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RELIABILITY ANALYSIS OF FATIGUE-SENSITIVE AIRCRAFT STRUCTURES UNDER RANDOM LOADING AND PERIODIC INSPECTION

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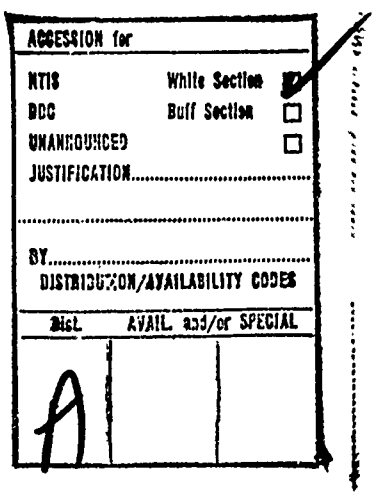

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A reliability analysis of fatigue-sensitive structures, based on the application of random vibration theory, is presented. Operational Service loads, composed of ground loads, ground-air-ground loads, and gust loads, are all random in nature. The fatigue process involved here consists of crack initiation, crack propagation and strength degradation. The time to crack initiation and the ultimate strength are random variables. After a fatigue crack is initiated, fracture mechanics is applied to predict crack propagation

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Abstract continued:

under random loading. While the fatigue crack is propagating, the residual strength of the structure decreases progressively, thus increasing the failure rate with time. The aircraft structure is subjected to periodic inspection in service. When a fatigue crack is detected during inspection, the implicated component is either repaired or replaced, so that both the static and the fatigue strength are renewed. Such a renewal process is taken into account in the present analysis. The detection of an existing fatigue crack during inspection is also a random variable which depends on the resolution capability of the particular technique employed and the size of the existing crack. Taking into account all the random variables as well as the random loadings, the solution for the probability of first failure in a fleet of aircraft is derived. Finally, numerical examples are given to demonstrate the effect of inspection and fleet size on the fleet reliability.

FOREWORD

This report was prepared by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under Project No. 7351, "Metallic Materials", Task No. 735106, "Behavior of Metals".

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ABSTRACT

An improved reliability analysis of fatigue-sensitive aircraft structures is presented which accounts for the effects of realistic operational loading inputs, inspection frequency and damaged part renewal on the subsequent probability of first failure in a fleet of aircraft. Furthermore, the analysis provides an approach to conducting trade-offs between given fleet reliability levels and the associated costs of the necessary inspection and maintenance procedures.

The analysis is based on the application of random vibration theory. Operational service loads, composed of ground loads, ground-air-ground loads and gust loads, are all random in nature. The fatigue process involved here consists of crack initiation, crack propagation and strength degradation. The time to crack initiation and the ultimate strength are also random variables. After a fatigue crack is initiated, fracture mechanics is applied to predict crack propagation under random loading. While the fatigue crack is propagating, the residual strength of the structure decreases progressively, thus increasing the failure rate with time. The aircraft structure is subjected to periodic inspection in service. When a fatigue crack is detected during inspection, the implicated component is either repaired or replaced, so that both the static and the fatigue strength are renewed. Such a renewal process is taken into account in the present analysis. The detection of an existing fatigue crack during inspection is also a random variable which depends on the resolution capability of the particular technique employed, the thoroughness of the inspection and the size of the existing crack. Taking into account all the random variables as well as all the random loadings, the solution for the probability of first failure in a fleet of aircraft is derived. Further, an inspection frequency optimization is formulated based on the concept of cost of failure. Finally, numerical examples are given to demonstrate the effect of inspection on the fleet reliability.

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I

INTRODUCTION

Fatigue has been a problem in the design of many structures in mechanical engineering, e.g., turbine blades, propeller shafts; in aeronautical engineering, e.g., aircraft structures, and in civil engineering, e.g., buildings, highway and railroad bridges, etc. The problem of fatigue is further complicated by the fact that most of the loading inputs to these structures in service are random in nature [e.g., Refs. 1-6]. Typical examples are gust and maneuver loads on aircraft, [e.g. Refs. 7-13], wind and earthquake forces on buildings, traffic loading to bridges, etc., to mention just a few.

Fatigue damage is revealed in a structure by the initiation of a visible crack. It has been a practice, e.g., on railroad bridges and aircraft structures, to periodically inspect fatigue-sensitive structures in order to detect such cracks and to repair or replace the cracked components [e.g., Refs. 14-17]. Inspection is an important procedure to increase the reliability of fatigue-critical structures. Hence, reliability analysis of fatigue-sensitive structures, under random loading and periodic inspection, is of practical importance, and is the primary concern of this study. In addition, an inspection frequency optimization is formulated, based on minimization of the expected cost of failure. Although the application of reliability analysis to aircraft structures is emphasized, the approach discussed in this report is equally

applicable to other fatigue-sensitive structures, e.g., civil and mechanical engineering structures, under random loading.

The specific type of random loading considered herein is a flight-by-flight loading to transport-type aircraft (bombers, tankers, etc.). It consists of ground loads, ground-air-ground loads and gust loads, which are all random in nature. The ultimate strength of the structure is also a random variable with certain statistical variability [e.g. Refs. 18-19]. Failure occurs as soon as the strength, either the ultimate strength or the residual strength after crack initiation, is exceeded by the random load level. This is referred to as the first-passage or first-excursion failure in random vibration [e.g. Refs. 20-23].

The fatigue process considered consists of (i) crack initiation, (ii) crack propagation and (iii) strength degradation. The time to crack initiation is a random variable and is assumed to have a two-parameter Weibull distribution [Refs. 24-25]. After the fatigue crack is initiated, fracture mechanics is applied to estimate crack propagation under random loading, where the statistics of rise and fall of random loading plays an important role [Refs.26-29]. While the crack is propagating, the ultimate strength is reduced progressively. As a result, the residual strength of a cracked structure decreases, thus increasing the failure rate (or risk function) in time [Ref. 30].

The residual strength after crack initiation is related either to the ultimate strength and the crack size through the Griffith-Irwin equation for non redunant structures [e.g. Refs. 31-32] or is determined by testing and analysis for redundant structures [e.g., Refs. 14,15,33-36]. With the concept of fail-safe design, fatigue crack propagation will be arrested by the "crack stopper"; thus the fail-safe crack size defines the maximum crack allowable in the structure.

The inspection is performed at periodic intervals in order to detect the fatigue crack if it exists. When a crack is detected, the cracked component is repaired or replaced so that both the residual strength and the fatigue strength of the component are renewed. This renewal process is taken into account in the present reliability analysis. During inspection, however, the fatigue crack may not be detected. The detection of an existing crack is also a random variable, which depends on the resolution capability of a particular method or technique employed for inspection. The probability of crack detection, in general, is an increasing function of the existing crack size [Refs.32,37].

Taking into account all the random variables and random loadings described above, the solution for the probability of failure is derived through application of the conditional probability theory. Then, an inspection frequency optimization is formulated on the basis of the expected-cost-of failure concept [Refs.38,39]. The optimum inspection frequency is determined, to minimize the expected cost of failure, while the constraint on the structural reliability is satisfied.

II

RANDOM LOADING

Consider a designed flight-by-flight random loading history (see Fig. 1), where each flight has three different characteristics: (1) ground loads $S_g(t)$, (2) gust loads $S(t)$ and (3) ground-air-ground loads Z. This specific type of random loading has been used for the design of transport-type aircraft (bombers, logistic aircraft, etc.).

The ground loads $S_g(t)$, resulting from landing and take-off, have been modeled as a random process (see Ref. 1). They produce compressive stresses in the fatigue critical component, and have some effect on the fatigue life. It has been observed in fatigue experiments, that when the general loading range of a specimen is in tension, the introduction of occasional high level loads results in a prolongation of the fatigue life due to the effect of beneficial residual stresses. This beneficial effect, however, is eliminated when compressive stressess are introduced in the loading history. Hence, the existence of the ground loads $S_g(t)$ eliminates the possible beneficial effect due to occasional high gust loads.

In each flight, there is one cycle of ground-air-ground load Z, which is also a random variable over the life of the aircraft. The magnitude or range of this load cycle is so large that it has a profound effect on the fatigue life of the aircraft structure.

The catastrophic failure of the structure is essentially due to gust loading, since failure occurs when the ultimate strength (or the residual strength after crack initiation) is exceeded. The gust loading S(t), modeled as a stationary composite Gaussian process [Refs. 7-13], will be adopted herein and is described briefly in the following:

The gust loading S(t) consists of a series of turbulence patches modeled as stationary Gaussian random processes $S(t,\sigma_i)$, i=1,2,..., where σ_i is the standard deviation. The power spectral densities $G_i(\omega)$ for $S(t,\sigma_i)$, i=1,2... are identical when normalized with respect to σ_i^2, i.e., $G_i(\omega)/\sigma_i^2$ is invariant for all i=1,2,...

The expected number of upcrossings (or upcrossing rate) per load cycle $\nu^+(R_0,\sigma_i)$ across a threshold R_0, the ultimate strength, by $S(t,\sigma_i)$ is well-known,[e.g. Ref. 1],

$$\nu^+(R_0,\sigma_i) = \exp\left\{-(R_0-X_0)^2/2\sigma_i^2\right\} \tag{1}$$

where X_0 is the average value of $S(t,\sigma_i)$, which is equal to the stress associated with one g loading [see Fig. 1]. The standard deviations σ_i, i=1,2... are assumed to be statistically independent and identically distributed random variables with an half-normal distribution [e.g., Refs 12-13]

$$f_\sigma(x) = P_1(2/\pi\sigma_{c1}^2)^{1/2}\, e^{-x^2/2\sigma_{c1}^2} + P_2(2/\pi\sigma_{c2}^2)^{1/2}\, e^{-x^2/2\sigma_{c2}^2} \tag{2}$$

where $f_\sigma(x)$ is the density function of σ_i, P_1 and P_2 represent the fractions of nonstorm turbulence (clear air) and the thunderstorm turbulence, respectively, with associated intensities σ_{c1} and σ_{c2}. Hence, $P_1 + P_2 = 1$. Parameters $P_1, P_2, \sigma_{c1}/B$ and σ_{c2}/B are referred to as turbulence field parameters and are specified in Ref. 7 for various altitudes, where B represents the structural characteristics [see Ref. 12]. A unified approach for the determination of these parameters from the measured turbulence data has recently been proposed in Refs. 12-13.

Therefore, the average number of upcrossings per cycle (one cycle is defined as one upcrossing of the mean, X_0) by the S(t) process) is obtained as

$$\nu^+(R_0) = \int_0^\infty \nu^+(R_0, x) f_\sigma(x)\, dx$$

$$= P_1\, e^{-(R_0 - X_0)/\sigma_{c1}} + P_2\, e^{-(R_0 - X_0)/\sigma_{c2}} \qquad (3)$$

Thus the upcrossing rate of a threshold R_0 is an exponential function. This has been verified by extensive turbulence field data [Refs. 7-13], and Eq. 3 has been used in the current U.S. Air Force specification [Ref. 7] for aircraft structural design for atmospheric turbulence [Refs. 12-13]. The gust process S(t) thus defined is referred to as a composite Gaussian process.

It should be mentioned that the first term in Eq. 3 represents the contribution from clear-air turbulence and $P_1 >> P_2, \sigma_{c2} > \sigma_{c1}$.

Therefore, it is primarily responsible for the fatigue initiation and crack propagation. In the current practice in random fatigue testing, the second term is usually disregarded [e.g. Refs. 15,39]. The second term, representing the contribution from storm turbulence with large intensity σ_{c2} is primarily responsible for the excursion or exceedance of the ultimate strength or the residual strength of the structure. Eq. 3 will be used later for computing the failure rate (or risk function) in order to estimate the structural reliability.

We digress here to comment that the fact that this turbulence model results in an exponential exceedance (Eq. 3), does not imply that it is the only feasible model. This is very important, since other models of random processes may also produce an exponential exceedance such as Eq. 3. The model is employed herein for expediency in view of the fact that no simpler model for gust loading for the purpose of implimentation in design, exists in the literature. An exploratory nonstationary model for gust loading, recently proposed by Lin [Refs. 2-3], should be mentioned.

III

MATERIALS/STRUCTURAL PERFORMANCE AND INSPECTION

3.1 Fatigue Crack Initiation

When a structure is subjected to cyclic loading for some time, a fatigue crack will be initiated first. This initiated crack will propagate progressively until a critical crack size is reached and fracture occurs. It is well-known that the fatigue process consists of crack initation, crack propagation and final fracture. It has been shown that the statistical distribution of the time T to fatigue crack initiation for the critical components of aircraft structures can be represented by a two-parameter Weibull distribution [e.g., Refs. 15,24].

$$W(t) = \frac{1}{\beta}\left(\frac{t}{\beta}\right)^{\alpha-1} e^{-(t/\beta)^{\alpha}} \qquad (4)$$

where α is the shape parameter and β the scale parameter. These parameters should be estimated from the test results of both the coupon specimens and the full scale structure under flight-by-flight loading shown in Fig. 1 [see Refs. 15,40-42]. If the result of the full-scale test is not available, an alternate approach is to estimate the parameter β by use of the cumulative damage hypothesis and the S-N curve.

3.2 Crack Propagation Under Random Loading

Once the fatigue crack is initiated and has a detectable size, say 0.02", fracture mechanics can be applied to predict the crack

propagation under random loading. The applicability of fracture mechanics requires that the crack size should be large compared to the plastic zone at the crack tip. For most materials, such as aluminum, this requirement is satisfied for a detectable crack size, say 0.02"[see Ref. 32]. Therefore, the power law of crack propagation under Gaussian random loading, which has been verified experimentally [see Refs. 26-28], will be used,

$$da/dn = c\ \overline{\Delta K^b} \tag{5}$$

where a is the crack size, da/dn is the rate of crack propagation per cycle, ΔK is the range of stress intensity factor, and b and c are material constants. $\overline{\Delta K^b}$ is the average of the bth power of the stress intensity factor range. For aluminum under random loading, b=4 seems to be appropriate [Refs. 27-28]. For the sake of simplicity of presentation, we shall set b=4, realizing that when b is different from 4 for other materials, the approach discussed herein remains valid and it does not involve any difficulty to account for it. Hence,

$$\overline{\Delta K^4} = \overline{S^4}\ a^2 \tag{6}$$

where $\overline{S^4}$ is the average of the fourth power of the rise and fall of the composite Gaussian process S(t). Approximate methods for estimating $\overline{S^4}$ from the power spectral density are available in

Refs. 26-29, and are summarized in the Appendix.

Thus,

$$da/dn = c\,\bar{S}^4\,a^2 \tag{7}$$

Integrating Eq. 7 from the initial crack size a_0 to the crack size a (t), after t flight hours, one obtains,

$$a(t) = a_0/\,[1-ta_0cQ]$$
$$Q = N_0\,[\bar{S}^4 + \bar{Z}^4/N_a] \tag{8}$$

in which N_0 is the number of gust load cycles per flight hour and $\bar{Z}^4$ is the average of the fourth power of the ground-air-ground cycle. N_a is the number of gust load cycles per flight. In Eq. 8, the contribution to the crack propagation from both the gust load S(t) and the ground-air-ground cycle Z have been taken into account. The ground oad $S_g(t)$, producing a compressive stress, (Fig.1) seems not to make a significant contribution to the crack propagation, except that it eliminates the beneficial effect resulting from the occasional high loads S(t), and hence it is omitted in Eq. 8 [see Section VIII for discussion].

3.3 Residual Strength

After a fatigue crack is initiated in the structure, the ultimate strength decreases due to the presence of the crack.

Based on fracture mechanics, the relationship between the residual strength R of a structure containing a crack, and the crack size a is given by the Griffith-Irwin equation,

$$K_c = R\sqrt{\pi a/2} \tag{9}$$

in which K_c is the critical stress intensity factor (fracture toughness), which is a material constant. The relationship, Eq. 9, holds up to the point A (see Fig. 2) where R is equal to the ultimate strength R_0. Thus, the strength (residual) of the critical component of the structure follows the curve A-B-C as shown in Fig. 2. As a result, there is a critical crack size a_c (point B) beyond which the strength R_0 starts to decrease following Eq. 9. This critical crack size a_c is a very important parameter in selecting or comparing candidate materials for a particular structure [see Ref. 32].

Let t_c be the time (flight hours) required to reach a_c after crack initiation. Then, it follows from Eq. 8 that

$$t_c = [1 - (a_0/a_c)]/c a_0 Q \tag{10}$$

Let $R(t_m)$ be the residual strength at t_m flight hours after a_c, i.e., the residual strength at $t=t_c+t_m$ after crack initiation. Then, integrating Eq. 7 from a_c to $a(t_m)$ that is the crack size associated with the residual strength $R(t_m)$, and using Eq. 9, one

obtains

$$R(t_m) = R_0[1-a_c\ c\ Q\ t_m]^{1/2} \qquad (11)$$

in which R_0 is the ultimate strength (see Fig. 2).

In order to prevent the crack from propagating to a catastrophic size, it has been a design practice to provide crack-stoppers in the structure, which will arrest the crack. This practice is called fail-safe design. If a_s denotes the distance between adjacent fail-safe stoppers, then it is the maximum crack size allowable in the structure, and the minimum residual strength at this crack size can be obtained from Eq. 9 (see Fig. 2).

Thus far, the residual strength of a cracked structure is obtained from the Griffith-Irwin equation (Eq. 9). It applies to nonredundant structures [e.g., Ref. 30]. Many structures, however, are designed with high redundancy. Under this circumstance, the residual strength of the cracked structure no longer follows Eq. 9, but depends on the particular design and has to be obtained by analysis and testing [e.g. Refs.,14,15,33-36]. As a result, it is not possible to discuss the residual strength of a highly redundant cracked structure in general. However, the general trend is for the residual strength to be a monotonically decreasing function of the flight hours or the crack size.

Let $R_{0\xi}$ be the residual strength at the fail-safe crack size

a_s which is determined from analysis and testing. In view of the form of Eqs. 9 and 11 as well as the test results [e.g., Refs. 14-15, 33-36], a possible model for the residual strength $R(t_n)$ at t_n flight hours after crack initiation is suggested as follows;

$$R(t_n) = R_0 \left\{1 - (1-\xi)\left(\frac{a(t_n) - a_0}{a_s - a_0}\right)^{1/2}\right\} \qquad (12)$$

where $a(t_n)$ is the crack size at t_n and is computed from Eq. 8.

3.4 Periodic Inspection and Crack Detection

In the preceding section, the fatigue damage is expressed in terms of the fatigue crack size $a(t_n)$ (Eq. 8), which increases monotonically with respect to flight hours t_n, and hence the residual strength $R(t_n)$ (Eq. 12) decreases. The purpose of the periodic inspection is to detect the fatigue cracks. If a fatigue crack is detected, it is repaired and the strength of the component is renewed, thus increasing the structural reliability.

The probability of detecting a fatigue crack in the critical component depends on (i) the probability of inspecting the cracked detail (correct location) in the component and (ii) the resolution capability of the crack detection method used for the inspection, [Refs. 32,36,37]. Let U_1 be the probability of inspecting the cracked detail and $U_2(a)$ be the probability of detecting an existing crack of length a when the cracked detail is inspected. U_1 depends on the thoroughness of inspecting all the details and

$U_2(a)$ depends on the resolution capability of a particular detection method used for inspection as well as the existing crack size a. A typical example for the detection probability $U_2(a)$ is given by Fig. 3 where the method of dye penetrant is employed [Ref. 32]. In general, $U_2(a)$ is a monotonically increasing function of the crack size a. It is reasonable to assume a minimum crack size a_1 below which the crack cannot be detected and a maximum crack size a_2 beyond which it can certainly be detected. Hence, a possible model for $U_2(a)$ is:

$$\begin{aligned} U_2(a) &= 0 && a < a_1 \\ &= [(a-a_1)/(a_2-a_1)]^m && a_1 < a < a_2 \\ &= 1 && a_2 < a \end{aligned} \tag{13}$$

where m is a dimensionless parameter. It can be observed from Fig. 3 that m=1, a_1=0.02" and a_2=0.3" for the dye penetrant method. Consequently, when a crack of length a exists in the structure, the probability of detecting it, denoted by F[a], is the product of U_1 and $U_2(a)$,

$$F[a] = U_1 U_2(a) \tag{14}$$

and the probability of not detecting the crack $F^*[a]$ is equal to 1-F[a].

IV
CONDITIONAL FAILURE RATE (RISK FUNCTION)

As mentioned previously, catastrophic failure occurs as soon as the ultimate strength R_0 (or the residual strength after crack initiation $R(t_n)$) is exceeded by the gust load. It can be observed from Fig. 1 that the problem is a first-passage problem with one-sided threshold [Ref. 1,20-23]. The average failure rate (or risk function) per load cycle for the threshold R_0 denoted by $h_0(R_0)$, is therefore [Refs. 1,22],

$$h_0(R_0) = \nu^+(R_0)/M_c \tag{15}$$

where $\nu^+(R_0)$ is the upcrossing rate given by Eq. 3, and $M_c \geq 1$ is the average clumpsize [Refs. 1,22]. For most structures, particularly for aircraft structures, the threshold R_0 is very high compared to σ_{c2} so that the events of excursion (or exceedance) are statistically independent, and hence $M_c = 1$. We shall set $M_c = 1$ realizing that such an approximation is conservative [Ref. 1].

The ultimate strength R_0 for most structures is a random variable [see Refs. 18-19]. For aircraft structures, data has been compiled in Ref. 18 where a Weibull distribution with the shape parameter equal to 19 has been proposed. Therefore, the failure rate h_0 per flight hour follows from Eq. 15 as

$$h_0 = N_0 \int_0^\infty \nu^+(x) f_{R_0}(x)\,dx \tag{16}$$

where $f_{R_0}(x)$ is the probability density of R_0 and $\nu^+(x)$ is given by Eq. 3.

Following Ref. 18, that the statistical distribution of the ultimate strength is a Weibull distribution with the shape parameter α_0 and the scale parameter β_0, we obtain the failure rate h_0, by substituting Eq. 3 into Eq. 16 and by making appropriate transformations, as follows:

$$h_0 = \sum_{i=1}^{2} P_i N_0 \int_0^{\infty} e^{-x} \left\{1-\exp\left[-\frac{x+\nu_{oi}}{\nu_{ci}}\right]^{\alpha_0}\right\} dx \qquad (17)$$

in which

$$\begin{aligned} \nu_{ci} &= \beta_0/\sigma_{ci} \\ \nu_{oi} &= X_0/\sigma_{ci} \end{aligned} \qquad i=1,2 \qquad (18)$$

The failure rate h_0 obtained above is the conditional failure rate, the condition being that the fatigue crack has not been initiated.

Let $h(t_n)$ be the failure rate at t_n flight hours after crack initiation; at this time, the residual strength $R(t_n)$ is given by Eq.12. Then, it can easily be shown that $h(t_n)$ can be computed from Eq. 17 where β_0 appearing in Eq. 18 should be replaced by $\beta_0\gamma_n$ (see Eq. 12),

$$\gamma_n = 1 - (1 - \xi)\left[\frac{a(t_n) - a_0}{a_s - a_0}\right]^{1/2} \tag{19}$$

When the residual strength follows Eq. 11, it is obvious that $h(t_n) = h_0$ for $t_n \leqslant t_c$, where t_c is given by Eq. 10. For $t_n > t_c$, $h(t_n)$ can also be computed from Eq. 17, where β_0 appearing in Eq. 18 should be replaced by $\beta_0 \gamma_n^*$ (see Eq. 11),

$$\gamma_n^* = [1 - a_c c Q(t_n - t_c)]^{1/2} \tag{20}$$

If the statistical distribution of the ultimate strength R_0 is assumed to be normal with a mean value μ_0 and a coefficient of variation V_0 (dispersion), the failure rate $h(t_n)$ can be obtained in a closed form as follows:

$$h(t_n) = \sum_{i=1}^{2} P_i N_0 \left\{\frac{1}{2}\,\mathrm{erf}(\eta_i/\sqrt{2}r_i) + \frac{1}{2}\, e^{-(2\eta_i - r_i^2)/2}\left[1+\mathrm{erf}\left(\frac{\eta_i - r_i^2}{\sqrt{2}\, r_i}\right)\right]\right\} \tag{21}$$

in which

$$\eta_i = (\gamma_n \mu_0/\sigma_{ci}) - (X_0/\sigma_{ci})$$
$$r_i = V_0 \gamma_n \mu_0/\sigma_{ci} \qquad i = 1,2. \tag{22}$$

where γ_n is given by Eq. 19. The failure rate h_0 before crack initiation can be computed from Eq. 21 where γ_n appearing in Eq. 22 is

1.0. For the case where the residual strength follows Eq. 11, γ_n should be replaced by γ_n^* (see Eq. 20).

V

PROBABILITY OF FAILURE UNDER PERIODIC INSPECTION

Having obtained the conditional failure rates h_0 and $h(t_n)$, representing the failure rate before crack initiation and the failure rate at t_n (flight hours) after crack initiation, respectively, we are in a position to derive the probability of structural failure under periodic inspection. Since the time to crack initiation is a random variable, the following formula for the conditional probability will be used frequently,

$$P[A] = \int_0^\infty P[A|t]\ W(t)dt \tag{23}$$

Where P[A] is the probability of failure, W(t)dt is the probability of crack initiation in [t,t+dt] (flight hours) and W(t) is given by Eq. 4. P[A|t] is the probability of failure under the condition that the crack is initiated at time t. Furthermore, if the total failure rate within an interval of time is denoted by K, then the probability of failure P_f in that time interval is

$$P_f = 1 - e^{-K} \tag{24}$$

Let P_0 be the probability of failure within the intended service life T(flight hours) without inspection. Then, it follows from Eq. 23 and 24 that

$$P_0 = \int_0^T \left\{1-e^{-th_0-\int_0^{T-t} h(t)dt}\right\} W(t)\, dt$$

$$+ \int_T^\infty \left\{1-e^{-Th_0}\right\} W(t)\, dt \qquad (25)$$

The first term is the probability of failure under the condition that the fatigue crack is initiated at time t in [0,T]. The second term represents the probability of failure when the fatigue crack is initiated after the service life T, in which case the total failure rate is Th_0.

Define $H(t_n)$ as the summation of failure rate from the crack initiation to t_n flight hours after crack initiation,

$$H(t_n) = \int_0^{t_n} h(t)\, dt \qquad (26)$$

Then, with the aid of Eq. 4, P_0 can be written as

$$P_0 = 1-e^{-Th_0-(T/\beta)^\alpha} - \int_0^T W(t)\, e^{-th_0-H(T-t)}\, dt \qquad (27)$$

Suppose the structure undergoes a periodic inspection at each T_0 flight hours [see Fig. 4]. Let P_j^* be the probability of failure in j service intervals $[0,jT_0]$ under the condition that the crack is initiated after j-1th inspection. Then, it follows from Eq. 23 that

$$P_j^* = \int_0^{T_0} W[(j-1)T_0+t] \left\{1-e^{-h_0[(j-1)T_0+t]-H(T_0-t)}\right\} dt$$
$$+ \int_{jT_0}^{\infty} W(t) \left\{1-e^{-jT_0h_0}\right\} dt \qquad j=1,2...$$

in which the first term denotes the failure probability in $[0,jT_0]$, when the fatigue crack is initiated in the j th service interval $[(j-1)T_0,jT_0]$, and the second term denotes the failure probability when the fatigue crack is initiated after j T_0.

With the aid of Eq. 4, P_j^* can be simplified as follows;

$$P_j^* = e^{-[(j-1)T_0/\beta]^\alpha} - e^{-[jT_0/\beta]^\alpha - jT_0h_0}$$
$$- \int_0^{T_0} W[(j-1)T_0+t]\, e^{-h_0[(j-1)T_0+t]-H(T_0-t)} dt \qquad (28)$$
$$j=1,2,...$$

Let P(j) be the probability of failure within j service intervals $[0,jT_0]$ under periodic inspection. It is obvious that the probability of failure within the first service interval P(1) is equal to P_1^*, i.e.,

$$P(1) = P_1^* \qquad (29)$$

and the total failure rate in this time interval denoted by K_1, follows from Eqs 24 and 29,

$$K_1 = -\ln [1-P(1)] \tag{30}$$

The probability of failure in the first two service intervals $[0,2T_0]$ can be written as

$$P(2) = P_2^* + \int_0^{T_0} q_{12}(t)W(t)dt \tag{31}$$

where P_2^* is the contribution from the event of crack initiation after the 1st inspection given by Eq. 28, and the second term on the right-hand side is the contribution from the event of crack initiation in the first service interval $[0,T_0]$. $q_{12}(t)$ is the failure probability under the condition that the crack is initiated at time t, which consists of two parts,

$$q_{12}(t) = F[a(T_0-t)]\ C_{12}^{(1)}(t) + F^*[a(T_0-t)]V_{12}(t) \tag{32}$$

in which $a(T_0-t)$ is the crack size at the first inspection time T_0 [see Fig. 4]. $F[a(T_0-t)]$ is the probability that this crack is detected at T_0 and $F^*[a(T_0-t)]=1-F[a(T_0-t)]$ is the probability of not detecting the crack at T_0. Both $a(T_0-t)$ and $F[a(T_0-t)]$ are computed from Eqs. 8 and 14, respectively.

$V_{12}(t)$ is the failure probability in $[0,2T_0]$ under the condition that the crack, initiated at time t, is not detected at the first inspection. Hence,

$$v_{12}(t) = 1 - \exp\left[-h_0 t - H(2T_0 - t)\right] \tag{33}$$

The term $C_{12}^{(1)}(t)$ denotes the failure probability in $[0,2T_0$ under the condition that the crack, initiated at time t, is detected at the first inspection.

$$C_{12}^{(1)}(t) = 1-\exp\left[- h_0 t - H(T_0 - t) - K_1\right] \tag{34}$$

where $h_0 t + H(T_0 - t)$ is the total failure rate in $[0,T_0]$ and K_1 is the (renewal) total failure rate in $[T_0,2T_0]$, which is the same as the failure rate for P(t) (Eq. 30), because the crack is detected and the renewal process for the structure occurs after the first inspection.

The probability of failure within $[0,3T_0]$ can be written as follows;

$$P(3) = P_3^* + \int_0^{T_0} q_{13}(t)W(t)dt + \int_0^{T_0} q_{23}(t)W(T_0+t)dt \tag{35}$$

in which the second and the third terms are the failure probabilities contributed by the events of crack initiation in the first service interval and in the second service interval, respectively.

The failure probability $q_{13}(t)$, under the condition that the crack is initiated at time t (in the first service interval), consists of three parts,

$$q_{13}(t) = F[a(T_0-t)]C_{13}^{(1)}(t) + F^*[a(T_0-t)]F[a(2T_0-t)]C_{13}^{(2)}(t)$$

$$+ F^*[a(T_0-t)]F^*[a(2T_0-t)]V_{13}(t) \qquad (36)$$

where $a(2T_0-t)$ is the crack size at the second inspection time $2T_0$, when the crack is initiated at time t. Eq. 36 is self-explanatory. The first term is the failure probability contributed by the event of crack detection at the first inspection time. The second term is contributed by the event that the crack is not detected by the first inspection but by the second inspection. The third term is contributed by the event that the crack is not detected by both inspections. Hence.

$$V_{13}(t) = 1 - \exp\left[-h_0 t - H(3T_0-t)\right]$$

$$C_{13}^{(2)}(t) = 1 - \exp\left[-h_0 t - H(2T_0-t) - K_1\right] \qquad (37)$$

$$C_{13}^{(1)}(t) = 1 - \exp\left[-h_0 t - H(T_0-t) - K_2\right]$$

where K_2 is the total renewal failure rate in $[T_0, 3T_0]$, which is the same as that for P(2),

$$K_2 = -\ln[1 - P(2)] \qquad (38)$$

The failure probability $q_{23}(t)$ (Eq. 35), under the condition that the crack is initiated at time $T_0 + 1$, consists of two parts,

$$q_{23}(t) = F[a(T_0-t)]\ C_{23}^{(1)}(t) + F^*[a(T_0-t)]V_{23}(t) \tag{39}$$

where the first term denotes the failure probability when the crack is detected, and the second term when the crack is not detected at the second inspection time,

$$V_{23}(t) = 1 - \exp\left[-h_0(T_0+t)-H(2T_0-t)\right]$$
$$C_{23}^{(1)}(t) = 1 - \exp\left[-h_0(T_0+t)-H(T_0-t)-K_1\right] \tag{40}$$

In a similar fashion, the general solution for the probability of failure within j service intervals $[0,jT_0]$ can be obtained recursively as follows:

$$P(j) = P_j^* + \sum_{i=1}^{j-1} \int_0^{T_0} q_{ij}(t)W[(i-1)T_0+t]dt$$
$$j=2,3,\ldots$$
$$i=1,2,\ldots j-1$$

$$q_{ij}(t) = F[a(T_0-t)]C_{ij}^{(1)}(t) + \left\{\prod_{k=1}^{j-i} F^*[a(kT_0-t)]\right\} V_{ij}(t) \tag{41}$$
$$+\delta_{j-i-2} \sum_{k=2}^{j-i} \left\{\prod_{m=1}^{k-1} F^*[a(mT_0-t)]\right\} F[a(kT_0-t)]\ C_{ij}^{(k)}(t)$$

$$V_{ij}(t) = 1 - \exp\left\{-h_0[(i-1)T_0+t]-H[(j-i+1)T_0-t]\right\}$$

$$C_{ij}^{(k)}(t) = 1 - \exp\left\{-h_0[(i-1)T_0+t] -H(kT_0-t)-K_{j-i-k+1}\right\}$$

$$k=1,2,..,(j-i)$$

$$K_k = -\ln [1-P(k)] \qquad (41\text{-a})$$

where $\delta_{j-i-2} = 1$ if $j-i-2 \geq 0$, and $S_{j-i-2} = 0$ otherwise.

The probability of failure derived in Eq. 41 holds for a single airplane. For a fleet of M airplanes, the fleet reliability is defined as the probability of no failure at all [Refs. 24-25]. Since the material/structural performance parameters, such as ultimate strength, fatigue crack initiation, crack propagation; residual strength, etc. are statistically independent for each airplane, and since the random loads experienced by each airplane are also statistically independent, the event of failure of each airplane is statistically independent. Hence, the fleet reliability in j service intervals $[0,jT_0]$, denoted by $R_m(j)$, is $R_M(j) = [1-P(j)]^M$ and the probability of first failure in a fleet of M airplane is

$$P_f(j) = 1-R_M(j) = 1 - [1-P(j)]^M \qquad (42)$$

VI

NUMERICAL EXAMPLE AND COMPUTATIONAL PROCEDURE

A numerical example close to the real situation is given herein to demonstrate the approach proposed in this study. The parameters associated with the gust loading [Ref. 7] are as follows; $p_1 = 99.5\%$, $p_2 = 0.5\%$, $\sigma_{c1} = 0.07g$, $\sigma_{c2} = 0.18g$, where $1g = 10ksi$ (see Eq. 3). This loading spectrum is plotted in Fig. 5. It is assumed that each flight is of two hours duration and in one flight hour the structure is subjected to 600 load cycles, i.e. $N_0 = 600$, $N_a = 1200$ (see Eq. 8). The average fourth power of the ground-air-ground cycle is $\overline{Z^4} = (1.5g)^4$, and the initial crack size at crack initiation is $a_0 = 0.04"$(Eq. 8). The shape parameter for crack initiation is $\alpha = 4$ and the scale parameter $\beta = 30{,}000$ hours (Eq. 4). The material of the critical component is aluminum. The mean value of the ultimate strength R_0 is $\mu_0 = 5.7g$ and the dispersion is $V_0 = 5.6\%$ (see Eq. 21). The critical stress intensity factor $K_c = 75\ ksi\sqrt{in.}$ The fail-safe crack size at which the crack is arrested by the crack stoppers $a_s = 7"$ and the residual strength at a_s is equal to 43% of the ultimate strength, i.e., $\xi = 0.43$ (Eq. 19). The thresholds for crack detection are $a_1 = 0.02"$, $a_2 = 2"$ and the inspection quality $m = 0.2$ (see Eq. 13). Further assume that every detail in the critical component is inspected at the inspection time, i.e., $U_1 = 1.0$ (Eq. 14). The crack propagation factor under Gaussian random loading $C = 0.6\times10^{-7} ksi\sqrt{in}$ is taken from the test result of Refs. 27-28 (see Eq. 8). The design service life for the airplane is $T = 15{,}000$ flight hours. The power spectral density of the response due to gust loads is such that A=115(Eq.A-2 and Eq.A-5).

With all the input parameters given above, the computational procedure is summarized as follows:

(i) Compute the crack size, a(t), after crack initiation,using Eq. 8. Some results are shown in Fig. 6.

(ii) Compute the residual strength, R(t), after crack initiation, using Eq. 11 or 12. Some results using Eq. 12 are plotted in Fig. 6.

(iii) Compute the conditional failure rates h_0 and h(t) using either Eqs. 17-20 or Eqs. 21-22. Some results using Eqs. 21-22 are plotted in Fig. 6.

(iv) Compute the cumulative failure rate H(t) using Eq. 26.

(v) Compute the detection probability F[a(t)]using Eqs. 13-14, where a(t) has been evaluated in the procedure(i).

(vi) Compute P_j^* using Eq. 28 for j=1,2,...N.

(vii) Compute the failure probability P(j) in $[0,jT_0]$ for j=2,...N using Eqs. 41.

Results for the first failure probability $P_f(j)$ (Eq. 42) for a fleet of 50 airplanes as a function of service flight hours are plotted in Fig. 7 for different number of inspections. The entire computation takes 2 minutes on a CDC-6600 computer.

It is very interesting to note that the curve for the failure probability under no inspection, N=0, consists of two segments with completely different characteristics. The failure rate in the first segment from 0 to 5,000 flight hours is essentially h_0. This can be visualized from the fact that even though the fatigue crack is

initiated at the initial service time t=0, it takes approximately 4,700 hours for the crack to reach the fail-safe crack size a_s as clearly shown in Fig. 6. The failure in this region is essentially attributed to the exceedance of the ultimate strength R_0. As a result, inspection in this time interval [0 to 5,000 hours] has little effect in respect to an improvement of the fleet reliability. It can be observed from Fig. 7 that all the curves coincide in this region.

The second segment is in the region from 6,000-15,000 flight hours. In this region, the crack initiated in the early service hours has reached its fail-safe crack size and hence failure is essentially attributed to the exceedance of the residual strength ξR_0. Therefore, the failure rate is much higher than h_0 (see Fig. 6). This is a typical characteristic of the progressive fatigue damage effect. Because of the existence of the fatigue crack, the inspection in this region has a significant effect on the fleet reliability as clearly shown in Fig. 7.

Consequently, inspection at later service times is much more efficient than at the early service time. This, however, is only true if we are confident of the loading, material/structural fatige performance and the structural analysis. Otherwise, the early service time inspection is still desirable, because it will enable one to discover any deficiencies in the design of the airplane, and to detect if other uncontrollable factors, such as manufacturing variability, corrosion etc., have a significant effect on the fatigue crack initiation. It is the current practice to perform early

inspection so that necessary action, e.g., redesign, can be taken if unexpected fatigue cracks are detected in the early service life.

As mentioned previously, the purpose of inspection is to detect the cracked detail and repair it. Therefore, the ultimate benefit one can achieve through the inspection is to maintain the airplane in a crack-free condition. Under the crack-free condition, the failure rate is h_0. This ultimate improvement is shown in Fig. 7 by the curve associated with 39 inspections. It can be observed that this curve is practically the extension of the first segment of the curve N=0, i.e., failure results from the exceedance of the ultimate strength R_0. As a result, the number of inspections beyond this limit results in no benefit at all. This can be observed from Fig. 7 where the curve associated with 49 inspections practically coincides with the curve associated with 39 inspections.

The probability of first failure P_f in the intended service life of 15,000 flight hours for a fleet of M airplanes is plotted in Fig. 8 as a function of the number of inspections N, for different fleet size M. It indicates clearly the effect of both the inspection and the fleet size on the fleet reliability or the probability of first failure.

VII
OPTIMUM INSPECTION

Thus far, we have observed that the nondestructive inspection has a significant effect on the fleet reliability. In particular, the fleet reliablility increases as the inspection frequency or the inspection quality increases. However, the cost of inspection and maintenance increases also as the frequency or the quality of the inspection increases. As a result, there is a trade-off between the fleet reliability and the cost of maintenance. In this connection, there are a number of variables which can be adjusted in such a way that an objective function can be optimized. These variables are, for instance, the number of inspections N, the inspection quality (see Eq. 13) or the safety factor ν_{ci} (see Eq. 18) for the structural design of the component, etc. For the sake of simplicity in presenting the basic idea, we shall consider the trade-off for the number of inspections only, realizing that the trade-off for other variables can be made in a similar fashion.

The objective function to be minimized is the "expected cost" [Refs.38,39] while, at the same time, a prescribed level of fleet reliability is maintained. The expected cost C^* consists of the expected cost of inspection and the expected cost of failure of airplanes,

$$C^* = NMC_I + C_f M P(N) \tag{43}$$

in which N is the total number of inspections; M is the fleet size;

C_I is the cost of one inspection and repair for one airplane, which depends on a particular inspection quality given by Eqs. 13. P(N) is the probability of failure of one airplane in the design service life under N inspections, and MP(N) is the expected number of airplanes to fail during the design service life. C_f is the cost of failure of one airplane. P(N) is computed from Eq. 41. The first term in Eq. 43 denotes the expected cost of inspection and the second term denotes the expected cost of failure. Note that the first term is zero if no inspection is performed (N=0). The inspection cost increases as N increases but the cost of failure decreases, since P(N) decreases (see Fig. 8).

Meanwhile the probability of first failure P_f (see Eq. 42) should be lower than a prescribed level P_f^*.

$$P_f \leq P_f^* \tag{44}$$

Dividing Eq. 43 by MC_f, one obtains

$$C_r = \gamma N + P(N) \tag{45}$$

where $C_r = C^*/MC_f$ is the relative cost to be minimized and,

$$\gamma = C_I / C_f \tag{46}$$

is the ratio of the cost of one inspection for one airplane to the cost of failing one airplane during the service life. Hence, γ is the relative importance of the inspection cost compared to the cost of failure. It is also an important parameter for the determination of the optimum inspection frequency N^*.

The optimum inspection frequency N^* is obtained by minimizing the relative cost C_r given by Eq. 45, and meanwhile Eq. 44 is satisfied. The techniques for obtaining the optimum solution with the constraint given by Eq. 44 are available in the literature and will not be discussed herein. In Fig. 9, the cost C_r is plotted against N for various values of γ. The dashed curve, connecting all the minima, represents the possible optimum solutions for the inspection frequency. For instance, if the fleet size is M = 1 and $P_f^* = 0.01$ (see Fig. 8), the dashed curve in Fig. 9 represents the optimum solution for the inspection frequency. It can be observed that the smaller the value of γ is, the higher will be the optimum number of inspections.

In the optimization process, it is not necessary to estimate the absolute values of both the cost of inspection C_I and the cost of failure C_f. All one has to estimate is γ which is the relative importance of C_I to C_f. For instance, if the system is very expensive, such as the space-shuttle, or if its failure has a serious consequence such as loss of the system and/or loss of property and lives, γ will be very small and hence the optimum frequency N^* is higher.

VIII

DISCUSSION AND CONCLUSIONS

A reliability analysis for fatigue-sensitive aircraft structures based on the theory of random vibration has been presented. Flight loadings encountered by aircraft are random processes. The ultimate strength and the time to fatigue crack initiation are random variables. After a fatigue crack is initiated, fracture mechanics is applied for predicting the crack propagation under random loading, and the residual strength, a random variable, decreases as the fatigue crack propagates thus increasing the failure rate in time. The aircraft is also subjected to periodic inspections, wherein the detection of a fatigue crack is also a random variable that depends on the thoroughness of the inspection as well as the resolution capability of a particular technique used for inspection. When a fatigue crack is detected during the inspection, the cracked component is either repaired or replaced so that both the fatigue strength and the residual strength are renewed. Taking into account the renewal process, random loadings, and various random variables, the solution for the first failure of a fleet of airplanes is derived. The importance of inspection for improving the aircraft reliability and the influence of the inspection frequency and fleet size on the fleet reliability have been demonstrated in detail by a numerical example.

An optimization scheme for the inspection frequency has been formulated on the basis of the expected-cost-of-failure concept. The optimum inspection frequency is determined by minimizing the

expected cost while the constraint on the structural reliability is satisfied. It has been shown that the optimum inspection frequency increases as the relative importance of the cost of inspection compared to the cost of failure becomes smaller, and vice versa.

In the development of this report, various assumptions and restrictions have been made which can be relaxed in a more extensive subsequent study. Nevertheless, it is believed that the results presented herein are representative, and would not undergo major qualitative changes if these assumptions were relaxed, although quantitative changes would be expected. The assumptions, restrictions and their implications are discussed below.

The first restriction is that the flight-by-flight random loading considered is valid only for the design of transport-type aircraft, (e.g. bombers, tankers, high altitude logistic aircraft, etc.). For fighter aircraft, maneuver loading, in addition to gust loading, is the major cause of fatigue damage. The occurrence of the maneuver load is a random event and the resulting loading history is a random process. To date, the maneuver loading has not be characterized as a stochastic process as it should be [see Ref. 43]. Data, such as the exceedance curves or peak counts, for fighter aircraft has been available, and it exhibits asymmetric characteristics, rather than the symmetric characteristics of gust loading given by Eq. 3, where the upcrossing rate is the same as the downcrossing rate. This is a clear indication that the maneuver load is non-

Gaussian in nature and warrants a further study for the statistical characterization of such a random process. The study will also lead to a realistic and reasonable simulation technique to generate random sample functions (load histories) for fatigue tests. Gust loading, modeled as a composite Gaussian random process S(t), is employed for expendiency in this study. Although the assumption is believed to be reasonable, there are some indications that the gust loading may not be Gaussian in nature. As mentioned before, further study is needed, e.g., Refs. 2-3.

The number of load cycles N_0 per flight hour and the number of flight hours per flight, or N_a, are considered as deterministic parameters. These,in principle, should be treated as random variables. Since, however, the average number of load cycles per flight hour is large, the effect of their randomness on the final reliability estimate is believed not to be significant. The ground load $S_g(t)$ has been excluded from the crack propagation prediction in Eq. 8, on the rationale that it produces compressive stresses and that its influence is to eliminate the beneficial effect resulting from the occasional high gust loads. If there is any evidence or belief that it should be taken into account in the crack propagation equation, then it is a simple matter to include an extra term $N_g \overline{S_g^4} / N_a$ in Eq. 8, where N_g is the number of ground load cycles per flight, and $\overline{S_g^4}$ is the average of the fourth power of the rise and fall of the random ground load $S_g(t)$.

In predicting the crack propagation under Gaussian random loading, see Eq. 5, the crack propagation factor c has been considered as a deterministic parameter, because it is believed that its statistical dispersion can be neglected, as is indicated by limited experimental data [Refs. 27-28]. In fact, the variability of c reflects the statistical variability of fatigue behavior of materials in response to the random loading. This is because the statistics of the random loading, i.e., $\overline{S^4}$, has been taken into account. It has been observed from an extensive data base that the statistical dispersion of fatigue life under spectrum loading is much smaller than that under constant amplitude loading. The dispersion under random loading is even less than that under spectrum loading. Although it may be justified to neglect the statistical dispersion of c [see Refs. 27-28], extensive data is needed for further verification.

When c is considered a random variable, the situation can be handled as follows: The crack length $a(t_n)$ at t_n flight hours after crack initiation becomes a random variable and $a(t_n)$ appearing in the formulas for detection probability, Eq. 13-14, can be approximated by the average value of $a(t_n)$, which is computed from Eq. 8 with c replaced by its mean value. The statistical distribution for the residual strength $R(t_n)$, given by Eq. 11 or 12 will no longer be the same as that of the ultimate strength R_0. However, the distribution of $R(t_n)$ can be obtained from the distribution of R_0 and c at least numerically although the numerical computation for

the failure rate $h(t_n)$ will be very involved.

Both the ultimate strength R_0 and the residual strength $R(t_n)$ after crack initiation have been treated as random variables as they should be [see Ref. 18]. This fact is important and should be emphasized, since their statistical variability is disregarded by many. Our computational experience indicates that there is a significant difference in failure rates, h_0 and $h(t_n)$, and hence in the failure probability, when they are treated as deterministic quantities. The difference in the failure rates, h_0 and $h(t_n)$, ranges from one to two orders of magnitude higher for the case where R_0 and $R(t_n)$, are considered as random variables. As a result, failure rates are very unconservative without treating R_0 and $R(t_n)$ as random variables.

For the sake of simplicity of the presentation, only the failure probability under periodic inspection is derived. The inspection may not be periodic. The solution for failure probability under nonperiodic inspections can be derived easily in a similar fashion as discussed, except that the total renewal failure rates K_j, $j = 1,2...$ (see Eq. 41-a) have to be evaluated separately; since Eq. 41-a no longer holds. The evaluation of K_j involves no analytical difficulty.

Only a concept of optimization for inspection frequency, based on the cost of failure is formulated in this study. There are, in fact, a variety of problems associated with the inspection

optimization for aircraft structures, which have not been touched herein and will be reported later. For some types of military aircraft where the critical component is integrated into the entire structure, its replacement means the replacement of the entire wing. Therefore, the cost of replacement is significantly higher than the cost of inspection and both costs should be considered as different variables. It has been indicated in the numerical example that the inspection at the later time of the service life is much more efficient. Consequently, it is possible to adjust or vary the lengths of the inspection intervals, e.g., longer inspection intervals in the early life time and shorter inspection intervals at the later service life, so that the maximum benefit can be achieved. The possibility of using or combining various inspection qualities or techniques to achieve either a maximum utility or a maximum improvement of fleet reliability deserves further study. In all, the trade-off between replacement, repair, inspection quality, inspection interval, inspection frequency, retirement of aircraft, intended service life, etc. presents a broad spectrum of very interesting problems for further study.

Finally several statistical variables have not been accounted for in the present study, because of the lack of statistical information. Typical examples are (i) the statistical variability in aircraft performance resulting from the statistical variability of manufacturing, (ii) the statistical variability of environmental effects such as stress-corrosion, corrosion fatigue, buffeting

effects etc., and (iii) the probability of making errors in the structural analysis and in loading prediction, resulting from a lack of sufficient information. These random variables should be taken into consideration in the reliability analysis of aircraft structures, when their statistical background information becomes available. This has also been pointed out by Crichlow [Ref. 42].

APPENDIX

STATISTICS OF RISE AND FALL OF RANDOM PROCESSES

The technique proposed in Refs. 26-27 for evaluating the statistics of rise and fall of a stationary Gaussian random process $S(t,\sigma_i)$ is rather cumbersome. However, a simpler approximation has been suggested in Ref. 29 as follows:

$$\overline{S^4}\ (\sigma_i) = A\ \sigma_i^4 \tag{A-1}$$

$$A = 16 + 12\pi\ {}_2F_1(-\tfrac{1}{2},-\tfrac{3}{2};\ 1;k_0^2)$$

$$+24\ {}_2F_1(-1,-1;1;k_0^2) \tag{A-2}$$

where $S^4(\sigma_i)$ is the average of the fourth power of rise and fall of the Gaussian process $S(t,\sigma_i)$ and ${}_2F_1(.)$ is the hypergeometric function,

$$k_0^2 = k_0^2(\tau) = \rho^2(\tau) + \lambda^2(\tau) \tag{A-3}$$

$$\lambda^2(\tau) = \int_0^\infty G^*(\omega)\cos\ \omega\tau\ d\omega$$

$$\rho^2(\tau) = \int_0^\infty G^*(\omega)\ \sin\ \omega\ \tau\ d\omega \tag{A-4}$$

$$\tau = \pi/\omega_o$$

in which ω_o is the representative frequency of $S(t)$, and $G^*(\omega)$ is the normalized one-sided power spectral density of $S(t,\sigma_i)$,

which is identical for all i = 1,2,... as described in the text.

Therefore $\overline{S^4}$ for the composite Gaussian process S(t) can be obtained from Eq. A-1 and Eq. 2 as follows;

$$\overline{S^4} = \int_0^\infty \overline{S^4}(x) \quad f_\sigma(x) \quad dx = 3A \ (P_1 \ \sigma_{c1}^4 + P_2 \ \sigma_{c2}^4) \qquad \text{(A-5)}$$

where A is given by Eq. A-2.

REFERENCES

1. Lin, Y.K., "Probabilistic Theory of Structural Dynamics," McGraw-Hill, 1967.

2. Lin, Y.K., and Howell, L.J., "Response of Flight Vehicles to Nonstationary Turbulence," AIAA Journal, Vol. 9, No. 11, 1971, pp. 2201-2207.

3. Lin, Y.K., and Fujimori, Y., "Analysis of Flight Vehicles Response to Nonstationary Turbulence Including Wing Bending Flexibility," AIAA Journal, Vol. 11, No. 3, 1973, pp. 334-339.

4. Shinozuka, M., "Structural Reliability Under Condition of Fatigue and Ultimate Load Failure," Air Force Materials Laboratory, AFML-TR-68-234, 1968.

5. Shinozuka, M., "Probability of Structural Failure Under Random Loadings," Journal of the Engineering Mechanics Division, ASCE, Vol. 90, No. EM5, 1964, pp. 147-155

6. Shinozuka, M., "Fatigue Analysis of Highway Bridges," Proc. of Japan Soc. of Civil Engineering, No. 28, 1972, pp. 137-148.

7. Military Specification MIL-A-008861A (USAF),"Airplane Strength and Rigidity, Flight Loads", March 1971.

8. Press, H., Meadows, M.T. and Hadlock, I., "A Reevaluation of Data on Atmospheric Turbulence and Airplane Gust Loads for Application in Spectral Calculation," NASA Report 1272, 1956

9. Houbolt, J.C., et al, "Dynamic Response of Airplanes to Atmospheric Turbulence Including Flight Data on Input and Response," NACA Report, TR R-199, Langley Research Center, Virginia, 1964.

10. Houbolt, J.C., "Design Manual for Vertical Gust Based on Power Spectral Techniques", AFFDL-TR-70-106, 1970.

11. Hoblit, F.M. et al, "Development of a Power Spectral Gust Design Procedure for Civil Aircraft, "FAA-AD 5-53, 1966.

12. Neulieb, R.L., Garrison, J.N., and Golden, D.J., "Atmospheric Turbulence Field Parameters Determination," Air Force Flight Dynamic Lab, AFFDL-TR-72-51, 1972.

13. McClosky, J.N., et al, "Statistical Analysis of LO-LOCAT Turbulence Data for Use in Development of Revised Gust Data," Air Force Flight Dynamic Lab., AFFDL-TR-71-29, 1971.

14. Eggwertz, S., and Lindsjo, G., "Study of Inspection Intervals for Fail-Safe Structures," FFA Report 120, Aeronautical Research Institute of Sweden, Stockholm, 1970.

15. Eggwertz, S., "Investigation of Fatigue Life and Residual Strength of Wing Panel for Reliability Purposes", in Probabilistic Aspects of Fatigue, edited by Heller, R.A., ASTM STP-511, 1971, pp. 75-105.

16. Whittaker, I.C. and Saunders, S.C., "Exploratory Development on Application of Reliability Analysis to Aircraft Structures Considering Interaction of Fatigue Cumulative Damage and Ultimate Strength", Air Force Materials Lab., AFML-TR-72-283, 1973.

17. Whittaker, I.C., and Saunders, S.C., "Application of Reliability Analysis to Aircraft Structures Subjected to Fatigue Crack Growth and Periodic Inspection", Air Force Materials Lab., AFML-TR-73-92, 1973.

18. Freudenthal, A.M., and Wang, P.Y., "Ultimate Strength Analysis," Air Force Materials Lab., AFML-TR-69-60, 1969.

19. Freudenthal, A.M., Garrelts, J.M., and Shinozuka, M., "Analysis of Structural Safety," Journal of the Structural Division, ASCE, Vol. 92, No. STI, 1966, pp. 276-325.

20. Lin, Y.K., "First-Excursion Failure of Randomly Excited Structures II," AIAA Journal, Vol. 8, No. 4, 1970, pp. 720-725.

21. Lin, Y.K., "First-Excursion Failure of Randomly Excited Structures II," AIAA Journal, Vol. 8, No. 10, 1970, pp. 720-725.

22. Yang, J.N. and Shinozuka, M., "On the First Excursion Probability in Stationary Narrow-Band Random Vibration II," Journal of Applied Mechanics, ASME, Vol. 39, 1972, pp. 733-738.

23. Yang, J.N., "First-Excursion Probability in Nonstationary Random Vibration," Journal of Sound and Vibration, Vol. 27, 1973, pp. 165-182.

24. Freudenthal, A.M., "Reliability Analysis Based on Time to First Failure," Conference Paper presented at the Fifth I.C. A.F. Symposium, Australia, 1967.

25. Freudenthal, A.M., "The Expected Time to First Failure", Air Force Materials Laboratory, AFML-TR-66-37, 1965.

26. Rice, J.R., and Beer, F.P., "On the Distribution of Rises and Falls in a Continuous Random Process", Journal of Basic Engineering, ASME, 1965, pp. 398-404.

27. Rice, J.R., Beer, F.P., and Paris, P.C., "On the Prediction of Some Random Loading Characteristics Relevant to Fatigue," Proc. of the Second International Conf. on Acoustical Fatigue in Aerospace Structures, Syracuse Univ. Press, 1965, pp. 111-143.

28. Paris, P.C., "The Fracture Mechanics Approach to Fatigue," Proc. of the 18th Sagamore Army Materials Research Conf., Syracuse Univ. Press, 1964, pp. 107-127.

29. Yang, J.N., "Statistics of Random Loading Relevant to Fatigue", paper to be presented at the National Conference on Structural Engineering, ASCE, to be held in Cincinnati, Ohio, April 1974, also to appear in Journal of Eng. Mech. Division, ASCE, June 1974.

30. Yang, J.N., and Heer, E., "Reliability of Randomly Excited Structures," AIAA Journal, Vol. 9, No. 7, 1971, pp. 1262-1268.

31. Tiffany, C.F., et al, "Some Fracture Considerations in the Design and Analysis of Spacecraft Pressure Vessels," American Society for Metals, ASM Technical Report, C6-2.3, 1966.

32. Wood, H.A., "The Use of Fracture Mechanics Principles in the Design and Analysis of Damage Tolerant Aircraft Structures," in Fatigue Life Prediction for Aircraft Structures and Materials, AGARD-LS-62, 1973, pp. 4.1-4.3.

33. Whaley, R.E., et al, "Fatigue Crack Propagation and Residual-Static-Strength Results on Full-Scale Transport Airplanes Wings", NACA-TN-3847, 1956.

34. Hardrath, H.F., and Whaley, R.E., "Fatigue-Crack Propagation and Residual-Static-Strength of Build-Up Structures," NACA-TN-4012, 1957.

35. Snider, H.L., et al, "Residual Strength and Crack Propagation Test on C-130 Airplane Center Wings with Service-Imposed Fatigue Damage," NASA CR-2075, 1972.

36. Schütz, I.W., "Fatigue Life Prediction-A Somewhat Optimistic View of the Problem," in Fatigue Life Prediction for Aircraft Structures and Materials, AGARD-LS-62, 1973, pp. 10.1-10.31.

37. Knorr, E., "Reliability of the Detection of Flaws and the Determination of Flaw Size", in AGARD Fracture Mechanics Survey, edited by H. Liebowitz.

38. Shinozuka, M., and Yang, J.N., "Optimum Structural Design Based on Reliability and Proof Load Test", Annals of Assurance Science, Proc. of Reliability and Maintainability, Vol. 8, 1969, pp. 375-391.

39. Heer, E., and Yang, J.N., "Structural Optimization Based on Fracture Mechanics and Reliability Criteria", AIAA Journal, Vol. 9, No. 5, 1971, pp. 621-628.

40. Schütz, D., "Standardized Flight-by-Flight Test Program for Fatigue Evaluation of Wing Components of Transport Aircraft", paper presented at the 7th ICAF Symposium, London, 1973.

41. Development of Statistical Fatigue Failure Characteristics of 2024-T3 Aluminum Alloy Material Under Simulated Flight-by-Flight Loading", to appear as Air Force Materials Lab. Technical Report, April 1974.

42. Freudenthal, A.M., and Schueller, G.I., "Scatter Factors and Reliability of Aircraft Structures", The George Washington University, Technical Report NASA Grant NGR - 09-010-058, 1973.

43. Buxbaum, O., "Methods of Stress Measurement Analysis for Fatigue Life Evaluation", in Fatigue Life Prediction for Aircraft Structures and Materials, AGARD-LS-62, 1973, pp. 2.1-2.19.

44. Crichlow, W.J., "On Fatigue Analysis and Testing for the Design of Airframes", in Fatigue Life Prediction for Aircraft Structures and Materials, AGARD-LS-62, 1973, pp. 6.1-6.36.

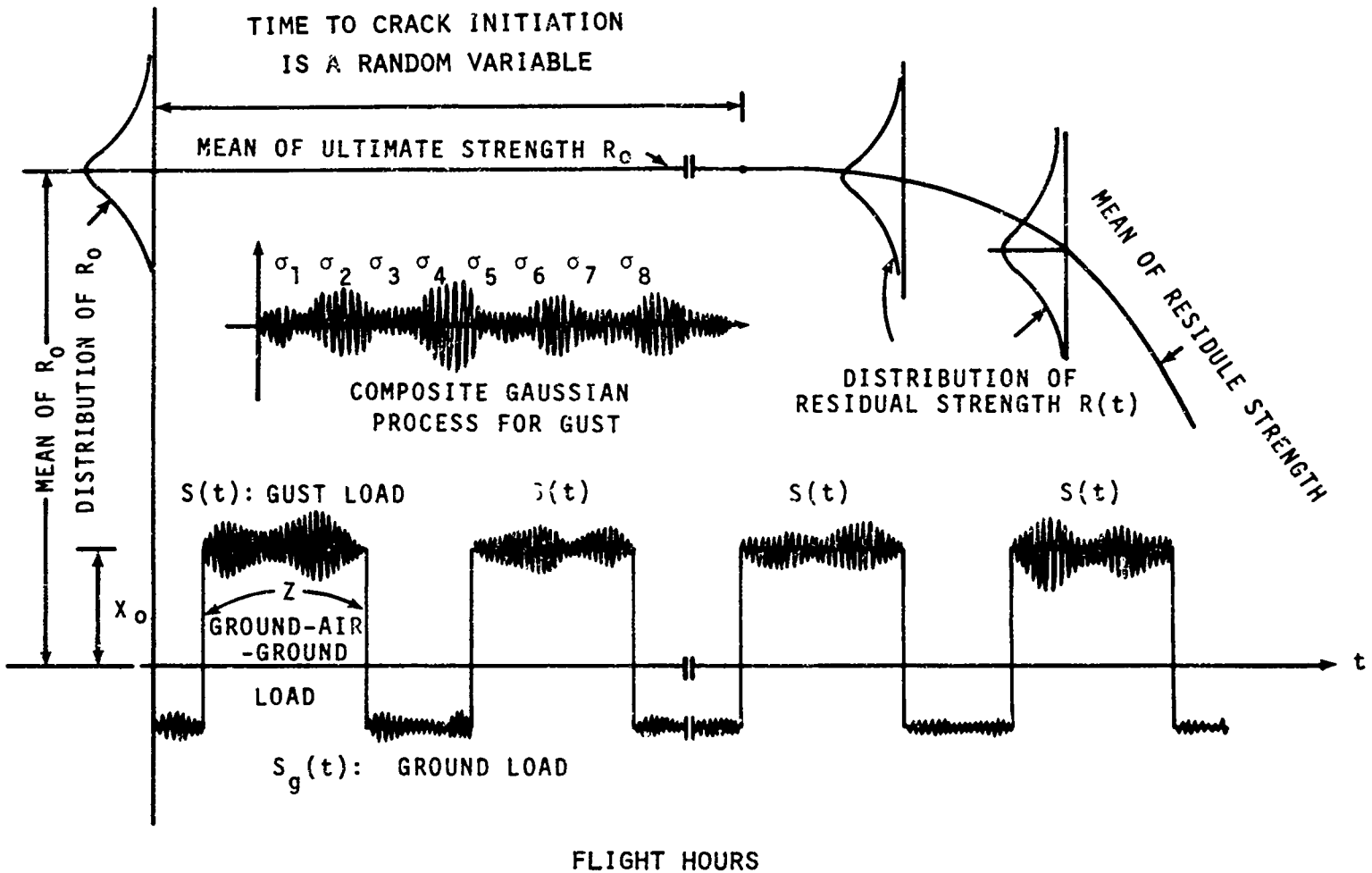

Fig. 1 - Profile of Flight-by-Flight Load Spectrum, Ultimate Strength and Residual Strength

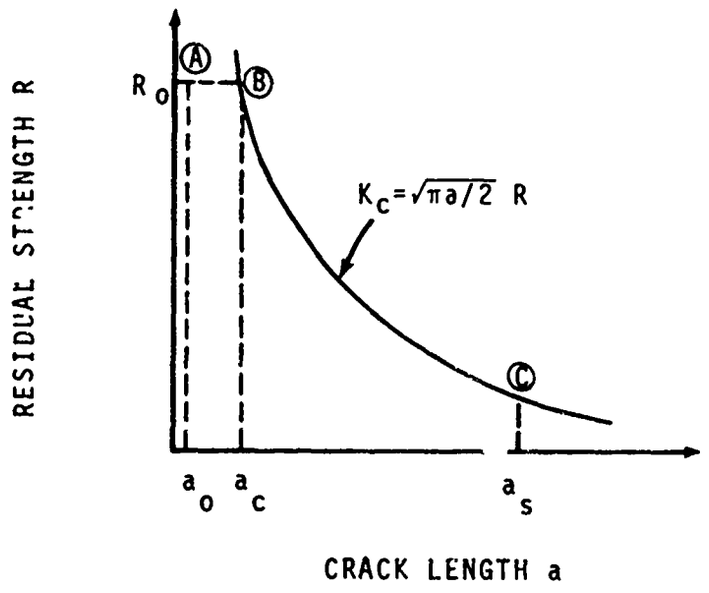

Fig. 2 - Relationship Between Residual Strength R and Crack Size a

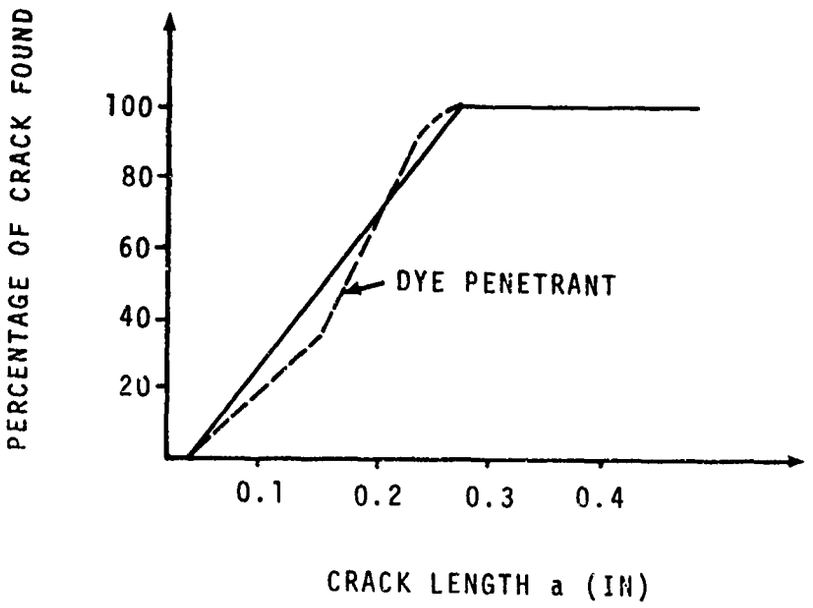

Fig. 3 - Demonstration of Crack Detection Capability

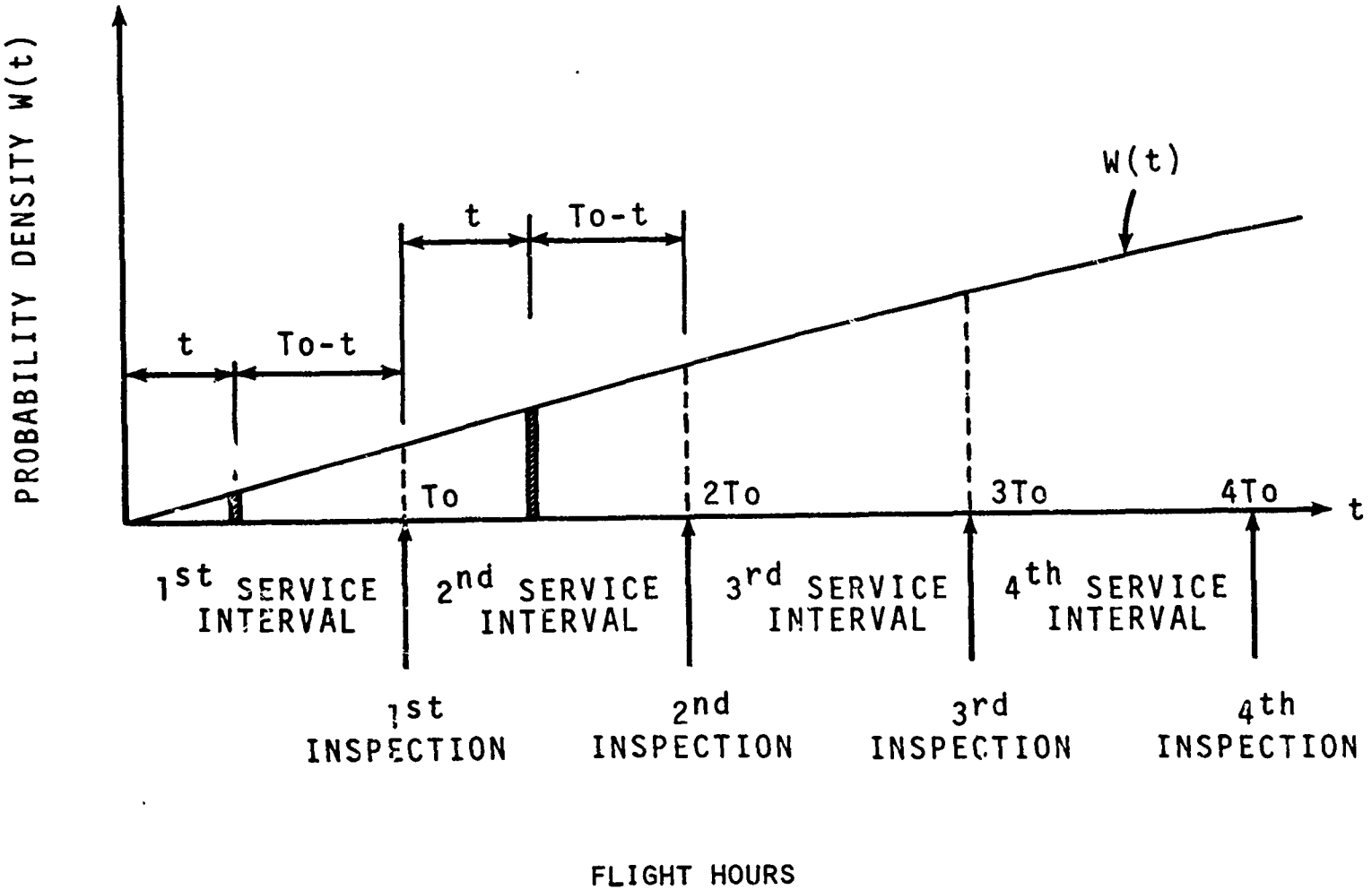

Fig. 4 - Periodic Inspection, Service Interval and Probability Density for Crack Initiation

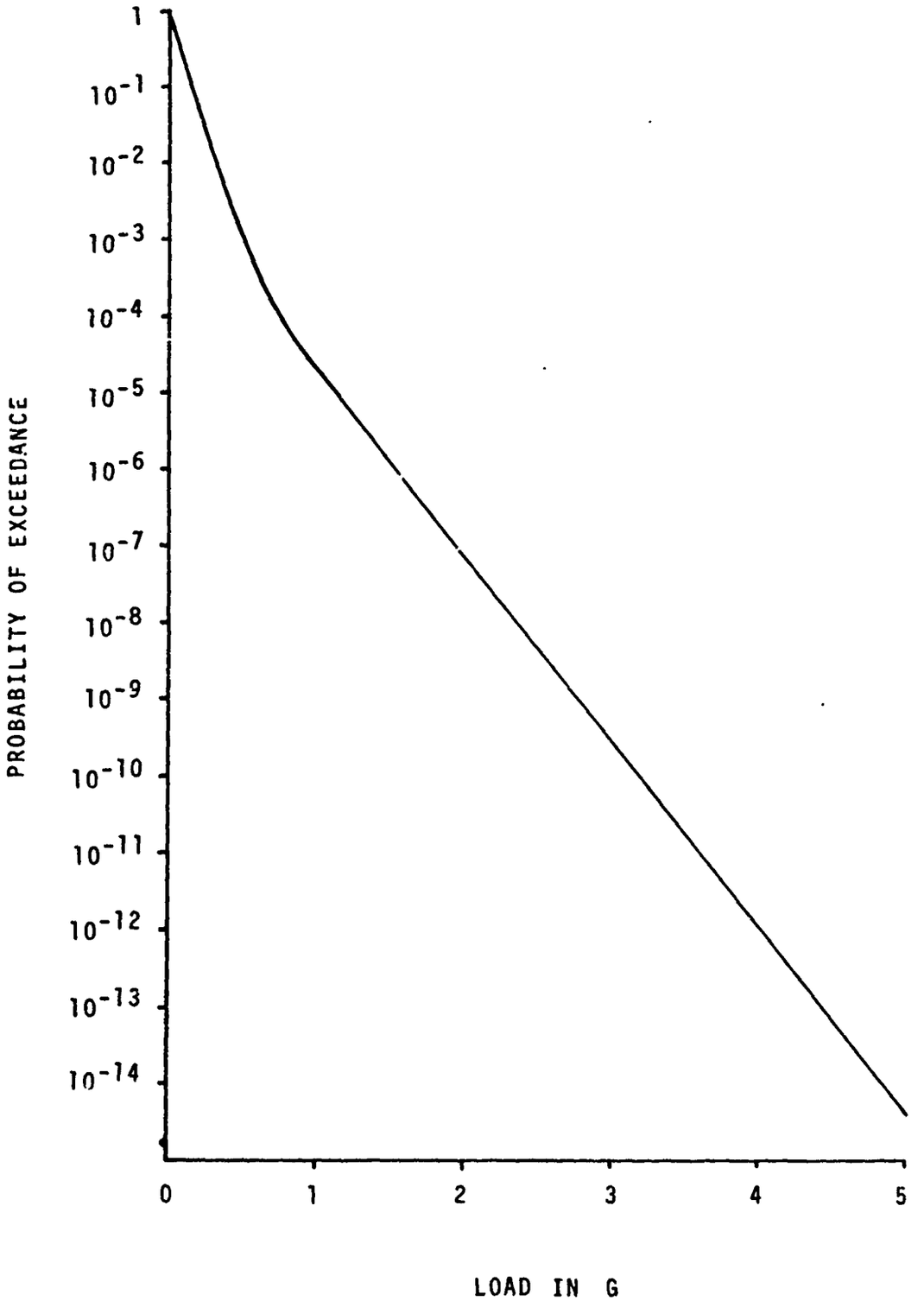

Fig. 5 - Exceedence Curve for Gust Loading

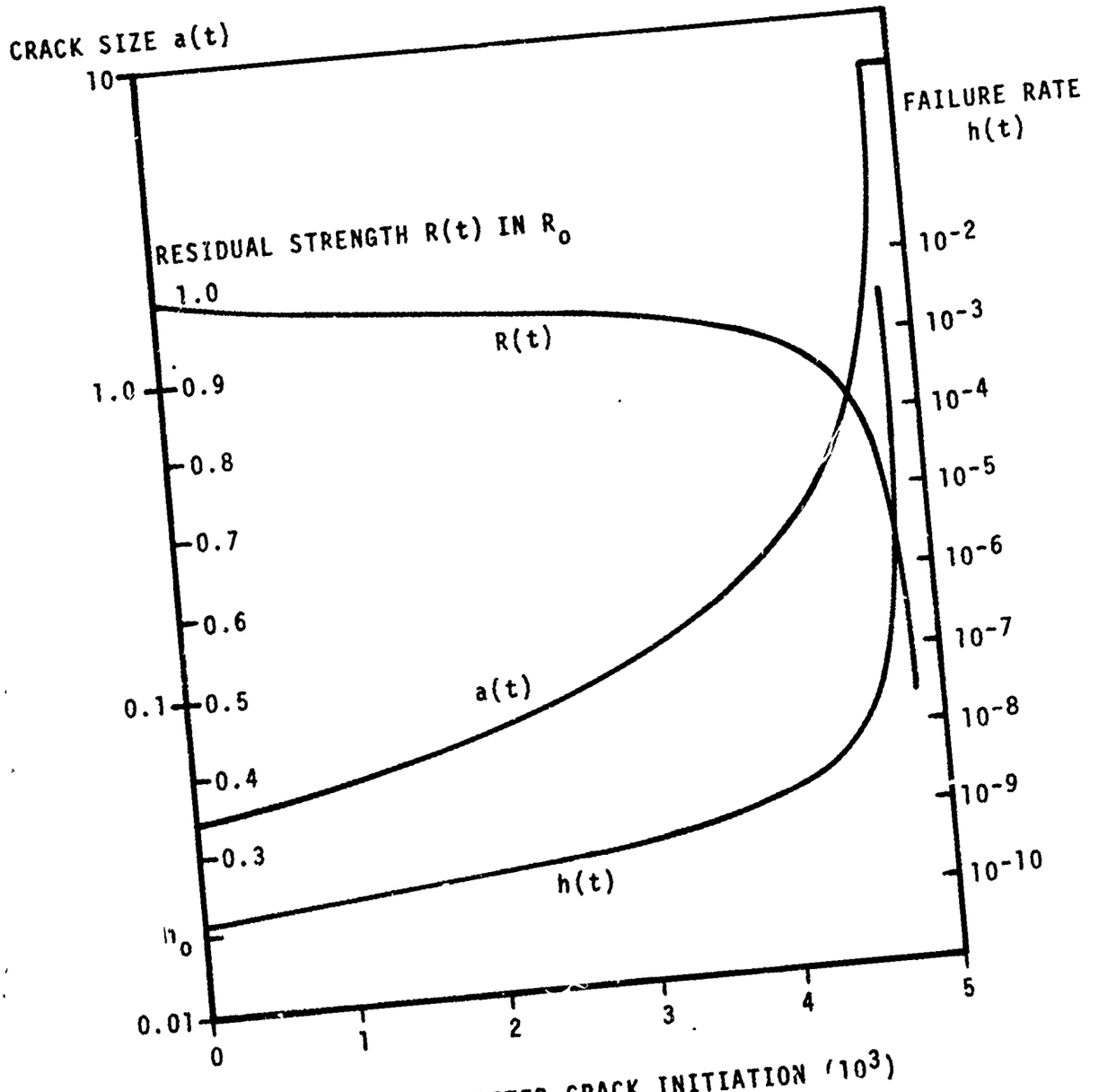

Fig. 6 - Crack Length a(t), Residual Strength R(t) and Failure Rate h(t) After Crack Initiation

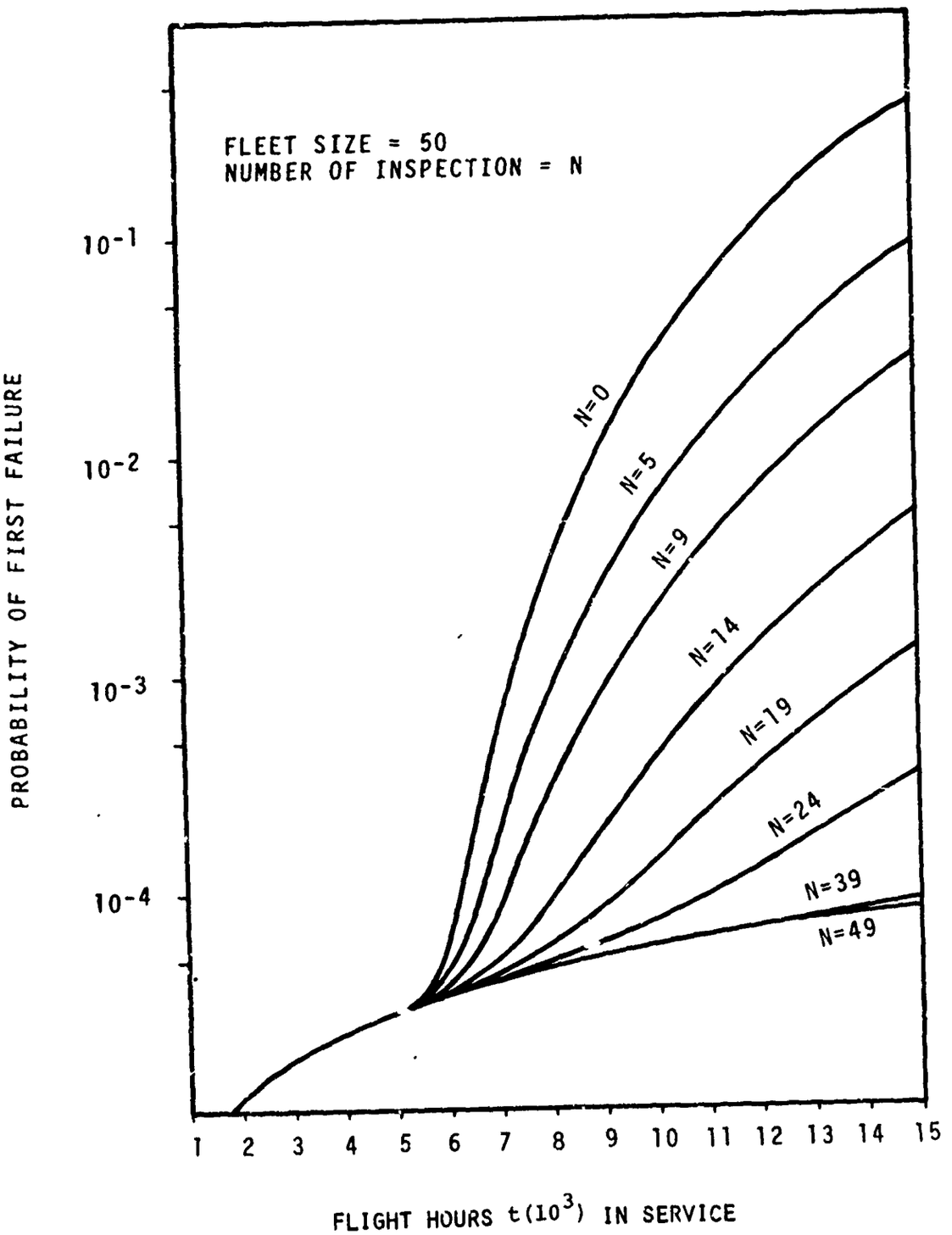

Fig. 7 - Probability of First Failure in a Fleet of 50 Airplanes vs. Service Time and Number of Inspections

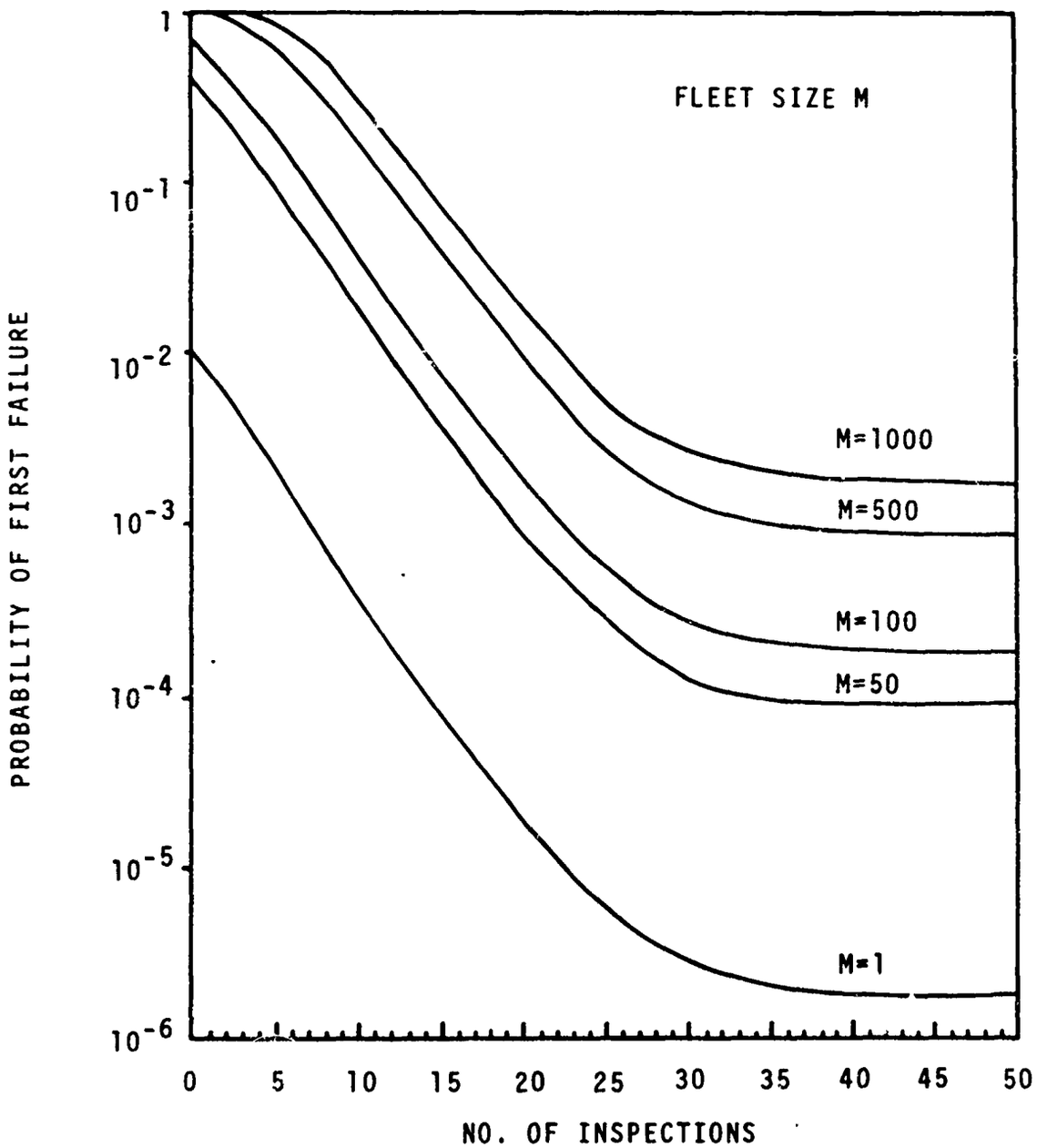

Fig. 8 - Probability of First Failure vs. Number of Inspections and Fleet Size

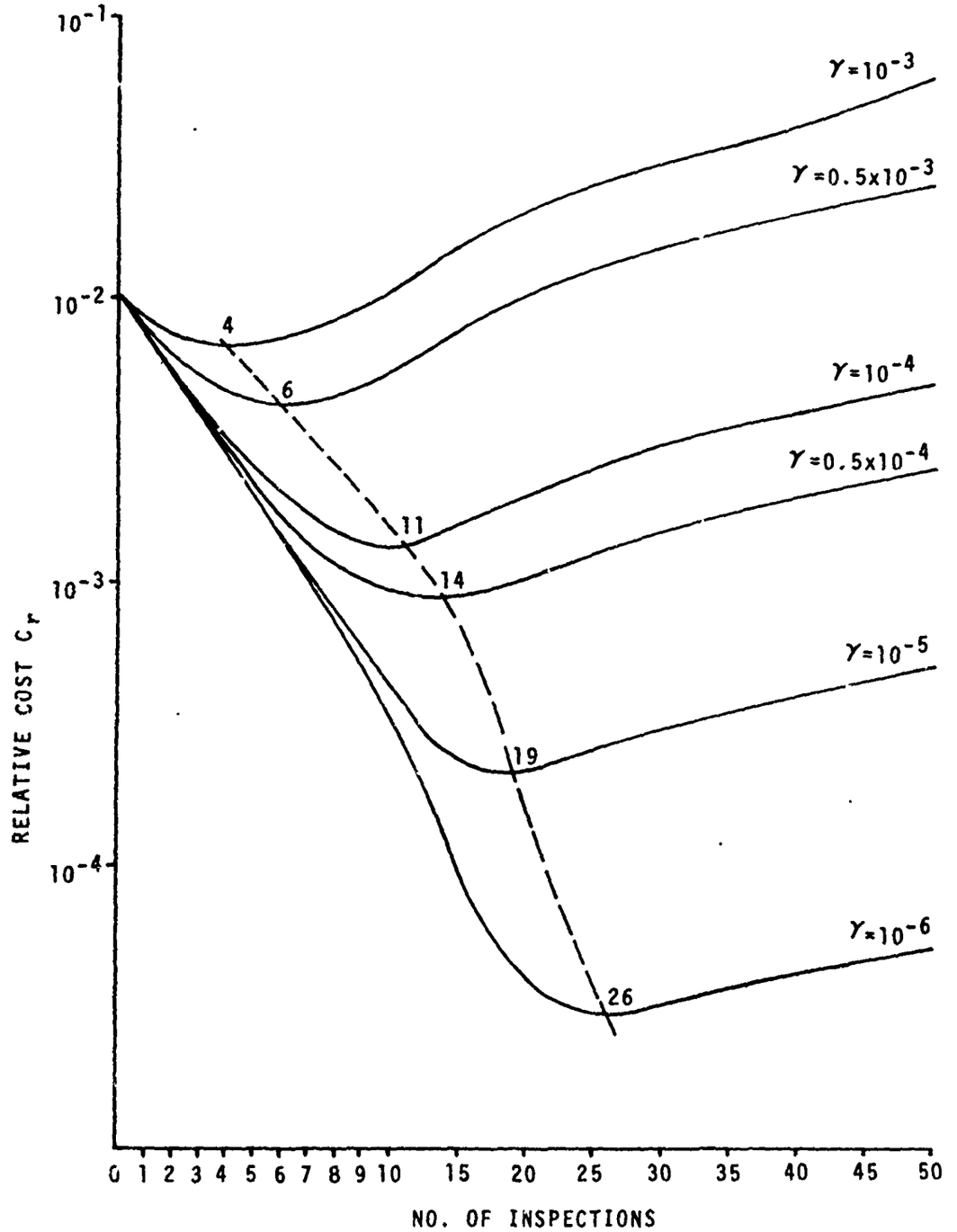

Fig. 9 - Relative Cost C_r vs. Number of Inspections, and Optimum Inspection Frequency

*U.S.Government Printing Office: 1974 — 758-434/555