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# IMAGE FORMATION BY MEANS OF SPATIAL INTENSITY CORRELATIONS

Paul H. Deitz

Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland

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PAUL H. DEITZ, PhD
US ARMY BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

A large number of the Army's present and future optical systems
have as their primary task the image formation of a distant object.
In virtually all of these systems, atmospheric turbulence plays a role
in limiting the image quality. This is so because the atmospheric
medium is inhomogeneous in temperature and, hence, in refractive
index. Thus, virtually all military optical systems must operate in
media that vary randomly in space and time. Even nonimage-forming
optical devices such as laser rangefinders[1] are affected by the
grosser manifestations of turbulence which take the form of energy
redistribution[2] within the optical beam and random pointing of the
beam.[3] However, for device; such as telescopes that are used to
form an image, the random modulation of the optical phase as light
traverses the medium acts in a more subtle way to lower the resolving
ability of an instrument below its *in vacuo* (diffraction-limited) per-
formance level.

Because of the deleterious effect of turbulence on the perform-
ance of image-forming instruments, we have reexamined the techniques
of image formation in terms of a new approach to a problem that has
afflicted astronomers for over two centuries. We will review the
basic mathematical relationships central to classical optical imagery
along with the constraints intrinsic to the detection of radiation at
optical wavelengths. It will be shown that the only parameter acces-
sible to measure is the intensity (mean square of the electric field)
and that this parameter corresponds to a second-order correlation of
the electric field.

Having developed the basis of classical imagery, we will intro-
duce the notion of higher-order correlations of optical fields (fourth,

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sixth, etc.) and discuss the ways in which these measures relate to standard image formation with which we are familiar. Specifically, by some manipulation of the far-field intensity distribution of a source, we wish to infer the intensity distribution over the source itself. We will discuss some of the constraints of higher-order correlation techniques, the potential they hold for reducing the effects of transmission in the atmosphere, and a basic experiment that illustrates the mathematical relationships.

In this paper we intend to present a mathematical framework upon which we can hang physical and intuitive arguments. Although extensive mathematical treatment will be avoided, annotation has been provided for those who might wish to explore these topics in detail.

## 1.   THE HUYGENS-FRESNEL EQUATION

We begin our discussion of optical imaging by examining the coordinate planes of Fig. 1. The object to be imaged here is found in the $\xi$-$\eta$ plane at the left-hand portion of the figure and is bounded by the aperture denoted $\Sigma$. In the general case, the object may be a primary source of light or a secondary surface from which light is scattered. The object is described by a two-dimensional source, since an extension along the axis of propagation can be accounted for, here, by an equivalent field at the plane of radiation. It is our interest to describe the nature of the electric field as it propagates from the $\xi$-$\eta$ to the x-y plane, a distance, $R_o$, away. For the moment, we are concerned only with the form of the electric field $V(x,y)$ as it relates to the field at the object, $V(\xi,\eta)$. The particular length between specific points in the two planes is indicated by the path $R(\xi,\eta,x,y)$.

Upon realizing that each point in the aperture radiates a spherical wave to the right, we can write the Huygens-Fresnel principle[4] for paraxial waves for which

$$V(x,y;t) = \iint\limits_{-\infty}^{\infty} V(\xi,\eta;t) \frac{\exp[ikR(\xi,\eta;x,y)]}{i\lambda R(\xi,\eta;x,y)} \, d\xi d\eta, \qquad (1)$$

where $k = 2\pi/\lambda$, and $\lambda$ equals the wavelength of the radiation. Equation (1) states that the electric field $V(x,y;t)$ is formed of a superposition (linear summation) of waves emanating from each point within the aperture, properly phase-shifted according to the exponential term and diluted by the 1/R expression. Now

$$R(\xi,\eta;x,y) = [R_o^2 + (x-\xi)^2 + (y-\eta)^2]^{\frac{1}{2}} \qquad (2a)$$

$$\simeq R_o \left\{ 1 + \frac{1}{2} \left( \frac{x - \xi}{R_o} \right)^2 + \frac{1}{2} \left( \frac{y - \eta}{R_o} \right)^2 \right\}, \tag{2b}$$

when approximated by the first two terms of the binomial[5] (the Fresnel approximation) expansion. Using Eq. (2b) in Eq. (1) and making the far-field (Fraunhofer) approximation $[R_o \gg k(\xi^2 + \eta^2)_{max}/2]$, we can write

$$V(x,y;t) = \frac{\exp[ik(R_o + (x^2 + y^2)/2R_o)]}{i\lambda R_o}$$

$$\times \iint\limits_{-\infty}^{\infty} V(\xi, \eta; t) \, \exp\left[ -i \, \frac{k}{R_o} \, (x\xi + y\eta) \right] d\xi \, d\eta. \tag{3}$$

Apart from the coefficients before the integral, Eq. (3) shows that the electric fields in an aperture and the far field are related by a spatial Fourier transform operation. This property is basic to the ensuing work involving intensity correlations with quasimonochromatic light. We note that Eq. (3) is linear; that is, the two-dimensional integral operator on $V(\xi, \eta; t)$ is linear. Therefore, if either $V(\xi, \eta; t)$ or $V(x,y;t)$ is known, the other is specified through a linear (and hence, invertible) transformation.

## 2. THE VAN CITTERT – ZERNIKE THEOREM

Although Eq. (3) holds for radiation of general frequencies, the relationship it expresses is somewhat academic here from the standpoint that it describes electric-field quantities that are unmeasurable at optical wavelengths. No known detector can follow oscillations at frequencies of $10^{14}$ hz. At optical frequencies, the parameter accessible to measurement is the intensity, the mean square of the electric field averaged over many oscillations. It is this constraint that has prompted many investigators to couch optical theory in the form of correlations of field quantities. The best known of these correlations is called the mutual coherence function, $\Gamma(\underline{x}_1, \underline{x}_2, \tau)$, where

$$\Gamma(\underline{x}_1, \underline{x}_2, \tau) \equiv \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} V(\underline{x}_1, t + \tau) V^*(\underline{x}_2, t) dt, \tag{4a}$$

$$= \overline{V(\underline{x}_1, t + \tau) V^*(\underline{x}_2, t)}, \tag{4b}$$

and $\tau$ is the time delay between the instantaneous product of the electric fields at the points $\underline{x}_1$ and $\underline{x}_2$ (and $\underline{x}_i = x_i, y_i$). A special case

of the mutual coherence function results when the time delay is set to zero or

$$\Gamma(\underline{x}_1, \underline{x}_2, \tau)_{\tau=0} \equiv J(\underline{x}_1, \underline{x}_2), \tag{5}$$

where $J(\underline{x}_1, \underline{x}_2)$ is called the mutual intensity function.

We wish to calculate the mutual intensity in the far field of a spatially incoherent source. Using Eqs. (5), (4b), and (3), we write

$$\overline{V(x_1, y_1; t) V^*(x_2, y_2; t)} = J(x_1, y_1; x_2, y_2) \tag{6a}$$

$$= \frac{\exp\left[\frac{ik}{2R_o} (x_1^2 - x_2^2 + y_1^2 - y_2^2)\right]}{(\overline{\lambda} R_o)^2} \iiiint\limits_{-\infty}^{\infty} \overline{V(\xi_1, \eta_1; t) V^*(\xi_2, \eta_2; t)}$$

$$\cdot \exp\left[-i\frac{\overline{k}}{R_o} (x_1 \xi_1 - x_2 \xi_2 + y_1 \eta_1 - y_2 \eta_2)\right] d\xi_1 d\xi_2 d\eta_1 d\eta_2, \tag{6b}$$

where the processes of temporal averaging and spatial integration have been interchanged and the radiation assumed quasimonochromatic ($\nu_{min} > \nu_{max} - \nu_{min}$) so that the wavelength dependence can be approximated by the mean wavelength. Since the source is spatially incoherent the mutual intensity takes the form[6]

$$\overline{V(\xi_1, \eta_1; t) V^*(\xi_2, \eta_2; t)} = J(\xi_1, \eta_1; \xi_2, \eta_2), \tag{7a}$$

$$= I(\xi, \eta) \delta(\xi_1 - \xi_2) \delta(\eta_1 - \eta_2). \tag{7b}$$

Physically, Eq. (7b) implies that the time fluctuations of the electric fields at two non-identical points in the source plane are completely uncorrelated;* equivalently, the total power measured at a point in the far field is simply the sum of the squared electric fields from each differential element of the source, taken with the proper phase delay and attenuation.

If Eq. (7b) is used in Eq. (6), the mutual intensity collapses to a single area integral, giving

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*The overbar of Eq. (7a) by the definition of Eq. (4) indicates a time average. Although spatial incoherence is usually defined by a time-averaging process, it can also be defined in terms of a spatial average in the source plane.

$$J(x_1,y_1;x_2,y_2) = \frac{\exp\left[i \frac{\overline{k}}{2R_o} (x_1^2 - x_2^2 + y_1^2 - y_2^2)\right]}{(\overline{\lambda}R_o)^2}$$

$$\times \iint\limits_{-\infty}^{\infty} I(\xi,\eta) \exp\left\{-i \frac{\overline{k}}{R_o} [(x_1 - x_2)\xi + (y_1 - y_2)\eta]\right\} d\xi d\eta. \qquad (8)$$

Although usually presented in a normalized form,[7] Eq. (8) is a statement of the van Cittert - Zernike theorem which, aside from the accompanying coefficient, shows that the mutual intensity in the far field of a spatially incoherent source is given by the Fourier transform of the intensity across that source.

The mutual intensity function of Eq. (8) is directly accessible through measurement using a technique known as Young's double pinhole experiment. Figure 1 shows an opaque surface erected in the x-y plane. Two pinholes are made in the surface. A distance behind the surface a fringe pattern can be observed on the u-v plane. The fringe contrast relates to the amplitude of $J(x_1,y_1;x_2,y_2)$, and the fringe shift corresponds to the relative phase between the radiation on the pinholes. If this measurement is made for all pinhole spacings and orientations in the x-y plane, $J(x_1,y_1;x_2,y_2)$ is completely specified and can be Fourier transformed to derive $I(\xi,\eta)$, the intensity on the object. This theorem is basic to most optical imaging, since lenses effect the inverse transformation of Eq. (8) to give a scaled distribution of the object irradiance. Referring again to Fig. 1, we note that a lens has been placed its own focal length from both the x-y and u-v planes. We assert without proof (see Ref. 4, p. 83) that in this configuration there is a Fourier transform relation between these two planes due to the operation of the lens on the radiation from the x-y surface similar to the relationship between the $\xi$-$\eta$ and x-y planes located a great distance apart. Thus, under the assumptions given in the mathematical treatment above, the electric field V(u,v) is a scaled distribution of the field $V(\xi,\eta)$. The inversion of the u-v axes is a mathematical convenience (see Ref. 4, p. 167).

By the scaling property from one domain to another intrinsic to the Fourier transform operation, large detail in the source is given by small separations in the far field and vice versa. Thus, imaging with a finite aperture implies a finite limit to the high-frequency detail resolvable on the source. This justifies the well-known description of lenses as low-pass filters. Also, since there are many more pairs of points within an aperture corresponding to small separations than to large, there is a built-in redundancy weighted in favor of low-frequency resolution.

Equation (8), although a statement of second-order correlation, is basic also to fourth-order correlation, as we shall see later.

## 3. THE EFFECTS OF COHERENCE

At this point in the development, it is important that we concern ourselves with the coherence properties of optical sources.[8,9] We again refer to Fig. 1. Let us imagine that there is a single point scatterer at the object ($\xi$-$\eta$) plane illuminated by an optical source. The field at the x-y plane (plane of observation) will be, of course, a spherical wave. If we begin to insert arbitrarily other point scatterers at the object plane, the electric field at the x-y plane will be the sum of all the individual contributors as determined by geometry per Eq. (3). At a particular instant of time, the contributions from the various scatterers will add constructively at some points on the x-y plane, while at other points they may not. But, as indicated earlier, the electric field is not accessible to measurement; the intensity (mean square of the electric field) must be averaged over many cycles of field oscillation. If the optical bandwidth of the light illuminating the scatterers is sufficiently narrow that the relative phase across the wavefront remains constant during the period of observation, then the intensity pattern that would be observed during just a few temporal cycles of oscillation will remain fixed during the total period of observation. This pattern is called a speckle pattern. If, on the other hand, the bandwidth of the source is broadened, then the speckle pattern begins to change with time and gradually washes out unless the period of observation is shortened accordingly.

Fluctuations of the speckle pattern in the time domain relate to the temporal coherence properties of the source. Fluctuations of the speckle pattern in space (across the x-y plane) relate to the source size and are termed spatial coherence effects. It is these latter coherence properties that are embodied in the mutual intensity function of Eq. (8) and relate directly to the parameter of interest, $I(\xi)$, the intensity distribution over the source plane. But the ability to record the speckle pattern without its being washed out depends on the source bandwidth and detection time in the x-y plane.

Speckle patterns are easily observed by eye when laser radiation is scattered from a rough surface. However, with thermal sources great care must be taken to utilize even a fraction of the speckle signal that is usually time-variant with extremely short time constants.

### 4. A THEOREM CONCERNING THE FOURTH-ORDER GAUSSIAN RANDOM PROCESS

We have seen in Eq. (7) the way in which a time average can be used to impose a condition on the correlation of an electric-field pair. No particular assumption was made about the statistics of the field variables. However, it is well known that classical thermal sources exhibit statistical fluctuations (temporally) that are gaussian in nature. Hodara[10] has asserted that lasers with but a few axial modes are, to a good approximation, gaussian as well. However, Troup[11] has argued that gaussian statistics are achieved only in the limit of a large number of axial modes.

The argument for gaussian statistics can be extended to the spatial domain as well. It has been argued that[12] the received field at any point in the far zone (as described above in Section 3) consists of a sum of random-amplitude, random-phase, complex phasors contributed by the elementary scatterers. If the size of the scattering area is large enough to include many point scatterers (or there are enough elementary coherence areas composing the source), the Central Limit Theorem may be used to conclude that the electric field in the detection plane is a gaussian random process in a spatial sense.

A well-known property of gaussian statistics is that all higher-order moments are representable in terms of the first and second.[13] In particular, the fourth-order correlation of electric fields (second-order correlation of intensities) can be shown to be[14]

$$\overline{I_1 I_2} = \overline{I_1}\ \overline{I_2} + |\Gamma_{12}|^2 , \qquad (9)$$

where the defining relation of Eq. (4a) has been used. Equation (9) therefore describes the relationship between intensity correlations and field correlations for a process that is gaussian in the time domain. We note that, in general, the field correlation is a complex quantity, so that only the relationship between the intensity correlation and the modulus of the field correlation is implied. It reveals the underlying principle by which intensity correlations in the far field may be used to infer the accompanying field correlations and hence, through Eq. (8), to gain knowledge of the intensity distribution at the source.

### 5. ASSUMPTIONS BASIC TO STATISTICAL AVERAGING

We are now in a position to examine the nature of the averaging processes fundamental to a number of optical processing schemes. Figure 2 represents an ensemble of similar experiments. As in Section 3, we consider the intensity distribution in the $\underline{x}$-plane to be made over

an observation time short compared with any temporal fluctuations in
intensity. The amplitude distribution over the source plane is identi-
cal from one sample to the next. The pair of receiver points in the
detection plane also remains constant. However, each source plane ex-
hibits a statistically similar, but independent phase mapping across
its extent. This insures that the speckle (intensity) patterns observed
at the receiver ($\underline{x}$) planes remain statistically identical as well. In
the language of probability theory, each pair of measurements in the
$\underline{x}$-plane represents one particular sample in the outcome space of the
experiment.

We now turn our attention to the classic relation of imaging,
Eq. (8). It must be recognized that there are two fundamental assump-
tions basic to its development which were not explicitly mentioned.
First, the mutual coherence function of Eq. (4) is fundamentally defined
in terms of an ensemble average. This is the average that would be
calculated if the total outcome space for a given experiment were known
for a given space-time set of boundary conditions. In practice, the
ensemble average is never measurable because of a lack of access to an
unlimited number of experimental configurations operating under identi-
cal circumstances. Instead, the ergodic hypothesis is invoked: that
is, if one particular experiment is performed under unchanging condi-
tions (stationarity), then the average that is taken in the domain of
stationary conditions is *assumed* equivalent to the ensemble (true)
average. The working definition of Eq. (4) necessarily assumes that
the ensemble average can be replaced by a time average because of a
condition of stationarity in the time domain, the domain in which the
averaging is made. Equation (8) is then a time average over many
instantaneous products of electric field. For this situation, Fig. 2
can represent a time series of field pairs (i,j,...n) which are sam-
pled, multiplied, and averaged.

b. FOURTH-ORDER CORRELATIONS IN THE TIME DOMAIN

Until about two decades ago, essentially all imaging was accom-
plished by means of second-order correlations taken over a time average
long compared with the coherence time of the radiation. About that
time, two astronomers, Hanbury Brown and Twiss,[15,16] were searching
for a technique to infer the diameter of stars which would not have the
sensitivity to atmospheric turbulence and instrument vibration charac-
teristic of the stellar interferometry of Michelson and Pease.[17] To
that end, they were first to suggest the use of fourth-order field cor-
relations. The key to this approach is embodied in Eq. (9).

Although Hanbury Brown and Twiss did not explicitly use the
gaussian theorem of Eq. (9), it is, in fact, intrinsic to their

mathematical development. Their result shows, nevertheless, that the time-averaged, two-point correlation of intensities in the far field of a spatially incoherent source is proportional to the square of the Fourier transform of the source intensity distribution. We note as before that $\Gamma_{12}$ is, in general, a complex function which, in fact, represents the amplitude spectrum of the source intensity distribution. Since in Eq. (9) the mutual coherence function is squared, this method yields the power spectrum of the source intensity distribution. Also, since the relative magnitudes of the real and imaginary components of the amplitude spectrum are unknown, no linear transformation is possible to infer the intensity distribution of the source itself.

Turning again to Fig. 2, the Hanbury Brown-Twiss experiment can be understood by letting each member of the ensemble represent a pair of intensity measurements made in a time less than the coherence time of the radiation. As many intensity products are averaged in time, the function approaches the ensemble average.

Although this method of imagery yields information only about the object intensity power spectrum, it has proved useful in the measurement of star diameters,[18] for which only the first zero crossing of the Fourier transform need be known. Since the optical phase is discarded immediately upon detection, the instrumentation is not only insensitive to mechanical vibrations but to turbulence-induced phase fluctuations as well. The primary limitation to the method is the low level of radiation that is detected. As a result, this approach is limited to relatively intense stars to overcome the severe signal-to-noise problem. Analyses of this problem have been given by Gamo[20] and Twiss.[21]

## 7. INTENSITY CORRELATION IN THE SPATIAL DOMAIN

With the utilization of the laser, the experimental constraints are quite different. The signal-to-noise ratio can be increased typically by six orders of magnitude.[22] With a view to exploiting this property, Deitz and Carlson[23] have recently investigated the potential for intensity correlation in terrestrial imagery. In many situations of interest to the Army, the availability of laser illumination greatly reduces the constraints due to noise limitations. In addition, since ground-to-ground imagery suffers from the greatest deterioration in image quality because of turbulence, it offers the greatest potential for improvement.

Although the details of this work can be found elsewhere,[23] we will again turn to Fig. 2 for the basis of this recent approach. In the case of spatial intensity interferometry, the signal in the x-plane

is detected over a large area, rather than at just two points. Conceptually, a large piece of film is placed in the $\underline{x}$-plane. The signal is recorded spatially over a time short compared with the coherence time of the radiation and is then autocorrelated. This operation corresponds to an average taken in the space domain. Here, through the assumption of spatial stationarity (and a sufficiently large area of spatial averaging), the spatial average is assumed equivalent to the ensemble average.

Although the method of signal processing is very different from the Hanbury Brown - Twiss experiment, the final mathematical relations are similar. The principal result[23] shows that the spatial intensity correlation in the far field of a source can be written

$$\langle I(\underline{x}_1,T)I(\underline{x}_2,T) \rangle = \frac{1}{(cr)^4} \left| \int_0^\infty \omega^2 d\omega \int_{-\infty}^\infty \text{sinc}(\rho T)H(\rho)d\rho \right. \tag{10a}$$

$$\times \iint_{-\infty}^\infty I(\underline{\xi},\omega) \exp\left[-i\frac{k}{r}(\underline{x}_1-\underline{x}_2)\cdot\underline{\xi}\right]d\underline{\xi}$$

$$\left. \times \iint_{-\infty}^\infty C(\underline{f}) \exp\left[-i\frac{k}{2r}(\underline{x}_1+\underline{x}_2)\cdot\underline{f}\right]d\underline{f} \right|^2$$

$$\alpha \left| \hat{I}\left[\frac{k}{r}(\underline{x}_1-\underline{x}_2)\right] \right|^2 \left| \hat{C}\left[\frac{k}{2r}(\underline{x}_1+\underline{x}_2)\right] \right|^2. \tag{10b}$$

The angle brackets indicate a spatial average, c is the vacuum velocity of light, r is the average distance between object and detection planes, and $\omega$ is the angular frequency of radiation. The parameter $\rho$ is the difference frequency between modes in the incident radiation, the H function is defined in Ref. 23 and relates to the temporal spectrum of the carrier, and k is the wave number of light $(2\pi/\lambda)$. Equation (10) shows that the spatial intensity correlation is proportional to two functions. The first is the square of the Fourier transform of the source intensity distribution as related in Eq. (9) by a temporal averaging process. The second term, $C(\underline{f})$, is a normalized phase correlation function[12] describing the coherence interval over a rough surface. In order to describe a spatially incoherent surface, $C(\underline{f})$ (which is defined by a spatial average) is often allowed to assume the role of a delta function. In that idealized limit, its transform becomes a constant and does not band limit the detectable spatial frequency spectrum of the object.[23]

## 8. AN EXPERIMENT IN SPATIAL INTENSITY INTERFEROMETRY

To illustrate the results of Eq. (10), a pair of crossed Ronchi rulings, shown in Fig. 3, were used as the object in the following

experiment.[*]  First, in order to demonstrate the Fourier transform
relation of the Huygens-Fresnel equation [Eq. (3)], the object of
Fig. 3 was transilluminated.  In the far field, the intensity distri-
bution was recorded and is shown in Fig. 4.  This signal corresponds
to the square of electric field represented by Eq. (3); equivalently,
the signal of Fig. 4 is the power spectrum of the electric field dis-
tribution over the object of Fig. 3.

To illustrate the results of intensity correlation, a section
of ground glass was inserted (to gain spatial incoherence) adjacent to
the object of Fig. 3 and again illuminated by a laser.  The far-field
speckle pattern that resulted is shown in Fig. 5.  The pattern appears
random.  The mathematical effect of the ground glass has been to multi-
ply the object field, $V(\xi, \eta)$, inside the integral of Eq. (3) by a random
phase function.

Next, the speckle pattern of Fig. 5 was used to make an identi-
cal pair of optical transparencies.  These were then used in an optical
autocorrelator to record the result shown in Fig. 6.  As described by
the relation of Eq. (10), Fig. 6 gives the power spectral density of
the intensity distribution over the object.  Since the electric field
amplitude and the intensity over the object of Fig. 3 are related by a
squaring process, the results of Figs. 4 and 6 are similar in the man-
ner indicated by Eqs. (3) and (10).

## 9.  THE INVERSION OF THE POWER SPECTRUM

As we have shown in the previous sections, intensity correla-
tions lead only to the power spectral density of the source intensity
or, equivalently, the modulus of the Fourier transform.  Depending upon
the application, the power spectral density or its Fourier transform,
the correlation function, may be the parameter sought for system
use.[24]  This is often the case in a target-guidance or recognition
problem.  However, without the phase information, the Fourier inversion
cannot be taken to derive the source distribution itself.

In the experiments of Hanbury Brown and Twiss, the loss of phase
is not a serious limitation since their objective is simply the meas-
urement of star diameters.  If a circular disk is used as a model for
a star, the object is known, *a priori*, to be symmetrical.  Thus the
spatial transform of the (real) intensity is pure real.  For this situ-
ation, the phase of the transform is zero or $\pi$ for all spatial wave
numbers, and the square root of the power spectrum can be taken (with
a sign ambiguity) to derive the spatial transform itself.

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In the application of intensity interferometric techniques to terrestrial imaging systems, though, the loss of phase is a serious limitation to the method. A number of authors have addressed themselves to the problem of phase recovery.[25] A general solution to this problem, however, has never been found.

Deitz and Carlson have recently proposed a scheme[26] whereby the incoming electric field is preprocessed before intensity detection and autocorrelation. By this method, the source is effectively symmetrized in its intensity distribution. If a function is real and exhibits even symmetry,[27] its Fourier transform must be pure real. Thus, even though the power spectrum is finally derived by the measurement method, its accompanying amplitude spectrum is known (by virtue of the preprocessing) to be pure real. Thus, the square root of the power spectrum can be taken to infer the amplitude spectrum. There is a critical choice of signs, however, to be made in the square root process. The utility of the method appears to rest with the degree to which system noise can be precluded from interferring with those decisions.[26]

## 10. CONCLUSIONS

There are a number of special benefits from detecting images by the technique of intensity correlation. (1) The method is relatively insensitive to the effects of atmospheric scintillation.[19] (2) Because the signal is detected in the spatial-transform domain, high-frequency detail about the scattering surface translates to large spatial lags in the far field. This result could be particularly important at frequencies where detector resolution is not well developed. (3) A special advantage to intensity interferometry in the spatial domain is the utilization of gaussian statistics in the spatial (not temporal) sense. By this method, sources with non-gaussian time statistics (such as single-axial-mode lasers) can be utilized. (4) Still another advantage of spatial detection is that images of moving surfaces can be formed using brief exposures.

Against these benefits must be weighed the limitations of intensity interferometry. The primary factor in this respect rests upon the greatly diluted energy density at the plane of detection. Because the signals are of such relatively low intensity, they tend to be masked by noise. The limitation can result from quantum noise in the carrier, detector noise, or stray signals from unwanted background. It remains to be seen whether these limitations can be successfully overcome so that the potential benefits of intensity correlation can ultimately be realized.

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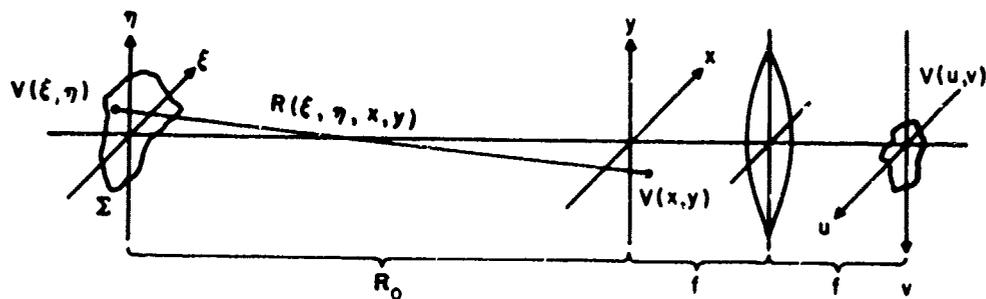
Fig. 1. Coordinate axes for the object (ξ-η) and primary detection (x-y) planes. In addition, a lens is placed its focal length behind the x-y plane. Finally, a focal length behind the lens is the u-v plane.
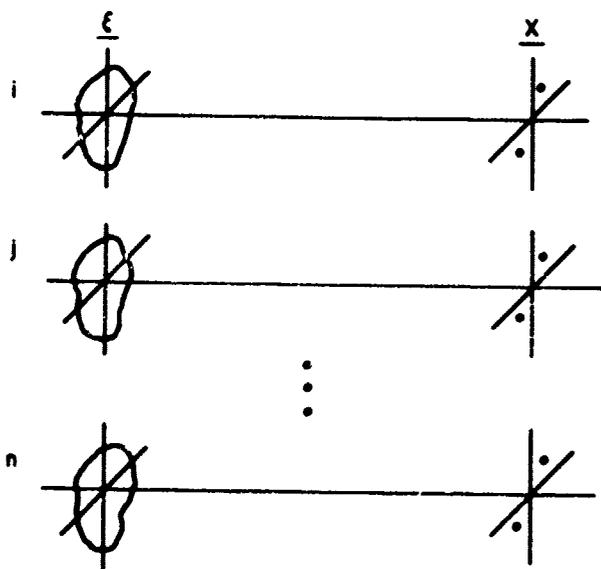


Fig. 2. An ensemble of similar experiments. Source amplitude and receiver points are identical. Random phase variations over each source plane are statistically independent.
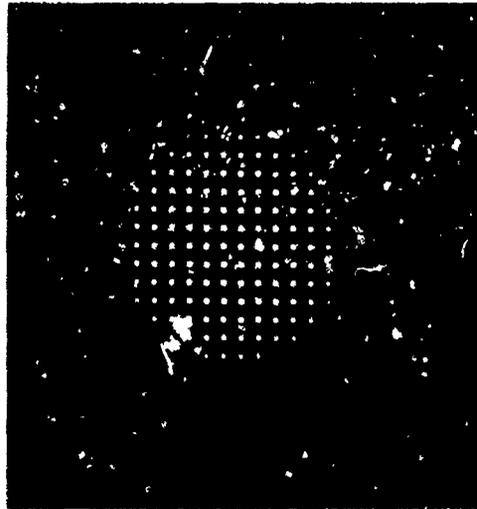
Fig. 3.  Optical Transparency
Used as Object



Fig. 4.  Spatial Power Spectral
Density of the Electric Field
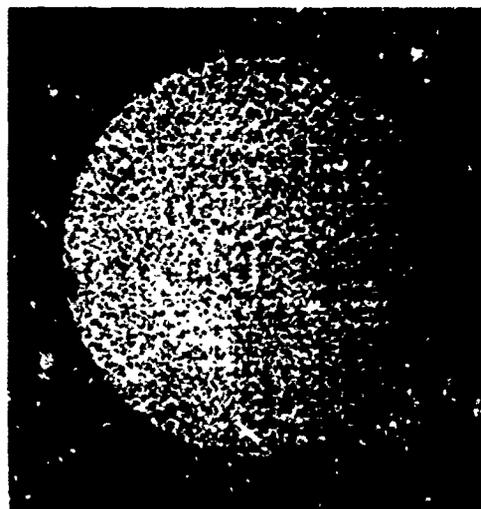Distribution of Fig. 3



Fig. 5.  Far-Field Speckle Pattern
of Transparency in Fig. 3 When
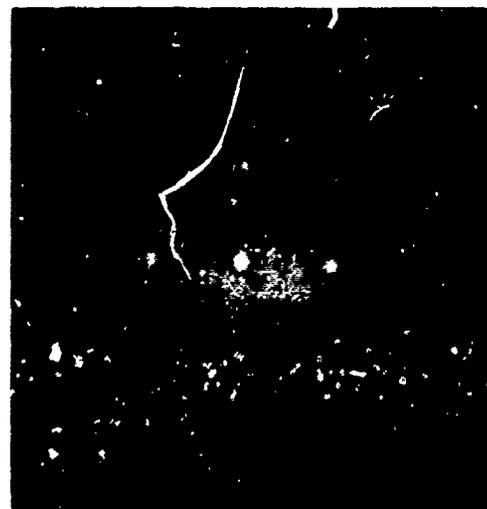Used with Random Phase Screen



Fig. 6.  Spatial Power Spectral
Density of the Intensity Dis-
tribution of Fig. 5