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# COMPUTATIONAL METHODS FOR DETERMINING 1 DEGREE X 1 DEGREE MEAN GRAVITY ANOMALIES AND THEIR ACCURACIES

Defense Mapping Agency Aerospace Center
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# PREFACE

GENERAL: This publication is one of a series of Reference Publications on achievements related to the fields of mapping, charting, and geodesy, and their related arts and sciences. Each Reference Publication is written by a Defense Mapping Agency Aerospace Center technician qualified by training and experience to contribute knowledge and technology to the selected subject.

PURPOSE: To contribute technical information to the field of geodesy by describing the computational methods used by DMAAC to determine 1° x 1° mean free-air gravity anomalies and their accuracies.

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# ABSTRACT

This publication contains the explanations of the methods used in computing the 1° x 1° mean free-air gravity anomaly $(\overline{\Delta g_f})$ values and their accuracies to be published in DMAAC Reference Publication No. 73-0002.

The information contained in this report is in two parts. Part I identifies each computational method, gives a written definition, and furnishes the equations with a sample computation. When and how the method should be used and any strengths and shortcomings of the method are addressed. Part II identifies the method used to arrive at an accuracy value for each computational method, gives a written definition with associated equations, and provides a sample computation. Shortcomings of the methods are stated.

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# PART I

# COMPUTATIONAL METHODS FOR DETERMINING MEAN FREE-AIR GRAVITY ANOMALIES

# COMPUTATIONAL METHODS FOR DETERMINING 1° x 1° MEAN GRAVITY ANOMALIES AND THEIR ACCURACIES

## INTRODUCTION

In the ideal situation, a mean free-air gravity anomaly ($\overline{\Delta g}_f$) for an area would be obtained from an infinite number of well distributed gravity observations. There are no areas where the density and distribution of gravity observations permit this ideal situation. In most cases, the gravity observations are either concentrated in local areas or along transportation routes so that it is necessary to compute the mean free-air gravity anomaly for an area from gravity observational data with irregular distribution. This necessitates using various methods to compute $\overline{\Delta g}_f$.

The $\overline{\Delta g}_f$ values are computed for all areas of the world where sufficient gravity data is available. In areas where there is a requirement for a $\overline{\Delta g}_f$ and there is sparse or no gravity coverage, the $\overline{\Delta g}_f$ is determined using gravity-geophysical correlation techniques. The $\overline{\Delta g}_f$ values represent the mean values of the anomalous gravity within the surface areas of various required sizes, and are developed directly from observed gravity data whenever possible. In ocean areas the $\overline{\Delta g}_f$ is computed from point free-air gravity anomalies. Over land areas it is computed using point Bouguer gravity anomalies and mean elevations.

All gravity data held by the DOD Gravity Library is referenced to the International Gravity Standardization Net, 1971 (IGSN 71), and gravity anomalies are computed using the Gravity Formula for the Geodetic Reference System 1967 (GFGRS 67).

All computed $\overline{\Delta g}_f$ values and supporting information are recorded in punch card form and stored on magnetic tape. The following is a list of codes which indicate the method used to compute/predict the 1° x 1° mean free-air gravity anomaly:

3 - Bouguer anomaly map estimates.

4 - Free-air anomaly map estimates.

6 - Geophysical correlation.

8 - Interpolated or extrapolated.

9 - Mean anomalies received from other organizations.

A - Average of smaller size quads.

M - Modified average free-air.

## MEAN GRAVITY ANOMALY COMPUTATION

### 1. Bouguer Gravity Anomaly Map Estimates

In this method, a rectangular grid with compartments of desired size is overlayed on a contoured Bouguer gravity anomaly map. The mean Bouguer gravity anomaly ($\overline{\Delta g}_B$) is estimated for each rectangular compartment.

The $\overline{\Delta g}_f$ is computed using the following equation:

$$\overline{\Delta g}_f = \overline{\Delta g}_B + 0.1119\ \bar{h}_m$$

where:

$\bar{h}_m$ = the estimated mean elevation of the compartment in meters.

0.1119 mgal/m = the effect of the Bouguer plate.

This method is used in areas with insufficient point gravity data in the DOD Gravity Library for direct computation provided a Bouguer gravity anomaly map is available.

Example:

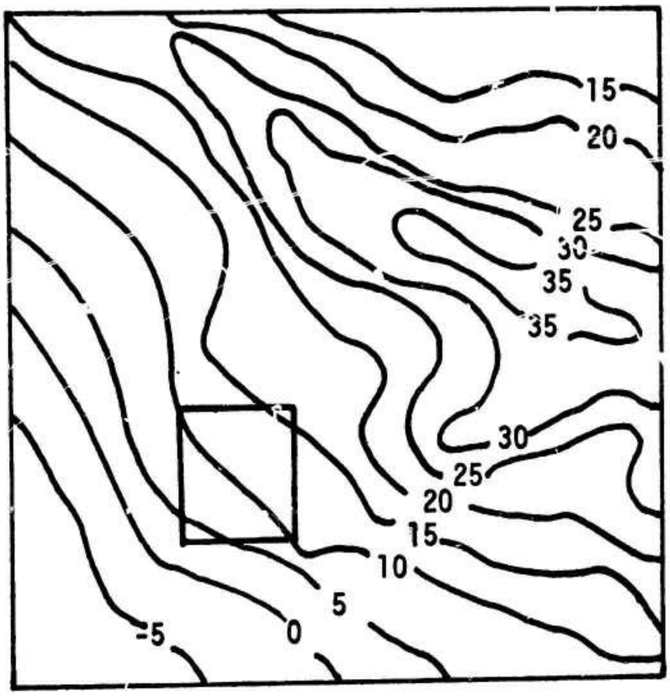


Figure 1. Contours of Bouguer Gravity Anomalies

The $\overline{\Delta g_B}$ of the inset block is 10 mgals, and the mean elevation estimated from a topographic map is 50 meters. The $\overline{\Delta g_f}$ is:

$$\overline{\Delta g_f} = 10 \text{ mgals} + (0.1119 \text{ mgal/m})(50\text{m})$$

$$\overline{\Delta g_f} = 10 \text{ mgals} + 5.6 \text{ mgals}$$

$$\overline{\Delta g_f} = 15.6 \text{ mgals.}$$

The Bouguer gravity anomaly map method is slow and prone to human error. The estimates made by the analyst are subjective, and since this is basically a manual method, blunders are possible. Grids must be made for each map used and the grid size varies for each latitude band.

## 2. Free-Air Gravity Anomaly Map Estimates

This method is mechanically the same as the Bouguer Gravity Anomaly Map Estimates method. A rectangular grid with compartments of desired size is overlayed on a contoured free-air gravity anomaly map. The $\overline{\Delta g_f}$ is estimated for each rectangular compartment. This method is used for areas with little or no point gravity data in the DOD gravity Library if a free-air gravity anomaly map is available. The $\Delta g_f$ of the inset block in Figure 2 is 20 mgals.

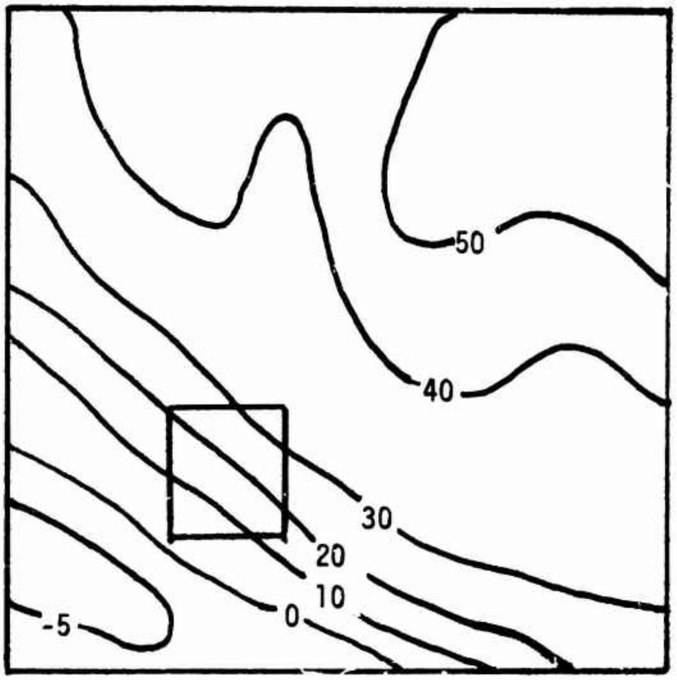


Figure 2. Contours of Free-Air Gravity Anomalies

This method has all of the disadvantages of the Bouguer Gravity Anomaly Map method plus the disadvantage of steep gradients inherent in the dependence of the free-air gravity anomaly on elevation. Special attention has to be given to bathymetric features, such as trenches, island arcs, etc., in ocean areas which usually have strong fluctuating gravity anomaly fields that may not be reflected in a free-air gravity anomaly map because of the poor distribution of point observations. The use of this method is restricted to water areas and flat land areas.

### 3. Geophysical Correlation

Geophysical correlations are used to compute mean free-air gravity anomalies in gravimetrically deficient land and water areas. The many different techniques used are being documented in "Gravity Correlation Study: Manual of 1° x 1° Mean Gravity Anomaly Prediction for Continental Areas," in preparation. These values are recomputed and replaced using another technique as soon as sufficient gravity data becomes available.

### 4. Interpolation and Extrapolation

This method is primarily used in ocean areas with insufficient point gravity anomaly data to compute a $\overline{\Delta g_f}$. The point gravity anomaly can generally be correlated with the bathymetric features in surveyed areas, and the same correlation applied in unsurveyed areas with similar bathymetric features. The systematic relation to water depth is generally linear and the different bathymetric features (islands, trenches, sea mounts, etc.) have a characteristic influence on the point gravity anomaly.

The analyst's knowledge of the expected behavior of the gravity field is used to aid in contouring or to continue contours with respect to bathymetric features. The $\overline{\Delta g}_f$ is then estimated from the contours.

Example:

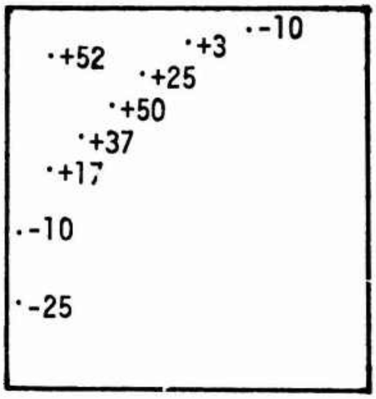


Figure 3. Free-Air Gravity Anomalies

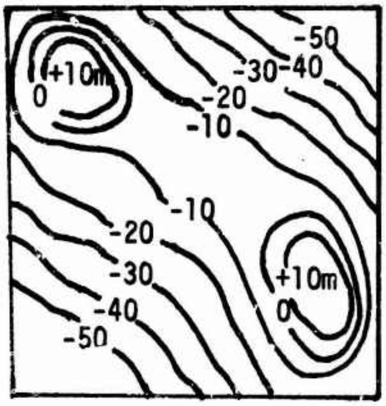


Figure 4. Bathymetry

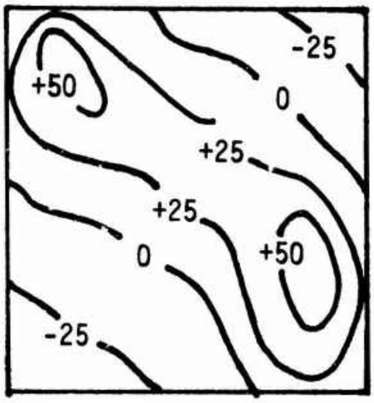


Figure 5. Extrapolated Anomaly Field

The diagrams illustrate the general method of extrapolating a gravity anomaly field. Starting with point values from observations (Figure 3) and the bathymetric information (Figure 4), the gravity anomaly field for the quad is extrapolated (Figure 5) using a linear relationship between the point gravity anomaly and bathymetry.

This method can be used on land with $\Delta g_B$ as point data instead of the $\Delta g_f$ because of the smoother field. The method is at best an approximation, and is basically a manual process. These values are recomputed and replaced using another technique as soon as sufficient gravity data becomes available.

5. Mean Anomalies Received from Other Organizations

These values may be mean Bouguer or free-air gravity anomalies. The methods used for computing the mean gravity anomaly ($\overline{\Delta g}$) values and their accuracies vary. Many times the methods used are either not documented or not available. If there is gravity data available in the DOD Gravity Library that falls within the area, an attempt is made to evaluate the $\overline{\Delta g}$ values. This is usually done by simple comparison of the values. Many times, this comparison is not possible because of lack of data, and the $\overline{\Delta g}$ is accepted simply because there is nothing better.

6. Average of Smaller Size Quads

In areas where the gravity field has been studied in detail, $\overline{\Delta g}_f$ values of small size surface elements are formed. In this method, the $\overline{\Delta g}_f$ values of the small size surface elements are averaged to form the representative mean of a larger surface element.

This method is always used where smaller size means are available.

The general equation is:

$$(\overline{\Delta g_f})_1 = \frac{1}{n} \sum_{i=1}^{n} (\overline{\Delta g_f})_i$$

where:

$(\overline{\Delta g_f})_1$ = mean gravity anomaly of the large surface element.

$(\overline{\Delta g_f})_i$ = mean gravity anomaly of the small surface element.

n = number of small surface elements within the large surface element.

In Figure 6, the block has been divided into 5' x 5' elements, and the $\overline{\Delta g_f}$ for each 5' x 5' element shown. The $\overline{\Delta g_f}$ for the 1° x 1° is +25 mgals using:

$$(\overline{\Delta g_f})_{1° \times 1°} = \frac{1}{144} \sum_{i=1}^{144} (\overline{\Delta g_f})_i$$

To produce the basic 5' x 5' $\overline{\Delta g_f}$ values, an automated method which predicts a 5' x 5' $\overline{\Delta g_f}$ and its accuracy by using point gravity data is used. The basic equation for the least-squares

5'

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| -21 | -11 | 3 | 23 | 27 | 38 | 47 | 45 | 39 | 33 | 19 | 18 |
| -22 | - 6 | 13 | 30 | 37 | 49 | 57 | 53 | 41 | 33 | 18 | 14 |
| -17 | 1 | 19 | 31 | 42 | 47 | 58 | 50 | 37 | 20 | 13 | 11 |
| - 7 | 14 | 31 | 36 | 45 | 45 | 54 | 50 | 29 | 23 | 7 | -5 |
| 11 | 33 | 39 | 48 | 45 | 44 | 47 | 47 | 45 | 19 | -5 | 0 |
| 13 | 48 | 58 | 57 | 46 | 48 | 45 | 37 | 32 | 8 | 0 | 1 |
| 27 | 37 | 55 | 52 | 45 | 50 | 47 | 39 | 22 | 18 | 9 | 3 |
| 35 | 46 | 46 | 41 | 40 | 44 | 40 | 33 | 29 | 20 | 1 | -8 |
| 32 | 39 | 41 | 39 | 44 | 46 | 40 | 27 | 38 | 8 | -6 | -13 |
| 39 | 39 | 37 | 38 | 34 | 41 | 43 | 38 | 37 | 17 | -19 | -20 |
| 39 | 42 | 30 | 19 | 12 | 25 | 43 | 37 | 16 | -18 | -28 | -29 |
| 39 | 39 | 28 | 23 | 1 | 5 | 6 | 8 | -22 | -28 | -32 | -28 |

5'

1°

1°

Figure 6. 5' x 5' Mean Free-Air Gravity Anomalies

prediction is:

$$\Delta g_p = [C_{p1}, C_{p2} \cdot \cdot \cdot C_{pn}] \begin{bmatrix} C_{11} & C_{12} & \cdot & \cdot & \cdot & C_{1n} \\ C_{21} & C_{22} & \cdot & \cdot & \cdot & C_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ C_{n1} & C_{n2} & \cdot & \cdot & \cdot & C_{nn} \end{bmatrix}^{-1} \begin{bmatrix} \Delta g_1 \\ \Delta g_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta g_n \end{bmatrix}$$

where:

$\Delta g_p$ = predicted gravity anomaly at a point

$\Delta g_i$ = given gravity anomaly

$C_{11} = C_{22} = C_{33}$ = variance of gravity anomalies

$C_{ik}$ = covariance of gravity anomalies located at points i and k.

This equation is derived and thoroughly discussed by Moritz [2]. Rapp [3], [4], [5] developed a computer program from these equations which is now used to predict most 5' x 5' mean anomalies (either Bouguer or free-air). These references contain numerical examples with discussion of the application of this method.

There are basically two methods of prediction:

a. The $\overline{\Delta g}_f$ predicted directly from point free-air gravity anomalies. This method is principally used to obtain predictions at sea or in poorly mapped land areas where 5' x 5' mean elevations are unavailable.

b. The $\overline{\Delta g_f}$ obtained from elevation corrected, predicted $\overline{\Delta g_B}$. The $\overline{\Delta g_B}$ predictions are based on Bouguer point gravity anomalies. The 5' x 5' mean elevations are multiplied by the constant 0.1119 mgal/meter and added to the predicted $\overline{\Delta g_B}$ to obtain the predicted $\overline{\Delta g_f}$. This method is always used in land areas where 5' x 5' mean elevations are available.

Two sets of covariances are computed for a 5° x 5° area. One set is for east-west profiles while the other set is for north-south profiles. These two sets are combined into one system of covariances by fitting the data in a least squares sense to a tenth order polynomial in r, the separation of the anomalies, as described by Rapp [4], [5]. This may be written as:

$$C(r) = C_0 + C_1 r + C_2 r^2 + . . . C_{10} r^{10}.$$

If a coefficient set has not been computed for the area to be predicted, a computed set for a similar area is substituted. The choice of which coefficient set to use is based on a comparison of the magnitude of the variance ($C_0$) to the magnitude of the variability of the gravity anomaly field. The $C_0$ term produced by a smooth gravity field is numarically smaller than the $C_0$ term produced by a rough gravity anomaly field.

The point gravity anomalies used for prediction are geographically separated by a minimum of 0.4 minutes in latitude and longitude. This is necessary to prevent singular conditions occurring in the matrix inversion, Rapp [3]. Point gravity data can be deleted, scaled, or translated thus allowing for the elimination of known systematic errors. A predetermined acceptance or rejection criterion expressed as a multiple of the standard deviation (e.g., $2\sigma$) of a single observation computed from the Bouguer or free-air point gravity anomalies is used to eliminate extreme point gravity anomaly values from those points used in the prediction. For example, if ten points are to be used in the prediction and one additional point is to be considered, the eleven closest points are located, the standard deviation of a single observation is computed using either the free-air or Bouguer point gravity anomalies, and the point which exceeds the rejection criterion is rejected. Where one or more of the elevations of the closest points is less than or equal to zero, the error and rejection criteria are computed using the free-air point gravity anomalies. In all other cases, it is computed using the Bouguer point gravity anomalies.

After the 5' x 5' $\overline{\Delta g}_f$ values have been predicted, they are placed in a master file. The values are than averaged to produce the 1° x 1° $\overline{\Delta g}_f$.

This method is considered to be the best for 5' x 5' prediction, provided data to compute the necessary statistical coefficients is available. The gravity data in areas where this method is used should have a fair distribution and can have any random pattern. The method works equally well in ocean and land areas.

The 1° x 1° $\overline{\Delta g}_f$ values determined by this method replace all other $\overline{\Delta g}_f$ values previously computed in an area. The disadvantage in using this method is the slow process of determining the $\overline{\Delta g}_f$ values for the small size surface elements.

### 7. Modified Average Free-Air

This is an automated method which is used on a worldwide basis to compute 1° x 1° $\overline{\Delta g}_f$ values. In each 1° x 1° area, the simple average gravity anomaly for each 10' x 10' area which contains point gravity data is computed. The 10' x 10' gravity anomalies are then averaged to obtain 30' x 30' mean gravity anomalies. These 30' x 30' mean gravity anomalies are averaged for the 1° x 1° modified average gravity anomaly.

$$(\overline{\Delta g}_f)_{10' \times 10'} = \frac{1}{n} \sum_{i=1}^{n} (\overline{\Delta g}_f)_i$$

$$(\overline{\Delta g}_f)_{30' \times 30'} = \frac{1}{k} \sum_{i=1}^{k} (\overline{\Delta g}_f)_{10' \times 10'}$$

$$(\overline{\Delta g}_f)_{1° \times 1°} = \frac{1}{j} \sum_{i=1}^{j} (\overline{\Delta g}_f)_{30' \times 30'}$$

where:

n = number of gravity anomalies in a 10' x 10' square.

k = number of non-empty 10' squares in a 30' square.

j = number of non-empty 30' squares in a 1° square.

In Figure 7, the 1° x 1° block has been divided into 10' x 10' blocks, and the point gravity anomalies have been plotted and contoured at a 10 mgal interval.

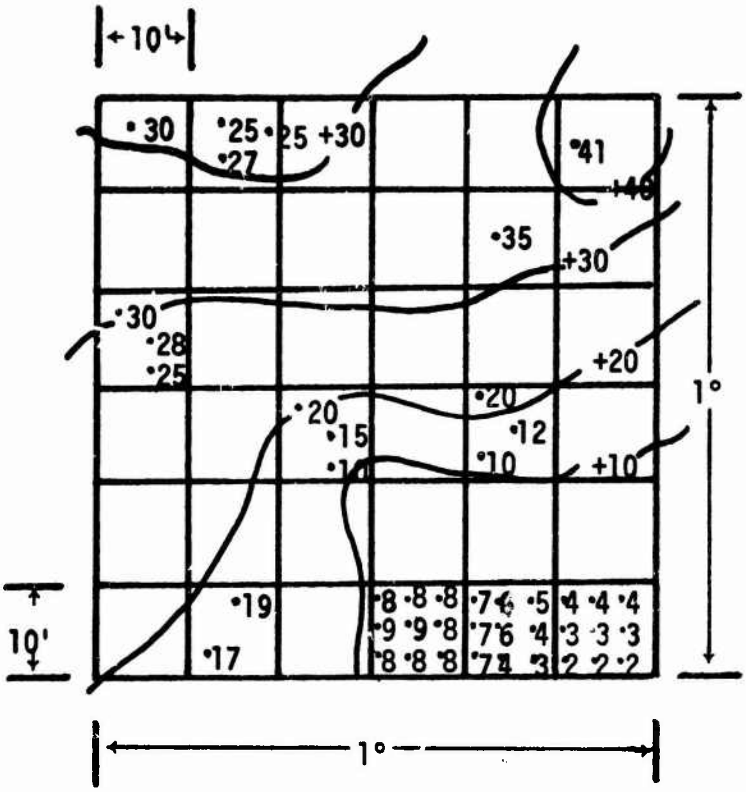


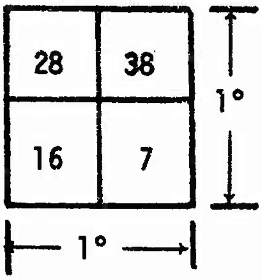
Figure 7. Point Free-Air Gravity Anomalies

Figure 8 shows the simple average for each 10' x 10' block using the data in Figure 7.

| | | | | | |
|---|---|---|---|---|---|
| 30 | 26 | | | | 41 |
| | | | | 35 | |
| 28 | | | | | |
| | | 15 | | 14 | |
| | | | | | |
| | 18 | | 8 | 5 | 3 |

Figure 8. 10' x 10' Mean Free-Air Gravity Anomalies

The 10' x 10' blocks in Figure 8 are averaged for the 30' x 30' blocks shown in Figure 9.



| | |
|---|---|
| 28 | 38 |
| 16 | 7 |

Figure 9. 30' x 30' Mean Free-Air Gravity Anomalies

The modified average value for the 1° x 1° block is:

$$\frac{1}{4}\sum_{1}^{4}(\overline{\Delta g}_f)_{30' \times 30'} = \frac{1}{4}(89) = 22 \text{ mgals.}$$

The example can be used to illustrate how the distribution of the point gravity data and the method of computation can affect the final $\overline{\Delta g}_f$ of the block. The simple average of the points in Figure 7 is 12 mgals which is too small a value for the mean. When using the simple average, too much weight is given to the densely surveyed area in the lower right corner. The average of the 10' x 10' blocks in Figure 8 is 20 mgals. Here, the mean is too low due to the distribution of the 10' x 10' blocks which contain gravity data. By comparing the computed modified average (22 mgals) with the contours of the example (Figure 7), the modified average best describes the $\overline{\Delta g}_f$ of the block.

Satisfactory results are usually obtained from the modified average method. In areas where the distribution is very poor and/or where the point count is less than 5, the computed 1° x 1° $\Delta g_f$ may have to be adjusted in order to be more compatible with the physical situation.

No $\overline{\Delta g}_f$ values are extrapolated or interpolated for the 10' x 10' blocks which do not contain gravity data. Data which is immediately adjacent to the 1° x 1° block is not used, and no mathematical correlation to the 1° x 1° blocks surrounding the area is attempted here. The 1° x 1° $\overline{\Delta g}_f$ values are visually inspected to insure that they are generally compatible with surrounding values and the physical conditions of the area.

This method is used to rapidly incorporate point gravity data into the worldwide 1° x 1° gravity anomaly file. The modified average values are replaced in areas where the gravity field is studied more intensely using other computational techniques.

Most 1° x 1° $\overline{\Delta g}_f$ values have been computed using the Modified Average Free-Air method. The approximate number and percentage of the 1° x 1° $\overline{\Delta g}_f$ values computed by the various methods are tabulated in Table 1.

## TABLE 1

## NUMBER AND PERCENTAGE OF WORLDWIDE 1° x 1° $\Delta\bar{g}_f$ VALUES COMPUTED BY EACH METHOD

| METHOD | NUMBER | PERCENTAGE |
|---|---|---|
| Bouguer Gravity Anomaly Map Estimates | 857 | 3.1 |
| Free-Air Anomaly Map Estimates | 692 | 2.5 |
| Geophysical Correlation | 5691 | 20.7 |
| Interpolation or Extrapolation | 228 | 1.0 |
| Mean Anomalies Received from Other Organizations | 2045 | 7.4 |
| Average of Smaller Size Quads | 948 | 3.4 |
| Modified Average Free-Air | 16980 | 61.9 |
| TOTAL | 27441 | 100.0 |

# PART II

# COMPUTATIONAL METHODS FOR DETERMINING THE ACCURACY OF A MEAN FREE-AIR GRAVITY ANOMALY

# GENERAL ERROR EQUATION

The standard error ($\sigma_{\overline{\Delta g}}$) of a computed or predicted mean gravity anomaly is in general determined by:

$$\sigma^2_{\overline{\Delta g}} = E_p^{\,2} + E_{\overline{h}}^{\,2} + E_r^{\,2}$$

where:

$E_p$ = error in the point gravity anomalies which are used in the mean gravity anomaly ($\overline{\Delta g}$) computation or prediction.

$E_{\overline{h}}$ = error in the mean elevation value (land).

$E_r$ = error of representation which is a function of the density and distribution of the point anomalies and the gravity gradient.

## 1. Error in the Point Gravity Anomalies

The master point gravity anomaly file contains both evaluated and unevaluated gravity data. Surveying organizations transmit gravity data which may or may not contain an estimate of the survey error ($E_p$). When gravity data is received, it usually is immediately placed in the $\Delta g$ file. After being placed in the $\Delta g$ file, the source may be evaluated and assigned an $E_p$ value, or a previously determined $E_p$ value may be changed.

When computing or predicting $\overline{\Delta g}_f$ values using an evaluated source, $E_p$ is usually obtained directly from the $\Delta g$ file. When using an unevaluated source, $E_p$ is determined empirically by comparing $\Delta g$ values of the unevaluated source with $\Delta g$ values of adjacent source(s).

Usually, a computer print plot of the $\Delta g_B$ values is used for comparison purposes. Since the $\Delta g$ file contains both evaluated and unevaluated sources, the empirically determined $E_p$ is used when an unevaluated source is encountered. If sufficient evaluated data is available for prediction, the unevaluated data is deleted. In computing or predicting $\overline{\Delta g_f}$ values, $E_p$ is always set equal to or greater than one milligal even though the source is evaluated to be better than one milligal. This is because the smallest final accuracy recorded for a $\overline{\Delta g_f}$ value is $\pm 1$ mgal.

2. Error in the Mean Elevations

Mean elevation errors ($E_{\bar{e}}$) are obtained directly from the master mean elevation file. The mean elevation accuracies are determined at the time the mean elevations are determined. The mean elevation error is converted to milligals ($E_{\bar{h}} = 0.1119 \times E_{\bar{e}}$). When the $\overline{\Delta g_f}$ is computed/predicted directly on the geopotential surface (ocean area), $\overline{\Delta g_f}$ values are used directly. Normally, the stated accuracies of the mean elevation values are accepted. However, if there are indications that a mean elevation or its stated accuracy is questionable, the mean elevation or its stated accuracy is changed accordingly.

3. Error Representation (Prediction)

$E_r$ is the error of a computed or predicted mean gravity anomaly without any consideration for errors due to inaccuracies in $\Delta g$ values, mean elevations, or similar quantities. It is dependent only upon the density and distribution of the point gravity anomaly data used to compute or predict the mean gravity anomaly and the gradient of the gravity anomaly field.

## ACCURACY COMPUTATION

### 1. Bouguer Gravity Anomaly Map Estimates

The accuracy determination for this method is subject to many variables which in most cases can only be estimated. These variables include:

- Accuracy of Bouguer Gravity Anomaly Map
    - Accuracy of point source
    - Accuracy of map compilation
- Accuracy of Estimation
    - Distribution and density of point control
    - Gravity anomaly field gradient
    - Quad size

The accuracy of the map depends on the accuracy of the point gravity data used for control, and the method and care used in producing the map. In most cases, these accuracy values are not indicated on the maps and are not obtainable from other sources. Therefore, the analyst must estimate some accuracy value for the map based mainly on his experience. Evaluated data can sometimes be used to evaluate a map if the data lies within or adjacent to the area.

For the estimation of the accuracy of the estimated $\overline{\Delta g}_B$, the analyst has a little more control. If the points are plotted, their distribution and density will help in the accuracy estimation. If they are not, the detail in the contours may indicate point distribution. The contribution to the error of estimation by the gradient of the field must be considered in conjunction with the quad size. This error of estimation is made at the same time as the mean gravity anomaly is estimated, and there is no general equation used. The root mean square of the Bouguer gravity anomaly map accuracy and the accuracy of estimation is the final accuracy for the 1° x 1° quad.

## 2. Free-Air Gravity Anomaly Map Estimates

The accuracy determination for this method, like that for the Bouguer Map Estimation Method, is subject to many variables which in most cases can only be estimated.

The general problems here are the same as those in the Bouguer Map Estimation Method discussed in Part II, Section 1. The major difference here is the much steeper gradients, and the inherent difficulty in estimating the contribution of this steeper gradient to the accuracy estimation.

## 3. Geophysical Correlation

The many different methods used to compute the accuracies of geophysically correlated $\overline{\Delta g}_f$ values are recorded in the document referenced in Part I, Section 3.

## 4. Interpolation or Extrapolation

The accuracy of an interpolated or extrapolated gravity anomaly is at best an estimation made by the analyst based on the particular situation and his experience. The amount, distribution, and quality of observed gravity data, and the bathymetric or topographic characteristics of the area are considered. In many cases, a blanket error is assigned to areas of rather large extent.

### 5. Mean Anomalies Received from Other Organizations

The methods and data used by other organizations to compute $\overline{\Delta g}$ values and their related accuracies often are not documented or available. If evaluated gravity data in the DOD Gravity Library lies within the area, an attempt is made to validate the accuracy values furnished by the source. Usually this is not possible because of the lack of gravity data, and the furnished accuracy values are recorded.

### 6. Average of Smaller Size Quads

The general form of the empirical equation for the accuracy of a $\overline{\Delta g}_f$ made up of the average of smaller quads is:

$$\sigma_1 = \left[ \frac{\sum(\sigma_i)^2}{N(\sqrt{n})} \right]^{1/2}$$

where:

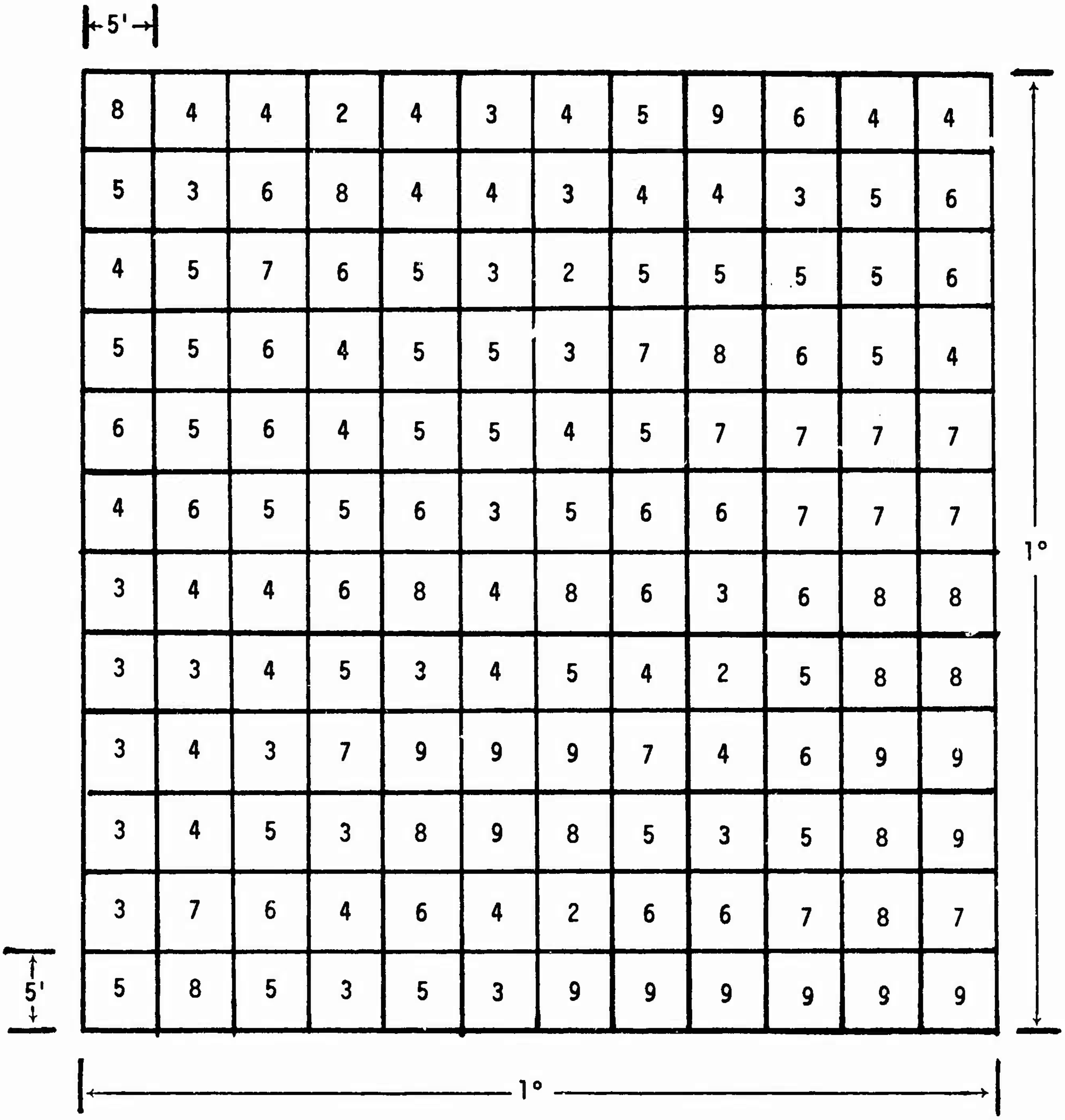
$\sigma_1$ = error of large surface element.

$\sigma_i$ = error of small surface element.

N = number of small surface elements.

n = number of small surface elements with gravity data.

In Figure 10, the 1° x 1° block has been divided into 5' x 5' elements, and the accuracy of the mean gravity anomaly for each 5' x 5' element is shown.



| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 4 | 4 | 2 | 4 | 3 | 4 | 5 | 9 | 6 | 4 | 4 |
| 5 | 3 | 6 | 8 | 4 | 4 | 3 | 4 | 4 | 3 | 5 | 6 |
| 4 | 5 | 7 | 6 | 5 | 3 | 2 | 5 | 5 | 5 | 5 | 6 |
| 5 | 5 | 6 | 4 | 5 | 5 | 3 | 7 | 8 | 6 | 5 | 4 |
| 6 | 5 | 6 | 4 | 5 | 5 | 4 | 5 | 7 | 7 | 7 | 7 |
| 4 | 6 | 5 | 5 | 6 | 3 | 5 | 6 | 6 | 7 | 7 | 7 |
| 3 | 4 | 4 | 6 | 8 | 4 | 8 | 6 | 3 | 6 | 8 | 8 |
| 3 | 3 | 4 | 5 | 3 | 4 | 5 | 4 | 2 | 5 | 8 | 8 |
| 3 | 4 | 3 | 7 | 9 | 9 | 9 | 7 | 4 | 6 | 9 | 9 |
| 3 | 4 | 5 | 3 | 8 | 9 | 8 | 5 | 3 | 5 | 8 | 9 |
| 3 | 7 | 6 | 4 | 6 | 4 | 2 | 6 | 6 | 7 | 8 | 7 |
| 5 | 8 | 5 | 3 | 5 | 3 | 9 | 9 | 9 | 9 | 9 | 9 |

Figure 10. Accuracy of 5' x 5' Mean Gravity Anomalies

The $\sigma_1$ for the 1° x 1° is ±1.7 mgals using:

$$\sigma_{1° \times 1°} = \left[ \frac{\sum(\sigma_{5' \times 5'})^2}{144(12)} \right]^{1/2}$$

This accuracy would then be recorded as ±2 mgals. Since the 5' x 5' $\overline{\Delta g}_f$ values have been studied in detail and edited, the 1° x 1° $\overline{\Delta g}_f$ values and their accuracies can be considered the best available.

The basic equation for the mean square error of least squares prediction is used to compute the error of the 5' x 5' $\overline{\Delta g}_f$ (Moritz [2]).

$$m_p^2 = C_o - [C_{p1}, C_{p2} \cdots C_{pn}] \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{bmatrix}^{-1} \begin{bmatrix} C_{p1} \\ C_{p2} \\ \cdot \\ \cdot \\ \cdot \\ C_{pn} \end{bmatrix}$$

where:

$C_o = C_{11} = C_{22}$ = variance of gravity anomalies

$C_{ik}$ = covariance of gravity anomalies located at points i and k.

The localized error covariances are discussed in Part I, Section 6. The computation of the covariances is fairly straightforward, though lengthy. The specific computations needed for the covariance computation have been fully described in Rapp [3], [4], and [5]. The screening and elimination of erroneous point gravity data and methods to eliminate systematic errors are addressed in Part I, Section 6. This method is considered the best and is used to predict most 5' x 5' $\overline{\Delta g}_f$ values and their accuracies.

## 7. Modified Average Free-Air

In the computation of the standard deviation of the modified average, 10' x 10' blocks with gravity data are considered independent, and those 10' x 10' blocks without gravity data are considered dependent. The general form of the equation is:

$$\sigma_{1^\circ \times 1^\circ} = \frac{1}{36}\left[\left(n(\text{RMS})\right)^2 + (36-n)\ (M)^2\right]^{1/2}$$

where:

$$\text{RMS} = \left[\frac{\sum_{i=1}^{n} m_i^2}{n}\right]^{1/2}$$

m = a dependent error = error of a blank 10' x 10' block = ±19 mgal.

M = an independent error = error of a 10' x 10' block with gravity data = ±2 mgal.

n = number of 10' x 10' blocks without gravity data.

Using these values, the preceding general equation reduces to:

$$\sigma_{1° \times 1°} = \frac{1}{36} \left[(n \cdot 19)^2 + (36-n)\ (2)^2\right]^{1/2}$$

In Figure 11, the 10' x 10' blocks marked with X contain point gravity data. The number of blocks without point data is 25.

| | | | | | |
|---|---|---|---|---|---|
| X | X | | | | X |
| | | | | X | |
| X | | | | | |
| | | X | | X | |
| | | | | | |
| | X | | X | X | X |

Figure 11. 10' x 10' Blocks Containing Point Gravity Data

$$\sigma_{1° \times 1°} = \left[\frac{[25(19)]^2 + (11)(2)^2}{36}\right]^{1/2}$$

$$\sigma_{1° \times 1°} = \left[\frac{225625 + 44}{36}\right]^{1/2}$$

$$\sigma_{1° \times 1°} = \pm 13 \text{ mgals.}$$

Since there are no $\overline{\Delta g}_f$ values extrapolated or interpolated for the blank 10' x 10' blocks, the errors for these blank blocks are likewise not extrapolated or interpolated. Instead, a blanket value of $\pm 19$ mgals is used. This value was taken from Heiskanen and Moritz [1], pg 279, who stated that the error variance for a 1° x 1° block with one gravity station is 356 $\text{mgals}^2$. The independent blocks have been assigned a blanket standard deviation of $\pm 2$ mgals. Again, this method was developed in order to produce a 1° x 1° working field from worldwide point data. As areas are more intensely studied, these modified averages are replaced.

## SUMMARY

This publication describes the methods presently used by DMAAC to determine mean gravity anomalies and their accuracies. The definition, equations, sample computation, advantages, and disadvantages of each method are discussed.

Research to find better methods to compute mean anomalies and their accuracies is a continuing assignment. Some of the equations now in use have been empirically derived. As research progresses, the current methods may be discontinued or altered to incorporate the newly developed improvements.

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