_3	a_4	a_5	a_6		
Upper level	+	+	+	+	+	+		
Lower level	−	−	−	−	−	−		
Interval of variation	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6		
Testing sequence								
4	−	−	−	−	−	−	y_4	$\dot{y}_4$
8	+	−	−	+	+	−	y_8	$\dot{y}_8$
2	−	+	−	+	−	+	y_2	$\dot{y}_2$
1	+	+	−	−	+	+	y_1	$\dot{y}_1$
6	−	−	+	−	+	+	y_6	$\dot{y}_6$
3	+	−	+	+	−	+	y_3	$\dot{y}_3$
7	−	+	+	+	+	−	y_7	$\dot{y}_7$
5	+	−	+	−	−	−	y_5	$\dot{y}_5$
Regression coefficient	b_1	b_2	b_3	b_4	b_5	b_6		
Step of variation	$\Delta_1 b_1$	$\Delta_2 b_2$	$\Delta_3 b_3$	$\Delta_4 b_4$	$\Delta_5 b_5$	$\Delta_6 b_6$		
Planned and monitoring tests								
9	← $a_i+b_i\Delta_i$ →						y_9	$\dot{y}_9$
10	← $a_i+2\Delta_i b_i$ →						y_{10}	$\dot{y}_{10}$
11	← $a_i+3\Delta_i b_i$ →						y_{11}	$\dot{y}_{11}$
12	← $a_i+4\Delta_i b_i$ →						y_{12}	$\dot{y}_{12}$

Then is carried out the following stage of planning for the purpose of motion along the surface of the response in the direction of the gradient of linear approximation. For the purpose of the testing of the reliability of the model a number of monitoring tests is accomplished. Their results are compared with the calculated results. With considerable disagreement the regression coefficients are more precisely formulated by the successive approximation, and the new direction of the gradient for motion along the surface of the response is selected.

Sample size, as a rule, is limited to not more than twenty articles.

In the analysis of the test results from the viewpoint of the evaluation of accuracy, the testing of the convergence of experimental and theoretical data is important. In this case the basic error when evaluating the reliability is determined by the sample size presented for the tests. The method of steep ascending along the surface of the response is usually formulated as the problem of the determination of extreme conditions. In our case this solution is used for the search and description of a certain local zone of the model by means of linear approximation. Knowledge of the model and composition of the significant factors makes it possible to establish the degree of the effect each of them on the reliability of the tested article.

§ 6. METHODS OF THE WEIGHTED TESTS

With the final development of complex technical systems with a very large number of elements, when in brief periods it is necessary to insure the high requirements with respect to reliability and lifetime, the methods of accelerated tests become virtually uniquely possible. In this case one can speak about the shortening of the time of the tests because of an increase in the failure rate.

A peculiar variety of accelerated tests consists of weighted tests which differ by the increased level of the load of the operational characteristics. In connection with the ZhRD such parameters can be combustion-chamber pressure and the fuel-component ratio. Correspondingly, the increased level of the load for the ZhRD means boosting according to the conditions and thrust up to the boundary conditions. The latter determine those maximum levels of the basic parameters higher than which the engine is inefficient. The nature of the loading can be sufficiently diverse and changed in accordance with the test program according to different laws. The program of the weighted tests can be characterized by the emergence of the engine into the nominal mode with the subsequent boosting up to boundary conditions or emergence into the forced system, passing by the nominal values of the parameters. In the first case the emergence into the assigned system is reached by an adjustment of the engine to the rated conditions, and its further boosting occurs as a result of the start of the control system.

In the second case the final mode is provided only by the adjustment of the engine at the extreme values of the parameters, which are then maintained constant during the entire time of the tests.

This program, in providing the tests for boundary conditions, can be realized in such a case when they are previously known.

Besides this some forms of the possible programs of the weighted tests can be represented by curves each of which determines the exponential, linear and stepped laws of loading, respectively. It should be noted that the maximum level of boosting must not be determined by the boundary conditions: it can be below, but, as a rule, exceeds the upper limit of control x_{max} assigned in technical assignment for development.

A change in parameter x in accordance with the program can provide for the simultaneous variation and other factors. However, their number must not be large, since this complicates the analysis of the test results. As a rule, within the limits of one program of the weighted tests, the number of variable factors does not exceed two. Let us allow that there is a certain system whose efficiency is determined by limits of the control of the two variables x_I and x_{II}. Each of them has three assigned levels of variation - minimum (min), nominal (nom) and maximum (max). By transferring them to the curve (Fig. 4.8), it is possible to obtain a working square with the vertexes a_1, b_1, a_1', b_1'. The point A determines the nominal mode. In all probability, depending on the design features the emergence of the system into this system can be characterized by the curve aA or a'A. It is easy to explain this phenomenon in a particular case.

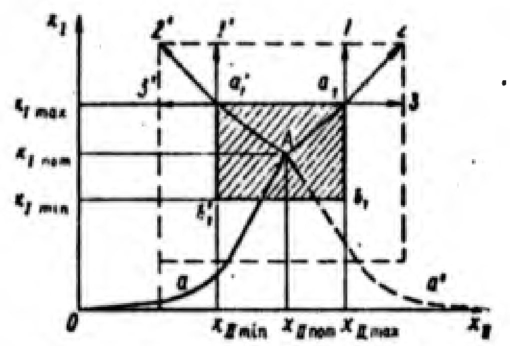


Figure 4.8. Operating modes of the dynamic system under boundary conditions.

Let us assume that the factor x_I characterizes the combustion chamber pressure and factor x_{II} - the fuel component ratio, then curve aA is the characteristic of activation of the engine with the advance of the fuel in the combustion chamber, and curve a'A - with the advance of the oxidizer. In the first case the limits of a change in parameter x_{II} from zero to x_{IImax} $(0<x_{II}<x_{IImax})$, and in the second - parameter x_{II} changes in the range of infinity to x_{IImin} $(\infty>x_{II}>x_{IImin})$.

After the system emerges into nominal mode in accordance with the test program, further boosting of the parameters occurs.

As is known, it occurs in the direction of the characteristic Aa_1. But in a number of cases the boundary conditions can prove to be nearer with motion in the direction Aa_1'. In this case the vertexes of square a_1 and a_1' denote the assigned maximum combinations of factors $x_{Imax} \cdot x_{IImax}$ and $x_{Imax} \cdot x_{IImin}$, respectively.

Further from each vertex of square the boosting before the boundary conditions can be conducted in three directions. On Fig. 4.8 they are respectively designated by numbers 1, 2, 3 and 1', 2', 3'.

In parametrical form this can be characterized in a following manner:

mode 1 is determined by the combination of factors $x_{I\,\Pi} \cdot x_{IImax}$;
mode 2 - by the combination $x_{I\,\Pi} \cdot (x_{II\,\Pi})_{max}$;
mode 3 - by the combination $x_{Imax} \cdot (x_{II\,\Pi})_{max}$;
mode 1' - by the combination $x_{I\,\Pi} \cdot x_{IImin}$;
mode 2' - by the combination $x_{I\,\Pi} \cdot (x_{II\,\Pi})_{min}$;
mode 3' - by the combination $x_{Imax}(x_{II\,\Pi})_{min}$,

where subscript "Π" denotes the maximum permissible level of variation for the parameter x_{II} to the side of boosting $(x_{II\,\Pi})_{max}$ and to the side of throttling $(x_{II\,\Pi})_{min}$; for x_I - only to the side of boosting $x_{I\,\Pi}$. Modes 2 and 2' are respectively characterized by a simultaneous change in both parameters up to the boundary conditions. Unlike them, modes 1, 3 and 1', 3' are characterized by the fixed level of one of the factors using the variable second parameter.

In summation, beyond the limits of the working square there can be obtained another external square, which characterizes the maximum levels of the factors with which the final development of engine or the limits of its efficiency are conducted.

The square in Fig. 4.8 considered is conditionally noted by a dashed line. However, in practice the external "square", determined by boundary conditions, does not have so correct a geometric shape. The parametrical field of the efficiency can take different contours. As an example let us point out in Fig. 4.9 where the zones of unstable operation are shaded.

It is quite obvious that the value of the field of the efficiency allows the designer to make a confident selection of the operational parameters and characteristics of the engine which ensure the high level of the quality of final development.

The practical realization of the experimental program depends on a wide range of the control of the basic parameters and the possibility of providing repeated switch-ons of the engine. In all the remaining cases when it is not possible to test the engine repeatedly, it is necessary to test a large quantity of regular articles, which increases the total periods of final development and material expenditures.

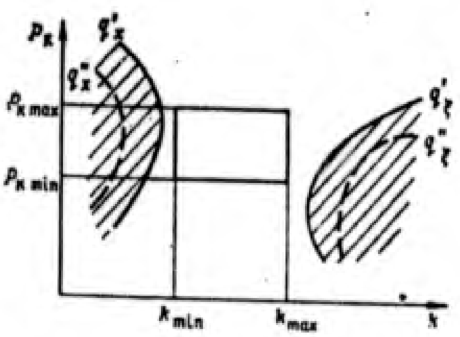


Figure 4.9. Qualitative pattern of the distribution of boundaries of the efficiency of the ZhRD.

The zones of the incapacity for work of the engine noted in Fig. 4.9 are characterized by the increased intensity of the failures. One should understand this so that the probability of failure-free operation on the boundary of this zone is low but always equal to zero. Most frequently it is determined by the failure type the probabilities of the appearance of which in this case are designated by q_x and q_ξ.

With distance into the depth of the zone, the failure probability increases. In summation, it is virtually possible to define the boundaries equal to the failure probability:

$$q_i' < q_i'' \text{ and } q_i'' < q_i'''.$$

As a whole this representation of the zone of incapacity for work reminds one of the geodetic image of intersected terrain and can be described mathematically as the surface of response. Similar boundaries of equal probability exist in the field between the working "square" and the designated zone. But only in this case is the failure probability lower than that permissible and as a whole satisfies the assigned requirements with respect to reliability. However, the region of the efficiency, as a rule, is determined by the limited quantity of the tests; therefore, in all cases it is expedient in its determination to introduce confidence limits and not to be satisfied by random results.

With the experimental final development the empirical search for the boundaries of the efficiency is very complex in a technical respect and causes the mass of the difficulties, which, first of all, entail the individual adjustment of the engine to the basic parameters under the boundary conditions, which sometimes it requires special assembly, the selection of assemblies with maximum characteristics, and so on. Therefore, the use here of mathematical methods, which make it possible to formalize the process in the form of the surface of the response, deserves the most serious attention. By means of them, according to the small number of tests, it is possible to determine the mathematical model of the investigated process and the limits of equal failure probability and finally carry out the forecasting of the expected results.

CHAPTER V

APPLIED METHODS OF RELIABILITY THEORY IN SYSTEMS OF ONE-TIME USE

§ 1. BASIC CRITERIA AND STATISTICAL MODELS OF RELIABILITY

The brief presentation of the applied methods of the reliability theory in systems of one-time use requires the refinement of some positions for the statement of the problem and determination of the basic indices. This is necessary for using the fundamental theorems and laws in the planning of tests, the processing of results and the evaluation of the technical state of the articles in different stages of development.

Let us agree to understand by systems and elements of one-time use as the unrestorable systems and their elements in the form of units, assemblies and articles of a different form of technical equipment, including rocket and space technology. Unlike them, the systems of repeated use are restorable since they allow the repair and replacement of malfunctioned subassemblies, units, and so on. Thus, for instance, the electropneumatic valve (EPK) [ЭПК] is a typical element of repeated action. At the same time it can be applied in systems of one-time use of the rocket type. The nature of the operation of the EPK is retained independently of the object of the use, but the principle of its operation for each of these systems is completely different. If at thermo-electric power station the EKP is the restorable element of the system, then on rockets it is impossible to carry this out in the process of the use (this is admissible with storage in arsenals, on training articles). The same refers also to elements of single action of the type of burst diaphragms. Incidentally it should be noted that their use on systems of repeated use is very limited.

The reliability as a complex characteristic determines the property of the article to remain operable under the assigned conditions. In this case the quantitative measure of reliability should be understood as the probability of failure-free operation throughout the period of functioning under the assigned operating conditions with any possible combinations of external and internal factors.

The estimate of reliability is characterized, first of all, by the determination of the probability of the onset of the event of interest to us.

Checking of reliability is the checking of the accomplishing of requirements with respect to reliability on the part of the customer on the basis of confirmed methods of quantitative indices.

The provision for reliability is the realization of the plan on the basis of plans for the conducting of experimental works, procedures for final development, various kinds of programs and other documents, which facilitate the execution during the established periods of the technical assignment for the system as a whole.

Let us assume that with tests of a large number of articles under some fixed conditions the failures were revealed. In the reliability theory this phenomenon is characterized as the flow of failures. The latter are subordinated to definite laws which depend on the design and technological features of the article, the physical properties of the processes, and also the operating modes or test conditions. These laws, expressed by mathematical symbols, are conventionally designated as models of failures. In the

For the hypergeometric distribution the binomial is a special case.

Taking into account that a large quantity of investigations [21, 76] is devoted to this question, we will be limited to that which was given above. The possibility of the practical use to evaluate the reliability of the qualitative models will be examined in the following paragraph. A common shortcoming in these distributions is the following. They do not consider the physical features of the investigated processes, the time and forms of the accrued operating time, reasons for failures, but only very fact of their manifestation. Therefore, in the examination of results of the tests much valuable information about the behavior of the physical parameters is lost. Correspondingly, for the confirmation of acceptable levels of reliability the extremely large volume of statistical data is always required.

However, with the aid of qualitative models the specific problems, whose solution by other methods is not always possible, can be solved. For example, if articles of mass or large-scale production must undergo the destructive forms of checking, then to evaluate the quality of commodity production qualitative models can be used.

Such methods have an extensive application [63, 88]. If the production is small-scale or even unit, then to evaluate the quality and reliability of these articles it is necessary to search for already other, possibly, physical methods.

§ 2. ANALYSIS OF THE RELIABILITY OF ESTIMATES IN THE QUANTITATIVE EXAMINATION OF RELIABILITY

The determination of quantitative indices is the result of the processing of the accumulated information. The calculation is preceded by the collection, systematization and classification of reliability theory and mathematical statistics, they are known as the distribution laws.

The quantitative reliability indices can be obtained only on the basis of the knowledge of distribution laws of failures. The latter are established by the processing of statistical data. If one speaks about the quantitative reliability indices of elements and systems and not about for which conditions they are suitable, then these conditions are implied. If the test conditions are not determined, the quantitative reliability indices have no practical meaning. They are valid only for that form of information and those conditions under which they are obtained. An exception should be considered the "accelerated" or "weighted" types of tests, about which we were speaking above.

The forecasting of reliability on the basis of statistical estimates can be conducted in the direction of their refinement as a result of a change in the volume of information but not its form or test conditions. More accurately, if according to tests of a certain sampling or totality of the articles the estimate of reliability P is obtained, then with a definite reliability it is possible to confirm that on the average it will be preserved for future tests of the regular articles carried out under the same conditions. But if the conditions were changed, or they are not stable, then it is not necessary to speak about correctness of the prediction.

Nevertheless, there is a vast class of systems and elements whose efficiency is provided within the sufficiently wide limits of a change in the environmental factors, i.e., the oscillation in the known boundaries of the test conditions will not attract a considerable change in the reliability indices. In the specific case this conclusion requires experimental confirmation. If this is so, then the appropriate conclusion is made about the insensitivity of the articles to a change in test conditions, and their reliability can be estimated according to the entire totality of statistical data.

As the basic criterion used for estimating the reliability of the mechanical systems of one-time use, one should consider the probability of failure-free operation

$$P(x)=\int_x^{\infty} f(x)\,dx,$$

where f(x) is the density of distribution of parameter x or the failure rate.

Unlike the two named criteria P(x) and f(x), the failure rate λ(x) has a very limited use for the stationary flows of failures in which

$$\lambda(x)=\text{const.}$$

The failure rate is the characteristic of reliability and quantitatively is equal to the ratio of distribution density f(x) to the probability of failure-free operation P(x):

$$\lambda(x)=\frac{f(x)}{P(x)}$$

In any case the probability of failure-free operation can be defined as the function of the failure rate:

$$P(x)=\exp\left[-\int_0^x \lambda(x)\,dx\right]. \qquad (5.1)$$

From equation (5.1) it is easy to obtain the expression for the special case when λ(x)=const:

$$P(x)=\exp(-\lambda x).$$

Correspondingly, for the distribution density

$$f(x)=\lambda\cdot\exp(-\lambda x).$$

This special case is the exponential law for the stationary flows of failures.

If the flow is characterized by condition $\lambda(x)\neq$const and has a density of distribution of the form

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x-x_0)^2}{2\sigma^2}\right],$$

then it follows the normal distribution, in this case x_0 and σ are distribution parameters (x_0 - mathematical expectation; σ - rms deviation). The random variable x changes within limits of

$$-\infty<x<+\infty.$$

The probability of failure-free operation will be equal to

$$P(x)=\text{Fo}\left(\frac{x_0-x}{\sigma}\right),$$

where Fo is the Laplace function.

If value x characterizes the time of normal functioning or another parameter of accrued operating time, then, naturally, the lower limit of its change is limited to zero:

$$0<x<+\infty.$$

Taking this into account the law itself is somewhat modified, which in this case one should consider truncated by magnitude of the coefficient

$$\left[\frac{1}{Fo\left(\frac{x_0}{\sigma}\right)}\right].$$

Then the expression for P(x) will take the form

$$P(x)=\frac{Fo\left(\frac{x_0-x}{\sigma}\right)}{Fo\left(\frac{x_0}{\sigma}\right)}.$$

The accuracy of such estimates, as will be shown below, to a great degree depends on the volume of the tests. In such a case, when they are small, the Student distribution can be used [76]. The parameters of this distribution agree well with the normal with the quantity of tests being more than thirty ($R \geq 30$). The characteristics (x_0-x/σ) and (x_0/σ) are the quantiles of normal distribution. For them there are Tables [76] which make it possible to estimate operational by the probability of failure-free operation.

Thus we examined very briefly two forms of distributions - exponential and truncated normal. Their empirical investigation prompted an additional two very important properties: the exponential law describes only the sudden and the truncated normal the gradual failures.

However, in practice in the majority of the cases with tests of elements and systems of one sampling, both types of failures together are encountered, but the parts of their manifestation are different. Then the solution is accepted as the following. The test results are strictly demarcated and classified according to types of failures, and are estimated separately the characteristics of reliability in relation to the sudden P(x) and gradual P(x) failures. The total probability of the failure-free operation is equal to the product:

$$P(x) = P(x)_{\text{вн}} P(x)_{\text{нео}}$$

and portions of their manifestation c_1 and c_2 in total are equal to unity. But if the distributions of failures and portions of their manifestation c_1 and c_2 are known, then to evaluate the probability of the failure-free operation the superpositions of two laws can be used, and in this case the condition

$$c_1 + c_2 = 1.$$

is observed. In our case

$$P(x) = c_1 \exp(-\lambda x) + c_2 \frac{F_0\left(\frac{x_0 - x}{a}\right)}{F_0\left(\frac{x_0}{a}\right)}.$$

However, for the practice of the final development of elements of the complex technical systems, portions of the distribution of failures are previously unknown. Therefore the use of this form of the superposition of laws not always is represented possible.

From the viewpoint of the empirical investigation of the distribution of failures, for a number of the mechanical systems the law of Weibull deserves serious attention [37]:

$$P(x) = \exp\left(-\frac{x^\beta}{x_0}\right).$$

The feature of this distribution lies in the fact that it is the generalizing feature for such distributions as the exponential (with β=1.C) Rayleigh (with β=2.0) and the truncated normal (approximately).

Figure 5.1 gives the family of curves of the probability distribution of the operation without breakdown for the Weibull law at different values of the β index. From the curves it is evident

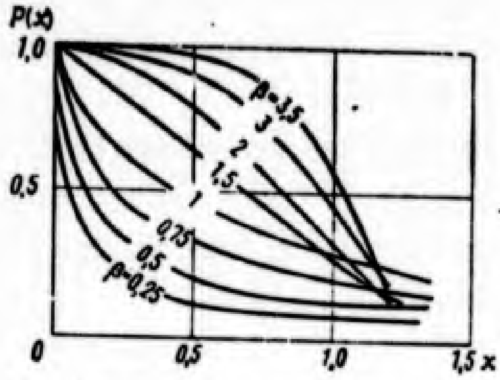


Figure 5.1. Probability distribution of operation without breakdown for the Weibull law.

that with an increase in the parameter β, and namely when β≥1.5, the sign of the curvature is changed.

At values β=3.25 and above the Weibull distribution has a sufficiently good convergence with the truncated normal distribution law. The degree of this convergence increases in proportion to an increase in β. Quantitatively an increase in index β for the truncated normal distribution denotes a decrease in the coefficient of variation.

If one takes into account that the exponential law describes sufficiently well the sudden, and the truncated normal the gradual failures, then it becomes obvious that the distribution index characterizes a change in the failure type. As the investigations show, the Weibull distribution at values of β≤1.0 describes the sudden and at β≥3.25 only the gradual failures. If this is so, then with a change in the index β in the interval of 1.0 to 3.25 the Weibull distribution can describe the different combinations of each failure, i.e., sudden and gradual. The portion of some and others depends on the value of the index β: the nearer to unity, the higher the portion of random failures, and, on the contrary, with an increase in it the portion of sudden failures is decreased, and that of gradual failures is increased. The portion of gradual failures predominates at values of β>1.5. This means that the Weibull distribution in a number of cases can replace the superpositions of two laws - the truncated normal and exponential. Therefore, the need for the division of failures in the processing of experimental data into sudden and gradual is eliminated, and, consequently, the possibility of the errors connected with the acceptance of the subjective solutions in the classification of the failures is eliminated.

The considered distribution laws describe the different flows of failures for the fixed test conditions. They are valid for determining the physical processing. But, strictly speaking, for mechanical systems and their elements the operating modes are changed in time. For the majority of the mechanical systems and their elements there are modes of switch-on, which are steady-state, or sustainer modes in accordance with the predetermined program of tests and finally modes of switch-off. For any system it is possible to determine the transient and fixed operating modes. But in a number of cases, in accordance with these modes, the physical processes themselves are changed. As an example it suffices to indicate the operation of the combustion chamber of the ZhRD [ЖРД - liquid-propellant rocket engine] or at least such a simple element of the pneudraulic system as the EPK.

The latter in the mode "Normally closed" accomplishes functions of the closing element. In the "Switch-on" mode it passes a definite gas consumption. In both cases the physical processes are different and different structural elements of the EPK, in this case as systems operate.

For a combustion chamber the processes of starting and operation under sustained conditions in no way correspond to one another in relation to the carburetion, fuel combustion, and dynamic and thermal loads. In virtue of this, the failures characteristic to these modes must be different. Therefore, they cannot be described by any one of the distributions examined above, eliminating the superpositions of several laws. As a whole this is confirmed by the analysis and processing of statistical data. The theoretical distribution for flows of failures, obtained by the processing of a large number of tests, in similar cases has a bend.

On Fig. 5.2 the latter corresponds to the moment of time a. As can be seen from the figure, line 1' corresponds to the exponential law (λ=const) and curve 2' - to the period of manifestation of the abrasion failures described by the truncated normal law ($\lambda \neq$const). Curves 1 and 2, in judging from the nature of their change, in all probability, correspond to the two Weibull distributions, when $\beta_1<1.0$ $\beta_2>1.0$, or to the Weibull distribution when $\beta<1.0$ and the truncated normal law. The given dependences can be respectively described mathematically by the superposition of two laws or more precisely by the product of the conditional probabilities:

for 1'-2'

$$P(t) = [\exp(-\lambda t_1)]\,Fo\left(\frac{t_{02} - t_2}{\sigma_2}\right);$$

for 1-2

$$P(t) = \exp\left[-\left(\frac{t_1^{\beta_1}}{t_{01}} + \frac{t_2^{\beta_2}}{t_{02}}\right)\right],$$

where t_1 has limits of a change in parameter t from zero to the moment of time a $(0 \leqslant t_1 \leqslant a)$; t_2 is changed within limits of the moment of time a to infinity $(a \leqslant t_2 \leqslant \infty)$; β_1; β_2 and t_{01}; t_{02} - parameters of the Weibull distribution and the mathematical expectations corresponding to periods t_1 and t_2.

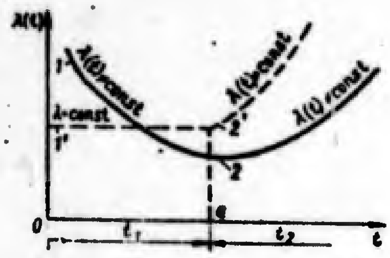


Figure 5.2. Superpositions of two distribution laws for the failure rate.

Both distributions are superpositions of the two laws. If one considers that the exponential and truncated normal distributions, with the appropriate values of the parameters, are sufficiently accurately described by the Weibull distribution, then the superposition of the first distribution is virtually a special case of the second. The latter for a certain quantity of modes can be presented in a more general form:

$$P(x_i < x) = \exp\left(-\sum_{i=1}^{d} \frac{x_i^{\beta_i}}{x_{0i}}\right).$$

In summation, we obtained a new distribution for any quantity of superpositions of the Weibull law. It can be applied for the study of the flow of failures of elements of complex systems which operate in several consecutive modes.

For this distribution the probability density is equal to

$$f(x_i < x) = \sum_{i=1}^{d} \frac{\beta_i}{x_{0i}} x_i^{\beta_i - 1} \exp\left(-\sum_{i=1}^{d} \frac{x_i^{\beta_i}}{x_{0i}}\right).$$

and the failure rate

$$\lambda(x_i < x) = \sum_{i=1}^{d} \frac{\beta_i}{x_{0i}} x_i^{\beta_i - 1}.$$

Thus the examined distribution is a multiparametric law capable of describing on the plane of the combination virtually any flows of failures which characterize the different operating modes of elements of complex systems. In this case to estimate the lower boundary of the total probability of failure-free operation in accordance with the central limit theorem, there can be used the expression of the form

$$P(x_i < x)_{\text{ниж}} = \bar{P}(x_i < x) - t_\alpha \sigma_{P(x_i<x)},$$

where $\sigma_{P(x_i<x)}$ is the total rms deviation of the estimate of composite probability determined by the formula

$$\sigma^2_{P(x_i<x)} = \left\{\sum_{i=1}^{d} x_i^{2\beta_i}\left[\sigma^2_{\left(\frac{1}{x_0}\right)_i} + \left(\frac{\ln x_i}{\bar{x}_{0i}}\right)^2 \sigma^2_{\beta_i} + 2\frac{\ln x_i}{\bar{x}_{0i}} \operatorname{cov}\left[\left(\frac{1}{x_0}\right)_i; \beta_i\right]\right\} \exp\left(-2\sum_{i=1}^{d}\frac{x_i^{\beta_i}}{\bar{x}_{0i}}\right)\right].$$

where $\sigma_{\left(\frac{1}{x_0}\right)_i}, \sigma_{\beta_i}$ and $\operatorname{cov}\left[\left(\frac{1}{x_0}\right)_i; \beta_i\right]$ are, respectively, the rms deviations and the covariance of the instantaneous values of the distribution parameters $\left(\frac{1}{x_0}\right)_i$ and β_i, which characterize the separate operating modes and flows to failures characteristic to them.

Further let us examine one of possible examples for the purpose of the practical use of the examined law.

Let us assume that the system operates on three modes. The first system is continued from zero to 0.2 $(0 \leqslant x_1 \leqslant 0{,}2)$, the second from 0.2 to 2.0 $(0{,}2 \leqslant x_2 \leqslant 2{,}0)$ and the third from 2.0 to x $(2{,}0 \leqslant x_3 \leqslant x)$. The interconnection of the mathematical expectation x_0 with mean statistical value of x_{cp} is determined in accordance with the expression

$$x_{cp} = x^{\frac{1}{\beta}} \cdot \Gamma\left(\frac{1}{\beta} + 1\right).$$

The characteristic x_{cp} is taken as a scale factor, and in our case for all modes $x_{cp}=5.0$.

As follows from equation (5.2), parameters x_0 and β are connected with each other. Even the insignificant change in parameter β substantially affects the value of the mathematical expectation. Thus, for instance, if we accept that $\beta=2$, then with $x_{cp}=5$ the mathematical expectation $x_0=31.8$, with $\beta=3$ the mathematical expectations is almost an order higher ($x_0=176$), and with $\beta=4$, $x_0=954$. This indicates that for the Weibull distribution parameter β is determining not only quantitatively but also qualitatively. Actually the value of parameter β indicates the form of the flow of failures.

Together with the examined models, in the theory of reliability and mathematical statistics other forms of distributions can be applied, but the field of their use is considerably narrower. The indicated distributions consider quantitative information in the form of operation time x.

Nevertheless, there are statistical models which make it possible to consider the qualitative information. The most widespread in practice are the hypergeometric distribution, binomial law and Poisson's law [21, 76].

Let us assume that there is a certain totality of articles N from which there are no flaws M. We want to estimate the quality of this totality from tests of the sampling R. Then the probability that with tests R of m articles there will be no flaws is determined in accordance with the hypergeometric distribution by the formula

$$P_{R,m}=\frac{(N-R)!\,(N-M)!\,R!\,m!}{(M-m)!\,(R-m)!\,((N-M-R+m)!\,N!\,m!}.$$

statistical data. The reliability of the obtained reliability indices is determined by the accuracy of these data.

The method of determination of the quantitative indices depends on the form of information, and the accuracy of the estimate basically depends on the volume of statistical data. The estimate of the accuracy can be conducted by different methods, in particular, by means of confidence limits or rms deviations.

In specialized literature most frequently examined are questions of provision and checking of the reliability of elements of radio-electronics and electrotechnical equipment. This is explained by their mass nature and specific nature of design. The mass production of the elements makes it possible to accumulate and process the vast information, on the basis of which the failure types, the laws of their distribution, characteristics of reliability, etc. are established quite accurately. The specific feature of the design includes the fact that the electronic systems, in their majority, consist of standard elements, the characteristics and conditions of the tests of which are previously known. According to the λ-characteristics of these elements, for the assigned test conditions it is possible to estimate the reliability of the equipment even at such early stages of development as design.

The applicability of these methods in their majority is based on the use of the exponential distribution law for which the failure rate does not depend on the time of operation time (λ=const).

In all the remaining cases when the distribution of failures of the elements differs from the exponential ($\lambda \neq$const), the use of the λ-characteristics leads to serious errors.

The reliability of the system is determined largely by its parameters and test conditions. For any equipment they are stipulated in design, technological and operational technical documentations. For electronic equipment the λ-characteristics are obtained for the fixed conditions under which the systems operate. This is why their use for estimating the reliability of such systems is substantiated.

For nonelectric systems, in particular, mechanical, the specific nature of the design and production is different. The system consists, basically, not of standard elements, but of assemblies and subassemblies specially designed for this system. In a number of cases there are no analogs and prototypes. When they do exist, their operational characteristics can differ many times from those assigned in value, and the design and flow scheme differs by the degree of perfection.

Let us examine the example: the ZhRD is designed on the cryogenic oxidizer-oxygen of up to 1000 t in thrust ($0.98 \cdot 10^7$ N) and the combustion chamber pressure of 300 kgf/cm^2. It is required to estimate its reliability in the planning stage. The statement of the problem is substantiated, especially when several design concepts are designed and it is required to select most ideal according to considerations of reliability. However, in all probability it is impossible to expect soon the appearance of reliable methods for this estimate. Nevertheless, in published literature the λ-characteristics of mechanical rocket systems and elements are sometimes given [16]. We will speak about them somewhat later, and now let us examine the possibility of applying λ-characteristics for the solution to our problem.

Without going into technicalities, and only applying known mathematical procedures, it is possible to obtain λ-characteristics for ZhRD elements according to the test results of analog systems. In connection with the solution to our problem, a number of known types of ZhRD which operate on liquid oxygen is related to them.

The analysis and systematization of the statistical data do not make it possible to find the engine-analogs close to the designed version according to the parameters and the design and technological solution. If we adhere to the structural solutions of engine F-1, then the design of an engine with 1000 t of thrust ($1.02 \cdot 10^5$ N) and pressure p_H=300 kgf/cm^2 in all probability cannot be realized at present. The reason for this, first of all, is the complexity of providing high-frequency stability of the combustion chamber, to say nothing of the fact that with such parameters it is inexpedient to develop the engine according to the scheme "liquid+liquid" due to the high losses of specific thrust on the TNA [THA - turbopump unit]. The creation at present of an engine 1000 t in thrust pressure $p_{\kappa}=300$ kgf/cm^2 will require the development of the appropriate fields of technology.

Therefore, the ZhRD considered in our example must largely differ from its prototypes, first of all, in design relation [101, 102].

The analog systems also are distinguished one from another not only in power engineering, design and technological characteristics, but also in the kind of fuel. This places in doubt an attempt to generalize the test results of the ZhRD of the different period of developments and different ultimate purpose.

An attempt at the calculation of the design reliability of the ZhRD according to λ-characteristics leads to the fact that the obtained quantitative indices for the different design versions of the engines of 680 to 1000 t in thrust ($0.67 \cdot 10^7$-$0.98 \cdot 10^7$ N) in practice do not differ from each other (a thrust of 680 tons corresponds to the engine-analog F-1). This paradox is explained by the fact that the λ-characteristics do not consider the design and technological features of the promising schemes. But since information about their efficiency in the early stages of the plan is absent, then quantitatively the reliability cannot be determined. Therefore, the calculations of reliability based on λ-characteristics of previous developments cannot be reliable.

That stated above is not found in contradiction with the known works with respect to reliability. As an example it is possible to give the following affirmation [37]: "... usually only with electronic and electromechanical equipment is it possible to gather sufficient information and obtain from it the substantiated analytical dependences .... Equipment elements of the nonelectronic type for the most part are developed specially for the specific system with its functional scheme and technical specifications. Specifically, these are turbines, combustion chambers, gas generators and other elements of the ZhRD .... In these and similar examples the information about failures of elements is usually very limited and is related only to this specific system. Consequently, it cannot be applied to other systems of this type."

This is entirely related to the λ-characteristics of mechanical systems. If such characteristics are given, then they are suitable for the calculation of the reliability of only those systems according to test results of which are obtained when the distribution law of failures is exponential, i.e., λ=const.

If the discussion is about the reliability of the reliability indices, based on the qualitative forms of information, then it largely depends not only on volume and quality of the initial data, but also on the objectivity of the experts who generate the statistical estimates. The fact is that the reliability of reliability indices in this case is based on methods of the classification of results of tests which, as a rule, are reduced to the simplest division of tests into examined and unexamined with the subsequent division of examined tests into successful and failure. However, as practice shows, the approach to the evaluation of test results for different groups of specialists in a number of cases can be different. This is most easily shown in following examples.

Let us assume that it is required to estimate the probability of the faultless operation of an automatic machine tool, which makes retainer screws of one standard dimension. To do this, a sufficiently large sample of the articles (one or several) is taken, and by means of continuous checking their quality is checked. If the normal production process is examined, then all the articles are considered to be examined. The articles which do not satisfy the technical specifications are classified as "failures." In this case the uniqueness of the solution is provided by the method of checking.

Another matter is when the test results of complex systems are analyzed. For example, on the test stand N engines are tested. The visual inspection and subsequent flaw detection showed that in a number of cases there was observed the erosion of walls of the combustion chamber or small burnouts of the nozzle ("streaks"). The noted defects did not lead to a change in the specifications of the article. If this happened in flight, the final goal would be fulfilled, and the defects remained unnoticed. How does one classify these cases: as defects or failures? The opinions here can be contradictory: some of the researchers confirm that these are defects which did not lead to any noticeable consequences; others consider that these are failures, since they are connected with the disturbance of the completeness of material part.

In a number of cases by references to technical specifications and records such defects will be referred to failures, although these solutions are to a greater degree, more juridicial than technical.

There are many similar examples and by no means always are they connected with the distrubance of the completeness of the material part. But it is clear that for complex systems the diversity of the defects and failures is such that virtually they cannot be specified entirely by the corresponding procedures. This can lead to the acceptance of subjective or even volitional solutions.

At the same time less important is the question concerning the evaluation of the impressiveness of the initial information for the elimination of failures after the manufactured modifications. If they are successful, then the previously recorded failures from the general totality of examined tests can be excluded, for example, when the form and cause of the failure are unambiguously established and the physical methods of checking confirm the effectiveness of the adopted design modification. But in a number of cases the reason for the failure can be explained by a whole number of hypotheses, according to each of which the corresponding modifications are carried out. As a whole for checking their effectiveness, a large volume of tests will be required.

If the checking of the effectiveness of the modifications is conducted only by the processing of the qualitative information, the volume of check tests is increased. Therefore, in practice widely a study of the physical processes for the purpose of an explanation of the physical causes for failure is conducted. In this case the quantity of check tests is sharply decreased.

In summation, it can be concluded that the use for the reliability evaluation of qualitative forms of information requires, first of all, the entire substantiated approach to the methods of generalization and classification of results of the tests and the knowledge of the physical causes of failures according to which the modifications are produced. The indispensable section of any procedures must be the clear rules of the classification of test results for the evaluation of their impressiveness: examined and unexamined successful and failure.

With the summation of the initial statistical data or the generalization of quantitative reliability indices for different test programs, it is necessary to focus serious attention on the conditions under which they are obtained. As a rule, it is impossible to achieve the full identity or reproducibility of the conditions of full-scale tests in the process of final development. Moreover, the purposes and programs of the tests themselves are different. Therefore, in practice frequently used is the procedure for the supplement of statistical data, which assumes the usual summation of a certain quantity of successful tests with the regular if the first were carried out in knowingly more severe conditions. The successful tests of the articles in the weighted system are considered as examined and are equated to the regular, and failures from further examination are eliminated.

The processing of statistical data in the process of experimental final development should be conducted virtually continuously in proportion to their admission.

The reliability indices cannot be more precise than the information on the basis of which they are obtained [37]. Consequently, they cannot communicate or predetermine more greatly than the test results themselves.

The primary information is most reliable, although it contains measuring errors; the results of its processing are supplemented by errors in the systematic plan, and the conclusions made include even elements of a subjective approach to the evaluation of the obtained data.

§ 3. DETERMINATION OF RELIABILITY INDICES ON THE BASIS OF QUALITATIVE INFORMATION. METHODS OF THE PROCESSING OF STATISTICAL DATA

Let us examine some methods based on information processing classified according to qualitative characteristic "yes-no" or "success-failure." The methods do not consider the physical features of the failures and the nature and periods of their manifestation.

For this form of information the distribution function is discrete. Therefore, methods of the evaluation of reliability based on this type of information, subsequently will also be called discrete. Serving as the initial information for the calculation is a quantity of conducted observations (or tests) and a number of successes (or failures). Discrete methods virtually in equal measure can be used to evaluate the reliability of the simplest elements and most complex systems.

Subsequently, let us pass to the examination of the mathematical methods of the evaluation of reliability, considering that the number of successful tests and quantity of failures are known reliably to us. The difference between the total number of the tests and the quantity of successful results is equal to the number of failures.

From the probability theory it is known that the ratio of a certain quantity of successful tests m to the total number of tests N, carried out under specific conditions, can be characterized by a certain probability of failure-free operation P. For a small number of tests this ratio will be the random variable which can be changed within very wide limits:

$$0 \leqslant P < 1.$$

In this case the characteristic $\overline{P} = \frac{m}{N}$ is the point evaluation of reliability in the form of the mean statistical probability of failure-free operation. In the probability theory this ratio (m/N) is more frequently called the frequency or frequency of the event.

With respect to the true value of probability of failure-free operation P, the characteristic $\overline{P}$ will be the unbiased estimate of reliability.

The characteristic P is equal to the mathematical expectation of probability with a large number of tests. With the limited volume of production the difference should be considered as the error $(\overline{P} - P)$ of the probability evaluation of failure-free operation with the assigned confidence coefficient α:

$$\alpha = \text{Bep}\{(\overline{P} - \varepsilon) < P < (\overline{P} + \varepsilon)\}.$$

From this expression it follows that when evaluating the reliability the true value of probability is located in the certain confidence interval for which $(\bar{P}-\varepsilon)$ characterizes the lower and $(\bar{P}+\varepsilon)$ the upper limits.

The distributions of statistical probabilities can be different. But in practice more frequently than others the binomial and normal distribution laws are used. According to binomial distribution the upper and lower confidence limits of the probability of failure-free operation are determined respectively from equations [13]:

$$\left.\begin{aligned} \sum_{m=k}^{N} \frac{N!}{m!\,(N-m)!} P^{m}(1-P)^{N-m} = \frac{\alpha}{2}, \\ \sum_{m=0}^{N-k} \frac{N!}{m!\,(N-m)!} P^{m}(1-P)^{N-m} = \frac{\alpha}{2}. \end{aligned}\right\} \qquad (5.2)$$

The common sense of the interval estimation entails the fact that with the limited number of tests the unknown value of probability P is located within the certain interval whose boundaries are the lower and upper confidence limits of the probability of failure-free operation $P_{\text{ниж}}$ and $P_{\text{верх}}$, respectively.

On Fig. 5.3 for the frequencies $\bar{P}_1$ and $\bar{P}_2$ the confidence intervals are shown in the form of shaded areas whose limits asymptotically approach the values $\bar{P}_1$ and $\bar{P}_2$ with an infinitely large number of tests N. The quantitative value of levels of limits $P_{\text{ниж}}$ and $P_{\text{верх}}$, as can be seen from equations (5.2), depends not only on the frequency $\bar{P}$ and the volume of the tests N, but also on the confidence coefficient α.

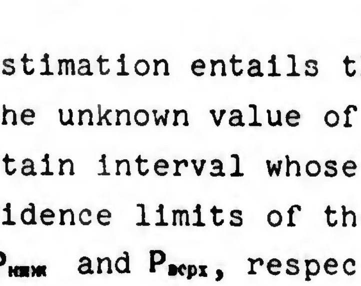


Figure 5.3. Confidence intervals of probabilities of failure-free operation $\bar{P}_1$ and $\bar{P}_2$.

If with the same number of tests the confidence coefficient α for any reason descends, the confidence interval correspondingly becomes narrow and vice versa. With reliable tests, independently of the N number, the upper limit of the confidence interval is always equal to unity.

In practice to estimate the reliability, it sometimes proves to be sufficient to use only one lower limit of the confidence interval. In this case the estimate with the aid of the upper limit is omitted.

This means that we are satisfied with the estimate during which the unknown value of probability P is not lower than the lower confidence limit $P_{ниж}$.

Thus, the concept of unilateral estimate is introduced by means of the lower confidence limit with the appropriate unilateral confidence coefficient α_1.

The interconnection of the bilateral α and unilateral α_1 confidence coefficients is established by the relation [13]

$$\alpha=2\alpha_1-1.$$

Figure 5.4 gives graphic dependences for the binomial distribution which establish the interconnection of the unilateral lower confidence limit of the probability of failure-free operation with the number of tests N and quantity of failures m_0 when α_1=0.90 and 0.95. These dependences are very convenient for practical use. Calculations show that at values of $\bar{P}N$ and $(1-\bar{P})N$, larger than four, for statistical estimates of the probability of failure-free operation the normal distribution can be used with a sufficient degree of accuracy [13]. In this case the confidence limits of probability P are determined on the basis of expression [13]:

$$P=\frac{1}{\left(1+\frac{1}{N}t_\alpha^2\right)}\left\{\overline{P}+\frac{t_\alpha^2}{2N}\pm t_\alpha\sqrt{\frac{\overline{P}(1-\overline{P})}{N}+\frac{t_\alpha^2}{4N^2}}\right\}. \tag{5.3}$$

where t_α is the coefficient which characterizes the scattering of the parameters of the normal distribution at the assigned confidence coefficient.

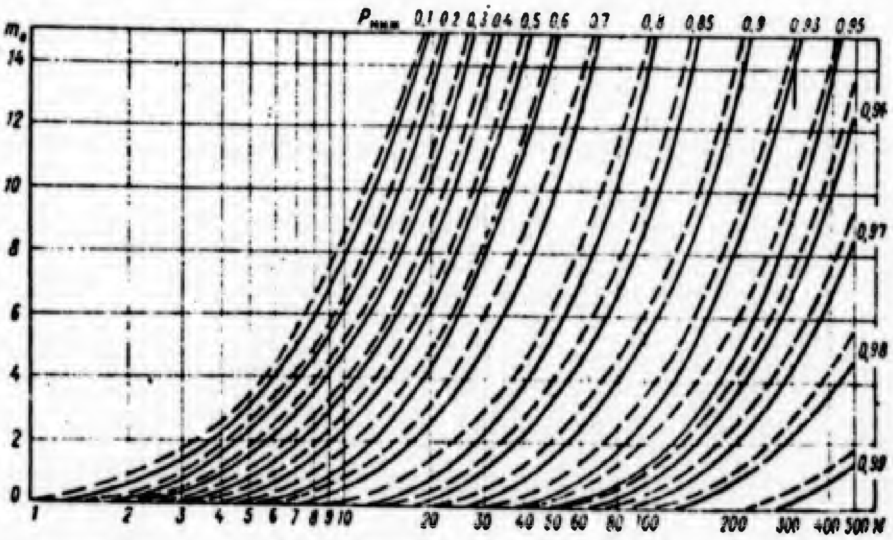


Figure 5.4. Dependence of the unilateral lower confidence limit of the probability of failure-free operation (binomial distribution) $P_{нижн}$ on the number of tests N and quantity of failures m_0 with α_1=0.90 (dashed line) and α_1=0.95 (solid lines).

The solution to equations (5.3) with a plus sign gives an estimate for the upper limit and with a sinus sign - for the lower limit. Similar solutions can be obtained also by means of the hypergeometric distribution and Poisson's law.

Let us examine now some features of discrete estimates which are encountered in practice. For this we will again turn to Fig. 5.3.

If in the process of the tests frequency $\overline{P}$ remains stable, the estimates of probability P vary monotonically in accordance with the confidence limits within the limits of one shaded zone. But if in the process of the tests the frequency $\bar{P}$ is unstable, the probability estimates vary not monotonically but discretely, since the estimates of probability of this zone convert into the other. Thus, for instance, after finishing the first series of tests N_1, we will obtain the statistical probability of survival $\bar{P}_1$ by which the reliability estimate P is located in the interval of 1-1 (see Fig. 5.3). Then the tests continued and wit. their total number N_2 the quantity of failures was increased. As a whole this led to a deterioration in the test results, in connection with which the frequency $\bar{P}_1$ was lowered to the level $\bar{P}_2 (\bar{P}_1 < \bar{P}_2)$. Correspondingly on the same level α the estimate of reliability P was moved into interval 2-2.

Quantitatively let us show this in an example. When $N_1=10$ and $P_1=1.0$ ($m_0=0$) the lower limit of the probability of failure-free operation with $\alpha_1=0.90$ composed $P_{\text{нижн}}=0.80$. The obtained estimate makes it possible to confirm that with this outcome of tests the determined level of reliability with $\alpha_1=0.90$ is not below 80%. But let us assure that after this an additional two tests were conducted, and the failures were obtained, i.e., $m_0=2$, $N_2=12$ and $\bar{P}=0.83$. These results correspond to the level $P_{\text{нижн}}=62\%$ when $\alpha_1=0.90$. Thus our prediction about the fact that the reliability of the tested articles is not below 80% was not confirmed, since $P_{\text{нижн}}=62\%$. Strictly speaking, the prediction will be accurate under the supplementary condition that the frequency $\bar{P}$ in the process of the tests is not changed. As follows from the experiment of the conduct of the experimental works, with any number of tests there cannot be such guarantee for the future.

Consequently, with the small number of tests such predictions of reliability are barely reliable, but with an increase in the number of tests the experimental frequency becomes ever stabler. For articles with a high degree of reliability at values of

$N \approx 100$ the frequency $\bar{P}$, as a rule, varies within limits of several percentages. This means that by these methods the relatively reliable estimates of reliability can be obtained only with a very large number of tests. Thus, for instance, if it is necessary to confirm the level of reliability of 0.99 with $\alpha = 0.95$, not less than 298 reliable tests will be required (see Fig. 5.4). In the presence of one failure the quantity of the tests is increased by more than 450 and so on.

For the confirmation of the level of reliability 0.995 when $\alpha = 0.95$ the quantity of reliable tests is already equal to 598.

The given examples show how unreal the use of discrete methods are for estimating the operation of the highly reliable, expensive articles manufactured in small batches. This immediately raises the question concerning the urgency of the development of quantitative methods of determination by which the physical features of processes, the entire course of a change in the fundamental characteristics of the article, and not the single fact of the onset of failures.

§ 4. DETERMINATION OF RELIABILITY INDICES ON THE BASIS OF QUANTITATIVE INFORMATION. METHODS OF THE PROCESSING OF STATISTICAL DATA

The quantitative information about the reliability of the systems and their elements is the result of planned tests in all stages of design final development and the test result in the process of production and operation. Used as initial data, as a rule, are the different forms of accrued operating times up to failure for the time of operation without breakdown during tests for service life, the quantity of cycles, and so on.

After a careful analysis and the classification of statistical data, the totality of the articles tested according to the same program can be separated. It is more rational if such tests are planned in the process of the final development.

A certain totality of articles characterized by the accrued operating time, measured by characteristic x is examined.

Then all the possible results can be subdivided into three groups.

1. All the tested articles of certain sampling R have an accrued operating time up to failure which corresponds to a series of experimental values $x_1, x_2, x_3, \ldots, x_i, \ldots, x_R$. This sampling, in accordance with the obtained results, is considered complete.

2. With tests according to one program, part of the sampling r is brought to failure. The remaining part (R-r) has an accrued operating time not less than x_a, but failures did not begin. Unlike the complete one, this accrued operating time is considered truncated. According to the program for reliable tests, the accrued operating time x_a must exceed the accrued operating time of the articles r by failure. For the procedure of the processing of test results, this condition in principle is not necessary. However, a rational program must be constructed in the following way. Tests of the entire sampling R are planned up to the accrued operating time x_a, and failures of separate articles can begin earlier ($x_i \leqslant x_a$). In the sum of such articles there must be r. After the achievement of accrued operating time x_a further tests are ceased.

However, in a number of cases the tests can be continued. Then this is equivalent to an increase in accrued operating time x_a up to a certain value of x_A.

3. With the article test of sampling R according to the predetermined program up to the accrued operating time x_a, the failures did not generally appear. With this outcome, by decision of the director of the works, the tests can be continued up to

accrued operating time x_A. In a number of cases, if this value is not specified by special conditions, it can be accepted arbitrarily, for example, equal to $x_A \geqslant (1.5\text{-}2.0)x_a$. It is natural that the continuation of the program can lead to other results, identical complete or truncated samples.

However, with tests of highly reliable articles with a long technical service life, the outcome can remain as before, i.e., the failures cannot begin. In this case, despite the general, as it were, favorable result, the failures or "weak" places in the design will remain those which were not revealed. Consequently, such tests will not bear fully the proper information necessary in the stage of design final development.

Practice showed that one should consider as rational the outcome which makes it possible to establish the forms of possible failures and reasons for their appearance which then should be removed in the process of modifications. The information about failures is a guarantee of the provision of high levels of reliability.

After the realization of the programs and classification of the test results, their pocessing is conducted. In evaluating the quantitative reliability indices one of the primary problems is the determination of the laws governing the distribution of failures by a checking of the conformity of the experimental and theoretical data.

In such a case when the volume of information is sufficiently great (on the order of hundreds and more observations), the actual distribution law of failures on the basis of statistic studies can be determined [13]. In this case the method of the construction of histograms is most acceptable.

In the processing of unique and expensive articles, the tests, as a rule, are conducted up to the total loss of the efficiency of the assemblies and units. If the tests according to one of the experimental programs are concluded with the breakdown of separate elements of the system, the latter undergo dismantling and flaw detection, and the assemblies and units, which maintained the efficiency, enter again into the next assembly for the preparation of the following program. Then the tests are continued until they are finished by failure or flaw detection.

Thus the quantities of the tests and articles in the sampling do not always correspond to each other. Therefore, one should speak about the test as about the fact of the realization of the program and about the article - as about the object undergoing an experimental check. Consequently, the volume of the tests cannot correspond to the quantity of articles in the sampling. Therefore, when we speak about the establishment of the form of distribution, we must proceed from the information obtained from tests of a large number of articles.

In the study of random processes the more accurate their laws are exhibited, the greater the volume of information. In this case the procedure for the processing of experimental data must eliminate the secondary phenomena barely inherent to the investigated process which are characteristic for one limited aggregate of the articles. For this purpose in mathematical statistics the methods of equalization or smoothing of the statistical distribution by means of analytical dependences are widely applied.

Thus with a large number of experiments the statistical data, for the purpose of generalization, can be subjected to the appropriate ranking, which arranges them in statistical series. For this the entire range of measurement of the random variable x is divided into 10-20 digits (the more uniform and more reliable the experimental data, the greater the number of digits). The values of digits can be identical or different, and this is explained by the features of the statistical series in each specific case. If the range of a change in the random variable x is presented graphically in the form of a linear axis, then the appropriate number of digits will fill the entire range equal to value $(x_{max}-x_{min})$. Correspondingly, each digit $(x_{i+1}-x_i)$ will be characterized by a definite quantity of observations m_i of the random variable x.

This makes it possible to establish for each digit its experimental frequency

$$\bar{P}_i = \frac{m_i}{R}.$$

If now the values of the statistical series are transferred to a curve of form $\bar{P}(x)$, then a histogram will be obtained. In this case the value of the frequency $\bar{P}_{[(i+1)-i]}$ determines the rank of the digit. As an example Fig. 5.5 shows the probability distribution of an operation without breakdown for a certain selection of the articles depending on the time of the tests. As is evident, the values of the digits are not equal to each other. For quite clear reasons the first and last digits (0-1) and (9-10) should not be divided into smaller ones, since a change in frequency $\bar{P}_i$ in each of the ranges occurs very slowly. The midpoints of the digits are connected by a smooth curve, which in this case is the statistical distribution function. The more the volume of experimental data, the more accurate its form, and with this the portion is decreased and lost is the weight of the phenomena not inherent to the established law, i.e., the phenomena which were randomly revealed in this totality of statistical information.

If sample volume is limited, one can speak not about the establishment of the form of the distribution function, but about the checking of the conformity of the test results to one of the taken theoretical distributions. For example, if the quantity of the articles tested is equal to 10--5, then used as the criteria of agreement are the criteria of Pearson χ^2 and Kolmogorov [76].

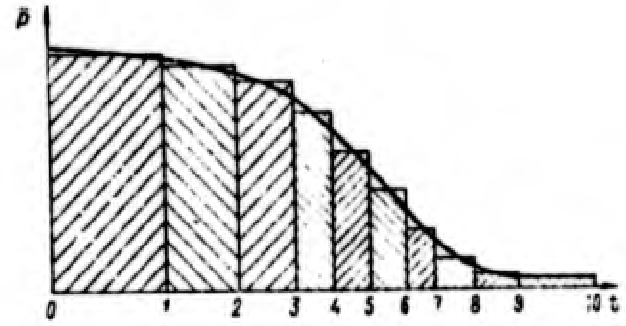


Figure 5.5. Probability distribution of operation without breakdown for a certain sampling of articles depending on time (histogram of tests).

An important stage is the determination of distribution parameters according to experimental data. Very convenient in this respect is the graph-analytic method in which the function of the taken theoretical distribution by logarithmic operation is converted into a straight line. This method should be considered as approximate. Thus, for instance, for the Weibull distribution the function of probability of failure-free operation can be thus converted:

$$z(x) = \ln\{-\ln[\bar{P}(x)]\} = -\ln x_0 + \beta \ln x. \quad (5.4)$$

In this case the frequency $\bar{P}$ is a function of parameter x. Let us examine equation (5.4). It is easy to note that this is the equation of a straight line which intercepts from the $z(x)$ axis the segment equal to $-\ln x_0$. For the purpose of the determination of parameters x_0 and β from experimental data, the graphic dependence $z(x)=f(\ln x_i)$ (Fig. 5.6) is constructed. Similar dependences can also be obtained for other distributions. For example, for the exponential law function z(x) takes the form of a straight line passing through the beginning of the coordinates:

$$z(x) = \ln P(x) = -\frac{x}{x_0},$$

or

$$z(x) = -\lambda x.$$

In this case the λ-characteristic is equal to the tangent of the angle between the approximating straight line and x axis:

$$\lambda = -\frac{z(x)}{x}.$$

However, not in all cases can the experimental distribution be sufficiently well described by any of the laws in the form of a straight line.

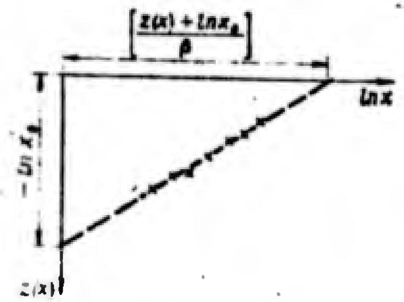


Figure 5.6. The determination of parameters x_0 and β.

On Fig. 5.7 approximated by the exponential law in the form of a straight line 1 is only a group of experimental points, and the others conditionally lie on line 2. This is possible to explain by the fact that the distribution shown, in all probability, is the superposition of two exponential laws with failure rates of λ_1 and λ_2. But this does not eliminate the possibility of the use for checking the agreement of the experimental and theoretical data of distributions of other forms.

The examined graph-analytic method of the determination of parameters of distributions is approximate and does not provide for an estimate of their accuracy.

In principle the degree of the agreement of the experimental data with the approximating straight line can be determined by means of Pearson's χ^2 and Kolmogorov's criteria [76].

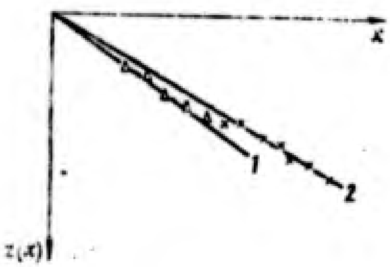


Figure 5.7. Distribution in the form of the superposition of two exponential laws.

More precise estimates of the parameters can be obtained by means of the function of plausibility L(x). This function characterizes the probability of the distribution of experimental data and in a number of sources is accepted as the sum of the densities and sometimes as the sum of their logarithms. The partial derivative of the function of plausibility, equated to zero, determines the maximum of the plausibility:

$$\frac{\partial L(x)}{\partial x}=0.$$

This condition is used to estimate the values of the very parameters of distribution x_0, σ or β.

The indicated property of the function of plausibility is widely used in mathematical statistics. Therefore, for the convenience in differentiation it is sometimes expedient to represent the function of plausibility in the form of a sum of logarithms of the densities of distribution.

Let us now examine a concrete example. If there are experimental data on tests of the sampling with the most probable distribution density f(x), which correspond to the accrued operating time up to failure (case 1), then

$$L(x)=\sum_{i=1}^{R}\ln f(x_i).$$

But if tests up to failure are alternated with reliable tests (case 2), then the function of plausibility assumes the form

$$L(x)=\sum_{i=1}^{r}\ln f(x_i)+(R-r)\ln P(x_a).$$

As one would expect, a further solution depends on the form of the distribution. Thus, for instance, for the Weibull distribution the function of plausibility can be represented

for the complete sampling

$$L(x)=R(\ln\beta-\ln x_0)+(\beta-1)\sum_{i=1}^{R}\ln x_i-\frac{1}{x_0}\sum_{i=1}^{R}x_i^{\beta};$$

for the truncated sampling

$$L(x) = r(\ln\beta - \ln x_0) + (\beta - 1)\sum_{i=1}^{r} \ln x_i - \frac{1}{x_0}\sum_{i=1}^{r} x_i^{\beta} - (R-r)\frac{x_a^{\beta}}{x_0}.$$

Further, depending on the test results, let us conduct the partial differentiation of the corresponding form of the function L(x) for the purpose of the determination of the values of parameters x_0 and β:

$$\frac{\partial L(x)}{\partial\left(\frac{1}{x_0}\right)} = 0 \text{ and } \frac{\partial L(x)}{\partial\beta} = 0.$$

Incidentally, let us note that it is convenient to produce the differentiation of function L(x) for the Weibull distribution not from parameter x_0 but from its reciprocal value $(1/x_0)$.

As a result we will obtain the dependences which make it possible to determine the mean statistical estimates of the parameters of distribution:

for the complete sampling $\bar{x}_0 = \frac{1}{R}\sum_{i=1}^{R} x_i^{\bar{\beta}}$,

$$\bar{\beta} = \frac{R}{\frac{1}{\bar{x}_0}\sum_{i=1}^{R} x_i^{\bar{\beta}} \ln x_i - \sum_{i=1}^{R} \ln x_i};$$

for the truncated sampling $\bar{x}_0 = \frac{1}{r}\sum_{i=1}^{r} x_i^{\bar{\beta}} + \frac{R-r}{r} x_a^{\bar{\beta}}$,

$$\bar{\beta} = \frac{R}{\frac{1}{x_0}\sum_{i=1}^{r} x_i^{\bar{\beta}} \ln x_i - \sum_{i=1}^{r} \ln x_i + \frac{R-r}{\bar{x}_0} x_a^{\bar{\beta}} \ln x_a}. \qquad (5.5)$$

These dependences, despite their complexity, are easily solved by trial and error or by the graph-analytic method.

The statistical estimates for the truncated normal and other laws of one- or biparametric distributions can be similarly obtained. Since at first we were assigned the form of the distribution, then an indispensable consequence of such estimates must be the checking of the agreement of theoretical and experimental data. The methods of checkings by means of the Pearson χ^2 and Kolmogorov compatibility tests are well-known [71, 76].

If by means of one of the distributions the acceptable convergence with the experimental data is reached, then the estimate of the confidence limits of the probability of failure-free operation can be conducted. The probability in this case can be used to estimate quantitative reliability indices. In practice, together with interval estimates, used as measures of accuracy is the standard deviation of probability σ_P.

For the complete selection of the tests, let us show one of the methods of the estimates for the probability of survival in an example of the Weibull distribution as one of a total in the reliability theory. This distribution is biparametric; therefore, for determining the rms deviation of probability σ_P it is required to find not only the parameters of x_0 and β, but also the covariance $\mathrm{cov}\left[\left(\frac{1}{x_0}\right);\beta\right]$ and the rms deviation of the parameters $(1/x_0)$ and β, $\sigma(1/x_0)$ and σ_β respectively.

For this purpose let us compose the dependences for the extreme values of function L(x), i.e., conditions for the maximum of plausibility:

$$\frac{\partial L(x)}{\partial\left(\frac{1}{x_0}\right)}=0 \text{ and } \frac{\partial L(x)}{\partial\beta}=0.$$

In accordance with expressions (5.5) let us produce the statistical estimates of parameters $\bar{x}_0$ and $\bar{\beta}$ and find the second derivatives:

$$\frac{\partial^2 L(x)}{\partial\left(\frac{1}{x_0}\right)^2}=-R\bar{x}_0^2,$$

$$\frac{\partial^2 L(x)}{\partial \beta^2} = -\frac{R}{\beta^2} - \frac{1}{\bar{x}_0}\sum_{i=1}^{R} x_i^{\beta} \ln^2 x_i,$$

$$\frac{\partial^2 L(x)}{\partial\left(\frac{1}{x_0}\right)\partial\beta} = -\sum_{i=1}^{R} x_i^{\beta} \ln x_i.$$

Further let us form the matrices:

$$A = \begin{vmatrix} \dfrac{\partial^2 L(x)}{\partial \beta^2}; & \dfrac{\partial^2 L(x)}{\partial\left(\frac{1}{x_0}\right)\partial\beta} \\ \dfrac{\partial^2 L(x)}{\partial\left(\frac{1}{x_0}\right)\partial\beta}; & \dfrac{\partial^2 L(x)}{\partial\left(\frac{1}{x_0}\right)^2} \end{vmatrix},$$

$$B = \begin{vmatrix} \sigma_\beta^2; & \mathrm{cov}\left[\left(\frac{1}{x_0}\right); \beta\right] \\ \mathrm{cov}\left[\left(\frac{1}{x_0}\right); \beta\right]; & \sigma^2_{\left(\frac{1}{x_0}\right)} \end{vmatrix}.$$

If condition $B=-[A^{-1}]$ is correct, then

$$B = \frac{1}{\det A}\begin{vmatrix} R\bar{x}_0^2; & \left(-\sum_{i=1}^{R} x_i^{\beta}\ln x_i\right) \\ \left(-\sum_{i=1}^{R} x_i^{\beta}\ln x_i\right); & \left(\frac{R}{\beta^2} + \frac{1}{\bar{x}_0}\sum_{i=1}^{R} x_i^{\beta}\ln^2 x_i\right) \end{vmatrix},$$

where the determinant A is equal to

$$\det A = \left(\frac{R}{\beta^2} + \frac{1}{\bar{x}_0}\sum_{i=1}^{R} x_i^{\beta}\ln^2 x\right) R\bar{x}_0^2 - \left(\sum_{i=1}^{R} x_i^{\beta}\ln x_i\right)^2.$$

Hence we obtain the dependences:

$$\sigma^2_{\left(\frac{1}{x_0}\right)} = \frac{1}{\det A}\left(\frac{R}{\beta^2} + \frac{1}{\bar{x}_0}\sum_{i=1}^{R} x_i^{\beta}\ln^2 x_i\right),$$

$$\sigma_\beta^2 = \frac{R\bar{x}_0^2}{\det A},$$

$$\mathrm{cov}\left[\left(\frac{1}{x_0}\right), \beta\right] = -\frac{1}{\det A}\sum_{i=1}^{R} x_i^{\beta}\ln x_i.$$

Further with the enlistment of the method of linearization of function P(x), which depends on several variables, it is possible to determine the rms deviation of the probability σ_P:

$$\sigma_p^2 = x^{2\beta}\left\{\frac{\sigma_\beta^2 \ln^2 x}{\bar{x}_0^2} + \sigma^2_{\left(\frac{1}{x_0}\right)} + \frac{2}{\bar{x}_0}\operatorname{cov}\left[\left(\frac{1}{x_0}\right); \beta\right]\ln x\right\} \times$$
$$\times \exp\left(-\frac{2x^\beta}{\bar{x}_0}\right). \qquad (5.6)$$

For articles with a reliability level of not below 0.95, with a sufficient degree of accuracy it is possible to use the following dependence for determining the lower limit of probability:

$$P(x)_{\text{ниж}} = \bar{P}(x) - t_a \sigma_P$$

In this case we used an expression for the normal distribution law. In principle a similar estimate of the lower limit of probability is approximate, since the Weibull distribution is replaced by the normal distribution. However, calculations show that this assumption is correct for articles with high reliability for which in this case a quantitative estimate of the reliability with the confirmation of the assigned levels is conducted. With the considered levels the index of distribution β, as a rule, is sufficiently great ($\beta \geqslant 3.25$), and therefore both laws (the truncated normal and Weibull), as was shown above, agree sufficiently well with each other.

The presented mathematical apparatus can seem to be sufficiently bulky for rough calculations. Approximate estimates for the high levels of reliability, when virtually only gradual failures occur, can be conducted by means of dependences for the normal truncated distribution.

If the reliability level supposedly is unknown to us or random failures occur, quite good estimates can be obtained virtually always with the aid of the Weibull distribution. In this case it is expedient to program the calculated dependences for the ETsVM [ЭЦВМ - digital computer].

With reliable tests (case 3) the total operating time of the articles is characterized as

$$\bar{x}_0 = x_a R.$$

In this case the failures did not begin, and therefore, not only their form, but also the nature of distribution was unknown. Only the accrued operating time and sample volume are virtually known. Therefore, in practice in similar cases in the determination of the quantitative reliability indices, the exponential law which does not give high estimates is usually used.

Then the point value of the probability of failure-free operation can be determined on the basis of expression

$$\bar{P}(x) = \exp\left(-\frac{x}{x_a R}\right).$$

When evaluating the lower limit the mean statistical value of the accrued operating time must be decreased by the appropriate coefficient. It depends only on confidence probability and is quantitatively equal to $\overline{\ln (1-\alpha)}$.

Under this condition the lower limit of the probability of failure-free operation for the sampling with reliable tests is determined from the equation

$$P(x)_{\text{ниж}} = \exp\left[-\frac{x \ln(1-\alpha)}{x_a R}\right]. \quad (5.7)$$

With complete or truncated samples, when the flow of failures follows the exponential law, the mean statistical value of the accrued operating time is equal to the ratio of the total period of tests to the quantity of the revealed random failures –

$$\bar{x}_0 = \frac{1}{m_0} \sum_{i=1}^{n} x_i.$$

Then the point value of probability $\bar{P}(x)$ is equal to

$$\bar{P}(x) = \exp\left(-\frac{x}{\bar{x}_0}\right). \quad (5.8)$$

Expression (5.8), as one would expect, characterizes the special case of the Weibull distribution at the value of the parameter $\beta=1$.

Correspondingly, the estimate for the lower limit of probability P(x) is defined as

$$P(x)_{\text{ниж}} = \exp\left(-\frac{x}{\bar{x}_0 r_2}\right).$$

where coefficient r_2 is the function of the number of failures m_0 and confidence probability α. The quantitative values of coefficient r_2 are in Table [76].

The examined methods of the determination of the quantitative reliability indices provide for the use of the initial information on results of finishing tests and reliability tests of elements and systems of repeated action. In the specific cases accepted as the parameters of accrued operating time can be the time of operation without breakdown, the quantity of cycles up to failure and other parameters which determine the flows of sudden and gradual failures under fixed test conditions. The indispensable conditions of such estimates are: the statistical independence of the sampling with which the articles for the tests were not selected in a special way, and the strict fixation of the conditions of their conducting. The obtained reliability indices must be valid for conditions under which the initial information is obtained.

§ 5. PROCESSING OF TEST RESULTS FOR ARTICLES OF SINGLE AND REPEATED ACTION IN THE CASE OF NORMAL DISTRIBUTION

Test results of articles of single and repeated action can follow normal distribution. In the first case this is connected with parameter distribution according to the "load-strength" scheme, and in the second - in the presence only of gradual failures. In connection with the latter this means that the article in a design respect is developed, the sudden failures are absent, and the limit of the efficiency is determined by the wear, losses of strength as a result of fatigue effects and similar reasons. The failures of elements and systems of single action are characterized by the absence of their function at a given moment of time.

As one would expect, methods of the processing of test results for both types of articles differ somewhat; therefore, let us examine them in turn.

1. Processing of test results of elements and systems of single action. The quantitative reliability indices for articles of single action can be determined in the presence of the appropriate information on the basis of a comparison of parameters which characterize the "load" on the stop element and its "strength." In this case for the mean statistical estimates of the probability of failure-free operation, in accordance with the normal distribution, the dependence (4.2) can be used. In this case the parameters x_0', x_0^* and σ_1, σ_2 are the mean statistical estimates of mathematical expectations and rms deviations of the parameters of distribution. The values of interval estimations of probability P in each specific case depend on the volume of information and confidence level α with which the calculation is conducted.

The initial data are determined from tests of samplings R of similar articles under fixed conditions close to full-scale.

With the test data the selections which determine the "load" ($R_н$) and "strength" ($R_п$) can be equal ($R=R_н=R_п$) or $R_н \neq R_п$. In summation, after the completion of the program according to each of the controllable parameters which characterize the "load" and "strength", a statistical series of test results can be comprised:

$$\begin{gathered} p_{н_1};\ p_{н_2};\ p_{н_3};\ldots p_{н_i};\ldots p_{н_R}; \\ p_{п_1};\ p_{п_2};\ p_{п_3};\ldots p_{п_j};\ldots p_{п_R}. \end{gathered} \tag{5.9}$$

The pressure (or force) of loading and loss of strength can be attributed to the number of controllable parameters. The point values of the parameters of distribution for the "load" and "strength", respectively, are determined from the equations:

$$\bar{p}_н = \frac{1}{R_н}\sum_{i=1}^{R_н} p_{н_i},\quad \sigma_н^2 = \frac{1}{R-1}\sum_{i=1}^{R_н}(\bar{p}_н - p_{н_i})^2,$$

$$\bar{p}_п = \frac{1}{R_п}\sum_{j=1}^{R_п} p_{п_j},\quad \sigma_п^2 = \frac{1}{R_п-1}\sum_{j=1}^{R_п}(\bar{p}_п - p_{п_j})^2.$$

Further we find the point value of the quantile of the normal distribution

$$\bar{U}_p = \frac{\bar{p}_п - \bar{p}_н}{\sqrt{\sigma_п^2 + \sigma_н^2}}$$

and its lower boundary, respectively,

$$U_{p_{нижн}} = \bar{U}_p - t_\alpha \sigma_U.$$

To evaluate the accuracy of the determination of the quantile by means of the rms deviation σ_U, we use the function of linearization.

Taking this into account, we obtain

$$\sigma_U^2 = \left(\frac{\partial U_p}{\partial \bar{p}_п}\right)^2 \sigma_п^2 + \left(\frac{\partial U_p}{\partial \bar{p}_н}\right)^2 \sigma_н^2 + \left[\frac{\partial U_p}{\partial (\sigma_п^2)}\right]^2 \sigma^2_{\sigma_п^2} + \left[\frac{\partial U_p}{\partial (\sigma_н^2)}\right]^2 \sigma^2_{\sigma_н^2} \quad (5.10)$$

In this case $\sigma_{\sigma_п^2}$ and $\sigma_{\sigma_н^2}$ are the rms deviations of the estimates of dispersions $\sigma_п^2$ and $\sigma_н^2$.

By solving equation (5.10), we obtain

$$\sigma_U^2 = \frac{\left(\frac{\sigma_п^2}{R_п} + \frac{\sigma_н^2}{R_н}\right) + \frac{\bar{U}_p^2}{4}\left(\frac{\frac{\sigma_п^4}{R_п-1} + \frac{\sigma_н^4}{R_н-1}}{\sigma_п^2 + \sigma_н^2}\right)}{\sigma_п^2 + \sigma_н^2}. \quad (5.11)$$

Now there are all initial data for determining the lower limit of the probability of failure-free operation:

$$P_{\text{ниж}} = Fo\left(\bar{U}_p - t_\alpha \sigma_U\right).$$

As can be seen from equation (5.11), the magnitude of deviation σ_U is greatly affected by the sample volumes $R_п$ and $R_н$. The less they are, the more the error in the estimate of the accuracy of the probability of failure-free operation.

Questions of the effect of the sample volume R on quantitative reliability indices will be examined below.

2. Processing of test results of elements and systems of repeated action. In an analysis of test results of elements and systems of repeated action, the failures of which follow a normal distribution, one can speak only about the complete and truncated

samples. Reliable tests are not examined, since according to these test results the parameters of normal distribution, in particular, the rms time scatter of the onset of failures, cannot be established.

For the complete sampling when the parameter of the mean time between failures for all articles R is known, the parameters of normal distribution x_0 and σ can be determined by the equations

$$\bar{x}_0 = \frac{1}{R}\sum_{i=1}^{R} x_i, \quad \sigma^2 = \frac{1}{R-1}\sum_{i=1}^{R}(\bar{x}_0 - x_i)^2.$$

Further, according to the known formula (4.1), let us estimate the point value of the probability of failure-free operation.

In this case we use the truncated normal distribution. Correspondingly, the point value of the quantile will be equal to

$$\bar{U} = \frac{\bar{x}_0 - x}{\sigma},$$

and its lower limit

$$U_{\text{ниж}} = \bar{U} - t_{\alpha}\sigma_U,$$

where σ_U is the rms deviation.

Then for determining the dispersion σ_U^2, we will use the function of linearization. Taking this into account, we will obtain the expression

$$\sigma_U^2 = \left(\frac{\partial U}{\partial x_0}\right)^2 \sigma^2 + \left(\frac{\partial U}{\partial \sigma}\right)^2 \sigma_\sigma^2.$$

By solving it, it is possible to find the dependence

$$\sigma_U^2 = \frac{1}{R} + \frac{\bar{U}^2}{2(R-1)}.$$

Thus all the initial data for estimating the lower limit of probability of failure-free operation are obtained:

$$P(x)_{min} = \frac{Fo\left(\bar{U} - t_\alpha \sigma_U\right)}{Fo\left(\bar{U}_{x=0} - t_\alpha \sigma_U\right)}. \quad (5.12)$$

In equation (5.12) the coefficient of truncation is calculated from the lower limit of the quantile $(\bar{U} - t_\alpha \sigma_U)$ at the value of x equal to zero (x=0). In the case of the truncated sample the distribution parameters can be determined on the basis of the function of plausibility:

$$L(x) = \sum_{i=1}^{r} \ln\left[\frac{1}{\sigma} f\left(\frac{x_i - \bar{x}_0}{\sigma}\right) + (R-r) \ln\left[1 - Fo\left(\frac{x_e - \bar{x}_0}{\sigma}\right)\right]\right]. \quad (5.13)$$

The solution for the given case is given in work [76]; therefore, let us indicate only the working formulas for the system of equations according to which the calculation for the purpose of the determination of parameters x_0 and σ is conducted:

$$y = \frac{\frac{1}{r}\sum_{i=1}^{r} x_i^2 - \left(\frac{1}{r}\sum_{i=1}^{r} x_i\right)^2}{\left(x_e - \frac{1}{r}\sum_{i=1}^{r} x_i\right)^2},$$

$$y = \frac{1 + \left(\frac{R}{r} - 1\right) k \cdot \frac{f(k)}{Fo(k)} - \left[\left(\frac{R}{r} - 1\right)\frac{f(k)}{Fo(k)}\right]^2}{\left[\left(\frac{R}{r} - 1\right)\frac{f(k)}{Fo(k)} - k\right]^2}. \quad (5.14)$$

where $k = \frac{\bar{x}_0 - x_e}{\sigma}$.

Subsequently, to estimate the probability of failure-free operation, relations (5.12) and (5.14) can be used.

From an analysis of equations (5.13) and (5.14), it follows that the accuracy of the estimates of parameters x_0 and σ to a great degree depends on the relationship R/r, i.e., on the degree of truncation of the complete selection.

In each specific case, besides parameters r and R, the accuracy depends also on values x_0, x_a and σ. Nevertheless, in the solution of a series of practical problems it is established that the acceptable accuracy of the estimates can be obtained with the quantity of articles $r \geq 0.5$ R.

As a result of the processing of statistical data from tests, of articles which belong to one general population, a quantitative estimate of reliability is obtained. If the sampling of R is limited, then the corresponding indices make it possible to judge, basically, about the degree of the design perfection of the articles and only to a small degree about the quality of their production. To estimate the quality of the production processes, the information on tests of a large quantity of articles is required.

After the completion of a definite experimental program the considered estimate of reliability, as a rule, is not final, especially when this program was preceded by a large volume of the design, finishing and other types of the tests. Then the computed values of the obtained indices must be more precisely formulated. This is reached by the simple summation of the statistical series (5.9), if the test conditions are identical, or by the genealization of very estimates of probability P(x) obtained for the different general populations of the articles. The methods of generalization of such estimates are sufficiently different; therefore, in each specific case it is required to connect them with the physical sense of the problem.

The basic approach to the generalization of the statistical estimates, which belong to different samplings or several test series, is presented in works [55, 76]. The initial processing of the statistical data is conducted for the purpose of checking the belonging to one general totality. Only under this condition can the test results and estimates themselves be generalized. However, the purely mathematical approach to the checking of the belonging to one totality of different test programs does not always give the proper results and sometimes can contradict the physical sense of the problem. In this case only the degree of the distinction in parameters x_0 and σ for different programs is indeed considered.

If the parameters of the accrued operating time x_0 and σ are close, this still does not mean that test conditions are identical. There can be a completely different composition of the external factors, and then the results similar in significance for two programs can be obtained because of the redistribution of the degree of effect of the factors. Finally, similar results can be obtained randomly as a result of a small sampling R for one of the programs.

Consequently, in order to avoid serious errors in similar problems, it is necessary always to consider the physical side of the problem. This is its kind of checking of the statistical model and its results for adequacy.

§ 6. DETERMINATION OF THE MINIMAL VOLUME OF THE SAMPLING OF ARTICLES IN RELIABILITY TESTS

By examining the different methods of the processing of tests, we ascertained that the sample volume depends on the form of information, the required or estimated level of reliability, the form of distribution, and the degree of truncation of the sampling and confidence coefficient. One should understand by the form of information not only its qualitative, but also quantitative composition: for example, the degree of commensurability of the mathematical expectation of accrued operating time x_0 with the rms deviation σ and the fixed value of accrued operating time x. It is also important whether or not tests up to failure or the volume of the tests are carried out. This significantly affects not only the accuracy of the determination of the distribution parameters, but also which form of the law one should apply in the processing of the experimental data.

The form of the distribution for this type of articles can prove to be known, but if in the tests there were no failures and the parameters of distribution are unknown, then there is accepted the exponential law, which gives knowingly low estimates in comparison, for example, with the normal at values of x less than x_0.

Used as measures of accuracy of the estimates of quantitative reliability indices (for such distributions as normal, exponential, Weibull, etc.) can be the magnitude of deviation of the probability

$$\varepsilon = \bar{P}(x) - P(x)_{нижн}.$$

For the normal and Weibull distribution, this concretely means that

$$\varepsilon = t_{\alpha} \sigma_{p},$$

i.e., at the assigned level of confidence the accuracy of the estimates of probability P(x) is determined by its rms deviation σ_P or by the rms deviation of the quantile σ_U for the normal distribution. If we are speaking about elements and systems of single action, then by means of equation (5.9) it is possible to calculate and construct the graphic dependence (Fig. 5.8), which makes it possible to establish the sample volume R with the specific relationships of parameters $\sigma_н$ and $\sigma_н$ for which the deviation is minimal. The calculations are carried out when the sample volumes for the elements

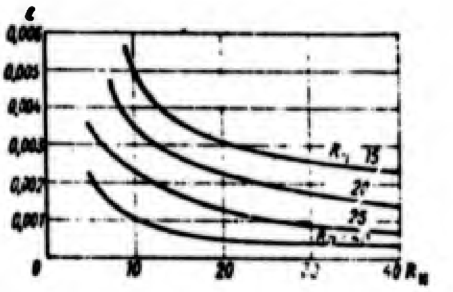


Figure 5.8. Error in determination of probability for the model "load-strength" $\sigma_{н}$=0.5 σ; $\alpha_{н}$=0.95.

which characterize the "load" and "strength" are equal ($R_{п}=R_{н}$). If the volumes are not equal, then even a considerable increase in one of them (when the second is small) will not lead to a considerable increase in the accuracy of the estimates P(x). If one sampling approaches infinity ($R_{п} \to \infty$), then the accuracy of the estimates of the quantile and probability (P(x) will be determined by the volume of the second selection. And if it is small, then the error will be considerable. The aforesaid is correct when values $\sigma_{п}$ and $R_{н}$ are approximately of the same order. Taking this into account, in practice one usually strives that the samplings $R_{п}$ and $R_{н}$ be as equal as possible.

In speaking about the elements of single action, it is necessary to keep in mind, first of all, the elements of the automation, which we consider as highly reliable articles with sufficiently high quantitative values of quantiles ($U \geq 3.0$). The calculations conducted taking into account this observation showed that the magnitude of deviation of probability ε, other conditions being equal, is noticeably affected by the sample volume at values of R<15-20. In all probability, this value should be accepted for the minimum sample size. In each specific case the sample volume can be more precisely formulated, but this will not lead to a substantial change.

It must be especially noted that at the low levels of reliability ($U<3.0$) the optimum of the curves on Fig. 5.8 becomes implicit. However, this does not mean that in this case one should increase the sample volume for the purpose of the "confirmation" of the high levels of reliability. On the contrary, efforts must be directed at the design finishing of the articles.

In proportion to the increase in reliability level, the optimum of the curves in Fig. 5.8 becomes ever more obvious and they approach the coordinate axes. If the "load" exceeds the "strength" many times, then a sufficiently high level of reliability can be achieved even with tests of literally unit articles (on the order of three). However, in this case it is recommended to increase the sample volume at least up to ten (R_{min}=10), since at very small R the technological presence of the made batches is lost. This can lead to the fact that under adverse conditions the unit success of production for a prolonged period can be ascribed to the quality of the articles of this design.

The aforesaid is largely correct for articles of repeated action. Only in this case should one take into consideration the appropriate relationships of parameters of distribution x_0 and σ, on the one hand, and the programmed accrued operating time x, on the other.

The established minimum sample volume R_{min}=15-20 is obtained by means of an analysis of parameters of normal distribution, which in equal measure is used for the processing of the test results of elements and systems both of single and repeated action. The calculations showed that the thus established volume R_{min} virtually in equal measure can be related to tests of each type of articles.

In speaking about the accuracy of the estimates obtained from tests of elements and systems of repeated action, we are forced again to examine three outcomes: 1) tests of complete sampling; 2) tests of truncated sampling; 3) reliable tests.

Independently of the results, the processing of the first two outcomes can be conducted in accordance with the normal law and the Weibull distribution.

Let us examine the results of the test of complete sampling R in this case in connection with the Weibull distribution, since for the normal distribution law they are known to us. For this we will turn to equation (5.6), from which it follows that the value of the rms deviation of probability σ_P is determined, basically, by the programmed value of accrued operating time x and the parameters of distribution x_0 and β. It is known that the effect of parameter β is the determining one for this form of distribution; nevertheless, the actual deviation of the probability ε in each concrete case depends on the totality of the characteristics x, $\bar{x}_0$ and $\bar{\beta}$. The latter two, in turn, are functions of the sample volume R. Taking this into account the calculations of the magnitude of deviation ε were performed for the defined values of probability P(x), which characterized the effect of values x, $\bar{x}_0$ and $\bar{\beta}$ in the complex on ε.

Obtained as a result are data (Fig. 5.9) which make it possible to estimate the error in determination of the probability of failure-free operation for the Weibull distribution, depending on the sample volume R. From Fig. 5.9 it follows that for the most probable levels of reliability the minimum sample volume R_{min} consists on the average of 15-20 articles. With an increase in the parameters of distribution $\bar{x}_0$ and $\bar{\beta}$, which lead to an increase in the reliability, the sample volume decreases. However, it must not be less than approximately ten ($R_{min} \approx 10$), taking into account the technological presence of batches with serial production.

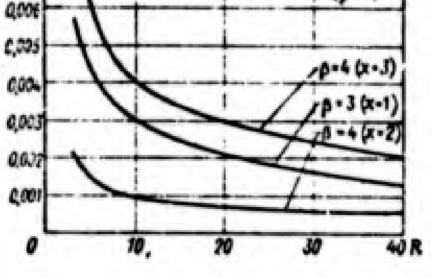


Figure 5.9. Computed values of errors in the determination of probability for the Weibull distribution (x_{cp}=5.0; α=0.95).

In the case of truncated sampling, the minimum sample volume is noticeably affected also by the characteristics R/r and x_a. With an increase in x_a the parameter of distribution β falls, and this means a lowering in the reliability level and, therefore, an increase in the minimum sample volume. Unsatisfactory accuracy of the estimates can be obtained if the coefficient of truncation of the sampling (R/r) is relatively great. Comparatively acceptable results can be obtained at values of $R/r<2.0$.

In the case of reliable tests the minimum sample volume can be obtained on the basis of expression (5.7). More accurately

$$R_{\min} x_a = -\frac{x \ln(1-\alpha)}{\ln P(x)_{\min}}. \qquad (5.15)$$

In this case for the minimum sample volume the dependence is linear, since in each concrete case the right side of equation (5.15) will be constant. In this case the value R_{min} is inversely proportional to the parameter of the accrued operating time x_a: the more it is, the less the sample volume.

For highly reliable articles in the limit it can also be reduced to a very small number, but this does not mean that in practice the sample volume should be taken less than approximately 10 in virtue of that given above ($R_{min} \geq 10$ is recommended). Equation (5.15) symbolizes the condition of ergodicity, according to which an increase in the accrued operating time is several times equivalent to a decrease in as many times of the sample volume: for example, the tests of 1000 articles during one minute are equivalent to tests of 10 articles during 100 minutes. In the case of the exponential distribution of failures, such tests make it possible to estimate the design characteristics of the article, but this does not mean that in this case it is possible to estimate the component of reliability, which characterizes the quality of production.

§ 7. STRUCTURAL SCHEMES OF THE RELIABILITY OF COMPLEX TECHNICAL SYSTEMS. ESTIMATE OF THE RELIABILITY OF SYSTEMS ACCORDING TO THEIR ELEMENTS

The block diagram of reliability reflects the connection between the system's elements in the process of its operation and conditions the degree of the effect of these elements on the efficiency of system as a whole. The structural scheme is determined by the design scheme of the article and is constructed according to the principle of the connection of elements-links into structural circuits. The latter establish the order of the arrangement and the order of functioning of elements in the system. The structural circuits can consist of series, parallel and series-parallel connections of the elements. Schematically these forms of connections are shown on Fig. 5.10. Let us examine some features of each of them.

The simplest analysis of the series form of connection of the elements shows that the breakdown of any link of the structural circuit is equivalent to the failure of the entire system. In this case the reliability of the system is determined by its "weakest" link independently of the reliability level of the remaining elements. For this reason the series connection is used only for the highly reliable systems when it is not possible to use the redundancy [standby or back-up] of the "weakest" link according to design or other considerations. Correspondingly, the reliability of the elements must be not less than one order higher than the requirements of the system as a whole.

The connection in parallel of the elements is designed so that failure of one of the links of the series circuit would not lead to failure of the system. An example of the series-parallel or combined connection scheme of the elements can be called a series form of connection with the redundancy of the "weakest" link.

In connection with mechanical systems, the series connection of the elements is most widely accepted. Such systems are used on the basis of the producing of highly reliable elements.

The parallel connection schemes of elements in the design of mechanical systems are encountered considerably more rarely than are the series. Nevertheless, separate schemes of the connection in parallel or the redundancy of self-contained units of the ZhRD in the DU [ДУ - engine installation] of the rocket stage are known. Combined schemes are found much more frequently, for example, in pneudraulic systems with twofold or even triple redundancy of the instruction sensors, most responsible EPK and so on.

If for scheme I (see Fig. 5.10) conditional probabilities of the failure-free operation of elements P_i (or elements themselves) are independent variables, then the total probability of the normal functioning of the system, which consists of l elements, is determined by the product

$$P_1 = \prod_{i=1}^{l} P_i.$$

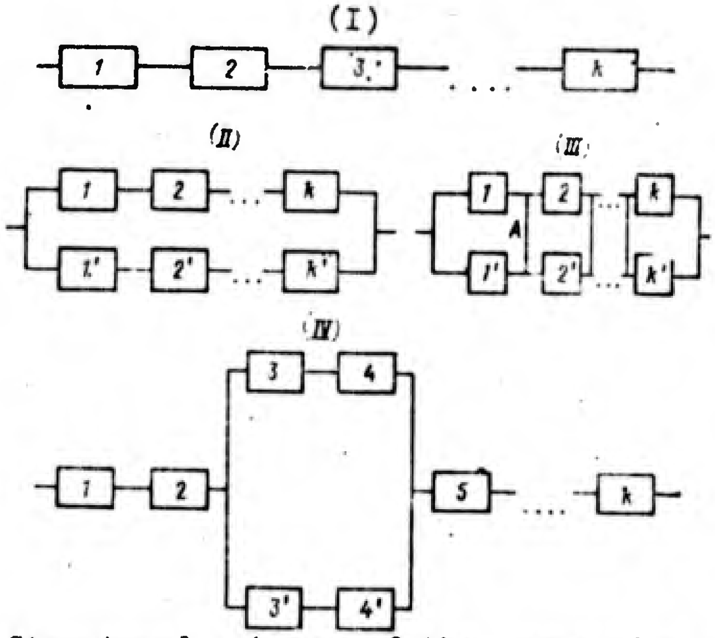


Figure 5.10. Structural schemes of the connection of the elements: I - series; II - parallel; III - parallel with additional couplings; IV - combined.

Correspondingly, the probability of survival of the system by scheme II, which consists of d identical parallel circuits, each of which has k elements, is determined on the basis of the dependence

$$P_{\text{за}} = 1 - \left(1 - \prod_{j=1}^{k} P_j\right)^{d}. \quad (5.16)$$

If in this system the redundancy is accomplished by dissimilar circuits, the dependence (5.16) should be written thus:

$$P_{\text{за}} = 1 - \left[\left(1 - \prod_{j=1}^{k} P_j\right)\left(1 - \prod_{i=1}^{k} P_i\right) \cdots \left(1 - \prod_{t=1}^{k} P_t\right)\right].$$

In the extreme case this scheme provides normal functioning if only one circuit is maintained. The quantity of malfunctioned elements in the remaining circuits of the system is not important.

For the majority of the mechanical systems the normal functioning of the scheme with redundancy is possible when the element of the malfunctioned circuit or the entire circuit will subsequently be opened. In a whole number of cases this condition is necessary, since the elements of the malfunctioned circuit can affect the system; for example, they can cause explosion, fire, leakage of the propellant components or gases, and secondary failures, or they can lead to a reduction in the parameters of the system, and so on.

In the implementation of scheme II failure of the system is possible if only one element in each standby circuit will fail. The simultaneous failure of elements in each circuit is unlikely, and the successive breakdown of all standby circuits is possible, for example, due to the overload of the "weakest" link of each circuit. It is natural that the onset of this event is highly improbable for the system finished in a structural respect.

An example of the use of scheme II, taking into account some features, it is possible to consider the scheme of the engine installation of the first stage of the carrier rocket "Saturn-1" [16]. The DU of this stage consists of eight autonomous ZhRD of the N-1 type at the time when for the execution of the flight program only six are sufficient. Thus, two ZhRD are found in "hot" reserve. The term "hot" or "loaded" (unlike "cold" or "unloaded") is applied to those forms of standby circuits which are included in the operation simultaneously with the basic circuits but are not connected to the basic when there is a breakdown.

Since the quantity of standby ZhRD of the examined DU does not correspond to the number of basic (two standby on six basic), then this standby in a certain type is "sliding." But this term can completely correspond to this scheme if any of the spare units could replace any of the malfunctioned basic ZhRD. The scheme of redundancy of the DU of the stage of "Saturn-1" is somewhat different: with the appearance of an emergency situation of one of eight ZhRD by the system of the emergency protection system (SAZ) [CA3] two instructions - for shutting off the emergency unit and the ZhRD diametrically opposite to it even if it functions normally. This measure is completely justified from the viewpoint of stabilization of the rocket flight and provision for the coaxiality of thrust of the DU.

Let us examine the scheme of redundancy from the viewpoint of the probabilistic estimate of the events which ensure the successful execution of the flight program. If we take the probability of failure-free operation of the autonomous ZhRD to be equal to P_0, then the event is characterized by the fact that in flight all eight engines of the DU worked out the positioned time normally. Conditionally let us write this in the form of a series of the particular outcomes:

Event No. 1: $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$.

In this case the probability of failure-free operation of the DU prior to the beginning of the tests is equal to the value

$P_{ду} = P_0^8$. But together with event No. 1 there can occur events connected with the breakdown of any of eight ZhRD, and let us agree to note these by a line on top, for example:

Event No. 2: $\bar{A}_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$.

. 3 event is characterized by engine failure with the ordinal number 1. the following event - by engine failure with the ordinal number 2 and so on. This can be written as the enumeration of events with the failure of one ZhRD:

Event No. 3: $A_1, \bar{A}_2, A_3, A_4, A_5, A_6, A_7, A_8$.

Event No. 4: $A_1, A_2, \bar{A}_3, A_4, A_5, A_6, A_7, A_8$.

Event No. 5: $A_1, A_2, A_3, \bar{A}_4, A_5, A_6, A_7, A_8$.

Event No. 6: $A_1, A_2, A_3, A_4, \bar{A}_5, A_6, A_7, A_8$.

Event No. 7: $A_1, A_2, A_3, A_4, A_5, \bar{A}_6, A_7, A_8$.

Event No. 8: $A_1, A_2, A_3, A_4, A_5, A_6, \bar{A}_7, A_8$.

Event No. 9: $A_1, A_2, A_3, A_4, A_5, A_6, A_7, \bar{A}_8$.

For the last eight named events, which are characterized by the failure of one ZhRD, the probability of the failure-free operation of DU will be defined in accordance with the binomial distribution as

$$P_{ду_1} = C_8^7 (1 - P_0)^{8-7} P_0^7,$$

or

$$P_{ду_1} = 8(1 - P_0) P_0^7. \quad (5.17)$$

On the right side of this equation the coefficient, equal to eight, denotes the quantity of events with the considered outcome, a power of the term $(1-P_0)$ is equal to the number of failures permissible in this case, and P_0 is the probability of failure-free operation of the remaining engines. The subsequent arrangement of favorable events is the simultaneous manifestation of failures of two diametrically opposite engines. In this case they are characterized in the following way:

Event No. 10: $\bar{A}_1, A_2, A_3, A_4, \bar{A}_5, A_6, A_7, A_8$.

Event No. 11: $A_1, \bar{A}_2, A_3, A_4, A_5, \bar{A}_6, A_7, A_8$.

Event No. 12: $A_1, A_2, \bar{A}_3, A_4, A_5, A_6, \bar{A}_7, A_8$.

Event No. 13: $A_1, A_2, A_3, \bar{A}_4, A_5, A_6, A_7, \bar{A}_8$.

For the last four events the probability of failure-free operation will be defined as

$$P_{\text{ДУ}_3} = 4(1 - P_0)^2 P_0^6. \qquad (5.18)$$

The coefficient 4 on the right side of this equation does not determine the total number of combinations according to the binomial distribution, since we examine only the successes which make it possible to fulfill the flight mission. Therefore, when evaluating the probability of the onset of each of the events of interest to us, in the formula for the binomial distribution instead of the number of combinations C_i it is necessary to take the numbers of variants for this outcome. Prior to the first event when all engines are reliable, the quantity of variants $C_1=1$. For the second event the failure of only one of the eight engines is possible, the quantity of variants $C_2=8$, and for the third favorable event when failures of the two diametrically opposite engines are allowed $C_3=4$.

In summation, for any quantity of favorable events permissible by the design scheme, the probability of failure-free operation of the DU with redundancy is equal to

$$P_{ДУ}=\sum_{i=0}^{D} C_i(1-P_0)^i P_0^{d-i},$$

where i is the quantity of failures permissible for each event Д; d - the quantity of self-contained units in the DU.

In our concrete example

$$P_{ДУ}=P_0^8+8(1-P_0)P_0^7+4(1-P_0)^2P_0^6. \quad (5.19)$$

However, the reliability of this equation is based on the assumption that the reliability of the system of switching devices and monitoring of the emergency state of the units is equal to unity. In practice this is by no means so - the monitoring of the onset of emergency situations of the ZhRD is one of the most complex problems. And since the reasons for the failures can be different, it is necessary to place on each self-contained unit a large number of checking elements. With an increase in their number, obviously, the reliability of the very system of emergency protection decreases. For this purpose let us introduce the characteristic of the probability of failure-free operation of the system of emergency protection $P_{САЗ}$. The latter in a complex with the DU in the general structural scheme of reliability of the system are in series connection. Therefore, expression (5.19) will take the form

$$P_{ДУ}=P_{САЗ}\left[\sum_{i=0}^{D} C_i(1-P_0)^i P_0^{d-i}\right] \quad (5.20)$$

and, correspondingly, in the examined example

$$P_{ДУ}=P_{САЗ}\left[P_0^8+8(1-P_0)P_0^7+4(1-P_0)^2P_0^6\right].$$

Quantitatively this means that if for conditions of the considered problem we accept the reliability of the autonomous engine as P_0=0.99, and for the system of emergency protection $P_{САЗ}$=0.995, then the probability of the failure-free operation of the redundant system $P_{ДУ}$=0.99.

At the same time probability of failure-free operation of the non-standby DU, which consists of eight ZhRD, is equal to $P_{ДУ}$=0.92, and of six ZhRD - $P_{ДУ}$=0.94. Hence it follows that the presence of the system of redundancy (two engines out of eight) increases the reliability of the DU from 0.94 to 0.99. The aforesaid will be valid under the condition when the transfer from eight ZhRD to six and, connected with it, the boosting of each self-contained unit in thrust, does not lead to a reduction in the reliability of the DU (P_0=const).

As a whole, in speaking about the redundancy of such systems as the ZhRD, it is necessary to keep in mind that the basic requirement for using even the most ideal design schemes is the presence of the reliable emergency protection system (SAZ) [CA3]. As it already follows from expression (5.20), the low reliability of the SAZ elements can sharply lower the total effect of redundancy. Moreover, as a result of the delivery of false instructions about the beginning of the emergency situation, the normally functioning engines can be turned off. In summation, this can lead not to an increase in the reliability of the DU as a result of the redundancy, but to its decrease.

If now we again turn to the example examined above, then by calculation it is easy to establish that with $P_{САЗ}$ =0.945 the emergency protection system reduces the effect of redundancy to zero. More precisely, in this case the reliability of the DU, which consists of six standby units, is equal to the reliability of a DU with eight ZhRD with two standby. If the reliability of the SAZ is below the level of 0.945 (with P_0=0.99), then the effect from redundancy becomes negative. This is easy to trace on Fig. 5.11, where the effect of the reliability of the SAZ on the reliability of the DU is shown as a whole for the concrete scheme of redundancy. Represented on the axis of the ordinates is a complex characteristic in the form of the ratio of reliability indices of the DU with redundancy $P_{ДУ(рез)}$ (two standby ZhRD out of eight) without it.

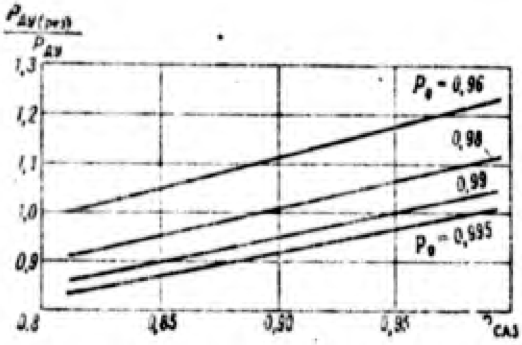


Figure 5.11. Effect of the reliability of the emergency protection system $P_{САЗ}$ on the reliability of the engine installation.

From an analysis of the curves it follows that the presence of a highly reliable SAZ makes it possible to attain a positive effect from the redundancy ($P_{ДУ(рез)}/P_{ДУ}=1.0$) even at the relatively low level of reliability of the autonomous ZhRD.

In the case of a reduction in the reliability level of the SAZ, a positive effect from redundancy is reached only because of an increase in the reliability of the engines.

As a whole the reliability of the standby DU cannot be higher than the reliability of the SAZ.

We will now turn again to Fig. 5.10. In speaking about the redundancy with respect to scheme II, we examined the full redundancy of the circuits in the block diagram. In our example this circuit was the autonomous system of the ZhRD.

A theoretical shortcoming in scheme II is the fact that the separate circuit elements, being in series connection, are not backed up by elements of other circuits. More ideal in this sense will be scheme III. It allows breakdown of the separate elements of different circuits, which does not reduce the system to failures. If in scheme II breakdown of elements 1 and 2 means complete failure, then in scheme III the efficiency of the system is provided for. In this case the full probability of failure-free operation of the system will be determined according to the equation

$$P_{III} = \prod_{j=1}^{k} [1 - (1 - P_j)^q]. \qquad (5.21)$$

For a quantitative evaluation of the effectiveness of the scheme of redundancy, let us examine the following example. Let us assume that the system consists of two doubled equally reliable elements connected by schemes II and III (see Fig. 5.10).

If the reliability of the elements is characterized by index P=0.90, for the system which consists of four elements, P_{II}=0.96 and P_{III}=0.98.

Thus the introduction into the block diagram of reliability of the given system of only one additional coupling A (scheme III) decreases the failure probability by virtually two times.

In connection with the DU, which consists of a series of self-contained units, this denotes the connection between them of "inputs" and "outputs" of a series of basic assemblies of standby ZhRD. More precisely, with the implementation of scheme III it is necessary to connect the pump pressure lines along the line of the fuel and oxidizer, respectively, the gas generator, and so on.

However, the appearance of such connections introduces the portion of the cross correlation between the self-contained units of the ZhRD and the need for switching-off devices virtually on all the standby circuit elements. This immediately places in doubt the advisability of the practical realization for the sustainer DU of the redundancy scheme III, which is very theoretically ideal. For the separate assemblies of the pneudraulic scheme the implementation of scheme III in all probability does not cause any special difficulties and in a number of cases can prove to be advisable.

Let us now examine some features of the series-parallel connection schemes of elements (scheme IV, see Fig. 5.10). Virtually these are the most frequently encountered systems with redundancy. In designing or developing this system, the designer, first of all, solves the problem concerning the redundancy of unreliable, small-scale elements. The redundancy of bulky, large-size elements and units is conducted, as a rule, only in exceptional cases. Most frequently the provision of the required levels of reliability of the systems is achieved by means of finishing up to the required levels of the reliability of the elements themselves (or units) whose redundancy is made difficult by the design or other considerations.

The mathematical description of the structural series-parallel schemes of reliability is accomplished by means of the multiplication of the probabilities which characterize the individual sections of the circuit with the series and parallel connection of the elements. In summation, in connection with scheme IV (see Fig. 5.10) we can write

$$P_{IV} = \prod_{i=1}^{l} P_i \left\{1 - \left(1 - \prod_{j=1}^{k} P_j\right)^d\right\}.$$

More complex cases of the connection of elements into series-parallel or combined schemes can be encountered, but the principle of the solution to the problem remains as before.

Thus, we examined the schemes with the "hot" redundancy of the elements. In the design and final development of these schemes, as a rule, the preference over schemes with "cold" redundancy is given up. The latter is explained by the fact that the "cold" redundancy differs by the complexity of the structural solution, requires the supplementary connecting devices and increases the passive weight of the article.

But, on the other hand, prior to the moment of the inclusion into the operation of the element of "cold" redundancy its operating service life is retained. This is the main advantage of the cold redundancy over the "hot." In all probability the ZhRD cannot be referred to such systems, since the main course of failures of elements of the ZhRD appears in the starting and transient conditions, but while operating in flight within limits of the time of the flight program it is small [54]. At the same time the ZhRD, in remaining sensitive to the starting, depending on the pressure of tank pressurization, the ambient temperature, field of gravitation, axial g-forces and so on can prove to be not equally reliable in the scheme with "cold" redundancy for various conditions of starting with all diversity of the characteristics and combinations of the external and intrinsic factors. The final development of the engine, equally reliable for so broad a range of variation of environmental factors and test conditions, is a sufficiently complex task.

The intermediate between the "cold" and "hot" redundancy is the "alleviated" standby. Before there is a breakdown in the basic element, the standby element is in the "on" position but operates in a reduced "alleviated" mode.

It goes without saying that under conditions of this loading there is a certain probability of the onset of failure of the standby element, but we keep in mind that it is low. An example of the practical use of the "alleviated" standby in rocket engineering is the DU, which consists of D autonomous ZhRD, the d of which is in "hot" reserve. Correspondingly, on the rocket the stated problem can be solved by means of (D-d) engines. If all the engines operate, the gross thrust of the DU can exceed that calculated. In order that this does not happen, their throttling is conducted. As a result all units of the DU operate in the state of "alleviated" standby with respect to each other.

With the breakdown of one or several engines (not exceeding d standby units) the emergency units are disconnected, but the basic ones are boosted up to the programmed value of thrust. But this understanding of the "alleviated" standby in application to the ZhRD undoubtedly to a certain degree is conditional, since the range of stable throttling of the ZhRD in thrust is relatively small. In summation, it is not difficult to note that the examined example of redundancy of the ZhRD in the DU combines in itself the elements of the "alleviated" and "sliding" standby, although as a whole this standby is "hot."

In conclusion let us note that the use of the redundancy of elements for mechanical systems for the purpose of the provision of high levels of reliability must closely be connected with the reliable estimate of the reliability of composite elements. Only under this condition is it possible to speak about advantages and shortcomings in those or other block diagrams. If in the process of final development the reliability of one of the elements (including the SAZ) is not provided for, then this can radically change the system's characteristics and even the design itself: for example, it is necessary to use redundancy where it was not provided for earlier. On the contrary, upon reaching the higher levels of reliability it is possible to forego the redundancy of the "weakest" link, and so on.

Because of this the structural scheme of reliability proves to be closely related to the reliability programs of the system's elements, and, therefore, in the concluding development stages of these elements it must constantly be more precisely formulated.

On the basis of the equation of the structural scheme of reliability, the point estimate of the probability of failure-free operation of the entire system can be obtained. The accuracy of this probability can be estimated by the linearization of function $P_{ДУ}$:

$$\sigma^2_{P_{ДУ}} = \sum_{i=1}^{l}\left(\frac{\partial P_{ДУ}}{\partial P_i}\right)^2 \sigma^2_{P_i} + \sum_{j=1}^{k}\left(\frac{\partial P_{ДУ}}{\partial P_j}\right)^2 \sigma^2_{P_j}.$$

Further, by converting to the determination of the lower boundary of the probability of failure-free operation of the system, we will use the known expression

$$P_{ДУ_{нижн}} = \bar{P}_{ДУ} - t_{\alpha}\sigma_{P_{ДУ}}.$$

The obtained estimate is that final criterion from which the degree of perfection of the experimental works is judged. With the high qualification and great experience of the designers, technologists and testers, the experimental final development concludes, as a rule, with the complete correcting of failures in all the systems.

As a result of this statistic on tests of the elements, units and systems of the DU of regular design, it proves to be that represented by reliable tests. For the given case the determination of quantitative reliability indices of the system from tests of its elements is reduced to the known solution of R. A. Mirny and A. D. Solovyev [28].

The system is represented in the form of a certain quantity of consecutive statically independent elements, which have the levels of reliability

$$P_1, P_2, P_3, \ldots, P_k.$$

Then the reliability of the system is defined as the product

$$P_\Sigma = P_1 P_2 P_3, \ldots, P_k. \quad (5.22)$$

If we now agree that the quantity of tests for the first type elements is N_1, for the second - N_2, for the third - N_3, etc., then the probability of the fact that during the tests there will occur not one failure will be respectively equal to

$$P_1^{N_1} P_2^{N_2} P_3^{N_3}, \ldots, P_k^{N_k} > 1-\alpha. \quad (5.23)$$

The obtained estimate must correspond to the confidence level $(1-\alpha)$ with which the calculation is conducted.

Now let us assume that for a certain ith type of elements the quantity of tests proved to be minimum

$$N_1 > N_i;\ N_2 > N_i;\ N_3 > N_i; \ldots N_k > N_i.$$

Then, by using expression (5.23) for the condition of equality, it is possible to obtain

$$P_1 P_2 P_3 \ldots P_k = (1-\alpha)^{\frac{1}{N_i}} P_1^{1-\frac{N_1}{N_i}} P_2^{1-\frac{N_2}{N_i}} \ldots P_k^{1-\frac{N_k}{N_i}}. \quad (5.24)$$

By accepting our condition (N_i=min), it is easy to establish that all the values of powers of the second, third, etc. terms of equation (5.24), except P_i, are negative. And this means that the minimum values of probabilities $P_1 = P_2 = P_3 \ldots = P_k = 1.0$, since in the limit they cannot be more than unity. Then, taking into account expression (5.22), we obtain

$$P_{\Sigma_{\text{мин}}} = (1-\alpha)^{\frac{1}{N_i}}.$$

or

$$P_{\Sigma_{\text{мин}}} = P_i.$$

Actually this means that in the case of reliable tests, the reliability of the system, which consists of a certain number of elements, is determined by the minimum reliability index of one of the elements for which the quantity of the tests is less than the others.

CHAPTER VI

EQUATIONS OF DYNAMICS OF BASIC ASSEMBLIES OF LIQUID-PROPELLANT ROCKET ENGINES

§ 1. DYNAMIC PROCESSES IN LIQUID-PROPELLANT ROCKET ENGINES

The contemporary ZhRD [ЖРД - liquid-propellant rocket engine] installation is an involved complex of interdependent assemblies and systems which ensure the obtaining of the assigned basic parameters on the flight vehicle.

The ZhRD operate in steady-state and unsteady (transient) modes.

Belonging to the steady-state should be modes of the main, preliminary and final stages. In these cases the parameters of the engine remain virtually constant in time. Deviations of the parameters from the potential values are insignificant and can be disregarded.

In the mode of the main stage (nominal mode of the operation) the engine operates at the most prolonged time and at total power. In connection with it the basic design parameters of the engine are selected.

At present the theory of the operating conditions which occur in the assemblies of the engine installation in the steady-state mode has been developed sufficiently fully and makes it possible to design characteristics of the ZhRD and adjust it to the assigned mode.

In transient modes the engine installations operate upon starting, the switching of stages, with disconnection, with automatic control, and so on. In emergency situations, i.e., with the appearance of a malfunction, in engine there also appears the transient process which leads to a considerable change in the operating mode of the engine installation.

The dependences which connect the parameters of the engine installation with operation in transient modes are called dynamic characteristics.

The dynamic processes which occur in the engine installation are complex and are characterized by a large quantity of interacting connections and factors and by the rapid occurrence of the processes: for example, such a complex process as the starting of the engine does not exceed 2-3 s. In connection with this, the dynamic processes which occur in engine plants have still been insufficient ; although recently there has appeared literature which has touched upon problems of dynamics [16, 40, 44, 74].

In the process of the producing the ZhRD a whole number of problems connected with the dynamics of the engine is encountered. For the engine itself these are the stability of the engine together with the internal engine control system, an explanation of the reasons for the emergency outcomes of the tests, the organization of starting of the engine, transient processes with a change in the engine operating mode.

Interest in the investigations in the field of dynamics of the ZhRD was increased substantially upon transition from engines of the "open" design to engines with the afterburning of producer gas ("closed" design).

A considerable decrease in the time constant of the ZhRD with the afterburning of the producer gas led to the fact that the engine itself as a dynamic control object was changed, and the interconnection of the operating processes in all the assemblies was reinforced. The final development of engines of "closed" designs caused a whole number of important difficulties with the organization of reliable starting and the stabilization of the control system of the engine. The high static, and mainly dynamic loads, on the casings, impellers, turbine, shaft and other parts of the TNA [THA - turbopump assembly] led to the most diverse breakages of these parts.

For a theoretical study of the dynamic characteristics, the engine installation is examined in the form of a mathematical model represented by the system of differential equations which describe the operating processes in separate assemblies of the engine installation.

The unknown time functions are the basic parameters of the engine installation. The number of equations must be equal to the number of unknown parameters of the ZhRD, i.e., the system of equations must be closed.

Each type of transient conditions (starting, cutoff, transient processes during emergency situations) has its own specific features and must find reflection in a mathematical model either by means of the addition of a number of equations or their modification in the general model of the engine installation. Furthermore, depending on the concretely assigned problem, the form of the equations of the assemblies of engine installation can be different. Thus in the study of conditions of start-up, cutoff and emergency situations when the parameters considerably differ from values which correspond to the steady-state conditions, it is necessary to use nonlinear differential equations. In the solution to problems on the control of the ZhRD, when small deviations in parameters of the engine installation are observed, nonlinear equations of assemblies can be used in linearized form.

§ 2. EQUATION OF DYNAMICS OF THE COMBUSTION CHAMBER

We will consider the chamber as a gas tank. The processes in the combustion chamber of the ZhRD (Fig. 6.1) can be affected by a change in the per-second mass flow rates of the oxidizer (G_o) and fuel (G_r) and in the chamber of the ZhRD with the afterburning of the producer gas (Fig. 6.2), furthermore, even by the flow rate of the producer gas - G_ϕ. These parameters are input signals of the chamber. The output signal of the chamber as a gas volume is the pressure in the chamber.

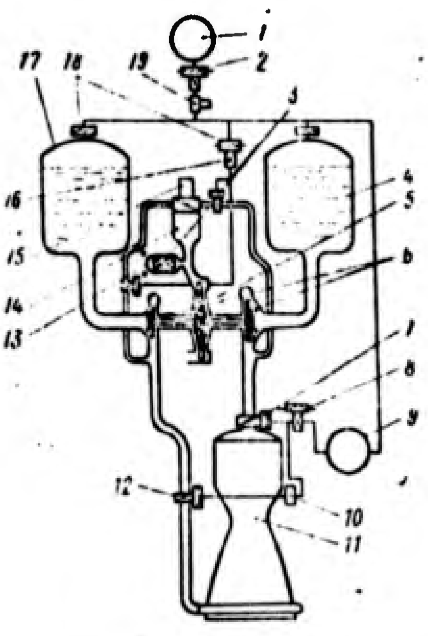


Figure 6.1. Diagram of a ZhRD with ejection of the producer gas: 1 - gas pressure reservoir; 2, 8, 16 - electropneumatic valves; 3 - air operated valves; 4 - oxidizer tank; 5 - turbine; 6 - oxidizer and fuel pumps; 7 - main oxidizer valve; 9 - low-pressure accumulator; 10 - jet; 11 - combustion chamber; 12 - main fuel valve; 13 - pyrostarter; 14 - liquid-gas generator; 15 - valve; 17 - fuel tank; 18 - diaphragm; 19 - gas pressure reducer.

In the study of the dynamics of the operation of the combustion chamber the following processes are examined:

1) the transformation of liquid components into gas combustion products;

2) the storage of gases in the combustion-chamber volume;

3) outflow of gas from the combustion chamber.

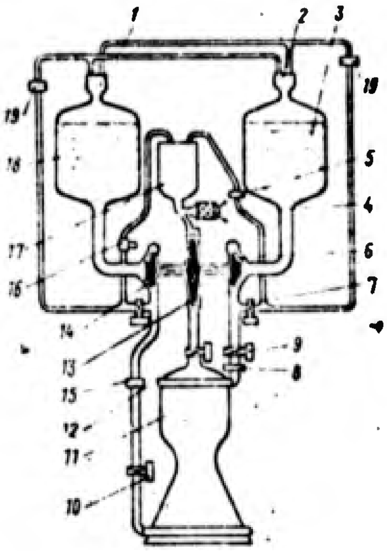


Figure 6.2. Diagram of a ZhRD with the afterburning of producer gas: 1, 2 - gas generators for tank pressurization; 3 - oxidizer tank; 4 - pyrostarter; 5, 8, 15 - throttle disks; 6 - oxidizer pump; 7 and 12 - valves; 9, 10 - main valves of oxidizer and fuel; 11 - combustion chamber; 13 - turbine; 14 - fuel pump; 16 - regulator; 17 - gas generator; 18 - fuel tank; 19 - diaphragms.

The physical and chemical nature of intrachamber processes is very complex, and it is difficult to describe them accurately by methods of mathematical analysis. Therefore, in deriving the equation of the combustion chamber let us make the following assumptions [40, 16].

1. According to the lapse of time of the conversion of $\tau_{пр}$, the fuel which entered into the combustion chamber completely and instantly converted into combustion gases, i.e., the real curve of the burnout is replaced by a stepped curve (Fig. 6.3.).

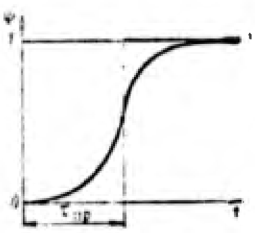


Figure 6.3. Curve of burnout.

2. At each moment of time t the gas pressure $p_к$ is equal for all points of the volume of the chamber up to the nozzle entry, and the mass of the gas in this volume changes as one whole, i.e., the acoustic effects and hydraulic friction of the gas volume can be disregarded. Furthermore, it is also accepted that the efficiency of the gases $R_к T_к$ is identical for all points of the chamber volume independently of the specific nature of local conditions of the combustion process.

3. The combustion products are considered as an ideal gas, i.e., for them the equation of Clapeyron-Mendeleyev is correct.

When making these assumptions the dynamics of the gas flow in the combustion chamber is described by the equation of mass balance. At the given instant t the mass rate of formation of the gas in the combustion chamber must be equal to the mass exhaust gas velocity from the chamber and the rate of storage of the gas mass in the combustion chamber itself.

On the basis of the law of conservation of the substance, it is possible to confirm that a change in the weight content of the gases in the volume dQ during an infinitesimal time dt is equal to the difference in weights of the supplied fuel $G_m dt$ and the gas which escaped from the tank Gdt:

$$dQ = G_m dt - G dt. \qquad (6.1)$$

In connection with the presence of conversion time τ_{np}, in the process of the conversion of fuel into the combustion gases, the total amount of fuel which burned down during the time interval from 0 to t is

$$\int_0^t G_m dt = \int_0^{t-\tau_{np}} [G_0 + G_r] dt. \qquad (6.2)$$

The upper limit of the integral, which is on the right side of equation (6.2), is equal to $(t-\tau_{np})$, since the particles of fuel which entered after a moment of time $(t-\tau_{np})$, by the moment of time t still do not take part in the combustion process. Hence, differentiating with respect to t, we obtain

$$G_m = \left(1 - \frac{d\tau_{np}}{dt}\right) [G_0 (t - \tau_{np}) + G_r (t - \tau_{np})]. \qquad (6.3)$$

According to the first assumption τ_{np}=const, i.e., $d\tau_{np}/dt=0$.

Then, substituting expression (6.3) into the equation of mass balance, we obtain

$$\frac{dQ}{dt}=G_0(t-\tau_{np})+G_r(t-\tau_{np})-G. \quad (6.4)$$

We determine the quantity of gas Q in the combustion-chamber volume $V_к$ according to equation of state

$$Q=\frac{p_к V_к}{R_к T_к}. \quad (6.5)$$

Having differentiated equation (6.5), we obtain

$$\frac{dQ}{dt}=\frac{V_к}{R_к T_к}\cdot\frac{dp_к}{dt}-\frac{p_к V_к}{(R_к T_к)^2}\frac{d(R_к T_к)}{dt}. \quad (6.6)$$

The flow rate of gas through the nozzle of the combustion chamber is determined by the dependence

$$G=A_к\frac{p_к F_{кр}}{\sqrt{R_к T_к}}, \quad (6.7)$$

where $A_к=\sqrt{\varkappa g\left(\frac{2}{\varkappa+1}\right)^{\frac{\varkappa+1}{\varkappa-1}}}$;

κ - the average index of the expansion of gas in the nozzle.

Having substituted expressions (6.6) and (6.7) into equation (6.4), we obtain the equation of dynamics of the combustion chamber

$$\frac{V_к}{R_к T_к}\frac{dp_к}{dt}-\frac{p_к V_к}{(R_к T_к)^2}\frac{d(R_к T_к)}{dT}=G_0(t-\tau_{np})+G_r(t-\tau_{np})-$$
$$-A_к\frac{p_к F_{кр}}{\sqrt{R_к T_к}}. \quad (6.8)$$

The efficiency of the gas $R_к T_к$ and the index of the process of expansion κ are complex functions which depend for this kind of fuel on the relationship of the propellant components k and combustion-chamber pressure $p_к$: $R_к T_к = R_к T_к(k, p_к)$; $\kappa = \kappa(k, p_к)$, and the determining effect is exerted by the fuel component ratio $k = G_0/G_г$. The derivative of the efficiency of gas necessary for equation (6.8) takes the form

$$\frac{d(R_к T_к)}{dt} = \frac{\partial(R_к T_к)}{\partial p_к}\frac{dp_к}{dt} + \frac{\partial(R_к T_к)}{\partial k}\frac{dk}{dt}.$$

The local derivative dk/dt characterizes a change in the relationship of the components with time at the inlet of the combustion chamber and is determined by the operating mode of the feed system [44]:

$$\frac{dk}{dt} = \frac{G_г \frac{dG_о}{dt} - G_о \frac{dG_г}{dt}}{G_г^2}.$$

In afterburners of the producer gas the equation of the dynamics of the combustion chamber retains the form of equation (6.8) with introduction into its right side of the new member

$$\frac{V_к}{R_к T_к} \cdot \frac{dp_к}{dt} - \frac{p_к V_к}{(R_к T_к)^2}\frac{d(R_к T_к)}{dt} = G_о(t - \tau_{пр}) + G_г(t - \tau_{пр}) + G_ф - A_к \frac{p_к F_{кр}}{\sqrt{R_к T_к}},$$

where $G_ф$ is the flow rate of the producer gas which enters through the gas injectors into the combustion chamber at a given instant.

The relationship of the propellant components in the combustion chamber in this case is determined by the expression

$$k = \frac{G_0(t - \tau_{пр}) + \frac{k'}{k'+1} G_ф}{G_г(t - \tau_{пр}) + \frac{1}{k'+1} G_ф},$$

where k' is the relationship of the propellant components in the gas generator.

Let us agree subsequently that all parameters of the gas generator will be noted by one prime.

The equation of dynamics of the gas generator is derived in a way similar to the equation of the combustion chamber and takes the form of (6.8). One should only keep in mind that the outflow of gas from the gas generator can be subcritical, and the flow rate of the gas through the turbine depending on flow conditions of the gas will be determined:

- for the supercritical outflow

$$G_{\text{т}} = A' \frac{p_{\text{к}}' F_{\text{кр}}'}{\sqrt{RT}};$$

- for the subcritical outflow

$$G_{\text{т}} = \sqrt{2g \frac{\varkappa'}{\varkappa' + 1} \frac{(p_{\text{к}}')^2 F_{\text{кр}}}{RT} \left[\left(\frac{p_2}{p_{\text{к}}'} \right)^{\frac{2}{\varkappa'}} - \left(\frac{p_2}{p_{\text{к}}'} \right)^{\frac{\varkappa'+1}{\varkappa'}} \right]},$$

where p_2 is the pressure behind the turbine.

In designs of the ZhRD with the afterburning of the producer gas, there is a number of gas main lines through which the gas is fed from the turbine to the combustion chamber and from the cooling jacket (when using low-boiling components as the liquid coolant) to other engine components.

In relation to dynamics the gas main lines are similar to the combustion chamber and gas generator, i.e., these are gas volumes with a changing inlet temperature. Therefore, their equations virtually coincide. Only in the term which gives the rate of discharge of the gas entering into the main line will naturally not be the delay which characterizes the conversion time of the liquid into the gas.

The equation of the gas main line takes the form

$$\frac{V_{\text{м}}}{R_{\text{м}}T_{\text{м}}}\frac{dp_{\text{м}}}{dt} - \frac{V_{\text{м}}p_{\text{м}}}{(R_{\text{м}}T_{\text{м}})^2}\frac{d(R_{\text{м}}T_{\text{м}})}{dT} = G_{\text{г}} - G_{\text{ф}},$$

where $p_{\text{м}}$ - the pressure in the gas main line;

$V_{\text{м}}$ - volume of the gas main line;

$R_{\text{м}}T_{\text{м}}$ - efficiency of gas in the gas line;

$$R_{\text{м}}T_{\text{м}} = R'T' - \frac{\varkappa' - 1}{\varkappa'}\frac{c_{\text{ад}}^2 \eta_{\text{т}}}{2g},$$

where $c_{\text{ад}}$ - the adiabatic exhaust velocity of the gas from nozzles of the turbine;

$\eta_{\text{т}}$ - efficiency of the turbine.

The equations derived for the gas volumes are valid for the majority of the cases of computation of the dynamic characteristics of the ZhRD, since in the gas volumes of the ZhRD the Mach number usually does not exceed 0.2-0.3. Consequently, these equations are valid in the range of frequencies which are of interest for the dynamics of the entire engine (up to 30-50 Hz).

The limiting parameter for using the derived formulas in this frequency range is the Mach number. When M>0.3-0.4 it is necessary to use the accurate solutions of equations for the volume as distributed parameters.

§ 3. EQUATION OF DYNAMICS OF HYDRAULIC LINES

Hydraulic lines include a system of pipelines with the elements of control (throttles, valves, regulators, etc.) arranged in them and serve for the feed of components from the tanks to the pumps of the turbopump unit and from pumps to the combustion chamber and gas generator.

The series arrangement of fuel tanks in large carrier rockets is conditioned by the need for the feed of one of the propellant components to the engine along a long tube [23]. A change in the momentum of the liquid in the main line, cuased by a break in the diaphragm by the opening or closing of the fuel valves, is expressed in the appearance of pressure peaks as with hydraulic impact. Such phenomena are observed with the starting and cutoff of the engines and during emergency situations.

The switchings of the controls in hydrualic lines from pumps to the combustion chambers and the appearance of malfunctions of hydraulic lines cause transient processes in the pipelines connected with changes in pressure and rate of the fluid, which are accompanied by the appearances of peaks and troughs of pressures at inlets into the pumps.

The inertia properties of the fluid especially greatly appear in the trnasient processes with small cross sections and long length of the pipelines.

It cannot be considered that the changes in pressure and rate of the fluid occur simultaneously in all sections of the pipeline. The disturbance of the initial state in any cross section of the pipeline causes the emergence and propagation of elastic waves over the entire pipeline. With long pipelines and for the duration of the transient process close to the duration of the phase of the hydraulic impact, the investigation of the transfer phenomena in the hydraulic lines is carried out on the basis of the wave theory [64].

For the compilation of the equation of dynamics of the pipeline, let us make the following assumptions.

1. The fluid is incompressible, and its motion is one-dimensional. The pipeline is absolutely rigid. In this case

the fluid flow rate along all cross sections of the pipeline is constant ($G=G(t)$).

2. Translational acceleration for all particles of the fluid in the pipeline is identical and equal to the acceleration of the center of gravity of the rocket j. The rotation of the rocket around its axis is disregarded.

In this case the projection of the total particle acceleration of the fluid on the axis of the pipeline S, directed in the direction of the steady motion of the fluid will be equal to (Fig. 6.4)

$$j_a = j_i - j\cos\alpha_j,$$

where j_i is the projection of the relative acceleration of the particle of the fluid;

α_j - the angle between the projection of the relative acceleration and the direction of acceleration of the center of gravity of the rocket.

The cross-sectional area of the pipeline on the section between cross sections i-i and i'-i' is equal to F=const, so that the rate of the fluid relative to the walls of the pipeline is

$$w_i = \frac{G}{\gamma F_i} \tag{6.9}$$

and the relative acceleration on this section is equal for all particles of the fluid to

$$j_i = \frac{dw_i}{dt} = \frac{1}{\gamma F_i}\frac{dG}{dt}. \tag{6.10}$$

3. On the basic section of the pipeline with length l_1 between cross sections i-i and i'-i', the forces of friction are absent, and all the hydraulic frictions are concentrated on the output section of the pipeline (i+1)-(i+1), i'-i' and are considered to be the reduced coefficient of hydraulic losses ξ.

Figure 6.4. Derivation of the equation of dynamics of the hydraulic line.

4. With unsteady flows to the fluid it is possible to use the Bernoulli equation, as is done in work [32]:

$$p_i' + \frac{\gamma w_i^2}{2g} = p_{i+1} + \frac{\gamma w_{i+1}^2}{2g} + \xi\frac{\gamma w_{i+1}^2}{2g}. \qquad (6.11)$$

Strictly speaking, the Bernoulli equation is correct for the steady flows. As the justification of the assumption made serves the small length of the section, i.e., the small mass and rapid response of the fluid, included between the cross sections i'-i' and (i+1)-(i+1).

The rate of the fluid in the cross section (i+1) is equal to

$$w_{i+1} = \frac{G}{\gamma F_{i+1}}. \qquad (6.12)$$

On the basis of the first three assumptions, it is possible to draw the conclusion that the fluid in the main section of the pipeline moves forward as a solid body. Therefore, the flow equation can be written on the basis of the second Newton law:

$$\frac{Q_i}{g} j_a = p_i F_i - p_i' F_i + Q_i \cos\alpha_g, \qquad (6.13)$$

where Q_i is the gravity of the fuel equal to

$$Q_i = \gamma l_i F_i; \qquad (6.14)$$

α_g - the angle between the directions of the relative acceleration and G force.

Let us substitute expressions (6.9) and (6.12) into equation (6.11)

$$p_i' + \frac{\gamma G^2}{\gamma^2 F_i^2 2g} = p_{i+1} + \frac{\gamma G^2}{\gamma^2 F_{i+1}^2 2g} + \xi\frac{\gamma G^2}{2g\,\gamma^2 F_{i+1}^2}.$$

whence

$$p_i' = p_{i+1} + \frac{G_2}{2\gamma g}\left[\frac{1+\xi}{F_{i+1}^2} - \frac{1}{F_i^2}\right]. \qquad (6.15)$$

After substituting expressions (6.10) and (6.15) into equation (6.13) and after dividing into F_i, we obtain

$$\frac{Q_i}{g\gamma F_i^2}\frac{dG}{dt} + \frac{1}{2g\gamma}\left[\frac{1+\xi}{F_{i+1}^2} - \frac{1}{F_i^2}\right]G^2 = p_i - p_{i+1} + \frac{Q_i}{gF_i}(g\cos\alpha_g - j_p\cos\alpha_j) \qquad (6.16)$$

After substituting expression (6.14) into equation (6.16), we obtain the equation of dynamics of the pipeline

$$\frac{l}{gF_i}\frac{dG}{dt} + aG^2 = p_i - p_{i+1} + p_{Mi},$$

where a - the coefficient of hydraulic friction;

$$a = \frac{1}{2g\gamma}\left[\frac{1+\xi}{F_{i+1}^2} - \frac{1}{F_i^2}\right];$$

$p_{M\,i}$ - the pressure of the mass forces of the fluid column on the cross-sectional area i'-i';

$$p_{Mi} = \frac{Q_i}{gF_i}(g\cos\alpha_g - j_p\cos\alpha_j).$$

The ratio $l/gF=b$ is called the coefficient of inertia losses. Taking this into account the equation of dynamics of the pipeline can be written in the form

$$b\frac{dG}{dt} + aG^2 = p_i - p_{i+1} + p_{Mi}. \qquad (6.17)$$

In steady-state conditions $dG/dt=0$; therefore, the equation statics of the pipeline is obtained from (6.17) in the form

$$aG^2 = p_i - p_{i+1} + p_{Mi}.$$

By means of expression (6.13) according to pressure differential and flow rate G in known steady-state conditions the following parameter is determined:

$$Q=\frac{p_i+p_{\text{н}i}-p_{i+1}}{G^2}.$$

For the hydraulic line of invariable geometry the coefficient of hydraulic friction is a constant value (Q=const) not dependent on the operating mode.

For the hydraulic line with the controls the equation of dynamics of the hydraulic line is written similar to the uncontrolled elements; only in this case the coefficient of hydraulic friction of the main line is a variable value which depends on the movement of the controlling element $x_{p.o}$ (for example, the movement of the needle of a piston of a regulator, movement of the disk of a valve, etc.):

$$p_i-p_{i+1}+p_{\text{н}i}=a(x_{p.o})G^2+b\,\frac{dG}{dt}.$$

§ 4. EQUATION OF DYNAMICS OF A CENTRIFUGAL PUMP

The basic assemblies of a liquid-propellant rocket engine which determine the configuration of the engine section of the rocket are the combustion chamber and turbopump unit: the design of the combustion chamber determines the requirements for the fuel, specific thrust and thrust of the rocket, and the turbopump unit, i.e., requirements for the power-supply system [23]. The power-supply system includes pumps, elements of pressurization, hydraulic lines, elements of automation and tanks.

One of the most important assemblies of the power-supply system with unloaded tanks which ensure the motion of the propellant components along the hydraulic lines is the centrifugal pump. The pump is driven by a turbine and provides with nominal revolutions the necessary pressure behind the pump and the required flow rate of the component.

The equation of the centrifugal pump must determine the connection between the revolutions of the TNA and the fluid flow rate with the pressure and power input created by the pumps.

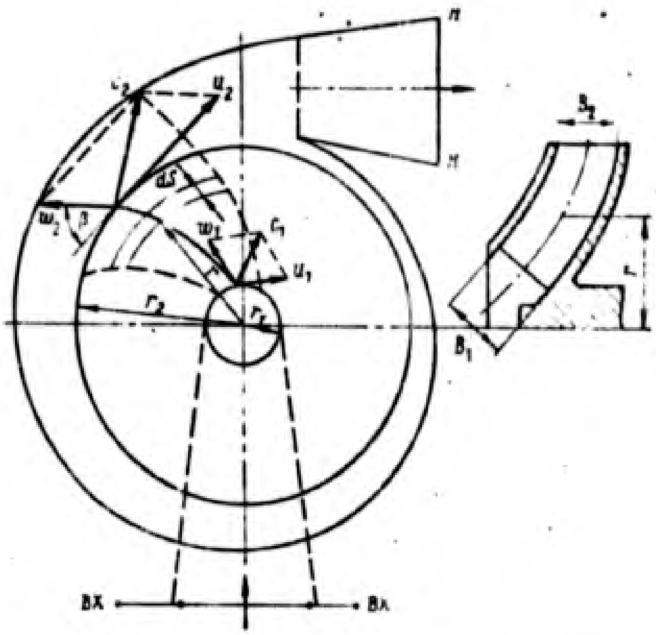


Figure 6.5. Derivation of the equation of the dynamics of a pump.

Let us conduct the derivation of the equation in connection with a design diagram (Fig. 6.5).

According to the principle of the conservation of energy, the power developed by the impeller of the pump is consumed for an increase in the energy of the flowing fluid flow. The power of the blade wheel can be determined by the angular velocity and the magnitude of the moment of the force of interaction of the wheel with the fluid flow, i.e., according to the torque of the wheel [38].

The determination of the resulting moment of the interaction of the blade wheel with the fluid flow is a problem of hydrodynamics which, generally speaking, can be solved only after an analysis of the phenomena within the region of the wheel, which is associated with a whole number of difficulties and requires considerable outlining of the real phenomenon.

However, a solution can be made on the basis of the equation of momentum, and the states of the flow within the wheel are not examined and remain only states on boundaries, i.e., before and after the wheel.

The power of the forces which act on the fluid in the impeller is determined by the torque of the wheel and the work of forces of pressure at the inlet and output from the wheel [16]:

$$N = M\omega + p_1F_1w_1 - p_2F_2w_2, \qquad (6.18)$$

where N - the power;

M - the torque applied to the shaft;

ω - angular velocity;

p_1, p_2 - pressures at the inlet and at outlet from the rotor wheel;

w_1, w_2 - relative rates of motion of the fluid in the wheel (at the inlet and outlet, respectively);

F_1, F_2 - areas of channels at the inlet and outlet from the rotor wheel.

The moment of momentum of the fluid, which takes place in the impeller vanes, relative to the axis of rotation,

$$M_{\kappa} = \int_{s_1}^{s_2} rc_u dm,$$

where r - the radius of this circular element in the fluid flow;

c_u - the projection of the absolute velocity on the circular velocity: $c_u = u - w \cos \beta$;

β - angle between the relative and peripheral velocities;

dm - elementary mass equal to $dm = \rho F ds$;

F - area of the channel between the impeller vanes;

ds - elementary length of the way of the fluid in the wheel.

If a force acts on a body, then as a result of a change in the speed of the body, the momentum and, consequently, the moment of momentum are changed. According to the theorem about the moment of momentum, a per-second change in the moment of momentum equal to the moment of the external forces which act on this body, i.e., the time derivative of the moment of momentum of the fluid is equal to the torque applied to the shaft:

$$M = \frac{dM_x}{dt} = \frac{d}{dt}\int_{s_1}^{s_2} r(u - w\cos\beta)\rho F ds.$$

Taking into account the fact that ds/dt-w, we obtain

$$M = r(u - w\cos\beta)\rho F w\Big|_1^2 + \int_{s_1}^{s_2} r\left(\frac{du}{dt} - \frac{dw}{dt}\cos\beta\right)\rho F ds. \tag{6.19}$$

After substituting equation (6.19) into (6.18), we obtain the expression for the power of the forces which act on fluid in the impeller, taking into account that u=rω

$$N = u(u - w\cos\beta)\rho F w\Big|_1^2 + \int_{s_1}^{s_2} u\rho F ds\left(\frac{du}{dt} - \frac{dw}{dt}\cos\beta\right) + \\ + p_1F_1w_1 - p_2F_2w_2. \tag{6.20}$$

On the other hand, the power is expressed as a change in the kinetic energy per unit time:

$$N = \frac{dE}{dt} = \frac{d}{dt}\int_{s_1}^{s_2} \frac{c^2}{2} dm.$$

Since $c^2=w^2+u^2-2uw\cos\beta$, then

$$N = \frac{1}{2}(w^2 + u^2 - 2uw\cos\beta)\Big|_1^2 + \int_{s_1}^{s_2}\Big[w\frac{dw}{dt} + u\frac{du}{dt} - \\ - \left(w\frac{du}{dt} + u\frac{dw}{dt}\right)\cos\beta\Big]\rho F dS. \tag{6.21}$$

After equating expressions (6.20) and (6.21) and using the equation of continuity $w_1F_1=w_2F_2=\text{const}$, after some transformations we will obtain the equation of theoretical pressure, developed with the impeller of the pump:

$$p_2 - p_1 = \frac{\rho}{2}\left[(u_2^2 - w_2^2) - (u_1^2 - w_1^2)\right] - \\ - \rho \int_{s_1}^{s_2} \left(\frac{dw}{dt} - \frac{du}{dt}\cos\beta \right) ds. \qquad (6.22)$$

Here the index "1" denotes the parameters at the inlet into the wheel; index "2" - at the outlet from the wheel.

A pressure increase in the inlet branch pipe can be determined by equation (6.22), taking into account that in the branch pipe $u=0$, $w=c$;

$$p_1 - p_{вх} = \frac{\rho}{2}(c_1^2 - c_{вх}^2) - \rho \int_{вх}^{1} \frac{dc}{dt}\, ds.$$

Since $G=\gamma Fc$,

$$p_1 - p_{вх} = \frac{\rho}{2}(c_1^2 - c_{вх}^2) - \frac{1}{g}\frac{dG}{dt}\int_{вх}^{1} \frac{ds}{F_{вх}}. \qquad (6.23)$$

The pressure increase in the branch device is determined similarly:

$$p_н - p_2 = \frac{\rho}{2}(c_2^2 - c_н^2) - \frac{1}{g}\frac{dG}{dt}\int_{2}^{н} \frac{ds}{F_2}, \qquad (6.24)$$

The obtained equations (6.22), (6.23) and (6.24) make it possible to determine the full pressure increment in the pump:

$$p_н - p_0 = \frac{\rho}{2}\left[(c_2^2 - c_н^2) + (u_2^2 - w_2^2) - (c_1^2 - c_{вх}^2) - (u_1^2 - w_1^2) - \right. \\ - \rho \int_{s_1}^{s_2} \left(\frac{dw}{dt} - \frac{du}{dt}\cos\beta \right) - \frac{1}{g}\frac{dG}{dt}\int_{вх}^{1} \frac{ds}{F_{вх}} - \frac{1}{g}\frac{dG}{dt}\int_{2}^{н} \frac{ds}{F_2}. \qquad (6.25)$$

In deriving equation (6.25) losses of the head in pump as a result of the fluid friction against the walls, the turning of the flow, losses to a finite number of blades, etc. were not considered. Hydraulic losses are proportional to the square of the flow rate; therefore, we will consider them by the introduction of the loss factor ξ_H.

Let us determine the integrals which enter into the expression of the full pressure increment in the pump (6.25), which consider the dynamic pressure components created by the pump taking into account

$$w = \frac{G}{2\pi r \varphi\rho b_{2H} \sin\beta}, \quad \frac{du}{dt} = \frac{\pi D}{60} \frac{dn}{dt},$$
$$\frac{dw}{dt} = \frac{1}{2\pi r \rho\varphi b_{2H} \sin\beta} \frac{dG}{dt},$$

where b_{2_H} - the width of blades at the output from the wheel;

ϕ - the coefficient of constraint equal to $1-\delta_H i/2\pi r \sin\beta$;

δ_H - the thickness of impeller vane;

i - the number of the blades.

Let us replace the calculation of the integrals by the numerical integration for the sections of the center line of the blades:

$$I_1 = \rho\int_{s_1}^{s_2}\left(\frac{dw}{dt} - \frac{du}{dt}\cos\beta\right) ds = \rho\int_{s_1}^{s_2}\frac{dw}{dt} ds - \rho\int_{s_1}^{s_2}\frac{du}{dt}\cos\beta\, ds =$$
$$= \frac{dG}{dt}\sum_{i=1}^{l}\frac{\Delta s_i}{2\pi r_i \rho\varphi_i b_{2H}\sin\beta_i} - \frac{\pi\rho}{60}\sum_{i=1}^{l} D_i \Delta s_i \cos\beta_i \frac{dn}{dt}, \qquad (6.26)$$

where i is the number of the sections.

Determined similarly are the integrals

$$I_2 = \frac{1}{g} \cdot \frac{dG}{dt} \int_{\text{вх}}^{1} \frac{ds}{F} = \frac{dG}{dt} \sum_{i=1}^{m} \frac{\Delta s_i}{gF_i}, \tag{6.27}$$

$$I_3 = \rho \int_{2}^{н} \frac{dw}{dt}\, ds = \frac{dG}{dt} \sum_{i=1}^{m} \frac{\rho \Delta s_i}{F_{iн}}. \tag{6.28}$$

After substituting expressions (6.26), (6.27) and (6.28) into equation (6.25) and taking into account that

$$c_1^2 = u_1^2 - 2u_1 w_1 \cos\beta_1 + w_1^2, \quad c_{\text{вх}} = \frac{G}{\rho F_{\text{вх}}},$$

$$c_{н} = \frac{G}{\rho F_2}; \quad w = \frac{G}{2\pi\rho r b_{2н} \varphi \sin\beta}, \quad u = \frac{\pi D n}{60},$$

we obtain

$$p_{н} - p_{\text{вх}} = d_1 n^2 - d_2 n G - d_3 G^2 - D_{н} \frac{dG}{dt} + E_{н} \frac{dn}{dt}. \tag{6.25a}$$

The coefficient d_1 characterizes the effect on the pump head of dimensions of the rotor wheel, d_2 - geometry of the flow part, d_3 - hydraulic losses and the effect of inlet and exhaust pipes.

The coefficients D_H and E_H determine the inertia components of pressure according to flow rate and the number of revolutions:

$$d_1 = \frac{2{,}75 \cdot 10^{-3}}{1 + n_л} \rho D_1^2 \left(\frac{D_2^2}{D_1^2} - 1 \right),$$

$$d_2 = \frac{1{,}67 \cdot 10^{-2}}{1 + n_л} \left(\frac{1}{\varphi_2 b_2 \operatorname{tg}\beta_2} - \frac{1}{\varphi_1 b_1 \operatorname{tg}\beta_1} \right),$$

$$d_3 = \xi_{н} + \frac{1}{2\rho} \left(\frac{1}{F_{н}^2} - \frac{1}{F_{\text{вх}}^2} \right),$$

$$D_{н} = \sum \frac{\Delta s_{iн}}{2\pi r_i \rho \varphi b_{2н} \sin\beta_i} + \sum \frac{\Delta s_{i\text{вх}}}{gF_{i\text{вх}}} + \sum \frac{\Delta s_i}{gF_{iн}},$$

$$E_{н} = \frac{\pi\rho}{60 g} \sum D_i \Delta s_i \cos\beta_i,$$

where $n_л$ is the coefficient of the reduction in pressure due to the transition to the finite number of blades.

With the lowering of the pressure at the inlet into the pump, the centrifugal pumps begin operating in the mode of partial cavitation.

Cavitation is a disturbance of the continuity of fluid flow caused by the appearance in it of bubbles or cavities filled with vapor or gas, which appear in those places of the fluid flow where the static pressure falls below the pressure of steam generation [16, 19, 49]. In the majority of the cases the gas liberation from the solution does not play a significant role. In this case the cavitation is frequently called steam.

In the centrifugal pump the true cavitation appears on the blade of the rotor wheel usually near its inlet edge. The pressure here is considerably lower than the pressure in the suction pipe of the pump due to a local increase in the velocity with inleakage to the blade and due to hydrualic losses in the feed.

In the operation on the gasified components it is necessary to deal with the flow of liquid to which the gas bubbles are "mixed". With the passage of the bubbles through the region of reduced pressure there occurs an intense increase in them and, therefore, an increase in the volume concentration of the gas. A sharp disturbance in the operating mode of the pump can occur as a result of the fact that in the center section of the wheel there is an accumulation of gas which was separated from the fluid under the action of centrifugal forces. This phenomenon is called gas cavitation.

In centrifugal pumps the cavitation is accompanied by a loss in pressure, power and efficiency.

The very approximately cavitation operating mode of the pumps can be considered by the introduction into the pump performance of the coefficient which characterizes the depth of cavitation:

$$H_н = H'_н \cdot \varepsilon_н, \quad \eta_н = \eta'_н \cdot \varepsilon_н,$$

where $H_н$ - the pump head in the presence of cavitation;

$H'_н$ - the pressure created by the pump in the absence of cavitation in m, $H'_н = p_н - p_{вх}/\gamma = p/\gamma$

Function $\varepsilon_н$ is dimensionless and can take any values within limits of 0 to 1.

With the full separation $\varepsilon_н = 0$, and in the full absence of the cavitation $\varepsilon_н = 1$.

Function $\varepsilon_н$ depends on the pressure at the inlet into the pump $p_{вх}$, flow rate of the components, pressure of elasticity of vapors of the component at the given temperature $p_{упр}$ and the number of revolutions of the pump.

The static characteristic of the cavitation function $\varepsilon_н$ is obtained from the experimental cavitation characteristic of the pump.

The cavitation characteristics are the dependence of pressure $H_н$, flow rate and $\eta_н$ of the pump on pressure at the inlet $p_{вх}$ with n=const (Fig. 6.6).

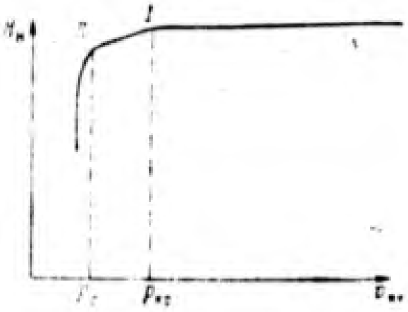


Figure 6.6. Cavitation characteristic of the pump.

According to the cavitation characteristic two characteristic points are determined. The salient point I characterizes the appearance of partial cavitation during a decompression at the inlet into the pump up to a definite value. This value of pressure at the inlet into the pump with which the pump begins to cavitize will be called critical. Since the beginning of cavitation is not always sufficiently distinctly outlined according to the characteristic, there is usually established the conditional boundary of the beginning of cavitation by which a drop in pressure of 2-3% is understood.

With a lowering in pressure at the inlet into pump down to a definite value, which corresponds to the stalling of the operation of the pump p_c, on the cavitation characteristic of the pump the salient point II appears; in this case the pressure at the outlet begins to fall very sharply.

The depth of the cavitation depends on the tangent of the slope angle of the descending branch of the cavitation characteristic (i.e., the straight line which connects the salient points I and II).

Let us construct the pump characteristic in coordinates which determine the appearance of cavitation $H_н/n^2$, G/n, $p_{кр}-p_{вх}$ (Fig. 6.7).

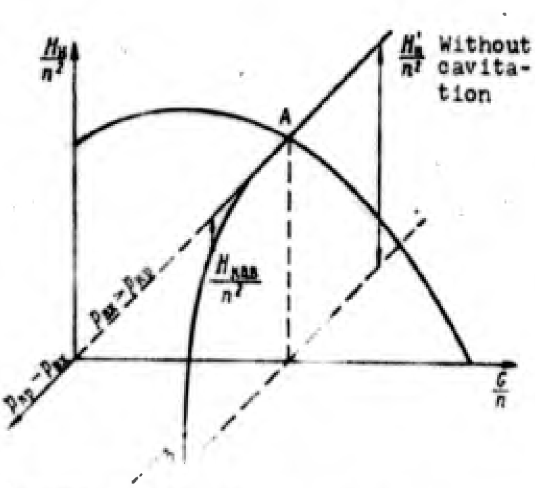


Figure 6.7. Pump characteristics in coordinates which determine the appearance of cavitation, $H_н/n^2$; G/n; $p_{кр}-p_{вх}$.

The cavitation characteristic will be located in parallel to the axis $p_{кр}-p_{вх}$. To the right of point A the zone without cavitation will be located. Point A corresponds to a lowering of the pressure at the inlet into the pump up to the critical value when $p_{вх}=p_{кр}$

The pressure created by the pump with cavitation $H_н$ will be less than the pressure created by the normally operating pump in the absence of cavitation by the magnitude of losses of pressure with cavitation:

$$H_н = H'_н - H_{кав} \qquad (6.29)$$

The locus of the points which correspond to losses of the head to cavitation with a decrease in the pressure at the inlet into the pump is a parabola of the form

$$H_{кав} = ax^2. \qquad (6.29a)$$

Let us determine coefficient according to the coordinates of point c, which corresponds to the complete stalling of the operation of the pump:

$$a = \frac{H'_н}{(p_{кр} - p_c)^2}.$$

Then expression (6.29a) will take the form

$$H_{кав} = \frac{H'_н}{(p_{кр} - p_c)^2}(p_{кр} - p_{вх})^2.$$

Let us substitute the expression for losses of the head to cavitation into equation (6.29) and obtain

$$H_н = H'_н - \frac{H'_н}{(p_{кр} - p_c)^2}(p_{кр} - p_{вх})^2 = H'_н\left[1 - \frac{(p_{кр} - p_{вх})^2}{(p_{кр} - p_c)^2}\right].$$

Hence we obtain the expression for determining the static depth of cavitation $\varepsilon_{н.ст}$:

$$\varepsilon_{к.ст} = 1 - \frac{(p_{кр} - p_{вх})^2}{(p_{кр} - p_c)^2}.$$

With complete stalling $p_{вх}=p_c$ and $\varepsilon_н=0$. With $p_{вх}=p_{кр}$ the cavitation is absent: $\varepsilon_н=1$.

If one assumes that a drop in pressure and efficiency of the pumps follow dips in pressures not instantly but with a delay, then the dynamics of the propagation of cavitation phenomena can be described in the following way:

$$\frac{d\varepsilon_{\kappa}}{dt}=\frac{\varepsilon_{\kappa.ст}-\varepsilon_{\kappa}}{\tau_{н}},$$

where τ_{H} is retention time of fluid in the pump:

$$\tau_{н}=\frac{G_{н}\sqrt{\varepsilon_{\kappa}}}{G_{н}}.$$

The weight volume of the fluid in the pump $G_{H}=V_{H}\gamma$, where V_{H} is the volume of the pump.

Factor $\sqrt{\varepsilon_{\kappa}}$ considers that the presence of bubbles decreases the volume of the pump.

§ 5. EQUATION OF DYNAMICS OF THE TURBOPUMP UNIT

It is accepted to call the equation of dynamics of the turbopump unit the equation of motion of its rotor, which is formed on the basis of d'Alembert's principle:

$$J\frac{d\omega}{dt}=M_{т}-\sum_{i=1}M_{нi},$$

where M_{T} - the torque of the turbine;

$M_{H\,i}$ - the moment of the ith pump required for its rotation;

J - the moment of inertia of the rotor of the TNA, which is added from the moments of inertia of the turbine rotor J_{T}, the rotor wheels of the pump J_{H}, the fluid which is located in the wheels $J_{Ж}$, and the fluid column J_{CT}:

$$J=J_{т}+\sum(J_{н}+J_{ж})+J_{ст};$$

ω - the angular rate of rotation of the rotor:

$$\omega=\frac{2\pi n}{60}.$$

Taking into account that M=N/ω, where N is the power, the equation of the TNA can be written thus:

$$A_1 n \frac{dn}{dt} = N_т - \sum_i N_{нi},$$

where $A_1 = J \frac{\pi^2}{900}$.

§ 6. EQUATIONS OF POWER OF THE TURBINE AND PUMPS

The power of the turbine N_T is equal to the product of the flow rate of gas through the turbine G_T by the efficiency of the turbine η_T and by the adiabatic work of the expansion of gas in the nozzles of the turbine L_T:

$$N_т = G_т L_т \eta_т, \quad L_т = \frac{c_{ад}^2}{2g}.$$

The adiabatic exhaust gas velocity from nozzles of the turbine depends on the expansion ratio of the gas $\varepsilon = p_2/p_T$, where p_2 is the gas pressure behind the turbine; p_T is the gas pressure in front of the turbine.

With subcritical outflow when $\varepsilon > \varepsilon_{кр} = \left(\frac{2}{\varkappa+1}\right)^{\frac{\varkappa}{\varkappa-1}}$:

$$c_{ад} = \sqrt{2g \frac{\varkappa'}{\varkappa'-1} R_т T_т \left[1 - (\varepsilon)^{\frac{\varkappa'-1}{\varkappa'}}\right]}.$$

The critical exhaust gas velocity from nozzles of the turbine is determined from the equation

$$c_{кр} = \sqrt{2g \frac{\varkappa'}{\varkappa'+1} R_т T_т}\ ,$$

where $R_T T_T$ is the efficiency of the gas at the inlet into the turbine.

If we do not consider the retention time of the gas in the gas generator, then

$$R_T T_T = R'T'.$$

The adiabatic velocity is connected with the critical by the relation $c_{ад} = \lambda c_{кр}$, where the reduced velocity λ is determined by the expansion ratio ε and is connected with the Mach number by expression

$$\lambda = \sqrt{\frac{\varkappa+1}{2}} \cdot \frac{M}{\sqrt{1+\frac{\varkappa-1}{2}M^2}}.$$

The gas flow rate through the turbine also depends on the expansion ratio of the gas in the nozzle of the turbine.

When $\varepsilon > \varepsilon_{кр}$ the gas flow rate through the turbine is determined from the equation

$$G_T = \mu F_T \sqrt{2g\frac{\varkappa'}{\varkappa'-1}\frac{p_T^2}{R_T T_T}\left[(\varepsilon)^{\frac{2}{\varkappa'}} - (\varepsilon)^{\frac{\varkappa'+1}{\varkappa'}}\right]},$$

where F_T - the total area of the nozzles of the turbine;

μ - the coefficient of flow rate.

With the critical outflow

$$G_T = \mu F_T \sqrt{\varkappa' g\left(\frac{2}{\varkappa'+1}\right)^{\frac{\varkappa'+1}{\varkappa'-1}} \frac{p_T^2}{R_T T_T}}.$$

The overall efficiency of the turbine is a function of ratio $n/c_{ад}$ (Fig. 6.8). When necessary this dependence can be approximated by a polynomial of the third power:

$$\eta_T = a_T\frac{n}{c_{ад}} + b_T\left(\frac{n}{c_{ад}}\right)^2 + c_T\left(\frac{n}{c_{ад}}\right)^3.$$

The power expended for rotation of the pump depends on the pressure created by the pump, flow rate and efficiency

$$N_{\tau}=\frac{pQ}{\eta_{\text{н}\tau}}.$$

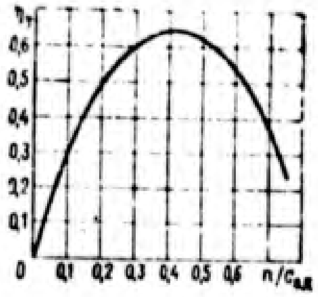


Figure 6.8. Dependence of the overall efficiency of the turbine on relation $n/c_{\text{ад}}$.

§ 7. EQUATIONS OF DYNAMICS OF ASSEMBLIES OF AUTOMATION OF THE ZhRD

The efficiency of the ZhRD is determined by the correct function of its separate systems, subassemblies and assemblies, including the assemblies of automation.

The internal relationships between parameters of different elements of the ZhRD define its properties as the object of automatic control. Automatic devices impose on elements of the engine installation new relationships necessary for providing the required operating modes of the ZhRD.

The engine installation in the process of the operation undergoes the effect of various kinds of external and internal perturbation factors, which lead to a disturbance of modes of operation, and with the unsatisfactory final development of the engine - to failures [16].

To maintain the output parameters of the engine installation constant or changing according to the assigned law, independently of the perturbation factors, the control of the engine installation is accomplished. Contemporary ZhRD are equipped with diverse (according to the form of the fulfilled operations) automation assemblies.

There has been widespread use of assemblies of automation which operate by the "closed-opened" scheme. They affect the engine and change its mode upon a signal from without or from the object itself. As an example it is possible to cite the work of different valves, cutoff pyrotechnic valves and air-operated valves (see Fig. 6.12), and bypass valves of the gas past the turbine [77].

Fuel valves are intended for a change in the propellant component flow: with their aid in full or in part the flow area of the corresponding main lines is opened or closed.

The design of the valves must eliminate the possibility of the incidence of fuel and oxidizer into the combustion chamber and gas generator of the ZhRD prior to the beginning of the starting and after the cutoff of the engine, and also provide for the possibility of the transition from one mode to the other.

The use for the engine control of pyro-, electro- or pneumatic automatics is determined, first of all, by the purpose of the engine and the type of flight vehicle onto which this engine, and it depends on the requirement for a one-time or repeated action of the valves. The use of pyrotechnical automatics on the engine provides high airtightness and allows prolonged storage of the flight vehicle in the fueled state, but it does not make it possible to control the operation of the assemblies after their installation on the engine. Furthermore, the use of pyrotechnical automatics conditions the operating speed, the simplicity of design of the valve itself, the simplicity of the design of the engine which uses this automation, and the minimum time spread of the function of the valve.

The design of the ZhRD with the use of pneumatic automatics is more flexible in the final development, since it makes it possible to change the time characteristics of the processes of the feed of components into the combustion chamber, providing a smoother or more sharp growth in pressure, and makes it possible after the elimination of the defect to produce restart without the replacement of the assemblies.

The bypass valve of the gas is normally a closed valve. With the starting and under the conditions of the main stage it is closed. With the conversion of the engine into the mode of the final stage, the valve is opened on instruction from the rocket control system. The flow rate of the gases through the turbine decreases, and the thrust of the engine is decreased.

The operation of such automatic devices occurs without an opposite effect of the object on the automatic device after its operation.

Automatic devices called regulators have had widespread use in ZhRD, and these maintain with a definite accuracy the assigned mode of its operation.

The regulator measures the deviation of the controllable parameter from the rated value and acts on the controlled object until its deviation is eliminated (Fig. 6.9). Consequently, the control system is a closed system with negative feedback.

The ZhRD as a controlled object can have several controllable parameters for the execution of assigned task with maximum efficiency.

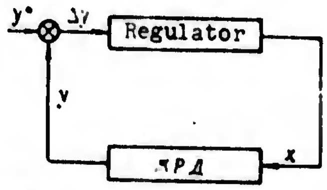
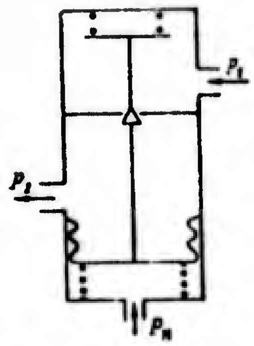


Figure 6.9. Block diagram of the control system.

The basic parameters of the engine installation are the thrust and specific thrust, the magnitudes of which with the assigned geometries of the engine chamber and pressure in the nozzle section, are determined only by the pressure in the chamber and the relationship of the propellant components:

$$P=P(p_{\kappa},\ k);\quad P_1=P_1(p_{\kappa},\ k).$$

Consequently, the combustion-chamber pressure and the relationship of the propellant components are the basic controllable



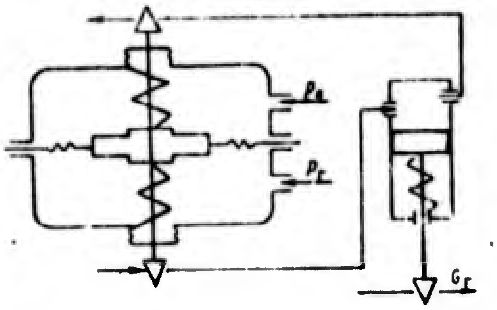


Figure 6.10 Figure 6.11

Figure 6.10. Schematic diagram of a pressure regulator.

Figure 6.11. Schematic diagram of a regulator of the ratio of propellant components.

parameters of the engine installation, and for maintaining these parameters constant pressure regulators (Fig. 6.10) and regulators of the ratio of propellant components (stabilizers) are used (Fig. 6.11).

Besides these basic controllable parameters, in engine installation there can be others. The selection of these parameters is dictated by the need of providing for heat resistance of the turbine (maintaining of the constancy of the relationship of propellant components in the gas generator) or of the minimum guaranteed fuel residues with the cutoff of the engine installation (provision for the minimum error in levels in the fuel tanks).

Widespread use has been obtained also of automatic devices which disconnect the engine installation with the appearance of a danger of its breaking down: systems of emergency engine cutoff and systems of emergency protection system of the engine. For example, on the central ZhRD F-1 of the first stage of the space vehicle Saturn-5 there is the system of the cutoff of oxygen into the engine with the appearance of excessive vibration [18, 79, 99].

The equation of motion of the controlling element of the automatic device is called the differential equation which determines the interconnection between the movement of the controlling element with time x(t) and the parameters of the hydraulic line and appropriate automatic device which ensures this movement. This equation is simply called the equation of the appropriate automatic device.

The equation of motion of any mechanical system is written on the basis of d'Alembert's principle:

$$m\frac{d^2x}{dt^2}=\sum_i P_i, \qquad (6.30)$$

where m - the reduced mass of the moving parts (taking into account the mass of elastic elements and the hydrodynamic drag of the medium);

$\sum_i P_i$ - the sum of the forces which act on the mobile system;

x - movement of the controlling element.

The sum of the forces which act on the system in equation (6.30) is determined by the design of the appropriate controlling device. The equation of motion of the controlling element and pressure regulator (see Fig. 6.10) is written in the following way:

$$m\frac{d^2x}{dt^2}+f_т\frac{dx}{dt}=p_н F_{c.эф}+P_{c0}-cx-p_2(F_{c.эф}-F_{шт})-p_1 F_x,$$

where $p_н$ - the pressure of the adjustment of the regulator;

p_1 and p_2 - inlet and outlet pressures of the hydraulic line, respectively;

$F_{c.эф}$ - active area of the bellows;

$F_{шт}$ - area of the rod of the controlling element;

F_x - current area of the needle of the controlling element;

P_{c0} - total initial compression of the spring and bellows at x=0;

c - total rigidity of the spring and bellows.

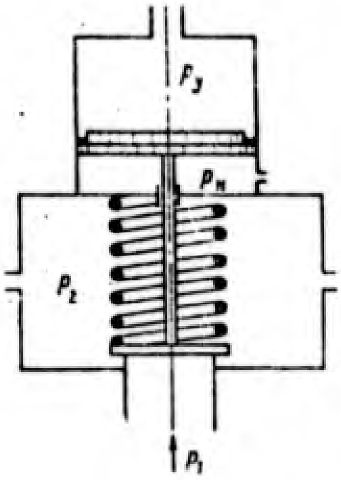


Figure 6.12. Schematic diagram of an air operated valve.

For the air-operated valve (Fig. 6.12) the equation of motion of the valve disk is written in the form of

$$m\frac{d^2x}{dt^2} = p_1F_1 - p_2(F_1 - F_3) - (p_y - p_{\text{н}})F_y - P_{c0} - cx - f_{\text{т}}\frac{dx}{dt},$$

where p_1 and p_2 - the pressures of the components at inlet and outlet from the valve;

F_1 - area of the valve seat;

F_3 - area of the piston plunger;

p_y - pressure of controlling gas on the piston;

$p_{\text{н}}$ - ambient pressure

F_y - area of the piston;

P_{c0} - spring force with a closed valve (initial compression of the spring);

c - spring rigidity.

§ 8. MODELS OF FAILURES OF ZhRD WITH THE APPEARANCE OF MALFUNCTIONS

The appearance in the engine installation of malfunctions of any form causes a transient process [9, 94].

The method of the calculation of transient processes with the appearance of malfunctions (during emergency situations) must be distinguished by the peculiar flexibility, since it must make it possible to model both the normal and emergency operation of the engine and investigate the engine operation with malfunctions of different forms, which can appear in different places of the hydraulic lines of the ZhRD of different designs.

Each malfunction introduces its own features into the course of the transient process and requires a definite account in the general model of the engine. In connection with this, the method of the calculation of emergency situations must include the general system of nonlinear differential equations, which describe the dynamics of the engine, and for an examination of the emergency situations it is necessary to consider the specific nature of the physical processes in each concrete situation, which in a mathematical description can lead to definite changes in this system of equations:

- to changes in separate coefficients of the equations which describe the dynamics of separate engine components;

- to the modification of some equations which describe the dynamics of separate elements;

- to the introduction of new additional equations.

We will form the models of failures of the ZhRD on the basis of the defined concepts about the physical picture and quantitative changes which occur in each separate engine component with the appearance of a malfunction.

The full or partial overlap of the hydraulic line is modeled by an increase in the coefficient of hydraulic friction of the defined section of the main line, which corresponds to an increase in the pressure differential on this section as a result of a decrease in its flow area. With a gradual overlap of the main line the law of the increase with time of the coefficient of hydraulic friction of the defined section of the main line is assigned.

With the emergence of the unsealing of gas lines as a result of the destruction or burnout through the formed opening, the gas begins to escape into the ambient medium. Modeling of the failure of this type provides for the modification of the equation of the gas line, which is expressed in the appearance of a new term which reflects the gas escape. The ambient pressure during bench and flight tests p≤1 kgf/cm^2, and therefore the supercritical outflow of gas from the opening formed in the tank will always be realized:

$$G_y = \frac{\mu F_{отв} p_x}{\sqrt{RT}} \left(\frac{2}{\varkappa+1}\right)^{\frac{\varkappa+1}{2(\varkappa-1)}} \sqrt{\varkappa g}, \qquad (6.31)$$

where $F_{отв}$ - the area of the opening which was formed;

μ - coefficient of the flow rate of gas with the outflow from the opening.

In connection with this, with the gas escape the equation of the gas generator, for example, is written in the form of

$$\frac{V'_x}{R'T}\frac{dp'_x}{dt} - \frac{V'_x p'_x}{(R'T)^2}\frac{d(R'T)}{dt} = g_o(t-\tau_o) + g_r(t-\tau_r) -$$
$$- G_r - \frac{\mu F_{отв} p'_x}{\sqrt{R'T}} \left(\frac{2}{\varkappa+1}\right)^{\frac{\varkappa'+1}{2(\varkappa'-1)}} \sqrt{\varkappa' g}. \qquad (6.32)$$

The equations of other gas lines with the emergence of leakage are similarly written.

With the emergencies caused by a change in the quality of carburetion, the real specific impulse of the combustion-- chamber pressure is decreased as a result of a decrease in $\phi_к=\beta/\beta^*$ - the coefficient of the chamber, where β* is the theoretical specific impulse of pressure. In this case it is necessary to recalculate the dependence $\beta = f(k)$ or $RT = f(k)$, taking into account the understating of the value of the coefficient of the chamber.

It is possible to carry out the theoretical studies connected with the cavitation stalling of the operation of the pump in such a case when in the general system of equations of dynamics of the engine the feed lines from the tank to the inlet into the pump and also the dynamics of the cavitation phenomena are taken into account. The approximate account of cavitation in this case gives only a qualitative picture. In order to obtain a quantitative agreement with the experiment, it is necessary to consider the dimensions of the cavitation pockets and the dependence of the volumes of the cavitation pockets on the number of cavitation (pressure at the inlet into the pump).

If cavitation stalling is preceded by any other malfunction, the overlap of the feed line or leakage and, as a result, the lowering of pressure at the inlet into the pump and the pressure created by the pump, then in the system of equations of dynamics of the engine these initial causes must be reflected: it is necessary either to increase the value of the coefficient of hydraulic friction of the feed line or introduce an additional equation of the hydraulic line of leakage, which appeared as a result of the destruction of the main line.

Breakages of bearings of the pumps and the rubbing of the floating rings lead to an increase in the moment of resistance of the pump, which can be obtained because of a reduction in the efficiency of the pump. This form of malfunctions will be simulated by the lowering of the efficiency of the pumps, assigning $\eta_H = 0.8\ \eta_H^*$ (where η_H^* is the nominal value of efficiency of this pump).

Malfunctions connected with defects of the pump itself - breakages in the impellers and worm conveyors, lead to a change in the geometry of the flow part and, consequently, to a change in the head characteristic of the pump.

Malfunctions of the turbine (fusing of the turbine blades, burnout of the rotor of the turbine) produce a change in the geometric dimensions of channels of the nozzles, change the conditions of the gas-dynamic flow in the nozzles of the turbine, which leads to a change in the exhaust gas velocity from the nozzles of the turbine and as a final result causes a lowering of the efficiency of the turbine and decrease in the available power of the turbine. With the mathematical description of a similar type of malfunctions, it is possible to assign a change in the total area of the nozzles of the turbine (μF_T) and a decrease in the efficiency of the turbine.

The destruction of the shroud of the turbine rotor causes an abrupt deceleration of the rotor of the TNA due to an increase in the moment of friction on the shaft of the turbine and reduction of η_T due to the appearance of additional tip losses and an increase in losses to the overflowing through the increased (due to separation of the shroud) clearance. This form of malfunctions can be considered by the lowering of the efficiency of the turbine or by the introduction of an additional term into the equation of the TNA:

$$A_1 n \frac{dn}{dt} = N_T - N_0 - N_r - N_{tp}. \qquad (6.33)$$

With breakage in the spring the operation of the TNA can be described by two equations:

$$A_1 n_1 \frac{dn_1}{dt} = N_T - N_0,$$

$$A_2 n_2 \frac{dn_2}{dt} = -N_r.$$

At the initial moment $n_1=n_2$, which corresponds to the value of the number of revolutions of the TNA shaft up to the emergence of a malfunction.

With the breaking of the hydraulic line or with the appearance of leakage at any place appears new flow rate (leakage), which is a time varying value and determined by the dimension of the opening through which there occurs leakage of the component and by the pressure differential which ensures the outflow. With the breaking of the hydraulic line the outflow occurs into the ambient medium with pressure p_a, which is in general the value of the variable dependent on the lift of the article $p_a = f(H)$.

The position of the breaking will be considered as a new assembly in the hydraulic system of the engine.

To determine the value of the flow escaping from the main line $G_y = f(t)$, to equations of the main lines of a normally operating engine it is necessary to add one additional equation, which reflects the seeming appearance of a new hydraulic line (tank is the position of the break).

In the formulation of this equation, besides the hydraulic friction of the circuit of the engine to the position of breaking, it is necessary to consider the fluid resistance with outflow from the opening. The coefficient of hydraulic friction in this case will be defined as

$$a_y = \frac{1}{(\mu F_{отв})^2 2g\gamma}, \qquad (6.34)$$

where $F_{отв}$ - the area of the opening which was formed in the breaking;

μ - the flow coefficient with the outflow of fluid from the opening.

In the equation of the hydraulic line in which the breaking occurred, the losses on the section from the tank up to the place of breaking must be determined by taking into account the flow escaping from the main line, since up to the place of the breaking there occurs the total flow $G = G_y + G_1$, which includes the flow escaping from the main line and the flow through the main line of the engine after the place of the break. Since the hydraulic lines of the engine can be a multi-assembly branched system of pipelines, the flow escaping from the main line must be considered also in the equations of those main lines of the engine which have a common section of the pipelines with the main line in which there occurred the break, which corresponds to added losses of pressure on these sections, and also in equations of power and pressure created by the pump.

CHAPTER VII

STUDY OF EMERGENCY SITUATIONS OF LIQUID-PROPELLANT ROCKET ENGINES

§ 1. EXPLANATION OF CAUSES FOR THE EMERGENCY OUTCOMES OF TESTS OF LIQUID-PROPELLANT ROCKET ENGINES

We will understand as an emergency situation of the ZhRD [ЖРД - liquid-propellant rocket engines] the appearance of any malfunction which causes a transient process and the establishing of a new mode distinct from the preceding one.

An experiment of the final development and operation of the ZhRD shows that the establishment of reasons for emergency outcomes of tests in each concrete case requires the conducting of special investigations of results of measurements, the material part etc., and sometimes also special experiments whose purpose is the reproduction of the nature of the emergency for determining the causes which induced it [6, 9, 12, 94].

The establishment of the reason for the emergency outcome of the flight test of the ZhRD is a more complex problem than the bench test in connection with the less volume of information, the difficulties in the detection of the material part of the engine, and also due to deformations and destructions with the impact of the rocket.

For the establishment of the reason for the emergency outcome of testing, at present a whole number of the works is being carried out, and these include an analysis of results of measurements, an inspection and flaw detection check of the material part, a check of the industrial processes of the manufacture of assemblies and subassemblies, experimental works, an analysis of bench and flight tests which precede that given, calculated works, and a simulation of the malfunctions assumed to be on the test stand.

In an explanation of the reason for the emergency outcome, data of the results of measurements are the basic source of information about the engine which underwent tests. On the basis of an analysis of results of these measurements, assumptions about the reason for the emergency are made [6, 9, 18, 19].

The presence of the material part facilitates the search. As a result of an inspection of the material part some assumptions about the causes of the emergency made on the basis of an analysis of the measurements can be immediately rejected.

An inspection of the material part of the engines of stage I of the rockets makes it possible to draw the conclusion about the presence of high-frequency pulsations in the combustion chamber, burnouts of the fire wall of the combustion chamber, gas generator and other engine components, the fusion and erosion of metal at the intake pipes of the pumps, the incorrect expansion of the diaphragms, the destruction of separate assemblies of the engine which occurred prior to the impact of the article.

If at the end of the engine operation of stage II failure occurred, then almost eliminated is the possibility of the establishment of the reason for the state of the material part and even the detection of the residues of the material part is extremely difficult [19, 79].

Thus in flight tests the data of telemetering measurements are the starting point in the establishment of the reason for the emergency outcome.

In the course of the explanation of the reason for the emergency outcome of the flight test, there are carried out experimental works which include the flaw detection of the assemblies delivered from the point of impact of the rocket, investigations in laboratories and autonomous tests of subassemblies and assemblies, hydro- and pneumatic tests before the destruction of the unit assemblies of the engine of the same period of manufacture as the unit assembly of the engine which underwent flight tests, the possibility of the destruction of separate subassemblies with the static feed of the pressure and with the simulation of the dynamics of filling of the pipelines is determined, and the strength of the bellows, pumps, quality of the subassemblies of seals and coatings is checked [84].

In the explanation of the reason for abnormal flight test, one analyzes the data of static firing tests, which makes it possible sometimes to establish the reasons for the emergency outcome in flight. For this we select and analyze those bench tests in which the same pattern of the course of the processes as during the flight test was recorded [6, 9, 94].

An important link in the explanation of the reason for the emergency outcome of the test is the calculations whose role increases in the analysis of reasons for the emergency operation of the engine during flight tests. They assume to be the linking of the basic parameters of the ZhRD for the purpose of the confirmation of the correctness of results of measurements and the analytical studies of the transient processes and parameters of the steady-state engine operating mode under the assumption of a defined malfunction.

The systematically correctly conducted analytical investigation facilitates the operation on the explanation of the reason for the emergency outcome of the testing. It makes it possible to narrow the sphere of the search for the possible reasons for emergencies, reject the invalid versions and retain the most probable, and sometimes (with a sufficient quantity of measurements) unambiguously establish the reason for the emergency outcome [7, 36, 39].

If the nature of the transient process and parameters of the new steady-state mode, recorded with emergency testing, correspond to the parameters and nature of the transient process, obtained by calculation under the assumption of a definite malfunction, the conclusion that this malfunction with a full-scale test was the reason for its emergency outcome is made.

In certain cases during bench tests on the state of material part and results of the measurements, it is also impossible to reveal the reason for the emergency outcome. Then it is necessary to conduct analytical investigations, which make it possible to make a more detailed analysis of the test results. Specifically, the conducting of similar calculated investigations can prove to be useful in the explanation of the initial cause which caused the secondary defects of the material part, which, in turn, led to the emergency outcome of the testing.

If it is necessary to simulate the emergency situation, the results of the calculated works make it possible to conduct a special adjustment of the experimental engine.

The reproduction under the bench-test conditions of the nature of the transient process, which arose in the engine operation during flight tests (simulation), is the concluding stage of the establishment of the reason for the emergency outcome of the testing. If the nature of a change in the system of the experimental engine with the simulation of the defined malfunction and the parameters of its steady-state mode correspond to the nature and parameters recorded during the flight test, then on the basis of the conducted experimental works the conclusion that this malfunction during the flight test was the reason for its emergency outcome is made [6, 9, 94].

Simulation under the bench conditions of different types of malfunctions requires test conducting according to specific programs and sometimes serious modifications of the experimental engine, which would make it possible to reproduce a malfunction.

The leakage of the main line can be simulated by installation on the pipeline, where leakage is assumed, of additional pyrotechnic valves. By the operation of these pyrotechnic valves for a specified second of engine operation leakage of the main line is created. On the test stand an overflow line for the removal of the component is connected to these pipelines. For the measurement of the flow which escapes the main line through the pyrotechnic valves in discharge line, flow meters are installed.

The shutting off of the main line can be simulated by the installation of a throttle or a supplementary discharging nozzle into the main line in which the malfunction is assumed. The simulation of complete shutoff can be insured with the operation of the pyrotechnic valve of the cutoff of the component.

The reproduction of malfunctions of pumps, assemblies of automation and turbines is more complex. It requires design modifications of these assemblies and their fulfillment with the necessary defects for the present case.

In certain cases the simulation under bench-test conditions is the only method of the establishment of the reason for the emergency outcome of the flight test of the ZhRD.

In conclusion let us give several examples from practice with respect to the malfunctions of the engine installation, an explanation of their reasons and the methods of elimination. Thus according to results of the measurements it was established that during aircraft design tests of engine installations of the lunar expeditionary module (LEM-3) of the space vehicle Apollo-9, from the very beginning of the first starting of the engine installation of the landing stage there is no fuel-tank pressurization as a result of the clogging of the helium main line (Fig. 7.1). This led to the disturbance of the feed system (pressure in system was lowered). The clogging of the helium line was eliminated only after 5 seconds of operation under conditions of full thrust, whereupon the pressures in the engine installation were raised to their normal levels [9, 84, 94].

The temperature sensors of the helium system showed that at approximately the 35th second the temperature at the inlet into the regulator and also the temperature at the inlet and exit of the internal (helium-helium) heat exchanger were sharply lowered. This sudden inflow of cold helium indicated the elimination of the clogging of the helium line. Then the temperature returned to the normal level.

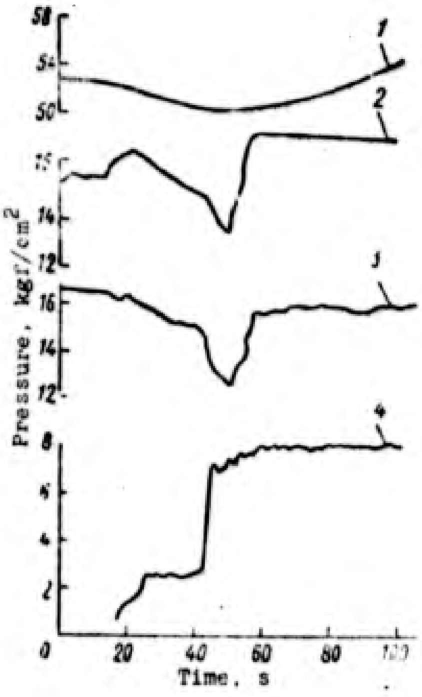


Figure 7.1. Pressures in the engine installation of the landing stage of LEM-3 (beginning of operation): 1 - helium in small tank; 2 - helium at output from unit of regulators; 3 - fuel at engine inlet; 4 - combustion chamber.

Subsequent research of the causes of the malfunction of an engine plant, conducted at the Kennedy Space Center, in particular, the testing of the technology of the servicing of the small tank with helium, showed that the pressure in the helium line can be lowered in certain cases down to zero, and this can lead to inleakage into the main line of air from the ambient medium.

Reproduction of a malfunction in the experimental model of the lunar expeditionary module during bench tests showed that the presence of vacuum in the helium line for approximately 20 minutes is sufficient in order to freeze and clog the heat exchanger. Figure 7.2.gives the comparison of flight test data of the LEM-3 and bench tests of the experimental model [9, 84].

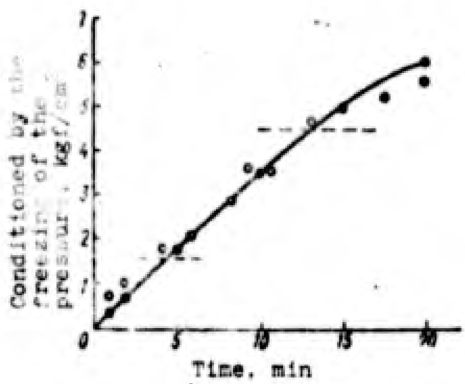


Figure 7.2. Results of simulation of the freezing of the heat exchange with the inleakage of air: o - LEM-3; ● - the experimental model of LEM; --- - modes of the experiment.

During the second starting of the engine installation of the landing stage of the lunar expeditionary module the crew of the space vehicle Apollo-9 noted unstable engine operation at the level of 27% of thrust, which lasted for several seconds. During the explanation of the reason for the malfunction, the simulation in the bench-test conditions of the nature of the operation during flight tests was carried out. Bench tests of the experimental model were carried out with the intentional feed of helium into the engine. A comparison of the transient processes which occurred in the flight of the space vehicle Apollo-9 and bench tests with the helium feed confirms the correctness of the established reason of the malfunction (Fig. 7.3). A growth in the pressure in the main lines was caused by the operation of the stabilizers of flow in the cavitation mode. As a result of the large liberated volume in the fuel tanks (60%) and also the transverse and angular accelerations of the space vehicle, the helium, apparently, entered into the intake devices and then the engine. However, bench tests showed that the incidence of the helium into the engine does not cause any additional breakdowns [9, 84, 94].

At present works devoted to the study of the emergency states of the separate systems of the rocket and to the development of systems of the automatic recognition of forms of emergencies during bench and flight tests were conducted [81, 97, 98].

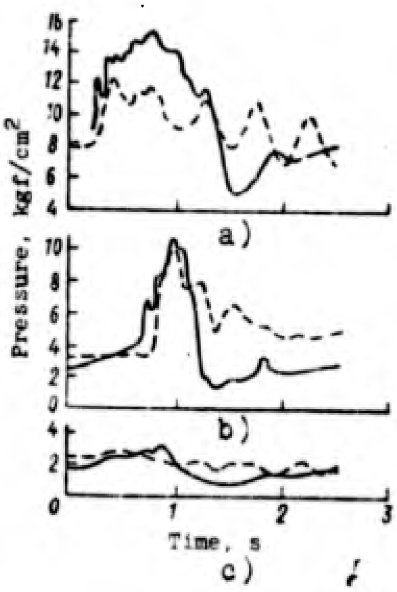


Figure 7.3. Instability of engine operation of the landing stage: a) pressure of oxidizer at inlet into injector head; b) pressure of fuel at inlet into injector head; c) pressure in the chamber; —— - LEM-3; --- - ground tests with helium feed.

In reports [81, 97] it is indicated that one of the basic problems of rocket engineering connected with the further development of space flights is the creation of systems of automatic recognition and warding off of different forms of emergencies in flight. Several experimental complexes, which consist of mathematical models of separate systems and are located in the stage of experimental final development are developed.

The system "Mark 7A" acting at present is intended for the forecasting and modeling of different flight states and characteristics of the engine installation of I stage of booster Saturn-5. The system also makes it possible to simulate failures.

In essence the mathematical model "Mark 7A" describes the steady-state mode of operation of the engine installation with the normalized values for the transient processes. Utilized in the model are influence coefficients (engine characteristics in partial derivatives). With the aid of these the engine characteristics in off-design conditions are calculated. Included in the model are the corresponding models of all the systems which affect the intake parameters. According to the program the thrust, the per-second propellant component flow rate, pressure, levels of components in the tanks, the time of cutoff of the engine, and so on are calculated [7, 98].

The use of systems of automatic recognition of forms of emergencies is promising, since in the conducting of flight tests this system will make it possible to estimate rapidly the nature of the emergency and make a decision on the advisability of continuation or stopping of the flight, and also to get rid of the need for the search and transportation of remainders of material part of the emergency engine and reproduction in bench-test conditions of the nature of emergency.

§ 2. METHOD OF CALCULATION OF PARAMETERS OF THE STEADY-STATE MODE OF THE ZhRD IN EMERGENCY SITUATIONS

Let us conduct the modeling of the operation of the ZhRD [7, 39] on the basis of an analysis of its structure, taking into account the physical processes in the engine and also the basic forms of malfunctions and damages which can occur during the operation.

1. System of Equations Which Describes the Engine Operation in Steady-State Conditions

Let us examine the fairly complicated design of a 4-chamber engine (Fig. 7.4) such that the majority of the designs of existing ZhRD, which operate by an open circuit, would represent special cases of that examined.

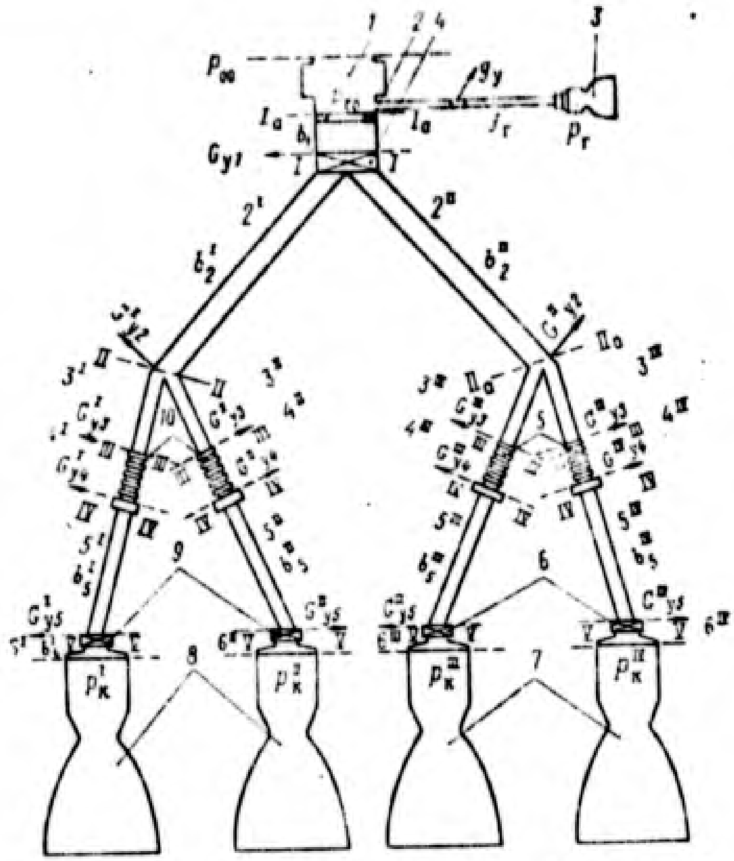


Figure 7.4. Design diagram of hydraulic lines of the oxidizer: 1 - pump; 2 - flow disk; 3 - gas generator; 4 - valve; 5 - bellows; 6, 9 - cut-off valves; 7, 8 - combustion chamber.

This eliminates the need for the solution and creation of programs for each specific design with a less number of chambers, since in such case they are obtained by simplification of the main program.

Let us write the system of nonlinear equations which describe the engine operation in steady-state conditions. Let us provide for the possibility of the appearance of leakage on the main lines.

The equations of statics are easily obtained from the equations of the dynamics of main assemblies of the ZhRD if we equate the time derivatives in them to zero (see Chapter VI).

For example, the head of the pumps, on the basis of equation (6.25a), can be written in the form

$$p_0 = d_1 n^2 - d_2 n G_0 - d_3 G_0^2.$$

The power equation after elementary transformations will take the form

$$N_0 = d_4 n^3 + d_5 n^2 G_0.$$

The equation of the turbine

$$N_T = G_T (d_6 n - d_7 n^2), \quad G_T = g_0 + g_r.$$

where g_0 and g_r are flow rates of components through the gas generator.

In the steady-state condition dn/dt=0 and the TNA [THA - turbopump unit] equation will be written as follows:

$$N_T = N_0 + N_r.$$

Equation of the combustion chamber

$$p_K = l(G_0 + G_r),$$

where $l = \beta / F_{KP}$ (β - the specific impulse of the combustion-chamber pressure). For this kind of fuel and constant quality of the operating conditions, β depends on k and p_K. However, since β depends on pressure in the chamber considerably more weakly than on the fuel component ratio, then the effect of p_K is disregarded.

The equation of the gas generator is

$$p'_K = l'(g_0 + g_r), \quad l' = \frac{\beta'}{F'_{KP}}.$$

The equations of the hydraulic lines of the gas generator are

$$p_o + p_{вх.о} = p'_к + \xi_1 g_{o\Sigma}^2 + \xi_2 g_o^2, \quad p_г + p_{вх.г} = p'_к + \xi_{1г} g_{г\Sigma}^2 + \xi_{2г} g_г^2,$$

$$g_{o\Sigma} = g_o + g_y, \quad g_{г\Sigma} = g_г + g_{гy},$$

where ξ_1, $\xi_{1г}$ are coefficients of hydraulic friction of sections of the main lines of the oxidizer and fuel up to the place of the break;

ξ_2, $\xi_{2г}$ - the coefficients of hydraulic friction of sections of the main lines from the place of the break up to the gas generator;

$p_{вх.о}$ - pressure at the inlet into the oxidizer pump;

$p_{вх.г}$ - pressure at the inlet into the fuel pump.

The hydraulic line from the outlet of the pump to the combustion chambers, which is a multi-assembly branched system, is divided into sections of simple pipelines. Section 1-1 is from the outlet of the pump to the valve, with flow G_{o1}, coefficients of hydraulic friction a_1 and leakage G_{y1}. Sections 2^I and 2^{II} are from the valve up to the cross section II and IIa. The flow, coefficients of hydraulic friction and leakage on section 2^I are designated G^I_{o2}, a^I_2 and G^I_{y2}; and, respectively, on section 2^{II} - G^{II}_{o2}, a^{II}_2 and G^{II}_{y2}. The main lines of the combustion chambers are divided into four sections: 3, 4, 5, and 6. The flow, coefficients of hydraulic friction and leakage of components on these sections will be designated by the indices "I, II, III, and IV", which correspond to the numbers of the combustion chambers. For example, the flow, coefficient of hydraulic friction and leakage on section 3 of combustion chamber I will be designated G^I_{o3}, a^I_3, G^I_{y3}; of combustion chamber IV - G^{IV}_{o3}, a^{IV}_3 and G^{IV}_{y3}.

Pressures in combustion chambers I-IV will be designated

$$p^I_к, p^{II}_к, p^{III}_к, p^{IV}_к.$$

Then the equations of the steady-state motion of the propellant components along main lines in connection with the engine, the design scheme of main lines of which is represented in Fig. 7.4, are written in the following way:

$$p_o + p_{вх.о} = p_к^{I} + a_1 G_{o1}^2 + a_2^{I}(G_{o2}^{I})^2 + a_3^{I}(G_{o3}^{I})^2 + a_4^{I}(G_{o4}^{I})^2 + a_5^{I}(G_{o5}^{I})^2 + a_6^{I}(G_o^{I})^2.$$

$$p_o + p_{вх.о} = p_к^{II} + a_1 G_{o1}^2 + a_2^{I}(G_{o2}^{I})^2 + a_3^{II}(G_{o3}^{II})^2 + a_4^{II}(G_{o4}^{II})^2 + a_5^{II}(G_{o5}^{II})^2 + a_6^{II}(G_o^{II})^2.$$

$$p_o + p_{вх.о} = p_к^{III} + a_1 G_{o1}^2 + a_2^{II}(G_{o2}^{II})^2 + a_3^{III}(G_{o3}^{III})^2 + a_4^{III}(G_{o4}^{III})^2 + a_5^{III}(G_{o5}^{III})^2 + a_6^{III}(G_o^{III})^2.$$

$$p_o + p_{вх.о} = p_к^{IV} + a_1 G_{o1}^2 + a_2^{II}(G_{o2}^{II})^2 + a_3^{IV}(G_{o3}^{IV})^2 + a_4^{IV}(G_{o4}^{IV})^2 + a_5^{IV}(G_{o5}^{IV})^2 + a_6^{IV}(G_o^{IV})^2.$$

The hydraulic lines of the fuel represent the same multi-assembly branched system and are divided into the same sections as is the main oxidizer line. Equations of main lines of the fuel of the combustion chamber will be written similar to equations of the main line of the oxidizer, but we designate the coefficient of hydraulic friction of the fuel main line by $a_г$.

Besides these equations one should write still the equations of the balance of flows at the branch points:

$$G_o^{I} + G_{y5}^{I} = G_{o5}^{I}, \quad G_{o4}^{II} + G_{y3}^{II} = G_{o3}^{II}, \quad G_o^{IV} + G_5^{IV} = G_{o5}^{IV},$$

$$G_{o5}^{I} + G_{y4}^{I} = G_{o4}^{I}, \quad G_{o3}^{I} + G_{o3}^{II} + G_{y2}^{I} = G_{o2}^{I}; \quad G_{o5}^{IV} + G_{y4}^{IV} = G_{o4}^{IV},$$

$$G_{o4}^{I} + G_{y3}^{I} = G_{o3}^{I}, \quad G_o^{III} + G_{y5}^{III} = G_{o5}^{III}, \quad G_{o4}^{IV} + G_{y3}^{IV} = G_{o3}^{IV},$$

$$G_o^{II} + G_{y5}^{II} = G_{o5}^{II}, \quad G_{o5}^{III} + G_{y4}^{III} = G_{o4}^{III}, \quad G_{o3}^{III} + G_{o3}^{IV} + G_{y2}^{II} = G_{o2}^{II},$$

$$G_{o5}^{II} + G_{y4}^{II} = G_{o4}^{II}, \quad G_{o4}^{III} + G_{y3}^{III} = G_{o3}^{III}, \quad G_{o2}^{I} + G_{o2}^{II} + G_{y1} = G_{o1}.$$

Thus the system of equations describing the engine operation in steady-state conditions takes the following form:

$$p_o = d_1 n^2 - d_2 n G_o - d_3 G_o^2, \quad (7.1) \qquad p_г = d_{1г} n^2 - d_{2г} n G_г - d_{3г} G_г^2, \quad (7.2)$$

$$N_o = d_4 n^3 + d_5 n^2 G_o, \quad (7.3) \qquad N_г = d_{4г} n^3 + d_{5г} n^2 G_г, \quad (7.4)$$

$$N_т = G_т (d_6 n - d_7 n^2), \quad (7.5) \qquad N_т = N_o + N_г, \quad (7.6)$$

$$p_к^{I} = l^{I}(G_o^{I} + G_г^{I}), \quad (7.7) \qquad p_к^{II} = l^{II}(G_o^{II} + G_г^{II}), \quad (7.8)$$

$$p_к^{III} = l^{III}(G_o^{III} + G_г^{III}), \quad (7.9) \qquad p_к^{IV} = l^{IV}(G_o^{IV} + G_г^{IV}). \quad (7.10)$$

$$p_o + p_{вх.о} = p_к^I + a_1 G_{o1}^2 + a_2^I (G_{o2}^I)^2 + a_3^I (G_{o3}^I)^2 + + a_4^I (G_{o4}^I)^2 + a_{o5}^I (G_{o5}^I)^2 + a_6^I (G_o^I)^2, \quad (7.11)$$

$$p_o + p_{вх.о} = p_к^{II} + a_1 G_{o1}^2 + a_2^I (G_{o2}^I)^2 + a_3^{II} (G_{o3}^{II})^2 + + a_4^{II} (G_{o4}^{II})^2 + a_5^{II} (G_{o5}^{II})^2 + a_6^{II} (G_o^{II})^2, \quad (7.12)$$

$$p_o + p_{вх.о} = p_к^{III} + a_1 G_{o1}^2 + a_2^{II} (G_{o2}^{II})^2 + a_3^{III} (G_{o3}^{III})^2 + + a_4^{III} (G_{o4}^{III})^2 + a_5^{III} (G_{o5}^{III})^2 + a_6^{III} (G_o^{III})^2, \quad (7.13)$$

$$p_o + p_{вх.о} = p_к^{IV} + a_1 G_{o1}^2 + a_2^{II} (G_{o2}^{II})^2 + a_3^{IV} (G_{o3}^{IV})^2 + + a_4^{IV} (G_{o4}^{IV})^2 + a_5^{IV} (G_{o5}^{IV})^2 + a_6^{IV} (G_o^{IV})^2, \quad (7.14)$$

$$p_r + p_{вх.r} = p_к^I + a_{1r} G_{r1}^2 + a_{2r}^I (G_{r2}^I)^2 + a_{3r}^I (G_{r3}^I)^2 + + a_{4r}^I (G_{r4}^I)^2 + a_{5r}^I (G_{r5}^I)^2 + a_{6r}^I (G_r^I)^2, \quad (7.15)$$

$$p_r + p_{вх.r} = p_к^{II} + a_{1r} G_{r1}^2 + a_{2r}^I (G_{r2}^I)^2 + a_{3r}^{II} (G_{r3}^{II})^2 + + a_{4r}^{II} (G_{r4}^{II})^2 + a_{5r}^{II} (G_{r5}^{II})^2 + a_{6r}^{II} (G_r^{II})^2, \quad (7.16)$$

$$p_r + p_{вх.r} = p_к^{III} + a_{1r} G_{r1}^2 + a_{2r}^{II} (G_{r2}^{II})^2 + a_{3r}^{III} (G_{r3}^{III})^2 + = a_{4r}^{III} (G_{r4}^{III})^2 + a_{5r}^{III} (G_{r5}^{III})^2 + a_{6r}^{III} (G_r^{III})^2, \quad (7.17)$$

$$p_r + p_{вх.r} = p_к^{IV} + a_{1r} G_{r1}^2 + a_{2r}^{II} (G_{r2}^{II})^2 + a_{3r}^{IV} (G_{r3}^{IV})^2 + + a_{4r}^{IV} (G_{r4}^{IV})^2 + a_{5r}^{IV} (G_{r5}^{IV})^2 + a_{6r}^{IV} (G_r^{IV})^2, \quad (7.18)$$

$$p_к' = l' (g_o + g_r), \quad (7.19)$$

$$p_o + p_{вх.о} = p_к' + \xi_1 g_{o\Sigma}^2 + \xi_r g_o^2, \quad (7.20)$$

$$p_r + p_{вх.r} = p_к' + \xi_{1r} g_{r\Sigma}^2 + \xi_{2r} g_r^2. \quad (7.21)$$

$$G_{o5}^I + G_{y4}^I = G_{o4}^I, \quad (7.22) \qquad G_o^I + G_{y5}^I = G_{o5}^I, \quad (7.23)$$

$$G_o^{II} + G_{y5}^{II} = G_{o5}^{II}, \quad (7.24) \qquad G_{o4}^I + G_{y3}^I = G_{o3}^I, \quad (7.25)$$

$$G_{o4}^{II} + G_{y3}^{II} = G_{o3}^{II}, \quad (7.26) \qquad G_{o5}^{II} + G_{y4}^{II} = G_{o4}^{II}, \quad (7.27)$$

$$G_o^{III} + G_{y5}^{III} = G_{o5}^{III}, \quad (7.28) \qquad G_{o3}^I + G_{o3}^{II} + G_{y2}^I = G_{o2}^I, \quad (7.29)$$

$$G_{o4}^{III} + G_{y3}^{III} = G_{o3}^{III}, \quad (7.30) \qquad G_{o5}^{III} + G_{y4}^{III} = G_{o4}^{III}, \quad (7.31)$$

$$G_{o5}^{IV} + G_{y4}^{IV} = G_{o4}^{IV}, \quad (7.32) \qquad G_o^{IV} + G_{y5}^{IV} = G_{o5}^{IV}, \quad (7.33)$$

$$G_{o3}^{III} + G_{o3}^{IV} + G_{y2}^{II} = G_{o2}^{II}, \quad (7.34) \qquad G_{o4}^{IV} + G_{y3}^{IV} = G_{o3}^{IV}, \quad (7.35)$$

$$g_{o\Sigma} = g_o + g_y, \quad (7.36) \qquad G_{o2}^I + G_{o2}^{II} + G_{y1} = G_{o1}, \quad (7.37)$$

$$G_r^I + G_{y5r}^I = G_{r5}^I, \quad (7.38) \qquad G_{o1} + g_{o\Sigma} = G_o, \quad (7.39)$$

$$G_{r4}^I + G_{y3r}^I = G_{r3}^I, \quad (7.40) \qquad G_{r5}^I + G_{y4r}^I = G_{r4}^I, \quad (7.41)$$

$$G_{r5}^{II} + G_{y4r}^{II} = G_{r4}^{II}, \quad (7.42) \qquad G_r^{II} + G_{y5r}^{II} = G_{r5}^{II}, \quad (7.43)$$

$$G_{r3}^I + G_{r3}^{II} + G_{y2r}^I = G_{r2}^I, \quad (7.44) \qquad G_{r4}^{II} + G_{y3r}^{II} = G_{r3}^{II}, \quad (7.45)$$

$$G_{r5}^{III} + G_{y4r}^{III} = G_{r4}^{III}, \quad (7.46) \qquad G_r^{III} + G_{y5r}^{III} = G_{r5}^{III}, \quad (7.47)$$

$$G_r^{IV} + G_{y5r}^{IV} = G_{r5}^{IV}, \quad (7.48) \qquad G_{r4}^{III} + G_{y3r}^{III} = G_{r3}^{III}, \quad (7.49)$$

$$G_{r4}^{IV} + G_{y3r}^{IV} = G_{r3}^{IV}, \quad (7.50) \qquad G_{r5}^{IV} + G_{y4r}^{IV} = G_{r4}^{IV}, \quad (7.51)$$

$$G_{r2}^I + G_{r2}^{II} + G_{y1r} = G_{r1}, \quad (7.52) \qquad G_{r3}^{III} + G_{r3}^{IV} + G_{y2r}^{II} = G_{r2}^{II}, \quad (7.53)$$

$$G_{r1} + g_{r\Sigma} = G_r, \quad (7.54) \qquad g_{r\Sigma} = g_r + g_{y.r}, \quad (7.55)$$

2. Solution of the System

The given system of equations is written for a maximum quantity of the possible places of leakage.

As a rule, at a definite point in time only one malfunction can arise; and therefore the system of equations in each concrete case will be considerably simplified.

First let us give the solution for the general case, i.e., with a simultaneous account of the following forms of malfunctions: 1) the leakage of the main line; 2) the closing (clogging) of the main line; 3) the malfunction of the turbine. Then from the general solution it will be possible to obtain the quotients which correspond to one or two of the enumerated malfunctions.

The initial data for the solution are: 1) the pressure and power characteristics of the pumps; 2) the coefficients of hydraulic friction of all sections of the main lines; 3) the flows of components G_o, G_r, g_o and g_r, which correspond to the steady-state condition of the engine, which preceded the appearance of the malfunctions; 4) the values of the fuel component ratios in combustion chambers and gas generator k^{I}, k^{II}, k^{III}, k^{IV}, k'; 5) the coefficients which show in which relationship there occurs the distribution of flows of components along the main lines of the combustion chambers:

$$A=\frac{G^{I}_{o2}}{G^{II}_{o2}}, \qquad (7.56)$$

$$B=\frac{G^{I}_{r2}}{G^{II}_{2r}}. \qquad (7.57)$$

Prior to the appearance of malfunctions along main lines of the combustion chambers, the distribution of the flows is proportional, i.e., A=B=1.

Furthermore, the form of the malfunction must be assigned, namely: the flow of leakage with unsealing, the values of coefficients of hydraulic friction, which corresponds to the closing of the main lines, and so on.

According to these values which determine the malfunction, the connecting of the basic parameters of the engine by the method of iterations will be accomplished.

By solving equations (7.3), (7.4), (7.5) and (7.6) together, let us determine the values of numbers of revolutions of the TNA:

$$n = \frac{-(d_5 G_0 + d_{5r} G_r + d_7 g_0 + d_{7r} g_r)}{2(d_4 + d_{4r})} + \sqrt{\left[\frac{d_5 G_0 + d_{5r} G_r + d_7 g_0 + d_{7r} g_r}{2(d_4 + d_{4r})}\right]^2 + \frac{g_0 C_6 + g_r C_{6r}}{d_4 + d_{4r}}}. \qquad (7.58)$$

From equations (7.1) and (7.2) we find

$$p_0 = d_1 n^2 - d_2 n G_0 - d_3 G_0^2, \qquad (7.59)$$

$$p_r = d_{1r} n^2 - d_{2r} n G_r - d_{3r} G_r^2. \qquad (7.60)$$

Let us substitute the expression for $p'_к$ from (7.19) into equation (7.20), and, taking into account that $g_{0\Sigma} = g_0 + g_y$, and $g_г = g_0/k'$, we obtain

$$g_0 = -\frac{I'(1+k') + 2\xi_1 g_y k'}{2(\xi_1 + \xi_2)k'} + \sqrt{\left[\frac{I'(1-k') + 2\xi_1 g_y k'}{2k'(\xi_1 + \xi_2)}\right]^2 + \frac{p_0 + p_{вх.о} - \xi_1 g_y^2}{\xi_1 + \xi_2}}. \qquad (7.61)$$

Let us substitute (7.19) and (7.54) into equation (7.21), and, taking into account that $g_0 = k' g_г$, we obtain

$$g_r = -\frac{I'(1-k') + 2\xi_{1r} g_{y.r}}{2(\xi_{1r} + \xi_{2r})} + \sqrt{\left[\frac{I'(1+k') + 2\xi_{1r} g_{y.r}}{2(\xi_{1r} + \xi_{2r})}\right]^2 + \frac{p_r + p_{вх.r} - \xi_{1r} g_{y.r}^2}{\xi_{1r} + \xi_{2r}}}. \qquad (7.62)$$

The pressure at inlets into the pumps $p_{вх.о}$ and $p_{вх.г}$ in the calculations are taken as being constant.

In terms of the obtained values $g_о$ and $g_г$, we determine the new values

$$k', g_{oΣ}, g_{гΣ}$$

$$k = \frac{g_о}{g_г}, \quad \begin{matrix} g_{oΣ} = g_о + g_{у} \\ g_{гΣ} = g_г + g_{у.г} \end{matrix}$$

Utilizing expressions of balance of the flows at the branch points of (7.36)-(7.38), let us determine the sum of the flows which go to the combustion chambers:

$$G'_{o2} + G''_{o2} = G_о - G_{у1} - g_о - g_г \qquad (7.63)$$

In general the distribution of the flows along the combustion chambers will be nonuniform ($G^{I}_{o2} \neq G^{II}_{o2}$). Utilizing the coefficient which shows in which relationship there occurs the distribution of the flow of the component along two chambers (7.56), the flow G^{I}_{o2} in terms of G^{II}_{o2} is expressed:

$$G'_{o2} = AG''_{o2}.$$

After substituting the obtained expression for G^{I}_{o2} into equation (7.63), we obtain

$$AG''_{o2} + G''_{o2} = G_о - G_{у1} - g_о - g_г$$

Let us solve this equation relative to G''_{o2}:

$$G''_{o2} = \frac{G_о - G_{у1} - g_о - g_г}{A + 1}. \qquad (7.64)$$

Taking equation (7.64) into account, the expression for G'_{o2} will take the form

$$G^{I}_{o2} = \frac{A(G_o - G_{y1} - g_o - g_y)}{A+1}. \quad (7.65)$$

Now it is possible to turn to a determination of the flows which enter into the combustion chambers. Let us substitute into equation (7.11) the expression for p^{I}_{κ} (7.7) and G^{I}_{o2} from equation (7.65) and the equations of the balance of flows (7.22)-(7.24) and obtain, taking into account $k^{I}=G^{I}_{o}/G^{I}_{\Gamma}$, the following:

$$l^{I}\left(\frac{1+k^{I}}{k^{I}}\right)G^{I}_{o} + [a^{I}_{3} + a^{I}_{4} + a^{I}_{5} + a^{I}_{6}](G^{I}_{o})^{2} + \varphi^{I}G^{I}_{o} - \alpha^{I} = 0, \quad (7.66)$$

where
$$\varphi^{I} = 2a^{I}_{3}(G^{I}_{y5} + G^{I}_{y4} + G^{I}_{y3}) + 2a^{I}_{4}(G^{I}_{y5} + G^{I}_{y4}) + 2a^{I}_{5}G^{I}_{y5};$$
$$\alpha^{I} = p_o + p_{\text{вх.о}} - a_1(G_o - G_{y1} - g_o - g_y)^{2} - a^{I}_{2}\left[\frac{A(G_o - G_{y1} - g_o - g_y)}{A+1}\right]^{2} - a^{I}_{3}(G^{I}_{y5} + G^{I}_{y4} + G^{I}_{y3})^{2} - a^{I}_{4}(G^{I}_{y5} + G^{I}_{y4})^{2} - a^{I}_{5}(G^{I}_{y5})^{2}.$$

The solution to equation (7.66) makes it possible to determine the oxidizer flow, which enters into combustion chamber I:

$$G^{I}_{o} = -\frac{l^{I}(1+k^{I}) + \varphi^{I}k^{I}}{2k^{I}(a^{I}_{3} + a^{I}_{4} + a^{I}_{5} + a^{I}_{6})} + \sqrt{\left[\frac{l^{I}(1+k^{I}) + \varphi^{I}k^{I}}{2k^{I}(a^{I}_{3} + a^{I}_{4} + a^{I}_{5} + a^{I}_{6})}\right]^{2} + \frac{\alpha^{I}}{a^{I}_{3} + a^{I}_{4} + a^{I}_{5} + a^{I}_{6}}}.$$

By using equation (7.12), the expression for p^{II}_{κ} (7.8) and G^{I}_{o2} from equation (7.65) and the equations of the balance of flows (7.25-7.27), taking into account $k^{II}=G^{II}_{o}/G^{II}_{\Gamma}$ we will obtain

$$l^{II}\left(\frac{1+k^{II}}{k^{II}}\right)G^{II}_{o} + (a^{II}_{3} + a^{II}_{4} + a^{II}_{5} + a^{II}_{6})(G^{II}_{o})^{2} + \varphi^{II}G^{II}_{o} - \alpha^{II} = 0, \quad (7.67)$$

where
$$\varphi^{II} = 2a^{II}_{3}(G^{II}_{y5} + G^{II}_{y4} + G^{II}_{y3}) + 2a^{II}_{4}(G^{II}_{y5} + G^{II}_{y4}) + 2a^{II}_{5}(G^{II}_{y5});$$
$$\alpha^{II} = p_o + p_{\text{вх.о}} - a_1(G_o - G_{y1} - g_o - g_y)^{2} - a^{I}_{2} \times \left[\frac{A(G_o - G_{y1} - g_o - g_y)}{A+1}\right]^{2} - a^{II}_{3}(G^{II}_{y5} + G^{II}_{y4} + G^{II}_{y3})^{2} - a^{II}_{4}(G^{II}_{y5} + G^{II}_{y4})^{2} - a^{II}_{5}(G^{II}_{y5})^{2}.$$

Let us determine the oxidizer flow which enters into combustion chamber II by solving equation (7.67):

$$G_o^{II} = -\frac{l^{II}(1+k^{II})+\varphi^{II}k^{II}}{2k^{II}(a_3^{II}+a_4^{II}+a_5^{II}+a_6^{II})} + \\ + \sqrt{\left[\frac{l^{II}(1+k^{II})+\varphi^{II}k^{II}}{2k^{II}(a_3^{II}+a_4^{II}+a_5^{II}+a_6^{II})}\right]^2 + \frac{\alpha^{II}}{a_3^{II}+a_4^{II}+a_5^{II}+a_6^{II}}}.$$

The oxidizer flows which enter into combustion chambers III and IV can be defined by the same way as flows of chambers I and II. But for determination of G_o^{III} it is necessary to use equations (7.13), (7.9), (7.29)-(7.31) and (7.64):

$$G_o^{III} = -\frac{l^{III}(1+k^{III})+\varphi^{III}k^{III}}{2k^{III}(a_3^{III}+a_4^{III}+a_5^{III}+a_6^{III})} + \\ + \sqrt{\left[\frac{l^{III}(1+k^{III})+\varphi^{III}k^{III}}{2k^{III}(a_3^{III}+a_4^{III}+a_5^{III}+a_6^{III})}\right]^2 + \frac{\alpha^{III}}{a_3^{III}+a_4^{III}+a_5^{III}+a_6^{III}}},$$

where

$$\varphi^{III} = 2a_3^{III}(G_{y5}^{III}+G_{y4}^{III}+G_{y3}^{III}) + 2a_4^{III}(G_{y5}^{III}+G_{y4}^{III}) + 2a_5^{III}G_{y5}^{III};$$

$$\alpha^{III} = p_o + p_{вх.о} - a_1(G_o - G_{y1} - g_o - g_y)^2 - \\ - a_2^{II}\left[\frac{(G_o - G_{y1} - g_o - g_y)}{A+1}\right]^2 - \\ - a_3^{III}(G_{y5}^{III}+G_{y4}^{III}+G_{y3}^{III})^2 - a_4^{III}(G_{y5}^{III}+G_{y4}^{III})^2 - a_5^{III}(G_{y5}^{III})^2.$$

Fuel flow along the main lines of combustion chambers is determined in a similar manner. By utilizing equations of the balance of flow for the main fuel lines (7.53)-(7.55), let us determine

$$G_o^{IV} = -\frac{l^{IV}(1+k^{IV})+\varphi^{IV}k^{IV}}{2k^{IV}(a_3^{IV}+a_4^{IV}+a_5^{IV}+a_6^{IV})} + \\ + \sqrt{\left[\frac{l^{IV}(1+k^{IV})+\varphi^{IV}k^{IV}}{2k^{IV}(a_3^{IV}+a_4^{IV}+a_5^{IV}+a_6^{IV})}\right]^2 + \frac{\alpha^{IV}}{a_3^{IV}+a_4^{IV}+a_5^{IV}+a_6^{IV}}},$$

where

$$\varphi^{IV} = 2a_3^{IV}(G_{y5}^{IV} + G_{y4}^{IV} + G_{y3}^{IV}) + 2a_4^{IV}(G_{y5}^{IV} + G_{y4}^{IV}) + 2a_5^{IV} G_{y5}^{IV};$$

$$\alpha^{IV} = p_0 + p_{ex.c} - a_1(G_0 - G_{y1} - g_0 - g_y)^2 -$$
$$- a_2^{II}\left[\frac{(G_0 - G_{y1} - g_0 - g_y)}{A+1}\right]^2 -$$
$$- a_3^{IV}(G_{y5}^{IV} + G_{y4}^{IV} + G_{y3}^{IV})^2 - a_4^{IV}(G_{y5}^{IV} + G_{y4}^{IV})^2 - a_5^{IV}(G_{y5}^{IV})^2.$$

Fuel flow along the main lines of combustion chambers is determined in a similar manner. By utilizing equations of the balance of flow for the main fuel lines (7.53)-(7.55), let us determine

$$G_{r2}^{I} + G_{r2}^{II} = G_r - G_{y1r} - g_r - g_{y.r} \qquad (7.68)$$

According to expression (7.57) the distribution of fuel flow along sections of main lines of the two combustion chambers will be determined by the coefficient B:

$$G_{r2}^{I} = G_{r2}^{II} B.$$

Let us substitute the obtained expression into equation (7.68) and solve the latter relative to G_{r2}^{II}:

$$G_{r2}^{II} = \frac{G_r - G_{y1r} - g_r - g_{y.r}}{B+1}. \qquad (7.69)$$

Then $G_{r2}^{I} = \frac{B(G_r - G_{y1r} - g_r - g_{y.r})}{B+1}$. (7.70)

In order to determine the flow of the fuel which enters into combustion chamber I, equations (7.7) and (7.70) must be substituted into equation (7.15). Taking into account (7.39)-(7.41)

$$G_r^{I} = -\frac{l^{I}(1 + k^{I}) + \psi^{I}}{2k^{I}(a_{3r}^{I} + a_{4r}^{I} + a_{5r}^{I} + a_{6r}^{I})} +$$
$$+ \sqrt{\left[\frac{l^{I}(1 + k^{I}) + \psi^{I}}{2(a_{3r}^{I} + a_{4r}^{I} + a_{5r}^{I} + a_{6r}^{I})}\right]^2 + \frac{\beta^{I}}{a_{3r}^{I} + a_{4r}^{I} + a_{5r}^{I} + a_{6r}^{I}}}, \qquad (7.71)$$

where

$$\psi^{I} = 2a^{I}_{3r}(G^{I}_{y5r} + G^{I}_{y4r} + G^{I}_{y3r}) + 2a^{I}_{4r}(G^{I}_{y5r} + G^{I}_{y4r}) + 2a^{I}_{5r}G^{I}_{y5r};$$

$$\beta^{I} = p_r + p_{\text{вх}.r} - a_{1r}(G_r - G_{y1r} - g_r - g_{y.r})^2 - a^{I}_{2r}\left[\frac{B(G_r - G_{y1r} - g_r - g_{y.r})}{B+1}\right]^2 - a^{I}_{3r}(G^{I}_{y5r} + G^{I}_{y4r} + G^{I}_{y3r})^2 - a^{I}_{4r}(G^{I}_{y5r} + G^{I}_{y4r})^2 - a^{I}_{5r}(G^{I}_{y5r})^2.$$

By solving together equations (7.16), (7.8), (7.70) and (7.42)-(7.44), let us determine the flow which enters into combustion chamber II:

$$G^{II}_r = -\frac{l^{II}(1 + k^{II}) + \psi^{II}}{2(a^{II}_{3r} + a^{II}_{4r} + a^{II}_{5r} + a^{II}_{6r})} + \sqrt{\left[\frac{l^{II}(1 + k^{II}) + \psi^{II}}{2(a^{II}_{3r} + a^{II}_{4r} + a^{II}_{5r} + a^{II}_{6r})}\right]^2 + \frac{\beta^{II}}{a^{II}_{3r} + a^{II}_{4r} + a^{II}_{5r} + a^{II}_{6r}}},$$

where

$$\psi^{II} = 2a^{II}_{3r}(G^{II}_{y5r} + G^{II}_{y4r} + G^{II}_{y3r}) + 2a^{II}_{4r}(G^{II}_{y5r} + G^{II}_{y4r}) + 2a^{II}_{5r}G^{II}_{y5r};$$

$$\beta^{II} = p_r + p_{\text{вх}.r} - a_{1r}(G_r - G_{y1r} - g_r - g_{y.r})^2 - a^{I}_{2r}\left[\frac{B(G_r - G_{y1r} - g_r - g_{y.r})}{B+1}\right]^2 - a^{II}_{3r}(G^{II}_{y5r} + G^{II}_{y4r} + G^{II}_{y3r})^2 - a^{II}_{4r}(G^{II}_{y5r} + G^{II}_{y4r})^2 - a^{II}_{5r}(G^{II}_{y5r})^2.$$

We determine the fuel flow which enters into combustion chamber III by utilizing equations (7.17), (7.9), (7.46)-(7.48), and (7.69) and in combustion chamber IV, on the basis of equations (7.18), (7.10), (7.49-7.51), and (7.69):

$$G^{III}_r = -\frac{l^{III}(1 + k^{III}) + \psi^{III}}{2(a^{III}_{3r} + a^{III}_{4r} + a^{III}_{5r} + a^{III}_{6r})} + \sqrt{\left[\frac{l^{III}(1 + k^{III}) + \psi^{III}}{2(a^{III}_{3r} + a^{III}_{4r} + a^{III}_{5r} + a^{III}_{6r})}\right]^2 + \frac{\beta^{III}}{a^{III}_{3r} + a^{III}_{4r} + a^{III}_{5r} + a^{III}_{6r}}}, \quad (7.72)$$

where

$$\psi^{III} = 2a^{III}_{3r}(G^{III}_{y5r} + G^{III}_{y4r} + G^{III}_{y3r}) + 2a^{III}_{4r}(G^{III}_{y5r} + G^{III}_{y4r}) + 2a^{III}_{5r}G^{III}_{y6r};$$

$$\beta^{III} = p_r + p_{вх.г} - a_{1г}(G_г - G_{y1г} - g_г - g_{y.г})^2 -$$
$$- a^{II}_{3г}\left[\frac{G_г - G_{y1г} - g_г - g_{y.г}}{B+1}\right]^2 -$$
$$- a^{III}_{3г}(G^{III}_{y5г} + G^{III}_{y4г} + G^{III}_{y3г})^2 - a^{III}_{4г}(G^{III}_{y5г} + G^{III}_{y4г})^2 - a^{III}_{5г}(G^{III}_{y5г})^2.$$

The expression for the fuel flow of combustion chamber IV has the form of (7.72), and only the index "III" with the coefficients of this equation must be replaced by "IV".

By knowing the values of the oxidizer flows along the main lines of each combustion chamber, it is possible to determine by (7.28) and (7.35) new total flows by two combustion chambers

$$G^{I}_{o3} + G^{II}_{o3} + G^{I}_{y2} = G^{I}_{o2},\ G^{III}_{o3} + G^{IV}_{o3} + G^{II}_{y2} = G^{II}_{o2}$$

and the new value of the coefficient, which shows in which relationship the component distribution along the main lines of the combustion chambers occurs (7.56).

We will note the new values of the parameters necessary for the following approximation by index "(н)":

$$A_{(н)} = \frac{G^{I}_{o2}}{G^{II}_{o2}}.$$

Similarly, for the main fuel lines:

$$G^{I}_{г3} + G^{II}_{г3} + G^{I}_{y2г} = G^{I}_{г2};\quad B_{(н)} = \frac{G^{I}_{г2}}{G^{II}_{г2}}.$$
$$G^{III}_{г3} + G^{IV}_{г3} + G^{II}_{y2г} = G^{II}_{г2};$$

Let us determine the new values of the oxidizer and fuel flows by utilizing equations (7.36), (7.38), (7.53) and (7.55):

$$G_o = G^{I}_{o2} + G^{II}_{o2} + G_{y.},\quad G^{I}_{г1} = G^{I}_{г2} + G^{II}_{г2} + G_{y.},$$
$$G_{o(н)} = G_o + g_{.г},\quad G_{г(н)} = G_{г1} + g_{г.г}$$

Let us accept the average values of the established and obtained quantities as the initial data for the following approximation:

$$\frac{G_{o} + G_{o(\text{н})}}{2} = G_{o}, \quad \frac{G_{\text{г}} + G_{\text{г}(\text{н})}}{2} = G_{\text{г}}$$

$$\frac{g_{o} + g_{o(\text{н})}}{2} = g_{o}, \quad \frac{g_{\text{г}} + g_{\text{г}(\text{н})}}{2} = g_{\text{г}}$$

$$\frac{A + A_{(\text{н})}}{2} = A, \quad \frac{B + B_{(\text{н})}}{2} = B.$$

To obtain the solution with the necessary accuracy ε, it is necessary to repeat the approximations until the difference between the established and obtained values in this approximation becomes less than the assigned magnitude ε.

3. Methods of the Assignment of Factors of Emergency Situations

With the emergence of unsealing of the main flow lines, the value of the leakage is assigned in a definite ratio of the total flow of this component.

All the remaining values of leakages along the main lines of this combustion chamber, other combustion chambers and the gas generator in this case are equated to zero.

The partial closing (obstruction) of the hydraulic line is assigned by the increased (in comparison with the nominal) value of the coefficient of hydraulic friction of its corresponding section.

With the full closing (unsealing) of the main hydraulic fuel line of combustion chamber I, the pressure in this combustion chamber must be equated to the pressure of the saturated vapors of the oxidizer $p_{H}^{I}=p_{S}$. With the full closing (unsealing) of the hydraulic line of the oxidizer of combustion chamber I, the pressure in it is determined as

$$p_{\text{к}}^{\text{I}} = \frac{\beta^{\text{I}}}{F_{\text{кр}}} G_{\text{г}}^{\text{I}},\ \beta^{\text{I}} = \beta\,(k^{\text{I}} = 0).$$

The oxidizer flow with the full closing of the main line of combustion chamber I must be equated to zero $G_{o}^{I}=0$, and it is necessary to eliminate in the solution to the system of equations the determination of the value of this flow.

With the complete destruction the coefficients of hydraulic friction of sections of the pipelines after the place of destruction should be equated to zero. The coefficient of hydraulic friction of the section up to the place of destruction must be increased by the value of the drag coefficient with outflow from the opening:

$$a_{\text{у}} = \frac{1}{\mu^2 F_{\text{отв}}^2 2g\gamma}.$$

The appearance of a malfunction of the turbine must be considered by a change in coefficients c_6 and c_7 in the expression for the power of the turbine.

Thus, the static procedure allows calculating for the following forms of malfunctions of the ZhRD: the closing of the main line, leakage of the main lines, and malfunctions of the turbine. However, the use of this procedure does not make it possible to carry out investigations with the malfunctions of the pumps caused by the cavitation stalling of their operation, since the solution of the system was carried out at the constant values of pressures at the inlet into the pump.

Figure 7.5 gives the dependences of the basic parameters of the ZhRD on the magnitude of unsealing of the hydraulic line of the fuel $G_{\text{г.у}}$ obtained by calculation and also during the bench tests which simulate the emergency situation.

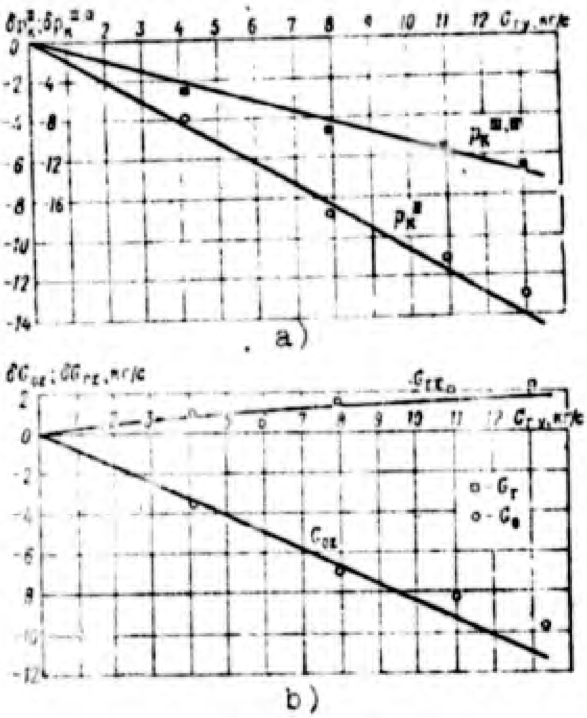


Figure 7.5. Dependence of the basic parameters on the degree of unsealing of the main line: a) combustion-chamber pressure; b) flows of components through the pumps; --- - computed values; o■o- experimental points.
Abbreviation: кг/с = kg/s.

However, sometimes with the emergence of malfunctions of different forms in the engine, identical values of the parameters which characterize the operating mode are established. In such cases the establishment of the cause of the emergency situation on the basis of the conducted static computations is not possible. Then it is expedient to conduct calculations of the transient processes for an explanation of the reason for the emergency, comparing not only the parameters of the steady-state system but also the nature of the transient process.

In the various forms of malfunctions the transient processes in a quantitative respect significantly differ from one another. Figure 7.6 gives combined graphs of transient processes according to the basic parameters of the engine for the cases of closing, leakage of the hydraulic line of the oxidizer and lowering of the efficiency of the oxidizer pump. It is evident that from the parameters G_o - the flow into the chamber, and $G_{\Gamma\Sigma}$ - fuel flow through the pump, the transient processes considerably differ from one another, whereas the parameters of the steady-state mode with these malfunctions are virtually not different.

§ 3. METHOD OF CALCULATION OF TRANSIENT PROCESSES IN THE ZhRD IN EMERGENCY SITUATIONS

The method of the calculation of emergency situations is based on the solution of the system of nonlinear differential

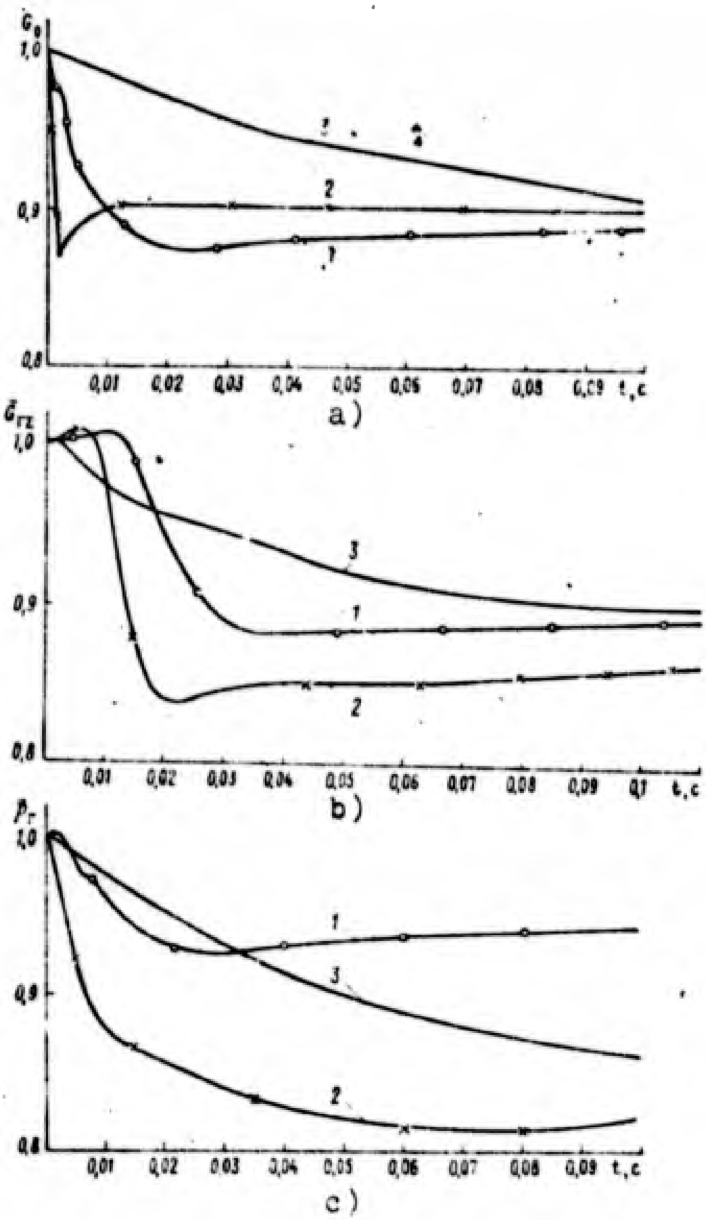


Figure 7.6. Comparison of transient processes of the ZhRD which appear with the different malfunctions: a) oxidizer flow; b) fuel flow through the pump; c) the head of fuel pump; 1 - closing of the hydraulic line; 2 - leakage of the hydraulic line; 3 - decrease in efficiency of the pump.

equations, which describe the dynamics of the engine by taking into account the models of failures with the appearance of malfunctions. It is obvious that it is difficult to solve this problem without the aid of a digital computer.

From the experiment of bench and flight tests of the ZhRD it is known that the simultaneous emergence of two independent failures of the engine is highly improbable. However, the system of equations and program on the computer is expediently formulated for the simultaneous implementation of all forms of malfunctions in order to avoid the alterations of the program in each concrete case. With this setting the concrete variant is obtained from the total system automatically by the assignment of definite initial and boundary conditions.

The solution to the system of nonlinear differential equations can be carried out by the numerical integration for the method of Euler or Runge-Kutta.

The initial conditions in the calculation of the transient processes which appear in the engine during emergency situations are determined by the parameters of the steady-state mode which preceded the emergency situation of the engine and by the concrete conditions of the emergency situation: the value of the reduction in efficiency, the dimension of the formed opening $\mu F_{ОТВ}$ with the leakage of the hydraulic and gas lines, and so on.

The closing of the main line, which occurs according to the stepped law, is assigned as being increased in comparison with the nominal coefficient of the hydraulic friction of the defined section of the hydraulic line. With the gradual closing of the main line the law of the time build-up of the coefficient of hydraulic friction of the defined section of the main line is assigned.

In the case of the burnout of gas channels the area of the formed opening $\mu F_{ОТВ}$ is assigned. If the burnouts of the gas lines are absent, this product is equal to zero. In this case automatic transition to the normal operation of the engine will be accomplished, since with $G_y=0$ the equation of the gas tank is equivalent to the equation of the normally operating gas generator.

Since the hydraulic lines of the engine are a multi-assembly branched system of pipelines, a relocation of the destruction of the hydraulic line of the engine, connected with the transference through the subassembly (point of the selection of any flow to a definite engine assembly), will compulsorily require the introduction of changes into the program. In order to avoid this, in the program it is necessary to provide for places of the break after each junction of the hydraulic line. This leads to the fact that the design scheme is more branched than the real hydraulic scheme of the engine, since new branches at places of possible breaks are added.

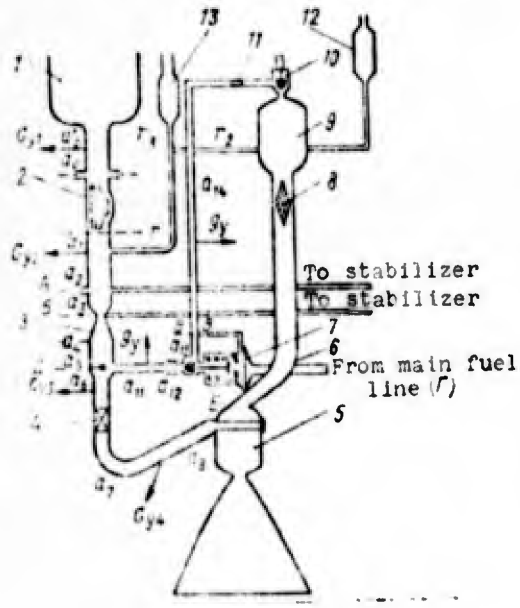


Figure 7.7. Design diagram of hydraulic oxidizer lines of the engine with afterburning of the producer gas: 1 - oxidizer tank; 2 - pump; 3 - Venturi tube; 4 - cut-off valve; 5 - combustion chamber; 6 - gas main; 7 - pressure stabilizer; 8 - turbine; 9 - gas generator; 10 - cut-off valve; 11 - discharging nozzle; 12 - gas generator of the pressurization of the fuel tank; 13 - gas generator of the pressurization of the oxidizer tank.

Special difficulties with hydraulic lines appear because the system of equations for them must be solved together in order to find in an explicit form the values of derivatives in terms of the flow. For example, the equations of hydraulic lines of the oxidizer, the design scheme of which is shown on Fig. 7.7, will be written in the following way:

1. $$p_{б.о} - a'_0 G^2_{o\Sigma} - b'_0 \frac{dG_{o\Sigma}}{dt} - a_0 (G_{o\Sigma} - G_{y1})^2 - b_0 \left(\frac{dG_{o\Sigma}}{dt} - \frac{dG_{y1}}{dt} \right) + p_0 - a_1 (G_{o\Sigma} - G_{y1})^2 - b_1 \left(\frac{dG_{o\Sigma}}{dt} - \frac{dG_{y1}}{dt} \right) - (a_2 + a_3 + a_4 + a_5)(G_0 + G_{y4} + G_{y3} + g_\Sigma)^2 - (b_2 + b_3 + b_4 + b_5) \times$$

$$\times\left(\frac{dG_0}{dt}+\frac{dG_{y4}}{dt}+\frac{dG_{y3}}{dt}+\frac{dg_\Sigma}{dt}\right)-a_6(G_0+G_{y4}+G_{y3})^2-$$
$$-b_6\left(\frac{dG_0}{dt}+\frac{dG_{y4}}{dt}+\frac{dG_{y3}}{dt}\right)-a_7(G_0+G_{y4})^2-$$
$$-b_7\left(\frac{dG_0}{dt}+\frac{dG_{y4}}{dt}\right)-a_8G_0^2-b_8\frac{dG_0}{dt}=p_\kappa$$

where $$G_{0\Sigma}=G_0+G_{y4}+G_{y3}+G_{y2}+G_{y1}+g_\Sigma+g_{\Sigma r.r},$$
$$g_\Sigma=g_0+g_y,\quad g_{\Sigma r.r}=g_{o.o}+g_{o.r}.$$

2. $$p_{6.0}-a_0'G_{0\Sigma}^2-b_0'\frac{dG_{0\Sigma}}{dt}-a_0(G_{0\Sigma}-G_{y1})^2-b_0\left(\frac{dG_{0\Sigma}}{dt}-\right.$$
$$\left.-\frac{dG_{y1}}{dt}\right)+p_0-a_1(G_{0\Sigma}-G_{y1})^2-b_1\left(\frac{dG_{0\Sigma}}{dt}-\frac{dG_{y1}}{dt}\right)-$$
$$-(a_2+a_3+a_4+a_5)(G_0+G_{y4}+G_{y3}+g_\Sigma)^2-(b_2+b_3+b_4+b_5)\times$$
$$\times\left(\frac{dG_0}{dt}+\frac{dG_{y4}}{dt}+\frac{dG_{y3}}{dt}+\frac{dg_\Sigma}{dt}\right)-a_7(G_0+G_{y4})^2-b_7\left(\frac{dG_0}{dt}+\right.$$
$$\left.+\frac{dG_{y4}}{dt}\right)-S_4G_{y4}^2-S_4'\frac{dG_{y4}}{dt}-p_4-b_6\left(\frac{dG_0}{dt}+\frac{dG_{y4}}{dt}+\frac{dG_{y3}}{dt}\right)-$$
$$-a_6(G_0+G_{y4}+G_{y3})^2=0,$$

3. $$p_{6.0}-a_0'G_{0\Sigma}^2-b_0'\frac{dG_{0\Sigma}}{dt}-a_0(G_{0\Sigma}-G_{y1})^2-b_0\times$$
$$\times\left(\frac{dG_{0\Sigma}}{dt}-\frac{dG_{y1}}{dt}\right)+p_0-a_1(G_{0\Sigma}-G_{y1})^2-b_1\left(\frac{dG_{0\Sigma}}{dt}-\right.$$
$$\left.-\frac{dG_{y4}}{dt}\right)-(a_2+a_3+a_4+a_5)(G_0+G_{y4}+G_{y3}+g_\Sigma)^2-$$
$$-(b_2+b_3+b_4+b_5)\left(\frac{dG_0}{dt}+\frac{dG_{y3}}{dt}+\frac{dG_{y4}}{dt}+\frac{dg_\Sigma}{dt}\right)-a_{11}g_\Sigma^2-$$
$$-b_{11}\frac{dg_\Sigma}{dt}-a_{12}(g_0+\varepsilon g_y)^2-b_{12}\left(\frac{dg_0}{dt}+\varepsilon\frac{dg_y}{dt}\right)-$$
$$-\Delta p_{cr}\left(\frac{g_0+\varepsilon g_y}{g_0^\circ}\right)^2-a_{13}(g_0+\varepsilon g_y)^2-b_{13}\left(\frac{dg_0}{dt}+\varepsilon\frac{dg_y}{dt}\right)-$$
$$-a_{14}g_0^2-b_{14}\frac{dg_0}{dt}-p_\kappa'=0,$$

4. $$p_{6.0}-a_0'G_{0\Sigma}^2-b_0'\frac{dG_{0\Sigma}}{dt}+a_0(G_{0\Sigma}-G_{y1})^2-b_0\left(\frac{dG_{0\Sigma}}{dt}-\frac{dG_{y1}}{dt}\right)-$$
$$-(b_2+b_3+b_4+b_5)\left(\frac{dG_0}{dt}+\frac{dG_{y4}}{dt}+\frac{dG_{y3}}{dt}+\frac{dg_\Sigma}{dt}\right)-$$
$$-(a_2+a_3+a_4+a_5)(G_0+G_{y3}+G_{y4}+g_\Sigma)^2-a_{11}g_\Sigma^2-b_{11}\frac{dg_\Sigma}{dt}-$$
$$-\varepsilon(a_{12}+a_{13})(g_0+g_y)^2-\varepsilon(b_{12}+b_{13})\left(\frac{dg_0}{dt}+\frac{dg_y}{dt}\right)-$$
$$-\varepsilon\Delta p_{cr}\left(\frac{g_0+g_y}{g_0^\circ}\right)^2-S_{11}g_y^2-S_{11}'\frac{dg_y}{dt}-p_{11}+p_0-$$

$$-a_1(G_{o2}-G_{y1})^2-b_1\left(\frac{dG_{o2}}{dt}-\frac{dG_{y1}}{dt}\right)=0,$$

5. $$p_{6.o}-a_0'G_{o2}^2-b_c'\frac{dG_{o2}}{dt}-a_0(G_{o2}-G_{y1})^2-b_0\left(\frac{dG_{o2}}{dt}-\frac{dG_{y1}}{dt}\right)+$$
$$+p_o-(b_2+b_3+b_4+b_5)\left(\frac{dG_o}{dt}+\frac{dG_{y4}}{dt}+\frac{dG_{y3}}{dt}+\frac{dg_2}{dt}\right)-$$
$$-(a_2+a_3+a_4+a_5)(G_o+G_{y4}+G_{y3}+g_2)^2-S_3G_{y3}^2-$$
$$-S_3'\frac{dG_{y3}}{dt}-p_3-a_1(G_{o2}-G_{y1})^2-b_1\left(\frac{dG_{o2}}{dt}-\frac{dG_{y1}}{dt}\right)-$$
$$-b_4\left(\frac{dG_o}{dt}+\frac{dG_{y4}}{dt}+\frac{dG_{y3}}{dt}\right)-a_3(G_o+G_{y3}+G_{y4})^2=0,$$

6. $$p_{6.o}-a_0'G_{o2}^2-b_0'\frac{dG_{o2}}{dt}-a_0(G_{o2}-G_{y1})^2-$$
$$-b_0\left(\frac{dG_{o2}}{dt}-\frac{dG_{y1}}{dt}\right)+p_o-a_1(G_{o2}-G_{y1})^2-b_1\left(\frac{dG_{o2}}{dt}-\right.$$
$$\left.-\frac{dG_{y1}}{dt}\right)-S_2G_{y2}^2-S_2'\frac{dG_{y2}}{dt}-p_2=0,$$

7. $$p_{6.o}-a_0'G_{o2}^2-b_0'\frac{dG_{o2}}{dt}-a_0(G_{o2}-G_{y1})^2-$$
$$-b_0\left(\frac{dG_{o2}}{dt}-\frac{dG_{y1}}{dt}\right)+p_o-a_1(G_{o2}-G_{y1})^2-b_1\left(\frac{dG_{o2}}{dt}-\right.$$
$$\left.-\frac{dG_{y1}}{dt}\right)+p_o-a_1(G_{o2}-G_{y1})^2-b_1\left(\frac{dG_{o2}}{dt}-\frac{dG_{y1}}{dt}\right)-$$
$$-r_1(g_{o.o}+g_{o.r})^2-r_1'\left(\frac{dg_{o.o}}{dt}+\frac{dg_{o.r}}{dt}\right)-r_2g_{o.r}^2-r_2'\frac{dg_{o.r}}{dt}-p_{r.r}=0,$$

8. $$p_{6.o}-a_0'G_{o2}^2-b_0'\frac{dG_{o2}}{dt}-a_0(G_{o2}-G_{y1})^2-b_0\left(\frac{dG_{o2}}{dt}-\right.$$
$$\left.-\frac{dG_{y1}}{dt}\right)+p_o-a_1(G_{o2}-G_{y1})^2-b_1\left(\frac{dG_{o2}}{dt}-\frac{dG_{y1}}{dt}\right)-$$
$$-r_1(g_{o.o}+g_{o.r})^2-r_1'\left(\frac{dg_{o.o}}{dt}+\frac{dg_{o.r}}{dt}\right)-r_3g_{o.o}^2-$$
$$-r_3'\frac{dg_{o.o}}{dt}-p_{r.o}=0,$$

9. $$p_{6.o}-a_0'G_{o2}^2-b_0'\frac{dG_{o2}}{dt}-S_1G_{y1}^2-S_1'\frac{dG_{y1}}{dt}-p_1=0.$$

where $G_{y\,i}$ - the flow of leakage at different places of the hydraulic lines of the combustion chamber;

g_y - flow of leakage along the main line of the gas generator;

p_o - pressure created by the oxidizer pump;

a_1, b_1 - coefficients of hydraulic friction and inertia losses of different sections of the hydraulic lines;

Δp_{cp} - pressure differential on the controlling element of the stabilizer of pressure along the main line of the gas generator;

r_1, r_2, r_3 - coefficients of hydraulic friction of main lines of gas generators of tank pressurization;

r_1', r_2', r_3' - coefficients of the inertia losses of hydraulic lines of gas generators of tank pressurization;

S_j - coefficient of hydraulic friction of the outflow of the component G_y from the formed opening;

S_j' - coefficient of inertia losses with the outflow of the component G_y from the formed opening;

$g_{o.o}$, $g_{o.r}$ - flow of the oxidizer drawn off to the gas generators of pressurization;

$p_1 \ldots p_4$, p_{11} - pressure of ambient medium where the outflow of the component with the appearance of leakage occurs;

$p_{r.o}$, $p_{r.r}$ - pressures in the gas generators of the tank pressurization of the oxidizer and fuel;

g_o^* - flow of oxidizer into the gas generator in the nominal mode;

ε - the conditional coefficient which makes it possible to provide for the appearance of only one leakage along the main line of the gas generator (instead of two) and decrease the quantity of equations. In this case, if the leakage appears before the stabilizer, we assume that $\varepsilon=0$, and if on the section after the stabilizer (after point B), we assume that $\varepsilon=1$.

Let us introduce the notations:

$$f_1 = p_{б.о} - a_0' G_{o2}^2 - a_n (G_{o2} - G_{y1})^2_1 + p_0 - a_1 (G_{o2} - G_{y1})^2 -$$
$$- (a_2 + a_3 + a_4 + a_5)(G_0 + G_{y4} + G_{y3} + g_2)^2 - a_6 (G_0 + G_{y4} + G_{y3})^2 -$$
$$- a_7 (G_0 + G_{y4})^2 - a_8 G_0^2 - p_{к},$$

$$f_3 = p_{0.0} - a_0' G_{02}^2 - a_0 (G_0 - G_{y1})^2 + p_0 - a_1 (G_{02} - G_{y1})^2 -$$
$$- (a_2 + a_3 + a_4 + a_5) (G_0 + G_{y4} + G_{y3} + g_2)^2 - a_6 (G_0 + G_{y4} + G_{y3})^2 -$$
$$- a_7 (G_0 + G_{y4})^2 - S_4 G_{y4}^2 - p_4.$$
$$f_3 = p_{0.0} - a_0' G_{02}^2 - a_0 (G_0 - G_{y1})^2 + p_0 - a_1 (G_{02} - G_{y1})^2 -$$
$$- (a_2 + a_3 + a_4 + a_5) (G_0 + G_{y4} + G_{y3} + g_2)^2 - a_{11} (g_0 + g_y)^2 -$$
$$- (a_{12} + a_{13}) (g_0 + \epsilon g_y)^2 - \Delta p_{c7} \left(\frac{g_0 + g_y}{g_0^*} \right)^2 - a_{14} g_0^2 - p_k',$$
$$f_4 = p_{0.0} - a_0' G_{02}^2 - a_0 (G - G_{y1})^2 + p_0 - a_1 (G_{02} - G_{y1})^2 -$$
$$- (a_2 + a_3 + a_4 + a_5) (G_0 + G_{y4} + G_{y3} + g_2)^2 - a_{11} (g_0 + g_y)^2 -$$
$$- (a_{12} + a_{13}) \cdot \epsilon (g_0 + g_y)^2 - \Delta p_{c7} \left(\frac{g_0 + g_y}{g_0^*} \right)^2 - S_{11} - p_{11},$$
$$f_5 = p_{0.0} - a_0' G_{02}^2 - a_0 (G_0 - G_{y1})^2 + p_0 - a_1 (G_{02} - G_{y1})^2 -$$
$$- (a_2 + a_3 + a_4 + a_5) (G_0 + G_{y4} + G_{y3} + g_2)^2 -$$
$$- a_6 (G_0 + G_{y4} + G_{y3})^2 - S_3 G_{y3}^2 - p_3,$$
$$f_6 = p_{0.0} - a_0' G_{02}^2 - a_0 (G_{02} - G_{y1})^2 + p_0 - a_1 (G_{02} - G_{y1})^2 -$$
$$- S_2 G_{y2}^2 - p_2,$$
$$f_7 = p_{0.0} - a_0' G_{02}^2 - a_0 (G_{02} - G_{y1})^2 + p_0 - a_1 (G_{02} - G_{y1})^2 -$$
$$- r_1 (g_{0.0} + g_{0.r})^2 - r_3 g_{0.r}^2 - p_{r.r},$$
$$f_8 = p_{0.0} - a_0' G_{02}^2 - a_0 (G_{02} - G_{y1})^2 + p_0 - a_1 (G_{02} - G_{y1})^2 -$$
$$- r_1 (g_{0.0} + g_{0.r})^2 - r_3 g_{0.0}^2 - p_{r.0},$$
$$f_9 = p_{0.0} - a_0' G_{02}^2 - S_1 G_{y1}^2 - p_1.$$

Then

$$(b_8 + b_7 + b_6 + b_5 + b_4 + b_3 + b_2 + b_1 + b_0 + b_0') \frac{dG_0}{dt} + (b_7 + b_6 +$$
$$+ b_5 + b_4 + b_3 + b_2 + b_1 + b_0 + b_0') \frac{dG_{y4}}{dt} + (b_5 + b_4 + b_3 + b_2 + b_1 +$$
$$+ b_0 + b_0') \left(\frac{dg_0}{dt} + \frac{dg_y}{dt} \right) + (b_6 + b_5 + b_4 + b_3 + b_2 + b_1 + b_0 +$$
$$+ b_0') \frac{dG_{y3}}{dt} + (b_1 + b_0 + b_0') \left(\frac{dG_{y2}}{dt} + \frac{dg_{0.0}}{dt} + \frac{dg_{0.r}}{dt} \right) +$$
$$+ b_0' \frac{dG_{y1}}{dt} = f_1, \qquad (7.73)$$

$$(S_4' + b_7 + b_6 + b_5 + b_4 + b_3 + b_2 + b_1 + b_0 + b_0') \frac{dG_{y4}}{dt} + (b_7 + b_6 +$$
$$+ b_5 + b_4 + b_3 + b_2 + b_1 + b_0 + b_0') \frac{dG_0}{dt} + (b_5 + b_4 + b_3 + b_2 + b_1 +$$
$$+ b_0 + b_0') \left(\frac{dg_0}{dt} + \frac{dg_y}{dt} \right) + (b_6 + b_5 + b_4 + b_3 + b_2 + b_1 + b_0 +$$

$$+ b'_0)\frac{dG_{y3}}{dt} + (b_1 + b_0 + b'_0)\left(\frac{dG_{y2}}{dt} + \frac{dg_{o.o}}{dt} + \frac{dg_{o.r}}{dt}\right) +$$

$$+ b'_0\frac{dG_{y1}}{dt} = f_{9} \qquad (7.74)$$

$$(b_{14} + b_{13} + b_{12} + b_{11} + b_8 + b_4 + b_3 + b_2 + b_1 + b_0 + b'_0)\frac{dg_o}{dt} +$$

$$+ (\varepsilon b_{13} + \varepsilon b_{12} + b_{11} + b_8 + b_4 + b_3 + b_2 + b_1 + b_0 + b'_0)\frac{dg_y}{dt} +$$

$$+ (b_8 + b_4 + b_3 + b_2 + b_1 + b_0 + b'_0)\frac{dG_{y3}}{dt} + (b_1 + b_0 + b'_0) \times$$

$$\times \left(\frac{dG_{y2}}{dt} + \frac{dg_{o.o}}{dt} + \frac{dg_{o.r}}{dt}\right)' + b'_0\frac{dG_{y1}}{dt} = f_{8} \qquad (7.75)$$

$$(S'_{11} + \varepsilon b_{13} + b_{12} + b_{11} + b_8 + b_4 + b_3 + b_2 + b_1 + b_0 + b'_0)\frac{dg_y}{dt} +$$

$$+ (b_{13} + b_{12} + b_{11} + b_8 + b_4 + b_3 + b_2 + b_1 + b_0 + b'_0)\frac{dg_o}{dt} +$$

$$+ (b_8 + b_6 + b_4 + b_3 + b_1 + b_0 + b'_0)\left(\frac{dG_o}{dt} + \frac{dG_{y4}}{dt}\right) + (b_8 + b_4 +$$

$$+ b_3 + b_2 + b_1 + b_0 + b'_0)\frac{dG_{y3}}{dt} + (b_1 + b_0 + b'_0)\left(\frac{dG_{y2}}{dt} + \frac{dg_{o.o}}{dt} +\right.$$

$$\left. + \frac{dg_{or}}{dt}\right) + b'_0\frac{dG_{y1}}{dt} = f_{6} \qquad (7.76)$$

$$(S'_8 + b_8 + b_6 + b_4 + b_3 + b_1 + b_0 + b'_0)\frac{dG_{y3}}{dt} + (b_8 + b_6 + b_4 + b_3 +$$

$$+ b_2 + b_1 + b_0 + b'_0)\left(\frac{dG_o}{dt} + \frac{dG_{y4}}{dt}\right) + (b_8 + b_4 + b_3 + b_2 + b_1 +$$

$$+ b_0 + b'_0)\left(\frac{dg_o}{dt} + \frac{dg_y}{dt}\right) + (b_1 + b_0 + b'_0)\left(\frac{dG_{y2}}{dt} + \frac{dg_{o.o}}{dt} +\right.$$

$$\left. + \frac{dg_{o.r}}{dt}\right) + b'_0\frac{dG_{y1}}{dt} = f_{8} \qquad (7.77)$$

$$(S'_2 + b_1 + b_0 + b'_0)\frac{dG_{y2}}{dt} + (b_1 + b_0 + b'_0)\left(\frac{dG_o}{dt} + \frac{dG_{y4}}{dt} + \frac{dG_{y3}}{dt} +\right.$$

$$\left. + \frac{dg_o}{dt} + \frac{dg_y}{dt} + \frac{dg_{o.o}}{dt} + \frac{dg_{o.r}}{dt}\right) + b_0\frac{dG_{y1}}{dt} = f_{6} \qquad (7.78)$$

$$(r'_2 + r'_1 + b_1 + b_0 + b'_0)\frac{dg_{o.r}}{dt} + (r'_1 + b_1 + b_0 + b'_0)\frac{dg_{o.o}}{dt} +$$

$$+ (b_1 + b_0 + b'_1)\left(\frac{dG_o}{dt} + \frac{dG_{y4}}{dt} + \frac{dG_{y3}}{dt} + \frac{dg_o}{dt} + \frac{dG_{y2}}{dt} +\right.$$

$$\left. + \frac{dg_y}{dt}\right) + b'_0\frac{dG_{y1}}{dt} = f_{7} \qquad (7.79)$$

$$(r'_3 + r'_1 + b_1 + b_0 + b'_0)\frac{dg_{o.o}}{dt} + (r'_1 + b_1 + b_0 + b'_0)\frac{dg_{o.r}}{dt} +$$

$$+ (b'_0 + b_0 + b_1)\left(\frac{dG_o}{dt} + \frac{dG_{y4}}{dt} + \frac{dG_{y3}}{dt} + \frac{dg_o}{dt} + \frac{dG_{y2}}{dt} +\right.$$

$$\left. + \frac{dg_y}{dt}\right) + b'_0\frac{dG_{y1}}{dt} = f_{8} \qquad (7.80)$$

$$(S_1' + b_0')\frac{dG_{y1}}{dt} + b_0'\left(\frac{dG_0}{dt} + \frac{dG_{y2}}{dt} + \frac{dG_{y3}}{dt} + \frac{dG_{y4}}{dt} + \right.$$

$$\left. + \frac{dg_0}{dt} + \frac{dg_y}{dt} + \frac{dg_{o.o}}{dt} + \frac{dg_{o.r}}{dt}\right) = f_v \tag{7.81}$$

Since the system of equations of the hydraulic lines is linear relative to the derivatives, its solution is found by means of matrices. The matrix is composed of the totality of the coefficients of inertia losses of individual sections of the hydraulic lines (7.73)-(7.81). The column vector is composed of right sides of differential equations of the hydraulic lines where these sides are designated by coefficients f_1, f_2, ..., f.

The program is composed for the maximum quantity of branches of the hydraulic line corresponding to the matrix of the 12th order. The logic subprogram, which automatically prevents the unnecessary equations and makes up the matrices of the necessary order is provided for. It is as though the operations of this subprogram "feel" the order of the matrices by means of special coefficients. Each coefficient corresponds to a definite hydraulic line and can take only two values: 1 or 0 with the presence or absence of the main line, respectively.

If all coefficients are equal to unity, then in the program no changes occur.

The logic program analyzes the system of equations of the hydraulic lines by operating with the coefficients and produces changes in the very program of the solution to the matrix, leading it in conformity with the initial matrix of the necessary order.

The program of the calculation of transient processes during emergency situations is composed for the computer and consists of two autonomous programs: the engine proper and the regulating unit. Therefore, in the case of a change in the operating principle of the controlling elements the main program remains without changes, since in it only output parameters of the regulators - pressure differentials on the controlling elements, participate.

§ 4. TRANSIENT PROCESSES WITH THE CLOSING OF THE HYDRAULIC CIRCUIT

Any malfunction leads to disruption of the operating mode (emergency situation) of the ZhRD and causes transient process in it.

For each class of engines its course of the transient process with the emergence of the malfunction is characteristic, although it is possible to distinguish some general laws characteristic to engines of different designs.

The nature of the course of transient processes in the emergency situation can be determined by carrying out the calculations under the assumption of a defined malfunction according to the procedure given in § 3.

An analysis of these processes will be carried out according to the parameters measured during bench and flight tests ($p_к$, $p'_к$, $p_м$, n, $p_о$, $p_г$, $G_{о\Sigma}$, $G_{г\Sigma}$, $G_о$, g, etc.) and not measured but making it possible to understand deeply the reason for a change in the parameters in the emergency situation and facilitating the analysis ($N_о$, $N_г$, $N_т$, k, k', the movement of the controlling elements x the expansion ratio of the gas on the turbine $\pi_т$).

In many respects the engine operation is determined by the specificity of the controlling elements. An analysis of results of the calculations shows that the controlling elements affect the quantitative and qualitative sides of the transient process during emergency situations. Therefore, it is expedient to carry out an analysis of the emergency situations both without taking into account the controlling elements and taking into account the concrete control system. This will make it possible to explain the effect of the operation of the concrete control system on the nature of the transient processes and parameters of the new steady-state system in typical emergency situations. Therefore, the initial problem of the investigation is to explain the nature of the transient process which appears during the emergency situation, the determination of the effect of the degree of malfunction and the place of the emergent malfunction on the nature of the transient process, and the operating mode of the uncontrolled engine with a reducing gas generator.

The emergence of the same form of malfunctions at different places of the hydraulic lines can cause a completely different qualitative course of the transient process.

With the overlap of the hydraulic lines, the determining factor is the place of its emergence: prior to the branch point, along the main line of the combustion chamber of gas generator. The quantitative and qualitative effect of closing will be completely determined by the place in which blockage of main line occurred.

Let us examine the transient processes which appear with the closing of the main line of the oxidizer in the uncontrolled engine (see Fig. 7.7). (The stabilizers in this case were considered as permanent resistances).

The nature of the transient process, which appears as a result of the closing of the section of the hydraulic line of the oxidizer up to the branch point to the gas generator (by scheme Д), for this engine is shown on graphs (Fig. 7.8). Plotted along the axis of the abscissas on these graphic representations was the time, and along the axis of the ordinates - the relative values which are the ratio of the instantaneous to the nominal value of the corresponding parameter:

$$\bar{p}_к = \frac{p_к}{p_к^{\circ}},\ \bar{p}'_к = \frac{p'_к}{p'^{\circ}_к};\ \bar{p}_о = \frac{p_о}{p_о^{\circ}},\ \bar{n} = \frac{n}{n^{\circ}},$$

$$\bar{G}_о = \frac{G_о}{G_о^{\circ}},\ \bar{p}_г = \frac{p_г}{p_г^{\circ}},\ \bar{G}_г = \frac{G_г}{G_г^{\circ}}\ \text{etc.}$$

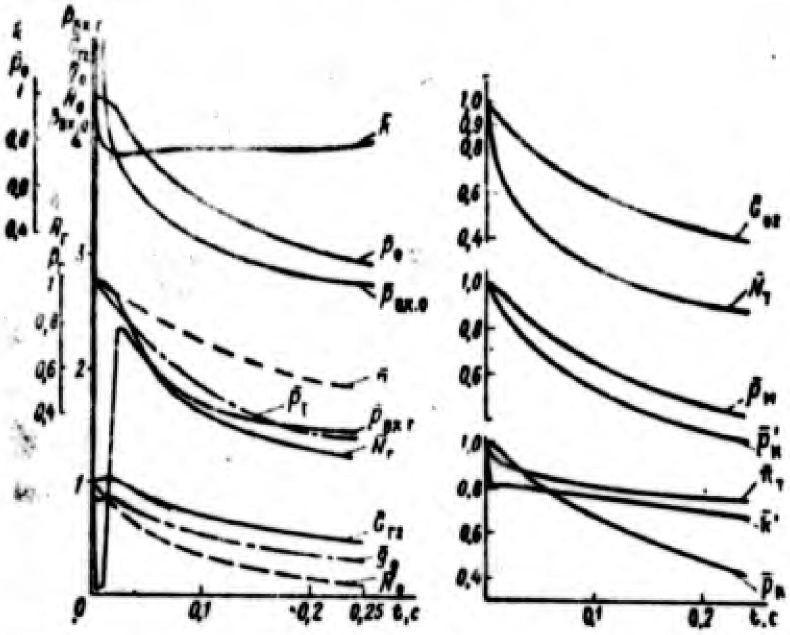


Figure 7.8. Closing of the section of the hydraulic line of the oxidizer from the pump to the branch point up to the gas generator of the uncontrolled ZhRD. Abbreviation: c = s.

It is evident that in the considered case a sharp decrease in the oxidizer flow in the engine and, consequently, in the flows through the combustion chamber and gas generator is observed. This leads to a decrease in the values of the fuel component ratios in the combustion chamber and gas generator (k and k').

Since the gas generator operates in the section of the sharp dependence of the efficiency of the gas on the relationship of the components, and the combustion chamber - in its sloping part, then $dR'T'/dk' > dRT/dk$. Consequently, the initial lowering of pressure in the gas generator will be more.

A decrease in the value π_T with a simultaneous reduction in the flow of gas through the turbine leads to a decrease in the available power of the turbine N_T. In engines without the afterburning of the producer gas a decrease in N_T is connected only with a decrease in the flow of the components which enter into the gas generator, since the combustion-chamber pressure in this case is not a counterpressure for the turbine.

A decrease in the oxidizer consumption unloads the pump which operates on this component and leads to a lowering of the power consumed by it.

The effect of N_T is more considerable. A sharp decrease in the revolution number of the turbopump unit occurs. As a result of this the pump heads, propellant component flow, and so on are decreased.

Due to an increase in the pressure differential along the main line (as a result of a sudden drop in pressure in the gas generator), initially the fuel flow increases, but a reduction in the pump head subsequently leads to a decrease in the fuel flow. As a result the reduced mode in comparison with the initial mode is established.

With the overlap of the main line of the oxidizer the pressure at the inlet into the pump of the oxidizer is sharply increased. At this moment at the inlet into the fuel pump there occurs a short-term lowering of the pressure with further sharp increase up to the values which exceed the nominal.

The rate of descent in the parameters and also the duration and magnitude of "peaks" and "troughs" of pressures at inlets into the pumps are determined by the closing of the main line.

Now let us turn to the examination of the steady-state operating mode of the engine. The greater the degree of closing, the lower the values of parameters which characterize the new operating mode, whereupon the values of the parameters of the new steady-state system to a considerable degree will be to determined by a decrease in the efficiency of the gas in the gas generator, i.e., by the dependence of RT=f(k).

The nature of the transient process, which appears in the engine with the closing of the main line of the oxidizer after the point of selection (Д) to the gas generator, is completely different than that with the closing of this main line up to the point of selection to the gas generator. In this case the design of engine operation, with afterburning or without afterburning of the producer gas, is important.

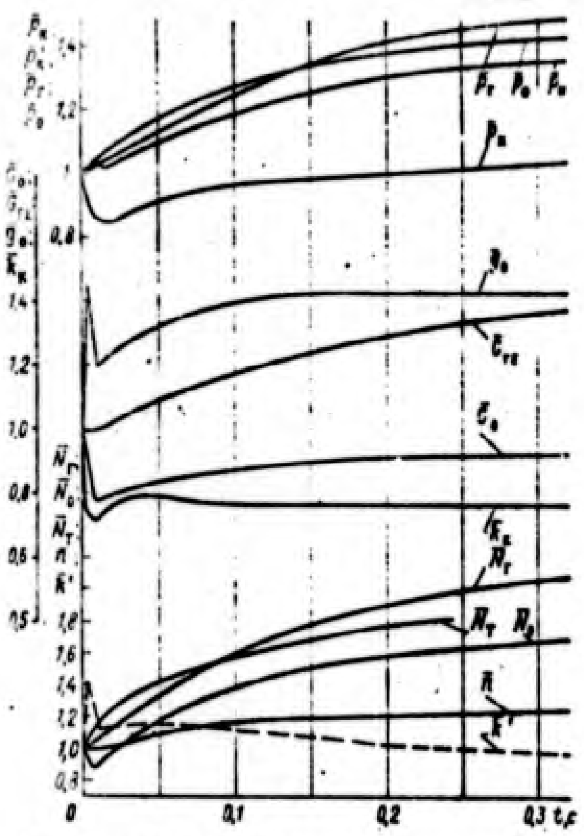


Figure 7.9. Transient process with the closing of the section of the hydraulic line of the oxidizer of the combustion chamber.
Abbreviation: c = s.

It is possible to note the following course of the transient process in the closing of sections 6-8 of the main line of the oxidizer of the combustion chamber (after point Д) of the engine with afterburning of the producer gas (Fig. 7.9).

As a result of the closing of the hydraulic line of the oxidizer of the chamber the flow into the combustion chamber is sharply decreased, and the flow through the gas generator is increased. This leads to a reduction in pressure in the combustion chamber and a pressure increase in the gas generator due to an initial increase in flow rate and an increase in the fuel component ratio k', which leads to an increase in the efficiency of the gas R'T'.

Since the combustion-chamber pressure is a counterpressure for the turbine, the lowering of it causes an increase in the expansion ratio of the gas on the turbine π_T and, connected with it, the available power of the turbine N_T.

An increase in N_T and also a reduction in the power input of the pump of the oxidizer, due to a decrease in the oxidizer flow, lead to an increase in the number of revolutions of the TNA. A consequence of this is the increase in all the basic parameters of the engine: the pump heads of the oxidizer and fuel, pressure in the gas generator, and fuel flow. After the initial rapid decrease, the oxidizer flow through the pump begins to increase, and this leads to an increase in the flow rate through the chamber and gas generator.

The engine emerges into the new steady-state conditions forced with respect to the initial conditions.

According to results of calculations of transient processes at different values of the closing of the main line of the oxidizer of the combustion chamber, let us construct the dependences of basic parameters of the steady-state mode of the ZhRD on the closing of main line - the ratio of the additional drop on the defined section to the combustion-chamber pressure $\left(\delta\overline{\Delta p}=\frac{\delta\Delta p}{\overset{\circ}{p}_\kappa}\right)$ (Fig. 7.10).

An analysis of these dependences shows that with an increase in the closing of the main line of the combustion chamber, the parameters of the steady-state mode of the ZhRD behave dissimilarly.

The parameters of the turbopump unit, the number of revolutions, pressures, and also pressure in the gas generator and the fuel flow monotonically increase with an increase in the closing. The rate of increase of these parameters decreases with an increase in the closing, which is connected with a decrease in the value of the correlation coefficient of the components and efficiency of the gas in the gas generator.

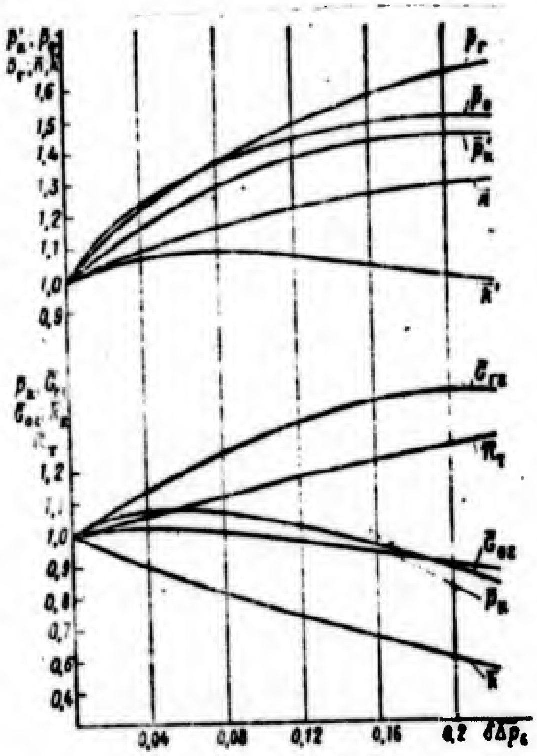


Figure 7.10. Dependence of the basic parameters of the ZhRD on the closing of the main oxidizer line of the combustion chamber.

Despite a continuous increase in the number of revolutions and pressures, the oxidizer flow through the pump and combustion chamber first increases up to a definite value of the closing and then begins to decrease. A decrease in the oxidizer flow in this case is connected with the operation of the pump of the oxidizer in the mode of partial cavitation in steady-state conditions. An increase in revolutions of the TNA leads to an increase in the critical pressure $p_{кр}$ at which the pump head according to the cavitation characteristic begins to drop. With an increase in the closing of the section of the hydraulic line of the oxidizer and an increase in the number of revolutions of the TNA, the depth of cavitation ($\varepsilon_к$) is increased. A sharp reduction in the power input of the pump of the oxidizer in this case is not observed due to a drop in the efficiency of the pump with a decrease in the ratio G/n. Thus, the boosting of the engine with an afterburner of the producer gas can be obtained with a definite closing. With a further increase in the closing, despite an increase in parameters of the TNA a lowering of the pressure in the combustion chamber is observed.

But if on the engine large cavitation reserves of the pumps will be provided for, then with an increase in the closing of the hydraulic line of the oxidizer in front of the combustion chamber boosting from all parameters of the engine will be observed.

With the closing of the main line of the fuel of the gas generator (the design scheme of the main line is shown on Fig. 7.11), it is possible to note the following features of the transient process (Fig. 7.12). A decrease in the fuel flow in the main line of the gas generator with closing causes an increase in the correlation coefficient of components k' and an increase in the efficiency of the gas in the gas generator (R'T'). However, a reduction in the total flow rate in this case exerts a greater effect than does an increase in the efficiency of the gas and causes a lowering of the pressure in the gas generator (in the considered range of the dependence (R'T')=f(k')). A relative change in the total flow of the components into combustion chamber in this case is much less than the flow into the gas generator, owing to which the combustion-chamber pressure falls less than that in the gas generator.

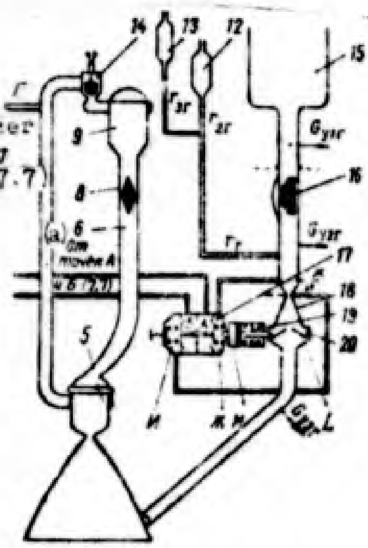


Figure 7.11. Design diagram of hydraulic lines of the fuel of the engine with afterburning of the producer gas: 5 - combustion chamber; 6 - gas line; 8 - turbine; 9 - gas generator; 12 - gas generator of pressurization of the fuel tank; 13 - gas generator of pressurization of the oxidizer tank; 14 - cut-off valve; 15 - fuel tank; 16 - pump; 17 - valve; 18 - Venturi tube; 19 - servo-throttle; 20 - grid of servo-throttle.
Key: (a) From points A and B.

As has already been stated, a decrease in the expansion ratio of the gas on the turbine π_T, together with a reduction in the flow of the gas through the turbine, causes a rapid decay in the available power of the turbine N_T. But in this case the power input of the fuel pump N_Γ decreases due to a decrease in the flow.

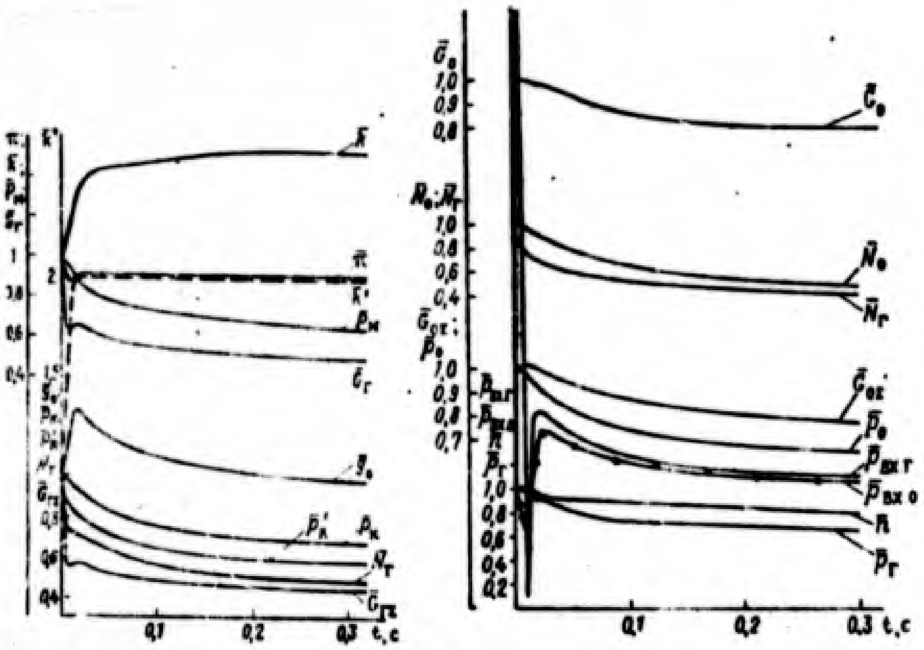


Figure 7.12. Transient processes with the closing of the main fuel line of the gas generator.
Abbreviation: c = s.

In connection with this, the number of revolutions of the TNA [THA - turbopump unit] begins to decrease, but the rate of this decrease is weakened by a reduction in the power input of the fuel pump.

A decrease in the revolution number of the TNA entails a reduction in pressures and flows of the pump and, consequently, pressures in the combustion chamber and gas generator. There is a decrease in the flow of the oxidizer into the gas generator, which in the beginning of the transient process sharply increases as a result of an increase in the drop along its main line from the pump to the gas generator.

Thus, as a result of the closing of the main fuel line of the ZhRD with the afterburning of the reducing producer gas, the engine operating mode is reduced. Occurring similarly is the transient process with the closing of the hydraulic line of fuel of the gas generator in the ZhRD, which operates by the scheme with the ejection of the producer gas. Parameters of the new steady-state operating mode of the engine are realized with increased values of the fuel component ratios in the gas generator and combustion chamber.

The study of transient processes with different closings makes it possible to draw the conclusion that the nature of the transient process in practice does not depend on the closing. With an increase in the closing of the hydraulic fuel line of the engine with the reducing gas generator, the engine operating mode is reduced. The behavior of the oxidizer flow along the main line of the gas generator and pressures at inlets into the pumps are the exceptions.

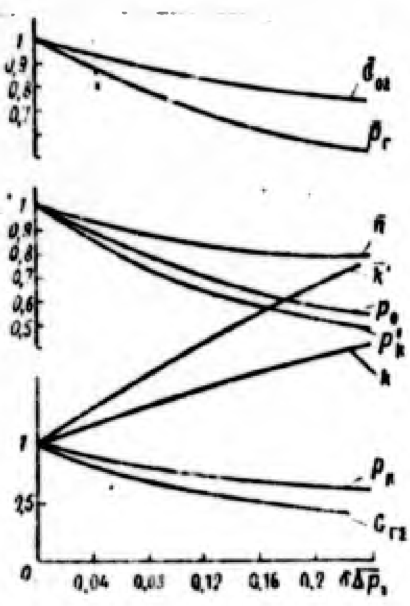


Figure 7.13. Effect of the degree of closing of the main fuel line on ZhRD parameters.

Let us examine the steady-state modes of the engine.

Figure 7.13 gives the dependences of the basic parameters of the steady-state operating mode of the engine on the degree of closing of the main fuel line ($\delta\overline{\Delta p}_1$). It is evident that the rate of the change in the parameters with an increase in the closing is variable: it is maximum at small values of closing and decreases with an increase in it. This is connected with an increase in the operation of the gas in the gas generator R'T' due to an increase in the relationship of components k', which parries a decrease in the total flow through the gas generator. This fact explains the comparatively high operating mode of the ZhRD at large values of the closing of the hydraulic fuel line. But with an increase in the closing the danger of burnouts of the gas lines increases due to an increase in the temperature of the producer gas.

§ 5. TRANSIENT PROCESSES WITH LEAKAGE OF THE HYDRAULIC LINES

The leakage of the main line can lead to a considerable change in the engine operating mode and sometimes to the complete cessation of the operation.

The leakage is characterized by the appearance of a new flow, i.e., leakage. The rate of an increase in this flow is very great due to the large gradients operable on the formed openings. In the initial period of leakage (approximately after 0.01 s) the flow through the formed opening reaches the maximum value. A further decrease in the engine operating mode causes a decrease in the value of leakage. Thus, the appearance of leakage occurs with the initial jump, and the larger this value, the more formed the opening.

The appearance of the flow of leakage leads to the fact that the coefficient of weight ratio of the propellant components in the engine considerably differs from the relationship of components in the combustion chamber. It is logical that the total flow of the components in the engine is greater than the sum of the flows in the combustion chamber and gas generator.

The nature of the transient process which appears during the appearance of leakage of the hydraulic main fuel line of the gas generator is shown in Fig. 7.14. A decrease in the flow rate in the main line of the gas generator $\overline{G}_{\Gamma}$ causes a rapid decrease in pressure in the gas generator despite an increase in the efficiency of the producer gas.

A relative change in the total flow rate in the combustion chamber is less than that in the gas generator, and in connection with this a lowering of the pressure in the combustion chamber is less considerable. The process is accompanied by a decrease in the expansion ratio in nozzles of the turbine $\overline{\pi}_{T}$. The joint effect

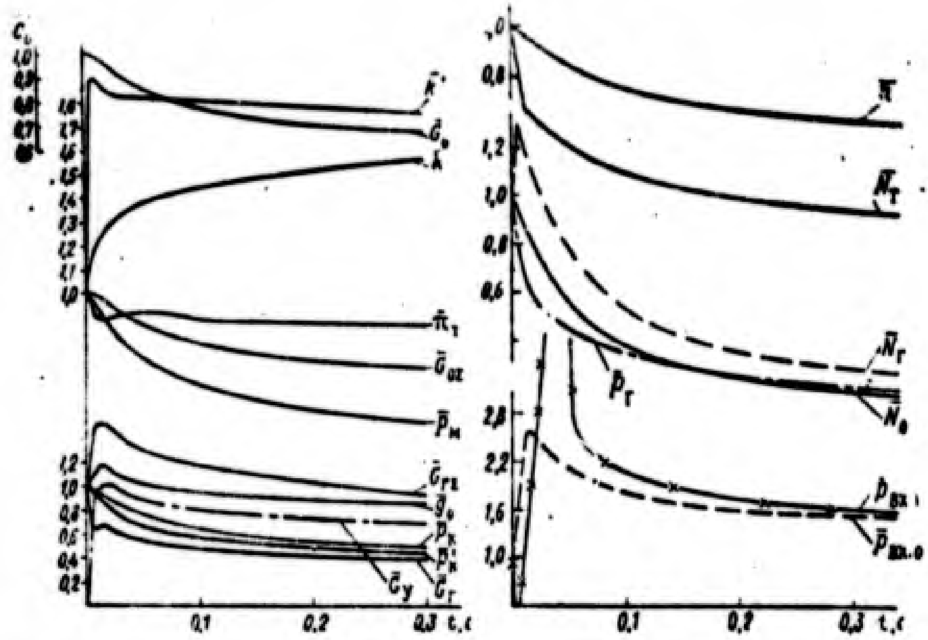


Figure 7.14. Transient process with leakage of the hydraulic fuel line.
Abbreviation: c = s.

of a decrease in the flow of gas through the turbine and $\bar{\pi}_T$ causes a rapid decrease in the available power of the turbine $\bar{N}_T$.

The appearance of leakage leads to an increase in the flow of fuel $\bar{G}_{\Gamma\Sigma}$ through the pump and, consequently, to an increase in the power input of the pump $\bar{N}_{\Gamma}$.

Thus, if with the closing of the main line a decrease in the available power of the turbine was somewhat compensated for by a decrease in the power input of one of the pumps, then with leakage a change in one and another parameter is directed to the side of a decrease in the number of revolutions of the TNA. In connection with this, the rate of the change in the number of revolutions $\bar{n}$ with leakage is more than that with the closing of the main line (see Fig. 7.12). The engine emerges into the new steady-state conditions, which is characterized by the reduced values of the parameters in comparison with the nominal.

The destruction of the hydraulic line is accompanied by a sharp decrease in the pressure at the input into the pump of this component with the subsequent increase and "jump" higher than the nominal. By the decrease in the value of pressure at the inlet into the pump up to the critical the short-term operation of the pump in the mode of partial cavitation is determined. At the inlet into the pump of the second component at this moment, a pressure increase with a subsequent smooth drop is observed.

On the basis of results of calculations of transient processes at different values of leakage, let us construct the dependences of parameters of the steady-state operating mode of the ZhRD on the degree of leakage, which we will characterize by the dimension of the formed opening (Fig. 7.15). An analysis of these dependences shows that with an increase in the leakage of the main fuel line of the gas generator the engine operating mode is reduced. The value of leakage is determined by the dimension of the formed opening $\mu F_{отв}$. The flow of leakage on the graphs was determined by the ratio of that leakage flowing to the maximum value in the given transient process:

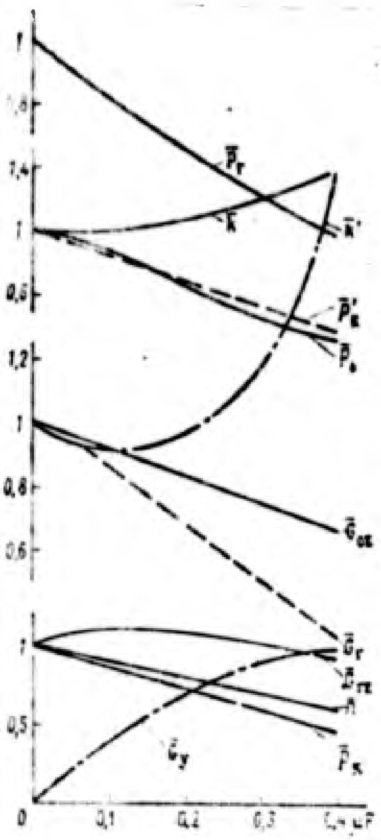


Figure 7.15. Dependence of the basic parameters of the ZhRD on the degree of leakage of the hydraulic fuel line.

$$\overline{G}_y = \frac{G_y}{G_{y\,max}}.$$

However, the dependence $\overline{G}_y = f(\mu F_{отв})$ has a maximum. A further increase in $\mu F_{отв}$ causes an initial increase in the flow of leakage, but as a result of a considerable reduction in the engine operating mode the flow of leakage is decreased.

An increase in temperature in the gas generator due to an increase in k' with considerable leakage is dangerous for material part of the engine due to the possibility of burnouts of gas lines.

With the emergence of leakage of the hydraulic the oxidizer line, just as with overlap, the nature of the transient process is determined by the place of the emergence of the malfunction.

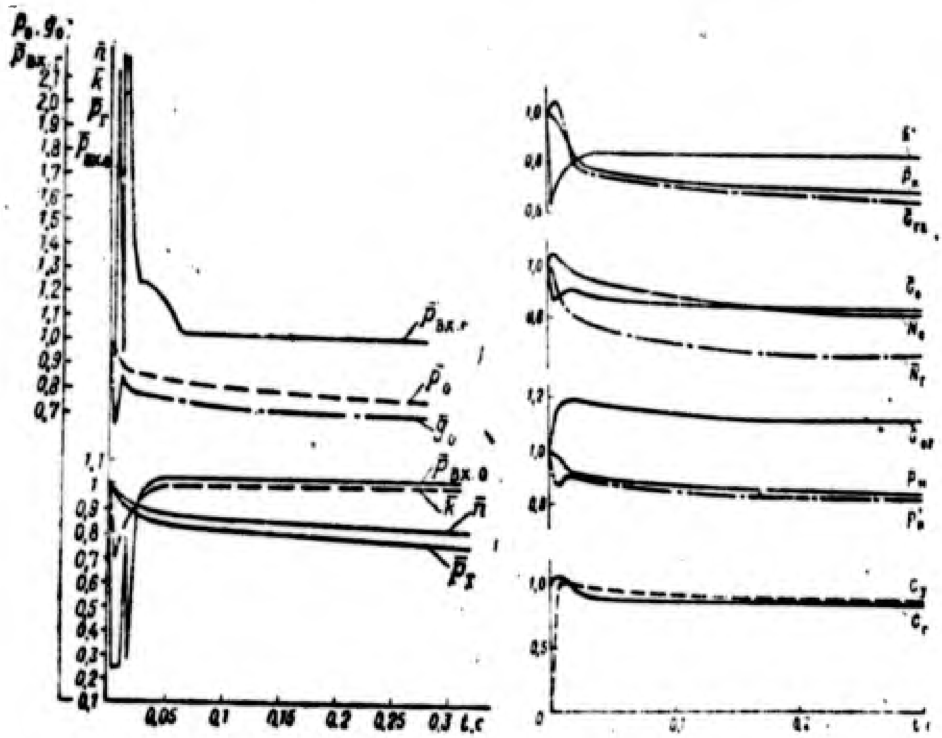


Figure 7.16. Transient process with leakage of the section of the hydraulic line of the oxidizer up to the exhaust point to the gas generator. Abbreviation: c = s.

The transient process with leakage of the section of the main line up to the exhaust point to the gas generator is shown in Fig. 7.16. It is characterized by a reduction in the basic parameters of the operating mode of the ZhRD.

Figure 7.16 shows that there occurs simultaneously a decrease in the oxidizer flow to the combustion chamber and gas generator, but due to the appearance of leakage the total flow through the pump increases, and the power input of the oxidizer pump is increased. A reduction in the oxidizer flow along the main line of the gas generator leads to a sharp decrease in the correlation coefficient of the components and the efficiency of the gas in the gas generator, which causes a rapid decrease in the pressure in the gas generator.

As a result of an increase in the pressure differential along the main fuel line, due to pressure decrease in the gas generator, the beginning of the transient process is accompanied by an increase in the fuel flow of the gas generator.

A decrease in the correlation coefficient of the components in the combustion chamber does not cause a considerable decrease in efficienty RT. Therefore, in the chamber a smooth pressure decrease is observed. The engine emerges into the new steady-state conditions, decreased in comparison with the initial.

A distinctive feature of the transient process with leakage of the main oxidizer line of the gas generator is a decrease in the total flow through this pump as a result of the relatively low values of leakage and nominal oxidizer flow to the gas generator ($g_c < 4\% \ G_o$). The appearance of leakage along the main line of the gas generator is completely compensated for by an insignificant decrease in the flow in the combustion chamber. The flow through the pump in this case is initially also decreased, and the power input of the oxidizer pump is decreased.

Thus, if with the leakage of the section of the main line up to the exhaust point to the gas generator the flow through the pump of the oxidizer increased, and this led to a sharp pressure decrease at its inlet, then with leakage of the gas-producing hydraulic line itself these parameters are changed in an opposite manner.

An analysis of parameters of the steady-state operating mode of the ZhRD at different values of leakage shows that with an increase in leakage the engine operating mode is reduced.

A reduction in the engine operating mode is accompanied by an increase in the rate of decrease in the parameters, an increase in the depth and duration of "peaks" and "troughs" of the inlet pressures, and an increase in the initial jump in the flow of leakage.

With the emergence of the leakage of the hydraulic line of the oxidizer before the combustion chamber after point Д, the nature of the transient process qualitatively differs from the transient process with leakage of the main oxidizer line up to it or the main line of the gas generator (11-14). These distinctions are connected with the design of the engine installation and with the direct effect of the combustion chamber on the operation of the turbine in engines with afterburning of the producer gas. This leakage causes a sharp decrease in the flow and correlation coefficient of components in the combustion chamber, which is accompanied by a pressure drop in the chamber (Fig. 7.17). It somewhat also decreases the oxidizer flow in the main line of the gas generator and leads to a decrease in the pressure in it. But this decrease is substantially less than that in the combustion chamber. The expansion ratio of the gas in nozzles of the turbine increases, and its available power increases. This causes an increase in the number of revolutions of the TNA and heads of the oxidizer and fuel pumps. The flows of the components through the pumps are increased.

Because of this the pressure in the gas generator and combustion chamber increases. But if pressure in the gas generator, in increasing, exceeds the initial value (oxidizer and fuel flows along the main line of the gas generator exceed the nominal), then the combustion-chamber pressure does not reach the initial value, since the oxidizer flow into the combustion chamber remains below the value which preceded the appearance of the malfunction.

Consequently, with leakage of the hydraulic line of the oxidizer in front of the combustion chamber in engines with afterburning of the reducing producer gas, the boosting of the engine operating mode from parameters of the turbopump unit occurs: $\overline{p}_{о}$, $\overline{p}_{г}$, $\overline{n}$, $\overline{p}'_{к}$, $\overline{N}_{о}$, $\overline{N}_{г}$, $\overline{N}_{т}$, and also $\overline{G}_{о\Sigma}$ and $\overline{G}_{г\Sigma}$. At the same time

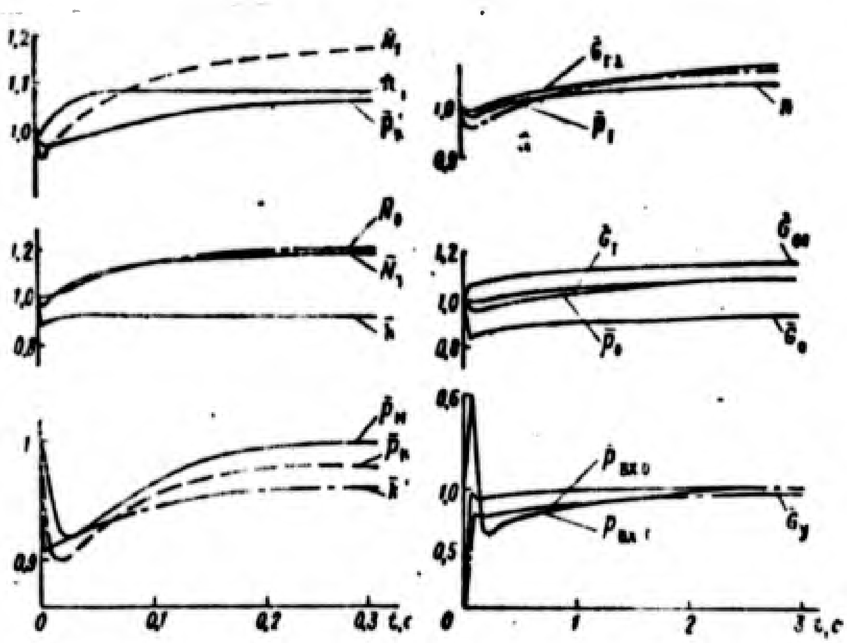


Figure 7.17. Transient process with leakage of the section of the hydraulic oxidizer line of the combustion chamber. Abbreviation: c = s.

the oxidizer flow $\bar{G}_o$ and combustion-chamber pressure are decreased. This course of the transient process is characteristic only to engines with afterburning of the producer gas. In engines with the ejection of the producer gas the throttling of the operating mode occurs.

Let us examine the steady-state operating mode. By utilizing results of calculations of transient processes at different values of $\mu F_{отв}$, let us construct the dependences of parameters of the steady-state operating mode of the ZhRD on the degree of leakage (Fig. 7.18). An analysis of these dependences shows that with an increase in the leakage of the main oxidizer line of the combustion chamber, the pressure in it is reduced. The parameters of the turbopump unit ($\bar{n}$, $\bar{p}_o$, $\bar{p}_г$, $\bar{p}'_н$) depend on the dimension of the formed opening: up to a definite value of $\mu F_{отв}$ they are increased and then decreased. This is explained by the sharp decrease in value of R'T' due to the appropriate reduction in the fuel component ratio in the gas generator k' and also by the operation of the oxidizer pump in the system of partial cavitation.

§ 6. TRANSIENT PROCESSES WITH LEAKAGE OF THE GAS VOLUMES OR MAIN LINES

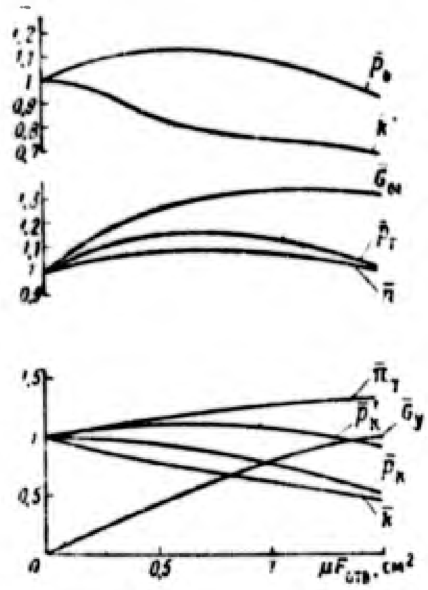


Figure 7.18. Dependence of basic parameters of the ZhRD on the degree of leakage of the hydraulic oxidizer line of the combustion chamber.

It is necessary to distinguish two cases with respect to the location of this gas volume: 1) up to the turbine, for example, the defect of the gas generator; 2) after the turbine (burnout of the gas conductor).

Leakage of the gas generator can begin due to the breakaway of branch pipes of the measurement of pressure in the gas generator and burnouts of the walls of the uncooled gas generator. The gas escape leads to a lowering in the pressure in the volume, a decrease in the flow rate of the gas, expansion ratio of it on the turbine and its available power. As a result a reduced (throttled) engine operating mode begins. Since from the gas generator there occurs leakage of the reducing gas, the correlation coefficient of the components in the combustion chamber somewhat increases with a constant correlation coefficient of the components in the engine.

Leakage of gas conductors after the turbine decreases the pressure in them. Since this pressure is counterpressure for the turbine, the flow rate through which was not changed, the expansion ratio of the gas in the turbine and its available power will be increased. In summation, the engine is boosted relative to the mode which preceded the appearance of the considered malfunction.

§ 7. TRANSIENT PROCESSES WITH THE MALFUNCTION OF PUMPS

Let us examine the emergency situations caused by the abnormal operation of the pumps. The initial cause of this abnormality can be the external and internal factors which act negatively on the operating mode of the pump. The number of environmental factors includes malfunctions in delivery pipe (its closing or the disturbance of airtightness), an increase in the number of revolutions of the turbopump unit, and the gas concentration of the components. These factors cause cavitation stalling of the operation of the pumps. The internal factors include defects in the pump itself: breakage of impellers and worm conveyors, the rubbing of floating rings, and breakage of bearings. They can be accompanied by explosions and the conflagration of the separate subassemblies of the pump.

This form of malfunctions presents the greatest difficulties in the explanation of reasons for the emergency outcomes of the tests. Even when by calculation it is possible to obtain the nature of the transient process similar to that which occurred during a full-scale test, there remains unexplained the question of whether or not this malfunction will lead to an explosion. With an explanation of such malfunctions, usually required is the simulation under bench-test conditions of the emergency situation, which occurred in a full-scale test.

The nature of the transient process with cavitation stalling of the operation of the pump due to the partial closing of the feed flow main line before the inlet into the oxidizer pump is shown in Fig. 7.19.

When the inlet pressure becomes lower than the critical, at which the pump begins to cavitate, the pressure created by it drops.

A distinctive feature of the transient process in this case is the initial increase in the number of revolutions of the TNA, the pressure of the noncavitating fuel pump and fuel flow through it due to a reduction in the load on the TNA as a result of a sharp decrease in the power consumed by the oxidizer pump.

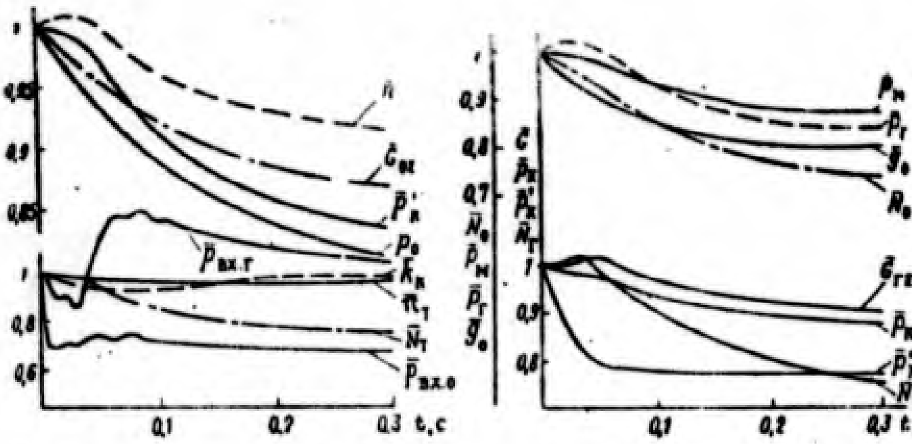


Figure 7.19. Transient process with the closing of the main oxidizer feed line. Abbreviation: c = s.

A decrease in the oxidizer flow into the gas generator leads to a decrease in the correlation coefficient of the components and, therefore, the efficiency of the gas R'T'. Therefore, despite the initial increase in the fuel flow, the pressure in the gas generator $\bar{p}'_{\kappa}$ falls. This, in turn, causes a lowerin in pressure inthe combustion chamber, however, less intense than that in the gas generator, since the dependence RT=f(k) on the section of the combustion chamber operation is very flat, and the efficiency of the gas is changed weakly. There is a decrease in the number of revolutions of the TNA and fuel pump head, and a further decrease in the pressure created by the oxidizer pump occurs. Established in the engine is a mode decreased in comparison with the initial mode.

The characteristic parameters, with respect to a change in which it is possible to judge the presence of the cavitation stalling of the operation of the pump as a result of the closing of feed line, are $\bar{p}_{o}$, $\bar{p}_{\Gamma}$, $\bar{n}$, $\bar{G}_{o}$, $\bar{G}_{\Gamma}$, $\bar{p}_{\kappa}$, $\bar{p}_{вх.о}$, and $\bar{p}_{вх.\Gamma}$. The transient process in this case is accompanied by a reduction in the flow of the cavitating pump. If with the closing of main line behind the pump such behavior of the flow caused the initial "peak" in pressure and a pressure increase at the inlet into pump in steady-state conditions, then with the closing of the main feed line the pressure at the inlet into pump is sharply decreased and remains below the initial in steady-state conditions. The transient process with respect to the inlet pressures is accompanied by fluctuations in pressure of small amplitude.

This behavior of pressure at the inlet into the pump will be recorded during the engine tests if the closing of the main line occurs before the site of installation of the sensor of the pressure measurement. But if the closing of the main feed line occurs after the place of placing of the sensor, then the pressure sensor will record the initial pressure increase. Recordings of the pressure sensor in this case will not reflect a real change in the pressure at the inlet into the pump.

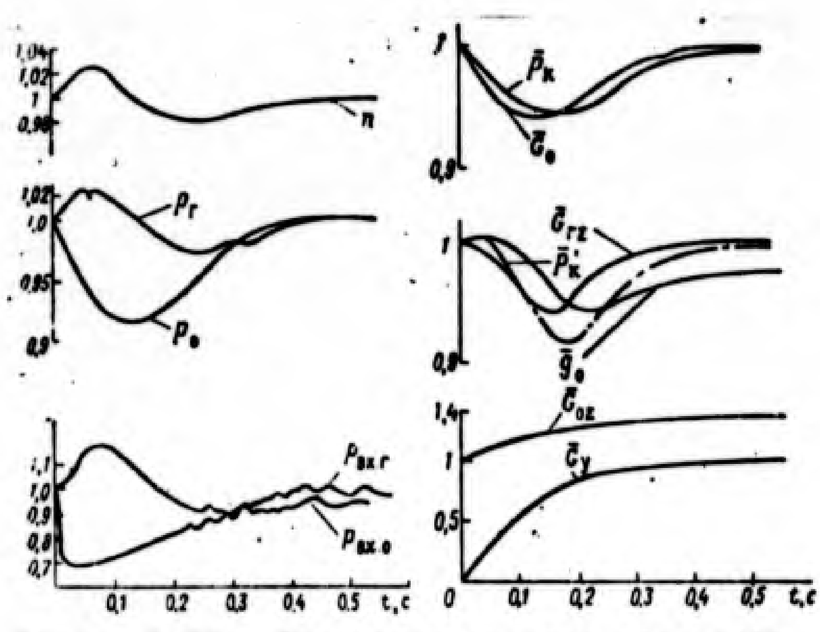


Figure 7.20. Transient process with leakage of the main feed line.
Abbreviation: c = s.

The nature of the transient process with cavitation stalling of the pump, caused by the leakage of the main feed oxidizer line, is shown in Fig. 7.20. Here a reduction in the mode occurs both as a result of a decrease in pressure due to cavitation with a reduction in the inlet pressure below the critical and as a result of a decrease in the flow through the oxidizer pump due to the appearing leakage of the component before the pump.

A sharp decrease in the pressure and flow through the oxidizer pump leads to a reduction in the power input of the pump. Because of this in the beginning of the transient process the number of revolutions of the TNA, the pressure created by the fuel pump, and fuel flow along main lines of the engine are increased.

A decrease in the relationship of components in the gas generator and in the total flow through the gas generator causes a pressure decay in it and a reduction in the number of revolutions of the TNA, the pump heads and combustion-chamber pressure.

A decrease in the pressure at the inlet into the oxidizer pump due to the appearing leakage causes an increase in the pressure differential in main feed line from the tank to the inlet into the pump. This leads to an increase in the flow from the tank.

But if with leakage of the main line after the pump the beginning of the transient process was accompanied by a sharp increase in the flow of the leakage, and subsequently this expenditure was decreased, then with the appearance of leakage from the main feed line an increase in this flow up to a definite value throughout the transient process occurs. The total flow from the tank is similarly changed.

A decrease in the flow after the pump leads to a pressure increase at the inlet into the oxidizer pump. This causes an increase in pressure, since the pump transfers over to the section of the cavitation characteristic during which the very weak dependence of pressure on the pressure at the inlet is observed.

An increase in the pressure and flow through the oxidizer pump leads to an increase in the flow rate and pressure of the combustion chamber and flow rate and pressure of the gas generator. As a result the numbers of revolutions of the TNA and heads of the oxidizer and fuel pumps are increased.

With the small hydraulic friction of the section of the main line in which the destruction occurred, the new steady-state conditions do not differ from the initial, since with an increase in the total flow rate from the tank the hydraulic losses in the section up to the location of destruction remain comparatively small.

Malfunctions of an internal nature caused by breakdowns of the worm conveyors and impellers of the pumps lead to a change in the geometry of the flow part of the pump and, consequently, to a change in the head characteristic of the pump. It is possible to note the distinctive feature of these kinds of malfunctions: the pressure and flow rate through the pump do not correspond to the number of revolutions of the TNA. Therefore, if the nonconformity of pressure and flow rate through the pump to the number of revolutions of the TNA is noted (the pump operates not according to the design characteristics), then it is possible to assume that in the flow part of the pump the breakages occurred.

Malfunctions connected with breakages in the bearings of the pumps and the rubbing of the floating rings lead to an increase in the moment of resistance of the pump. These forms of malfunctions can be simulated by a decrease in the efficiency of the pumps. The approximate nature of the transient process in the case of a decrease in the efficiency of the fuel pump is shown on Fig. 7.21. A sharp increase in the power input of the pump due to a reduction the efficiency of the pump leads to a reduction in the revolution number of the TNA. This, in turn, causes a decrease in pressures of the propellant components created by the pumps.

A reduction in the oxidizer and fuel flows leads to a decrease in the pressure in the gas generator and combustion chamber. It should be noted that there subsequently occurs a disproportionate decrease in the oxidizer and fuel flows, and the fuel flow drops more intensely; and therefore the ratio of components in the combustion chamber is increased.

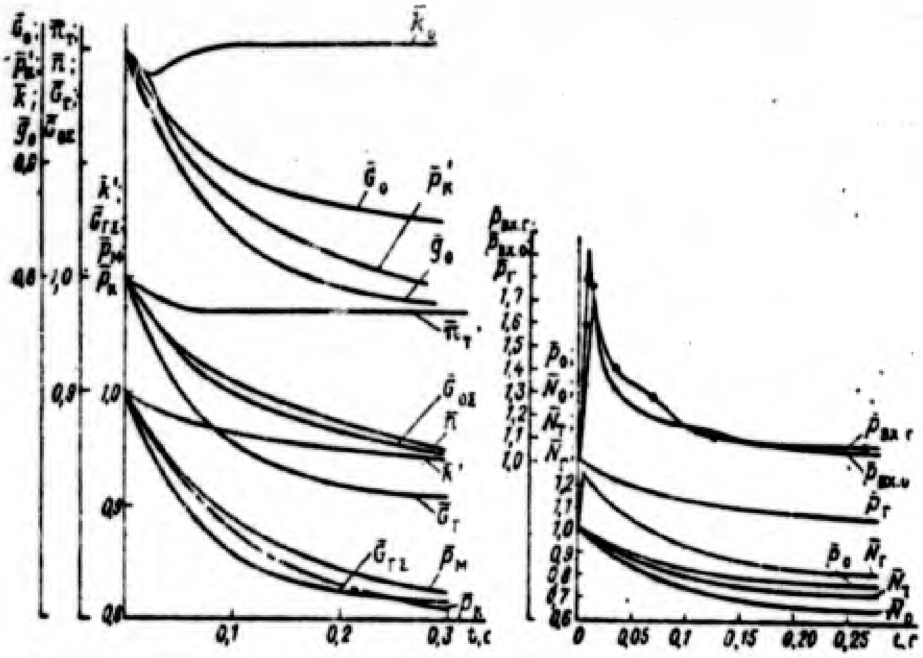


Figure 7.21. Transient process with a decrease in efficiency of the pump.
Abbreviation: c = s.

The steady-state mode, which corresponds to the considered form of malfunction, is decreased in comparison with that which preceded its appearance.

The characteristic feature of the transient process in the reduction of the efficiency of the pumps is the fact that in the beginning of the transient process the rate of decrease in the number of revolutions of the TNA exceeds the rate of descent in pressure in the gas generator. A reduction in heads of the oxidizer and fuel pumps is synchronous. These parameters are most sensitive to these forms of malfunctions.

The passage of the transient process with a decrease in the efficiency of the oxidizer pump is similar to that examined above.

With a considerable reduction in the efficiency of the oxidizer or fuel pumps, the transient process occurs with an increase in the coefficient of the weight ratio of the propellant components in the gas generator. This leads to an increase in temperature in the gas generator and can be the reason for the burnouts of the gas main lines of the engine.

§ 8. EFFECT OF THE CONTROL SYSTEM OF THE ZhRD ON THE NATURE OF TRANSIENT PROCESSES DURING EMERGENCY SITUATIONS

The operation of the engine installation, its mode parameters and the nature of the transient process depend substantially on the operation of the control system used on the engine.

For an explanation of the effect of the control system on the nature of the transient process and the mode parameters of the ZhRD, in emergency situations calculations of the transient processes caused by different malfunctions in the engine with afterburning of the reducing producer gas with a specific control system were carried out. The calculations were conducted over a wide range of changes in the parameters characterizing the malfunction: $\mu F_{\text{отв}}$, $\delta\overline{\Delta p}$, η, and others, which lead to a considerable change in the engine operating mode (up to 80%). A comparison of results of these calculations with results of calculations of emergency situations in an uncontrolled engine of similar design (see § 4-7) made it possible to conduct an analysis of the operation of the control system with different emergency situations and explain its effect on the nature of the transient processes and parameters of the new steady-state mode.

Since with the emergence of emergency situations, as a rule, there occurs a considerable change in the ZhRD mode, in the study of the operation of controllable engines a sufficient differential on the controlling elements for the final development of the disturbance acquires special importance. This requires a necessary account of limitations on the movement of the control element, which reflect the real operation of the control system of the engine.

An investigation of the effect of the control elements on the operation of the engine installation during emergency situations is conducted in an example of the operation of the system of the internal engine stabilization of the relationships of the propellant components.

The stabilization of the relationship of components on the considered design of the engine (see Figs. 7.7 and 7.11) is provided for by the operation of two stabilizers. The stabilizer of the flow includes flow-meter devices: Venturi tube 3 and 18, servothrottle 19 and valve 17. It provides on the main fuel line of the engine installation a flow proportional to oxidizer flow through the Venturi tube 3, maintaining a constant coefficient of the weight ratio of components in the engine $k_{дв}$=const.

The pressure stabilizer 7 maintains pressure $p_{н.ст}$ at point В of the bleed of feedback behind itself on the main oxidizer line of the gas generator (see Fig. 7.7) equal to the control pressure on the main fuel line ($p_{унр.ст}$) (at point Г in Fig. 7.11). With the equality of pressures ($p_{н.ст}=p_{унр.ст}$) and the equality of differentials on sections of the oxidizer and fuel main lines from the exhaust points В and Г up to the gas generator (the necessary differentials are provided for by the selection of the discharging nozzle 11 with adjustment of the engine), the pressure stabilizer will maintain the constant relationship of the propellant components in the gas generator.

An analysis of the operation of the stabilization system will be carried out with the enlistment of the following parameters: movements of the controlling elements of stabilizers of flow rate and pressure ($x_с$ and $x_н$), differentials on the controlling elements of the stabilizer of flow rate ($\Delta p_с$) and pressure stabilizer ($\Delta p_н$). An increase in differentials $\Delta p_с$ and $\Delta p_н$ correspond to an increase $x_с$ and $x_н$.

The conducted investigations show that the nature of the transient process and parameters of the new steady-state system with the emergency situations of the engine with the stabilization system of the relationship of the propellant components depend on the place of origin of the malfunction - up to or after the exhaust points to the controlling elements. Thus with the appearance of leakage after the Venturi tube on the main line of the combustion chamber and gas-generator oxidizer, the flow through the Venturi tube is increased, in connection with which the differential on tube 3 increases (see Fig. 7.7), the diaphragms of the slide valve is moved toward the nozzle, increasing the differential on the nozzle-valve (Fig. 7.22); the pressure above the piston of the servothrottle is reduced (Fig. 7.23). The piston begins to be moved toward the opening (x_c is decreased), decreasing the differential on the controlling element of the flow stabilizer - grid of the servothrottle. The fuel flow along the main line also increases.

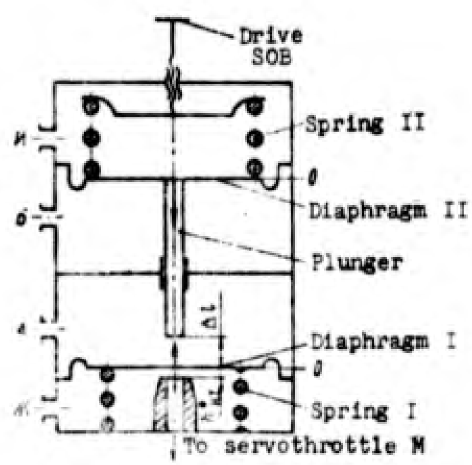


Figure 7.23. Slide valve.

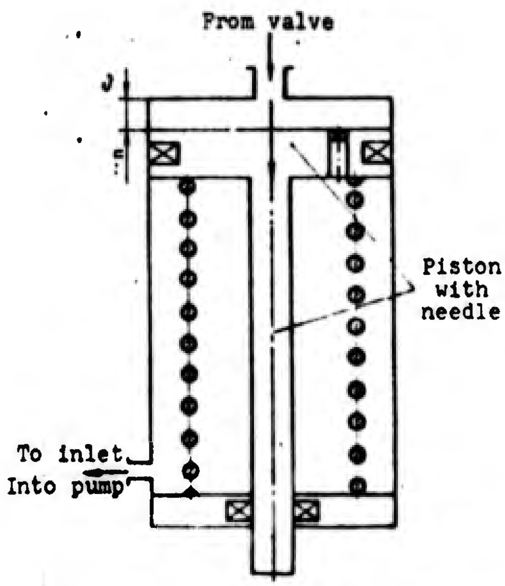


Figure 7.23. Servothrottle.

The controlling pressure ($p_{упр.ст}$) for the stabilizer installed on the main line of the oxidizer of the gas generator is the pressure bled on the main fuel line of the gas generator. An increase in this pressure leads to the movement of the diaphragm of the stabilizer toward the nozzle and an increase in the differential on nozzle-valve (Fig. 7.24). Because of this the pressure above the piston in cavity E is decreased. The piston begins to be moved toward the detent of minimum friction of the main oxidizer line. The differential on the needle of the pressure stabilizer is decreased.

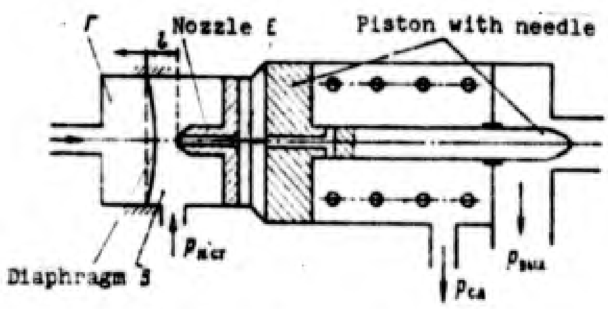


Figure 7.24. Pressure stabilizer.

Because of the work of the controlling elements, which decrease the differentials along the main lines of the gas generator, in the considered emergency situation the boosting of the engine operating mode from all the basic parameters (in the range of the control).

If the closing or leakage appear behind exhaust point B to the pressure stabilizer, the ratio of the propellant components on the gas generator is immediately disturbed. In this case the emergence of the malfunction on the main oxidizer line leads to a sharp reduction in the engine operating mode, and the operation of the pressure stabilizer in this case aggravates the exhaust point Г of the control pressure on the pressure stabilizer (in the range control), the boosting of the engine operating mode as a result of the "opening" of the pressure stabilizer due to the relatively high pressure $p_{упр.г}$ (at point Г), an increase in the oxidizer flow and an increase in the efficiency of gas in the gas generator occur.

With the closing of the hydraulic fuel line up to the exhaust point Г of the control pressure on the pressure stabilizer the beginning of the transient process is accompanied by a decrease in the basic parameters (excluding k and k'). If the emergent malfunction is found in the range of the control of the engine, after the initial reduction in parameters of the ZhRD their increase up to values which preceded the emergency situation is provided. This course of the transient process is connected with the movement of the piston of the flow stabilizer toward the detent of the minimum friction and a decrease in the differential on the grid of the servothrottle of the stabilizer. As a result of the closing of the main line as soon as the available differential on the controlling element is exceeded the piston of the flow stabilizer will reach the detent of minimum friction ($x_c=0$). The parameters of the steady-state engine operating mode $\bar{p}_к$, $\bar{p}'_к$, $\bar{p}_o$, $\bar{p}_г$, $\bar{n}$, $\bar{G}_{o\Sigma}$, and $\bar{G}_{г\Sigma}$ will begin to be decreased (Fig. 7.25).

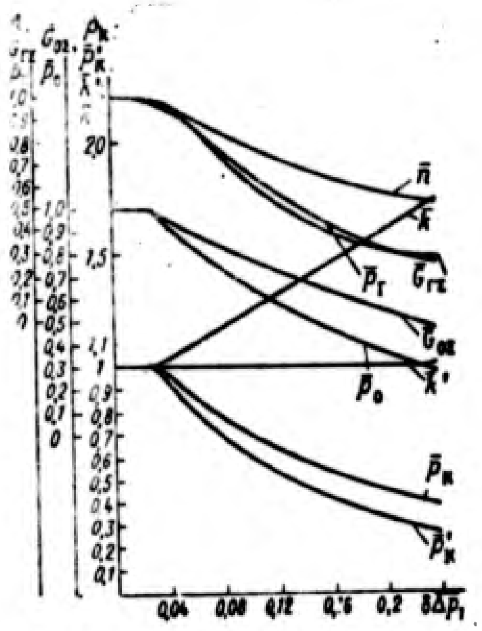


Figure 7.25. Effect of degree of closing of the main fuel line on parameters of the controllable ZhRD.

In certain cases (with the malfunctions of the pumps, the closing of the hydraulic line of the oxidizer up to the point Д (see Fig. 7.7), the closing of the main fuel line up to point Г (see Fig. 7.11) of the bleed of the control pressure to the stabilizer, the leakage of the main fuel and oxidizer line to the Venturi tubes) this control system does not affect the nature of the transient process. But because of the operation of the flow and pressure stabilizers (in the range of control) the operating mode of the ZhRD, which is established with these malfunctions, is substantially increased. Thus, for instance, with the identical closing of the section of the hydraulic oxidizer line to the Venturi tube ($\delta\overline{\Delta p}=0.4$), the operating mode of the uncontrolled engine is $p_к=0.2p^*_к$, $p'_к=0.13(p'_к)^*$, the operating mode of the adjustable engine is $p_к=0.9p^*_к$, $p'_к=0.9p^*_к$.

With the appearance of malfunctions of the type of closing and leakage of the section of the hydraulic line of the combustion chamber and the leakage of the section of the main line of the gas generator up to the stabilizer of pressure, the operation of the stabilization system of the ratio of the components leads to a qualitatively new course of transient processes.

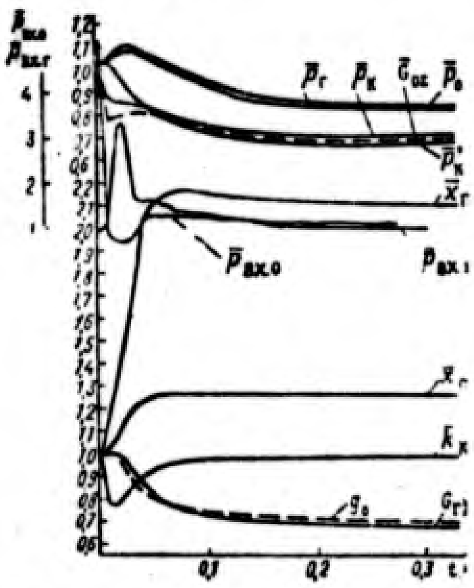


Figure 7.26. Transient process with the overlap of the section of the hydraulic oxidizer line of the combustion chamber of the controllable ZhRD.

Thus, with the closing of the hydraulic oxidizer line of the combustion chamber, the nature of the transient process which appears in the controllable engine (Fig. 7.26) is completely different than that in the uncontrollable. The beginning of the transient process occurs similarly. But the flow stabilizer follows the oxidizer flow through the Venturi tube, which is decreased as as result of the closing, increases the differential on the grid of the servothrottle, and decreases the fuel flow. In accordance with a decrease in control pressure ($p_{\text{унр.г}}$) at point Г, the piston of the stabilizer 7 (see Fig. 7.7) is also moved toward the detent of maximum friction, increasing the differential on the controlling element. The oxidizer flow along the main line of the gas generator is decreased. This leads to a lowering of the pressure in the gas generator, the number of revolutions of the TNA and so on. Consequently, after the initial boosting the parameters of the ZhRD are decreased.

The appearance of leakage of the hydraulic oxidizer of the combustion chamber (in the range of control) leads to the boosting of the engine operating mode from all the basic parameters. This occurs because of the operation of the flow and pressure stabilizers, which ensure in this emergency situation an increase in the oxidizer and fuel flow in main lines of the gas generator (Fig. 7.27).

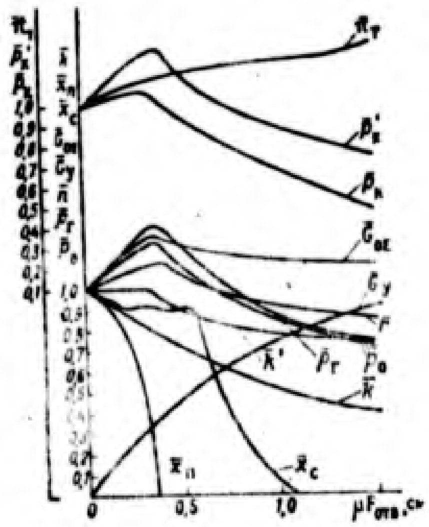


Figure 7.27. Dependence of basic parameters of the controllable ZhRD on the degree of leakage of the hydraulic oxidizer line of the combustion chamber.

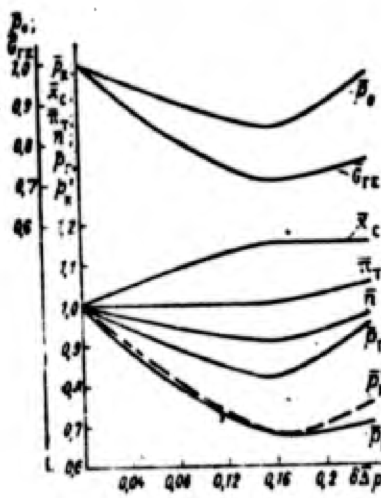


Figure 7.28. Dependence of the basic parameters of the controllable ZhRD on the closing of the main oxidizer line of the combustion chamber.

The effect of the degree of malfunction of those emergency situations in which the operation of the controlling elements causes a qualitatively new course of the transient process leads to the ambiguity of the behavior of the basic parameters of the steady-state system of the controllable engine. For example, the parameters of the new steady-state system up to a definite closing of the section of the hydraulic oxidizer line of the combustion chamber are decreased, and with its further increase they begin to increase (Fig. 7.28). This ambiguity is connected with achievement by the control element of the stabilizer of the flow of the position of the "detent", which corresponds to the maximum friction (x_c=const). In this case the ratio of the propellant components on the engine is disturbed, and the expansion ratio of the gas on the turbine and the available power of the turbine are increased.

CHAPTER VIII

MODELING OF PHYSICAL PROCESSES IN THE FINAL DEVELOPMENT OF ELEMENTS AND SYSTEMS OF THE ENGINE INSTALLATION

The theory of similarity finds wide practical application in the physical modeling of different processes. In this case the essence of the latter entails the fact that the investigated process is studied not on the full-scale object but on its physical model. The obtained results can be transferred to the real process by means of the corresponding conversion when the model is similar to the natural one.

§ 1. APPLICATION OF THE THEORY OF SIMILARITY TO THE EXPERIMENT

Any physical experiment is limited to the empirical study of the specific article according to the definite previously confirmed program. However, the purposes which in this case are posed cover a wider circle of problems than the determination of parameters or properties of the investigated unit article. As a rule, one strives to extend the test results to the totality of the articles measured by the entire volume of the commodity production or to the investigated process as a whole. Together with mathematical statistics, the theory of similarity serves as this purpose.

The theory of similarity determines that region or section of the surface of response to which it is possible to extend the results of the unit experiment. Moreover, the possibility to compose the system of integral equations in criterial form which describe the simulatable process is created.

The most important, in a practical respect, field of application of the theory of similarity consists in the development on its basis of methods of physical modeling. Ensuing directly from the third theorem are the conditions and prerequisites which determine the similarity of the model to full-scale. The initial stage of the modeling is the establishment of conditions of uniqueness, i.e., the determining parameters and similarity criteria. In the study of the process on the reduced model on the basis of the equality by the similarity criterion, its geometric dimensions are determined, and the working drawings, on which the model is prepared, are compiled. The parameters which enter into the condition of uniqueness for the model and full-scale must be proportional.

According to the third theorem, the model is similar to nature if the following three conditions are observed:

1) the geometric similarity of the model and full-scale is provided;

2) the physical constants of the model are proportional to the appropriate constants of the natural process, including the boundary conditions;

3) the corresponding similarity criteria for the full-scale and the model are equal to each other.

It is completely natural that a precise observance of all conditions of similarity is possible only in very rare events, and, therefore, in practice all the more frequently one resorts to methods of the approximate similarity with the modeling of complex physical processes [3, 29].

Models with transparent walls play an important role in the study of aero-, gas- and hydrodynamic processes. In this case it is possbile to observe the physical pattern of the course of the processes and, if necessary, record it with the aid of a camera or movie camera. This method finds wide practical use in different fields of technology.

In the region of the experimental final development of the assemblies of the ZhRD [ЖРД - liquid-propellant rocket engine], for example, at the Lewis Research Center (NASA) aerodynamic investigations of turbines of the oxygen-hydrogen engine M-1 on scale models were carried out [102]. Experiments were carried out on physical models made in scales of 0.450 and 0.646 with respect to full-scale. Used as the working medium was air. For visual observations of the flow, a transparent model of the collector of the turbine was made. Investigations were conducted by the method of photographing of smoke flow, which makes it possible to estimate the uniformity of the flow of the working body along the channels of the collector, the absence of circulation and similar phenomena, which are very difficult to be determined by any other method. As a result was selected the optimum shape of the turbine blades, the values of efficiency were determined, and the interconnection between losses in total pressure and rates of flow in the inlet branch pipe was established.

At present, with the aid of the physical models, it is possible to study the hydrodynamics of the gas flow and the convection heat transfer virtually of any thermal apparatus and engine, including the ZhRD. However, the latter is the totality of complex systems of a different class, and the methods of final development used for the combustion chambers and ZhGG [ЖГГ - liquid-gas generators] must be different for fuel pumps or feed systems. For this reason cannot be developed are single methods of modeling of all systems of the ZhRD. In each concrete case, in connection with each class of systems and assemblies, they will be different.

Let us examine now the problem of providing the similarity of centrifugal pumps by utilizing fundamental therorems of similarity. The design and final development of high-speed pumps for high-flow rates and pressures can require the creation of scale models just as for turbines [102]. Simultaneously with this, the problem of the selection of dimensions of the model under the condition of similarity to its full-scale will arise. As was already mentioned above, full similarity assumes to be, first of all, the geometric similarity, i.e., the ratio of the corresponding dimensions in accordance with the established scale. At the same time for centrifugal pumps the kinematic and dynamic similarity, i.e., the geometric similarity of the velocity triangles, pressures, flow rates, powers, etc., must be provided.

If we introduce the notations for the geometric dimensions of the model and full-scale (index "м" refers to the model, index "н" - to full-scale).

$$a_м, b_м, \ldots, l_м; \quad a_н, b_н, \ldots, l_н,$$

then the constant of geometric similarity C_l will be equal to the relations

$$C_l = \frac{a_м}{a_н} = \frac{b_м}{b_н} = \ldots = \frac{l_м}{l_н}. \tag{8.1}$$

Introducing the notations for power N, volumetric efficiency Q, pressure H, the number of revolutions of the pumps n, and the circular u, relative w and absolute c velocities, we obtain the constants of the kinematic and dynamic similarity:

$$C_u = \frac{c_м}{c_н} = \frac{w_м}{w_н} = \frac{u_м}{u_н},$$
$$C_H = \frac{H_м}{H_н}, \quad C_Q = \frac{Q_м}{Q_н}, \quad C_N = \frac{N_м}{N_н}. \tag{8.2}$$

Under the condition of equality for the model and full-scale of the hydraulic and overall efficiency η, we obtain

$$\frac{N_{\text{м}}}{N_{\text{н}}}=\frac{H_{\text{м}}}{H_{\text{н}}}\cdot\frac{Q_{\text{м}}}{Q_{\text{н}}}, \tag{8.3}$$

since in the general

$$N=\frac{H\cdot Q}{\eta}\gamma,$$

where γ is the specific mass of the liquid.

Further, by introducing the notation for angular velocity ω and radius of the wheel r, we will obtain, taking into account the relation u=ωr, the following expression for the constant of kinematic similarity:

$$C_{\text{к}}=\frac{r_{\text{м}}\cdot\omega_{\text{м}}}{r_{\text{н}}\cdot\omega_{\text{н}}}=C_l\frac{n_{\text{м}}}{n_{\text{н}}}. \tag{8.4}$$

Since the volumetric efficiency is expressed by dependence Q=2πrbc, then

$$\frac{Q_{\text{м}}}{Q_{\text{н}}}=C_l\frac{n_{\text{м}}}{n_{\text{н}}}. \tag{8.5}$$

The given formulas establish the relationships between volumetric efficiencies, velocities, numbers of revolutions and geometric dimensions of the model and full-scale.

Similarly dependences for pressures and powers can be established:

$$\frac{H_{\text{м}}}{H_{\text{н}}}=C_l^2\left(\frac{n_{\text{м}}}{n_{\text{н}}}\right)^2,$$
$$\frac{N_{\text{м}}}{N_{\text{н}}}=C_l^5\left(\frac{n_{\text{м}}}{n_{\text{н}}}\right)^3. \tag{8.6}$$

Finally, on the basis of equations (8.5) and (8.6) there can be obtained the expression for the power-speed coefficient, which is the universal characteristic of similarity of the designed centrifugal pumps of a certain standard model, which has an efficiency identical to full-scale. These questions are examined in detail in work [16].

Relations (8.1)-(8.6) allow in the initial stages the conducting of an autonomous final development of the pumps of unique ZhRD virtually on small-size models. If we are speaking about tests of a centrifugal pump with a productivity of more than 1 m^3/s, then its final development, from all points of view, is expendiently begun with a design and test of the physical models made in the considerably reduced dimensions. As a result of modeling the optimum dimensions and shapes of the basic elements can be determined: rotor wheel, collector, blades, etc. and also the basic parameters of the pump, including its efficiency.

Thus, fulfilled in the process of modeling is the whole complex of investigations, which complete, basically, the range of problems solvable with designing and autonomous final development. Further experiments have already been carried out on the full-scale object. Their purpose consists, mainly, in the testing of the results of modeling, which all the same one should consider to be approximate.

§ 2. PHYSICAL MODELING WITH THE FINAL DEVELOPMENT OF ELEMENTS AND SYSTEMS OF THE ENGINE INSTALLATION

Independently of the complexity of the experimental programs, the quantity of expensive tests must be reduced to a minimum, and the total effectiveness of the works must be as complete as possible. Therefore, experimenter searches for the possibility of the replacement of the complex program of a simpler one. The reasons for this replacement there can be several, and the main one lies in the fact that in order to insure a reduction in the cost of the finishing works and eliminate the need for the creation of an expensive experimental base.

One of the known means of the solution to the problem entails the use of methods of physical modeling by which the real process is investigated with the aid of physical models. This method allows the possibility of the appearance of errors similar to errors of the approximation, which estimate the degree of conformity of theoretical and experimental data with methods of mathematical modeling.

The replacement of full-scale by a model does not eliminate the possibility of the fact that the simulatable process will virtually always differ from the full-scale process. Nevertheless, the criterial similarity of full-scale and the model must be provided.

Physical modeling is based on an experiment, and only an experiment most fully makes it possible to carry out an inspection of the criterial similarity of the model and full-scale. By accepting the physical model, it is not possible to use its results without an estimate of the degree of conformity of the simulatable and full-scale processes. In practice this conformity can be realized by means of the theory of similitude.

By examining the concrete problem, it is necessary to determine the purpose and up to the possible limits narrow the representation of the physical process, which would correspond to conditions of the problem of program of study. More precisely, it is necessary to proceed along the path of the creation of a model which reflects not the entire complex of the physical phenomena which occur in full-scale, but only the one main side.

Let us give the simplest example. It is necessary to carry out a hydraulic flow test of the assemblies for determining the adjustment precision ZhRD on the basic parameters. The propellant components are aggressive; therefore, the flow test of assemblies on full-scale components requires the creation of special protective means and test stands, and it is also undesirable according to technological and other considerations. It is proposed to carry out hydraulic tests on a neutral, model fluid, for example, water. Then there arises the question as to what measure this will affect the accuracy of adjustment of the engine. The answer depends on the accuracy of the provision for criterial similarity.

Let us examine the second example. Let us assume that we are interested in the atomization and carburetion of the propellant components of a newly designed injection head of the combustion chamber of a ZhRD of high thrust. It is obvious that this statement of the problem in the early stages of the investigation does not completely require experimentation in full-scale dimensions. It is more expedient in this stage to be limited to the conducting of model experiments. In order to do this, it is necessary quite substantially to select the physical models and, for example, injectors, groups of injectors, honeycomb cells, precombustion chambers, model fluids, and so on. After the completion of the experimental program, results of the investigation are transferred to the full-scale object.

It is quite obvious that hydrotests of the full-scale injector assemblies of combustion chambers of the ZhRD of high thrust will require the creation of a powerful hydraulic test-stand with the productivity of the order of several tons of fluid per second at high operating pressures and large expenditures of time and resources for the manufacture of different design variants of heads and so on.

An experimental study of the atomization of components by means of physical models does not represent a peculiar complexity and can be realized under laboratory conditions. In this case an investigation can be carried out with the a more complete account of the effect of various kinds of factors, such as counterpressure, the rarefaction of the medium, etc., which in tests of full-scale objects under bench-test conditions is sufficiently difficult to carry out.

More important, from an economic point of view, are examples of the use of separate models in complex tests of systems of the ZhRD and DU [ДУ - engine installation]. Here the total effect appears more noticeable, the more complex system itself is and the more expensive its final development.

The method of system testing with the enlistment of physical models of separate elements allows in a number of cases to transfer the complex experiment from fire test-stands into laboratory conditions. In this case by means of modeling the degree of the effect of extreme values of parameters of separate elements and input factors on output system performances is determined. This makes it possible to complete the search for extreme test conditions, estimate the possibility of the appearance of emergency situations and soundly determine the extreme values of the parameters for the completion elements.

It was noted above that the ZhRD is a very dynamic system with a developed degree of correlation relations between the elements, as a result of which their autonomous final development in a number of cases with respect to effectiveness can be reduced to zero. An increase in the effectiveness of the final development of greatly correlated elements requires the conducting of complex tests of these elements in the composition of the entire system, and this means an increase in the quantity of expensive test of such systems as the ZhRD and DU. For this reason for units and assemblies subjected to interaction and which have strong correlation relations, it is expedient to carry out tests with physical models of contiguous elements of the systems. With the satisfactory similarity of the physical models to their prototypes in the process of functioning, the degree of the interaction and correlation of the elements is retained, and, therefore, the conditions of autonomous final development prove to be approximate to conditions of complex tests of this element in the composition of the entire system.

In connection with this, in the practice of the final development of the ZhRD problems of the modeling of a somewhat different plan are encountered. For example, introduced into system is the model of the element not for the study of the full-scale processes but for the retention of the degree of its correlation with adjacent elements of the system. Let us assume that the system consists of two elements 1 and 2 correlated with each other.

By virtue of this, as was shown above, the autonomous final development of element 1 can prove to be barely effective. To conduct its final development in the composition of the entire system is irrational. Then there arises the question, is it not possible to replace element 2 with its physical model. The only condition of this replacement must be the observance of test conditions of element 1 in a complex with model 2. Thus for the correlated system the problem of the modeling of test conditions in no way includes the provision for the similarity of physical processes between full-scale elements and their models. Subsequently, we will consider the purpose of this modeling to be the provision for the correlation similarity between the developed and model elements.

As an example let us examine the tests of the feed system of an engine of open circuit with the physical model of the combustion chamber. In this case at the output of the pressure main lines along the fuel and oxidizer lines there are throttle disks, which ensure the total pressure differentials and per-second propellant component flows equivalent to the full-scale. In this example used as the physical model of combustion chamber are throttle disks. As a whole the considered model provides the dynamic functioning of all assemblies of the ZhRD (excluding the chamber) virtually without the combustion of the propellant, if we do not consider the relatively low flow for the provision of operation of the ZhGG. This testing can be carried out under laboratory conditions. It does not require the automation of the complex firing tests of the engine for the final development of the feed system of a complex with the generator. In this case it is possible to study in the dynamics the transient and steady-state processes, the questions of the flow-rate control, the change in thrust, and so on.

With the provision of the parametric similarity of hydraulic channels of this type, tests on the informativeness are not inferior to full-scale tests, excluding the intrachamber processes and problems of the interference of the chamber and feed system. In the considered case for the test work, powerful fire test-stands are not required. Consequently, the model tests can be conducted independently of the other more complex programs. The advantage of the considered tests is also the fact that they made it possible to carry out the autonomous final development of the basic assemblies (for example, the TNA [THA - turbopump assembly]) as if in a complex with the whole "engine."

Thus the correlations between the testable assemblies are retained, and at the same time the final development of some elements (units) does not depend on the degree of readiness of others for tests.

Together with tests of the engine without a combustion chamber (model - throttle flanges), also others are used. For example, there are tests of a full-scale combustion chamber with the pressure feed system of the propellant components. In this case the physical model is the feed system criterially and in a correlation manner similar to the full-scale feed system of the engine jointly with the TNA and automation. Tests of the DU with the simulation of pressurized systems, autonomous tests of pumps, autonomous tests of the TNA and so on are similar.

Together with the models examined above in the practice of the final development combined models, which are the synthesis of the physical and mathematical modeling can be used. Their sense includes the fact that the physical model, which has a real physical "inlet", feeds the output signal to the converting device, which imparts an analytical form to it in the usual digital code. Further the transformed signal enters into the calculating program, which describes the mathematical model of the investigated process.

The combined modeling is an effective means for the final development of elements of the complex dynamic systems, if the insufficiently studied or new physical processes are investigated when it is not possible to recreate a reliable mathematical model of this element (unit), since the composition of the significant factors is unknown, and the necessary information is absent.

Statistical studies showed that the more extensive the experimental studies with the bench-test final development of the elements (units) and systems are conducted, the higher the quality and reliability of the articles. Modeling does not eliminate the need for the conducting of complex full-scale tests of the system as a whole. But in this case they are carried out in a considerably less volume than that which the autonomous final development requires.

The purpose of the complex tests, besides the checking of the accomplishing of the requirements of the technical assignment for the development of the system, consists in an efficiency test with the interaction of the elements and an estimate of the correctness of the selection of the basic similarity criteria with the modeling of the physical processes.

The physical modeling closely adjoins the design modeling whose the purpose is for an explanation of the reason for the emergency (failure) to recreate the design model of the occurred event.

Design modeling requires a precise repoduction of all the conditions and aspects of the tests (design, technology of manufacture, process of preparation and experimentation). The design model requires the retention and recording of not only external and internal relationships, but also the surrounding situation. The less the information about the reason for the event, the more precisely it must be reproduced. It is natural that this is required, first of all, for an explanation of precise reasons for emergency or failure. Design modeling is a peculiar demonstration of the event of interest to us and this means the irrefutable proof of its initial cause.

The design modeling is based on the experiment and is used in exceptional cases when as a result of an emergency (failure) the material part was not retained entirely, or its state is such that the reason for unfavorable outcome remained unknown. The reconstruction of design models, which correspond to the onset of an emergency situation in the operation of the ZhRD, and especially test conditions is an extremely laborious problem. In this case the main difficulty is explained by the fact that the volume of information in this situation, as a rule, is very limited. In such cases one usually proceeds along the path of the realization of a certain quantity of design models, which correspond to several most probable hypotheses. Such measures partially speed up the realization of the total program, but this approach makes it possible to investigate a certain complex of the possible emergency outcomes of the ZhRD under conditions of tests similar to the boundary tests.

For additional information the inspection of remainders of the material part is very important: the state of the injector head assembly of the chamber, gas generator, turbine rotor, housings of the assemblies, impellers of the pumps, end position of the actuating elements and assemblies of automation, cables, instruction instruments, and so on. If an emergency arose in flight, then the destruction of the remainders of the material part when falling back to the earth is an obstruction for the execution of this operation. In such case all efforts of the group of an analysis must be directed toward the investigation of the telemetering data, which correspond to the moment of the onset of the emergency state. A great advantage of this investigation should be considered the comparative analysis of telemetering recordings of emergency starting with successful tests carried out under approximately the same conditions. A comparison must be carried out virtually according to all the controllable parameters, and all the distinctions in the behavior of the parameters must be recorded.

The generalized data on the investigation of the reason for a failure lie as the basis of the test program of the design model which reproduces the emergency situation. If in this case it will be established that there are several reasons, then the effect of each of them for the purpose of determining the resulting conditions and unfavorable combinations of the basic factors is investigated. In such cases the quantity of tests of the design models cannot be restricted to one experiment, but several of them are carried out in accordance with the factor plans given above. Together with the generalized data, the factor plans make it possible to create a rational program of experimental researches.

§ 3. PROVISION FOR THE SIMILARITY OF DYNAMIC PROCESSES

The final development of the complex technical systems requires the modeling of not only static (establish) processes but mainly their dynamics. The dynamic processes are characterized by a variable number of revolutions of the TNA, the unsteady motion of the working medium, liquid or gas, and also the unstable evaporation and burning of propellant components [40]. The dynamic processes are described by the equations of dynamics or are represented by dynamic characteristics (see Chapter VI). These equations establish the interconnection between the basic parameters of the engine under transient conditions of operation when all the parameters are changed in time.

The dynamic modeling must include the study of the transient processes: starting, shuting off, modes of control, and different pulsating modes which appear in the feed system, the gas generator and combustion chamber. In addition to this, with the final development of the engine plants it is necessary to insure the dynamic similarity of bench-test conditions to flight conditions and so on. Dynamic modeling can be conducted only on the basis of the provision for similarity of the model to actual flight.

The dynamic processes of the ZhRD are complex and studied the least of all. But the processes which take place in the period of the starting of the ZhRD are especially complex and little investigated. This period is accompanied by the complex interaction of different transient processes (mechanical, hydraulic, gas-dynamic, physicochemical, thermal, etc.). It is impossible to present many of them in a quantitative or parametrical form. Consequently, it is impossible to describe qualitatively enough this process by the appropriate equations of dynamics. The study of the intrachamber processes of the ZhRD by means of analytical methods in a whole number of cases does not lead to success, especially if we are speaking about the combustion stability, the selection of optimum characteristics of the chamber, and so on. For example, in the equation of dynamics of the chamber not all the physical features of dynamics of the intrachamber processes (laws of the entrance of propellant components into the chamber, the propagation of the burning front, the acoustic characteristics of the chamber, the motion of shock waves, etc.) are considered. The design characteristics, combustion chamber configuration, the presence of curtain belts, and many others are also not considered. Taking this into consideration, the examined equation should be considered as approximate, and therefore its use for the derivation of similarity conditions must be limited.

However, if we assume that the equation of dynamics of the chamber completely reflects the investigated real process, then its recording in criterial form changes little. An attempt at the dimensional analysis of all the known variables leads to many different solutions and to the intuitive evaluation of the significance of separate factors. In this case the screening of "trivial" variables also leads to additional errors.

Therefore, new ways of provision and inspection of the similarity of systems with dynamic modeling are necessary. One of them, obviously, is the refusal to determine of the similarity criteria in the study of dynamic processes of complex systems. For this reason in the final development of the intrachamber processes of the ZhRD, the basic and most reliable remains as yet the empirical search, which is based on the experiment and modeling. If the simulatable processes from a physical point of view are uniform and conditioned by the effect of the same composition of the significant factors, then the inspection of the provision for dynamic similarity can be produced by means of a comparison of characteristics which determine the output quality of the process.

For example, let us assume that it is necessary to estimate the flight and engineering characteristics of the ZhRD according to results of bench tests. In this case the flying engine is "full-scale", and its bench-test version - the "model". It is natural that the primary problem should be considered as the provision on the test-stand of the simulation of conditions of flight tests.

In this case for the designer and experimenter, from the viewpoint of the perfection of the model (test-stand) and the provision for its similarity to full-scale, there is a wide sphere of action. It includes the matching of the bench-test dynamic characteristics to flight. In such a case when we are interested in the characteristic of the output of the engine to the nominal (sustainer) system, then prior to the beginning of the flight tests accepted as full-scale tests can be the characteristic obtained from results of bench tests of the DU in the composition of a stage.

Virtually this means that full-scale tests are carried out on a limited quantity of articles at the extreme values of the factors. As a result the telemetering recording of the investigated parameters (Fig. 8.1) can be obtained. In this case characteristic 1 is obtained according to results of full-scale tests, and characteristic 2 under the same conditions corresponds to bench tests.

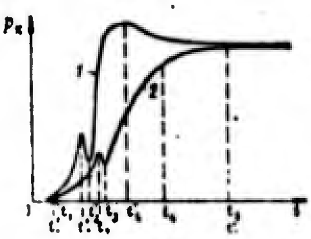


Figure 8.1. Dynamic characteristics of the output of the engine on the nominal (sustainer) operating mode: 1 - under flight conditions; 2 - under bench-test conditions.

Subsequently, both characteristics are compared. If in this case their scales correspond to one another, then a checking of the convergence of the "model" with "full-scale" can be provided for by means of a matching of the oscillograms.

By means of a comparison and analysis of characteristics 1 and 2, it is possible to establish that the emergence of the engine into actual conditions is more stressed and rigid than it is on the test stand. This is indicated by the higher rate of pressure buildup $p_к$ in accordance with characteristic 1 and the jump in pressure higher than the nominal level. Under bench-test conditions the emergence into the mode is smoother, without a jump in pressure and, therefore, less stressed. This means that with bench tests the starting of the engine must be characterized by a comparatively less failure rate. From the viewpoint of the quality of the final development the latter is inadmissible. As a whole this will require the modification of the bench-test feed systems.

It is possible to speak about the similarity of systems or conditions of tests if in both cases for the "model" and "full-scale" the failure rates will be equal to ($\lambda_M = \lambda_H$). For the examined distributions this means the equality of the probabilities of the failure-free operation and their frequencies is:

$$P(x)_M = P(x)_H. \quad (8.7)$$

Since in this case only transient processes are examined, then accepted as the distribution law of failures can be, with a sufficient degree of reliability, the Weibull law. The possibility of this assumption was substantiated in Chapter V. Taking this into account, equality (8.7) must provide the convergence of results of the processing of statistical data for test conditions of the "model" and "full-scale." In this case for an evaluation of the convergence, the grapho-analytic method, based on the logarithmic operation of the distribution function examined in Chapter V, can be proposed. The convergence of results of the processing of statistical data for the "model" and "full-scale" in effect denotes the equality of the corresponding distribution parameters x_0 and β. The latter in this case for the "model" and "full-scale" must be equal or similar to each other:

$$x_{0M} \cong x_{0H}; \ \beta_M \cong \beta_H.$$

The absence of full equality with the observance of similarity conditions of processes can be explained only by the limitedness of the volume of the tests. In accordance with the method of processing, the test results of both combinations of articles can be represented in a similar way, as is shown in Fig. 8.2. It is evident that when a similarity of the "model" and "full-scale" is present, their statistical data lie on one straight line. In the absence of the similarity of dynamic processes of the "model" and "full-scale", the parameters of the Weibull distribution x_0 and β must differ from each other, namely:

$$x_{0м} \neq x_{0н} \text{ and } \beta_м \neq \beta_н .$$

Correspondingly, final test results of the "model" and "full-scale" cannot be similar; therefore, separate points are placed on different lines (Fig. 8.3).

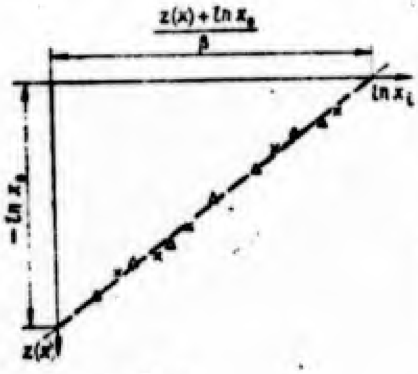


Figure 8.2

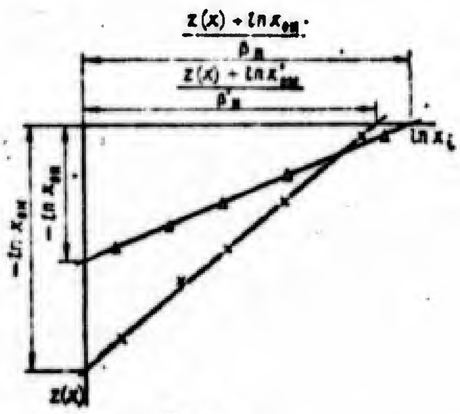


Figure 8.3

Figure 8.2. Grapho-analytic method of the evaluation of the convergence of the dynamic characteristics of the "model" and "full-scale" (case of the presence of similarity): × - "model"; Δ - "full-scale".

Figure 8.3. Grapho-analytic method of the evaluation of the convergence of the dynamic characteristics of the "model" and "full-scale" (case of the absence of similarity): × - "model"; Δ - "full-scale".

The account given above should be considered as the method of the checking of the similarity of dynamic processes, which is based on the processing of statistical data. The method establishes the statistical similarity of the dynamic transient processes. In this case it is possible to speak about the similarity conditionally: it is more correct to speak about the conformity of the processes or their identity. A shortcoming of this method, just as of other statistical methods, should be considered to be their sensitivity to the volume of information.

The statistical "criteria" of the identity of dynamic processes should be applied when reliable physical methods of the recording of dynamic characteristics, in particular, the telemetering recordings of the output parameters of the system, are absent. In the presence of such parameters a check of the dynamic similarity of the investigated processes, as was noted above, can be carried out by the matching or imposition of oscillograms of the "model" and "full-scale". This method can be considered sufficiently objective even in the presence of single experiments. In this case, of course, there must actually be provided the reproducibility of the dynamic characteristics of the investigated processes with repeated tests. This is virtually possible even for random processes with the known composition and fixed levels of the significant factors which affect the "inlet" of the system.

The indispensable condition of the checking of the similarity of the dynamic processes should be considered as their full conformity from the viewpoint of the provision of the equality of levels of input factors for the "model" and "full-scale."

As a quantitative measure of the convergence of the dynamic characteristics of the "model" and "full-scale" the known Pearson chi-square compatibility test can be proposed. Kolmogorov's criterion in existing form [76], unfortunately, cannot be used, since for determining the empirical distribution function it provides for a finite number of realizations of the "random variable." In our case this number will always be very large, close to infinity, since the recording of the dynamic characteristic under bench conditions is conducted by monitoring sensors continuously, while under flight conditions - with a sufficiently high frequency of interrogation.

Kolmogorov's criterion can be applicable if as the realization of the dynamic characteristic we take not the compatible points of curves of the "model" and "full-scale", but, for example, their separate compatible sections. In connection with the dynamic characteristics of the starting, depicted on Fig. 8.1, such sections can be the intervals: $(0-t_1)$ and $(0-t_1')$; (t_1-t_2) and

$(t'_1-t'_2)$, (t_2-t_3) and $(t'_2-t'_3)$; (t_3-t_4) and $(t'_3-t'_4)$; (t_4-t_5) and $(t'_4-t'_5)$, respectively, for the "full-scale" and "model" (with prime).

Then the factor which determines the convergence of the two dynamic characteristics is the maximum difference in the control parameters of the "model" and "full-scale" for each of the sections. In the considered case this is the maximum difference in pressures in the combustion chambers:

$$D_{max} = (p_{к_м} - p_{к_н})_{max}.$$

The evaluation of the agreement of characteristics of the investigated processes is determined from the probability P:

$$P = \text{Вер}\{D_{max}\sqrt{R}\}, \quad (8.8)$$

where R is the number of realizations equal for the considered example to the number of intervals (R=5).

As follows from expression (8.8), the considered probability depends on two parameters: D_{max} and R. If the first, when evaluating the agreement, is determined unambiguously, then the parameter R to any degree should be considered as conditional.

A similar condition exists for the Pearson's criterion-χ^2. It is connected with the arbitrary selection of the number of intervals into which the investigated characteristic of the dynamic process is divided. For the purpose of the achievement of uniqueness, when using criterion χ it is proposed to divide each of the sections into 3-4 intervals, which are determined by taking into account the physics of the phenomena.

The considered evaluation of the identity of the dynamic processes is of practical importance. With its aid the functional similarity of the systems is determined. In the examined specific case it is possible to estimate how fully the bench-test conditions correspond to the flight conditions.

If conditions of identity are not provided for, modification of the model, taking into account the physical features of the real process must be conducted. In this case the level of knowledge of the physics of the phenomena and the experimenter's experience and intuition are important. As regards the intuitive approach to the solution of those or other questions, in the most responsible and complex cases it is expedient to apply the method of expert evaluations. With a sufficient volume of statistical data, the solutions on the modification of the model can be accepted on the basis of the optimization of particular mathematical models in a linear variant.

One additional condition, which has in a number of cases decisive importance for practice, is the presence of physical limitations imposed on the investigated process. The fact is that, having achieved the functional similarity of the two dynamic processes, it is not possible to eliminate the natural scattering of the parameters due to errors in the adjustment, technological factors of the first and second kind, random phenomena, measuring errors, and so on.

If the experiment is conducted under the boundary conditions or in the extreme zone, there is the possibility of a qualitative change in any of the parameters determining the conditions of "inlet" and emergence beyond the limits by which the physical state of the working medium or design elements is changed. The dynamic characteristic which determines a change in the pressure of one of the propellant components at the inlet into the pump of the engine can serve as an example. For a certain conditional ZhRD it takes the form shown on Fig. 8.4. The moment of time 0 determines the instruction for the starting of the DU and the beginning of tank pressurization. The moment of time t_1 is the opening of the main valve and the beginning of filling of the free cavities of the engine. This process is characterized by a sharp pressure decay. In proportion to the filling of the cavities, the process is restored. It is characterized by the turbulent flow of fluid under the action of increasing pressure. However, the combustion of the propellant components in the combustion chamber creates a sharp resistance to the flow and a shock wave, which is characterized by the appearance of a "peak" of pressure at the moment of time t_3. After the "peak" there follows drop t_4. In summation, the emergent oscillating process attenuates by the moment of time t_5. Further fluctuations in pressure are observed, and from the moment of time t_6 the process can be considered as being established.

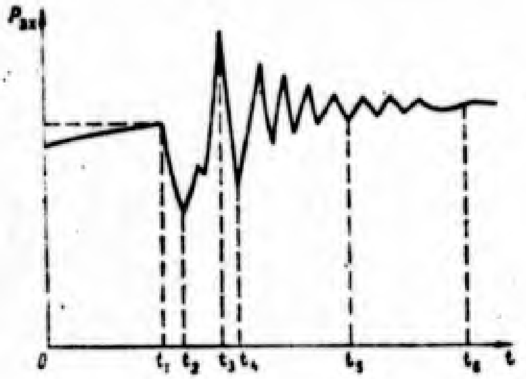


Figure 8.4. Dynamic pressure characteristic of the fluid at the inlet into the pump.

The physical limitations imposed on the considered dynamic characteristic include the fact that in the pressence of troughs of pressure at moments of time t_2 and t_4 the level can be lower than the elasticity of the vapors of the components. Then the corresponding points of time, especially t_2, will be characterized by the effervescence of the component. As a result into the TNA and combustion chamber there will begin to enter a vapor-liquid emulsion capable of causing unsteady combustion, fluctuations, and so on. If this is possible in limitating cases with full-scale tests, then the indicated phenomena can not always be inherent in the model. In summation, the functional similarity of the dynamic processes from a mathematical point of view can be provided, and boundary conditions for the model are not observed. Therefore, it is necessary, together with the mathematical evaluation of the identity of the processes, to produce a physical inspection of the boundary conditions and as required to impose the appropriate limitations on the model and full-scale.

The question of the identity of the complex dynamic processes also in many respects is determined by the possibilities of the measuring system and by sensors and their feeders. In a number of cases the latter can be dampers which "extinguish" the fluctuations and "peaks" of pressure in the oscillograms and thus distort the real process.

Finally, one should indicate one more condition on the providing of a functional similarity of the dynamic systems. It entails the use for the recording of the characteristic of similar processes of single metering equipment, one type of sensors and identical form and length of the feeders.

If examined is the complex dynamic process of which the analytical relationships between the parameters of the system are always clear, then it is most convenient to present it by the model of the "black box." In accordance with this, in the solution to the problem of providing for the dynamic similarity, we will have two models: one for the full-scale and the second for model processes (Fig. 8.5).

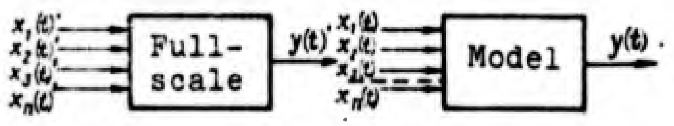


Figure 8.5. Diagrams of the "black box" for full-scale and model processes.

It is known that both processes have an identical physical full-scale, and the inlet and outlet characteristics are identical. The presence of the known "inlet" and of "outlet" allows, in accordance with the methods given in Chapter V, determining the mathematical model of each of the processes, i.e., the establishment of the missing functional relationships between the parameters of the systems.

The form of these relationships can be expressed by the polynomials:

- for the full-scale

$$y_n = \sum_{i=0}^{n'} b_i' x_i' + \sum_{i<j}^{n'} b_{ij}' x_i' x_j' + \ldots;$$

- for the model

$$y_m = \sum_{i=0}^{n'} b_i x_i + \sum_{i<j}^{n'} b_{ij} x_i x_j + \ldots,$$

where with the prime there are the parameters of "full-scale", without the prime - the "model."

If now we accept conditions for the "outlet" and "inlet":

$$n_1' = n, \quad x_1' = x_1, \quad x_2' = x_2, \ldots x_n' = x_n, \quad y_m = y_n,$$

then correspondingly the regression coefficients will be equal

$$b_0' = b_0, \; b_1' = b_1, \; b_2' = b_2 \text{ etc.}$$

As a result we can obtain conditions for the functional similarity of the dynamic processes:

$$y = \sum_{i=0}^{n} b_i x_i + \sum_{i<j}^{n} b_{ij} x_i x_j + \ldots = \text{idem}.$$

On the basis of this it is possible to draw the conclusion: if the phenomena have an identical physical full-scale, then the investigated dynamic processes will be functionally similar.

This position at present is quite fully checked experimentally. It found wide application in practice in test procedures of the ZhRD.

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