AD-786 928

TROPOSPHERIC RANGE ERROR CORRECTIONS FOR THE NAVSTAR SYSTEM

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L. G. Hanscom Field, Massachusetts

16 April 1974

Unclassified
Security Classification

AD-786 928

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Air Force Cambridge Research Laboratories (LZN) L. G. Hanscom Field Bedford, Massachusetts 01730	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE
TROPOSPHERIC RANGE ERROR CORRECTIONS FOR THE NAVSTAR SYSTEM

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Scientific. Interim.

5. AUTHOR(S) (First name, middle initial, last name)
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6. REPORT DATE 16 April 1974	7a. TOTAL NO. OF PAGES 13	7b. NO. OF REFS 3

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT, TASK, WORK UNIT NOS. 86820201	AFCRL-TR-74-0198
c. DOD ELEMENT 62101F	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d. DOD SUBELEMENT 681000	AFSG, 286

10. DISTRIBUTION STATEMENT
Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES TECH, OTHER	12. SPONSORING MILITARY ACTIVITY Air Force Cambridge Research Laboratories (LZN) L. G. Hanscom Field Bedford, Massachusetts 01730

13. ABSTRACT

NAVSTAR is a triservice satellite navigation system designed to provide the means for military aircraft, ships, and ground users throughout the world to obtain precise position and velocity information at any time of day, in any kind of weather. Since the user's position is measured by the time delays between his location and several satellites, the additional time delay due to tropospheric refraction must be taken into account. For angles above 5° the time delay, or corresponding range error, can be estimated from the local surface refractivity, if known.

The worldwide statistics of surface refractivity analyzed in this report show that refractivity is correlated with site latitude, height above sea level, and season, all of which should be readily known. Regression lines for range error corrections based on these parameters are derived, and the accuracy of the corrections are estimated. At 5°, the minimum elevation angle at which NAVSTAR is intended to be used, the range error is about 80 ft. As the elevation angle is increased, the range error decreases approximately as the cosecant of the elevation angle and approaches a minimum value of about 7 ft at zenith. Range error corrections based on a global average surface refractivity of 324.8 N units have a standard error of about 8 percent of the total refractive range error. Corrections based on a regression model that is a function of site latitude, height above sea level, season, and elevation angle, have a standard error of 6 percent of the total range error. If the surface refractivity at the user's location is known, the standard error of the range error correction is only about 3.7 percent of the total correction.

DD FORM 1 NOV 65 1473

Unclassified
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Tropospheric refraction						
Navigation satellite system						
Global surface refractivity statistics						
Air traffic control						
Tropospheric range error						

Contents

Tables

Tropospheric Range Error Corrections for the NAVSTAR System

1. INTRODUCTION

At the request of the Space and Missile Systems Organization (SAMSO), AFCRL (LZN) is providing these worldwide tropospheric refractive range error corrections for the NAVSTAR system [formerly Defense Navigation Satellite System (DNSS)].

As projected, NAVSTAR will allow military (and commercial) aircraft, ships, and ground users throughout the world to obtain precise position and velocity information at any time of day during all kinds of weather conditions. The system will consist of several constellations of satellites generating pseudorandom noise-coded signals. The time delays of the signals will enable the user to ascertain his range to each satellite, and thereby obtain sufficient information to determine his position and velocity.

The accuracy of any system tends to be limited by tropospheric refraction, which produces two main effects on radiowaves: angular bending and time delay. Angular bending—deviation of the ray path from a straight line—is due to variability of the refractive index with position in the atmosphere. The delay in the arrival time of a signal occurs primarily because the index of refraction of the atmosphere is greater than unity, and is to a lesser extent due to lengthening of the path by angular bending.

(Received for publication 15 April 1974)

Thus, if the distance from a ground position or an aircraft to a satellite is to be accurately determined, the time delay produced by tropospheric refraction must be taken into account. This time delay is often represented in terms of an equivalent range error (1 nsec ≈ 1 ft). The tropospheric range error typically has a maximum value of about 300 ft for a ray path whose initial elevation angle is 0°. As the elevation angle is increased, the range error decreases approximately with the cosecant of the elevation angle, approaching a minimum value of about 7 ft at zenith. At an elevation angle of 5°, the minimum angle at which NAVSTAR is intended to be used, the range error is about 80 ft.

2. APPROACH

In the most accurate of the several methods used for computing range error, the error is calculated either by ray-tracing techniques or by numerical integration, based on an actual refractivity profile along the ray path or on a vertical profile at the observer's position. If an actual profile is not available, then a representative refractivity model must be resorted to.

The statistics of refractivity as a function of altitude for climatologically diverse regions show that in the first kilometer, refractivity decreases approximately linearly; above that altitude, it decreases exponentially. It has also been found that the variation of refractivity reaches a minimum at an altitude of 9 km. These findings form the basis of the widely used CRPL Reference Atmosphere—1958 Refractivity Model.[1] The model consists of a three-section refractivity profile: a linear decrease from the surface to 1 km; an exponential decrease from 1 km to 9 km, at which height it is assumed to have a value of 105 N units; and an exponential decrease throughout the remainder of the lower atmosphere. The only variable that must be specified by the user is the actual surface refractivity value to which the three-piece profile is attached. Thus, if the surface refractivity for a region is known, an estimate of the range error can be calculated. Other refractivity models have been used but they are for the most part a variation of the CRPL Reference Atmosphere.

3. RESULTS

Representative surface refractivity values were determined from seasonal 5-yr average surface refractivity data for 268 stations scattered throughout the

1. Bean, B.R., and Dutton, E.J. (1966) Radio Meteorology, Dover Publications, New York.

world. These were used with CRPL profiles to compute range error values by ray-tracing. Algebraic expressions representing the variation of range error as a function of surface refractivity N_s, height above sea level h, and apparent elevation angle θ, were derived for defining the basic correction algorithm for the NAVSTAR system. Care was taken to avoid using transcendental and trigonometric expressions that would have added considerably to the complexity of the digital processor required in the field. Hence, the algebraic expressions that were derived included only the operations of addition, subtraction, multiplication, and minimal division.

3.1 Range Error Function

The range error ΔR can be expressed as a product of three factor functions, as follows:

$$\Delta R(\theta, h, N_s) = G(\theta)\, H(h)\, F(h, N_s) \quad \text{(feet)} , \tag{1}$$

where $G(\theta)$ and $H(h)$ are functions of only the elevation angle θ and the observer's height h, and the function $F(h, N_s)$ is dependent only on h and the surface refractivity N_s. The factor functions, chosen to be of polynomial form, are:

$$G(\theta) = (g_0 + g_1\theta^{-1} + g_2\theta^{-2} + g_3\theta^{-3})\,[g_4 + g_6(\theta - g_5)^2] , \tag{2}$$

where θ, in degrees, is valid only for the range $\theta \geq 5^{\circ}$, and

$$\begin{aligned}
g_0 &= 0.1556 ,\\
g_1 &= 138.8926 ,\\
g_2 &= -105.0574 ,\\
g_3 &= 31.5070 ,\\
g_4 &= 1.0000 ,\\
g_5 &= 30.0000 ,\\
g_6 &= 1.0 \times 10^{-4} ;
\end{aligned}$$

$$H(h) = \left[(b_0 + b_1(h + 2.630)^{-1} + b_2(h + 2.630)^{-2} + b_3(h + 2.630)^{-3})\right] , \tag{3}$$

where h is in thousands of feet, and

$$b_o = 0.00970 ,$$
$$b_1 = -2.08809 ,$$
$$b_2 = 122.73592 ,$$
$$b_3 = -703.82166 ;$$

$$F(h, N_s) = c_o\left[\frac{c_1}{h+c_o} + c_2 (h+c_o) + c_3 N_s - c_4\right]\left[1 - c_5(N_s - c_6)^2\right] , \tag{4}$$

where

$$c_o = 3.28084 ,$$
$$c_1 = 2.07800 ,$$
$$c_2 = 0.30480 ,$$
$$c_3 = 0.00423 ,$$
$$c_4 = 1.33333 ,$$
$$c_5 = 1.41723 \times 10^{-6} ,$$
$$c_6 = 315.00000 .$$

Note that although G, H, and F contain a total of seven implied divisions, only three are actually required: $(1/\theta)$, $1/(h+2.630)$, and $1/(h+c_2)$. Once these have been carried out the remaining terms are expressed as products of the pertinent factors, requiring only the operations of multiplication and addition.

3.2 Refractivity Model

The range errors caused by tropospheric refraction can be determined by means of Eqs. 1 through 4 if the NAVSTAR user knows the elevation angle between his position and the satellite, his height above sea level, and the local surface refractivity N_s. If he does not know the local N_s, he must estimate it.

A statistical analysis of our surface refractivity data showed that the average global surface refractivity is 324.8 N units and that the standard deviation of this sample is 30.1 N units. A further analysis of the data showed that N_s correlates with site latitude, height above sea level, and season. The following regression line of N_s as a function of these parameters can therefore be derived:

$$N_s(h, L, M) = \alpha_o - \alpha_1 h - \alpha_2 L + \alpha_3 hs^2 + \alpha_4 Ls^2 + \alpha_5 hc + \alpha_6 Lc , \tag{5}$$

where h is in feet, L is in degrees, M is the calendar month, and

$$\alpha_0 = 369.03000 \,,$$
$$\alpha_1 = -0.01053 \,,$$
$$\alpha_2 = -0.92442 \,,$$
$$\alpha_3 = 0.00160 \,,$$
$$\alpha_4 = 0.19361 \,,$$
$$\alpha_5 = 0.00063 \,,$$
$$\alpha_6 = -0.05958 \,,$$

and

$$s = \sin\left(\frac{\pi}{12} M\right) \,,$$

$$c = \cos\left(\frac{\pi}{12} M\right) \,.$$

This expression describes the worldwide variation of N_s and has a standard error of ± 17.0 N units, approximately ± 5 percent of the mean N_s value.

It was also disclosed that as a factor in predicting N_s, knowledge of just the season alone is as valuable as knowledge narrowed to a specific month. The season was effectively used as a variable in the expression for the regression line, with the following assignment of M values.

Season	M Value
Winter	1.5
Spring	4.5
Summer	7.5
Fall	10.5

With this model for N_s, the basic algorithm for estimating tropospheric range errors ceases to be a function of N_s and becomes a function of θ, h, L, and M, denoted

$$\Delta R(\theta, h, L, M) = G(\theta)\, H(h)\, F(h, L, M) \,, \tag{6}$$

where

$$F(h, L, M) = c_0 \left[\frac{c_1}{h+c_0} + c_2(h+c_0) + c_3 N_s(h, L, M) - c_4\right] \cdot \left\{ 1 - c_5 \left[N_s(h, L, M) - c_6\right]^2 \right\} . \tag{7}$$

3.3 Accuracy of Range Error Correction

The standard errors in range error, calculated from the standard errors in surface refractivity by using Eq. (6), are listed in Table 1 as a function of surface refractivity N_s and elevation angle θ. These tabulations represent only the uncertainties that are due to the statistical variation in surface refractivity. Even when the surface refractivity is known, the model has an inherent error, which must also be included. Unfortunately, the accuracy with which range error can be corrected on the basis of the CRPL Reference Atmosphere—1958 is extremely difficult to determine.

Table 1. Uncertainty in Range Error Due to Uncertainty in Surface Refractivity (ft/N units)

θ^{o} \ N_s	240	260	280	300	320	340	360	380	400
4	0.197	0.188	0.178	0.168	0.159	0.149	0.139	0.130	0.120
5	0.146	0.140	0.135	0.129	0.123	0.117	0.112	0.106	0.100
6	0.116	0.112	0.108	0.104	0.101	0.097	0.093	0.089	0.085
7	0.096	0.093	0.090	0.088	0.085	0.082	0.080	0.077	0.074
8	0.082	0.080	0.078	0.076	0.074	0.072	0.070	0.068	0.066
9	0.071	0.070	0.068	0.066	0.065	0.064	0.062	0.061	0.059
10	0.063	0.062	0.061	0.059	0.058	0.057	0.056	0.055	0.054
12	0.051	0.051	0.050	0.049	0.048	0.048	0.047	0.046	0.045
14	0.043	0.043	0.042	0.042	0.041	0.041	0.040	0.040	0.039
16	0.038	0.037	0.037	0.036	0.036	0.036	0.035	0.035	0.034
18	0.033	0.033	0.033	0.032	0.032	0.032	0.031	0.031	0.031
20	0.030	0.030	0.029	0.029	0.029	0.029	0.028	0.028	0.028
25	0.024	0.024	0.024	0.023	0.023	0.023	0.023	0.023	0.023
30	0.020	0.020	0.020	0.020	0.020	0.020	0.019	0.019	0.019
40	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
50	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013
60	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
70	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
80	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
90	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010

Norton[2] has calculated range errors for the CRPL Standard Sample and plotted the standard deviation of the normalized range error $\Delta R_e/R_e$ for cases where the surface refractivity is both known and unknown. If it is assumed that the standard deviation of the range error for the CRPL Reference Atmosphere—1958 is the same as for the CRPL Standard Sample, then the accuracy of the range error correction for a horizontally stratified atmosphere can be estimated.[3] Errors due to atmospheric anomalies such as a nonstratified atmosphere, clouds, and precipitation, must also be taken into account; these are estimated to be a maximum of about 2 percent of the total range error.

The standard errors of range error corrections are listed in Table 2 as a function of elevation angle and number of parameters available. It is seen that if N_s is known, the standard error is approximately 3.7 percent of the total value of the range error correction. If N_s is not known but the latitude, height above sea level, and season are known, then the standard error increases to slightly more than 6 percent. If the latitude of the observer is unknown, the standard error is about 7 percent. If only the global average surface refractivity of 324.8 N units is used, the standard error increases to about 8 percent of the range error correction.

4. SUMMARY

Tropospheric range error corrections based on a global average surface refractivity of 324.8 N units have a standard error of about 8 percent of the total refractive range error. Since the range error from the surface to a target above the troposphere varies from about 80 ft for an elevation angle of 5° to about 7 ft for angles close to zenith, standard errors of about 6.4 ft to 0.6 ft would be expected if all systems distributed throughout the world used corrections based on elevation angle only.

It has been shown statistically that range error corrections based on a regression model that is a function of site latitude, height above sea level, season, and elevation angle, show a standard error of approximately 6 percent of the total range error. Thus, a 25 percent improvement in the accuracy of the range error correction can be obtained simply by using parameters that can readily be ascertained for any user location. The range error correction can be further

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2. Norton, K.A. (1964) Effects of tropospheric refraction in earth-space links, Proc. Second Tropospheric Refraction Effects Technical Review Meeting, Electronic Systems Division, TDR-64-103, 1:155-193.

3. Altshuler, E.E. (1971) Corrections for Tropospheric Range Error, AFCRL-71-0419.

Table 2. Standard Deviation in Range Error Corrections as a Function of Information Available (ft)

θ	A	B	C	D	Avg ΔR_e
5	2.96	5.11	5.91	6.75	83.8
6	2.60	4.36	5.02	5.72	71.2
7	2.30	3.79	4.34	4.92	61.8
8	2.05	3.35	3.83	4.33	54.6
9	1.86	3.00	3.42	3.86	48.9
10	1.68	2.70	3.07	3.47	44.2
12	1.42	2.26	2.57	2.90	37.2
14	1.22	1.94	2.20	2.48	32.2
16	1.07	1.70	1.93	2.18	28.3
18	0.95	1.51	1.72	1.94	25.3
20	0.86	1.37	1.56	1.75	22.9
25	0.70	1.10	1.25	1.41	18.6
30	0.58	0.93	1.06	1.20	15.8
40	0.43	0.69	0.79	0.89	12.3
50	0.37	0.60	0.68	0.77	10.3
60	0.33	0.51	0.59	0.67	9.1
70	0.30	0.48	0.54	0.61	8.4
80	0.29	0.47	0.53	0.60	8.0
90	0.29	0.47	0.53	0.60	7.9
$\sigma_{\Delta R_e} / \Delta R_e$	0.037	0.061	0.070	0.080	

A: N_s known

B: N_s unknown; season, height, and latitude known

C: N_s unknown; season and height known

D: No information

improved if the surface refractivity at the user location is known; the standard error of the range error correction for this case is only about 3.7 percent of the total correction.

It should be evident that very accurate range error corrections will be required only for those locations where the satellites are at low elevation angles since the total range error decreases as the elevation angle increases. Also, the range error decreases as the user's height above sea level increases since the dense region of the troposphere, which produces most of the additional time delay, is below the user.

From Table 2 it is apparent that the range error is very strongly a function of elevation angle. In the work reported here the assumption has been that the user knows the precise elevation angle between his location and the satellite. For low elevation angles this is particularly important because the range correction at those angles changes rapidly. How accurately the position can be determined will therefore be very much dependent on geographic location.

It must be emphasized that our results have been directed toward, and are mainly applicable to, the nonstationary case. If the user is confined to a region within which the climatology does not vary appreciably from point to point, then more accurate range error corrections can be derived from refractivity statistics for that region.

AFCRL-TR-74-0198
AIR FORCE SURVEYS IN GEOPHYSICS, NO. 286

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Errata

Page 7 - Change Equation (3) to read

$$H(h) = [(b_o + b_1(h + 8.6286)^{-1} + b_2(h + 8.6286)^{-2} + b_3(h + 8.6286)^{-3})] , \quad (3)$$

Page 8 - Change Line 9 to read

$$c_1 = 6.81758$$

Page 8 - Change Line 16 to read

three are actually required: $(1/\theta)$, $1/(h+8.6286)$, and $1/(h+c_2)$. Once these have

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