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LARGE DEFLECTIONS OF THIN ELASTIC PLATES

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М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

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RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

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rot	curl
lg	log

LARGE DEFLECTIONS OF THIN ELASTIC PLATES

V. V. Nekhotyaev and A. V. Sachenkov

The wide application of thin-walled constructions, elements of which are plates of different configurations, requires the development of theoretical and experimental methods of their calculation with deflections, comparable with the thickness.

In the article ~~is~~ given a summary of the main scarce works, dedicated to the study of large deflections of plates, subjected to the action of evenly distributed transverse load. In mathematical relation these problems are reduced to the solution of Karman nonlinear equations, the precise solution of which is known only for an evenly loaded circular plate with restrained contour. This solution, given by Way, is given in [5].

Other problems about large deflections of plates, which are examined by the authors of the works given in the bibliography, are solved, as a rule, by known approximation methods. The results obtained on the basis of these methods almost always give good agreement with the available experimental results. This suggests the possibility of the effective use of the theoretical-experimental method for the solution of similar problems. In accordance with this in the last paragraph of the article there are provided the results of the experimental study carried out by the authors of the large deflections of free rectangular and parallelogram plates under the action of uniform transverse load. The results are illustrated by graphs.

In the conclusion of the article we focus attention on the need for the advisable and economically substantiated application of different methods of solution of the problems. It is completely clear that there is an essential difference in whether that or another problem is implemented during several days, months, etc., or an uncontrolled large time interval is expended for its solution.

Therefore the problem of the substantiation of the application of one method or the other, the problem of the comparison of the possibilities of mathematical and physical experiments are an exceptionally urgent problem, since it is connected with the question concerning the rise of productivity of labor.

Designations:

- h - thickness of plate;
- a, b - sides of rectangular plate;
- x, y, z - Cartesian coordinate system;
- q - intensity of transverse load;
- u, v - displacement of points of the middle surface of the plate; along axes x, y respectively;
- W - deflection of arbitrary point of the middle surface;
- f - deflection;
- σ_{xm}; σ_{ym} - normal stresses in the middle surface of the plate or shell along lines x, y;
- τ - tangential stresses in the middle surface;
- $\sigma_{xи}$; $\sigma_{yи}$ - normal stresses, which correspond to the bending moments;
- $\tau_{и}$ - tangential stress, which corresponds to torque;

E - elastic modulus of material;

μ - Poisson ratio;

$=Eh^3/12(1-\mu^2)$ - cylindrical rigidity;

λ - relationship of sides of the plate;

$q^* = \frac{qb^4}{Eh^4}$ - dimensionless parameter of load;

$W_0^* = f/h$ - parameter of deflection in the center;

$\sigma^* = \frac{\sigma b^2}{Eh^2}$ - parameter of stress;

$\sigma_i = \sigma_{im} + \sigma_{iи}$ - total stress, where i=x, y.

§ 1. Rectangular plates

The state of the question concerning large deflections of rectangular plates under the action of uniform transverse load is sufficiently fully reflected in [1, 5]. Concerning the historical side of the question it should be noted that the first work, connected with the study of powerful cylindrical bending of a long band (plate), belongs to Russian naval engineer I. G. Bubnov. The solution of this problem is precise and is given in the mentioned works [1, 5]. For plates with finite relationship of sides there are no mathematically precise solutions. The common methods of solution of these problems are variational methods, finite-difference and the method of collocation, the last of which began to be widely applied in connection with the implementation of calculation by electronic digital computer into practice.

In [1] the problem of bending of rectangular plate is solved by A. Feppl's method of "imposition" of solutions for rigid and absolutely flexible plates. Special attention in this case is given to the case of hinged supported square plate with fixed edges, whereupon it is considered that the edges of the plate remain rectilinear in the process of deformation.

The expression for deflection is selected in the form of double trigonometric series:

$$W = f_{11} \sin\frac{\pi x}{a} \cdot \sin\frac{\pi y}{b} + f_{12} \sin\frac{\pi x}{a} \cdot \sin\frac{2\pi y}{b} + f_{21} \sin\frac{2\pi x}{a} \cdot \sin\frac{\pi y}{b}, \ldots \quad (1)$$

or

$$W = \sum_{m=1,2\ldots} \sum_{n=1,2\ldots} f_{mn} \cdot \sin\frac{m\pi x}{a} \cdot \sin\frac{n\pi y}{b}.$$

f_{mn} - are independent indeterminate parameters.

Each of the terms of this series satisfies the boundary conditions of hinged support. Naturally, the solution of the problem is more precise, the more terms of the series that are kept. However the obtaining of refined results in comparison with the case where deflection is selected in the first approximation,

$$W = f_{11} \cdot \sin\frac{\pi x}{a} \cdot \sin\frac{\pi y}{b} \quad (2)$$

is quite difficult, since instead of one cubic equation it is necessary to deal with a system of cubic equations, the number of which is equal to the number of approximations.

The solution of the problem is somewhat facilitated in the case of a square plate, since in view of the symmetry of the bent surface the number of equations is decreased.

a) Square plate with freely shifting edges.

As computations showed, the account of additional terms in (1) introduces insignificant correction with respect to the first approximation. For example, with $q^*=500$ the additional coefficient greatest in value $f_{13}=f_{31}$ is 7% of the main f_{11}, and the remaining coefficients do not exceed 3%. The selection of the expression for deflection in the form (1) is explained by the fact that the curvature of the bent surface of the plate in the central part is less than the curvature calculated according to the first approximation (2). With increase of the load the curvature of the central part is gradually decreased. Figure 1 gives the graph of the dependence of dimensionless parameters of deflection in the center of the plate, where curve 1 represents the first approximation, curve 2 - the refined solution, the shaded part - the region of experimental values for plates with bending edges.

The observed disagreements of theory and experiment are explained by the fact that in the theoretical solution the edges of the plate remain rectilinear in the process of deformation, and in the experiment the edges of plates could be bent in the plane of the supporting contour. This is confirmed by the theoretical solution of R. Kaiser, pertaining to the case of freely bending edges [22].

In figure 1 with $q^*=117$ and $W_0^*=2.5$ this solution is marked by point 3.

The graphs of deflection-stress dependences in the middle surface (membrane stresses) and deflection-bending stress are given in figures 2-3, from which it is evident that in the case of the first approximation (curves 1) stresses in the middle surface reach maximum in the center of the plate (elongation along axes x and y) and at the corner (compression in both directions), where the absolute values of stresses at these points are identical. From the results of the refined solution (curves 2) it is evident that the stress in the corner increases with increase of the deflection somewhat faster than in the center.

From graphs it follows that the dangerous zone with small deflections is located in the center of the plate, and with large - in the corner. Data of experiment confirm this. Further the problem is examined for the case where the edges of the plate

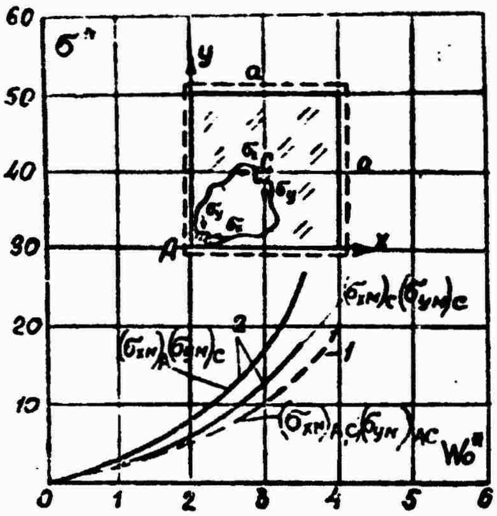


Figure 1

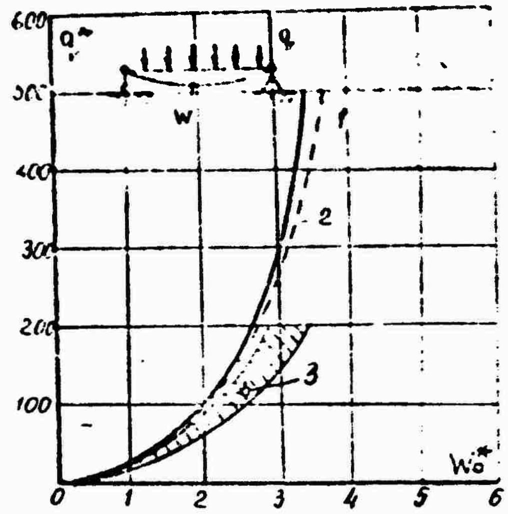


Figure 2

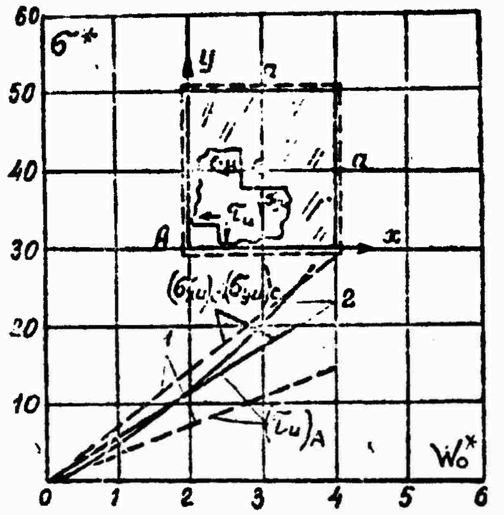


Figure 3

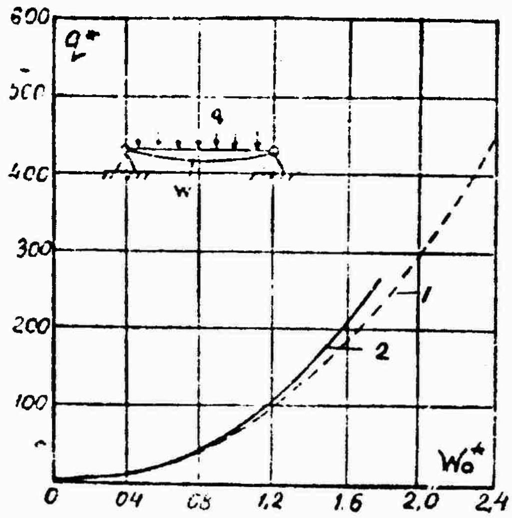


Figure 4

cannot be brought together. Figure 4-6 gives the graphs of dependence: deflection-load, deflection-stress of middle surface and deflection-bending stress.

From these graphs it is evident that in this case with large deflections ($W_0^*>1.7$) the stresses in the corner of the plate become dangerous. With certain load, the stresses in the middle surface of the plate with restrained edges are considerably more than in the case of free edges.

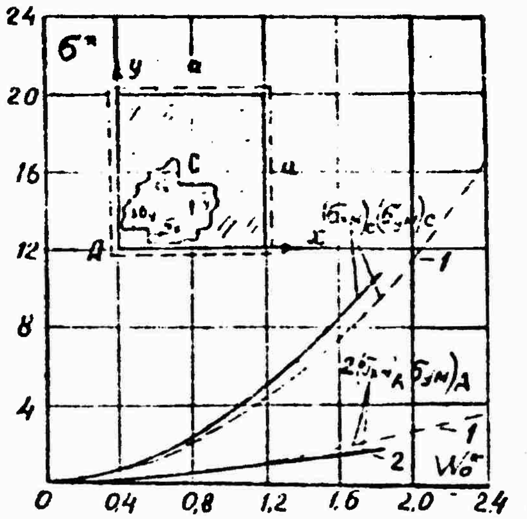


Figure 5

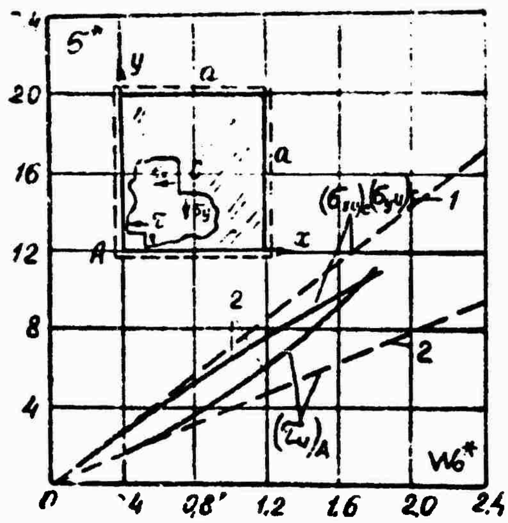


Figure 6

b) <u>rectangular plate with restrained edges</u>.

Here it is assumed, as in the case of hinged support, that the edges of the plate remain rectilinear in the process of deformation.

Deflection in the first approximation is selected in the form:

$$W = f \cdot \sin^2 \alpha x \cdot \sin^2 \beta y, \tag{3}$$

where

$$\alpha = \frac{\pi}{a}, \quad \beta = \frac{\pi}{b}.$$

This leads to satisfactory results during the determination of deflection of the plate. However, this expression poorly covers the true character of the bent surface, particularly for restrained edges. Therefore for the restrained plate it was suggested to retain the scheme of solution for hinged support. The expression for deflection is presented in form (1). The effect of reaction moments is covered by the addition to a prescribed uniform load of some fictitious load, distributed at the edges of the plate. The additional load is represented by trigonometric series.

The obtained results are given in figures 7, 8, 9, where dotted curves 1 represent the first approximation, and solid 2 - the refined solution. Moreover two cases of restraint are examined - sliding and rigid.

As we see from the graph, the bending stresses in the center are determined already in the first approximation quite well, and the stresses in the middle of the side in the direction perpendicular to the edge turn out to be highly underestimated. This is explained by the fact that with increase of deflection the curvature of the section of the plate at the edges increases, and in the center somewhat drops. It is obvious that for a rectangular plate the stresses in the middle of the long side reach the yield point the earliest. In this case there occurs partial break of the plate at the edges. Subsequently the plate should work with restraint on the edges close to hinged support. This is confirmed by experimental data. Although the expression for deflection (3) does not correspond to the true shape of the bent surface, the obtained calculated equation leads, as is evident from figure 10, to reliable results even with very large deflections. In this figure the small circles note the results of the experiment with square duralumin plate, whose deflections reached 12h.

Work [5] gives the solution, arrived at by the energy method, for restrained plate.

The numerical values of all parameters were calculated for different intensities of load q and different ratios of the sides of the plate - namely for $b/a=1$, $2/3$, $1/2$ with $\mu=0.3$. These results are illustrated in figure 11 by graphs of dependence load-deflection.

By the addition of membrane stresses and bending stresses we obtain combined (or total) stresses. The maximum values of these stresses appear in the middle of the long sides. Data on them are given in figure 12.

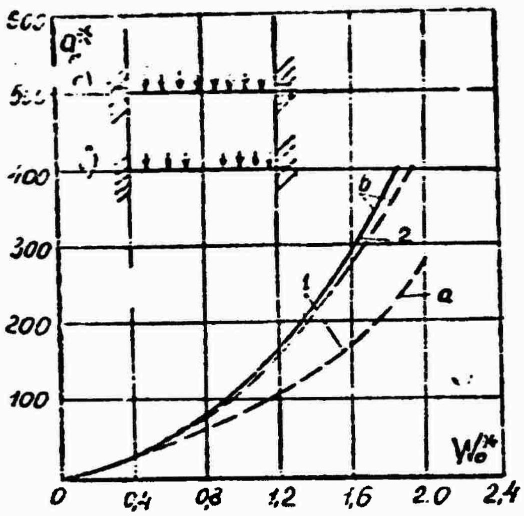


Figure 7

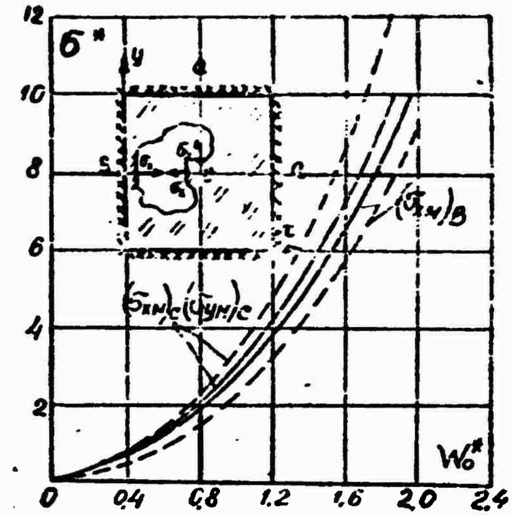


Figure 8

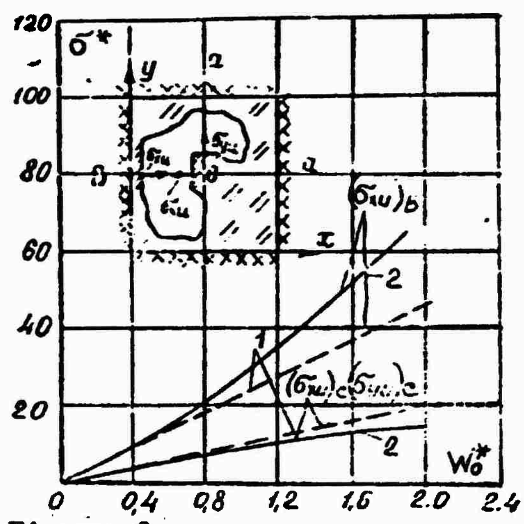


Figure 9

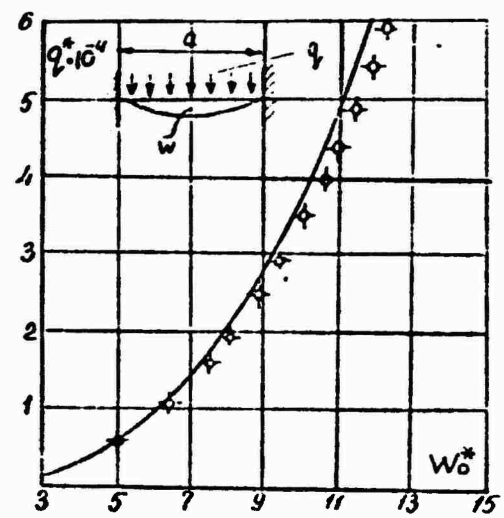


Figure 10

In works [2], [3], [4] for the solution of problems about large deflections of rectangular plates there is applied the finite-difference method or the grid method. The solution of nonlinear problems by this method leads to systems of nonlinear difference equations, of comparatively high order. The solution of such systems by manual means is an extremely laborious matter. Therefore in the examination of nonlinear problems earlier, before the implementation of ETsVM [ЭЦВМ - Electronic digital computer], preference was given to variational methods, mainly, the methods of Ritz, Ritz-Papkovich and Bubnov-Galerkin, since these methods with successfully selected approximating functions permit being limited to a smaller quantity of unknown parameters and are reduced to systems of nonlinear equations of lower order.

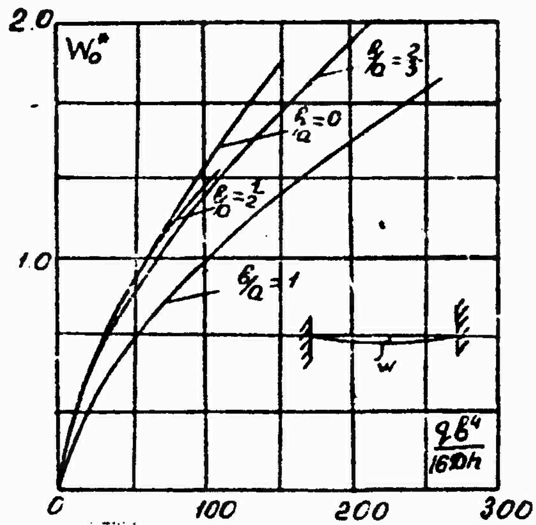


Figure 11

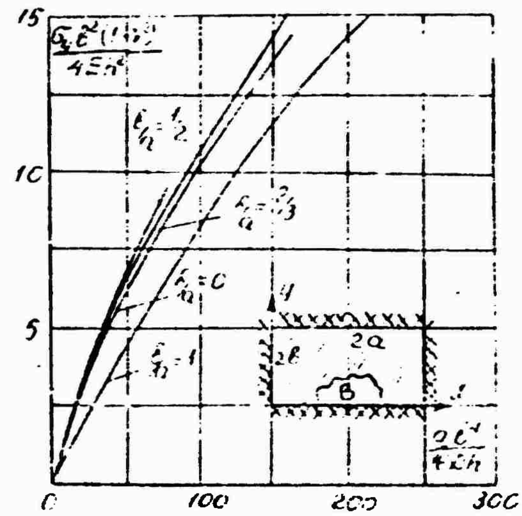


Figure 12

However with the development of computer technology the position is changed. Since during the solution of nonlinear problems by the finite-difference method the preparatory time for machine calculation is minimal in comparison with other methods, in this sense the grid method is the most commonly used at present.

In work [4] there is applied a modification of the finite-difference method, so-called finite-difference method of increased accuracy.

With the aid of this method (calculation was performed on ETsVM Strela) results were obtained for flexible rectangular plates with hinged edges, and also for the case of two restrained and two others hinged and for rigid restraint of edges. Here we give results only for evenly distributed load q.

a) <u>Hinged attachment of edges</u>. Figures 13-14 give the graphs of the basic dependences, obtained by the finite-difference method

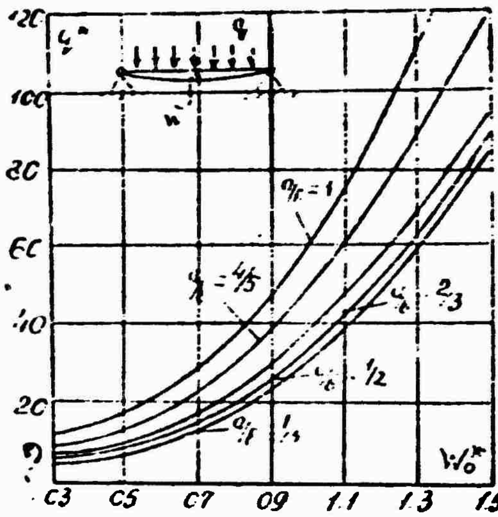


Figure 13

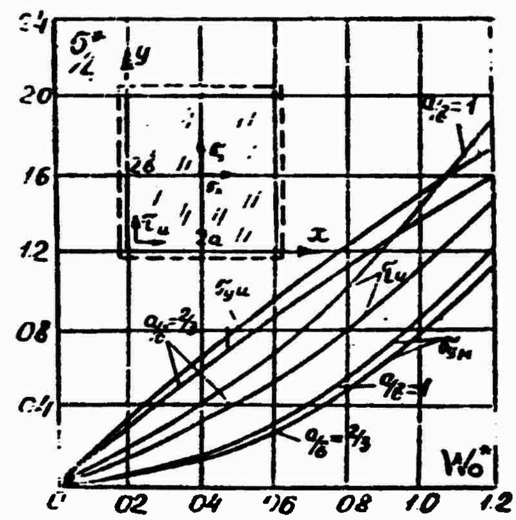


Figure 14

of increased accuracy. For a square plate the solution by this method well agrees with the known solution of S. Levy, provided in work [1]. In figure 15 the dotted curve 2 depicts the Levy solution, and solid 1 - solution by this method.

Disagreement in the amount of deflection in the center is less than 1%, and in the amount of maximum total stresses approximately 3%. This disagreement is explained by some difference in boundary conditions.

b) Rigid restraint of edges. Figure 16 gives the results obtained with the aid of the finite-difference method of increased accuracy for different ratios of sides. Results given in figures 17-19 also well agree with previously known solutions, for example, with the solution of S. Levy [1]. In figures the Levy solution is represented by dotted line 2, and solid line 1 - the solution obtained by this method. With $q^*=400$ the disagreement in the amount of deflection in the center is 3.7%, and in the amount of total stresses on the edge, at point B with $W_0^*=1.6$ - approximately 3%.

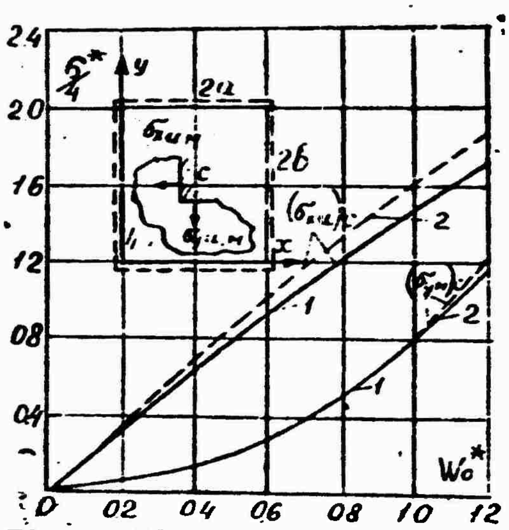


Figure 15

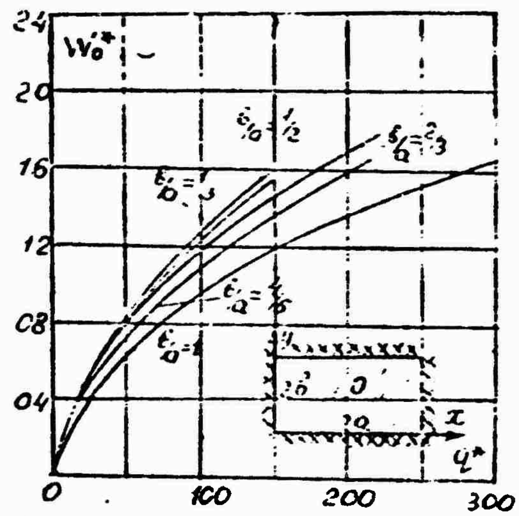
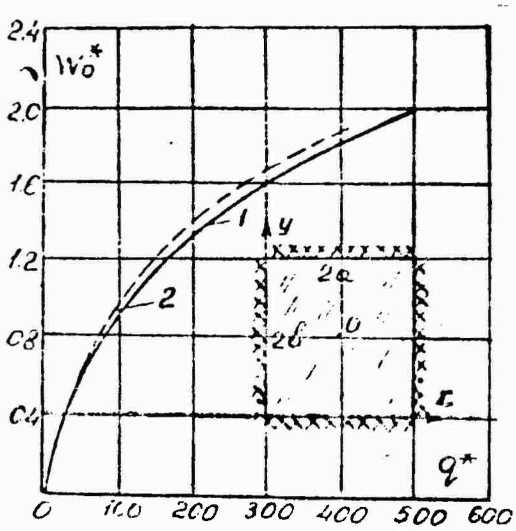


Figure 16

c) <u>Case of combined boundary conditions</u>. 1. Long edges are restrained, short are hinged. 2. Long are hinged, short are restrained.

Figures 20-21 give the graphs of dependences load-deflection for cases 1 and 2 with different ratios of sides λ.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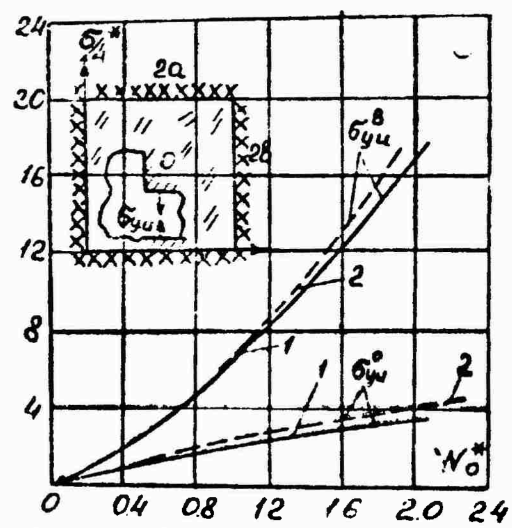
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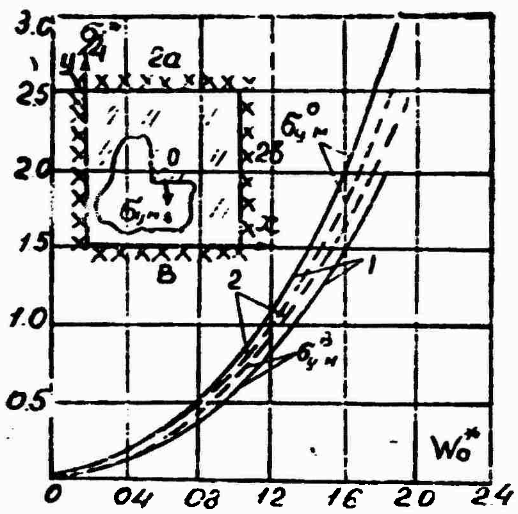

Figure 19

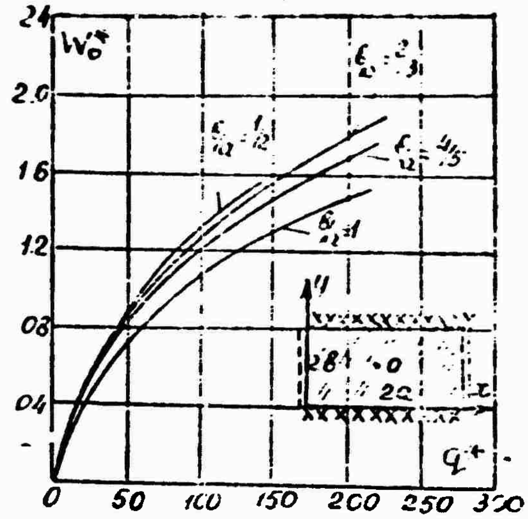

Figure 20

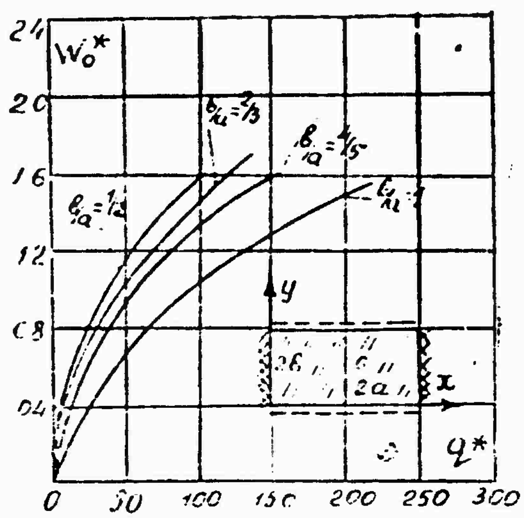

Figure 21

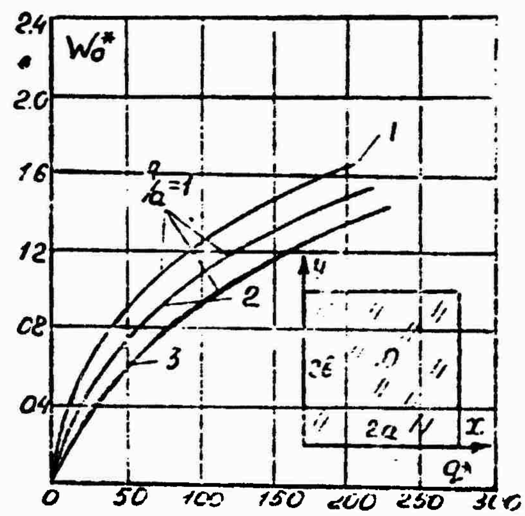

Figure 22

The solutions are obtained also with the aid of the finite-difference method of increased accuracy.

In figure 22 for a square plate there is given a graphic comparison of results for three types of boundary conditions, obtained by the same method. Curve 1 in this case corresponds to hinged attachment, curve 2 - combined and curve 3 - to rigid attachment.

It is noted that for a certain initial range of loading the difference in the method of attachment of edges is substantially reflected on the amount of deflection in the center.

With further increase of the intensity of load membrane stresses begin to play an increasingly larger role and the difference in the methods of attachment of edges will affect deflection less.

In work [3] there is examined the problem of large deflections of rectangular plates, supported on flexible nonstretchable edges, under the action of loads of different type. The solution is conducted by the method described in [4]. Computations are done on ETsVM "Strela". Here we record results only for case q=const.

Table 1 gives the results of computation for different ratios of sides of the plate λ.

Figures 23-24 depict the relationships between the parameters of maximum deflection and load, and also the maximum total stress in the center of the plate and load for λ=1, 0.5.

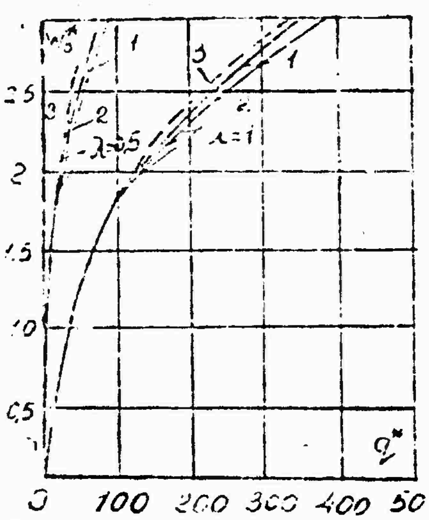

Figure 23

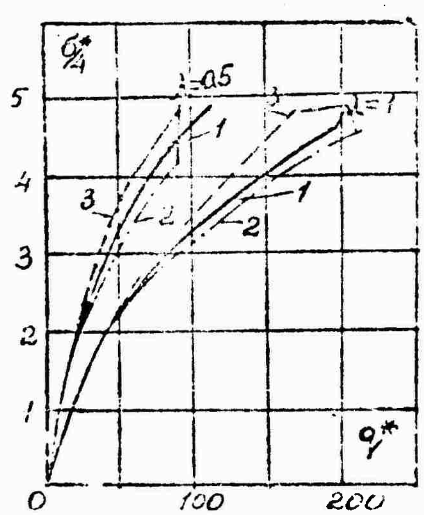

Figure 24

Curves 1 correspond to data of table 1, curves 2 - to the solution obtained by the energy method, curves 3 - to the solution by the Bubnov-Galerkin method. There is noted good agreement of results. In work [2], just as in [4], for the solution of problems an EVM [ЭВМ - Electronic computer] is utilized. Fundamental equations and boundary conditions are represented in finite differences. The solution is done by successive approximations. The effect of grid spacing on the results of computations is studied. Tables and graphs are given, which contain the amounts of deflections, stresses in the middle surface and bending stresses at some characteristic points. The obtained data are compared with the results of solutions of the same problems by other methods.

§ 2. Circular and semicircular plates

To the designations, taken in the previous paragraph, let us add the following:

r - radius of plate; $\sigma_{rи}$ - radial bending stress, σ_{rm} - radial membrane stresses.

Let us give some results on the study of large deflections of a circular plate, which is under the action of an evenly distributed load.

In work [15] the precise solution is given for evenly loaded circular plate with restrained outline. For the solution of this problem S. Way used power series.

In literature the greatest number of solutions is devoted to the problem of large deflections of evenly loaded circular plate with rigidly restrained edges. Part of these solutions, done by the perturbation method, is given in [6].

In [4] this problem is solved by the finite-difference method of increased accuracy and by the collocation method. Moreover it is shown that the solution by the collocation method, taking into account "equivalent correction" can virtually be considered as precise, and its agreement with the solution in finite differences confirms the reliability of the latter.

Figures 25-26 give graphic comparison of the solutions presented in [4] and [6]. The solid curve on the graphs indicates the solution by the finite-difference and collocation method, and dotted curves are obtained by the perturbation method.

The error of solution, obtained in [6], with dimensionless parameter of deflection in the center of the plate $W_0^*=1.2$ becomes especially considerable when determining the stresses, which is evident from the graph in figure 26, where stress-deflection curves are depicted. For example, according to [6] the bending stress in the center with $W_0^*>2.4$ becomes negative. This explicit nonconformity with reality was noted by A. S. Vol'mir in [1]. Further, according to [6] beginning with $W_0^*=2.8$ the membrane stresses on the edge become greater than in the center, which also contradicts other solutions.

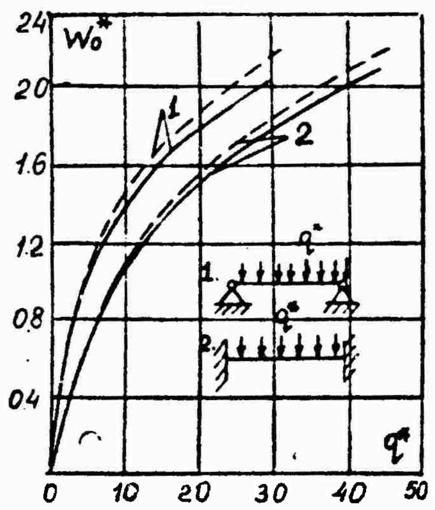

Figure 25

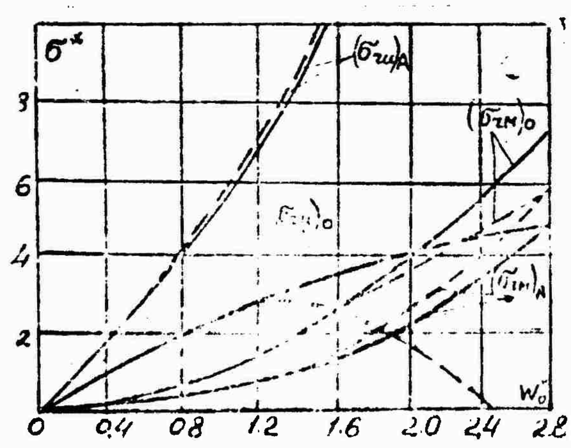

Figure 26

Satisfactory results during the solution of these problems are given by the energy method, used by S. P. Timoshenko and described in [5].

Some experimental data for circular plates are given in [16]. The relationship between the deflection of restrained duralumin plates and load is depicted on figure 27. Curve 1 in this figure represents the first approximation by the perturbation method, 2 - refined solution by this same method, 3 - refined solution of Nadai, obtained in [19]. The points in figure 27 mark the data of experiment for a duralumin plate with thickness h=0.76 mm, the blackened point is in the plastic deformation area.

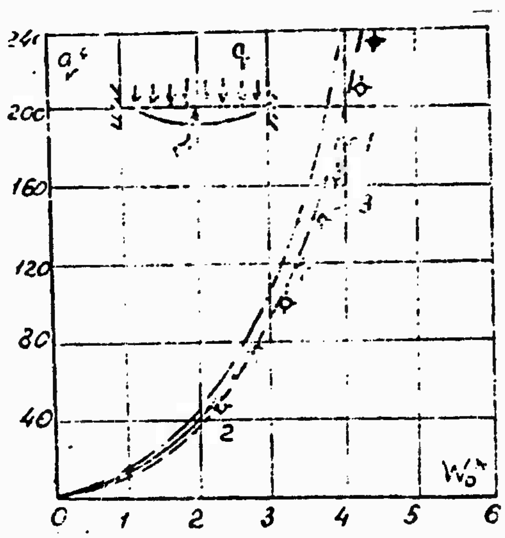

Figure 27

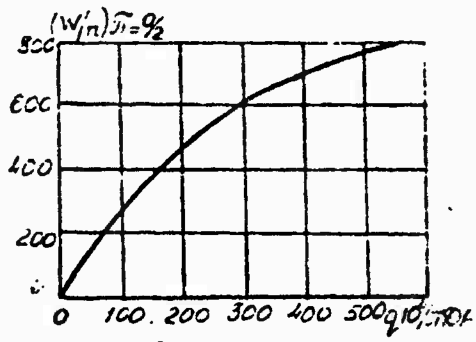

Figure 28

Let us note that the refined solutions will somewhat depart from experimental data. This is explained by the fact that in the experiment there is observed some slippage of the plate along the edge, which appears with considerable loads.

In [7] there is solved the problem of large deflections of a semicircular plate, restrained along the maximum diameter, under uniform load. The approximate equations, which the author uses, were derived by Berger in his work [8]. Figure 28 gives the graph of relationship load-maximum deflection.

§ 3. Parallelogram plates

Let us introduce additional designations: α, β - oblique-angled coordinates; θ - bevel angle of plate; ξ, η - dimensionless oblique-angled coordinates; 2a, 2b - sides of plate along axes α, β respectively.

Unlike the circular and rectangular plates the elastic behavior of these plates is completely insufficiently investigated as a result of the complexity of the mathematical model. For calculation of oblique-angled plates with restrained edges with large deflections in [11] there is applied a simple and quite precise method, based on the use of a small parameter. The presentation of this method can be found in [1] and [13].

Calculation was done by ETsVM. The computations were done with accuracy to 16 significant digits. The obtained results are represented in the form of graphs for different ratios of sides and bevel angles. Figures 29-31 give the graphs of dependences of the dimensionless parameter of load and dimensionless parameter of deflection for λ=0.5, 0.66, 1.

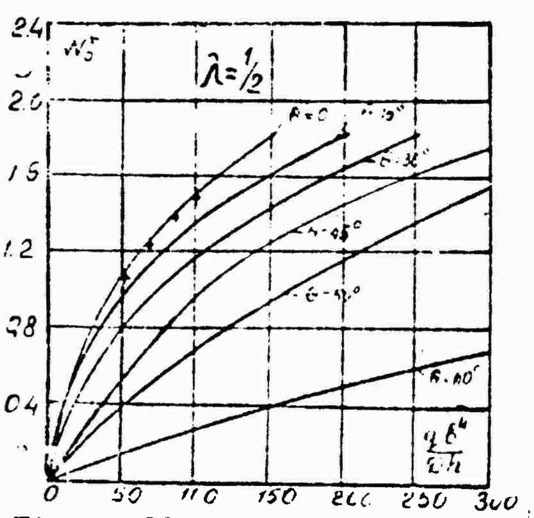

Figure 29

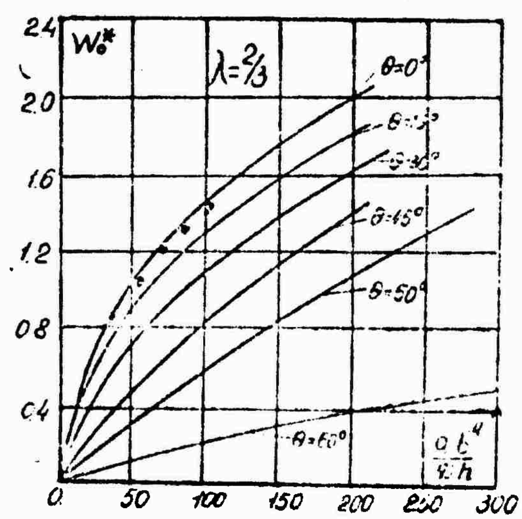

Figure 30

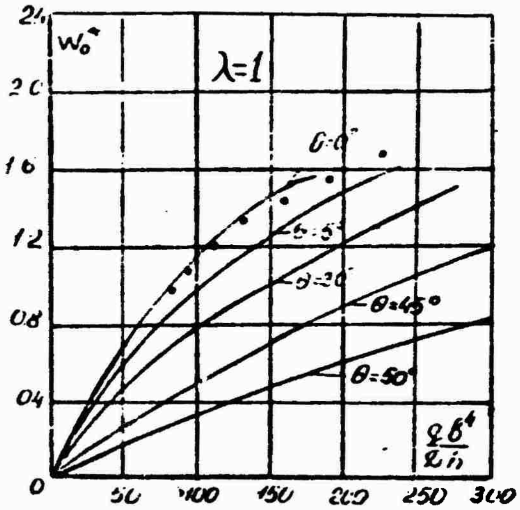

Figure 31

The poisson ratio was taken as μ=0.333. In these figures the points designate the results of Way [21] and comparison is given with the solution obtained by the perturbation method for θ=0.

Good agreement of theoretical and experimental results is revealed for all the examined ratios of the sides of the plate with small, and also with large deflections. The difference, by affirmation of the authors, does not exceed 5% for λ=1 and q*=200. This, although insignificant disagreement, the authors explain by the fact that the accepted expressions for displacement components u, v, w reflect only the polar symmetry, whereas for a rectangular plate there is required satisfaction of the condition of square symmetry. The effect of the difference of symmetry on the behavior of the plate under load is apparently strengthened with increase of the ratio of sides.

Further it is noted that the effect of nonlinear terms on deflection is decreased with increase of the bevel angle. Thus, for any value of the ratio of sides of the plate the curves of large deflections develop a tendency toward transformation into linear functions with large bevel angles. Figures 32-37 for different λ show a change of the greatest principal stresses in the center of the plate (32-34) and at the edge - at point A (35-36, 37), where the solid line corresponds to bending stress, broken line - membrane and points designate the results of Way [21]. In the limiting case with θ=0 and for all the examined ratios of sides of the plates the bending

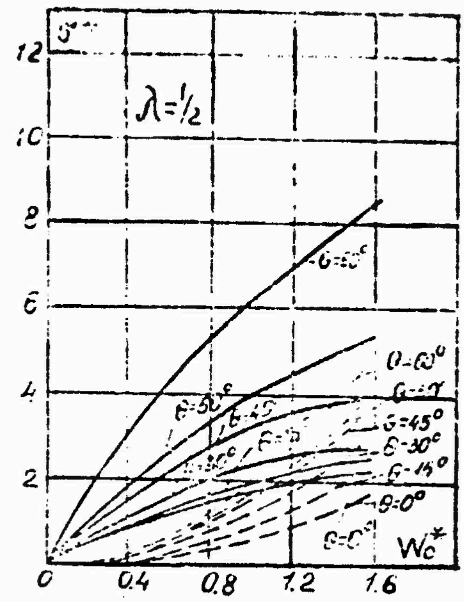

Figure 32

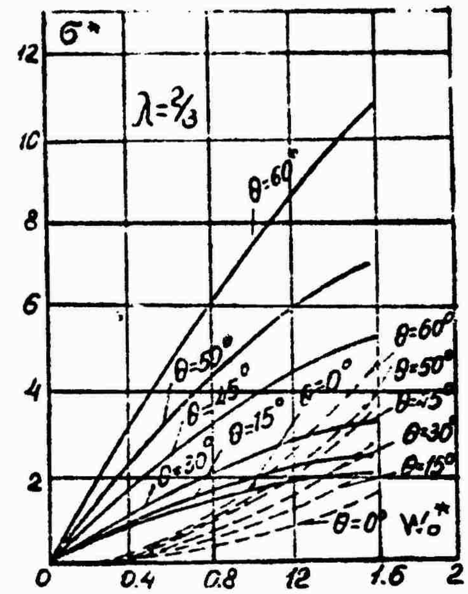

Figure 33

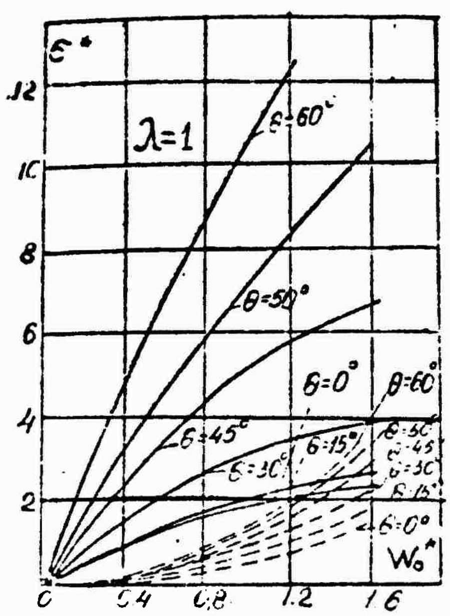

Figure 34

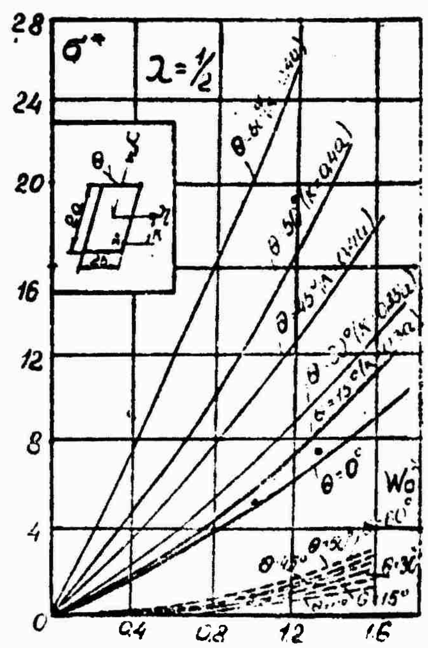

Figure 35

stresses on the edges well agree with the results of Way. However, Way's values of the corresponding boundary membrane stresses are 30% higher. This nonconformity is explained partially by the form of approximation in Way's "energy" solution, which Timoshenko also indicates [5]. This is partially explained also by the

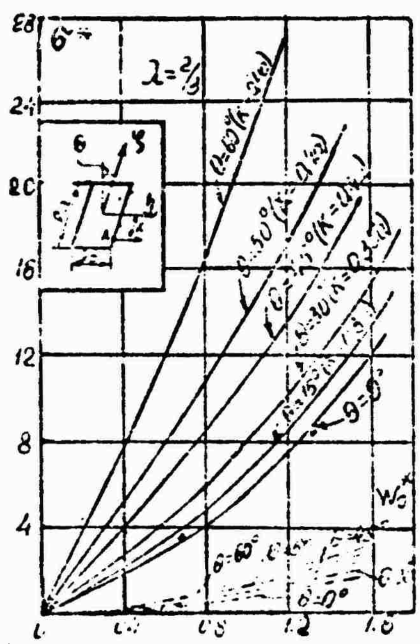

Figure 36

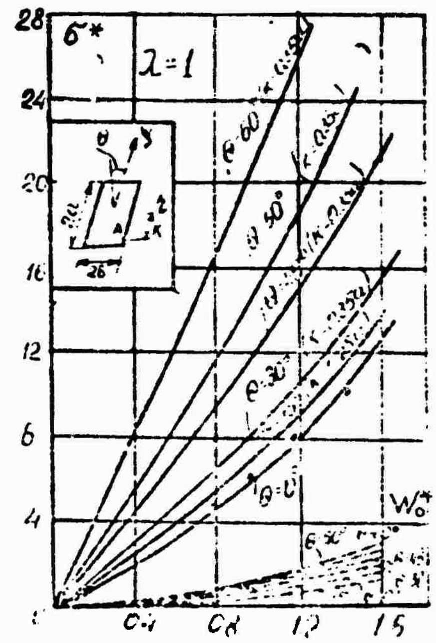

Figure 37

difference in the character of symmetry for problems of oblique-angled and rectangular plates.

From the analysis of graphic data it follows that the greatest total stress appears at the point of the edge of the larger side of the plate, where this point is shifted toward an obtuse angle. Stress in this case increases with increase of the bevel angle. The latter is also valid with respect to membrane stresses on the edges and in the center of the plate; however, they react quite weakly to a change of the ratio of sides.

For any geometry of the plate the membrane stresses in the center invariably prove to be greater than the same stresses which act on the edges. But their value all the same is low in comparison with the greatest total stress, which appears at the edge of the plate.

§ 4. Elliptical plates

Works [12], [14] are dedicated to the study of large deflections of restrained elliptical plates of constant thickness.

In [14] this problem is solved by Ritz's method, where there are examined limiting cases of an elliptical plate - circular plate and infinite band, for which precise solutions are known. Maximum deflections, as total stresses in the center and on the edge, decrease in proportion to transfer from the band through the ellipse to the circular plate, considering the width of the plates constant (figure 38). In this figure the solid curves are obtained by Ritz's method, dot-dash correspond to precise solution.

It is established that the relationship between the boundary stress on semiminor axis (maximum stresses in the plate) and the central deflection in practice does not depend on the proportions of the elliptical plate. Consequently, the maximum total stress can be determined from one curve for this load on an elliptical plate of arbitrary dimensions, if deflection in the center is known (figure 39).

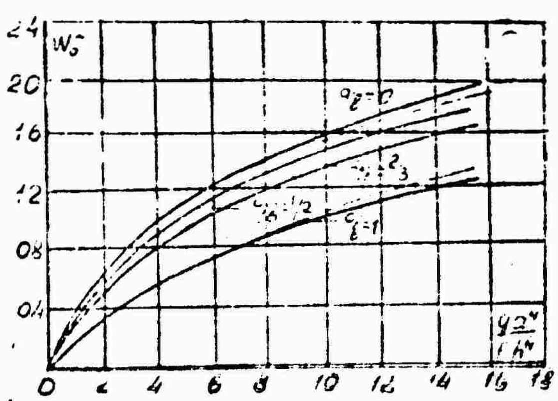

Figure 38

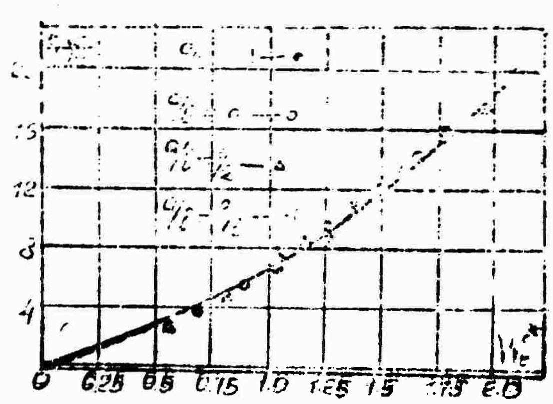

Figure 39

In this figure the blackened point marks the result for a circle, points - results for infinite band, triangle - for an ellipse with ratio of semi-axes a/b=1/2, and for ellipse a/b=2/3 - asterisks. Figures 40-41 give the graphs of relationships between the dimensionless parameter of load and the total dimensionless stress in the center and on the edge respectively. Stress on the edge turns out to be the greatest in this case.

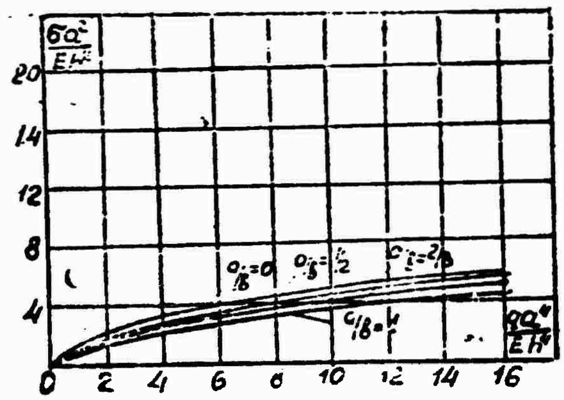

Figure 40

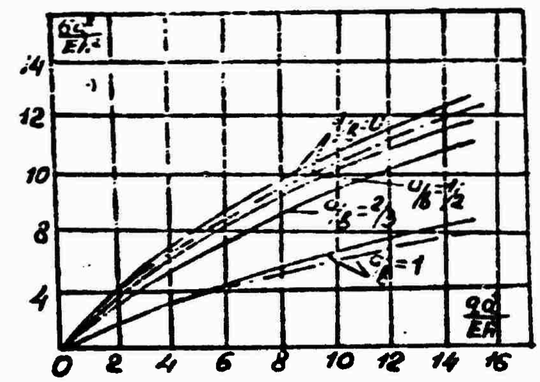

Figure 41

In [12] there was applied the small parameter method or Poincare's perturbation method for solution of the problem of large deflections of an elliptical plate with ratio of semi-axes a/b=1/2. Figure 32 gives the comparison of experimental results (they are designated by small circles), with the results of [14] (dot-and-dash line) and [12] (solid line). The difference of the theoretical results from experimental is explained here by some difference in boundary conditions, since in the experiment it is not possible to create absolutely rigid restraint.

Figure 43 gives the same graphic comparison of results for boundary stresses on the semiminor axis of ellipse. Here the dot-dash curve represents the results of work [14], and solid - the results of work [12].

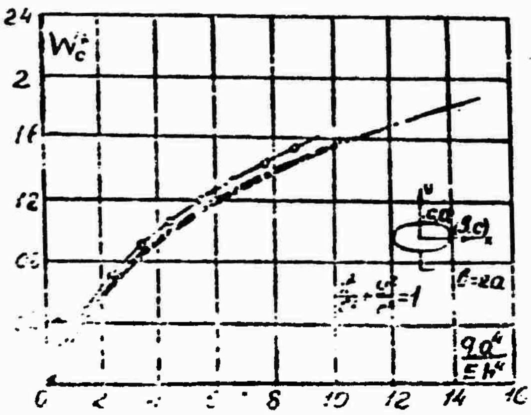

Figure 42

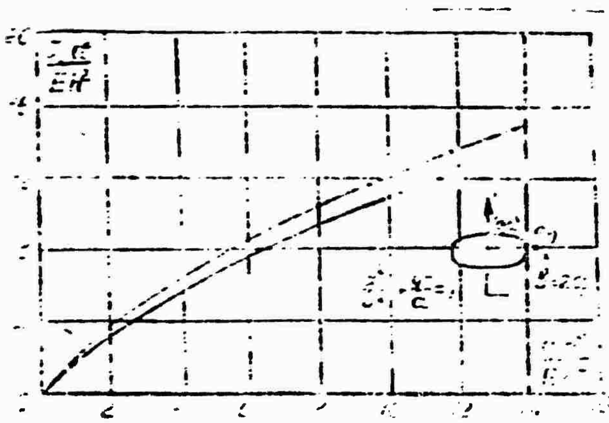

Figure 43

§ 5. Trapezoidal plates

Designations:

b - height of trapezoid; a - half the lower base; c - half the upper base; θ - bevel angle of trapezoid; $\gamma=\frac{c}{b}$; $\lambda=\frac{h}{b}$ - dimensionless parameters; $W^{*}_{max}=\frac{W_{max}}{h}$ - dimensionless parameter of maximum deflection.

From the studies of the elastic behavior of trapezoidal plates under the action of uniform transverse pressure the work [9], dedicated to the practical method of calculation of plates and mildly sloping shells supported on trapezoidal or triangular layout, deserves attention. For the solution of this problem in the article there is utilized the method of successive loadings, proposed in 1958 by Prof. V. Z. Vlasov. However, the function of deflections taken in this solution, which corresponds to the elastic line of a beam with restrained ends, can give satisfactory results only for trapezoidal plates, close to rectangular. Furthermore the calculation is done only for the case of sliding restraint, usually difficultly realized in practice.

Because of this in work [20] for this problem it is proposed to use the energy method. Here there is given a practical example of calculation of a plate, for which $\mu=0.3$, $\theta=30°$, $\gamma=0.25$.

The results are given in the form of a graph for dimensionless parameter of load and dimensionless parameter of deflection in Figure 44.

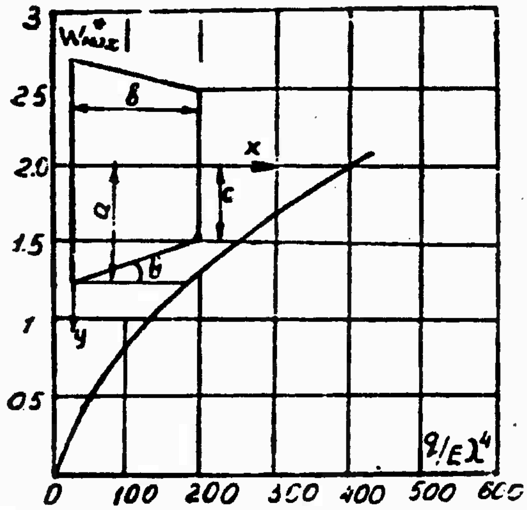

Figure 44

§ 6. Experimental study of large deflections of freely supported rectangular and parallelogram plates

The short survey of works, done in the previous paragraphs, makes it possible to formulate the following conclusions.

1. In essence the theoretical studies of large deflections of flat plates, made until recently, are scarce and they all, even with the active use of contemporary computer technology, are connected with the production of very laborious computational work. This developed the nature of these problems. Therefore the authors for the most part during solution are limited predominantly to some one form of restraint of the plate, assuming in this case that the deflections do not exceed two-three thicknesses.

2. Of all the types of flat plates, which are under the action of transverse load, at present the rectangular plate is the most examined. However, even here the question concerning the effect of boundary conditions remains not entirely investigated.

3. Up to now the problems of large deflections of orthotropic (or generally anisotropic) plates remain not studied.

4. The experimental works available in literature on flat plates are also scarce, whereupon in essence in all works the role of experiment is auxiliary, checkout with respect to theory. The comparative data of theory and experiment, provided by different authors usually, as a rule, will agree with each other. This is explained by the fact that in experiment with flat plates it is comparatively easy to eliminate the reasons which cause its antagonism with theory. These reasons should include two - nonconformity of theoretical boundary conditions with the conditions in the experiment and, finally, initial imperfections. By virtue of this the thought about the use of experiment with flat plates as an active means of the solution of mathematically formulated problems is natural. The question concerning the application of this, so-called theoretical-experimental method, to flat plates was considered in article [23].

In this article it was shown that with uniform boundary conditions and fixed Poisson ratio in the experiment with plates there is required the observence only of conditions of geometric similarity of the model and nature. The volume of the experiment in this case, necessary for solution of the problem, of the solution valid in the entire range of change of the basic geometric parameters of the plate, turns out to be minimum, and the experiment itself easily realizable in practice.

This fact makes the physical experiment not only competitive with mathematical, but often a more powerful and more economical means of solution of the problems. The clarification of this question has important value.

In accordance with the conclusions of the mentioned article [23] we have made an experimental study of the bending of freely supported rectangular and parallelogram plates under the action of uniform transverse load. In this paragraph a short description of it is given and the results are in the form of graphs.

In the laboratory of the mechanics of shells of KGU [КГУ - Kazan' State University im. V. I. Ul'yanov (Lenin)] there were tested rectangular plates with ratio of sides λ=1, 1.5, 2.5 and parallelogram plates with λ=1, 1.5, 2 for different bevel angles $\theta=30°$, 45°, 60°.

The plates were manufactured from material of brand D16AT $E=0.7 \cdot 10^6$ kg/cm^2, $\mu=0.33$. The thickness of the plates was measured by a vertical optical range finder IZV-2. There were tested rectangular plates of thickness h=0.96 mm, and parallelogram h=0.493 mm. Deflection in the center was eliminated with the aid of a dial indicator with scale graduation 0.01 mm. Pressure was measured with the aid of a specimen vacuum gauge with scale graduation 0.01 kg/cm^2.

For rectangular plates the results are given in the form of graphs in figures 45-46. The points designate the results of experiment.

Figure 45 for λ=1 gives comparison of the results of our experiment with the experimental data given in [22]. The blackened point in this figure corresponds to the Kaiser theoretical solution, which was mentioned in § 1. The agreement of theoretical result with experimental attests to the satisfactory conformity of the experimentally realizable boundary conditions with theoretical.

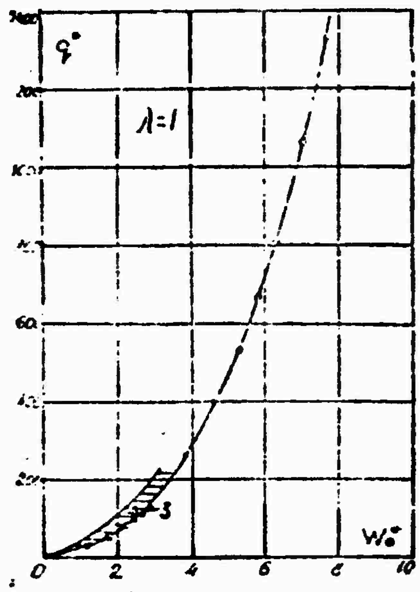

Figure 45

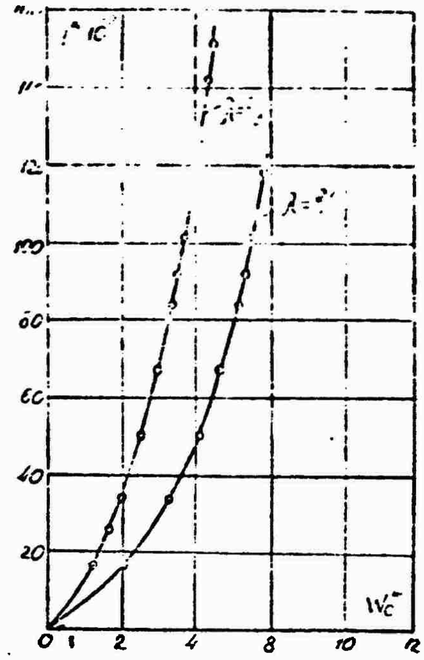

Figure 46

By the way, the establishment of this type of conformity is the basic difficulty of the experiment with plates, since in a technical respect usually the elimination of the effect of initial inaccuracies does not present great difficulties.

Figures 47-49 give the results of experiment for parallelogram plates. The points in these figures mark the results of experiment. In this case in the expression for q*, as in § 3, b - half of the smaller side. The effect of the edges of the plate, protruding above the support, on deflection was investigated. The case was examined where $\lambda=1$, $\theta=60°$. Relative width of the protruding edges was taken equal to $\lambda_1=l_1/h=3.04$ with $l_1=1.5$ mm and $\lambda_1=l_1/h=5.06$ with $l_1=2.5$ mm.

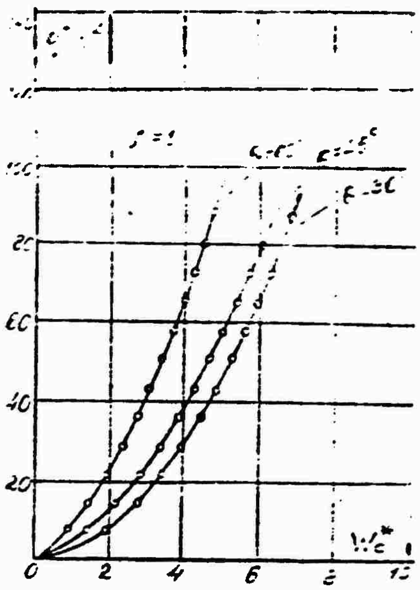

Figure 47

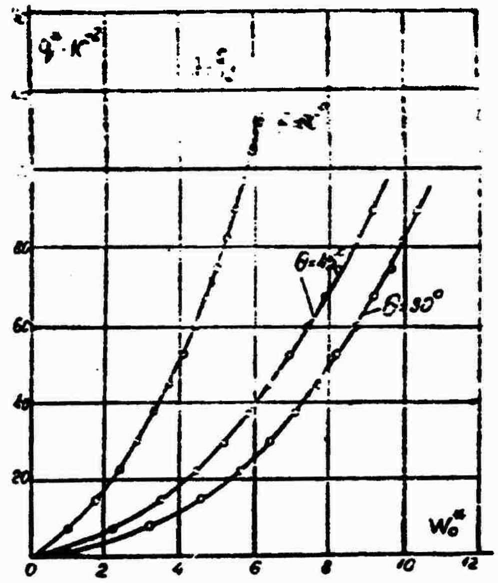

Figure 48

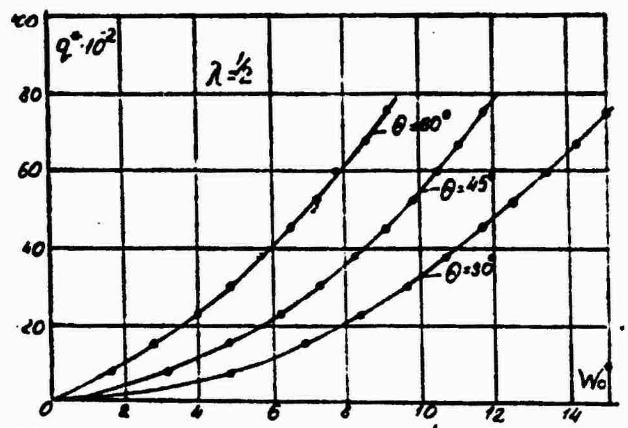

Figure 49

In the experiment there was compared the value of maximum deflections with identical loads. The results in the second case turned out to be 1.5-2% understated in comparison with the first.

BIBLIOGRAPHY

1. Вольмир А. С. Гибкие пластины и оболочки. 1956.

2. Вольмир А. С., Бирkган С. М. Применение ЭВЦМ при решении задач больших прогибов прямоугольных пластин. „Изв. АН СССР", ОТН, №2, 1959.

3. Антова Ф. С. Большие прогибы гибких пластинок опертых на гибкие нерастяжимые ребра. Сб. „Исследования по теории пластин и оболочек", №4, Казань, 1966.

4. Корнишин М. С. Нелинейные задачи теории пластин и пологих оболочек и методы их решения. „Наука", 1964.
5. Тимошенко С. П., Войновский-Кригер С. Пластинки и оболочки. „Наука", 1966.
6. Теория гибких пластинок. Перевод с китайского под редакцией А. С. Вольмира. М., 1956.
7. Banerjee B. Large deflection of a semicircular plate under a uniform load. „Bull. Acad. Polon. Sci Ser Scitech", 1967, 15, №3.
8. Berger H. M. A new approach to an analysis of large deflection of plates. „J. Appl. Mech.", 22, 1955.
9. Соколова Г. А. Расчет пластинок и оболочек с трапециевидным и треугольным контуром при конечных прогибах. „Изв. высш. учеб. заведения. Строительство и архитектура". Новосибирск, №4, 1962.
10. Муштари Х. М., Галимов К. З. Нелинейная теория упругих оболочек. Казань, Татгиз, 1957.
11. Kennedy J. B. and S. Ng. Linear and nonlinear analyses of skewed plates. „Trans ASME", 1967, E34, №2.
12. Nash W. A. and Cooley I. D. Large deflection of a clamped elliptical plate subjected to uniform pressure. „Trans ASME", 1959, E26, №2.
13. Колтунов М. А., Огибалов П. М. Пластины и оболочки. Изд. МГУ, 1969.

14. Weil N. A., Newmark N. M. Large deflections of elliptical plates. „Journal of Appl. Mechanics", vol. 23, „Trans ASME", vol. 78, 1956.
15. Way S. Bending of circular plate with large deflection ASME. Transaction. Applied Mechanics, 56, 1934.
16. Pherson A. Mc., Bamberg W., Levy S. Normal pressure tests of circular plates with clamped edges. „Rep NACA", N 744, 1942.
17. Недумов Н. В. Расчет пластинок в форме трапеции при заделке по всему контуру. Сб. „Расчет на прочность", вып. 5, Машгиз, 1960.
18. Levy S. Large deflection theory for rectangular plates. „Proceedings of Symposia in applied mathematics", vol. 1, N. Y., 1949.
19. Nadai A. Die elastischen Platten. Berlin, 1925.
20. Недумов Н. В. Большие прогибы трапециевидных пластин с заделанными краями. „Прочность и устойчивость элементов тонкостенных конструкций", №2, М., „Машиностроение", 1967.
21. Way S. Uniformy, loaded rectangular plates with large deflections proceedings of the fifth. „International Congress for Applied Mechanics".
22. Kaiser R. Recherische und experimentalle Ermittlung der Durchdiegung von quadratishen Platten. „Zs. f. angew. Math und Mech.", 16, №2, 1936.
23. Коноплев Ю. Г., Саченков А. В. Исследование прочности и устойчивости пологих сферических оболочек под действием локальных нагрузок. Казань, Сб. „Исследования по теории пластин и оболочек". №5, 1967.
24. Андреева Л. Е. Расчет мембран с пологой трапециевидной гофрировкой. Расчеты на прочность маш. констр. Машгиз, 1953.