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# TECHNIQUES FOR THE ASSESSMENT OF WORTH

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and

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SYSTEMS INTEGRATION & COMMAND/CONTROL TECHNICAL AREA



U. S. Army

Research Institute for the Behavioral and Social Sciences

August 1974

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J. E. UHLANER  
Technical Director

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Subjective worth forms the basis for the selection of future alternatives and the evaluation of past actions. During the past few years several formal methods of worth assessment have been developed. These methods provide an explicit basis for incorporating military judgment into the assessment process in a manner designed to produce consistent and valid assessment. Seven general methods for worth assessment are reviewed. Step-by-step procedures, a specific example, and variations of each method are provided. The description of each method includes training and equipment requirements, possible problems, type of results, the		

20. advantages and disadvantages, methods for checking accuracy, and sources of information. The methods cover four areas of assessment: those which yield worth on an ordinal scale, those which yield numerical values directly, the gamble methods involving probabilities, and the multivariate methods for dealing with multiple or dependent factors.



# TECHNIQUES FOR THE ASSESSMENT OF WORTH

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## FOREWORD

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The Systems Integration & Command/Control Technical Area of the U. S. Army Research Institute for the Behavioral and Social Sciences (ARI) is concerned with human information analysis and subsequent product utilization in intelligence systems. The objective is to provide both technological advances in human/machine aided tactical intelligence information processing and the translation of these advances in support of intelligence systems, design decisions, and the formulation of doctrine and procedures. The entire research effort is responsive to requirements of RDTE Project 20162101A754, "Intelligence Information Processing," FY 1974 Work Program and to special requirements of the U. S. Army Combat Arms Development Agency and the Intelligence Center and School.

Achievement of the required technology is often inhibited by a lack of understanding of fundamental principles. Where this is true, the requirement is to increase the scientific basis underlying the state of the art. The present publication provides a review and analysis of one such area--the assessment of subjective value or worth. A functional assessment technology is needed to support efforts such as determining the value of intelligence data and improving intelligence collection procedures.

ARI research in this area is conducted as an in-house research effort augmented by contracts with organizations selected as having unique capabilities and facilities for research in a specific area. The present study was conducted jointly by personnel of the Army Research Institute and the Industrial Engineering Department of the University of Wisconsin at Madison.



J. E. UHLANER,  
Technical Director

# TECHNIQUES FOR THE ASSESSMENT OF WORTH

## BRIEF

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### Requirement:

To analyze and evaluate methods for translating qualitative subjective impressions of worth into consistent, meaningful, and uniformly understandable quantitative evaluations. Worth assessment is a vital ingredient of advanced intelligence information processing methods.

### Procedure:

Worth assessment involves the entire process of identifying, measuring, and combining factors to create a conscious, articulated worth structure as a basis for making decisions. The mathematical theory of worth assessment has been well established and a large number of models and methods have been developed. This study reviews, classifies, and presents in usable form the important methods for measuring worth.

Assessment methods were classified according to their major characteristics--use of probability, type of judgment required, number and type of factors involved, type of output needed. Four classes of methods were differentiated: the ordinal, which yields an ordinal preference scale; the direct, which yields numerical preferences; the gamble, using probability; and the multivariate, for multiple or dependent factors. Within these four classes, seven general methods were reviewed in detail; a specific example of each was given, with step-by-step procedures and possible variations, training and equipment needed, advantages and disadvantages, possible pitfalls, type of results, ways of checking accuracy, and sources of further information.

### Findings:

No single method has been found to be the best for every situation; in different situations different methods are optimally effective, depending on the requirements both of the problem and of the decision maker. The assessment method chosen must meet the requirements of the specific situation. A summary table comparing a variety of dimensions of the seven general methods is presented as an aid for making such a choice.

### Utilization of Findings:

Methods of assessing subjective worth provide explicit procedures for incorporating military judgment into formal quantitative expressions of worth. For example, in establishing information acquisition priorities and managing collection assets, worth assessment procedures can provide consistent and easily communicated quantitative evaluations. This data then can be used with both existing and advanced methods of collection management to improve the responsiveness of the intelligence system.

# TECHNIQUES FOR THE ASSESSMENT OF WORTH

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# TECHNIQUES FOR THE ASSESSMENT OF WORTH

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## INTRODUCTION

Subjective worth forms the basis for the selection of future alternatives and the evaluation of the past actions. Each time an automobile is purchased, a house is bought, a course of action taken, subjective worth is used as a basis. It is the intuitive feeling by which one differentiates between a "good deal" and a "poor deal" when selecting an automobile, or by which the pros and cons of a prospective course of action are incorporated into a single evaluation of its worth.

Techniques for measuring subjective worth have been utilized for many years and extend back to the eighteenth century. In the last two or three decades there has been an explosive growth in the interest and research concerned with the theory and assessment of subjective worth. This growth has been stimulated by the use of the axiomatic method in developing axioms of preference or worth which guarantee, in a formal mathematical sense, the ability to assess subjective worth numerically.<sup>1</sup> Concurrent with the development of theory a number of procedures for assessing subjective worth have been developed. Although many of these procedures have been developed directly from theories of preference and worth (e.g., the gamble methods), many are direct extensions of classical psychophysical methods.<sup>2,3</sup> With the exception of a comprehensive but short review by Fishburn,<sup>4</sup> there has been no attempt to review the many procedures now available for assessing subjective worth. This paper provides such a review. The review focuses on how to measure worth rather than on the theory underlying worth assessment or models for combining and synthesizing component values in a single worth for an object or event.

Worth refers to the subjective or intuitive values with which a person evaluates an object or course of action. People appear to evaluate objects or courses of action on the basis of numerous criteria, many of which are not quantifiable in objective terms. Criteria such as "importance" or "undesirability" are related to, yet quite different from, the "real world objective" attributes usually associated with the objects or actions (e.g., cost, speed, color, size). These subjective criteria are combined by

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<sup>1</sup> Fishburn, P. Utility theory for decision making. New York: Wiley, 1970.

<sup>2</sup> Guilford, J. P. Psychometric methods. New York: McGraw-Hill, 1954.

<sup>3</sup> Torgerson, W. S. Theory and methods of scaling. New York: Wiley, 1958.

<sup>4</sup> Fishburn, P. Methods of estimating additive utilities. Management Science, 1967, 13, 435-453.

the decision maker into an intuitive value scale by which the worth of objects or actions are evaluated. This intuitive process is sometimes referred to as military judgment, engineering judgment, or experience. Worth assessment refers to the process of transforming and measuring these subjective scale values onto an objective, real world scale.<sup>5,6</sup>

Worth assessment serves several purposes. First, in conjunction with appropriate models,<sup>7</sup> it can be used to predict the decision behavior of individuals. Consumer demand for a product will depend on the consumer's value for that product. A decision-maker's demand for information will depend on the value he places on the information. Worth assessment can aid in prescribing decisions. When the number of decision alternatives becomes exceedingly large and cumbersome, techniques can be used to decompose the problem into manageable portions, assess the worth of these smaller problems, and then recombine these worths to prescribe that course of action which maximizes total worth.<sup>8</sup>

Worth assessment is also a method of communicating individual or group values. This can be especially useful in communicating policy from superiors to subordinates. Summers, Taliaferro and Fletcher,<sup>9</sup> in a study involving subjects learning another person's (target's) policy, "found that information afforded by a mathematical analysis of the target's policy was substantially more beneficial to the learner than was information provided by the target himself." In addition to this type of individual communication, worth assessment can be used when a group worth function is needed. Since people characteristically vary tremendously in their use of superlatives and adjectives, analytical worth assessment can provide a common medium of communication, more homogeneous than verbal description.

The procedures of worth assessment are the focus of this paper. The following section provides a brief overview of the worth assessment process. A system for classifying the methods is provided in the third section.

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<sup>5</sup> Stevens, S. S. A metric for the social consensus. Science, 1966, 151, 530-541.

<sup>6</sup> Stevens, S. S. Issues in psychophysical measurement. Psychological Review, 1971, 78, 426-450.

<sup>7</sup> E.g., Fishburn, P. Decision and value theory. New York: Wiley, 1964.

<sup>8</sup> Huber, G. Multi-attribute utility models: A review of field and field-like studies. Management Science, 1974, in press.

<sup>9</sup> Summers, D. A., J. D. Taliaferro, and D. J. Fletcher. Judgment policy and interpersonal learning. Behavioral Science, 1970, 15, 514-521.



In the fourth section, an example problem is given which will subsequently be used to illustrate each of the techniques of worth assessment--the ordinal, direct, gamble, and multivariate methods which are discussed in the next four sections. The final section is a brief summary. It should be noted that although some of the material presented in the fifth through eighth sections is new, most of the paper is tutorial in nature.

### THE WORTH ASSESSMENT PROCESS

The assessment of worth involves the entire process of identifying, measuring, and combining attributes to create a conscious, well-defined, easily articulated worth structure which can form the basis for evaluation and for decision. The worth assessment process has the following general steps:

1. Identify "worth to whom," the individual or organization whose subjective worth is to be assessed.
2. Determine the scope of the problem and identify the objectives, purposes, or uses of the objects or events whose worths are to be assessed.
3. Identify the set of alternatives or options to be evaluated.
4. Determine the relevant dimensions of value (or the attributes) of the objects or events to be assessed.
5. Develop physical measures for each attribute or factor.
6. Choose an appropriate technique for assessing the worth of each attribute or factor.
7. Assess the value of each alternative on each dimension of value.
8. Choose an assessment model.
9. Evaluate each alternative using this model.
10. Select the "best" alternative.

It is not the intent to describe these steps in detail or to analyze in general the decision process; rather, the intent is to provide the context within which worth assessment is conducted.

The first step in worth assessment is to determine whose value judgments are to be measured. The outcome of an assessment will be critically dependent upon that determination. Although an individual may be asked to make judgment from the point of view of another clearly defined person, the result will be inextricably confounded with the values of the individual whose judgments are used.<sup>10</sup> For example, in selecting an automobile, the worth assigned to different alternatives will depend upon whether the subjective worth of a salesman, a mechanic, or a prospective buyer is assessed. Worth measured in terms of an organization's values is often beset with similar difficulties. The judgments of different individuals will reflect different points of view and different values. In large organizations it may even be difficult to identify the decision makers whose values are sought.

The worth of an object, activity, or alternative is related to the extent to which such is perceived as satisfying clearly stated objectives. An alternative may have several objectives. For example, in selecting an automobile, subjective worth may be based on a consideration of its use for business, pleasure, and commutation. The purpose influences the resulting worths. A sleek sports car may have a greater subjective worth than a sedate sedan when evaluated as a pleasure vehicle; however, the sedan may have a higher worth as a business vehicle. A clear statement of objectives or purposes is necessary to establish the basis for worth assessment.<sup>11</sup>

Often the goals may be uncertain.<sup>12</sup> The decision maker may be unable to communicate his explicit goals; he may only mention some of them, if he feels that publicly divulging others may involve ridicule; or he may not be aware of his goals or of which goals are applicable in a given situation. This is a pervasive problem. If the criterion with respect to which worth is to be assessed is unclear, the results of any assessment procedure will likewise be unclear.

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<sup>10</sup> Ginsberg, A. Decision analysis in clinical patient management with application to the pleural-effusion syndrome. Rand Corp. Memo R-751-RC/NLM, July 1971.

<sup>11</sup> Pardee, F. S., C. T. Phillips, and K. Smith. Measurement and evaluation of alternative regional transportation mixes: Vol II, Methodology. Rand Corp. Report RM-6324-DOT, August 1970.

<sup>12</sup> Miller, J. R., III. A systematic procedure for assessing the worth of complex alternatives. USAF ESD-TR-67-90, November 1967. (AD 662 001)



The third step is to identify the complete set of alternatives or options to be assessed. In most contexts these will be obvious. However, care should be taken that all the feasible alternatives are identified and that the alternatives identified are in fact feasible.

The first three steps in the worth assessment process define the scope of the assessment procedure. They answer the questions: Whose worth? For what purpose? Of what objects or activities? The fourth step is to identify the relevant attributes or dimensions of value of the alternatives to be assessed, to decompose the alternatives into a set of critical dimensions that influence the worth of the alternatives. Dimensions of worth are the attributes, factors, or features of the plausible alternatives that are desired or valued by the decision maker; they are the dimensions of performance of the alternatives in satisfying the objective.<sup>13</sup> It should be noted that a complex problem can be decomposed in more than one way. Figure 1 illustrates one set of attributes which might be used in assessing the worth of an automobile. The question of which of the various possible decompositions is more natural, easier to obtain, and leads to the best results cannot be settled on a priori ground. However, some desirable properties of a set of attributes can be identified.

First, the set should be complete and exhaustive. All of the attributes relevant to the final decision should be represented by items on the list. This guarantees that no important dimensions of value are overlooked in the assessment procedure.

Second, all of the attributes should be within the experience and comprehension of the decision maker. Individuals are unable to consider more than a few value dimensions at a time and thus may ignore potentially relevant information. However, if the decomposition is too fine-grained or care is not taken, the simpler dimensions of value may be incomprehensible to the decision maker. In assessing the worth of an automobile, power as measured by rated horsepower would be meaningful to most individuals, whereas such attributes as brake horsepower and thermal efficiency would be incomprehensible.

Third, the dimensions of value should generally be mutually exclusive and independent. That is, each attribute should be stated in such a way as not to include any other attribute, either in whole or in part, and the worth of an attribute at any level should not vary as a function of

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<sup>13</sup> The terms attribute and factor are used interchangeably in this report to refer to a value dimension.

ATTRIBUTEMEASURE

Cost

Dollars

Model Type

Two-door Hardtop  
Two-door Coupe  
Four-door Hardtop  
Station Wagon  
Convertible

Power

Horsepower

Efficiency

Miles per Gallon

Cost of Repairs

Relative Consumer Index

Reliability

Years Between Failures (%)

Availability of Accessories

Number Possible

Size

Square Feet

Serviceability

Number of Dealers in Area

Handling

Consumer Index

Workmanship

Subjective Evaluation

Warranty

Coverage

Attractiveness

Subjective Evaluation

Figure 1. Automobile attributes

the levels of the other attributes. This permits the use of additive assessment models<sup>14</sup> and reduces the number of attributes to be evaluated. Note that although additive assessment models are the most commonly used, other techniques are available.<sup>15, 16, 17</sup>

There are no formal algorithms to aid in defining the set of value dimensions. Several quasi-mathematical approaches exist for identifying a set of dimensions for a psychological space (e.g., factor analysis, multidimensional scaling). However, these are cumbersome and time consuming; in addition, they usually require some sort of overall judgment of similarity or worth and thus are not well suited for the worth assessment problem. The identification of value dimensions is a critical step and one to which considerable time and effort should be devoted in any worth assessment problem.<sup>18</sup>

Once an acceptable list of relevant attributes has been constructed, the fifth step in the procedure is to develop a measure for each attribute or dimension. The term "physical measure" is used to refer to any tangible reading or concrete observation that can be extracted from the real world. It is any unit of measurement which can adequately reflect the plausible range of values of a given attribute. This measure serves to link the subjective worths of the decision maker to the set of alternatives being evaluated.<sup>19</sup> Note that the level of measurement may sometimes be only nominal or the identity of different values of the attribute, for example, model type of an automobile (Figure 1).

It should be noted that the use of decomposed worth assessment procedures has been implicit in this discussion of dimensions of value--that is,

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<sup>14</sup> Fishburn, 1970, op. cit.

<sup>15</sup> Fischer, G. W. Multi-dimensional value assessment for decision making. University of Michigan, Engineering Psychology Laboratory Technical Report 037230-2-T, June 1972. (a)

<sup>16</sup> Fishburn, 1970, op. cit.

<sup>17</sup> Huber, 1974, op. cit.

<sup>18</sup> Cf. O'Connor, M. F. The assessment of worth for multi-attributed alternatives: A survey. Unpublished report, University of Michigan. 1972.

<sup>19</sup> Miller, J. R. Assessing alternative transportation systems. Rand Corp. Report RM5865-DOT, April 1969. (PB 185 167)



procedures in which the worth of each attribute is evaluated or assessed separately and then synthesized into a global assessment using mathematical techniques.<sup>20</sup> However, intuitive assessment procedures can be used, in which the synthesis of attribute information into a global worth value is entirely subjective. In both mathematical and intuitive synthesis of value information, it is essential that agreement be reached on the dimensions of value and their associated measures for the set of alternatives to be evaluated. This serves to make explicit the bases of the subjective worth judgments.

The next steps in worth assessment are to select a technique of worth assessment and to assess attribute worths. A summary of factors to be considered in selecting a method for worth assessment is presented following the detailed explanation of each of the methods. All of the methods for assessing worth depend on the decision maker being able to delve into his feelings and somehow order or quantify these feelings. However, these subjective impressions are not likely to be stable over time and are dependent upon the short-term and long-term states of the individual decision maker--his motivation, job stress, time pressure, etc.<sup>21</sup> The decision analyst or researcher himself may induce problems by exerting social pressure, consciously or unconsciously,<sup>22,23</sup> or by neglecting attributes he feels are not critical.

The eighth step in the worth assessment process is the selection of an assessment model. In many cases this will be an additive assessment model, as it is usually found that worth is insensitive to the algebraic form of the model.<sup>24</sup> However, other models are available (e.g., multiplicative models). These assessment models generally synthesize the component values into a single worth for each alternative, but this is not always necessary or even desirable. Thus, there are several classes of models such as lexicography in which the assessment of an alternative is based directly on the component values.<sup>25,26</sup> The mathematics of assessment models

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<sup>20</sup> Cf. Fischer, 1972a, op. cit.; Fishburn, 1970, op. cit.

<sup>21</sup> Fishburn, 1964, op. cit.

<sup>22</sup> Suchman, F. Evaluative research. New York: Russell Sage Foundation, 1967.

<sup>23</sup> Zaleska, M. and N. Kogan. Level of risk selected by individuals and groups when deciding for self and for others. Sociometry, 1971, 34, 198-213.

<sup>24</sup> Fischer, 1972a, op. cit.

<sup>25</sup> Cf. Fishburn, 1970, op. cit.

<sup>26</sup> MacCrimmon, K. R. Decision making among multi-attribute alternatives: A survey and consolidated approach. Rand Corp. Memo RM-4823-ARPA, December 1968.

are well developed. The question of when the models will work is a procedural one, and a review of these procedures is outside the scope of this report.<sup>27</sup>

Evaluation of the worth of each alternative and selection of the "best" alternative are the final steps in the worth assessment process. These depend on the specific assessment model selected and the objectives of the assessment. The result is an evaluation of the worth of alternatives and an explicit basis for decision.

To summarize, the steps in the process of worth assessment are designed to capture the essence of a problem and to attain some reasonable degree of measurability. The judgments asked of a decision maker are designed to reflect the context of the real-world decision. Hypothetical value judgments will yield hypothetical worth assessments and invalidate the entire decision-making process.<sup>28, 29</sup>

A final issue which must be addressed is the validity of worth assessment. Worth judgments cannot be confirmed by empirical test. There is neither a right nor a wrong worth function; worth exists in the minds of men to be accepted or rejected by other men. The only external evaluation of the results of worth assessment is informed opinion. The internal consistency of worth judgments and the accurate prediction of decisions<sup>30</sup> indicate that the methods of worth assessment are descriptive of human performance, and consistency can be evaluated using the checking techniques discussed with each method. However, there are no criteria or standards for evaluating the validity of the worth judgments themselves.

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<sup>27</sup> Cf. Fischer, 1972a, op. cit.; Fishburn, 1970, op. cit.; Huber, 1974, op. cit.

<sup>28</sup> Fishburn, 1964, op. cit.

<sup>29</sup> Slovic, P. and S. Lichtenstein. Relative importance of probabilities and payoffs in risk taking. Journal of Experimental Psychology Monograph, 1968, 78 (No. 3, Part 2).

<sup>30</sup> E.g. Hurst, P. and S. Siegel. Prediction of decisions from a higher ordered metric scale of utility. Journal of Experimental Psychology, 1956, 52, 138-144.



## THE CLASSIFICATION SYSTEM

In order to facilitate a comparison of the characteristics, inputs, processes, and output of the methods, each worth assessment method has been classified on the basis of the five attributes shown in Figure 2. This classification is a modification of that used by Fishburn.<sup>31</sup>

The first attribute describes the extent to which the decision maker must understand the concept of probability. Methods which use probability are generally based on the classical axioms of utility.<sup>32</sup> Methods which do not use probability are generally not based on a set of axiomatics of choice, but rather on extensions of the classical techniques of psychophysics.<sup>33</sup>

The second attribute describes the character of the response expected from the decision maker to questions by the analyst. Preference judgments (e.g., "I prefer #10 to #1.") are relatively easy to make; indifference judgments (e.g., "I am indifferent between receiving #2 and #10.") are slightly more demanding; while quantitative judgments (e.g., "Receiving two apples is four times better than receiving one orange.") are generally the most difficult.<sup>34</sup>

The third attribute describes the number of factors involved in a worth judgment. In this paper, a factor, attribute, or factor level refers to a dimension or specific point on a dimension of the object or event whose worth is being assessed. This attribute describes roughly how many different factor dimensions the decision maker must simultaneously consider when making a judgment. In general, multiple factors increase the complexity of the worth assessment task.

Attribute four simply denotes whether a factor can be described as discrete (e.g., number of rooms in a house) or continuous (velocity of an airplane). For many factors, the distinction between discrete and continuous becomes vague. For example, the cost of a house might be viewed as either a discrete or continuous variable.

Finally, the fifth attribute in the classification system describes the output or the product of the method. The output may only be a ranking of the attributes (e.g., two apples are preferred to one orange which is preferred to one apple). Approximated numerical worth describes those

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<sup>31</sup> Fishburn, 1967, op. cit.

<sup>32</sup> Von Neumann, J. and O. Morgenstern. Theory of games and economic behavior. New York: Wiley, 1944.

<sup>33</sup> Torgerson, 1958, op. cit.

<sup>34</sup> Cf: Torgerson, 1958, op. cit.; Guilford, 1954, op. cit.

## A/B/C/D/E

- A: N: PROBABILITIES ARE NOT USED  
P: PROBABILITIES ARE USED  
P 1/2: PROBABILITIES OF ONE HALF ONLY ARE USED
- B: C: DEPENDS UPON PREFERENCE JUDGEMENTS  
I: DEPENDS UPON INDIFFERENCE JUDGEMENTS  
Q: DEPENDS UPON QUANTITATIVE JUDGEMENTS
- C: NUMBER OF FACTORS INVOLVED IN ANY ONE JUDGEMENT
- D: C: BEST WITH CONTINUOUS FACTORS  
D: BEST WITH DISCRETE FACTORS  
E: USABLE WITH EITHER CONTINUOUS OR DISCRETE FACTORS
- E: R: OUTPUT IS RANKING OF WORTH  
A: OUTPUT IS APPROXIMATED NUMERICAL WORTH  
B: OUTPUT IS BOUNDED NUMERICAL WORTH  
N: OUTPUT IS DIRECT NUMERICAL WORTH

Figure 2. Worth assessment classification system

outputs where numbers are assigned to attributes on a somewhat arbitrary basis which reflects at least their preference ordering. Equal intervals is one such technique. In the above example, for instance, although two apples, one orange, and one apple lie on some worth continuum, with equal intervals it is assumed that they lie equidistant from each other (Figure 3). The numbers assigned to their respective worths will reflect this assumption. As shown in Figure 3, the numbers 10, 20 and 30 might be assigned these attributes as a measure of their respective worths. Bounded numerical worth refers to the assignment of ranges to the worth of an attribute. The output is in the form of two numbers, one representing the lowest value and the other the highest value of worth.

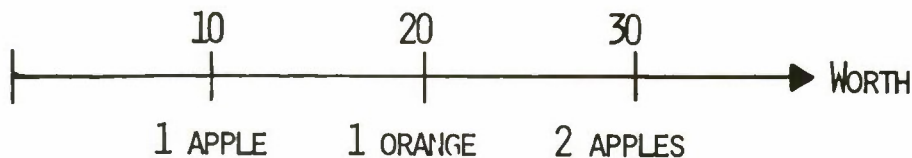


Figure 3. A scale of worth

Continuing the previous example, the value of one apple might be given as 7-16, one orange as 19-22, and two apples as 22-35. These bounds give an indication of the extreme points that the worth of an attribute may take. Lastly, direct numerical worth refers to worth indices generated directly from the decision maker's numerical judgments. For instance, in the example, if the decision maker states that one orange is 1.5 times better than one apple and that two apples are 2 times better than one orange, and if we make the worth of one apple equal to 10, then one orange will be worth 15 and two apples worth 30: a simple algebraic manipulation.

#### EXAMPLE PROBLEM

Each of the worth assessment techniques will be illustrated with an example, consistent from technique to technique. The example takes the form of assisting an individual (the decision maker) with purchasing an automobile. He knows that he wants to buy an automobile, but because of the many factors or attributes involved, he is unable to make a choice. He has come to the worth assessment researcher for assistance. The task of the researcher will be to help the decision maker assess his worths so that the possible purchases can be evaluated. Although in many situations the decision maker and worth assessment researcher will be the same person, the two roles have been separated to help clarify the two different perspectives. The decision maker can often assist himself by using the same techniques.



The important attributes involved in the decision, according to the decision maker, have been ascertained, and measures of each specified (Figure 1). For simplicity, the first six attributes will be used in assessment method examples. Cost will be assumed to be any dollar amount between zero dollars and \$4000, the upper limit having been specified by the decision maker. In this case, Cost may be thought of as a continuous factor. Model Type, a discrete factor, can take on any of the five types shown in Figure 1. The acceptable range for Horsepower has been set at 50-350 hp, and Miles Per Gallon between 10 and 30 mpg. These four ranges have been specified by the decision maker; an auto having an attribute which falls outside any one of these limits will not be considered. Relative Cost of Repairs can vary between arbitrary indices of 0 and 100, based upon data from a national consumer organization. And finally, Reliability can vary between 0 and 100%, based on information from the consumer organization. These six attributes will be used in describing the assessment methods.

#### METHODS OF ORDINAL WORTH ASSESSMENT

Two methods of ordinal worth assessment will be discussed--ranking and equivalence grouping. The term ordinal is used here because the results of the methods are ordinal preferences. That is, the results show the order or direction of preference, as opposed to the magnitude of preference. This is sufficient information for several decision models;<sup>35</sup> however, in many situations worth must be specified on an interval scale (showing relative magnitudes of preferences). In order to use the ordinal methods in these cases, the researcher and/or the decision maker must make assumptions regarding the translation of the ordinal results to an interval scale. The assumption of equal intervals is often made. In any case, the resulting numerical worths on an interval scale will only approximate the decision maker's true preference magnitudes. The fact that only approximate numerical worth can be generated is the main shortcoming of these methods.

The main advantage of the ordinal methods is that only qualitative judgments are required of the decision maker. Since this type of judgment is commonly made, ordinal methods are generally intuitively appealing to decision makers.

The two methods can be used with one or more factors or attributes. For instance, the methods could be used to look at a pair of factors, say Model Type and Horsepower. In this case a factor level would be for instance a "two hundred horsepower convertible." The number of factors

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<sup>35</sup> Cf. MacCrimmon, 1968, op. cit.

to be assessed increases rapidly as attributes are grouped, since all possible combinations of factor levels must be assessed. However, if there are a small number of alternatives to choose from, then grouping is not prohibitive. The ordinal methods are described using single factors.

#### Ranking Methods

The ranking methods are the simplest worth assessment procedures. They require the decision maker to order the factors from most preferred to least preferred. Ranking is often used in conjunction with other methods which give more precise numerical worth estimates. The method:

- N - uses no probabilities
- C - depends upon preference judgments
- 2 - involves two factor levels at a time
- E - is usable with either continuous or discrete factors
- R(A) - provides a ranking of worth or approximated numerical worth

Process Example. Ranking is a simple one-step process, but there are many variations. As an example, consider the ranking of various models when buying an automobile. The decision maker is presented a list of the alternative models (Figure 4) and then given the following instructions:

"The automobile you are to purchase will be one of the five models shown. Consider which model you would prefer. Place a number one next to that model which you most prefer, and a number two next to that model which you prefer next. Continue in this manner until you have placed a number five to the model least preferred."

These ranks or numbers can be used directly in several decision strategies such as lexicography or dominance.<sup>36</sup> The ranking tells us that one model is preferred to another, but does not tell us how much more it is preferred.

The five automobile models may be thought of as points on a continuum of worth, with the order of the points known but the distances between them unknown. If the researcher is willing to make some assumptions, he can approximate these distances (by using equal interval assumptions, for instance) and use the ranking information in a higher order decision model. This may be necessary when none of the more powerful worth assessment techniques can be used. Such an approximate worth might not reflect the decision maker's true worth perspective, however, and should be used with caution.

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<sup>36</sup> See MacCrimmon, 1968, op. cit., for a discussion of various decision strategies.



	<u>RANK</u>
TWO-DOOR HARDTOP	_____
TWO-DOOR COUPE	_____
FOUR-DOOR HARDTOP	_____
STATION WAGON	_____
CONVERTIBLE	_____

Figure 4. Example list of factor levels to be ranked.

Variations. Instead of presenting a list, it is sometimes advantageous to place each alternative on a card and have the decision maker sort the cards, placing the most preferred on the top of the pile and so on until the least preferred is the bottom card. This facilitates reviewing the rank order by the decision maker.

If the number of alternatives is large the decision maker can be asked to first divide the cards into a row of several piles, where all the cards in any one pile are more preferred than any cards in the other piles to the right. In other words, the pile farthest to the right will contain the least preferred factors and the pile farthest to the left will contain the most preferred. The cards in each pile are then ranked, and the piles are combined to form a complete ranking.

A natural correspondence between worth and factor level exists for many continuous factors. For instance, higher cost is usually associated with lower worth. In these cases only the most preferred and least preferred factor levels need be found. Then, given any two points the one closest to the most preferred level will be preferred to the other. Worth can then be approximated as a function of the factor levels, if the researcher is willing to make further assumptions.

Pair comparisons is a popular technique for obtaining a rank order.<sup>37</sup> The decision maker is presented with a series of pairs of alternatives and is asked to indicate which alternative is preferred. Enough pairs are presented to insure a complete ordering. Figure 5 illustrates one method of pair comparisons.

PLACE AN 'X' NEXT TO THE MODEL IN EACH PAIR THAT YOU WOULD PREFER TO HAVE. CONSIDER ONLY MODEL TYPE.

STATION WAGON \_\_\_\_\_, . . . . . CONVERTIBLE X

TWO-DOOR HARDTOP \_\_\_\_\_, . . . TWO-DOOR COUPE X

CONVERTIBLE X, . . . . . TWO-DOOR COUPE \_\_\_\_\_

FOUR-DOOR HARDTOP X, . . . STATION WAGON \_\_\_\_\_

TWO-DOOR HARDTOP X, . FOUR-DOOR HARDTOP \_\_\_\_\_

Figure 5. Sample pair comparison work sheet

From the information shown we can deduce, given no intransitivities, that the order of preference from most to least is convertible, two-door coupe, two-door hardtop, four-door hardtop and station wagon. The decision maker must be transitive in his preferences to use pair comparisons.

Characteristics. Cautions: Two problems might occur with this method. First, the decision maker may be indifferent between two factor levels and unable to state a clear preference. If this happens, assigning the same approximate numerical worth to each will avoid the problem. Alternatively, the decision maker can be forced to arbitrarily assign one factor level a rank higher than the other. The second problem concerns intransitivity and is most likely to occur with pair comparisons. For example, if in

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<sup>37</sup> Torgerson, 1958, op. cit.

Figure 5 the first factor level in each pair was preferred, then it could be inferred that a station wagon is preferred to a four-door hardtop and at the same time a four-door hardtop is preferred to a station wagon. This type of impasse should be discussed with the decision maker and rectified by him.

Second, the reader should be cautioned to not place too much faith in approximate numerical worths obtained with this method. These numerical values should be used explicitly in a decision model only if the decision is relatively insensitive to small variations in the worth of the factor, as is often the case.<sup>38</sup>

Equipment: Depending upon the variation being used, either paper and pencil or cards are necessary with these methods.

Results: The output of this method is a ranking of the alternatives or an approximate numerical representation of the rank order. Approximate numerical worth can be used in numerical utility models, but with the cautions previously mentioned.

Advantages. The method is the easiest for the decision maker of the several we will consider. It requires no training, little equipment, and is intuitively appealing, since rankings (preference statements) are common judgments for people to make.

Disadvantages. The main disadvantage of the method is that only approximate numerical worth can be obtained, and because of this it may not be applicable in many situations. In these cases, ranking must be augmented with another method to get more precise worth numbers.

Checking Techniques. After the ranking has been established, the decision maker can be presented with several additional choice preference situations. His preferences in each pair should conform to the expected preference. For instance, if we assume the decision maker has given us the information in Figure 5, he might be asked:

"Do you prefer a station wagon or a two-door hardtop?"

Based on the ranking developed from Figure 5 he should be expected to prefer the two-door hardtop. Any discrepancy found using the additional pairs should be cleared up with the decision maker.

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<sup>38</sup> Von Winterfeldt, D. and W. Edwards. Costs and payoffs in perceptual research. In E. C. Carterette and M. P. Friedman (Eds.), Handbook of Perception. New York: Academic Press, in press.



Sources of Information. The theory and general discussions of the method can be found in Guilford,<sup>39</sup> Thurstone,<sup>40</sup> and Torgerson.<sup>41</sup> The basic method was used by Gulliksen<sup>42</sup> for determining the values of various meats. In his study, subjects were asked to state preferences between Tongue, Pork, Lamb, Beef, and Steak, as well as between pairs of the meats. For example, the subjects were asked to choose the most preferred in each of the following three pairs:

1. Port - Beef
2. Pork and Beef - Steak
3. Pork and Steak - Tongue and Beef

It was assumed that the subjects must eat both meats in a composite alternative. The subjects rated all possible pairs except those in which the same meat appeared in both of the choices. The resulting relations were solved using a least squares technique to get scale values.

By looking at pairs of meats it was found that some meats seemed to have positive worth, while others had negative worth. That is, the value of a Steak and Tongue composite was higher than Tongue alone, the value of Steak and Lamb was greater than Lamb alone, etc., indicating that Steak added worth when used in a composite. On the other hand, the value of Pork and Beef was lower than Beef alone, the value of Pork and Lamb was lower than Lamb alone, etc., thus indicating that Pork detracts from composite value and thus has a negative value. This seems to indicate the existence of negative worth.

Bechtel<sup>43,44</sup> used pair comparison to develop similarity ratings of words and phrases, and describes several methods for obtaining approximate numerical worth. Aumann and Kruskal<sup>45</sup> used pair comparison to obtain an analytic allocation strategy for naval equipment.

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<sup>39</sup> Guilford, 1954, op. cit.

<sup>40</sup> Thurstone, L. L. The measurement of values. Chicago: University of Chicago Press, 1959.

<sup>41</sup> Torgerson, 1958, op. cit.

<sup>42</sup> Gulliksen, H. Measurement of subjective values. Psychometrika, 1956, 21, 229-244.

<sup>43</sup> Bechtel, G. Comparative scaling of unidimensional discrimination and similarity data. Psychometrika, 1966, 31, 75-84.

<sup>44</sup> Bechtel, G. Folded and unfolded scaling from preferential paired comparisons. Journal of Mathematical Psychology, 1968, 2, 333-357.

<sup>45</sup> Aumann, R. J. and J. B. Kruskal. Assigning quantitative values to qualitative factors in the naval electronics problem. Naval Research Logistics Quarterly, 1959, 6, 1-16.

## Equivalence Grouping

This method involves the classification by the decision maker of the worth of levels of a factor into fixed categories. One factor level at a time is rated onto a category rating scale. Only approximate numerical worth is possible with this method. The method:

- N - uses no probabilities
- I - depends on indifference judgments
- 1 - involves one factor level at a time
- D - works best with discrete factors
- A - provides approximated numerical worth

Process Example. Step 1: Select all the factors and factor levels whose worths are to be assessed. As an example, let us assume that the factor to be rated is the model of car: the states (or factor levels) are two-door hardtop, two-door coupe, convertible, station wagon and four-door hardtop.

Step 2: Choose a rating scale such as a Likert or Thurstone scale. In our example we shall choose a seven-point Likert scale. A discussion of the various scales is given below.

Step 3: Provide cues for the scale. The words or cues on a rating scale serve two purposes: They help give the decision maker a sense of what the scale represents, and they help distinguish between different categories. The choice of words used is important. In general, the words should be clear to both the researcher and decision maker and should be relevant to the situation. It is assumed, and the researcher must ensure, that the words across the scale are in a natural rank order. For instance, if the decision maker believes that a "highly desirable" factor is better than a "most desirable" factor (see Figure 6), any inference drawn from his groupings will be invalid, since the equivalence grouping assumes that "most desirable" is better than "highly desirable."

An alternative to naming all categories is to name only the extreme ones. This indicates the range of possible responses but does nothing to distinguish between adjacent categories. Guilford<sup>46</sup> discusses more completely the issues involved in constructing such rating scales.

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<sup>46</sup> Guilford, 1954, op. cit.



	MOST DESIRABLE	HIGHLY DESIRABLE	DESIRABLE	INDIFFERENT	UNDESIRABLE	HIGHLY UNDESIRABLE	MOST UNDESIRABLE
TWO-DOOR HARDTOP							
TWO-DOOR COUPE							
CONVERTIBLE							
STATION WAGON							
FOUR-DOOR HARDTOP							

Figure 6. Sample questionnaire for equivalence grouping

Step 4: Frame the factors for which the worths are to be assessed and the rating scale in a format like that shown in Figure 6. The decision maker is then told:

"This questionnaire is for determining your preferences for the different automobile models listed below. For the purposes here, please consider only your preference for model type, and disregard any other factors. Please mark your preference for each model on the scale provided."

Step 5: Assign approximate worth numbers to the categories. This step is necessary only if an interval scale of worth is needed. The assignment of approximate worth numbers depends upon the assumption of equal intervals. This assumption allows the categories to be assigned consecutive integers representing their place along the scale. These numbers are often included on the questionnaire with the cues.

In the example, we assumed equal intervals and assigned the numbers 7, 6, 5, 4, 3, 2, and 1 to the categories from left to right. Thus, each factor level rated as most desirable will be assigned a worth of 7; those rated highly desirable a 6, etc. The number assigned to a factor level will represent its approximate numerical worth.

Variations. The scale need not have exactly seven intervals. Five, seven, nine and eleven are most often used, but as few as two and as many as twenty-one intervals have been reported.<sup>47</sup> The optimal number of intervals depends upon two factors. First, as the number of intervals increases it becomes increasingly more difficult for the decision maker to perceive differences between categories and hence to rate the factor levels. Second, more intervals allow a more precise breakdown of values, as long as the decision maker is able to perceive accurately the differences between categories. The discrimination of the decision maker and the precision of the resulting worth must be balanced according to the particular situation.

A scale may be replaced by containers labeled by the points on the scale. Factor levels can be placed on cards and the cards sorted into the appropriate containers.

Characteristics. Cautions: Although the researcher may perceive the distances between successive points to be equal, the decision maker may perceive them to be unequal. For example, he might disagree that the difference between Most Desirable and Highly Desirable is the same as the difference between Indifferent and Undesirable. If the researcher suspects this to be the case, he should ask the decision maker to rate the relative worth of the various points on the scale by another method.

Equipment: Paper and pencil is required unless the container variation is used. When this method is used for the assessment of worth by a large number of decision makers, preprinted questionnaires may be necessary.

Results: The output of this method is numbers that approximate the worth of given factor levels. The range of the worth values is arbitrary and assigned by the researcher based on his needs.

The results are often different from those obtained using ranking methods because some categories may contain several factor levels and others may contain none. Depending upon the number of categories, the decision maker may be able to give more information than in the ranking procedures. Specifically, he can give some indications of worth magnitude by how close

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<sup>47</sup> Guilford, 1954, op. cit.

or far away two factors are placed. On the other hand, when there are few categories he may not be able to give as much information as with the ranking methods, since each category will contain many factor levels and rankings within categories are unknown.

**Information Needed:** A precise statement of the factors to be assessed is the only information needed for this method.

**Advantages.** This is one of the easiest methods of assessment and is widely used. It requires no additional skills on the part of the decision maker.

**Disadvantages.** It is difficult to obtain very precise worth assessments by this method since numerical worth values can only be approximated by an interval.

**Checking Techniques.** After the factors have been placed in equivalence groups, the decision maker can be presented with choice preference situations (see Figure 5) involving factor levels in different groups. The preferred factor level should be that indicated by the equivalence grouping. Alternatively, he can be asked to rank order the factor levels, which can then be used to check the equivalence grouping. Finally, factor lists can contain repeated factor levels which should all be grouped into the same category.

**Sources of Information.** Guilford<sup>48</sup> and Thurstone<sup>49</sup> deal with this technique. Vroom and Deci,<sup>50</sup> in a more theoretical treatment, describe the use of five and eleven interval scales to measure importance of attributes of jobs.

Slovic<sup>51</sup> used the method in a study of stockbrokers' decision making. The purpose was to discover how two stockbrokers used information when deciding to buy stocks. The subjects rated the growth potential of 128 hypothetical stocks described by 11 factors normally associated with stock, such as price/earnings ratio, past year's performance, and volume trend. Each factor was described by only two levels, such as up or down for trends, and high or low for yields. This was done to simplify the procedure.

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<sup>48</sup> Guilford, 1954, op. cit.

<sup>49</sup> Thurstone, 1959, op. cit.

<sup>50</sup> Vroom, V. H. and E. Deci. The stability of post-decision dissonance: A follow-up study of the job attitudes of business school graduates. Organization Behavior and Human Performance, 1971, 6, 36-49.

<sup>51</sup> Slovic, P. Analyzing the expert judge: A descriptive study of a stockbroker's decision process. Journal of Applied Psychology, 1969, 53, 255-263.



The stockbrokers were presented with the hypothetical companies and asked to make recommendations on a nine point rating scale. Categories 1, 4, 5, 6, and 9 were labeled respectively "strong recommendation not to buy," "slight recommendation not to buy," "neutral," "slight recommendation to buy" and "strong recommendation to buy." The category numbers were used directly in an analysis of variance to determine the effects of each factor and factor pair on the rating of the companies' stock. This procedure is the reverse of that shown in our example; Slovic analyzed decisions to find worth factors, and in our example worth factors are analyzed to determine a decision.

In other studies, Goldberg<sup>52</sup> used an eleven-point scale for assessing diagnostic judgments, and Stagner<sup>53</sup> used seven point scales for determining corporate decision making practices. In a comparison of various scaling techniques, Patrick, Bush and Chen<sup>54</sup> used an eleven point scale as one method. It was found to correlate highly with other methods. They have also used this approach in developing health status indices.<sup>55</sup> Cooper<sup>56</sup> used the method in studies of preference for various soft drinks. An extensive program of research in the area of food preference as well as methods for statistical inference is reviewed by Bock and Jones.<sup>57</sup>

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<sup>52</sup> Goldberg, L. Five models of clinical judgment: An empirical comparison between linear and nonlinear representations of the human inference process. Organizational Behavior and Human Performance, 1971, 6, 458-479.

<sup>53</sup> Stagner, R. Corporate decision making: An empirical study. Journal of Applied Psychology, 1969, 53, 1-13.

<sup>54</sup> Patrick, D., J. Bush, and M. Chen. Comparing three methods for measuring social preference for levels of function. Unpublished report. University of California at San Diego, 1971.

<sup>55</sup> Bush, J., M. Chen, and D. Patrick. Cost effectiveness using a health status index: Analysis of the New York State phenylketonuria screening program. Unpublished report, University of California at San Diego, Department of Community Medicine, September 1972.

<sup>56</sup> Cooper, L. G. Metric multidimensional scaling and the concept of preference. Los Angeles: Western Management Science Institute, Working Paper 163. October 1970.

<sup>57</sup> Bock, R. D. and L. V. Jones. The measurement and prediction of judgment and choice. San Francisco: Holden-Day, 1968.



## DIRECT METHODS OF WORTH ASSESSMENT

This section deals with methods of worth assessment in which the decision maker is required to make quantitative judgments rather than qualitative judgments. The different techniques of direct magnitude estimation can be categorized into three classes; first are those methods where the range of the scale is defined by the researcher. In these methods the extreme factor levels are usually anchored at the extreme worth levels, although only the scale endpoints need to be specified. The range of the worth scale and its zero point are arbitrarily set by the researcher, and hence the decision maker specifies his estimates of worth entirely on an interval scale. These methods will be called "double anchored estimation methods."

A second class consists of those methods where the researcher provides only a single reference point. The resulting worth scale, while still an interval scale, will have a range determined by the decision maker. These methods allow the decision maker to give more information, but for the same reason may be more difficult for him to comprehend. These methods will be called "single anchor estimation methods."

The third class of direct estimation methods are those in which no reference point or range is specified by either the decision maker or the researcher. No research has been done on the use of these methods for worth assessment. There is some question as to whether these theoretical techniques are appropriate for worth assessment, and they will not be discussed in this paper.

Because the direct estimation methods do not correspond to the classical axioms of utility (gamble methods), the validity of their results has been questioned. Beach<sup>58</sup> investigated the relationship between the direct methods and the methods which follow the axioms of utility. Choices among bets were predicted correctly in about 94% of the cases, using information previously found using the direct methods. However, about two-thirds of the subjects had not performed in the gamble methods according to the axioms and had to be dropped from the study. Of the remaining subjects about half showed statistically significant correlations between direct worth and worth derived from the gamble methods. The study did not attempt to determine which was the best method of worth assessment, so the inconsistencies could be a result of variances within both methods. Fischer,<sup>59</sup> however, found high correlations between the direct methods and the axiomatic gamble methods. These two studies indicate that the direct methods may yield results comparable to the axiomatic methods.

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<sup>58</sup> Beach, B. Direct and indirect methods for measuring utility. Unpublished research report, University of Washington, Department of Psychology, July 1972.

<sup>59</sup> Fischer, G. W. Four methods for assessing multi-attribute utilities: An experimental validation. University of Michigan, Engineering Psychology Laboratory Technical Report 037230-6-T, September 1972. (b)

The main disadvantage of the direct methods is that they require numbers from the decision maker instead of preference judgments. Most people are familiar with making judgments of preference or indifference, but may be uncomfortable having to give numbers. It often requires training before they accept the procedures.

Direct methods have been used extensively, mainly because of their ease and speed of application. A worth function can be generated for a factor very quickly and without burdensome calculations. The direct methods are integral to many systems of worth assessment.<sup>60, 61, 62</sup>

#### Double Anchored Estimation Method

Double anchored estimation requires the decision maker to give a number which directly represents the worth of a factor level. The worths assessed may be in terms of a reference factor level and may involve the use of interval scales. The method:

- N - uses no probabilities
- Q - depends on quantitative judgments
- 3 - involves three factor levels at a time
- E - works with either continuous or discrete factors
- N - provides direct numerical worth

Process Example. Step 1: Rank the factor levels, or for continuous factors specify the most preferred and least preferred levels. For our example we will use efficiency of the automobile measured in miles per gallon, a continuous factor. Assume that the decision maker has said that 30 miles per gallon is the most preferred level for this factor and that 10 miles per gallon is the least preferred.

Step 2: Anchor the extreme values and obtain worths. The two extreme factor levels can be assigned the extreme values of the worth function. In our example, worth will be specified on a one-to-ten scale. We therefore assume a value of 1 for the least preferred factor level and 10 for

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<sup>60</sup> Churchman, C. W. and R. L. Ackoff. An approximate measure of value. Operations Research, 1954, 2, 172-187.

<sup>61</sup> Edwards, W. Social utilities. In Proceedings of the Symposium on Decision and Risk Analysis--Powerful New Tools for Management. Annapolis, Md.: U. S. Naval Academy. June 1971.

<sup>62</sup> Stevens, 1966, op. cit.

the most preferred. The decision maker is then presented with a list of the factor levels, or sample of factor levels in the case of continuous attributes, similar to Figure 7. He is then asked to:

"Indicate in the space provided a number between one and 10 which indicates your relative preference for the factor level in relationship to the first and last factor levels which have been arbitrarily assigned 10 and 1, respectively."

<u>MILES PER GALLON</u>	<u>SATISFACTION</u>
30	<u>10</u>
27	—
24	—
19	—
14	—
10	<u>1</u>

Figure 7. Example direct magnitude estimation work sheet.

The numbers specified by the decision maker can be used as direct numerical worth. They are direct estimates of the decision maker's true worth. In the case of continuous factors, as in our example, graphing the points estimated may be useful for interpolation. Let us assume that the decision maker specified the numbers 9, 7.5, 4 and 1.5 in the spaces in Figure 7, top to bottom. These points could then be graphed and a curve estimated as shown in Figure 8. This graph may also be useful feedback for the decision maker.

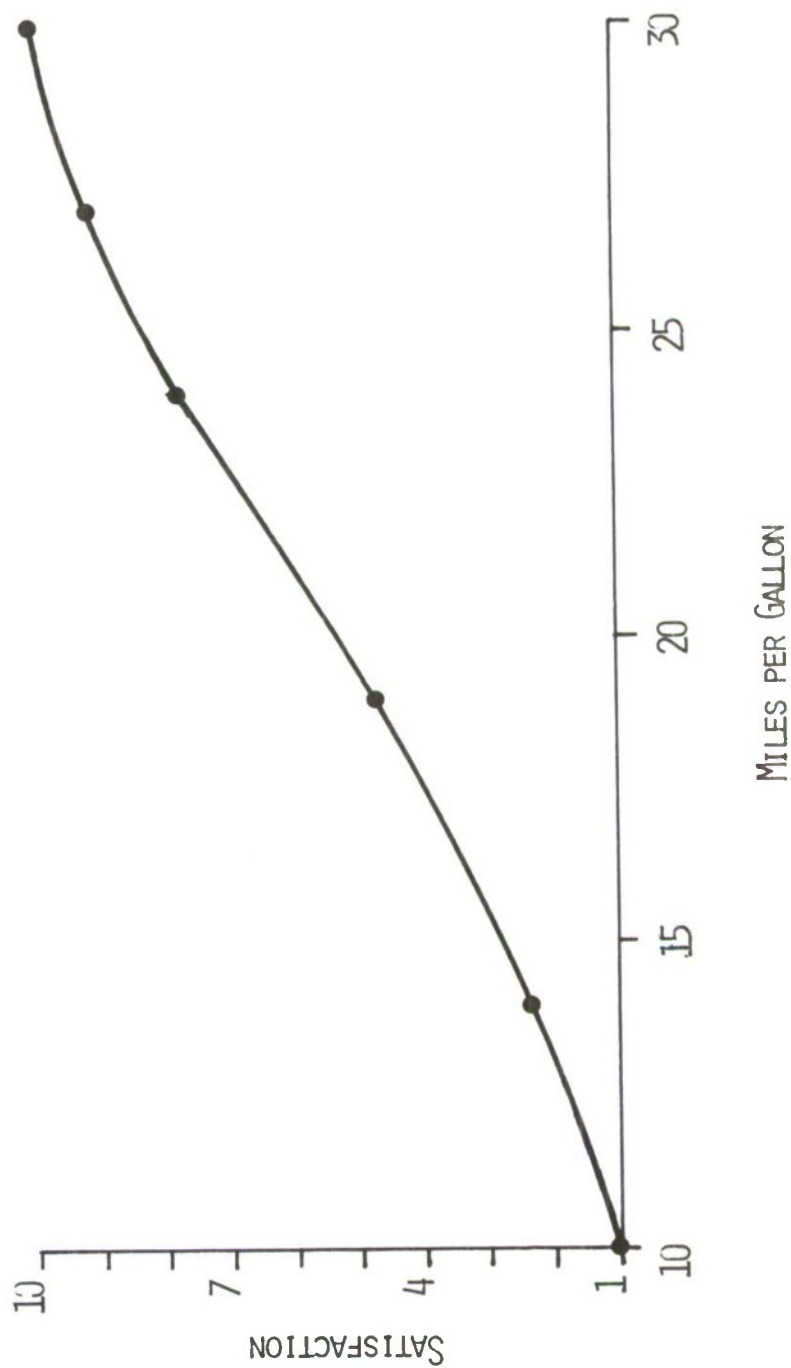


Figure 8. Worth curve of direct estimates



Variations. A simple variation is to list the factor levels next to an appropriate scale and have the decision maker draw a line from each factor to a point on the scale representing his worth or satisfaction. The two extreme attribute levels are anchored to the extreme worth values: 30 miles per gallon to 10 on the scale, and 10 miles per gallon to one (Figure 9).

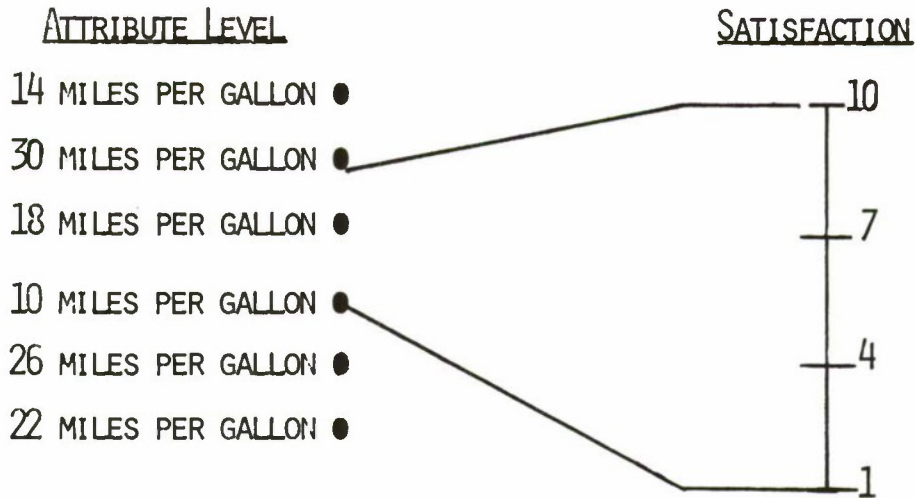


Figure 9. Estimation on a scale

Note that the factors have not been placed in rank order. The order can be set arbitrarily.

The scale may be annotated with descriptive phrases to aid the decision maker. Figure 10 shows a 10-point scale annotated with various levels of satisfaction. This may help the decision maker quantify his subjective feelings.

Another variation useful with continuous factors involves having the decision maker draw a picture of his worth function. A graph is constructed with the factor levels on one axis and the worth scale on the other (Figure 11). The extreme worth values, one and ten in the figure, are anchored to the extreme factor levels, ten and 30 miles per gallon. The decision maker is then asked to draw a curve connecting these two points which indicates his satisfaction or preference with the intermediate points.

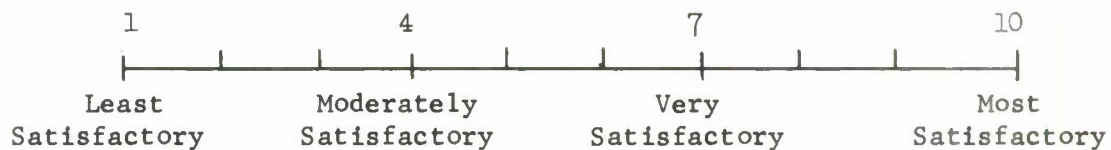


Figure 10. Annotated estimation scale

The researcher can assist the decision maker with this task by coaching and by providing feedback. He can tell the decision maker that if he feels there is little difference in preference between two points on the miles per gallon scale, then the line he draws between should be relatively flat, and if there is a big difference in preference the line should be steep. After the graph is drawn the researcher can critique it with the decision maker. For instance, given the curve in Figure 11, it can be noted that the decision maker saw little difference in preference between 26 and 30 miles per gallon. With this type of assistance the decision maker should be able to modify the curve until it accurately reflects his worth function.

For continuous factors, worth can be assessed by the method of bisection. The decision maker specifies the most desirable and least desirable factor levels, which are assigned the extreme worth values. He is then asked to specify a point which would yield satisfaction halfway between the two points. For instance, he might be asked:

"Assume that 30 miles per gallon is worth 50 and 10 miles per gallon is worth 0. How many miles per gallon would be worth 25?"

The question is repeated for other pairs of points until the function can be estimated.

Characteristics. Cautions: The decision maker may have a hard time comprehending what is required of him, since quantifying subjective feelings is not often done. The researcher must insure that he is well trained prior to an actual assessment session. In addition, using a terminology meaningful to the decision maker will increase his confidence. For instance, in our examples, "satisfaction" was used rather than "worth" because it seemed more appropriate to the situation. The researcher must insure that the decision maker understands the procedure. Practice before an actual assessment session may help the decision maker understand what is required of him.

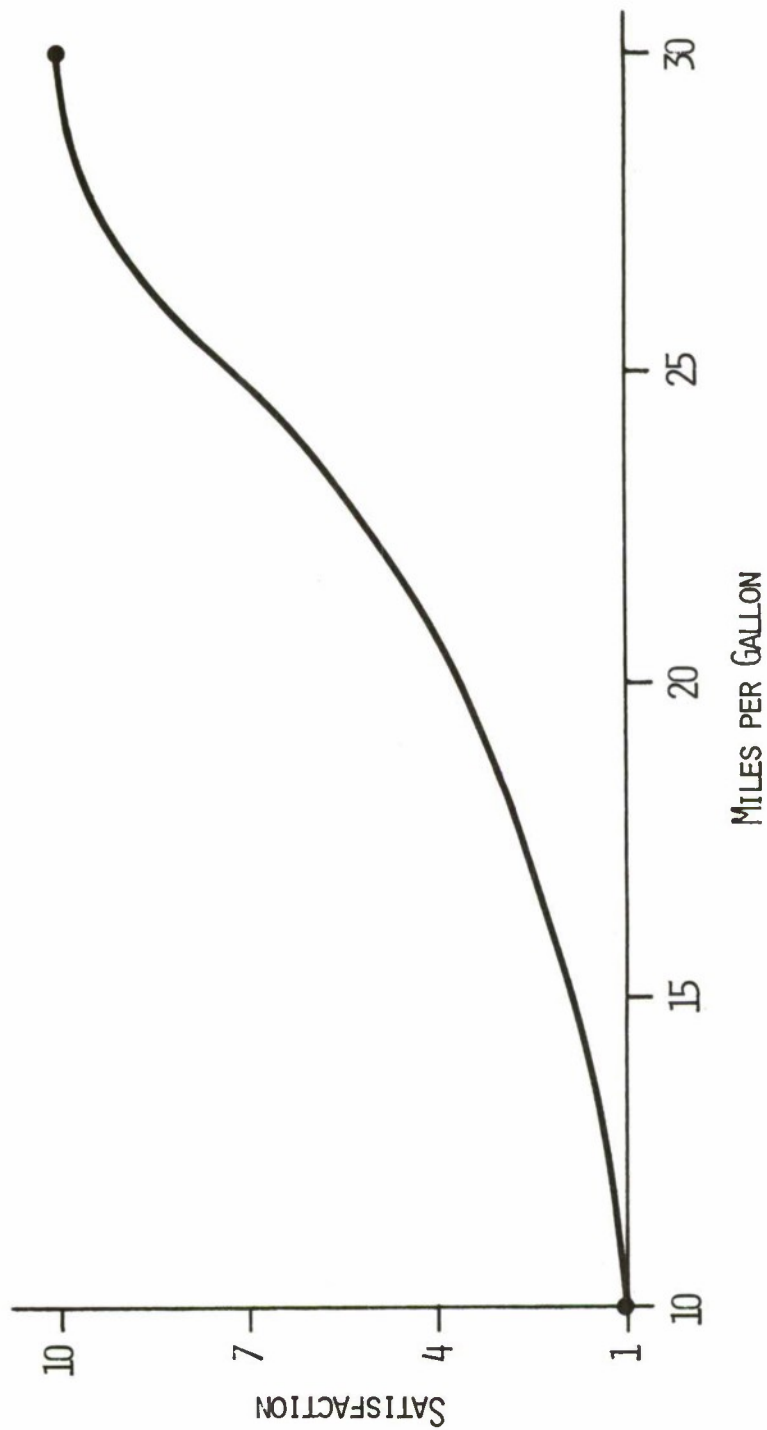


Figure 11. Graphical variation work sheet

Problems can also arise if a factor level is anchored to an endpoint of the scale before ranking is done. As an example, assume when assessing the worth of horsepower that the researcher anchors the maximum horsepower to the high end of the scale. The decision maker might actually feel that a low horsepower is most satisfactory but could not indicate this on the scale, since no scale remained above the anchor point. In this case, the researcher assumed that more horsepower was more satisfactory and has forced the decision maker to give worths that are unrealistic to him.

It should be noted that two distinctly different techniques have been mentioned here. The researcher can specify a factor value and ask the decision maker to specify a worth, either directly or indirectly; or he can specify a worth and ask the decision maker to give a factor value. These two different techniques may yield slightly different results.

Equipment: Paper and pencil is all that is required. Preprinted scales may be useful.

Results: The output of this method is numerical worth that can be used directly in utility models. In the case of continuous factors, the numbers can be used to estimate a continuous worth function.

Information Needed: The researcher must know the nature of the attribute under study, its measure, and the range of the measure. Information regarding the total decision problem is also helpful in order to make the presentation more meaningful to the problem at hand.

Training Required: Since this method may be difficult for the decision maker, he should be given a chance to practice it before attempting a final assessment.

Advantages. Direct magnitude estimation is a quick method of assessing worth. The decision maker is provided with reference points to make his task easier, and results can be used immediately. The method is flexible and can be modified to fit specific situations.

Disadvantages. The method does not meet the axioms of classical utility theory<sup>63</sup> since it requires direct quantification of subjective feelings. This may cast doubts upon the validity of its results, although the literature<sup>64</sup> indicates that the direct method yields results comparable with the more axiomatic methods.

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<sup>63</sup> Cf. Von Neumann and Morgenstern, 1944, op. cit.

<sup>64</sup> Beach, 1972, op. cit.; Fischer, 1972b, op. cit.



Checking Techniques. The results can be checked easily by performing the method twice, at different times. The degree to which the two sets of results agree gives an indication of their stability and accuracy. If large variances occur between sessions more detailed assessment is warranted.

If the factor under analysis is continuous, the dispersion of worth points from the estimated curve gives an indication of the accuracy. After the curve has been estimated, the decision maker can be asked to give the worth of points on the curve not already assessed. If he gives worths far from the expected value using the curve, he should be consulted about the discrepancy.

Sources of Information. The theory behind direct magnitude estimation and the bisection variation can be found in Torgerson.<sup>65</sup> The method has been described in Huber,<sup>66</sup> Eckenrode,<sup>67</sup> and Pardee and Phillips.<sup>68</sup> Huber, Sahney and Ford<sup>69</sup> found the method useful in a study of hospital ward evaluations. They used scales augmented with verbal descriptors (see Figure 10). Hoepfl and Huber<sup>70</sup> used 100-point scales with the method in a study involving evaluation of instructors, and Huber, Daneshgar and Ford<sup>71</sup> used similar scales in a study evaluating prospective jobs. Sellin and Wolfgang<sup>72</sup> used this method in evaluating the judged seriousness of crimes.

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<sup>65</sup> Torgerson, 1958, op. cit.

<sup>66</sup> Huber, 1974, op. cit.

<sup>67</sup> Eckenrode, R. Weighting multiple criteria. Management Science, 1965, 12, 180-192.

<sup>68</sup> Pardee, F. S. and C. T. Phillips. The utility of utility theory in regional transportation mix analysis. Presented at the 36th National Meeting of the Operations Research Society of America, Miami, August 1970. (AD 711-497)

<sup>69</sup> Huber, G., V. Sahney and D. Ford. A study of subjective evaluation models. Behavioral Science, 1969, 14, 483-489.

<sup>70</sup> Hoepfl, R. and G. Huber. A study of self-explicated utility models. Behavioral Science, 1970, 15, 408-414.

<sup>71</sup> Huber, G., R. Daneshgar and D. Ford. An empirical comparison of five utility models for predicting job preferences. Organizational Behavior and Human Performance, 1971, 6, 267-282.

<sup>72</sup> Sellin, T. and M. E. Wolfgang. The measurement of delinquency. New York: Wiley, 1964.

The graphical variation was tested by Pai, Gustafson and Kiner<sup>73</sup> and was used by Gustafson et al.<sup>74</sup> to develop a burn severity index. Five factors were developed which appeared important in evaluating burn severity: size of full thickness burn, size of partial thickness burn, age, past medical history, and location of burn. A severity function was developed for each of these factors, the factors were weighted, and their weighted sum was used as a burn severity index. The first three factors can be described as continuous and were analyzed by the graphical method. Figure 12 is an example of a graph drawn by a physician to show the "satisfaction function" of age on severity of burn. The graphs drawn by several physicians were combined to get a severity function of each of these factors. The other two factors and the factor weights were developed using a single anchored estimation technique. The model developed correlated well with both physician estimates and survival rate.

Davidson<sup>75</sup> describes the technique as used in land-use decision making. Levine et al.<sup>76</sup> and McKendry et al.<sup>77</sup> both used direct estimation methods in their studies of surveillance information error costs. Miller<sup>78</sup> has suggested the use of the graphical variation in R&D project evaluation. Direct magnitude estimation is used as one part of the Churchman-Ackoff method of worth assessment<sup>79</sup> and was used by Dalkey, Brown and Cochran<sup>80</sup> in explicating subjects' self-ratings in a Delphi study.

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<sup>73</sup> Pai, G., D. H. Gustafson and G. Kiner. Comparison of three non-risk methods for determining a preference function for money. Unpublished report, University of Wisconsin, January 1971.

<sup>74</sup> Gustafson, D. H., I. Feller, K. Crane, and D. Holloway. A decision theory approach to measuring severity in illness. Unpublished report, University of Wisconsin, 1971.

<sup>75</sup> Davidson, F. Utility and decision theory in a planning environment. Unpublished report, University of Virginia, McIntire School of Commerce, 1970.

<sup>76</sup> Levine, J. M., J. B. Feallock, R. Sadacca and R. Andrews. Method for quantifying subjective costs of large numbers of image interpretation errors. ARI Technical Research Note 218, November 1969. (AD 704 706)

<sup>77</sup> McKendry, J. M., P. C. Harrison, A. H. Birnbaum and R. Sadacca. Estimating the value of surveillance information using error cost matrices. ARI Technical Research Note 184. June 1967. (AD 667 390)

<sup>78</sup> Miller, 1967, op. cit.; 1969, op. cit.

<sup>79</sup> Churchman and Ackoff, 1954, op. cit.

<sup>80</sup> Dalkey, N., B. Brown and S. Cochran. The Delphi method: Use of self ratings to improve group estimates. Rand Corp. Memo RM-6115-PR, November 1969. (AD 698 735)

## Single Anchored Estimation Method

The basic method described here is based on ratio comparisons of two factors. Each factor level is compared to a standard, and the decision maker is asked to give a number which represents the ratio of the worths. The method:

- N - uses no probabilities
- Q - depends upon quantitative judgments
- 2 - involves two factor levels at a time
- E - works with either continuous or discrete factors
- N - provides direct numerical worth

Process Example. Step 1: First, the decision maker must rank order the factor levels under consideration. For continuous factors, he should specify the least and most desirable levels the factors can take. As an example, assume that the relative worths of five models are needed in a decision to purchase an automobile (Figure 4). Assume that the decision maker has said that his preference is in the order of convertible, two-door coupe, two-door hardtop, four-door hardtop and station wagon, with convertible being the most preferable.

Step 2: Compare the worths of all factor levels with a randomly chosen level one by one. One factor level is chosen to act as the reference. The other factor levels are compared to this factor level in an arbitrary order. Step 1 indicated which of the pair is preferred; in this step the decision maker is asked to specify how much more it is preferred in terms of a ratio. In the example, assuming the two-door hardtop is the reference factor, the decision maker would be asked:

"How much more is a convertible worth to you compared to a two-door hardtop?" (i.e., twice? three times? one and one-half times?)

He might answer that a convertible is worth one and one-half times as much as a two-door hardtop to him. This procedure is repeated for all the other factors.

Step 3: Assign the reference factor level an arbitrary worth. In our example, a two-door hardtop will be assigned a worth of 50 by the researcher. Then numerical points can be assigned for each of the other factor levels according to the ratio of the worth of those factor levels to the reference factor level. In our example, convertible would be assigned a worth of  $1.5 \times 50 = 75$ .

Once the points for all the factors have been assigned, they could be represented on a scale to provide the decision maker with visual feedback. Figure 13 shows a possible scale where convertible is clearly most preferred, station wagon is clearly least preferred and the other three levels are grouped in the middle.



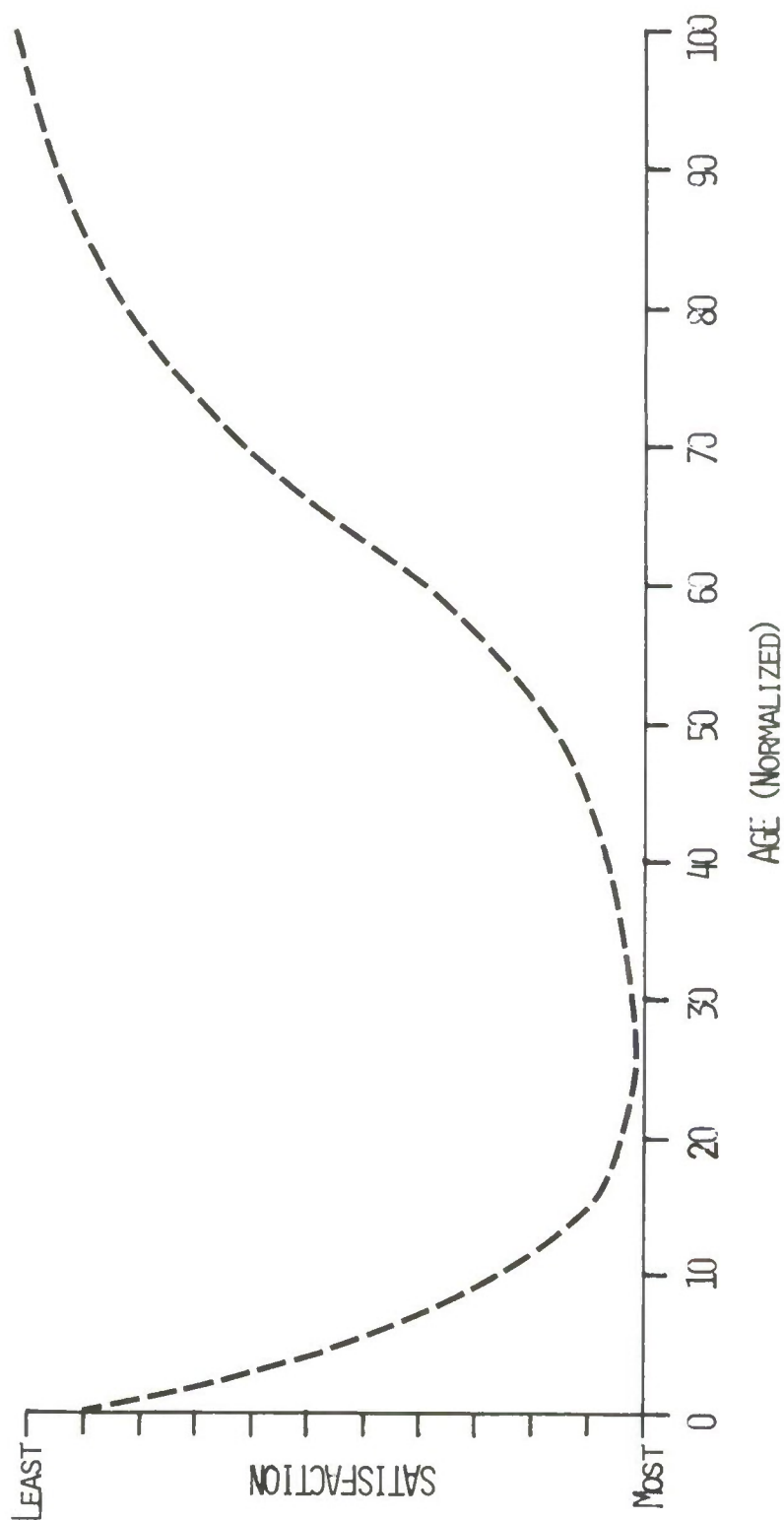


Figure 12. Worth function drawn by a physician for developing a severity of burn index.  
 (From Gustafson, D. H., I. Feller, K. Crane, and D. Holloway. A decision theory approach to measuring severity in illness. Unpublished report, University of Wisconsin, 1971.)

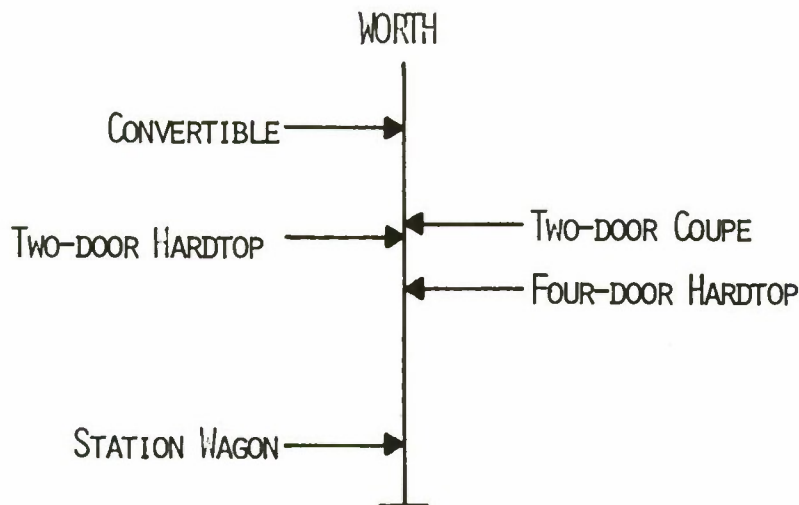


Figure 13. Example worth scale

Variations. One variation involves estimating factor level magnitudes rather than worth ratios. This can be used only with continuous factors. Using miles per gallon as an example: first, 10 miles per gallon would be arbitrarily assigned a worth--assume a value of 10. The decision maker is then asked to specify how many miles per gallon would be twice as satisfactory as 10 miles per gallon. This factor level is then assigned a value of 20. The process is then repeated asking for a value three times as satisfactory as 10 miles per gallon, or twice as satisfactory as the last specified value. These various points can then be plotted and a curve estimated. A similar variation involving a gamble method is described in the next section.

Most of the variations described in the double anchored estimation section can be modified and used with the single anchored estimation method. However, care must be taken to insure that the decision maker is not constrained by the given scale. For instance, assume that a reference level is anchored to nine on a ten-point scale. If the decision maker feels that some other level is three times as satisfactory as the reference, he will be unable to show this on the scale. The use of log scales may help alleviate this problem,<sup>81</sup> since a wide range of values are possible while small differences can still be noted (Figure 14).

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<sup>81</sup> Gustafson et al., 1971, op. cit.

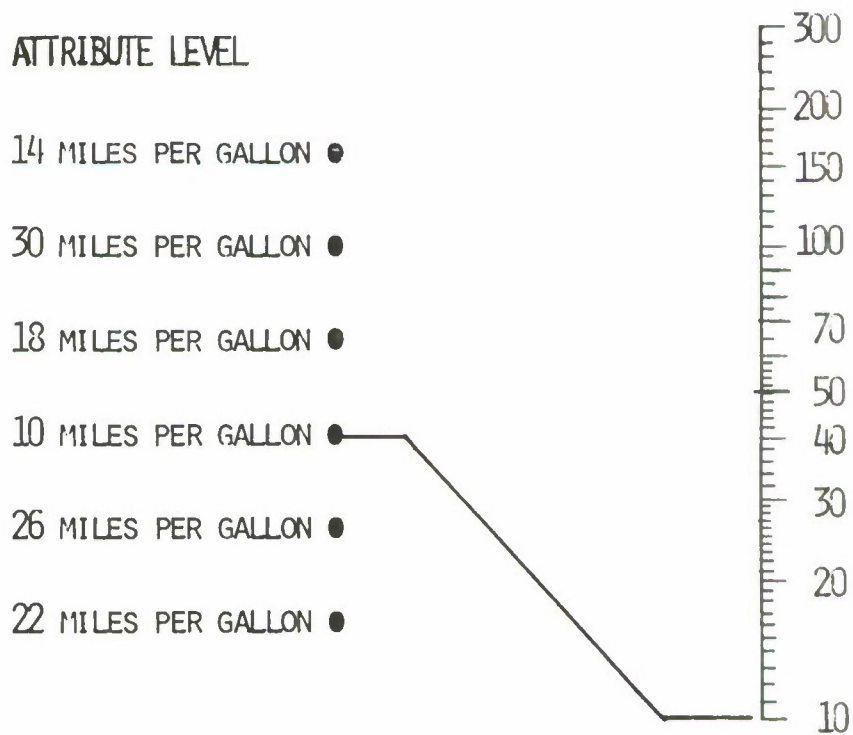


Figure 14. Single anchored estimation on a log scale



Characteristics. Cautions: This method is only useful when the decision maker is familiar with the estimation of ratios. For example, when asked to compare a two-door hardtop with a convertible, he might estimate the worth as 80% of that of a convertible. But when asked the question the other way around, he might estimate that a convertible is worth 1.5 times as much as a two-door hardtop. Note that if a convertible is assumed to have a worth of 100, the worth of the two-door hardtop is 80 in the first case and 67 in the second. Training in the use of ratios may be needed to prevent such errors as an order bias.

Results: The output of this method is a direct numerical worth of factor levels on an arbitrary scale. The endpoints will vary according to the one point fixed by the researcher.

Information Needed: The only information needed is the factor levels whose worths are to be measured (if discrete) and the range of values in the case of continuous factors.

Equipment: No special equipment is needed for this procedure. Paper and pencil may be required in some of the variations.

Advantages. Ease of Application: Once accepted, this is probably one of the easiest methods of estimating the worth of factor levels where probability is not involved. It demands very little of the researcher or decision maker.

No Fixed Scales: Since the decision maker is not forced to evaluate the relative worths of different factor levels on a fixed scale (for example, on a 10-100 scale), there is no "endpoint problem," i.e., the decision maker would not have to feel uneasy about choosing values near 10 or 100.

Disadvantages. Acceptability: This method is quite unpopular with decision makers. Many feel more uncomfortable estimating in ratios than in any other mode. This can be alleviated by training and discussion but this is a slow, time-consuming process.

Accuracy: The decision maker estimating the ratios may not be able to judge ratios well. Some people tend to overestimate ratios while others tend to underestimate them. Results of a study in the area of subjective probability estimation<sup>82</sup> where actuarial data exist to check assessment validity indicate that this type of range error may be no greater and possibly less than with other direct estimation techniques.

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<sup>82</sup> Stauss, F. Comparison of five estimation techniques for subjective probability. Unpublished Master's Thesis, University of Wisconsin, Department of Industrial Engineering, 1972.

**Sensitivity:** This method is not very sensitive to small differences in ratios. For example, the decision maker may not be able to differentiate between 14/16 and 15/16 (about a six-point difference on a 100-point scale). The result is a certain lack of resolution or degree of error inherent in the method.

**Checking Techniques.** One way to check on the decision maker's estimation is to have him evaluate all (or a subset of) the factor levels again, this time relative to some other factor level other than the original reference point. In our example, the decision maker might be asked to estimate the relative worth of a station wagon compared to a four-door hardtop. These new estimates should be nearly the same ratio as the ratio of the worths developed originally. If the researcher finds gross inconsistencies, he should point them out to the decision maker and ask him to reconsider. This procedure could be repeated as many times as desired using different factors as bases.

**Source of Information.** The theoretical background for this method is found in Ekman,<sup>83</sup> Coombs and Komorita<sup>84</sup> and Coombs.<sup>85</sup> Gustafson et al.<sup>86</sup> used the method in developing the severity-of-burn index mentioned earlier, in which two of the five factors assessed were evaluated using a single anchored estimation technique--location of burn and past medical history. To evaluate the effects of location, various locations were written on cards and the decision makers sorted these in order of importance. They then indicated how much more satisfied they would be with a patient burned in the first location than with a patient burned in each of the other locations. They gave responses on a log scale. The worst location was then arbitrarily assigned a value of 100 and every other location was valued in relation to this. In a similar manner, past medical history and the criteria weights were assessed. The model, which included the three factors evaluated using a double anchored estimation technique, correlated highly with both survival rate and physician estimates.

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<sup>83</sup> Ekman, G. Two generalized ratio scaling methods. Journal of Psychology, 1958, 45, 287-295.

<sup>84</sup> Coombs, C. H. and S. S. Komorita. Measuring utility of money through decisions. American Journal of Psychology, 1958, 71, 383-389.

<sup>85</sup> Coombs, C. H. Inconsistency of preferences: A test of unfolding theory. In W. Edwards and A. Tversky (Ed.), Decision Making. Baltimore: Penguin, 1967.

<sup>86</sup> Gustafson et al., 1971, op. cit.



Klahr<sup>87</sup> used a variant of this method for use with a model to predict college admission decisions. Pai, Gustafson and Kiner<sup>88</sup> found the results of this method correlated highly with the results of other methods. Huber and Gustafson<sup>89</sup> used the method on several attributes for developing severity of illness indices. In another example, Beach<sup>90</sup> used the method in a comparison of direct and indirect worth assessment techniques. Gustafson, Kramer and Pai<sup>91</sup> used the method in a study of research and development project selection strategies, and Stimson<sup>92</sup> employed the technique in his studies to improve the decision making process for funding public health grants. Finally, Galanter<sup>93</sup> described a variation of the method.

#### GAMBLE METHODS OF WORTH ASSESSMENT

The methods using gambles as a basis generally follow the axioms of utility theory<sup>94</sup> and therefore have been quite popular. These axioms detail a set of rules which, if followed in choice situations, will guarantee the existence of a worth function which describes the decision maker.

A gamble or wager is defined as a situation where each one of a set of outcomes can occur with a given probability. For instance, winning \$10 or losing \$5 based on the flip of a coin is a wager where the two outcomes, "win \$10" and "lose \$5" occur with probabilities of 1/2. The methods rely on the decision maker's ability to choose between such a wager and an alternative with no risk involved, a "sure thing." Extending the above wager, the decision maker would choose between (a) a wager where the probability of winning \$10 is 1/2 and the probability of losing \$5 is 1/2; or (b) winning \$2 for sure.

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<sup>87</sup> Klahr, D. Decision making in a complex environment: The use of similarity judgments to predict preferences. Management Science, 1969, 15, 595-618.

<sup>88</sup> Pai, Gustafson, and Kiner, 1971, op. cit.

<sup>89</sup> Huber, G. and D. H. Gustafson. Some efforts to apply behavioral decision theory in the medical care field. Unpublished report, University of Wisconsin, September 1971.

<sup>90</sup> Beach, 1972, op. cit.

<sup>91</sup> Gustafson, D. H., G. Kramer, and G. Pai. A weighted aggregate approach to R&D project selection. AIIE Transactions, 1971, 3, 22-31.

<sup>92</sup> Stimson, D. H. Utility measurement in public health decision making. Management Science, 1969, 16, 17-30.

<sup>93</sup> Galanter, F. The direct measurement of utility and subjective probability. American Journal of Psychology, 1962, 75, 208-220.

<sup>94</sup> Von Neumann and Morgenstern, 1944, op. cit.



There are two different ways for using these choices to develop worth functions. First, the probabilities in the wager can be varied until the decision maker is indifferent between wager and sure-thing. At this point, it is assumed that the expected worth of the wager and worth of the sure-thing are equal. These methods will be called the variable probability methods. Second, the probabilities in the wager can be held constant and one of the outcomes can be varied until the decision maker is indifferent. These will be called the constant probability methods.

The fact that the methods are generally axiomatically valid has been their main advantage. In addition, they incorporate the notion of risk in the analysis, which many believe makes the method more realistic, since most real world situations involve a certain amount of uncertainty.

The main disadvantage of these methods is that they are difficult and time-consuming to administer. Many different wagers must be constructed and analyzed, and the decision maker is forced to evaluate rather complex situations and to evaluate them in a specific manner. That is, he must follow the axioms and try to maximize his expected worth.<sup>95</sup>

The methods involve presenting many hypothetical situations which may appear to be very game-like to the decision maker. This may jeopardize his serious participation in the methods, and training and support may be necessary to overcome the problem.

#### Variable Probability Method

This method involves the comparison of wagers by the decision maker to generate his worth function. Probabilities in the wagers are varied until the decision maker is indifferent between the wager and a sure-thing option. These indifference points define a worth function on an arbitrary interval scale. The method:

- P - used probabilities
- I - depends upon indifference judgments
- 3 - involves three factor levels at a time
- E - works with either continuous or discrete factors
- N - provides direct numerical worth

Process Example. Step 1: The decision maker must rank order the levels of discrete factors, or at least specify a most preferred and least preferred factor level. For continuous factors, only the most and least preferred factor levels are necessary. An example, assume that the worths of

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<sup>95</sup> See Edwards, W. Utility, subjective probability, their interaction and variance preferences. Journal of Conflict Resolution, 1962, 6, 42-51.

various models are needed in a decision to purchase an automobile. There are five possible models: station wagon, convertible, two-door coupe, two-door hardtop and four-door hardtop. Assume that the decision maker said he most prefers a convertible and least prefers a station wagon.

Step 2: Choose a scale and assign numerical values to the most preferred and least preferred factor levels. The researcher should determine the required range of the worth scale. This is arbitrary and chosen for convenience, since points on the scale have no physical meaning; only the distances between points are useful. Assume that in our example a scale from +10 to -10 was chosen, with -10 being least preferred and +10 being most preferred. Thus, a convertible will be assigned a worth of +10 and a station wagon -10. It should be noted that any range of values is justifiable. For instance, 0-100 is often used.

Step 3: Construct a wager and present a choice situation to the decision maker. A wager is constructed which involves the most preferred and least preferred factor levels. The wager might be getting a convertible with a probability of .1 or getting a station wagon with a probability of .9. The decision maker is then presented with a choice, either the wager or one of the other factor levels as a sure-thing, and asked which he prefers. In our example he would be asked:

"Would you prefer to accept a wager where the probability of getting a convertible is .1 and the probability of getting a station wagon is .9, or would you prefer to have a two-door coupe for sure?"

The objective of this step is to find a point at which the decision maker is indifferent between the wager and the sure-thing. If he clearly prefers one or the other, the probabilities must be varied until he is indifferent, i.e., feels that the wager and sure-thing are equally acceptable. Changing the probabilities will change the expected worth of the wager. If the probability of getting the most preferred factor level is increased (with a similar decrease in probability of the least preferred factor level) the expected worth of the wager will increase. Therefore, if the sure-thing is preferred, it is worth more than the wager. The probability of the most preferred factor level (a convertible in this example) must be increased to increase the expected worth of the wager. If the wager is preferred, its expected worth must be reduced by decreasing the probability of the most preferred factor level and increasing the probability of the least preferred factor level. In this manner, the expected worth of the wager is modified until the decision maker believes it to be equal to the worth of the sure-thing option.

At the indifference point, the decision maker feels that the expected worth of the wager and the worth of the sure-thing are equal, or at least so close as to be indistinguishable. In other words, the chances of getting the most preferred factor level (convertible) are just high enough and the chances of getting the least preferred factor level (station wagon) are just low enough so that it is almost "worth it" to take the gamble rather than settle for an intermediate factor level (two-door coupe).



Let us assume that the decision maker is indifferent when the probability of a convertible is .95 and the probability of a station wagon is .05. The worth of a two-door coupe is then equal to .95 times the worth of a convertible plus .05 times the worth of a station wagon. In symbols this is:

$$W(\text{two-door coupe}) = .95 W(\text{convertible}) + .05 W(\text{station wagon}) \quad (1)$$

Since we assigned values to the worths of a convertible and station wagon in Step 2, we can calculate the relative value of the two-door coupe.

$$W(\text{two-door coupe}) = .95(10) + .05(-10) = 9.0 \quad (2)$$

Step 4: Vary the sure-thing. A worth is assigned to the sure-thing in Step 3, so that step is repeated using a different sure-thing outcome. This is continued until all factor levels other than most and least preferred have been the sure-thing option and assigned a worth. In the case of continuous factors, the process is continued until enough values have been generated to estimate the relationship between factor levels and worth, one point being determined for each replication of Step 3. In our example, Step 3 must be repeated twice, once using two-door hardtop and once using four-door hardtop as the sure-thing options. Assume that the decision maker felt that the two-door hardtop was indifferent to the wager when the probability of getting a convertible was .7 and a four-door hardtop was indifferent to the wager when the probability was .25. We can calculate from this the worth of these models, or +4 and -5, respectively:

$$W(\text{two-door hardtop}) = .7(10) + .3(-10) = 4.0 \quad (3)$$

$$W(\text{four-door hardtop}) = .25(10) + .75(-10) = -5.0$$

It should be noted that using zero as one end of the scale reduces the number of calculations, since either the worth of the most preferred or worth of the least preferred factor level will be zero and can be left out of the calculations. For continuous factors, the points can be plotted and a curve estimated.

Variations. In constructing choices for the decision maker it is possible to use other than the most and least preferred factor levels in the wagers. The only requirement is that the sure-thing be more preferred than one of the wager factor levels and less preferred than the other. It can be easily seen that if the sure-thing is preferred to both factor levels in the wager, it will always be preferred to the wager regardless of the probabilities. The opposite holds true if both wager factor levels are preferred to the sure-thing. No point of indifference can ever be found in either case. One common method is to use three adjacent factor levels in rank order. Using four-door hardtop and two-door coupe in the wager against two-door hardtop as a sure-thing would be such a situation. The solution of the resulting worth relationships (Equation 1) will not contain common terms and solving them will be slightly more difficult.



Characteristics. Cautions: This method is applicable only with subjects who are familiar with or capable of understanding probability. If this condition is violated, the resulting worth values will be questionable. For example, assume that a decision maker values two-door hardtop, 10; two-door coupe, 30; and convertible, 60; and we present him with the following choice:

"Would you prefer a wager where the probability of getting a convertible is .7 and the probability of getting a two-door hardtop is .3, or would you prefer a two-door coupe for sure?"

He may underestimate the meaning of .7 and overestimate what is meant by .3 and be indifferent between the choice. This would lead us to believe that he values a two-door coupe at 45, though he really values it at 30. Training the decision maker in the use of probability usually reduces this problem.

Another aspect of this method that may cause problems is that the probabilities in the wager must add to one. In other words, the wager must be exhaustive. For instance, if the two probabilities added to .7, there would be a probability of .3 that something other than the two outcomes would occur. Since the 'third' outcome is undefined, errors will occur.

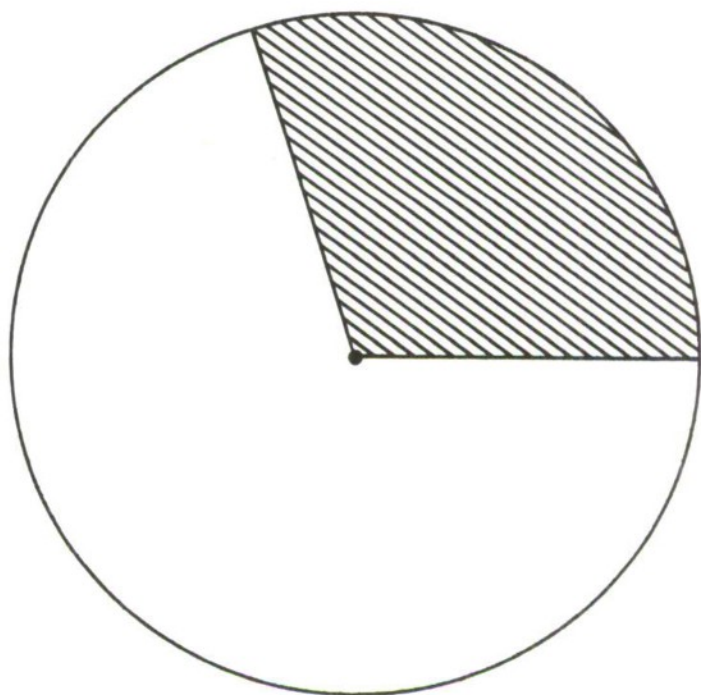
The "indifference point" is sometimes an interval; the decision maker may be indifferent over a relatively wide range of probabilities. In these cases it is worthwhile to continue varying the probabilities even though an indifferent situation has been found. In this way an approximate range of indifference can be found and its central point used in the worth calculations.<sup>96</sup>

Finally, the decision maker may like to gamble or be afraid of gambling. His decision would be influenced by this attitude toward risk.

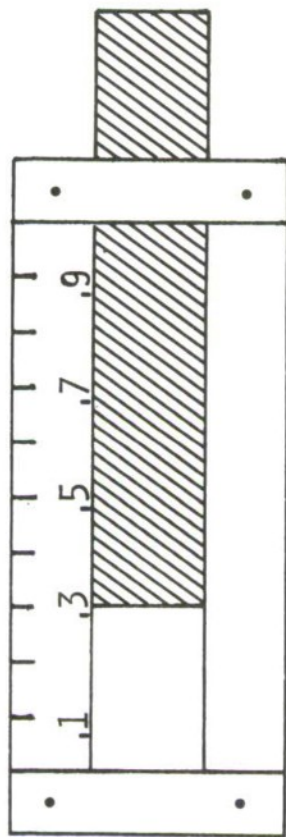
Equipment: A device for graphically portraying the probabilities to the decision maker is recommended, since there are problems with using stated probabilities. Figure 15 shows two different types: the probability wheel (a) and the probability bar (b). The wheel is constructed using two colored circles that are interwoven and pinned at the center so they can be rotated relative to each other to vary the size of the darkened area. In Figure 15, the wheel shows graphically the wager probabilities of .30 and .70. The probability bar works like a slide rule, except that the relative sizes of the two colors represent probabilities for the decision maker.

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<sup>96</sup> Beach, 1972, op. cit.



(A) PROBABILITY WHEEL



(B) PROBABILITY BAR

Figure 15. Devices for portraying probability

**Results:** The output of this method is numbers whose values represent the relative worth of the factor levels. The range and origin of the numbers are arbitrarily set by the researcher. If the factor is continuous, the method will generate points from which the worth curve can be estimated.

**Information Needed:** The range and precise description of the factor being studied is the main information needed. It is recommended that the decision maker's ability with probability be at least estimated before attempting this method. This can be done most directly by asking the decision maker for his own self-evaluation, or alternatively by having him judge known probabilities.

**Training Required:** Allowing the decision maker to practice the use of this method before an actual assessment may be necessary to insure he understands the technique. Training him in the meaning of probability may be beneficial. Hoffman and Peterson<sup>97</sup> describe a method for training people to be good probability assessors.

**Advantages.** Axiomatic: The method is theoretically appealing since it is derived from the axioms of utility.<sup>98</sup>

**Disadvantages.** Probability Interpretation: The main problem with the method is that different individuals may interpret or react differently to the same probability. People appear to make choices in wager conditions on the basis of perceived probabilities, rather than the stated objective probabilities.<sup>99,100</sup> Since worths are generated using objective probabilities, bias is introduced into the method according to how the decision maker perceived the probabilities. Fishburn<sup>101</sup> has suggested that a subjective function of objective probability should be determined for the decision maker, and that this be used in the method when calculating worths.

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<sup>97</sup>Hoffman, J. and C. Peterson. A scoring rule to train probability assessors. University of Michigan, Engineering Laboratory Technical Report 037230-4-T, September 1972.

<sup>98</sup>Von Neumann and Morgenstern, 1944, op. cit.

<sup>99</sup>Edwards, W. Probability-preference among bets with differing expected values. American Journal of Psychology, 1954, 67, 56-57.

<sup>100</sup>Edwards, 1962, op. cit.; Slovic and Lichtenstein, 1968, op. cit.

<sup>101</sup>Fishburn, 1964, op. cit.



Inveterate Gamblers: Some people enjoy gambling and will tend to choose a gamble over a sure-thing, even when the sure-thing is obviously worth more. The thrill of the gamble tends to increase the worth of the factors within. If the decision maker being studied is this type, this method will be strictly inappropriate. In the opposite extreme, a decision maker may be adverse to risk and always choose the sure-thing, again invalidating the method.

Acceptability: Decision makers may rebel at the idea of playing a 'game.' If forced to continue they may not concentrate on the task. The method should be used primarily in situations where the hypothetical situation posed is realistic.

Checking Techniques. There are two basic techniques for checking the worths generated by this method. The first involves presenting the decision maker with redundant wagers after worth has been assessed. For instance, assuming we have completed the assessment of model worth, we might ask the decision maker:

"Would you prefer to accept a chance wager where the probability of getting a two-door coupe is .8 and the probability of getting a four-door hardtop is .2, or would you prefer a two-door hardtop for sure?"

He should prefer the wager since we assume the expected worth of the wager is .2 times the worth of a four-door hardtop plus .8 times the worth of a two-door coupe, or 6.2 (Equation 1) and the worth of a two-door hardtop is 4. If the decision maker does not prefer the wager, the discrepancy must be cleared up before using the worths.

The second method of checking is to measure the "width" of the indifference point. Specifically, there will usually be a range of probabilities in the wager within which the decision maker is indifferent because of his inability to perceive precisely his own values and the probabilities presented. A wide range for the indifference point may indicate that the decision maker is not sure about the worths and may change as his values strengthen. The decision maker may have to be questioned further if the points are wide, so that he may increase his confidence in his own values.

Source of Information. The general theory for this method was developed by Von Neumann and Morganstern<sup>102</sup> and has been described by Mosteller and

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<sup>102</sup> Von Neumann and Morganstern, 1944, op. cit.

Nogee,<sup>103</sup> Raiffa,<sup>104</sup> and Swalm.<sup>105</sup> The Decision Analysis Group of the Stanford Research Institute (especially Carl Spetzler) makes extensive use of this method using the probability wheel (Figure 15).

Marquardt, Makens and Larzelere<sup>106</sup> used the method to determine the worth of brand names on turkeys. They compared a common brand-name turkey with an unknown turkey. The subjects were told that both turkeys were worth \$4.00. The subjects were asked to choose between two situations. In the first they would receive the brand name turkey eight times out of ten or two dollars two times out of ten. (This was the "wager" option.) In the second they received the unknown turkey for sure. Each subject was asked to make a choice in eleven such situations with the probability varied from ten out of ten to zero out of ten for getting the brand name turkey. The probability at which the subject switched to the unknown turkey was used as the indifference point. The hypothesis that people would not pay more for a brand name turkey was statistically rejected; some proportion of people will pay more for brand name products.

Finally, the method has been researched extensively.<sup>107</sup>

#### Constant Probability Method

The constant probability method involves wagers where the probabilities in the wager are held constant and factor levels are varied until an indifference point is found. The method:

- P 1/2 - uses constant probabilities
- I - depends upon indifference judgments
- 3(4) - involves three or four factor levels at a time
- C - works best with continuous factors
- N - provides direct numerical worth

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<sup>103</sup> Mosteller, F. and P. Nogee. An experimental measurement of utility. Journal of Political Economics, 1951, 59, 371-404.

<sup>104</sup> Raiffa, H. Preferences for multi-attributed alternatives. Rand Corp. Memo RM-5868-DOT-RC, April 1968.

<sup>105</sup> Swalm, R. O. Utility theory--insights into risk taking. Harvard Business Review, 1966, 44, 123-136.

<sup>106</sup> Marquardt, R., J. Makens, and H. Larzelere. Measuring the utility added by branding and grading. Journal of Marketing Research, 1965, 2, 45-50.

<sup>107</sup> Beach, 1972, op. cit.; Edwards, 1954, op. cit.; Slovic and Lichtenstein, 1968, op. cit.

Process Example. Step 1: Rank the factor levels. This method requires that factor level and preference be related. If they are not related some other worth assessment method must be used. For instance, if the decision maker prefers a middle factor level to both extremes, he will have a non-monotonic worth function and this method would be inappropriate. If the factors are continuous and monotonic, only the most and least preferred levels need be specified. As an example, consider the assessment of worth for horsepower. The range of acceptable horsepower has been set at 50 to 350 horsepower. The decision maker has said that 50 horsepower is least preferred and 350 horsepower is the most preferred level. Assume that the worth is desired on a 0 to 100 scale. Thus we assign a worth of 0 to 50 horsepower and a worth of 100 to 350 horsepower.

Step 2: Construct a choice wager and present this to the decision maker. The wager is set up with the most and least preferred factor levels, each having a probability of one-half of occurrence. The decision maker is asked whether he prefers the wager or some other factor level for sure. In our example he might be asked:

"Would you prefer to accept a wager where the probability of getting 350 horsepower is one-half and the probability of getting 50 horsepower is one-half, or would you prefer to have 200 horsepower for sure?"

Step 3: Vary the sure-thing until an indifference point is found. Assume that when presented the above choice the decision maker preferred the wager. In order to make the sure-thing more attractive it is changed from 200 to 250 horsepower, and the choice is presented again.

We will assume the decision maker is indifferent at this point. We can therefore determine a worth value for 250 horsepower. Because we are using probabilities of one-half, its worth will be midway between the worths of the factor levels in the wagers; thus it is worth 50. In symbols this is:

$$\begin{aligned} W(250 \text{ hp}) &= .5 W(50 \text{ hp}) + .5 W(350 \text{ hp}) = .5 (0) + .5 (100) \\ &= 50 \end{aligned} \tag{4}$$

Step 4: Vary the wager. We construct a new wager using two of the points already determined. For instance, if the next wager involves 250 horsepower and 350 horsepower, we find the worth point midway between them. Step 2 and 3 are repeated using this new wager.

As points are found they could be plotted so as to estimate the worth curve (Figure 16).



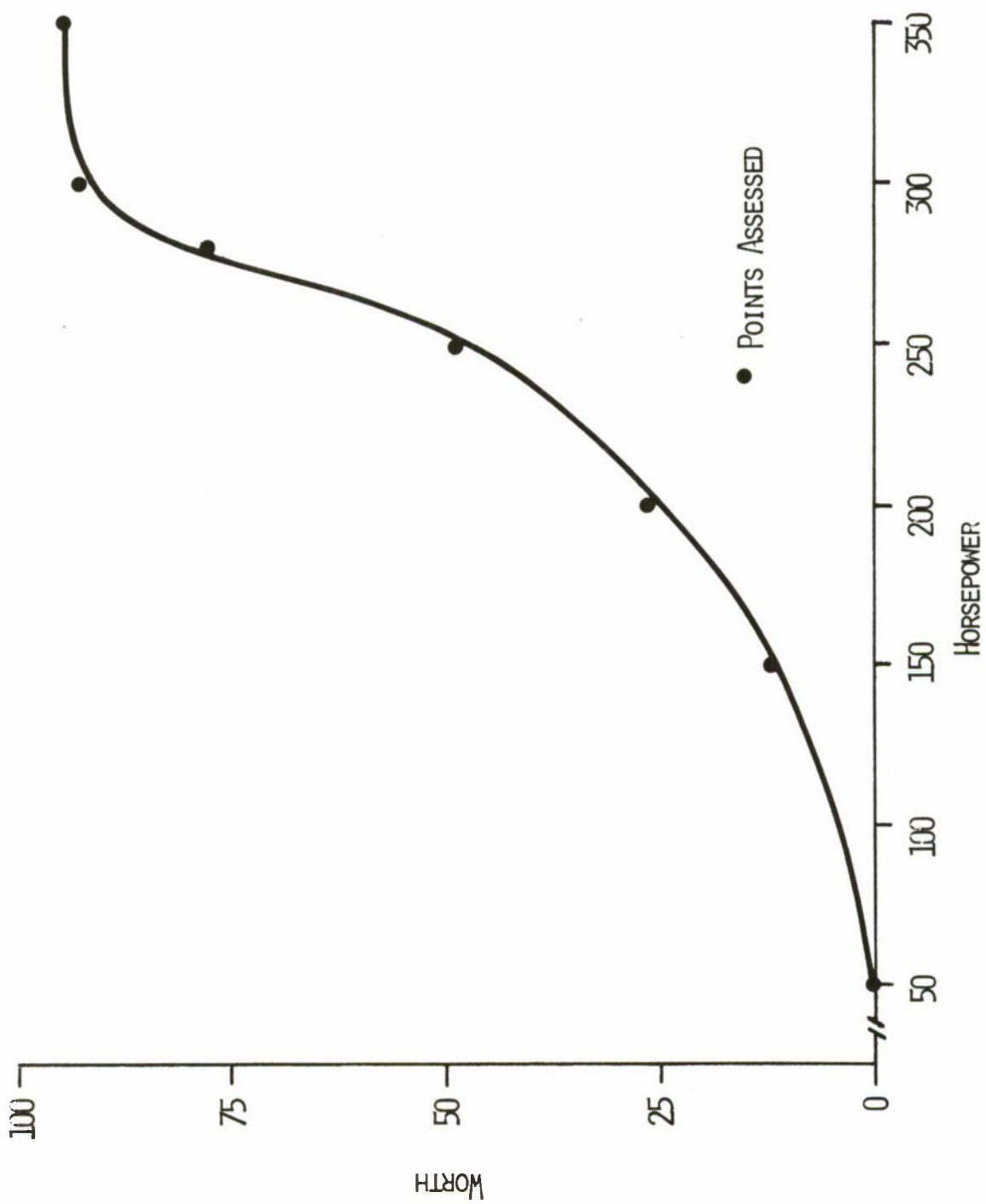


Figure 16. Worth curve estimated for horsepower

Variations. One of the major complaints associated with the gamble methods is that one option involves risk while the other is a sure-thing. For instance, if the decision maker is averse to risk, he will always choose the sure-thing. To alleviate this type of problem we can have the decision maker choose between two wagers. Although this is more difficult for both researcher and decision maker, response bias can be reduced with proper administration.

The method described in steps one through three above is done once so that three points are known. Using these three points we can ask the decision maker:

"Would you prefer a wager where the probability of getting 250 horsepower is .80 and the probability of getting 50 horsepower is .20, or;

"Would you prefer a wager where the probability of getting 200 horsepower is .80 and the probability of getting 350 horsepower is .20, or are you indifferent between the wagers?"

The unknown factor in the second wager is varied to make that wager more or less attractive. In this case, 200 horsepower would be changed since we have worths for 50, 250 and 350 horsepower. The factor is varied until an indifference point is found. Assuming that the decision maker is indifferent to the choice as stated, we can calculate the worth of 200 horsepower. The expected worth of the first wager is known to be 40 ( $0 \times .20 + 50 \times .80$ ), so the worth of 200 is found by solving the following for  $W(200)$ , read "worth of 200":

$$.80 \times W(200) + .20 \times 100 = 40$$

$$W(200) = 20 / .80 = 25$$

Thus, the worth of 200 horsepower is 25.

A point above 250 horsepower can be found by setting one wager as 250 horsepower with probability of .80 and 350 horsepower with .20, and setting the second as 200 horsepower with probability of .20 and 300 horsepower with .80. The 300 horsepower is varied to find the indifference point. The process is repeated until enough points are found to estimate a curve.

Finally, one variation involves direct magnitude estimation in conjunction with this gamble method. The decision maker estimates directly the value of the sure-thing that makes it indifferent to the wager. For instance, he might be asked:

"The horsepower of the automobile you will purchase will either be 50 horsepower with a probability of one-half, or 350 horsepower, also with a probability of one-half. How many horsepower for sure would make you indifferent to the wager? That is, it would not matter to you whether the horsepower you specify is used for sure?"

These indifference points are then used in a manner similar to the basic method. This variation speeds up the method considerably.

Characteristics. Cautions: All of the cautions of the variable probability method apply. Some of the problems with the use of probabilities are reduced with this method because probabilities are not varied, and generally only the probability of one-half is used which is familiar to most people. The probabilities of course must add to one in the wagers, and the situations portrayed should be as realistic as possible.

Results: The result of this method is numerical worth derived directly from the indifference points.

Information Needed: The specific description of the factor being assessed is all that is necessary. However, information about the decision maker's expertise with probabilities would be helpful. If he does not understand the concept, a different assessment technique should be used.

Training Required: Same as variable probability method.

Advantages. The advantages of this method are the same as with the variable probability methods. However, this method may reduce biases caused by probability preferences since all wagers contain the same probabilities. Hence, any bias will be constant and should not affect the interval worth scale.

Disadvantages. The problems of inveterate gamblers and acceptability previously described in the variable probability method are also disadvantages in this method.

Checking Techniques. The best method of validation involves presenting the decision maker with redundant choice situations. For instance, given the worth function in Figure 16, we could ask the decision maker:

"Would you prefer a wager where the probability of getting a car with 250 horsepower is one-half, and for a car with 350 horsepower is also one-half, or would you prefer to have a car with 300 horsepower for sure?"

The decision maker should prefer the sure-thing, since the expected worth of the wager is 75 and the worth of the sure-thing is about 90 (see Figure 16).



Sources of Information. The theory for this method can be found in Mosteller and Nogee<sup>108</sup> and in Raiffa,<sup>109</sup> who used the method in transportation system studies. The method is described in Hurst and Siegel<sup>110</sup> and Suppes and Walsh<sup>111</sup> although different techniques were used to obtain numerical worth. Royden, Suppes and Walsh,<sup>112</sup> Conrath and Deci,<sup>113</sup> and Siegel<sup>114</sup> have used a variant of this method. The method and several variations are described by Ackoff, Gupta and Minas.<sup>115</sup>

Finally, the variation involving direct estimation was used by Tversky<sup>116</sup> in a study of additive utility, and was tested by Becker, DeGroot and Marschak.<sup>117</sup> They presented subjects with wagers involving money and asked them for the lowest amount of money they would accept in lieu of the wager. The task was structured so that it was in the subject's interest to give the cash equivalent of the wager as his selling price, and this price was used as the indifference point in the method. The main finding of the study was that subjects become more consistent and closer to theoretical behavior with practice. This supports the contention that training will improve the gamble methods.

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<sup>108</sup> Mosteller and Nogee, 1951, op. cit.

<sup>109</sup> Raiffa, 1968, op. cit.

<sup>110</sup> Hurst and Siegel, 1956, op. cit.

<sup>111</sup> Suppes, P. and K. Walsh. A non-linear model for the experimental measurement of utility. Behavioral Science, 1959, 4, 204-211.

<sup>112</sup> Royden, H., P. Suppes and K. Walsh. A model for the experimental measurement of the utility of gambling. Behavioral Science, 1959, 4, 11-18.

<sup>113</sup> Conrath, D. and F. Deci. The determination and scaling of a bivariate utility function. Behavioral Science, 1969, 14, 316-327.

<sup>114</sup> Siegel, S. A method for obtaining an ordered metric scale. Psychometrika, 1959, 21, 207-216.

<sup>115</sup> Ackoff, R. L., S. Gupta and J. Minas. Scientific method: Optimizing applied research decisions. New York: Wiley, 1962.

<sup>116</sup> Tversky, A. Utility theory and additivity analysis of risky choices. Journal of Experimental Psychology, 1967, 75, 27-36.

<sup>117</sup> Becker, G., M. DeGroot and J. Marschak. Measuring utility by a single-response sequential method. Behavioral Science, 1964, 9, 226-232.

## MULTIVARIATE METHODS OF WORTH ASSESSMENT

The multivariate methods are those which are specifically designed to consider more than one factor at a time. In particular, they are well suited for analyzing dependent factors, although they are not restricted to this type. When developing a list of attributes it is often impossible to obtain a set of attributes which is both exhaustive and independent. In our example problem, for instance, we included the attributes Cost of Repairs and Reliability which are valuewise dependent. (The attribute Warranty, which is probably also dependent with these, will not be considered in this discussion.)

These factors should not be assessed as separate attributes; rather, one of the several ways of handling such interdependent factors should be used. First, all possible (or a set of all possible) combinations of the factor levels could each be assessed pairwise as a single factor using one of the preceding methods. However, for factors with several levels, the number of possible combinations is too large to make this strategy feasible.

Second, the attributes could be combined through analysis into a single attribute. For instance, the inverse of Reliability can be thought of as number of breakdowns per year. This multiplied by Cost of Repairs gives an expected repair cost per year which could be used instead of the two separate factors of Reliability and Cost of Repairs. The problem with this is that the assumed dependence accounted for may or may not reflect the dependence seen by the decision maker. However, this technique can be a powerful tool in obtaining a single factor from a set of dependent factors.

Third, the two dependent factors can be assessed using multivariate techniques. This strategy allows the assessment of very complex dependencies and can be used where the other two strategies cannot. The indifference curve or trade-off method for doing this will be described in this section. Its main weak point is that it is a long and tedious process. However, it can yield much information on complex dependencies that cannot be found by any other method.

### Indifference Curve Method

The indifference curve method is used when two factors are involved and their joint worth is to be assessed. The factors can be dependent or independent, but there must be a monotonic relationship between worth and factor level for each factor. The method can be used with more than two factors but becomes very difficult. It is based on the trade-off between worths of continuous factors but can be used with discrete factors if they can be approximated as continuous.

- N - uses no probabilities
- I - depends upon indifference judgments
- 2 - involves two factor levels at a time
- C - works best with continuous factors
- R(N) - provides ranking of factors or direct numerical worth

Process Example. Step 1: Identify an indifference plane. The initial step is to construct a graph, or plane, such that the axes represent the two factors under study. The range of values for each factor should contain all logically possible factor values.

As an example we will use Cost of Repairs and Reliability as our factors. They can be assumed to be dependent since high cost of repairs with high reliability might be as preferred as low cost of repairs and low reliability. Figure 17 shows the indifference plane for this example, using a 0 - 100 scale for each factor.

Step 2: Determine an indifference curve. A point on the plane is picked at random to become a reference point (for example, 40% reliable and a relative repair cost of 30). The point should be in a part of the graph which seems logical to the decision maker. If the point is at an extreme (zero cost, zero reliability, for instance) he may have trouble visualizing what it means. Other points on the indifference plane are then compared to this point. The researcher asks the decision maker to state a preference between the reference point and a comparison point. For instance, in our example he might be asked:

"Would you prefer to have a car which is 40% reliable and has a relative repair cost of 30 or would you prefer to have one which is 50% reliable and has a relative repair cost of 50, or are you indifferent between the two?"

If a preference is expressed, new comparison points are chosen until he is indifferent between the two. The two points then lie on the same indifference curve; i.e., have the same worth. This procedure is continued until sufficient indifference points are found to describe a curve.

There are several techniques for choosing new comparison points. Assume, in the above example, that worth on the indifference plane increases from bottom right to upper left. Then, if the reference point is preferred, the new comparison point should be to the left and/or above the previous comparison point, hence increasing its worth. Should the comparison point be preferred to the reference point, the new comparison point chosen would be below and/or to the right of the previous one.



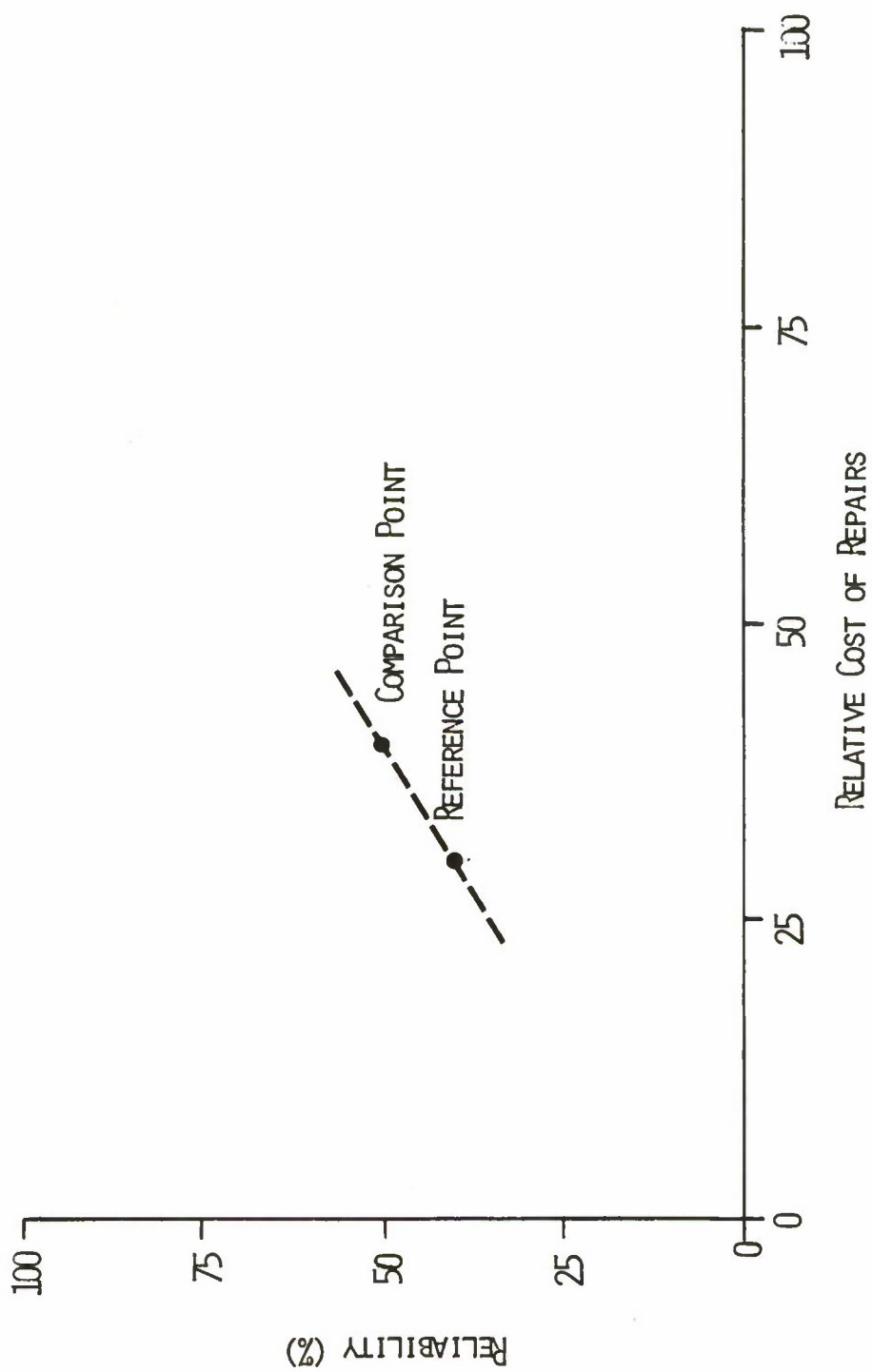


Figure 17. Partially completed indifference curve

For example, assume that the decision maker preferred the reference point to the comparison point, but was indifferent when the comparison point was (40,50). The indifference plane at this point is shown in Figure 17.

Once an indifference point is found, a new comparison point is chosen so as to define more of the indifference curve. The next point to check in the example might be a relative cost of 60 and 80% reliability. The objective is to generate enough indifference points to estimate an indifference curve. Assume that the decision maker was indifferent between the reference point (30,40) and comparison points (10,30), (40,50), (50,60) and (70,95). With these five points we can estimate an indifference curve as shown in Figure 18.

Step 3: Choose a new reference point. Once an indifference curve has been determined a new reference point is chosen. This new point will form the basis for another curve on the plane. Step 2 is then repeated for each new reference point chosen. The final outcome of this step is a set of curves which cover several possible combinations of the two factors (Figure 19). A total of six reference points were chosen, yielding the six curves shown in Figure 19. All of the points on a given curve have the same worth.

Step 4: Determine the worths associated with the curves. Up to this point we have determined that the points on different curves have different worths but we have not determined explicitly how much more one point is worth than another. One point on each curve is chosen from which worths can be assessed. The reference points make good choices, since the decision maker is more familiar with them than with other points on the curves. The worths of these points (six in our example) are determined using one of the other methods described in this paper. This step will not be illustrated here, but assume that the worth of each curve is as shown in Figure 19, the lowest curve being worth 30 and the highest worth 80. The worth of any point on the plane can now be determined by interpolation, if we assume linearity between curves. This assumption should not introduce any serious error, but should be used with caution.

Variations. It is possible to have the decision maker draw indifference curves directly. Since the process of determining curves can become very time-consuming, it may sometimes be advantageous after he has helped construct one or two curves and is familiar with the procedure to have him draw the curves directly through specified reference points. He must understand what the curves represent and how they are derived before this direct technique is attempted.

One variation allows direct numerical worth of independent factors to be determined directly from an indifference plane. This variation is sometimes called the trade-off method. Two independent factors are assessed using the indifference method described above, so that two indifference curves are determined. The two curves should be close together and cover as much of the factor ranges as possible. As an

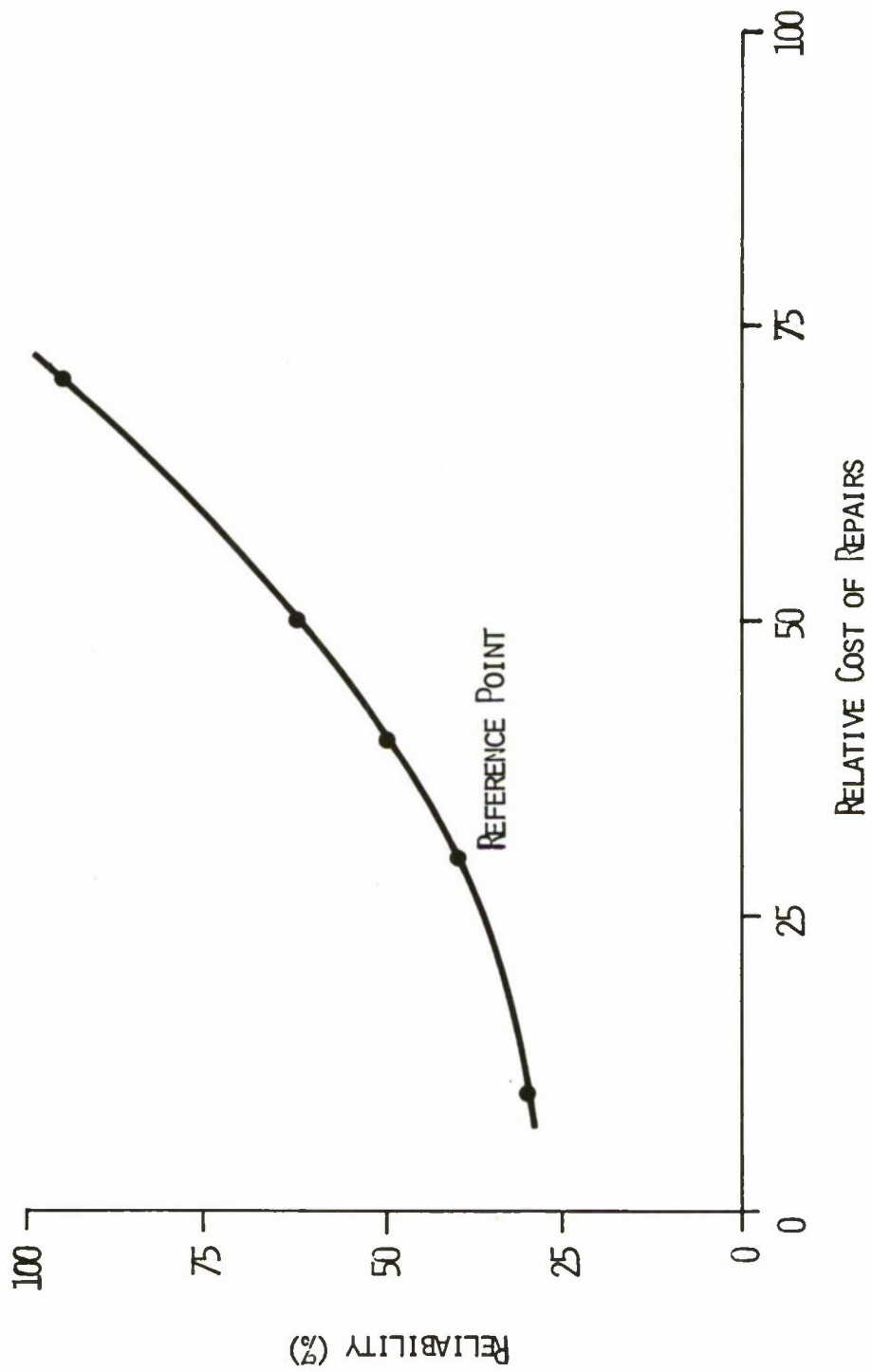


Figure 18. Single indifference curve



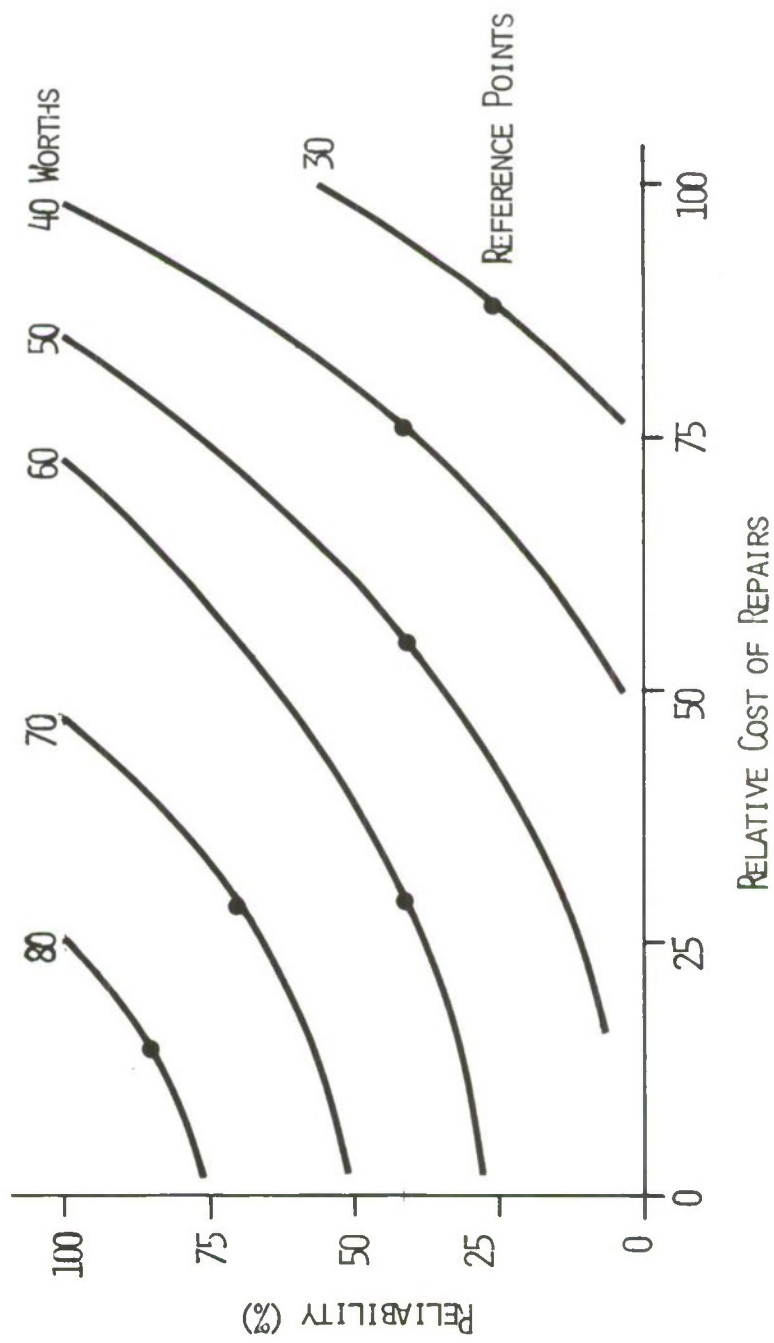


Figure 19. Completed indifference map (with worths)

example assume that two curves have been found on the Horsepower/Miles per Gallon indifference plane. The two curves are shown in the upper right corner of Figure 20. A "staircase" is drawn between the curves, each stair or rise parallel to one of the axes. Since the worth of each curve is constant over its length, the distances between "stairs" (shown with dotted lines to the axes) represent equal intervals of worth. For instance, the difference in worth between 50 horsepower and 200 horsepower is the same as the difference in worth between 200 horsepower and 240 horsepower. Assuming that these differences are worth 1.0, we can graph the functions of the two individual factors (Figure 20). This method allows joint assessment of two independent factors, which may be meaningful in many cases, while still allowing the worths of individual factors to be determined.

Characteristics. Cautions: The indifference region may be wide, and it may be hard to estimate a single curve. To illustrate, it will be assumed that a decision maker's indifference plane (given a reference point) can be divided into three areas: an indifference area, a reference-point-preferred area and a comparative-point-preferred area. Any comparative point in the indifference area will be indifferent in terms of worth to the reference point, and in the other two areas either the comparative or the reference point will be preferred. If the indifference region is wide the indifference points may not fall into a smooth curve; in Figure 21 the indifference points do not form a smooth curve. Although the problem will seldom be as severe as illustrated, the solution is the same. A curve can be drawn which most closely follows the points and passes through the indifference region.

Equipment: Graph paper and pen or pencils are required.

Results: The result of this method is a description of the worth of two factors. The first three steps will provide enough information to state a preference relationship between any two factor combinations. If Step 4 is completed, a quantitative worth value can be estimated for any logical combination of the two factors. These relationships will depend upon any interpolation rules that are used.

Information Needed: Since the method is primarily applicable with two factors, the range and precise description of both are needed.

Advantages. Ability to assess dependent factors: By far the major advantage of this method is the fact that it can assess the worth of factors which are dependent. While other methods could be used to assess joint worth, this method is clearly the most straightforward for this purpose.

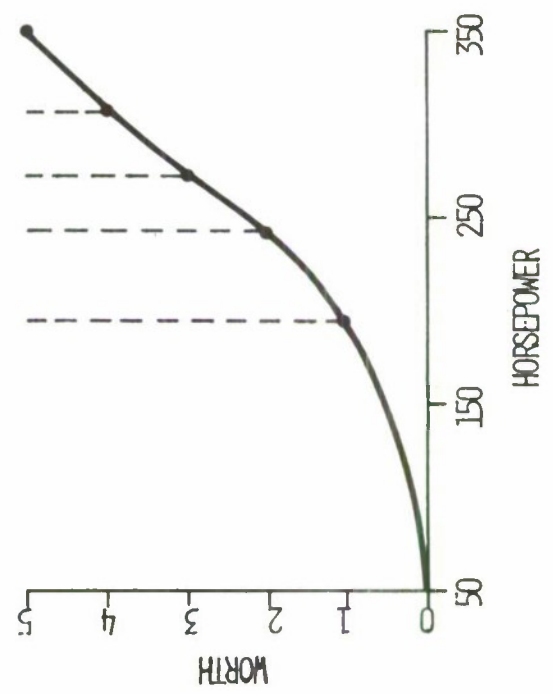
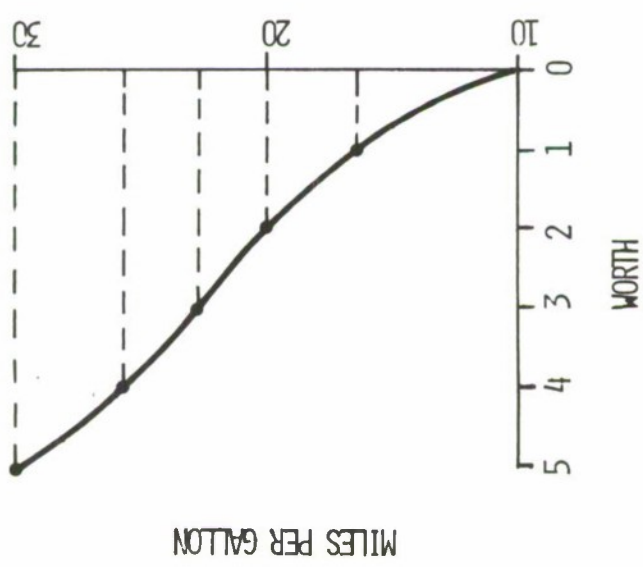
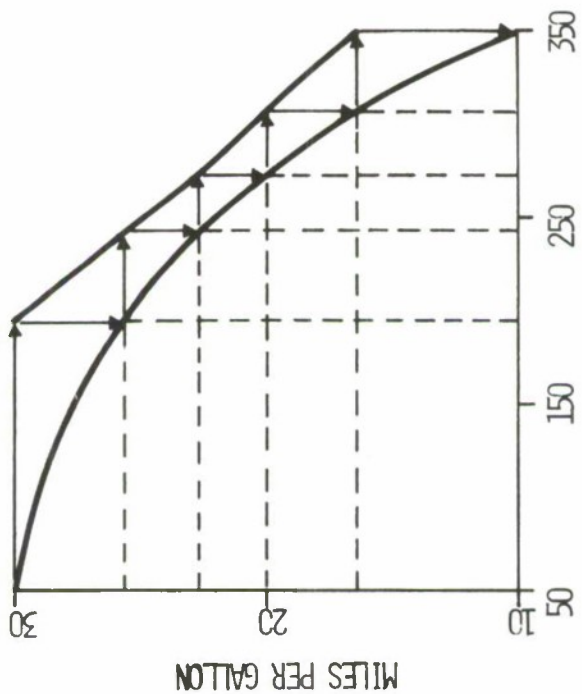


Figure 20. Determining worth functions of independent factors from the indifference plane



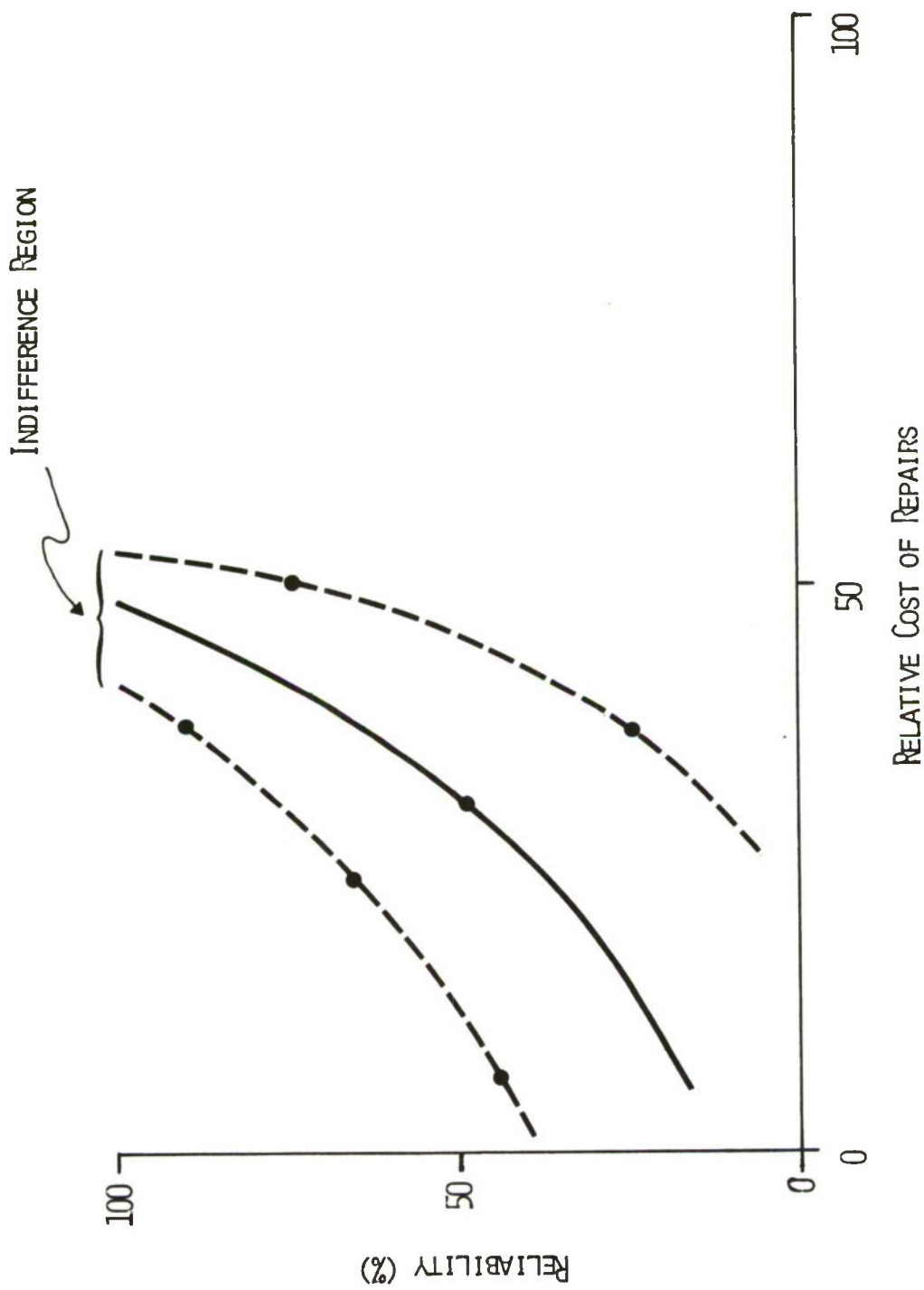


Figure 21. Example of a wide indifference region

Disadvantages. This method involves a long tedious procedure, which may tax the decision maker. The variation which attempts to minimize this problem unfortunately may also introduce error into the assessment. The only solution to the problem is to minimize the number of estimated points per curve and the total number of curves in the plane. There are no set rules to follow in this respect. However, complex curves require more points than simple relationships, and more curves make interpolation easier. In addition, as the researcher becomes more familiar with the technique he will be able to zero in on difference points more rapidly, so practice sessions are useful before attempting a final assessment with a decision maker.

Checking Techniques. Determining whether the indifference curves are logically consistent is the easiest checking technique. The indifference curves should not cross, although they may touch. If the curves cross this implies that the intersection point of the curves is indifferent to points in four different directions, which implies contradictory preferences. Touching is permitted because even though the curves touch they can be assumed to be a small finite distance apart. Touching will most often occur at the ends of the indifference curves. The curves should also be checked for unusual kinks or curves, and the decision maker questioned about any irregularities.

The other checking technique involves presenting the decision maker with random choice preference situations after (or possibly before) the curves have been completed. In our example (Figure 19) he may be asked whether he prefers an auto which has a relative cost of repair of 50 and 30% reliability or an auto which has a relative cost of repair of 90 at 60% reliability. He should prefer the former (50,30). If he prefers the other, the discrepancy should be cleared up.

Sources of Information. This method of worth assessment has been documented by MacCrimmon and Toda<sup>118</sup> and Toda.<sup>119</sup> MacCrimmon<sup>120</sup> describes

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<sup>118</sup> MacCrimmon, K. R. and M. Toda. The experimental determination of indifference curves. Los Angeles: Western Management Science Institute, Working Paper 124, 1968.

<sup>119</sup> Toda, M. Indifference map method for estimating utility functions. Hokkaido Report of Psychology, HRP-1-71-12, August 1971.

<sup>120</sup> MacCrimmon, K. R. Improving the system design and evaluation process by the use of trade-off information: An application to northeast corridor transportation planning. Rand Corp. Memo RM-5877-DOT, April 1969. (PB 185 166)

the use of the method in transportation system planning. Slovic<sup>121</sup> used the technique in an experiment involving trade-offs between cigarettes and money. The trade-off variation is described by Fishburn.<sup>122</sup>

## SUMMARY

Seven general methods for assessing the worth of decision alternatives have been reviewed. All the techniques have been used in practice and in some cases compared with each other.<sup>123,124</sup> In different situations different methods have been found to be most effective, and no single method has been found to be better across all situations. One question, however, remains: Which method should be used in a given situation? Two facts form a loose strategy for choosing a method: the method must meet the requirements of the situation, and it must meet the requirements of the decision maker.

First, the situational requirements must be considered. The researcher should determine the type of output that is needed. For instance, it would serve no function to determine numerical worth if only a ranking of alternatives is required. The inputs should be specified. Are they discrete or continuous? Can they logically be handled independently? For instance, if independence is justified any of the methods are applicable. If not, the indifference curve method or other multivariate method must be used. The researcher must also decide whether the method must conform to an axiom set, or whether one of the more ad hoc procedures will suffice.

Second, the method chosen should be tried on a small scale in order to determine if it is acceptable to the decision maker. Do the scenarios or questions seem logical and realistic for the situation? Is the method comfortable and acceptable for both the researcher and decision maker? Can the method be used with the given resources of time, manpower, etc.? Finally, are the results of the trial reproducible? Are they accurate?

Although there will usually be many more situational requirements to consider or questions to answer, the method chosen must satisfy the requirements and must work in the given situation. Satisfying these two criteria may involve trade-offs or conceivably may require that a method be modified in order to find the best possible worth assessment method.

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<sup>121</sup> Slovic, P. Consistency of choice among equally valued alternatives. Proceedings of the 76th Annual Convention of the American Psychological Association, 1968, 3, 57-59.

<sup>122</sup> Fishburn, P. Independence, trade-offs and transformations in bivariate utility functions. Management Science, 1965, 11, 792-801.

<sup>123</sup> Eckenrode, 1965, op. cit.

<sup>124</sup> Bartlett, C., F. Heerman and S. Rettig. A comparison of six different scaling techniques. Journal of Social Psychology, 1960, 51, 343-348.



Figure 22 presents a summary comparison of the methods described in this paper. No attempt is made to describe variations of methods since the same characteristics apply to each. Detailed discussion of the entries in the matrix can be found in the appropriate section of the text.

CHARACTERISTICS	TECHNIQUES				INDIFFERENCE METHODS
	RANKING METHODS	EQUIVALENCE GROUPING	DIRECT METHODS	GAMBLE METHODS	
Probabilities required	No	No	No	Yes	No
Response required	Preference judgments	Indifference judgments	Qualitative judgments	Indifference judgments usually, but may be a preference judgment	Indifference judgments
Number of factor levels considered	Two	One	Two/Three	Three/Four	Two
Continuous or discrete factors	Discrete only	Discrete only	Either	Either	Continuous
Output	Relative worth	Approximate numerical worth	Numerical worth	Numerical worth	Ranking
Ease of use	Very simple	Very simple	Hard to do for some people	Tedious	Tedious
Training of decision maker	Almost none	Almost none	Moderate	Extensive	Little
Reaction of decision makers	High acceptability	High acceptability	Often requires selling for acceptability	Sometimes frustrating	Can be slow and taxing
Speed	Very quick	Very quick	Moderate	Slow	Slow
Data processing	Very little	Very little	Usually none	Moderate	Can be extensive
Unique worth assignment	No	No	Yes	Yes	Yes
Accuracy (Consistency)	High, if only a few alternatives	High if only a few alternatives	Moderate	Accuracy is a function of patience, range of indifference and understanding of probabilities and risk properties	Possible wide indifference area
Skill and experience required of analyst	Little	Little	Moderate	High	High

Figure 22. Comparison of worth assessment methods

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