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A PRACTICAL ON-LINE FILTER TO PROCESS GYROCOMPASS DATA

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The problem of determining true azimuth from noisy gyrocompass data is presented. First, an adaptive filter is shown to be able to correct the gyrocompass output for environmental effects. Second, it is demonstrated that a low-pass digital filter is able to reduce the measurement noise in the raw data so that it can be properly passed through the adaptive estimator. Computer simulations are presented that indicate that the proposed digital and adaptive filter perform as anticipated.

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Introduction and Problem Statement

The basic problem addressed in this paper is that of providing an accurate real time estimate of a heading direction on a reference test pad. This situation is symbolically depicted in Figure 1. In this figure

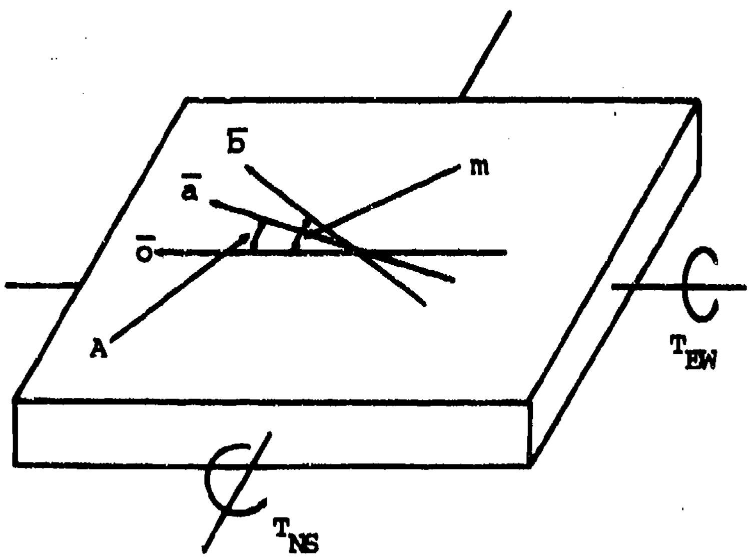

Figure 1. Symbolic Definition of Problem Variables

Note: The main concepts in this report were presented at the 1973 AIAA Guidance and Control Conference, Key Biscayne, Florida, 20-22 August. This report is a revised and corrected form of AIAA Paper No. 73-841.

the three dimensional slab represents the test pad to which is fixed an azimuth measuring device. The function of this device is to measure the angle between a fixed inertial direction and some arbitrary pad reference. Let us assume that the quantity $\bar{a}$ is the desired fixed heading vector and that it is located at an angle A from the pad reference heading vector $\bar{o}$. Because the instrument is sensitive to other variables, we cannot measure A directly but must obtain an estimate of the $\bar{a}$ direction by measuring the angle m formed between the reference $\bar{o}$ and the indicated heading vector $\bar{b}$.

Much work has been done in modeling the error sources involved in this process [1,2]. It is not the aim of this paper to derive a new error model but rather to demonstrate a method by which a reference heading can be obtained from raw gyrocompass data. Only a simplified error model will be presented but the proposed technique can be used equally well with any error model.

In order to demonstrate the proposed method let us assume that only the platform North-South tilt, T_{NS}, and East-West tilt rate, $\dot{T}_{EW}$, are modelled as error sources. Further it is assumed that the difference between the angle m and the angle A is a linear function of the modelled error sources. Thus we can approximate the mathematical relationship between A and m by

$$m = A + C_1 T_{NS} + C_2 \dot{T}_{EW} . \qquad (1)$$

Note that in Equation (1) the coefficients C_1 and C_2 must be determined along with the heading A.

A problem arises in that even in the best of error models other variables not modelled may affect the difference between the measurement m and the heading angle A. That is to say, an equation in the form of (1) is most likely not an accurate model of the measurement process. In order to account for these model inaccuracies we can assume that the quantity A is variable with time. If A were assumed to be a constant quantity along with the coefficients C_1 and C_2 then there would be little hope of obtaining long term, real time estimates of the heading direction. At best all that could be obtained would be average estimates of A for discrete time intervals. Further, if a standard Kalman filter were used to try to track these parameters the filter output would actually diverge [3]. Thus the problem is now to estimate the constant error model coefficients and to track the variable A_k, where now A_k represents the value of A at the kth time instant.

A further problem occurs in that the measurement of m will also be corrupted with measurement noise. The processing technique developed must also account for this fact. Thus Equation (1) can be more accurately represented by

$$m_k = A_k + C_1 T_{NS_k} + C_2 T_{EW_k} + v_k . \qquad (2)$$

In the above equation v_k represents the noise and the subscript k indicates that the quantities so subscripted are time varying and that their value at the kth measurement is $(\cdot)_k$.

Our task is thus to develop a method by which the measurements m_k, T_{NS_k} and $\dot{T}_{EW_k}$ can be processed so as to yield accurate estimates of the coefficients C_1 and C_2 while also tracking a time variable quantity A_k. We will now proceed to derive a method which can perform the above task.

Method of Approach

Before proceeding to develop the method used to solve the proposed problem, Equation (2) will be rewritten here in a more general form. To do this the parameters to be estimated will be written as the column vector x_k, namely

$$x_k^T = \left[A_k, C_1, C_2\right] . \qquad (3)$$

Also the measurement matrix will be defined as a row vector $M(k)$, that is,

$$M(k) = \left[1, T_{NS_k}, \dot{T}_{EW_k}\right] . \qquad (4)$$

Now using the symbol y_k to represent the measurement m_k, Equation (2) can be written as

$$y_k = M(k)\ x_k + v_k . \qquad (5)$$

Equation (5) will be referred to as the measurement equation. In the above equation, v_k is the random measurement noise with zero mean and covariance $E\left\{v_k v_k^T\right\} = R$. In the above it is assumed that the measurement equation is scalar. If more than one heading measurement is to be processed at one time this approach is still applicable though appropriate changes must be made to the equations throughout this paper.

It should be noted here that any error model equation can be used as long as the resultant measurement equation is of the same form as Equation (5).

In general the value of the state vector at time t_{k+1} can be related to its value at time t_k by means of the state transition matrix $\phi(k+1/k)$. This allows us to write

$$x_{k+1} = \phi(k+1/k)\, x_k + \Gamma(k)\, w_k . \tag{6}$$

Equation (6) will be referred to as the state equation. In this equation w_k is the system noise and $\Gamma(k)$ is the system noise coefficient matrix at time t_k. The system noise coefficient matrix relates the effect that the system noise has upon the states. The quantity w_k is a random variable with mean zero and covariance given by $E\left\{w_k w_k^T\right\} = q$.

For the present problem it is assumed that the states are constant between measurements, thus $\phi(k+1/k) = I$. Because it was assumed that the system noise is only in the first state, $\Gamma(k)$ can be given as $\Gamma^T(k) = [1, 0, 0]$. Thus all quantities in the state and measurement equations are specified.

The object of the approach presented below is to allow for variations in the parameter A by estimating the system noise covariance q. This will be done by using the residual between the actual measurement and a predicted measurement that is based upon past values of the states [4,5].

Assume that we have processed the measurement y_k and now wish to bring the filter and states forward to time t_{k+1}. In the processes of going forward in time we wish to use that q value which yields the predicted residual, $r(k+1/k)$, that is the most probable. In other words, find q according to

$$\max_{q\geq 0} \; p[r(k+1/k)] \tag{7}$$

where $p[\quad]$ is the probability density function. The q that maximizes (7) will be denoted by $\hat{q}_k$.

Let us now define the predicted residual by

$$r(k+1/k) = y_{k+1} - M(k+1)\,\hat{x}(k+1/k) \; . \tag{8}$$

In this equation $\hat{x}(k+1/k)$ is the estimate of the state at time t_{k+1} given the measurements up to y_k. The mean of the predicted residual can now be given by

$$E[r(k+1/k)] = E[y_{k+1} - M(k+1)\ \hat{x}(k+1/k)]\ ,$$
$$= M(k+1)\ E[x_{k+1} - \hat{x}(k+1/k)] + E[v_{k+1}]\ ,$$
$$= 0\ . \qquad (9)$$

This is by virtue of the fact that the expected value of the measurement noise is assumed to be zero as is the expected value of $[x_{k+1} - \hat{x}(k+1/k)]$, the error between the predicted state and the true state.

Further the variance of the residual can be given by

$$E\left[r(k+1/k)r^T(k+1/k)\right] =$$
$$E\left\{\left[y_{k+1} - M(k+1)\hat{x}(k+1/k)\right]\left[y_{k+1} - M(k+1)\hat{x}(k+1)\right]^T\right\}\ ,$$

or

$$E\left[r(k+1/k)r^T(k+1/k)\right] =$$
$$E\left\{\left\{M(k+1)\left[x_{k+1} - \hat{x}(k+1/k)\right] + v_{k+1}\right\}\left\{M(k+1)\left[x_{k+1} - \hat{x}(k\ 1/k)\right] + v_{k+1}\right\}^T\right\}\ . \qquad (10)$$

Letting ε_{k+1} be the error in the estimate at time t_{k+1}, that is, $\varepsilon_{k+1} = x_{k+1} - \hat{x}(k+1/k)$, equation (1) can be simplified to yield

$$E\left[r(k+1/k)r^T(k+1/k)\right] =$$
$$E\left\{\left[M(k+1)\varepsilon_{k+1} + v_{k+1}\right]\left[M(k+1)\varepsilon_{k+1} + v_{k+1}\right]^T\right\}$$

or

$$E\left[r(k+1/k)\,r^T(k+1/k)\right] =$$

$$M(k+1)\ E\left\{\epsilon_{k+1}\ \epsilon_{k+1}^T\right\}\ M^T(k+1) + E\left\{v_{k+1}\ v_{k+1}^T\right\}$$

$$+\ M(k+1)\ E\left\{\epsilon_{k+1}\ v_{k+1}^T\right\} + E\left\{v_{k+1}\ \epsilon_{k+1}^T\right\}\ M^T(k+1)\ .$$

The residual's variance can now be written as

$$E\left[r(k+1/k)\,r^T(k+1/k)\right] =$$

$$M(k+1)\ P(k+1/k)\ M^T(k+1) + R(k+1)\ , \tag{11}$$

where the state error covariance matrix is denoted as

$$P(k+1/k) = E\left\{\epsilon_{k+1}\ \epsilon_{k+1}^T\right\}\ . \tag{12}$$

Recall from the equations for the basic Kalman filter that the system noise covariance enters into the calculation of $P(k+1/k)$ [6]. This relationship can be given by

$$P(k+1/k) = \phi(k+1/k)\ P(k/k)\ \phi^T(k+1/k) + q\ \Gamma(k)\ \Gamma^T(k)\ , \tag{13}$$

or for this problem

$$P(k+1/k) = P(k/k) + \begin{pmatrix} q & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\ . \tag{14}$$

Now if we assume a gaussian distribution of the residuals for the scalar measurement case

$$p\left[r(k+1/k)\right] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{r^2}{\sigma^2}} \tag{15}$$

where

$$\sigma^2 = E\left[r(k+1/k)\ r^T(k+1/k)\right] \tag{16}$$

and

$$r^2 = r^T(k+1/k)\ r(k+1/k)\ . \tag{17}$$

Thus the variance σ^2 is a function of q and hence (15) is also dependent on the quantity q.

This fact allows us to minimize Equation (15) with respect to q by differentiating it with respect to σ^2 because $\frac{\partial(\sigma^2)}{\partial q}$ is constant. Proceeding with the differentiation and equating the result to zero we get

$$\frac{\partial p(r)}{\partial\sigma^2} = \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{r^2}{\sigma^2}}\right)\left(\frac{r^2}{2\sigma^4}\right) - \frac{1}{2\sigma^2\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{r^2}{\sigma^2}} = 0\ .$$

This implies that

$$\frac{r^2}{2\sigma^4} - \frac{1}{2\sigma^2} = 0$$

which says that the maximum probability occurs when

$$r^2 = \sigma^2$$

or equally when

$$r^2(k+1/k) = E\left[r^2(k+1/k)\right] . \qquad (18)$$

Now using Equation (13) in (11) we can get

$$E\left[r^2(k+1/k)\right] =$$

$$M(k+1)\ \phi(k+1/k)\ P(k/k)\ \phi^T(k+1/k)\ M^T(k+1)$$

$$+ q\ M(k+1)\ \Gamma(k)\ \Gamma^T(k)\ M^T(k+1) + R(k+1) . \qquad (19)$$

Letting

$$E\left[r^2(k+1/k)q = 0\right] =$$

$$M(k+1)\ \phi(k+1/k)\ P(k/k)\ \phi^T(k+1/k)\ M^T(k+1) + R(k+1) ,$$

we can get from (18) and (19)

$$\hat{q}_k = \begin{cases} \dfrac{r^2(k+1/k) - E[r^2(k+1/k)\ q = 0]}{M(k+1)\ \Gamma(k)\ \Gamma^T(k)\ M^T(k+1)} & \text{if} > 0 \\ 0 & \text{otherwise.} \end{cases} \quad (20)$$

where $\hat{q}_k$ must be > 0 from the physical meaning of the variance.

Equation (20) can now be simplified for the specific problem described. This simplification allows us to write

$$M(k+1)\ \Gamma(k)\ \Gamma^T(k)\ M^T(k+1) =$$

$$\left[1, T_{NS}, \dot{T}_{EW}\right]_k \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [1\ 0\ 0] \begin{bmatrix} 1 \\ T_{NS} \\ \dot{T}_{EW} \end{bmatrix}_k = 1$$

and

$$E\left[r(k+1/k)q = 0\right] = M(k+1)\ P(k/k)\ M^T(k+1) + R(k+1).$$

So

$$\hat{q}_k = \begin{cases} r^2(k+1/k) - M(k+1)P(k/k)M^T(k+1) - R(k+1) & \text{if} > 0 \\ 0 & \text{otherwise.} \end{cases} \quad (21)$$

If preferred Equation (22) can also be expressed in terms of the measurement y_{k+1} as

$$\hat{q}_k = \begin{cases} \left[y_{k+1} - M(k+1)\hat{x}(k+1/k)\right]^2 - M(k+1)P(k/k)M^T(k+1) - R(k+1) & \text{if} > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

Referring to Equation (20) we note that $\hat{q}_k$ is directly proportional to the excess of the residual squared over the predicted value of the residual squared where the predicted value is based upon the assumption that the system noise is zero. If the residual squared is greater than the value predicted under the assumption of no system noise, then a positive value of $\hat{q}_k$ is generated which indicates that the no noise assumption was most likely false. This value of $\hat{q}_k$ is then used in later processings to produce more consistency between the residuals and their predicted statistics.

The filter adapts to the system noise level as follows. If Equation (20) yields a non-positive value for $\hat{q}_k$ then the residuals are within their 1σ values and the assumption of no input noise was probably true. The residuals are thus behaving according to their statistics, are relatively small and the filter is operating satisfactorily. On the other hand if the residuals are larger than the predicted 1σ values, the filter is actually diverging. This then generates a positive $\hat{q}_k$ value which then causes $P(k+1/k)$ to increase as seen from Equation (14).

The Kalman gain can be given by [7]

$$K(k+1) = P(k+1/k)\, M^T(k+1) \left[M(k+1)\, P(k+1/k)\, M^T(k+1) + R(k+1)\right]^{-1} . \tag{23}$$

We note that increasing $P(k+1/k)$ causes the gain to increase. The increased gain then causes the filter to become more sensitive to the latest data. This allows the filter to follow parameter changes that become evident as later data is processed. This is in contrast to the standard filter that tries to fit constant parameter values to all of the data and which in truth is biased towards earlier data. This bias comes about from the fact that filter gain decreases as more and more data are processed and $P(k+1/k)$ decreases [5].

The approach presented can be used processing one residual at a time or processing many residuals at once. This later approach is more statistically significant although does complicate the filter computations. Because the value of $\hat{q}_k$ responds to large measurement noises as well as to large system noises, a statistical approach to calculate $\hat{q}_k$ may be preferred. To process many residuals, we use the sample mean given by

$$\bar{r} = \frac{1}{N}\left[\sum_{i=1}^{N} \frac{r(i+1/i)}{R^{1/2}(i+1)}\right]. \tag{24}$$

This value is then used in the equations for $\hat{q}_k$ with appropriate modifications.

The main advantage of using an adaptive filter is that it allows the estimates of the states to follow the variations in the true states.

Thus the filter should yield varying parameters that track the true parameters accurately, rapidly and with minimal computational burden.

Figure 2 shows the basic flow of information within the filter. It should be noted that this differs from the standard Kalman filter

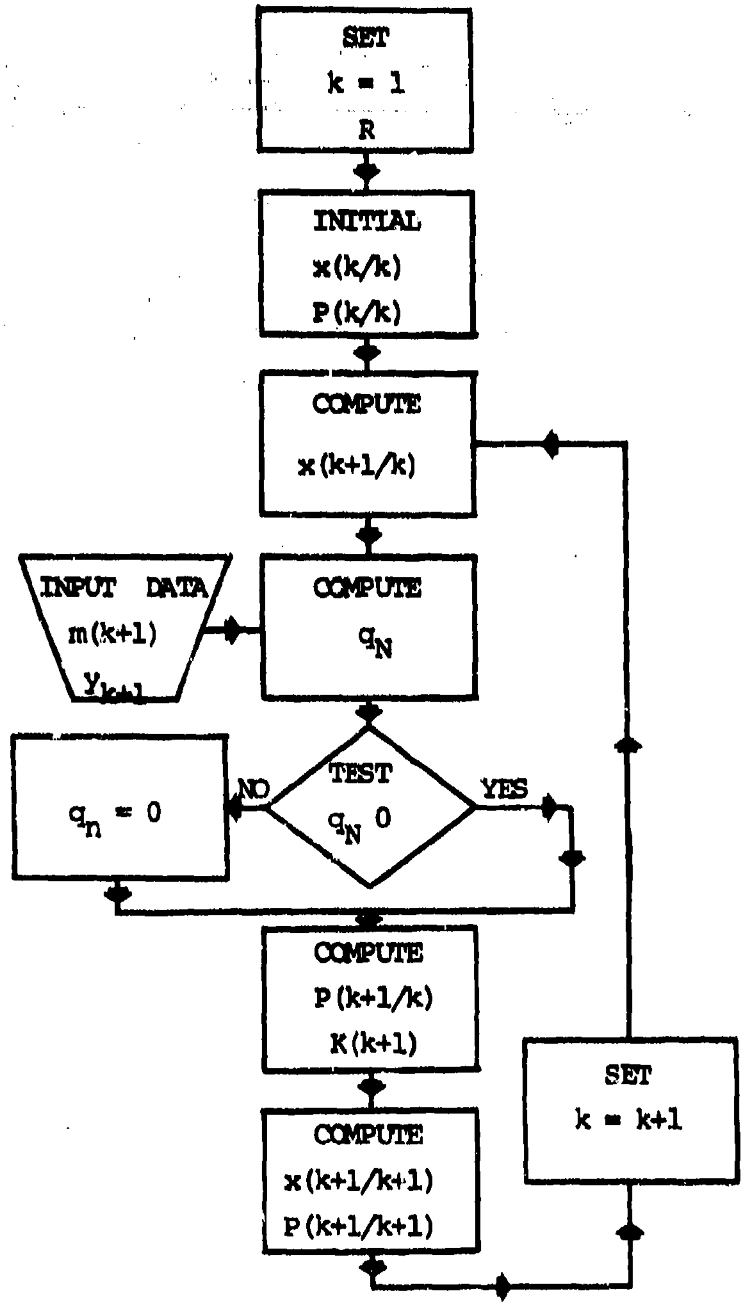

Figure 2. Adaptive Filter Information Flowgram

only in the blocks that calculate $\hat{q}_N$ and that update $P(k+1/k)$ if a positive $\hat{q}_N$ is found.

To test the ability of the adaptive filter to follow the variable heading A_k a sample program was simulated using the Fortran IV language on a Burroughs B6700 computer. For this simulation it was assumed that the variation in A_k was sinusoidal with a mean of 15.0 and a peak to peak variation of 6.0. The period of this signal was taken to be 24 hours. The tilt and tilt rate used had a period of 1 hour and varied as the sine and cosine respectively. Both were assumed to have a zero mean with peak to peak variations in the tilt of 2.6 and in the tilt rate of 1.6. The true values of C_1 and C_2 were chosen to be 0.4 and 0.2.

For this simulation three residuals were averaged and three measurements were processed simultaneously. Each time a new measurement was available the oldest measurement was discarded and the three newest measurements were processed. In this manner a new estimate of A_k could be obtained each time a data point was available. It was also assumed that the measured heading, tilt and tilt rate were sampled every 10.5 minutes.

The results obtained when the above case was simulated with measurement noise free data being processed through the filter are shown in Figure 3. For the simulation shown the initial values of A_k, C_1 and C_2 were chosen as 12.0, 0.1, and 0.0. In this figure we see plotted the noise free measurement m_k, the true heading A_T, and the heading estimate obtained from the adaptive filter, A_F. Figure 4 shows the

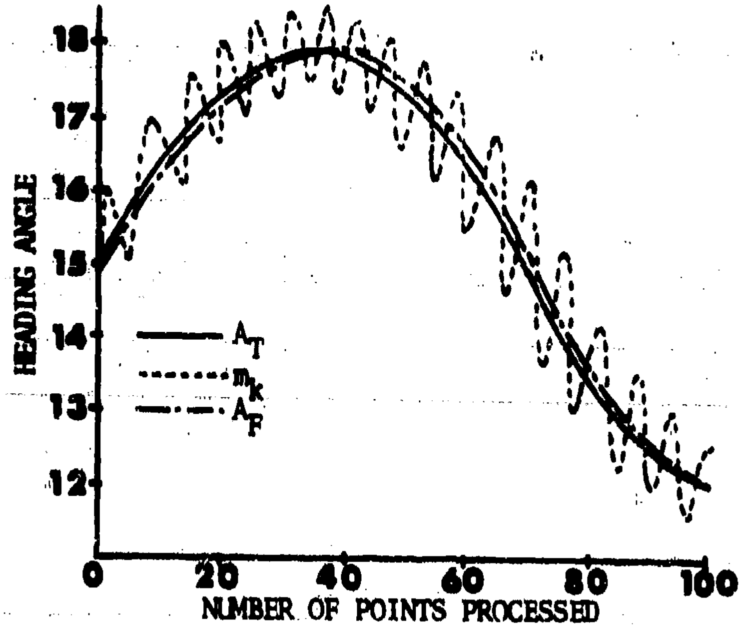

Figure 3. Results of Processing True Data through the Adaptive Filter

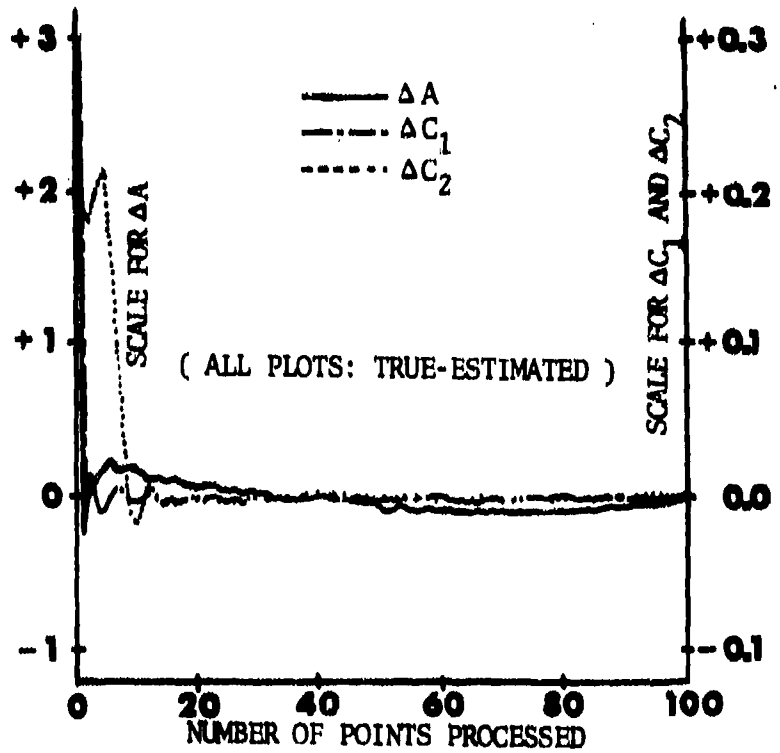

Figure 4. Error in Parameter Estimates using True Data

error in estimating A_k, C_1 and C_2 as a function of the number of points processed. As seen from these two figures the adaptive filter is able to give quite accurate results.

On the other hand when the same situation is simulated using measurements that are corrupted with random noise the filter does not perform as well. In Figure 5 we see the results when the measurement noise has a standard deviation of 1.0. Here the noisy measurement m_k

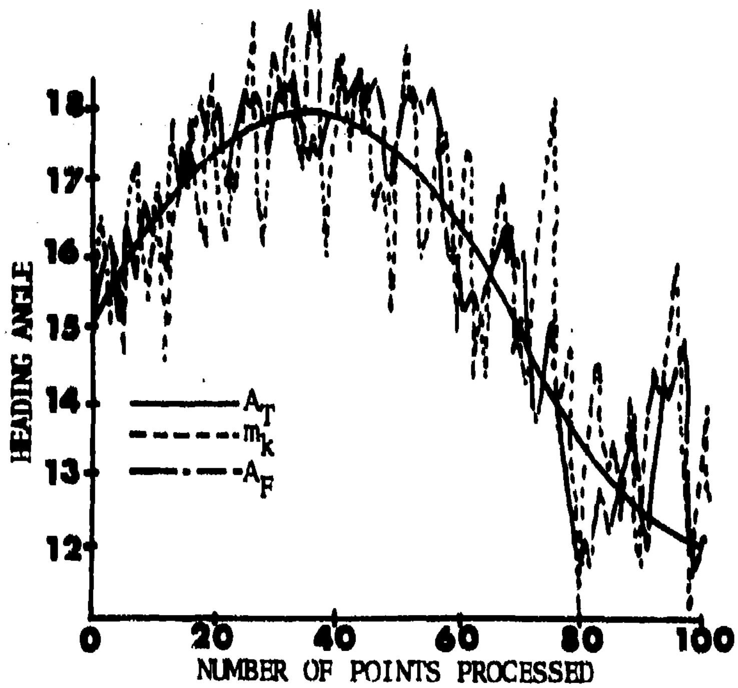

Figure 5. Results of Processing Noisy Data through the Adaptive Filter

is shown along with the true and estimated headings A_T and A_F. As seen in this figure the adaptive filter does not perform satisfactorily for this large a measurement noise. Even using the averaged residual the filter still reacts to the measurement noise as well as to the actual variations in the heading.

In order for this filter to behave properly, the noise level of the input signal must be relatively small. The next portion of this paper shows how this secondary task can be accomplished.

Noting that in general the measurement signal will be of a lower frequency than the measurement noise, it appears that a low-pass filter is needed. This type of filter will allow low frequency signals to pass almost unattenuated while attenuating the higher frequency components. Many different types of digital filters have been derived that are able to perform this task. A large number of these are the extension of analog filters into the digital domain [8,9].

Each of the available filter types has advantages and disadvantages. For this application a Butterworth filter will be chosen because it is monotonic in both passband and in stopband. This type of filter is able to be represented in digital form by the following squared magnitude function

$$\left|H\left(e^{j\omega T}\right)\right|^2 = \frac{1}{1 + \dfrac{\tan^{2n}(\omega T/2)}{\tan^{2n}(\omega_c T/2)}} \quad . \tag{25}$$

In this equation n is the number of poles of the filter and T is the time between samples. The cutoff frequency, ω_c, is the frequency at which the filter gain falls off to 3 db.

Figure 6 shows the gain of the digital Butterworth filter as a function of frequency for various values of n. While at first it may appear that a large value of n is desirable it should be pointed out that the phase difference between the signal and the filter output increases with increasing n. Thus the value of n should not be any larger than necessary if real time data processing is desired.

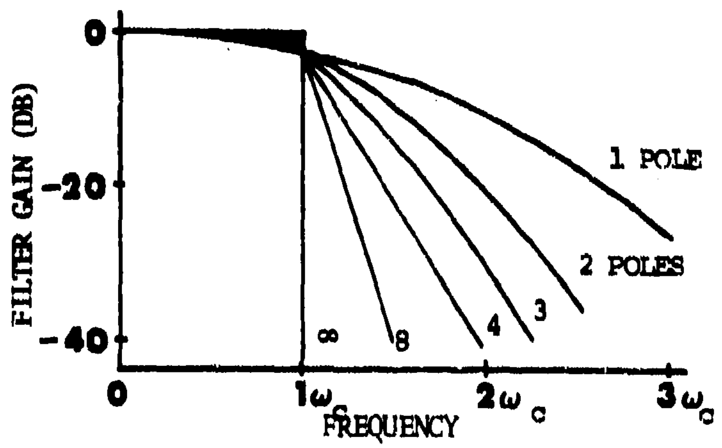

Figure 6. Digital Butterworth Filter Gain versus Frequency for a Variety of Poles

In order to determine the minimum number of poles to be used in the filter the desired operating criteria have to be specified. We will assume that the highest signal we want to pass through the filter is 1 cycle/hr., corresponding to the tilt and tilt rate effects. Thus the cutoff frequency will be 1 cycle/hr. or 27.78×10^{-5} Hz. Assuming that that we have a sample every 10.5 minutes gives a sampling rate of 158.73×10^{-5} Hz. Further it is specified that at least a 40 db drop is required at 2 cycles/hr. This implies that the squared magnitude function equal 10^{-4} at 55.56×10^{-5} Hz. Using these numbers in Equation (25), n is readily calculated to be 3.95, which implies that a 4 pole Butterworth digital filter can perform the required task.

Proceeding with the design of the filter we first express the general fourth order Butterworth filter as [10]

$$H(s) = \frac{\text{constant}}{s^4 + 2.6\ s^3 + 3.4\ s^2 + 2.6\ s + 1} . \tag{26}$$

Replacing s by s/a, where a will equal $\tan(\omega_c T/2)$, Equation (26) can be rewritten

$$H(s) = \frac{a^4}{s^4 + 2.6\ as^3 + 3.4\ a^2s^2 + 2.6\ a^3s + a^4} . \tag{27}$$

In this form the constant has been adjusted so that the gain at $s = 0$ is equal to unity.

Equations (26) and (27) are still in analog form and must now be transformed into the digital domain. This is accomplished by letting $s = (z-1)/(z+1)$. This transformation allows the filter transfer function to be written as

$$H(z) = \frac{a^4(z^4 + 4z^3 + 6z^2 + 4z + 1)}{\alpha z^4 + \beta z^3 + \gamma z^2 + \delta z + \varepsilon} . \tag{28}$$

The Greek symbols used in Equation (28) are defined by

$$
\begin{aligned}
\alpha &= 1 + 2.6\,a + 3.4\,a^2 + 2.6\,a^3 + a^4 \\
\beta &= -4 - 5.2\,a + 5.2\,a^3 + 4\,a^4 \\
\gamma &= 6 - 6.8\,a^2 + 6\,a^4 \\
\delta &= -4 + 5.2\,a - 5.2\,a^3 + 4\,a^4 \\
\varepsilon &= 1 - 2.6\,a + 3.4\,a^2 - 2.6\,a^3 + a^4
\end{aligned}
\tag{29}
$$

and were obtained after algebraic simplification of the resulting equation when the s to z transform was used in Equation (27).

In order to express Equation (28) in terms of measurement sample times a few more substitutions are required. First Equation (28) is multiplied by z^{-4}/z^{-4} to obtain

$$H(z) = \frac{a^4(1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4})}{\alpha + \beta z^{-1} + \gamma z^{-2} + \delta z^{-3} + \varepsilon z^{-4}} . \tag{30}$$

Recalling that the transfer function relates the output of a system to its input, Equation (30) can be written as

$$\frac{Y_f(z)}{Y_m(z)} = \frac{a^4(1 + 4z^{-1} + 6z^{-2} + 4z^{-3} + z^{-4})}{\alpha + \beta z^{-1} + \gamma z^{-2} + \delta z^{-3} + \varepsilon z^{-4}} . \tag{31}$$

where $Y_f(z)$ is the filter output and $Y_m(z)$ is the filter input.

Equation (31) now relates the output of the filter to the input in the z plane. The transformation to the time sampled domain is now rather straightforward.

This process is begun by expressing Equation (31) in block diagram form. Figure 7 is the result of performing a direct transform from the

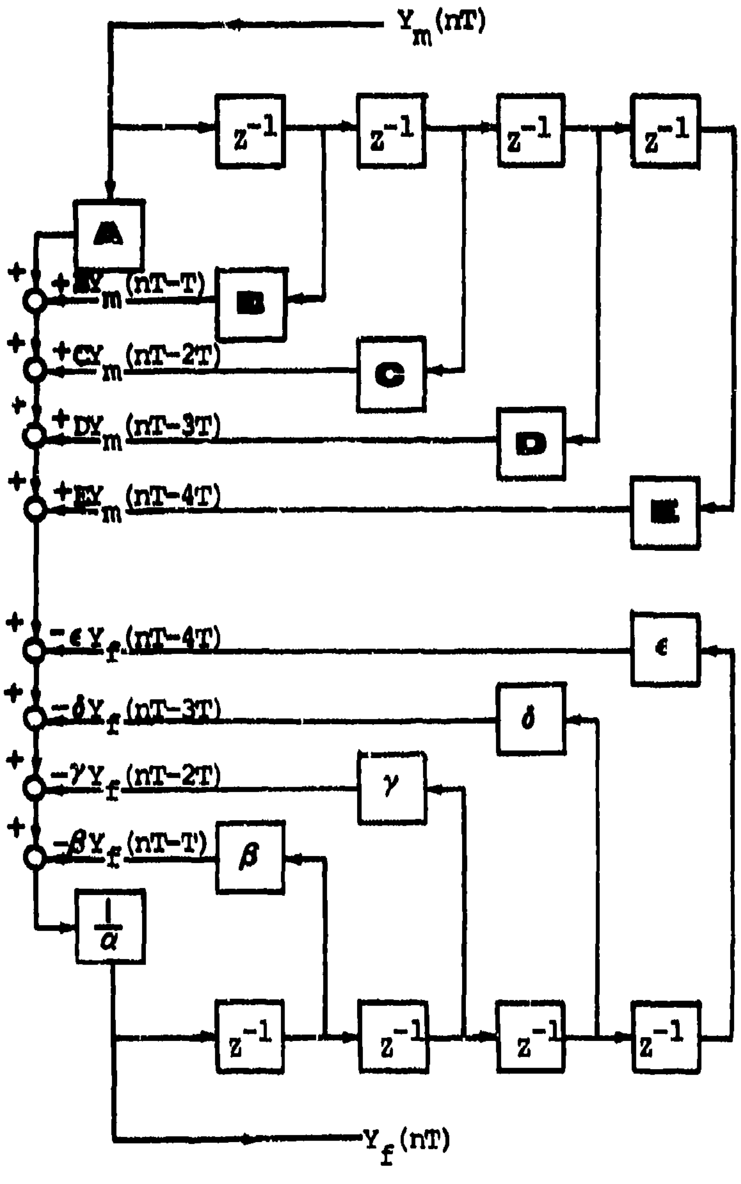

Figure 7. Block Diagram of Digital Filter

equation of the transfer function to its block diagram form. Also in this figure it is shown how the z^{-1} block relates to the samples, that is, a block whose transfer function is z^{-1} represents a delay of one time sample. This block diagram form now allows the equation that relates the filtered output to the input to be written as

$$\alpha Y_f(nT) + \beta Y_f(nT-T) + \gamma Y_f(nT-2T) + \delta Y_f(nT-3T) + \epsilon Y_f(nT-4T) =$$
$$AY_m(nT) + BY_m(nT-T) + CY_m(nT-2T) + DY_m(nT-3T) + EY_m(nT-4T) \ , \tag{32}$$

where the symbols used are defined below.

$$A = a^4$$
$$B = 4\ a^4$$
$$C = 6\ a^4$$
$$D = 4\ a^4$$
$$E = a^4$$
$$\alpha = 1 + 2.6\ a + 3.4\ a^2 + 2.6\ a^3 + a^4$$
$$\beta = -\ 4 - 5.2\ a + 5.2\ a^3 + 4\ a^4$$
$$\gamma = 6 - 6.8\ a^2 + 6\ a^4$$
$$\delta = -\ 4 + 5.2\ a - 5.2\ a^3 + 4\ a^4$$
$$\epsilon = 1 - 2.6\ a + 3.4\ a^2 - 2.6\ a^3 + a^4$$

It should be noted that Equation (32) is the filter realized in direct form and may not be the best to use for the processing of actual data. The subject of how to realize the filter as well as more details on the s and z transform are covered in many texts and the interested reader is referred to some of them [11,12,13,14].

Equation (32) indicates that the present filter output $Y_f(nT)$ is related to the last four filter outputs as well as to the last four measurements as well as the present measurement $Y_m(nT)$. It is this dependence on past measurements as well as past outputs that introduces the increasing phase difference between the input and output. The order of the filter determines how many past data values are to be used. It is for this reason that the order of the filter should be kept relatively small. Also the lower the filter order, the easier it is to program.

The results obtained when noisy data were passed through the digital filter just presented are shown in Figure 8. This figure shows the true

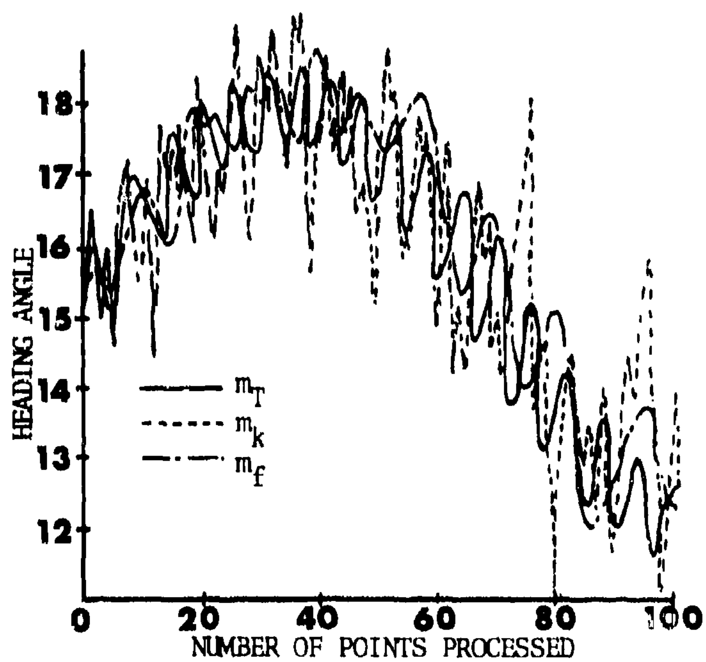

Figure 8. Results of Processing Noisy Data through the Digital Filter

signal, m_T, the measured signal, m_k, and the filtered output, m_f. It should be noted that the filtered output is a large improvement over the raw data in indicating the true signal. The phase difference is also evident in this figure.

Results and Conclusions

The results obtained when the noisy data is first passed through the digital filter and then through the adaptive estimator are presented in Figure 9. In this figure we see the actual heading angle, A_T, the

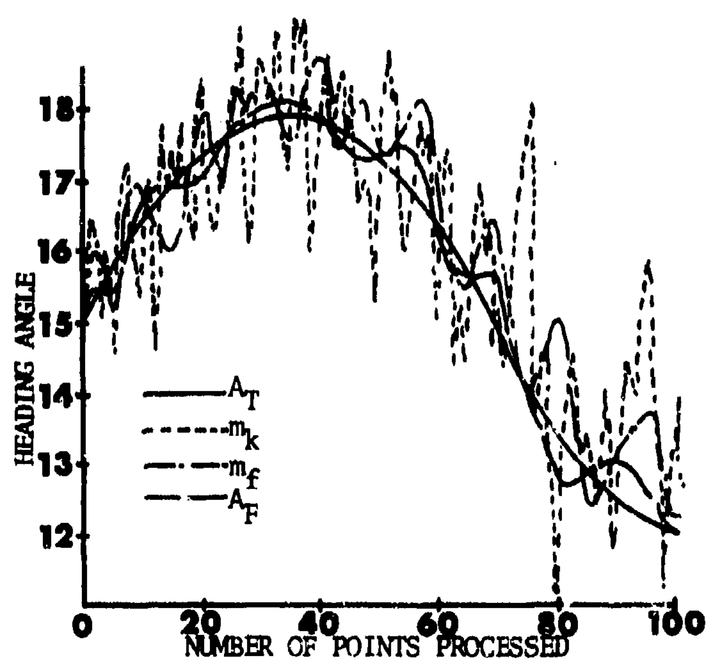

Figure 9. Results of Processing Noisy Data through both Filters

actual measurement, m_k, the filtered measurement m_f and the estimated heading angle A_F. Comparing this figure with Figure 5 it is noted that digital processing of the measurement is indeed an improvement over passing the raw data directly into the adaptive estimator.

Using actual gyrocompass data has been investigated and some results are presented in Figure 10. This figure shows the results of passing normalized raw gyrocompass data through the digital filter as well as

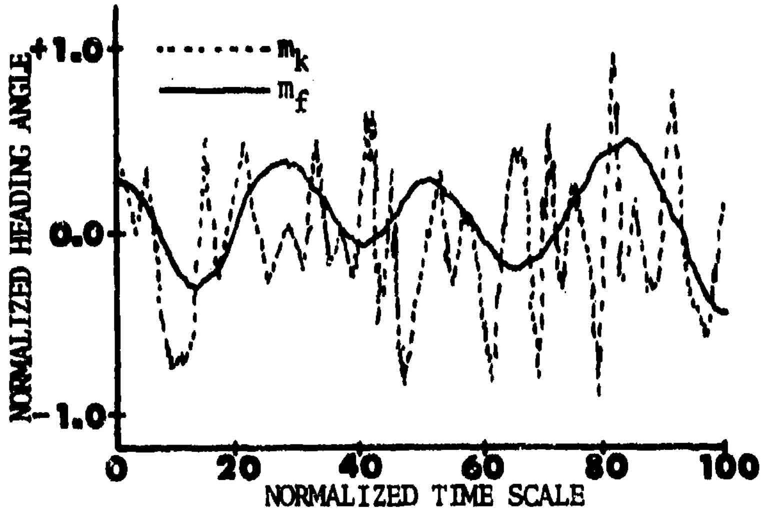

Figure 10. Results of Processing Actual Gyrocompass Data through the Digital Filter

depicting the normalized raw data. No statement can be made as to how well this approximates the true heading angle because this fact is not continuously known. The passing of this digitally filtered data through the adaptive filter could not be accomplished because the concurrent tilt and tilt rate information was not available.

Based upon the results obtained using the simulated data, where a comparison can be made to the truth, it appears that the proposed process is able to satisfactorily track a variable heading while simultaneously solving for error model coefficients. This system can be satisfactorily used for real time data processing because the total time required to digitally filter and then to adaptively estimate the parameters is much less than the time between samples. Consequently, this analytical technique should be of great value in the determination of a continuous heading reference that can be used as a laboratory standard.

Theoretically, this procedure can be modified to further improve the azimuth measurement accuracy of a gyrocompassing system. The provision for the incorporation of additional measurable error model terms allows for this amelioration.

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