AEDC-TR-74-35

Cy. 2

REGULAR NORMAL TRANSMITTANCE OF ROUGHENED DIELECTRIC INTERFACES: COMPARISON OF THEORY AND DATA

A. M. Smith
ARO, Inc.

VON KÁRMÁN GAS DYNAMICS FACILITY
ARNOLD ENGINEERING DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
ARNOLD AIR FORCE STATION, TENNESSEE 37389

September 1974

Final Report for Period January 1, 1973 – January 4, 1974

Prepared for

ARNOLD ENGINEERING DEVELOPMENT CENTER
ARNOLD AIR FORCE STATION, TENNESSEE 37389

APPROVAL STATEMENT

This technical report has been reviewed and is approved.

MELVIN L. GUIOU
Capt, USAF
Research and Development
Division
Directorate of Technology

ROBERT O. DIETZ
Director of Technology

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AEDC-TR-74-35	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) REGULAR NORMAL TRANSMITTANCE OF ROUGHENED DIALECTRIC INTERFACES: COMPARISON OF THEORY AND DATA		5. TYPE OF REPORT & PERIOD COVERED Final Report – January 1, 1973–January 4, 1974 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) A. M. Smith, ARO, Inc.		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Arnold Engineering Development Center Arnold Air Force Station, Tennessee 37389		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element 65802F
11. CONTROLLING OFFICE NAME AND ADDRESS Arnold Engineering Development Center Arnold Air Force Station, Tennessee 37389		12. REPORT DATE September 1974 13. NUMBER OF PAGES 24
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Available in DDC.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

dielectrics
refractivity
transmission data
wavelength
normal density functions
rock salt
absorption
scattering
surface roughness

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The theory for the regular normal transmittance of a rough dielectric interface is compared with experimental results. Relative regular transmission data from a very novel experiment involving variation of interface refractive index ratio and root-mean-square (rms) roughness at a single wavelength are found to be in excellent agreement with the theoretical transmittance relation for a rough dielectric interface having a Gaussian distribution of surface heights. Also, absolute spectral transmission

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. Continued.

measurements for rock-salt (NaCl) windows roughened on both faces are found to be in good agreement with the regular transmittance theory at wavelengths wavelengths where absorption and surface scattering are negligible.

PREFACE

The research reported herein was conducted at the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC). The results were obtained by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of AEDC, AFSC, Arnold Air Force Station, Tennessee. The research was conducted under ARO Project Nos. VF202 and VF402. The manuscript (ARO Control No. ARO-VKF-TR-74-21) was submitted for publication on February 20, 1974.

CONTENTS

		Page
1.0	INTRODUCTION	5
2.0	THEORY	
	2.1 Rough Dielectric Interface	5
	2.2 Dielectric Plates with Rough Interfaces	8
3.0	EXPERIMENTAL DATA FOR ROUGH GLASS INTERFACES	10
4.0	COMPARISON OF THEORY AND DATA	
	4.1 Rough Glass Interfaces	12
	4.2 Rough NaCl Plates	16
5.0	CONCLUSIONS	20
	REFERENCES	20

ILLUSTRATIONS

Figure		
1.	Sketch of Rough Dielectric Interface	6
2.	Sketch of Dielectric Plate Roughened on Both Faces	9
3.	Schematic of Experimental Surface System for Roughened Glass Samples	11
4.	Graphical Comparison of Regular Transmittance Theory and Data for Different Glass Interface Roughnesses	13
5.	Computer Comparison of Regular Transmittance Theory and Data (Ref. 14) for a Glass Interface Roughness of 0.520 μ	14
6.	Computer Comparison of Regular Transmittance Theory and Data (Ref. 14) for a Glass Interface Roughness of 0.645 μ	14
7.	Computer Comparison of Regular Transmittance Theory and Data (Ref. 14) for a Glass Interface Roughness of 0.819 μ	15

Figure Page

8. Computer Comparison of Regular Transmittance Theory and Data (Ref. 14) for a Glass Interface Roughness of $0.869\ \mu$ 15

9. Graphical Comparison of Theory and Data for the Relative Regular Normal Transmittance of Rough Glass Interfaces 17

10. Comparison of Theory and Data for the Absolute Normal Transmittance of Roughened NaC1 Windows with Effective Surface Roughness, σ_e, Taken as a Parameter 17

11. Comparison of Theory and Data for the Absolute Normal Transmittance of Roughened NaC1 Windows . . . 19

TABLES

1. Experimental Regular Transmittance Results (Ref. 14), $\tau_R(\psi = 0°, n_1, n_2, \sigma/\lambda)/T(\psi = 0°, n_2/n_1)$, for Roughened Glass Interfaces, $n_1 = 1.530$, $\lambda = 0.546\ \mu$. 12

2. Numerical Comparison of Regular Transmittance Theory, Eq. (8), and Data (Ref. 14), $\tau_R(\psi = 0°, n_1, n_2, \sigma/\lambda)$ $/T(\psi = 0°, n_2/n_1)$ for various Glass Interface Roughnesses. . 16

NOMENCLATURE

1.0 INTRODUCTION

In the past there have been numerous studies of specular, i.e., regular, reflection from rough metal and dielectric surfaces (Refs. 1 through 12). However, the regular transmittance of such surfaces, i.e., interfaces, has not been studied until recently and then only briefly in a paper dealing primarily with the anomalous refraction phenomena occurring in the bidirectional transmittance of roughened dielectrics (Ref. 13). The major objective of this report is to compare the theory presented in Ref. 13 for regular normal transmittance of a rough dielectric interface with some experimental transmission data (Ref. 14) which recently came to the author's attention. A secondary objective is to compare regular normal transmittance theory with absolute spectral transmission measurements (Ref. 1) for plates of rock-salt (NaCl) crystals roughened on both faces.

2.0 THEORY

2.1 ROUGH DIELECTRIC INTERFACE

Following the approach of Ref. 13, the theoretical expression for the relative, regular normal transmittance of a rough interface having some distribution of local surface heights can be written as

$$\frac{\tau_R(\psi = 0°, n_1, n_2, \sigma/\lambda)}{T(\psi = 0°, n_2/n_1)} = \frac{I_{t,R}(\psi = 0°)}{I_{t,0}(\psi = 0°)} \tag{1}$$

Here, $\tau_R(\psi = 0°, n_1, n_2, \sigma/\lambda)$ is the regular normal transmittance of the rough dielectric interface which is sketched in Fig. 1, $T(\psi = 0°, n_2/n_1)$ is the regular normal transmittance for a smooth interface of the same material, n_1 and n_2 are the refractive indices (relative to air) of the media on opposite sides of the interface, λ is the radiation wavelength in air, σ is the root-mean-square (rms) value of the surface heights $\zeta(x)$ of the rough interface (see Fig. 1), $\psi = 0°$ is the zenith incidence angle of the irradiance, $I_{t,R}(0)$ is the radiant intensity regularly transmitted through the rough interface, and $I_{t,0}(0)$ is that transmitted through the smooth interface. Since the intensity-solid angle product of a transmitted wave is proportional to the square of the modulus of its amplitude, Eq. (1) may be written as

$$\frac{r_R(\psi = 0^\circ, n_1, n_2, \sigma/\lambda)}{T(\psi = 0^\circ, n_2/n_1)} = |E_R|^2 / |E_o|^2 \qquad (2)$$

Here, E_R is the amplitude of the beam regularly transmitted through the rough interface, and E_o is the amplitude that would be obtained when the interface is smooth.

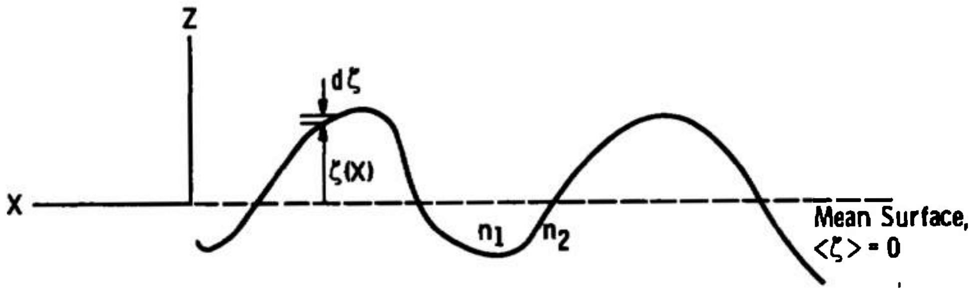

Figure 1. Sketch of rough dielectric interface.

Now the amplitude E_R can be related to E_o by noting that the elementary waves resulting from transmission through the individual elements of the rough interface should be summed with respect to their phases to form one resultant transmitted wave. Taking the mean plane of the interface, $<\zeta> = 0$, as the reference plane, it can be shown that $2\pi(n_1 - n_2)\zeta/\lambda$ is the relative phase of an elementary wave transmitted in the normal direction from a surface element located at height ζ above $<\zeta> = 0$. Summing up the elementary waves from all the surface elements with respect to their phases, it is found that the resultant transmitted wave in the normal direction has the amplitude

$$E_R = |E_o| \int_{-\infty}^{\infty} w(\zeta) \exp\left[-i(2\pi/\lambda)(n_1 - n_2)\zeta\right] d\zeta \qquad (3)$$

where $w(\zeta)$ is the distribution function specifying how surface elements of the rough interface are distributed with height ζ from the mean plane. This distribution function is mathematically defined by the relation

$$w(\zeta)d\zeta = dA(\zeta)/A \qquad (4)$$

where $dA(\zeta)$ is the projected aggregate area of surface elements lying in the differential layer, $d\zeta$, between ζ and $\zeta + d\zeta$ and A is the projection of the area of all the illuminated rough surface elements onto the mean plane.

Introducing Eq. (3) into Eq. (2) and performing the indicated operations yields the following general relation for the relative regular normal transmittance of a rough dielectric interface:

$$\frac{\tau_R(\psi = 0^\circ, n_1, n_2, \sigma/\lambda)}{T(\psi = 0^\circ, n_2/n_1)} = \left|\int_{-\infty}^{\infty} w(\zeta) \exp\left[-i(2\pi/\lambda)(n_1 - n_2)\zeta\right] d\zeta\right|^2 = \left|g[(n_1 - n_2)/\lambda]\right|^2 \quad (5)$$

From Eq. (5) it is seen that the relative, regular normal transmittance as a function of $(n_1 - n_2)/\lambda$ is the squared modulus of the exponential Fourier transform $g[(n_1 - n_2)/\lambda]$ of the surface height distribution function, $w(\zeta)$. The usefulness of this relationship is immediately apparent when one considers the possibility of determining the surface height distribution function directly from the measured regular transmittance by means of Fourier inversion of Eq. (5). Note that it is possible to interpret $(n_1 - n_2)/\lambda$ in Eq. (5) as the reciprocal of an effective wavelength, λ_e, as defined by

$$1/\lambda_e \equiv 1/\lambda_1 - 1/\lambda_2 \quad (6)$$

where

$$\lambda_1 \equiv \lambda/n_1$$

and

$$\lambda_2 \equiv \lambda/n_2$$

If $w(\zeta)$ in Eq. (5) is taken to be the Gaussian distribution function,

$$w(\zeta) = \left[1/\sigma(2\pi)^{1/2}\right] \exp\left(-\zeta^2/2\sigma^2\right) \quad (7)$$

and the indicated mathematical operations are carried out, then the relative regular transmittance for normal irradiance on a rough Gaussian interface is given by the relation

$$\frac{\tau_R(\psi = 0^\circ, n_1, n_2, \sigma/\lambda)}{T(\psi = 0^\circ, n_2/n_1)} = \exp -\left[2\pi(\sigma/\lambda)(n_1 - n_2)\right]^2 \quad (8)$$

Introducing Eq. (6) into Eq. (8) yields the expression

$$\frac{\tau_R(\psi = 0^\circ, n_1, n_2, \sigma/\lambda)}{T(\psi = 0^\circ, n_2/n_1)} = \exp\left[-(2\pi\sigma/\lambda_e)^2\right] \quad (9)$$

which allows the interpretation that the relative, regular transmittance of a rough Gaussian interface is functionally dependent only on the ratio of surface roughness, σ, to effective wavelength, λ_e.

2.2 DIELECTRIC PLATES WITH ROUGH INTERFACES

Figure 2 shows a sketch of a dielectric plate roughened on both faces with the rms surface roughness of face 1, σ_1, being taken unequal to that of face 2, σ_2. Thus, the regular normal transmittance, τ_{R1}, and reflectance, ρ_{R1}, for interface 1 are not equal to those of interface 2, τ_{R2} and ρ_{R2}. The geometrical thickness of the plate is d, the refractive index is n_1, and the absorption coefficient, which ranges from small to negligible for the wavelengths considered in this investigation, is k_1. Following the approach of Ref. 15, the absolute, regular normal transmittance, t_R, of the roughened dielectric plate can be obtained by solving the one-dimensional radiative transport equation with appropriately modified intensity boundary conditions for the rough interfaces. The technique of solution (Ref. 15) is elementary and results in the relation

$$t_R = \frac{\tau_{R1}\tau_{R2} \exp(-k_1 d)}{1 - \rho_{R1}\rho_{R2} \exp(-2k_1 d)} \tag{10}$$

Assuming that the rough interfaces of the dielectric plate in Fig. 2 have a Gaussian distribution of surface heights, τ_{R1} and τ_{R2} in Eq. (10) can be expressed as

$$\tau_{R1} = T(\psi = 0°, n_1) \exp\{-[2\pi(n_1 - 1)(\sigma_1/\lambda)]^2\} \tag{11}$$

and

$$\tau_{R2} = T(\psi = 0°, 1/n_1) \exp\{-[2\pi(n_1 - 1)(\sigma_2/\lambda)]^2\} \tag{12}$$

by use of Eq. (8). Also, the regular internal reflectances of the rough Gaussian interfaces, ρ_{R1} and ρ_{R2}, can be expressed as (Ref. 2)

$$\rho_{R1} = F(\psi = 0°, 1/n_1) \exp[-(4\pi n_1\sigma_1/\lambda)^2] \tag{13}$$

and

$$\rho_{R2} = F(\psi = 0°, 1/n_1) \exp[-(4\pi n_1\sigma_2/\lambda)^2] \tag{14}$$

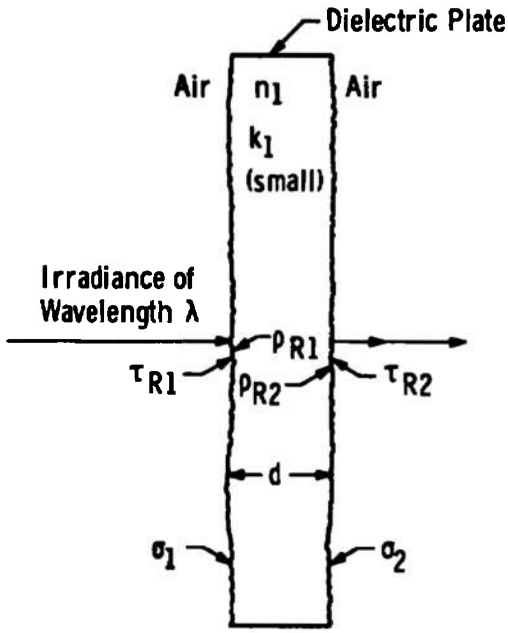

Figure 2. Sketch of dielectric plate roughened on both faces.

where $F(\psi = 0°, 1/n_1)$ is the regular normal internal reflectance for a smooth interface of the same material. Introducing Eqs. (11 through 14) into Eq. (10) yields

$$t_R = \frac{T^2(0) \exp -[(2\pi/\lambda)(n_1 - 1)]^2(\sigma_1^2 + \sigma_2^2) \exp - k_1 d}{1 - F^2(0) \exp -[(4\pi/\lambda)n_1]^2(\sigma_1^2 + \sigma_2^2) \exp - 2k_1 d} \tag{15}$$

where the fact that $T(\psi = 0°, 1/n_1) = T(\psi = 0°, n_1) \equiv T(0)$ and $F(\psi = 0°, 1/n_1) = F(\psi = 0°, n_1) \equiv F(0)$ has been utilized. Note in Eq. (15) that the mean-square surface roughnesses of the two interfaces are additive in the argument of the exponential function. Hence, this allows the equation to also be interpreted as representing the absolute regular normal transmittance of a dielectric plate that is smooth on one face and rough on the other with the latter having an rms surface roughness equal in value to $(\sigma_1^2 + \sigma_2^2)^{1/2}$.

The additivity of the mean-square roughnesses in the exponentials of Eq. (15) also allows this equation to be expressed in a form which would represent the absolute regular normal transmittance of a dielectric plate that is equally rough on both faces with each interface having an effective rms surface roughness

$$\sigma_e \equiv [(\sigma_1^2 + \sigma_2^2)/2]^{1/2} \tag{16}$$

Equation (15) then may be written as

$$t_R = \frac{T^2(0) \exp -2[2\pi(n_1 - 1)(\sigma_e/\lambda)]^2 \exp -k_1 d}{1 - F^2(0) \exp -2[4\pi n_1(\sigma_e/\lambda)]^2 \exp -2k_1 d} \tag{17}$$

Equation (15) or (17) can be greatly simplified for this investigation by noting that the multiple reflections term in the denominator can be neglected relative to unity because

$$F^2(0) \exp -2[4\pi n_1(\sigma_e/\lambda)]^2 \exp -2k_1 d \leq F^2(0) \tag{18}$$

and

$$F^2(0) \leq [(n_1 - 1)/(n_1 + 1)]^4 \approx 1.6 \times 10^{-3} \tag{19}$$

for the refractive index values of interest (Ref. 16), $n_1 \approx 1.5$. Thus, Eq. (17) becomes

$$t_R = T^2(0) \exp -2[2\pi(n_1 - 1)(\sigma_e/\lambda)]^2 \exp -k_1 d \tag{20}$$

Equation (17) can be even further simplified by noting that $\exp(-k_1 d) \approx 1$ since k_1 for the dielectric plate is essentially negligible at all except the longest wavelengths of interest here. Hence, Eq. (17) finally simplifies to

$$t_R = T^2(0) \exp -2[2\pi(n_1 - 1)(\sigma_e/\lambda)]^2 \tag{21}$$

It should be noted that Eqs. (1 through 3, 5, 8 through 12, 15, 17, 20 and 21) are applicable only for the regular normal transmittance and do not account for any normal transmittance contribution attributable to radiation scattered into the normal transmission direction.

3.0 EXPERIMENTAL DATA FOR ROUGH GLASS INTERFACES

The relative regular normal transmittance data to be compared with the theoretical relationship in Eq. (8) were the results of a very novel experiment (Ref. 14) involving variation of interface refractive index ratio and rms roughness at a single wavelength. Glass with zero absorption and a known refractive index, $n_1 = 1.530$, for this wavelength, $\lambda = 0.546\ \mu$, was used as the dielectric. Figure 3 shows a schematic of the experimental surface system. The surface of the glass plate on which the radiation was incident from air was highly polished, whereas the surface on the opposite side of the plate was mechanically roughened. This rough side of the plate was bounded by a liquid which was introduced into the narrow gap existing between the rough surface and a highly polished cover glass slide. The radiation transmitted through this surface system was measured for various liquids having different known refractive indices and zero absorption at $\lambda = 0.546\ \mu$. Thus, the relative refractive index n_2/n_1 was varied over wide limits, whereas the surface height distribution function and the radiation wavelength were held constant. The refractive indices of the liquids, which consisted of aqueous solutions of glycerin, were n_2 = 1.333, 1.354, 1.364, 1.384, 1.396, 1.417, 1.432, 1.456 and 1.470. These measurements were repeated for glass plates of the same material having different interface roughnesses, including the special case of a highly polished interface. The measurements for the polished interface were used to normalize the data for the roughened interfaces. Abrasives with grain sizes of 10 μ (M-10), 14 μ (M-14), 20 μ (M-20), and 28 μ (M-28) were used to mechanically prepare the roughened interfaces. In all the measurements, the regular component of the normal transmitted radiation was separated from the scattered radiation contribution, and only the former was reported. Table 1 shows the measurement results for the four different roughened samples, M-10, M-14, M-20, and M-28.

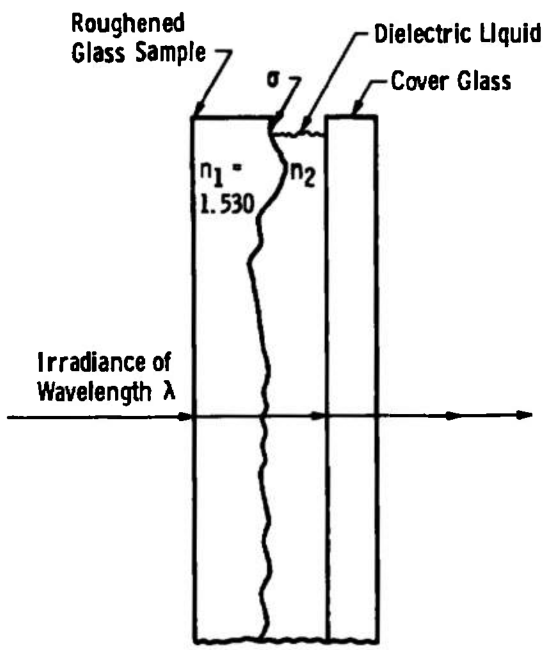

Figure 3. Schematic of experimental surface system for roughened glass samples.

Table 1. Experimental Regular Transmittance Results (Ref. 14), $\tau_R(\psi=0°,n_1,n_2,\sigma/\lambda)/T(\psi=0°,n_2/n_1)$, for Roughened Glass Interfaces, $n_1 = 1.530$, $\lambda = 0.546\ \mu$.

n_2	M-10	M-14	M-20	M-28
1.333	0.269	0.118	0.0306	0.0186
1.354	0.332	0.189	0.0670	0.0465
1.364	0.358	0.212	0.0816	0.0745
1.384	0.444	0.302	0.151	0.113
1.396	0.507	0.360	0.206	0.156
1.417	0.623	0.520	0.315	0.282
1.432	0.694	0.574	0.443	0.386
1.456	0.840	0.768	0.647	0.608
1.470	0.870	0.850	0.760	0.720

4.0 COMPARISON OF THEORY AND DATA

4.1 ROUGH GLASS INTERFACES

Figure 4 shows as a function of $[2\pi(n_1 - n_2)/\lambda]^2$ the graphical comparison of the theoretical transmittance in Eq. (8) with the data of Ref. 14 for the relative regular normal transmittance of the roughened glass interfaces. The different interface roughnesses are taken as parameters with the values of σ indicated on the graph determined from the square root of the negative slopes of the theoretical curves through the experimental data. The data for samples M-10, M-14, M-20, and M-28 are respectively represented by the circles, inverted triangles, squares, and upright triangles.

Figures 5 through 8 show for each of the respective glass interface roughnesses the computer plots of the least-mean-squares fit of the theory to the data. It is seen from each of these figures that there is very good agreement between the theoretical relation for the regular normal transmittance, Eq. (8), and the experimental data. Table 2 shows a numerical comparison of this transmittance theory and experimental data and gives an excellent illustration of just how good the agreement is.

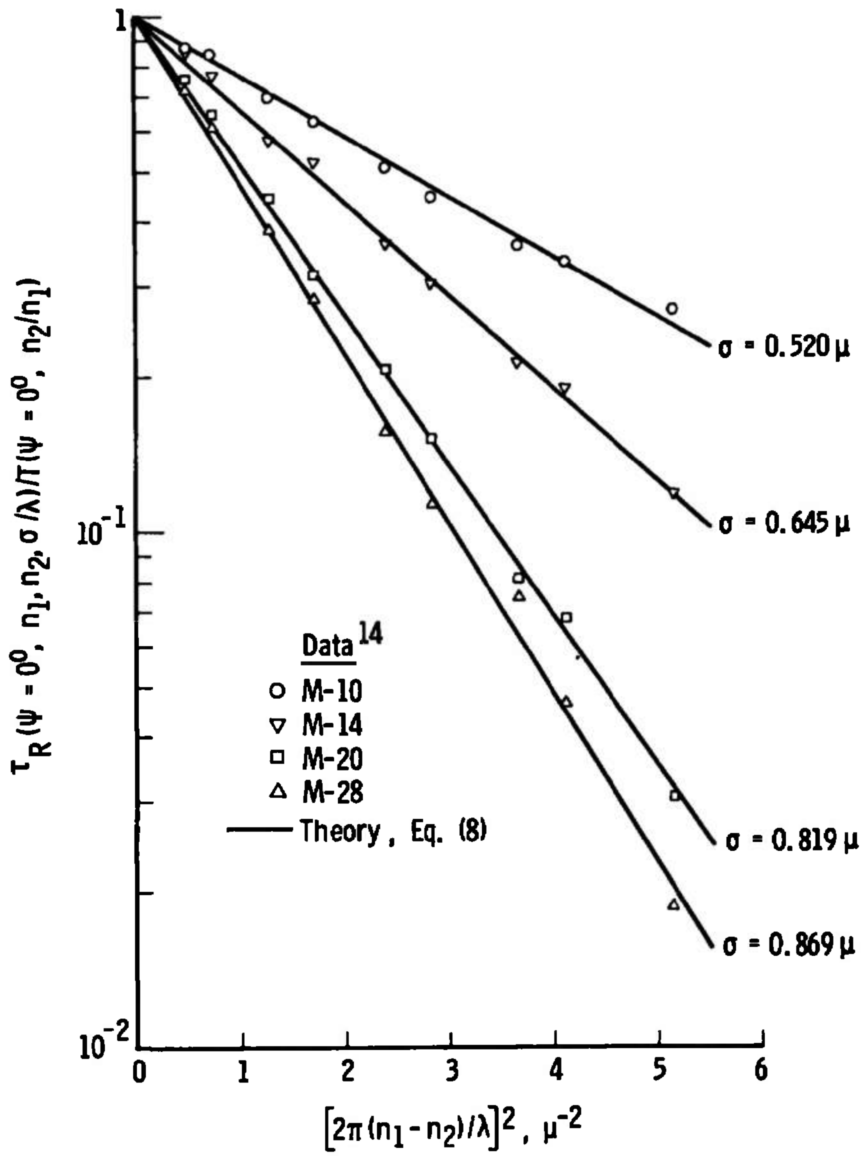

Figure 4. Graphical comparison of regular transmittance theory and data for different glass interface roughnesses.

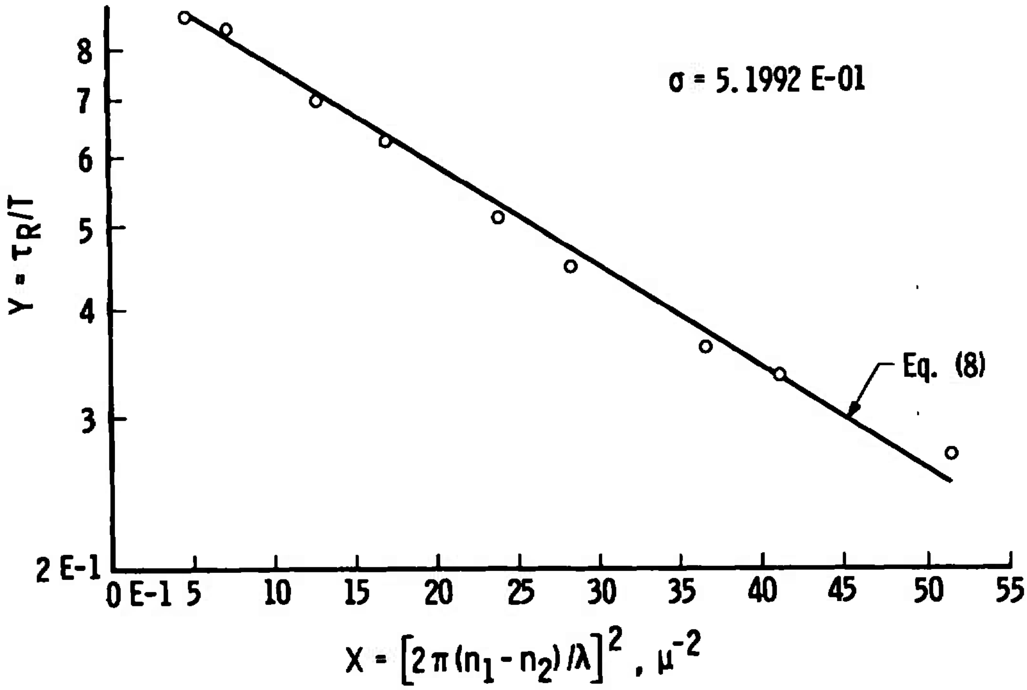

Figure 5. Computer comparison of regular transmittance theory and data (Ref. 14) for a glass interface roughness of 0.520 μ.

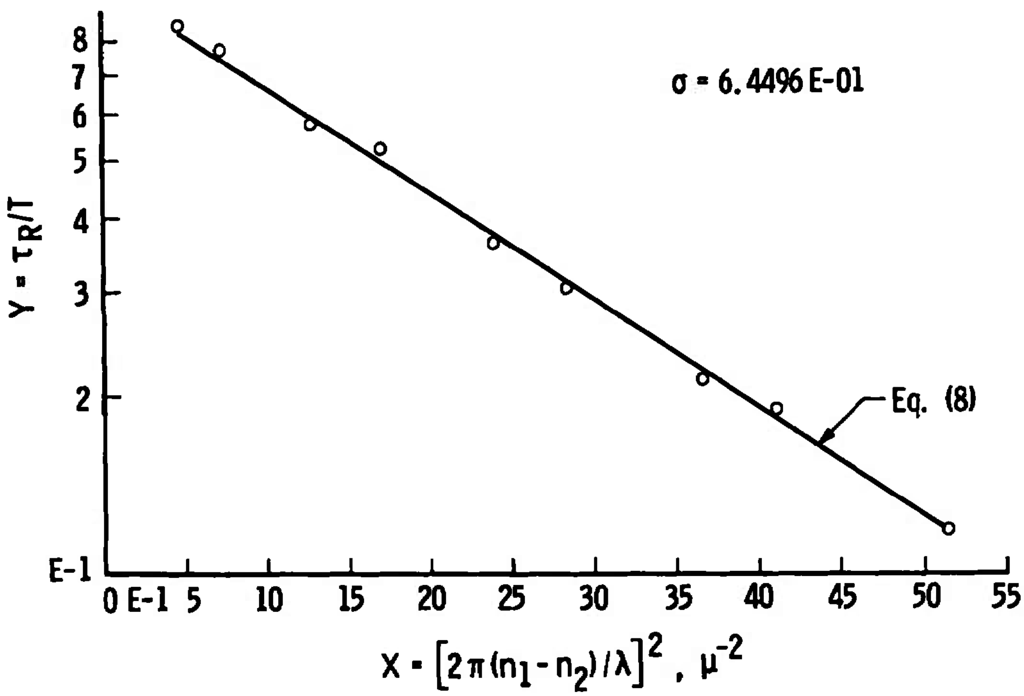

Figure 6. Computer comparison of regular transmittance theory and data (Ref. 14) for a glass interface roughness of 0.645 μ.

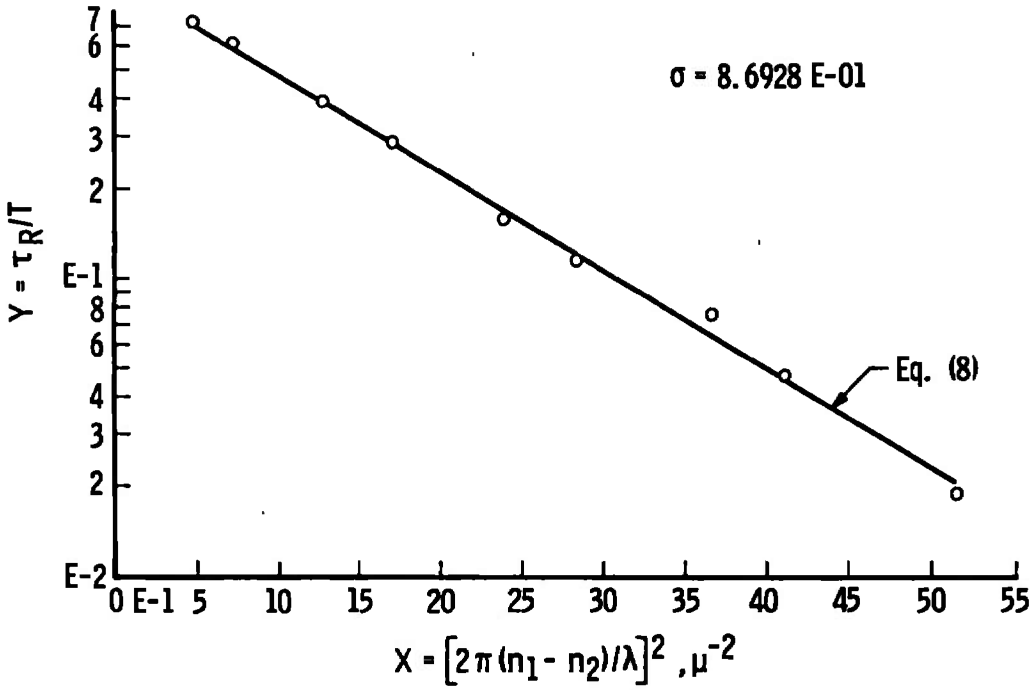

Figure 7. Computer comparison of regular transmittance theory and data (Ref. 14) for a glass interface roughness of 0.819 μ.

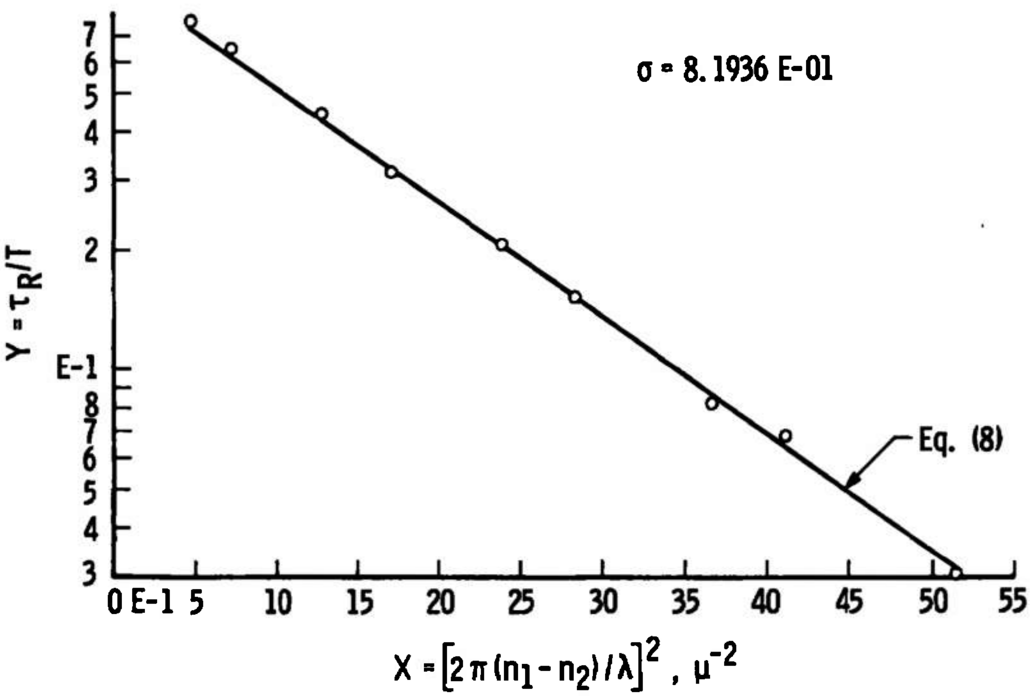

Figure 8. Computer comparison of regular transmittance theory and data (Ref. 14) for a glass interface roughness of 0.869 μ.

Table 2. Numerical Comparison of Regular Transmittance Theory, Eq. (8), and Data (Ref. 14), τ_R (ψ=0°, n_1,n_2, σ/λ)/T(ψ=0°,n_2/n_1) for Various Glass Interface Roughnesses.

$\left[\frac{2\pi(n_1-n_2)}{\lambda}\right]^2$, μ^{-2}	M-10 data	M-10 theory (σ=0.520 μ)	M-14 data	M-14 theory (σ=0.645 μ)	M-20 data	M-20 theory (σ=0.819 μ)	M-28 data	M-28 theory (σ=0.869 μ)
0.4767	0.870	0.879	0.850	0.820	0.760	0.726	0.720	0.698
0.7252	0.840	0.822	0.768	0.740	0.647	0.615	0.608	0.578
1.2713	0.694	0.709	0.574	0.589	0.443	0.426	0.386	0.383
1.6910	0.623	0.633	0.520	0.495	0.315	0.321	0.282	0.279
2.3778	0.507	0.526	0.360	0.372	0.206	0.203	0.156	0.166
2.8228	0.444	0.466	0.302	0.309	0.151	0.150	0.113	0.118
3.6492	0.358	0.373	0.212	0.219	0.0816	0.0863	0.0745	0.0634
4.1020	0.332	0.330	0.189	0.182	0.0679	0.0637	0.0465	0.0451
5.1395	0.269	0.249	0.118	0.118	0.0306	0.0317	0.0186	0.0206

Figure 9 shows as a function of $[2\pi\sigma(n_1 - n_2)/\lambda]^2$ the comparison of theory and data for the relative regular normal transmittance of all four glass interface roughnesses. As in Fig. 4, the data for samples M-10, M-14, M-20, and M-28 are respectively represented by circles, inverted triangles, squares, and upright triangles. It is seen that all the transmittance measurements are in excellent agreement with the single theoretical curve representing the regular normal transmittance of rough dielectric interfaces. Thus, the relative regular normal transmittances of the rough glass interfaces are an exponential function of the similarity variable $[2\pi\sigma(n_1 - n_2)/\lambda]^2$ which implies that the local surface height distributions were Gaussian.

4.2 ROUGH NaCl PLATES

Figure 10 shows as a function of $[2\pi(n_1 - 1)/\lambda]^2$ the absolute normal transmittance measurements performed by Gorton (Ref. 1) on plates of rock-salt crystal having both faces roughened. The experimental data for one sample is represented by the squares, whereas that for the other sample is represented by the circles. The effective surface roughness, σ_e, of the rough interfaces of a NaCl plate was determined by matching the theoretical regular transmittance relation in Eq. (21) to the linear portion of the experimental absolute transmittance data curve as plotted on semilog graph paper. For the NaCl plates, n_1 was taken from Ref. 16. The σ_e for the rough interfaces of one NaCl plate was 0.397 μm, that for the interfaces of the other was 0.448 μ.

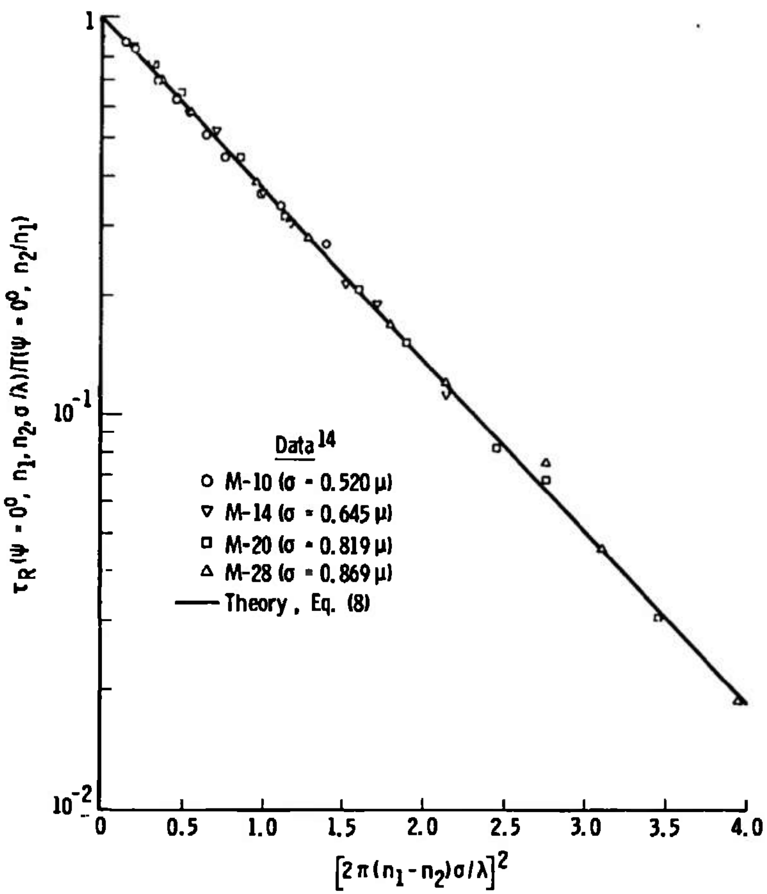

Figure 9. Graphical comparison of theory and data for the relative regular normal transmittance of rough glass interfaces.

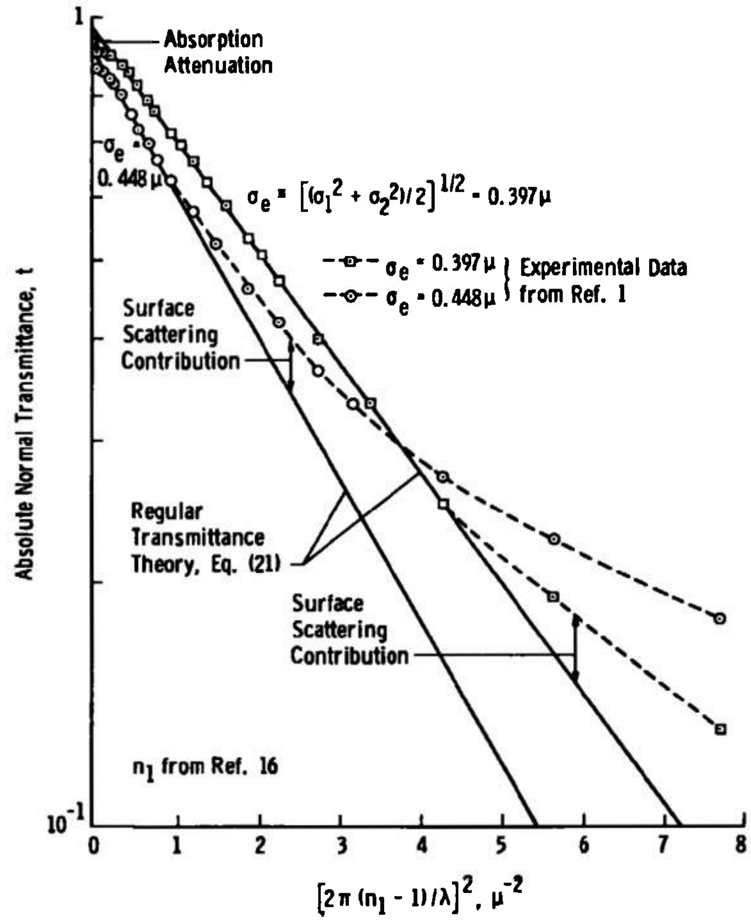

Figure 10. Comparison of theory and data for the absolute normal transmittance of roughened NaCl windows with effective surfaces roughness, σ_e, taken as a parameter.

It is seen from Fig. 10 that there is excellent agreement between the regular transmittance theory and the absolute transmittance data for the $\sigma_e = 0.397\ \mu$ sample except at very small and very large values of the argument $[2\pi(n_1 - 1)/\lambda]^2$. The deviation of the data from the theory at small values of $[2\pi(n_1 - 1)/\lambda]^2$ for the $\sigma_e = 0.397$-μ sample, and also the $\sigma_e = 0.448$-μ sample, is speculated to be a result of a small amount of absorption attenuation by the NaCl plates for wavelengths in the range of 8 to 13 μ. If such is the case, this deviation would not appear for relative transmittance measurements of the NaCl plates.

The fact that the absolute transmittance data for the $\sigma_e = 0.397$-μm sample in Fig. 10 is greater than the theoretical transmittance of Eq. (21) at very large values of $[2\pi(n_1 - 1)/\lambda]^2$ is probably a result of surface scattering at the rough interfaces of the NaCl plate. As noted earlier, Eq. (21) is applicable only for the regular normal transmittance and does not account for any normal transmittance contribution attributable to radiation scattered in the normal transmission direction. For the shorter wavelengths, the surface scattering contribution to the absolute normal transmittance of NaCl plates would be significant relative to the regular transmittance component. It appears from Fig. 10 that the surface scattering contribution to the absolute transmittance data is appreciable for the $\sigma_e = 0.397$-μ sample until $\lambda \geq 1.6\ \mu$. For the $\sigma_e = 0.448$-μ sample, the surface scattering contribution to the absolute normal transmittance is significant until $\lambda \geq 3.4\ \mu$. The surface scattering contribution to the absolute transmittance for the $\sigma_e = 0.448$-μ sample is appreciable at larger wavelengths than for the $\sigma_e = 0.397$-μ sample because its interfaces are rougher. For the same reason, the magnitude of the surface scattering contribution to the absolute transmittance at a given wavelength is much greater for the $\sigma_e = 0.448$-μ sample. Because of this fact, the absolute normal transmittance of the $\sigma_e = 0.448$-μ sample exceeds that of the $\sigma_e = 0.397$-μ sample for $\lambda \lesssim 1.7\ \mu$.

Figure 11 presents the absolute normal transmittance measurements for the roughened NaCl plates as a function of $[2\pi(n_1 - 1)(\sigma_1^2 + \sigma_2^2)^{1/2}/\lambda]^2$. As in Fig. 10, the squares and circles, respectively, represent the experimental data for the $\sigma_e = 0.397$-μ and 0.448-μ samples. It is observed in Fig. 11 that the regular transmittance theory of Eq. (21) agrees very well with the linear portion of the absolute transmittance data curve for each sample as plotted on semilog graph paper. However, the experimental data for the $\sigma_e = 0.397$-μ and 0.448-μ roughened NaCl plates do not collapse into a single curve in the linear region. It is speculated that this is a result of an error in the measurements for $\sigma_e = 0.397$-μ sample. The speculation is based on the fact that for a zero value of the argument $[2\pi(n_1 - 1)(\sigma_1^2 + \sigma_2^2)^{1/2}/\lambda]^2$, the theoretical regular transmittance curve through the experimental data intercepts the ordinate at a value which, from Eq. (21), represents the absolute transmittance of the NaCl plate with both interfaces smooth, i.e., $\sigma_e = 0$. For the $\sigma_e = 0.397$-μ sample, the value of the ordinate intercept is approximately 0.98, whereas for the $\sigma_e = 0.448$-μ sample it is about 0.92. This latter value is essentially equal to the absolute normal transmittance commonly measured for highly polished NaCl windows in the 1 to 13-μ wavelength range, 0.92 to 0.93. However, the value of 0.98 as determined for the $\sigma_e = 0.397$-μ sample is about 0.05 to 0.06 too high for the experimental transmittance of polished NaCl windows and also exceeds the theoretical transmittance of NaCl windows with Fresnel interfaces by about the same amount. Thus, it appears that the absolute transmittance measurements of Ref. 1 for the $\sigma_e = 0.397$-μ sample are in error by approximately +0.05.

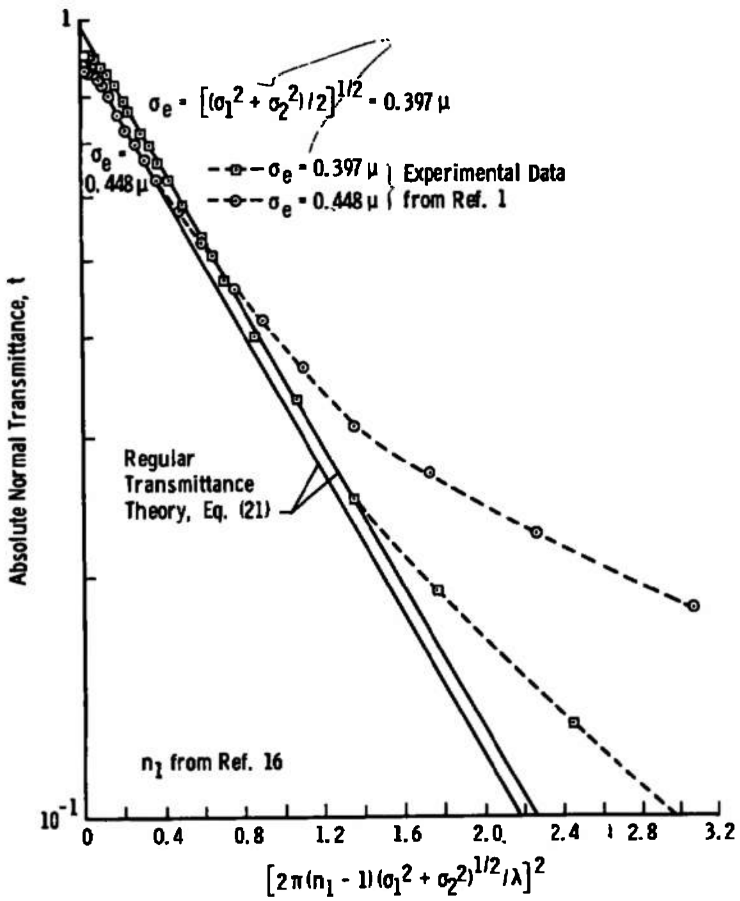

Figure 11. Comparison of theory and data for the absolute normal transmittance of roughened NaCl windows.

For each sample in Fig. 11, the deviation between the absolute normal transmittance data and the regular transmittance theory of Eq. (21) at very small as well as large values of the argument $[2\pi(n_1 - 1)(\sigma_1^2 + \sigma_2^2)^{1/2}/\lambda]^2$ is a result of the same effects described earlier in the discussion associated with Fig. 10. Of course, it should be noted that for a given value of $[2\pi(n_1 - 1)(\sigma_1^2 + \sigma_2^2)^{1/2}/\lambda]^2$, the surface scattering contribution for the σ_e = 0.448-μ sample in Fig. 11 is much greater than for the σ_e = 0.397-μ sample. This much larger surface scattering contribution for the σ_e = 0.448-μ sample at a given $[2\pi(n_1 - 1)(\sigma_1^2 + \sigma_2^2)^{1/2}/\lambda]^2$ should not be directly attributed to the fact that the interfaces of the surface scattering contribution have a higher rms surface roughness. Instead, this is probably a result of the interfaces having a greater rms slope than the interfaces of the σ_e = 0.397-μ sample.

5.0 CONCLUSIONS

From the excellent agreement exhibited between the theory and data presented in Figs. 4 through 9 and Table 2, it is concluded that the theoretical relationship for the relative normal regular transmittance of a rough dielectric interface, Eq. (8), is correct and can be used to correlate experimental transmission data for rough glass interfaces whose surface height distributions are Gaussian. From Figs. 10 and 11, it is concluded that the regular normal transmittance theory for roughened dielectric plates, Eq. (21), is in good agreement with absolute spectral transmission measurements for roughened NaCl windows at wavelenghts where absorption and surface scattering effects are negligible.

REFERENCES

1. Gorton, A. F. "Reflection From, and Transmission Through, Rough Surfaces." Physical Review, Vol. VII, Ser. 2, 1916, pp. 66-78.

2. Chinmayanandam, T. K. "On the Specular Reflection from Rough Surfaces." Physical Review, Vol. XIII, Ser. 2, 1919, pp. 96-101.

3. Davies, H. "The Reflection of Electromagnetic Waves from a Rough Surface." Proceedings of the Institution of Electrical Engineers, Vol. 101, 1954, pp. 209-214.

4. Bennett, H. E. and Porteus, J. O. "Relation Between Surface Roughness and Specular Reflectance at Normal Incidence." Journal of the Optical Society of America, Vol. 51, 1961, pp. 123-129.

5. Beckmann, P. and Spizzichino, A. The Scattering of Electromagnetic Waves from Rough Surfaces. Macmillan, New York, 1963.

6. Bennett, H. E. "Specular Reflectance of Aluminized Ground Glass and the Height Distribution of Surface Irregularities." Journal of the Optical Society of America, Vol. 53, No. 12, December 1963, pp. 1389-1394.

7. Birkebak, R. C. and Eckert, E. R. G. "Effects of Roughness of Metal Surfaces on Angular Distribution of Monochromatic Reflected Radiation." Journal of Heat Transfer, Vol. 87, Series C, 1965, pp. 85-94.

8. Torrance, K. E. and Sparrow, E. M. "Biangular Reflectance of an Electric Nonconductor as a Function of Wavelength and Surface Roughness." Journal of Heat Transfer, Vol. 87, Series C, 1965, pp. 283-292.

9. Love, T. J. and Francis, R. E. "Experimental Determination of Reflectance Function for Type 302 Stainless Steel." AIAA Progress in Astronautics and Aeronautics: Thermophysics of Spacecraft and Planetary Bodies, Vol. 20, edited by G. B. Heller, Academic Press, New York, 1967, pp. 115-135.

10. Houchens, A. F., and Hering, R. G. "Bidirectional Reflectance of Rough Metal Surfaces." AIAA Progress in Astronautics and Aeronautics: Thermophysics of Spacecraft and Planetary Bodies. Vol. 20, Edited by G. B. Heller, Academic Press, New York, 1967, pp. 65-89.

11. Hensler, D. H. "Light Scattering from Fused Polycrystalline Aluminum Oxide Surfaces." Applied Optics, Vol. 11, No. 11, 1972, pp. 2522-2528.

12. Smith, T. F. and Hering, R. G. "Surface Roughness Effects on Bidirectional Reflectance." AIAA Paper 73-152, AIAA 11th Aerospace Sciences Meeting, Washington, D. C., January 10-12, 1973.

13. Smith, A. M. and Müller, P. R. "Anomalous Refraction Maxima in the Bidirectional Transmittances of Roughened Dielectric Surfaces." AIAA Progress in Astronautics and Aeronautics: Thermal Control and Radiation, Vol. 31, edited by C. L. Tien, MIT Press, Cambridge, Mass., 1973, pp. 419-433.

14. Polyanskii, V. K. and Kotlyavova, I. B. "Cooperative Effects on Rough Surface in Transmitted Radiation." Optics and Spectroscopy, Vol. 30, No. 6, June 1971, pp. 609-611.

15. Francis, John E. and Love, Tom J. "Effect of Optical Thickness on the Directional Transmittance and Emittance of a Dielectric." Journal of the Optical Society of America, Vol. 56, No. 6, June 1966, pp. 779-782.

16. Ballard, S. S., Browder, J. S., and Ebersole, J. F. "Refractive Index of Special Crystals and Certain Glasses." American Institute of Physics Handbook, Ed. 3, edited by Dwight E. Gray, McGraw-Hill, New York, Sec. 6-b, p. 6-43.

NOMENCLATURE

A	Projection of the area of all the illuminated rough surface elements onto the mean plane
d	Geometrical thickness of the plate
$dA(\zeta)$	Projected aggregate area of surface elements lying in the differential layer between ζ and $\zeta + d\zeta$
$d\zeta$	Differential layer
E_o	Amplitude that would be obtained when the interface is smooth
E_R	Amplitude of beam regularly transmitted through the rough interface
$I_{t,o}(0)$	Radiant intensity transmitted through the smooth surface
$I_{t,R}(0)$	Radiant intensity regularly transmitted through the rough surface
k_1	Absorption coefficient
n_1, n_2	Refractive indices (relative to air) of the media on opposite sides of the interface

T	Regular normal transmittance for a smooth interface
t_r	Absolute regular normal transmittance
$w(\zeta)$	Distribution function
$\zeta(x)$	Surface height
λ	Radiation wavelength in air
λ_e	Effective wavelength
ρ_R	Reflectance
σ	Root-mean-square value of the surface heights
τ_R	Regular normal transmittance of the rough dielectric interface sketched in Fig. 1
ψ	Zenith incidence angle of the irradiance