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SENSITIVITY OF CONFLICT SIMULATION MODELS TO CHANGES IN VARIABLE PARAMETERS

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In 1973, the RAND Corporation developed a computer program for the United States Air Force to study the problem of the escalation of conflict in a tactical nuclear environment. As in any computer program, real life entities and activities were reduced to quantities and formulas. The usefulness of such models and the relevancy of the findings are related to the accuracy of the selection of such parameters. An indication of the accuracy required can be obtained by performing a sensitivity analysis on the model in question. No such sensitivity analysis had been performed on the RAND model. It was our original intent to perform such a sensitivity analysis on the force values used in the RAND model. Before the sensitivity analysis could be performed it was necessary for us to translate the JOSS based computer program into FORTRAN. At the request of Mr. R. F. Robinson, AF/SAG, sponsor of the original study, we conducted a sensitivity analysis simultaneously on both force values and effectiveness coefficients. We concluded that the model was indeed sensitive to the selection of the force values and effectiveness coefficients.

SLSR 27-74B

SENSITIVITY OF CONFLICT SIMULATION MODELS TO CHANGES IN VARIABLE PARAMETERS

A Thesis

Presented to the Faculty of the School of Systems and Logistics

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Logistics Management

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August 1974

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MASTER OF SCIENCE IN LOGISTICS MANAGEMENT

DATE: 14 August 1974

Edward J. Fisher

COMMITTEE CHAIRMAN

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DEFINITION OF TERMS

For the purposes of this study the authors have defined the following terms:

Critical Time of Escalation (Tc)--The last possible time at which Blue can escalate and still win the war. (From RAND study.)

Game--A mathematical model used to represent a particular situation of conflict between two or more opponents in which the outcome or payoff of the conflict depends upon the strategies independently selected by the players.

Game Theory--The use of mathematical models to study conflict situations.

Effectiveness Coefficients--The rate at which a specific type of force can destroy a given type of enemy force.

Force Utility--A single integrated measure of force capability, the will to use that capability, and the anticipated effectiveness of that capability.

Lanchester Equations--Differential equations formulated to calculate force levels as a function of time for given initial force levels and attrition rates.

Minimax Principle--The choice of a strategy designed to minimize one's own losses regardless of the particular strategy selected by the opposition.

Payoff--The outcome of a conflict for a specific combination of opposing strategies measured in terms of force utility.

Payoff Matrix--The complete set of payoff values for all possible combinations of opposing strategies.

Sensitivity--A change in Blue's critical time of escalation greater than or equal to Ø.1 day per a change of 100 units of the force level.

Strategy--A set of decisions or decision formulation guidelines outlining specific courses-of-action for all possible situations to be faced in the play of the game.

Warfare:

Conventional--Warfare limited to non-nuclear forces.

Nuclear (All-out)--Warfare including nuclear weapons with no restrictions on weapon size or type of target.

Nuclear (Limited)--Warfare limited to the use of nuclear weapons of less than 20 kilotons against a limited number of military targets.

CHAPTER I

INTRODUCTION

Problem Statement

Mathematical modeling is the primary means for studying the escalation of tactical nuclear warfare. The strength of the findings using such models is directly related to the integrity of the model and the values of the parameters used therein. The sensitivity of the model to changes in initial parameters provides some measure of the criticality of the parameters selected.

In early 1973 the RAND Corporation submitted a draft study (3) to the United States Air Force concerning some methods of examining the effects of escalation of nuclear warfare using such a model. However, little thought was given to the selection of the original parameters. Knowledge of the sensitivity of the model to changes in original parameters would provide a measure of the usefulness of the results. Prior to this report no such sensitivity analysis had been performed on the RAND model.

Background

National Goals

As the United States moves toward detente with the Union of Soviet Socialist Republics (USSR), national strategy should be formulated carefully. Although detente with the

Communist world might be a reality, hopes for a lasting peace should be tempered with caution. Recent publicity given to detente does not mean that it has become more important than the fundamental goal of security through strength (8:21). Americans should not think that the Soviets have abandoned their previous national objectives and goals. The Soviets' recent capability (since first-stage agreement on the Strategic Arms Limitation Talks--SALT) of deploying multiple, independently targetable re-entry vehicles should leave no doubt as to their objective of ". . . establishing clear military superiority . . . [8:3]." Those who advocate reduction of United States' armed forces and abandonment of the North Atlantic Treaty Organization (NATO), simply because the USSR has agreed to sit down and talk with the United States, have oversimplified the problem. The United States should remain militarily strong--consistent with national goals and objectives. President Nixon's strategy of realistic deterrence is applicable; military strength should be an intrinsic ingredient of any scheme for detente of world peace (8:155). Detente, as an approach to protect a nation's interests without resort to force, might not achieve its purpose unless it is supported by military strength.

> It is precisely this strength which safeguards a nation's interests and enables it to head off crises, instead of having to pay the terrible costs of war involved in actually applying this force once its threat has become clear. Power can be employed in administering pressure upon an opponent, forcing him to be conciliatory if he wishes to avoid a clash. Military strength, in short, can act as an incentive for an adversary to compromise; for if he is unwilling to be conciliatory, he is faced with the prospect of defeat in battle [25:12].

Military Strength

America's military strength performs a dual role in the balance of international power (21:868). During periods of peace a force's value is the weight it adds to negotiations as a potential threat. During war a force's value is its ability to defeat the enemy. Military power of any nation is a function of several factors. Nuclear capability has become an increasingly important factor in the military power of both the United States and the USSR (8:1). The importance of nuclear capability to the NATO alliance should not be underrated (20:10). United States' participation in NATO is important since the military strength of the Soviet Union and its Warsaw Pact allies is inconsistent with a relaxation of tensions (8:3).

The essence of NATO is a pledge by 15 nations for a mutual defensive alliance--". . . NATO has neither offensive nor aggressive intentions [29:459]." The alliance is built around three components: conventional forces, tactical nuclear forces, and strategic nuclear forces (29:460). The conventional forces represent the commitment to the alliance (29:460).

> The tactical nuclear forces provide the firepower flexibility so urgent to the NATO strategy. . . . The option for controlled and selective use of tactical nuclear weapons is a powerful deterrent. It is also a necessity, if we are to defeat any major aggression in Europe [29:460].

Strategic nuclear capability represents the ". . . ultimate deterrent and, if necessary, the ultimate defense [29:460]." As recently as 1972 U.S. officials have stressed the need to strengthen two of the components in order to increase the credibility of NATO to deter war--specifically conventional forces and tactical nuclear weapons (2:17). Reliance on nuclear weapons as a basis for NATO strategy was firmly established by the time the Kennedy administration took office (12:6). The Nixon Doctrine, which includes a national strategy of flexible response, depends on maintaining NATO force levels which, in turn, depend upon the level of United States forces in Europe (8:52). If the United States numerical force level is reduced there should be some off-setting factor--namely, an increase in nuclear capability, an increase in troop force levels from other NATO countries, an increase in troop efficiency (per capita productivity), or some combination of the three (5:27).

> For example, within the context of Mutual and Balanced Force Reduction, a reduction in the number of U.S. troops in Europe, carefully worked out and counterbalanced by technology, might not be a bad thing. Increases in firepower and mobility and accuracy of weapons is a logical American substitute for sheer numbers . . . [15:51].

Nuclear capability, and the will to use that capability, has become an intrinsic part of the USSR-United States balance of power.

> The fear of mutual destruction through nuclear war has imposed effective restraints upon the foreign policies of the superpowers in two respects: the avoidance of direct military confrontation and, when it inadvertently occurs, its speedy liquidation [18:430].

<u>Nuclear Capability</u>

"At the center of the swirling controversies involving national security in 1974 will be the little-noticed issue of tactical nuclear weapons . . . [19:16]." This issue will surface during talks on mutual force reduction, SALT II, and during debate over the U.S. defense budget (19:16). Although SALT I was an attempt to limit the proliferation of nuclear weapons (1:15), there is no assurance that the Soviets have limited the production or development of nuclear capability (8:9). Subsequently, the mutual respect for deterrence is based on a perceived balance of power and represents what can be described as an unstable equilibrium (13:437). "Tactical nuclear weapons are clearly needed so long as the enemy has them at his disposal [28:48]." There is no absolute, objective, or quantitative method for actually determining whether a balance actually exists. "It is this psychological conviction that a nuclear war is a genocidal and suicidal absurdity which has preserved the peace and at least a modicum of order in the relations of the superpowers [18:437]."

At the present time the U.S. nuclear forces supporting NATO consist of about: 190,000 ground forces, 7,000 nuclear warheads, 2,100 tanks, and 400 tactical aircraft (27:17).

> The major argument for relying on tactical nuclear weapons has always been and continues to be based on the assumption that NATO cannot defend conventionally against vastly superior Warsaw Pact forces [12:6].

Tactical nuclear war takes on importance because ". . . under alliance counterforce, and strategic parity, the true problem of defense lies on the tactical battlefield [10:7]." Current NATO policy also seens to indicate a willingness to use tactical nuclear weapons in response to non-nuclear attack (12:3).

Although tests have been made on the effectiveness of nuclear weapons, the relative importance of nuclear capability, as it relates to foreign policy and national power, is abstract and subject to conjecture (3:vii,1). A perceived potential for destruction is the key to the present unstable equilibrium mentioned previously. "Tactical nuclear warfare is a deterrent to tactical war as powerful as strategic nuclear warfare has been . . . to strategic war [10: 8]." Any knowledge acquired pertaining to the specific value of tactical nuclear capabilities should contribute toward stabilizing the current political situation. More reliance on concrete knowledge and less reliance on perceptions should lead toward better foundations for world peace, even though that world peace may essentially be based on military strength.

In order to gain additional knowledge of the values associated with military strengths, nuclear capabilities, and nuclear exchanges as they relate to foreign policy and international alliances, various theories should be tested. It is not feasible to actually test these theories under circumstances of armed conflict leading to nuclear war. Therefore, in order to derive some meaningful conclusions from any theory concerning the value of military forces, nuclear capability, and nuclear exchanges some alternative method should be employed that can simulate a conflict environment. A feasible alternative is mathematical modeling.

Game Theory

Mathematical modeling becomes appropriate in those cases involving decisions based upon some formal or systematic approach to decision making.

> Proponents of greater reliance on . . . tactical nuclear weapons tend to look upon the problem of escalation as one that can be scientifically calculated through the use of certain analytical tools, including, for example, systems analysis, game theory, and simulation [12:9].

The solution of a problem requiring the analysis of a situation involving two or more opposing sides, normally referred to as conflict situations, is aptly addressed through a mathematical technique known as game theory. "Game theory is a collection of mathematical models formulated to study decision making in situations involving conflict and cooperation [14:3]." A detailed description of game theory was contributed by John Von Neumann. Although his ideas were first published in 1928, the most extensive account appeared in 1944 titled "Theory of Games and Economic Behavior," co-authored by Askar Morgenstern. The theory is based on intricate laws of strategy: how to adopt the best course of action to avoid defeat by the opponent; how to obtain the most favorable outcome in a poor situation; and how to avoid the least favorable outcome in a good situation. In a situation that offers no clear-cut approach, game theory attempts to show how to find the strategy that will come closest to minimizing maximum possible losses--the principle of minimax.

> A game occurs when each participant in a situation has an objective which may not coincide with the objectives of other participants, and when each of the

> participants controls some, but not all, of the controllable variables of the action and the outputs. . . . Conflict between opposing military forces is the outstanding instance of a game [19:10-1].

Payoffs

Conflict arises when adversaries have different preferences. If military confrontations can be quantified, in terms of opposing force levels or force values, game theory can be used to study possible effects of relative military strengths and strategy choices. Game theory ". . . describes in detail the potential payoffs . . . and points out how one should act in order to arrive at the best possible outcome in light of the options open to one's opponents [14:3]." The objective of this approach is to estimate optimum force levels, force compositions, and strategies. In the case of a military conflict, a payoff represents a perceived value to the commander. This value, stated as a single number, further represents a merging together of a set of possible outcomes in terms of the forces involved, the environment surrounding the conflict, and the options available to the commander. Therefore, force utility may be defined as a single integrated measure of force capability, the will to use that capability, and the anticipated effectiveness of that capability. Certain assumptions must be made when designing the model, albeit the number and type of assumptions may limit the relevance of the findings. Therefore, the usefulness of the findings depends upon the integrity of the model and the assumptions necessary for its creation.

Lanchester's Equations

The results of each side's strategy selection should be translated into a payoff in order to establish a game matrix. ". . . A critical point in Game Theory so far as its application to real-life conflict situations is concerned, is reached when we try to fill in the boxes with the values of the payoff [31:21]." In 1916 F. W. Lanchester published his Aircraft In Warfare. In this book Lanchester applied differential calculus to the problem of force attrition as a result of military contact. "Lanchester's work and extensions of his results have been nominated for use as attrition models in studies of conventional land, naval, and aerial battles and guerrilla warfare [13:5]." If military capability can be equated to a rate at which one force can destroy another, Lanchester's differential equations can be used to represent this capability. Although Lanchester assumed an average combat effectiveness for the force as a whole, he ". . . pointed out that force-size itself is an exceedingly popular and much used measure to explain why the outcome of battle goes the way it does, and upon which to base expectancy of its future outcome [9:15-9]." Mathematically these attrition rates are expressed as a function of forces committed against a particular target and the degree of effectiveness against the target at various levels of escalation. For example, Blue air forces may have effectiveness coefficients (the number of Reds that one Blue can kill per unit time) of 1.3 against Red ground forces and 1.0

against Red air forces in a conventional war and effectiveness coefficients of 17.8 against Red ground forces and 0.6 against Red air forces in a nuclear war. Once the opposing force capabilities are represented mathematically a computer can be used to simulate the conflict. An elementary discussion of Lanchester's theory and the derivation of the differential equations can be found in Appendix A.

RAND Study

To make an adequate and practical determination of the relative value of military forces and nuclear capability ". . . it is desirable to have quantitative estimates of the marginal values of competing force types [21:868]." Thomas A. Brown, Selmer Johnson, and Mel Dresher of the RAND Corporation used such a model (hereafter referred to as the BJD model) for determining optimum strategies and conflict outcomes in a study for the United States Air Force. The specific purpose of the study was

> . . . to develop an analytic means for obtaining insights into the relative capabilities of theater nuclear forces, some of the interactions with conventional forces, and an ability to assess aspects of theater deterrence and military stability [7:1].

In creating the model, the opposing forces were divided into ground, air, and missile capabilities. No subdivisions of these capabilities were considered, and the force structure was assumed to be fixed once conflict began. Ground and air forces were assumed to possess both conventional and nuclear capability. Missile forces, however, were configured strictly as nuclear weapons. The model itself was a highly aggregated Lanchester model and consequently had ". . . the disadvantage that every effectiveness factor [was] a study in itself [3:2]." Effectiveness coefficients were critical to the play of the game. Their arbitrary selection negated the value of the game theory approach to the problem. Furthermore, the RAND model did not provide for force reinforcement nor did the model allow for reconfiguration of forces (converting a conventional capability to a nuclear capability) after conflict began (3:6-8). Each force had the option of four different strategies (Figure 1).

Simulation of military conflict using Lanchester's theory involves the use of the average quantitative effects that determine the outcome of the battle because of the importance of sheer force size and an average attrition rate (9:2-12). The use of analytical models to represent something as dynamic as warfare can be unreliable unless the results are considered tentative and analyzed qualitatively as well as quantitatively. Therefore, the validity of any model depends, in large part, upon the proper selection of assumptions and the proper relationships between components of the model. One method for testing the validity of conflict simulation models is to vary one or more of the principle parameters of the model and then analyze the outcome to see if the results are consistent and plausible. In other words, a sensitivity analysis is required if the model is to be accepted as a reliable representation of the situation under study. If the outcome is highly dependent upon

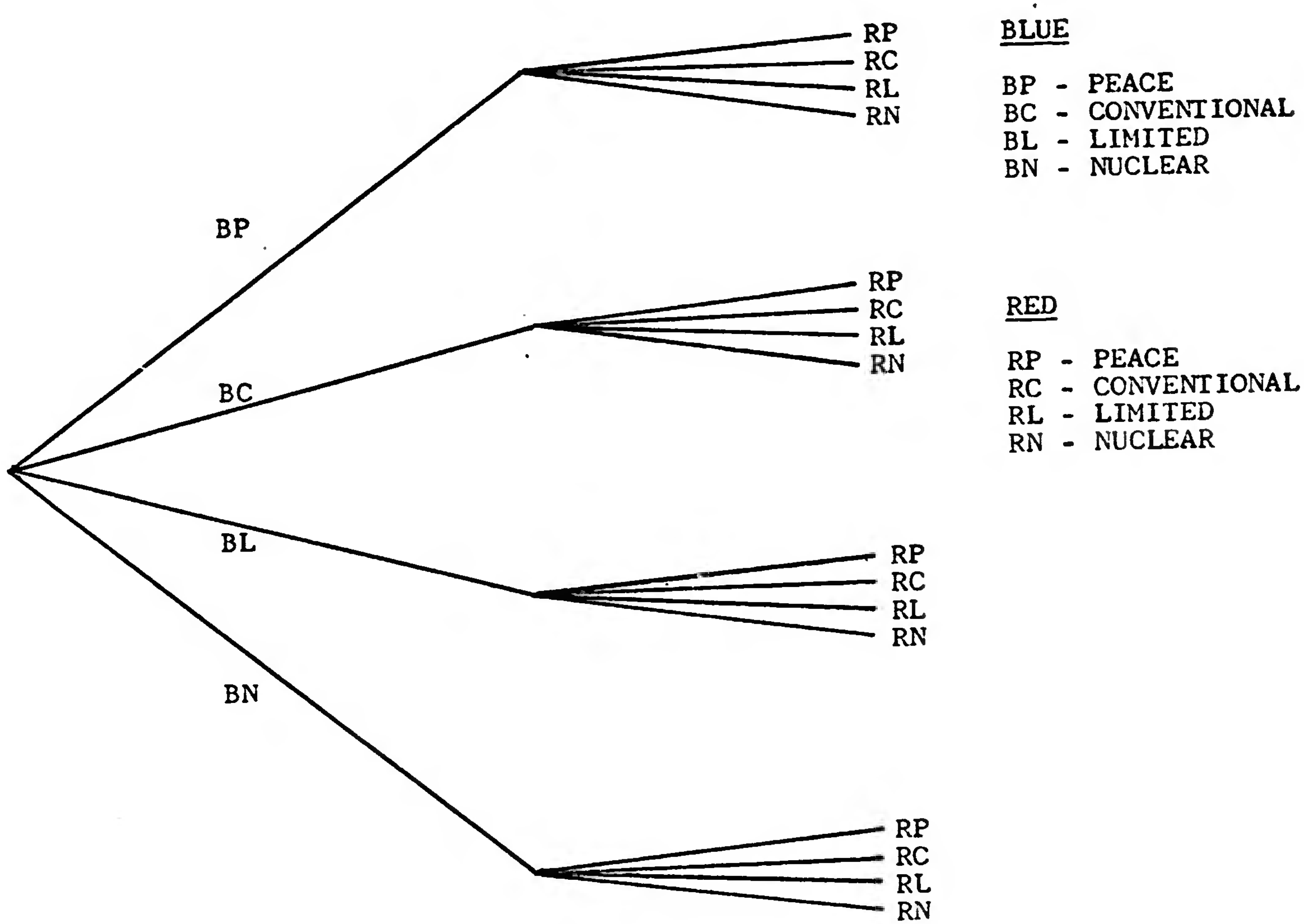

Figure 1

Force Strategies

selection of one of the principle parameters this fact should be discovered and isolated so that the results of the simulation can be analyzed accordingly.

> A good system study will include sensitivity tests on the assumptions in order to find out which ones really affect the outcome and to what extent. This enables the analyst to determine where further investigation of assumptions is needed and to call the attention of the decision-maker to possible dangers that might be present [22:296].

Accordingly, we tested the BJD model through a sensitivity analysis of the conflict outcomes by varying the force values used as parameters in the original study. The values selected to represent the respective forces were considered principle parameters since those values combined with the appropriate effectiveness factors primarily determined the outcome of the conflict (3:8). Therefore, the following objective was established as the basis for this study for determining the criticality of force values.

Research Objective

To determine the sensitivity of conflict outcomes to changes in initial force values when using the BJD model to simulate conflict.

Research Question

Is the simulated conflict outcome, using the BJD model, sensitive to changes in the force values used as parameters for the model?

CHAPTER II

AN ESCALATION MODEL

A nation's military capability is directly related to the overall amount of national resources allocated to it. In effect, once this figure is established, an upper bound has been placed upon the effectiveness of a nation's forces. Additional factors include the specific types of forces to be employed and the percentage of available resources allocated to each. In the RAND Study, force composition was limited to mixtures of ground, air, and missile forces (3:vii). This assumption agreed with the intended purpose of the BJD model; the study of escalation in a tactical nuclear environment. Given this classification of forces, the amount of national resources expended on any one of these types of forces (air, ground, missile) restricts the amount available for the other two types of forces. Once a determination has been made as to how available resources will be divided among the three force types, a force mix has been defined. Thus each specific force mix can be identified with specific percentage figures. The total set of all possible force mixes available to either side, subject to the overall resource allocation constraint, is represented in Figure 2.

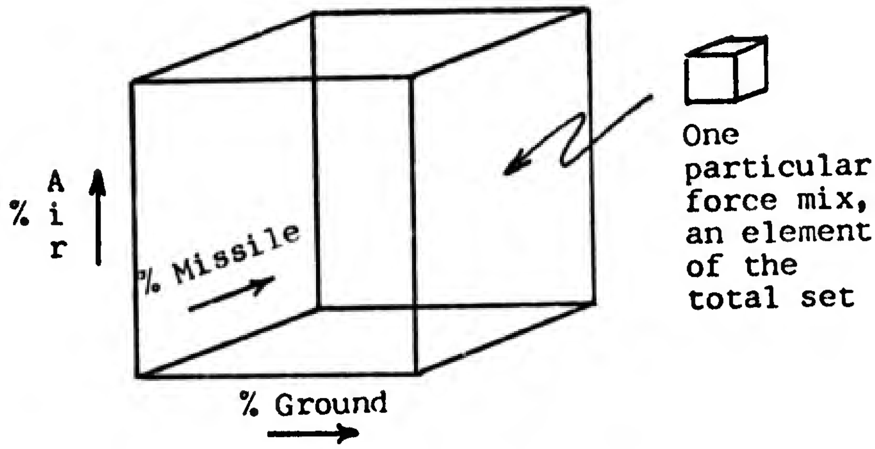

Figure 2

Set of Possible Force Mixes
Available to Either Side

The selection of a particular force mix by each of the opponents establishes a preconflict posture unique to that particular set of forces chosen. For an established set of force mixes the outcome or payoff will depend upon the strategies selected, the effectiveness coefficients of the two sides, and the times at which the opponents choose to escalate.

The RAND study initially assumed a force mix (air, ground, and missile) for Blue forces and Red forces as shown in Table 1 (3:11). The effectiveness coefficients used in the study are shown in Table 2 (3:11).

The particular values chosen ". . . enable Blue to win a nuclear war if T(B) [time of escalation for Blue] = T(R) [time of escalation for Red] = Ø. Red will win a conventional war where neither side goes nuclear [3:20]." As a result, the study suggests that in order to win, Blue must

Table 1

Initial Force Values (Utility)--RAND Study

	BLUE	RED
Aircraft	B(1) = 1100	R(1) = 900
Ground Forces	B(2) = 3006	R(2) = 6618
Missiles	B(3) = 180	R(3) = 300

Table 2

Effective Coefficients for Weapon Type
Versus Weapon Type--RAND Study

	BLUE			RED		
Nuclear War						
Weapon / Target	Air	Ground	Missiles	Air	Ground	Missiles
Air	30.0	13.2	1.6	15.0	13.2	0.8
Ground	0	0.8	0	0	0.1	0
Missiles	15.0	15.0	1.0	15.0	15.0	1.0
Conventional War						
Weapon / Target	Air	Ground	Missiles	Air	Ground	Missiles
Air	0.4	1.32	0.2	0.7	1.32	0.2
Ground	0	0.08	0	0	0.02	0
Missiles	0	0	0	0	0	0

escalate to the nuclear level prior to some specific point in the conflict. A question specifically addressed by RAND dealt with the determination of this critical time of escalation, defined as ". . . the last moment that Blue can escalate from conventional to nuclear war and still win the war . . . [3;20]." It is at this critical time of escalation that Blue loses his nuclear supremacy. Thus, it serves as ". . . a measure of stability because it measures the security, or ruggedness, of Blue's nuclear deterrent [3;viii]." To test the sensitivity of this critical time of escalation an initial force value was arbitrarily (3;22) increased by 100 units, e.g., B(1) = 1100 + 100 = 1200, while the remainder, B(2), B(3), R(1), R(2), R(3), remained unchanged. This represented a second conflict situation. For the third case B(2) was increased by 100 while the remainder were kept at their original values (B(1) reset to 1100). The resulting total sample of force mixes used by RAND for the study was limited to the seven force mixes shown in Table 3. The results of the original RAND study are listed in Table 4. The study, limited to the seven cases listed in Table 3, seemed to indicate that, for the values chosen, a critical time of escalation did exist and was indeed sensitive (i.e., caused a change in critical time of escalation greater than or equal to .1 days or 2.4 hours) to changes in certain force values (air and missile forces) and was not sensitive to changes in other force values (ground forces).

Table 3

Sample of Initial Force Values (Utility)--RAND Study

FORCE MIX		BLUE	RED
1	Aircraft	B(1) = 1100	R(1) = 900
	Ground Forces	B(2) = 3006	R(2) = 6618
	Missiles	B(3) = 180	R(3) = 300
2	Aircraft	B(1) = 1200*	R(1) = 900
	Ground Forces	B(2) = 3006	R(2) = 6618
	Missiles	B(3) = 180	R(3) = 300
3	Aircraft	B(1) = 1100	R(1) = 900
	Ground Forces	B(2) = 3106*	R(2) = 6618
	Missiles	B(3) = 180	R(3) = 300
4	Aircraft	B(1) = 1100	R(1) = 900
	Ground Forces	B(2) = 3006	R(2) = 6618
	Missiles	B(3) = 280*	R(3) = 300
5	Aircraft	B(1) = 1100	R(1) = 1000*
	Ground Forces	B(2) = 3006	R(2) = 6618
	Missiles	B(3) = 180	R(3) = 300
6	Aircraft	B(1) = 1100	R(1) = 900
	Ground Forces	B(2) = 3006	R(2) = 6718*
	Missiles	B(3) = 180	R(3) = 300
7	Aircraft	B(1) = 1100	R(1) = 900
	Ground Forces	B(2) = 3006	R(2) = 6618
	Missiles	B(3) = 180	R(3) = 400*

*Indicates the force value that was increased

Table 4

Force Mix	Tc (Days)	Delta Tc (Days)
1	1.19662	---
2	1.67052	+.47390
3	1.26585	+.07923
4	1.73303	+.53641
5	.95877	-.23785
6	1.17273	-.02389
7	.48238	-.71424

It was our intent to expand upon the original sensitivity analysis. To do so a separate set of force values were to be selected as a base case study and the force values allowed to vary as in the original study.

Instruments

The RAND study dealing with the sensitivity of the critical time of escalation included Lanchester equations to calculate the outcome of the conflicts (Appendix A). The RAND analysts varied the force levels while holding the effectiveness coefficients and the strategy set constant. Since the computer facilities available to us, the Air Force Institute of Technology, School of Systems and Logistics, CREATE system (Computer Resources for Engineering and Simulation Training and Education) did not use the JOSS (Johnniac Open Shop System) computer language, we translated the BJD program into FORTRAN in order to expand upon the study (Appendix B). The FORTRAN program was then used to replicate the values obtained in the RAND study in order to verify the integrity of the translation. This translated program was the instrument used to test for the existence of a critical time of escalation and its sensitivity to changes in the force values chosen.

Once the findings obtained in the RAND study with the original force values and effectiveness coefficients were replicated, the original force values were replaced with the values furnished us by Mr. R. F. Robinson, AF/SAG, sponsor of the RAND study (24).

The strategy sets, missile depletion rate, effectiveness coefficients, and delay in time of escalation for Red were to be identical to the RAND study. As in the original study Blue's target priorities were air, then missile, and then ground. Red's target priorities were air, and then ground. Both missile utilization rates were set equal to the number of missiles on hand at the outset. That is, all missile forces were to be consumed in one day of conflict regardless of the number on hand at the start of the battle. Finally, the delay in the time of escalation for Red was kept at .1 days or 2.4 hours.

Study Design

In testing the sensitivity of the BJD model we used the force values shown in Table 5 as a base case. These values were chosen to replace the original values used in the RAND study on the recommendation of Mr. Robinson. As previously mentioned, the effectiveness coefficients, conflict scenario, and delay in time of escalation for Red were to be identical to that of the original study.

Table 5

Force Values (Utility)--Current Study

	Blue		Red	
	Base Value	Range	Base Value	Range
Aircraft	3,200	2,800-4,000	2,400	1,900-3,200
Ground Forces	12,000	6,000-18,000	19,000	9,000-30,000
Missiles	180	0-1,100	300	0-3,000

The RAND study limited its investigation of the sensitivity of the BJD model to single excursions of 100 units added separately to each of the force values. We intended to investigate a much wider range as indicated in Table 5. For those cases where a critical time of escalation did exist the values were to be incremented in steps of 100 units at a time. As in the original study only one force value was allowed to vary at a time. When the results of a specific force value's excursions over its particular range were determined, the force value was again fixed at its original base case value for the excursions of other values. The data for cases under study was collected and presented in a format similar to Table 6.

Table 6

Data Collection Format for Sensitivity Analysis

Force Value Modified	Change in Value	New Value	Value of Critical Time of Escalation	Change From Critical Time of Escalation at Base Case
B(1)	⋮ -200 -100 +100 +200 ⋮			
B(2)	⋮ -200 -100 +100 +200 ⋮			
B(3)	⋮ -200 -100 +100 +200 ⋮			
R(1)	⋮ -200 -100 +100 +200 ⋮			
R(2)	⋮ -200 -100 +100 +200 ⋮			
R(3)	⋮ -200 -100 +100 +200 ⋮			

Assumptions

1. A critical time of escalation does exist for the force values and effectiveness coefficients chosen.

2. Payoffs for conflict simulation can be measured numerically.

3. Both sides selected their strategies in a rational manner so as to optimize their resulting payoffs.

4. Forces could be divided into only three separate categories--ground, air, and missile.

5. Ground and air forces could be used at all levels of conflict.

6. Missile forces could be used only at the limited nuclear and nuclear conflict level.

7. Missile forces were neither used nor attacked in conventional war.

8. Ground and air forces were attrited only by enemy action and not by use.

9. Ground forces were used only against ground forces.

10. There was a delay time of .1 days (held constant for all situations) before changes in strategy could be effected.

11. Once conflict was initiated forces could not be reinforced or reconfigured.

12. Once conflict was initiated only the possibility of further escalation was considered. Therefore, no provision was made for unilateral de-escalation.

Limitations

As in the RAND study the sensitivity analysis on the force values was conducted with a specified battle scenario, set effectiveness coefficients, and a fixed delay in time of escalation for Red.

CHAPTER III

RESULTS

Program Translation

The translation of the BJD computer program from JOSS (4,11) into GE FORTRAN IV (26) was the first step required in our thesis effort. (A computer listing of the FORTRAN version appears in Appendix B.) As previously mentioned this was necessary since the facilities available to us did not include the JOSS computer language.

In the process of translating, several changes were made to the original BJD program. For example, a number of modifications were made to adapt the BJD program to FORTRAN logic. These modifications consisted mainly of restructuring the order in which operations were carried out. The numbers used within the program to identify the various sections of the translated version correspond to the numbered sections of the original program. While this practice resulted in a nonsequential numbering of the sections, it did facilitate cross-referencing the translated program with the original JOSS version. Additionally, variable names used within the program were changed to make them more meaningful and thereby make the program logic easier to follow. For example, the expression B(1), used in the original version to stand for Blue's air strength was changed to BLUEFOR (1); the letter N, used in the BJD model to represent the rate at which Red's missile forces are depleted was changed to REDMSLR. Additionally, we added a number of comment statements throughout to explain the internal logic of the program and the calculations carried out in the various sections. It should be emphasized that the changes made in no way compromised the integrity of the original program. The formulas used within the translated version and the results obtained with it for the original force values and effectiveness coefficients were identical to those used in the original study.

Verification

In order to verify the integrity of the translation the two examples discussed in the original study were duplicated using the translated program and the results compared to those of the original study. In doing so, the force values and effectiveness coefficients used were identical to those used in the original study.

In part one of the verification process the program was used in the standard fashion. Times of escalation for both Blue and Red along with the respective strategies were inserted in the same order as in the original example. The intermediate findings and the final results (shown in Appendix C) were identical to the original example.

In the second portion of the verification process the example dealing with the calculation of the critical time of escalation for Blue was duplicated. Here again the results were identical to those obtained in the original study (Appendix D).

Once the program had been validated, the computer program was modified to include a subroutine which automatically computed the critical time of escalation. This was in contrast to the original BJD model where the user was required to input different times, using trial and error, until the critical time of escalation was found. The subroutine used in the modified version of the computer program is in Appendix E.

Sensitivity

Original Variables

Since the sensitivity of Blue's critical time of escalation to the force values used was our primary interest, the sensitivity analysis of the original study was expanded upon in conjunction with the verification process. This was done simply by expanding the range over which the force values were allowed to change.

The limits of the expanded range, established in accordance with guidance furnished by Mr. R. F. Robinson, AF/SAG (24), were one-half the base case value and twice the base case value or that point beyond which a critical time of escalation did not exist, whichever came first. The results of this expanded analysis on the original force values and effectiveness coefficients are presented in Table 7. As in the original study changes in air and missile force levels produced a change in the critical time of escalation of greater than .1 days per 100 units of change in the force

Table 7

Expanded Sensitivity Analysis on the Original Force Values and Effectiveness Coefficients

Force Value Modified*	Change in Value	New Value	Value of Critical Time of Escalation	Change From Critical Time of Escalation at Base Case
B(1) (1100)	- 500	600	0.059	-1.138
	- 200	900	0.632	-0.565
	- 100	1000	0.881	-0.316
	+ 100	1200	1.670	+0.473
	+ 150	1250	2.100	+0.903
B(2) (3006)	-1506	1500	0.553	-0.644
	- 200	2806	1.068	-0.129
	- 100	2906	1.130	-0.067
	+ 100	3106	1.266	+0.069
	+ 200	3206	1.338	+0.141
	+2994	6000	5.156	+3.959
B(3) (180)	- 90	90	0.840	-0.357
	+ 100	280	1.733	+0.536
	+ 180	360	2.311	+1.114
R(1) (900)	- 125	825	1.505	+0.308
	- 100	800	1.672	+0.475
	+ 100	1000	0.959	-0.238
	+ 200	1100	0.802	-0.395
	+ 900	1800	0.341	-0.856
R(2) (6618)	-3318	3300	2.405	+1.208
	- 200	6418	1.245	+0.048
	- 100	6518	1.220	+0.023
	+ 100	6718	1.173	-0.024
	+ 300	6918	1.126	-0.071
	+6382	13000	0.550	-0.647
R(3) (300)	- 150	150	2.927	+1.730
	- 100	200	2.232	+1.035
	+ 100	400	0.482	-0.715
	+ 190	490	0.001	-1.196

Critical time of escalation in the original base case = 1.19662

*all other values are unchanged from those shown level. Changes in the ground forces produced a change in the critical time of escalation of less than .1 days per 100 units of change in the force level. According to the definition of sensitivity we used (a change in critical time of escalation greater than or equal to .1 days per a change in the force level of 100 units), the BJD model was found to be sensitive to changes of air and missile forces and not sensitive to changes in ground forces. This finding confirmed the results of the RAND study.

New Variables

Having translated the BJD program, verified the integrity of the translation, and replicated and expanded upon the sensitivity analysis of the original force values, we were ready to conduct our study, a sensitivity analysis on a different set of force values. The new force values (Table 5, p. 21) which formed the basis of our study were furnished by Mr. Robinson, AF/SAG (24). While it was our original intent as outlined in the study design of Chapter II to change only the force values, Mr. Robinson requested that the effectiveness coefficients used in the program also be changed. The new effectiveness coefficients are listed in Table 8. At his request both the force values and effectiveness coefficients were changed simultaneously. Having done so, an attempt was then made to determine the critical time of escalation for the base case. In the process, times of escalations for Blue ranging from 0 to 999 were used. In all cases the outcome was the same--Blue won (Table 9). In

Table 8

Effectiveness Coefficients for Weapon Type Versus Weapon Type--Current Study

Conventional War	Blue			Red		
Target						
Weapon	Air	Ground	Missiles	Air	Ground	Missiles
Air	0.12	0.7	0.02	0.08	0.02	0.01
Ground	0	0.08	0	0	0.08	0
Missiles	0	0	0	0	0	0
Nuclear War						
Target						
Weapon	Air	Ground	Missiles	Air	Ground	Missiles
Air	1.1	2.5	1	0.8	2.5	1
Ground	0	0.8	0	0	0.08	0
Missiles	1.0	2.0	0.8	1.2	8.0	1.2

Table 9

Results of the Sensitivity Analysis Using the New Force Values and Effectiveness Coefficients

Case	Blue Force Values			Red Force Values			Existence of Tc		Remarks
	Air	Ground	Missile	Air	Ground	Missile	Yes	No	
Base	3,200	12,000	180	2,400	19,000	300		*	*
1	2,800							*	*
2		6,000						*	*
3			0					*	*
4				3,200				*	*
5					30,000			*	*
6						3,000		*	**(5.817)
7	2,800	6,000						*	*
8		6,000	0					*	*
9	2,800		0					*	*
10				3,200	30,000			*	*
11					30,000	3,000		*	**(5.817)
12				3,200		3,000		*	**(31.030)
13	2,800	6,000	0					*	*
14				3,200	30,000	3,000		*	**(31.030)
15	2,800	6,000	0	3,200	30,000	3,000		*	**(112.540)

*Blue always wins

**Blue wins if he delays his time of escalation to beyond the time indicated or does not escalate at all no case, regardless of the time selected for Blue to escalate, could Red win the conflict. Since the final outcome (i.e., whether or not Blue would win) was found to be independent of Blue's time of escalation, no critical time of escalation could be established for the set of force values and effectiveness coefficients under study.

This clearly violated one of the assumptions made at the outset of the study, i.e., a critical time of escalation does exist for the force values and effectiveness coefficients chosen. This being the case there was little value in testing for the existence of a critical time of escalation for force values for Blue greater than the base case or for force values for Red less than the base case. Clearly, any such changes to the force values made in these directions would only strengthen Blue's position. Indeed our primary objective now became one of locating a case that would admit to the existence of a critical time of escalation for Blue. To do so required that we either weaken Blue's position or strengthen Red's position or some combination of the two changes.

In an attempt to do so, Blue's force values were decreased to the lower limits as shown in Table 5 (p. 21 Blue's force values were varied singularly, in pairs, and finally all three at a time. Blue's force values were then returned to their base case values and Red's force values raised to the upper limits shown in Table 5 (p. 21). Again Red's force values were varied singularly, in pairs, and three at a time. In a final attempt to locate a critical time of escalation all of Blue's force values were dropped to the lower limits and all of Red's force values were raised to the upper limit of the range. The results of the various trial cases is depicted in Table 9. As indicated in Table 9 no case could be found that would admit to the existence of a critical time of escalation. This being the case, no analysis on the sensitivity of Blue's critical time of escalation to the force values used was possible.

CHAPTER IV

DISCUSSION AND CONCLUSIONS

Discussion

The outcome of the conflict simulation was significantly affected by the changes to the variables, as requested by Mr. Robinson (24), and an underlying nuclear superiority enjoyed by the Blue forces. The contrast between the results of our thesis and the study conducted by RAND can be attributed to three factors: (1) the changes made to the force values, (2) the changes made to the effectiveness coefficients, and (3) Blue's nuclear superiority.

Force Values

As previously discussed, there were significant changes made in the force values used for this study as contrasted to those force values used in the RAND study. As shown in Table 10 the ratios of Blue air to Red air and Blue ground to Red ground were increased, and, in addition, the ground forces for both sides became a larger proportion of each total force. These changes, coupled with the changes in the effectiveness coefficients (to be discussed next), resulted in the conflict outcome as explained in the previous chapter. Furthermore, the increase in the overall force ratio--from 0.55:1 to 0.71:1--and the change in effectiveness coefficients resulted in a situation where Blue could never

Table 10

Comparison of Changes in Force Values

	Old Values	% of Total	Ratio of Blue to Red	New Values	% of Total	Ratio of Blue to Red
Blue						
Air	1100	25.7	1.2	3200	20.8	1.33
Ground	3006	70.1	.45	12000	78.0	.63
Missile	180	4.2	.6	180	1.2	.6
Total	4286		.55	15380		.71
Red						
Air	900	11.6		2400	11.1	
Ground	6618	84.6		19000	87.6	
Missile	300	3.8		300	1.3	
Total	7818			21700		

lose a conflict at the base case regardless of the time of escalation.

Effectiveness Coefficients

The changes made to the effectiveness coefficients have been discussed several times previously but, for the sake of clarity, the changes are summarized and shown in Table 11. The important thing to note is not the absolute changes for any particular force but the change in the relationship that now exists between two opposing forces. For example, Table 11 shows that the Blue effectiveness coefficient for air versus air in nuclear conflict was changed from 30.0 to 1.1. However, it is more important to note in Table 12 that the previous effectiveness coefficient for Blue air versus Red air was twice the coefficient for Red air versus Blue air (30.0 compared to 15.0) but the new coefficients are 1.1 for Blue and 0.8 for Red--a Blue to Red ratio of 1.375. This indicates the two air forces, in a nuclear conflict, are much closer in effectiveness than previously.

Another significant change is the difference in effectiveness for ground versus ground--nuclear conflict--with the new effectiveness coefficients. The new values now indicate that Blue ground forces are ten times as effective as Red ground forces in a nuclear conflict.

In case of a conventional conflict, the new values indicate another significant change in the case of air versus air. Before the change, Red was more effective (Blue--0.4, Red--0.7: a ratio of 0.571:1), but after the change Blue has

Table 11

Summary of Changes to Effectiveness Coefficients

Conflict Configuration	Nuclear				Conventional			
	Blue		Red		Blue		Red	
	Old	New	Old	New	Old	New	Old	New
Air vs Air	30.0	1.1	15.0	.8	0.4	0.12	0.7	0.08
Air vs Ground	13.2	2.5	13.2	2.5	1.32	0.7	1.32	0.02
Air vs Missile	1.6	1.0	0.8	1.0	0.2	0.02	0.2	0.01
Ground vs Air	C	0	0	0	0	0	0	0
Ground vs Ground	0.8	0.8	0.1	0.08	0.08	0.08	0.02	0.08
Ground vs Missile	0	0	0	0	0	0	0	0
Missile vs Air	15.0	1.0	15.0	1.2	0	0	0	0
Missile vs Ground	15.0	2.0	15.0	8.0	0	0	0	0
Missile vs Missile	1.0	0.8	1.0	1.2	0	0	0	0

Table 12

Ratio of Blue to Red Effectiveness Coefficients--Original and New

Conflict Configuration	Nuclear		Conventional	
	Original	New	Original	New
Air vs Air	2	1.375	0.571	1.5
Air vs Ground	1	1	1	35
Air vs Missile	2	1	1	2
Ground vs Ground	8	10	4	1
Missile vs Air	1	0.833	-	-
Missile vs Ground	1	0.25	-	-
Missile vs Missile	1	0.667	-	-

become more effective (Blue--0.12, Red--0.08; a ratio of 1.5:1). Also the effectiveness of Blue's air against Red's ground, instead of being the same, is now 35 times more effective than Red's air against Blue's ground.

The results of the changes for nuclear conflict are shown in Figures 3 and 4. These figures show the comparative loss rates for the two forces using the original variables (Figure 3) and the new variables (Figure 4). Since the critical time of escalation in the RAND study was 1.2 days (3:22), the conflict is simulated and losses shown for the first day of conflict. This was done to provide a uniform time interval for the sake of comparing the situation shown in Figures 3, 4, and 5. At the nuclear level of conflict we found that Blue encounters a higher rate of loss than before, but still had an overwhelming nuclear advantage. At a conventional level of conflict the biggest change occurs in the area of air effectiveness. This is depicted in Figure 5. As can be seen in this figure, there is a complete turnaround in the air against air battle. Furthermore we found (Table 13) that at a conventional level of conflict with the new effectiveness coefficients, Red's air forces attrite to zero at time 7.27 days leaving Blue with a force value of 2530 for air. At time 9.6 days Blue's ground forces have been reduced to zero by Red's ground forces leaving each side with its missiles, which do not engage in a conventional conflict. Therefore, Blue ultimately wins by using his remaining air against Red's ground units which can not attack air in this

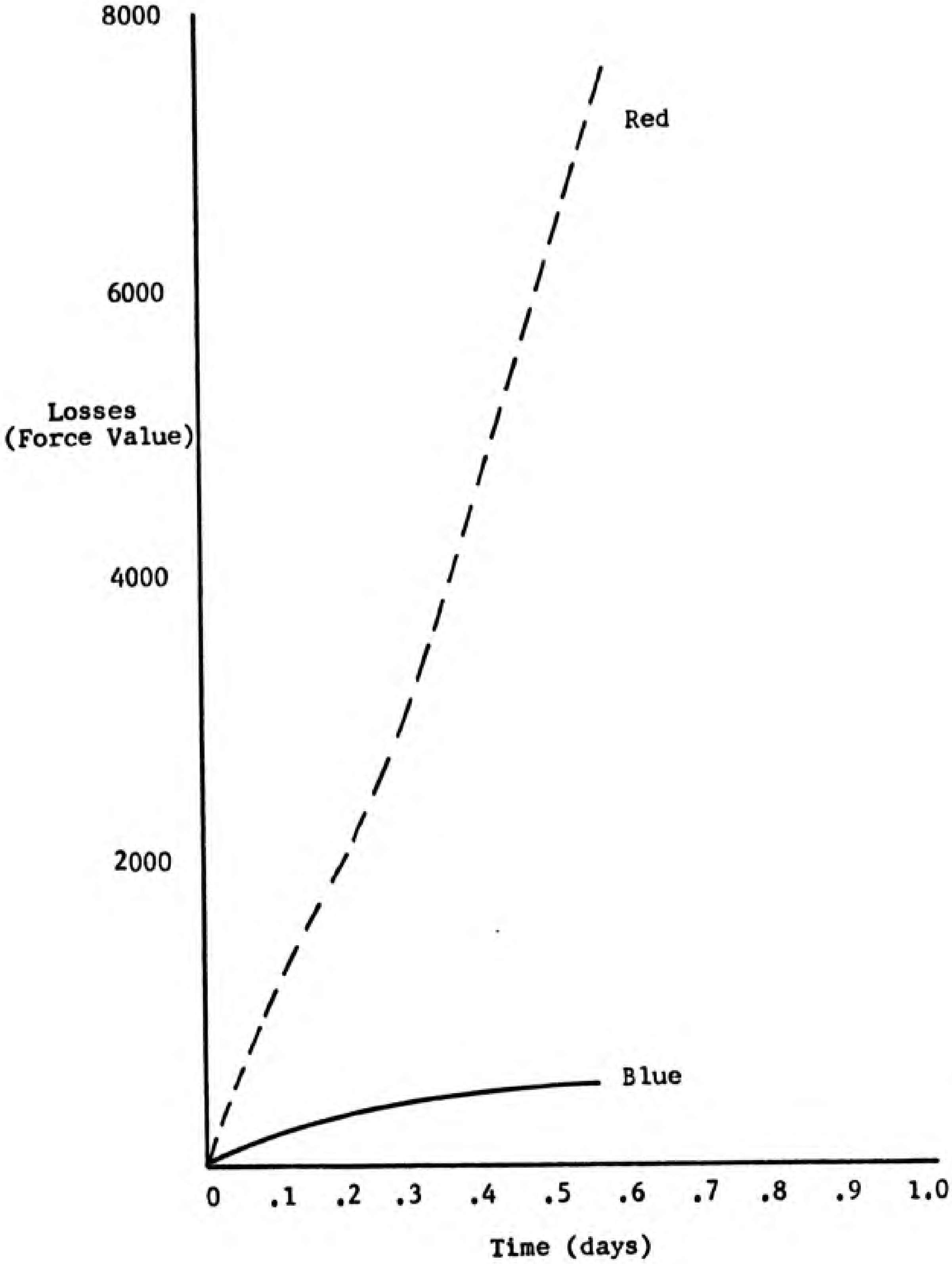

Figure 3

Cumulative Losses for Nuclear Conflict
With Original Variables

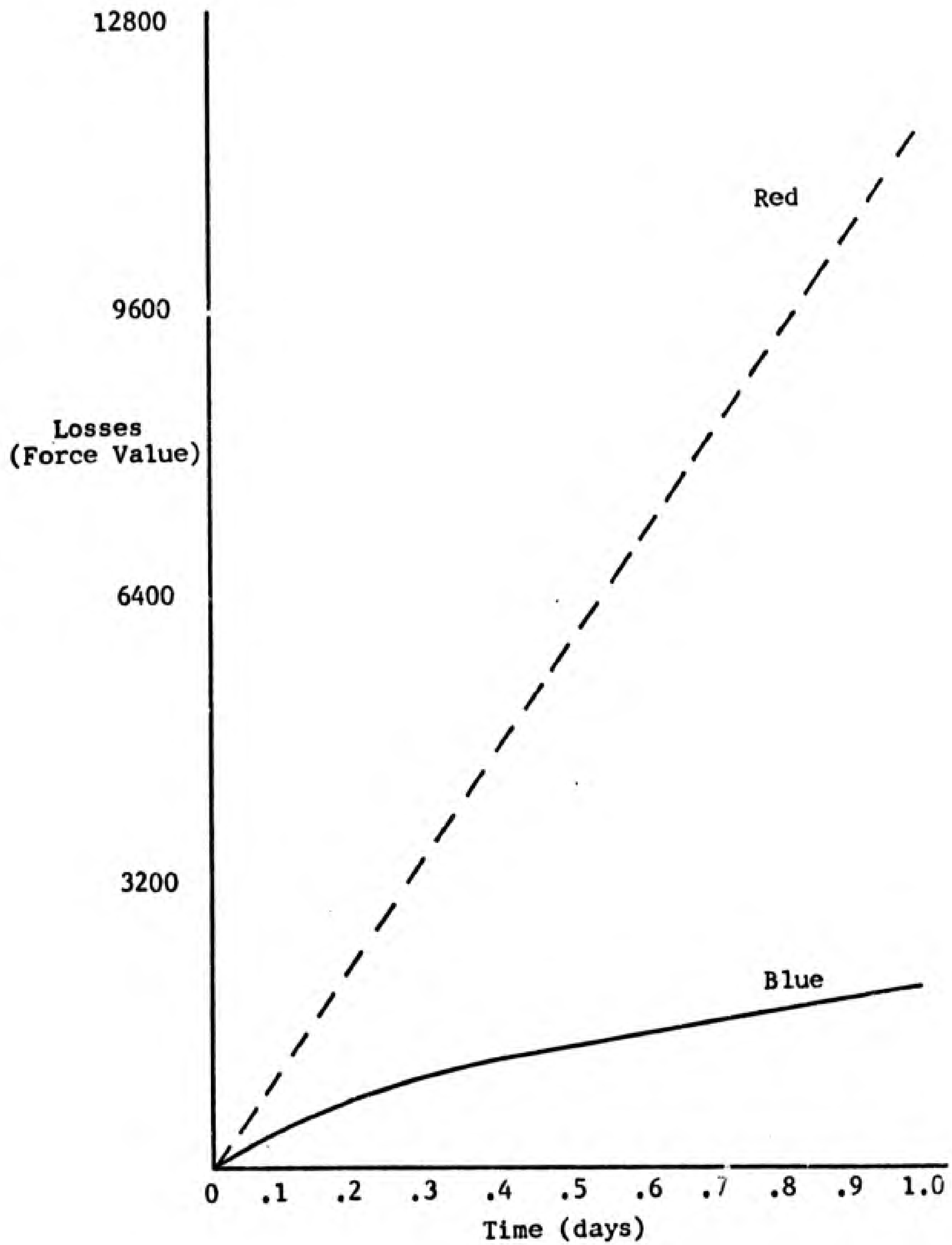

Figure 4

Cumulative Losses for Nuclear Conflict
With New Variables

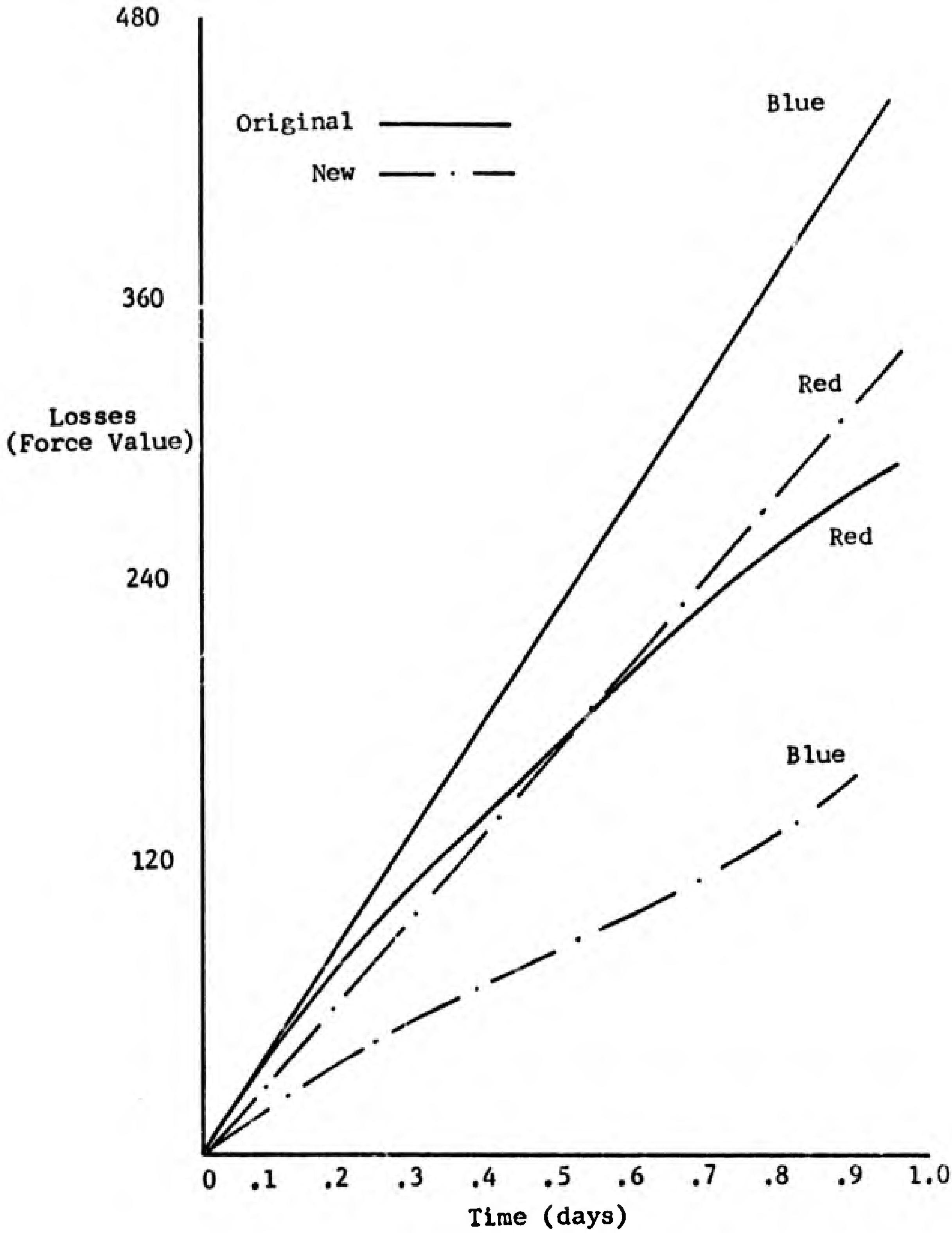

Figure 5

Comparison of Original and New Variables
for Air Losses for Conventional Conflict

Table 13

Results of a Conventional Conflict Using the New Variables

Time (Days)		Air	Ground	Missile*	Allocation
0	Blue	3200	12000	180	Air vs Air, Ground vs Ground
	Red	2400	19000	300	Air vs Air, Ground vs Ground
7.27405	Blue	2503	2399	180	Air vs Ground, Ground vs Ground
	Red	0	14925	300	Ground vs Ground
9.63707	Blue	2530	0	180	Air vs Ground
	Red	0	10527	300	No option at conventional level

*Missiles are not used in a conventional conflict.

model. This is another contrast to the RAND study where Blue always lost a conventional or limited nuclear conflict. As stated before, this represents the most significant change made to the model and explains why Blue can always win a conflict which uses the new base values with proper timing of his escalation.

Nuclear Superiority

The nuclear superiority of Blue, both before the changes and after the changes, is shown in Figures 3 and 4. Although Blue's loss rate is higher with the new values, his nuclear effectiveness against Red's forces is overwhelming. Even though Red has some advantage in the missile area (see Table 11) this is overcome by day 0.827 when Red missiles have been totally destroyed or depleted. The results of a nuclear conflict using the new values are shown in Table 14. The force allocations used for this simulation were basically the same as used in the RAND study.

The significance of this is that Blue is now (with the modified values) able to withstand a conventional attack (Table 13) by Red until Red's capability to respond with nuclear air weapons has been eroded. If Blue waits until this point to escalate to nuclear war he speeds his victory and minimizes his losses. This is a result of Red having a reduced nuclear (missile) capability with which to respond thereby reducing Blue's losses if Blue waits until this critical time of delay before escalating to a nuclear level.

Table 14

Nuclear Conflict--New Variables

Time (Days)		Air	Ground	Missile	Allocations
0	Blue	3200	12000	180	Air vs Air, Gr vs Gr, Msl vs Msl
	Red	2400	19000	300	Air vs Air, Gr vs Gr, Msl vs Msl
.33333	Blue	2707	11535	0	Air vs Air, Gr vs Gr,
	Red	1326	15864	152	Air vs Air, Gr vs Gr, Msl vs Air
.82554	Blue	2277	10999	0	Air vs Msl, Gr vs Gr
	Red	0	11433	4	Gr vs Gr, Msl vs Air
.82723	Blue	2276	10997	0	Air vs Gr, Gr vs Gr,
	Red	0	11418	0	Gr vs Gr
1.62604	Blue	2276	10634	0	
	Red	0	0	0	

Conclusions

We conclude that the outcome of simulated conflict using the BJD model is sensitive to changes in the force values and the effectiveness coefficients. Because of the nature of the changes requested by Mr. Robinson, force values and effectiveness coefficients concurrently, it is not possible to determine the effects each of the changes might have made individually. As mentioned in Chapter III, our original intent was to vary only the force values since we felt that a sensitivity analysis on the effectiveness coefficients was a study in and of itself.

In one important respect our findings were contrary to RAND's findings. Specifically, a critical time of escalation, in the sense defined by the RAND study, does not necessarily exist. As explained before, various changes to the force values and to the effectiveness coefficients can materially change the relationship between the Blue and Red forces. The changes requested by Mr. Robinson resulted in a situation where Blue can always win with proper timing of his escalation. Furthermore, if Blue delayed his escalation to the nuclear level until he had exhausted most of Red's nuclear potential, Blue could then escalate to the nuclear level and minimize his losses for that conflict. We defined this "time" as the critical time of delay in contrast to RAND's "critical time of escalation."

Nevertheless, when validating the translated computer program, trial runs were made on various force compositions using the original variables. As discussed in Chapter III, the outcome of these excursions showed that the conflict outcome of the original model was sensitive to changes in force values for air, somewhat sensitive to changes in force values for missiles, and not sensitive to changes in ground force values.

Recommendations

As a result of the findings of this study we recommend the following areas for additional research on the original RAND project.

1. Conduct a sensitivity analysis on the effectiveness coefficients holding the force values constant. We suggest that this be done using two base cases: (1) with the original force values used in the RAND study, and (2) with the new force values described in Mr. Robinson's memorandum (24).

2. Conduct a sensitivity analysis on the missile exchange rate. As explained in Chapter II, the present missile exchange rate is set to exhaust the missile forces in exactly one day regardless of the number on hand at the outset. This could be varied to allow for a greater exchange rate; e.g. all expended in one-half of a day, or reduced by allowing for expenditure over two or more days.

3. Conduct a sensitivity analysis on the one-tenth of a day delay time used for counter escalation. We believe that a response within less than two and one-half hours is not realistic. In the case of nuclear weapons, communications

alone could take considerably longer, e.g. six hours or more. The model should be analyzed with respect to what difference in outcome would result if the delay time were increased from .1 to .25 (or longer).

4. Conduct a study of changes in outcomes based on varying the scenario. The RAND study and our thesis assumed a one step escalation from conventional to nuclear conflict. The model is designed to handle a phased escalation (conventional-limited-nuclear) and the results of this should be analyzed accordingly.

APPENDICES

APPENDIX A

LANCHESTER THEORY

APPENDIX A

LANCHESTER THEORY

As an explanation of the derivation of Lanchester's Equations we extracted the following material from "A Lanchester Model for Air Battles," an unpublished Master's thesis by John H. Latchaw (13). This particular discussion was used because it represented the most concise, clear, and logical explanation of Lanchester's equations of all the references studied by us. We quoted verbatim except for changes to symbology on notation. These changes were made, where possible, so that the discussion would conform to the symbology and notation used in the RAND study.

Lanchester Equations

Lanchester (Aircraft In Warfare--1916) expresses his theory in mathematical statements by considering how force size varies during the course of battle. To accomplish this, he represents force size as a function of time and assumes that the battle is continuous and terminates with the annihilation of one force.

The equations include a number of inherent assumptions:

(1) Forces on either side are within range of all opposing forces.

(2) Each force type is internally homogeneous, being equally vulnerable and having equivalent effectiveness coefficients (though the vulnerability and/or effectiveness coefficients of one side can differ from that of the other).

(3) Both sides possess perfect information (i.e. are at all times aware of the location and condition [dead or alive] of all enemy forces).

(4) Fire is uniformly distributed over surviving units.

(5) No replacements are made after the conflict starts.

(6) Effectiveness coefficients remain constant throughout the conflict.

Let B represent the Blue force size at time t and R represent the opposing Red force size at time t. Let k be a parameter representing force B's combat loss rate per opposing combatant. That is, k expresses the number of B's fighting units which a member of the R force is capable of destroying per unit of time.

The parameter k is most frequently interpreted as the product of the rate of fire of a single R force fighting element and the probability of destroying a B force element with a single round of fire. Let l similarly represent force R's combat loss rate. The dot notation shall be used here . . . to denote differentiation with respect to time. Using the preceding conventions, a battle conforming to the Square Law conditions may be represented by the following set of differential equations.

$$\dot{B} = -kR \tag{1}$$

$$\dot{R} = -lB \tag{2}$$

A solution for this set of equations can be obtained in the following manner. Differentiate and rearrange equation (1).

$$\ddot{B} + k\dot{R} = 0 \tag{3}$$

Substitute -lB from equation (2) for R in equation (3).

$$\ddot{B} - klB = 0 \tag{4}$$

Solve equation (4) and let y represent $(kl)^{\frac{1}{2}}$.

$$B = c_1e^{yt} + c_2e^{-yt} \tag{5}$$

Similarly

$$R = k_1e^{yt} + k_2e^{-yt} \tag{6}$$

Let the initial force sizes at time zero be denoted by B_o for the Blue force and by R_o for the Red force. Substitute zero for t in equations (5) and (6).

$$B_o = C_1 + C_2 \tag{7}$$

$$R_o = K_1 + K_2 \tag{8}$$

Differentiate equation (5) and substitute the resulting expression for B in equation (1). Also substitute the right hand portion of equation (6) for R in equation (1).

$$c_1ye^{yt} - c_2ye^{-yt} = -k(K_1e^{Yt} + K_2e^{-Yt}) \quad (9)$$

When t equals zero, equation (9) becomes

$$y(c_1 - c_2) = -k(k_1 + k_2) \quad (10)$$

Similar substitutions from the derivative of equation (6) and equation (5) in (2) produce

$$y(k_1 - k_2) = -1(c_1 + c_2) \quad (11)$$

Using equations (10) and (8), one obtains

$$R_o = k_1 + k_2 = \frac{-y}{k}(c_1 - c_2) \quad (12)$$

Solving equations (7) and (12) simultaneously results in expressions for c_1 and c_2,

$$c_1 = \frac{B_o - R_o(k/y)}{2} \quad (13)$$

$$c_2 = \frac{B_o + R_o(k/y)}{2} \quad (14)$$

Using equations (11) and (7), one obtains

$$B_o = c_1 + c_2 = \frac{-y}{1}(k_1 - k_2) \quad (15)$$

Solving equations (8) and (15) simultaneously results in expressions for k_1 and k_2,

$$k_1 = \frac{R_o - B_o(1/y)}{2} \quad (16)$$

$$k_2 = \frac{R_o + B_o(1/y)}{2} \qquad (17)$$

Replacing y with $(kl)^{\frac{1}{2}}$ in equations (13), (14), (16), and (17) and substituting these expressions for the constants in equations (5) and (6) gives the desired results.

$$B = \frac{B_o - R_o \quad k/1}{2} e^{yt} + \frac{B_o + R_o \quad (k/1)^{\frac{1}{2}}}{2} e^{-yt} \qquad (18)$$

$$R = \frac{R_o - B_o \quad 1/k}{2} e^{yt} + \frac{R_o + B_o \quad (1/k)^{\frac{1}{2}}}{2} e^{-yt} \qquad (19)$$

Rearranging terms yields

$$B = B_o \frac{e^{yt} + e^{-yt}}{2} - R_o(k/1)^{\frac{1}{2}} \frac{e^{yt} - e^{-yt}}{2} \qquad (20)$$

$$R = R_o \frac{e^{yt} + e^{-yt}}{2} - B_o(1/k)^{\frac{1}{2}} \frac{eyt - e^{-yt}}{2} \qquad (21)$$

$$\text{Or } B = B_o \cosh yt - R_o(k/1)^{\frac{1}{2}} \sinh yt \qquad (22)$$

$$R = R_o \cosh yt - R_o(1/k)^{\frac{1}{2}} \sinh yt \qquad (23)$$

The ratio of k to 1 is termed the exchange rate and may be used to indicate which of the opposing forces has the greater effectiveness. Let E represent this exchange rate. A value of E greater than one would indicate that the R force has a superior edge in killing effectiveness, whereas a value less than one would indicate that the B force is superior in this respect. Let B_o and R_o represent the initial size of the Blue force and Red force respectively. A general solution for equations (1) and (2) yields an expression for the exchange rate.

$$(B_o^{\ 2} - B^2) = E(R_o^{\ 2} - R^2) \qquad (24)$$

When the opposing forces are quite similar in fighting elements and skills, then the corresponding loss rates may be considered equal. This is the case when two armies equipped with comparable weapons systems engage in conflict. Suppose that each force's loss rate is one tenth per unit of time ($k = .10$; $l = .10$). Then, the exchange rate equals one and the outcome is determined by the size of the forces which participate. For example, suppose the Blue force enters the battle with 1,400 armed men and the Red force begins with 1,000. Then the Red force will be annihilated and the Blue force will have approximately 980 survivors. These results may be obtained by setting R equal to zero and solving equation (24) for the correct value of B. The duration of the battle may be determined by setting equation (23) equal to zero and solving for t or, similarly, by setting equation (22) equal to 980 and solving for t. In the example, t is approximately 8.959 units of time. Equations (22) and (23) might also be used to predict respective force sizes for a given value of time.

If either force possesses an advantage in skill or equipment, then the exchange rate may be such that a numerically inferior force can achieve victory. Suppose, in the example above, that the combat loss rate for the Blue force is one tenth and that the Red force combat loss rate is one twentieth ($k = .10$; $l = .05$). Under these conditions, the Red force can achieve victory and count approximately 140 survivors.

A battle which conforms to the Linear Law conditions may be represented as shown by equations (25), (26), and (27). Here the combat loss rate k, may be interpreted as the Square Law parameter times the ratio of the average area presented by a B force element and the total area over which the fire of the R force is directed.

$$\dot{B} = -kBR \tag{25}$$

$$\dot{R} = -lBR \tag{26}$$

$$(B_o - B) = E(R_o - R) \tag{27}$$

When fire is directed toward an area, there is no destructive effect unless a vulnerable portion of the area is struck. During a siege in which the fighting elements are infantrymen, a member of the entrenched force will barricade himself to minimize his exposure to fire. The area a man presents as a vulnerable target could be approximately one square foot. Let B denote the size of the entrenched force and R denote the size of the attacking force. Let r_R be the rate of fire of an R force, P_R be the single shot kill probability for an R force element, k_B be the average area presented by a B force element and k_{tB} be the total area that is occupied by the R force. Equation (25) may be rewritten to display these factors which determine the value of the parameter, k.

$$\dot{B} = (r_R P_R R)\frac{(A_B B)}{A_{tB}} \tag{28}$$

The change in size for the B force in this, the Linear Law application, is equal to the change which would be experienced in a Square Law engagement times the proportion of the occupied area which is vulnerable. As one would expect, the Linear Law losses occur less rapidly than do losses for a Square Law battle involving the same forces.

It should be noted that it is assumed that B force elements are evenly distributed about their fortified area. If they are clustered and this is known to the R force, the attackers' fire will be concentrated toward this cluster. The effect of such an action will be a reduction of k_{tB} and an increase in the B force loss rate.

APPENDIX B

BJD MODEL--FORTRAN VERSION

APPENDIX B

BJD MODEL--FORTRAN VERSION

```
•@RUN •=(CORE=26)
C        THIS PROGRAM IS A TRANSLATION OF A PROGRAM WRITTEN BY
C        THOMAS A. BROWN,SELMER JOHNSON,AND MEL DRESHER OF THE
C        RAND CORPORATION. THE ORIGINAL COMPUTER PROGRAM,WRITTEN
C        IN JOSS,WAS DESIGNED TO STUDY THE ESCALATION OF CONFLICT
C        IN A TACTICAL NUCLEAR ENVIORNMENT.THE NUMBERED PARTS OF
C        THIS TRANSLATION REFER TO THE ORIGINAL JOSS PROGRAM AND
C        THEREFORE MAY APPEAR TO BE OUT OF SEQUENCE.
C        **********     **********     **********     **********
C
 DIMENSION BLUEFOR(3),REDFOR(3),TIESCAL(2),TOTBLFOR(3),TOTRDFOR(3)
 DIMENSION REDLOSS(3),BLUELOSS(3),U(3,3),V(3,3),L(3,3),K(3,3)
 DIMENSION Z(3),ZZ(3),BL(3),BK(3),C(3),D(3),X(3),Y(3),AA(3)
 DIMENSION H(3),IDA(3),OO(3),XX(3),YY(3)
 REAL K,L,IDA,JCODE
C
C
C*** 1 *** 1 *** 1 *** 1 *** 1 *** 1 *** 1 *** 1 *** 1 *** 1 *** 1 ***
C
C
C     PART 1 SETS THE VARIABLES TO ZERO TO START THE PROGRAM AND ASKS FOR
C     THE SELCODE.THIS CODE ALLOWS FOR AN OPTION OF RUNNING THE PROGRAM TO
C     CALCULATE THE TIME A FORCE IS REDUCED TO ZERO OR THE SECOND OPTION OF
C     COMPUTING THE CRITICAL TIME OF ESCALATION FOR BLUE.
C
C
 150 FORMAT("ENTER OPTION CODE 0 NORM,1 TIME,3 STOP")
 100 PRINT 150
 READ  SELCODE
 IF (SELCODE.EQ.3.0) GO TO 9999
 DO 110 I=1,3
 DO 110 J=1,3
 U(I,J)=0.0
 V(I,J)=0.0
 L(I,J)=0.0
```

```
      K(I,J)=0.0
  110 CONTINUE
      DO 120 I=1,3
      BLUEFOR(I)=0.0
      REDFOR(I)=0.0
      TOTBLFOR(I)=0.0
      TOTRDFOR(I)=0.0
      BLUELOSS(I)=0.0
      REDLOSS(I)=0.0
      AA(I)=0.0
      C(I)=0.0
      D(I)=0.0
      H(I)=0.0
      BL(I)=0.0
      BK(I)=0.0
      IDA(I)=0.0
      DD(I)=0.0
      X(I)=0.0
      XX(I)=0.0
      Y(I)=0.0
      YY(I)=0.0
      Z(I)=0.0
      ZZ(I)=0.0
  120 CONTINUE
      TIESCAL(1)=0.0
      TIESCAL(2)=0.0
      TNOW=0.0
      JCODE=0.0
      BLMSLFIR=0.0
      RMSLFIR=0.0
      E=0.0
      EE=0.0
      HH=0.0
C
C
```

```
C
C*** 2 *** 2 *** 2 *** 2 *** 2 *** 2 *** 2 *** 2 *** 2 *** 2 *** 2 **
C
C
C     PART 2 CONTAINS THE INITIAL FORCE VALUES, ASKS FOR THE TIMES OF ESCAL-
C     ATION,AND ASKS FOR THE INITIAL STRATEGIES.
C
C
      BLUEFOR(1) = 1100.
      BLUEFOR(2) = 3006.
      BLUEFOR(3) = 180.
      TOTBLFOR(1) = BLUEFOR(1) + BLUEFOR(2) + BLUEFOR(3)
      BMSLRATE = BLUEFOR(3)
      REDFOR(1) = 900.
      REDFOR(2) = 6618.
      REDFOR(3) = 300.
      TOTRDFOR(1) = REDFOR(1) + REDFOR(2) + REDFOR(3)
      RMSLRATE = REDFOR(3)
  250 FORMAT(1H0,33HENTER TIME OF ESCALATION FOR BLUE)
      PRINT 250
      READ   TIESCAL(1)
      IF(SELCODE.EQ.1.0) TIESCAL(2) = TIESCAL(1) + 0.1
      IF(SELCODE.EQ.1.0) GO TO 3300
  260 FORMAT(1H0,"ENTER TIME OF ESCALATION FOR RED")
      PRINT 260
      IF(SELCODE.EQ.0.0) READ      TIESCAL(2)
  270 FORMAT(///,"ENTER BLUE STRATEGY")
      PRINT 270
      READ  BLUSTRAT
  280 FORMAT(1H0,"ENTER RED STRATEGY")
      PRINT 280
      READ  REDSTRAT
C
C
C
```

```
C... 3 ... 3 ... 3 ... 3 ... 3 ... 3 ... 3 ... 3 ... 3 ... 3 ... 3 ...
C
C     IN PART 3 THE INITIAL FORCE VALUES ARE TYPED OUT AND THE PROGRAM THEN
C     ASKS FOR ANY CHANGE IN STRATEGIES.NEXT THE PROGRAM BRANCHES TO PART
C     4,5,6,OR 7 TO GET THE APPROPRIATE EFFECTIVENESS FACTORS BASED ON
C     THE CURRENT STRATEGY OF BLUE.
C
C
 330 FORMAT(///,9X,"TNOW",4X,"BLUE AIR",7X,"BLUE GROUND",7X,"BLUE MISSILES")
 300 PRINT330
 310 FORMAT(1H0,5X,F9.5,5X,F6.0,10X,F6.0,12X,F6.0,///)
 PRINT 310,TNOW,BLUEFOR(1),BLUEFOR(2),BLUEFOR(3)
 340 FORMAT (20X,"RED AIR",8X,"RED GROUND",8X,"RED MISSILES")
 PRINT 340
 320 FORMAT (1H0,19X,F6.0,10X,F6.0,12X,F6.0)
 PRINT 320,REDFOR(1),REDFOR(2),REDFOR(3)
 IF(TNOW.EQ.0.0)  GO TO 390
 IF(TNOW.GE.TIESCAL(1)) GO TO 350
 GO TO 360
 355 FORMAT(///,"ENTER BLUE STRATEGY")
 350 PRINT 355
 READ  BLUSTRAT
 360 IF (TNOW.GE.TIESCAL(2)) GO TO 370
 GO TO 390
 375 FORMAT(1H0,"ENTER RED STRATEGY")
 370 PRINT 375
 READ  REDSTRAT
  390 GO TO (400,500,600,700),BLUSTRAT
C
C
 ... 4 ... 4 ... 4 ... 4 ... 4 ... 4 ... 4 ... 4 ... 4 ... 4 ... 4 ...
C
C
C    PARTS 4 THRU 7 CONTAIN THE APPROPRIATE EFFECTIVENESS FACTORS FOR BLUE
C    FOR EACH OF THE 4 POSSIBLE STRATEGIES.
```

```
C
C
C                 ********************
C   BLUES EFFECTIVENESS COEFFICIENTS AT NUCLEAR LEVEL
C                 ********************
 400 L(1,1)=30.0
 L(1,2)=13.2
 L(1,3) = 1.6
  L(2,1) = 0.0
 L(2,2) = 0.8
 L(2,3) = 0.0
 L(3,1)=15.0
 L(3,2)=15.0
 L(3,3)=1.0
 FF=1.0
 GO TO 800
C
C
C*** 5 *** 5 *** 5 *** 5 *** 5 *** 5 *** 5 *** 5 *** 5 *** 5 *** 5 ***
C
C
C                 ********************
C   BLUE EFFECTIVENESS COEFF - LIMITED WAR LEVEL
C                 ********************
 500 L(1,1) = 30.0
 L(1,2)=1.32
 L(1,3)=0.2
 L(2,1) = 0.0
 L(2,2)=0.08
L(2,3) = 0.0
 L(3,1)=15.0
 L(3,2) = 0.0
 L(3,3) = 0.0
 FF=1.0
 GO TO 800
```

```
C
C
C*** 6 *** 6 *** 6 *** 6 *** 6 *** 6 *** 6 *** 6 *** 6 *** 6 *** 6 ***
C
C
C                 ********************
C  BLUE EFFECTIVENESS COEFF - CONVENTIONAL WAR LEVEL
C                 ********************
 600 L(1,1)=0.4
 L(1,2)=1.32
 L(1,3)=0.2
 L(2,1) = 0.0
 L(2,2)=0.08
 L(2,3) = 0.0
 L(3,1) = 0.0
 L(3,2) = 0.0
 L(3,3) = 0.0
 FF = 0.0
 GO TO 800
C
C
C*** 7 *** 7 *** 7 *** 7 *** 7 *** 7 *** 7 *** 7 *** 7 *** 7 *** 7 ***
C
C
C
C                 ********************
C  BLUE EFFECTIVENESS COEFF - PEACE
C                 ********************
 700 FF=0.0
 L(1,1)=1.0
C
C
C*** 8 *** 8 *** 8 *** 8 *** 8 *** 8 *** 8 *** 8 *** 8 *** 8 *** 8 ***
C
C
```

```
C    PART 8 BRANCHES THE PROGRAM TO PART 9,10,11,OR 12 TO SELECT THE EFFEC-
C    TIVENESS FACTORS BASED ON RED'S STRATEGY.
C
C
 800 GO TO (900,1000,1100,1200),REDSTRAT
C
C
C*** 9 *** 9 *** 9 *** 9 *** 9 *** 9 *** 9 *** 9 *** 9 *** 9 *** 9 ***
C
C
C    PARTS 9 THRU 12 CONTAIN THE APPROPRIATE EFFECTIVENESS FACTORS FOR RED
C    FOR EACH OF THE 4 POSSIBLE STRATEGIES.
C
C
C                  *********************
C RED EFFECTIVENESS COEFF - NUCLEAR WAR LEVEL
C                  *********************
 900 K(1,1)=15.0
 K(1,2)=13.2
 K(1,3)=0.8
 K(2,1) = 0.0
 K(2,2)=0.1
 K(2,3) = 0.0
 K(3,1)=15.0
 K(3,2)=15.0
 K(3,3)=1.0
 GG=1.0
  GO TO 1300
C
C
C*** 10 *** 10 *** 10 *** 10 *** 10 *** 10 *** 10 *** 10 *** 10 *** 10 ***
C
C
C                  *********************
C RED EFFECTIVENESS COEFF - LIMITED WAR LEVEL
```

```
C              ********************
 1000 K(1,1)=15.0
 K(1,2)=1.32
 K(1,3)=0.2
 K(2,1) = 0.0
 K(2,2)=0.02
 K(2,3) = 0.0
 K(3,1)=15.0
 K(3,2) = 0.0
 K(3,3) = 0.0
 GG=1.0
  GO TO 1300
C
C
C*** 11 *** 11 *** 11 *** 11 *** 11 *** 11 *** 11 *** 11 *** 11 *** 11
C
C
C              ********************
C  RED EFFECTIVENESS COEFF - CONVENTIONAL WAR LEVEL
C              ********************
 1100 K(1,1)=0.7
 K(1,2)=1.32
 K(1,3)=0.2
 K(2,1) = 0.0
 K(2,2)=0.02
 K(2,3) = 0.0
 K(3,1)  = 0.0
 K(3,2) = 0.0
 K(3,3) = 0.0
 GG=0.0
  GO TO 1300
C
C
C
C
```

```
C*** 12 *** 12 *** 12 *** 12 **** 12 *** 12 *** 12 *** 12 *** 12 *** 12
C
C              *********************
C  RED EFFECTIVENESS COEFF - PEACE
C              *********************
 1200 GG=0.0
 K(1,1)=1.0
C
C
C** 13/14 *** 13/14 *** 13/14 *** 13/14 *** 13/14 *** 13/14 *** 13/14
C
C
C   PART 13 ALLOCATES ALL GROUND FORCES AGAINST EACH OTHER FOR ALL OPTIONS.
C   FOLLOWING THIS THE OTHER FORCE ALLOCATIONS ARE READ IN ON A SELECTIVE
C   BASIS.PART 14 CHANGES THE ALLOCATIONS,IF NECESSARY, TO INSURE THAT
C   FORCES ARE NOT ALLOCATED AGAINST NON-EXISTENT FORCES.
C
C
 1300 U(2,2) = 1.0
 V(2,2) = 1.0
 IF(SELCODE.GT.0.0) GO TO 3500
 1320 FORMAT(///,"ENTER FORCE ALLOCATIONS FOR BLUE")
 PRINT 1320
 DO 1305 I=1,9
 READ  M,N,U(M,N)
 IF(M.EQ.99) GO TO 1310
 1305 CONTINUE
 1330 FORMAT(1H0,"ENTER FORCE ALLOCATIONS FOR RED")
 1310 PRINT 1330
 DO 1315 I=1,9
 READ  M,N,V(M,N)
 IF (M.EQ.99) GO TO 1470
 1315 CONTINUE
 1470 DO 1484 I=1,3,1
 DO 1484 J=1,3,1
```

```
 1474 IF(AMIN1(BLUEFOR(I),REDFOR(J),L(I,J)).EQ.0.0) U(I,J)=0.0
 1464 IF(AMIN1(REDFOR(I),BLUEFOR(J),K(I,J)).EQ.0.0) V(I,J) = 0.0
C
C
C** 15/16 *** 15/16 *** 15/16 *** 15/16 *** 15/16 *** 15/16 *** 15/16
C
C
C
C   PARTS 15 & 16 ESTABLISH A 'DO LOOP' FOR THE PURPOSE OF COMPUTING
C   FORCE ATTRITION.
C
C
   DO 2390  J = 1,3,1
  IF ( J.EQ.3)   GO TO 1700
   IF ( U(J,J)*V(J,J).GT.0.0 ) GO TO 2200
C
C
C*** 17 *** 17 *** 17 *** 17 *** 17 *** 17 *** 17 *** 17 *** 17 ***
C
C
C
C   PART 17 COMPUTES THE LINEAR ATTRITION FOR THE CASES WHERE ONE
C   FORCE IS NOT RETURNING THE FIRE OF THE OPPOSING LIKE FORCE.THIS PART
C   ALSO COMPUTES THE QUADRATIC TERM FOR THE CASE WHERE THERE IS A
C   BATTLE BETWEEN AN AIR FORCE AND THE OPPOSING MISSILE FORCE.
C
 1700 REDLOSS(J)=U(1,J)*L(1,J)*BLUEFOR(1)+U(2,J)*L(2,J)*BLUEFOR(2)
&             + U(3,J)*L(3,J)*RMSLRATE
 Z(J)=0.0
  IF (J.NE.3 ) GO TO 1750
 REDLOSS(3)=REDLOSS(3)+GG*RMSLRATE*(V(3,1)+V(3,2)+V(3,3))
 Z(3)=(0.5)*(V(3,1)*K(3,1)*RMSLRATE*U(1,3)*L(1,3))
 1750 BLUELOSS(J)=V(1,J)*K(1,J)*REDFOR(1)+V(2,J)*K(2,J)*REDFOR(2)
&          +V(3,J)*K(3,J)*RMSLRATE
 ZZ(J)=0.0
```

```
      IF ( J.NE.3 ) GO TO 1900
      BLUELOSS(3)=BLUELOSS(3)+FF*BMSLRATE*(U(3,1)+U(3,2)+U(3,3))
      ZZ(3)=(0.5)*( U(3,1)*L(3,1)*BMSLRATE*V(1,3)*K(1,3)   )
C
C
C*** 19 *** 19 *** 19 *** 19 *** 19 *** 19 *** 19 *** 19 *** 19 ***
C
C
C     PARTS 19 & 20 GIVE THE TIMES WHEN EACH WEAPON FORCE GOES TO ZERO.
C
C
 1900 IF ( (ZZ(J)+BLUELOSS(J)).EQ.0.0) H(J) = 10**7
      IF( (ZZ(J)+BLUELOSS(J)).EQ.0.0 ) GO TO 2000
      IF (ZZ(J).EQ.0.0)H(J) = BLUEFOR(J)/BLUELOSS(J)
      IF(ZZ(J).EQ.0.0) GO TO 2000
      IF(((BLUELOSS(J)*BLUELOSS(J))-4.0*ZZ(J)*BLUEFOR(J)).LT.0.0 ) H(J) = 10**7
      IF(((BLUELOSS(J)*BLUELOSS(J))-(4.0*ZZ(J)*BLUEFOR(J))).LT.0.0)GO TO 2000
      H(J)=(0.5/ZZ(J))*(BLUELOSS(J)-SQRT((BLUELOSS(J))**2-4*ZZ(J)*BLUEFOR(J)))
C
C
C*** 20 *** 20 *** 20 *** 20 *** 20 *** 20 *** 20 *** 20 *** 20 ***
C
C
C
 2000 IF ( (Z(J)+REDLOSS(J) ).EQ.0.0 )IDA(J) = 10**7
      IF(  (REDLOSS(J)+Z(J)).EQ.0.0 )GO TO 2390
      IF( Z(J).EQ.0.0 ) IDA(J) = REDFOR(J)/REDLOSS(J)
      IF(Z(J).EQ.0.0) GO TO 2390
      IF( ( (REDLOSS(J)*REDLOSS(J))-4.0*Z(J)*REDFOR(J)).LT.0.0) IDA(J)=10**7
      IF(  ( (REDLOSS(J)*REDLOSS(J))-4.0*Z(J)*REDFOR(J)).LT.0.0 )GO TO 2390
      IDA(J)=(0.5/Z(J))*(REDLOSS(J)-SQRT((REDLOSS(J))**2-4.0*Z(J)*REDFOR(J)))
      GO TO 2390
C
C
C
```

```
C*** 22 *** 22 *** 22 *** 22 *** 22 *** 22 *** 22 *** 22 *** 22 ***
C
C    PARTS 22 & 23 COMPUTE FORCE ATTRITION FOR THE CASES WHERE ONE FORCE
C    IS RETURNING THE FIRE OF THE OPPOSING LIKE FORCE.THESE PARTS ONLY
C    APPLY TO AIR AND GROUND FORCES.
C
C
  2200 BL(J)=SQRT(L(J,J))
 BK(J)=SQRT(K(J,J))
 C(J) = U(3,1)*L(3,1)*BMSLRATE
 D(J) = V(3,1)*K(3,1)*RMSLRATE
 IF(J.EQ.2) C(J) = U(1,2)*L(1,2)*BLUEFOR(1)*U(3,2)*L(3,2)*BMSLRATE
 IF(J.EQ.2) D(2) = V(1,2)*K(1,2)*REDFOR(1)*V(3,2)*K(3,2)*RMSLRATE
 C(J)=C(J)/BL(J)
 D(J)=D(J)/BK(J)
 X(J) = BL(J)*BLUEFOR(J) + C(J)
 Y(J)=BK(J)*REDFOR(J)+D(J)
 AA(J)=BL(J)*BK(J)
C
C
C*** 23 *** 23 *** 23 *** 23 *** 23 *** 23 *** 23 *** 23 *** 23 *** 23
C
C
C
 2300 IF(X(J).EQ.Y(J)) H(J)=10**7
 IF(X(J).EQ.Y(J)) GO TO  2360
 QQ(J)=0.0
 IF (  (C(J)*C(J) - X(J)*X(J) + Y(J)*Y(J)).LT.0.0   ) GO TO 2322
 QQ(J)=C(J) + SQRT(C(J)*C(J) - X(J)*X(J) + Y(J)*Y(J))
 2322 IF( (D(J)*D(J)+X(J)*X(J)-Y(J)*Y(J)).LT.0.0 ) GO TO 2340
  ROOT=SQRT( X(J)*X(J)-Y(J)*Y(J)+D(J)*D(J))
 QQ(J)=AMAX1( QQ(J),D(J)+ROOT )
 2340 H(J) = (1/AA(J))*(ALOG((X(J)+Y(J))/QQ(J)))
 2360 IDA(J) = H(J)
 2390 CONTINUE
```

```
C
C
C*** 18 *** 18 *** 18 *** 18 *** 18 *** 18 *** 18 *** 18 *** 18 ***
C
C
C    PART 18 GIVES THE TIME WHEN THE FIRST NEW WEAPON FORCE GOES TO ZERO.
C    THE TIME(HH) WHEN A NEW ALLOCATION IS CALLED FOR IS ALSO DETERMINED.
C
C
      HH=AMIN1( H(1),H(2),H(3),IDA(1),IDA(2),IDA(3) )
      IF(TNOW.LT.TIESCAL(1)) HH=AMIN1(HH,(TIESCAL(1)-TNOW))
      IF(TNOW.LT.TIESCAL(2)) HH=AMIN1(HH,TIESCAL(2)-TNOW)
      IF (JCODE.GT.0.0)  HH = AMIN1(HH,JCODE)
C
C
C
C*** 24 *** 24 *** 24 *** 24 *** 24 *** 24 *** 24 *** 24 *** 24 **
C
C
C    PART 24 UPDATES THE CURRENT TIME.
C
C
 2400 TNOW=TNOW+HH
      DO 2690 J = 1,3,1
C
C
C*** 25 *** 25 *** 25 *** 25 *** 25 *** 25 *** 25 *** 25 *** 25 ***
C
C
C    PARTS 25 & 26 GIVE THE NEW VALUES FOR THE BLUE AND RED FORCES AT THE
C    END OF THE CURRENT PERIOD.
C
C
      GO TO ( 2505,2505,2510 ),J
 2505 IF (U(J,J)*V(J,J).GT.0.0 ) GO TO 2600
```

```
 2510 IF (REDSTRAT.GT.2.0)GO TO 2550
 IF(REDSTRAT.EQ.1.0)GO TO 2540
 IF(J.NE.1) GO TO 2550
 2540 EE=EE+HH*(BLUELOSS(J)-HH*ZZ(J))
 IF(J.EQ.3) EE=EE-HH*RMSLRATE*(U(3,1)+U(3,2)+U(3,3))
 2550 IF(BLUSTRAT.GT.2.0) GO TO 2580
 IF(BLUSTRAT.EQ.1.0) GO TO 2570
 IF(J.NE.1) GO TO 2580
 2570 E=E+HH*(REDLOSS(J)-HH*Z(J))
 IF(J.EQ.3) E=E-HH*PMSLRATE*(V(3,1)+V(3,2)+V(3,3))
 2580 REDFOR(J)=REDFOR(J)-HH*(REDLOSS(J)-HH*Z(J))
 2590 BLUEFOR(J)=BLUEFOR(J)-HH*(BLUELOSS(J)-HH*ZZ(J))
 GO TO 2690
C
C
C*** 26 *** 26 *** 26 *** 26 *** 26 *** 26 *** 26 *** 26 *** 26 *** 26
C
C
C
 2600 ZEEK=0.5*(X(J)-Y(J))*(EXP(HH*AA(J)))
 XX(J)=((0.5)*(X(J)+Y(J))*(EXP(-HH*AA(J))) + ZEEK)
 YY(J)=((0.5)*(X(J)+Y(J))*(EXP(-HH*AA(J))) - ZEEK)
 IF(REDSTRAT.GT.2.0)   GO TO 2626
 IF(REDSTRAT.EQ.1.0) GO TO 2624
 IF (J.NE.1) GO TO 2626
 2624 EE=EE+BLUEFOR(J)-( XX(J)-C(J) )/BL(J)
  2626 IF(BLUSTRAT.GT.2.0) GO TO 2630
 IF(BLUSTRAT.EQ.1.0)GO TO  2629
 IF(J.NE.1) GO TO 2630
  2629 E=E + REDFOR(J) - ( YY(J)-D(J) )/BK(J)
 2630 BLUEFOR(J)=(XX(J)-C(J))/BL(J)
 REDFOR(J)=(YY(J)-D(J))/BK(J)
 2690 CONTINUE
 DO 2460 J=1,3,1
 2460 IF(BLUEFOR (J).LT.0.1)BLUEFOR(J)=0.0
```

```
 DO 2480  J=1,3,1
 2480 IF  (REDFOR(J).LT.0.1)    REDFOR(J) = 0.0
C
C
C... 27 ... 27 ... 27 ... 27 ... 27 ... 27 ... 27 ... 27 ... 27 ...
C
C
C   PART 27 DETERMINES THE TOTAL NUMBER OF MISSILES FIRED UP TO THE PRESENT
C   TIME AND ALSO THE NUCLEAR DAMAGE UP TO THE PRESENT TIME.IF ONE SIDE HAS
C   BEEN WIPED OUT, THE FINAL ACCOUNTING TYPED OUT,OTHERWISE THE PROGRAM
C   GOES BACK TO PART 3.
C
C
  2700 BLMSLFIR=BLMSLFIR+HH*RMSLRATE*( U(3,1)+U(3,2)+U(3,3) )
 RMSLFIR=RMSLFIR+HH*RMSLRATE*(V(3,1)+V(3,2)+V(3,3))
 TOTRDFOR(2)=REDFOR(1)+REDFOR(2)+REDFOR(3)
 TOTBLFOR(2)=BLUEFOR(1)+BLUEFOR(2)+BLUEFOR(3)
 IF(SELCODE.GT.0.0) GO TO 2792
 IF( (AMIN1(TOTBLFOR(2),TOTRDFOR(2))).GT.0.0) GO TO 300
 PRINT 330
 PRINT 310,TNOW,BLUEFOR(1),BLUEFOR(2),BLUEFOR(3)
 PRINT 340
 PRINT 320,REDFOR(1),REDFOR(2),REDFOR(3)
 2720 FORMAT(///,2X,"BLUE TOT DAMAGE",10X,"BLUE NUC DAMAGE",10X,"BLUE SURV")
 PRINT 2720
 2710 FORMAT(1H0,5X,F6.0,20X,F6.0,18X,F6.0)
 PRINT 2710,TOTBLFOR(1)-TOTBLFOR(2)-BLMSLFIR,EE,TOTBLFOR(2)
 2730 FORMAT(///,3X,"RED TOT DAMAGE",11X,"RED NUC DAMAGE",11X,"RED SURV")
 PRINT 2730
 PRINT 2710,TOTRDFOR(1)-TOTRDFOR(2)-RMSLFIR,E,TOTRDFOR(2)
 GO TO 9999
 2792 CONTINUE
 IF((AMIN1(TOTBLFOR(2),TOTRDFOR(2))).EQ.0.0) GO TO 3200
 GO TO 3300
C
```

```
C
C*** 32 *** 32 *** 32 *** 32 *** 32 *** 32 *** 32 *** 32 *** 32 ***
C
C
C     PART 32 IS PART OF THE OPTION FOR DETERMING CRITICAL TIME.THIS PART
C       TYPES OUT THE TIME OF ESCALATION USED AND THE REMAINING TOTAL
C    FORCE VALUE FOR BLUE AND RED.CONTROL IS THEN TRANSFERED TO THE BEGIN-
C    ING OF THE PROGRAM SO THAT A NEW TIME OF ESCALATION CAN BE USED.
C
C
 3210 FORMAT (1H0,3X,"TIME OF ESCALATION FOR BLUE",1X,F12.8)
 3220 FORMAT (1H0,3X,"TOTAL BLUE FORCES",2X,F6.0,5X,
& "TOTAL RED FORCES",2X,F6.0)
 3200 PRINT 3210,TIESCAL(1)
 PRINT 3220,TOTBLFOR(2),TOTRDFOR(2)
 GO TO 100
C
C
C*** 33 *** 33 *** 33 *** 33 *** 33 *** 33 *** 33 *** 33 *** 333 ***
C
C
C    PART 33 AUTOMATICALLY DETERMINES THE STRATEGIES FOR THIS OPTION.
C
C
 3300 BLUSTRAT=3.0
  IF(TNOW.GE.TIESCAL(1))  BLUSTRAT = 1.0
 REDSTRAT=3.0
 IF(TNOW.GE.TIESCAL(2))  REDSTRAT = 1.0
 GO TO 390
 3500 DO 3690 I=1,3,2
C
C
C
C
C
```

```
C*** 35/36 *** 35/36 *** 35/36 *** 35/35 *** 35/36 *** 35/36 *** 35/36
C
C
C    PARTS 35 & 36 AUTOMATICALLY DETERMINE THE FORCE ALLOCATIONS FOR
C    THIS OPTION.FROM HERE THE PROGRAM GOES TO PART 14 WHERE FORCE
C    ATTRITION IS COMPUTED AS IN THE OTHER OPTION.
C
C
 IF(REDFOR(1).GT.0.0) U(I,1) = 1.0
 IF(REDFOR(1).EQ.0.0) U(I,3) =1.0
 IF( (REDFOR(1)+REDFOR(3)).EQ.0.0) U(I,2) = 1.0
 IF( BLUEFOR(1).GT.0.0 ) V(I,1)=1.0
 IF(BLUEFOR(1).EQ.0.0) V(I,3)=1.0
 IF(   (BLUEFOR(1)+BLUEFOR(3)).EQ.0.0   ) V(I,2)=1.0
 IF( (AMAX1(BLUEFOR(1),REDFOR(1))).GT.0.0) GO TO 3690
 IF(BLUEFOR(2).GT.0.0) V(3,2)=1.0
 IF(BLUEFOR(2).GT.0.0)V(3,3)=0.0
 3690 CONTINUE
 GO TO 1470
C
C
C
 9999 STOP
 END
```

APPENDIX C

BJD MODEL--STANDARD PROGRAM VERIFICATION

APPENDIX C

BJD MODEL--STANDARD PROGRAM VERIFICATION

```
ENTER OPTION CODE: Ø NORMAL, 1 CRIT TI, 3 STOP
=Ø
ENTER TIME OF ESCALATION FOR BLUE
=1
ENTER TIME OF ESCALATION FOR RED
=1.1

ENTER BLUE STRATEGY
=3
ENTER RED STRATEGY
=3
      TNOW         BLUE AIR       BLUE GROUND       BLUE MISSILES
       Ø.           11ØØ.           3ØØ6.               18Ø.
                   RED AIR        RED GROUND        RED MISSILES
                    9ØØ.            6618.               3ØØ.

ENTER FORCE ALLOCATIONS FOR BLUE
=1,1,1
=99,Ø,
ENTER FORCE ALLOCATIONS FOR RED
=1,1,1
=99,Ø.
      TNOW         BLUE AIR       BLUE GROUND       BLUE MISSILES
    1.ØØØØØ         598.            2876.               18Ø.
                   RED AIR        RED GROUND        RED MISSILES
                    568.            6383.               3ØØ.

ENTER BLUE STRATEGY
=1
ENTER FORCE ALLOCATIONS FOR BLUE
=3,1,1
=99,Ø,
ENTER FORCE ALLOCATIONS FOR RED
=99,Ø,
      TNOW         BLUE AIR       BLUE GROUND       BLUE MISSILES
    1.Ø2768         592.            2872.               175.
                   RED AIR        RED GROUND        RED MISSILES
                     Ø.             6319.               3ØØ.

ENTER BLUE STRATEGY
=1
ENTER FORCE ALLOCATIONS FOR BLUE
=1,3,1
=3,3,1
=99,Ø,
```

```
ENTER FORCE ALLOCATIONS FOR RED
=99,Ø,
     TNOW        BLUE AIR      BLUE GROUND      BLUE MISSILES
    1.1ØØØØ        592.          2863.             162.
                 RED AIR       RED GROUND       RED MISSILES
                    Ø.           6153.             218.

ENTER BLUE STRATEGY
=1
ENTER RED STRAT
=1
ENTER FORCE ALLOCATIONS FOR BLUE
=1,3,1
=3,3,1
=99,Ø,
ENTER FORCE ALLOCATIONS FOR RED
=3,1,1
=99,Ø,
     TNOW        BLUE AIR      BLUE GROUND      BLUE MISSILES
    1.23163         Ø.           2784.             138.
                 RED AIR       RED GROUND       RED MISSILES
                    Ø.           5856.              93.

ENTER BLUE STRATEGY
=1
ENTER RED STRAT
=1
ENTER FORCE ALLOCATIONS FOR BLUE
=3,3,1
=99,Ø,
ENTER FORCE ALLOCATIONS FOR RED
=3,2,1
=99,Ø,
     TNOW        BLUE AIR      BLUE GROUND      BLUE MISSILES
    1.42516         Ø.           18Ø4.             1Ø3.
                 RED AIR       RED GROUND       RED MISSILES
                    Ø.           55Ø1.              Ø.

ENTER BLUE STRATEGY
=1
ENTER RED STRAT
=1
ENTER FORCE ALLOCATIONS FOR BLUE
=3,4,1
=99,Ø,
ENTER FORCE ALLOCATIONS FOR RED
==99,Ø,
     TNOW        BLUE AIR      BLUE GROUND      BLUE MISSILES
    2.ØØØØØ         Ø.           1555.              Ø.
                 RED AIR       RED GROUND       RED MISSILES
                    Ø.           3181.              Ø.
```

```
ENTER BLUE STRATEGY
=1
ENTER RED STRAT
=1
ENTER FORCE ALLOCATIONS FOR BLUE
=99,Ø,
ENTER FORCE ALLOCATIONS FOR RED
=99,Ø,
     TNOW          BLUE AIR        BLUE GROUND        BLUE MISSILES
    5.234Ø1          Ø.              1Ø74.                Ø.
                    RED AIR         RED GROUND         RED MISSILES
                     Ø.               Ø.                  Ø.

BLUE TOT DAMAGE          BLUE NUC DAMAGE           BLUE SURV
    3Ø32.                     2382.                  1Ø74.
 RED TOT DAMAGE           RED NUC DAMAGE            RED SURV
    772Ø.                     7153.                   Ø.
```

APPENDIX D

BJD MODEL--CRITICAL TIME OF ESCALATION VERIFICATION

APPENDIX D

BJD MODEL--CRITICAL TIME OF ESCALATION VERIFICATION

```
ENTER OPTION CODE:  Ø NORMAL, 1 CRIT TI, 3 STOP
=1
ENTER TIME OF ESCALATION FOR BLUE
=1
   TIME OF ESCALATION FOR BLUE   1.ØØØØØØØØ
   TOTAL BLUE FORCES    1Ø74.    TOTAL RED FORCES       Ø.

ENTER OPTION CODE:  Ø NORMAL, 1 CRIT TI, 3 STOP
=1
ENTER TIME OF ESCALATION FOR BLUE
=2
   TIME OF ESCALATION FOR BLUE   2.ØØØØØØØØ
   TOTAL BLUE FORCES       Ø.    TOTAL RED FORCES    41Ø5.

ENTER OPTION CODE:  Ø NORMAL, 1 CRIT TI, 3 STOP
=1
ENTER TIME OF ESCALATION FOR BLUE
=1.2
   TIME OF ESCALATION FOR BLUE   1.2ØØØØØØ
   TOTAL BLUE FORCES       Ø.    TOTAL RED FORCES     365.

ENTER OPTION CODE:  Ø NORMAL, 1 CRIT TI, 3 STOP
=1
ENTER TIME OF ESCALATION FOR BLUE
=1.19
   TIME OF ESCALATION FOR BLUE   1.19ØØØØØ
   TOTAL BLUE FORCES     182.    TOTAL RED FORCES       Ø.

ENTER OPTION CODE:  Ø NORMAL, 1 CRIT TI, 3 STOP
=1
ENTER TIME OF ESCALATION FOR BLUE
=1.196
   TIME OF ESCALATION FOR BLUE   1.196ØØØØ
   TOTAL BLUE FORCES      56.    TOTAL RED FORCES       Ø.

ENTER OPTION CODE:  Ø NORMAL, 1 CRIT TI, 3 STOP
=1
ENTER TIME OF ESCALATION FOR BLUE
=1.197
   TIME OF ESCALATION FOR BLUE   1.197ØØØØ
   TOTAL BLUE FORCES       Ø.    TOTAL RED FORCES     122.
```

```
ENTER OPTION CODE:  Ø NORMAL, 1 CRIT TI, 3 STOP
=1
ENTER TIME OF ESCALATION FOR BLUE
=1.1966
   TIME OF ESCALATION FOR BLUE    1.19660000
   TOTAL BLUE FORCES       1Ø.   TOTAL RED FORCES       Ø.

ENTER OPTION CODE:  Ø NORMAL, 1 CRIT TI, 3 STOP
=1
ENTER TIME OF ESCALATION FOR BLUE
=1.19662
   TIME OF ESCALATION FOR BLUE    1.19662ØØØ
   TOTAL BLUE FORCES        3.    TOTAL RED FORCES      Ø.

ENTER OPTION CODE:  Ø NORMAL, 1 CRIT TI, 3 STOP
=1
ENTER TIME OF ESCALATION FOR BLUE
=1.196622
   TIME OF ESCALATION FOR BLUE    1.196622ØØ
   TOTAL BLUE FORCES        Ø.    TOTAL RED FORCES      2.

ENTER OPTION CODE:  Ø NORMAL, 1 CRIT TI, 3 STOP
=1
ENTER TIME OF ESCALATION FOR BLUE
=1.1966219
   TIME OF ESCALATION FOR BLUE    1.19662189
   TOTAL BLUE FORCES        1.    TOTAL RED FORCES      Ø.

ENTER OPTION CODE:  Ø NORMAL, 1 CRIT TI, 3 STOP
=3
```

APPENDIX E

SUBROUTINE FOR COMPUTATION OF CRITICAL TIME OF ESCALATION

APPENDIX E

SUBROUTINE FOR COMPUTATION OF CRITICAL TIME OF ESCALATION

```
SUBROUTINE SEARCH (TIME,BLUE,RED,STARTI,DELTATI,*,*)
BLUE = AINT(BLUE)
RED = AINT(RED)
IF (TIME.NE.STARTI) GO TO 1
1 IF (BLUE-RED) 2,3,4
2 TIME = TIME-DELTATI
GO TO 5
4 TIME = TIME+DELTATI
5 DELTATI = DELTATI/2.Ø
RETURN1
3 RETURN2
END
```

SELECTED BIBLIOGRAPHY

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A. REFERENCES CITED

1. "Arming to Disarm in the Age of Detente," Time, February 11, 1974, pp. 15-24.

2. Beecher, William. "Over the Threshold," Army, XXII (July 1972), pp. 17-20.

3. Brown, Thomas A., Selmer Johnson, and Mel Dresher. "Some Methods for the Study of Escalation (Draft)." WN-8123-PR. Rand Corporation, Santa Monica, California. February 1973.

4. Bryan, G. E., and J. W. Smith. "JOSS Language: Apercu and Precis," RM-5377-PR/2, The Rand Corporation, Santa Monica, California, August 1967.

5. Brzezinski, Zbigniew. "America and Europe," Foreign Affairs, XLIX (October 1970), pp. 11-30.

6. Clausewitz, Carl Von. On War, ed. Anatol Rapoport. Middlesex, England: Penguin Books, Ltd., 1968.

7. Department of the Air Force. "Suggested AF/SAG Problem Statement." Undated informal paper outlining a suggested research/study project by Rand Corporation, Santa Monica, California.

8. Department of the Army. "Implications of the Nixon Doctrine for the Defense Planning Process." SSC-TN-8974-61. Stanford Research Institute, Washington, D.C., May 1972.

9. Department of the Army Pamphlet Number 70-5. Mathematics of Military Action, Operations and Systems. Washington, D.C.: Government Printing Office, January 1968.

10. Geneste, Marc E. "A Common Western Nuclear Doctrine?" Military Review, LI (September 1971), pp. 3-12.

11. Gimble, E. P. "JOSS: Problem Solving for Engineers," Memorandum RM-5322-PR, The Rand Corporation, Santa Monica, California, May 1967.

12. Gormley, Dennis M. "NATO's Tactical Nuclear Option: Past, Present, and Future," Military Review, LIII (September 1973), pp. 3-18.

13. Latchaw, John H. "A Lanchester Model for Air Battles." Unpublished Master's thesis, Air Force Institute of Technology, February 1972.

14. Lucas, William F. "An Overview of the Mathematical Theory of Games," Management Science, XVIII (January 1972), pp. 3-19.

15. Milton, T. R. "Behind NATO's Shiny Facade--A Troubled Future?" Air Force Magazine, LV (August 1972), pp. 49-52.

16. Montross, Lynn. War Through the Ages. New York: Harper and Brothers, 1960.

17. Moore, William C. "Mideast War's Lesson for U.S.," U.S. News and World Report, LXXVI (January 14, 1974).

18. Morgenthau, Hans J. "Changes and Chances in American-Soviet Relations," Foreign Affairs, XLIX (April 1971), pp. 429-441.

19. Norman, Lloyd. "The Reluctant Dragon: NATO's Fears and the Need for Nuclear Weapons," Army, XXIV (February 1974), pp. 16-21.

20. Partlow, Frank A., Jr. "NATO: Defense and Detente," Army, December 1972, pp. 8-14.

21. Pugh, George E., and John P. Mayberry. "Theory of Measures of Effectiveness for General Purpose Military Forces: Part I. A Zero-Sum Payoff Appropriate for Evaluating Combat Strategies," Operations Research, XXI (July-August 1973), pp. 867-885.

22. Quade, E. S., and W. I. Boucher (eds.). Systems Analysis and Policy Planning, Application in Defense. New York: American Elsevier Publishing Company, Inc., 1968.

23. Richardson, Lewis F. Statistics of Deadly Quarrels, ed. Quincy Wright and C. C. Lienau. Chicago: Quadrangle Books, Inc., 1960.

24. Robinson, R. F. "Suggestions for Test Data for a Nuclear Escalation Model," Memorandum, Headquarters, United States Air Force, Assistant Chief of Staff, Studies and Analysis (AF/SAG), April 1974.

25. Spanier, John W. American Foreign Policy Since World War II. New York: Frederick A. Praeger, 1962.

26. Thompson, Ivan B., and Daniel E. Reynolds. "Remote Computing." SLTR 8-71, School of Systems and Logistics, Air Force Institute of Technology, Wright-Patterson Air Force Base, August 1970.

27. "Time for a Pullback in Europe," Time, April 1, 1974, pp. 15-17.

28. Trettner, Heinz. "Tactical Nuclear Weapons for Europe," Military Review, LI (July 1971), pp. 43-49.

29. Wade, Horace M. "NATO--A Cornerstone," Ordnance, LVII (May-June 1973), pp. 459-461.

30. Weiss, Herbert K. "Lanchester-Type Models of Warfare," Operations Research, V (September-October, 1957), pp. 82-99.

31. Williams, J. D. The Compleat Strategyst. New York: McGraw-Hill Book Company, Inc., 1954.

32. Willoughby, Charles A. Maneuver In War. Harrisburg, Pennsylvania: The Military Service Publishing Company, 1939.

33. Wright, Quincy. A Study of War. Chicago: The University of Chicago Press, 1964.

B. RELATED SOURCES

Aumann, R. J., and M. Maschler. "Some Thoughts on the Minimax Principle," Management Science, XVIII (January 1972), pp. 54-63.

Barclay, C. N. "What Happens to Europe if the Americans Leave?" Army, XXIII (November 1973), pp. 7-11.

Berkovitz, Leonard D., and Melvin Dresher. "A Game Theory Analysis of Tactical Air War." P-1592. Rand Corporation, Santa Monica, California, January 1959.

Brown, Thomas A. "The Notion of Stability and its Relation to Selected Air Force Systems." WN-7441-PR. Rand Corporation, Santa Monica, California, September 1971.

Canby, Steven L. "NATO Muscle: More Shadow than Substance," Military Review, LIII (February 1973), pp. 65-74.

Cochard, Douglas D., and Robert E. Riggs. "A Force Survival Model for Analysis of Strategic Bomber Basing in the Prelaunch Survival Mode." Unpublished Master's thesis, Air Force Institute of Technology, September 1971.

Dalkey, Norman C. "Games and Simulations." P-2901. Rand Corporation, Santa Monica, California, April 1964.

Etzold, Thomas H., and Bruce W. Menning. "Force and Diplomacy in a Nuclear Age," Army, XXIV (January 1974), pp. 10-15.

Freund, John E. (trans.). Introduction to the Theory of Games by Ewald Burger. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963.

Friedman, Avner. Differential Games. New York: John Wiley and Sons, 1971.

Grubbs, Frank E., and John H. Shuford. "A New Formulation of Lanchester Combat Theory," Operations Research, XXI (July-August 1973), pp. 926-941.

Harsanyi, John C., and Reinhard Selten. "A Generalized Nash Solution for Two-Person Bargaining Games with Incomplete Information," Management Science, XVIII (January, 1972), pp. 80-106.

Hempley, Roy B. "Game Theoretic Models for a Limited, Tactical Warfare Engagement." Unpublished Master's thesis, Air Force Institute of Technology, June 1969.

Johnson, S. M. "A Game Solution of a Missile-Launching Problem with Hard and Soft Kill Probabilities." RM-2781, Rand Corporation, Santa Monica, California, August 1961.

________. "A Game Solution to a Missile Launching Scheduling Problem." RM-2398, Rand Corporation, Santa Monica, California, November 1959.

Kristian, Jerome, and Michael B. P. Slater (trans.). An Introduction to the Theory of Games, by E. S. Venttsel. Topics in Mathematics. Boston: D.C. Heath and Company, 1963.

Lucas, W. F. "An Overview of the Mathematical Theory of Games." Technical Report, Cornell University, Ithaca, New York, March 1971.

Mood, A. M., and R. D. Specht. "Gaming as a Technique of Analysis." P-579, Rand Corporation, Santa Monica, California, October 1954.

Owen, Guillermo. Game Theory. Philadelphia: W. B. Saunders Company, 1968.

________. "Multilinear Extensions of Games," Management Science, XVIII (January 1972), pp. 64-79.

Pugh, George E. "Theory of Measures of Effectiveness for General Purpose Military Forces: Part II. Lagrange Dynamic Programming in Time-Sequential Combat Games," Operations Research, XXI (July-August 1973), pp. 886-906.

________, and John P. Mayberry. "Analysis of General-Purpose Force Procurement." Technical Report, Lambda Corporation, Arlington, Virginia, August 16, 1971.

Schwodianer, Gerhard. "A Glossary of Game Theoretical Terms." Unpublished research report, New York University, June 1971.

Shubik, Martin. "On Gaming and Game Theory," Management Science, XVIII (January 1972), pp. 37-53.

________. "On the Scope of Gaming," Management Science, XVIII (January 1972), pp. 20-36.

Taylor, James G. "A Note on the Solution to Lanchester-Type Equations with Variable Coefficients," Operations Research, XIX (May-June 1971), pp. 709-712.

Von Neumann, John, and Oskar Morgenstern. Theory of Games and Economic Behavior. Princeton: Princeton University Press, 1953.

Weiner, M. G. "Gaming Limited War." P-2123, Rand Corporation, Santa Monica, California, October 1960.

Whetten, Lawrence L. "A European View of NATO Strategy," Military Review, LI (September 1971), pp. 25-37.

Williams, J. B. "Stability in Noncooperative Games," Operations Research, XIX (May-June 1971), pp. 774-783.

Wilson, Robert. "Computing Equilibria of Two-Person Games From the Extensive Form," Management Science, XVIII (March 1972), pp. 448-460.