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THE PRACTICAL IMPACT OF RECENT COMPUTER ADVANCES ON THE ANALYSIS AND DESIGN OF LARGE SCALE NETWORKS

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13. ABSTRACT New research results on the following major questions are reported: ARPANET growth, Impact of Satellite channels on packet network cost and performance, terminal oriented network cost and performance, packet switched network design, routing in packet networks, large network computations, broadcast packet system considerations, routing and acknowledgement schemes for broadcast systems; spread spectrum considerations for packet systems, broadcast packet techniques on cable.

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ARPANET, packet switching, computer networks, broadcast packet systems, cost, reliability, throughput, routing.

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The Practical Impact of Recent Computer Advances on the Analysis and Design of Large Scale Networks

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SUMMARY

Technical Problem

Network Analysis Corporation's contract with the Advanced Research Projects Agency has the following objectives:

- To determine the most economical and reliable configurations to meet growth requirements in the ARPANET.
- To study the properties of packet switched computer communication networks.
- To develop techniques for the analysis and design of large scale networks.
- To determine the cost/throughput/reliability characteristics of large packet-switched networks for application to Defense Department computer communication requirements.
- To apply recent computer advances, such as interactive display devices and distributed computing, to the analysis and design of large scale networks.

General Methodology

The approach to the solution of these problems has been the simultaneous

- study of fundamental network analysis and design issues
- development of efficient algorithms for large scale network analysis and design
- development of an interactive distributed display and computational system to deal with large-scale problems
- application of the new analysis and design tools to study cost and performance tradeoffs for large systems

Technical Results

In this report, the following major accomplishments are discussed:

- ARPANET designs with up to 57 nodes were derived.
- A study of the effect of point-to-point and broadcast satellite channels on ARPANET cost and throughput was completed.

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- The second phase of a study of terminal oriented network cost and performance was completed.
- A new large network design technique, based on "cut-saturation" was developed and found to be more cost effective than the branch exchange techniques currently in use.
- Investigations of large network routing schemes were continued, including new routing methods for both high bandwidth and for partitioned networks.
- A multiplexing experiment to obtain low cost leased line terminal access to ARPANET was successfully completed. Four CRT terminals are now operating on a single 4800 BPS line into ARPANET TIP.
- The first phase of an interactive network data handling system has been completed for an IMLAC display editing system for large network graphics.
- The definition of a network analysis problem solving system was completed. This includes system definitions for network data structure manipulation, network algorithm programming, and the first phase in the specification of a network programming language.
- The first phase of a detailed, event oriented simulation model to develop flow control and routing algorithms was completed for the packet radio system.
- Major progress was made in the development of flow control acknowledgement schemes and routing techniques for packet radio.
- A combinatorial model to study properties of message flow in packet radio networks was developed and extensively exercised.
- A spread spectrum model to investigate properties of the packet radio channels was developed.
- A study of the combination of packet broadcast techniques and two way coaxial cable systems for use in local distribution in urban and suburban environments was completed.

Department of Defense Implications

The Department of Defense has vital need for highly reliable and economical communications. The results described in this reporting period reinforce conclusions of earlier periods about the validity of packet switching for massive DOD data communications problems. A major portion of the cost of implementing this technology will occur in providing local access to the networks. Hence, the development of local and regional communication techniques must be given high priority. In addition, the initial results on the use of domestic satellites indicates that substantial savings can be accrued by their use in large scale for DOD data communications.

Implications for Further Research

Further research must continue to develop tools for the study of large network problems. These tools must be used to investigate tradeoffs between terminal and computer density, traffic variations, the effects of improved local access schemes such as packet radio, the use of domestic satellites in broadcast mode for backbone networks, and the effect of link and computer hardware variations in reliability on overall network performance. The potential of these networks to the DOD establishes a high priority for these studies.

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CHAPTER 1

ARPANET GROWTH

During this reporting period a total of ten new nodes (TIP and IMP sites) were proposed for addition to the ARPANET.

If the locations of all network nodes are known in advance, it is clearly most efficient to design the topological structure as a single global effort. However, in the ARPANET, as in most actual networks, node locations are added and modified on numerous occasions. On each such occasion, the topology could be completely reoptimized to determine a new set of link locations.

In practice, however, there is a long lead time between the ordering and the delivery of a link, and major topological modifications cannot be made without substantial difficulty. It is therefore prudent to add or delete nodes with as little disturbance as possible to the basic network structure, consistent with overall economic operation.

Figures (1), (2), (3), (4), and (5) show the present and proposed ARPANET's, derived using a mixed policy of minimum incremental cost and disturbance to the network. Figure (1) represents the present net, while Figure (2) includes those TIP's and IMP's under immediate consideration or already committed for connection to the ARPANET. Figure (3) contains, in addition, all the TIP's and IMP's that are still under active consideration. Figure (4) consists of additional nodes that have been proposed, but are not under active consideration. Figure (5) contains nodes identical to those in Figure (4), but shows, in addition, the several suggested additional lines that would be required for additional reliability if the Figure (4) nodes were to be connected to the ARPANET. Estimated line and modem costs and throughputs for the ARPANET at various stages of growth are given in Table 1. The costs and throughputs corresponding to Figures (1), (2), (3), (4), and (5) are given, respectively, in the last five lines of Table 1. Locations of the IMP's and TIP's are given in Tables 2(a), 2(b), 2(c), and 2(d).

FIGURE 1

ARPANET

(As of December, 1972)

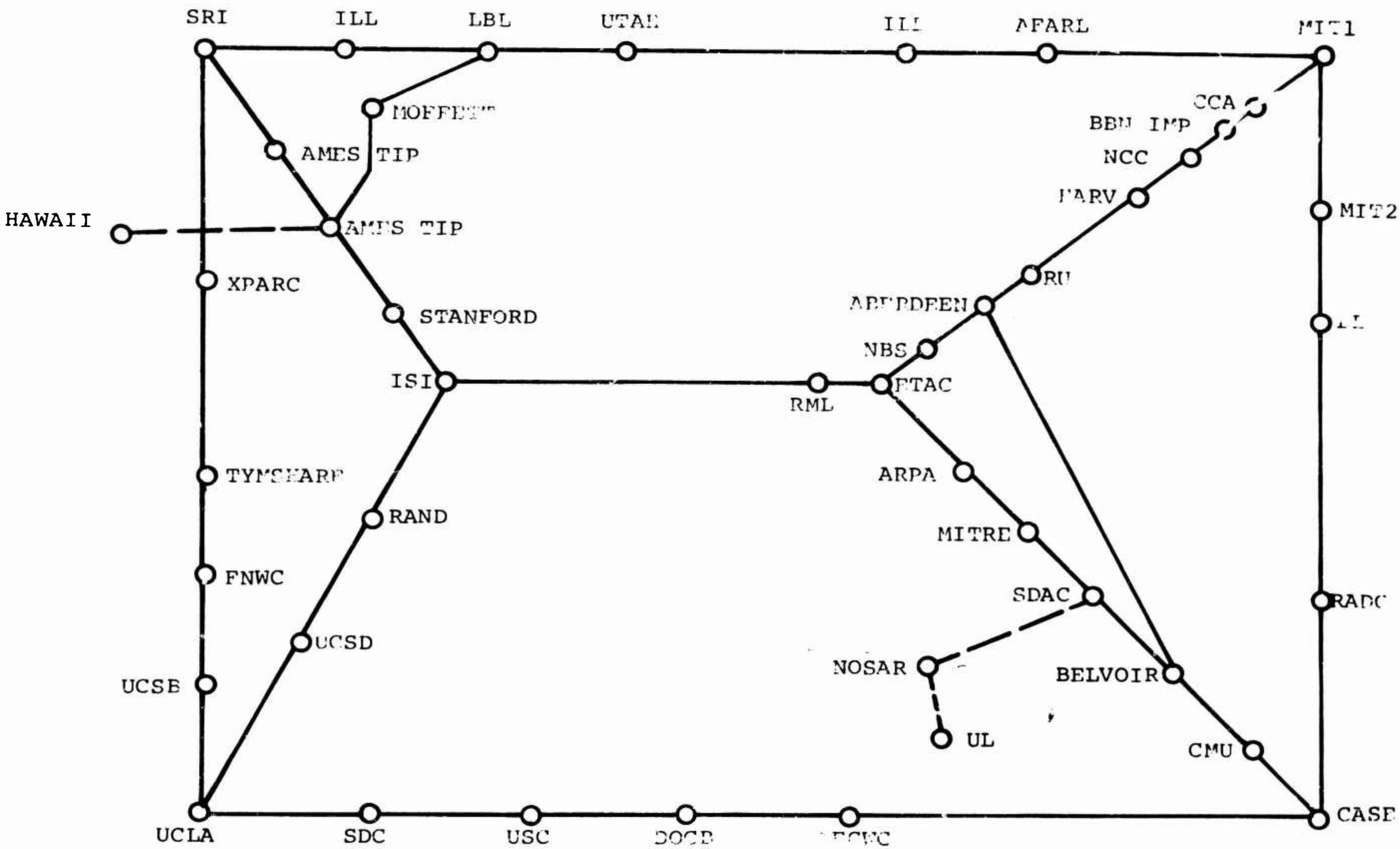
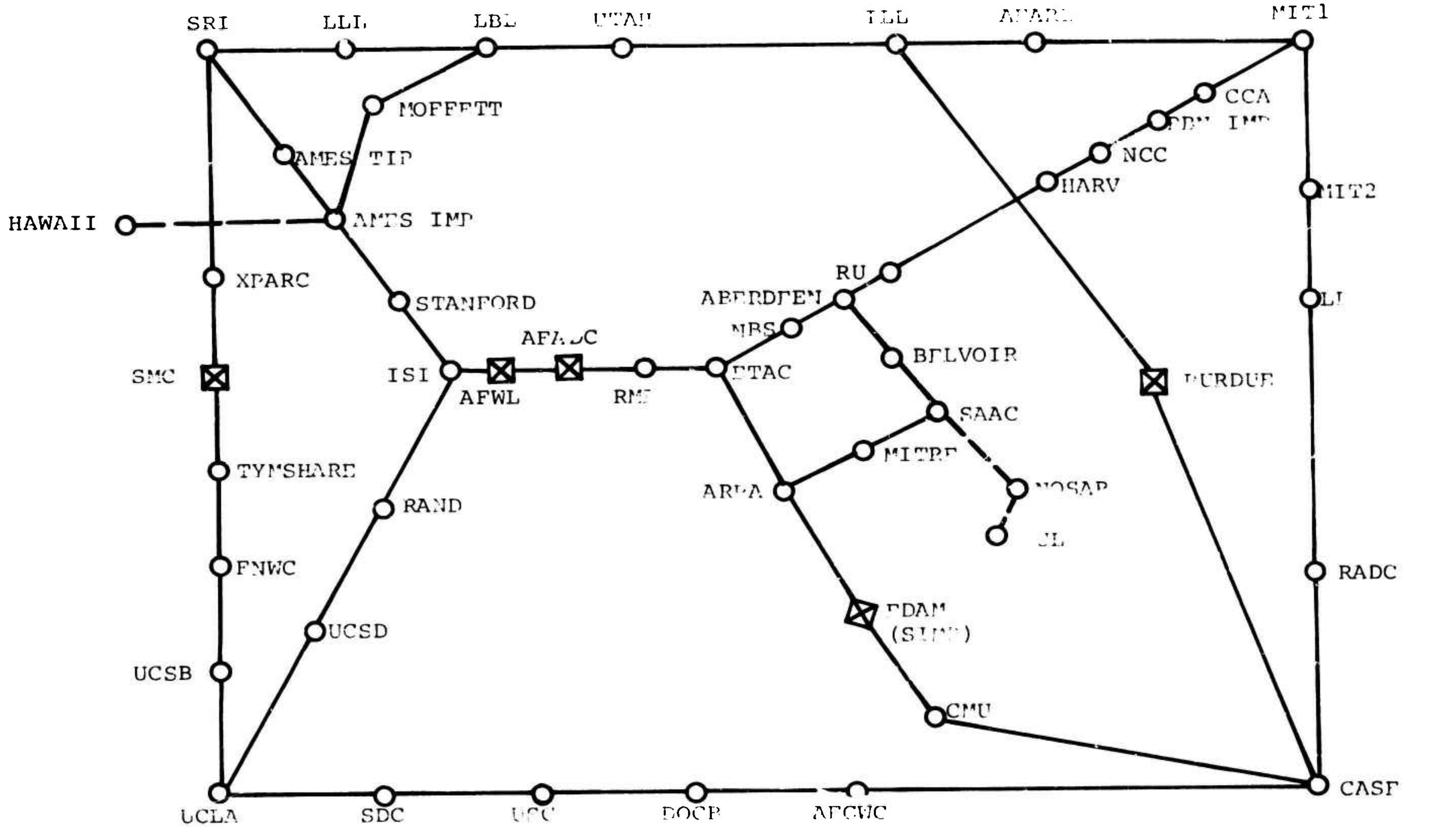


FIGURE 2

ARPANET

(Including Sites Under Immediate Consideration)

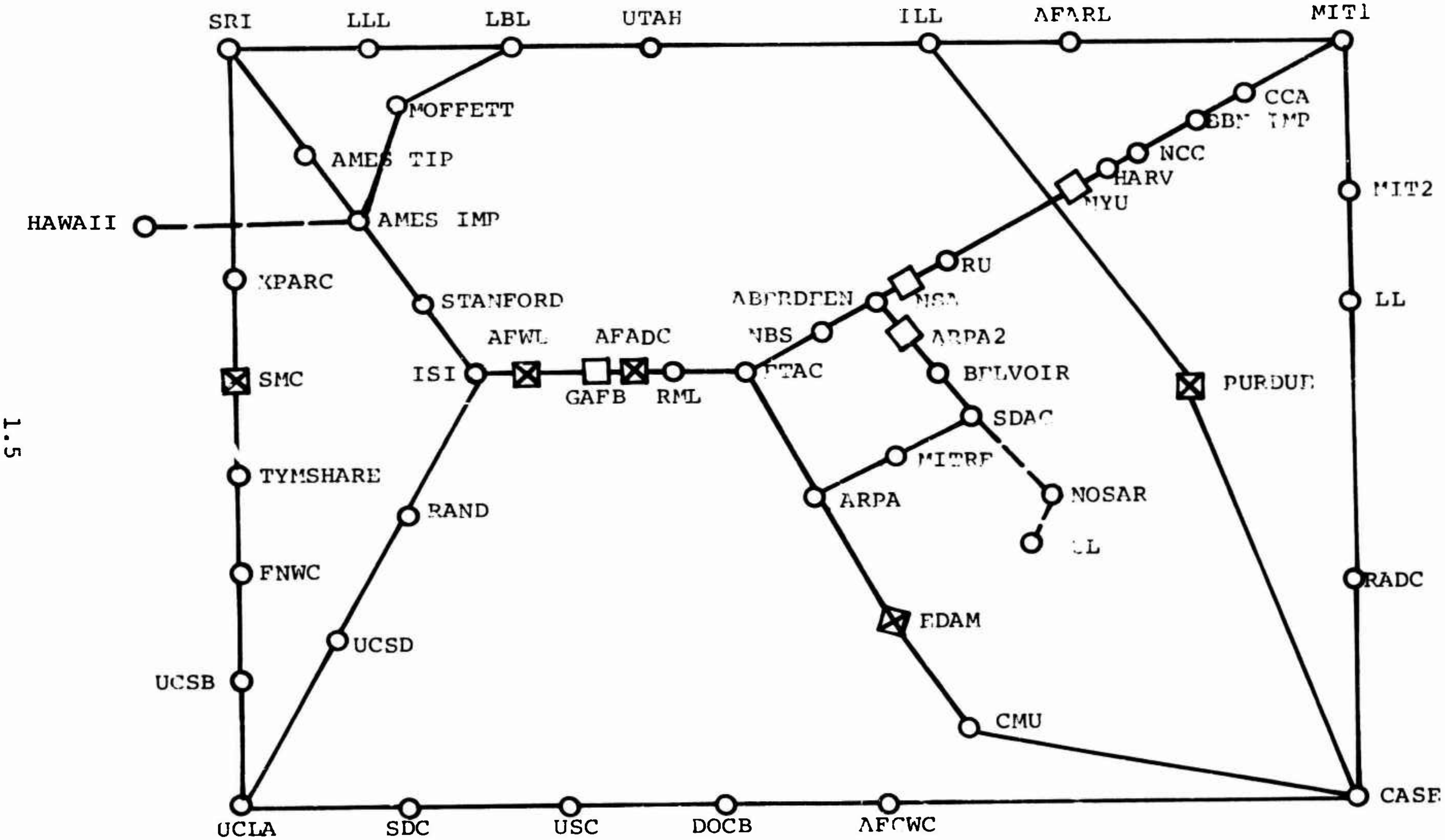


○ Site that is now or to be included in the ARPANET by December, 1973

☒ Site that is under immediate consideration or has been scheduled for connection

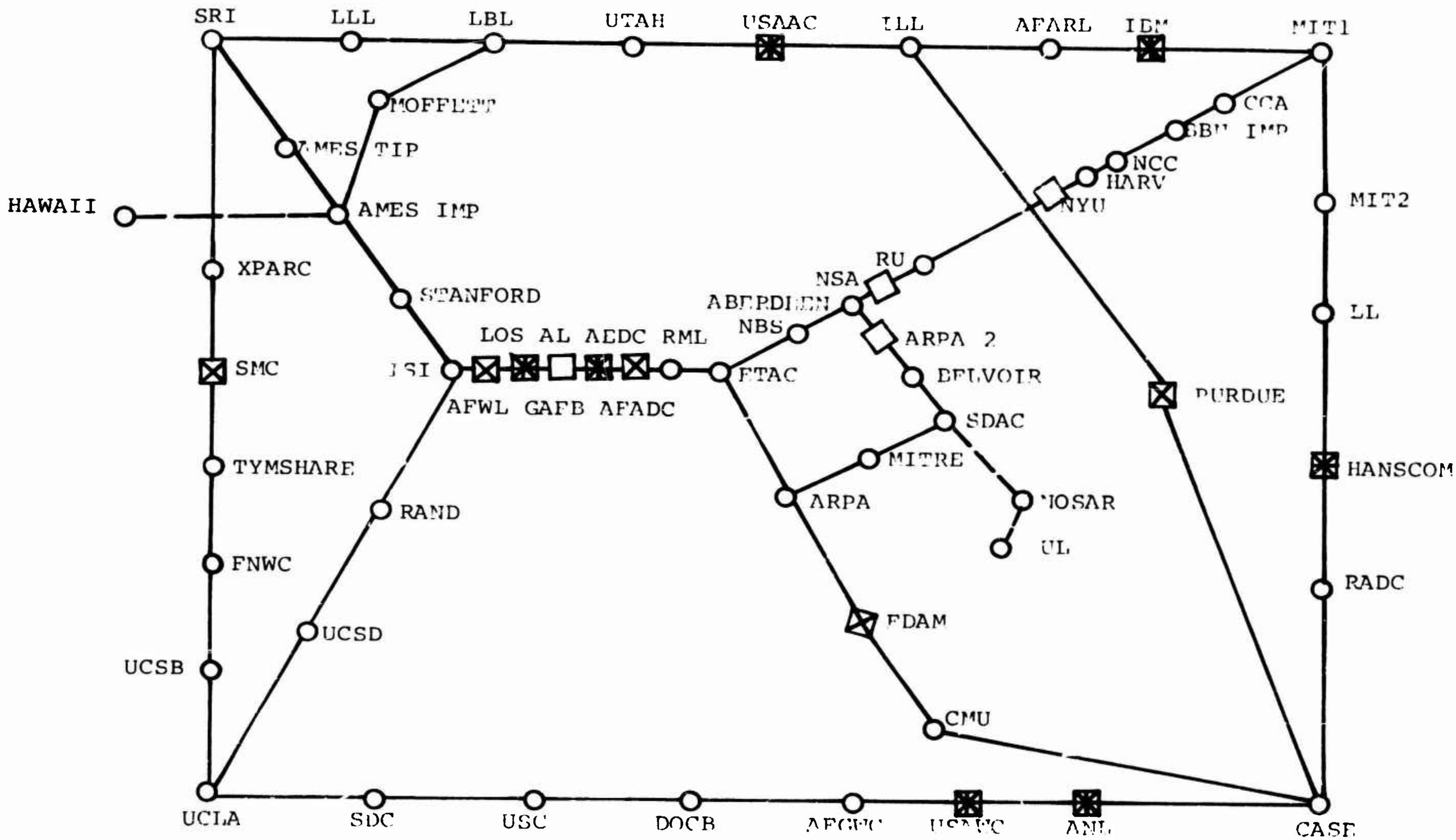
FIGURE 3

ARPANET
(Including Sites Under Active Consideration)



○ Site that is now or to be included in the ARPANET by December, 1973
□ Site that is under active consideration but not for immediate connection
⊠ Site that is under immediate consideration or has been scheduled for connection

FIGURE 4
ARPANET
(Including All Proposed Sites)



○ Site that is now or to be included in the ARPANET by December, 1973

☒ Site that is under immediate consideration or has been scheduled for connection

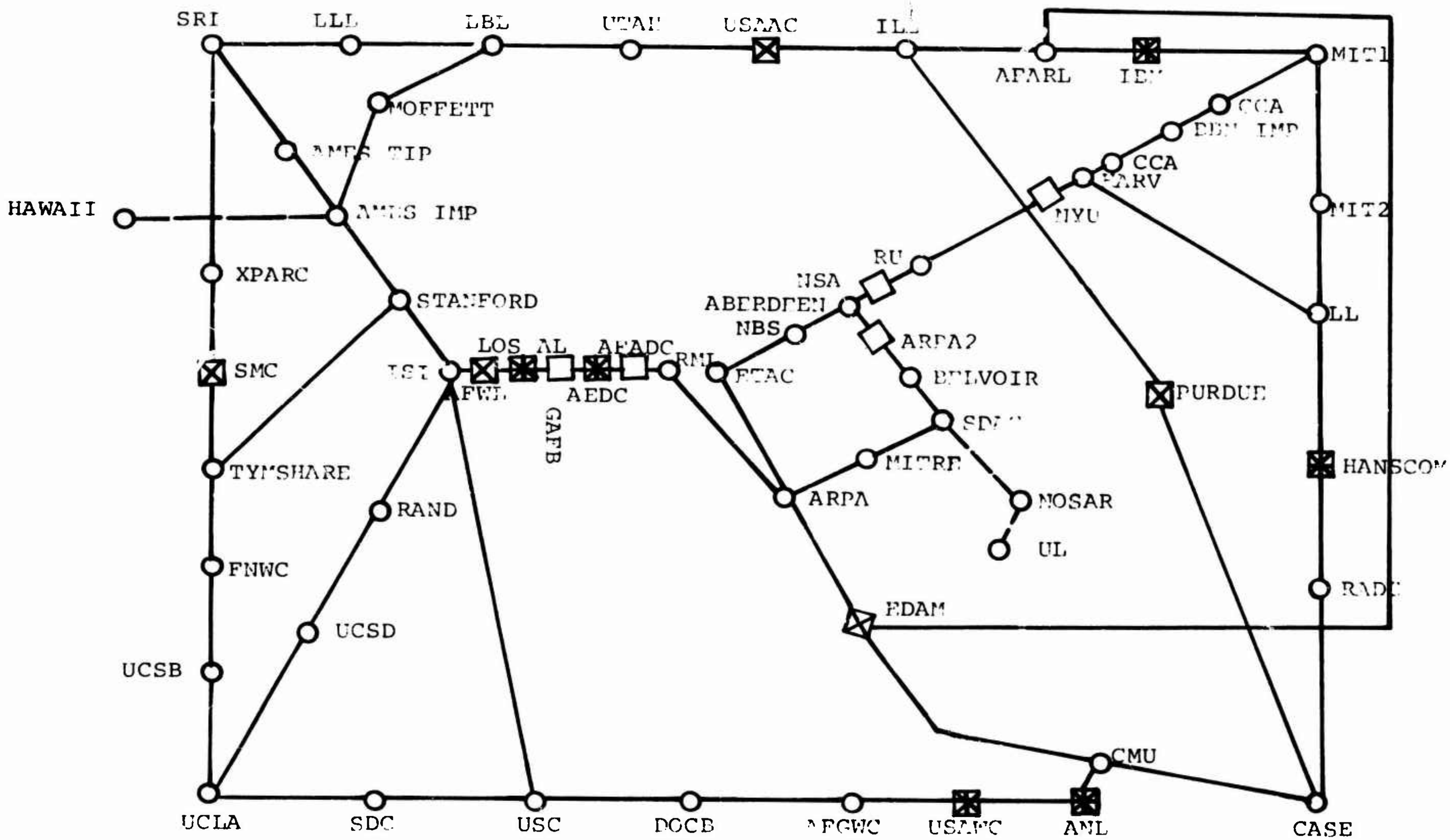
☐ Site that is under active consideration, but not for immediate connection

⊠ Site having been proposed, but not under active consideration

FIGURE 5

ARPANET

(Including All Proposed Sites, With Additional Lines For Reliability)



○ Site that is now or to be included in the ARPANET by December, 1973

⊠ Site that is under immediate consideration or having been scheduled for connection

□ Site under active consideration, but not for immediate connection

⊠ Site having been proposed, but not under active consideration

TABLE 1

ARPANET LINE COSTS AND THROUGHPUT

Number of Nodes	Yearly Line Cost (K$)	Throughput (Uniform Traffic) KBPS/Node	K Packet/ Day/Node*	M Packet/ Day *	Line Cost/ Node (K$)	Line Cost/ K Packet (cents)
14	605	12.2	1690	23.66	43.2	7
15	659	12.5	1730	25.95	43.9	7
18	792	14.2	1970	35.46	44.0	6
21	825	12.4	1710	35.91	39.3	6
23	849	11.9	1640	37.72	36.9	6
24	860	11.1	1530	36.72	35.8	6
26	810	11.6	1600	41.60	31.2	5
30	859	10.1	1400	42.00	28.6	6
33	886	9.3	1290	42.57	26.8	6
39	1,016	8.7	1210	47.19	26.1	6
40	1,022	8.5	1180	47.20	25.6	6
41	1,052	8.1	1130	46.33	25.6	6
46	1,140	7.3	1010	46.46	24.8	6
50	1,189	6.4	890	44.50	23.8	6
57 (63 lines)	1,288	5.2	720	41.04	22.6	6
57 (66 lines)	1,343	5.5	760	43.32	23.6	6

*Based on 24 hr/day operation.

TABLE 2(a)

LIST OF EXISTING TIP's AND IMP's

Abbreviation on Map	Center	Location
ABER	Aberdeen Proving Ground	Maryland
AMES IMP	N.A.S.A. Ames Research Center	Sunnyvale, Calif.
AMES TIP	N.A.S.A. Ames Research Center	Sunnyvale, Calif.
ARPA	Advanced Research Project Agency	Arlington, Va.
BBN	Bolt Beranek and Newmann	Cambridge, Mass.
NCC	Network Control Center, BBN	Cambridge, Mass.
BELV	Fort Belvoir	Virginia
CASE	Case Western Reserve	Cleveland, Ohio
CCA	Computer Corporation of America	Cambridge, Mass.
CMU	Carnegie-Mellon University	Pittsburgh, Pa.
ETAC	Environmental Technical Applications Center	Washington, D.C.
FNWC	Fleet Numerical Weather Control	Monterey, Calif.
AFGWC	Air Force Global Weather Central	Offutt A.F.B., Neb.
HARV	Harvard University	Cambridge, Mass.
ILL	University of Illinois	Urbana, Ill.
ISI	Information Sciences Institute	Los Angeles, Calif.
LL	MIT Lincoln Laboratories	Cambridge, Mass.
MIT1	MAL Project, Mass. Institute of Technology	Cambridge, Mass.
MIT2	Information Processing Center, Mass. Institute of Technology	Cambridge, Mass.
MITRE	MITRE Corporation	McLean, Va.
NBS	National Bureau of Standards	Washington, D.C.
RADC	Rome Air Development Center	Rome, N.Y.
RAND	Rand Corporation	Santa Monica, Calif.
SDAC	Seismic Data Analysis Center	Alexandria, Va.
SDC	System Development Corporation	Santa Monica, Calif.
SRI	Stanford Research Institute	Menlo Park, Calif.
STANFORD	Stanford University	Stanford, Calif.

TABLE 2(a) (cont'd)

Abbreviation on Map	Center	Location
UCLA	University of California, Los Angeles	Los Angeles, Calif.
UCSB	University of California, Santa Barbara	Santa Barbara, Calif.
UCSD	University of California, San Diego	San Diego, Calif.
USC	University of Southern Calif.	Los Angeles, Calif.
UTAH	University of Utah	Salt Lake City, Utah
XPARC	Xerox Palo Alto Research Center	Palo Alto, Calif.
DOCB	Dept. of Commerce, Boulder	Boulder, Colo.
TYMSHARE	Tymshare, Inc.	Cupertino, Calif.
MOFFETT	Moffett Field	Sunnyvale, Calif.
RML	Range Measurement Lab	Patrick AFB, Fla.
LLL	Lawrence Livemore Lab	Livemore, Calif.
LBL	Lawrence Berkeley Lab	Berkeley, Calif.
RU	Rutgers University	New Brunswick, N.J.
AFARL	Air Force Aeronautic Research Lab	Dayton, Ohio
NOSAR	Norway Seismic Array	Kjellas, Norway
HAWAII	University of Hawaii	Hawaii
UL	University of London	London, England

TABLE 2(b)

TIP's UNDER IMMEDIATE CONSIDERATION

Abbreviation on Map	Center	Location
AFWL	A.F. Weapons Lab	Albuquerque, N.M.
SMC	Stanford Medical Center	Stanford, Calif.
PURDUE	Purdue University	Lafayette, Ind.
AFADC	A. F. Armament Development Center	Fort Walton, Fla.
EDAM(SIMP)	COMSAT	Edam, W. Virginia

TABLE 2(c)

TIP's STILL UNDER ACTIVE CONSIDERATION

Abbreviation on Map	Center	Location
NYU	New York University	New York City, N.Y.
NSA	National Security Agency	Fort Meade, Maryland
GAFB	Gunther Air Force Base	Montgomery, Alabama
ARPA2	Advanced Research Project Agency (Second TIP)	Arlington Va.

TABLE 2(d)

SITES HAVING BEEN PROPOSED BUT NOT UNDER ACTIVE CONSIDERATION

Abbreviation on Map	Center	Location
USAWC	U.S. Army Weapons Command	Rock Island, Ill.
USAAC	U.S. Army Aviation Command	St. Louis, Mo.
ANL	Argonne National Lab	Argonne, Ill.
AEDC	Arnold Engineering Development Center	Tullahoma, Tenn.
HANSCOM	Hanscom Field	Hanscomfield, Mass.
LOS AL	Los Alamos National Lab	Los Alamos, N.M.

CHAPTER 2

THE IMPACT OF SATELLITE CHANNELS ON NETWORK COST AND PERFORMANCE

Part 1: Satellite Links in ARPANET

1. INTRODUCTION

This chapter contains the preliminary results of a cost-throughput study considering satellite links in ARPANET. Satellite bandwidth is leased from a common carrier, and satellite access is possible only through carrier's ground stations. Two schemes for satellite access are considered:

A. Point-to-Point connections consisting of full duplex channels between pairs of IMPs, say A and B, consisting of terrestrial connection from A to the ground station nearest to A, from B to the ground station nearest to B, and a satellite hop between the two ground stations. With the exception of cost and delay characteristics, this link can be modeled as a normal terrestrial link for routing and throughput computations.

B. ALOHA multiple access connection: SIMPs (satellite IMPs) are connected to the ground stations and to one or more network IMPs. Packets arriving at a SIMP from terrestrial links are transmitted on the satellite channel either in specific time slots (slotted ALOHA) or as soon as they reach the head of the SIMP transmitting queue (unslotted ALOHA). If two stations transmit simultaneously, the two packets interfere with each other and must be retransmitted. To compute delay, routing, and throughput, a special model of the ALOHA channel is developed in Section 4.

Several alternatives for ground station selection and satellite access implementation are considered for the present ARPANET, and the results are compared.

Although the emphasis in this report is in the area of ARPANET applications, the development of simple general network models for satellite channels and an exact solution of the ground station location problem in an idealized geometry is also considered. Future studies will extend this work into the area of very large networks and less idealized models.

2. PROBLEM AND MODEL FORMULATION

Two new variables, ground station location, and satellite access scheme must be considered when satellites are added to conventional terrestrial networks. The general design problem can then be formulated. The design problem is aimed at finding a minimum cost configuration, including a terrestrial network design, ground station locations with an appropriate access technique and network routing so that time delay and reliability constraints are satisfied.

2.1 SATELLITE FACILITIES: COST AND CHARACTERISTICS

The satellite facilities considered here include: satellite channel; ground station; SIMP; and line connections from SIMP to station, or from IMP to station.

The following costs are assumed:

A. Satellite Segment:

Bandwidth (Kbs)	Cost ($/mo.)
100 (50 full duplex)	2,500
460 (230 full duplex)	5,500
1,500	8,000

B. Local Loop (Station to SIMP, or station to central office):

Bandwidth (Kbs)	Cost ($/mo.)
50 (full duplex)	1,000
230 (full duplex)	1,300

(To connect an IMP to a ground station, IMP to central office and central office to station connections must be purchased).

C. SIMP

Two types of SIMPs are assumed: regular SIMP, with bandwidth greater than 1,500 kbs and cost = 5,500 \$/mo. This SIMP corresponds to the high speed version of IMP presently under development of BBN. It can support a combination of land traffic rates L and satellite traffic rate S such that:

$$L + 3S \leq 1,500$$

D. A small SIMP, with bandwidth = 600 kbs, and cost = 1,400 \$/mo. which is structurally similar to the H-316 IMP, and is presently being developed by BBN. The throughput equation is:

$$L + 3S \leq 600$$

2.2 SATELLITE ACCESS TECHNIQUES

Packet delay on the satellite channel, utilization, and optimal packet routing depend on the access technique used. Point to Point access divides satellite channel bandwidth into subchannels, each corresponding to a full duplex point to point connection between two given ground stations. Multiple access allows any station to communicate with all other stations using the satellite down link in a broadcast mode, transmitting simultaneously to all stations. The multiple access techniques are divided into: channel division techniques, where the channel is divided into frequency (e.g., FDMA) or time (e.g., TDMA) frames, and each frame is preassigned to a given ground station for transmission to the satellite; and channel contention techniques, where each ground station can compete for use of the total channel for transmission to the satellite (e.g., random access ALOHA schemes, reservation schemes, etc.).

It is also possible to implement hybrid access techniques, with one portion of the channel dedicated to point to point requirements, and the remaining portion used in a multiple access mode.

2.3 CHANNEL MODELS

Assume that packets arrive at the ground stations in a Poisson fashion and that packet length is exponentially distributed. The average packet delay for both point to point and multiple access channel division cases, has the same expression as the delay for terrestrial channels. In particular,

$$T_i = \frac{1}{\mu C_i} \frac{1}{1-f_i/C_i} + P_s \tag{1}$$

where:

T_i = delay on i^{th} channel (sec)

C_i = capacity of i^{th} subchannel (bits/sec)

f_i = rate (bits/sec) on i^{th} subchannel

μ = average packet length (bits)

P_s = satellite propagation delay ($\cong$ .27 sec)

The multiple access channel contention case is more difficult to analyze because of the possibility of interference from packets transmitted by different ground stations. Here, consideration is limited to the ALOHA case. The following assumptions are made:

1. The number of ground stations is very large (this leads to a worst case condition)

2. Packets are of constant length, equal to block size

3. Packet arrivals are Poisson distributed; and

4. If collision occurs, a packet is retransmitted after a random interval uniformly distributed between zero and k seconds.

Under these assumptions, the average packet delay T, the sum of propagation and retransmission delays is given in the case of <u>unslotted</u> ALOHA channel by:

$$T = P_s + \left(P_s + \frac{k}{2}\right)\left(\frac{1}{e^{-2G}} - 1\right) \tag{2}$$

where:

e^{-2G} = probability of non collision between two packets
G = f_g/C:global channel utilization
f_g = global satellite traffic (including retransmission)
C = satellite channel bandwidth

Since:

$$S = Ge^{-2G} \tag{3}$$

where:

S = f_s/C: effective channel utilization

f_s = effective satellite traffic (not including retransmissions),

T is uniquely determined given S.

The maximum channel utilization obtainable with unslotted ALOHA is 1/2e = .184. An exact closed form expression of T(S) is not possible. Therefore, a convenient analytical approximation is desirable. Assuming that $k << P_s$:

$$T \simeq P_s/e^{-2G} \tag{4}$$

Consider the following approximation $T_a(S)$ to eq. (4):

$$T_a(S) = P_s\,(1 - \frac{1}{e}) + \frac{P_s}{e}\,\frac{1}{1 - 2eS} \tag{5}$$

It can be easily verified that:

$T(o) = T_a(o)$; $T'(o) = T_a'(o)$. For S = .11 we have:

$T(.11) = 1.35\ P_s$; $T_a(.11) = 1.55\ P_s$.

This approximation was found to be sufficiently accurate for the purposes of the present study. Notice that Equation (5) can be rewritten as follows:

$$T_a(S) = P_s\,(1 - \frac{1}{e}) + \frac{P_s}{e}\,\frac{1}{1 - f_s/C'} \tag{6}$$

where:

$C' = \frac{C}{2e}$ = effective unslotted ALOHA bandwidth

If eq. (6) is compared with eq. (1), it is found that the unslotted ALOHA channel delay can be approximated by the delay of a M/M/1 channel with bandwidth = C' using proper packet length and propagation time.

A similar analysis is possible for the slotted ALOHA channel. The delay T_a is:

$$T_a = P_s\,(1 - \frac{1}{e}) + \frac{P_s}{e}\,\frac{1}{1 - f_s/C'} \tag{7}$$

where:

$C' = C/e$ = effective slotted ALOHA bandwidth.

2.4 NETWORK MODELS

As an example of the issues that must be handled to construct satellite network models under different access techniques, suppose a system contains four ground stations. There are several possible cases:

A. Point to Point Access

Suppose the total bandwidth C is divided into four full duplex point to point channels (A-B, A-D, C-B, C-D). Then, the equivalent network is represented in Figure 1, where each link has capacity $C_i = C/8$. Link delays are as in eq. (1).

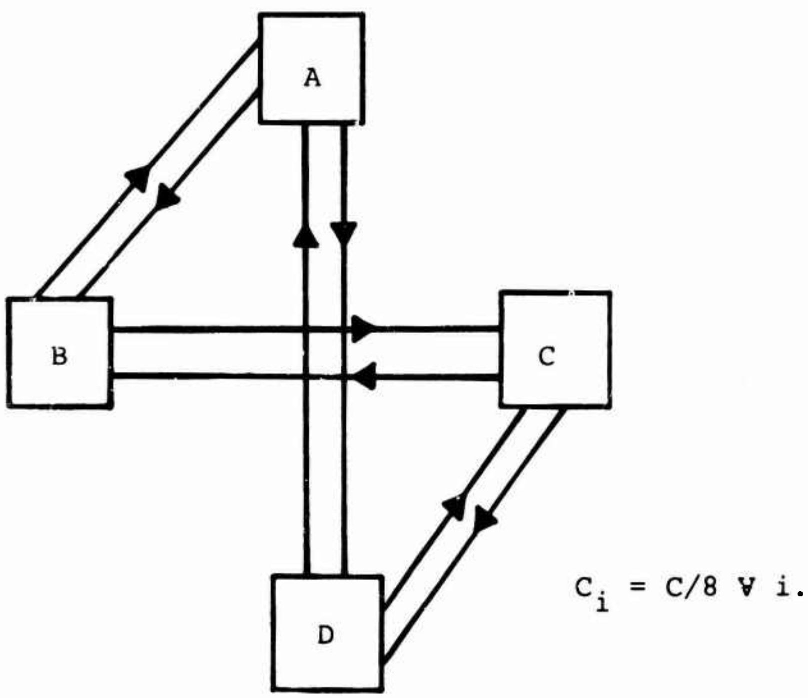


FIGURE 1 Point to Point

B. <u>Multiple Access (Channel Division)</u>

The equivalent model is shown in Figure 2, where the satellite is represented as a store and forward node. Up-link channels have capacity C/4, and delay as in eq. (1); down-link channels have infinite capacity and zero delay.

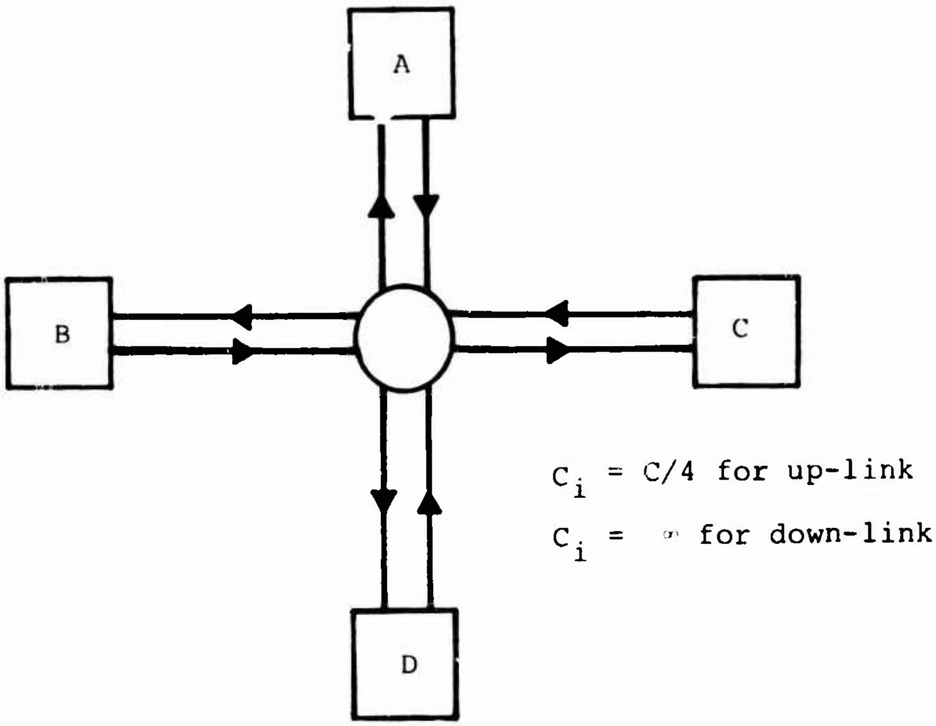


FIGURE 2 Multiple Access; Channel Division

C. <u>Multiple Access (ALOHA)</u>

The equivalent model is shown in Figure 3, where the satellite is represented by two store and forward nodes. The equivalent channel between the two nodes has capacity C' (where C' = C/2e for unslotted ALOHA and C' = C/e for slotted ALOHA) and delay given by eq. (6) or (7). All other links have infinite capacity and zero delay. To compute average delay, optimal routing policy, and throughput of a mixed terrestrial and satellite network, we replace the satellite links with the appropriate equivalent network and use traditional queueing formulae and routing algorithms.

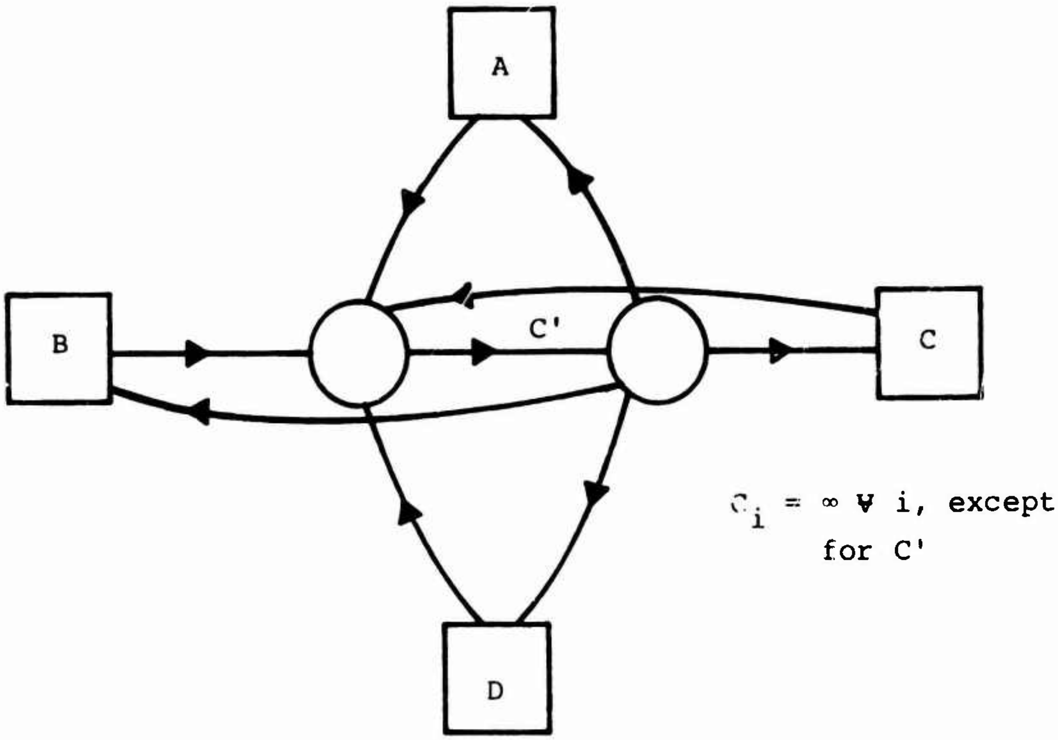


FIGURE 3 ALOHA Access

3. DELAY, ROUTING, AND THROUGHPUT ANALYSIS

Traditional ARPANET design requires an average delay $T \leq .200$ sec. If the same requirement was to be maintained after introduction of satellite links, then satellite utilization would be very small, since satellite propagation delay alone is equal to .270 sec. Thus, the problem should be approached by considering two classes of traffic, interactive and background. The .200 sec requirement is then applicable to the first class while larger delays are allowed for the second one. In this preliminary phase of research, the following simple approach is used. Initially, it is assumed that there is only one class of traffic. The delay on a satellite link is assumed to be given by:

$$1/\mu\ C_i \quad \frac{1}{1-f_i/C_i}$$

where C_i is a proper value of capacity dependent on the access scheme. The optimal routing problem under this assumption is used to find the maximum throughput that can be accommodated with an average delay = .200 sec. Finally, the amount of traffic that uses the satellite is computed and the actual satellite delay (using the exact delay expressions) is calculated. The final solution contains two components of traffic: terrestrial traffic, with average delay equal to .200 sec; and satellite traffic, with average delay equal to .200 sec. + satellite delay.

4. OPTIMAL NUMBER AND LOCATION OF GROUND STATIONS

4.1 GENERAL

The global satellite network design requires the selection of ground station numbers and locations. The optimal location problem is a difficult integer type problem, and no efficient general solution methods are currently available. However, in most practical applications, there are a limited number of locations at which ground stations exist or can be installed. In such cases, the location problem is efficiently solved by analyzing and comparing only a few reasonable alternatives.

In this section, we establish a generalized relationship between the optimal number of ground stations and other important design parameters, such as cost and throughput. In addition, the impact of the access scheme on total cost and number of stations is evaluated.

To carry out the analysis, a simple terrestrial network geometry is considered in which the nodes are uniformly distributed along a circle, and are connected in a loop with ground stations symmetrically located on the circle. Traffic requirements are spanned uniformly between all node pairs. Multiple access schemes are allowed as well as a point to point access scheme in which diametrically opposite ground stations are connected by point to point satellite links.

Traffic requirements between any two node pairs can either be accommodated along the terrestrial loop network, or can be routed to the nearest ground station and then transmitted over the satellite channel. This routing decision affects the total communications cost, since terrestrial and satellite links must be sized according to link traffic rate. In this study, the following assumptions are made: link capacities are continuous variables and any value of capacity can be purchased. The incremental cost of link capacity is constant for a given link. Thus, cost is a linear function of capacity. Finally, for each link an amount of capacity equal to the average link traffic rate is purchased. (i.e., queueing delay is disregarded). Using the above assumptions, it can be shown that the optimal routes are the shortest ones computed according to incremental link costs.

4.2 PROBLEM FORMULATION

The ground station location problem is formulated as follows:

Given: network geometry
traffic requirements
type of satellite access

Minimize: (over number of stations and routing) total communications cost

Such that: traffic requirements are satisfied

To solve the location problem, the number m of ground stations is first assumed to be given. Therefore, ground station locations are given by symmetry considerations. The minimum cost routing problem for the associated network configuration is then solved. Using reasonable approximations, a closed form solution for the minimum cost D(m) in terms of m is obtained. Next, the optimal value of m with a one-dimensional optimization on D(m) is calculated.

The following notation is used:

NN = Number of nodes

r = Radius of the circle on which nodes are located (miles)

m = Number of ground stations

D_{Gs} = Monthly cost of the ground station (including the cost of the connection into the terrestrial network as well as the cost of the station, and a function of the access scheme used)

d_s = Monthly cost of satellite bandwidth (\$/Kbs x month)

d_ℓ = Monthly cost of terrestrial bandwidth (\$/Kbs x mile x month)

$S \triangleq d_s/d_\ell$ = Equivalent terrestrial length of the satellite link (miles)

R = Traffic requirement (Kbs/node pair)

The results for the point to point and multiple access schemes are presented in the next sections.

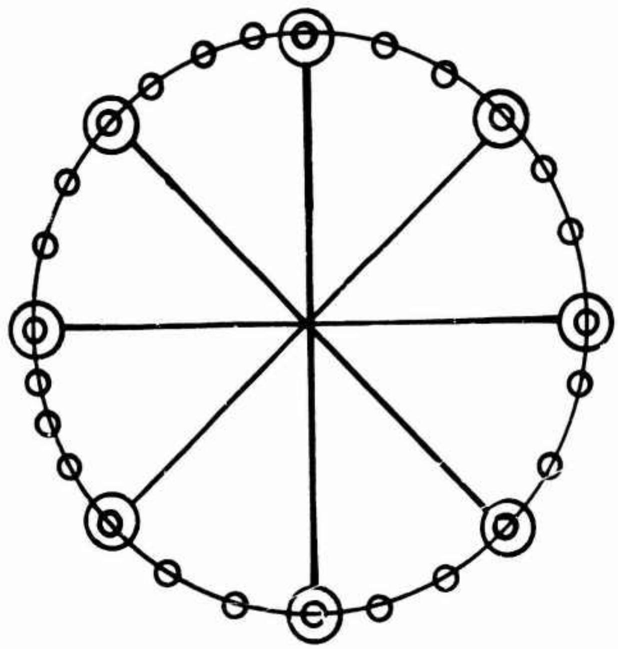


FIGURE 4 Point to Point Access

4.3 POINT TO POINT ACCESS

To obtain a simple analytical treatment when studying point to point satellite configurations, it is assumed that point to point links are established only between diametrically opposite ground stations (See Figure 4). Clearly, more complex point to point configurations can be proposed which would require a separate analysis for each case.

The following expressions of minimum cost D_t, corresponding to the minimum cost routing strategy, then apply:

- case of m = o (no ground stations)

$$D_t = NN^2 R d_\ell \frac{\pi}{2} r \tag{1}$$

- case of m = 2

$$D_t \cong 2D_{GS} + \frac{NN^2 R d_\ell}{8} (3\pi r + 2S) \tag{2}$$

- case of m > 2

$$D_t \cong mD_{GS} + \frac{NN^2 R d_\ell}{4} \left\{ \pi r \left(1 + \frac{1}{m}\right) + S\left(2 - \frac{1}{m}\right) \right\} \tag{3}$$

- optimal value of m (for m > 2):

$$m^* = \sqrt{\frac{NN^2 R d_\ell (\pi r - s)}{4D_{GS}}} \tag{4}$$

Notice from Equation (3) that the bandwidth cost, given by $D_t - mD_{GS}$, decreases with m until it reaches the lower bound $NN^2 R d_\ell (\pi r/4 + S/2)$, for $m \to \infty$. If S = o (i.e., satellite bandwidth has zero cost), the lower bound is exactly one half the cost without satellites (See Eq. (1)).

Equation (4) shows that m* critically depends on throughput (= $NN^2 R$), on D_{GS}, and on region size r; it does not critically depend on S. (Recall that typically s << r).

4.4 MULTIPLE ACCESS

Only the case $m \geq 3$ is considered because m = 2 can be better implemented with a point to point connection when requirements are assumed uniform. Then,

$$D_t \cong mD_{GS} + NN^2 \; R \; d_\ell \left[\frac{\pi r}{m} + S\left(1 - \frac{5}{4m}\right)\right] \tag{5}$$

$$m^* = \sqrt{\frac{NN^2 \; R \; d_\ell \; (2\pi r - \frac{5}{2} S)}{2D_{GS}}} \tag{6}$$

Notice from Equation (5) that the bandwidth cost $D_t - mD_{GS}$ decreases with m more rapidly than the corresponding cost of the point to point case, and becomes only satellite bandwidth cost for $m \to \infty$. The optimal number of stations m^* displays the same behavior as in the point to point case.

4.5 EXAMPLE

Let:

$$D_{GS} = \begin{cases} 7{,}000 \text{ \$/month (for multiple access)} \\ 1{,}300 \text{ \$/month (for point to point)} \end{cases}$$

$NN^2 \; R = 500$ Kbs

$d_\ell = .13$ \$/mile x Kbs

$d_s = 5.5$ \$/Kbs

$s = 40$ miles

$r = 500$ miles

The optimal number of ground stations, using point to point links is:

$$m^*_{pp} = \sqrt{\frac{500 \times .13 \times 1500}{4 \times 1300}} = 4.3 \simeq 4$$

Using multiple access,

$$m^*_{ma} = \sqrt{\frac{500 \times .13 \times 3000}{2 \times 7000}} = 3.7 \simeq 4$$

The corresponding minimum costs are:

1. Point to Point cost $(D_t)_{pp} = \underbrace{5{,}200}_{\text{Ground Station}} + \underbrace{30{,}000}_{\text{Bandwidth}} = 35{,}200$

2. Multiple Access cost $(D_t)_{ma} = \underbrace{28{,}000}_{\text{Ground Station}} + \underbrace{24{,}000}_{\text{Bandwidth}} = 52{,}000$

For this particular application, inspite of the fact that the multiple access scheme provides a much lower bandwidth cost, the point to point access is more economical because of the higher cost of multiple access ground stations.

5. ARPANET DESIGN USING SATELLITE LINKS

5.1 ALTERNATIVES FOR SATELLITE ACCESS IMPLEMENTATION

In this section, cost-throughput trade-offs for various satellite access schemes in a recent 43 node ARPANET configuration (See Figure 5) are evaluated. In particular, the following types of implementation are considered:

A. No satellite links. The capacity of the 43-node network is upgraded by introducing terrestrial links only. Cost and throughput trends are compared to those of the satellite case.

B. Three ground stations in San Francisco, Los Angeles, and Washington, D. C. are available for satellite access. Various configurations with two or three stations and with multiple or point to point access are considered. For the terrestrial configuration, two cases are considered: (1) terrestrial network unaltered; and (2) terrestrial network reoptimized.

C. Five ground stations in San Francisco, Los Angeles, Washington, D. C., New York, and Chicago are available for satellite access. In addition to the alternatives studied for the three ground station case, capacity reductions of terrestrial links from 50 Kbs to 19.2 and 9.6 Kbs are also allowed. These reductions are made possible by the larger number of stations available.

A hybrid implementation which includes both multiple and point to point access is also analyzed.

For each satellite alternative, a network optimization is carried out by first selecting satellite stations from the set of available locations. Ground station to network connections and, in the case of point to point access, satellite links are then assigned. In this step, the terrestrial network topology is optimized to achieve the best cost-throughput tradeoff.

5.2 RESULTS

Computed for each network configuration is: total cost; terrestrial cost of all terrestrial links; satellite cost; cost of SIMP (if applicable), connection from SIMP to station or from IMP to station, and satellite bandwidth; total throughput; traffic on satellite and satellite channel delay.

A. No Satellite Links

NETWORK CONFIGURATION	TOTAL COST K $/mo	TERR. COST K $/mo	SAT. COST K $/mo	THRU-PUT kbs	SAT. TRAFFIC kbs	SAT. DELAY sec
Present 43 node configuration (Figure 5)	93	93	--	447	--	--
Upgraded 43 node configur-ation (Figure 6)	112.9	112.9	--	635	--	--

B. Three Ground Stations Available

1. Unslotted ALOHA; unaltered terrestrial topology

NETWORK CONFIGURATION	TOTAL COST K $/mo	TERR. COST K $/mo	SAT. COST K $/mo	THRU-PUT kbs	SAT. TRAFFIC kbs	SAT. DELAY sec
3 Regular SIMP's (Figure 7)	128.300	99	29.3	670	214	.4
2 Regular SIMP's at Los Angeles and Wash., D.C. (Figure 8)	122.300	100	22.3	670	212	.4
3 Small SIMP's (Figure 9)	115.500	99.2	16.3	641	175	.4

2. Unslotted ALOHA; optimized terrestrial topology

NETWORK CONFIGURATION	TOTAL COST K $/mo	TERR. COST K $/mo	SAT. COST K $/mo	THRU-PUT kbs	SAT. TRAFFIC kbs	SAT. DELAY sec
2 Regular SIMP's at Los Angeles and Wash., D C. (Figure 10)	113.300	91	22.3	618	212	.4

3. Point to Point Access; unaltered terrestrial topology

NETWORK CONFIGURATION	TOTAL COST K $/mo	TERR. COST K $/mo	SAT. COST K $/mo	THRU-PUT kbs	SAT. TRAFFIC kbs	SAT. DELAY sec
2 Point to Point Links (Figure 11)	109.500	93	16.5	661	212	.27

4. Point to Point Access; optimized terrestrial topology

NETWORK CONFIGURATION	TOTAL COST K $/mo	TERR. COST K $/mo	SAT. COST K $/mo	THRU-PUT kbs	SAT. TRAFFIC kbs	SAT. DELAY sec
2 Point to Point Links (Figure 12)	101.6	85.1	16.5	745	320	.27
1 Point to Point Link of 230 kbs (Figure 13)	95.8	85.1	10.7	672	270	.27
1 Point to Point Link of 50 kbs (Figure 14)	91.3	85.1	6.2	347	92	.27

C. Five Ground Stations Available

1. Point to Point Access; optimized terrestrial topology

2 Point to Point Links (Figure 15)	91.9	75	16.9	654	330	.27

2. Slotted ALOHA Access; optimized terrestrial topology

3 Regular SIMP's at San Francisco, Chicago, Wash.,D.C. (Figure 16)	108.3	79	29.3	686	393	.5
5 Small SIMP's (Figure 17)	97.7	75.9	21.8	603	353	.5

3. Hybrid Access (unslotted ALOHA + 2 point to point links); optimized terrestrial topology

Hybrid configuration (Figure 18)	101.7	75.9	25.8	584	330	.4

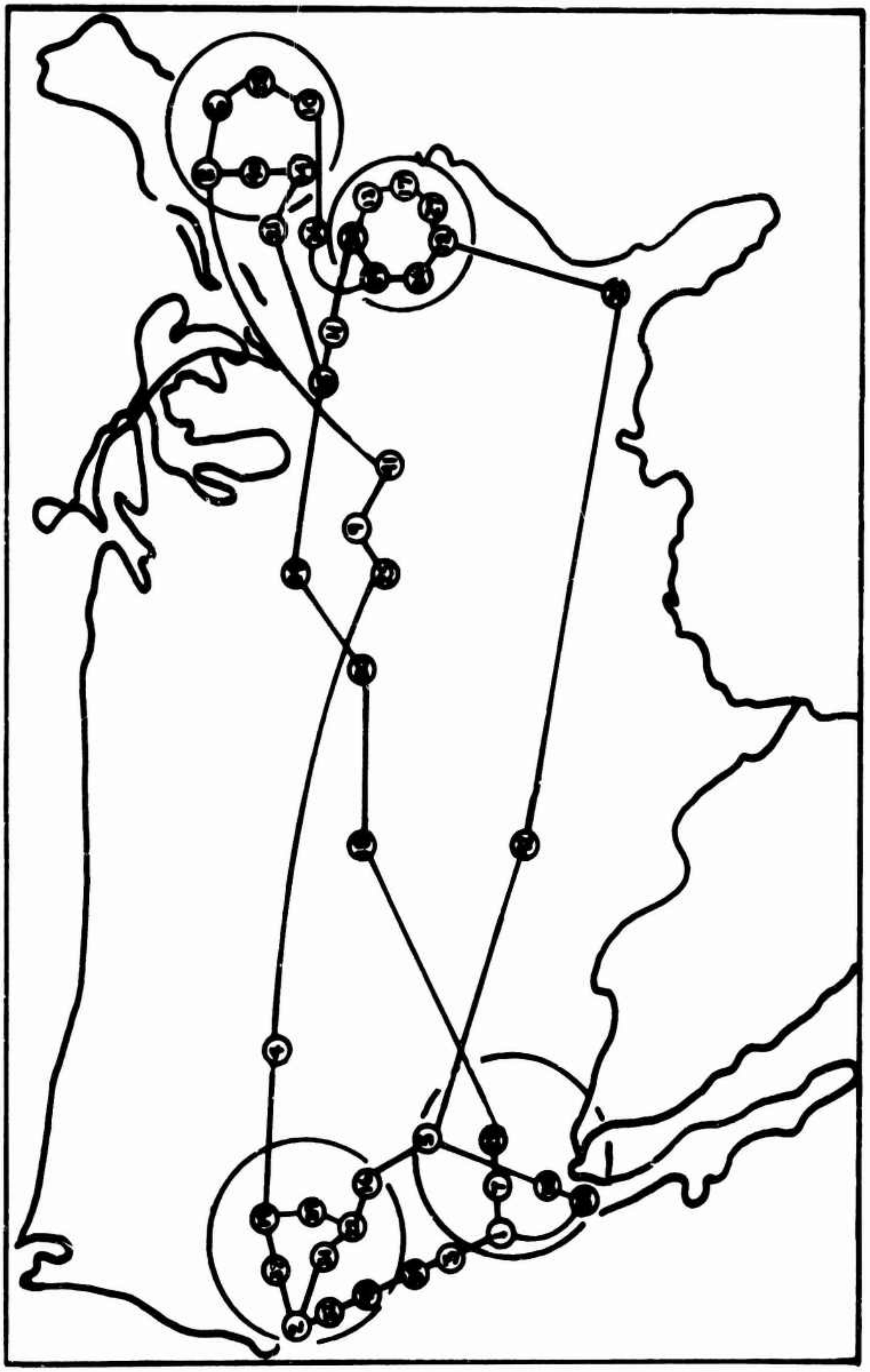


FIGURE 5
PRESENT ARPANET CONFIGURATION (OCTOBER, 1973)

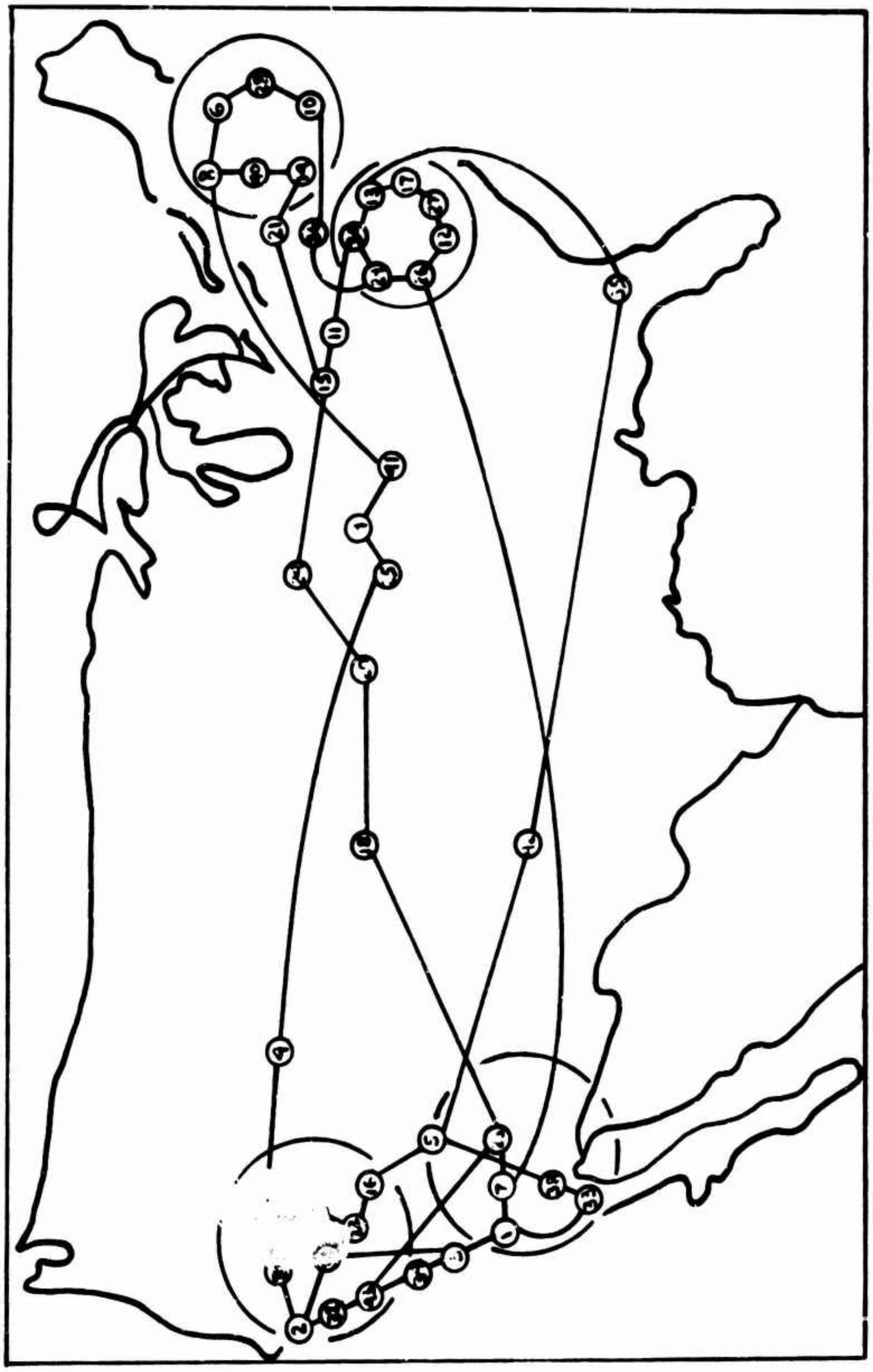


FIGURE 6

UPGRADE [illegible]

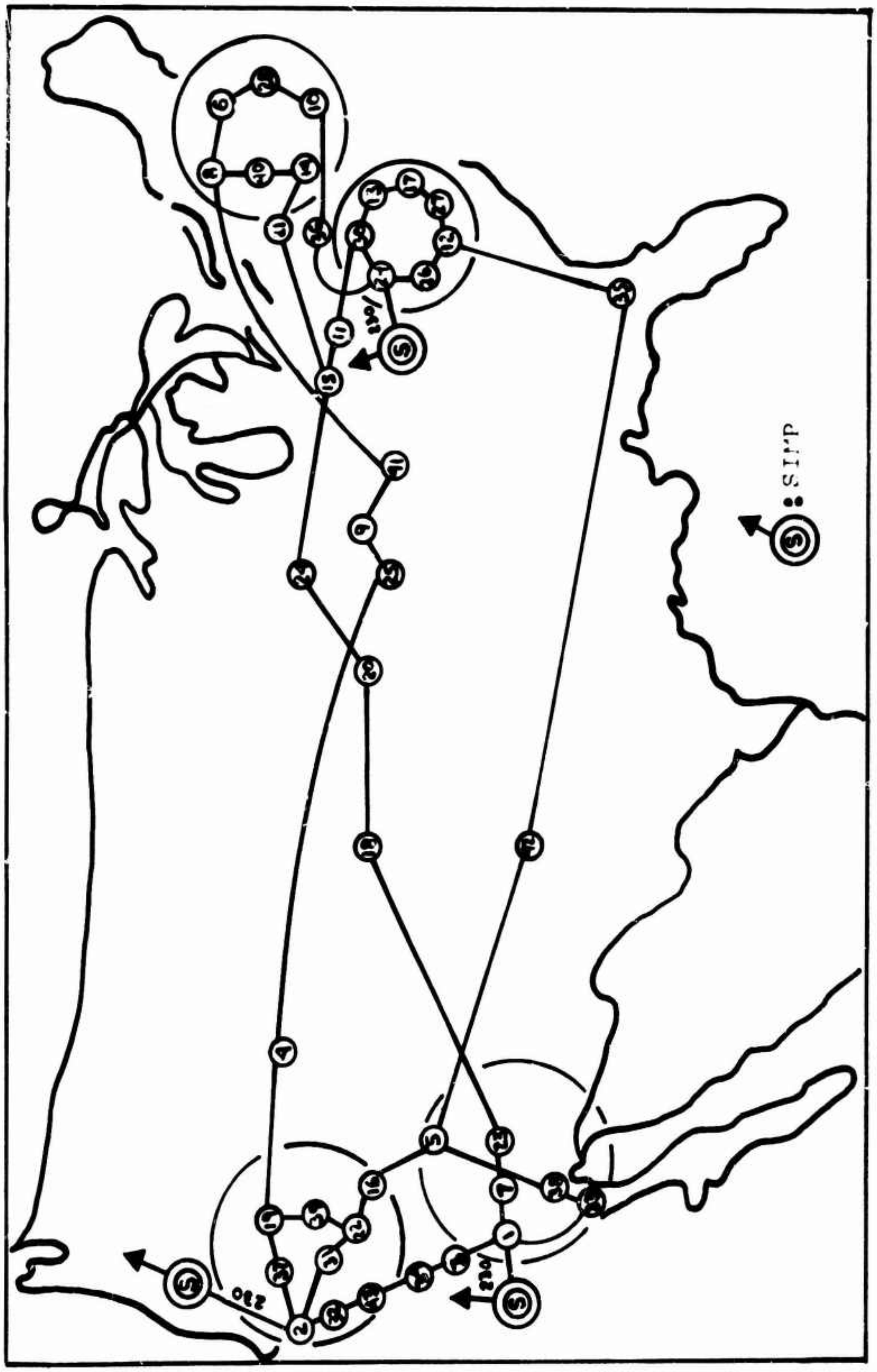


FIGURE 7

3 REGULAR [illegible] CONFIGURATION

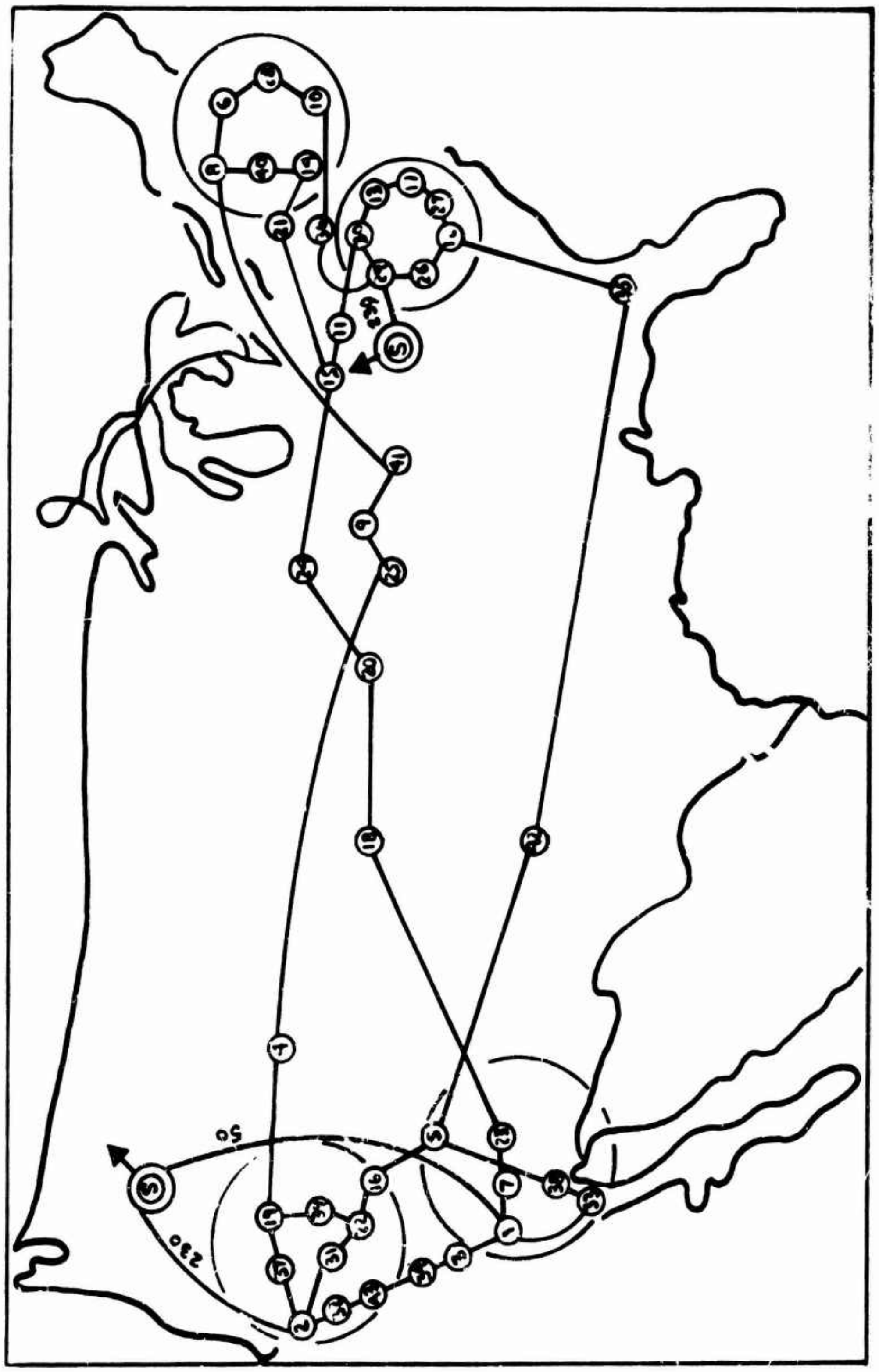


FIGURE 8

[illegible]

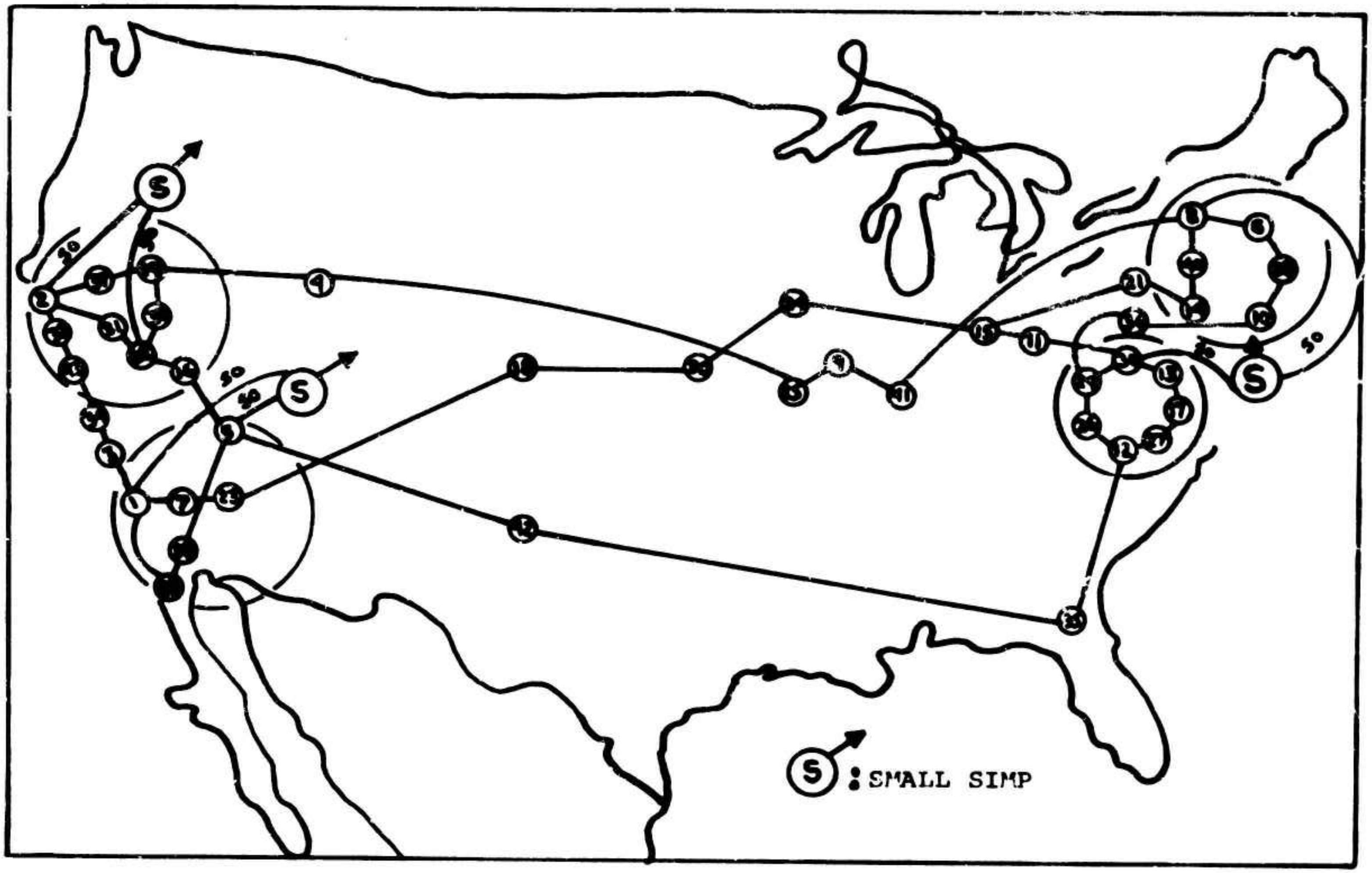


FIGURE 9
3 SMALL SIMP CONFIGURATION

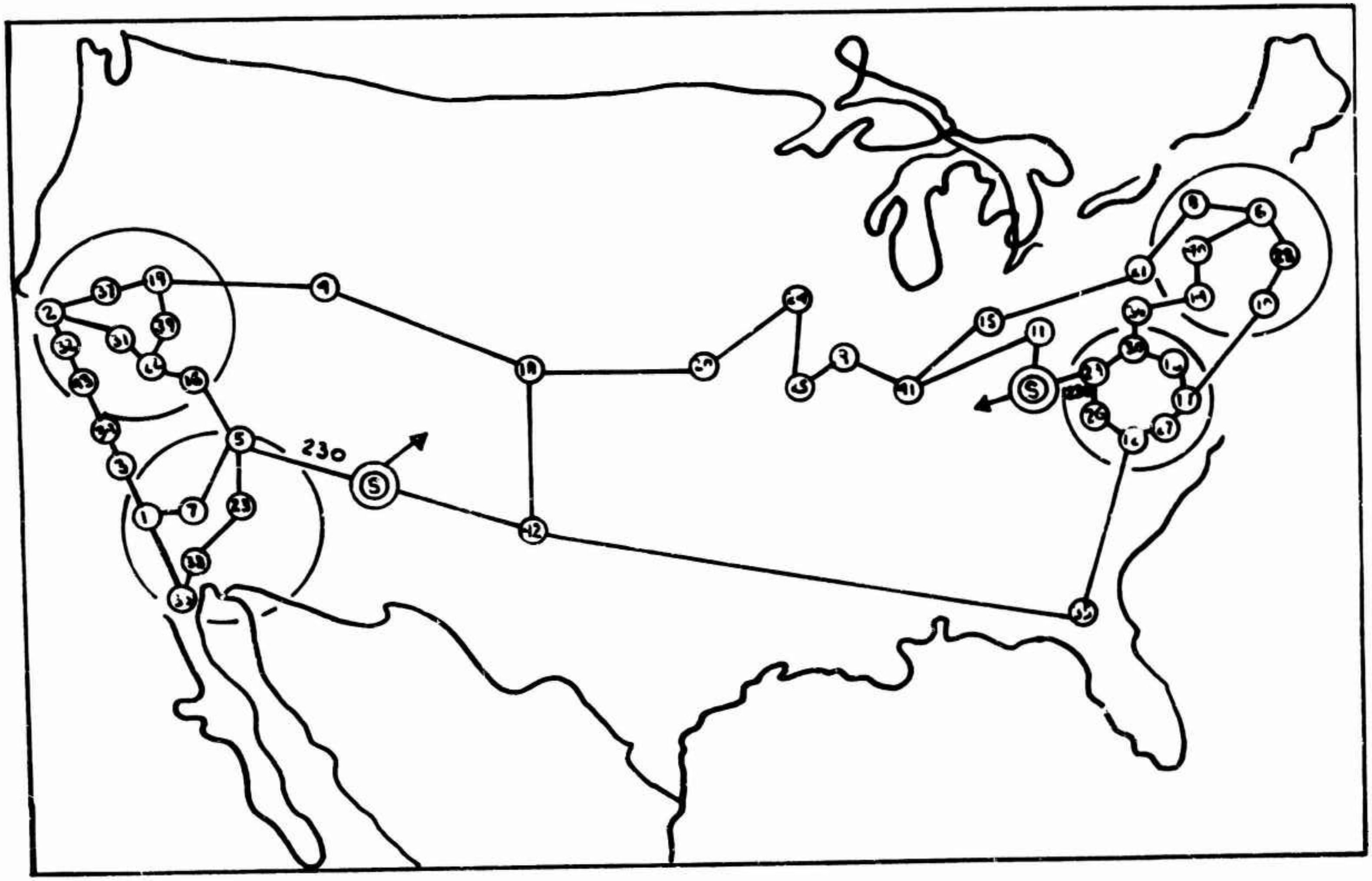


FIGURE 10

2 REGULAR STUBS; OPTIMIZED TOPOLOGY

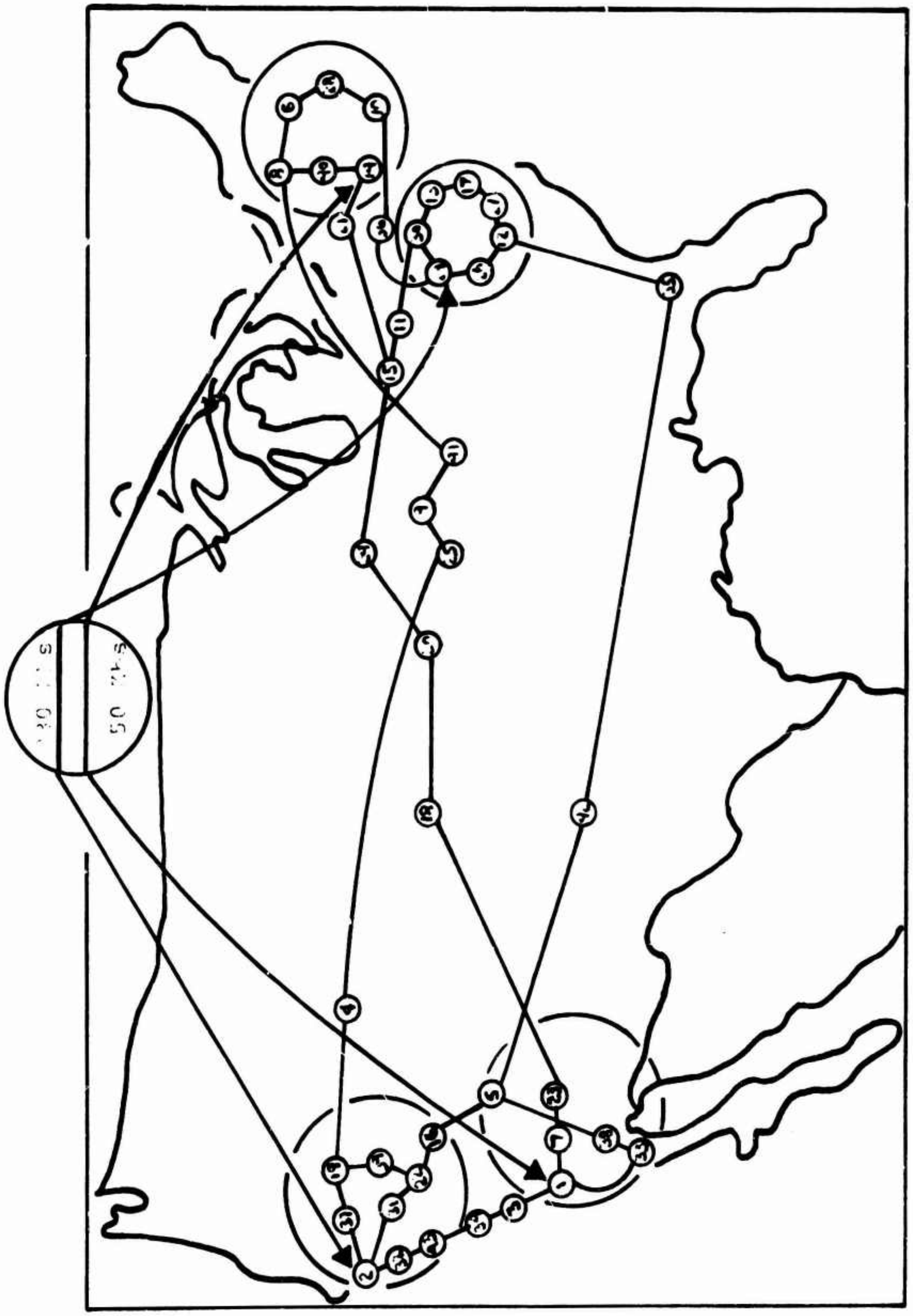


FIGURE 11

2 POINT TO POINT LINKS · UNALTERED TOPOLOGY

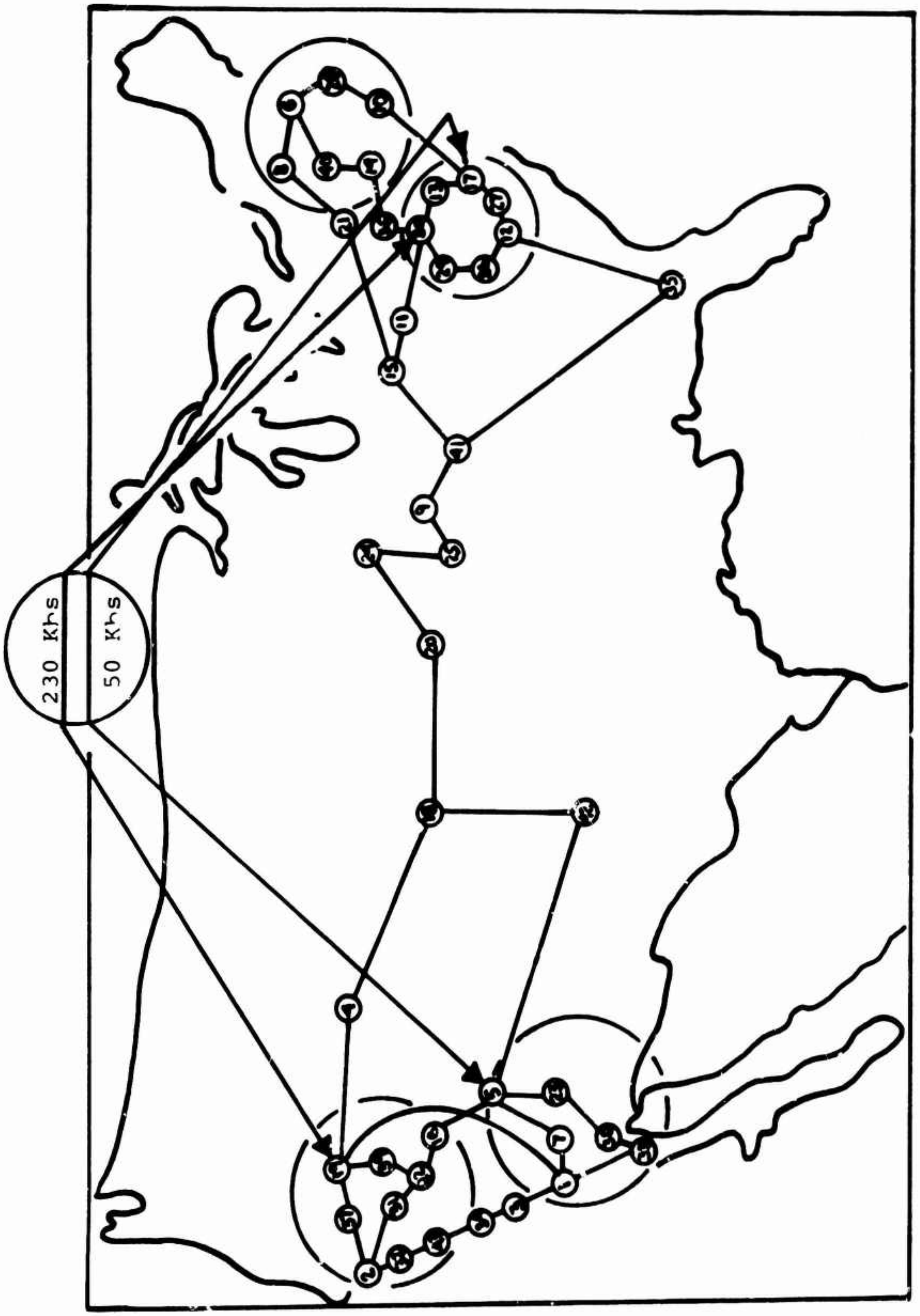


FIGURE 12

2 POINT TO POINT LINKS [illegible] TOPOLOGY

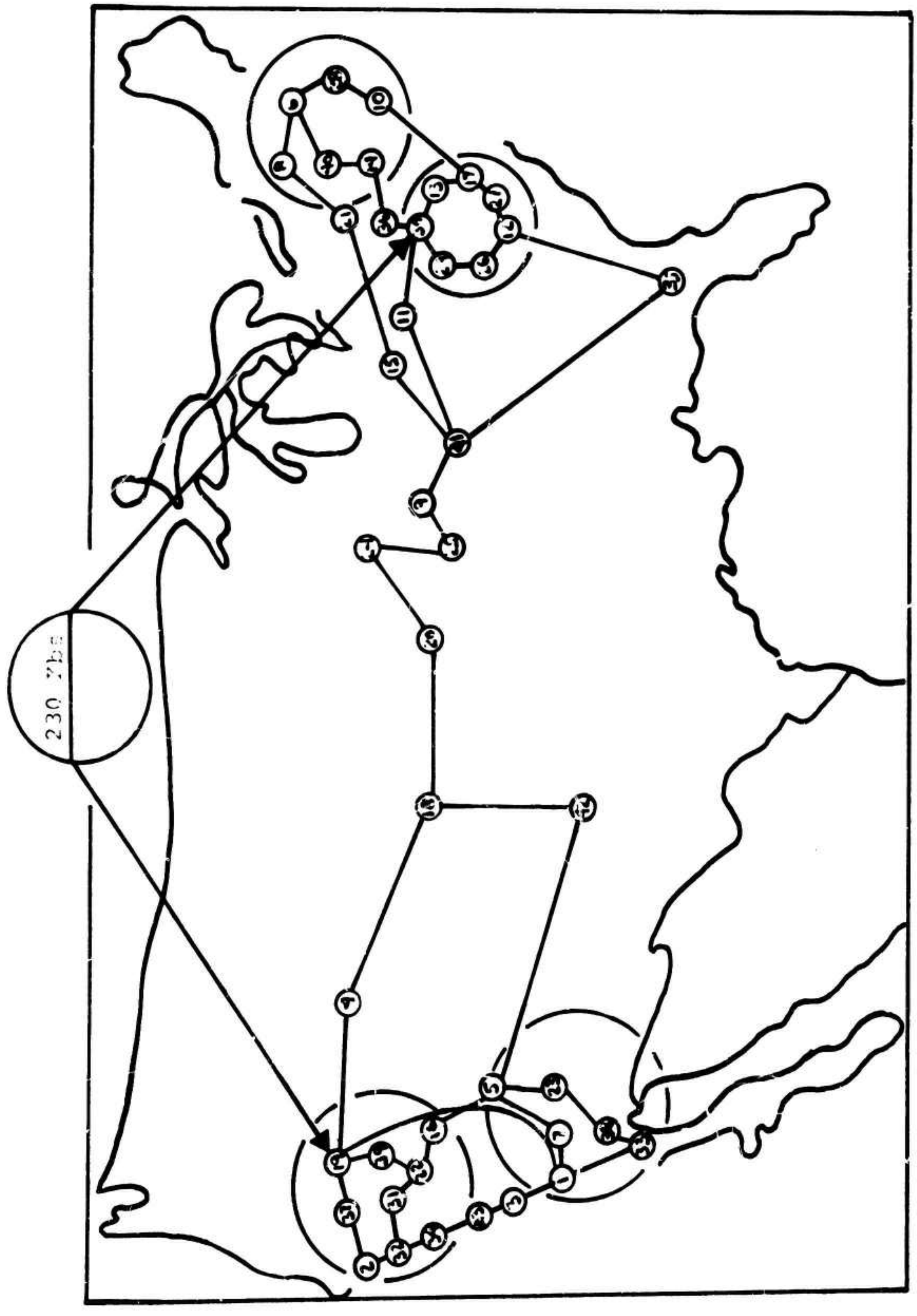


FIGURE 13

1 POINT TO POINT LINK · OPTIMIZED TOPOLOGY

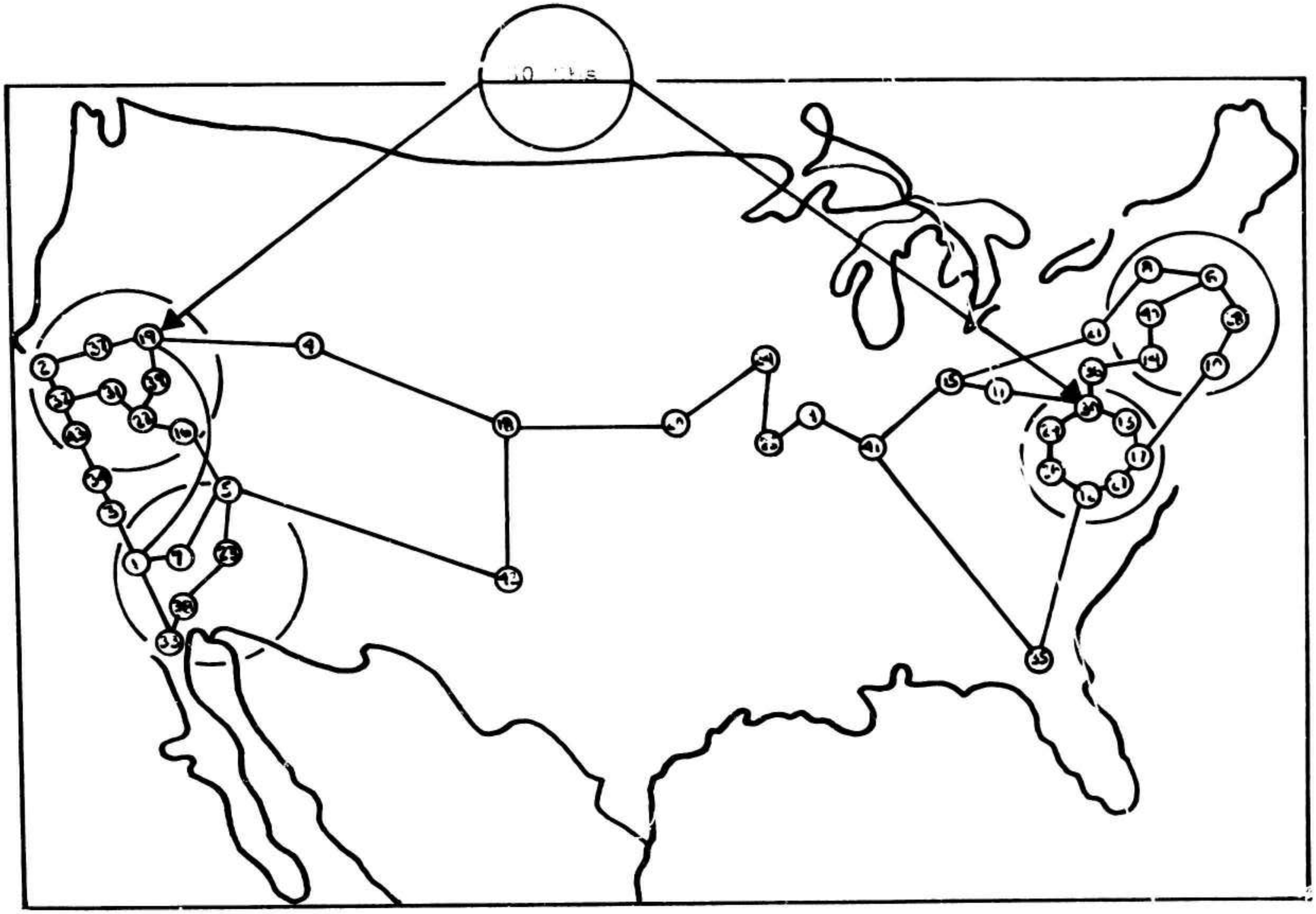


FIGURE 14

1 POINT TO POINT LINK

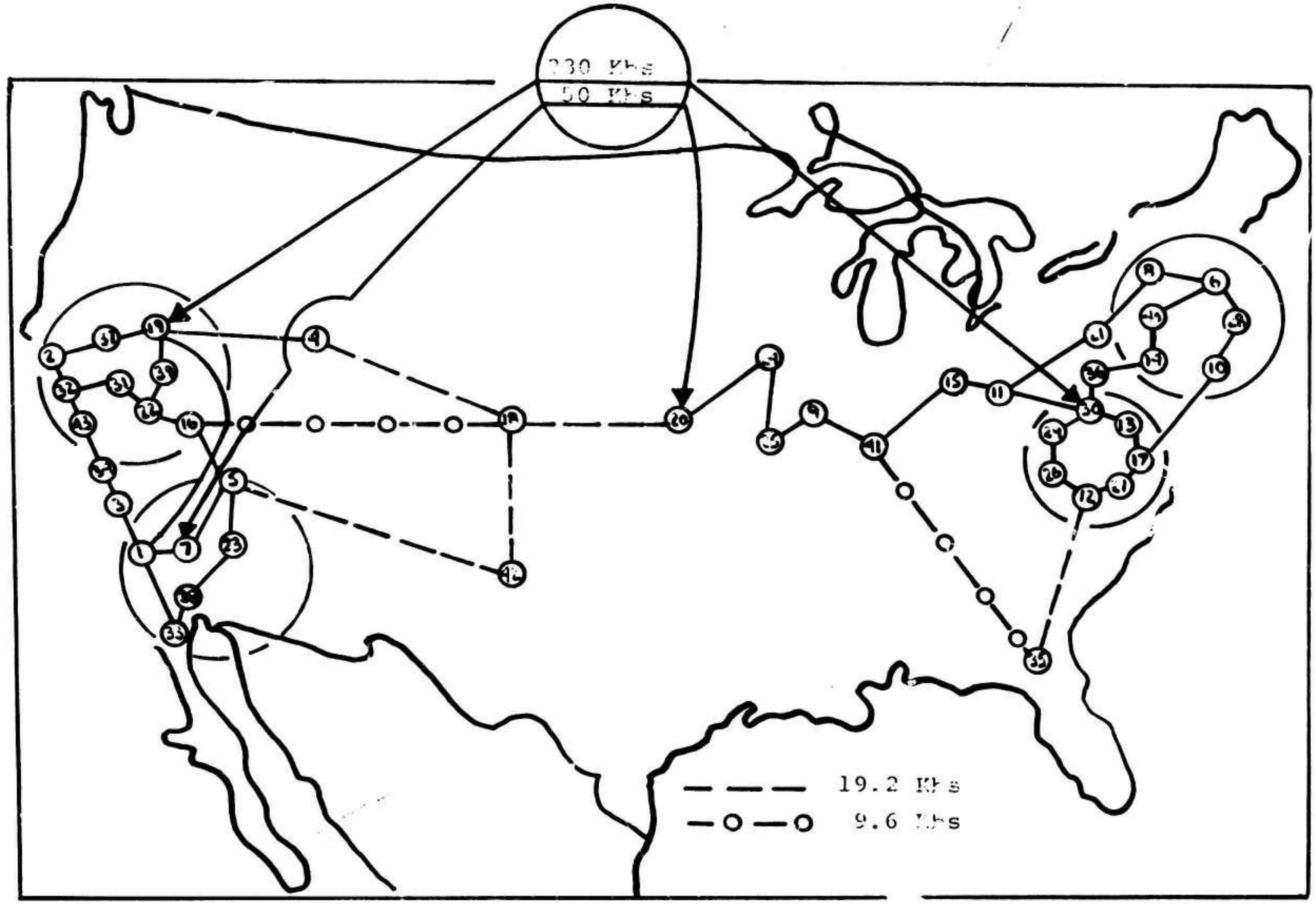


FIGURE 15

3. POINT TO POINT LINKS

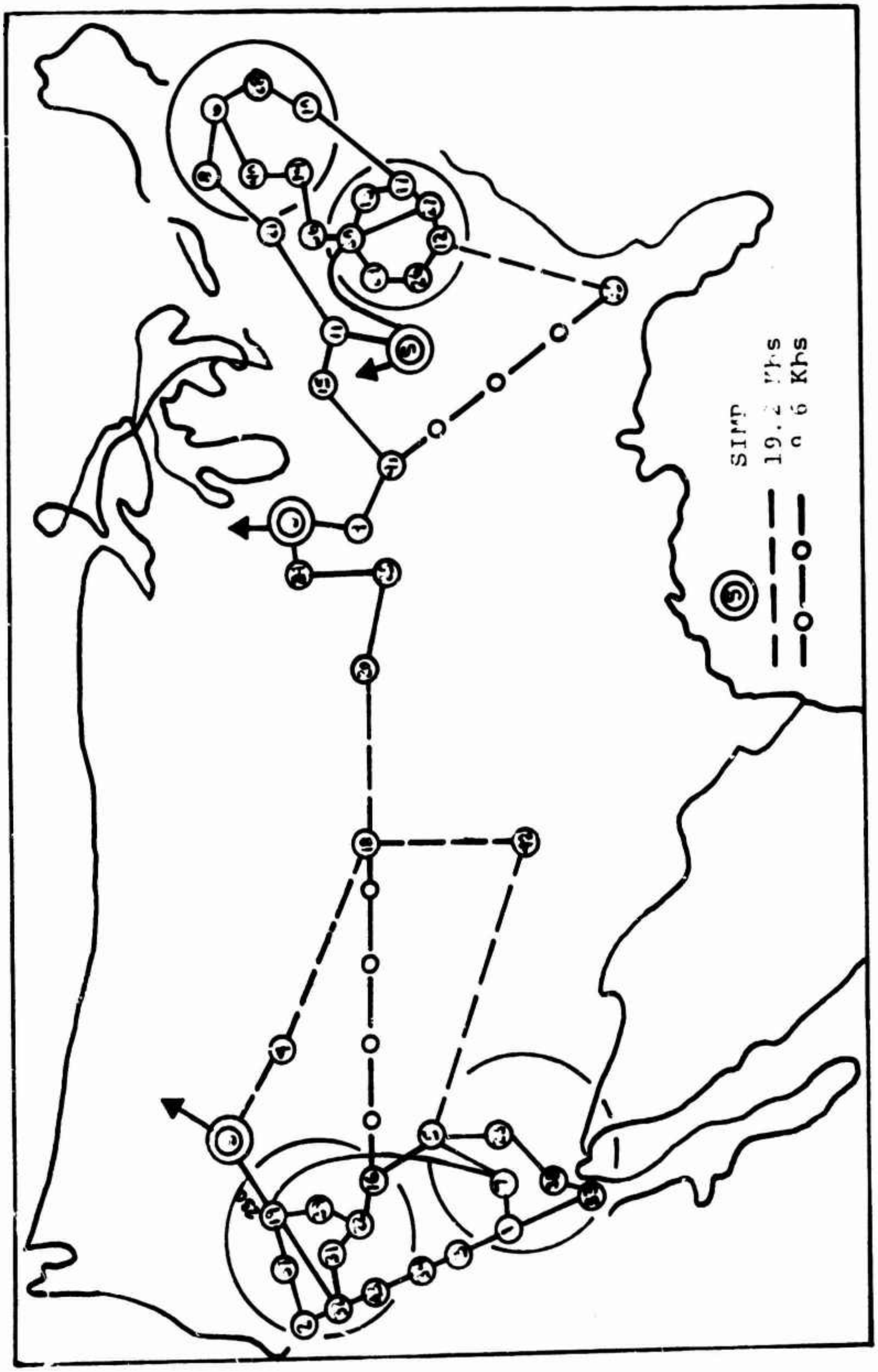


FIGURE 6
3 REGULAR SIMP's

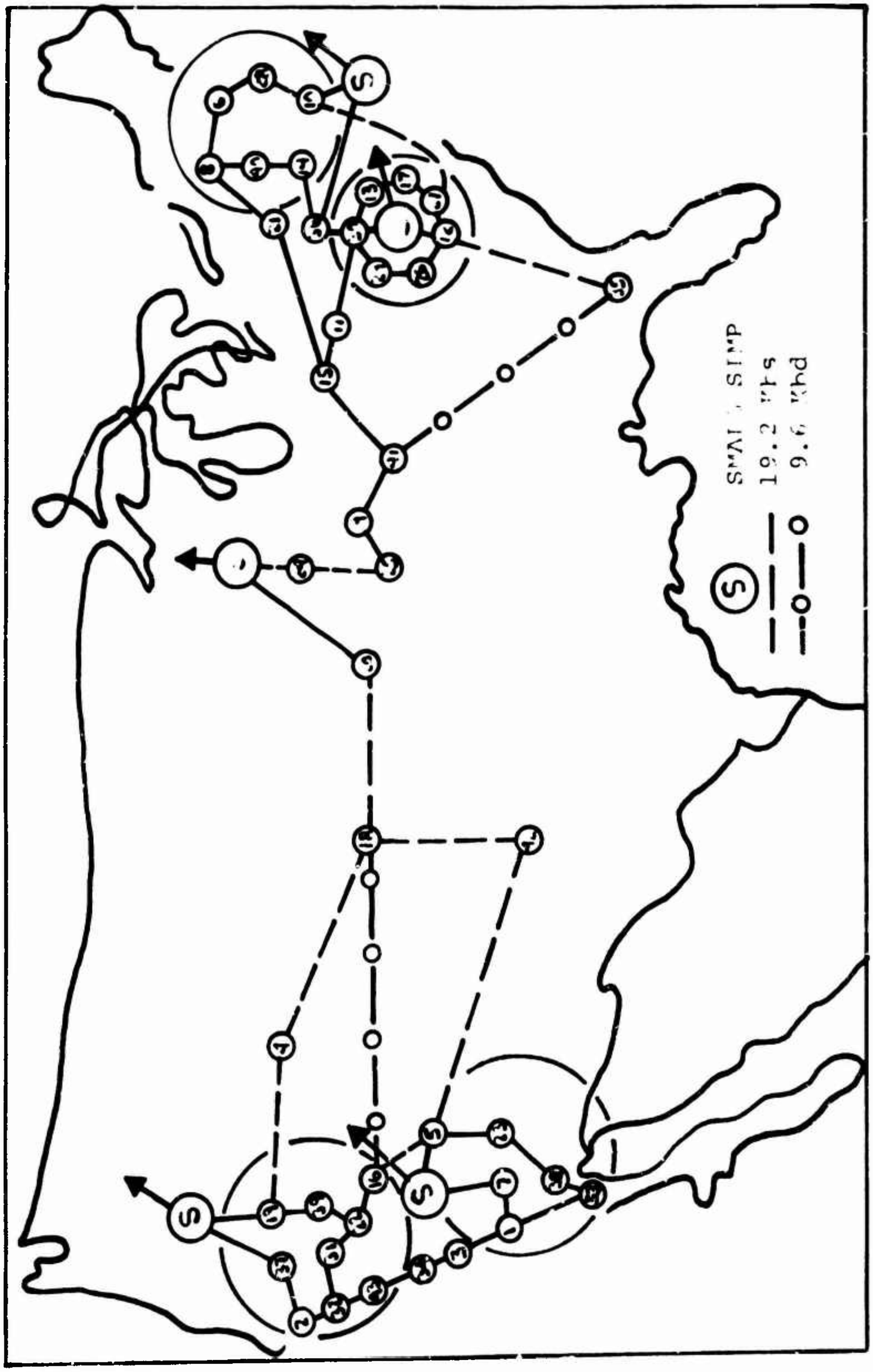


FIGURE 17

5 SMALL SIMP CONFIGURATION

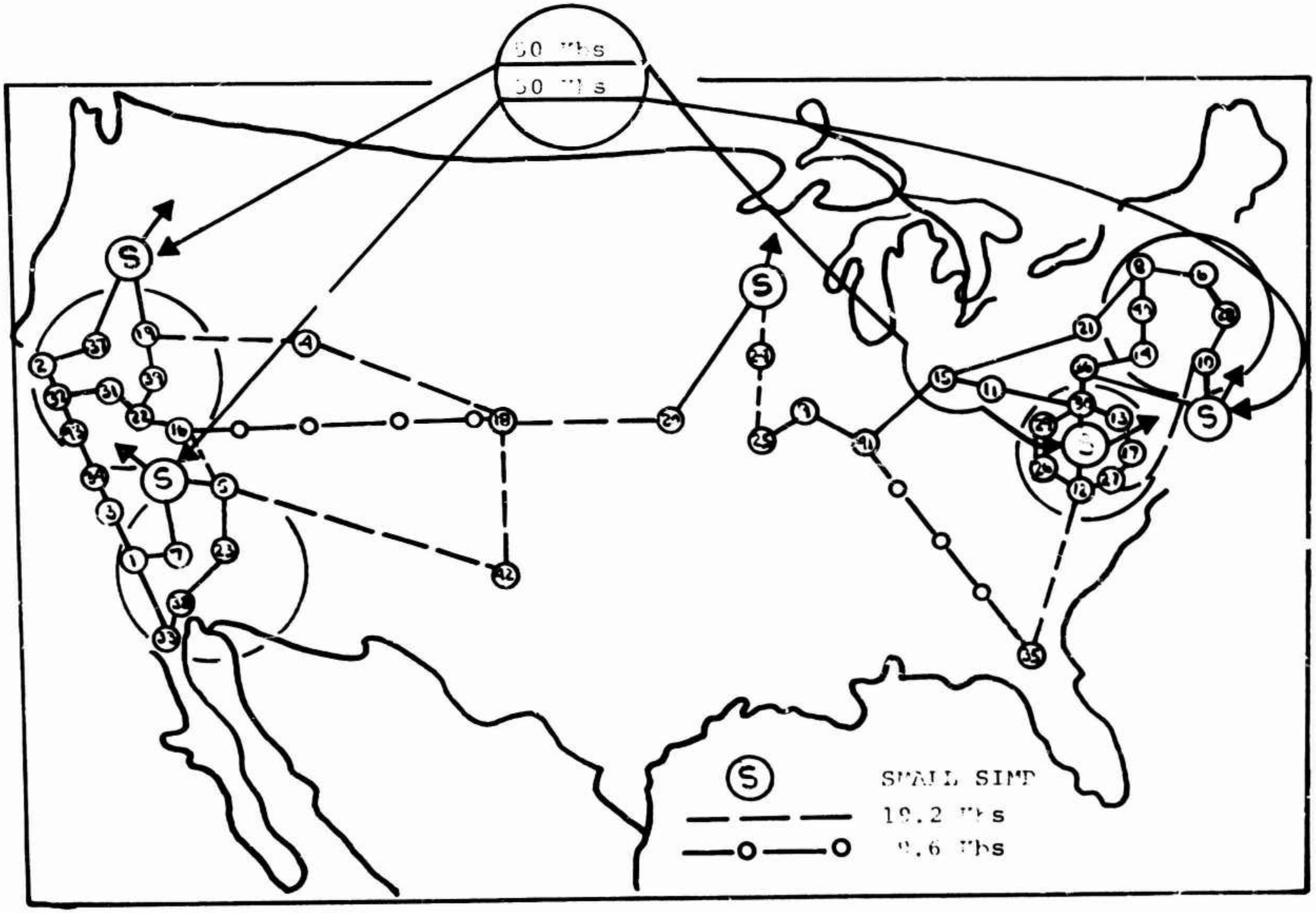


FIGURE 8

HYBRID CONFIGURATION (POINT TO POINT + UNSLOTTED ALOHA)

5.3 DISCUSSION

A general comment, valid for all the satellite configurations considered, is that satellite links are economical for throughput levels which are about 50% higher than the maximum throughput accommodated by the present terrestrial ARPANET configuration. However, changes in network topology and the reduction of some link capacities to 19.2 Kbps and 9.6 Kbps are required to take advantage of the economies available.

The fact that satellite links become attractive only for a relatively high throughput level is due to the high cost of network to ground station connections. However, many other factors, disregarded in this preliminary analysis, must be taken into consideration before general cost-performance trends are evident. In particular, the evaluation of point to point satellite link cost assumed that the standard IMP software can support satellite rates up to 230 Kbs. There are indications however that such a high rate will require modifications of the IMP hardware and software, and therefore will raise the cost of point to point links to the same levels as those of multiple access.

As for satellite bandwidth efficiency, it must be mentioned that, with additional software cost, reservation techniques for multiple access can be implemented on the SIMP; and such techniques can theoretically increase effective satellite bandwidth up to capacity. Furthermore, multiple access allocates satellite bandwidth dynamically, according to traffic pattern changes and, if needed, allows any two stations to use the full channel; while point to point access corresponds to a rigid bandwidth allocation between pairs of stations.

Considering the solutions obtained using five ground stations (case (c)) yield that performance is considerably better than with only three stations available. This is mainly due to the presence of a ground station in Chicago, which allows reduction of channel capacity on the cross country connections. Satellite traffic, on the other hand, is much higher in this case. Therefore, if we assume that the maximum available satellite bandwidth is 1.5 Mhz, we must use the slotted ALOHA access, which provides better bandwidth utilization, but requires synchronization techniques.

Cost and throughput performance for the hybrid configuration are only slightly inferior to that of the slotted ALOHA case (see C.3). However, the hybrid configuration uses an unslotted ALOHA scheme for the multiple access portion of the bandwidth, and, therefore, is simpler to implement than the slotted ALOHA scheme.

Satellite delays never exceed .5 sec. Therefore, the total average delay (averaged over satellite and terrestrial traffic) is always less than .5 sec.

The comparison of cost-throughput trends between implementations with and without satellite, when network throughput is increased, shows that satellite implementations can provide higher throughput at a lower cost, especially if the terrestrial network is reoptimized.

6. CONCLUSION AND FUTURE RESEARCH

The results of the present study show that satellite links can offer substantial savings to ARPANET growth, and can maintain average packet delay within acceptable limits. Therefore, they should be included in ARPANET short and long range plans.

Further research is required in the following areas:

1. Careful comparison of various access techniques (in particular, point to point and multiple access) using more accurate data for equipment cost and characteristics.

2. Development of optimal deterministic and adaptive routing techniques for two classes of traffic (interactive and batch), in the presence of satellite channels.

3. Study of the impact of type of satellite access on network reliability.

4. Study of the impact of satellite technology on large networks (i.e., satellite links in the highest hierarchical level; delay, routing, reliability implications, etc.).

5. Study of flow control schemes to prevent congestion (and therefore, very high delay) on the ALOHA channel.

CHAPTER 3

TERMINAL ORIENTED NETWORK COST AND PERFORMANCE-PART 2

1. INTRODUCTION

The interconnection of different time-sharing computers through a sophisticated communications subnet in the ARPANET gives terminal users access to a variety of time-sharing resources. Initially, ARPANET development was directed toward computer-computer communications and user protocols. Originally, only terminals connected directly to a computer in the network had access to the network. The successful completion of this initial phase led to a desire to complement resource development with increased user access. Many Terminal Interface Processors (TIP's) have already been installed and are currently in use, connecting users with a terminal, but with no local Host computer, to the network. Complimentary to the TIP has been the development of the ARPA Network Terminal System (ANTS), a terminal-access port designed to provide greater terminal variability and a degree of on-site processing for terminal users. The success of these developments, and the present interest in continued development, indicate a strong future for terminal access expansion.

The use of the ARPANET approach within the Defense Department would involve hundreds of Hosts accessed by tens of thousands of low speed terminals. Effective, economical terminal access to the ARPANET, and to similar networks, will depend on continued development of such facilities as TIP's and ANTS's, as well as on complimentary development of techniques for cost-effective utilization of these facilities. This chapter continues the study of this problem.

There are several ways to provide terminal access into the network. In particular, multidrop lines for connecting terminals to access ports and packet radio techniques provide potential low cost network access methods. It is necessary to investigate both of these schemes to evaluate the merits of each and to determine the conditions under which either may be preferable. It is not unreasonable to anticipate that both approaches may be applicable within the same network.

In Part 1 of Semiannual Report #1, an effective algorithm was described for the multidrop line-layout problem. The cost-effectiveness of the network design will depend not only on the line layout, but also on the number, location, and characteristics of the ports into the network. In this chapter, we provide the foundation for the investigation, development, and evaluation of design tools for the effective and efficient placement of ports for terminal access. Presently, efforts have been directed toward:

1) developing a model for use in terminal access design procedures, and
2) estimating the cost of terminal access, as a function of the number of terminals and traffic, using existing design techniques.

The models developed for use in terminal access design procedures will serve as a basis for estimating terminal access costs as a function of the number of terminals, traffic, and the particular design. The models are currently being used to estimate the cost of connecting terminals to the network through the use of TIP's and multidrop lines. The estimates cover a wide range of terminal numbers and traffic conditions, and are developed with the use of the previously reported multidrop line-layout algorithm and "engineering judgement" selection of TIP locations. These estimates will serve as a basis for comparison with other access approaches and as a measure of the effectiveness of new design tools.

2. PROBLEM EVOLUTION

The original ARPANET architecture was based on a sophisticated communications subnet composed of Interface Message Processors (IMP's) interconnected with 50 KBS lines. Each IMP served up to four Host computers, each of which was connected to its own set of terminals, as shown in Figure 1(a). The Terminal Interface Processor (TIP) is a modified version of the IMP with extended hardware

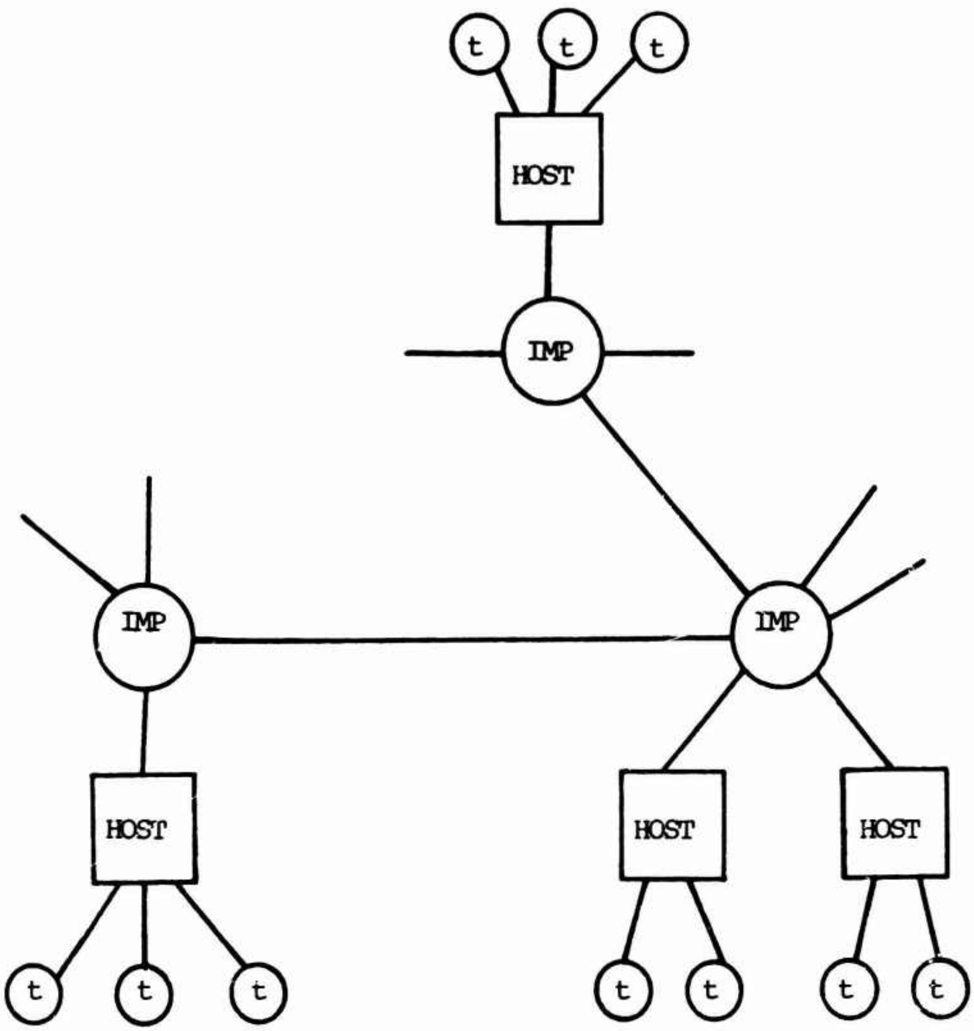


FIGURE 1(a)

ARPANET ARCHITECTURES

and software. This Terminal IMP, or TIP, serves in the communication subnet and has the ability to serve a Host computer in the same manner as an IMP, but differs from an IMP in that it also has the ability to connect directly with up to 63 terminals. It creates a new form of network architecture, as shown in Figure 1(b).

Although the TIP provides a means of direct access to the network by terminals, it does not provide local processing capabilities to terminals. To fill the gap between an IMP-Host combination and the direct access TIP, a "mini Host" computer system was developed. This system, the ARPA Network Terminal System (ANTS), provides a limited amount of on-site processing power and provides for a greater range of terminal variability. The ANTS is based on a PDP-11 minicomputer system which acts as a Host to interconnected IMP's or TIP's. Terminals are then connected to the PDP-11 which provides no resource to the network, but permits access by the terminals. The ANTS is a third variation of network architecture, as shown in Figure 1(c).

ANTS's and TIP's are the two types of terminal access ports currently used in the ARPANET, each of which must also serve as an ordinary message switching processor. Thus, such ports may be expected to be reasonably expensive and of high bandwidth. To interconnect a large number of interactive terminal users to the network, it is very attractive to consider such ports as the roots of centralized networks, composed of multidrop facilities for which cost-effective multiplexing and concentration techniques may be applicable. Such a network architecture is shown in Figure 2. The actual port for network entry may be an extended version of the TIP, a version of ANTS, or some new development. The design techniques developed in this study will only take into consideration the parameters characterizing the cost and constraints of the different facilities, not the internal characteristics of the facility itself.

To estimate the cost of terminal access to the network, it is assumed that continued development of terminal access ports will

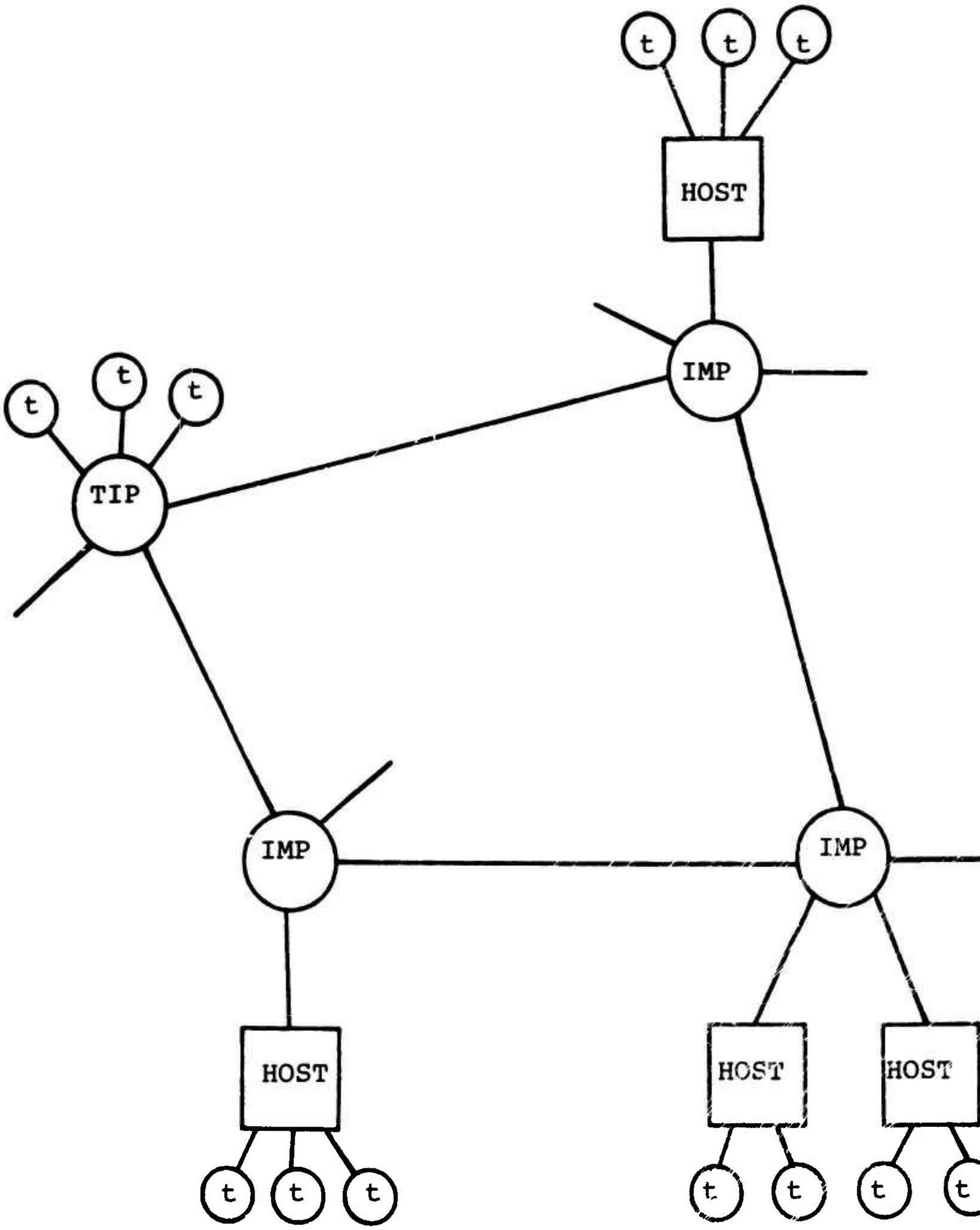


FIGURE 1(b)

ARPANET ARCHITECTURES

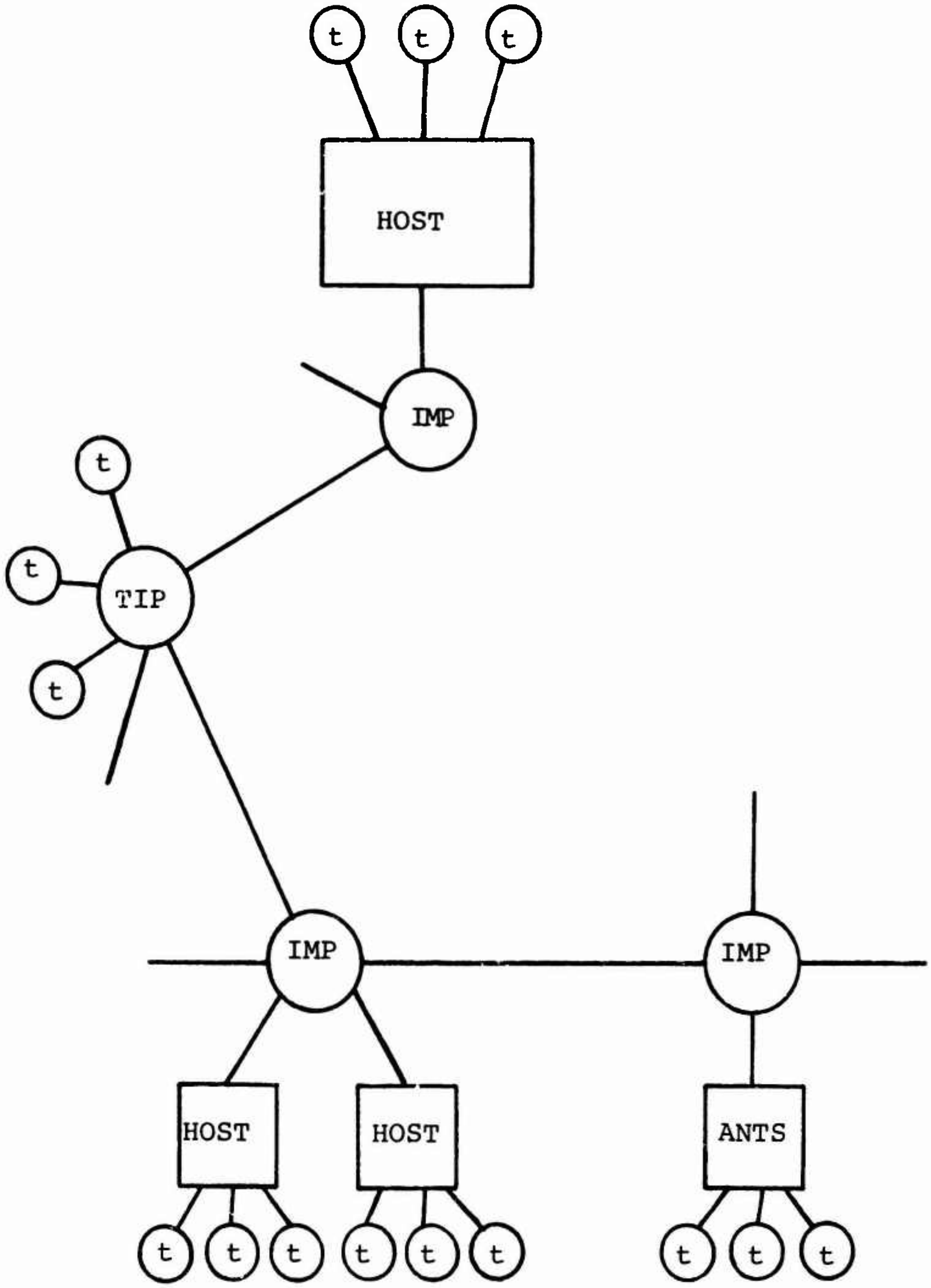


FIGURE 1(c)

ARPANET ARCHITECTURES

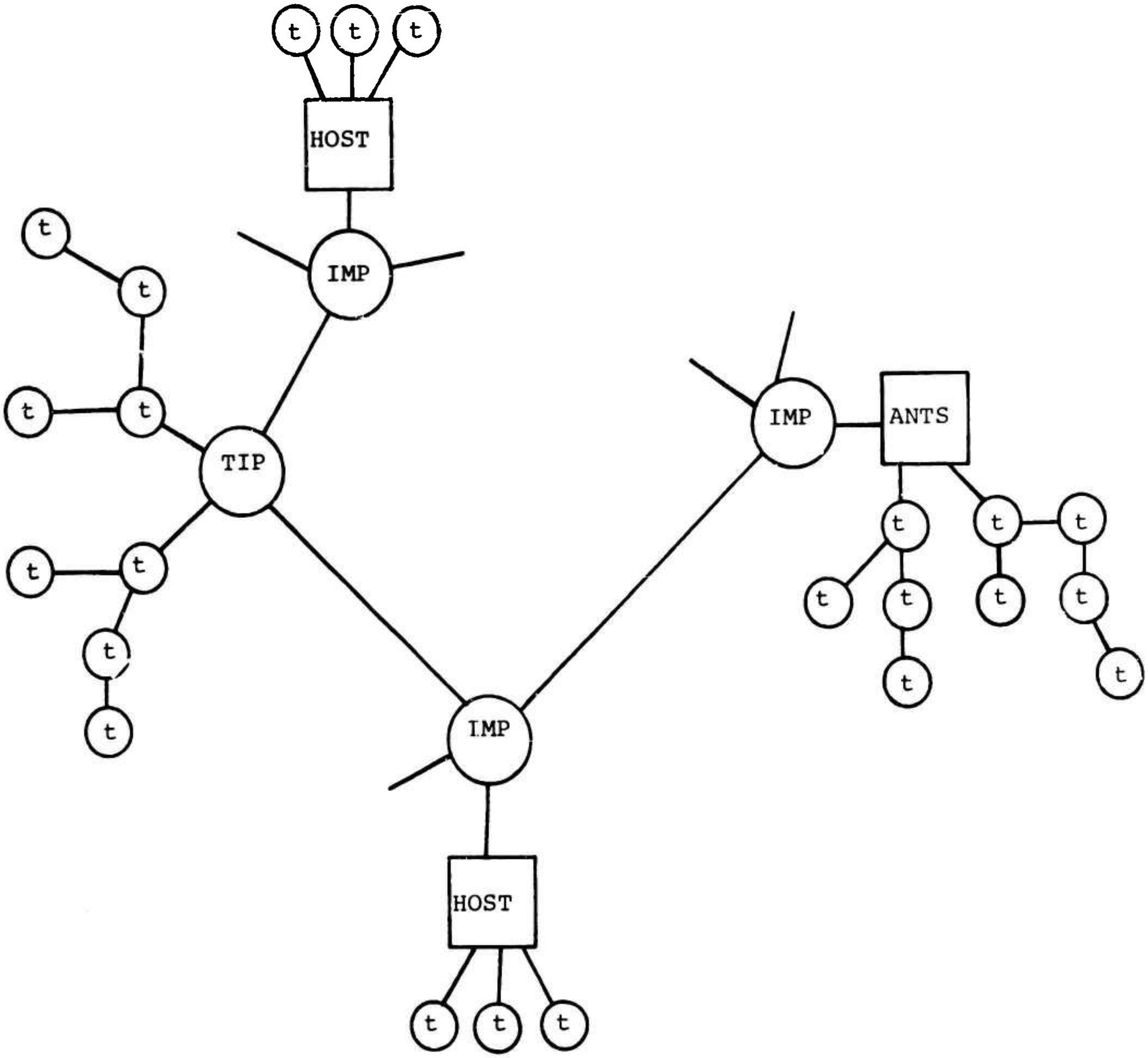


FIGURE 2

permit the use of the multidrop line architecture described above. Attention is therefore concentrated on the cost-performance aspects of this architecture and on the development of effective design techniques.

3. NETWORK MODELING FOR TERMINAL ACCESS

The process of investigating and developing approaches to the design of networks for terminal access requires, as a first step, the construction of an appropriate model. In this study, a primary goal is to determine the tradeoffs and parametric dependencies present in various approaches to terminal access. To compare these approaches, it is necessary to have a common data base to which each approach can be applied. Such a base has been constructed in the form of a model for the number and geographic distribution of terminals, for the terminal and terminal user, and for the multidrop communication lines. It is also necessary to model those components of a design peculiar to the particular approach considered. In this report we present a model for both the TIP system and the ANTS. The following describes these various models.

3.1 POPULATION

The cost of terminal access will depend on a variety of factors, including the number of terminals to be connected and their geographic distribution. To determine the parametric dependence of cost on the number of terminals, populations of from 100 to 10,000 terminals will be considered. The figure of 100 reflects the anticipated near-future requests for terminals. The figure of 10,000 reflects a realistic estimate of the number of terminals that can be expected to be served by a fully developed network. (The above consider only terminals without a local Host.)

To have a meaningful data base, it is necessary to geographically distribute the terminals in a sensible manner. Terminals were located on the basis of population density because of the success of this approach in previous NAC investigations. Figure 3 shows the placement of 1,000 terminals based on a distribution proportional to population. A rectangular region was determined for each city, or collection of cities, to reflect the feasibility of the region to support a population segment with access to urban facilities. Thus, consideration was given to natural geographical boundaries, such as mountains, lakes, and coast lines, to major roads in the area, to the number of nearby smaller communiti s, and to the natural pattern of urbanization between relatively close major population centers. Using this approach, 123 regions were defined, with varying sizes of approximately 70 miles square.

Once a number of terminals has been allocated to a region in proportion to population, the geographic positions of the terminals within the region are uniformly randomly distributed. With a large number of terminals, it is reasonable to anticipate that some of them may be located at points with no discernible geographic significance. Therefore, a fraction α of the terminals were located at random in a large geographic segment; east of Denver, west of Pittsburgh, north of Austin, and south of Milwaukee. The fraction α was selected on a sliding scale as shown below.

Number of Terminals	In Regions	Random
100	95%	5%
200	95%	5%
500	90%	10%
1,000	90%	10%
2,000	85%	15%
5,000	80%	20%
10,000	80%	20%

Thus, the data base developed will be used to evaluate various terminal access approaches and design procedures.

Locations of the 1000 nodes
Numbers indicate multiplicity of nodes
Nodes without numbers have multiplicity
one (simple nodes).

FIGURE 3.

3.2 TERMINAL - TERMINAL USER

Even though network resources in the ARPANET have been extended far beyond traditional time-sharing, the interactive user retains a significant role in network usage, and the extension of accessibility to a terminal basis will give even greater significance to terminal-computer traffic. To effectively design and evaluate terminal access networks, it is necessary to model the terminal traffic. Two of the few definitive papers on time-sharing modeling from a communications perspective have been written by Jackson and Stubbs [Jackson, 1969], and Fuchs and Jackson [Fuchs, 1970]. The following traffic characteristics of a terminal during a period of use are based on their results for time-sharing systems used in scientific applications, and extended in consideration of advances in terminal technology, higher speed lines being used, and more sophisticated time-sharing users and programs.

	Average Message Length	Minimum Average Traffic	Maximum Average Traffic
User Input	12.1 char	.1 char/sec	1 char/sec
Computer Response	52.8 char	1.0 char/sec	10 char/sec

The design of terminal access networks, and hence the cost, is anticipated to be dependent on the traffic level assumed for the terminals. Thus, in this study, traffic level will be varied to investigate this dependence, with a range of variation from the minimum to the maximum values indicated above. The minimum average traffic level reflects the results of the noted study for scientific applications using low speed facilities and ordinary time-sharing programs. The maximum average traffic level reflects an extension of these results in consideration of "smart-fast" terminals, higher speed communication facilities, more advanced time-sharing programs, and more sophisticated users. For comparison purposes, all network designs will be based on busy hour conditions of all terminals being active.

3.3 COMMUNICATION FACILITIES

The current ports for access by terminals to the ARPANET (TIP's and ANTS's) may connect terminals directly, or remotely through modems and phone lines. In this study, a large number of terminals serving interactive users are considered, and to economically connect all the terminals, multidrop lines will be used. The multidrop communications facility will be assumed to be a standard voice-grade line as described below.

3.3.1 MULTIDROP LINE

Capacity (full duplex)	1200 bps
Cost	$.50/mile + $40/drop

The cost is based on the Government rate of $.42/mile plus 20% for non-direct routing. It should be noted that in this model the number of drops on a line is restricted only by the traffic constraint. In reality, the number is often additionally restricted by telephone company practices. The effect of a more severe restriction is easily seen by simply assuming a correspondingly higher traffic level.

3.3.2 TIP

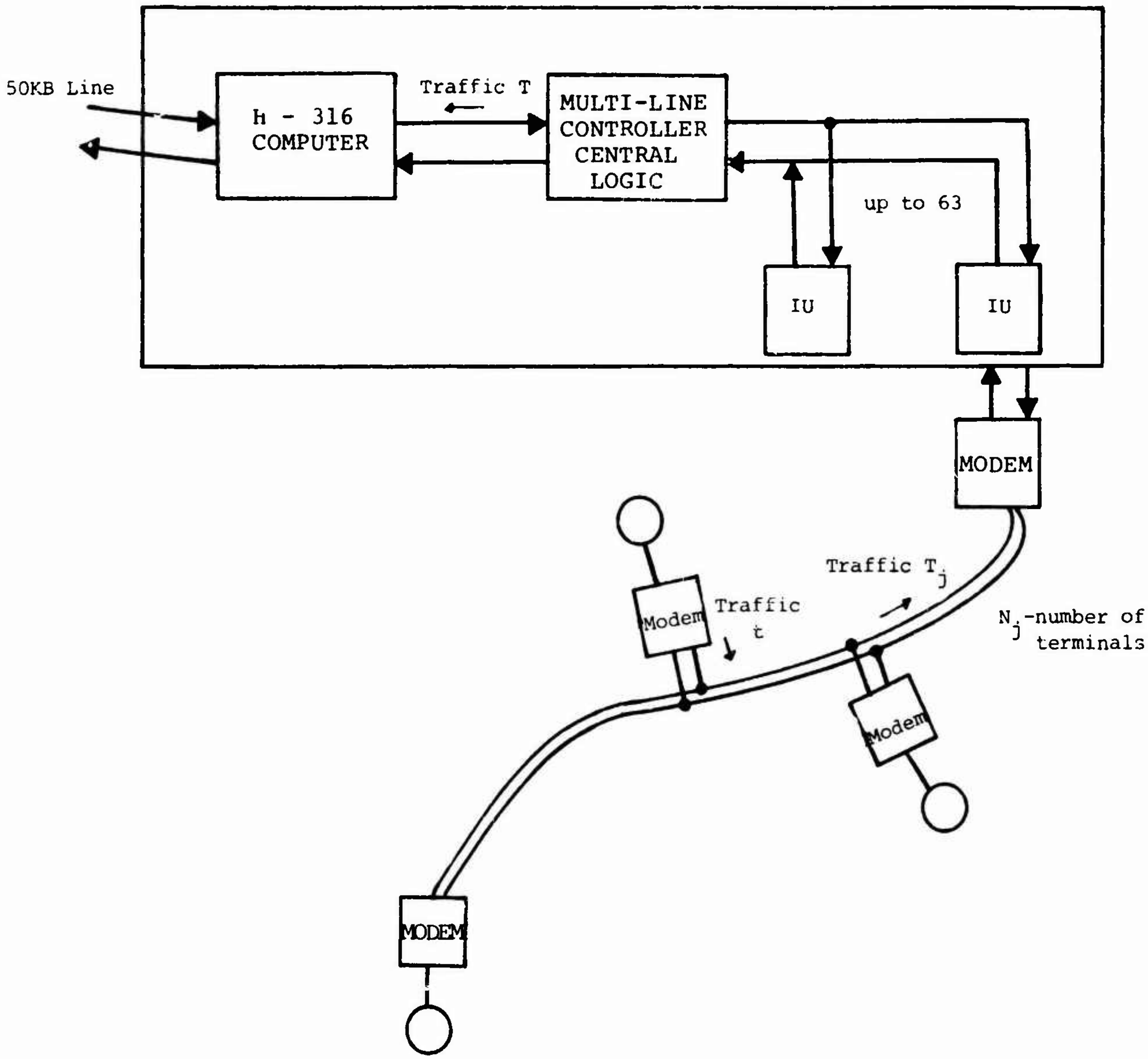
The first approach to be considered will be a TIP serving as the root of a centralized network of terminals. In this section we note the significant features of the TIP. In a later section we incorporate this description to construct a complete network model for design and cost evaluation.

The TIP, as described by Ornstein et. al. [Ornstein, 1972], is characterized in Figure 4. Its characteristics indicate:

1) The TIP has 63 terminal I/O slots.

2) Each slot can handle direct terminal connections or connections via modems.

3) Asynchronous data rates handled by the TIP include 1200, 1800, and 2400 bps.



50KB Line
H - 316
COMPUTER
Traffic T
MULTI-LINE
CONTROLLER
CENTRAL
LOGIC
up to 63
IU
IU
MODEM
Traffic T_j
N_j-number of
terminals
Modem
Traffic
t
Modem
MODEM

FIGURE 4

TIP

4) The TIP has a terminal program throughput of:

a) 100 Kbps one way traffic if messages are long ("many characters"), and

b) 5-10 Kbps if each terminal message is a single character.

5) The TIP uses 5% of its processing capacity to act as an IMP.

6) The TIP uses 10% of its processing capacity to field MLC interrupts.

7) The bandwidth capability of the TIP is summarized approximately by the formula.

$$P + H + 11T \leq 850$$

where P = total phone line traffic (Kbps)

H = total Host traffic (Kbps)

T = total terminal traffic (Kbps)

and full duplex units count twice baud rate, i.e. standard full duplex 50 Kbps phone line counts 100, and full duplex ASR - 33 counts as .220.

As noted, the TIP does not presently handle multidrop lines. Consequently, adjustments to its characteristics are required for the network model.

The above noted characteristics reflect TIP capacity as a message concentrator for terminals. Additional consideration must be given to TIP cost, which includes estimated rental rate, the cost of its interconnect to the ARPANET, and the cost of the modems necessary to connect terminals. Therefore, the total cost is a function of the TIP's geographical relationship to the rest of the ARPANET, the topology of its interconnection, and the number of modems required for terminal connections. These costs will be determined as follows:

50 Kbps line (ARPANET interconnect) $5/mile + $425/end
(based on current ARPANET experience)

1200 bps line (terminal connection) $17/modem
(current standard cost)

TIP rental $2500/month
(assumed TIP cost of $100,000 to be amortized over 5 years at 10% interest compounded quarterly)

3.3.3 ANTS

The ARPA Network Terminal System (ANTS) provides a powerful access port for terminals [Bouk, 1973]. The system is based on a PDP-11 minicomputer attached to an IMP (or TIP). The minicomputer acts as a full capacity Host for the IMP, and offers terminals some local processing power. The system has a higher degree of tailorability, providing access and control to a wider range of terminals and peripheral hardware than the TIP. Its increased flexibility makes the ANTS more difficult to evaluate for terminal access, and its cost effectiveness depends on the special terminals and local processing as well as on the number of ordinary, interactive terminals connected to it. At this point, the ANTS will be considered strictly for terminal access, and a system configuration oriented toward maximum terminal access bandwidth rather than local processing will be assumed. Estimates of significant ANTS characteristics are:

1) Terminal Bandwidth - 12.8 Kbps
2) Monthly rental - $1,000/month
3) 1200 bps line (terminal connection) $17/modem

If ANTS's are connected into the network at existing IMP's the only cost for the terminal port is the ANTS cost. However, the load of the ANTS on the IMP, coupled with the existing Host computer, must not exceed the capacity of the IMP. If ANTS's are located at new points requiring their own IMP's, the cost of the IMP's and their network interconnect cost must also be considered. Since ANTS's are Host computers to an IMP, up to four ANTS's can be connected to a single IMP.

4. CURRENT DESIGNS - TIP

This section presents preliminary estimates of the costs of terminal access based on using TIP's as roots of centralized networks. These estimates are made using the models developed in the previous section and the line-layout algorithm described in the previous report. The number and locations of TIP's were heuristically determined by visual inspection of the network configuration. The network model resulting from a combination of the model components presented in the previous section is described below.

4.1 NETWORK MODEL - TIP

As noted, the currently designed TIP has no provision for the support of multidrop lines. Both hardware and software modifications may be necessary for the acceptable addition of this capability. Significant requirements are line protocol for the multidrop lines and more extensive file manipulation resulting from the larger number of terminals. The line protocol must permit line utilization of approximately 50%, a conservative figure based on the use of ordinary polling techniques for multidrop lines. With the previously described traffic range of 10bps to 100bps, this gives a possible range of 6 to 60 terminals on a line. The sixty-three possible connected lines allow a maximum demand of 37.8 Kbps to be placed on a TIP by the terminals. Using the maximum demand figure in the TIP bandwidth formula indicates that such a TIP would have sufficient additional bandwidth to support a Host and also be connected to the ARPANET in a manner consistent with current practices. However, the number of terminals a TIP handles in the maximum demand case (378 - 3780) is far beyond the current maximum configuration (63). This increase in number should be anticipated as causing considerable additional overhead for file manipulation. Furthermore, additional overhead may be anticipated due to the burden of a multidrop line protocol. Under these conditions, the maximum number of terminals that a TIP can handle is assumed to be 630, one order of magnitude greater than its current direct connection capacity. This gives a network model as below.

TIP 1) up to 63 line connections
2) up to 630 terminals

Lines up to $\frac{600}{t}$ terminals/line
where t is the traffic/terminal in bps

4.2 DESIGN RESULTS - TIP

Cost is estimated as a function of the number of terminals and their traffic level, subject to fixed TIP locations; the multidrop line layout algorithm described in Semiannual Report #1 is applied to derive the multidrop line cost. In the table below, costs are given for a 100 terminal system at a traffic level of 100 bps each for different numbers of TIP's at different locations.

# TIP's	Locations	100 Terminals (100bps each) Monthly Line Costs	Monthly Line Costs And TIP Rental
1	Chicago	$14,007	$16,507
	Memphis	14,501	17,001
	New York	17,190	19,690
2	NY - LA	13,091	18,091
3	NY - LA - CHI	11,375	18,875
	NY - LA - MEM	11,302	18,802

These results show that a higher number of TIP's yields lower line costs, but not necessarily a lower total cost. Consequently, the number of TIP's is varied until a local minimum is reached. Table 1 gives preliminary estimates of the cost of terminal connection as a function of the number of terminals and the level of traffic. The size of the networks under consideration is at present being enlarged to refine these estimates. Results are shown as points connected by straight line segments in Figure 5. The curves suggest that for low numbers of terminals, and thus, low numbers of TIP's, the line constraints and TIP locations have significant impact on cost. For large numbers of terminals, and thus, larger numbers of TIP's, costs are less sensitive to TIP placement and line constraints. Simplified illustrations of several of the network designs are given in Figures 6 - 9. Note that for low traffic (10bps) it is cost effective to use as few TIP's as possible, while for high traffic (100bps), savings are achieved by using more than the minimum number of TIP's. (With low traffic, many terminals can be chained together on one line to economically connect distant terminals to a TIP. With high traffic, only a few terminals can be placed on a line, and distant terminals result in several long, uneconomical lines.) Since TIP's are relatively expensive when compared to conventional multiplexers, these simpler devices to achieve economy of scale will also be investigated as an alternative architecture.

5. COST-PERFORMANCE TRADEOFFS

Cost effective terminal access will depend on both an effective line layout algorithm to connect terminals with access ports, and on an effective port location algorithm to determine both the number and location of access ports. The line layout problem for a given set of ports has been effectively dealt with in Part 1 of Semiannual Report #1. Present research is aimed at developing a port location algorithm to be combined with the line layout algorithm for complete network design. The total algorithm will then be used to investigate network tradeoffs with up to 10,000 terminals and several hundred IMP's.

TABLE 1

PRELIMINARY TERMINAL - TIP EXPERIMENT RESULTS

Number of Terminals	Traffic (bps)			
	10	20	50	100
100	$ 13,095	$ 13,231	$ 15,146	$ 17,607
200	18,906	19,875	23,373	31,818
500	36,050	39,138	49,208	56,099
1,000	66,775	72,893	83,886	94,189
2,000	119,570	125,085	144,759	165,817

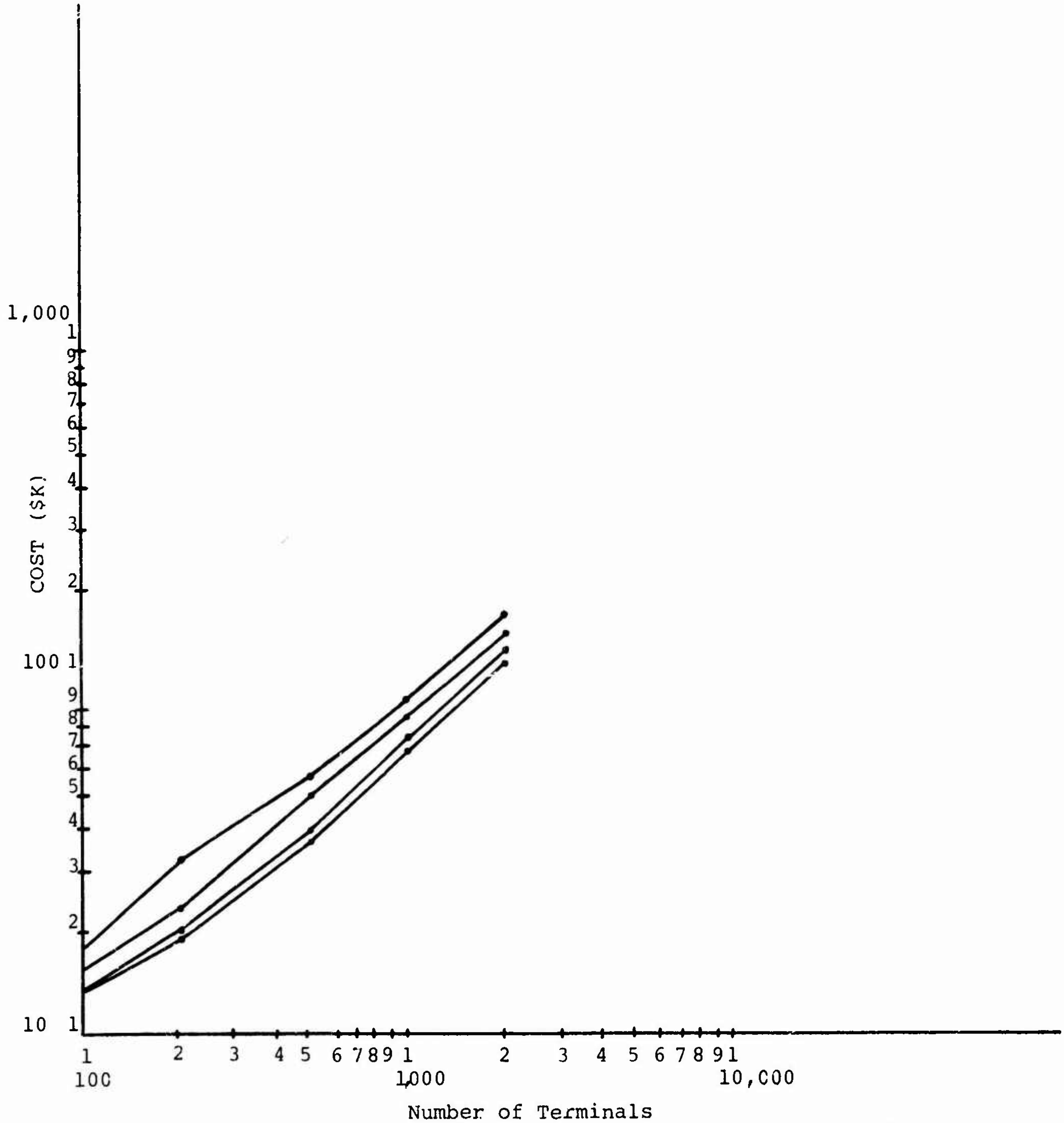


FIGURE 5

PRELIMINARY ESTIMATES OF TERMINAL CONNECTION COSTS

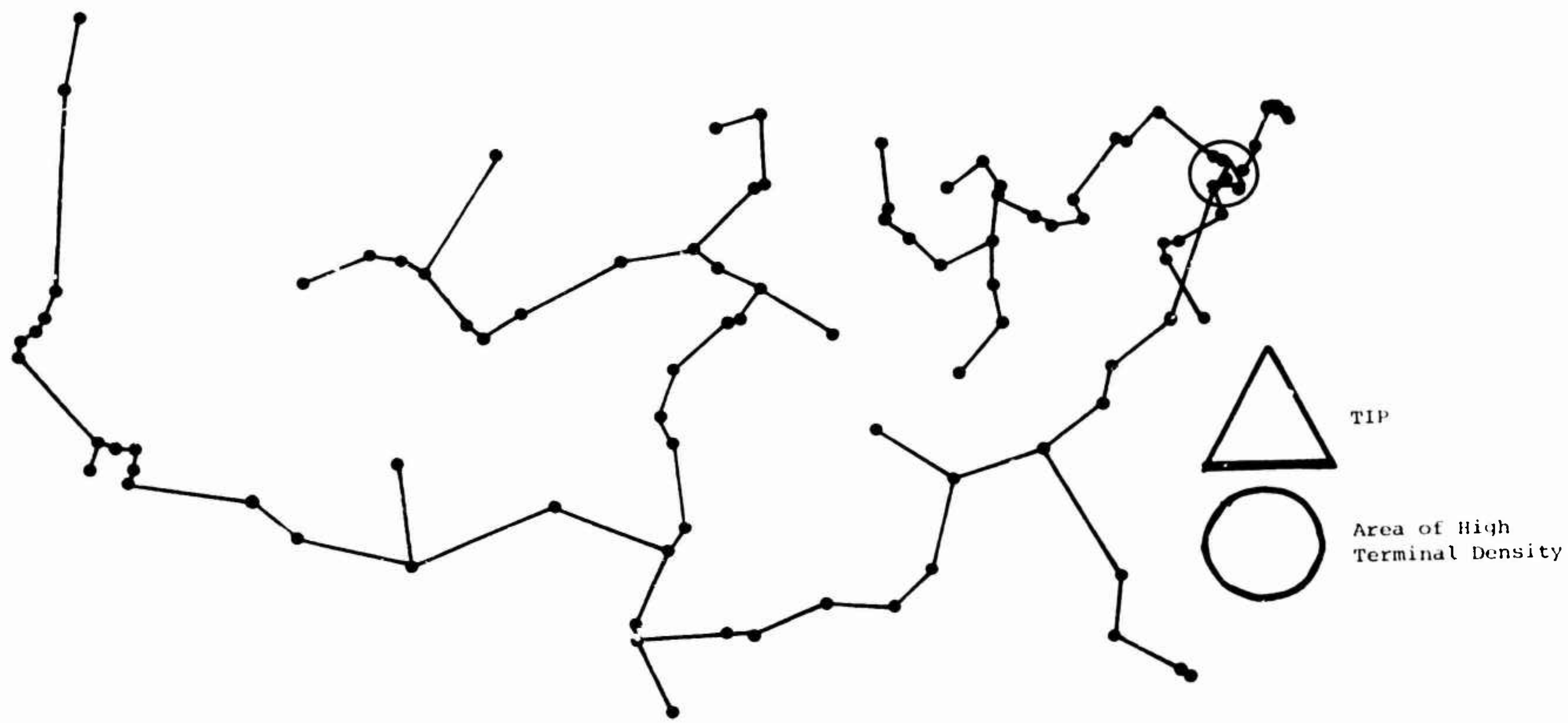


FIGURE 6

NETWORK DESIGN FOR 100 NODES, 10bps TRAFFIC, TIP IN N.Y.C.

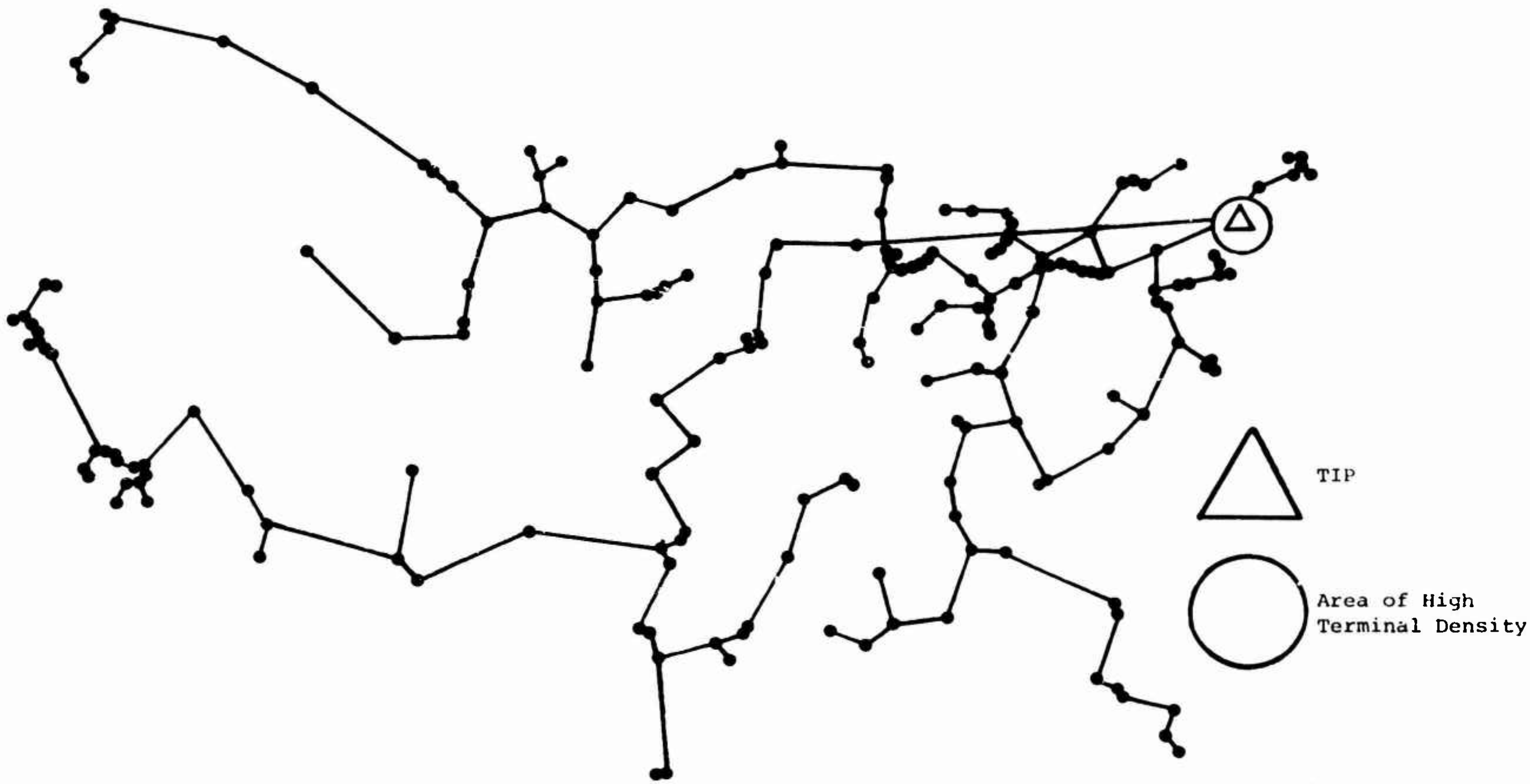


FIGURE 7

NETWORK DESIGN FOR 20? NODES, 10bps TRAFFIC, TIP IN N.Y.C.

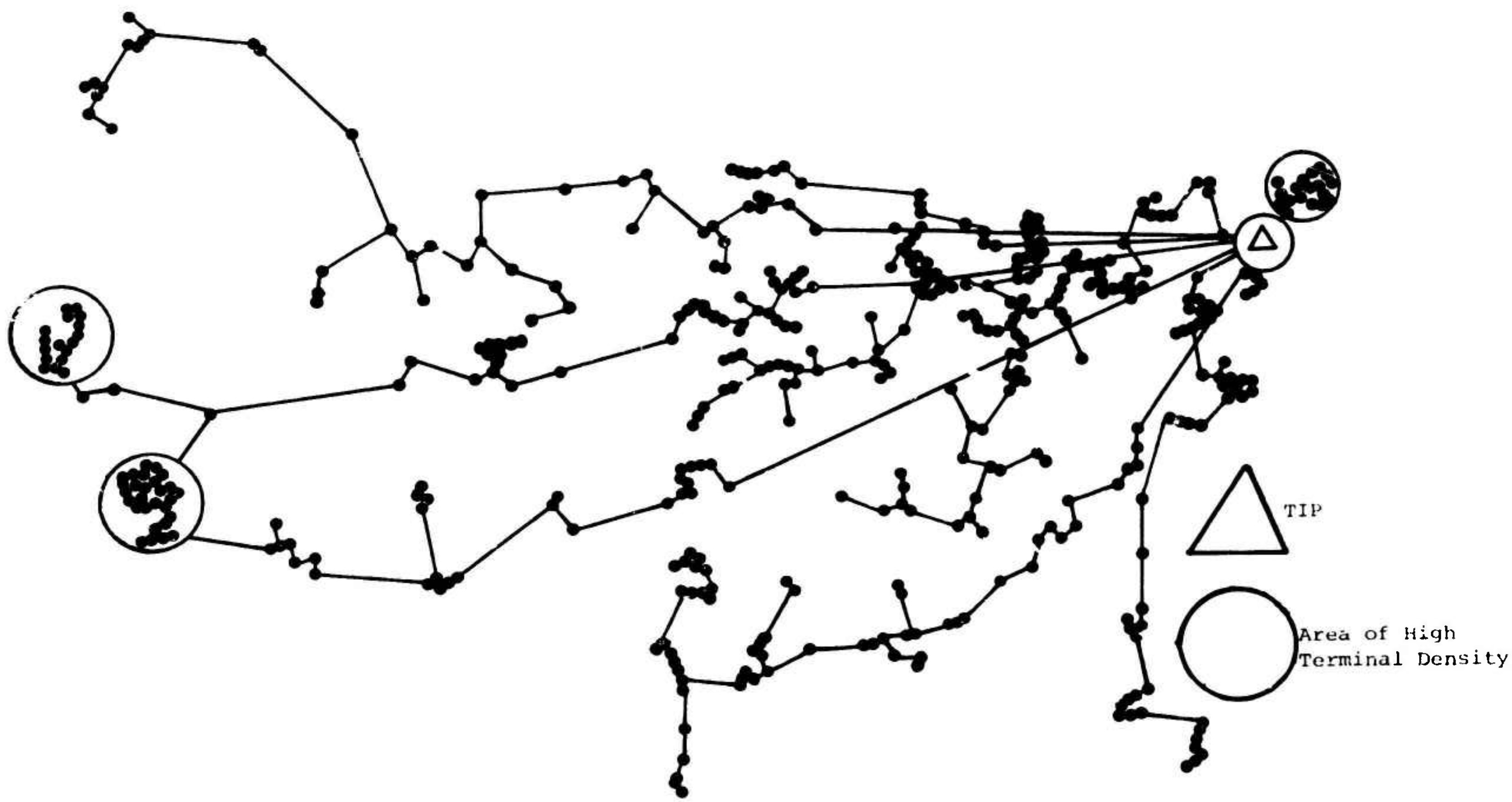


FIGURE 8

NETWORK DESIGN FOR 500 NODES, 10bps TRAFFIC, TIP IN N.Y.C.

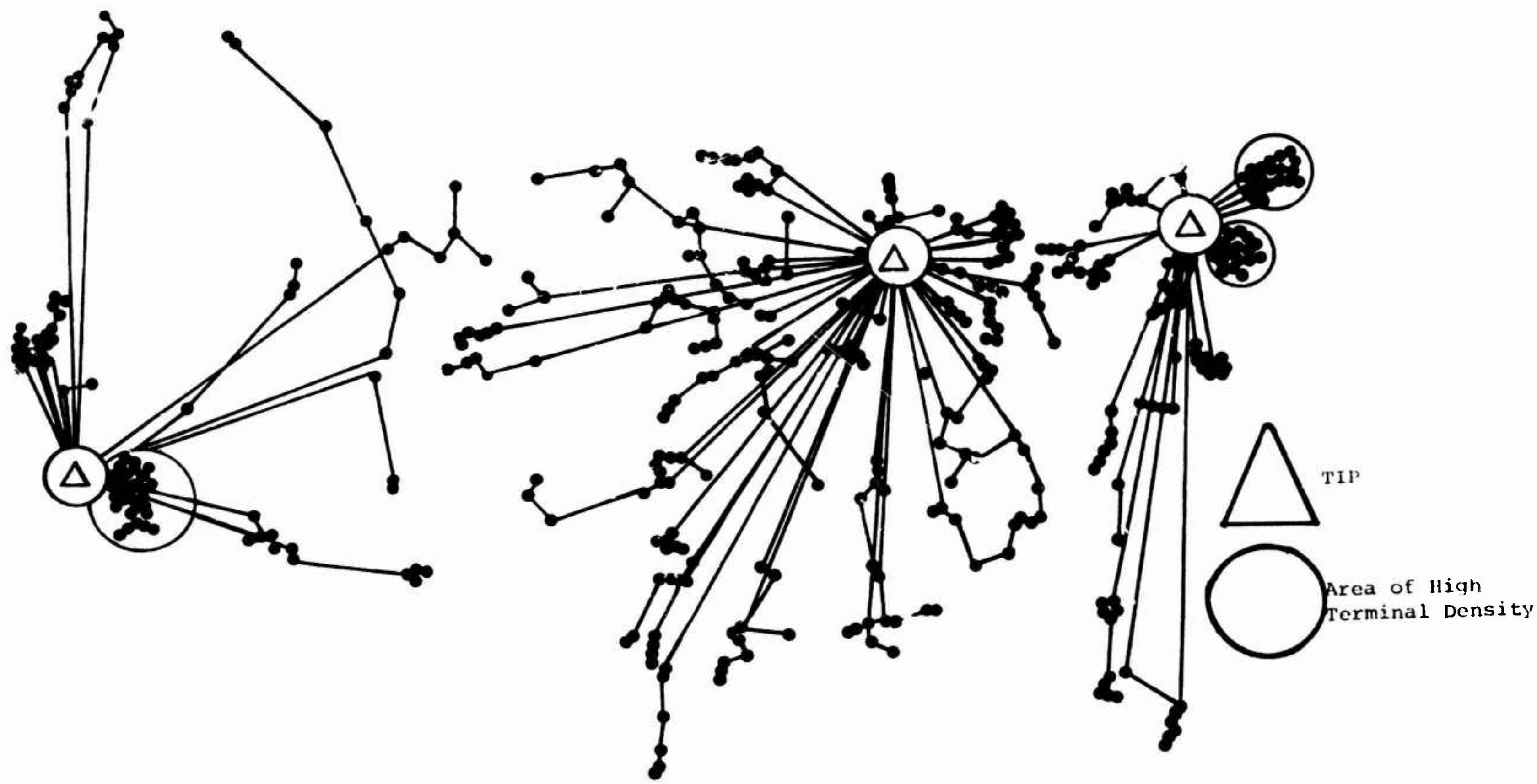


FIGURE 9

NETWORK DESIGN FOR 500 NODES, 100bps TRAFFIC, TIP's IN N.Y.C., CHICAGO, L.A.

CHAPTER 4

A CUT SATURATION ALGORITHM FOR TOPOLOGICAL DESIGN OF PACKET SWITCHED COMMUNICATION NETWORKS

PART I

1. INTRODUCTION

The topological design of packet switched communication networks was first discussed by Frank, Frisch, and Chou, [Frank, 1970], who described an effective method for finding low cost topologies based on branch exchange techniques. The overall design problem and the limitations of branch exchange techniques were described by Frank and Chou, [Frank, 1973]. It has become clear that as networks grow, new approaches are required to produce effective network designs.

In this chapter a new approach to the topological design problem is described. This approach is based on a fundamental limitation on packet switched network performance called cut saturation. The cut saturation property and its relation to network throughput and time delay were discussed by Frank, Kahn, and Kleinrock, [Frank, 1972], and an efficient routing technique based on this principle was reported by Chou and Frank, [Chou, 1972].

The problem addressed in this chapter is the design of low-cost packet switched network topologies with a fixed number of sites (Nodes) and with single fixed-capacity communication lines (Links). The approach taken is a modification of the Branch Exchange Method. The algorithm iteratively attempts to keep the network throughput within specified bounds while reducing the overall line cost and maintaining capacity, delay and reliability constraints.

The results show that:

(1) The cut saturation solutions are at least as good as the Branch Exchange Method (BXC).

(2) The cut saturation technique is computationally much more efficient than the BXC technique.

2. DESCRIPTION OF THE CUT SATURATION (CS) METHOD

Constraints on network designs are:

(1) A fixed number of sites

(2) All lines of one capacity (say 50 Kbps)

(3) Maximum average delay per packet (say 0.2 sec)

(4) At least 2-connectivity

The heart of the CS algorithm is its routing algorithm. The routing algorithm employed is an adaption of the Flow Deviation Method for solving non-linear, constrained, multi-commodity flow problems [Fratta,1973]. This algorithm, which has been described in detail elsewhere, provides optimal or near optimal link flows for a given traffic requirement, network topology, and link capacity allocation.

The object of the optimization is to achieve a desired network throughput at the lowest possible cost. Given a typical network configuration (usually, at least 2-connected, but much less than 3-connected) subject to the network constraints, the routing algorithm can be applied to obtain optimal link flows. With these link flows, some links will be highly utilized (80-90%), while others will be underutilized. If the links are ordered according to their utilization and then successively removed, the network will eventually be partitioned into two disjoint components of nodes. The minimal set of these highly utilized links that disconnects the network is called a saturated cutset.

This experiment was performed a number of times. It was continuously observed that the total traffic between these 2 components was usually very close to the sum of the flows in the saturated cutset links. In other words, if: ND_i = number of nodes in component i for i = 1,2; RE = node pair traffic requirement; NC = number of cutset links; and f_i = flow in cutset link i then,

$$ND_1 \times ND_2 \times RE \simeq \sum_{i=1}^{NC} f_i. \qquad (1)$$

Clearly, the saturated cutset imposes a physical limitation on the network throughput. In fact, a theoretical upper bound on RE, corresponding to fully saturated links (i.e. infinite delay) is as follows:

$$RE \leq \underset{S}{\overset{\text{MIN}}{\text{over}}} \frac{\sum_S C_i}{ND_1 \times ND_2}; \qquad (2)$$

where C_i = the capacity of cutset link k; S = any minimal cut; and ND_i = number of nodes in component i corresponding to cut S. Since the links in the saturated cutset have $f_i \simeq c_i$, from (1), it follows that:

$$RE \simeq \frac{\sum_{i=1}^{NC} C_i}{ND_1 \times ND_2}. \qquad (3)$$

Thus,

- The value of RE given by the routing algorithm is very close to the theoretical upper bound (i.e. near optimal).
- Improvements on RE can only be obtained by reinforcing the capacity of the cutset.

On the other hand, a new link introduced within a component will not significantly increase the capacity of the cutset. Therefore, if we want to introduce links that will increase the capacity of the cutset, we must consider only the potential links that join the 2 components. This guideline provides an efficient criterion for the selection of links to be introduced into the network.

3. CUT SATURATION ALGORITHM IMPLEMENTATION

The cut saturation algorithm consists of several basic sections:

(1) Routing - (performed after each network modification to generate new optimal link flows).

(2) Saturated Cutset Determination - (performed at each stage of routing).

(3) Add-Only - (Select the "Best" link to join the two components).

The Add-Only operation is key to the basic CS method.

(4) Delete-Only - (selects the best link for deletion from a highly connected topology).

(5) Perturbations - (combines Add-Only and Delete-Only operations).

(6) Chain collapsing - (replaces selected serial chains by a single equivalent link to improve efficiency of optimization).

3.1 THE ADD-ONLY OPERATION

The Add-Only algorithm determines the location of the link that will join the two components, K_1 and K_2. The links that must be considered for insertion correspond to the pairs (x,y), such that:

$$x \in K_1 , \quad y \in K_2$$

Using the cut saturation philosophy for throughput improvement while attempting to minimize the cost, a reasonable choice for the new link is the one that, while joining the two components, has the shortest distance, i.e. the smallest cost as shown in Figure 1.

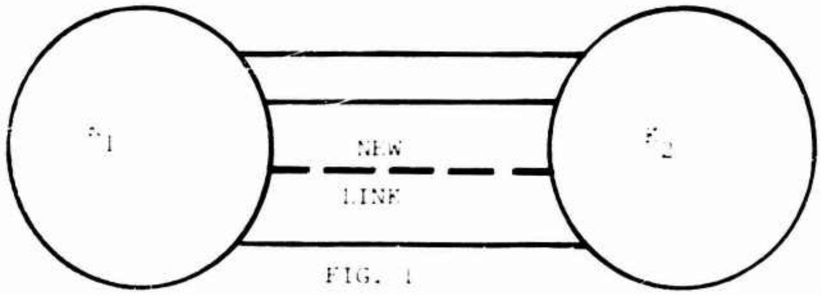


FIG. 1

Unfortunately, this modification tends to shift the cutset (see Figure 2) with little throughput improvement.

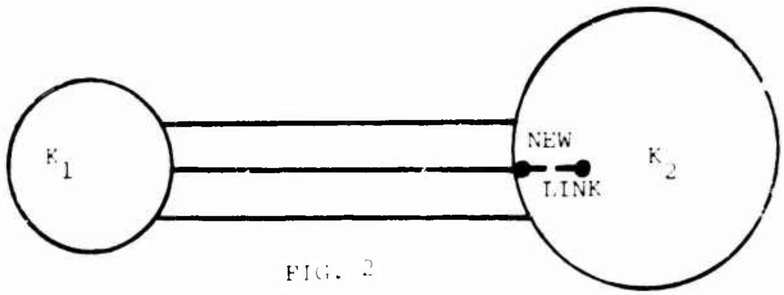


FIG. 2

This effect will be further discussed in a later section. A reasonable modification would be to connect the "centers of traffic" of the two components as shown in Figure 3.

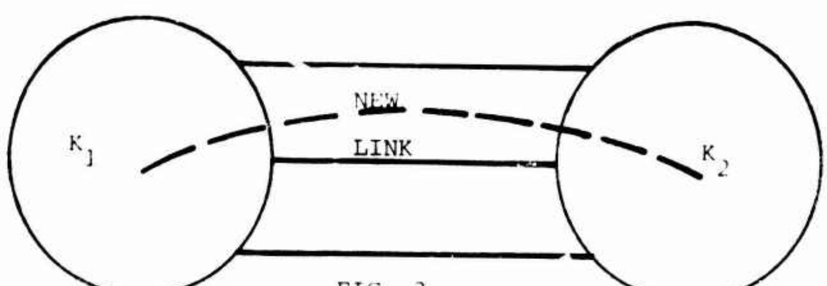


FIG. 3

Here the center of traffic is defined as the center of gravity of the component when each has node weight proportional to its traffic requirement. However, for networks of medium and large size, the centers of traffic are likely to be distant from the cutset, and thus, links joining them are costly.

A simple heuristic technique to obtain the center of traffic effect is called "Distance 2". This technique attempts to remove from the set of link insertion candidates the node pairs in which one of the nodes is close to the saturated cutset. The method retains nodes from each component that are at a distance of at least two links from nodes adjacent to the cutset. If this is not possible (because of small component size), nodes adjacent to (a unit distance of 1) the cutset nodes are retained. All nodes that do not satisfy this criterion are eliminated from the component (Figures 4 & 5).

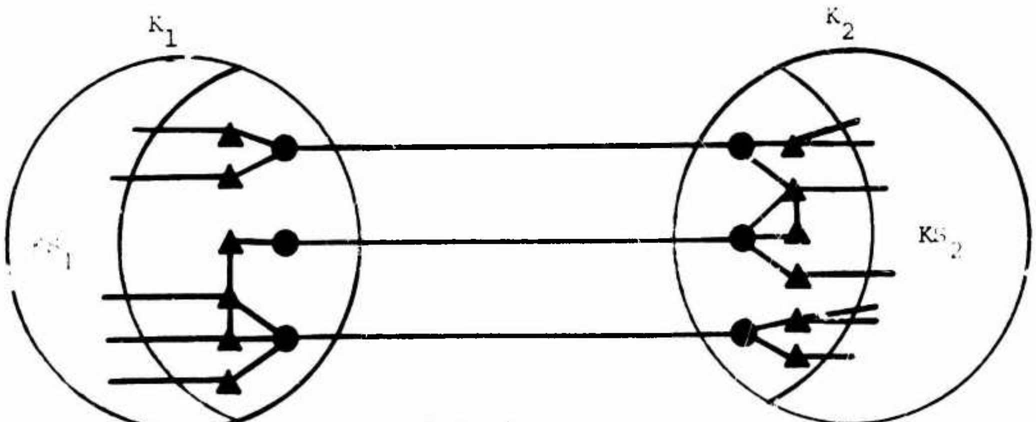


FIG. 4

where: ● = node that has a link in the cutset

▲ = node adjacent to ●

KS_i = revised component obtained from K_i consisting only of nodes that satisfy the distance 2 criterion

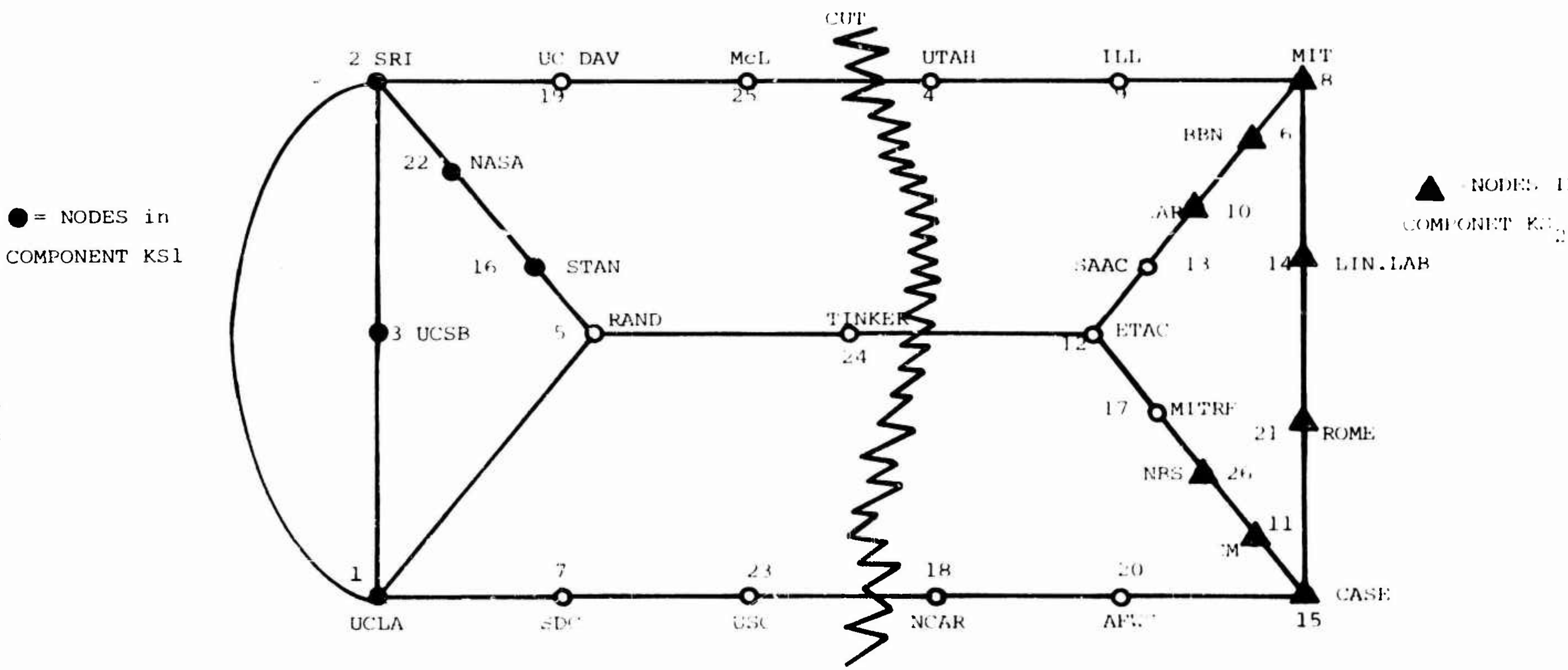


Figure 5 26 Nodes ARPA Network

Now, the only links considered for insertion are links, (x, y) such that:

$$x \varepsilon KS_1,\ y \varepsilon KS_2 .$$

The distance 2 criterion retains nodes that are at least 5 links distant from each other since

2	+ 2	+ 1	= 5
unit-link distance from cutset for KS_1	for KS_2	cutset link	

A specific 26-node example of the distance 2 criterion is given in Figure 5.

Another measure based on traffic considerations is called SDIS (I,J) which is an index of the saturation of the least saturated path, π(I,J) from node I to node J. This saturation of a path is defined as:

$$\text{SDIS}\ (I,J) \equiv \underset{\pi(I,J)}{\text{MIN for all}} \quad \sum_{\pi} \left(\frac{C_i}{(C_i - f_i)^2} \right)$$

where π(I,J) is any (I, J) path, and the sum is over all links in the particular path π(I,J). The values of all SDIS (I,J)'s are readily provided by the routing algorithm.

The insertion of a link between a node pair (I,J) with high SDIS (I,J) is likely to achieve good throughput improvement, because it provides an alternative to an already saturated path. Here again, as in the distance 2 criterion, the tradeoff between throughput improvement and link cost must be considered.

Several Add-Only optimizations for a 26-node, 30-link ARPANET topology were performed. In one, after ordering the potential links according to cost, at each iteration, the minimum cost links satisfying the distance 2 criterion were introduced.

In another after ordering the links according to the ratio cost/SDIS, the link minimizing the ratio was added. These results are shown in Figure 6 and further Add-Only results are presented in Section 4.2.

Surprisingly, the second run gave exactly the same results as the first one. Based on such findings, it was concluded that the two criteria are almost equivalent for the network sizes under consideration, and, therefore, the simpler distance 2 criterion was used for all further optimization.

3.2 THE DELETE-ONLY OPERATION

The Delete-Only operation begins with a highly connected topology and eliminates one link at each iteration, thus continuously reducing throughput and cost.

The criterion for link removal is based on link utilization and link cost. One approach is to remove the most expensive and underutilized link. That is, that link which when removed maximizes the quantity

$$E_i \triangleq (D_i \times \frac{(C_i - f_i)}{C_i}$$

is removed, where

D_i is the cost of link i; C_i is the capacity of link i; F_i is the flow in link i; and $\frac{C_i - f_i}{C_i}$ is the relative excess capacity.

If removal of this link results in the creation of a pendant or isolated node, the link is not removed. With this restriction, if the original network is 2-connected, the Delete

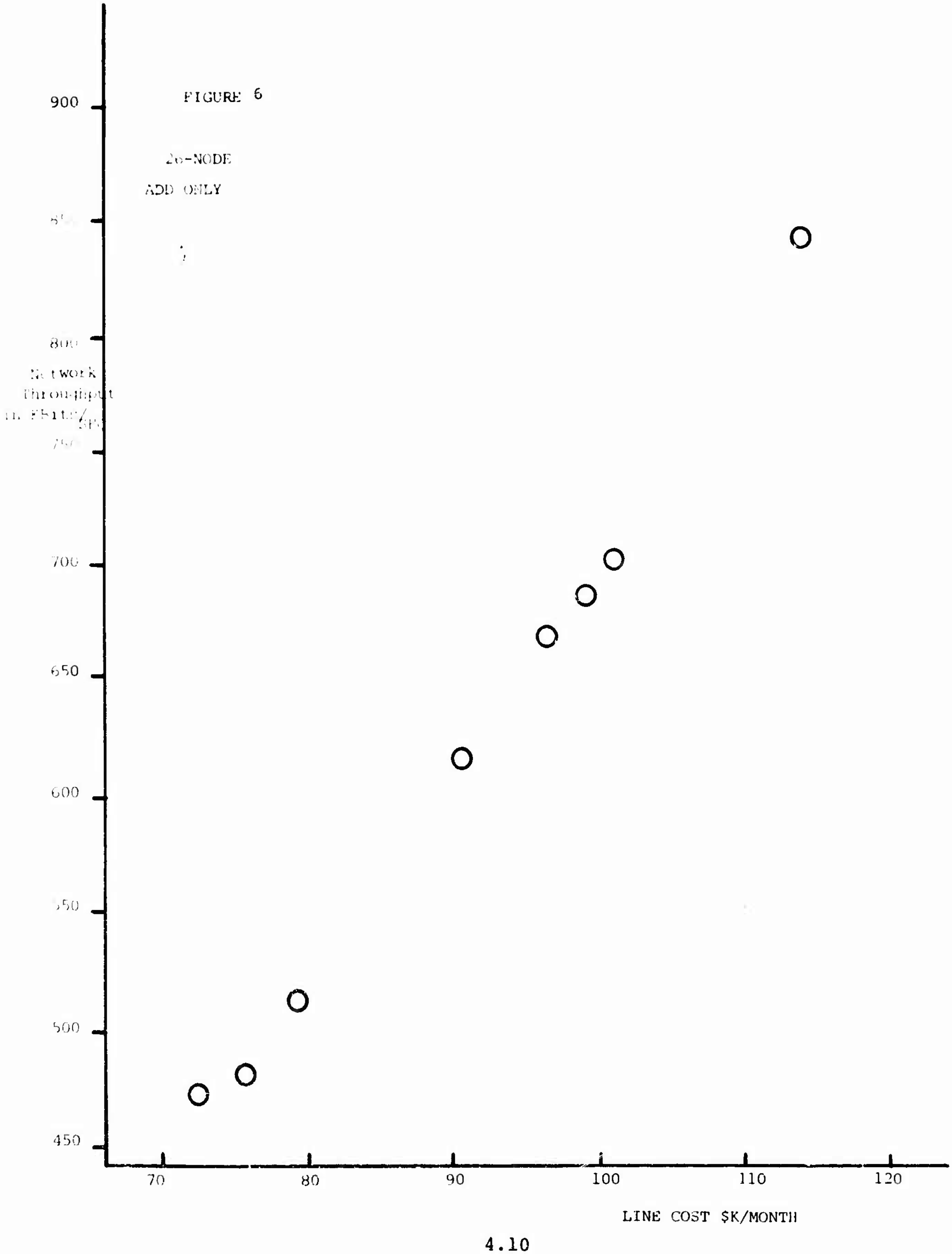


FIGURE 6
26-NODE
ADD ONLY
900
800
700
650
600
500
450
70
80
90
100
110
120
LINE COST $K/MONTH

Only method results in a 2-connected network; and if the network was connected, it remains connected. Finally, if a very high cost link in the saturated cutset leads to a maximum E, the link is not removed.

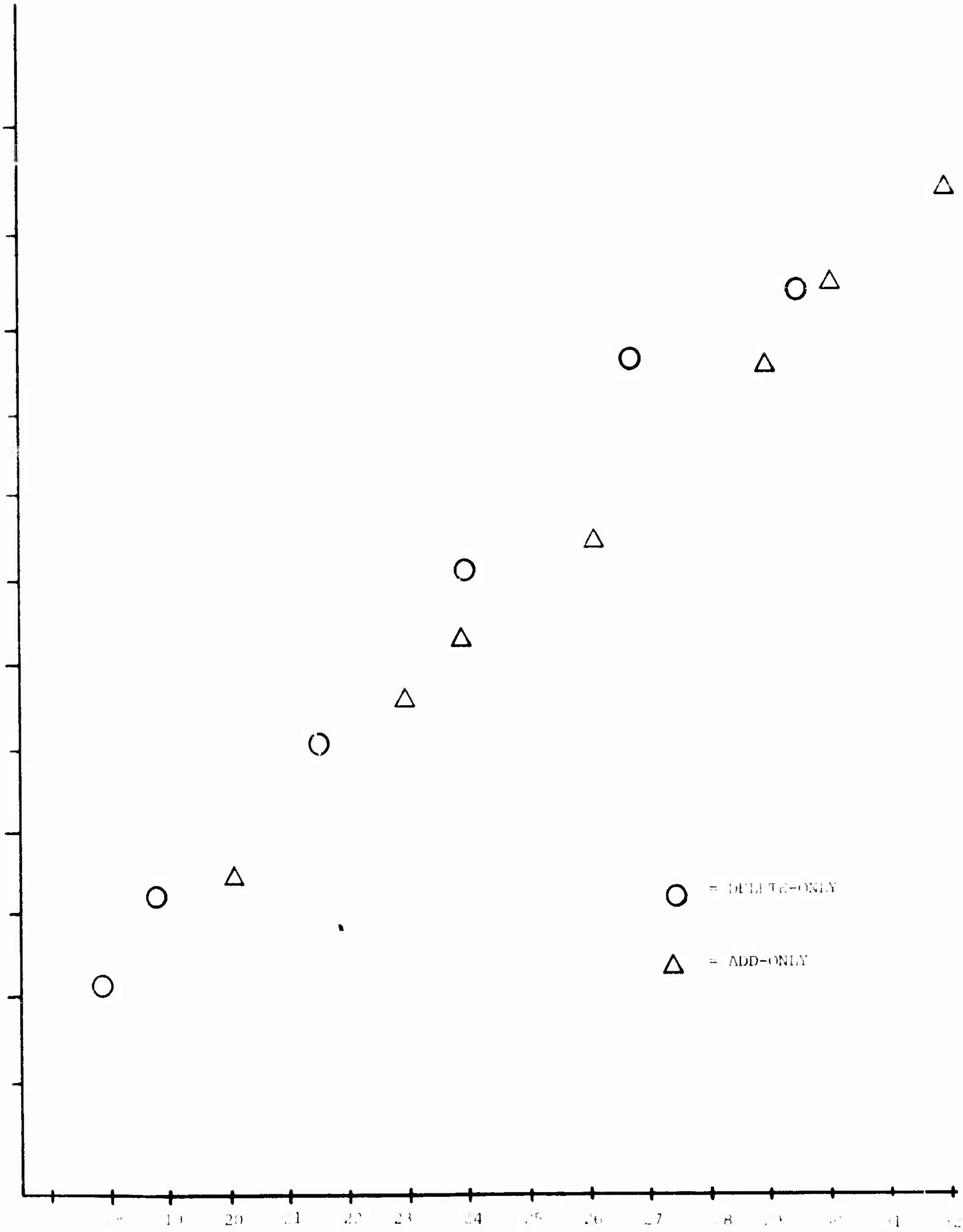
The results of Delete-Only and Add-Only experiments with 10 and 26 node networks are shown in Figures 7 and 8. Both delete runs were started from good, highly connected topologies. Considering that the two methods are conceptually very different, the fact that resulting cost-throughput solutions are in the same range leads to the conjecture that both methods are near optimal.

3.3 PERTURBATIONS

Once a configuration has achieved a required throughput, the designer would like to rearrange links so that throughput remains constant and cost is reduced. The branch exchange technique provides one such method.

The combination of the Add-Only and Delete-Only algorithms provides another technique which will be called Perturbation. Any exchange technique cannot be expected to keep throughput absolutely constant, and therefore, lower and upper bounds, within which throughput is allowed to vary for intermediate solutions (say $\pm$ 5% of the desired throughput), are specified.

The approach taken allows one link to be introduced according to the Add-Only criterion and one link to be removed according to the Delete-Only criterion. If throughput exceeds the upper bound, a Delete-Only is performed. Similarly, an Add-Only is performed when the throughput decreases beneath the lower bound. Therefore, starting with any link configuration, a desired throughput level is attained by either adding or removing links according to cost and cutset considerations. After reaching the desired level, links are rearranged so that cost is reduced and the desired throughput is retained.


= DELETE-ONLY
= ADD-ONLY
20
24

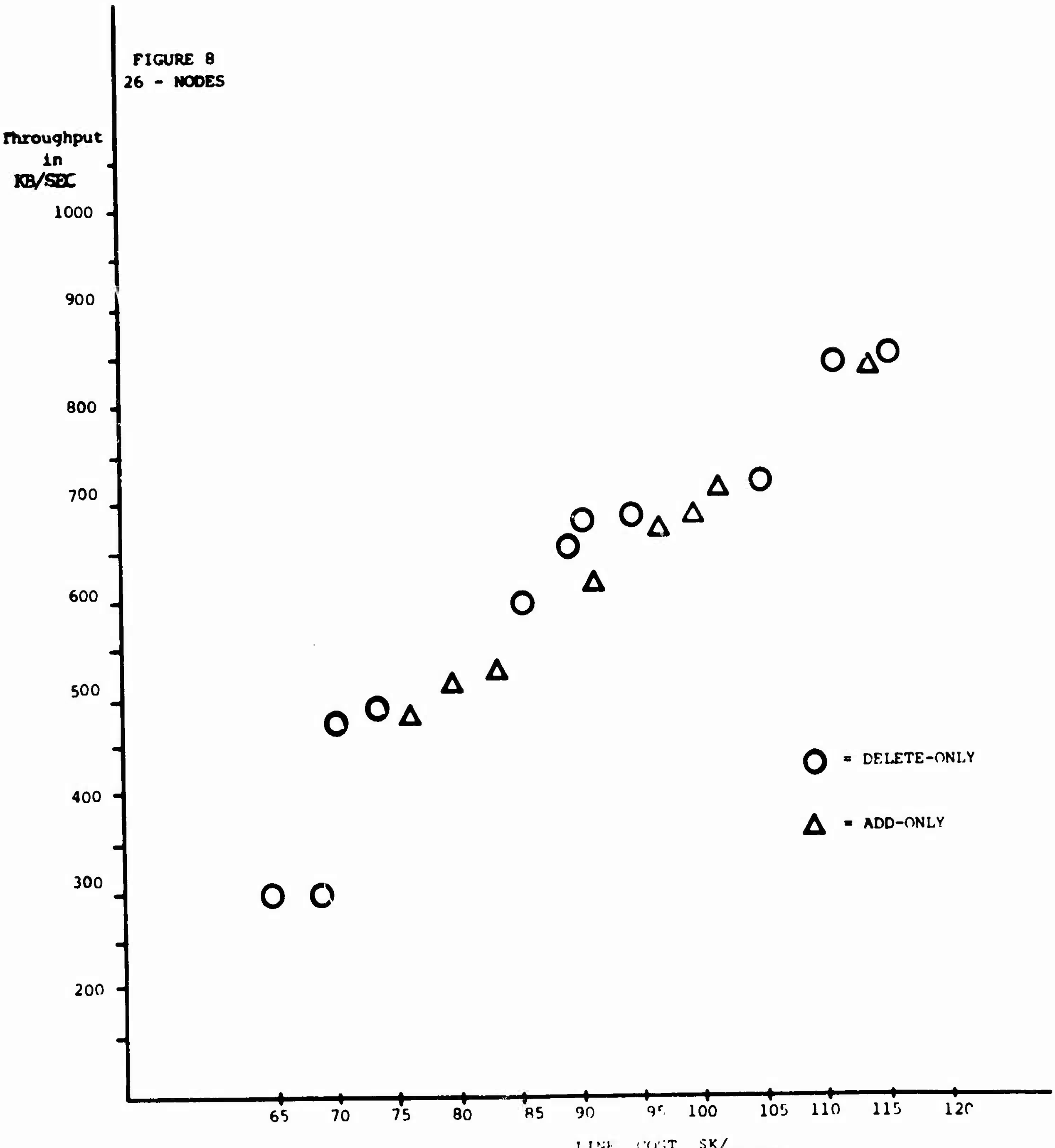


FIGURE 8
26 - NODES
Throughput in KB/SEC
1000
900
800
700
600
500
400
300
200
= DELETE-ONLY
= ADD-ONLY
65
70
75
80
85
90
95
100
105
110
115
120
LINE COST $K/MONTH

Also considered during perturbation, is the effect of <u>Domination</u>. (Network A dominates Network B, if $C_A \leq C_B$ and $R_A \geq R_B$, where C and R are cost and throughput, respectively). Each network, i, can be characterized by an ordered pair, P_i, (C_i, R_i). A list of the dominant points is kept to serve as a criterion for design modifications. In fact, during perturbation an intermediate network may be dominated by another previously generated network. In this case, the network is returned to the previous stage and the method is continued. A flow-chart for the perturbation method is given in Figure 9. An example of the application of the perturbation technique to a 26 node network is shown in Figure 10.

3.4 CHAINS COLLAPSING

Typical ARPANET configurations are at least 2-connected but much less than 3-connected. Consequently, there are many chains, with 4-5 serial nodes. The presence of chains can produce inefficiencies in the CS algorithm, as shown later in this section.

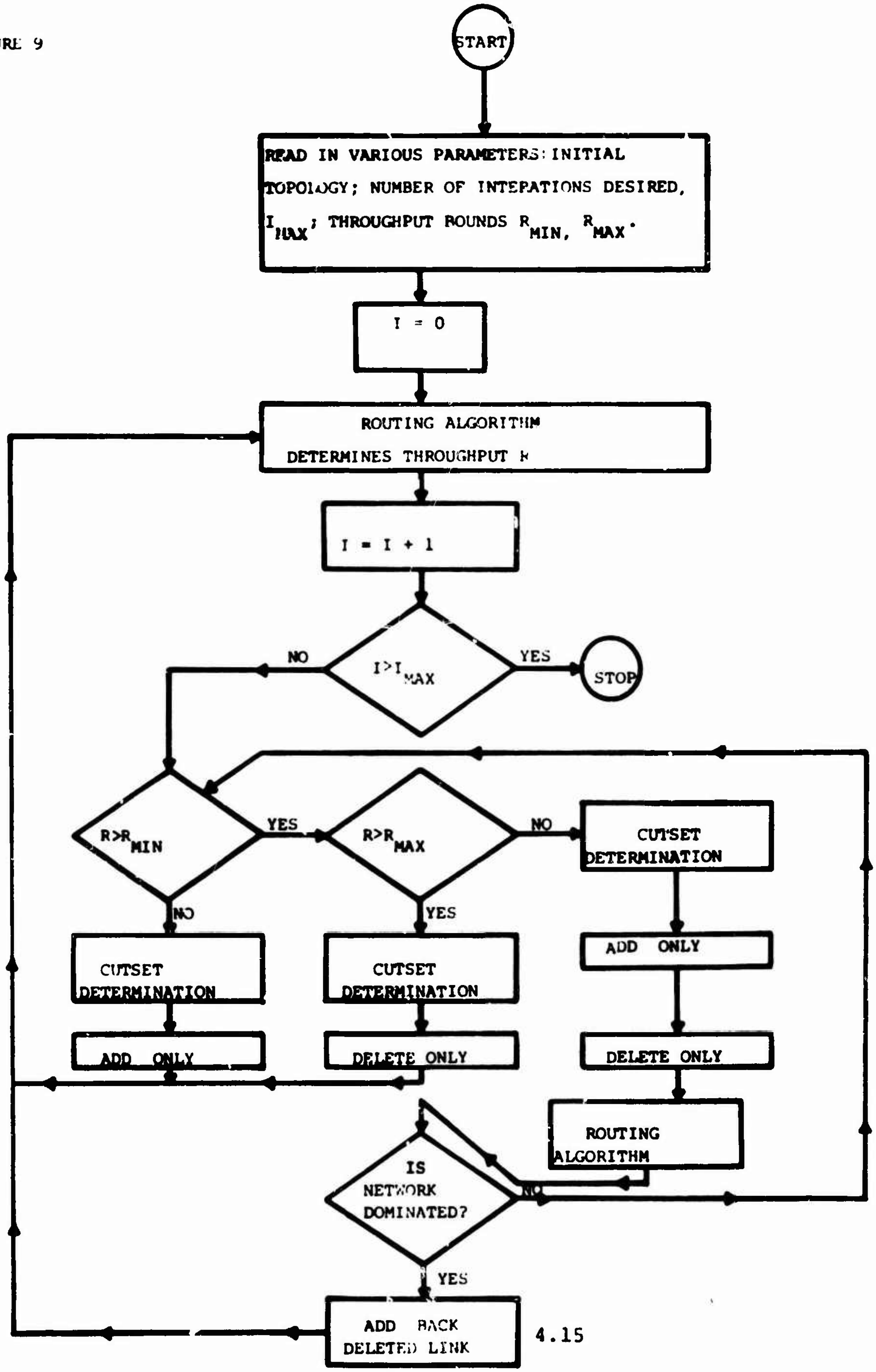
There are two types of traffic on a chain:

(1) <u>Internal Traffic</u>: corresponding to requirements from nodes internal to the chain to external nodes, and vice versa; and between internal nodes. This component of traffic varies from link to link along the chain.

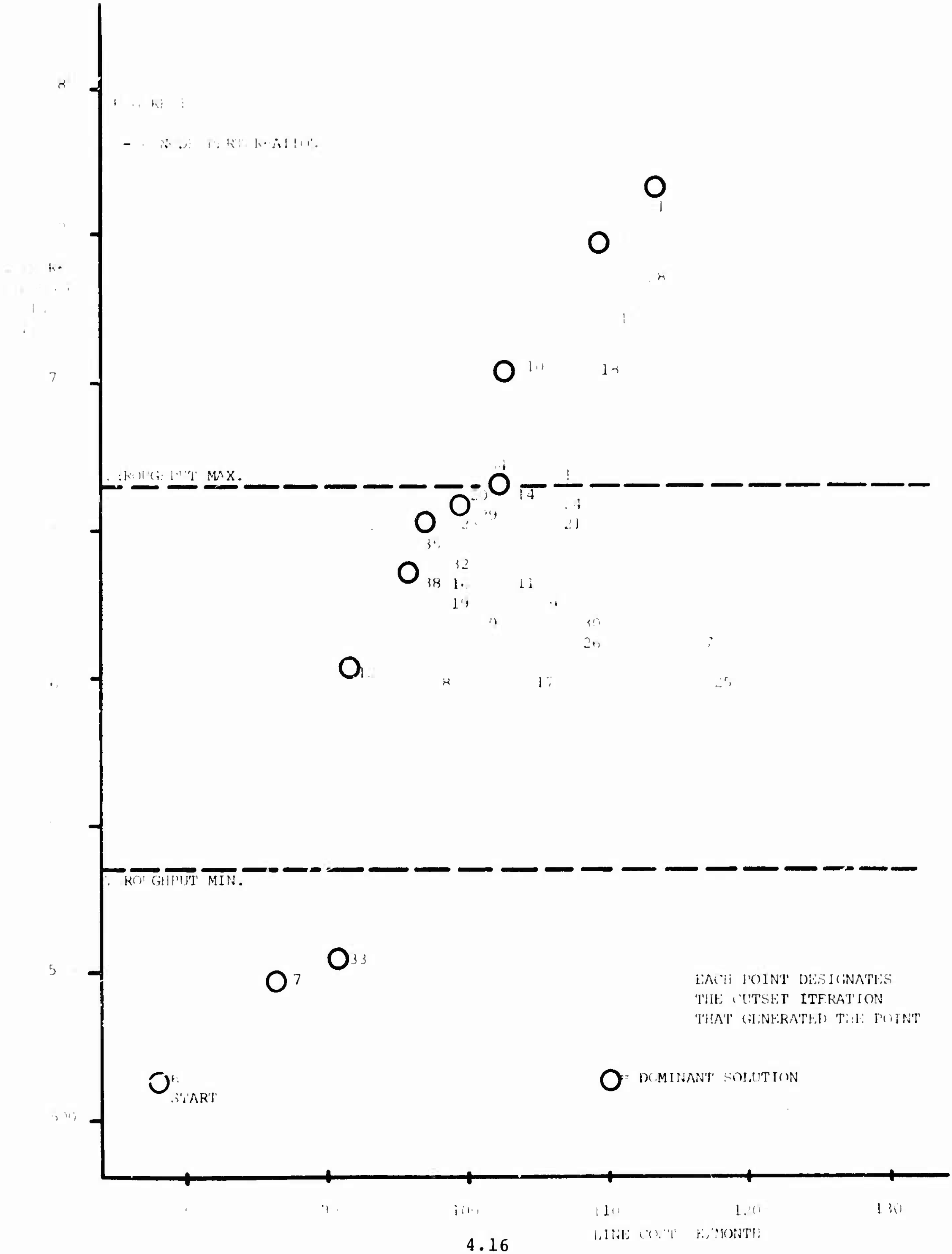
(2) <u>Transit Traffic</u>: corresponding to requirements between external nodes, which are routed along the chain. This traffic component is uniform along the chain.

The efficiency of the CS method is greatly enhanced if some of the chains are "collapsed" and replaced by a single equivalent link.

FIGURE 9


START
READ IN VARIOUS PARAMETERS: INITIAL TOPOLOGY; NUMBER OF INTERATIONS DESIRED, I_{MAX}; THROUGHPUT BOUNDS R_{MIN}, R_{MAX}.
I = 0
ROUTING ALGORITHM DETERMINES THROUGHPUT R
I = I + 1
$I > I_{MAX}$
NO
YES
STOP
$R > R_{MIN}$
YES
NO
$R > R_{MAX}$
NO
YES
CUTSET DETERMINATION
ADD ONLY
DELETE ONLY
CUTSET DETERMINATION
ADD ONLY
CUTSET DETERMINATION
DELETE ONLY
ROUTING ALGORITHM
IS NETWORK DOMINATED?
NO
YES
ADD BACK DELETED LINK

4.15



THROUGHPUT MAX.
ROUGHPUT MIN.
START
EACH POINT DESIGNATES
THE CUTSET ITERATION
THAT GENERATED THE POINT
= DOMINANT SOLUTION
LINE COST

In particular, chains with predominantly transit traffic should be collapsed.

Consider the network shown in Figure 11 with the saturated cutset indicated. Notice that each cutset link belongs to a different chain. It can be shown experimentally that the traffic on such chains is prevalently transit traffic. Therefore, all links in a chain carry the same amount of flow and are uniformly saturated. Suppose that the CS technique is applied. One of the two nodes between which a new link is inserted may be internal to the cutset chain (See Figure 12). The insertion then shifts the cut-set to some other link of the chain,with very little throughput improvement, (since all links were uniformly saturated). To avoid this, the algorithm is modifying to disregard saturated cut-set chain nodes during link insertion.

Consider the example in Figure 13. Since chain links tend to be uniformly saturated, the cutset shown in Figure 13 may be chosen by the CS method. This could happen, for example, if flows in the upper chain are slightly greater than those in the other chains. This selection is clearly improper, since little throughput improvement can be obtained by adding a link across this particular cutset. Again, to prevent such a situation, special attention must be given to chains during saturated cutset determination.

A remedy to the above situations is the chain collapsing technique, which replaces a chain by a single link with equivalent link flow equal to the maximum of all link flows in the chain (See Figure 14). This procedure is applied by the CS algorithm before cutset determination. After cutset determination and before link insertion, all collapsed chains are regenerated (i.e. the internal nodes are restored), except for the saturated cutset chains. For example, assuming that the saturated cutset in Figure 14 consists of the three intermediate chains, before link insertion, the network shown in Figure 15 is considered. The collapsing technique handles pathological situations, but there are cases when chains should not be collapsed. Typically, these are when internal traffic prevails over

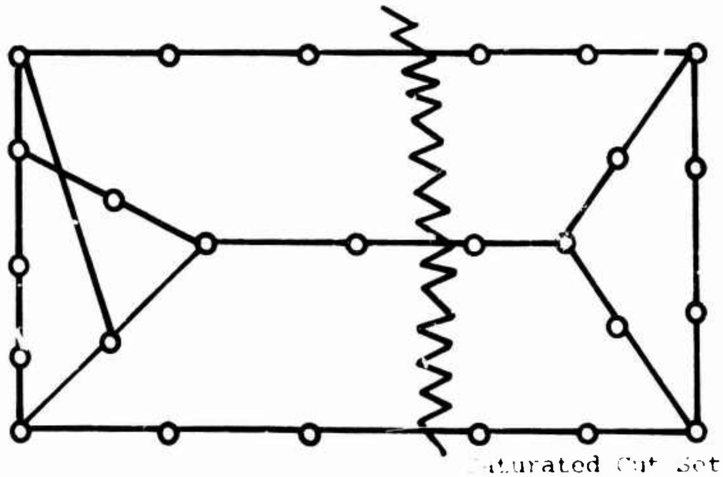
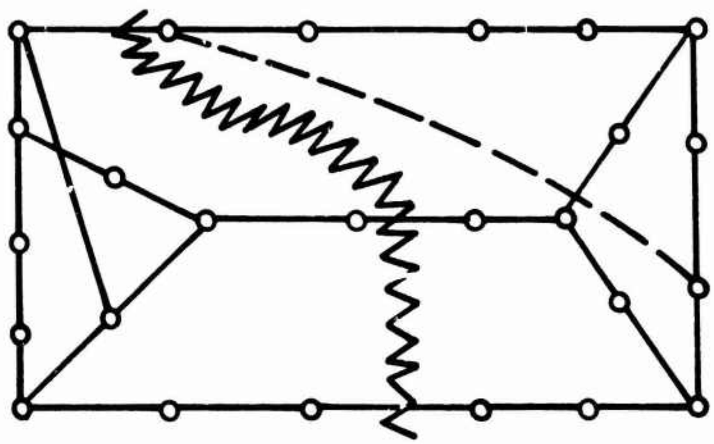


FIGURE 11



F

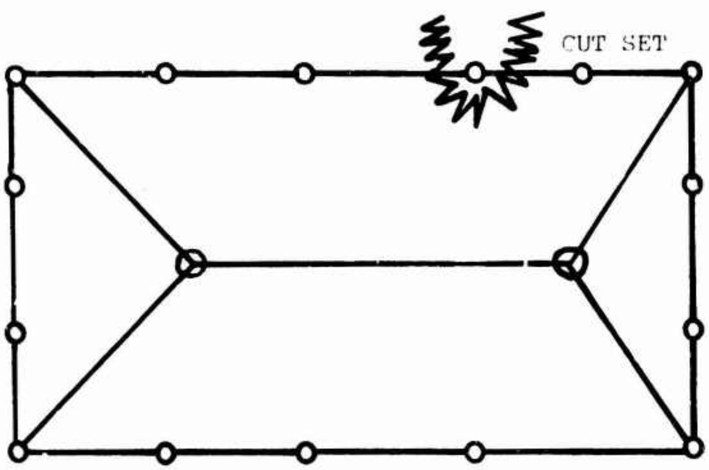


FIGURE 13

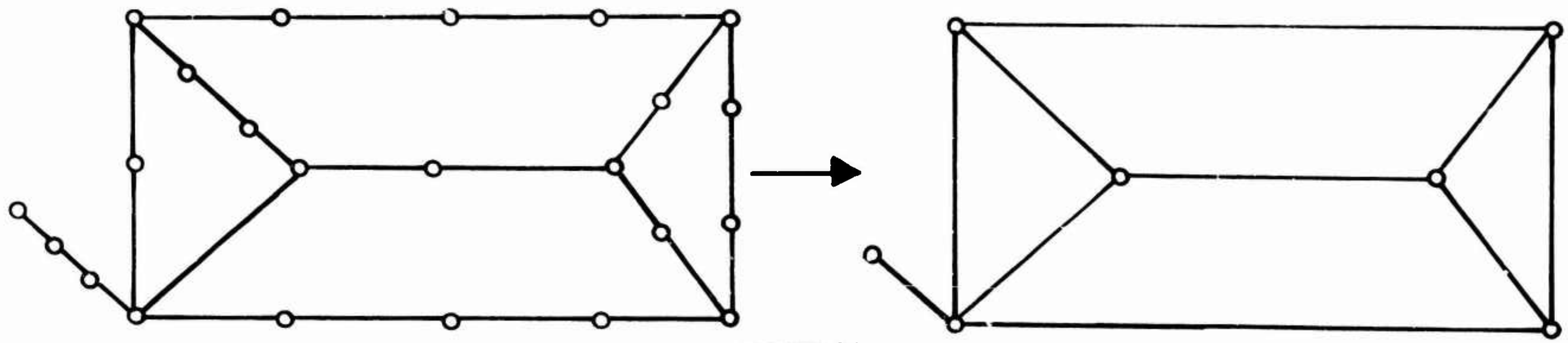
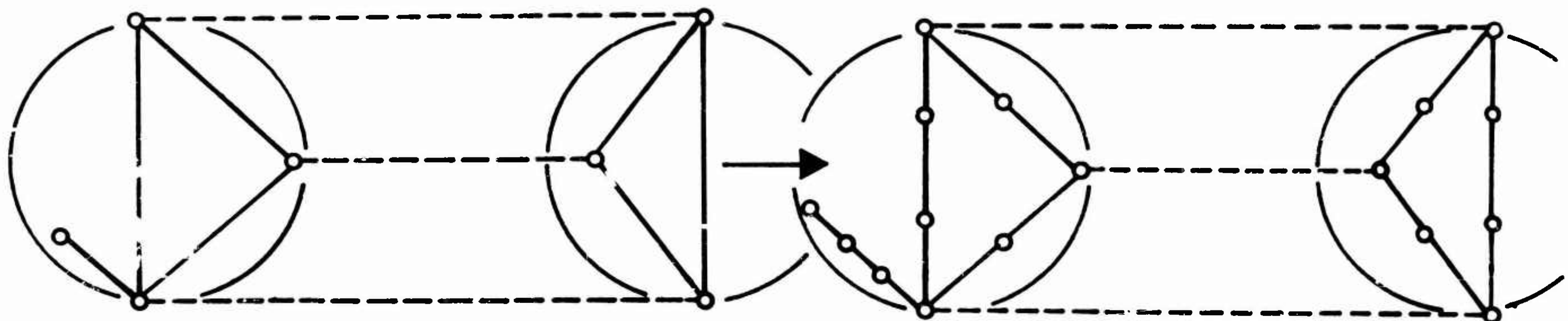


FIGURE 14



(a) (b)

FIGURE 15

transit traffic, and, therefore, link flow is not uniform along the chain. The following criterion, based on theoretical considerations and experimentation, is used to determine when a chain should be collapsed. A chain is <u>not collapsable</u> if: (1) the number of intermediate nodes in the chain is larger than NN/3, where NN is the total number of nodes; and (2) the ratio between maximum and minimum flow on the chain is larger than 1.5.

To demonstrate the impact of chain collapsing on the CS algorithm, consider the two 43 node ARPANET design examples shown in Figures 16 and 17. The first does not allow chain collapsing, while the second does.

no chain collapsing:

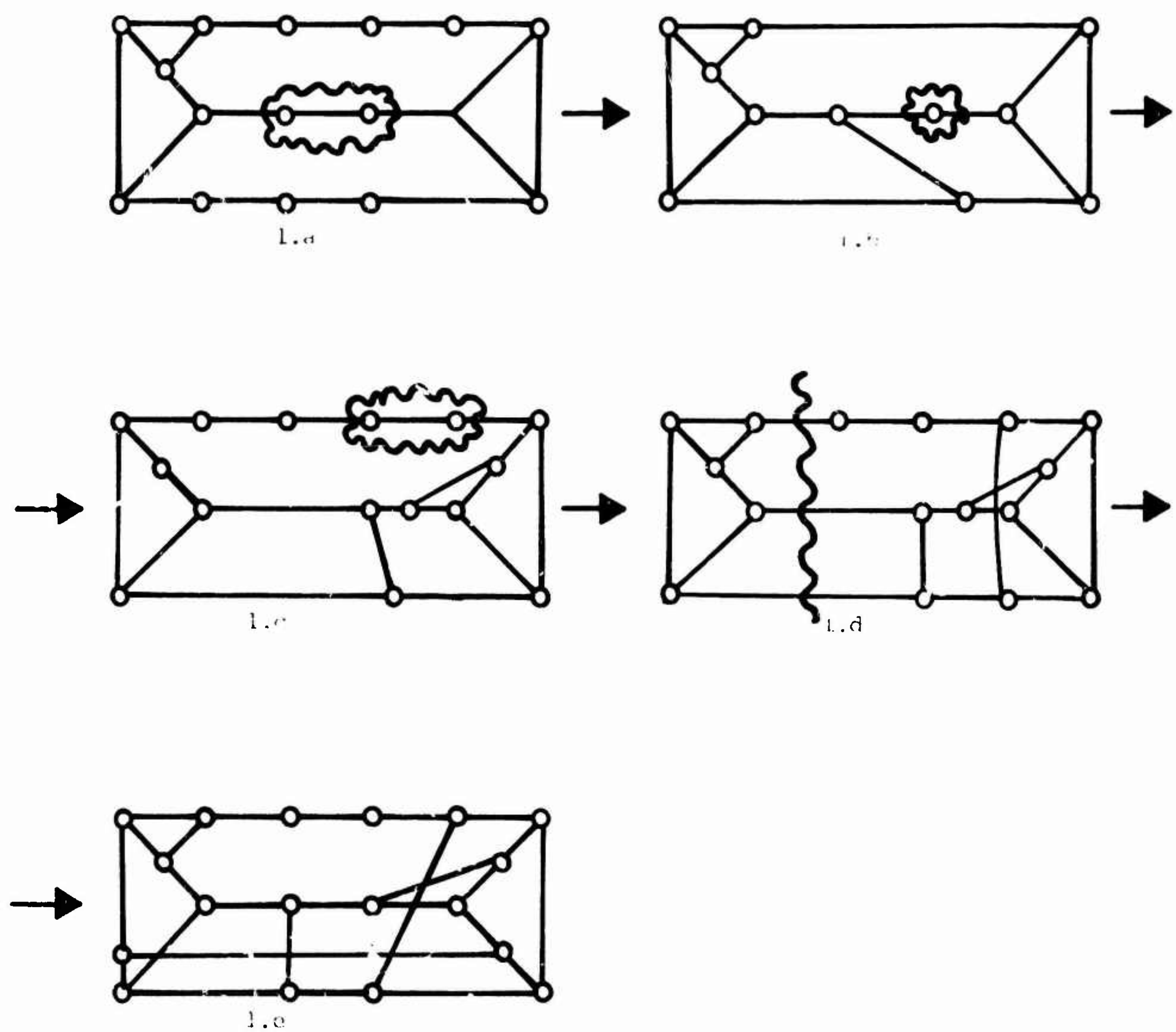
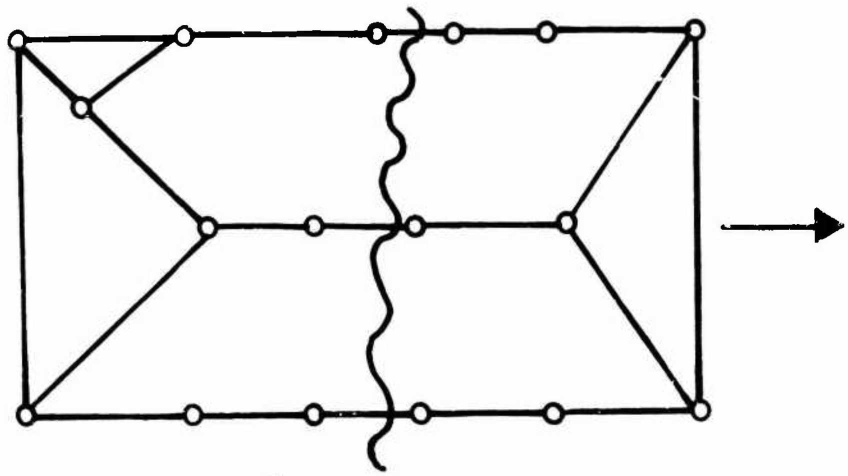
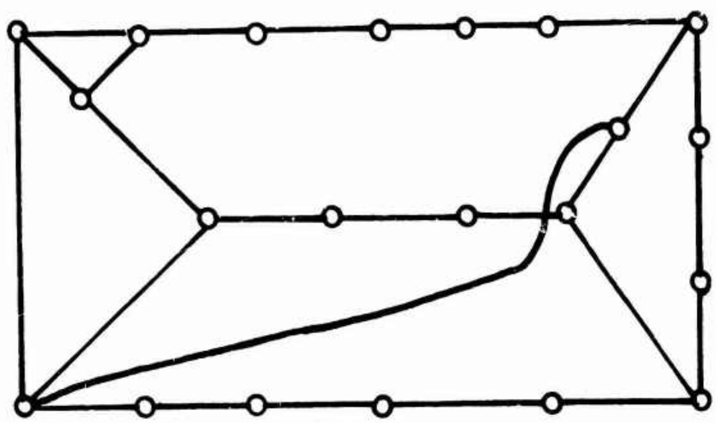


FIGURE 16

For the example of Figure 16, the saturated cutset is represented by the three cross country chains. However, because of the chain effect, the CS method is able to detect such a cutset only at step 1.d. The plot in Figure 18 shows cost and throughput for all the above steps; the largest improvement is obtained from 1.d to 1.e.



2.a



2.b

FIGURE 17

For the Figure 17 example, the proper cutset is detected at the first step, and significant throughput improvement is immediately achieved (See Figure 18). Notice from Figure 18 that the collapsing results are constantly better than the non-collapsing ones in the Add-Only phase, below the minimum throughput threshold. Above this threshold, branch deletions and additions are performed, and some of the bad initial choices of the non-collapsing algorithm are deleted. Therefore, the results of the two algorithms in that range are comparable.

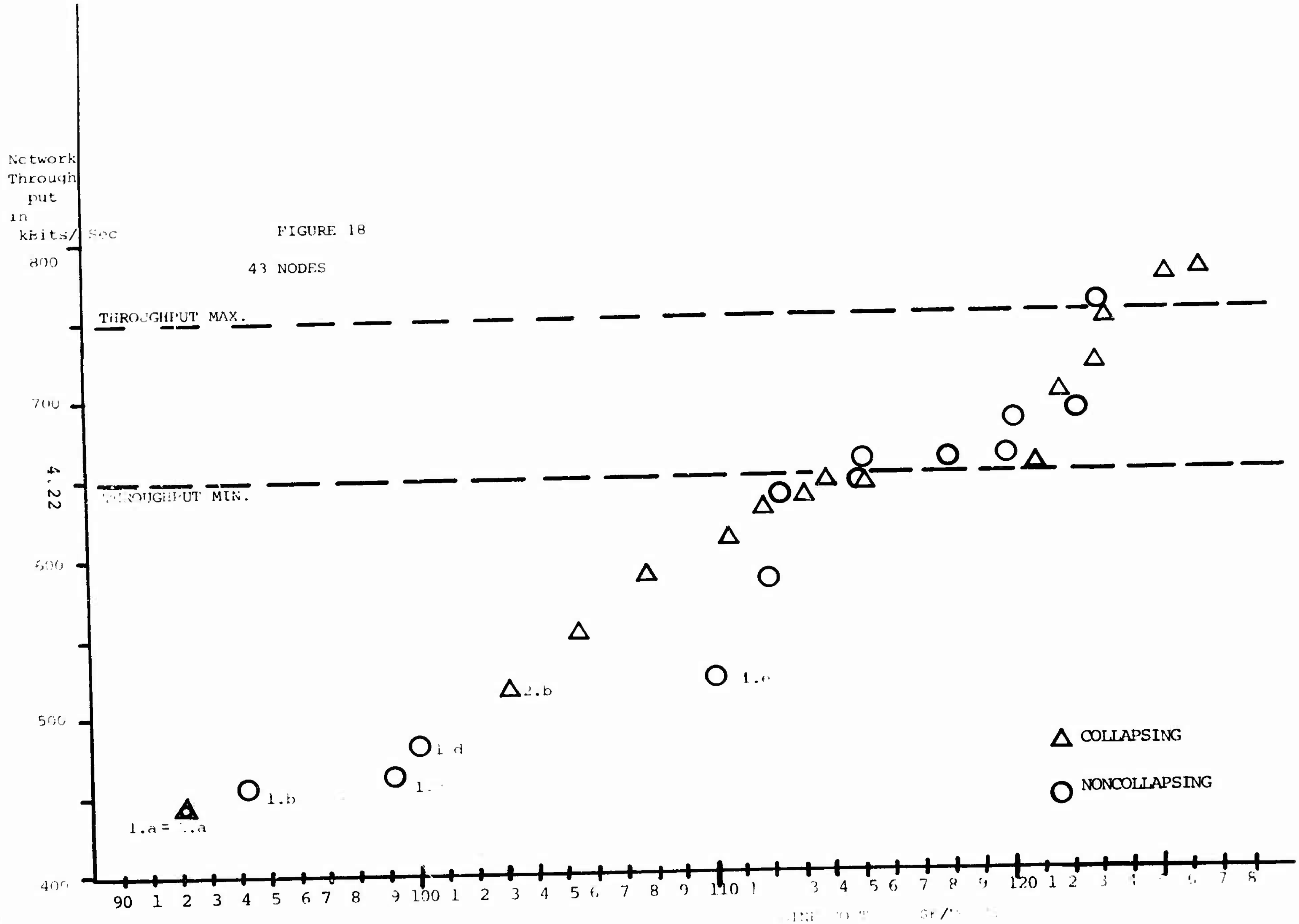


FIGURE 18
43 NODES
Network Through put in kBits/Sec
800
700
4.22
600
500
400
THROUGHPUT MAX.
THROUGHPUT MIN.
1.a = 2.a
1.b
1.d
2.b
1.e
COLLAPSING
NONCOLLAPSING
90 1 2 3 4 5 6 7 8 9 100 1 2 3 4 5 6 7 8 9 110 1 3 4 5 6 7 8 9 120 1 2 3 7 8

4. OTHER DESIGN CONSIDERATIONS

4.1 RELIABILITY

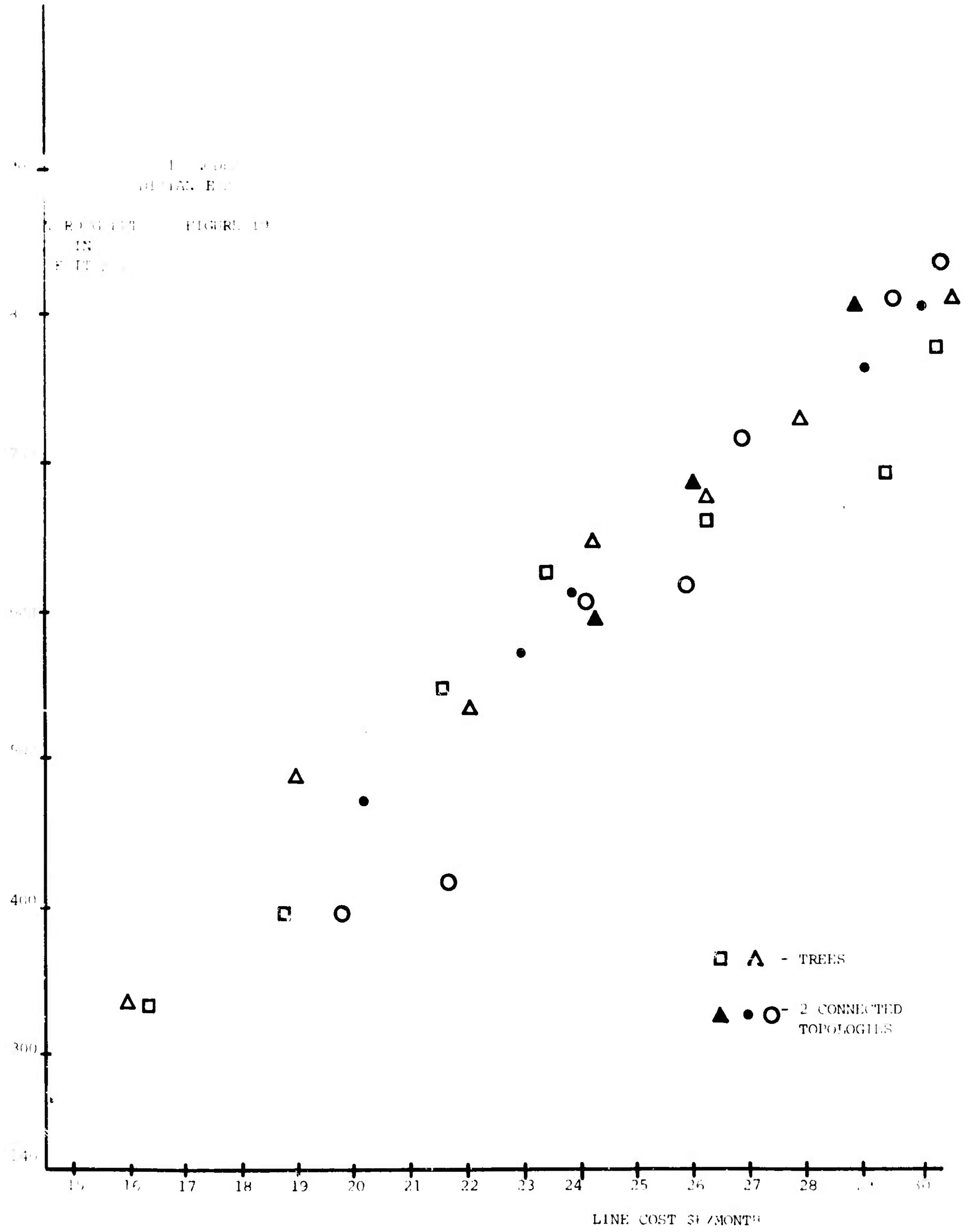
It has been shown in previous NAC reports that networks containing pendant nodes (nodes of degree 1) are not sufficiently reliable. No provision, however, is made in the CS algorithm to favor the connection of pendant nodes. Nevertheless, starting from network configurations of 10-26 nodes with many pendant nodes (trees), 2-connectivity is rapidly achieved.

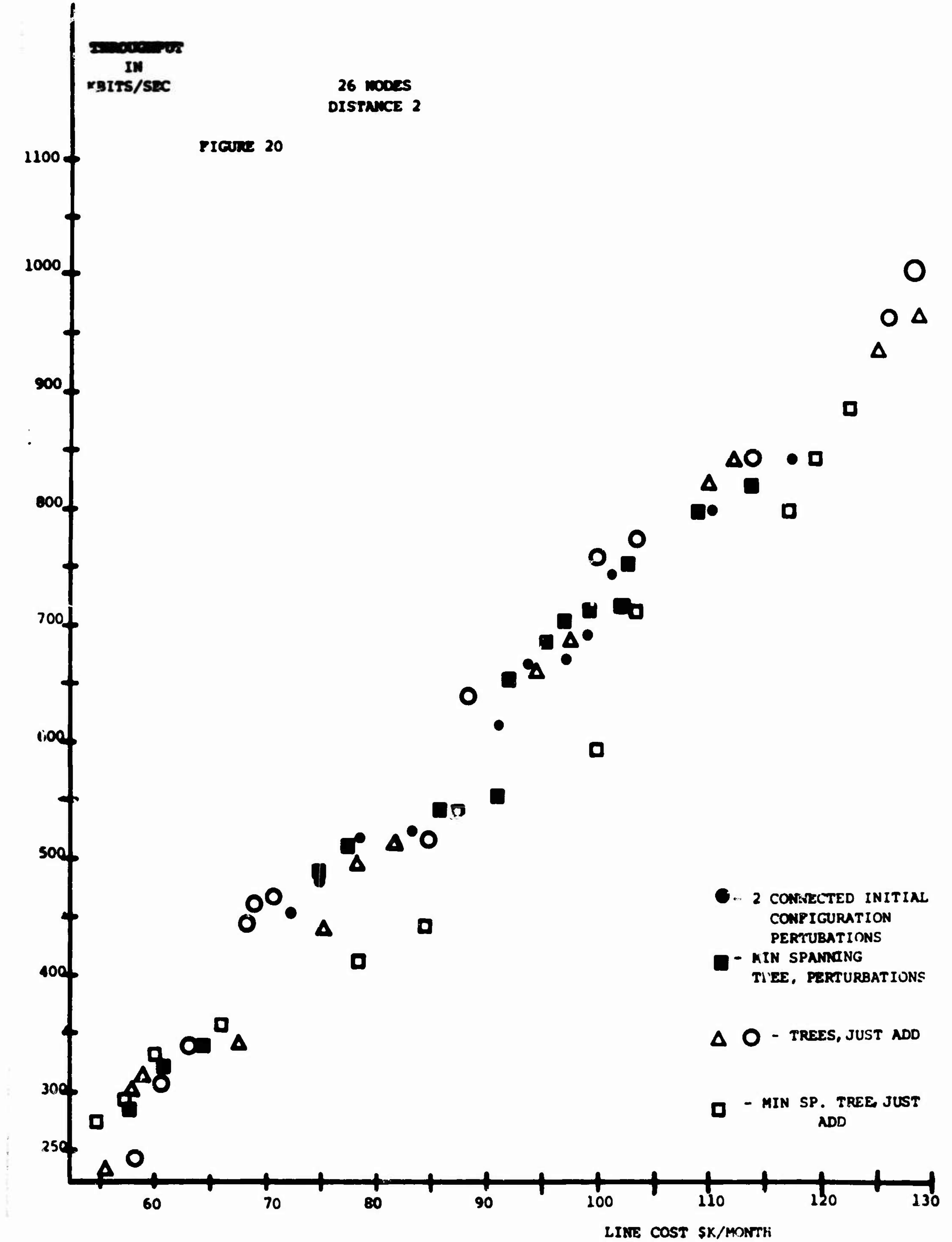
This can be attributed to the fact that links incident to pendant nodes become saturated early in the design algorithm. A pendant node link would constitute the cutset with the pendant node in one component and the rest of the network in the other component. The only alternative for the Add-Only algorithm, therefore, is to establish another connection between the pendant node and the rest of the network. Further study will focus on a better criterion to connect pendant nodes.

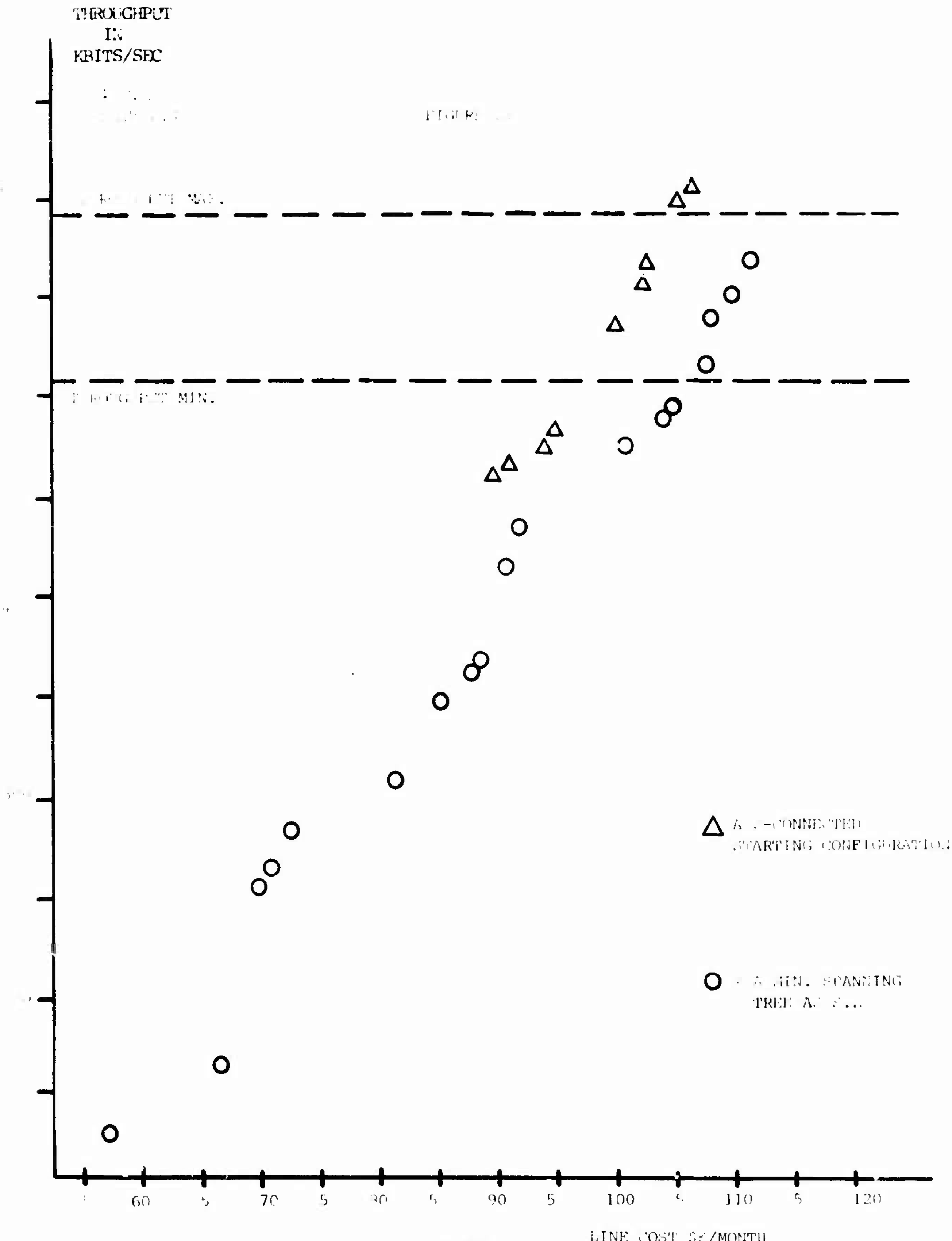
4.2 STARTING NETWORK CONFIGURATIONS

The input to the CS algorithm consists of various parameters (throughput thresholds, maximum number of iterations, time delay, tolerances, etc.). Of considerable importance is the initial network configuration. Several experiments using various starting topologies such as trees, mimimum spanning trees, and 2-connected topologies were performed. The results given in Figures 19, 20 and 21 show various solutions for 10, 26 and 40 node networks. The experiments for the 10 and 26 node case were run using the distance 2 criterion. The collapsing technique was used for the 40 node case.

The starting link configuation could be crucial to the performance of any design algorithms. The results, however, show that


400
300
17
18
19
20
21
22
23
24
25
27
28
LINE COST
TREES
2 CONNECTED TOPOLOGIES


THROUGHPUT IN KBITS/SEC
26 NODES
DISTANCE 2
FIGURE 20
1100
1000
900
800
700
600
500
400
300
250
60
70
80
90
100
110
120
130
LINE COST $K/MONTH
2 CONNECTED INITIAL CONFIGURATION PERTUBATIONS
MIN SPANNING TREE, PERTURBATIONS
- TREES, JUST ADD
- MIN SP. TREE, JUST ADD


THROUGHPUT
IN
KBITS/SEC
60
5
70
5
5
90
5
100
110
5
120
LINE COST

the CS algorithm is not sensitive to starting configuations, since cost-throughput curves for different initial topologies are close to each other. (The best starting topology seems to be a 2-connected one.)

Further sensitivity studies will be performed with other starting configuations such as star and loop topologies and fully-connected networks. The goal is to establish good standard initial topologies which depend on node location, reliability, etc., and to develop algorithms for the generation of such initial topologies to avoid the preliminary hand design.

5. COMPUTATIONAL REQUIREMENTS FOR THE CUT SATURATION DESIGN

5.1 EXECUTION TIME

The execution time per iteration (which includes routing and cutset modification) is dependent on the size of the network. In particular, the computational complexity, C_R of the flow deviation routing algorithm used is

$$C_R = \alpha(NN')^3 + \beta(NA)^2 \qquad (4)$$

where NN' = number of nodes with degree ≥ 3
NA = number of links; and α and β are proper coefficients which depend on code optimization and accuracy required. Faster routing algorithms can be used but this does not appear necessary at the present time.

The complexity C_s of the cutset modification algorithms is approximately

$$C_s \simeq \gamma\ (NA)^2 \qquad (5)$$

where γ is a proper coefficient which depends on code optimization.

In most practical applications, $NN' \ll$ total number of nodes. For sufficiently large networks, it appears that $\alpha\,(NN')^3 \ll (NA)^2$, and, therefore, the total complexity is $C \simeq \delta(NA)^2$ (6)

where $\delta = \beta + \gamma$.

Per-iteration running times on a CDC 6600 are presented in tables 1 - 3. Equation (6) can be verified from the results in those tables. Letting C be the running time, we evaluate δ as follows:

$$\delta \simeq \frac{C}{(NA)^2} .$$

FOR 10 NODES:

20 links: $\delta = \frac{.8}{(20)^2} = .0020.$

FOR 26 NODES:

30 links: $\delta = \frac{1.9}{(30)^2} = .0021$

39 links: $\delta = \frac{2.6}{(39)^2} = .0016$

FOR 40 NODES:

54 links: $\delta = \frac{4.4}{(54)^2} = .0015$

61 links: $\delta = \frac{6.2}{(61)^2} = .0017$

The relatively small range of fluctuation of the coefficient justifies the assumption in equation (6).

TABLE 1: 10 NODES

# Of Links	Routing	Cutset	Total
15	.3	.2	.5
16	.4	.3	.7
17	.4	.2	.6
18	.4	.3	.7
19	.5	.3	.8
20	.5	.3	.8

(In Seconds)

TABLE 2: 26 NODES

# Of Links	Routing	Cutset	Total
27	1.2	.5	1.7
30	1.2	.7	1.9
33	1.4	.5	1.9
36	1.5	.7	2.2
39	1.8	.8	2.6

(In Seconds)

TABLE 3: 40 NODES

# Of Links	Routing	Cutset	Total
52	3.1	1.1	4.2
54	3.2	1.2	4.4
56	3.3	1.3	4.6
58	3.6	1.4	5.0
61	4.7	1.5	6.2

(In Seconds)

5.2 CORE REQUIREMENTS

The amount of core M needed for the design algorithm corresponds to the sum of various node and link arrays and is given by:

$$M \approx 10(NN)^2 + 3(NA)^2 .$$

In addition, the code requires 10K words of core. On the CDC 6600, the algorithm accomodates a 60 node, 80 link network in 80 K words of core.

5.3 LARGE NETS

Computational and core requirements clearly set a limit to the size of the networks that can be solved directly with the present algorithm. Partitioning techniques are necessary for networks of 100 or more nodes.

6. COMPARISON BETWEEN THE CS ALGORITHM AND OTHER EXISTING METHODS

6.1 INTRODUCTION

To evaluate the efficiency of the CS algorithm, CS results were compared to: (1) Branch Exchange (BXC) algorithm results; (2) theoretical lower bounds; and (3) manual network design. The results show that the CS algorithm is more efficient than any other method and that it is indeed near-optimal.

6.2 COMPARISON WITH BXC ALGORITHM

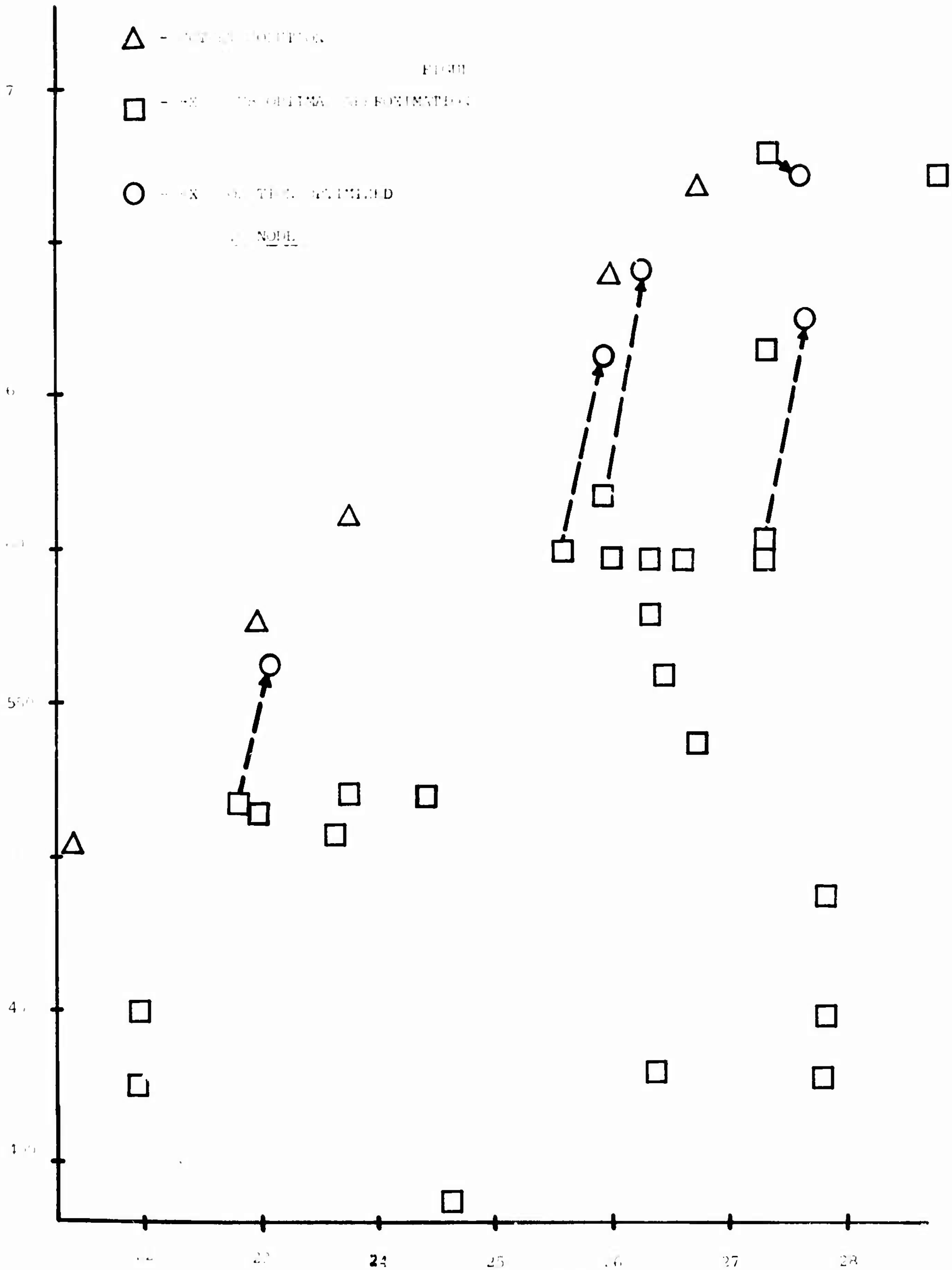
The BXC Algorithm is an iterative network design technique. At each iteration, links are added, deleted, or exchanged, and corresponding cost and throughput variations are computed. If the tradeoff between the cost/throughput variation is favorable, the topological modification (addition, deletion, or branch exchange) is accepted; otherwise it is refused. The procedure is exhaustive and terminates when no more improvement is possible.

The major difference between BXC and CS is that CS considers only those link insertions and deletions that are likely to yield a favorable cost/throughput tradeoff; therefore, the CS method can be expected to converge much more quickly to good solutions. Another drawback of the BXC algorithm is the inaccuracy of the throughput computation after each branch exchange: in fact, in order to cut down the computation time, the BXC algorithm applies a suboptimal routing technique, with throughput results 5% to 20% below optimum. This inaccuracy can be misleading in the search for improved network configurations and requires that, at the end of the BXC algorithm, a large number of candidate solutions be re-evaluated with an optimal routing algorithm.

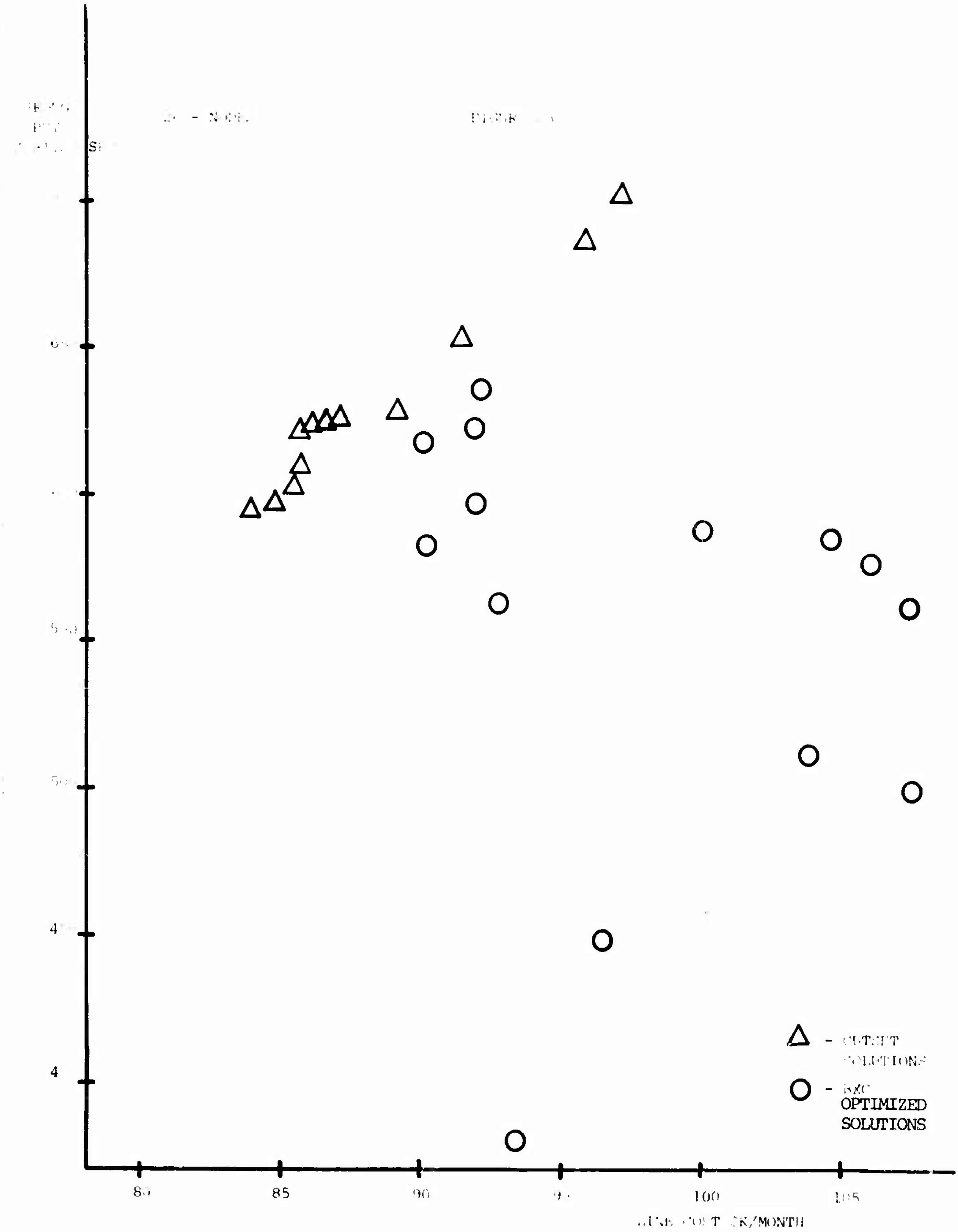
Three networks, with 10, 26, and 40 nodes, respectively, were designed using BXC, and the results are compared to CS results in Figures 22, 23, and 24. In no case was a BXC design better than a CS design. In addition, the BXC is much more time consuming than CS: for the 40 node example, the 11 BXC solutions required 450 seconds on a CDC 6600, while the 24 CS solutions required 118 seconds.

6.3 LOWER BOUNDS

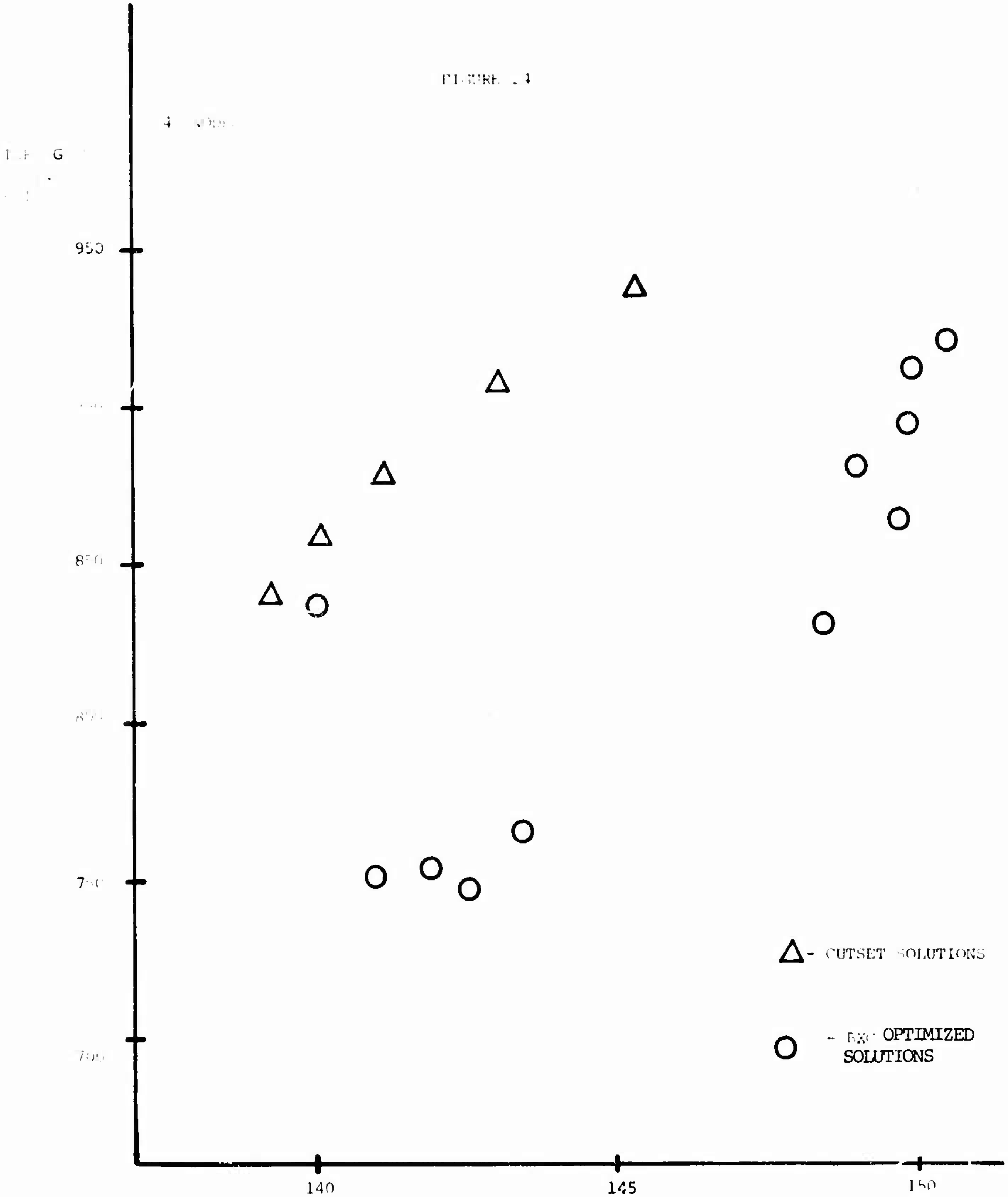
Lower bounds on the cost of the optimal solution can be obtained by approximating the link cost capacity functions with concave lower envelopes, and solving the associated concave multicommodity flow problem with the aid of mathematical programming techniques [Gerla, 1973]. In the specific case treated, the link cost function is a step function and is approximated with a properly defined concave curve as shown in Figure 25. Clearly, the minimum cost of the continuous, concave design problem is a lower bound to the cost of the real problem, because the concave link costs are always no greater than real link costs. Notice (Figures 26 and 27) that most CS solutions are within 5% to 10% of the lower bound.



4.32



4
85
90
100
OPTIMIZED
SOLUTIONS



950
850
750
140
145
150
LINE COST $K/MONTH
Δ- CUTSET SOLUTIONS
OPTIMIZED SOLUTIONS

Considering that: (1) the bound is not very tight, because concave costs are much lower than real costs; and (2) the throughput of the concave solutions is exact, while the throughput of the CS solution is within 2% to 5% of optimum; the strong conclusion is that the CS solutions are very near optimal.

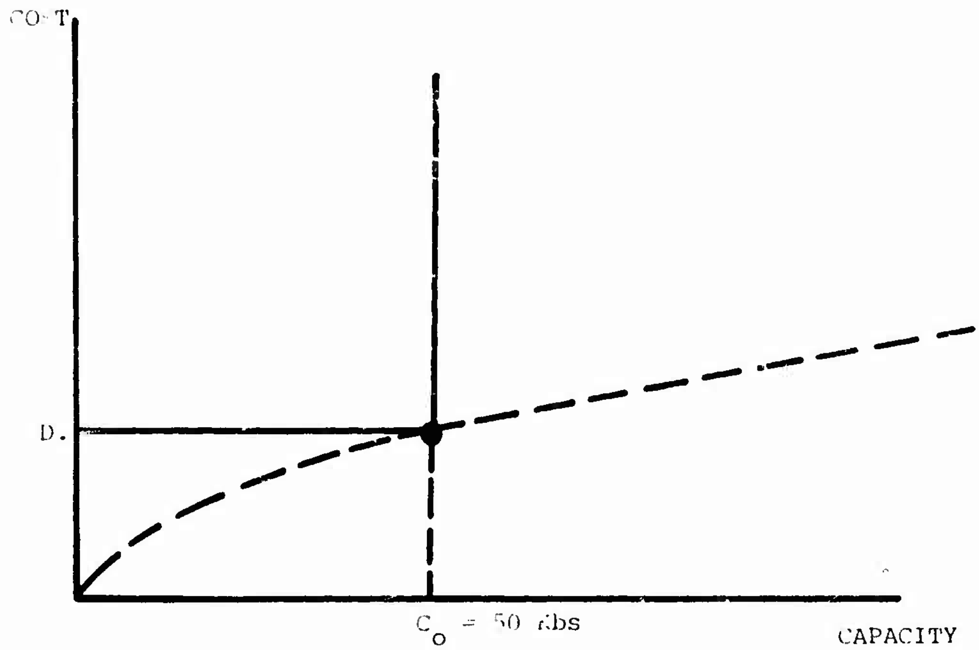


FIGURE 25

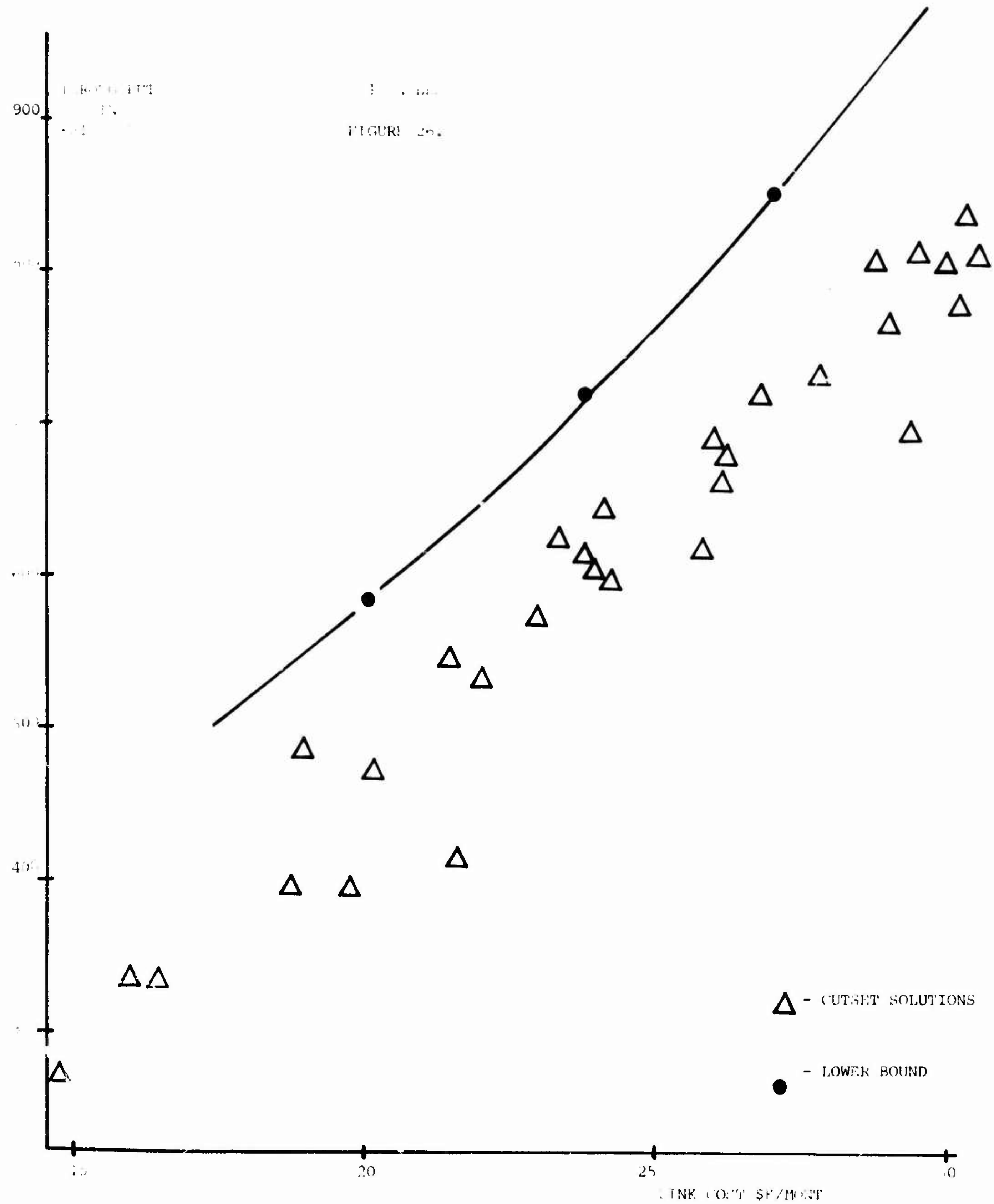


FIGURE
900
20
25
CUTSET SOLUTIONS
LOWER BOUND

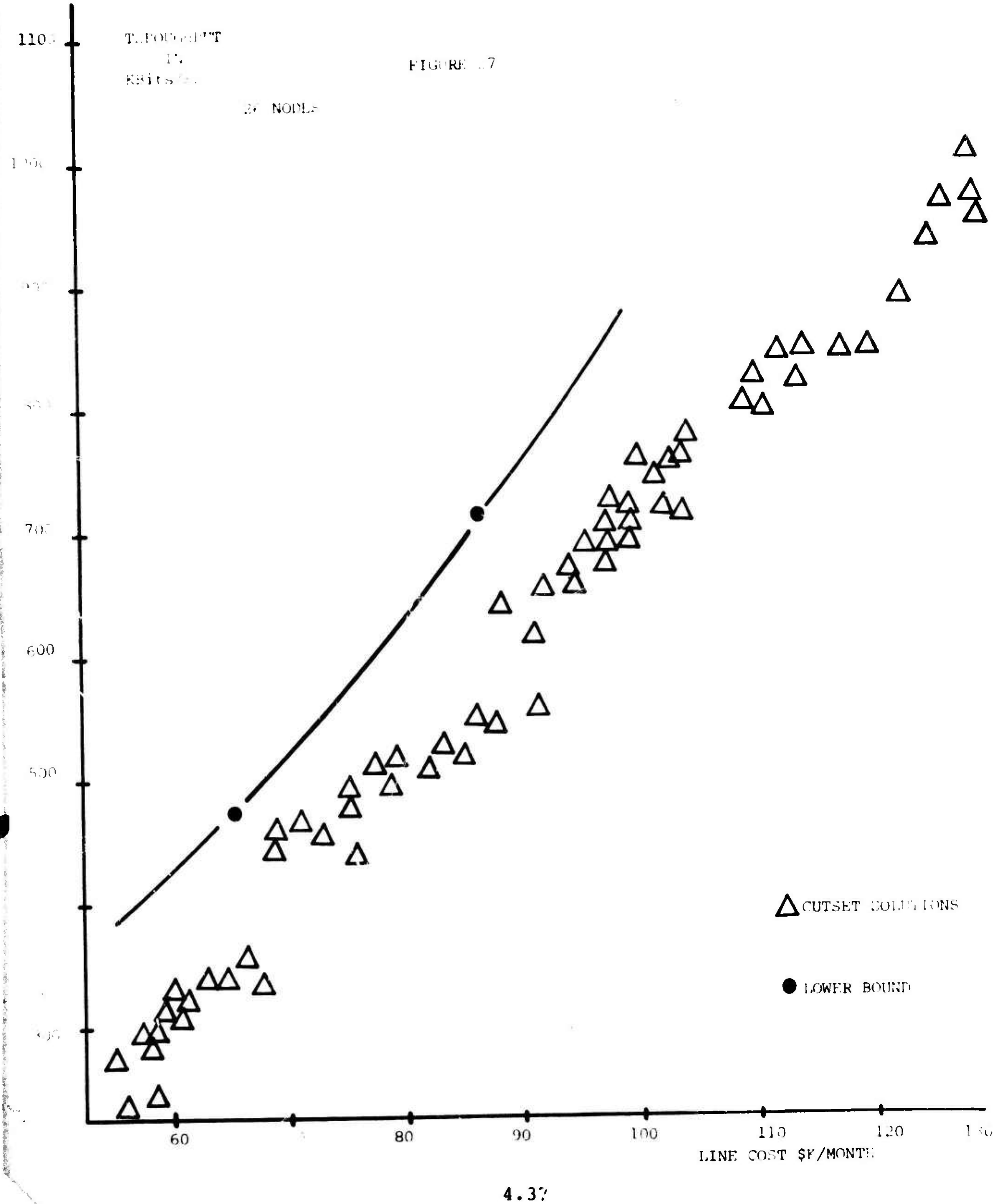


FIGURE .7
26 NODES
60
80
90
100
110
120
LINE COST $K/MONTH
CUTSET SOLUTIONS
LOWER BOUND
600

6.4 MANUAL DESIGN

In an attempt to improve CS solutions, topological modifications are often performed manually using, in addition to the link cost and link saturation information available to the CS method, human intuition. In most cases, however, generated solutions had either poorer performance than previous CS solutions, or were previously obtained CS solutions. A conclusion, therefore, is that manual interaction is not required for good cost-throughput results. Interaction is still necessary, however, at least at this stage of CS algorithm development, to deal with reliability issues that cannot be treated fully automatically (e.g. break long chains, etc.).

7. CONCLUSION AND FUTURE RESEARCH

The cut saturation algorithm described is a novel method for the topological design of distributed communication networks. A comparative analysis of this algorithm with respect to the Branch-Exchange Algorithm (another well known technique for distributed network design) shows that the former gives better results and is computationally more efficient than the latter. Furthermore, the comparison of CS solutions to theoretical lower bounds shows that the CS Algotithm is near optimal.

Although preliminary CS results were already very successful, there is ample space for further research to improve the present algorithm and extend its range of applications. In particular, required are: techniques for generating good starting topologies; considering a larger class of criteria for link insertion and deletion (which might include some measure of network reliability), performing more than one link addition and/or deletion per iteration; and providing interactive access to the design program via graphic terminals. In addition, the present CS algorithm will be extended so that it can be applied to problems with several levels of channel capacity and very large problems that require decomposition techniques.

CHAPTER 5

COST COMPUTATION FOR NEW LINE TARIFFS AND SERVICES

1. INTRODUCTION

With present line tariffs and network sizes, the cost of a line (or circuit) in any data or computer communications network depends mainly on the direct distance between the two end points of the line. Such is the case in the ARPANET. Thus far, it has been very simple to determine line costs. However, as new line tariffs are introduced, as new types of transmission services are made available, and as computer networks grow larger, the line cost calculation will no longer be straightforward.

With some of the proposed new tariffs, such as AT&T's proposed Hi/Lo density tariff, the line cost between a pair of locations depends on a set of parameters which may not relate to distance. Under the proposed new services, mainly digital transmission and domestic satellite communications, the user will be able to reduce cost by configuring networks in special ways. Finally, as a network's size grows, advantage can be made of line volume/discount in its own right. (At present, some networks achieve discounted rates on lines because they are part of much larger networks.)

The immediate impact of these tariffs is that new computational techniques must be developed to calculate line cost, to evaluate and compare different tariffs and services, and to configure least cost circuit routes.

On the surface, it may appear that each tariff requires a special cost optimization technique. However, many tariffs can be placed in a generalized cost structure that may be handled by the same optimization process. On the other hand, a design problem may involve more than one type of line cost structure. A general design goal is to allow the network designer to subdivide, on the basis of line tariffs, the various line cost problems into a small number of classes, each class corresponding to a different cost structure. For each cost structure, a different computational technique, general enough to handle all problems and tariffs corresponding to that cost structure, is developed. The global design program is obtained by combining the proper cost structure. For example, the line cost optimization for a 2-level satellite network would require 2 steps: one corresponding to the satellite cost structure, and one corresponding to the cost structure of the terrestrial subnetwork.

The purpose of this section is to classify possible cost structures and to propose outlines for future studies in developing computational techniques. The emphasis is on the application of domestic satellite communications to large computer networks.

2. NEW LINE TARIFFS AND THEIR IMPACT

There are many new tariffs being proposed by AT&T and various common carriers and domestic satellite companies. The following three tariffs are typical.

2.1 AT&T's PROPOSED HI/LO DENSITY TARIFF

Approximately 37 rate zones are defined to be high density locations, and the remaining are low density ones. The cost per channel-mile for a line connecting two high density locations is less than one third of the cost between a low density location and any other location. A low-to-low or low-to-high circuit can be implemented either by direct connection or by routing through high density locations.

Since most nodes in a computer network are likely to be located in or near populated areas, which usually are high density locations, the network's line cost will be lower under this proposed tariff. For nodes located in low density rate zones, the network designer must decide how to economically route circuits originating from these areas.

2.2 AT&T's PROPOSED DIGITAL DATA SERVICE (DDS)

This new data service, being developed by AT&T, is based on the T1 digital carrier network. Better quality and considerable economy can be obtained by transmitting data on the T1 rather than on traditional analog channels. Initially, DDS will be offered between 24 major cities, and will be extended to most of the 370 high density locations,mentioned before,at a later date. Different channel bandwidths can be leased at the following rates.

Bandwidth (Kbs)	Mileage Charge($/Mile x Mo.)	($/Mo.) Service Terminal
2.4	.45	140
4.8	.60	200
9.6	.90	280
56.0	4.50	500

From a location where DDP is not available, a customer can access the DDS network via a private analog channel of proper bandwidth and characteristics (series 3000, 5000, or 8000) and with adequate modems.

The cost for DDS's 56 KBPS line is less than that of a 50 KBPS line, even at the Government Telpak rates. Thus, the availability of DDS will have an immediate cost impact on the ARPANET. The greater impact will occur when T1 or T2 carriers are available for public service. Then, if a computer network is large enough, T1 carriers can be used to concentrate traffic, to channel high traffic volume, or to time-division-multiplex (TDM) a T1 carrier into sub-channels.

2.3 DOMESTIC SATELLITE SERVICE

Several companies (WU, Amersat, CML, RCA, GTE, AT&T) have been granted FCC approval to sell private satellite communication services in the U.S.A. Most satellite carriers will provide, in addition to the satellite channel, a terrestrial backbone network to facilitate satellite access and to improve overall reliability. Although most satellite tariffs are not yet definitive, it is anticipated that the total line cost will be given by the sum of the satellite and terrestrial cost components. In particular, the satellite segment (from antenna to antenna) will be much less expensive than a coast to coast terrestrial channel (e.g., a full duplex 56 KBS channel on the satellite will cost ≈ 500 $/Mo.). Different rates will apply, depending upon whether the customer provides his own ground stations, arranges for terrestrial access to the company's ground stations, or finally, uses the company's terrestrial network. Two general characteristics of satellite rates are: (1) rates not dependent on distance only, and (2) strong volume discount with respect to satellite bandwidth used.

3. LINE COST MODELS

In the leasing of communication facilities, the user is faced with a variety of alternatives differing in cost, quality of transmission, delay, etc. To achieve a minimum cost network design, all such alternatives must be carefully considered. It is practically impossible to develop computer programs for network design which would take into account all the available commercial offerings. The best approach is to classify such offerings into a limited number of very general cost structures and to develop efficient algorithms for each structure. Specific problems can be solved by properly varying the input parameters of each algorithm.

Four classes of cost structures are identified: distance dependent (DID) structures; location dependent (LOD) structures; volume discount (VOD) structures; and hierarchical structures. A description of the four classes follows.

(A) DID Structures

The cost per channel from point A to point B is a function of distance (A,B) only. It is independent of the specific locations of A & B, and of the number of channels (or bandwidths) from A to B. For practical purposes, circuits in the ARPANET can be estimated in this manner, even though the ARPANET's circuits are mostly routed through existing governmental Telpak circuits. Other examples of DID structure are the type 3000 tariff and the type 8000 tariff. If we restrict A and B to belonging to a privileged set of points, the Hi/Lo density tariff (where A and B are high density points) can also be considered as a DID structure.

B. LOD Structures

The cost per channel from A to B depends on the specific locations of A and B. It is independent, however, of the number of channels (or bandwidths) from A to B (no volume discount). A typical example of a LOD structure is the Hi/Lo density tariff. When either A or B or both are low density points, the rate depends not only on distance (A,B) but also on the geographical position of A and B with respect to high density points. In problems where only one channel of a given capacity must be allocated between A and B (and therefore, a volume discount does not apply), the DDS tariff, and in general all specialized and satellite carrier tariffs used in conjunction with AT&T tariff, can be considered as LOD structures. In fact, the cost of the channel will depend on the relative position of points A and B from the DDS network and on the special carrier network or terrestrial backbone network of the satellite company. In essence, this is a shortest path problem. Finding a least cost circuit between a node pair is equivalent to finding a shortest in cost path that connects the two. This path may contain other intermediate nodes if necessary.

C. VOD Structures

The cost of leasing an additional channel (or additional bandwidth) from A to B decreases with the volume of channels (or bandwidths), or channel-miles already leased from A to B. Furthermore, the cost depends on the distance between A and B (but does not depend on their specific locations). Examples of VOD structures are: the Telpak tariffs (where A and B can be any locations in the U. S. A.); the DDS tariff (where A and B belong to the DDS network); and the specialized carriers and satellite companies (where A and B belong to the respective networks). There is no exact method for solving optimization problems with such cost structures. A basic heuristic approach is to iteratively compute shortest routes, according to appropriate link costs (which change from iteration to iteration) and redistribute (or "deviate") requirements on such routes. The effect of such deviations is to achieve better economy of scale and therefore reduce cost.

Without going into detail, we simply mention that the efficiency of the VOD algorithms depends strongly on the nature of the cost-capacity functions of the links. In particular, link functions which are rather irregular, "neither concave nor convex," and with large capacity jumps (like the Telpak case shown in Figure 1) are in general difficult to handle during network design. On the other hand, link functions with small capacity jumps and which can be reasonably approximated by a concave curve (see Figure 2) lead to quite efficient algorithms.

Although optimal topology will depend on many factors (economy of scale, throughput level, node geographical locations, requirements, etc.), it is possible to anticipate that cost structures with strong economies of scale lead to tree topologies, while structures with mild economy of scale lead to highly connected topologies.

D. Hierarchical Structures (Partitioning and Node Location Problems)

Network partitioning consists of dividing the nodes into subsets and solving a separate design problem for each subset. This operation also requires the solution of a location problem,

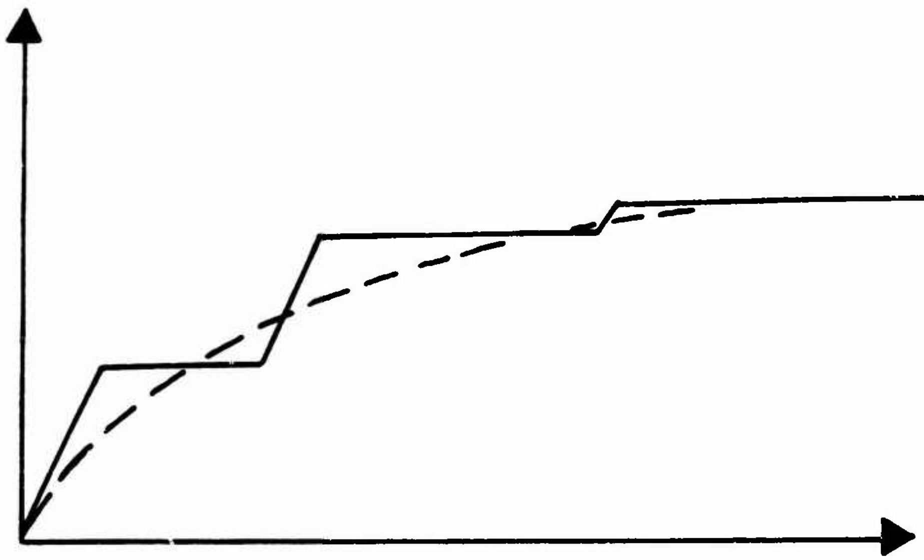


FIGURE 1

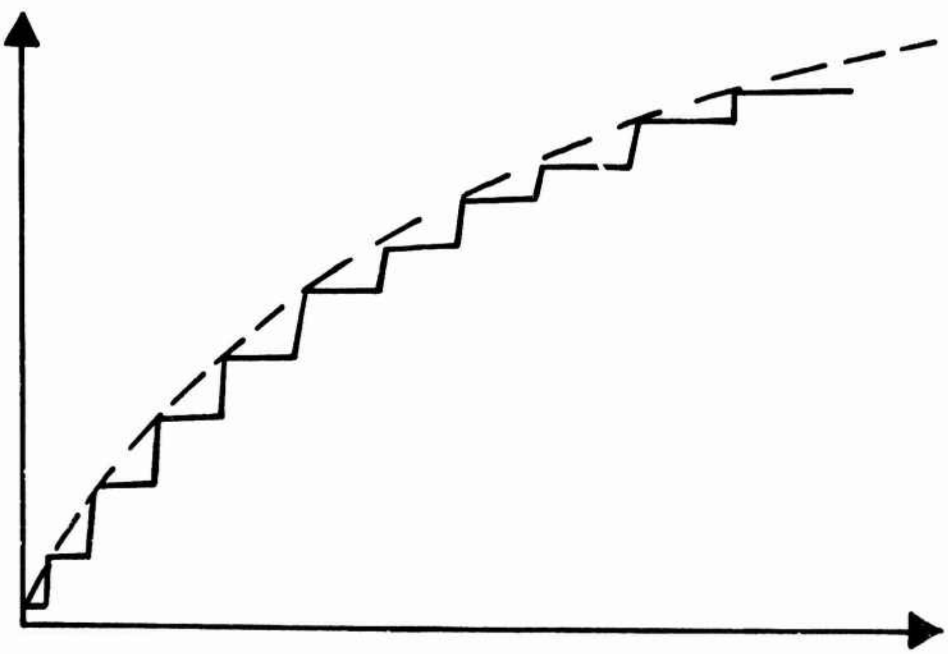


FIGURE 2

because partitions are connected to each other, or to a central node, through one or more "exchange" nodes whose locations must be optimally selected. Hierarchical structures and network partitioning are the natural consequences of a VOD cost structure or, more generally, of any economy of scale situation where it pays to implement a two-level hierarchical structure, with several low level networks and one high level network. Traffic between low level networks is sent to the exchanges, and from there to the high level network. The high level network links carry a high traffic volume and can achieve a better volume discount than can links in low level networks.

4. AN ALGORITHM FOR OPTIMIZING DOMESTIC SATELLITE COMMUNICATIONS NETWORKS FOR COMPUTER COMMUNICATIONS-PART 1

There are three possible ways to extract, from the satellite, channels to be used in a computer network: from a domestic satellite company's central offices; from a small "roof top" antenna at each terminal site (node); and from strategically located ground stations which are set up specially for the network. The first approach is treated in Chapter 2 of this report. The second approach will not be feasible in the near future, since current antennae (under 45 feet in diameter) can receive but cannot transmit adequately at high data rates. This subsection addresses the third approach.

In this approach, all four cost structures described in the last subsection are encountered. Nodes are partitioned, and a ground station is located in each partition. For the terrestrial network, "volume discounts" may be used to connect nodes to the ground stations; "location dependent structures" may be used to calculate line costs for the possible high/low density tariff, for connecting lines between different common carriers, and between private microwave links and common carriers; finally, costs for some lines can always be determined by direct distances (distance dependent structures).

To solve the problem stated above, a computer optimization program is necessary. Figure 3 shows a flow chart for a proposed program. The function of each block of the flow chart is described below. This description is intended to demonstrate the logical flow of the program, rather than to provide complete details for each block.

A. Data Base

The Data Base contains engineering data and cost information for termination and communication channels, it also contains V-H coordinates.

B. User Inputs

Information input by users includes:

- Network Information

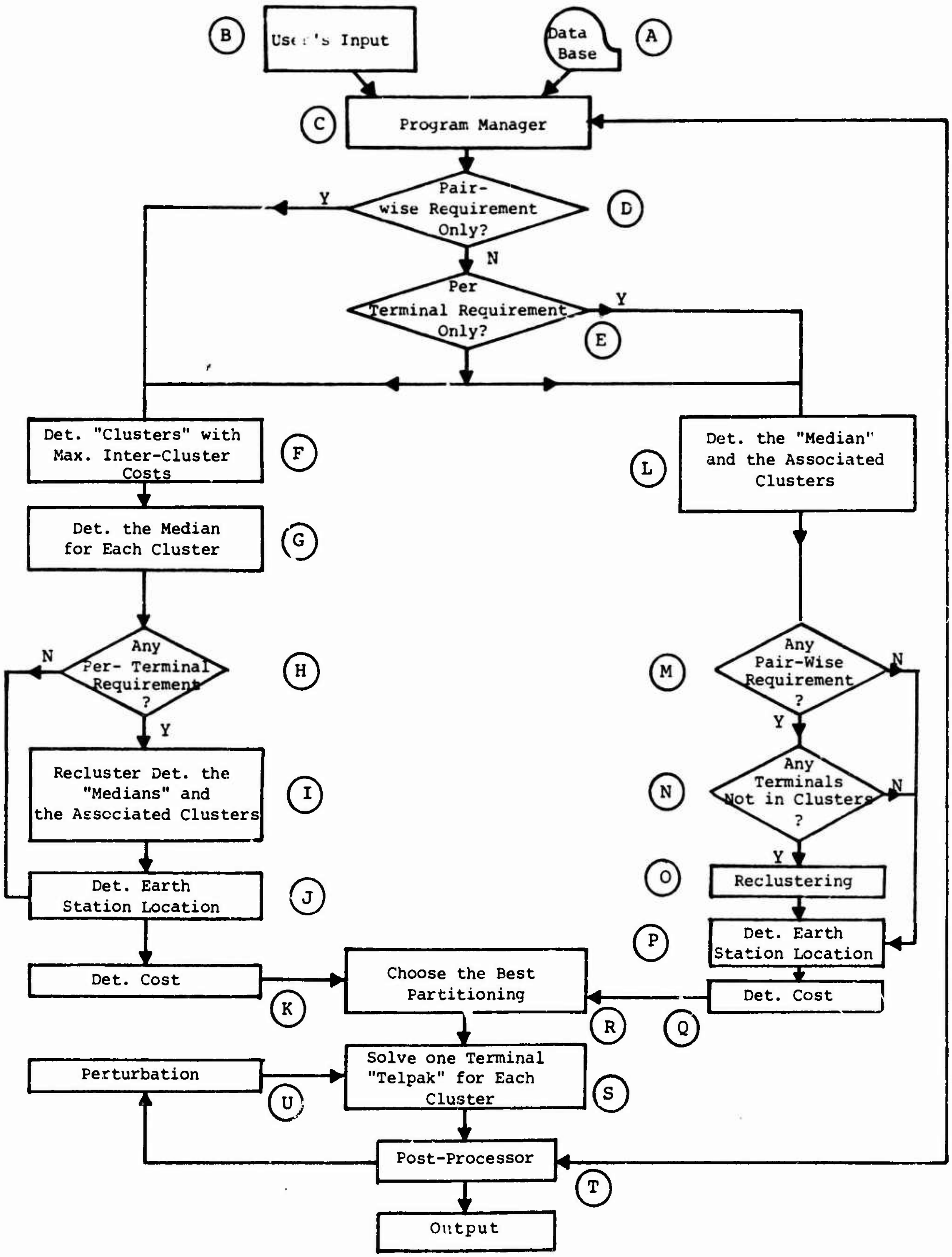
  This includes types of communication devices, engineering data, costs, and V-H coordinates, if not already in the data base.

- Traffic Requirements

  Traffic requirements can be expressed as number of accesses, number of channels, bandwidth requirements, or data bit rates at a terminal location or between a pair of locations. If available, traffic variations as a function of time should also be included.

- Design Restrictions

  Preassigned terrestrial links, preassigned earth stations sites, allowable earth station locations, and the number of earth stations are among possible constraints to be imposed on each design. Such restrictions may be the consequence of previous designs, or may be derived from other considerations.



B
User's Input
Data Base
A
C
Program Manager
Pair-wise Requirement Only?
D
Y
N
Per Terminal Requirement Only?
E
Y
Det. "Clusters" with Max. Inter-Cluster Costs
F
Det. the Median for Each Cluster
G
Any Per- Terminal Requirement ?
H
N
Y
Recluster Det. the "Medians" and the Associated Clusters
I
Det. Earth Station Location
J
Det. Cost
K
Det. the "Median" and the Associated Clusters
L
Any Pair-Wise Requirement ?
M
N
Y
Any Terminals Not in Clusters ?
N
N
Y
Reclustering
O
Det. Earth Station Location
P
Det. Cost
Q
Choose the Best Partitioning
R
Perturbation
U
Solve one Terminal "Telpak" for Each Cluster
S
Post-Processor
T
Output

FIGURE 3

5.10

- Other Relevant Data

C. Program Manager

The Program Manager manipulates user inputs and appropriate portions of the data base and creates a new data base in a format for use by the main body of the program.

D. Traffic Requirement Specifications

Traffic may be specified exclusively in terms of "traffic per terminal pair." In this case, the algorithm used to locate earth stations is different from the one used when traffic requirements are specified in other terms. When only terminal pair (i.e., point-to-point) requirements are specified, the sequence F, G, H, I, J, K, identified in the flow chart, is used to determine earth stations. This sequence represents the algorithm for this traffic requirement specification.

E. If requirements are specified in terms of total traffic at each terminal, the algorithm sequence L, M, N, O, P, Q, is used to determine earth station locations. If some requirements are specified in terms of "traffic per terminal pair," while others are specified in terms of "total traffic per terminal," two different approaches, represented by sequences F, G, H, I, J, K, and L, M, N, O, P, Q, are used to find locations for earth stations, and the least cost set is chosen as the final result.

F. Terminal Partitioning

The terminals involved at this stage are those whose traffic requirements are expressed in terms of "traffic per terminal pair." (There may be other terminals in the network whose traffic requirements are defined differently. These are considered for partitioning at a later stage.) If satellite links are not present, requirements can be satisfied by connecting a direct, least cost terrestrial link with a capacity equal to the traffic requirement for that pair. (Some of these links may be routed through Telpak circuits for cost reduction.) The goal of the design system is to reduce communications costs by replacing some of the terrestrial links with satellite links. If terminals of this terrestrial network are partitioned into clusters and one earth station is placed in each cluster, replacing only those terrestrial links interconnecting clusters may result in cost savings. Therefore, the more traffic between clusters, and the longer the links connecting the clusters, the larger the potential cost saving. That is, the higher the communications costs for terrestrial links interconnecting clusters, the higher the potential cost saving achieved by their replacement with satellite links. The steps involved at the present "terminal partitioning" stage involve the partitioning of the terminals of the terrestrial network in such a way that the costs for the terrestrial links connecting the terminals between different clusters is maximized. There is no exact method to achieve the above goal. However, there exists an iterative algorithm which gives very good results. The number of clusters, or the number of earth stations is not determined at this stage. It is defined at the Program Manager stage.

G. <u>Center of Gravities</u>

A good heuristic for locating earth stations is to place one at the center of gravity, or median, for each cluster. The object is to determine the center of gravity to locate earth stations so that the total terrestrial link cost for connecting terminals in the cluster directly to the station is theoretically minimum. Each link cost is a function of distance and traffic requirements. After all medians are found, one may discover that lower cost connections for some terminal pairs are direct point-to-point terrestrial links rather than satellite links. If such terminals exist, their requirements are deleted from consideration and a set of new centers is determined.

H. Checking Traffic Requirements

If there are no terminals whose traffic requirements are expressed in terms of "total traffic required at the terminal," the clustering is finished. Otherwise, these terminals must be added into the clusters generated at step G.

I. Reclustering

In this step, reclustering for the additional terminals and determination of new centers of gravity of the clusters is accomplished. The terminals defined in "H" are added to clusters obtained in "F", so that the total cost of connecting each of the terminals to its corresponding center of gravity is minimum. (Each of the connections is a function of distance and traffic requirements.)

J. Determining Earth Station Locations

The medians of the clusters are heuristically good locations for earth stations. In practice, there are restrictions on allowable locations for earth stations. The allowable location nearest to each median is chosen as the potential site for an earth station.

K. Determining Tentative Costs

The costs to connect each terminal to the nearest earth station through a terrestrial link with sufficient capacity to traffic requirements is determined.

L. Clustering and Center of Gravities

The definition of medians is stated in "G". Terminals are partitioned by some simple sets of rules into clusters. The number of clusters is equal to the number of earth stations supplied by the Program Manager. Terminals are iteratively shifted between clusters to minimize the total costs to connect each terminal to its nearest median with a terrestrial link satisfying the traffic requirements. At this stage, terminals whose traffic requirements are specified in terms of "traffic per terminal pair" are not included in the clustering process.

M. Checking Traffic Requirement Specifications

If there are no terminals whose traffic requirements are given in terms of "traffic per terminal pair" no further clustering process is needed. Otherwise, reclustering may be necessary.

N. Checking Requirements for Reclustering

If any of these terminal pairs with pair wise requirements are in the same cluster, or if any such terminals are not contained in any of the clusters, further clustering is necessary.

O. Terminal Reclustering

Each of the terminals not contained in any of the clusters is assigned to the cluster whose median is closest. If there is a direct traffic requirement between a pair of terminals, and if they are in a same cluster, the traffic requirement can be satisfied less expensively via terrestrial links only. These requirements are deleted from consideration. A new set of medians are then calculated.

P. Determining Earth Station Locations

Identical to Step J.

Q. Determinining Tentative Costs

Identical to Step K.

R. Choosing the Least Cost Partitioning

If only one type of traffic requirement is specified, there would be one set of earth station locations, and the program can proceed to the next block. If both types of traffic requirements are specified, there would be two sets of earth station locations. The total cost to connect terminals with terrestrial links directly to their associated stations is different for the two sets. The set of locations and their associated clusters corresponding to the lesser cost is chosen.

S. Solving the One Terminal Telpak Problem

The possibility exists that several terminals in the same cluster can be connected to the earth station by a shared wide band line. The cost optimization associated with this situation is called the "One Terminal Telpak Problem". The problem complexity is such that in general only heuristic solutions are possible.

T. Post-Processor

This portion of the Programming System determines whether the earth station location for each cluster should be perturbed by proceeding to block "U" or whether systems with a different number of earth stations should be evaluated by returning to block "C". If no further processing is needed, the results are placed into the desired format, a complete cost analysis is performed and the plotting routines are called.

U. Perturbation

With Telpak-like circuits included, the earth station locations determined earlier may not be the least cost choices. These locations are perturbed to test for possible cost reductions. Also, it may be less costly to connect some terminals near boundaries or the clusters to the earth station of a different cluster via a Telpak-like route.

CHAPTER 6

ROUTING CONSIDERATIONS FOR LARGE NETWORKS

1. INTRODUCTION

In this chapter, the computational aspects of the large network routing problem are considered. Both hierarchical and nonhierarchical networks are studied, and computation, storage and overhead traffic requirements are examined. Several hierarchical routing algorithms are proposed. These algorithms are based on a decomposition approach and provide significant savings in memory space and computation time when compared to other techniques which have been implemented or proposed. These algorithms can operate efficiently for distributed networks with 1000 or more nodes each of which is active in the routing process.

In a packet-switched communication network, the routing policy is defined as the set of rules that guide each packet through the network, along a route from source to destination. We distinguish two types of routing policies: deterministic policies, which implicitly assume time invariant input rates and a perfectly reliable network configuration; and adaptive policies, which are capable of adjusting to traffic fluctuations and network failures. The former type policy can be specified analytically and is mostly used for network analysis and design. Adaptive routing procedures protect against network failures and load fluctuations and thus are essential for traffic management in a network implementation.

An efficient routing policy should be able to fully utilize network capacity for any load pattern, by sending packets on minimum delay or maximum throughput paths and eventually distributing heavy traffic on multiple paths. However, the routing strategy alone cannot prevent network congestion if input traffic exceeds network capacity. Therefore, a flow control mechanism is needed to control input rates before congestion occurs.

Routing and flow control techniques have been designed and implemented for small and medium size networks (up to about 64 nodes), with satisfactory results. However, for networks of more than 100 nodes, existing techniques become inefficient, primarily because of computation time and memory space. In fact, the analytical solution of a deterministic routing problem with existing routing algorithms requires an amount of computation between $(NN)^2$ and $(NN)^3$ and a memory space on the order of $(NN)^2$, where NN is the number of nodes. Similarly, the implementation of the existing adaptive routing techniques for traffic management introduces an overhead proportional to $(NN)^2$. The adaptive routing program presently used in ARPANET, for instance, requires at each nodal processor a storage space (for routing table storage) and a processing time (for routing computation) both proportional to (NN); similarly, each node periodically transmits an amount of routing information proportional to (NN). The total overhead traffic, obtained by multiplying the overhead of each node times the number of nodes, is therefore proportional to $(NN)^2$.

The relation between network congestion and network size cannot be expressed in simple quantitative terms. However, it can be shown that in a large network under particular traffic conditions, present flow control procedures are not able to prevent congestion of regional or local areas.

New routing and flow control techniques are therefore required for large networks. Such techniques will, in general, use decomposition concepts, to reduce the large network problem to a set of dependent smaller problem, each solvable using the existing methods.

The overall solution should depend not only on network size, but also on network topological structure. In fact, some solutions appropriate for a distributed structure such as a grid topology might not be efficient for a hierarchical structure.

Hierarchical structures are particularly advantageous for large network analysis and design. Thus, in this paper we concentrate our discussion on various hierarchical routing algorithms.

2. HIERARCHICAL STRUCTURES

A variety of topological structures can be used for network design, ranging from hierarchical structures to uniformly distributed structures such as grids. The preliminary 1000 node network study in NAC's Semiannual Report #1 has shown that hierarchical structures are desirable for large networks because: (1) they are more economical; (2) they are easier to analyze and design; (3) they offer more flexibility in the choice of the system configuration, because different hierarchical levels can be implemented using different communication technologies and under different system requirements.

Because of the above factors, a hierarchical structure was selected for the present large network routing study. Without loss of generality, it is assumed that there are two hierarchical levels (the regional and the national level) consisting of m regional networks (with m nodes each) connected to each other by a national network. Each regional network is connected to the national network through two exchange nodes. Therefore, the national network has 2m nodes which are exchange nodes and belong both to regional and national networks.

It is also assumed that flows between two nodes belonging to the same region cannot be sent along routes containing nodes external to the region. The assumption is both physically realistic and also greatly simplifies both the deterministic and adaptive routing algorithms. While a variety of other assumptions are possible, reasonable ones do not seem to either substantially increase or decrease the difficulty of the routing problem.

3. DETERMINISTIC ROUTING POLICIES

The Deterministic Routing Problem. The optimal deterministic policy is generally defined as the routing policy that minimizes the average packet delay T for a given external traffic requirement, or, alternatively, as the policy that accommodates the maximum throughput, such that $T \leq T_{max}$, where T_{max} is the maximum admissible packet delay.

We assume that the basic traffic requirement is known and is given by an NN x NN matrix R°, where $R^{\circ}(i,j)$ is the average packet rate from source i to destination j. The traffic that the network can actually accommodate is given to $R \triangleq \rho R^{\circ}$, where ρ is the admissible traffic level. To maximize the throughput corresponds to maximize ρ.

Under appropriate assumptions, whose validity in practical network implementations has been verified experimentally, it is possible to obtain simple expressions of the delay T in terms of the average link flow rates. One such expression is;

$$T = \frac{1}{\gamma} \sum_{i=1}^{NA} \frac{f_i}{C_i - f_i} \qquad (1)$$

where: NA = number of links

γ = total external input rate [packet/sec]

f_i = average rate on channel i [bits/sec]

C_i = channel capacity of link i [bits/sec]

More detailed traffic models lead to more elaborate expressions of T: however, the conceptual difficulty of the routing problem remains the same.

Using the appropriate expression for T, the deterministic routing problem can be formulated as a convex multicommodity flow problem, whose solution, if it exists, is unique. Efficient algorithms exist for small and medium size networks. Typically, the routing policy for a 30 node network is obtained after 5 to 10 iterations, and requires 2 to 4 seconds CPU time on a large computer. However, storage requirements and computation time increase at a rate on the order of between $(NN)^2$ and $(NN)^3$. Therefore, such algorithms are not adequate for the solution of networks with several hundred nodes.

Hierarchical Routing Algorithms. If the network structure is hierarchical, it is possible to develop hierarchical routing algorithms with computing time proportional to $(NN)^2$ and memory space proportional to $(NN)^{3/2}$.

An example of hierarchical routing algorithm, based on the "flow deviation method" is discussed below. The algorithm, takes advantage of the fact tnat regional traffic can use only regional links, and it provides an exact solution to the problem of finding the routing that minimizes the average packet delay, given the external requirements.

HFD Algorithm:

Step (o) Let $\underline{f}$ be a feasible NA-dimensional link flow vector.

Let $T^\circ \triangleq T(\underline{f}^\circ)$: average delay corresponding to flow f°.

Let $n = 1$

Step (1) Compute the NA-dimensional length vector $\underline{\ell}^n$, where ℓ_i^n is the equivalent length of link i, as follows:

$$\ell_i^n \triangleq \left(\frac{\partial T}{\partial f_i}\right)_{\underline{f}=\underline{f}^{n-1}} \tag{2}$$

Using eq. (1), we have:

$$\ell_i^n = \frac{1}{\gamma} \frac{C_i}{(C_i - f_i^{n-1})^2} \tag{3}$$

(notice that ℓ_i^n increases with channel saturation)

Step (2) Compute Shortest Routes:
m shortest route problems (one for each region) are first solved. Using in part, the regional results, the shortest route problem for the national network is then computed.

Step (3) Assign link flows:
Flow requirements between node pairs are assigned to the shortest routes, as computed in Step (2). The assignment is performed in 3 phases: in phase (1), the equivalent national requirements (between different regions) are computed; in phase (2), flows are assigned to national links, and the equivalent regional requirements are computed; in phase (3), flows are assigned to regional links. The resulting flow is represented by an NA-dimensional vector $\underline{\phi}^n$.

Step (4) Minimize the delay:

$$\text{Let } T^n \triangleq \min_{\lambda} T\,(\lambda \underline{f}^{n-1} + (1-\lambda)\,\underline{\phi}^n) \tag{4}$$

$$\text{s.t. } 0 \leq \lambda \leq 1$$

Let $\bar{\lambda}$ be the minimizer of Eq (4); let $\underline{f}^{\,n} \triangleq \bar{\lambda}\underline{f}^{n-1} + (1-\bar{\lambda})\underline{\phi}^n$

Step (5) Stopping rule:
If $(T^{n-1} - T^n) \leq \varepsilon$, where ε is a proper positive tolerance, stop: $\underline{f}^n$ corresponds to the optimal deterministic routing, within tolerance ε.
Otherwise, let m = m + 1 and go to Step (1).

The HFD algorithm can be proven to converge to the optimal solution. Notice also that the assumption that regional traffic never leaves the region is satisfied by the regional flow assignment performed in Step (3).

Another hierarchical routing algorithm can be developed from the "cut-saturation" technique described in this report. The algorithm, called Hierarchical Cut-Saturation (HCS) algorithm, maximizes network throughput at saturation. The structure of HCS is very similar to that of HFD, and the fundamental steps are the same.

HSC Algorithm:

Step (0) Initialization:

Let $\rho^{\circ} = 0$

Let $\underline{f}^{\circ} = \underline{0}$

Let $n = 1$

Step (1) Compute equivalent length vector $\underline{\ell}^n$ as follows:

$$\ell_i^n = \begin{cases} 1 & \text{if link } i \text{ is not saturated} \\ \infty & \text{if link } i \text{ is saturated} \end{cases}$$

Step (2) Compute Shortest Routes: (Same as in HFD)

Step (3) Assign Link Flows: (Same as in HFD)

Step (4) Increase the Throughput:

Let $\underline{f}^n = \underline{f}^{n-1} + \delta\underline{\phi}^n$

Where δ is the largest positive coefficient such that one or more links, previously non-saturated, reach saturation.

Let $\rho^n = \rho^{n-1} + \delta$

Step (5) Stopping Rule:

Let S be the set of saturated links. If the removal of S makes the network disconnected, Stop. Otherwise, let n = n + 1 and go to Step (1).

Computational Considerations In nonhierarchical routing algorithms, the computational bottleneck is represented by the shortest route computation and by the flow assignment, the first requiring a number of operations between $(NN)^2$ and $(NN)^3$, depending on the degree of network connectivity, and the second requiring a number of operations proportional to $(NN)^2$.

In the two hierarchical algorithms, HFD and HCS, previously described, shortest route and flow assignment are still the computational bottlenecks. However, it can be easily seen that both shortest route computation and flow assignment require from $(NN)^{3/2}$ to $(NN)^2$ operations.

The total memory space needed to store routing, distance and requirement matrices is proportional to $(NN)^2$ in a nonhierarchical algorithm and to $(NN)^{3/2}$ in a hierarchical one.

The computation and storage reduction obtained with HFD and HCS allows the solution of the routing problem for 2-level hierarchical networks with a number of nodes on the order of one thousand, which could not possibly be attacked with the traditional routing algorithms. In the case of larger networks, three or more hierarchical levels must be considered, in order to obtain algorithms implementable on the computer.

4. ADAPTIVE ROUTING POLICIES

4.1 NON-HIERARCHICAL POLICY

Several types of nonhierarchical adaptive policies can be implemented on a packet switched communication network.

Here we use as a model the distributed adaptive policy presently used in the ARPANET.

In a nonhierarchical adaptive policy, each nodal processor i (i=1,...,NN) stores an NN x A_i delay Table $DT^{(i)}$, where A_i is the set of nodes adjacent to the node i. The entry $DT^{(i)}(k,\ell)$ is the estimated minimal delay from node i to destination k, if ℓ is chosen as the next node in the route to k. From $DT^{(i)}$, an NN dimenstional minimum delay vector $MDV^{(i)}$ is computed by the nodal processor as follows:

$$MDV^{(i)}(k) \triangleq \min_{\ell \varepsilon A_i} DT^{(i)}(k, \ell), \text{ for all } k=1,\ldots,NN.$$

$MDV^{(i)}(k)$ represents the minimum estimated delay from i to k, and the corresponding ℓ is the next node in the route to k. Periodically, each node asynchronously transmits the vector $MDV^{(i)}$ to its neighbors. Upon reception of neighbor's vectors, node i updates $DT^{(i)}$ as follows:

$$DT^{(i)}(k,\ell) = d(i,\ell) + MDV^{(\ell)}(k), \text{ for all } k=1,\ldots,NN$$

where $d(i,\ell)$ is the measured delay (queueing + transmission) on link (i,ℓ).

To perform the above operations, an amount of computation, a storage space and an exchange of routing information proportional to NN are required at each node.

4.2 A CENTRALIZED ADAPTIVE POLICY

The centralized adaptive policy here proposed is essentially a deterministic policy which is periodically updated according to load fluctuations and network failures. Thus, each node has a deterministic routing table; a Network Routing Center (NRC) collects network information, computes routing policy corrections according to such information, and transmits routing update messages to all nodes. For additional failure protection, the nodes are also equipped with Minimum Hop Tables.

Following is a description of possible specifications for a centralized adaptive policy implementation.

4.2.1 Basic Operations

Each node transmits to NRC asynchronously, every .5 seconds, the following information:

- number of packets transmitted on each output channel in the past .5 seconds.

- for each destination k, the number of packets directed to k, which were received from external sources (Host computers, terminals, etc.) in the past .5 seconds.

Using the information received from the nodes, a computer, available at the NRC site, evaluates channel and external input rates averaged over the past 10 seconds. Every 10 seconds the above data is fed to a routing program resident in core. The routing program can be the Flow Deviation algorithm: in such a case, the program computes, on the basis of channel traffic and external requirement, for each node-destination pair, the new route on which a fraction of the traffic, say α $(0 \leq \alpha \leq 1)$, must be "deviated", in order to improve network performance. At the end of the computation, which typically requires 100 to 300 msec., NRC delivers to each node a routing update message which contains:

- a vector of "next nodes", one per destination, to which a fraction α of the incoming traffic must be deviated.

- the parameter α.

Upon reception of the routing message, each node updates its routing table. Since the Flow Deviation algorithm is an optimal routing algorithm, the centralized adaptive policy converges to the deterministic policy in steady network conditions.

4.2.2 Line and Node Failures

In order to avoid congestion when a failure occurs in the 10-second interval between routing updates, a very simple local adaptive policy is provided as a backup. For example, the minimum hop number policy. During normal network operation (no failures) the min hop policy remains inoperative. As soon as a node experiences the failure of an output line or neighbor node, (1) it modifies its min hop numbers, according to the failures, (2) it transmits immediately the min hop table to the neighbors, (3) it switches from centralized adaptive to min hop mode of operation, i.e.,it routes the packets along min hop routes. Gradually, all nodes switch to min hop policy, until the NRC learns about the failure. New tables are then computed, which account for the failure, and the centralized adaptive policy is gradually restored. During the transient period following the failure, min hop and centralized adaptive policy co-exist in the network. It can be easily seen that such a situation is logically acceptable, and does not create severe performance degradation.

4.2.3 NRC Failures

The reliability of NRC can be improved by providing a backup NRC at another network site. When one NRC goes down, the backup takes over. Furthermore, if network failures isolate the NRC's so that they belong to two disconnected components, each component can be controlled by the respective NRC. Finally, if both NRC's go down, or if a network component becomes disconnected from both NRC's, packet routing is accomplished by the min hop policy.

4.2.4 <u>Centralized Versus Local Adaptive Policy</u>

Following are a few points of comparison between centralized adaptive policy (CAP) and local adaptive policy (LAP).

- Overhead traffic, due to transmission of network status and routing information, is approximately the same.

- Processor overhead, corresponding to routing and updating, is of the same order of magnitude for both systems (except that in LAP the routing computation is distributed among the nodes, while in CAP it is performed by NRC).

- In the transient period following each failure, CAP is less efficient than LAP. However, failures typically occur at the average rate of one every two hours, in a thirty node network. Therefore, the effect of failures on average CAP performance is not substantial.

- CAP has no loops and makes efficient use of alternate paths. LAP can generate loops and tends to use only one path at a time.

- CAP can perform a better and more selective flow control on the external input rates, than LAP, as it has the global knowledge of all external traffic requirements.

4.3 HIERARCHICAL POLICY

A hierarchical adaptive policy consists of the combination of several regional policies and one national policy, properly interfaced with one another at the exchange nodes. Packet routing within each regional network and in the national network is performed according to the traditional algorithms. Some new operations are required for the delivery of packets between different regions. An outline of the hierarchical routing procedure is given below.

- Regional Routing. Each node of a regional net uses and updates, in the traditional fashion, a regional routing table, where only destinations within the region are listed. Furthermore, the node receives a national minimum delay vector from each of the two exchange nodes, indicating the minimum delay from the exchange node to any region. Packets with destination within the region are routed according to regional routing tables. If a packet is directed to another region, the source node determines, after inspection of regional and national delay vectors, the exchange node which minimizes the sum of regional and national delay, and transmits the packet to it. From there on, the packet is handled by the national routing algorithm.

- National Routing. Each exchange node, in addition to regional routing and delay tables of the region to which it belongs, is equipped with national tables, which show routes and minimal delays to all other exchange nodes. Using the national delay table, each node computes the national minimum delay vector and propagates it among the nodes of its region. Furthermore, each exchange keeps track of the nodes of its region, that it can reach through a regional route, and stores the information in an m-dimensional regional connection vector. The i-th entry of the vector is 0 if regional node i is unreachable (because it is down or disconnected); it is 1 otherwise. As soon as a change in regional connection occurs, the exchange node updates its connection vector and sends an updated copy of it to the other exchange node in the same region. Each exchange node therefore stores two connection vectors, one indicating the regional nodes reachable from itself, the other indicating the regional nodes reachable from the other exchange node in the same region. From the inspection of these vectors, an exchange node, upon reception of a packet directed to a node in its region, determines whether; (1) the packet can be directly delivered to the regional node, or (2) must be routed through the other exchange node, or (3) cannot be delivered because the destination is not reachable from either exchange node. In the last case, the packet is discarded and a negative acknowledgment is sent to the source node.

The main difference between a nonhierarchical and a hierarchical policy is therefore represented by the existence, in the latter, of national minimum delay vectors in the regional nodes, to determine the shortest way out of the region, and of regional connection vectors in the exchange nodes, to ensure reliable delivery of packets between different regions, whenever a source to destination path exists.

4.4 COMPUTATIONAL CONSIDERATIONS

In the nonhierarchical case, the overhead (nodal processing, storage space, exchange of routing information) for each node is proportional to NN. In the hierarchical case, such an overhead is proportional to $\sqrt{NN}$, where $NN = m^2$, as usual. The overhead reduction is obviously considerable, and should allow the implementation of 2-level hierarchical adaptive routing algorithms on networks with on the order of a thousand nodes. For larger networks, it might be necessary to use hierarchical structures with 3 or more levels, and modify the routing algorithms accordingly.

5. CONCLUSION AND FUTURE RESEARCH

In this chapter, the computational aspects of large network routing have been considered and various hierarchical routing algorithms have been discussed. The algorithms are based on a decomposition approach and provide significant savings on memory space and computation time, with respect to the traditional techniques. Such savings allow the application of the algorithms to networks with a thousand or more nodes.

Many other large network routing aspects remain to be investigated including:

- Multilevel Structures. For very large networks, the computational reduction obtained with a 2-level topology might not be sufficient. In such a case, multilevel hierarchies must be investigated. Similarly, network reliability provided by two exchange nodes per region might not be adequate and configurations with 3 or more exchange nodes might be required. However, the selection of number of levels and number of exchange nodes will depend on many other design factors beside routing computational considerations.

- High Bandwidth Traffic. Channel capacities in regional networks are generally much smaller than channel capacities of the national network. Consequently, whenever a high bandwidth requirement arises between two nodes belonging to different regions, a bottleneck is most likely to occur in the two regional networks. Therefore, to improve high bandwidth performance, further research is required to develop efficient regional adaptive routing algorithms, which provide multiple routes between high throughput nodes and regional exchange nodes.

- Flow Control. As another consequence of lower regional capacities, regional networks can become congested much sooner than the national network. For instance, if several nodes belonging to different regions simultaneously send packets to the same destination, the destination region might become congested (and all the nodes in such a region unable to communicate with each other) much before the sources learn about it. The "flow control" technique currently implemented in ARPANET is based on a reassembly space reservation scheme. It is aimed at multipacket messages and cannot prevent regional congestion generated by single packet messages. Therefore, more efficient hierarchical flow control techniques should be investigated. Such techniques might involve metering of traffic entering each region, at the exchange nodes, and circulation among national nodes of traffic load tables, which reflect the load status of each region.

- Use of Different Communication Techniques. In the design of large, multilevel hierarchical structures it is often more economical to use different communication techniques at different levels. For example, one could conceive a 3-level system where the national level uses packet satellite communication, the regional level uses packet switching communications, and the local level uses broadcast radio techniques. New, more general techniques must therefore be investigated.

CHAPTER 7

ROUTING ALGORITHMS FOR HIGH BANDWIDTH TRAFFIC

1. INTRODUCTION

When a high volume data transfer must be performed between two high speed devices at two different ARPANET sites, the total time of the transfer, and therefore, the efficient utilization of the devices, can be considerably improved if data traffic between the two sites is routed along two or more routes, so as to best utilize the excess capacity of the network. This task, often referred to as "alternate routing", must be performed by the routing algorithm resident in each IMP.

2. DISTRIBUTED AND CENTRALIZED ROUTING ALGORITHMS

The routing algorithm presently used in ARPANET and the new routing algorithm proposed by BBN can be classified as distributed routing algorithms. Traffic is sent on shortest routes, computed according to some reasonable length criterion; the shortest route computation, however, is "distributed", in the sense that the computation at each node is based, in part, on the results of similar computations at neighbor nodes.

An alternative to the distributed algorithm is the centralized algorithm proposed earlier. Such algorithm assumes the existence of a network routing center that collects all appropriate traffic information, computes shortest routes between all node pairs and distributes routing information to all nodes.

Both distributed and centralized algorithms can be properly designed so that alternate routing is obtained. However, both have limitations; inparticular, distributed algorithms suffer the limitation of being "local", i.e. routing decision is based on local traffic information, or on global, but generally out of date traffic information; centralized routing algorithms can achieve a better global utilization of alternate routes, but cannot react rapidly enough to traffic changes. In the following we present some new concepts for the design of efficient high bandwidth routing algorithms which combine desirable characteristics of both approaches.

3. SOURCE AND DESTINATION ROUTING ALGORITHMS

Intermediate solutions between distributed and centralized routing scheme are represented by the source and the destination routing algorithms. In the two latter algorithms, each node collects traffic information from all the other nodes. Using such information, and having the global knowledge of network topology and channel capacities, the node evaluates appropriate equivalent lengths for all the links in the network.

Next, in the source algorithm, each node evaluates all shortest routes (according to the above lengths) from itself as a source to all the other nodes; in the destination algorithm each node evaluates all shortest routes from all the other nodes to itself as a destination. Alternatively, instead of shortest routes, maximal residual capacity routes can be determined.

At the end of the shortest route computation, each node propagates into the network the routing information in the form of a routing vector.

Efficient alternate routing is obtained by repeating such shortest route computations at a frequency that will depend on network saturation and presence of particular high bandwidth requirements. After each computation, new routes are obtained and the traffic is optimally distributed between new and preexisting routes.

As an example, assume that a high bandwidth requirement arises from source S to destination D. If source routing is used, the source node attempts to accomodate such a requirement on the (S,D) route (or routes) presently available. If there is no sufficient excess capacity on such routes, the source computes the new shortest route from S to D and accomodates additional traffic. Shortest route computations are repeated until the requirement is entirely accomodated or there are not more residual capacity routes between S and D. In either case, the source algorithm provides a smooth flow control on the input rate of the high bandwidth requirement.

4. COMPUTATIONAL CONSIDERATIONS

In the sequel, the question of the feasibility of source and destination routing implementations on minicomputers like those used in ARPANET is addressed. To answer such a question, the amount of computation and of memory space required by the algorithms is investigated. In particular, the two routines Shortest Route and Flow Assignment are described which are the backbone of both algorithms and account for most of the execution time and memory space requirement.

4.1 SHORTEST ROUTE ROUTINE

As mentioned before, the source routing algorithm requires the computation of all shortest routes from the given source to all the remaining nodes; conversely, the destination routing algorithm requires the computation of all shortest routes from all nodes to the given destination. In both cases a modified Dijkstra's Algorithm is used which applies Floyd's Treesort Algorithm for the sorting of the minimum distance node at each step. The computational complexity required to find all shortest routes is theoretically bounded by NN lg_2NN + NA (for comparison operations) and by NA (for addition operations), where NN is the number of nodes, NA the number of links. In practical cases such as ARPANET, the computational complexity with respect to comparison operations can be further reduced by treating the 2 and 3 degree nodes separately, and it approaches the lower bound NA.

The modified Dijkstra's Algorithm for the computation of shortest routes from source S to the other nodes is described below. A similar algorithm can be used to find shortest routes from all nodes to a given destination. Before introducing the algorithm, a few definitions are necessary.

4.1.1 HEAP

A list of $m=2^k-1$ numbers $n_1,\ldots,n_{2^k-1}$ can be identified with a rooted binary tree with k levels, where n_1 is at the root, n_2 and n_3 are at the next level, and in general, n_{2i} and n_{2i+1} at level i+1 are connected to n_i at level i. In figure 1 this mapping is illustrated for k=4.

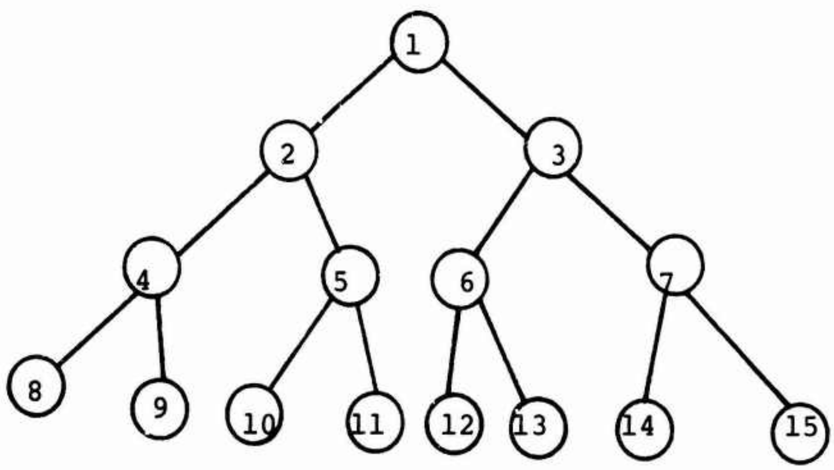


Figure 1

The list $L = [n_1, \ldots, n_{2^k-1})$ or equivalently the binary tree associated with it is called a <u>heap</u>, if $n_i \leq n_{2i}$ and $n_i \leq n_{2i+1} \forall i=1, \ldots 2^k-1$; i.e. the element at the top of the heap is the minimum of the list.

4.1.2 DATA STRUCTURE

DIS (I), ... NN, is the vector of shortest distances from S to each node I, and constitutes the heap of the modified Dijkstra's Algorithm.

D (IJ), IJ=1, ... NA, is the vector of link lengths. Assuming that the index IJ corresponds to the link (I,J), then D(IJ) is the length of link (I,J). NODE (K), K=1,...NN, is the vector of pointers from heap to list of nodes. PRE (I), I=1,...NN, is the vector of predecessors. In particular, PRE(I) is the node preceding I in the shortest route from S to I.

4.1.3 Modified Dijkstra's Algorithm

Step (0) Assume source node S is labeled as node 1.

Initialize:

$$\left.\begin{array}{r} \text{NODE (I)} = \text{I} \\ \text{DIS (I)} = \infty \end{array}\right\} \forall \text{ I} = 2, \ldots \text{NN}$$

NODE (1) = 1

DIS (1) = 0

Step (1) <u>Scan</u> the top node in the heap:

Let I = NODE (1)

For any unscanned node J adjacent to node I:

If DIS (J) > DIS (I) + D (IJ)

then: DIS (J) = DIS (I) + D(IJ)
PRE (J) = I
Do a treesort on the heap DIS

Step (2) Remove the top node. (which is now scanned).

If K is the lowest index in the heap such that DIS (NODE(K)) <∞, let:

NODE (1) = NODE (K)

NODE (K) = 0

Do a treesort

Step (3) If DIS (NODE (1)) = ∞, Stop.

Otherwise go to (1).

At the end of the algorithm, DIS (I) represents the shortest distance from S to I, for all I; PRE (I) represents the predecessor of I in the shortest route from S to I.

Notice that some minor modifications to the algorithm allow maximum residual capacity paths to be found instead of shortest paths. For example, in a maximum residual capacity algorithm, DIS (I) represents the residual capacity on the best path to node I; the top of the heap is the node which has currently the maximum residual capacity; the test in Step (1) becomes: DIS (J) < min (DIS (I), RS (IJ)) where RS (IJ) is the residual capacity of link (I ,J).

4.1.4 AMOUNT OF COMPUTATION REQUIRED BY DIJKSTRA'S ALGORITHM

The time consuming steps of the algorithm are Step (1) and Step (2). If k is assumed to be the number of unscanned nodes adjacent to the node presently scanned, and NX the total number of labeled and unscanned nodes (typically NX < NN), then

- Step (1) requires k additions; $\leq k(lg_2NX-2)$ comparisons; $\leq k(lg_2NN-1)$ interchanges.
- Step (2) requires $\leq 2lg_2NN-2$ comparisons, lg_2NN-1 interchanges.

Recalling that Step (1) and Step (2) are executed at most NN times, the upperbound on computational complexity is proportional to NN lg_2NN. Further computational reduction is obtained if the network contains several nodes of degree ≤ 3; in such case, it is possible to make the upperbound proportional to k_1 NN'lg_2NN' + k_2 NN" + k_3NA, where NN' is the number of nodes of degree >3, and NN" is the number of remaining nodes.

4.1.5 FLOW ASSIGNMENT ROUTINE

Let the NN-vector REQ (I) I=1, ...NN contain the flow requirements from S to all other nodes (such a vector is evaluated node S). Let us assume that the nodes are relabeled in the order they were removed from the heap, i.e. in the order of increasing DIS (I). Let the NA-vector FLOW (K), K-1, ...NA, contain the link flows obtained by assigning flow requirements to shortest route flows.

Step (0) Initially, FLOW (I) = 0, ∀ I=1, ...NA.

NL=NN

Step (1) NF = PRE (NL)

FLOW (FL)   REQ (NL) where FL is the index of link(NL,FL)

REQ (NF) = REQ (NF) + REQ (NL)

Step (2) If (NL.EQ.1) stop.

NL=NL-1

Go to (1).

The amount of computation required by the flow assignment is linear in NN.

4.1.6 STORAGE REQUIREMENT

The following arrays are required for the shortest route evaluation:

(1) NN-vector with node degrees

(2) NN-vector with node status (nodes up or down)

(3) NA-vector with link capacities

(4) NA-vector with link flows

(5) NA-vector with link lengths

(6) NA-vector with line status (lines up or down)

(7) NA-vector with adjacent nodes

(8) NN-vector with pointers from the list of nodes to the vector of adjacent nodes.

(9) NN-vector with shortest distances

(10) NN-vector of pointers to the heap

(11) NN-vector of pointers from the heap

In addition, the following arrays are required for the flow assignment:

(13) NN-vector with flow requirement from source S to all other nodes.

(14) NN-vector with the list of nodes in the order they were removed from the top of the heap.

(15) NA-vector with new link flows

4.1.7 ROUTING TRAFFIC OVERHEAD

Routing traffic corresponds to: (1) Line traffic information, which must be transmitted to each node, so that line "lenghts" can be evaluated and shortest routes computed; (2) routing information which is based on shortest route computation and is transmitted from the source (or the destination) node to all other nodes.

Line Traffic Information: each node evaluates average flows on its output channels and distributes such information to all other nodes in the network, using a propagation scheme.

Routing Information: each source (or destination) transmits the routing vector to all other nodes, using a propagation scheme.

5. CONCLUSION AND FUTURE RESEARCH

Source and destination routing algorithms are conceptually very simple, and can accomplish an efficient high bandwidth utilization. Their computational requirements are within the capability of minicomputers of the size of an ARPANET IMP.

Further research is required to develop a routing algorithm which implements the new concepts.

A SYSTEM FOR LARGE SCALE NETWORK COMPUTATIONS - PART 1

1. INTRODUCTION

Extensive experience in the design and implementation of sophisticated algorithms for network analysis applications has established the need for improved computational approaches to large scale network design. While rapid advances have been made in algorithm design using methods such as computational complexity analysis, non-linear programming theory, and decomposition techniques, the method of computation has remained relatively static. Remote job entry to a single large scale scientific computer in a batch mode has been the predominant approach to large network problems. This mode is very efficient for performing the algorithmic computations, but unfortunately, it is inadequate for human use. Consequently, the effectiveness of the remote job entry approach is limited. Time sharing, on the other hand, has many attractive user oriented features, but it does not offer an acceptable alternative because extensive network computations cannot be carried out on a time shared basis.

Developments in parallel processing, distributed computing, programming language theory, data structure design, and interactive graphics offer many opportunities to improve network computations. The application of these developments to practical large scale network computations has only recently begun. To collect and focus NAC's efforts in this area, the Laboratory for Large Scale Network Computations was established to study and develop mechanisms for carrying out computations for large scale network applications. A facility for such computations consists of computer hardware, systems and communication software, and the software implementing network algorithms. Section 2 describes the hardware resources of the laboratory, and Section 3 details the progress and plans for systems and communication software. Specific large scale network algorithms are described in the other chapters of this report.

2. HARDWARE RESOURCES FOR NETWORK COMPUTATIONS

2.1 NAC COMPUTER CENTER

The NAC computer center is built around three remote job entry terminals now used exclusively for access to CDC-6600 computers at various sites. Two of the terminals are CDC-200 User Terminals, both equipped with a medium speed card reader and line printer, and a hardwired controller. The third terminal is a Unitech UT-1 intelligent terminal built around a Data General 1220 Nova computer with high speed card reader, line printer, and 7 track tape drive. In addition, there are two large CalComp flat-bed plotters; one of which is connected online to one of the 200 User Terminals. The other is capable of being driven in either an online or an offline mode (through the tape drive) by the Unitech System. Additional computational equipment is in the laboratory itself.

2.2 LABORATORY HARDWARE

The first equipment for the laboratory was delivered in the fall of 1972 and consisted of:

1. An IMLAC PDS-1D programmable graphics display unit with 8K of memory, long vector hardware, cassette tape unit, and a "mouse" for graphic interactions;

2. A Texas Instruments 720 Silent Terminal; and

3. An Andersen-Jacobson combination 103 type modem and acoustic coupler.

In slightly over a year, two Infoton terminals have been added, and leased line connections to the ARPANET have been made to TIP's at CCA in Boston and at the National Bureau of Standards in

Washington. In July, 1973, NAC began an experiment to provide low cost terminal access to the ARPANET, even though the nearest TIP is over 200 miles away from NAC's facilities. In this experiment, several terminals were multiplexed onto a single voice grade line. Presently, the three CRT terminals operating at 1200 bps and the T.I. 720 at 300 bps, are multiplexed onto one voice grade line using a Bell 4800 bps modem. Since this is a new way of utilizing the ARPANET, the next section is devoted to describing in some detail, our experiences in connecting a multiplexed line into the net.

2.3 MULTIPLEXED ACCESS TO THE ARPANET TIP

Because NAC is distant from any ARPANET TIP site, terminal access through conventional dial-up or multiple leased lines to serve a number of users at NAC, is expensive. For example, in a single month of dial-up useage when NAC first began using ARPANET dial-up, communication charges were over 50% of total computation cost. To reduce communication costs, NAC was first connected to a TIP via a leased line by means of a Bell 103A2 modem with a ring circuit connection operating at a 300 baud maximum. This initial implementation was chosen because it was the only leased line configuration that had been connected to a TIP by late 1972.

As ARPANET usage grew, the following sequence of steps was planned to reduce network access costs.

1. A remote terminal was connected to the TIP without the complications of a ring-circuit.

2. Higher speed lines for use with CRT terminals could be utilized with appropriate modems.

3. To serve the users at NAC, lines could be multiplexed without the need for additional equipment beyond the multiplexers and appropriate modems.

A survey showed that none of the above steps had been attempted or implemented elsewhere. Hence, to increase the probability of success, the NAC implementation was sequential.

The first step was to determine whether higher speed leased lines could operate with the TIP under a simple connection strategy. A 1200 baud leased line was connected from NAC to the CCA-TIP with Bell 202R modems without the added complications of the multiplexer. Before this time, the 202R had not operated well with the TIP for other users, but this stage of the implementation went smoothly.

A 4800 baud line with Bell 208A, Dataphone 4800 modems was then installed. After some difficulties in testing it, due to voltage reductions in the New York area, the line was declared operational. A pair of T-4 Timeplex multiplexers were then connected at NAC and at the CCA-TIP site.

Initial tests proved that the multiplexer line configuration was successful; the TIP recognizing each line uniquely at the speeds required. Two problems were uncovered at this time.

1. The multiplexer was equipped with cards which require parity on data received.

2. The multiplexer was equipped to operate with attached modems rather than terminals.

The first problem caused certain ASCII characters to be acknowledged by the T-4 as control information. This was corrected by ordering different high speed cards which supplied parity for the internal operations of the T-4 and then removed the parity again when outputting the data.

The second problem has still not been satisfactorily settled, although patches have been made to temporarily circumvent the difficulty. Apparently, terminals connected directly to the T-4 do not supply all of the necessary EIA signals which recent TIP versions (TIP's 315 and 316) now require (e.g., carrier detect). After some experimenting, NAC found that by patching EIA signals, data-terminal-ready and request-to-send together the system operated successfully.

The multiplexed line configuration is highly cost effective. Cost for the line from NAC to CCA have two components.

1. The Inter-Exchange Connection (IXC) charge from NAC to the Telpak connection (approximately 8 miles) is $26.50/month.

2. There is a Telpak charge of $0.50/mile/month for the remaining air miles to CCA.

The equipment charges at both sites include the price of the modem and additional equipment at the specified site, and a charge for the equipment at the IXC/Telpak connection which is included in the equipment charge for one of the sites. The 103 A2 modem also had alternate voice capability. Table 1 shows a breakdown in cost per modem.

The total cost is the monthly cost for the modems and lines; the Telpak charges are conservatively computed assuming 300 air miles from Nassau County, New York to Boston, Massachusetts.

The cost of supporting the Timeplex T-4 multiplexer, Bell 208A modem, and leased line configuration is $522.90 monthly for the line plus a $1,610.00 purchase price for each T-4 unit. Without the multiplexer, three lines with 202R modems and one line with 103 A modems would cost $1,193.10 per month. Thus, within six months, both multiplexers have been completely paid for with the savings generated.

3. SOFTWARE FOR LARGE SCALE NETWORK COMPUTATIONS

3.1 A PROTOTYPE - THE NETWORK RELIABILITY ANALYZER

The first effort at developing improved large scale network software was aimed at a typical applied network problem-network reliability analysis. The system designed was used as a prototype. User experience was used to suggest improved computational techniques for large scale network problems and to evaluate the implementation of the conjectured improvements.

The abstract problem of network reliability analysis is to calculate the unknown reliability of a network given the reliaiblities of its nodes and links. The applied problem was the reliability analysis of data communication networks, in particular, the analysis of ARPANET. In the original topological design of the ARPANET [Roberts and Wessler, 1970], [Frank et al, 1970] network reliability was assured by requiring that at least two communication computers or telephone lines must fail before two functioning computers are disconnected from one another. As data was collected on the availability of lines and IMP's, the availability of the network became a parameter that was not only important but was also computable. In the first phase of the research, a very flexible simulation method was developed for performing this calculation [Van Slyke and Frank, 1972(a)], [Van Slyke and Frank, 1972(b)]. Since many samples must be taken in a simulation model to achieve a small variance, it is essential to make the sample determination as efficiently as possible. To do this, computational complexity analysis was brought to bear [Lawler, 1971], [Miller and Thatcher, 1972]. First, it was discovered that the sample determination is essentially equivalent to finding a minimum spanning tree on the network. In [Kershenbaum and Van Slyke, 1972], a careful analysis of the computational complexity of minimum spanning tree algorithms was carried out resulting in substantial improvements over traditional algorithms. Generalizations of these techniques of computational complexity analysis are being applied to other important areas of network analysis.

The result of this phase of the work was a very flexible and efficient computer algorithm written for remote job entry to a CDC-6600 computer. Much useful information was developed using this code. It was found that for networks the size of the current ARPANET the reliability implied by requiring at least two elements to fail before any disconnection yielded adequate available for the net; although it was also discovered that for larger nets, this approach was not adequate. Reconfigurations of ARPANET which dramatically increased reliability at very little cost were discovered using the program. Variants of the program were applied to a wide variety of communication networks including centralized networks and command and control networks [Frank, 1974].

At this point, efforts to more effectively use computational resources for solving network reliability problems were intensified. The first experiment along these lines was to make the program interactive so that analyzing the reliability of a sequence of variants of a basic configuration could be carried out conveniently. The first crude version of an interactive network analyzer was demonstrated at the International Conference on Computer Communication in Washington, D. C., October 24-26, 1972. The program was implemented on an IMLAC PDS-1D graphics display unit working with a PDP-10 with a TENEX Operating System. Communication was through the ARPANET. The user could type in a network of his chosing or start with a very small version of the ARPANET having 10 nodes. He could then edit the network by adding or deleting links. He could specify node and link failure probabilities either by assigning a common value to each node and another common value to each link or by assigning different values to each element.

Finally, a random number seed, the number of samples, and the range of variation for the probabilities had to be specified. After the simulation was completed, the user had a choice of tabular output, graphical output or both. The program then displayed the network analyzed so that further modifications could be made and another simulation carried out. The program had the capability simultaneously displaying the reliability curves resulting from three different simulations. Thus, one could easily compare the reliability of networks where reliabilities and/or topology were varied. This first version was well received at the conference. It was also considerably easier to use than the previous remote job entry version for demonstration problems. However, there were still several serious inadequacies:

1. The input was required to be in rather rigid format, making it almost impossible to enter any but the smallest of networks.

2. If any errors were made in entering the data, the program had to be restarted from the beginning.

3. If any portion of the network was modified, the entire display had to be redrawn (and retransmitted from the PDP-10).

4. Nodes could not be added or deleted from the network.

5. The program could only be operated on the IMLAC/PDP-10 hardware configuration.

6. The display of a net could not be modified to make it clearer.

7. Large networks had to be entered by the programmer and could not be saved. Network topologies could not be read in from a previously created file.

8. Operating the program was slow because of the necessity for elementary operations for use by beginners of the system.

9. Nodes and links could only be differentiated by sequential numbers assigned by the program rather than by labels or by varying symbols.

10. Since the simulation was being carried out on a time-shared computer, and the computation was extensive, it was only possible to analyze small networks using a small number of samples. For example, for the demonstrations at ICCC, even for 10 node networks and 100 samples, the delay due to competition with other users of the time sharing system could run to several minutes when the computer was heavily loaded.

11. The graphic display is vulnerable to line noise since it is connected to the time sharing computer by an asynchronous line with no error checking or facilities for retransmission. Moreover, the display vector commands are usually given relative to the end of the previous vector. Thus, if noise causes a translation of one vector it also causes a translation of all of the vectors dependent on it.

12. Very large networks could not be displayed (even if they could be read in - see 1, 2, 6), because of limitations on display screen size.

It is noteworthy that all of the above twelve difficulties could occur in <u>any application</u> of standard computational systems to large scale network computations. In the next phase of research, prototype computer utilities were developed which not only made the network reliability analyzer more effective, but which can also be used for better large scale network computations in general.

This effort culminated in early Summer, 1973 with the second version of an interactive network reliability analyzer. Many, but not all, of the twelve difficulties of the earlier version were eliminated by the creation of several network analysis utilities. Free format routines were created; opportunities for remodifying data without restarting were inserted; modification of the pictorial representation of a network could be made without completely redrawing the network; nodes could now be added, deleted, and moved on the display; a teletype version of the program was installed on the ARPANET for other users who did not necessarily have an IMLAC (This program was used successfully by at least one user at SRI); networks could be read from files, and, after computation, large networks could be saved on files automatically; and nodes could be characterized by alphanumeric lavels as well as by differing symbols, e.g., diamonds, boxes, triangles and circles. In addition a "mouse" capability for directly interacting with the graphic representation of a network was introduced.

Work currently in progress is to use distributed computing to combine the advantage of the raw computing power of basically remote job entry computers, such as the CDC-6600 and the IBM-360/91, with the flexibility and ease of use of interactive computing on time sharing computers. The basic configuration is to use a TTY, CRT, or graphics terminal together with a time sharing computer like the PDP-10 TENEX system for entering data and generating a remote job entry file for transmission to a large scale scientific machine which does the computation. The output is then returned to the time sharing computer for examination. Two network analysis programs, a program for the topological design of regional high density terminal oriented TIP networks, and a program for traffic routing on distributed communication networks, have been used in this manner. However, the programs were basically straightforward remote job entry codes without the interactive features developed for the network reliability analyzer.

The next version of the network reliability analyzer, which will be completed shortly, will combine the desirable features of the interactive version and the computational efficiencies of the first remote job entry version.

3.2 A SOFTWARE TASK FORCE

A review of the computation requirements for large network problems based on years of extensive overall experience in the area or large scale network computations and on the particular experience gained in building the various forms of the network reliability analyzer was begun in the Summer of 1973. Initial conclusions were that it was now possible to design a general computation system for large network problems. To this end in July of 1973, a task force was appointed to design a computation system that would exploit recent advances in distributed computation, interactive graphics, programming language theory, theory of data structures, and other modern computational resources for the more effective solution of large scale network problems.

Included in the mandate for the group were the following requirements and engineering constraints:

1. Ease of Use: The system should be equally accessible to users with little programming or network analysis experience and to experts. This is to be accomplished by;

    a. Several levels of documentation, ranging from one or two sheet explanations of the most useful and basic aspects of the system up to full documentation of the system which should suffice for expert programmers to recreate the entire system;

b. On line interactive aides which can inform users in real time how to use the system; and

c. Intelligently chosen defaults for options which will allow users to ignore the more erudite features of the system which they may not need.

2. Efficiency in the use of;

a. Communication Networks

b. Computer time (in execution of computing bounded programs).

c. User time and patience.

3. Capacity: The system should be capable of handling large scale networks.

4. Machine Independence: In order to take advantage of resource sharing on the ARPANET, the system (with slight and well defined modifications) should run on the major computers of the network.

5. Flexibility: It is most important that the system be developed in stages and that the result of each stage be useful and applicable without the need to wait for the system to be completed.

Required of the group in addition to a system design, was a schedule of implementation which would be gradually carried out with the most useful features implemented first so that practical and useful results would be apparent from the beginning. The design and the schedule of implementation should be motivated by requirements generated in practice based on the experience gained by the network reliability analyser.

3.3 PRELIMINARY RECOMMENDATIONS AND INITIAL IMPLEMENTATION

By the late fall of 1973, a consensus was beginning to emerge on the general outline of a large scale network computation system. It would consist of two languages - a Network Editing Language and a Network Language. The Network Language in turn has two parts - a Network Structure Language and a Network Programming Language.

As network problems get larger, necessary algorithmic calculations become more complicated, as expected. However, problems of data management grow much more dramatically and often become the major source of difficulty. Errors in input become inevitable while error detection and correction become much more difficult. In extreme cases, extensive computations are often rerun many times before trivial input errors with obvious effects on the output can be corrected. Thus, it was decided that the initial emphasis should be put on the Network Editing Language. Conceptually, the Network Editing Language is very much like a text editor except that the domain of application is networks rather than text. The main complications come form the variety of ways the network being modified can be displayed to the user. The network structure can be fed back to the user as a list of nodes, links and properties on a TTY of CRT alphanumeric display, or graphically on an interactive display device or flat-bed plotter. Text editors must often deal with more text than can be displayed at once. Equivalently, the Network Editor must deal with networks that are too large to be displayed in their entirety on a graphics terminal: Consequently, windowing and other graphics display techniques must be built into the editor. In the next section, the design of the first version of the

Network Editor is given. The Network Language is in a more rudimentary state. As mentioned before, it consists of two sub-languages, a Network Programming Language and a Network Structure Language. The Network Programming Language will be an extension of FORTRAN which admits network structures and operators resembling, for example, FGRAAL [Rheinboldt et al, 1972]. Most such languages are ineffective in an applicaitons environment because of their relatively rigid data structure. Either the structure is too complex, so that the program runs inefficiently, or it is too simple, in which case many applications do not fit gracefully to the language. The Network Structure Language, which can be thought of as an extension to the declarative statements in FORTRAN, allows one to use exactly that data structure which is appropriate to his problem.

4. THE NETWORK EDITING LANGUAGE

The Network Editing Language provides a set of commands by which the user can modify a network data structure. The network data base consists of nodes, links, and properties corresponding to nodes and links. The editing language must supply commands to add and delete nodes and links and assign values to their properties. It also supplies the necessary prompts to the user when input is required.

4.1 DESIGN

Three factors must be considered in the design of the command language: 1) simplicity, 2) consistency, and 3) economy. All are closely interconnected, and the command syntax must be written in such a way as to optimize each factor relative to the others. The goal is to minimize the number of commands, define each on the same syntactical base and limit the amount of input required for each command. Special features and options can be added for experienced users, but the basic command structure should be maintained.

4.1.1 Command Syntax

Commands are defined to be any input or sequence of inputs that modity the user data base. A command consists of two parts: 1) the Verb, and 2) the Agruments. The verb is a single character which will activate a process. Requiring the verb to appear before the arguments has two advantages:

1. Commands are easier to process and allow a variety of argument syntax, and

2. The program is able to respond to each succeeding input element in an appropriate manner and prompt the user if data is not entered.

The arguments are the necessary control information and data for editor use. An example of a command is 'A, N, 5' where 'A' is the verb to ADD and 'N' and '5' are the arguments specifying Nodes and the quantity 5. This command would add 5 nodes into the data base.

4.1.2 Command Structure

Commands must be simple enough for a new user to learn and concise enough to allow experienced users to omit reduntant control information. For example, consider the command to assign a value to a node property in the network. The basic command allows the user to enter a property identifier, the "pid," and the value to be assigned for a specified node, the "pval." Assume that nodes have 2 properties, their TYPE (pid = T), and LABEL (pid = L). The basic command allows the user to assign a property to a node, node 5 for example:

S, N, 5, T, 1
S, N, 5, L, NO5,

while the concise command allows listing the pid, pval pairs:

S, N, 5, T, 1, L, NO5.

Even the second form is still verbose if these properties have to be assigned to a large number of nodes. For this, a special command M(for MACRO) provides very concise commands for frequently performed operations on large data bases.

In the sections to follow, commands will be introduced on the most basic level, and additional features are discussed.

4.2 NOTATION

The following symbols and notations are used in command descriptions:

%	= =	carriage return
$	= =	escape
,	= =	comma
<	= =	prompt
<<	= =	continuation prompt
[]	= =	information enclosed in brackets is supplied by program.
b̸	= =	blank or space
#	= =	positive integer
pid	= =	property identifier
pval	= =	property value
N_i	= =	node label for node i
N_i, N_j	= =	node label pair defining link (N_i, N_j)
(//)	= =	delimits a repeated field in a macro command

4.2.1 Delimiters

The concept of a field is important to the command. The verb and each of its arguments in a command, is contained in a field. The field is delimited by a comma, a blank, or a carriage return which terminates the command. Taking the example from above:

S, N, 5, T, 1, L, NO5%

seven fields are specified; the first is the verb, and the rest are arguments. The final argument is delimited by the carriage return.

In the command descriptions, the comma will be used to delimit all fields except the last one. A blank can replace the comma (a string of blanks is considered as one) or can flank the comma for legibility. The blank cannot, however, replace a comma before a carriage return which specifies continuation of an argument list. Another set of delimiters will be discussed in Section 4.6.1. These will involve additional symbols which will not delimit the fields as such, but rather serve as guides for the user on inputting data.

4.3 COMMANDS: BASIC LEVEL

At the basic level, we are interested in supplying commands to do what is minimally required to enter a network definition. These commands are more suited to editing an existing data base, but could be used to enter a "small" network of say, 10 nodes and 20 links, each with a few properties. Primarily, they are presented to give the flavor of command structure and description techniques.

4.3.1 Add Command - 'A'

The 'A' command adds nodes and links to the network. The nodes are assigned default labels as they are added by which the user can reference that node in later commands.

[<] A, N, #%

will add '#' nodes to network.

[<] A, L, N_{i_1}, N_{j_1}, N_{i_2}, N_{j_2},..., N_{i_k}, N_{j_k} %

will add links N_{i_x}, N_{j_x}, x = 1, k, k ≥ 1 to network.

4.3.2 Delete Command - 'D'

The 'D' command deletes previously defined nodes and links from the network.

[<] D, N, N_1, N_2,..., N_k %

will delete nodes N_1, N_2,..., N_k, k ≥ 1

(Note that when a node is deleted, any links indicent to that node are also deleted.)

[<] D, L, N_{i_1}, N_{j_1},..., N_{i_k}, N_{j_k} %

will delete links (N_{i_x}, N_{j_x}), x ≥ 1

4.3.3 Set Command - 'S'

Now that the network has been defined, the user can assign values to those properties needed by the program. Assume that nodes have a property TYPE (pid = T), and links have a property LENGTH (pid = L).

To assign a type to node 5, enter

[<] S, N, 5, T, 2 %

In general,

[<] S, N, N_1, pid, pval % for nodes.

[<] S, L, N_{i_1}, N_{j_1}, pid, pval % for links.

To set the value of every node pid to the same pval, enter

[<] S, A, N, pid, pval %

and for links

[<] S, A, L, pid, pval %

4.3.4 Summary

We now have a set of commands to allow the user to enter node and link definitions and assign values to their properties. The keying in of input is considerable. In the next section, we will introduce new commands to perform more actions with less input.

4.4 COMMANDS: CONCISE LEVEL

This section will present commands, or extensions of previously defined commands which can be used to enter the network definition with less effort.

4.4.1 Continuation of Argument List

In commands which accepted a list as part of the arguments, the list was terminated by the first carriage return. By delimiting the last field prior to that carriage return with a comma, the program will read in another line of input and take it as a continuation of the argument list.

For example, if nodes had a few properties, then many nodes could be assigned values for their properties in a single command:

[<] S, N, N_1, pid_1, $pval_1$, pid_2, $pval_2$,%
[<<] N_2, pid_3, $pval_3$, pid_4, $pval_4$ %
[<]

Note the comma before the carriage return in the first line (which is required) and the subsequent double prompt in line 2.

In the next section, we see how a macro can remove the unnecessary pid's when they are the same for each node.

4.4.2 Add/Set Command - 'A, S'

Since most programs require properties associated with the nodes and links, a reasonable time to enter the values of those properties corresponding to that node or link is when it is initially defined. Thus, a new command is introduced which does two things:

1. It will add the element being described if it does not exist, and

2. It will assign the given values to that element.

For nodes, entering:

$$[<]\ A,\ S,\ N,\ N_1,\ pid_i,\ pval_i,\ i \geq 0$$

will add Node N_1, with property values $pval_i$, if N_1 does not already exist.

Similarly, for links:

$$[<]\ A,\ S,\ L,\ N_{i_1},\ N_{j_1},\ pid_k,\ pval_k,\ k \geq 0$$

will add link N_{i_1}, N_{j_1} if it does not already exist.

If the element does exist, it will be considered on error since this is an ADD command.

4.4.3 Summary

The amount of control information is now only a percentage of the total amount of data keyed in. Even this amount is much too large for a 1,000 node networks. Although it's impossible to completely eliminate the control information, we can reduce it considerably by using macros for frequently performed operations. Macros and variable length property values are the subject of the next section.

4.5 COMMANDS: EXPERIENCED USER LEVEL

For networks with nodes (or links) which all have the same properties, and values which must be assigned to the properties, it would be ideal to add the node, assign the values, and omit all control information. This is done by the MACRO command.

For some properties, the value assigned might be a combination of two or more values. For example, the LOCATION of a node might be its longitude and latitude. Longitude and Latitude are properties of LOCATION, not of the node, so that in response to a SET of property LOCATION for a node, the pval becomes a pair (longitude, latitude) which must now be added.

4.5.1 Macro Command - 'M'

In an effort to eliminate control information, the Macro command supplies a command skeleton with slots for required input. A macro is constructed by preceding a command by 'M'. The program will now save this input, and all further input will be controlled by this skeleton or format. Whenever a '*' is encountered, a value is taken from the input and the command processing continues. For example, suppose we wich to enter a large number of nodes with the following properties:

1. DEGREE pid = D
2. X-COOR. pid = X
3. Y-COOR. pid = y

We can use the Add/Set command:

```
[<] M, A, S, N, *, D, *, X, *, Y, * %
[Macro ready]

[<<] 1, 1, 120, 150 %

[<<] 2, 1, 135, 173 %

[<<] 7, 2, 200, 153 %
        .
        .
        .
        .
[<<] 88, 2, 350, 90 %

[<<] ↑Z %

[<]
```

The ↑Z (control - Z) terminates the macro and returns to the command mode. The macro defined above will input the lines following as if the values were entered with all the necessary control information:

```
[<] A, S, N, 1, D, 1, X, 120, Y, 150 %

[<] A, S, N, 2, D, 1, X, 135, Y, 1[illegible] %
        .
        .
        .
        .
```

An attempt must be made to keep the range of the macro restricted or else the system becomes unduly complex.

4.5.2 Multiple 'Pvals'

In some applications, a property may be considered as a group of values. For instance, if we assume nodes to have properties X-COOR. (X) and Y-COOR. (Y), the assignment command has two control arguments:

[<] S, N, N_1, X, pval, Y, pval

This could be extended to a Z-COOR. Therefore, one way to reduce the control information is to group the three (or more) properties under one property (say POSITION (P)).

[<] S, N, N_1, P, pval, pval, pval,...

This is a possible extension of the language. However, the macro feature should be able to handle this case whenever information of this type is to be entered:

[<] S, N, *, X, *, Y, *, Z, *

If editing must be done on these properties, they can be entered individually rather than grouped.

4.5.3 Argument List

The properties entered on a line of input may vary as to length; i.e., there may be a property which is a list of pvals. Suppose we consider nodes to have the property of the number of terminals (K) connected to it and a list of the terminals (T)

[<] S, N, 14, K, 3, T, T1, T, T2, T, T3

specifies that node 14 has 3 terminals, (T1, T2, T3) connected to it.

If this command is incorporated as a macro, a question arises as to how many slots to leave in the list. For this application, we introduce the list element in an argument list in a macro.

[·] M, S, N, *, K, *, (/T,*/)

where the (/T,*/) is the list element to be repeated 'N' times under program control.

4.6 OTHER DESIGN CONSIDERATIONS

This report has introduced the basic command syntax and the commands supplied to edit and initialize a network data base. Other areas in this editing language must still be considered, and the following sections we discuss a few of them.

4.6.1 Input Delimiters

To allow input to be more readable to the user, the following specifications are proposed:

1. Parentheses can delimit a node pair defining a link as (N_i, N_j).

2. A pid, pval pair can be entered, pid = pval,

3. A verb can be followed by a slash as

   [<] A/N, 5 %

   to highlight the 'A' as a verb.

Difficulties arise when programming such a syntactical scheme in a language such as FORTRAN if the delimiters are permitted to be entered optionally. Thus, as a matter of policy, these symbols will simply be considered blanks and become part of the delimiter separating the fields it lies between.

4.6.2 Input/Output Commands

Commands must be provided to permit the user to list on the teletype, the network definition, (or cause it to be displayed or plotted), and to save versions of the data base for subsequent sessions. This also implies a command to input the information saved from a previous session.

4.6.3 Error Detection and Handling

Three types of errors can occur while inputting information: 1) the user may mis-key a character (e.g., hitting a '0' instead of a '1'), 2) the user may enter an invalid command; and 3) the user may enter data which the program determines invalid.

The first case is the simplest and requires a special character such as ↑A to delete the most recently entered character.

The second error condition occurs when the program is expecting a certain input (such as the number of nodes to be added) or a set of inputs (such as 'N' or 'L' for a Delete command). The third condition is similar to the second in that the program detects an error which is found to be inconsistent with the data already available. It is not clear in the latter case whether the input on which the error was detected, or previously entered information, is the cause of the error. Procedures for handling these errors will be developed as a result of user experience (and misadventures).

4.6.4 Prompts to User

When the program is expecting input, there should be a short prompt available to be displayed to the user when the is unsure as to what is expected. This is especially desirable for a new user of the system, but is always an aid when input has become "messy" or the program must be continued.

CHAPTER 9

PACKET RADIO SYSTEM - NETWORK CONSIDERATIONS

1. INTRODUCTION

The Packet Radio System is a broadcast data network extending a point-to-point packet communication system (such as ARPANET [Roberts, 1973]) to provide local collection and distribution of data over large geographical areas. Since the system is wireless, it will be especially effective for mobile devices, devices for which the peak data rate requirements are much higher than average requirements, and devices for which hardwire connections are not feasible. The system is designed to be economical, reliable, secure, and conservative of spectrum. The properties and implementation of a Packet Radio System are being studied, under the direct guidance of ARPA, by Collins Radio Corporation, Network Analysis Corporation, Stanford Research Institute, the University of Hawaii, and the University of California at Los Angeles. An extensive discussion of the uses and need for the Packet Radio System is given by the project director, R. Kahn [Kahn, 1974].

In this chapter we discuss the network aspects of the Packet Radio System which must be taken into account in order to make the system workable. Any network consists of nodes and links. In the Packet Radio System, nodes correspond to communication devices, and links to transmission on the channel. These aspects are discussed in Sections 3 and 4. The geographic layout of the network is discussed in Section 5.

2. NETWORK OVERVIEW

There are three basic functional components of the Packet Radio System: the Packet Radio Terminal, the Packet Radio Station, and the Packet Radio Repeater. (See Figure 1) Packet Radio Terminals will be of various types, including personal digital terminals, TTY-like devices, unattended sensors, small computers, display printers, and position location devices.

The Packet Radio Station is the interface component between the broadcast system and the point-to-point network. It will have broadcast channels into the Packet Radio System and will have link channels into the point-to-point network. In addition, it will perform accounting, buffering, directory, and routing functions for the overall system.

The basic function of the Packet Radio Repeater is to extend the effective range of the terminals and the stations, especially in remote areas of low traffic, and thereby increase the average ratio of terminals to stations. A more detailed discussion of the network hardware functions can be found in Section 3 .

The devices (repeaters, stations, and terminals) of the Packet Radio System communicate in a broadcast mode using the Aloha random access method [Abramson, 1970] which is suitable

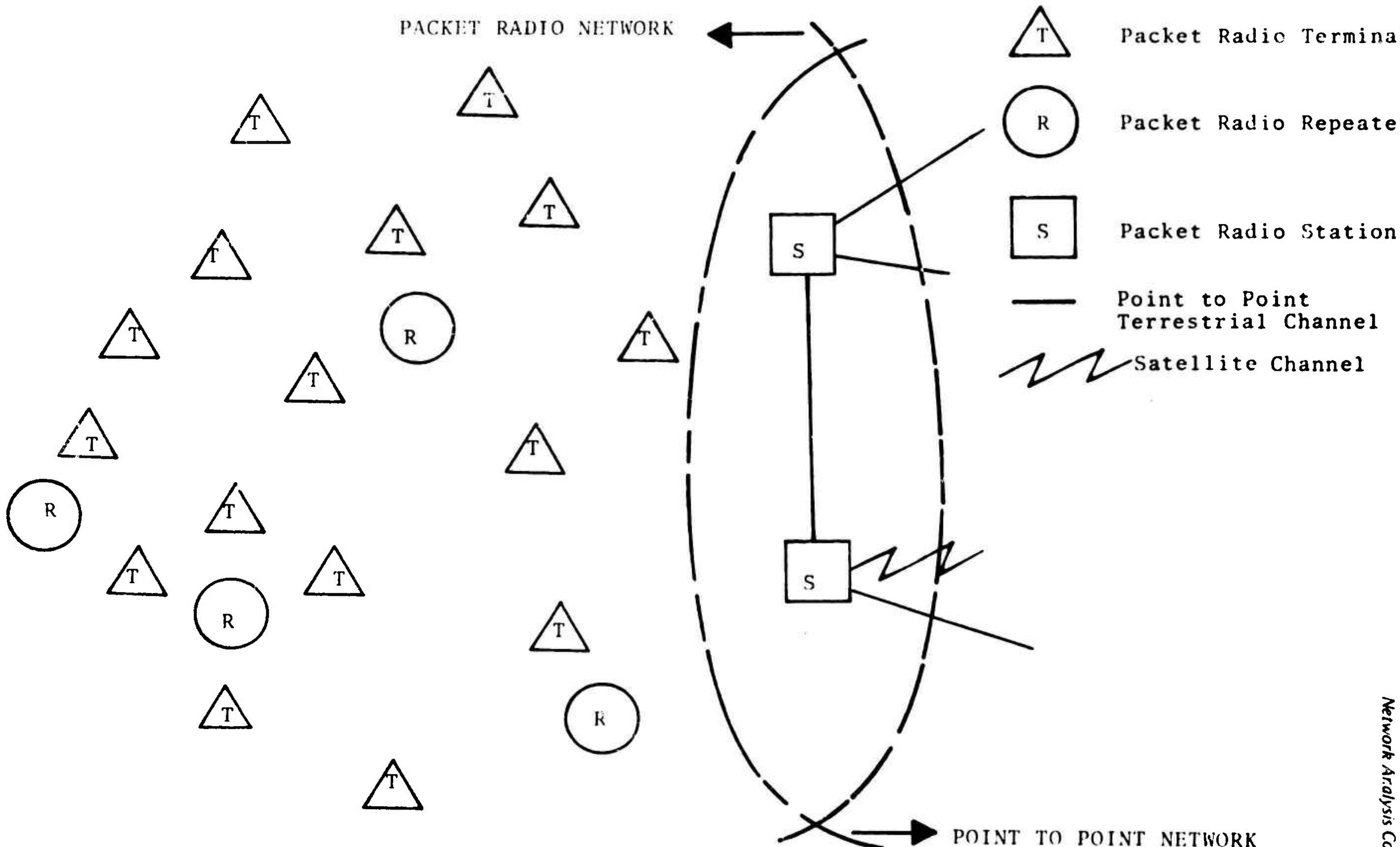


FIGURE 1

PACKET RADIO SYSTEM

for terminals [Roberts, 1972(a)] that:

(i) are mobile, so that a broadcasting mode is necessary,

(ii) are located in remote or hostile locations where hardwire connections are infeasible,

(iii) have a high ratio of peak bandwidth to average bandwidth requirements (because the Aloha method allows the dynamic allocation of channel capacity without centralized control), or

(iv) require little communication bandwidth so that hardwire connections are uneconomical.

The channel characteristics of the Packet Radio System are described more fully in Section IV.

Stations will be allocated on the basis of traffic. Thus, to first approximation, we can think of partitioning the area to be covered (e.g. the United States) into regions of equal traffic and allocate one station for each region. In regions of low traffic density, the station may not be in "line of sight" of all the terminals in the region; hence repeaters are used to relay the traffic to the station. Thus, repeaters correspond to a geographical partition of the area into sections small enough so that each terminal can communicate with a repeater and be relayed by it to a station.

In areas of high traffic, such as urban areas, repeaters may not be needed: in fact, the problem may be that a station can communicate with more terminals than it can handle. Broadcast of data in urban areas is also complicated by extreme multipath interference [Turin, 1972]. The rapidly expanding Cable Television (CATV) Systems within urban areas offer an attractive alternative to over-the-air broadcasting, except for mobile users who must use broadcast techniques. The same general Packet Radio concept can be applied to Packet Communication on CATV systems. This approach is explored in detail in [NAC, 1974(b)].

As a gross estimate of the size of such a system, suppose the 3,536,855 square miles of land area in the United States are to be covered. Then, if the average useful area that a repeater can cover is $\pi 10^2$ square miles, we would need approximately 11,258 repeaters.

In Table 1 a sample traffic distribution is given, based on one due to L. Roberts [1972(b)]. We assume that the number of the various types of terminals is proportional to population. Thus, in column (b) of Table 1 we give the relative number of various terminal types per N people, where N is an arbitrary constant of proportionality. In the table reasonable assumptions relative to the numbers of the different devices are made. For example, there are twice as many (100) TTY's and personal radios as position locators (50) per N people. Columns (a) through (h) specify the characteristics of the devices. Columns (i) and (j) give

TABLE 1--TEST TRAFFIC DISTRIBUTION[1]

(a) Traffic Type	(b) No./N[4] People	(c) Rate In Messages/ Second/ Device	(d) Rate Out Messages/ Second/ Device	(e) Message Size (without header bits)	(f) Fraction of Time Operating	(g) Device Peak Bit Rate In (bits/sec)	(h) Device Peak Bit Rate Out (bits/sec)	(i) Total Average Bit Rate In bxcxexf bits/sec exclusive of overhead	(j) Total Average Bit Rate bxdxexf bits/sec	(k) # of Bits Using 1000 + 250 Bit Packets[8] $\lceil \frac{e}{1000} \rceil \cdot 1250$ per device message (incl. overhead)	(l) Total Average Bit Rate In bxcxfxk (including overhead)	(m) Effective Total Traffic Out bxdxfk (including overhead)
Compressed Voice	33	2.5	2.5	1000[3]	.03	900	900	2475	2475	1250	3093.75	3093.75
TTY & Hand Held Data Terminals[2]	100[3]	1.	.1	100	.04[3]	300	300	400	40	1250	5000.	500.
Unattended Sensors	160	0	.01	50	1.	0	110	0	80	1250	0	2000.
Small Computer	1	10	1	1000	.05	50000	50000	500	50	1250	625	62.5
High Speed Display (3) Printers	1.6	10	0[3]	1000	.05	19200	0	800	0	1250	1000.	0.
Position Location[6]	50	0	.01	0	1.	110	110	0	0	250[6]	0	125.
TOTAL	345.6							4175	2645		9718.75	5781.25
								4175. + 2645. = 6820.			9718.75 + 5781.25 = 15500.	

Notes:
1. Traffic based on estimates by Larry Roberts 12/72.
2. Personal Radios added in with TTY.
3. Modification of Roberts estimate.
4. Number of devices assumed to be per N people.
5. $\lceil n \rceil$ is the smallest integer greater than or equal to n.
6. Position Location information assumed to be in packet header.
7. Message size assumed the same for incoming and outgoing messages.

the average rate of useful information being transmitted, while columns (l) and (m) give the rates when overheads due to headers and partially filled packets are included. Columns (g) and (h) are conservative estimates of the bandwidth required by each device if it had a dedicated channel with sufficient capacity to handle the peak rate of the device.

Based on the assumptions made, N people would generate 4175 bps into the terminals and 2645 bps out. This makes no allowances for a packet header, retransmissions, artificial traffic due to repeaters, acknowledgments, or other overhead.

We assume a one frequency system with 100 Kbps channel capacity, used in an unslotted Aloha random access mode (See Section IV). Using such a scheme, conflicts due to interference between packets reduces the maximum bandwidth to 1/2e of the nominal value or 18394 bps.*

From Table 1, we see that N people will generate 15500 bps, including overhead, given a packet size of 1000 bits of information plus 250 bits of header. Of the 15500 total bps, only 6820 bps is useful information; the rest is overhead--packet headers and wasted space from partly filled packets. Thus, the channel has a possible maximum effective utilization of $\frac{1}{2e}\left(\frac{6820}{15500}\right)$ = .08093, yielding for the 100 Kbps channel a maximum rate of useful information of 8093 bps.

---

*Recent studies indicate that a two data rate system, with high rate repeater to repeater channels and low rate terminal to repeater channels may be desirable.

It is instructive to compare the efficiency of an Aloha random access channel with schemes using dedicated channels for each device either by frequency or time multiplexing, even though the implementation scheme for a conventional time division or frequency multiplexed scheme on the scale of the Packet Radio System is not clear. The comparison given below is highly conservative since guard bands, packet headers, and synchronization requirements are ignored for the dedicated channel systems. Moreover, if time division multiplexing were used, system wide synchronization would be required. If such synchronization were obtained, the complexity of the hardware would be at least as great as that needed to implement a slotted Aloha scheme. Such a slotted Aloha scheme would lead to a gain by a factor of two in channel utilization over the unslotted Aloha channel. Finally, it is assumed for ease of analysis that the packets in the Aloha system must all be of equal size (the 1/e formula only applies to this case). In the actual system, packet size would vary - again increasing the utilization of the Aloha channel.

In columns (g) and (h) of Table 1 we list the estimated dedicated channel capacity required for each device. The total bandwidth required is then $\Sigma(b)(g+h)$ = 278720 bps. The total average rate of useful information carried by the channel is 6820 for an effective utilization of 6820/278720 = .0244. Thus, a very conservative estimate is that the Aloha system is 3 to 4 times as efficient as dedicated channel systems; and more realistically, we can expect at least an order of magnitude improvement in efficiency when hardware complexity and actual overhead factors are considered.

The appropriate choice of N to scale the traffic is arbitrary. Table 2 contains the number of stations, the number of people per 100 Kbps channel, the number of stations outside the large metropolitan areas, and the population density at which the population associated with a station covers the same area as a repeater. These numbers are all a function of N, assuming a 100 Kbps channel. In Table 3, the ratio of repeaters to stations is related directly to traffic density in bits per second per square mile.

For large N some stations would provide area coverage and thus replace repeaters. The extent of the replacement depends on the distribution of population density. The average population density in the U.S. is 57.6 people/square mile; the density in the states range from 0.5 people/square mile in Alaska to 953.1 people/square mile in New Jersey. The density in Manhattan is 67,808 people/square mile. At high traffic levels (N < 10,000 or traffic density > 100 bps/square mile) repeaters play a much smaller role, since the system is now traffic limited. In extreme cases, repeaters may not even be necessary. The 148 Standard Metropolitan Statistical Areas (SMSA) with more than

TABLE 2

N	Stations	People/ 100 Kbps Channel	Stations Outside SMSA's	Repeaters/ Station Outside SMSA's	Critical Population Density
100	1712013	118	638312	.02	.38
1000	171202	1186	63831	.16	3.78
10000	17121	11867	6383	1.65	37.77
100000	1713	118670	638	16.55	377.74
1000000	172	1186707	63	165.56	3777.41

TABLE 3

Traffic Density bps/sq. mi.	Sq. miles per channel (100 Kbps or 8093 bps effective)	No. of Repeaters (with 10 mile effective radius required per station)
1	8093	25
10	809.3	3
100	80.9	0
1000	8.1	0
10000	.8	0

200,000 population contain 127,417,000 of the 203,166,000 people in the United States in 301,661 square miles out of the 3,536,855 square miles of land area in the United States. Thus, the 148 SMSA's would correspond to enough area for 960 out of the 11258 repeaters, many of which could be replaced by the stations for area coverage. In the remaining part of the U.S., 10,298 repeaters would be required.

We can already draw several preliminary conclusions from this simple analysis. For low traffic levels (N > 100,000 or traffic density < 10 bps/sq. mi.), the number of stations << numbers of repeaters << number of terminals; hence, the assignment of functions to devices should be such that the terminal is as simple and cheap as possible, the repeater only slightly more sophisticated, and as many functions as possible should be delegated to the relatively few stations and the point-to-point packet communication network connected to them.

We now turn to a more detailed examination of the Packet Radio Network.

3. NETWORK NODES - TERMINALS, REPEATERS, STATIONS

In this section we discuss the devices' functional capabilities which are necessary for communication in the Packet Radio network.

Terminals: There are two categories of terminals; (i) those which usually await a response to a message they transmit (e.g. manually held radio terminals, small computers), and (ii) those which do not require such responses or acknowledgements (e.g. unattended sensors, position indicators). Some terminals in the former category will usually send and/or receive several packets in one message.

Necessary or desirable capabilities of a terminal:

(1) Ability to identify whether the packet is addressed to its ID.

(2) Calculation of packet checksum.

(3) Character generator logic.

(4) A random number generator for retransmission, or an assigned random number for this purpose.

(5) Capabilities related to packet routing such as; terminating retransmission when acknowledged, recording and using a specific ID of a repeater and/or station to be used for other packets of the same message, counting the number of retransmissions.

(6) Capabilities related to the response to previously determined types of error (see also [Roberts, 1972 (a)].

(7) For unattended terminals, capabilities by which a centralized control or a station will be able to identify whether the terminal is operative or dead.

Repeaters: Functional capabilities for repeaters include:

(1) Calculating packet checksum.

(2) Packet storage and retransmission.

(3) Capabilities by which a station can determine whether a particular repeater (or any repeater in a particular area) is operative or dead.

(4) Capabilities (1), (4), and (5) of terminals.

(5) Capabilities, dependent on the routing strategy, for calculating the most efficient next repeater on a transmission path to the station.

The routing functions in each of the three devices, Stations, Repeaters, and Terminals will, at least initially, be implemented in software on a micro-computer. The implementation of the routing algorithms will be described in a forthcoming report.

Stations: The station will have a broadcast channel for communication with the Packet Radio network, terrestrial channels for communication with the high speed point-to-point network, and possibly a satellite channel. Among the stations' functional capabilities are:

(1) Cryptographic apparatus suitable for handling sensitive and private messages.

(2) A directory of terminals and repeaters in its region.

(3) Operations necessary to convert packets from the

Packet Radio System into packets used in the point-to-point network and conversely.

(4) Storage buffers for packets received from terminals and for packets to be transmitted to terminals.

(5) Storage for character position information for active terminals.

(6) Character generation logic.

(7) Accounting capabilities.

(8) Capabilities related to routing of packets.

Adaptive Power

One can resolve conflicting power requirements between urban and rural use of terminals by having transmission power adapt to the local requirements. This is usually what people do when they speak and cannot be heard. Specifically, every device retransmits a packet with increased power until acknowledged, up to some maximum specified number of times. This increases its probability of being captured each time. (If the transmission is not successful because of collision with another packet, an increase in the transmission power of both transmitters will not resolve the problem.) Many aspects of this approach need study. General power considerations for the Packet Radio System are discussed in "Propagation Considerations and Power Budget" [Col, 1974].

4. NETWORK LINKS - THE CHANNEL

Communication between devices is by broadcast, using a variant of the Aloha random access method. Many aspects of the broadcast channel are of peripheral interest in the network design of the system; however, some factors are crucial to determine the behavior of the network. By Aloha transmission we mean the use of a channel which is randomly accessed by more than one user. In the simplest case users transmit equal size packets, each using a speed equal to the channel speed. Several modes of operation are possible. The two simplest are: a non-slotted (asynchronous) mode in which users can access the channel at any time, and a slotted (synchronous) mode in which users can access the channel only at the beginning of a slot of time duration equal to a packet transmission time. In the latter case, a form of synchronization is required since each user must determine the beginning time of each slot. The following theoretical results assume that, if two or more packets overlap, none is correctly received, and each must be retransmitted. Such a system is called a system without capture.

The simplest analytic results assume that there are an infinite number of users and that the point process of packet origination and the point process of packet originations plus retransmissions are Poisson with mean S and G, respectively; constant transmission time T for each packet is also assumed. Then, if a packet begins at some random time, the probability that it is correctly received (no overlapping, collision or conflict) in the nonsloted case is e^{-2GT}. The reception rate, equal to the origination rate (assuming that colliding packets are retransmitted until correctly received) is $S=G\ e^{-2GT}$. The effective channel utilization is $ST = GT\ e^{-2GT}$, and the maximum utilization is Max (S T) = 1/2e. For the slotted case, the probability of collision is e^{-GT} which leads to 1/e as the maximum utilization. GT, the channel traffic is equal to 1/2 and 1 at a maximum effective utilization for the non-slotted and slotted case, respectively [Abramson, 1970].

In the original Aloha system, implemented at the University of Hawaii [Abramson, 1973], a central station communicates with several remote sites. The system contains two channels - one for station to site traffic and the second for site to station traffic. This has several advantages for the Aloha system. First, the station broadcasts continuously to furnish synchronization between all sites. Second, station to site traffic is coordinated by the station so that messages <u>from</u> the station do not collide with one another. Thus, if the traffic from the station has a separate channel from the reverse traffic, retransmissions are substantially reduced. Allocating separate channels for inbound and outbound station traffic is not as attractive when repeaters and multiple stations are introduced. This channel allocation problem is examined in detail in a forthcoming report. Channel improvements also appear to be possible by using Spread Spectrum Coding, which offers the possibility of time capture. Competing packets arriving during the transmission time of the first may be ignored if their signal strength is not too great. When the transmitters are widely distributed, geometric or power capture is also possible [Roberts, 1972 (b)]. With or without spread spectrum a competing signal which is much weaker (further away), then the desired signal will not interfere. Both types of capture can give rise to performance superior to that predicted by the simple unslotted Aloha model. However, capture biases against more distant transmitters since the probability of a successful transmission to the station decreases as the distance from the station increases. Hence, the number of retransmissions increases as well as the delay. In Chapter 12, spread spectrum coding is evaluated as a function of:

(1) the packet arrival rate,

(2) the time bandwidth product K (extent of spectrum spreading),

(3) the signal to noise ratio SN,

(4) the power law assumed for the transmission power of a transmitter at distance r from the receiver, and of

(5) the multiplicity of receivers available at the receiving location.

Channel properties and spread spectrum techniques are discussed in detail in three reports by Stanford Research Institute, "Measurement and Propagation" [SRI, 1974a], "RF Capacity Considerations" [SRI, 1974b], and "Spread Spectrum" [SRI, 1974c].

5. NETWORK TOPOLOGY - REPEATER AND STATION LOCATION

Many more factors affect the location of repeaters and stations than the simple ones discussed in Section II. The calculations used to determine terminal to station fanouts and repeater to station fanouts neglect important considerations. Moreover, considering repeaters as area covers and stations as traffic covers neglects interactions between the two types of devices. In this section we identify complicating factors as well as indicate methods for choosing locations for repeaters and stations.

Factors affecting the location of repeaters and stations in addition to range and traffic are:

(i) Logistics: Some locations for repeaters may be preferable to others because of greater accessibility or more readily available power, eliminating the need for batteries (e.g. on telephone poles or near power lines). Stations should preferably be placed near existing facilities of the associated point-to-point network.

(ii) Reliability and redundancy: For many reasons, redundant repeaters and stations will be required. Since repeaters in remote areas will operate on batteries, it will be necessary to have sufficient redundancy so they need not be replaced immediately. Stations and repeaters will have intermittent and catastrophic failures for which backup is required. Extra repeaters are needed when line of sight to the primary repeater is locally blocked. Random variations in repeater and station manufacture and placement will cause inadequate performance. These factors will mandate a safety margin of redundant coverage in the design.

When a single channel is operated in an unslotted Aloha random access mode, no more than 1/2e of the bandwidth can be used, as discussed in the previous section. However, additional traffic is generated by repeaters, and conflicts created by transmissions between adjacent stations. Some sources of retransmissions are:

(a) For reliability several repeaters or stations must be within range of each terminal. If the repeaters retransmit every packet they receive, one message can generate an exponentially growing number of relayed messages. To prevent one message from saturating the network, traffic control is required. The discipline chosen and its efficiency will probably be the single most important system factor affecting system performance. Two types of undesirable routing through the repeaters can occur. First a message can circulate endlessly among the same group of repeaters if not controlled. Second, even if no message is propagated endlessly, a message can be propagated to a geometrically increasing number of new repeaters in a large network.

(b) For system reliability, more than one station must be able to transmit via repeaters to each terminal. Thus there can be conflicts between adjacent stations which reduces the useable bandwidth and also introduces coordination and routing problems.

(c) In general there will be many routes between any given terminal and any given station. Consequently, more conflicts can result than would be the case if the terminals communicated directly with a station.

Extraneous traffic can also be generated in the point-to-point network if several copies of the same message enter the network from different stations. These duplicates can be identified and eliminated either on entry to the stations or at the destination. In the latter case, traffic is artificially increased, and in the former case, additional computation must be performed by the stations to maintain coordination.

In a forthcoming report, several methods are described for locating repeaters to furnish reliable line of sight coverage of an area containing mobile or fixed terminals at unspecified locations. Repeaters must be located so that any terminal will be in line of sight of a repeater and "sufficient" connectivity will be assured. We wish to minimize the number of repeaters subject to this reliability of service constraint.

It is impractical to consider the infinite number of possible locations for repeaters and terminals. Thus, we limit consideration to finite sets of possible repeater and terminal locations. The choice of these sets will be of great computational importance and will probably be based on an adaptive selection process. For the present, we assume these sets are known and fixed.

Next, the possibility of line-of-sight transmission between pairs of points must be determined. In general, these procedures depend on many factors including effective earth radius; fresnel zones; weather conditions; design, height and orientation of the antenna; and topography between the two points. These factors are beyond the scope of the discussion here. Nevertheless we assume there are known functions which indicate if a repeater at a location can communicate with a terminal or a repeater at another location. More generally, we can consider the probability that the appropriate two locations can communicate. Among the initial problems is the proper choice of a reliability measure. We assume that the packet radio network is for local distribution - collection of data from terminals with small traffic rates compared to the channels' capacity so that throughput is not a major reliability consideration. That is, if any path through the network exists for a given pair of terminals we assume there is sufficient capacity for transmission. Possible measures of network reliability that have proven useful in the analysis of communication networks [Van Slyke and Frank, 1972] are the probability that all terminal pairs can communicate and the average fraction of terminal pairs which can communicate.

For the network design problem (as distinguished from the analysis problem) known probabilistic approaches appear inefficient from both computational and data collection points of view. This suggests that "deterministic" requirements such as "there exist K node disjoint paths between every terminal pair" should be considered in the design stage. This guarantees that at least

K repeaters or transmission links must fail before any terminal pair is disconnected. In a report in preparation, this problem is an amalgam of two well studied network problems, the set covering problem [Garfinkel and Nemhauser, 1972] [Roth, 1969] and the minimum cost redundant network problem [Steiglitz, Weiner and Kleitman, 1969].

CHAPTER 10

ROUTING AND ACKNOWLEDGEMENT SCHEME FOR THE PACKET RADIO SYSTEM

1. INTRODUCTION

In this chapter we discuss routing problems for broadcast oriented packet communication networks. Possible solutions to these problems are described and an approach, tested by simulation, is proposed for system operation.

There are basic differences between the packet radio network and existing point-to-point store and forward networks, such as the ARPANET. For example, the packet radio network serves mobile terminals; devices in the network share a common channel in a random access broadcast mode; and repeaters in this network will have significantly less storage and processing capabilities than the switching nodes in ARPANET like systems. Consequently, many of the routing techniques developed for the point-to-point networks are not directly applicable to the packet radio network.

The objective of the network is to distribute and collect traffic to and from terminals which have high ratios of peak to average traffic requirements. An initial test of the packet radio system is to serve as a local distribution system for traffic destined for the ARPANET from mobile sources.

The network consists of repeaters to provide area coverage and stations to provide traffic management and interfaces to other nets. Stations serve as a major source and sink for the packet radio net. There are many possible paths via repeaters over which a packet originating at a terminal may flow to reach a station. That is, a packet transmitted from a terminal can be received by several repeaters, and there may be several stages of transmission through repeaters before the packet is received by a station.

Some problems that arise in controlling traffic flow in a large scale broadcast network are:

(1) A packet transmitted can be received by many repeaters or stations or not be received by any.

(2) Many copies of the same packet can circulate in the broadcast network .

(3) Many copies of the same packet can enter the point-to-point network at different stations.

Indications of the consequences of not imposing a suitable flow control mechanism can be observed from combinatorial models analyzed in Chapter 11. In these ideal models, the repeaters are located at corner points of an infinite square grid and time is broken into unit intervals, each slotted into segments. A packet transmitted by a repeater can be received only by its four nearest neighbors. If a packet is correctly received by a repeater, it is retransmitted within the next unit interval of time at a random time slot within the interval. Suppose now that <u>a single packet</u> originates at the origin and that the transmission plus the propagation time falls within one unit interval of time. Then after n intervals of time:

(i) the number of repeaters which receive the packet for the first time, $B(n)$, is:

$$B(n) = 4n, \; n \geq 1 \quad B(0) = 1$$

(ii) the number or repeaters through which the packet passed,

$A(n)$, is:

$$A(n) = \sum_{j=0}^{n} B(j) = 2n^2 + 2n + 1, \; n \geq 0$$

(iii) if we assume that a repeater can receive and relay a large number of packets within the same time interval, the number of copies of the same packet received by a repeater at coordinates (d,j) after d + 2k units of time is:

$$N_j^d\ (d + 2k) = \binom{d+2k}{k+j} \binom{d+2k}{k} \xrightarrow{\text{for large } k} 2^{4k}$$

where d is the number of units of time that the packet requires to arrive from the origin to the repeater, and j is the horizontal number of units.

Unless adequate steps are taken, the explosive proliferation of redundant packets will severely limit the capacity of the system. One can now recognize two somewhat distinct routing and control problems:

(1) to ensure that a packet originating from a terminal arrives at a station, preferably using the most efficient (shortest) path; and

(2) to suppress copies of the same packet from being indefinitely repeated in the network, either by being propagated in endless cycles of repeaters or by being propagated for a very long distance.

In Section 2, we outline general techniques which can be combined to provide workable routing schemes. Acknowledgement schemes aimed at achieving high throughput and minimum delay are discussed in Section 3. Section 4 gives a detailed description of an efficient routing scheme. In Section 5, a method for repeater labeling to obtain efficient routing is proposed. Finally, some qualitative properties of the routing scheme proposed in Section 4, generated via a detailed simulation are given in Section 6.

2. POSSIBLE ROUTING TECHNIQUES

There are two key objectives in developing a routing procedure for the packet radio system. First, we must assure, with high probability, that a message launched into the net from an arbitrary point will reach its destination. Second, we must guarantee that a large number of messages will be able to be transmitted through the network with a relatively small time delay. The first goal may be thought of as a connectivity or reliability issue, while the second is an efficiency consideration.

A rudimentary, but workable, routing technique to achieve connectivity at low traffic levels can be simply constructed by using a maximum handover number [Boehm & Baron, 1964] and saving unique identifiers of packets at each repeater for specified periods of time. The handover number is used to guarantee that any packet cannot be indefinitely propagated in the net. Each time a packet is transmitted in the net, a handover number in the header is incremented by one. When the handover number reaches an assigned maximum, the packet is no longer repeated and that copy of the packet is dropped from the net. Thus, the packet is "aged" each time it is repeated until it reaches its destination or is dropped because of excessive age.

If the maximum handover number is set large, extensive artificial traffic may be generated in areas where there is a high density of repeaters. On the other hand, if it is set small, packets from remote areas may never arrive at stations. This problem can be resolved as follows: We assume that every repeater can calculate its approximate distance in numbers of hops to stations by observing response packets. (A labeling technique for this calculation is discussed in Section 5). The first repeater which received the packet from a terminal sets the maximum handover number based on its calculated distance from the station. The number is then decremented by one each time it is relayed through any other repeater. The packet is dropped when the number reduces to zero.

When a station transmits a packet, it will set the maximum hand-over number by "knowing" the approximate radius in "repeaters" in its region.

Even if a packet is dropped after a large number of trans-missions, local controls are needed to prevent packets from being successively "bounced" between two or a small number of repeaters which repeat everything they correctly receive. (Such a phenomena is called "cycling" or "looping.") A simple mechanism to prevent this occurrence is for repeaters to store for a fixed period of time entire packets, headers, or even a field within the header that uniquely identifies a packet. A repeater would then compare the identifier of any received packet against the identifiers in stor-age at the repeater. If a match occurred, the associated packet would not be repeated.

The time allotted for storage of any packet identifier would depend on the amount of available storage at a repeater and the number of bits required to uniquely identify the packet. For ex-ample, more than 4K packets could be uniquely identified with 12 bit words. Thus, 4K of storage could contain identifiers for more than 300 packets. With a 500 Kbps repeater to repeater common channel for broadcast and receive and 1,000 bit packets, this would be sufficient storage for over 1.5 seconds of transmission if the channel were used at full rate. Assuming a single hop would require about 20 milliseconds of transmission and retransmission time, a maximum hop number of 20 would guarantee that any packet would be dropped from the system because of an excessive number of retrans-missions long before it could return to a previously used repeater not containing the packet identifier.

The combination of loop prevention and packet ageing with otherwise indiscriminate repetition of packets by repeaters will guarantee that a packet travels, on every available path, a maximum distance away from its origin equal to its original handover number.

Thus, if the maximum handover number is larger than the minimum number of hops between the terminal and the nearest station, a packet accepted into the net should reach its destination. Unfortunately, with this scheme, copies of the packet will also reach many other points, with each repetition occupying valuable channel capacity. However, if those packets for which adequate capacity is not available are prevented from entering the net, the network will appear highly reliable to accepted packets.

The above routing scheme is an undirected, completely distributed procedure. Each repeater is in total control of packets sent to it, and the stations play no active part in the system's routing decisions. (They must still play a role in flow control.) In the above procedure, no advantage is taken of the fact that most traffic is destined for a station, either as a terminus or as an intermediate point for communication with the ARPANET. Also, the superior speed and memory space of the station is ignored. For efficiency, one is therefore led to investigate directed (hierarchical) routing procedures.

A directed routing procedure utilizes the stations to periodically structure the network for efficient flow paths. Stations periodically transmit routing packets called labels to repeaters to form, functionally, a hierarchical point-to-point network. Each label includes the following information: (i) a specific address of the repeater for routing purposes, (ii) the minimum number of hops to the nearest station, and (iii) the specific addresses of all repeaters on a shortest path to the station. In particular, the label contains the address of the repeater to which a packet should preferably be transmitted when destined to the station.

When relaying a packet to its destination, the repeater addresses the packet to the next repeater along the preferred path. Only this addressed repeater will repeat the packet and only when this mechanism fails will other repeaters relay the message. A detailed description of the directed routing technique proposed is given in Section 4. However, we first discuss acknowledgement structures for message flow since good acknowledgement schemes are an integral part of an efficient routing procedure.

3. ACKNOWLEDGEMENT CONSIDERATIONS

Acknowledgement procedures are necessary both as a guarantee that packets are not lost within the net and as a flow control mechanism to prevent retransmissions of packets from entering the net. Two types of acknowledgements are common in packet oriented systems:

1. Hop-by-Hop Acknowledgements (HBH Acks) are transmitted whenever a packet is received successfully by the next node on the transmission path.

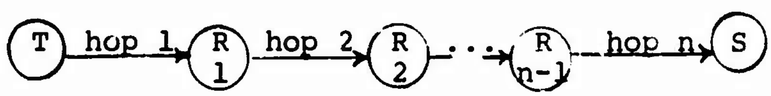
2. End-to-End Acknowledgements (ETE Acks) are transmitted whenever a packet correctly reaches its final destination within the network.

In a point-to-point oriented network such as the ARPANET, HBH Acks are used to transfer responsibility (and thus open buffer space) for the packet from the transmitting node to the receiving node. This Ack insures prompt retransmission should parity errors or relay IMP buffer congestion occur. The ETE Ack serves as a flow regulator between source and destination and as a signal to the sending node that the final destination node has correctly received the message. Thus, the message may be dropped from storage at its origin.

Both types of Ack's serve to ensure message integrity and reliability. If there is a high probability of error free transmission per hop and the nodes have sufficient storage, the Hop-by-Hop scheme is not needed for the above purpose. Without an HBH Ack scheme, one would retransmit the packet from its origin after a time out period expired. One introduced the HBH Ack to decrease the delay caused by retransmissions at the expense of added overhead for acknowledgements. In the ARPANET, this added overhead is kept small by "piggybacking" acknowledgements whenever possible on information packets flowing in the reverse direction. In the packet radio system, the overhead can be kept small by listening, whenever possible, for the next repetition of the packet on the common channel instead of generating a separate acknowledgement packet.

The value of an End-to-End acknowledgement is sufficiently great that it can be assumed present <u>a priori</u>. However, the additional use of a Hop-by-Hop acknowledgement is not as clear. Therefore, in this section, we examine the question of whether the ETE Ack is sufficient, or whether one needs a Hop-by-Hop (HBH) acknowledgement in addition. The problem is therefore whether an HBH Ack is superior to an ETE Ack with respect to throughput and delay, since the ETE Ack ensures message integrity. It is noted that the routing and flow control by devices in the network depend on the type of acknowledgement scheme used.

We consider a simple case where (n-1) repeaters separate the packet radio terminal from the destination station. Assuming that the terminal is at a distance of "one hop" from the first repeater, one obtains the following n-hop system:



A simple model is used to evaluate the total average delay that a packet encounters in the n-hop system when using HBH and ETE acknowledgement schemes. When the ETE acknowledgement scheme is used, every repeater transmits the packet a single time. If the packet does not reach the station, retransmission is originated by the terminal. The ETE acknowledgement is sent from the station. In the HBH scheme, repeaters store and retransmit the packet until positively acknowledged from the next repeater stage.

If, after a terminal (or a repeater in the HBH case) transmits the packet, an acknowledgement does not arrive within a specified period of time, it retransmits the packet. The waiting period is composed of the time for the acknowledgement to arrive when no conflicts occur plus a random time for avoiding repeated conflicts.

Two different schemes for ETE acknowledgement and one scheme for HBH acknowledgement are studied. Curves for the total average delay as a function of the number of hops and the probability of successful transmission per hop are obtained. Two cases are considered: One in which the probability of success is constant along the path and another in which the probability of success decreases linearly as the packet approaches the station. Finally, channel utilizations are compared when using ALOHA [Abramson; 1970, 1973] random access modes of operation.

It is demonstrated that the HBH scheme is superior in terms of delay or channel utilization. This conclusion becomes significant when the number of hops increases or when the probability of successful transmission is low. For example, in a five hop system, if the probability of success per hop is 0.7, then the total average delay is 12.5 and 53 packet transmission times for the HBH and ETE acknowledgement schemes, respectively.

The model used is based on [Kleinrock & Lam, 1973; Roberts]. The model is simplified, however, by assuming that the probability that a packet is blocked is the same when the packet is new or has been blocked any number of times before. Although the more general equations could have been written, the numerical solution is rather elaborate [Kleinrock & Lam, 1973] and seems unnecessary for this comparative study. It is further assumed that the probabilities of being blocked on different hops are mutually independent. The "total delay" is defined as the time between the transmission of the first bit by the terminal and the correct reception of the last bit of the packet by the station.

3.2 DELAY CONSIDERATIONS

The delay equations, normalized by the number of packet transmission times, are given by:

$$D(HBH) = (1 + \text{[illegible]}) \cdot n + (1+2\text{[illegible]}+\text{[illegible]}+\text{[illegible]})\left(\sum_{i=1}^{n} \frac{1-q_i}{q_i}\right) \qquad (1)$$

$$D_1(\mathrm{ETE}) = (1 + \beta) \cdot n + [(1+2\beta+\alpha)\cdot n+\delta] \left(\frac{1-Q}{Q}\right) \quad (2)$$

$$D_2(\mathrm{ETE}) = (1 + \beta) \cdot n + (1+2\beta+\alpha+\delta) \left(\frac{1-Q}{Q}\right) . \quad (3)$$

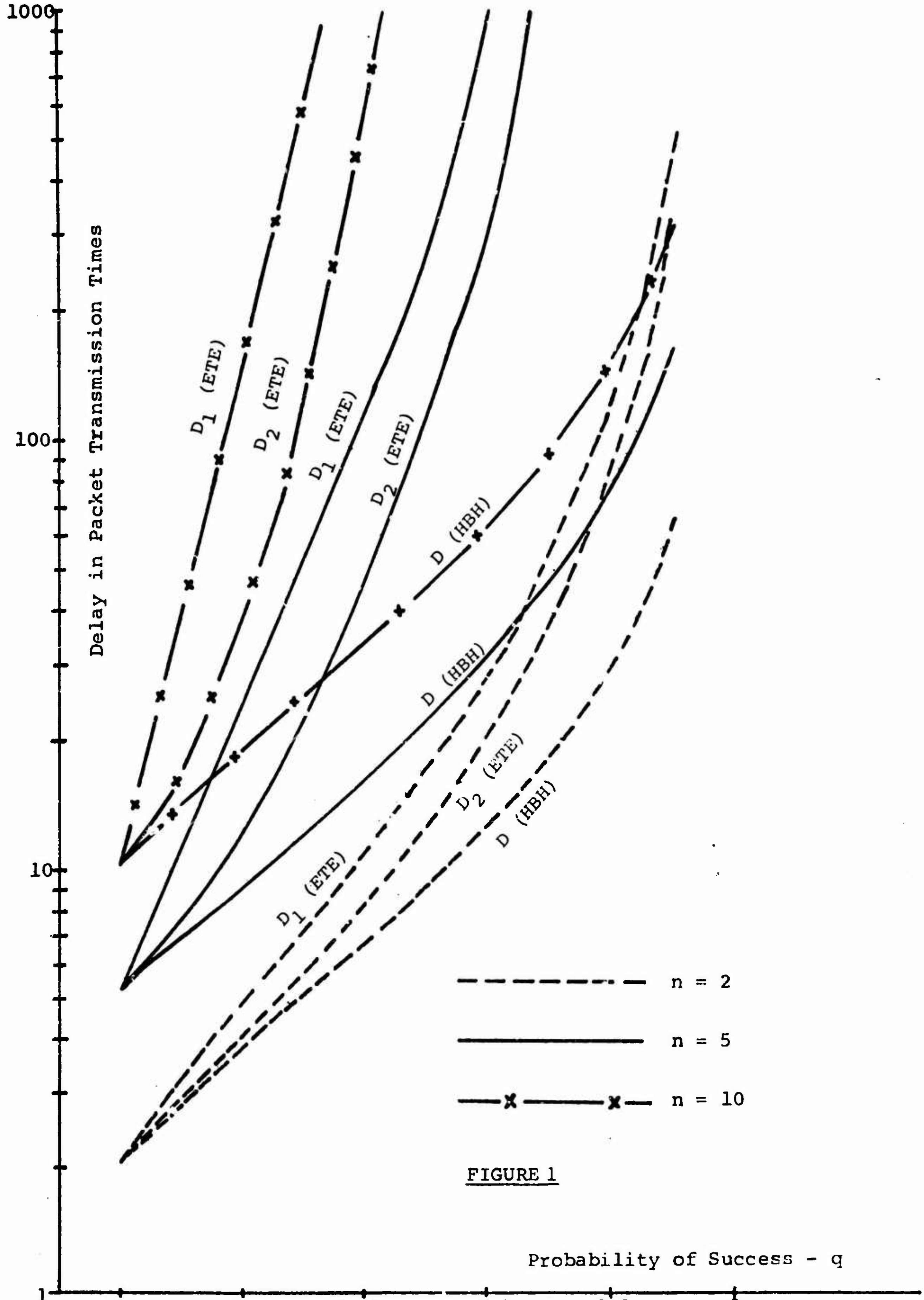
In these equations, α is the ratio of the acknowledgement transmission time to the packet transmission time; β is the ratio of the average propagation time per hop to the packet transmission time; and δ is the ratio of the average waiting time (beyond the minimum) for avoiding repeated conflicts, to the packet transmission time. The quantity q_i is the probability of successful transmission on hop i; and $Q = \prod_{i=1}^{n} q_i$.

As indicated before, two different cases for the ETE acknowledgement are considered. D_1(ETE) represents the delay when the terminal waits the expected time for the packet to reach the station and for the ETE acknowledgement to be received by the terminal before retransmitting the packet. D_2(ETE) is for the case in which the terminal retransmits after shorter periods of time because it anticipates a low probability of successful transmission Q. In particular, we examine the case in which the retransmission delay is the same as in the HBH method.

Figures 1, 2, and 3 show delay curves for the three acknowledgement schemes using the parameters $\alpha = 0.5$, $\beta = 0.02$, and $\delta = 2.0$. Figures 1 and 2 are for the case in which q is constant along the path.

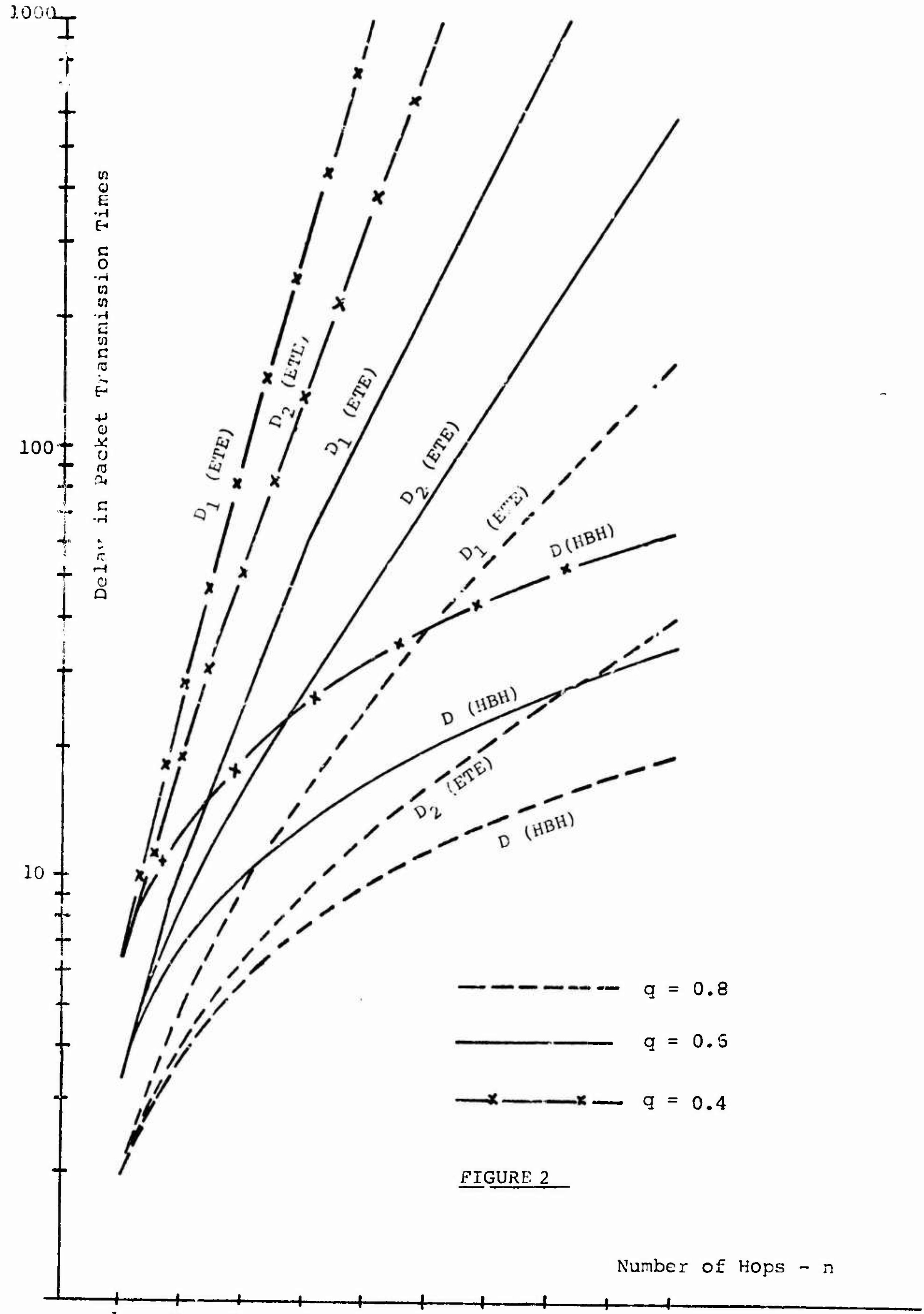
The curves show the delay as a function of the probability of successful transmission q rather than the channel utilization. Thus, they can be used for slotted or non-slotted ALOHA, or possibly for other access schemes.

It is evident from Figure 1 that the delays for the ETE acknowledgement schemes grow much more rapidly than delays for the HBH scheme. For example, in the 5-hop system, if the packet transmission



1000
100
10
1
Delay in Packet Transmission Times
D_1 (ETE)
D_2 (ETE)
D_1 (ETE)
D_2 (ETE)
D (HBH)
D (HBH)
D_1 (ETE)
D_2 (ETE)
D (HBH)
n = 2
n = 5
n = 10
Probability of Success - q
1.0
0.8
0.6
0.4
0.2
0

FIGURE 1



1000
100
10
Delay in Packet Transmission Times
D_1 (ETE)
D_2 (ETE)
D_1 (ETE)
D_2 (ETE)
D_1 (ETE)
D (HBH)
D (HBH)
D_2 (ETE)
D (HBH)
q = 0.8
q = 0.6
q = 0.4
Number of Hops - n
1
2
3
4
5
6
7
8
9
10

FIGURE 2

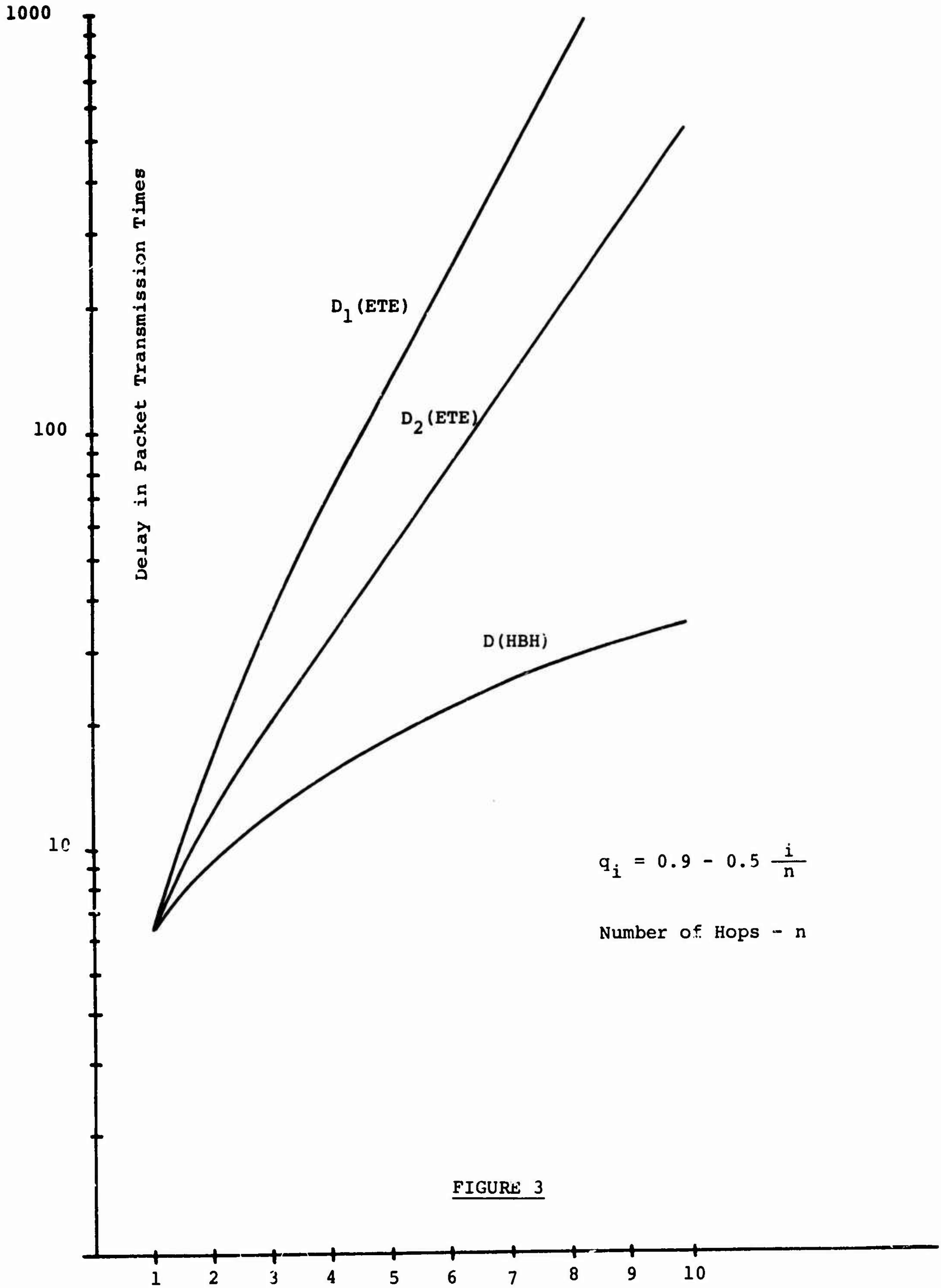

1000
100
10
Delay in Packet Transmission Times
D_1(ETE)
D_2(ETE)
D(HBH)
$q_i = 0.9 - 0.5 \frac{i}{n}$
Number of Hops - n
1
2
3
4
5
6
7
8
9
10

FIGURE 3

time is 10 msec, the average delays are 170 msec, 470 msec, and 1180 msec, for D(HBH), D_2(ETE), and D_1(ETE), respectively.

In practice, q will differ along the path. It is reasonable to assume that the probability of success, q, will decrease when the packet approaches the station. (Simulation results confirm this assumption.) When random access ALOHA systems are used, the practical range for q is from 1/e for which the effective utilization is maximum to 0.9 for which the utilization is 4.7% and 9.4% for the non-slotted and slotted case, respectively. We take a function of the form:

$$q_i = 0.9 - 0.5 \frac{i}{n} \; ; \; i = 1, 2, \ldots n \tag{4}$$

The normalized average delay as a function of n, with q_i, as in Equation (4) is shown in Figure 3.

3.3 EFFECT ON CHANNEL UTILIZATION

We now consider the effect of the acknowledgement scheme on the maximum utilization achievable when using slotted and non-slotted ALOHA random access schemes. To simplify the comparison, we take $\delta = 0$ (this affects the comparison with ETE scheme 1) and assume that q is constant along the path. It is further assumed that the arrival process, to each repeater, of new packets and new packets plus retransmissions are both Poisson with mean rates S and G, respectively and that the packet transmission time is one unit.

Given an n-hop system, suppose that one wants to use an ETE acknowledgement scheme such that the average delay equals that when using a HBH scheme. Equating (1) and (2), and (1) and (3), respectively, one obtains*:

---

* We use subscripts 1 and 2 to denote variables for ETE schemes 1 and 2, respectively; variables without a subscript will denote quantities related to the HBH scheme.

$$q_1 = q^{1/n} \; ; \; q_2 = \left(\frac{q}{n-(n-1)q}\right)^{1/n} \tag{5}$$

The relation between the channel traffics G for the acknowledgement schemes, when using slotted ALOHA are:

$$G_1 = \frac{G}{n} \; ; \; G_2 = \frac{Y^{-1}}{n} \tag{6}$$

where

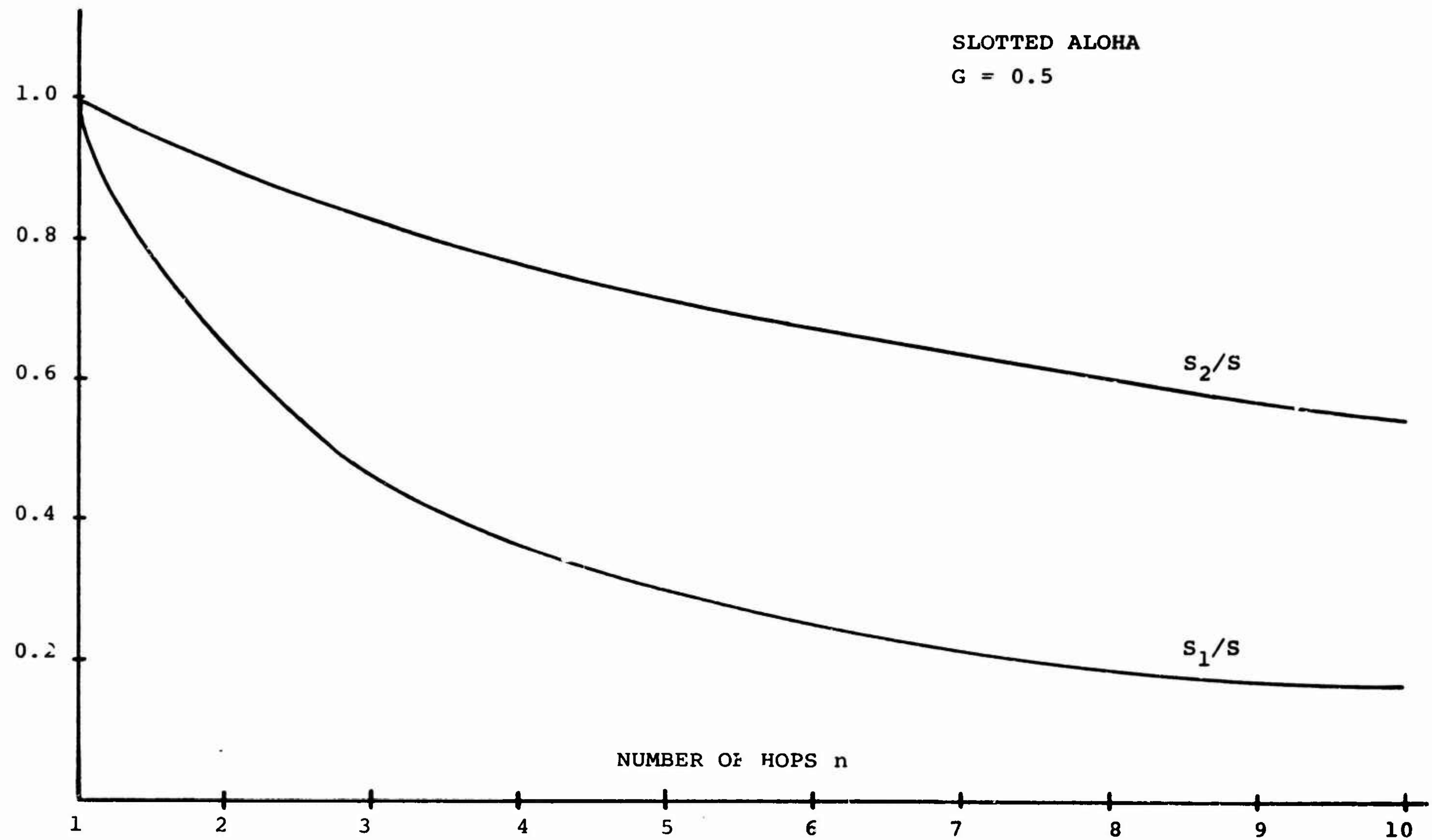
$$Y = \frac{e^{-G}}{n-(n-1)e^{-G}} \tag{7}$$

Consequently, the channel utilizations (or throughputs) S are related as follows:

$$\frac{S_1}{S} = \frac{1}{n} \; e^{-G(1-\frac{1}{n})} \tag{8}$$

$$\frac{S_2}{S} = \frac{e^{-G}}{nG} \; Y^{1/n} \; \ell n Y^{-1} \tag{9}$$

The ratios of utilization (Equations (8) and (9)) as a function of n are shown in Figure 4, for the case G = 0.5 which is equivalent to 30% utilization in the slotted ALOHA random access system.



SLOTTED ALOHA
G = 0.5
1.0
0.8
0.6
0.4
0.2
S2/S
S1/S
NUMBER OF HOPS n
1
2
3
4
5
6
7
8
9
10

FIGURE 4

4. A DIRECTED ROUTING PROCEDURE

In this section, a routing scheme is proposed aimed at achieving maximum throughput and minimum delay. This is obtained by using shortest path (minimum hop) routing from terminal to station and from station to terminal, and by preventing, where-ever possible duplicate copies of a packet from being circulated in the network. However, the routing procedure includes sufficient flexibility so that when the first choice shortest path cannot be used, the packet departs from this path and uses a shortest path from its new location. One pays overhead for this efficiency by "carrying" two labels in the packet header.

4.1 LABELING

The shortest path routing is obtained by labeling the repeaters to form, functionally, a hierarchical structure as shown in Figure 5. Each label includes the following information:
(i) a specific address of the repeater for routing purposes,
(ii) the minimum number of hops to the nearest station, and
(iii) the specific address of <u>all</u> repeaters on a shortest path to the station and the address of the repeater to which a packet has to be transmitted when destined to the station.

For simplicity, we describe routing for the case of a one station network. A label of repeater R_i of hierarchy level j will be denoted by L_{ij}; $i,j>1$. The station will have the label L_{11}. L°_{ij} will denote the label of the repeater which is the "nearest available" to the communicating terminal.

A label is composed of H subfields, where H is the maximum number of hierarchy levels (H-1 is the maximum number of hops on the shortest path between any repeater and the station). Every subfield has three possible entries, blank (BLK), a serial number (SER), or ALL. L_{ij} has j entries SER's and (H-j) BLK's as shown below:

1	2		j-1	j	j+1		H
SER	SER	. . . .	SER	SER	BLK	. . . .	BLK

$\underbrace{\qquad\qquad\qquad\qquad}_{\text{j serial numbers}}$ $\underbrace{\qquad\qquad\qquad}_{\text{(H-j) blanks}}$

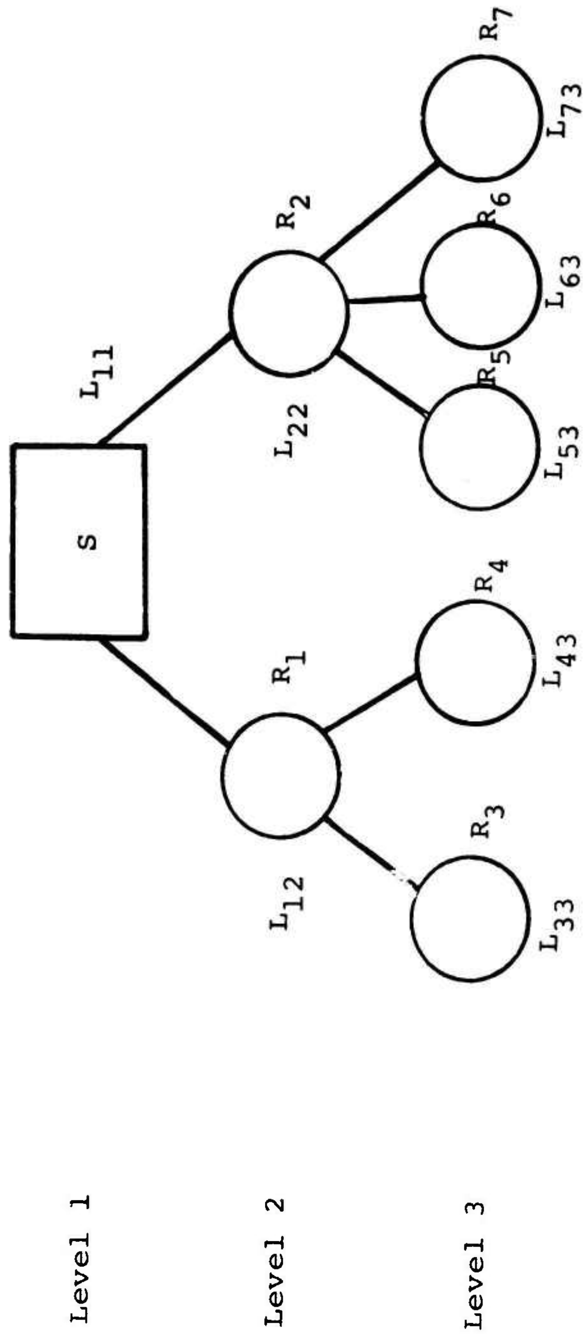

S
R_1
R_2
R_3
R_4
R_5
R_6
R_7
L_{11}
L_{12}
L_{22}
L_{33}
L_{43}
L_{53}
L_{63}
L_{73}
Level 1
Level 2
Level 3

FIGURE 5

We say that L_{ij} "homes" on L_{kp}, $h(L_{ij}) = L_{kp}$, if $p = j-1$ and the first j-1 subfields of both are identical. If two repeaters at level j home on the same repeater, their labels will differ only in the entry to subfield j.

As an example, if we use 3 bits per subfield, the labels of the station and the repeaters of the network shown in Figure 5 are as follows:

	Subfield 1	Subfield 2	Subfield 3
L_{11}	0 0 1	0 0 0	0 0 0
L_{12}	0 0 1	0 0 1	0 0 0
L_{22}	0 0 1	0 1 0	0 0 0
L_{33}	0 0 1	0 0 1	0 0 1
L_{43}	0 0 1	0 0 1	0 1 0
L_{53}	0 0 1	0 1 0	0 0 1
L_{63}	0 0 1	0 1 0	0 1 0
L_{73}	0 0 1	0 1 0	0 1 1

In this example, a subfield in which all bits are "0" is considered "blank." Note that all entries in Subfield 1 are the same since all repeaters home (eventually) on the same station.

4.2 ROUTING

The packet header, in both directions, will include the following routing information.

L_{kn}	L°_{ij}	OTHER HEADERS AND PACKET INFORMATION
TO	LABEL OF NEAREST REPEATER TO THE TERMINAL	

L_{kn} is the label of the repeater to which the packet is currently addressed. The complete packet will always be transmitted to a specific device; other devices which may receive the packet will drop it. The shortest path from a terminal to the station consists of L°_{ij}, $h(L^{\circ}_{ij})$, $h(h(L^{\circ}_{ij}))$, up to L_{11}, in the given order, and in the reverse order when routing from station to terminal. When a specific repeater along the shortest path is not known (by the terminal) or not available, then the terminal or repeater (which has the packet) will transmit only the header part of the packet, trying to identify a specific repeater. In that case, the label L_{kn} will include some entries ALL.

A. Routing from Terminal to Station

When a previously silent terminal begins to communicate, it first identifies a repeater or a station in its area. It transmits only the header part of the packet with all entries in L_{kn} set to ALL. The header is addressed to all repeaters and stations that can hear the terminal. A device which correctly receives this header substitutes its label in the space L_{kn} and repeats the header. This particular L_{kn} is also L°_{kn} and will be used by the terminal to transmit all packets during this period of communication. If a terminal is stationary, it can store this label for future transmissions. L°_{kn} begins to transmit the complete packet along the shortest path to the station.

Suppose that L_{ij} along the shortest path is not successful in transmitting the packet to $h(L_{ij})$. Then L_{ij} begins the search stage of trying to identify another repeater. In the first step, it tries to identify a repeater which is in level $p \le j-1$. This is done by using the following label:

1	2	3		j-1	j	j+1		
SER	ALL	ALL	. . .	ALL	BLK	BLK	. . .	BLK

The header is addressed to all repeaters in levels 2 to j-1, which eventually home on L_{11}. If this step is not successful, in the second (last) step, L_{ij} tries to identify any available repeater by using a label in which the first entry is SER and all other entries are ALL. When a specific repeater is identified and receives the packet, it transmits the packet on the shortest path from its location.

Note that if repeaters have sufficient storage, they can save alternative labels and thus reduce the necessity of searching for a specific repeater. Alternative solutions in which repeaters have multiple labels are also possible.

B. Routing from Station to Terminal

L°_{ij} contains sufficient information for shortest path routing to the terminal. Denote by h^{-1} the inverse of h and by $h^{-2} = h^{-1}(h^{-1})$, etc. The shortest path from station to terminal includes $h^{-(j-1)}(L^\circ_{ij})$, $h^{-(j-2)}(L^\circ_{ij})$, ... $h^{-1}(L^\circ_{ij})$, and L°_{ij}. If some L_{kp} is not successful in transmitting the packet along the shortest path, it begins the process of identifying another specific repeater. Note that when routing to the station, the next label is always a function of the label of the repeater that currently stores the packet. When routing to the terminal, the next label is a function of L°_{ij} and the hierarchy level of the repeater that currently stores the packet. Thus, when routing to the terminals, it will be useless to transmit the packet backwards, since it will usually arrive back at the current location; therefore it is more efficient to delay the packet. If, when routing to the terminal, a repeater on the shortest path is temporarily unreachable, the procedure attempts to by-pass this particular repeater and regain the original shortest path route. The labels that will be used by $h^{-3}(L^\circ_{ij})$ are shown below:

	1	2		j-3	j-2	j-1	j	j+1		
$h^{-3}(L^\circ_{ij})$	SER	SER		SER	BLK	BLK	BLK	BLK		BLK

	1	2		j-3	j-2	j-1	j	j+1		
$h^{-2}(L^\circ_{ij})$	SER	SER		SER	SER	BLK	BLK	BLK		BLK

	1	2		j-3	j-2	j-1	j	j+1		
SEARCH	SER	SER		SER	SER	ALL	BLK	BLK		BLK

All the entries SER are taken from L°_{ij}.

4.3 ACKNOWLEDGEMENTS

In Section 3, it has been shown that the use of HBH acknowledgements in the routing scheme in addition to the ETE acknowledgement is desirable. However, one can prevent specific acknowledgement packets from being transmitted by using the passive "echo" acknowledgement. This approach has other advantages as well, which will be described in the next section. Echo acknowledgement will be employed along the path. That is, the device transmitting the packet waits in a receive mode to receive this same packet when it is repeated by the next stage. The reception of the packet when transmitted by the next stage constitutes the acknowledgement, since it indicates that the next repeater stage has correctly received the packet and will store and retransmit it as necessary.

In fact, one has the option of adding parity bits after the header. In this event, it would be sufficient to "hear" the header of the packet, and thus, the header plus parity bits will constitute the acknowledgement. At the end of a path, the terminal or station will repeat the header. Note that the probability of correctly receiving an acknowledgement would be higher than the probability of correctly receiving a packet, due to the difference in transmission time. Furthermore, in many cases the station may correctly receive the header, whereas the entire packet is received in error. The information contained in the header can be used by the station for control purposes.

4.4 TRAFFIC CONTROL

The control procedures to be implemented would use control packets from stations to repeaters, from stations to terminals, and possibly from repeaters to terminals. Some of these may be implemented in the station - repeater protocol and relate to the initialization of repeaters, relabeling of repeaters under various overload conditions, activation and deactivation of repeaters, and so on. In this section, we discuss controls necessary for the routing scheme. These are:

(i) Initial search by the terminal.
(ii) Maximum handover number (MHN).
(iii) Maximum number of transmissions (MNT).

It was demonstrated in [Kleinrock & Lam; 1974] that after channel traffic exceeds a certain value, throughput reduces. If the number of retransmissions is not limited, the offered channel traffic will increase indefinitely and the throughput will reduce to zero. Thus, one problem is to prevent new traffic from entering the system when the system is congested. This control can be obtained by the search procedure which is used by terminals when entering the system. This control is "local" in the sense that it depends on the traffic level in the geographical neighborhood of the terminal. Terminals will nevertheless be able to enter the system when being "far" from stations, and the traffic introduced will propagate towards the stations.

The MHN is a control aimed at suppressing packets from being propagated in endless cycles of repeaters or being propagated over paths containing many hops. This may occur when packets depart from the shortest path. Furthermore, if the routing scheme used is relatively unsophisticated, the MHN will prevent the packet from arriving at remotely located stations. The MHN for a given packet depends on the number of hops between the originating terminal and the station with which it communicates (or vice versa). It will therefore be a function of the hierarchy level of the repeater with the label L°_{ij}.

The MNT is also a local control which reduces the traffic level when the system is congested by discarding a packet after a specified number of retransmissions have been attempted. It also prevents repeaters from indefinite transmission of a packet when surrounding repeaters are temporarily blocked and are unable to accept packets.

4.5 PACKET FORMAT

A possible packet format for performing the routing described is shown below:

HEADER	PACKET INFORMATION	PARITY

The header includes the following items:

T/F	C/I	DID	OID	L_{kn}	L°_{ij}	MHN	E/C

T/F - a bit, indicating whether the packet is addressed To station or From station.

C/I - a bit, indicating whether the packet is a Control packet or an Information packet.

DID - Destination address.

OID - Origination address.

L_{kn} - The label of the repeater to which the packet is currently addressed.

L°_{ij} - The label of the repeater "nearest" to the terminal which originated the packet or to which the packet is transmitted.

MHN - Maximum handover number.

E/C - An error or control message. If it is an information packet, the space may include a sequential number, specification of the packet number in the message, etc. If it is a "header packet," it may include an error message asking for retransmission of a certain number of packets. If it is a control packet from the station, this space may be used for the control message.

5. A PROCEDURE FOR REPEATER LABELING

In this section, we identify some of the problems of repeater labeling and propose one approach for the initial labeling of a repeater network. Assume that initially every station and repeater has a fixed ID, R_i. This ID will be used for labeling purposes, to identify whether the device is operative or dead, to activate and deactivate a repeater, and for other control purposes.

The station will determine and assign labels to all repeaters in its area in the initial labeling procedure. When more than one station operates in an area, the initial labeling will be done by the stations sequentially, and repeaters may be allowed to choose the home station according to the minimum number of hops.

First, it is necessary to specify two parameters: (i) the maximum number of subfields or hierarchy levels, say H, and (ii) the number of bits per subfield, say B. These parameters are to be the same for the entire broadcast network in order to have the same packet format when transmitting information. If some sections of the broadcast network are disjoint, it is sufficient that B x H be the same for the entire network. As indicated before, H-1 is the maximum number of hops that a packet will travel when using the shortest path route, and 2^B-2 is the maximum number of repeaters that can home on a single repeater or station (one label is needed for ALL and another for BLK). 2 x H x B is the number of bits in the header which will contain the routing information.

The initial labeling procedure is:

STEP I:

The station transmits a control packet to every repeater sequentially. This packet includes an MHN as well as another MHN to be used by the addressed repeater for its response packet. There is no directed routing at this stage; every repeater which correctly receives the control packet decrements its MHN, and stores and retransmits it until echo acknowledged by the next stage. The control packet is dropped when its MHN reduces to zero.

The repeater to which this packet is addressed transmits a response packet to the station using the assigned MHN. Every repeater which receives this packet will decrement the MHN and add its R_i in order.

The station may receive one response packet, several, or none. If no response packet is received, the station can try several more transmissions, each time increasing the MHN's, or conclude that the repeater is dead (this repeater can possibly be reached from another station).

STEP II:

The information acquired from the response packets is sufficient to determine a hierarchical labeling structure. In this step, the station processes the information and determines an "optimized" structure. The processing performed during this step is described in the next section.

STEP III:

In this step, the station tests the shortest path, particularly in the direction from station to repeaters, which was not tested before. The station transmits a control packet to every repeater, using its label. The station uses an MHN equal to the number of hops on the shortest path so that if this path is not possible the repeater will not be able to receive the packet. A repeater which receives this packet transmits a response to the station, which constitutes an ETE positive acknowledgement. If all repeaters have been successfully tested, the procedure ends; otherwise, the program returns to Step II for further processing.

5.1 AN ALGORITHM FOR DETERMINING THE LABELS

We describe a technique for processing the response packets and for determining the hierarchical labels in Step II of the labeling procedure. In general, the repeaters may be distributed at random locations, and the station may not know the geographical locations of repeaters. Furthermore, there may be more repeaters than the number needed for efficient routing. Ideally, one would want to obtain a network of repeaters which has the following properties:

(1) There should be a minimum number of hierarchy levels.

(2) There should be a shortest path from every repeater to the station.

(3) The entire area should be covered with a minimum number of repeaters.

(4) Every repeater should be able to transmit directly to at least j (say 2) other repeaters.

(5) The number of repeaters which home on one single repeater or station should be $\leq 2^B-2$.

A solution which satisfies all the requirements may not exist. For example, if more than 2^B-2 repeaters can directly reach the station and none can be deactivated, requirement (2) will not be satisfied.

Suppose that there are N-1 repeaters and one station denoted by R_1. The station first constructs a connectivity matrix $\underline{C} = (c_{ij})$, where,

$$c_{ij} = \begin{cases} 1 & \text{if device } j \text{ can hear } i \text{ directly} \\ 0 & \text{otherwise.} \end{cases} \qquad i,j = 1, 2, \ldots, N$$

$\underline{C}$ is constructed from the response packets in Step I. For example, if a response from R_i contains (in order) R_e, R_m, R_k, ..., then $c_{ie} = 1$, $c_{em} = 1$, $c_{mk} = 1$, ... One can see that the station does not have to transmit the first labeling packet to all repeaters, since it can learn about the functional location of some of the repeaters which were on the return path of previous control packets. Furthermore, the number of response packets to a control packet can be increased when the MHN assigned by the station is increased. Finally, we note that C is not necessarily symmetric.

The entries 1 in row i indicate the repeaters to which R_i can directly transmit, and the entries 1 in column i indicate the repeaters from which R_i can directly receive. The structure of the repeater network will be recorded by the vector $\underline{h}$, where h_j, $j = 1, \ldots, N$, indicates the repeater on which R_j homes.

Let S(m) denote the set of repeaters whose shortest path to the station includes exactly m hops. Assume that all repeaters in S(1), S(2), ..., and S(m), have been labeled (assigned home repeaters). We describe the labeling of (repeaters in) S(m+1) by (repeaters of) S(m). At every state k of labeling S(m+1) by S(m), we characterize repeaters of S(m) by:

$d_i(k)$ = The number in S(m+1) which have been labeled R_i (say, the degree of R_i at state k)

$v_i(k)$ = The number in S(m+1) which still can be labeled R_i

$f_i(k)$ = The potential degree of R_i at state k, i.e., $f_i(k) = d_i(k) + v_i(k)$.

Repeaters of S(m+1) will be characterized by:

u_j = The number of repeaters in S(m) which can label it.

At every state k, we distinguish among three disjoint subsets of S(m) and S(m+1):

$S_m^F(k) = \left\{R_i : v_i(k) = 0\right\}$ = cannot label more*

$S_m^L(k) = \left\{R_i : f_i(k) \leq 2^B - 2;\right.$ ordered according to increasing values of $d_i(k)$ and when the same then according to increasing values of $\left.v_i(k)\right\}$

$S_m^R(k) = \left\{R_i : f_i(k) > 2^B - 2;\right.$ ordered according to increasing values of $\left.v_i(k)\right\}$

$S_{m+1}^F(k) = \left\{R_i : h_i(k) > 0\right\}$ = already labeled (assume that $h_i(0) = 0$)

$S_{m+1}^L(k) = \left\{R_i : c_{ip} = 1\right.$ for some $R_p \varepsilon\, S_m^R(k)$; ordered according to decreasing values of the number of repeaters in $S_m^R(k)$ that can label them and when this is the same, then according to decreasing values of $\left.u_i\right\}$

$S_{m+1}^R(k)$ = The remaining repeaters of $S(m+1)$ ordered according to decreasing values of u_i

Note that $S_m^R(k)$ is the set which can potentially violate requirement (5), and $S_{m+1}^L(k)$ is the set to be labeled which may result in this violation. Therefore, one should try to label $S_{m+1}^L(k)$ by $S_m^L(k)$. When such a label is assigned, it decreases the values of $f_i(k)$ in $S_m^R(k)$. Furthermore, the orders of the subsets of $S(m)$ according to $d_i(k)$ are aimed at obtaining a network in which the repeaters of $S(m+1)$ are divided equally among repeaters of $S(m)$ (this was not specified in the requirements). The order of $S_{m+1}^L(k)$ is done so that (if possible) the repeater which can be labeled by the largest number of repeaters of $S_m^R(k)$ is labeled first.

The algorithm proceeds as follows:

---

* There may be repeaters in $S(m)$ for which $v_i(0) = 0$. It is convenient to refer to these as "end repeaters of level m" since no repeater will home on these.

STEP A:

Take the first of $S^L_{m+1}(k)$ and label it by one of $S^L_m(k)$ (beginning with the first). If it is labeled, evaluate the subsets of state k+1 and do the same; if not, take the next repeater of $S^L_{m+1}(k)$ and do the same.

STEP B:

(i) If $S^R_m(k)$ is empty, then $S^L_{m+1}(k)$ is also empty; complete the labeling of $S^R_{m+1}(k)$ by $S^L_m(k)$ using the procedure of Step A, then return to Step A.

(ii) If $S^R_m(k)$ is not empty, label one of $S^L_{m+1}(k)$ by $S^R_m(k)$ using the procedure of Step A, then return to Step A.

Note that if all of $S^R_m(k)$ becomes part of $S^L_m(k)$ at some state, the network produced satisfies requirements (1), (2), and (5); since (5) is satisfied by the last statement, (2) implies (1), and (2) is satisfied by the definition of S(m). If one of the above requirements must be violated, modification of (ii) in Step B of the algorithm is required.

The sets S(m), defined at the beginning of this section are constructed recursively as follows:

$$S(1) = \left\{R_k : c_{k1} = 1\right\}$$

$$\vdots$$

$$S(m+1) = \left\{R_k : R_k \not\in \bigcup_{p=1}^{m} S(p),\ c_{kj} = 1 \text{ for some } R_j \varepsilon S(m)\right\}$$

That is, to construct S(m+1) it is necessary to examine in the matrix $\underline{C}$ only the entries 1 in the columns which corresponds to repeaters of S(m) and choose the ones that have not been identified yet.

5.2 REMARKS

1. The shortest path label assignment does not, in general, correspond to physical distance. That is, the label assignment depends on the terrain as well

as possible variations in transmission power and reception sensitivity of devices. Thus, it is a functional rather than a geophysical assignment.

2. In the practical case, it may be necessary to label redundant repeaters and then deactivate them for use as stand-by repeaters. Furthermore, the procedure for this process should satisfy requirements (3) and (4) of the previous section.

6. SOME SIMULATION RESULTS

A computer program which simulates in detail the operation of the packet radio network and the proposed routing and control techniques is currently available and will be described in [NAC; 1974 (b)]. Some qualitative observations of the model's performance as related to routing are described in this section.

Figure 6 shows one network that has been extensively studied. The labels of repeaters in this network were assigned _a priori_, and the lines connecting the devices in Figure 7 signify the hierarchical structure created by the implementation of the directed routing technique. Terminal traffic is introduced into the system at random times, and originates at random locations on the plane. Once a terminal is introduced, it begins the search procedure, and the communication between a terminal and a station proceeds using the routing and control schemes described including the ETE and Echo HBH acknowledgements. Figure 6 shows the connectivity of the network simulated. That is, when a particular repeater transmits, all devices connected to it by line can receive the packet.

Simulation results demonstrate that the critical hop in the packet radio network is between the first level repeaters and the station. Thus, special attention should be given to the flow control design on this hop. In particular, repeater placement in the neighborhood of the station and the control of these repeaters by the station are significant. These repeaters also have higher power duty cycles, since they repeat all packets of repeaters which home on them.

It is also demonstrated that there is a higher probability of end-to-end successful transmission from station to terminal than from terminal to station. This is observed from the higher frequency of repeater searches and dropped packets when routing towards the station. One cause is that the station is the largest user and thus has higher probability of successful transmission over

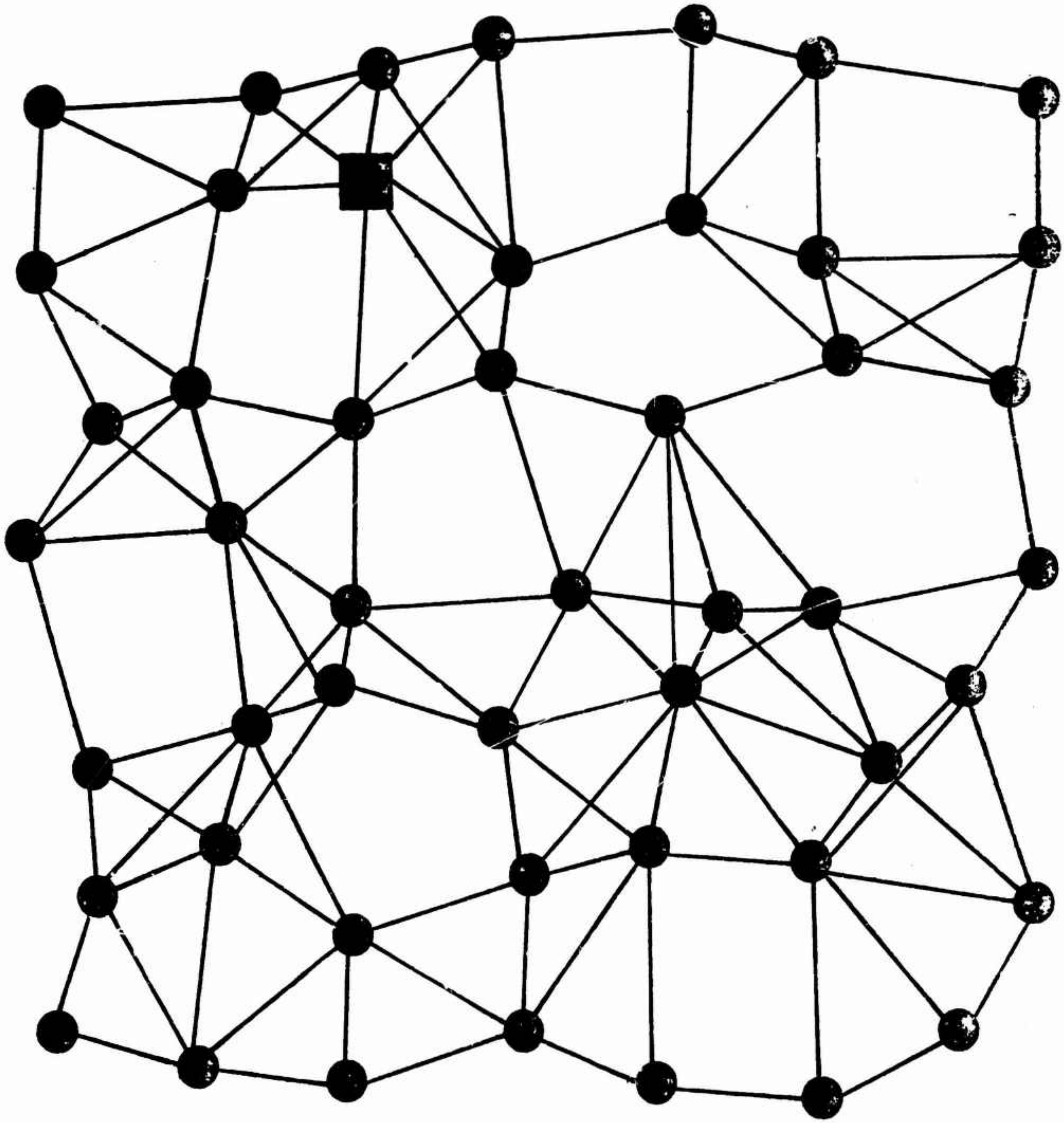


FIGURE 6

the critical hop because it manages its own traffic rather than competing with itself in a broadcast node.

The terminal simulated recognizes a packet addressed to it by checking a portion of the packet header not related to repeater labels. Consequently, the terminal can receive from a different repeater than the one to which it transmits. Since the response to a search packet by repeaters is randomized in time, the terminal frequently identifies a repeater which is not necessarily the nearest to the terminal or the nearest to the station. As a result, the path from the terminal to the station is not necessarily the same as from the station to the terminal. The latter is usually shorter. Such a case is illustrated in Figure 8. Here, the terminal will usually receive its packets from R1 at the time R1 transmits to R2. The echo by the terminal will usually acknowledge R1 and R2 simultaneously. Furthermore, R3 need not handle traffic to the terminal.

Figures 6 and 7 show that while the station has connectivity 7, only 4 of these repeaters are labeled as first level repeaters that home on the station. In particular, note that the station can hear all packets transmitted towards it by R26; however, these packets are addressed to R27. The station finally receives the packets from R27. Consequently, the station is busy a fraction of the time with non-useful traffic. This can be improved by changing the reception and transmission operation of stations. That is, the station can be made to receive from any repeater along the shortest path and to transmit to the repeater nearest to the terminal that it can reach. Another advantage of this type of station operation is that more repeaters can be placed in the neighborhood of the station and labeled arbitrarily. These repeaters may be required for area coverage or reliability considerations.

Other observations of the simulation show that some terminals are blocked when the system in their neighborhood is congested, and that a higher frequency of alternate routing occurs when the traffic offered to the system is increased.

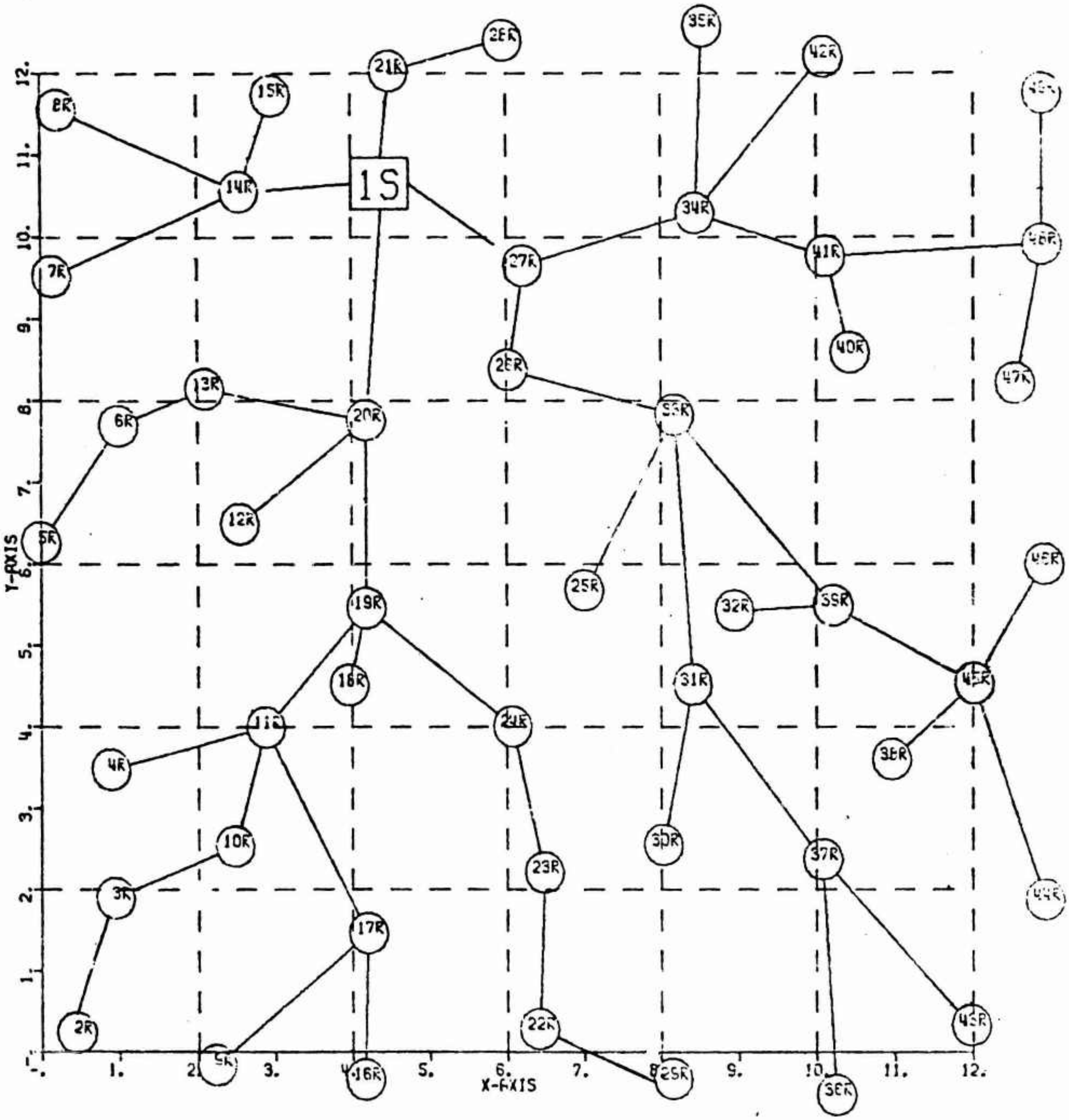


FIGURE 7

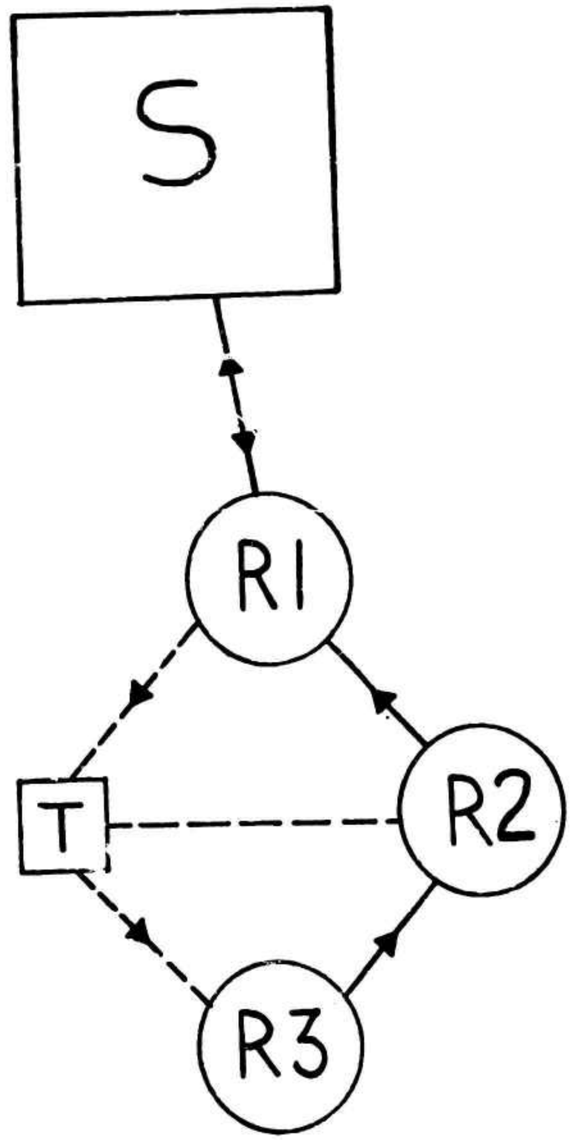


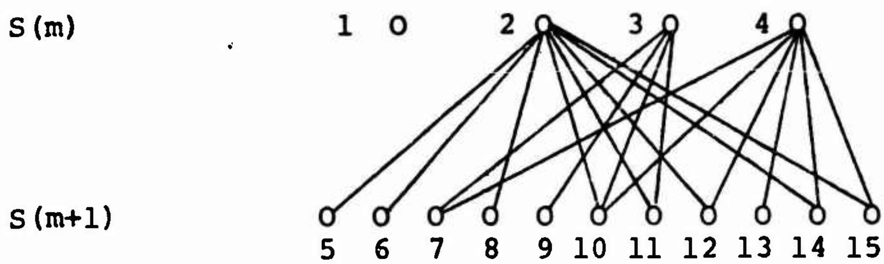
FIGURE 8

An extensive description of the simulation program and experiments is forthcoming [NAC; 1974(b)]. It is expected that the routing procedures described above will be modified as more experience is gathered. Further topics to be investigated include buffer management, flow control, and system initialization. These topics will be the subject of forthcoming reports.

7. APPENDIX: AN EXAMPLE OF REPEATER LABELING

The figure below shows the set S(m+1) to be labeled by S(m). The connection line between $R_i \varepsilon S(m+1)$ and $R_j \varepsilon S(m)$ was drawn to demonstrate that $c_{ij} = 1$, also B = 3 bits.

$f(0) = v(0) =$ 0 8 4 6



u_j = 1 1 2 1 1 3 2 2 1 2 2

$S_m^F(0) = \{R_1\}$ $\qquad$ $S_{m+1}^F(0) = \{\phi\}$

$S_m^L(0) = \{R_3, R_4\}$ $\qquad$ $S_{m+1}^L(0) = \{R_{10}, R_{11}, R_{12}, R_{14}, R_{15}, R_5, R_6, R_8\}$

$S_m^R(0) = \{R_2\}$ $\qquad$ $S_{m+1}^R(0) = \{R_7, R_9, R_{13}\}$

First label assigned is $h_{10} = R_3$, then,

$S_m^F(1) = \{R_1\}$ $\qquad$ $S_{m+1}^F(1) = \{R_{10}\}$

$S_m^L(1) = \{R_4, R_3\}$ $\qquad$ $S_{m+1}^L(1) = \{R_{11}, R_{12}, R_{14}, R_{15}, R_5, R_6, R_8\}$

$S_m^R(1) = \{R_2\}$ $\qquad$ $S_{m+1}^R(1) = \{R_7, R_9, R_{13}\}$

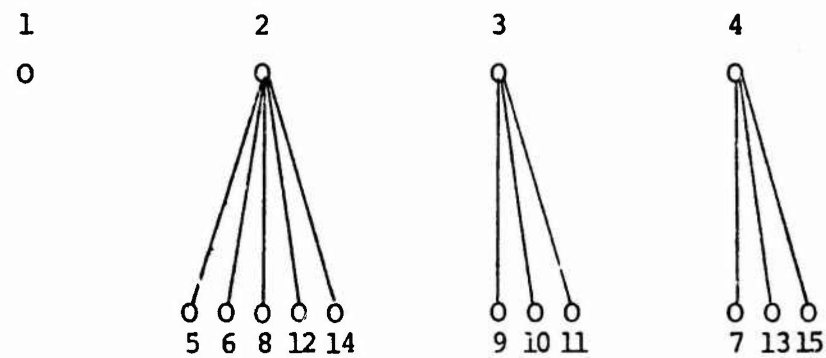
The second label is $h_{11} = R_3$, then,

$S_m^F(2) = \{R_1\}$ $\qquad$ $S_{m+1}^F(2) = \{R_{10}, R_{11}\}$

$S_m^L(2) = \{R_2, R_4, R_3\}$ $\qquad$ $S_{m+1}^L(2) = \{R_{11}, R_{12}, R_{14}, R_{15}, R_7, R_5, R_6, R_8, R_9, R_{13}\}$

$S_m^R(2) = \{\phi\}$ $\qquad$ $S_{m+1}^R(0) = \{\phi\}$

From now on, any labeling will satisfy requirement (5) since $S_m^R(2)$ is empty. The labeling by the algorithm proceeds as follows: $h_{12} = R_2$, $h_{14} = R_2$, $h_{15} = R_4$, $h_7 = R_4$, $h_5 = R_2$, $h_6 = R_2$, $h_8 = R_2$, $h_9 = R_3$, $h_{13} = R_4$. The final network is:



CHAPTER 11

COMBINATORIAL MODELS FOR ANALYSIS OF MESSAGE FLOW IN PACKET RADIO NETS

1. INTRODUCTION

This report is a summary of recent research on the macroscopic combinatorial analysis of message flow in packet radio nets. We begin by developing our so-called basic model of the repeater net, message origination and capture modes. The basic model assumes that all messages are transmitted in the direction of the fixed ground station or processing center.

The model is built in stages as more complex assumptions are made concerning design and operating modes of the repeater network. Initially, we assume an infinite repeater net at the lattice points of the usual Euclidean plane. The ground station is at the origin, and all messages which are repeated are accepted at each repeater. Messages originate at each point in discrete time independently at each repeater according to a Poisson distribution. All messages received at a repeater are received perfectly and repeated to those immediate neighbors which are one unit closer to the origin (ground station). Under these ideal conditions, we compute in closed form the number of messages received at the ground station, the number of unique messages received at the ground station, and their ratio, called the "inefficiency" of the system.

The next step in complexity of the model is to assume that not all messages which are received at a repeater are accepted. In this case, we specify two models for acceptance, called Type 1 and Type 2 slotting. We assume a time unit is broken into m-slots, and messages received are independently and at random assigned to one of the slots. In Type 1 slotting, the number of messages accepted is given by the number of slots with exactly one message. In Type 2 slotting, the number of messages accepted is given by the number of non-empty slots. Specific closed forms and asymptotic formulae are developed for Type 1 and Type 2 transfer functions (the functions P_{kj} which are the probability that j messages are accepted given that k arrive or are received). A combination theoretical-computer analysis is developed to obtain sample numerical data on survival of messages at the ground station as a function of various values of the parameters (number of slots, mean originations, configuration of repeaters, etc.). In particular, probability density functions for message arrivals or receptions and acceptances are obtained, and sample numerical data is given. The probability distributions for the number of copies of a single message which arrive at the origin are obtained by computer analysis of a set of derived difference equations. A special case of this probability distribution is the probability that at least one message gets through. Numerical data are given in terms of the parameters of the model.

The last stage of development of the so-called basic model is to include the possibility of retransmissions when a message is wiped out. Two types of retransmission modes are considered. In the first mode, retransmissions occur at the source of origination of the message after a time delay which depends on the distance to the origin or ground station. In the second mode, retransmissions occur at the point of wipeout of the message. Analyses are developed for the case of a single path from a repeater to the origin. Computer programs have been written which compute and study the number of retransmissions, delays, and bottlenecks. Some preliminary computer data has been obtained, and the results are summarized in Part J of Section II.

To summarize the development of the basic model:

The basic model was developed in three major stages:

A. <u>Perfect Reception at all Repeaters</u>

1. Number of messages at each repeater at each point in time

2. Number of distinct messages at each repeater at each point in time

B. Type I and Type 2 Slotting (capture model).

1. Closed form calculation of transfer functions
2. Survival probability of messages
3. Probability density functions of arrivals and receptions
4. Arrivals and receptions at origin at each point in time
5. Distribution of number of copies of a single message received at the origin when multiple routing is used and calculation of the probability that at least one message gets through.

C. Retransmissions

1. Number of retransmissions, acceptance and arrivals
2. Delays and average delays
3. Bottlenecks

Sections 1 through 11 summarize the advances in the basic model. The items with the capital letters are the problems solved in each stage of development of the model. All solutions are available in a computer programming package which will be described in the final section of this report.

In addition to the development of the so-called basic models a number of other questions and models were studied, mainly concerning message explosion with multiple routing schemes. The simplest such model was to assume that a single message is originated at the origin at time zero. This message is transmitted perfectly to each of the four nearest neighbors, each of which in turn send the message perfectly to each of their four nearest neighbors, and so on. For this simple scheme, we develop closed form formulae for the number of messages received at each repeater at each point in time assuming an infinite grid of repeaters. Some conjectures for the solution of the same problems in finite grids are made.

The above described simplified model was extended to assume that messages originate at each repeater according to a Poisson probability law at each point in time. Questions were asked and solved concerning message flow and various operating conditions of the repeaters. The different operating features which were considered are:

(i) No message can be repeated more than k-times.

(ii) If the same message arrives from different sources only onerepeat is made.

(iii) A repeater never repeats the same message except upon initial reception.

(iv) A combination of (ii) and (iii).

(v) A combination of (i) and (ii).

Closed form solutions are obtained for each case when repeaters are in an infinite network.

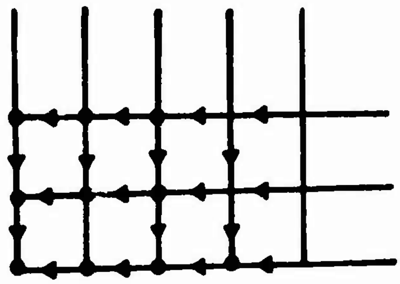
The last type of model considered, a very important part of the overall analysis of the packet radio configuration was to study the behaviour of a constant number of messages being sent <u>to repeaters</u> from the ground station. If this model is combined with the basic model of <u>inward</u> flow a model for messages being repeated back and forth from repeater to stations and back can be obtained and studied under various operating conditions.

Specifically, for flow of messages <u>from</u> the origin <u>to</u> the repeater, we assumed that a fixed number say J messages originate at the origin at each point in time. These messages are repeated to repeaters on paths which are the <u>duals</u> of the input paths to the station. We study how many messages get to repeaters at various distances from the ground station under the same two capture modes as we assumed for inward flow. Numerical data and graphs which resulted from computer analysis of the flow equations are given. The computer program is operable and can be used to develop other numerical data for a variety of combinations of values of the parameters.

2. THE BASIC MODEL: OUTLINE OF QUESTIONS

As mentioned earlier the basic model for messages going into a ground station from a repeater network is developed in three basic stages. The first and simplest stage assumes that messages are generated at each repater and repeated to repeaters one unit of distance closer to the origin. Specifically,

Assumption 1. Repeaters are located at the corner points of a square grid depicted as follows:



The arrows indicate the direction of flow of messages and the lower left hand corner represents the origin or ground station.

Assumption 2. Starting at time t=0 and at quantized time periods afterward (perhaps 1 second), t=0, 1, 2,..., messages originate at each repeater independently according to a Poisson probability law. That is, the probability that exactly k-messages originate at time t is given by $\frac{e^{-\lambda}\lambda^k}{k!}$; k=0, 1, 2,..., where λ is a constant which represents the mean or average number of originations.

Assumption 3. Allmmessages which arrive at any given repeater are repeated immediately to each repeater one unit of distance closer to the origin or ground station.

Problem 1. Under assumptions 1,2,3, compute $X_o(t)$ and $X_o^1(t)$ which are defined as the number of arrivals and the number of distinct arrivals at time t, respectively. Note that $X_o(t)$ and $X_o^1(t)$ are random variables. The quantity $X_o(t) \neq X_o^1(t)$ since multiple paths to the origin will produce a multiplicity of copies of each message.

Problem 2. Compute $N_o(t)$ and $N_o^1(t)$ which are the expected or average number of messages and distinct messages which arrive at the ground station at time t. Define the inefficiency of the system as the ratio $N_o(t)/N_o^1(t)$. Compute the inefficiency and the asymptotic inefficiency.

We can alter assumption 3. to model the situation where not all messages which arrive at a repeater are accepted. We will model the case of imperfect capture using two different modes. For the first mode of imperfect capture which we call Mode 1, we change assumption 3. to 3.1.

Assumption 3.1. Not all messages which arrrive at a repeater are captured or accepted. Each repeater has the capacity to accept at most m-messages. The arriving messages are independently and at random in one of m-slots. The number of messages accepted is given by the number of slots with exactly one message.

Assumption 4. Messages not accepted disappear from the system.

Problem 3. Under the assumptions 1, 2, 3.1, 4, compute the probability P_{kj} defined as the probability that exactly j messages are accepted given that k arrive, j=0,1,2,..., max(k,m). The quantity $P_{kj}=0$ for j>max(k,m) by assumption 3.1.

Problem 4. Under assumptions 1, 2, 3.1, 4, compute $X_o(t)$, $X_o^1(t)$, $N_o(t)$ and $N_o^1(t)$. Generate numerical data for a finite net of depth 5 repeaters in each direction for different numerical values of λ and m.

Problem 5. Study the relationships between arrivals and acceptances at the origin.

An alternative model for capture is given as assumption 3.2, called Mode 2 capture.

Problem 6. Under assumptions 1, 2, 3.2, 4, solve the analogues of Problems 3, 4, 5.

The next assumption that can be altered to refine the basic model concerns what happens to messages received but not accepted. We modify assumption four to allow two modes of retransmission of lost or erased messages.

Assumption 4.1. When a message is wiped out, it is retransmitted at its source of original transmission J(d) units of time after its original transmission.

Problem 7. Under assumptions 1, 2, 3.1, 4.1, compute the number of arrivals, acceptances,and retransmissions at each node or repeater for a variety of values of the parameters. Carry out the same calculations under assumptions 1, 2, 3.2, 4.1, i.e. change the capture mode to Mode 2.

Assumption 4.2. When a message is wiped out or erased, it is retransmitted from its point of erasure one time unit after erasure.

Problem 8. Solve Problem 7 under assumptions 1, 2, 3.1, 4.2 and 1, 2, 3.2, 4.2.

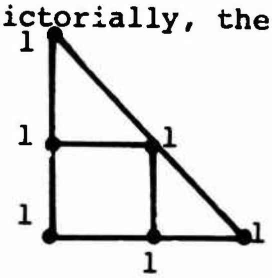
Problem 9. Under the sets of assumptions 1, 2, 3.1, 4.1; 1, 2, 3.1, 4.2; 1, 2, 3.2, 4.1; 1, 2, 3.2, 4.2 compute delays and average delays encountered by a message being repeated toward the origin.

3. THE NUMBER OF MESSAGES RECEIVED AT THE GROUND STATION

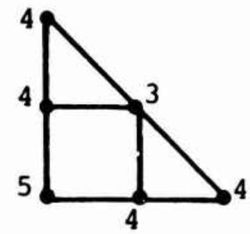
In this section, we begin our analysis of the basic model by solving problems one, two, and three. We study the Poisson case where messages arrive at each repeater according to a Poisson input. Since our interest is in the expected number of messages arriving at each repeater under perfect capture, we will assume that one message originates at each repeater at each point in time. It can be shown that we multiply the resultant numbers by λ, the mean number of Poisson arrivals. To fix ideas, we can plot a few time points and see by calculation how many messages arrive at each repeater. In the diagrams, the lower left-hand node represents the ground station or origin.

Pictorially, the propagation is as follows:

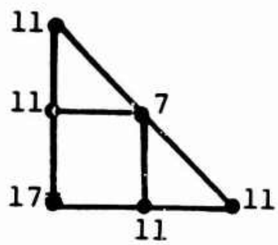
t = 0



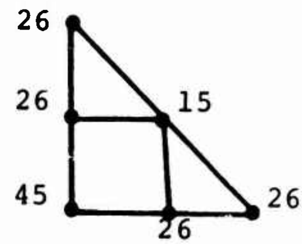
t = 1,



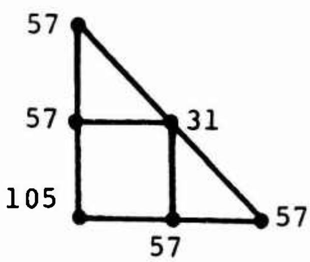
t = 2



t = 3,

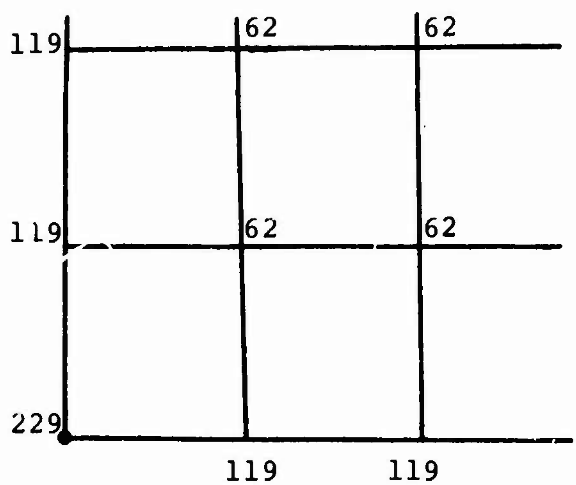


t = 4



Let $X_o(t)$ be the number of messages received at time t in the deterministic case.

t = 5



Let $X_j^d(t)$ be the number of messages received at a repeater at distance d, horizontal distance j (i.e. with coordinates (d,j)) at time t.

We have immediately;

$$X_o(t) = 4 \quad X_o^1(t-1) + 1, \qquad t \geq 1, \qquad \text{and}$$

$$X_o^1(t) = 2\ X_1^2(t-1) + X_o^2(t-1) + 1, \qquad \text{where}$$

$$X_1^2(t) = 2\ X_1^2(t-1) + 1$$

$$= 2\ X_1^2(t-2) + 2 + 1$$

$$=$$

$$= 2^{t+1}-1 .$$

This follows from the observation that repeaters at horizontal distance zero or d will receive the same number of messages for $d > 0$. Repeaters with coordinates (d,j), j=1, 2,..., d-1 will also receive the same number of messages.

Thus;

$$X_o^1(t) = 2(2^t-1) + X_o^1(t-1) + 1$$

$$= X_o^1(t-2) + 2^t + 2^{t+1}-2$$

$$=$$

$$= 1 + 2^2 + 2^3 + \ldots + 2^{t+1}-t .$$

We conclude that,

$$Xo(t) = 2^{t+3}-4t-7 \qquad t=0,\ 1,\ 2,\ldots.$$

Thus No(t) the expected number of messages received is $\lambda\ (2^{t+3}-4t-7)$.

The expected number of distinct messages received is $\lambda(2t^2+2t+1)$.

Therefore

$$\mathrm{Ieff}(t) = \frac{No(t)}{N_o^1(t)} = \frac{2^{t+3}-4t-7}{2t^2+2t+1} \sim \frac{2^{t+3}}{2t^2} = \frac{2^{t+2}}{t^2}$$

4. MESSAGE DISTRIBUTION WITH TYPE 1 SLOTTING

We derive formulae for P_{kj}, the probability that j messages are "received" given that k messages "arrive" at a repeater. Each message is assigned independently and at random to one of m-identical slots. The number of "received" messages is given by the number of slots with exactly one message.

To solve this problem, we consider the "ball in cell" model of placing k distinct balls into m-distinct cells. The quantity P_{kj} is the probability that exactly j cells have exactly one ball in each.

Let S be the set of all k-tuples where each component is one of the integers 1, 2,...,m, i.e., $S = \{(a_1, a_2,...,a_k)$: $a_i \in \{1, 2,...,m\}$, $i = 1, 2,...,k$. Let A_ν be the event(subset) where cell ν has exactly one ball, i.e. the subset of S of those sequences where the integer ν appears exactly once.

$$\text{Let} \sum_{i_1<i_2<i_3<\ldots<i_w} P_{i_1,i_2,\ldots,i_w} = S_w$$

where $P_{i_1,i_2,\ldots,i_w}$ is the probability $P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_w})$,

and the sum is over all subsets of integers of size w selected from [1, 2,...,m].

It is easy to compute S_w,

$$S_w = \sum_{i_1<i_2<\ldots<i_k} \frac{k(k-1)\ldots(k-w+1)}{m^k (m-w)^{w-k}} = \binom{m}{w} \frac{k!\,(m-w)^{k-w}}{m^k (k-w)!} \quad ;$$

for $w = 1, 2,...,\min(k,m)$, $S_w = 0$ for $w > \min(k,m)$.

By the well known variation of inclusion-exclusion.

$$P_{kj} = \sum_{j=0}^{m-j} (-1)^{\tau} \binom{j+\tau}{j} S_{j+\tau} ,$$

$$= \frac{1}{m^k} \sum_{\tau=0}^{m-j} (-1)^{\tau} \frac{\binom{j+\tau}{j}\binom{m}{j+\tau} k! \, (m-j-\tau)^{k-j-\tau}}{(k-j-\tau)!}$$

$$= \frac{\binom{m}{j}}{m^k} \sum_{\nu=0}^{m-j} (-1)^{\nu} \binom{m-j}{\nu} \frac{k!}{(k-j-\nu)!} (m-j-\nu)^{k-j-\nu}$$

when $j \leq \min(k,m)$. Actually, P_{kj} depends on m and should be written as $P_{kj}(m)$. Von-Mises has shown that when m is large, $P_{kj}(m)$ can be approximated by;

$$P_{kj}(m) = \frac{e^{-\lambda_1}(\lambda_1)^j}{j!} \quad ; \qquad \text{a Poisson variate with } \lambda_1 = ke^{-\frac{k}{m}}.$$

This is an approximation for the binomial with $p = e^{-\frac{k}{m}}$, $q = 1-e^{-\frac{k}{m}}$ when k is large, p small and kp moderate,

$$P_{kj} = \binom{k}{j} e^{-\frac{jk}{m}} (1-e^{-\frac{k}{m}})^{k-j} .$$

5. MESSAGE DISTRIBUTION WITH TYPE II SLOTTING

We derive formulae for P_{kj}^*, the probability that j messages are "received" given that k messages "arrive" at a repeater. Each message is assigned independently and at random to one of m-slots. The number of "received" messages is given by the number of slots with at least one message.

Again we use a "ball in cell" model and ask for the probability that exactly j boxes are not empty when we place k distinct balls in m-distinct boxes.

Using the same method of inclusion-exclusion as in section B, we find that;

$$P_{kj}^* = \binom{m}{j} \sum_{\nu=0}^{j} (-1)^{\nu} \binom{j}{\nu} \left(\frac{j-\nu}{m}\right)^k .$$

We can approximate P_{kj}^* by the same method as Section 4,

$$P_{kj}^* \sim \frac{e^{-\lambda_2} \cdot \lambda_2^{m-j}}{(m-j)!} \quad ; \quad 0 \leq j \leq \min(k,m)$$

where λ_2 is $me^{-\frac{k}{m}}$ when m,k are large.

6. SURVIVAL OF MESSAGES

Problem number three calls for the solution to the problem of finding the survival probability for a message which originates at a repeater with coordinates (j,d) at time t. We must first compute the probability that it is received at its original node. Let P^* be the probability that a given message which arrives at a repeater is received at that repeater. We must compute P^* under two modes or types of slotting. We call P_1^*, P_2^* the values of P^* under type I and II slotting respectively.

Let S_k be the set of (k+1) tuples formed from the integers 1, 2,...,m. Clearly, $n(S_k) = m^{k+1}$, i.e., the number of elements in S_k is m^{k+1}. Let $P^*_{k,1}$ be the probability that the message is received given that k-other messages arrive and type I slotting is used. Clearly, $P^*_{0,1} = 1$, and for $k > 0$,

$$P^*_{k,1} = \frac{m\text{(number of elements in } S_k \text{ with } \cdot \text{ in the } j^{th} \text{ position only)}}{m^{k+1}}$$

$$P^*_{k,1} = \frac{m}{m^{k+1}} [(m-1)^k] = \frac{(m-1)^k}{m^k} = (1-\frac{1}{m})^k$$

which is of course independent of time.

However, P^*_1 is dependent on time since it depends on the probability that k-other messages arrive: Therefore;

$$P^*_1(t) = \sum_{k=1}^{\infty} P^*_{k,1} \cdot P \text{ [exactly k other messages arrive at time t]}. \quad (1)$$

In principle $P^*_1(t)$ is computable from the distribution of arrivals at a given node at time t.

We must compute $P^*_2(t)$ for type II slotting. In this system, the given message is received if it is selected at random from the messages in its slot.

$$P^*_2(t) = \sum_{k=0}^{\infty} P\text{[Reception/k arrivals at t] } P\text{[k arrivals at t]}$$

$$= \sum_{k=0}^{\infty} \sum_{j=0}^{k} P\text{[Reception/k arrivals at t, j in same slot]}$$

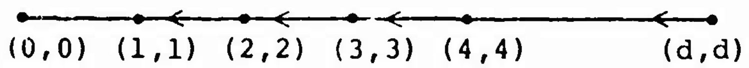
$$P\text{[j in same slot/k arrival]} \cdot P\text{[k arrivals at t]}.$$

$$= \sum_{k=0}^{\infty} \sum_{j=0}^{k} \frac{1}{j+1} P\text{[j in same slot/k arrivals]} \cdot P\text{[k arrivals at t]}.$$

$$P_2^*(t) = \sum_{k=0}^{\infty} \frac{m}{k+1} [1-(1-\frac{1}{m})^{k+1}] \cdot P[k \text{ arrivals at time } t]. \qquad (2)$$

Equations 1 and 2 can be solved once the distribution of message arrivals is determined.

To compute the solution to the problem of survival, let us, for simplicity, assume that the message arrives at a repeater with coordinates (d,d). Pictorially; the same problem for a repeater with coordinates (0,d).



Let $P_1(d,t)$ be the probability that a given message which originates at (d,d) at time t is received at the fixed station at (0,0) at time (t+d). Since the repeater at d-1 cannot "tell the difference" between a message arriving from (d,d) or originating, we can write:

$$P_1(d,t) = P_1(d-1,t+1) \cdot P[\text{message is initially received at } (d,d)].$$

The quantity, P[message is initially received at (d,d) was computed as (1) and (2) in the previous analysis for type I and type II slotting. Thus, the survival probabilities satisfy the fundamental difference equations for $P_1(d,t)$ and $P_2(d,t)$ the survival probabilities for type I and type II slotting respectively.

<u>FUNDAMENTAL DIFFERENCE EQUATIONS</u>:

$$P_1(d,t) = P_1(d-1,t+1) \sum_{k=0}^{\infty} (1-\frac{1}{m})^k \; P[X_d(t) = k]: \; t \geq 0 \quad d \geq 1 \quad (3)$$

with the initial condition,

$$P_1(0,t) = \sum_{k=0}^{\infty} (1-\frac{1}{m})^k \; P[X_d(t) = k]. \qquad (3a)$$

$$(5.4)\quad P_2(d,t) = P_2(d-1,t+1) \sum_{k=0}^{\infty} \frac{m}{k+1} \left[1-\left(1-\frac{1}{m}\right)^{k+1}\right] P[X_d(t)=k] \qquad (4)$$

for $t \geq 0 \quad d \geq 1$,

with the initial condition,

$$(5.4A)\quad P_2(0,t) = \sum_{k=0}^{\infty} \frac{m}{k+1} \left[1-\left(1-\frac{1}{m}\right)^{k+1}\right] P[X_{0,0}(t)=k]. \qquad (4a)$$

For t=0 all distributions $X_{j,d}(0)$ are Poisson with mean λ. Thus, we have theoretically solved the survival probability problem in terms of the message arrival distributions. We now turn our attention to the solution of problem 4.

7. DISTRIBUTION OF ARRIVALS AND RECEPTIONS

We now discuss the difficult problem of finding the distribution of arrivals and receptions at each repeater at each point in time. In sections 4 and 5 we have established formulae for P_{kj} which connect the arrival and reception distributions. It is possible to greatly simplify the problem by proving that there are only three random processes to determine.

A. $X_0(t)$ = arrivals at origin at time t.

B. $X_d(t)$ = arrivals at distance d from the origin on an axis at time t.

C. $X(t)$ = arrivals off the axis at time t.

The corresponding quantities $X_0^R(t)$, $X_d^R(t)$, $X^R(t)$ denote the reception distributions which can be obtained from the arrival distributions and P_{kj}. These remarks are justified by the following theorem.

<u>THEOREM</u>: A. $P[X_{0,d}(t)=k] = P[X_{0,e}(t)=k]$ for each $d,e \geq 0$, each $t \geq 0$ and k=0, 1, 2,...

B. $P[X_{j,d}(t)=k] = P[X_{\nu,w}(t)=k]$ for k=0, 1, 2,... for all pairs $j,\nu > 0, d,w > 0$.

C. $P[X_{0,d}^R(t)=k] = P[X_{0,\ell}^R(t)=k]$ for k=0, 1, 2,... for all pairs $d,\ell \geq 0$ and every t.

D. $P[X_{j,d}^R(t)=k] = P[X_{\nu,w}^R(t)=k]$ for k=0, 1, 2,... and all pairs $d,w > 0$ and $j,\nu > 0$.

PROOF: The proof follows by induction on t based on the facts that all repeaters start off with the identical Poisson process, the symmetry due to an unbounded region, and the relationships between $X_{j,d}(t)$ and $X^R_{j,d}(t)$.

Thus we need "only" to determine the distributions of the random process $X_0(t)$, $X_d(t)$ and $X(t)$; $t \geq 0$. The initial conditions are given by,

$$P[X_0(0)=k] = P[X_d(0)=k] = P[X(0)=k] = \frac{e^{-\lambda}\lambda^k}{k!} \qquad k=0,\ 1,\ 2,\ldots$$

For the "received" randon process,

$$P[X_0^R(0)=j] = P[X_d^R(0)=j] = P[X^R(t)=j]$$

$$= \sum_{k=0}^{\infty} P[X^R(0)=j \mid X(0)=k] \cdot P[X(0)=k]$$

$$= \sum_{k=j}^{\infty} P_{kj}\ P[X(0)=k]$$

$$= \sum_{k=j}^{\infty} P_{kj} \frac{e^{-\lambda}\lambda^k}{k!} = e^{-\lambda} \sum_{k=j}^{\infty} \frac{P_{kj}\lambda^k}{k!}$$

for type I slotting, and,

$$= e^{-\lambda} \sum_{k=j}^{\infty} \frac{P^*_{kj}\lambda^k}{k!} \qquad \text{for type II slotting.}$$

We can substitute the formulae for P_{kj} and P^*_{kj} from sections 4 and 5 into this equation, but the results are quite complicated.

The processes $X(t)$, $X_d(t)$ and $X_0(t)$ are each sequences of independent random variables over time, however, for each repeater, they depend on the values of earlier times at neighboring repeaters.

In fact, we can write the recursive equations,

$$X_0(t) = Y_1(t-1) + Y_2(t-1) + Y_3(t-1) + Y_4(t-1) + Y_5 \qquad (5)$$

Where Y_1, Y_2, Y_3, Y_4 are identically distributed (not independent) random variables with the same distribution as $X_1^R(t-1)$ and Y_5 is a Poisson random variable which is independent of Y_1, Y_2, Y_3, Y_4. It follows that,

$$E[X_0(t)] = 4E[X_1^R(t-1)] + \lambda \tag{5a}$$

Where E denotes expectation.

Similarly,

$$X(t) = W_1(t-1) + W_2(t-1) + W_3 \tag{6}$$

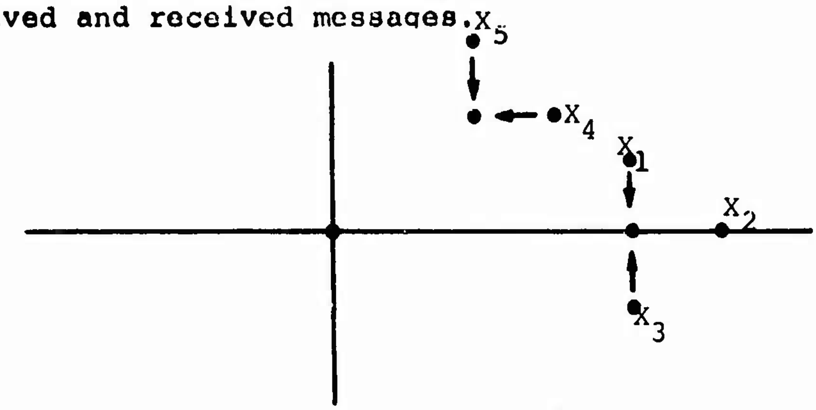
Where $W_1(t-1)$ and $W_2(t-1)$ have the same distribution as $X^R(t-1)$ but are dependent and W_3 is Poisson with mean λ and independent of W_1, W_2.

Finally;

$$X_d(t) = Z_1(t-1) + Z_2(t-1) + Z_3(t-1) + Z_4$$

where $Z_1(t-1)$ and $Z_2(t-1)$ have the same distribution as $X^R(t-1)$, $Z_3(t-1)$ has the same distribution as $X_d^R(t-1)$ and depends on Z_1, Z_2 while Z_4 is Poisson with mean λ and is dependent of Z_1, Z_2, Z_3.

There are essentially two types of repeaters in this model; those on an axis and those off the axis. Since neighboring repeaters have dependent arrival distributions, we shall consider two types of "clusters" of repeaters and their associated random distributions of arrived and received messages.



We define the following multivariate density functions:

$$f_t(Y_1, Y_2, Y_3) = P[X_1(t)=Y_1,\ X_2(t)=Y_2,\ X_3(t)=Y_3] \tag{7}$$

$$f_t^R(Z_1,Z_2,Z_3) = P[X_1^R(t)=Z_1,\ X_2^R(t)=Z_2,\ X_3^R(t)=Z_3] \tag{8}$$

$$g_t(W_1,W_2) = P[X_4(t)=W_1,\ X_5(t)=W_2] \tag{9}$$

$$g_t^R(U_1,U_2) = P[X_4^R(t)=U_1,\ X_5^R(t)=U_2] \tag{10}$$

where X_4,X_5 have the distributions of X as do X_1, X_2 while X_3 has the distribution of X_d. The initial conditions are:

$$f_0(Y_1,Y_2,Y_3) = \frac{e^{-\lambda}e^{-\lambda}e^{-\lambda}\lambda^{Y_1+Y_2+Y_3}}{Y_1!\,Y_2!\,Y_3!}\ ; \qquad Y_1,\ Y_2,\ Y_3 \geq 0 \tag{11}$$

$$g_0(W_1,W_2) = \frac{e^{-2\lambda}\lambda^{W_1+W_2}}{W_1!\ W_2!}\ , \quad W_1,\ W_2, \geq 0. \tag{12}$$

Under type I slotting, we can develop the following relations between the f's and g's.

$$f_t^R(Z_1,Z_2,Z_3) = \sum_{Y_1,Y_2,Y_3} P\ [(X_1^R(t)=Z_1,\ X_2^R(t)=Z_2,\ X_3^R(t)=Z_3 \,| \tag{13}$$

$$X_1(t)=Y_1,\ X_2(t)=Y_2,\ X_3(t)=Y_3] \tag{14}$$

$$\cdot\ P[X_1(t)=Y_1,\ X_2(t)=Y_2,\ X_3(t)=Y_3] \tag{15}$$

$$= \sum_{Y_1,Y_2,Y_3} \binom{Y_1}{X_1}\binom{Y_2}{X_2}\binom{Y_3}{X_3} e^{\frac{-(X_1Y_1+X_2Y_2+X_3Y_3)}{m}} \left(1-e^{-\frac{Y_1}{m}}\right)^{Y_1-X_1} \tag{16}$$

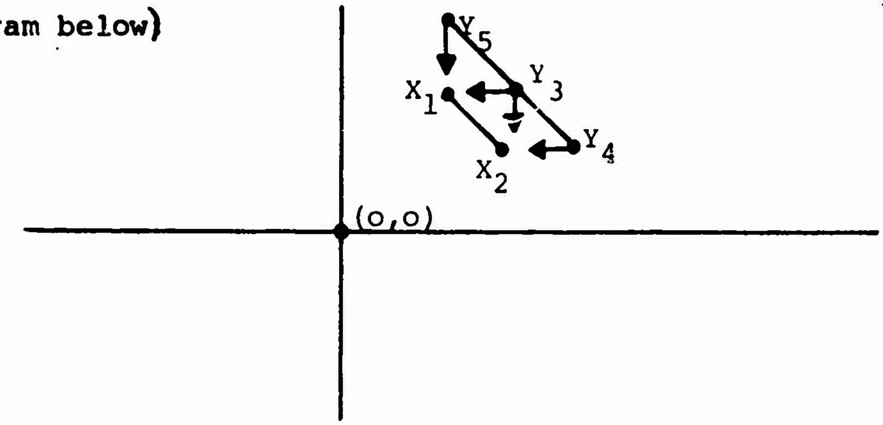
$$\left(1-e^{-\frac{Y_2}{m}}\right)^{Y_2-X_2} \left(1-e^{-\frac{Y_3}{m}}\right)^{Y_3-X_3} f_t(Y_1,Y_2,Y_3)\ . \tag{17}$$

$$g_t^R(U_1,U_2) = \sum_{W_1,W_2 \geq 0} P[X_4^R(t)=U_1,\ X_5^R(t)=U_2 \,|\, X_4(t)=W_1,\ X_5(t)=W_2] \tag{18}$$

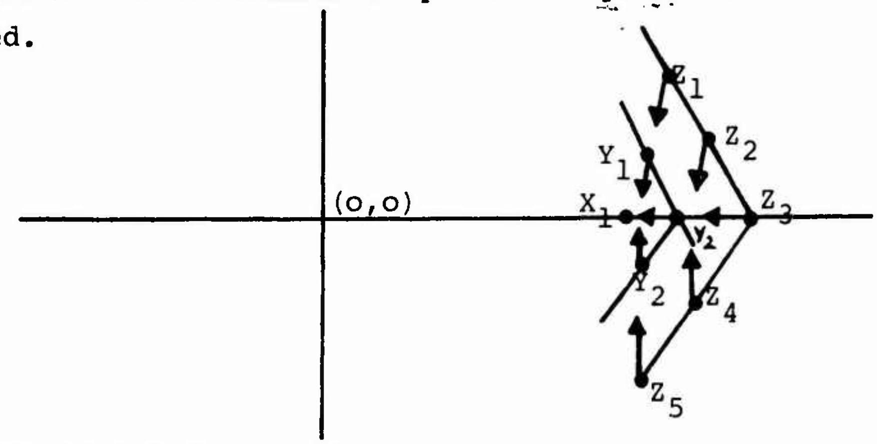
$$\cdot\ P[X_4(t)=W_1,\ X_5(t)=W_2].$$

$$= \sum_{W_1,W_2 \geq 0} \binom{W_1}{U_1}\binom{W_2}{U_2} e^{\frac{-(W_1U_1+W_2U_2)}{m}} \left(1-e^{-\frac{W_1}{m}}\right)^{W_1-U_1} \left(1-e^{-\frac{W_2}{m}}\right)^{W_2-U_2} \cdot g_t(W_1,W_2).$$

In order to determine the density functions (6.3, 6.4, 6.5, 6.6) we need equations relating these functions over time. In general, this is a difficult task since the distribution at say X_1, X_2 (refer to diagram below)



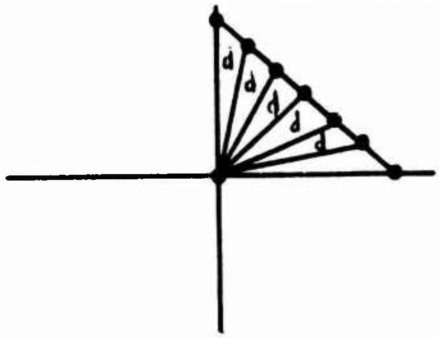
depends on the joint density of Y_3, Y_4, Y_5. Similarly, the density at points along the axis depend on distributions along a <u>wedge</u> which increase in the number of points (repeaters) which must be considered.



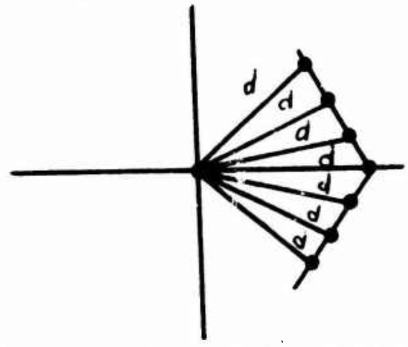
We are thus led to define two types of joint density functions of variable numbers of random variables.

We define the set of all points at the same distance from the origin as an <u>isodesic</u> set. Consecutive points on an <u>isodesic line</u>

are a sequence of points whose neighbors differ in distance by one unit from an axis. An <u>isodesic wedge</u> is a sequence of points on isodesic lines with a point on an axis and an equal number of points in neighboring quadrants separated by the axis.



isodesic line of 5 points



isodesic wedge of 7 points

For an isodesic line with k points at distance d define the joint density,

$\bar{f}_t(X_1,\dots,X_k)$ = P[the number of messages arriving at each of the k points is $X_1, X_2,\dots,X_k$ respectively]

(Note that the order and distance are unimportant due to symmetries).

Denote the received versions by $\bar{f}_t^R(X_1,\dots,X_k)$.

Similarly, for the wedge density define;

$\overset{>}{g}_t(X_1,\dots,X_k,Y,Z_1,\dots,Z_k)$ is the joint density of arriving messages at time t at the points along the wedges and

$\overset{>}{g}{}_t^R(X_1,\dots,X_k,Y,Z_1,\dots,Z_k)$ be the received joint densities. It is easy now to write difference equations over k and time (note that $k \le d-1$) otherwise f and g are not defined.

The initial conditions are independent over repeaters so that the joint densities are Poisson products.

$$\bar{f}_0(X_1,\dots,X_k) = \frac{e^{-k\lambda}\,\lambda^{\sum_{i=1}^{k} X_i}}{X_1!\,X_2!\cdots X_k!} \qquad \begin{array}{l} X_1,X_2,\dots,X_k \ge 0 \\ k=1,\dots,d-1\ . \end{array} \tag{15}$$

$$\overset{>}{g}_0(X_1,\ldots,X_k,Y,Z_1,\ldots,Z_k) = \frac{e^{-(2k+1)\lambda \sum_{i=1}^{k} X_i + \sum_{i=1}^{k} Z_i + Y}}{X_1!\ldots X_k!\; Y!\; Z_1!\ldots Z_k!}\;; \quad (16)$$

$$X_1,\ldots,X_k,Y,Z_1,\ldots,Z_k \geq 0$$

$$k=0,\ldots,d-1.$$

The received and arriving versions are connected by the following equations.

$$\bar{f}_t^R(X_1,\ldots,X_k) = \sum_{\substack{Z_i \geq 0 \\ i=1,2,\ldots,k}} \binom{Z_1}{X_1}\binom{Z_2}{X_2}\cdots\binom{Z_k}{X_k} e^{-(Z_1X_1+Z_2X_2+\ldots+Z_kX_k)}$$

$$\left(1-e^{-\frac{Z_1}{m}}\right)^{Z_1-X_1}\left(1-e^{-\frac{Z_2}{m}}\right)^{Z_2-X_2}\cdots\left(1-e^{-\frac{Z_k}{m}}\right)^{Z_k-X_k}$$

$$\bar{f}_t(Z_1,\ldots,Z_k)\,. \quad (17)$$

$$\overset{>}{g}{}_t^R(X_1,\ldots,X_k,Y,Z_1,\ldots,Z_k) = \sum_{\substack{Y_i \geq 0 \\ i=1,2,\ldots k}} \sum_{\substack{W_j \geq 0 \\ j=1,2,\ldots,k}} \sum_{r \geq 0}$$

$$\binom{Y_1}{X_1}\binom{Y_1}{X_2}\cdots\binom{Y_k}{X_k}\binom{r}{Y}\binom{W_1}{Z_1}\binom{W_2}{Z_2}\cdots\binom{W_k}{Z_k} e^{-\left(\sum_{i=1}^{k} X_iY_i + Y + \sum_{i=1}^{k} Z_iW_i\right)}$$

$$\left(1-e^{-\frac{Y_1}{m}}\right)^{Y_1-X_1}\left(1-e^{-\frac{Y_2}{m}}\right)^{Y_2-X_2}\cdots\left(1-e^{-\frac{Y_k}{m}}\right)^{Y_k-X_k}$$

$$\left(1-e^{-\frac{r}{m}}\right)^{r-y}\left(1-e^{-\frac{W_1}{m}}\right)^{W_1-Z_1}\left(1-e^{-\frac{W_2}{m}}\right)^{W_2-Z_2}\cdots\left(1-e^{-\frac{W_k}{m}}\right)^{W_k-Z_k}$$

$$\overset{>}{g}_t(Y_1,Y_2,\ldots,Y_k,r,W_1,\ldots,W_k)\,. \quad (18)$$

The difference equations over time are given by:

$$\bar{f}_t(X_1,\ldots,X_k) = \sum_{\substack{0 \le W_i \le m \\ i=1,2,\ldots,k+1}} \underset{\substack{\text{received at} \\ t-1}}{P[X_1,\ldots,X_k \mid W_1,\ldots,W_{k+1}]} \cdot \underset{\substack{\text{received at} \\ t-1}}{P[W_1,\ldots,W_k]}, \tag{19}$$

so that,

$$\bar{f}_t(X_1,\ldots,X_k) = \sum_{\substack{0 \le W_i \le m \\ i=1,2,\ldots,k+1}} P[W_1+W_2+Y_1=X_1] \cdot P[W_2+W_3+Y_2=X_2] \ldots \cdot P[W_k+W_{k+1}+Y_k=X_k] \quad \bar{f}^R_{t-1}(W_1,\ldots,W_{k+1})$$

$$= \sum_{\substack{0 \le W_i \le m \\ i=1,2,\ldots,k+1 \\ W_i+W_{i+1} \le X_i}} \frac{e^{-\lambda k} \lambda^{\sum_{i=1}^{k} X_i} \lambda^{-W_1-W_{k+1}} \lambda^{-2\sum_{j=2}^{k} W_j}}{(X_1-W_1-W_2)! \; (X_2-W_2-W_3)! \ldots (X_k-W_k-W_{k+1})!} \cdot \bar{f}^R_t(W_1,\ldots,W_{k+1}) \qquad k=1,\ldots,d-1. \tag{20}$$

Similarly for the "wedge" density functions;

$$\hat{g}_t(X_1,\ldots,X_k,Y,Z_1,\ldots,Z_k) = \sum_{W,U,V} P[W_1+W_2+Y_1=X_1] \cdot P[W_2+W_3+Y_2=X_2] \ldots \cdot P[W_k+W_{k+1}+Y_k=X_k] \; P[U+X_{k+1}+Z_1+S=Y] \cdot P[V_1+V_2+U_1=Z_1] \; P[V_2+V_3+U_2=Z_2] \ldots \cdot P[V_k+V_{k+1}+U_k=Z_k] \cdot g^R_{t-1}(W_1,\ldots,W_{k+1},U,V_1,\ldots,V_{k+1}). \tag{21}$$

$$= \sum_{\substack{0 \le W_i \le m \\ W_i + W_{i+1} \le X_i \\ 1 \le i \le k+1}} \; \sum_{0 \le U \le m} \; \sum_{\substack{0 \le V_i \le m \\ V_i + V_{i+1} \le Z_i \\ 1 \le i \le k+1}} e^{-(2k+1)\lambda} \lambda^{\sum_{i=1}^{k} X_i} \lambda^{\sum_{i=1}^{k} Z_i} \lambda^{-W_1 - W_{k+1}}$$

$$\lambda^{-V_1 - V_{k+1}} \lambda^{-2 \sum_{j=2}^{k} W_j} \lambda^{-2 \sum_{j=2}^{k} V_j} \lambda^{Y-U-X_{k+1}-Z_1} \cdot$$

$$\cdot \overset{>}{g}{}^{R}_{t-1}(W_1, \ldots, W_{k+1}, U, V_1, \ldots, V_{k+1})$$

all divided by,

$$(X_1-W_1-W_2)! \; (X_2-W_2-W_j)! \ldots (X_k-W_k-W_{k+1})! \; (Z_1-V_1-V_2)! \ldots (Z_k-V_k-V_{k+1})!$$

$$(Y-U-X_{k+1}-Z_1)!.$$

Theoretically, these equations can be solved since all initial condition (t=0) have been given and recurrences in time are derived.

8 . COMPUTER ANALYSIS WITH A CLOSED BOUNDARY

The results and complexities observed in sections 3 to 7 indicate the usefulness of computer analysis. An interesting and perhaps symbolic special case for computer analysis might be obtained by closing the boundary at d=5 steps from the origin. The distribution for each of the two modes (sections 4 and 5) can be computed and used as transfer functions from "arrived" to "received" messages. Of course, these transfers will take place by random sampling from those distributed and hence one less type of simulation will be required.

The computer analysis will begin once we have numbered the 61 repeaters which lie at a distance of five or less units from the origin. We do this in a counter clockwise direction beginning with d=0, d=1, d=2, d=3, d=4, d=5 and j coordinates 1, 2,...,4d d=0, 1, 2, 3, 4, 5 , as shown below in Figure 1.

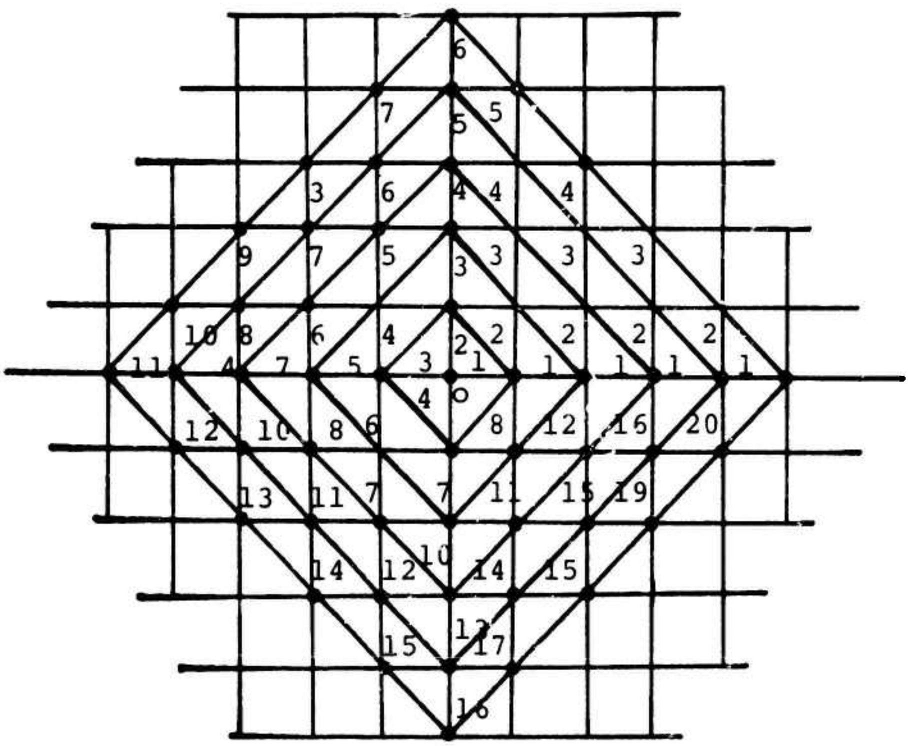


FIGURE 1

The repeaters at d=5 do not have messages arriving from other stations. They only receive their own traffic at Poisson rate. Repeaters at $d < 5$ which are on the "axes" (denoted by circles) have messages arriving from the three neighbors at d+1, as well as their own Poisson traffic.

Repeaters off the axes at distance $d < 5$ have input at each time point from the two neighbors at d+1 as well as their own Poisson traffic.

The network is activated at t=0 by having random Poisson arrivals with mean λ at each of the 61 repeaters. This input traffic at each repeater is converted to received messages in each of the two possible modes for different values of m by use of the "transfer functions".

$$(1)\quad P_{kj} = \frac{\binom{m}{j}}{m^k} \sum_{\nu=0}^{\min(k-j,m-j)} (-1)^{\nu} \cdot \binom{m-j}{\nu} \frac{k!}{(k-j-\nu)!} (m-j-\nu)^{k-j-\nu} \quad ;$$

$$(2)\quad P^*_{kj} = \binom{m}{j} \sum_{\nu=0}^{j} (-1)^{\nu} \binom{j}{\nu} \left(\frac{j-\nu}{m}\right)^k . \qquad j=0,\ 1,\ 2,\ldots,\min(k,m)$$

These calculations give us $P^R_{(d,j)}(0)$ for all repeaters with coordinates (d,j), d=1, 2, 3, 4, 5, j=1,...,4d, an (0,0) the station at the origin.

We can now determine message traffic at each repeater by using equations which describe message transmission in the direction of of the "origin".

<u>For Time t=1</u>

<u>When d=5; j=1, 2,...,20</u> , the repeaters at d=5 receive only their generated Poisson traffic. Thus, for time 1 we generate 61 Poisson traffic numbers which describe direct (i.e., at the source) message input. When $d \leq 4$, the repeater at coordinates (d,j) also receive traffic from its neighbors at further distance

by one unit. The following equations describe messages arriving at each repeater for arbitrary time $t>1$.

On the Axis:

$$P_{(0,0)}(t) = P^R_{(1,1)}(t-1)+P^R_{(1,2)}(t-1)+P^R_{(1,3)}(t-1)+P^R_{(1,4)}(t-1)+\text{Poisson}$$

At d=1 $(P_{1,1}, P_{1,2}, P_{1,3}, P_{1,4})$

$$P_{(1,1)}(t) = P^R_{(2,1)}(t-1)+P^R_{(2,2)}(t-1)+P^R_{(2,8)}(t-1)+\text{Poisson}$$

$$P_{(1,2)}(t) = P^R_{(2,2)}(t-1)+P^R_{(2,3)}(t-1)+P^R_{(2,4)}(t-1)+\text{Poisson}$$

$$P_{(1,3)}(t) = P^R_{(2,4)}(t-1)+P^R_{(2,5)}(t-1)+P^R_{(2,6)}(t-1)+\text{Poisson}$$

$$P_{(1,4)}(t) = P^R_{(2,6)}(t-1)+P^R_{(2,7)}(t-1)+P^R_{(2,8)}(t-1)+\text{Poisson}$$

At d=2 $P_{2,1}, P_{2,3}, P_{2,5}, P_{2,7}$

$$P_{(2,1)}(t) = P^R_{(3,1)}(t-1)+P^R_{(3,2)}(t-1)+P^R_{(3,12)}(t-1)+\text{Poisson}$$

$$P_{(2,3)}(t) = P_{(3,3)}(t-1)+P^R_{(3,4)}(t-1)+P^R_{(3,5)}(t-1)+\text{Poisson}$$

$$P_{(2,5)}(t) = P^R_{(3,6)}(t-1)+P^R_{(3,7)}(t-1+P^R_{(3,8)}(t-1)+\text{Poisson}$$

$$P_{(2,7)}(t) = P^R_{(3,9)}(t-1)+P^R_{(3,10)}(t-1)+P^R_{(3,11)}(t-1)+\text{Poisson}$$

At d=3 $P_{3,1}, P_{3,4}, P_{3,7}, P_{3,10}$

$$P_{(3,1)}(t) = P^R_{(4,1)}(t-1)+P^R_{(4,2)}(t-1)+P^R_{(4,16)}(t-1)+\text{Poisson}$$

$$P_{(3,4)}(t) = P^R_{(4,4)}(t-1)+P^R_{(4,5)}(t-1)+P^R_{(4,6)}(t-1)+\text{Poisson}$$

$$P_{(3,7)}(t) = P^R_{(4,8)}(t-1)+P^R_{(4,9)}(t-1)+P^R_{(4,10)}(t-1)+\text{Poisson}$$

$$P_{(3,10)}(t) = P^R_{(4,12)}(t-1)+P^R_{(4,13)}(t-1)+P^R_{(4,14)}(t-1)+\text{Poisson}$$

At d=4 $P_{(4,1)}(t)$, $P_{(4,5)}(t)$, $P_{(4,9)}(t)$, $P_{(4,13)}(t)$;

$$P_{(4,1)}(t) = P^R_{(5,1)}(t-1)+P^R_{(5,2)}(t-1)+P^R_{(5,20)}(t-1)+\text{Poisson}$$

$$P_{(4,5)}(t) = P^R_{(5,6)}(t-1)+P^R_{(5,5)}(t-1)+P^R_{(5,7)}(t-1)+\text{Poisson}$$

$$P_{(4,9)}(t) = P^R_{(5,10)}(t-1)+P^R_{(5,11)}(t-1)+P^R_{(5,12)}(t-1)+\text{Poisson}$$

$$P_{(4,13)}(t)= P^R_{(5,15)}(t-1)+P^R_{(5,16)}(t-1)+P^R_{(5,17)}(t-1)+\text{Poisson}$$

Off the Axes:

$$P_{(d,j)}(t) = P^R_{(d+1,j)}(t-1+P^R_{(d+1,j+1)}(t-1)+\text{Poisson};\quad j=2,3,\ldots,d;\ d=2,3,4.$$

$$P_{(d,j)}(t) = P^R_{(d+1,j+1)}(t-1)+P^R_{(d+1,j+2)}(t-1)+\text{Poisson};\quad j=d+2,d+3,..2d;\ d=2,3,4.$$

$$P_{(d,j)}(t) = P^R_{(d+1,j+2)}(t-1)+P^R_{(d+1,j+3)}(t-1)+\text{Poisson};\quad j=2d+2,..3d;\ d=2,3,4.$$

$$P_{(d,j)}(t) = P^R_{(d+1,j+3)}(t-1)+P^R_{(d+1,j+4)}(t-1)+\text{Poisson};\quad j=3d+2,..4d;\ d=2,3,4.$$

These equations relate arriving and received messages over neighboring time points and repeaters. Thus, the arriving number of messages can be computed in the grid at each point in time and each repeater.

In terms of a flow diagram, the procedure for analyzing this and all finite grids follows:

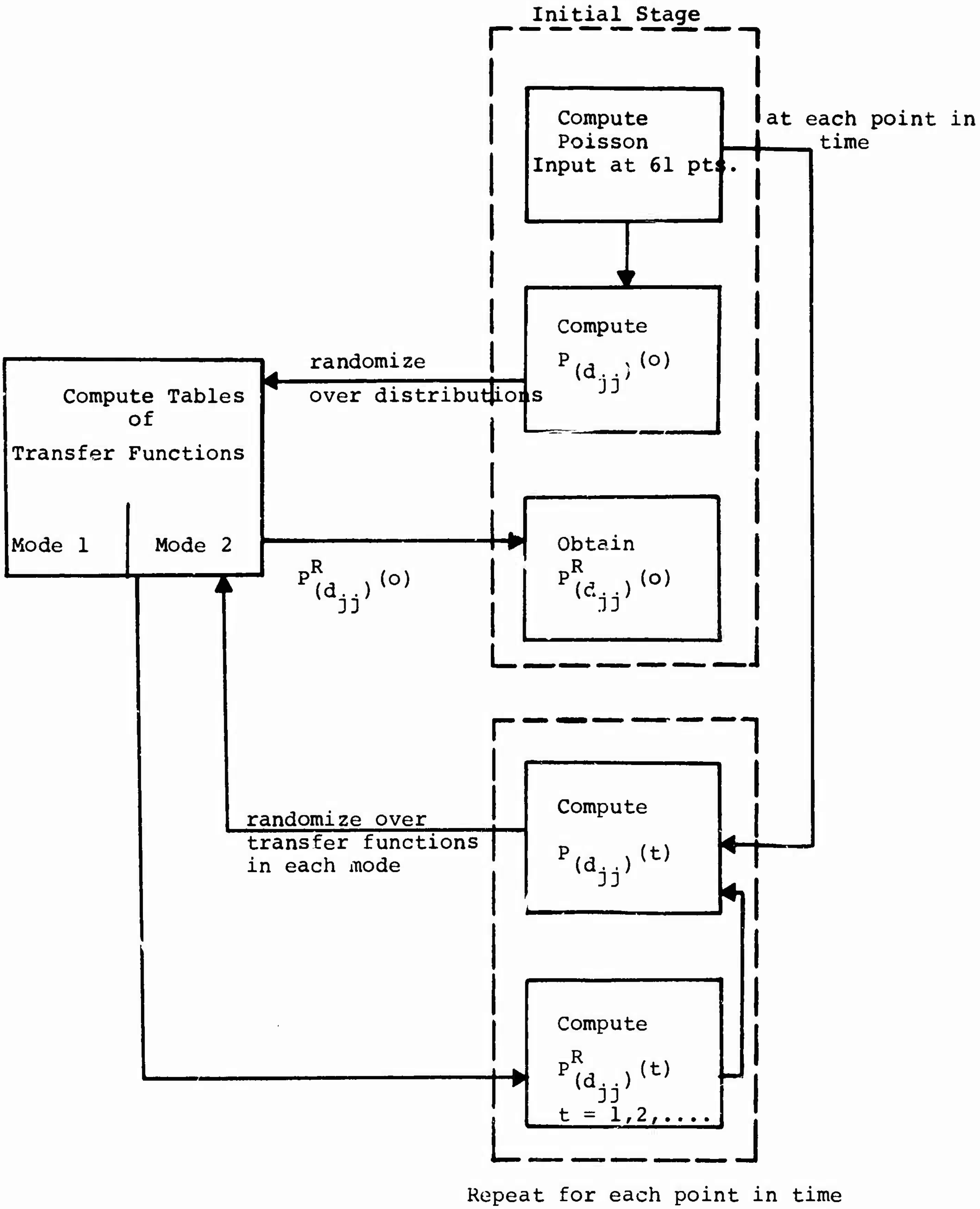

Initial Stage
Compute Poisson Input at 61 pts.
at each point in time
Compute $P_{(d_{jj})}(o)$
randomize over distributions
Compute Tables of Transfer Functions
Mode 1
Mode 2
$P^R_{(d_{jj})}(o)$
Obtain $P^R_{(d_{jj})}(o)$
Compute $P_{(d_{jj})}(t)$
randomize over transfer functions in each mode
Compute $P^R_{(d_{jj})}(t)$ $t = 1,2,....$
Repeat for each point in time

FIGURE 2

The parameters are, λ = mean Poisson arrival at each time point, at each repeater, m the number of slots in each mode one and two.

The output of the computer analysis is processed and presented in two forms, tabular and graphical. The tabular format is for each m and λ mode,

t	0	1	2	3	4	5	6	7	. . .
$P_{(0,0)}(t)$									. . .
$P^R_{(0,0)}(t)$									. . .
$P_{(1,1)}(t)$									. . .
$P^R_{(1,1)}(t)$									
$P_{(1,2)}(t)$									
$P^R_{(1,2)}(t)$									
⋮	⋮	⋮	⋮	⋮					

Various graphical analyses are also obtained.

A. A graph of arrived and received messages at the origin as a function of time for various values of m and λ.

B. A frequency histogram of arrivals off the axis. There are 24 points of the axis at distance 2, 3, 4,...

We take for each time t;

$$f(x) = \frac{\text{number of stations with x arrivals at time t}}{24} .$$

This is plotted for each time point.

C. The same histograms as in b except on the axis. There are 16 points on the axes at distances 1, 2, 3, 4.

D. The mean number of arrived and received messages $\hat{\lambda}(t)$ and $\hat{\lambda}^R(t)$ as a function of time on and off the axis. These are given by,

$$\hat{\lambda}(t) = \sum_{x=1} xf(x), \quad \hat{\lambda}^R(t) = \sum_{y=1} xf^R(x)$$

where f(x) if the frequency of arrivals and $f^R(x)$ is the frequency of received messages.

$$\hat{\lambda}_A(t) = \sum_{x=1} xf_A(x), \quad \hat{\lambda}_A{}^R(t) = \sum_{x=1} xf_A{}^R(x),$$

where $f_A(x)$ and $f_A{}^R(x)$ are frequencies on the axis of arriving and received messages. Some numerical results follow.

8.1 Summary of Initial Computer Analysis

Attached, are two curves which represent a summary of data compiled from a preliminary computer investigation of a closed grid network. The grid selected for initial analysis is the closed boundary grid at distance five. We combined computer runs with the closed form theoretical analyses of sections 4 and 5 of this report to obtain some observations of network behaviour.

The first six curves represent a study of messages arriving and being received at the origin (fixed ground station) as a function of time. We used 20 computer runs for each of the first fifty time units. In this initial study the number of slots was kept fixed at 100, but λ (the mean number of messages originating at a given repeater) was set at 10, 20 and 30. All calculations were carried out for mode 1 and mode 2.

The message flow and reception at the origin settle down at about t=4 and remained relatively constant. For $\lambda=10$ the number of arriving messages seemed to have a mean at about 155 and the number of received messages averaged to about 31. Since the system behaviour for $\lambda=10$, m=100 settled down so quickly it seems reasonable to combine all time point data past t=10 to estimate the probability density function of arrivals and receptions at the origin in each of modes 1 and 2 when $\lambda=10$. The curves would seem to indicate asymptotic Poisson behaviour with means about 31, 155 in mode 1 and about 100, 300, in mode 2 respectively. Saturation occurs quickly in mode 2 for $\lambda=10$ or more. These results are summarized in the last four curves of probability density functions.

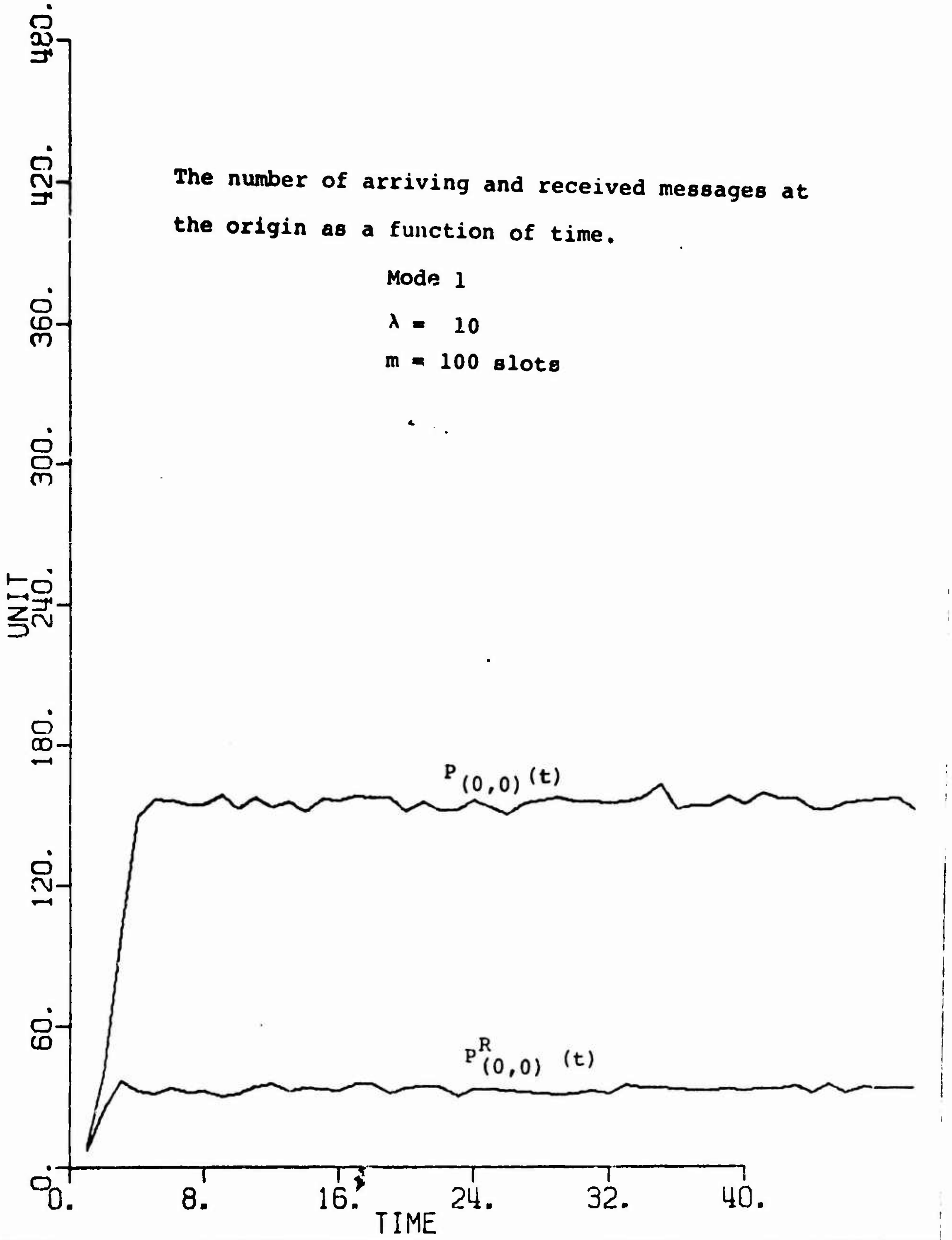

The number of arriving and received messages at the origin as a function of time.
Mode 1
λ = 10
m = 100 slots
$P_{(0,0)}(t)$
$P^R_{(0,0)}(t)$
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 3

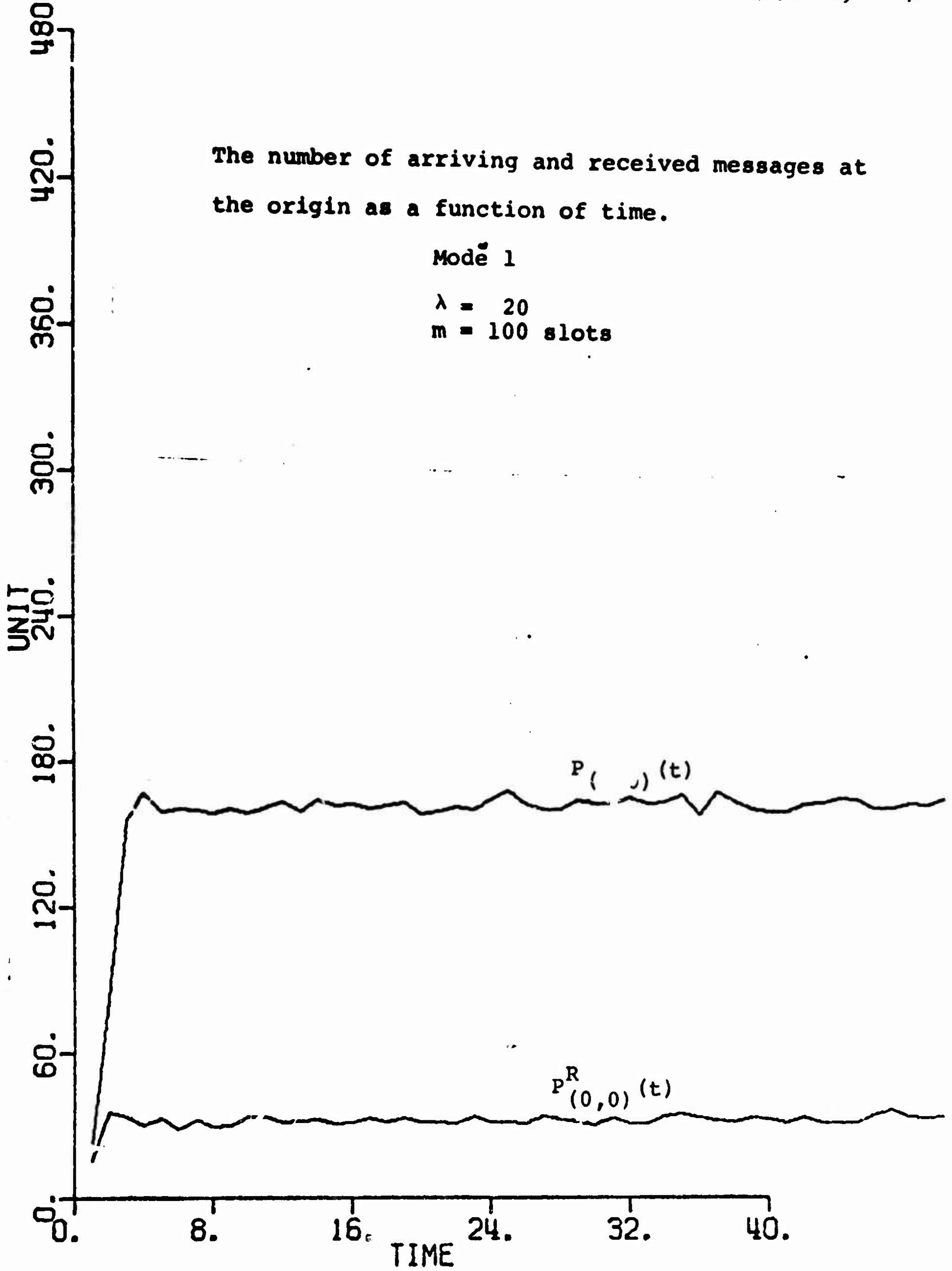

The number of arriving and received messages at
the origin as a function of time.
Mode 1
λ = 20
m = 100 slots
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
0.
8.
16.
24.
32.
40.
TIME
(t)
P^R (0,0) (t)

FIGURE 4

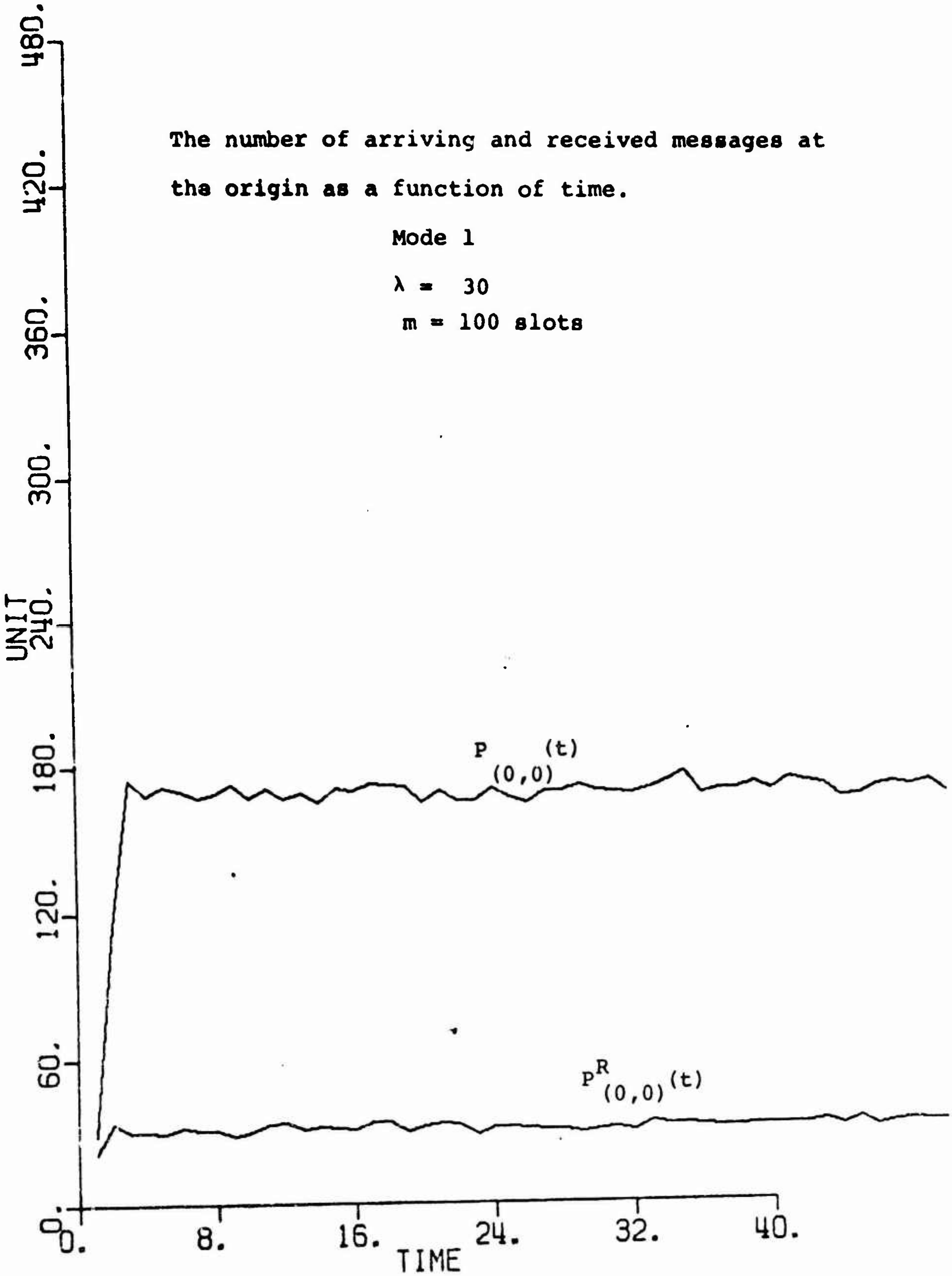

The number of arriving and received messages at
the origin as a function of time.
Mode 1
λ = 30
m = 100 slots
$P_{(0,0)}(t)$
$P^R_{(0,0)}(t)$
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 5

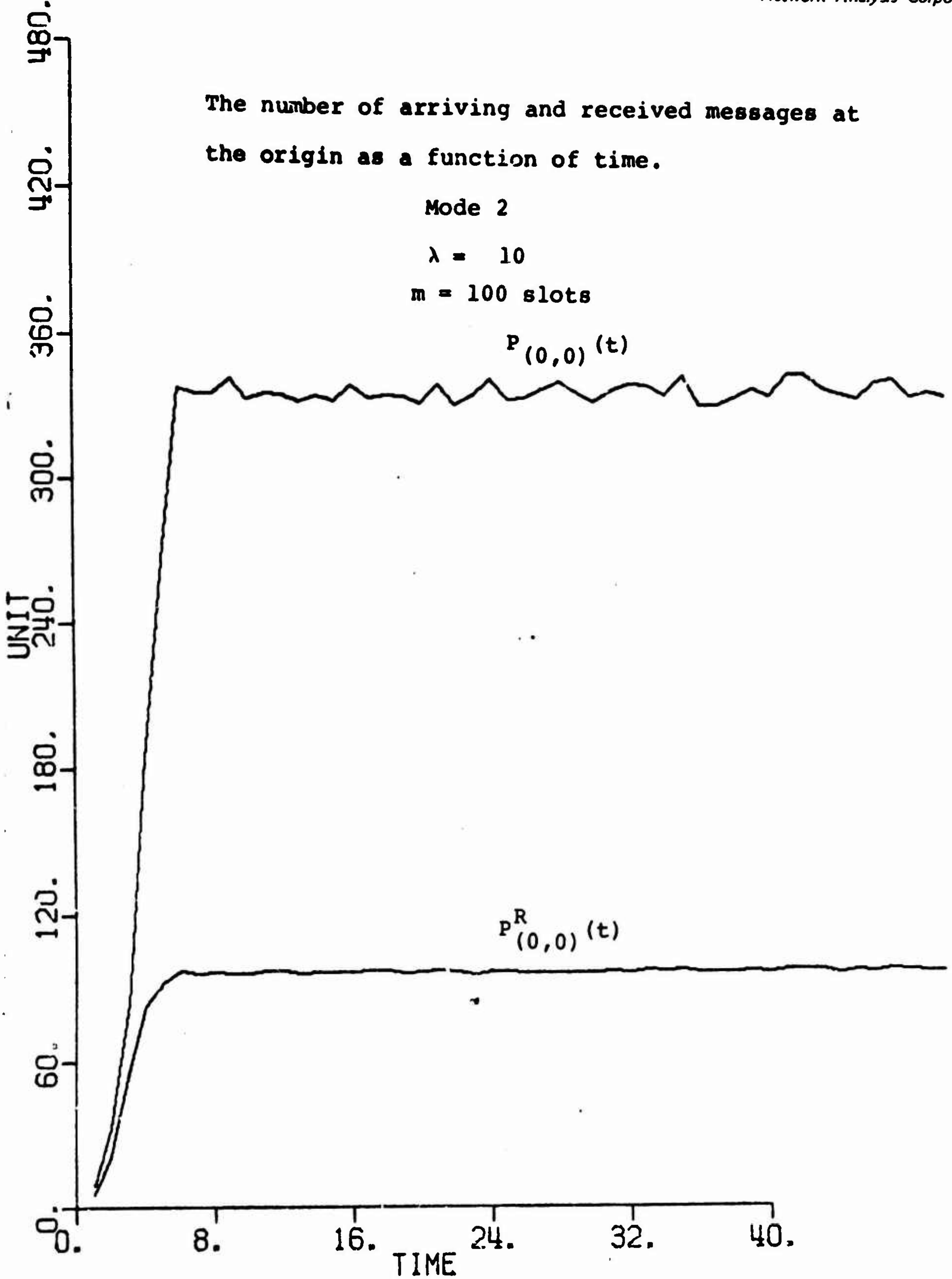

The number of arriving and received messages at the origin as a function of time.
Mode 2
λ = 10
m = 100 slots
P(0,0)(t)
P^R(0,0)(t)
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 6

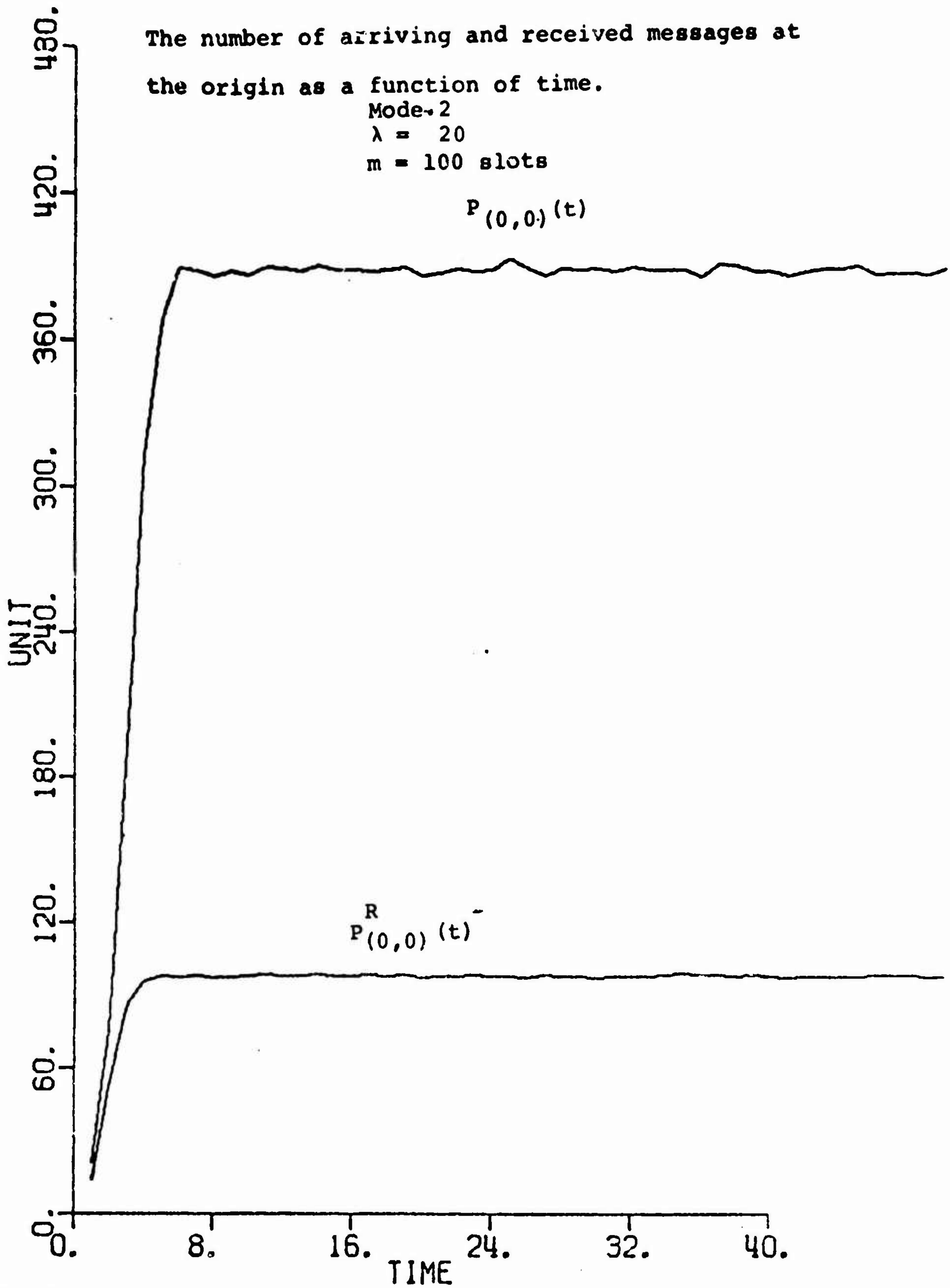

The number of arriving and received messages at
the origin as a function of time.
Mode- 2
λ = 20
m = 100 slots
$P_{(0,0)}(t)$
$P^R_{(0,0)}(t)$
430.
420.
360.
300.
240.
UNIT
180.
120.
60.
0.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 7

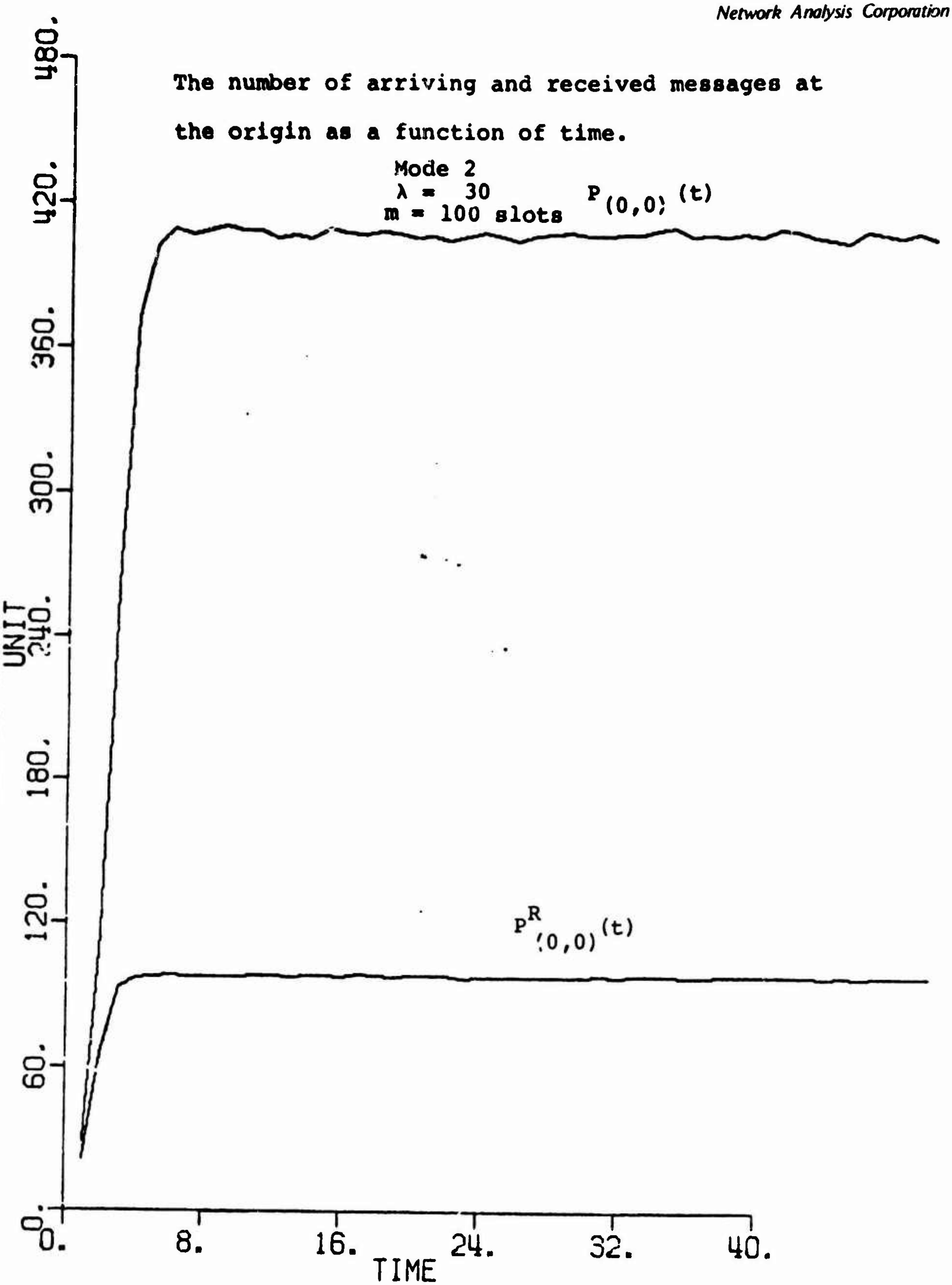

The number of arriving and received messages at
the origin as a function of time.
Mode 2
λ = 30
m = 100 slots
P(0,0)(t)
P^R(0,0)(t)
0.
60.
120.
180.
240.
300.
360.
420.
480.
UNIT
0.
8.
16.
24.
32.
40.
TIME

FIGURE 8

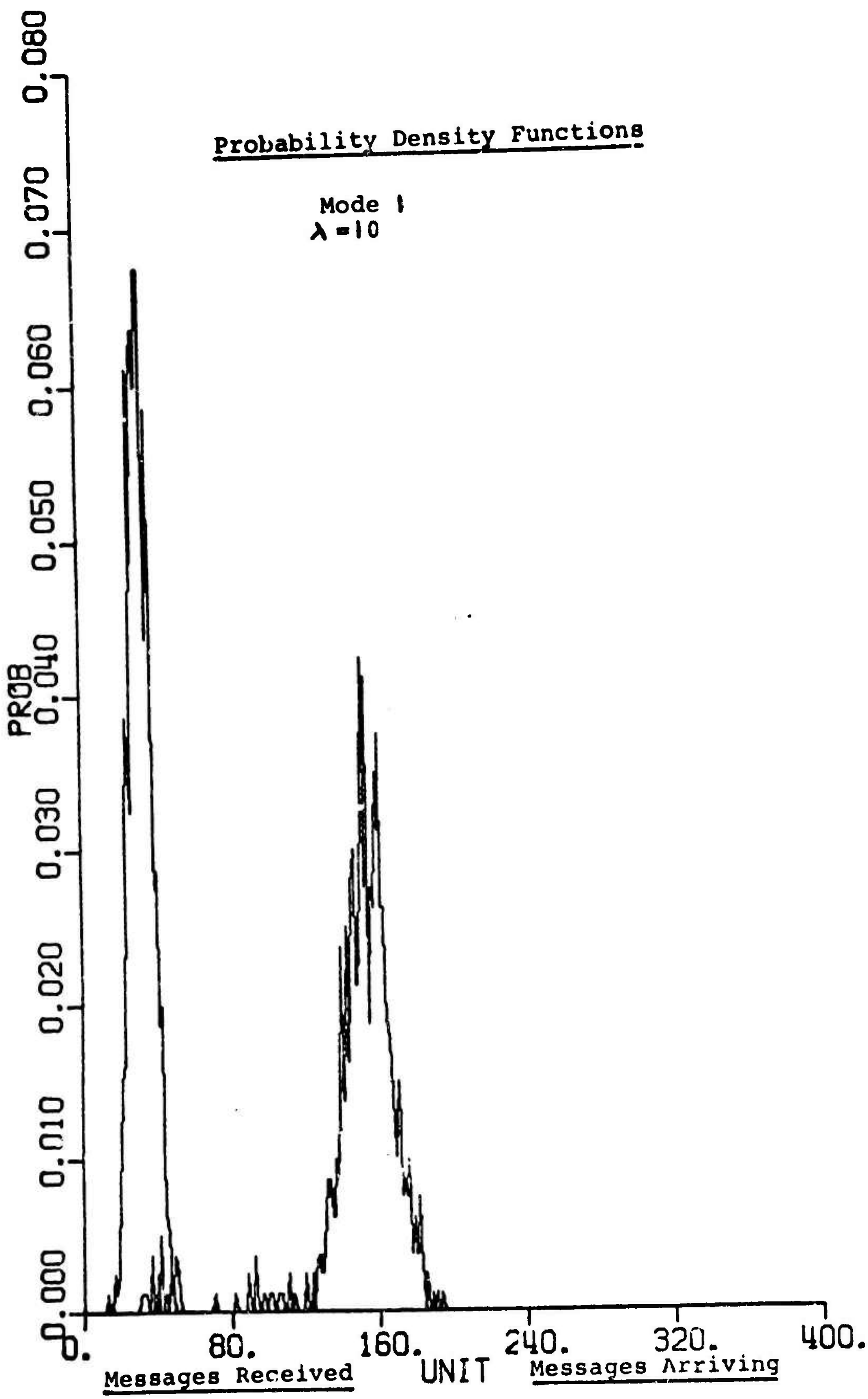

Probability Density Functions
Mode 1
λ =10
PROB
0.000
0.010
0.020
0.030
0.040
0.050
0.060
0.070
0.080
0.
80.
160.
240.
320.
400.
UNIT
Messages Received
Messages Arriving

FIGURE 9

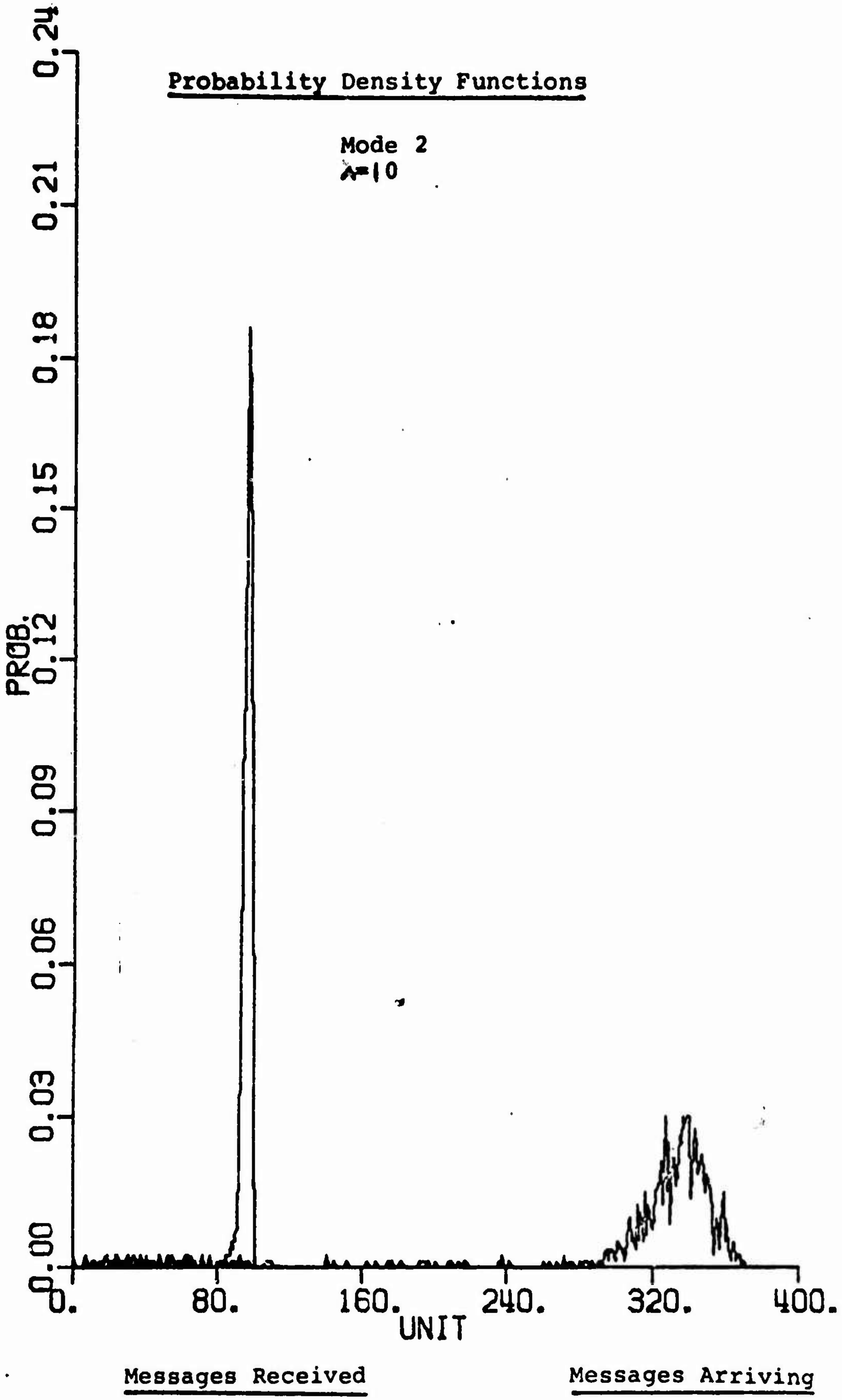

Probability Density Functions
Mode 2
λ=10
PROB.
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.21
0.24
0.
80.
160.
240.
320.
400.
UNIT
Messages Received
Messages Arriving

FIGURE 10

9. DYNAMICS OF A SINGLE MESSAGE ON ROUTE

In this section, we will develop the theoretical basis for a computer analysis of the dynamics of a single message originating at a repeater in the net and attempting to reach the ground station at the origin. The equations derived are directed towards a computer analysis. Let us assume that the given message originates at a repeater with coordinates (i,j) at time t. If the incoming and acceptance numbers at (k,j) at time t are respectively $X_{(i,j)}(t)$ and $X^A_{(i,j)}(t)$, we assume the given message is one of the $X_{(i,j)}(t)$ messages. Furthermore, we assume that each of the $X_{(i,j)}(t)$ messages is equally likely to be one of the accepted messages. Under these assumptions, it follows that at (i,j), there are two types of messages which have arrived. The first type is one message (the given one), the second type are $X_{(i,j)}(t)-1$ messages. The probability of acceptance at (i,j) is given by the hypergeometric probability density function:

$$\frac{\binom{X_{(ij)}(t)-1}{X^A_{(ij)}(t)-1}}{\binom{X_{(ij)}(t)}{X^A_{(ij)}(t)}} \qquad (22)$$

At each repeater on every path to the ground station the same analysis applies. At any given repeater, on the path, say with coordinates (k,e) there may be several copies of the original message which arrives.

Suppose (k,e) is on a path from (i,j) to (0,0) and the number of paths from (i,j) to (k,e) is w. Then at (k,e), at time t plus the distance from (i,j) to (k,e), either 0, 1, 2,..., up to w copies of the message may arrive. If d is the distance from (i,j) to (k,e) and at time (t + d), $X_{(k,e)}(t+d)$ and $X^{A}_{(k,e)}(t+d)$ messages respectively arrive and are accepted then we can compute the probability that exactly Z copies of the original messages are accepted. The computation of the required probabilities is a direct extension of

$$P\{\text{exactly } Z \text{ copies of original message is accepted at } (k,e) \text{ at time } t + d/\nu \text{ copies are amongst the arrivals}\}$$

$$= \frac{\binom{\nu}{Z}\binom{X_{(k,e)}(t+d)-\nu}{X^{A}_{(k,e)}(t+d)-Z}}{\binom{X_{(k,e)}(t+d)}{X^{A}_{(k,e)}(t+d)}} \quad ; \quad Z = 0, 1, 2, \ldots, \nu.$$

Equation(25) is valid at every repeater along every path from (i,j) to (0,0), and in particular at the origin. The only ingredient needed to apply the equations to a computer analysis and generate numerical values is a formula for the probability that exactly ν copies of the message arrive at each repeater. This formula can be obtained recursively using the idea of isodesic line and wedge joint density functions as developed in Section 7.

If a single copy of the given message is accepted at its origination repeater, it is:

a) repeated to each of two repeaters one unit closer to the origin if it is not on an axis;

b) repeated to the one repeater one unit closer to the origin if it is on an axis.

We will focus only on (a) since (b) is essentially identical as far as the analysis is concerned. The message, when accepted at its origination, is then repeated to repeaters at (i-1,j-1) and (i-1,j). Acceptances at (i-1,j-1) and (i-1,j) are determined according to Equation (23) The isodesic line joint density of receptions and acceptances are computed at (i-1,j-1) and (i-1,j). This joint density then determines arrivals and acceptances at (i-2,j-2), (i-2,j-1) and (i-2,j). The process then continues recursively until all computations are carried out at the origin.

9.1 Outline of Computer Analysis

In our computer analysis we used the above results to compute the probability distributions and mean value of the number of copies accepted at the origin of a single message which originates at distance of 5, 4, 3, 2, 1, 0 units from the ground station. For convenience and realism of the numerical results, we selected each originating repeater to have the maximum number of paths to the origin. The coordinate system we used for these calculations is given in Figure11, below:

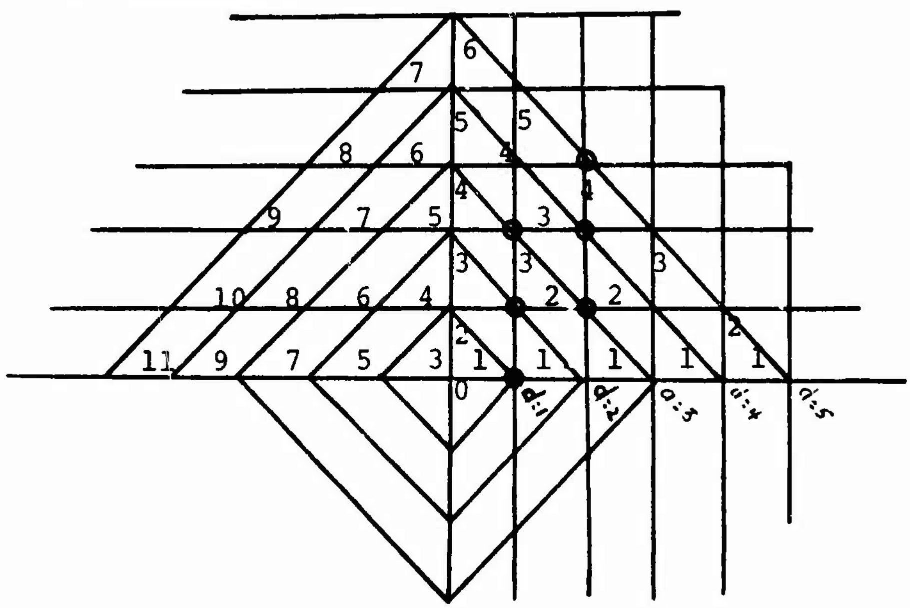


FIGURE 11

The repeaters selected for originating messages at distances 5,4,3,2,1,0 are respectively at (5,4), (4,3), (3,3), (2,2), (1,1), (0,0). The routes are designated in Figure 12, and the maximum number of copies of an originating message which can be received along each repeater on the route is given in Table 1 below. Note that the maximum number of possible copies is given by the number of paths from an originating repeater to the receiving repeater. Note in Table 1 that no copies can be received at a repeater further from the origin than the originator.

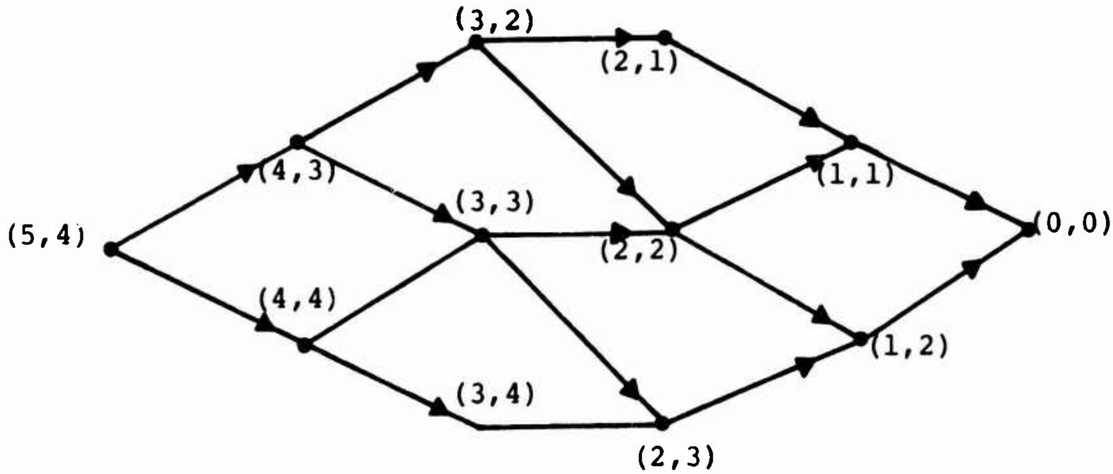


Fig. 12 Routing From (5,4) to (0,0)

	(0,0)	(1,1)	(1,2)	(2,1)	(2,2)	(2,3)	(3,2)	(3,3)	(3,4)	(4,3)	(4,4)	(5,4)
(0,0)	1	1	1	1	2	1	3	3	1	6	4	10
(1,1)		1	0	1	1	0	2	1	0	3	1	4
(1,2)			1	0	1	1	1	2	1	3	3	6
(2,1)				1	0	0	1	0	0	1	0	1
(2,2)					1	0	1	1	0	2	1	3
(2,3)						1	0	1	1	1	2	3
(3,2)							1	0	0	1	0	1
(3,3)								1	0	1	1	2
(3,4)									1	0	1	1
(4,3)										1	0	1
(4,4)											1	1
(5,4)												1

Table 1 Maximum Number of Copies - Between Two Repeaters

A. <u>The Equation for $Z_0(j;t)$</u>

Clearly $Z_0(j;t)$ is simply given by

$$Z_0(j;t) = \frac{A_{(0,0)}(1,j;t)}{A_{(0,0)}(0,0;t)} \; ; \; j = 0,\ 1; \quad t = 0,\ 1,\ 2,\ \dots,\ 40. \tag{24}$$

B. <u>The Equation for $Z_1(j;t)$</u>

$$Z_1(j;t) = \sum_{\mu=0}^{1} \binom{\mu}{j} \cdot \frac{A_{(0,0)}(\mu,j;t)}{A_{(0,0)}(0,0;t)} \cdot f_1^1(\mu;t-1) \tag{25}$$

for $j = 0,\ 1;\quad t = 1,\ 2,\ \dots,\ 40;$ where

$$f_1^1(j;t) = \frac{A_{(1,1)}(1,j;t)}{A_{(1,1)}(0,0;t)} \; ; \; j = 0,\ 1; \quad t = 0,\ 1,\ 2,\ \dots,\ 39.$$

C. <u>The Equation for $Z_2(j;t)$</u>

$$Z_2(j;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{1} f_2^2(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} \; ; \tag{26}$$

for $j = 0,\ 1,\ 2;\quad t = 2,\ 3,\ \dots,\ 40;$ where

$$f_2^2(i,j;t) = \sum_{\mu=0}^{1} f_1^2(\mu;t-1) \binom{\mu}{i} \binom{\mu}{j} \cdot \frac{A_{(1,1)}(\mu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu,j;t)}{A_{(1,2)}(0,0;t)} \; ;$$

for $t = 1,\ 2,\ \dots,\ 39;\quad i = 0,\ 1;\quad j = 0,\ 1;$ where

$$f_1^2(j;t) = \frac{A_{(2,2)}(1,j;t)}{A_{(2,2)}(0,0;t)} \; ; \; t = 0,\ 1,\ 2,\ \dots,\ 38; \quad j = 0,\ 1.$$

D. <u>The Equation for $Z_3(j;t)$</u>

$$Z_3(j;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{2} f_3^3(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} \; ; \tag{27}$$

for $t = 3,\ \dots,\ 40;\quad j = 0,\ 1,\ 2,\ 3;$ where

$$f_3^3(i,j;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{1} f_2^3(\mu,\nu;t-1) \binom{\mu}{i} \binom{\mu+\nu}{j} \cdot \frac{A_{(1,1)}(\mu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu+\nu,j;t)}{A_{(1,2)}(0,0;t)} \; ;$$

for $t = 2, \ldots, 39;\quad i = 0, 1;\quad j = 0, 1, 2;$ where

$$f_2^3(i,j;t) = \sum_{\mu=0}^{1} f_1^3(\mu;t-1) \binom{\mu}{i} \binom{\mu}{j} \cdot \frac{A_{(2,2)}(\mu,i;t)}{A_{(2,2)}(0,0;t)} \cdot \frac{A_{(2,3)}(\mu,j;t)}{A_{(2,3)}(0,0;t)} ;$$

for $t = 1, 2, \ldots, 38;\quad i = 0, 1;\quad j = 0, 1;$ where

$$f_1^3(j;t) = \frac{A_{(3,3)}(i,j;t)}{A_{(3,3)}(0,0;t)} ;\ t = 0, 1, 2, \ldots, 37;\ j = 0, 1.$$

E. The Equation for $Z_4(j;t)$

$$Z_4(j;t) = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} f_4^4(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} ;$$

for $t = 4, 5, \ldots, 40;\quad j = 0, 1, 2, \ldots, 16;$ where

$$f_4^4(i,j;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{2} \sum_{\rho=0}^{1} f_3^4(\nu,\mu,\rho;t-1) \binom{\nu+\mu}{i} \binom{\mu+\rho}{j} \cdot \frac{A_{(1,1)}(\mu+\nu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu+\rho,j;t)}{A_{(1,2)}(0,0;t)} ;$$

for $t = 3, 4, \ldots, 39;\quad i = 0, 1, 2, 3;\quad j = 0, 1, 2, 3;$ where

$$f_3^4(i,j,k;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{1} f_2^4(\mu,\nu;t-1) \binom{\mu}{i} \binom{\mu+\nu}{j} \binom{\nu}{k} \cdot \frac{A_{(2,1)}(\mu,i;t)}{A_{(2,1)}(0,0;t)} \cdot \frac{A_{(2,2)}(\mu+\nu,j;t)}{A_{(2,2)}(0,0;t)} \cdot \frac{A_{(2,3)}(\nu,k;t)}{A_{(2,3)}(0,0;t)} ;$$

for $t = 2, 3, \ldots, 38;\quad i = 0, 1;\quad j = 0, 1, 2;\quad k = 0, 1;$ where

$$f_2^4(i,j;t) = \sum_{\mu=0}^{1} f_1^4(\mu;t-1) \binom{\mu}{i} \binom{\mu}{j} \cdot \frac{A_{(3,2)}(\mu,i;t)}{A_{(3,2)}(0,0;t)} \cdot \frac{A_{(3,3)}(\mu,j;t)}{A_{(3,3)}(0,0;t)} ;$$

for $t = 1, 2, \ldots, 37;\quad i = 0, 1;\quad j = 0, 1;$ where

$$f_1^4(j;t) = \frac{A_{(4,3)}(1,j;t)}{A_{(4,3)}(0,0;t)} ;\ t = 0, 1, \ldots, 36;\ j = 0, 1. \tag{28}$$

The numbers in Table 1 give the upper limits of the summation for the possible copies of messages which can be received at each repeater of a single message originating at a repeater further from the origin but within the net of Figure 12. With the selected net and the numbers of Table 1, we can use the results of section 12 to obtain numerical data.

At time zero a random number of messages has arrived at each repeater. To compute the distribution of copies arriving at (0,0) from (5,4) we assume one of the messages arriving at (5,4) is singled out and followed along the route using the hypergeometric analysis of section 12. The procedure was used for $t = 0, 1, 2, \ldots, 40$ in conjuction with the random Poisson number generator developed and discussed earlier.

Specifically we seek to compute the five numbers:

$$
\begin{array}{lll}
Z_0(j;t); & j = 0, 1; & t = 0, 1, 2, \ldots, 40; \\
Z_1(j;t); & j = 0, 1; & t = 1, 2, \ldots, 40; \\
Z_2(j;t); & j = 0, 1, 2; & t = 2, 3, \ldots, 40; \\
Z_3(j;t); & j = 0, 1, 2, 3; & t = 3, 4, \ldots, 40; \\
Z_4(j;t); & j = 0, 1, 2, 3, 4, 5, 6; & t = 4, 5, 6, \ldots, 40; \\
Z_5(j;t); & j = 0, 1, 2, 3, \ldots, 10; & t = 5, 6, \ldots, 40;
\end{array}
$$

where $Z_k(j;t)$ is the probability that exactly j copies of a message originating at a repeater at distance k at time t-k, are accepted at the origin at time t. For the computer analysis we considered one repeater at each of the distances, as in Figure 11. The maximum j values are given by the first row of Table 1. Using the hypergeometric analyses the following equation can be used to compute each of the $Z_k(j;t)$; as a function of:

1) λ = mean number of originations at each repeater.
2) m = number of slots fixed at 100.
3) Each of two capture modes 1 and 2.

For ease of notation we denote:

$$A_{(ij)}(w,X;t) = \binom{X_{(ij)}(t)-w}{X^A_{(ij)}(t)-X}.$$

F. The Equation for $Z_5(j;t)$

(6.6) $$Z_5(j;t) = \sum_{\mu=0}^{4} \sum_{\nu=0}^{6} f_5^5(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} ;$$

for $t = 5, 6, \ldots, 40$; $j = 0, 1, 2, \ldots, 10$; where

$$f_5^5(i,j;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{3} \sum_{\rho=0}^{3} f_4^5(\nu,\mu,\rho;t-1) \binom{\mu+\nu}{i} \binom{\mu+\rho}{j} \cdot \frac{A_{(1,1)}(\mu+\nu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu+\rho,j;t)}{A_{(1,2)}(0,0;t)} ;$$

for $t = 4, 5, 6, \ldots, 39$; $i = 0, 1, 2, 3, 4$; $j = 0, 1, 2, \ldots, 6$;

where

$$f_4^5(i,j,k;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{2} \sum_{\rho=0}^{1} f_3^5(\nu,\mu,\rho;t-1) \binom{\nu}{i} \binom{\nu+\mu}{j} \binom{\mu+\rho}{k} \cdot \frac{A_{(2,1)}(\nu,i;t)}{A_{(2,1)}(0,0;t)} \cdot \frac{A_{(2,2)}(\nu+\mu,j;t)}{A_{(2,2)}(0,0;t)} \cdot \frac{A_{(2,3)}(\mu+\rho,k;t)}{A_{(2,3)}(0,0;t)} ;$$

for $t = 3, \ldots, 38$; $i = 0, 1$; $j = 0, 1, 2, 3$; $k = 0, 1, 2, 3$;

where

$$f_3^5(i,j,k;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{1} f_2^5(\mu,\nu;t-1) \binom{\mu}{i} \binom{\mu+\nu}{j} \binom{\nu}{k} \cdot \frac{A_{(3,2)}(\mu,i;t)}{A_{(3,2)}(0,0;t)} \cdot \frac{A_{(3,3)}(\mu+\nu,j;t)}{A_{(3,3)}(0,0;t)} \cdot \frac{A_{(3,4)}(\nu,k;t)}{A_{(3,4)}(0,0;t)} ;$$

for $t = 2, 3, \ldots, 37$; $i = 0, 1$; $j = 0, 1, 2$; $k = 0, 1$; where

$$f_2^5(i,j;t) = \sum_{\mu=0}^{1} f_1^5(\mu,t-1) \binom{\mu}{i} \binom{\nu}{j} \cdot \frac{A_{(4,3)}(\mu,i;t)}{A_{(4,3)}(0,0;t)} \cdot \frac{A_{(4,4)}(\mu,j;t)}{A_{(4,4)}(0,0;t)} ;$$

for t = 1, 2, 3, ..., 36; i = 0, 1; j = 0, 1; where

$$f_1^5(j;t) = \frac{A_{(5,4)}(1,j;t)}{A_{(5,4)}(0,0;t)}\ ; \quad t = 0, 1, 2, \ldots, 35; \quad j = 0, 1.$$

9.2 Probability of at Least One Message Getting Through

The first set of curves, Figures 13-18, plot the probability of at least one message getting through as a function of the mean number of originations at each repeater. There is one set of curves for each unit of distance d ranging from 0 to 5. Each figure contains one curve for mode 1 and one curve for mode 2. The number of slots was fixed at 100. The data for the curves is summarized in Table 2 below.

	λ=1		λ=3		λ=5	
distance	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0	.398	.589	.285	.421	.264	.431
1	.243	.434	.192	.273	.117	.321
2	.355	.614	.166	.341	.129	.326
3	.428	.695	.164	.358	.119	.285
4	.613	.874	.250	.534	.159	.271
5	.740	.967	.341	.692	.157	.198

Table 2: Probability That at Least One Message Gets Through

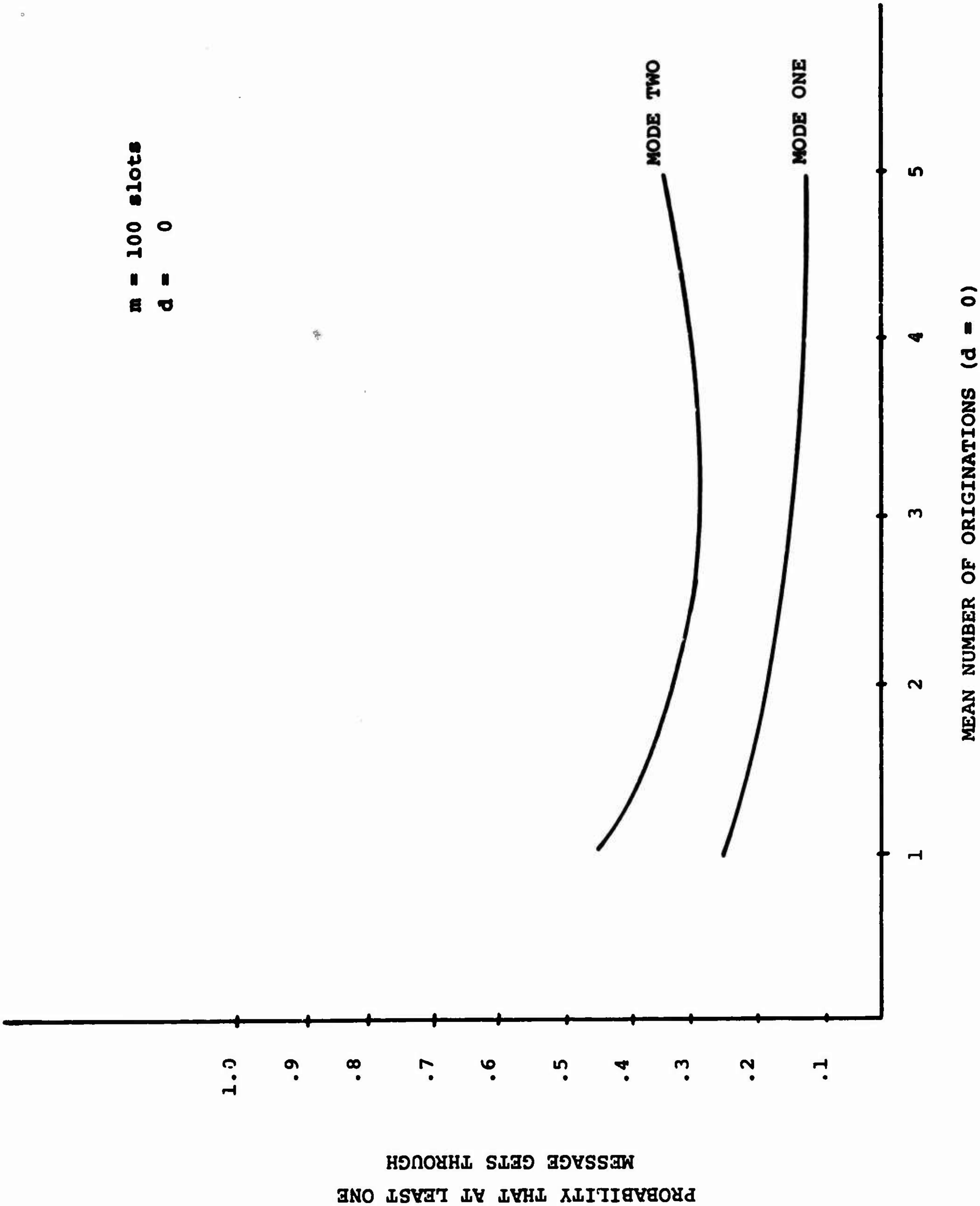

m = 100 slots
d = 0
MODE TWO
MODE ONE
1
2
3
4
5
MEAN NUMBER OF ORIGINATIONS (d = 0)
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH

FIGURE 13

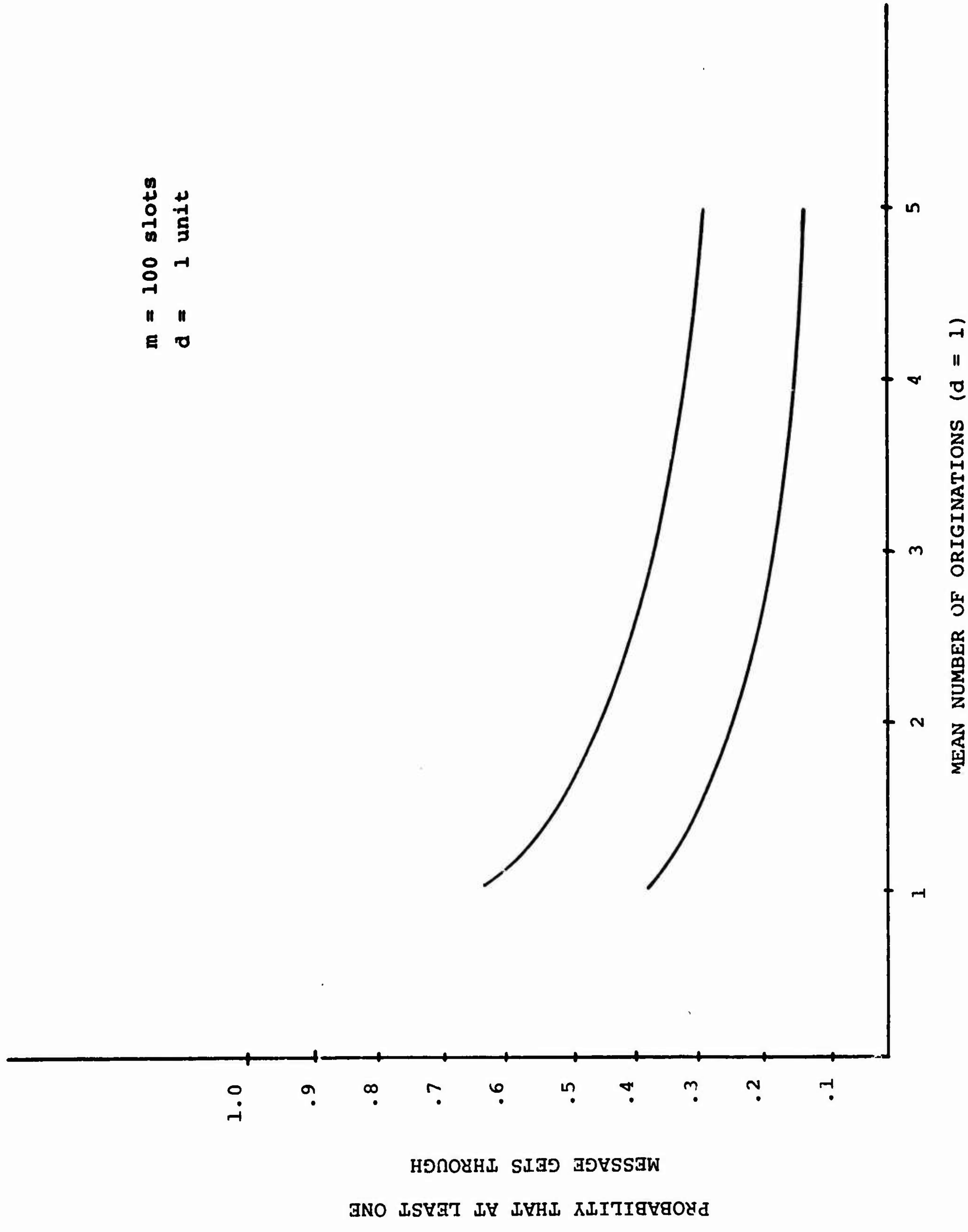

m = 100 slots
d = 1 unit
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
1
2
3
4
5
MEAN NUMBER OF ORIGINATIONS (d = 1)

FIGURE 14

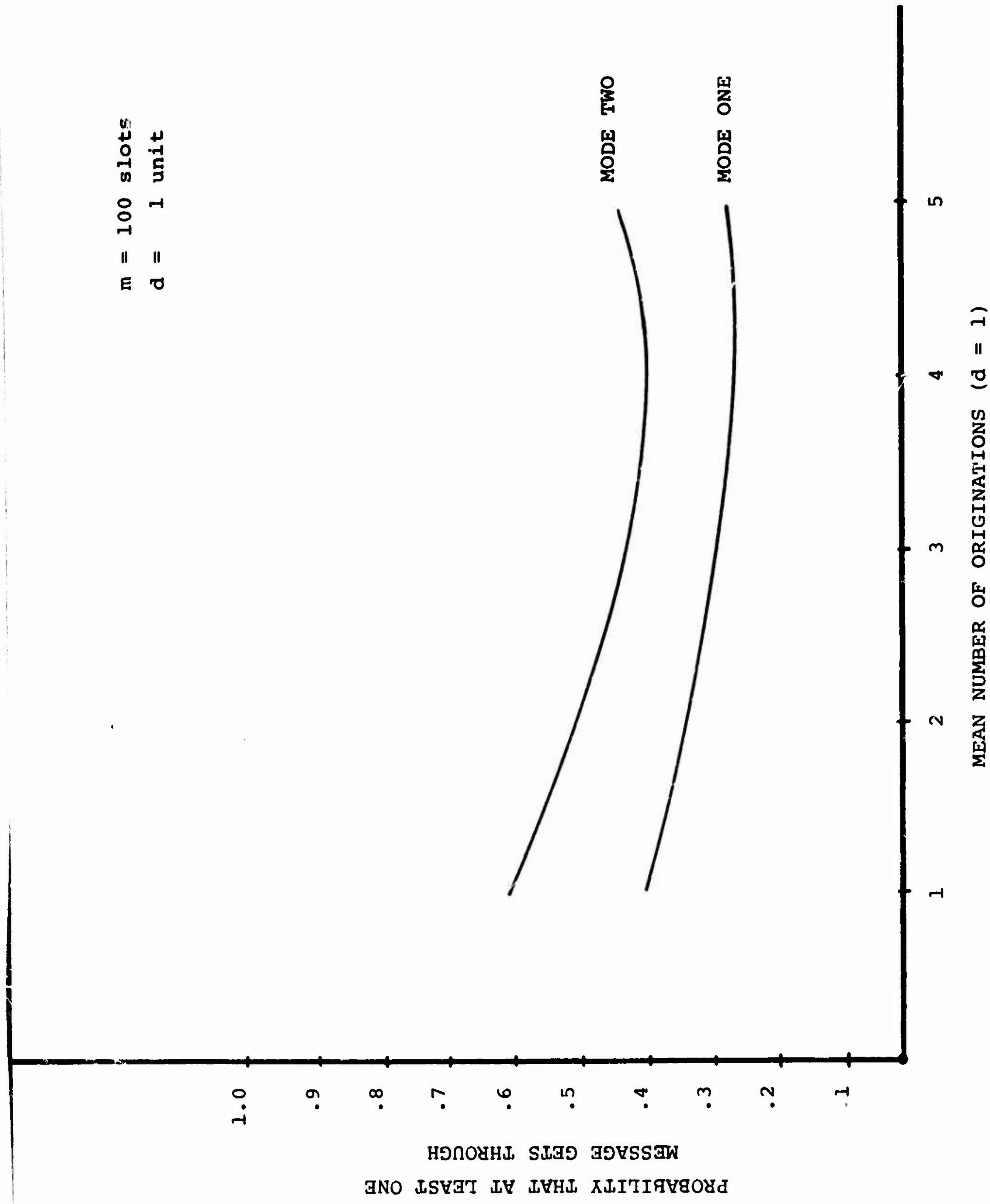

m = 100 slots
d = 1 unit
MODE TWO
MODE ONE
PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
1
2
3
4
5
MEAN NUMBER OF ORIGINATIONS (d = 1)

FIGURE 15

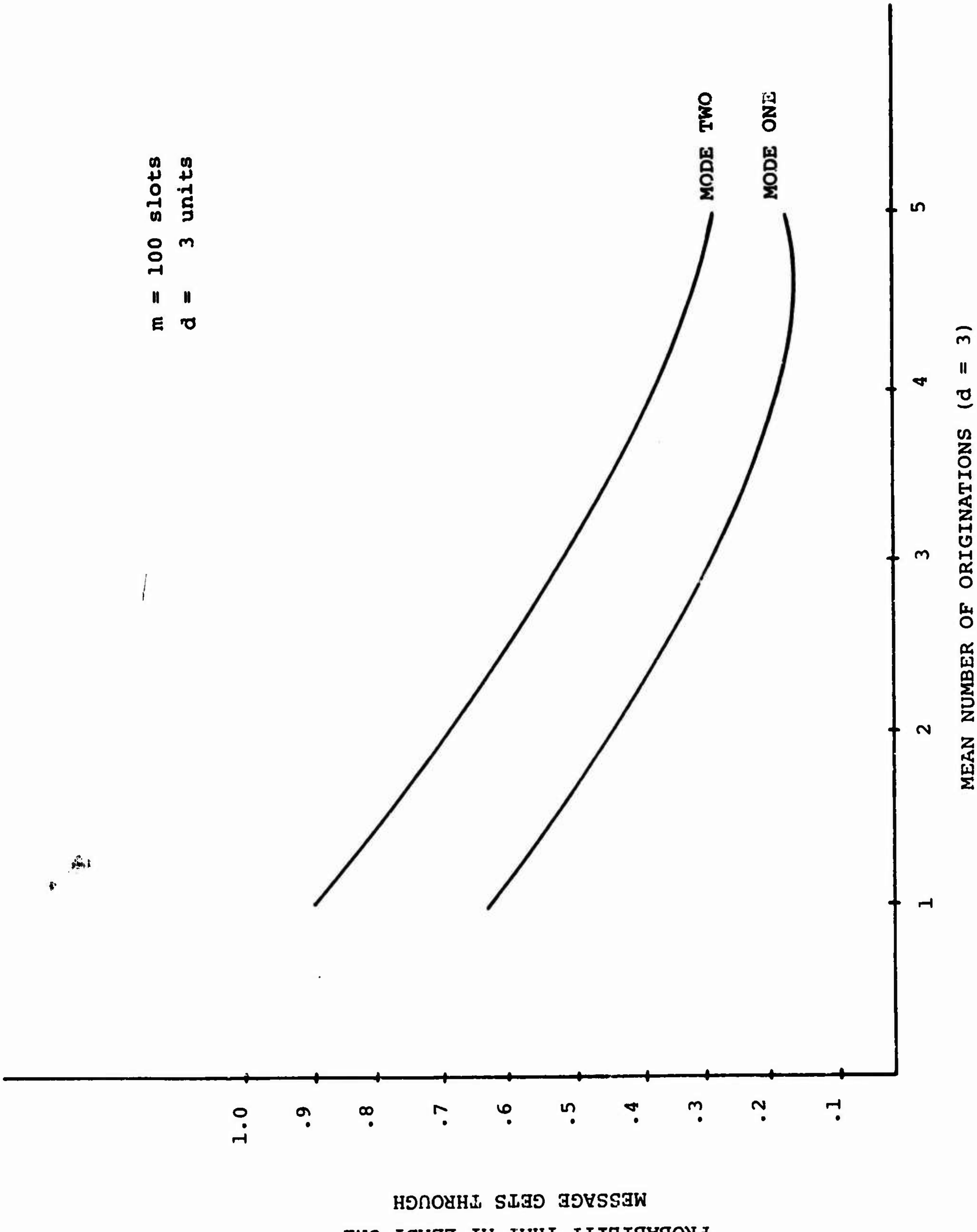

m = 100 slots
d = 3 units
MODE TWO
MODE ONE
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
1
2
3
4
5
PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH
MEAN NUMBER OF ORIGINATIONS (d = 3)

FIGURE 16

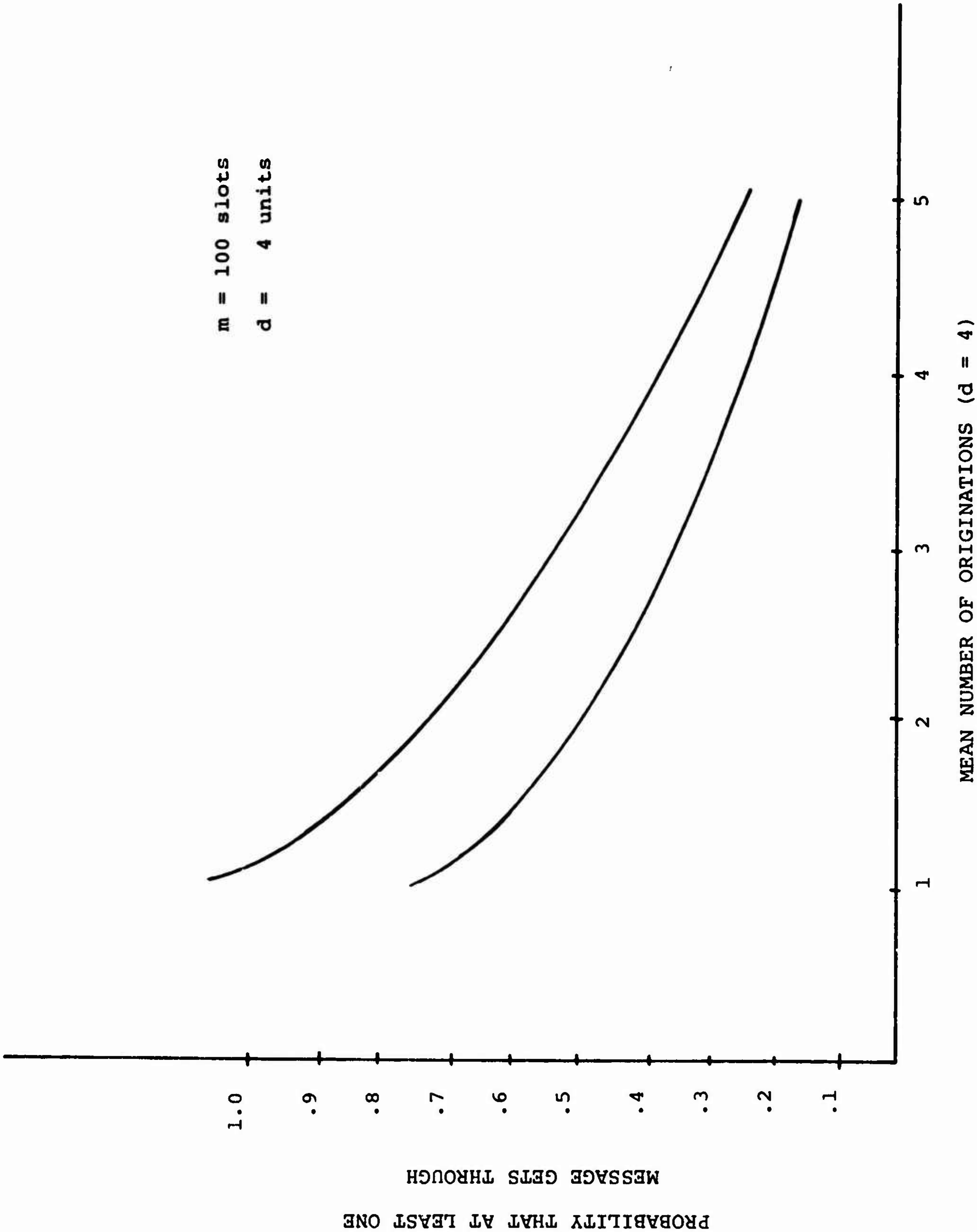

m = 100 slots
d = 4 units
MEAN NUMBER OF ORIGINATIONS (d = 4)
1
2
3
4
5
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1

FIGURE 17

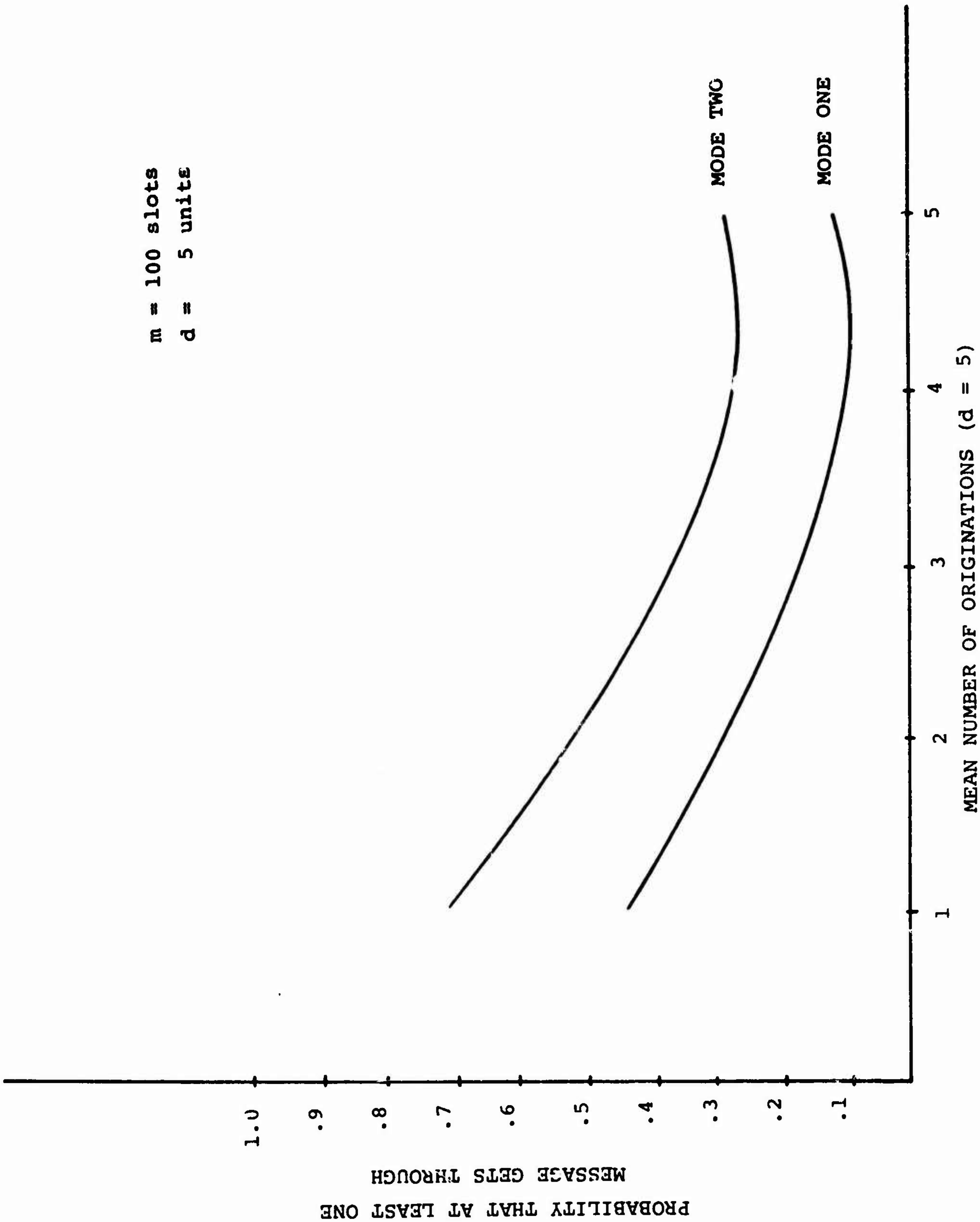

m = 100 slots
d = 5 units
MODE TWO
MODE ONE
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
1
2
3
4
5
PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH
MEAN NUMBER OF ORIGINATIONS (d = 5)

FIGURE 18

9.3 Distribution of Message Explosion as a Function of Slot Size and Mean Number of Originations

The equations for message explosion derived earlier in the report were used to obtain numerical data for message explosion. The results of the numerical analysis follow in Tables 3 through 26 and Figures 19,20, and 21.

9.4 Distributions of Copies Getting Through as a Function of Slot Size and Mean Originations

Tables 3-26 contain the probability distributions for the number of copies of a single message which are received at the origin (ground stations) for each distance (d=0,2,3,4,5,) of origination of the message. The tables vary according to mode (each of two modes), mean number of messages originating at each repeater (λ=1,3,5), and each of four slot sizes (m=25, 50, 75, 100). This produces a total of 4 x 3 x 2 = 24 tables.

In table 27, we summarize the results of the twenty-four tables by considering only the probability that at least one copy of the message gets through as a function of distance and the three parameters; mode, mean, and slot size.

The results of table 27 are presented pictorially in figures 19, 20, and 21 for distances of zero, two and four respectively of origination of the message.

Mode 1, λ= 1, m = 25

distance \ #	0	1	2	3
0	.786	.214		
1	.921	.079		
2	.900	.096	.004	
3	.897	.097	.005	
4	.831	.153	.015	.001
5				

Table 3

Mode 2, λ= 1, m = 25

distance \ #	0	1	2	3	4
0	.689	.311			
1	.834	.166			
2	.759	.219	.021		
3	.716	.245	.037	.002	
4	.554	.324	.103	.018	.002
5					

Table 4

Mode 1, $\lambda = 3$, $m = 25$

distance \ #	0	1	2
0	.805	.195	
1	.942	.058	
2	.952	.047	.001
3	.966	.033	.001
4	.953	.045	.002
5			

Table 5

Mode 2, $\lambda = 3$, $m = 25$

distance \ #	0	1	2	3
0	.739	.261		
1	.890	.102		
2	.887	.107	.006	
3	.892	.101	.007	
4	.826	.153	.019	.001
5				

Table 6

Mode 1, λ = 5, m = 25

distance \ #	0	1	2
0	.813	.187	
1	.951	.049	
2	.963	.036	.001
3	.979	.021	.001
4	.977	.022	.001
5			

Table 7

Mode 2, λ = 5, m = 25

distance \ #	0	1	2
0	.753	.247	
1	.914	.086	
2	.916	.080	.004
3	.930	.066	.004
4	.900	.091	.008
5			

Table 8

Mode 1, $\lambda = 1$, $m = 50$

distance \ #	0	1	2	3
0	.755	.245		
1	.878	.122		
2	.823	.166	.001	
3	.796	.185	.019	.001
4	.661	.272	.059	.007
5				

Table 9

Mode 2, $\lambda = 1$, $m = 50$

distance \ #	0	1	2	3	4	5
0	.605	.395				
1	.736	.264				
2	.602	.338	.060			
3	.531	.356	.103	.010		
4	.306	.360	.232	.083	.017	.002
5						

Table 10

Mode 1, $\lambda = 3$, $m = 50$

distance \ #	0	1	2
0	.788	.212	
1	.926	.074	
2	.921	.076	.003
3	.926	.070	.006
4	.882	.108	.009
5			

Table 11

Mode 2, $\lambda = 3$, $m = 50$

distance \ #	0	1	2	3	4
0	.709	.291			
1	.860	.140			
2	.816	.170	.014		
3	.800	.178	.021	.001	
4	.875	.258	.058	.008	.001
5					

Table 12

Mode 1, $\lambda = 5$, $m = 50$

distance \ #	0	1	2
0	.794	.206	
1	.937	.063	
2	.441	.057	.002
3	.954	.044	.002
4	.935	.062	.004
5			

Table 13

Mode 2, $\lambda = 5$, $m = 50$

distance \ #	0	1	2	3
0	.733	.267		
1	.890	.110		
2	.868	.124	.008	
3	.870	.120	.010	
4	.791	.180	.027	.002
5				

Table 14

Mode 1, $\lambda = 1$, $m = 75$

distance \ #	0	1	2	3	4
0	.711	.289			
1	.831	.169			
2	.742	.235	.023		
3	.868	.269	.043	.002	
4	.512	.342	.121	.023	.003
5					

Table 15

Mode 2, $\lambda = 1$, $m = 75$

distance \ #	0	1	2	3	4	5	6
0	.526	.474					
1	.655	.345					
2	.486	.408	.106				
3	.392	.408	.175	.025			
4	.184	.304	.295	.158	.049	.008	.001
5							

Table 16

Mode 1, $\lambda = 3$, $m = 75$

distance \ #	0	1	2	3
0	.782	.218		
1	.913	.087		
2	.895	.100	.005	
3	.891	.103	.006	
4	.818	.161	.019	.001
5				

Table 17

Mode 2, $\lambda = 3$, $m = 75$

distance \ #	0	1	2	3	4
0	.674	.326			
1	.815	.185			
2	.750	.225	.025		
3	.725	.233	.040	.002	
4	.561	.313	.103	.020	.002
5					

Table 18

Mode 1, $\lambda = 5$, $m = 75$

distance \ #	0	1	2
0	.788	.212	
1	.929	.071	
2	.924	.074	.003
3	.932	.065	.003
4	.894	.098	.008
5			

Table 19

Mode 2, $\lambda = 5$, $m = 75$

distance \ #	0	1	2	3
0	.711	.289		
1	.863	.137		
2	.819	.168	.013	
3	.818	.163	.019	.001
4	.703	.239	.051	.006
5				

Table 20

Mode 1, $\lambda = 1$, m = 100

distance #	0	1	2	3	4	5	6
0	.602	.398					
1	.757	.242					
2	.645	.302	.053				
3	.572	.325	.093	.013			
4	.387	.369	.184	.050	.008		
5	.260	.318	.250	.122	.039	.008	.003

Table 21

Mode 2, $\lambda = 1$, m = 100

distance #	0	1	2	3	4	5	6	7	8
0	.411	.589							
1	.566	.434							
2	.386	.433	.181						
3	.305	.399	.240	.056					
4	.126	.244	.308	.215	.087	.019	.002		
5	.003	.110	.200	.248	.211	.126	.054	.014	.001

Table 22

Mode 1, $\lambda = 3$, $m = 100$

distance \ #	0	1	2	3	4
0	.715	.285			
1	.808	.192			
2	.834	.143	.023		
3	836	.147	.016		
4	.750	.211	.035	.004	
5	.659	.267	.063	.010	.001

Table 23

Mode 2, $\lambda = 3$, $m = 100$

distance \ #	0	1	2	3	4	5	6
0	.579	.421					
1	.727	.273					
2	.659	.279	.062				
3	.642	.276	.073	.008			
4	.466	.338	.149	.040	.007	.001	
5	.308	.333	.223	.098	.030	.007	.001

Table 24

Mode 1, $\lambda = 5$, $m = 100$

distance \ #	0	1	2	3
0	.736	.264		
1	.803	.117		
2	.871	.119	.010	
3	.881	.110	.009	
4	.841	.140	.018	.001
5	.843	.133	.022	.002

Table 25

Mode 2, $\lambda = 5$, $m = 100$

distance \ #	0	1	2	3	4	5
0	.569	.431				
1	.679	.321				
2	.674	.283	.043			
3	.715	.214	.064	.007		
4	.729	.169	.076	.021	.004	
5	.802	.115	.056	.020	.005	.001

Table 26

	$\lambda = 1$								$\lambda = 3$								$\lambda = 5$							
	M = 25		M = 50		M = 75		M = 100		M = 25		M = 50		M = 75		M = 100		M = 25		M = 50		M = 75		M = 100	
d	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0	.214	.311	.245	.395	.289	.474	.398	.587	.195	.261	.212	.291	.218	.326	.285	.421	.187	.247	.206	.267	.212	.289	.264	.431
1	.079	.166	.122	.264	.269	.345	.243	.434	.058	.110	.074	.140	.087	.185	.192	.273	.049	.086	.063	.110	.071	.137	.197	.321
2	.100	.241	.177	.398	.258	.514	.355	.614	.048	.113	.079	.184	.105	.250	.166	.341	.037	.084	.059	.132	.076	.181	.129	.326
3	.102	.284	.204	.469	.304	.608	.428	.695	.034	.108	.074	.200	.109	.275	.164	.358	.021	.070	.046	.130	.068	.182	.119	.285
4	.169	.446	.339	.694	.488	.816	.613	.874	.047	.174	.118	.325	.182	.437	.250	.534	.023	.100	.065	.207	.106	.297	.159	.271
5							.740	.997							.341	.692							.157	.198

TABLE 27

THE PROBABILITY OF AT LEAST ONE MESSAGE GETTING THROUGH

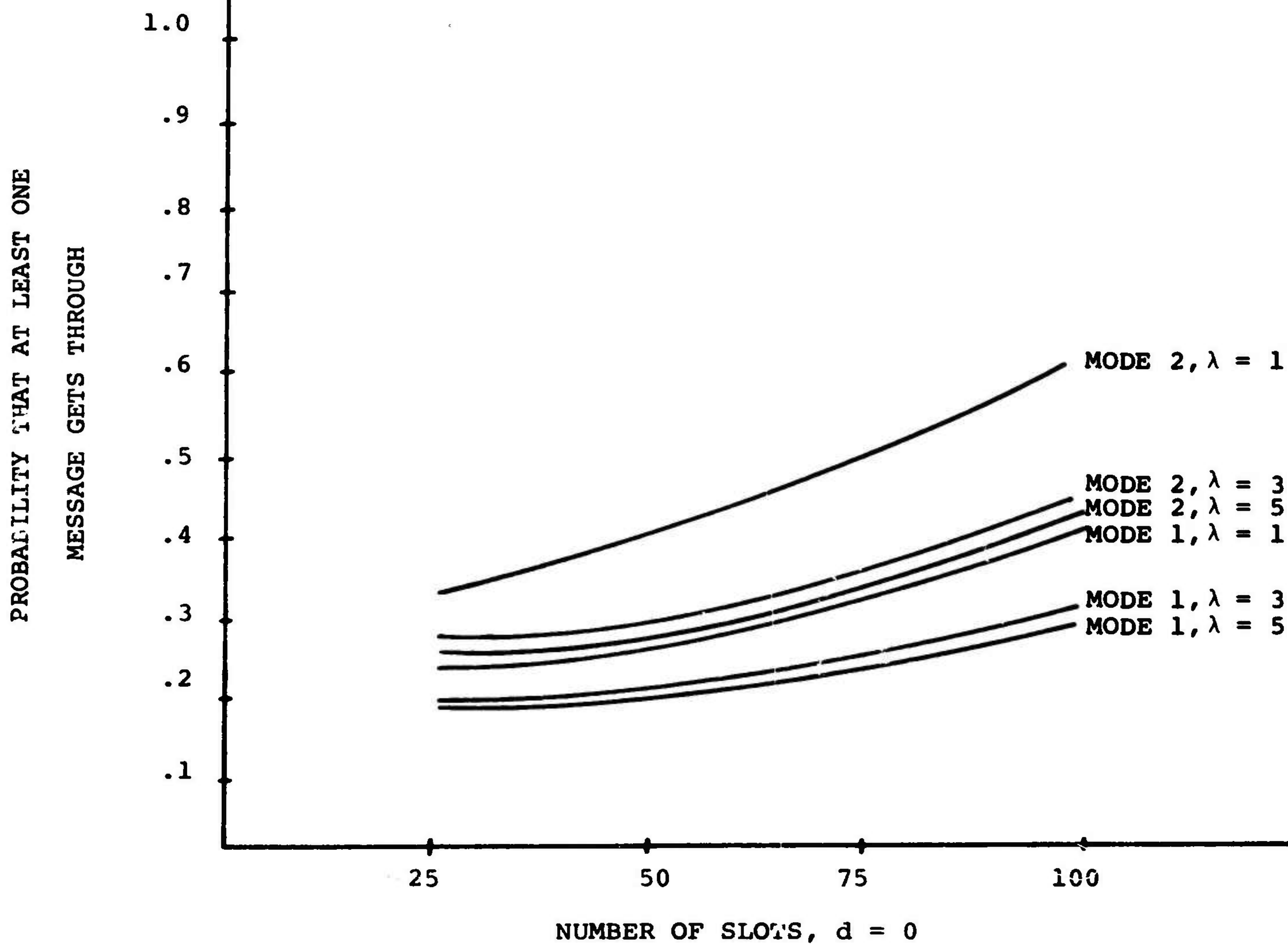

1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH
25
50
75
100
NUMBER OF SLOTS, d = 0
MODE 2, λ = 1
MODE 2, λ = 3
MODE 2, λ = 5
MODE 1, λ = 1
MODE 1, λ = 3
MODE 1, λ = 5

FIGURE 19

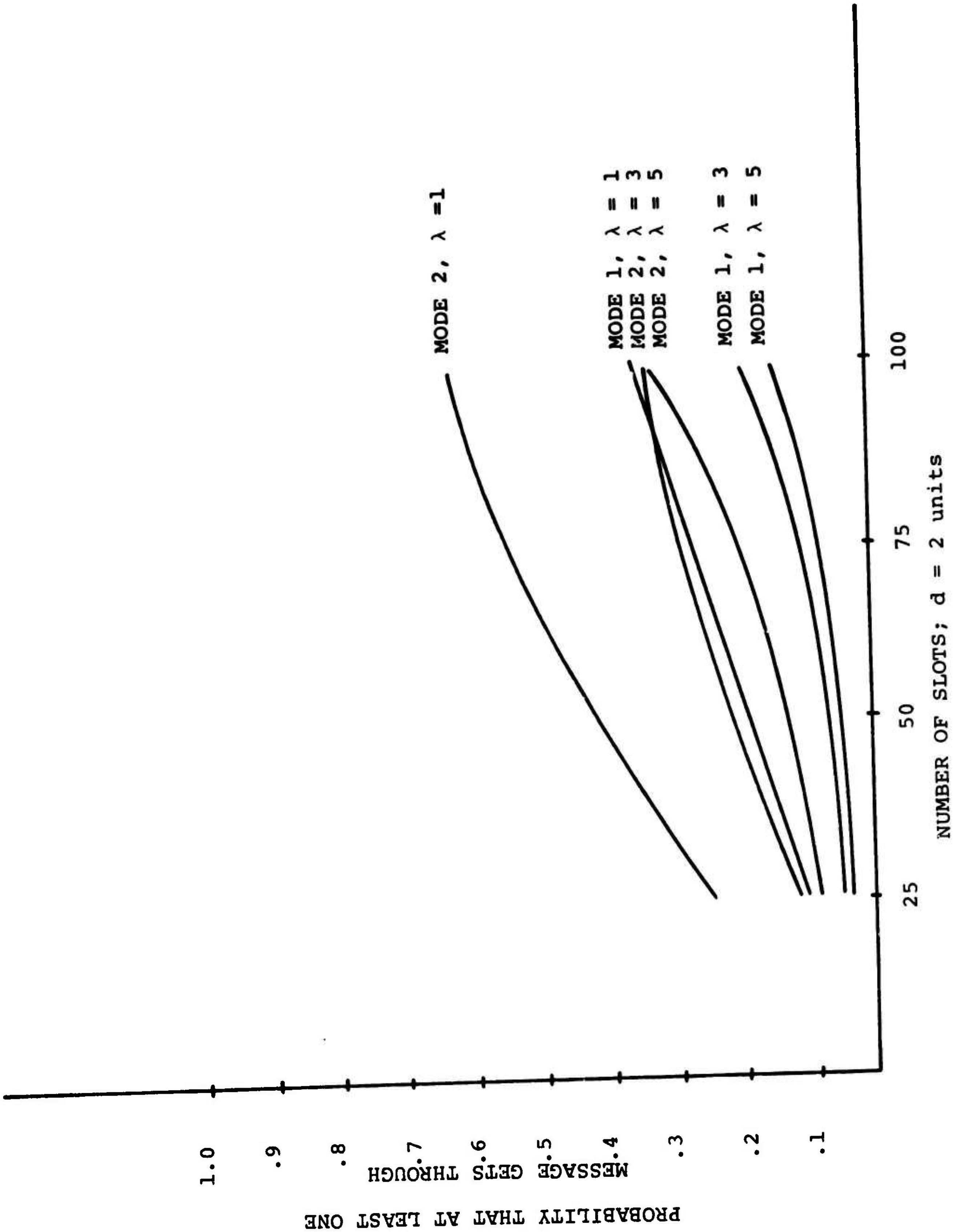

MODE 2, λ =1
MODE 1, λ = 1
MODE 2, λ = 3
MODE 2, λ = 5
MODE 1, λ = 3
MODE 1, λ = 5
25
50
75
100
NUMBER OF SLOTS; d = 2 units
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH

FIGURE 20

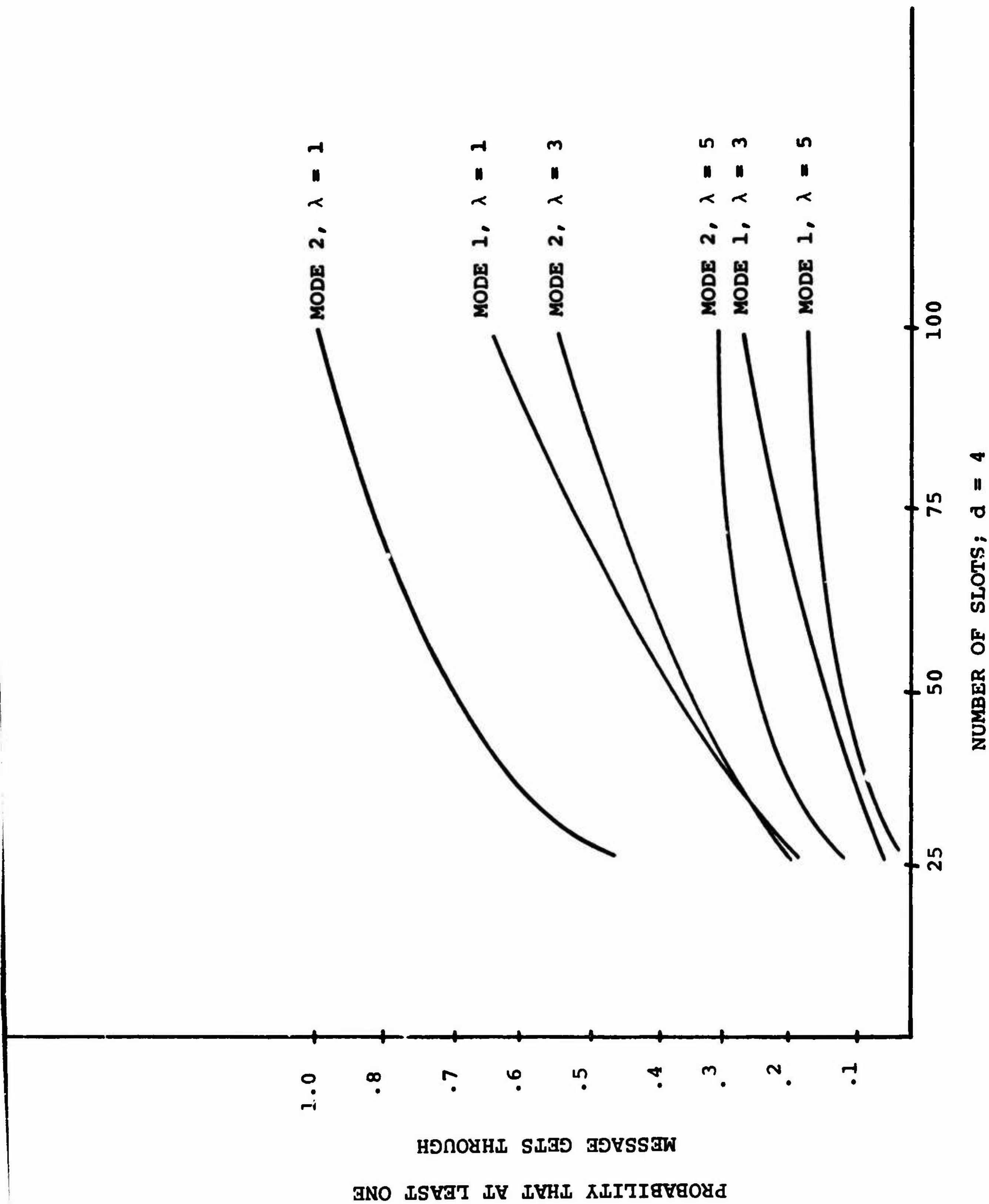

MODE 2, λ = 1
MODE 1, λ = 1
MODE 2, λ = 3
MODE 2, λ = 5
MODE 1, λ = 3
MODE 1, λ = 5
1.0
.8
.7
.6
.5
.4
.3
.2
.1
25
50
75
100
NUMBER OF SLOTS; d = 4
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH

FIGURE 21

10. SYSTEMS WITH RETRANSMISSIONS

10.1 Retransmission From Source of Origination

We now can extend the scope and generality of the basic model by including the possibility of retransmission of messages which are erased in random slotting or in technical language, "not captured". The notion of retransmission can be modelled in at least two ways. The first way, considered in this section is that when a message is wiped out, it is retransmitted from its source of origination after a fixed delay time J(d) which depends on the distance of origination from the ground station. The second type of retransmissions which we shall consider are retransmissions which occur at the point (repeater) of erasure at one time unit after wipeout. The latter type will be analyzed in Section 7.

To begin to develop programmable equations, we need some notation:

Let $X_{(i,j),(u,v)}(t)$ be the number of messages arriving at (i,j) at time t which originated at (u,v) at time t - (u-i). (recall that the first coordinate refers to distance from the origin).

Let $Y_{(i,j),(u,v)}(t)$ be the number of messages accepted at (i,j) at time t which originated at (u,v) at time t - (u-i).

Let $X_{(i,j)}(t)$ be the number of messages arriving at (i,j) at time t.

Let $Y_{(i,j)}(t)$ be the number of messages accepted at (i,j) at time t.

Let $Z_{(i,j)}(t)$ be the number of retransmissions at (i,j) at time t.

We develop equations to compute $Z_{(i,j)}(t)$. To begin, we have the following assumptions.

$$X_{(i,j),(i,j)}(t) = \text{Poisson Variate} + Z_{(i,j)}(t) \tag{30}$$

$Y_{(i,j),(i,j)}(t) = X_{(i,j)}(t)$ randomized over mode 1 or mode 2 distribution. (31)

Obviously;

$$X_{(i,j)}(t) = \sum_{(u,v)\varepsilon I(i,j)} X_{(i,j),(u,v)}(t) \tag{32}$$

Where $I_{(i,j)}$ is the set of all repeaters which arein the input set to (i,j), i.e., all repeaters for which there exists directed path to (i,j).

If we assume that each arriving message is equally likely to be accepted, it follows that:

$$Y_{(i,j),(u,v)}(t) = Y_{(i,j)}(t) \frac{X_{(i,j),(u,v)}(t)}{X_{(i,j)}(t)}, \tag{33}$$

and that

$$Z_{(i,j)}(t) = \sum_{\substack{(u,v) \text{ in} \\ O(i,j)}} \left[X_{(u,v),(i,j)}(t-J(i)) - Y_{(u,v),(i,j)}(t-J(i))\right], \tag{34}$$

where $O_{(i,j)}$ is the "outward set of (i,j)" defined as the set of all repeaters which receive messages from (i,j), including (i,j) itself. The quantity J(i) is the delay factor which depends on i the distancefrom the origin, but is independent of the source of message wipeout.

The only part of the equation (9.5) which is not accounted for is $X_{(u,v),(i,j)}(t)$. The next equation is obvious:

$$X_{(i,j),(u,v)}(t) = \sum_{\substack{(K,W) \text{ in} \\ II_{(i,j)}}} Y_{(K,W),(u,v)}(t-1); \tag{35}$$

where $II_{(i,j)}$ is the immediate input set to (i,j), that is those repeaters one unit of distance further than (i,j) which repeat to (i,j) in one time unit.

The equations (30) to (35) were successfully programmed for the square grid net of repeaters at the lattice points of the Euclidean plane, five units or less distance from the origin. This net has a total of 61 repeaters. We do not include numerical data since many time points must be computed to obtain meaningful steady state results. This can be done at any time since the program is available.

10.2. Retransmissions at Point of Loss

In this model we assume that retransmissions of wiped out messages occur at the point of wipeout one time unit later, independently of where the message originated. For this type of assumption, we need to compute, $Z_{(i,j),(u,v)}(s,t)$ the number of retransmissions at (i,j) at time t of messages which originated at (u,v) at time s, s=0, 1, 2,..., $t-|u-j|$. The quantity Z is computed for repeaters (u,v) in the input set to (i,j). The quantities $Z_{(i,j)}(t)$ and $Z_{(i,j),(u,v)}(t)$ are defined as in Section 13.

Since we are assuming that retransmissions occur at the point of wipeout, to compute delays we must keep track of time and place of origination of messages. We therefore define the quantities $X_{(i,j),(u,v)}(s,t)$ and $Y_{(i,j),(u,v)}(s,t)$ as the number of messages arriving and respectively accepted at (i,j) at time t which originated at (u,v) at time s.

According to our assumptions, the required quantity can be computed from:

$$Z_{(i,j),(u,v)}(s,t) = X_{(i,j),(u,v)}(s, t-1) - Y_{(i,j),(u,v)}(s, t-1) \qquad (36)$$

According to (5), we need to compute X and Y which can be done recursively.

We have:

$$X_{(i,j),(u,v)}(s,t) = \sum_{\substack{(k,l) \text{ in} \\ II(i,j)}} Y_{(k,l),(u,v)}(s,t-1) + Z_{(i,j),(u,v)}(s,t) \qquad (37)$$

and

$$Y_{(i,j),(u,v)}(s,t) = Y_{(i,j)}(t) \frac{X_{(i,j),(u,v)}(s,t)}{X_{(i,j)}(t)} \qquad (38)$$

As in the previous section $Y_{(i,j)}(t) = X_{(i,j)}(t)$ randomized over mode 1 or mode 2 distribution. The remaining quantity to compute is:

$$X_{(i,j)}(t) = \sum_{\substack{(u,v) \text{ in} \\ II(i,j) \text{ set}}} X_{(i,j),(u,v)}(s,t) + Z_{(i,j)}(t) \qquad (39)$$

These equations have been programmed for the grid of repeaters at the lattice points of the Euclidean plane, as earlier. The program is for the case where a repeater has a single fixed path to the origin or ground station. The program was run for ten time points using a single sample at each point. The numerical results are, therefore, subject to some variability. Some of the results of a single run are given in Tables 28 - 34. As is evident from the tables, saturation of the channel begins early for $\lambda = 5$. In Table 31 for example the probability that a message originating at d = 5 at time 1 gets to the origin with a delay of less than four time units is only about .44. For $\lambda = 5$ the situation deteriorates rapidly with time. To obtain a large set of representative data would require running the program for many time points, probably at least 15 or 20 for different values of λ and slot size. This can be done using the available program.

10.3. Delays and Average Delays as a Function of Distance

We can extend the calculations and analyses described in the previous two sections to include calculations of delay distributions and average delay. In addition to studying delays, we can develop equations to study bottlenecks in a given network. These formulae have been programmed and numerical results can be obtained.

Let $D_{(i,j)}(t)$ be the random variable delay of a message which originates at (i,j) at time t. We assume that the probability that a message is delayed by k-units of time is given by the proportion of

Retransmissions at Point of Wipeout

$\lambda=5$, m=100, Model

t \ d	0	1	2	3	4	5
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	6	6	0	0	6	0
4	6	11	6	1	3	0
5	43	26	18	1	1	0
6		19	19	7	2	0
7		66	29	13	0	0
8		91	47	3	0	0
9		67	67	1	0	0
10		151	64	9	2	0

TABLE 28

$\lambda=5$, m=100, Mode 2

t \ d	0	1	2	3	4	5
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	7	2	1	0	0	0
4	4	10	1	0	2	0
5	34	20	2	2	1	0
6		37	2	1	1	0
7		19	5	0	1	0
8		28	6	2	1	1
9		33	18	1	1	0
10		41	4	0	1	1

TABLE 29

Delay Probability Tables for a Message Being

Accepted d units from the Ground Station

$\lambda=5$, m=100, Mode 1

A Message Originating at d=5 at time 0.

delay \ dist	0	1	2	3	4	5
0	.159	.271	.524	.933	1.000	1.000
1	.235	.247	.325	.059		
2	.134	.157	.084	.004		
3	.097	.076	.028	.002		
4	.089	.045	.016	.001		
5		.041	.009			
6			.007			

TABLE 30

A Message Origination at d=4 at time 0.

delay \ dist	0	1	2	3	4
0	.249	.454	.727	1.000	1.000
1	.251	.221	.153		
2	.196	.123	.083		
3	.083	.070	.021		
4	.054	.033	.007		
5	.046	.018	.004		
6		.016	.002		
7			.002		

TABLE 31

A Message Originating at d=3 at time 0

delay \ dist	0	1	2	3
0	.916	.950	1.000	1.000
1	.036	.031		
2	.023	.010		
3	.013	.004		
4	.004	.002		
5	.003	.001		
6	.002	.001		
7		.001		

TABLE 32

A Message Originating at d=2 at time 0

delay \ dist	0	1	2
0	.823	1.000	1.000
1	.170		
2	.004		
3	.002		
4	.001		

TABLE 33

A Message Originating at d=1, at time 0

delay \ dist	0	1
0	.778	1.000
1	.183	
2	.038	
3	.001	

TABLE 34

THE NUMBER OF MESSAGES ACCEPTED AT THE ORIGIN

$\lambda = 5$, $m = 100$, mode 2

Originating at d = 5 at t = 0: 3 messages

Time of Acceptance at Origin	Number	Delay
t = 5	.814	0
t = 6	1.130	1
t = 7	.502	2
t = 8	.253	3
t = 9	.161	4
	2.860	

TABLE 35

Originating at d = 5 at t = 1: 6 messages

Time of Acceptance at Origin	Number	Delay
t = 6	2.010	0
t = 7	1.238	1
t = 8	.988	2
t = 9	.805	3
	5.061	

TABLE 36

Originating at d = 5 at t = 2: 2 messages

Time of Acceptance at Origin	Number	Delay
t = 7	.115	0
t = 8	.249	1
t = 9	.256	2
	.620	

TABLE 37

DELAY PROBABILITY TABLES FOR A MESSAGE

BEING ACCEPTED d UNITS FROM THE GROUND STATION

$\lambda = 5$, $m = 100$, mode 2

A Message Originating at d = 5 at time 0

Dist / Delay	0	1	2	3	4	5
0	.271	.496	.854	1.000	1.000	1.000
1	.377	.360	.140	-	-	-
2	.167	.071	.003	-	-	-
3	.084	.041	.003	-	-	-
4	.054	.018	-	-	-	-
5		.007	-	-	-	-

TABLE 38

A Message Originating at d = 4 at time 0

Dist / Delay	0	1	2	3	4	
0	.407	.452	.792	1.000	1.000	
1	.189	.300	.178	-	-	
2	.216	.178	.029	-	-	
3	.092	.035	.001	-	-	
4	.045	.020	.001	-	-	
5	.028	.008	-	-	-	
6	-	.003	-	-	-	

TABLE 39

A Message Originating at d = 3 at time 0

Dist / Delay	0	1	2	3
0	.511	.962	1.000	1.000
1	.425	.022	-	-
2	.031	.009	-	-
3	.020	.005	-	-
4	.007	.001	-	-
5	.003	.001	-	-
6	.002	-	-	-

TABLE 40

A Message Originating at d = 2 at time 0

Dist / Delay	0	1	2	3
0	.883	.923	1.000	
1	.061	.074	-	
2	.050	.002	-	
3	.003	-	-	
4	.[illegible]	-	-	
5	.001	-	-	

TABLE 41

A Message Originating at d = 1 at time 0

Dist / Delay	0	1		
0	.800	1.000		
1	.191	-		
2	.005	-		
3	.004	-		

TABLE 42

A Message Originating at d = 0 at time 0

Dist / Delay	0	1		
0	1.000			
1	.000			

TABLE 43

A Message Originating at d = 5 at time 1

Dist / Delay	0	1	2	3	4	5	
0	.335	.520	.718	.752	.846	1.000	
1	.206	.159	.116	.207	.132	-	
2	.165	.161	.125	.039	.020	-	
3	.134	.081	.024	.002	.002	-	
4	-	.036	.012	.003	-	-	
5	-	-	.003				

TABLE 44

PROBABILITY OF ZERO DELAY VS. DISTANCE OF ORIGINATION

MODE 1

t \ d	0	1	2	3	4	5
0	1.000	.778	.823	.916	.349	.159
1	.346	.329	.458	.125	.091	.123
2	.242	.229	,045	.085	.053	.047
3	.207	.043	.013	.058	.021	.024
4	.177	.033	.025	.021	.013	.037
5	.092	.019	.011	.006	.015	
6	.021	.005	.002	.008		
7	.029	.002	.001			
8	.031	.001				
9	.047					

TABLE 45

MODE 2

t \ d	0	1	2	3	4	5
0	1.000	.800	.883	.511	.407	.271
1	.373	.136	.118	.356	.170	.335
2	.333	.118	.102	.030	.180	.115
3	.100	.044	.033	.154	.066	.218
4	.112	.026	.061	.045	.109	.161
5	.002	.032	.005	.100	.073	
6	.021	.011	.022	.041		
7	.045	.022	.019			
8	.005	.018				
9	.021					

TABLE 46

messages which originate at time t which are delayed by k-units. If enough random samples are taken, this estimate becomes quite good. In notation, define $Y_{(o,o)\ (i,j)\ (s,t)}$ as the number of messages which are accepted at the ground station at time s, which originated at (i,j) at time t. Then,

$$P\{D_{(i,j)}(t)=k\} = \frac{Y_{(o,o),(i,j)\ (t+k+i,t)}}{O_{(i,j)}(t)}; \ k=0,\ 1,\ 2,\ldots, \tag{40}$$

When doing computer trials to determine 40 we can average over all 4i repeaters at distance i and compute the delay distribution as a function only of distance to the ground station, i.e.,

$$P\{D_i(t)=k\}=\frac{1}{4i}\sum_{j=1}^{4i} P\{D_{(i,j)}(t)=k\}, \text{ where} \tag{41}$$

$D_i(t)$ is the random variable delay of a message originating at distance i at time t.

To obtain a time invariant measure, we can average 41 over time and obtain the probability distribution of the random variable D_i and its expectation, given by:

$$E(D_i) = \sum_{k=o}^{\infty} k\ P\{D_i=k\} \tag{42}$$

The same kind of analysis can be used to study "bottlenecks." Let $D_{(i,j)}(w,t)$ be the random variable "delay of a message" originating at (i,j) at time t in getting w units from the ground station for w=0, 1, 2,..., i. As earlier we have:

$$P\{D_{(i,j)}(w,t)=k\} = \frac{Y_{(w,u),(i,j)}(t+k+i-w,\ t)}{O_{(i,j)}(t)}; \ k=\ 0,\ 1,\ 2,\ldots, \tag{43}$$

where (w,u) is the unique repeater on the path from (i,j) to (o,o) at distance w. Similarly as in 41 and 42 :

$$P\{D_i(w,t)=k\} = \frac{1}{4i}\sum_{j=1}^{4i} P\{D_{(i,j)}(w,t)=k\} \text{ and} \tag{44}$$

$$E\left[D_i(w,t)\right]=\sum_{k=1}^{\infty} K P\{D_i(w,t)=k\}. \tag{45}$$

11. MESSAGES OUTWARD FROM THE ORIGIN

11.1. A Single Message Originates at the Ground Station.

The next major part of our study is to model the situation when message flow is outward from the origin. We begin our study with the dynamics of the simple model where the repeaters are at the lattice points of the plane. A single message originates at the ground station at time t=0. Every message received by a repeater is accepted and perfectly retransmitted to each of its four nearest neighbors. We determine:

a) The number of repeaters which receive the message for the first time at time t: t = 0, 1, 2,....

b) The number of repeaters which have seen the single message by time t.

c) The number of copies of the single message received by any repeater at time t.

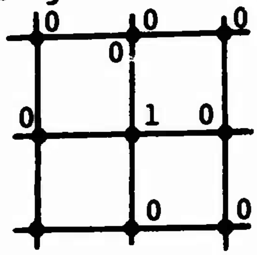
The assumptions are:

1) A single message arrives at a given node at time t = 0, no other messages are introduced into the network. We assume the message originated at the origin,(Cartesian coordinates (0,0)).

2) Message transmission is perfect, i.e., after one time unit each of the four neighbors to any repeater receive all messages transmitted by the repeater at the previous time point.

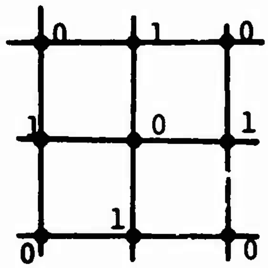
Some diagrams and numbers are helpful to fix ideas.

t = 0,

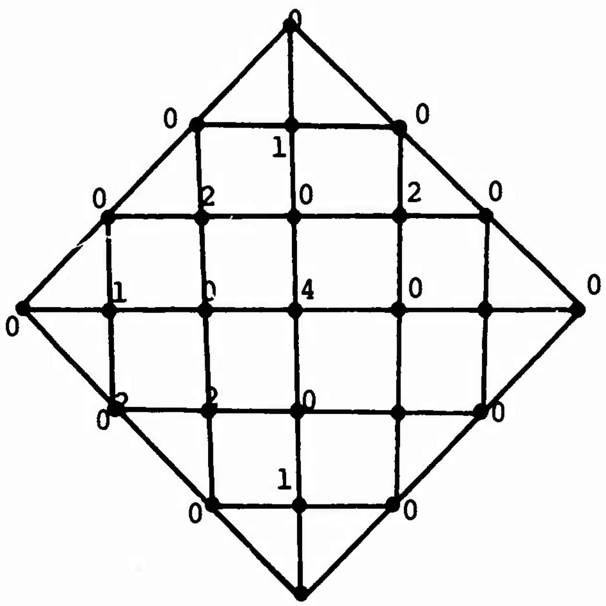


(one message at origin, no messages elsewhere)

t = 1,



(a single message at each of the four neighbors, no messages elsewhere)

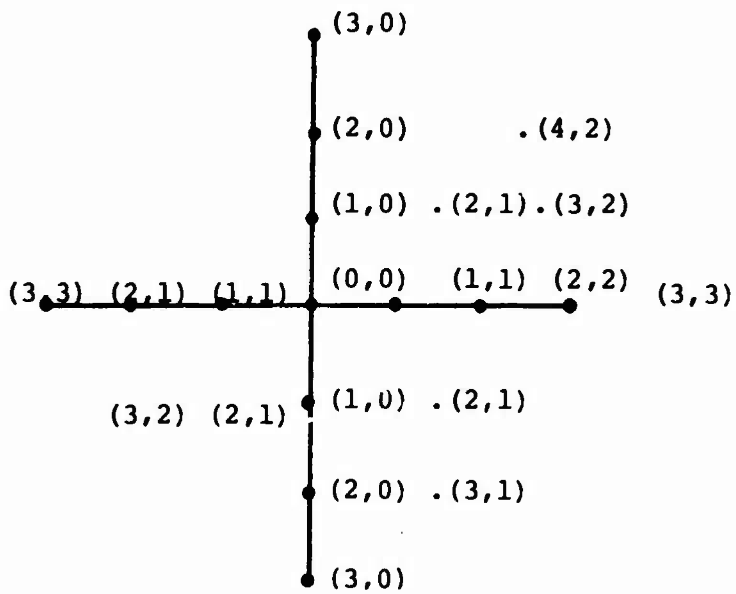


(4 messages at the origin, 0 Message at all repeaters 1 step from origin, 1 message at each of four repeaters 2 steps in either the horizontal or vertical directions, 2 messages at repeaters, 1 unit in horizontal direction and 1 unit in vertical direction.

By examining the diagrams we are led to introduce a coordinate system based on distance as measured in steps to reach a repeater and horizontal distance of the repeater from the origin. The quadrant symmetry of the model also indicates use of these coordinates.

The coordinates of a repeater are denoted by (d,j) where d is the distance of the repeater from the origin measured in minimum time units a message needs to arrive at the repeater from the origin; the second coordinate j is the horizontal distance of the repeater from the origin again measured in time units but only in the horizontal direction.

For example, we give some coordinates:



Some further notation which is necessary;

$B(t)$ = the number of repeaters which receive the message for the first time at time t.

Clearly $B(t)$ is the number of repeaters whose first coordinate is t. i.e. that are at distance t from the origin.

$A(t)$ = the number of repeaters which have seen the message by time t. Clearly $A(t) = \sum_{j=0}^{t} B(j)$.

$N_j^d(t)$ = number of copies of the message received by a given repeater at coordinates (d,j) at time t. Clearly

a) $N_j^d(t) = 0$ for $d > t$

b) $N_j^d(d+2k+1) = 0$ for $k = 1, 2, \ldots.$

Thus it is necessary to compute $N_j^d(d+2k)$ $k = 0, 1, 2, \ldots.$

A) Calculation of B(t):

The quantity B(t), (the number of repeaters at distance t from the origin is the number of repeaters which receive the message for the first time t, is easy to compute. This quantity is given by the number of integer solutions to $|i| + |j| = t$. Since a repeater is at distance d from the origin if and only if its' Cartesian coordinates (i,j) satisfy $|i| + |j| = t$, we can solve this equation and count solutions. Note that

$i=0$,	$j=t$	or $-t$	2 solutions
$i=1$,	$j=t-1$	or $-t+1$	2 solutions
$i=-1$,	$j=t-1$	or $-t+1$	2 solutions
$i=2$,	$j=t-2$	or $-t+2$	2 solutions
$i=-2$,	$j=t-2$	or $-t+2$	2 solutions
$\vdots$	$\vdots$	$\vdots$	
$i=t-1$,	$j=1$	or $j=-1$	. . .
$i=-t+1$,	$j=1$	or $j=-1$	. . .
$i=t$,	$j=0$		
$i=-t$,	$j=0$		

The number of solutions is, $B(0) = 1$

$$B(t) = 2 + 4(t-1) + 2 = 4t \text{ for } t \geq 1.$$

To compute A(t), we sum B(t) and obtain

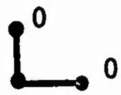
$$A(t) = \sum_{j=0}^{t} B(j) = 1 + \sum_{j=1}^{t} 4(j) = 1 + 4\sum_{j=1}^{t} j = 1 + 4\,\frac{t(t+1)}{2}$$

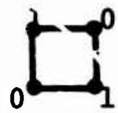
$$= 1 + 2t^2 + 2t = 2t^2 + 2t + 1$$

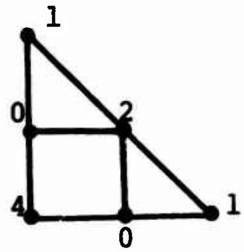
The rate at which A(t) the number of repeaters which receive the message by time t grows as $A'(t) = 4t + 2$ which is linear in t.

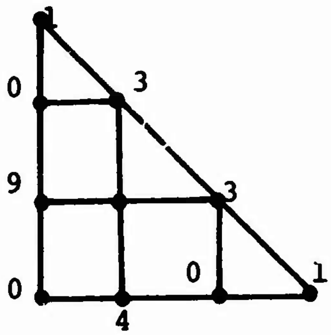
B. The quantity $N_j^d(d + 2k)$, $k = 0, 1, 2, \ldots$.

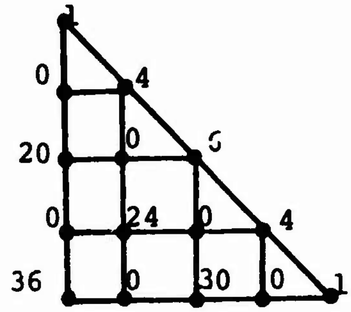
To compute the number of copies of the message received at a repeater with coordinates (d,j) at time $d + 2k$, $k = 0, 1, 2, \ldots$ we first draw some diagrams. Due to symmetry it suffices to examine only the first quadrant.

t = 0


t = 1


t = 2,


t = 3,


t = 4,


It seems clear from the diagram that a repeater with coordinates (d,j) will receive at $t=d$ the number of messages which is given by the binomial coefficient $\binom{d}{j}$. From the diagram we note the relationship of the outer edge to the d^{th} now of a Pascal triangle:

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1
\end{array}
$$

This result is also apparent from an argument based on the number of paths of a message from $(0,0)$ to (d,j). The number of messages received at a repeater with coordinates (d,j) is given by the number of paths from $(0,0)$ to (d,j) which is obviously $\binom{d}{j}$. To determine a general formula for $N_j^d(d + 2k)$ for $k > 1$, we can write and solve the appropriate difference equation.

$$N_j^d(d+2k) = N_{j-1}^{d-1}(d+2k-1) + N_{j+1}^{d+1}(d+2k-1)$$
$$+ N_j^{d+1}(d+2k-1) + N_j^{d-1}(d+2k-1)$$

for $k = 1, 2, \ldots,; d = 0, 1, 2, \ldots,; j = 0, 1, 2, \ldots, d$.

The initial conditions are

$$N_j^d(t) = 0 \quad \text{if } t < d,$$

$$N_j^d(d) = \binom{d}{j}.$$

The solution to this equation is given by $N_j^d(d+2k) = \binom{d+2k}{k}\binom{d+2k}{k+j}$

To check its validity note that the initial conditions are satisfied, and apply the well known definition of binomial coefficients;

$$\binom{n}{j} = \binom{n-1}{j-1} + \binom{n-1}{j}, \qquad n \geq 1 \quad j \geq 1.$$

$$N_{j-1}^{d-1}(d-1+2k) + N_{j+1}^{d+1}(d+1+2(k-1)) + N_j^{d+1}(d+1+2(k-1)) + N_j^{d-1}(d-1+2k)$$

$$= \binom{d+2k-1}{k}\binom{d+2k-1}{k+j-1} + \binom{d+2k-1}{k-1}\binom{d+2k-1}{k+j} + \binom{d+2k-1}{k-1}\binom{d+2k-1}{k+j-1} + \binom{d+2k-1}{k}\binom{d+2k-1}{k+j}$$

$$= \binom{d+2k-1}{k}\left[\binom{d+2k-1}{k+j-1} + \binom{d+2k-1}{k+j}\right] + \binom{d+2k-1}{k-1}\left[\binom{d+2k-1}{k+j} + \binom{d+2k-1}{k+j-1}\right]$$

$$= \binom{d+2k-1}{k}\binom{d+2k}{k+j} + \binom{d+2k-1}{k-1}\binom{d+2k}{k+j}$$

$$= \binom{d+2k}{k+j}\left[\binom{d+2k-1}{k} + \binom{d+2k-1}{k-1}\right] = \binom{d+2k}{k+j}\binom{d+2k}{k}$$

For k very large with respect to d we can use a stirling approximation to note the $N_j^d(d+2k) \sim 2^{4k}$ i.e. grows as 2^{4k}.

To summarize:

a) $B(t) = 4t\ ,\ t \geq 1 \quad B(0) = 1,$

b) $A(t) = 2t^2 + 2t+1 \quad t \geq 0,$

c) $N_j^d(d+2k) = \binom{d+2k}{k+j}\binom{d+2k}{k} \sim 2^{4k}$ for large k.

11.2. A Fixed Number of Messages Originate at the Ground Station and Subject to Non-Capture

The model of message flow from the ground station out to repeaters can be extended to allow the possibility of erasure or non-capture of messages.

We assume that at each point of time t, τ messages are being generated outward from the origin. Of messages accepted at each repeater, a fixed proportion k_o are addressed to that repeater and hence are not repeated. We study the distributions of the number of messages received and accepted at each repeater at each point in time, assuming an infinite net.

Recall $X_{(i,j)}^{(1)}(t)$ = number of messages arriving at a repeater with coordinates (i,j) at time t with Mode 1 capture.

$X_{(i,j)}^{A,(1)}(t)$ = number of messages accepted at a repeater with coordinates (i,j) at time t in Mode 1 capture. In Mode 2 capture, we use the same notation except that (1) as a superscript is replace by a (2).

A. In Mode One:

In Mode 1 capture, the relationship between arriving and accepted messages is described by the transfer function:

$$P_{kj} = \frac{\binom{m}{j}}{m^k} \sum_{\nu=0}^{\min(k,m)-j} (-1)^{\nu} \binom{m-j}{\nu} \frac{k!}{(k-j-\nu)!} (m-j-\nu)^{k-j-\nu} \tag{46}$$

when $j \leq \min(k,m)$, and zero otherwise.

We can study $X^{(1)}_{(ij)}(t)$ and $X^{A,(1)}_{(ij)}(t)$ recursively.

At the origin for each $t = 0, 1, 2, \ldots;$ $X^{(1)}_{(0,0)}(t) = \tau$, a fixed constant. Furthermore:

$$P\{X^{(1),A}_{(0,0)}(t) = w\} = P_{\tau,w} = \frac{\binom{m}{w}}{m^{\tau}} \sum_{\nu=0}^{\min(\tau,m)-w} (-1)^{\nu} \binom{m-w}{\nu} \frac{\tau!}{(\tau-w-\nu)!} (m-w-\nu)^{\tau-w-\nu} \tag{47}$$

If we assume $\tau \leq m$;

$$P\{X^{(1),A}_{(0,0)}(t) = w\} = \begin{cases} \dfrac{\binom{m}{w}}{m^{\tau}} \displaystyle\sum_{\nu=0}^{\tau-w} (-1)^{\nu} \binom{m-w}{\nu} \frac{\tau!}{(\tau-w-\nu)!} (m-w-\nu)^{\tau-w-\nu} & \text{if } w \leq \tau \\ 0 & \text{if } w > \tau. \end{cases} \tag{48}$$

At coordinates (1,1), (1,0) the distribution of $X_{(1,0)}(t)$ and $X_{(1,1)}(t)$ are identical, hence we write only $X^{(1)}_{(1,0)}(t)$.

We have:

$$X^{(1)}_{(1,0)}(t) = \begin{cases} X^{(1),A}_{(0,0)}(t-1) & t = 1, 2, \ldots; \\ 0 & \text{if } t = 0. \end{cases} \tag{49}$$

For the acceptance at $t = 1, 2, \ldots;$

$$P\{X^{A,(1)}_{(1,0)}(t) = j\} = \sum_{k=j}^{\tau} P_{kj} \cdot P\{X^{(1)}_{(1,0)}(t) = k\} \qquad j = 0, 1, 2, \ldots;$$

or recursively:

$$P\{X^{A,(1)}_{(1,0)}(t) = j\} = \begin{cases} \displaystyle\sum_{k=j}^{\tau} P_{kj} \cdot P\{X^{(1),A}_{(0,0)}(t-1) = k\} & \text{if } j \leq \tau; \\ 0 & \text{otherwise.} \end{cases} \tag{50}$$

Equation(50) is recursively solvable since $P\{X^{(1),A}_{(0,0)}(t-1) = k\}$ is given by (48)and P_{kj} is given by (46).

Now more generally, at a repeater at distance d with coordinates (d,j); $j \neq 0,d$. We have for $d = 2, 3, \ldots;$

$$X^{(1)}_{(d,j)}(t) = (1-k_0)\left[X^{(1),A}_{(d-1,j-1)}(t-1) + X^{(1),A}_{(d-1,j)}(t-1)\right]. \tag{51}$$

The acceptances are given by:

$$P\{X^{A,(1)}_{(d,j)}(t) = r\} = \sum_{k=r}^{(2m)} P_{k,r} \cdot P\{X^{(1)}_{(d,j)}(t) = k\}; \tag{52}$$

where P_{kr} is given by (1) and $P\{X^{(1)}_{(d,j)}(t) = k\}$ can be computed recursively from (51) using the notion of isodesic line joint densities. The equations for mode 2 analysis are identical except that P_{kj} is replaced by P^*_{kj}.

When $j = 0$ or d, i.e. the repeater is on the axis at distance d a simpler analysis unfolds. Since the random variables $X_{(d,0)}(t)$ and $X_{(d,d)}(t)$ have the same probability distribution, we write equations only for $X_{(d,0)}(t)$; $d \geq 2$ since $X_{(0,0)}(t)$ and $X_{(1,0)}(t)$ have already been determined.

$$X^{(1)}_{(d,0)}(t) = (1-k_0)\, X^{(1),A}_{(d-1,0)}(t-1); \qquad t = d, d+1, \ldots; \quad d = 2, 3, \ldots \; . \tag{53}$$

For the acceptances;

$$P\{X^{(1),A}_{(d,0)}(t) = j\} = \sum_{k=j}^{\infty} P_{kj}\, P\{X^{(1)}_{(d,0)}(t) = k\}, \qquad = 0, 1, 2, \ldots, m \tag{54}$$

The equations (46)through (54)can be used with computer generated data to study message arrivals and acceptances at each repeater.

In particular, we can write equations such as 46-54 for the closed net under consideration and obtain numerical data for flow from the ground station.

Let $X_{(i,j)}(t)$ be the number of messages received at (i,j) at time t and $Y_{(i,j)}(t)$ be the number of messages accepted at (i,j) at time t. We assume $X_{(o,o)}(t)=J$ a fixed constant for all time points. As in the inward model, $Y_{(i,j)}(t)$ is obtained from $X_{(i,j)}(t)$ by randomizing over either mode 1 or mode 2 slotting. When performing the calculations on the computer, we assumed a finite grid of 61 repeaters as earlier. However, now a repeater repeats messages to those repeaters which are one unit of distance further from the origin or ground station. Thus for example a repeater in a quadrant repeats to its two further neighbors, while a repeater on the axis repeats to the one repeater which is one unit further.

The specific equations used for the first quadrant calculations follow, we assume that $\frac{1}{61}$ of those messages accepted are addressed to each repeater and hence not repeated. The calculations were carried out in each of the two slotting modes.

Step 1. Set $X_{(o,o)}(t) = J = 80,\ t = 1, 2, \ldots, 35$.

Step 2. Compute $Y_{(o,o)}(t)$ by randomizing over the transfer distribution in each of two modes.

Step 3. Compute $X_{(1,1)}(t)$ and $X_{(1,2)}(t)$ from:

$$X_{(1,1)}(t) = X_{(1,2)}(t) = \frac{60}{61}\ Y_{(o,o)}(t-1).$$

Step 4. Compute $Y_{(1,1)}(t)$ and $Y_{(1,2)}(t)$ in each of two modes.

Step 5. For $t = 2, 3, 4, \ldots, 37$; Compute $X_{(2,1)}(t)$, $X_{(2,2)}(t)$ and $X_{(2,3)}$ from:

$$X_{(2,1)}(t) = \frac{60}{61} Y_{(1,1)}(t-1)$$

$$X_{(2,2)}(t) = \frac{60}{61} (Y_{(1,1)}(t-1) + Y_{(1,2)}(t-1))$$

$$X_{(2,3)}(t) = \frac{60}{61} Y_{(1,2)}(t-1).$$

Step 6. Compute $Y_{(2,1)}(t)$, $Y_{(2,2)}(t)$ and $Y_{(2,3)}(t)$ by randomizing in each slotting mode.

Step 7. Compute $X_{(3,2)}(t)$, $X_{(3,3)}(t)$, $X_{(3,4)}(t)$ from:

$$X_{(3,2)}(t) = \frac{60}{61}\left[Y_{(2,1)}(t-1) + Y_{(2,2)}(t-1)\right]$$

$$X_{(3,3)}(t) = \frac{60}{61}\left[Y_{(2,2)}(t-1) + Y_{(2,3)}(t-1)\right]$$

$$X_{(3,4)}(t) = \frac{60}{61} Y_{(2,3)}(t-1).$$

Step 8. Compute $Y_{(3,2)}(t)$, $Y_{(3,3)}(t)$, $Y_{(3,4)}(t)$ by randomizing in mode 1 and mode 2.

Step 9. Compute $X_{(4,3)}(t)$ and $Y_{(4,4)}(t)$ by randomizing.

Step 10. Compute $X_{(5,4)}(t)$ from:

$$X_{(5,4)}(t) = \frac{60}{61}\left[Y_{(4,3)}(t-1) + Y_{(4,4)}(t-1)\right].$$

Step 11. Compute $Y_{(5,4)}(t)$ by randomizing and print out X's and Y's for $t = 1, 2, \ldots, 40$. The numerical data is summarized in Table 26 and the accompanying Figure 10.

11.3 Messages Coming Outward From the Origin

It is assumed that at each point in time eighty (80) messages originate at the origin (ground station). The messages are repeated outwards to the various repeaters. At each point in time each repeater sends all but a fixed proportion of its accepted message to each neighboring repeater, one unit of distance further from the ground station. The fixed proportion not repeated is 1/61 of the number of accepted messages, which are assumed to be addressed to the given repeater. The number of slots is fixed at 100.

In table 47 we summarize the result of this calculation by giving the average number of messages accepted and arriving as a function of distance and mode. The numerical data is displayed graphically in figure 22.

DISTANCE	MODE 1		MODE 2	
	ARRIVING	ACCEPTED	ARRIVING	ACCEPTED
0	80	33	80	52
1	32	21	51	36
2	27	19	47	33
3	32	22	57	41
4	46	30	87	58
5	58	31	114	66

TABLE 47

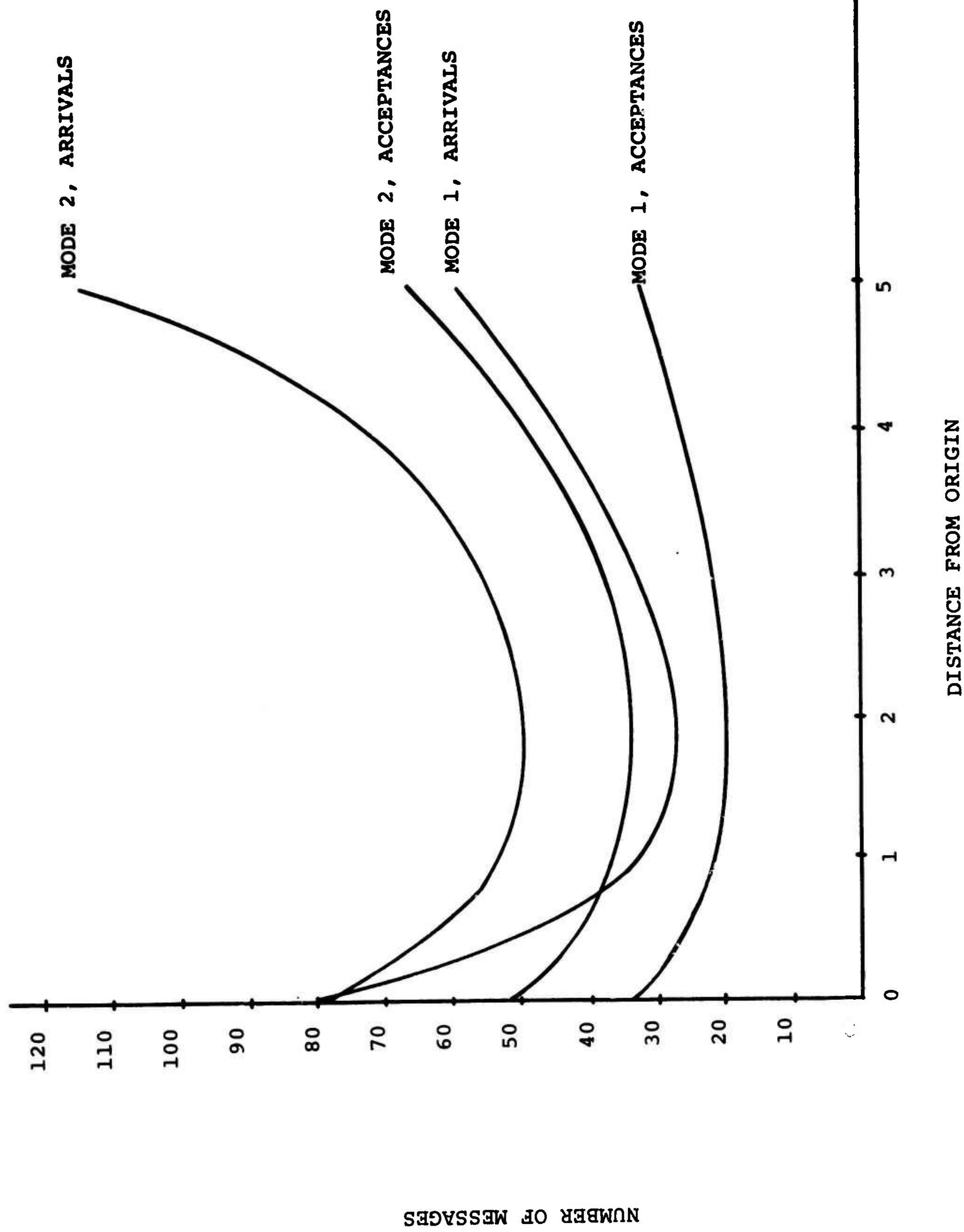

MODE 2, ARRIVALS
MODE 2, ACCEPTANCES
MODE 1, ARRIVALS
MODE 1, ACCEPTANCES
0
1
2
3
4
5
DISTANCE FROM ORIGIN
10
20
30
40
50
60
70
80
90
100
110
120
NUMBER OF MESSAGES

FIGURE 22

12. OTHER MODELS

A number of models other than the "basic model" were considered. The results for quantities of interest were obtained in closed forms under the assumption of an infinite grid. One example of such a model was described in Section 3 for messages repeated only toward the ground station. Of course that was part of our "basic model". In the process of developing the results of Section 3, we assumed that a single message originated at each repeater at each point in time and multiplied the results by the mean, τ, to obtain average flows. We will now "justify" that calculation and study uninhibited passage of messages in each direction in an infinite grid. Our new assumptions are:

1) At each point in time, starting at t=0, messages originate at each repeater according to a Poisson distribution with mean λ.

2) The arrivals (originations) at each repeater are <u>independent</u> over time and different repeaters.

The probability that exactly j new messages originate at any time point at any repeater is

$$\frac{e^{-\lambda}\lambda^{j}}{j!}, \quad j = 0, 1, 2, \ldots.$$

We compute formulas for;

a) $N_o(t)$ = average number of messages which arrive at the origin at time t. Since all repeaters are statistically identical there is no loss in generality in studying message flow at the origin.

b) $N'_o(t)$ = Average number of distinct messages which arrive at the origin at time t for the first time.

c) Ieff(t)=Inefficiency of the network defined

by;

$$\text{Ieff}(t) = \frac{No(t)}{N'_o(t)} = \frac{\text{average number of messages}}{\text{average number of new messages for the 1st time.}}$$

This is a measure of inefficiency since the larger Ieff, the more inefficient the system.

The actual number of messages which arrive at the origin at time t is a random variable. In fact it is a sum of a large number (when t is fairly large) of independent random variables. The summand random variables can loosely be described as the contribution to message flow at the origin arising out of some number of messages originating at each repeater at each point in time.

To compute $N_o(t)$ we can sum up all the contributions. This is interesting but tedious. A simpler method is to compute $X_o(t)$ which we define as the number of messages arriving at the origin at time t in a <u>deterministic</u> model obtained by assuming that at each point in time, at each repeater, exactly one new message originates. It will then follow that

$$N_o(t) = \lambda X_o(t).$$

Similarly, if we define $X'_o(t)$ to be the number of distinct messages that arrive at the origin at time t in the deterministic model it will follow that

$$N'_o(t) = \lambda X'_o(t).$$

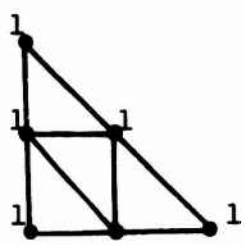
We indicate an armwaving proof of the first assertion. The quantity No(t) is a sum of average contribution to the flow at the origin at time t, as a result of messages originating at repeaters less than t units in distance and times earlier than t. The average contribution from each repeater is a <u>constant</u> (not with time) at each fixed time point and repeater, multiplied by λ, the average generation rate. (the constant is given by the calculations in section II and depends on the coordinates (d,j) and time. Thus λ factors from the sum and $N_o(t)$ is λ multiplied by the total flow resulting from a single message originating at each repeater at each point in time.

We now make the specific assumption.

At each point in time and at each repeater, a single new message is originated.

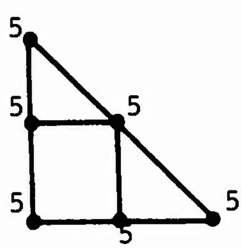
Under this model to compute $X_o(t)$ and hence $N_o(t)$ is trivial. To fix ideas we depict the situation at three time points

t = 0



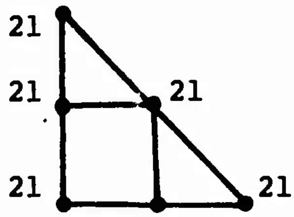
At time zero one message originates at each repeater, hence the message flow is $X_o(0)=1$.

t=1



At time 1 each repeater receives 5 messages,one from each of four nearest neighbors and one new message.

$t = 2,$



At time t = 2 each repeater receives 5 messages from each of its four nearest neighbors and one new message for a total of 21.

To compute $X_o(t)$ in general we note that each repeater is statistically identical in terms of message flow. Hence,

$$X_o(t) = 4X_o(t-1) + 1 \qquad t \geq 1$$

$$X_o(0) = 1.$$

This difference equation is trivial to solve and hence,

$$X_o(t) = \frac{4^{t+1}-1}{3} \qquad t \geq 0 .$$

Thus

$$N_o(t) = \frac{\lambda}{3} \; (4^{t+1}-1)$$

To compute $N_o'(t)$ we consider the same deterministic model and compute $X_o'(t)$, the number of distinct messages which arrive at the origin for the first time at time t. It is easy to compute $X_o'(t)$ from the following table by summing contributions.

Time of Origination	Distance from Origin	Number of Messages
o	t	4t
1	t-1	4(t-1)
2	t-2	4(t-2)
⋮	⋮	⋮
t-1	1	4
t	0	1

The first column is the time the message first appeared in the system if it is received by the origin for the first time at time t. The second column indicates the distance from the origin that the message originated. The third column indicates the number of message originated at that time and distance which are received at the origin at time t. Thus,

$$X'_o(t) = 4t + 4(t-1) + .. +4+1 = \sum_{j=1}^{t} 4j+1$$
$$= 2t^2 + 2t +1, \quad t \geq 0 .$$

Thus,

$$N'_o\ (t) = \lambda(2t^2 + 2t +1)$$

The inefficiency of this "undamped" network is

$$Ieff\ (t) = \frac{\frac{\lambda}{3}(4^{t+1}-1)}{\lambda(2t^2 +2t+1} = \frac{4^{t+1}-1}{3(2t^2+2t+1)} \sim \frac{4^{t+1}}{6t^2}$$

The inefficiency grows rapidly with time for this undamped system.

We can now put restrictions on the operating characteristics of the repeaters and message flow.

12.1. No Message Can Be Transmitted More Than k Times

In this mode it is assumed that each message has a counting feature whereby each time the message is repeated the counter is updated by one unit. When the counter reaches the number k the message is no longer repeated and disappears from the system.

In this mode we compute;

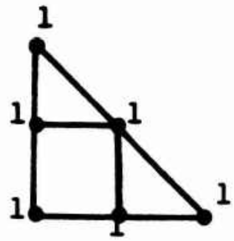
a) $N_o(t)$ = average number of messages received by the origin at time t.

b) $N_o'(t)$ = average number of distinct messages received at the origin for the first time at time t.

$$I_{eff}(t) = \frac{N_o(t)}{N_o'(t)}$$

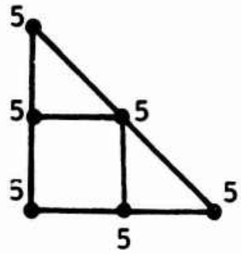
Once again by the argument presented in the previous section it suffices to consider the deterministic model and to compute $X_o(t)$ and $X_o'(t)$. To fix ideas we diagram the first 6 time points, inherently assuming $k \geq 5$.

t = 0,



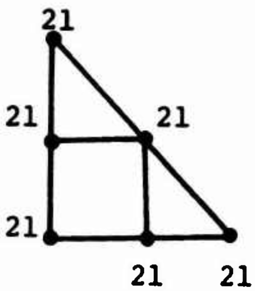
one message at each repeater,
all of age zero

t = 1,



five messages each repeater:
one of age zero
four of age one

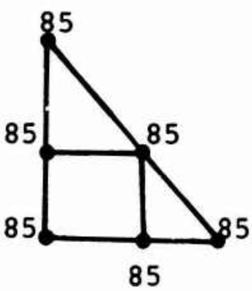
t = 2,



21 messages at each repeater

1 of age zero
4 of age one
16 of age two

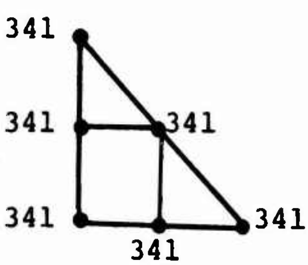
t = 3,



85 messages at each repeater

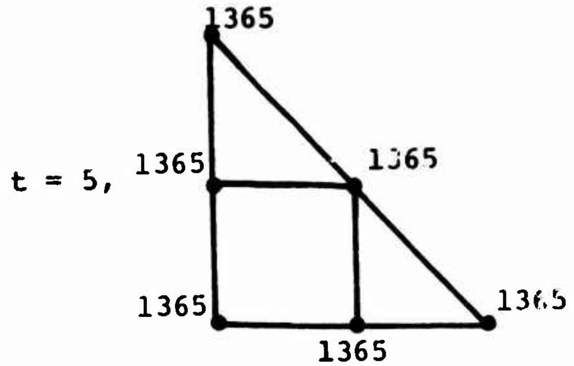
64 of age three
16 of age two
4 of age one
1 of age zero

t = 4,



341 messages at each repeater

256 of age four
64 of age three
16 of age two
4 of age one
1 of age zero



1365 Messages	
1024	of age 5
256	of age 4
64	of age 3
16	of age 2
4	of age 1
1	of age 0
1365	TOTAL

Let $X_o^{>k}(t)$ be the number of messages arriving at the origin at time t whose age is less than k. That is, those that will be transmitted. Since the flow at all repeaters is statistically identical:

$$X_o(t) = 4X_o^{>k}(t-1) + 1 , \qquad t \geq 1 .$$

The number of messages received at the origin at time t is four times the number that will be transmitted by any of the four nearest neighbors plus one new one. Once again this is trivial difference equation to solve in k. We obtain as a solution:

$$X_o(t) = \frac{4^{k+1}-1}{3} \qquad t \geq k,$$

$$X_o(t) = \frac{4^{t+1}-1}{3} \qquad t \leq k .$$

Hence by the arguments above;

$$N_o(t) = \begin{cases} \lambda\dfrac{[4^{k+1}-1]}{3} & t \geq k \\ \lambda\dfrac{[4^{t+1}-1]}{3} & t < k, \end{cases}$$

A similar analysis shows that,

$$N_o'(t) = \begin{cases} 2k^2 + 2k + 1, & t \geq k, \\ 2t^2 + 2t + 1 & t \leq k. \end{cases}$$

Thus:

$$\text{Ieff}(t) = \frac{\frac{\lambda}{3}\,[4^{k+1} - 1]}{\lambda(2k^2 + 2t + 1)} \qquad t \geq k.$$

$$= \frac{4^{k+1} - 1}{3(2k^2 + 2k + 1)} \sim \frac{4^{k+1}}{6k^2} = \frac{2 \cdot 4^k}{3k^2}.$$

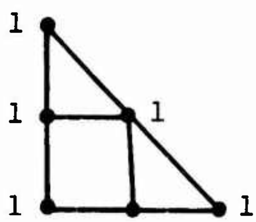
In tabular form for k=1, 2, 3, 4, 5 the inefficiencies are given

Ieff	1	1.62	3.40	8.3	22.4	....
k	1	2	3	4	5	....

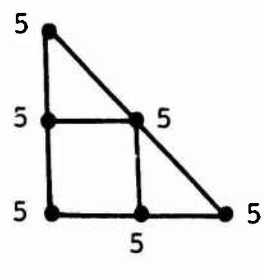
12.2 If the Same Message Arrives From Different Sources Only one Transmission is Made.

In this mode of operation a repeater has the ability to compare messages which arrive at the same instant. We may consider this mode to be a "memory" type system of length of time "one unit" or instantaneous. We seek to compute $N_o(t)$ and $N_o'(t)$. Again we consider the deterministic model. To fix ideas let us examine some early time points and compute multiplicities. All repeaters in this case also have statistically identical flows.

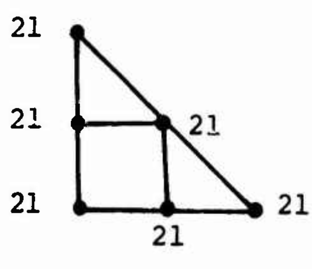
t = 0,



t = 1,



t = 2,



At time t = 2 all repeaters receive 21 messages, and they are statistically identical. Of the 21 total messages received at the origin only 14 are distinct.

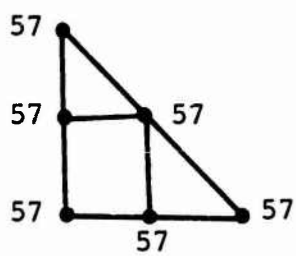
We can partition this total using a table as follows:

Origin of Message	Time of Origin	Number Distinct
(0,0)	t=2	1
(1,1), (1,0)	t=1	4
(2,0), (2,1), 2,2)	t=0	8
(0,0)	t=0	1

The total distinct is 14.

Hence the diagram for t=3 is,

t = 3,



of the 57 = 4·14 +1, one is new and 14 came from each of four nearest neighbors.

The same method can be used to compute $X_o(2t)$ and $X_o(2t-1)$ in general. Let $X_o^d(2t)$ and $X_o^d(2t-1)$ be the number of distinct messages received at the origin at time (2t) and (2t-1) respectively. Clearly, these quantities satisfy;

$$X_o(2t) = 4X_o^d(2t-1) + 1 \quad \text{and}$$

$$X_o(2t+1) = 4X_o^d(2t) + 1.$$

Thus it suffices to compute $X_o^d(2t)$ and $X_o^d(2t-1)$.

To compute $X_o^d(2t)$ and $X_o^d(2t-1)$ we decompose each as follows:

$$X_o^d(2t) = X_o^{d,1}(2t) + X_o^{d,2}(2t) + X_o^{d,3}(2t) + \ldots + X_o^{d,t+1}(2t),$$

$$X_o^d(2t-1) = X_o^{d,1}(2t-1) + X_o^{d,2}(2t-1) + \ldots + X_o^{d,t}(2t-1).$$

Where $X_o^{d,v}(t)$ = number of distinct messages received at the origin at time t for the v^{th} time. The quantities $X_o^{d,v}(t)$ are computed by the following table, which essentially decomposes $X_o^{d,v}(t)$ by distance and time of origination of each message contributing to $X_o^{d,v}(t)$. For $X_o^d(2t-1)$;

1^{st} time -	time of origin.	0	1	• • •	2t-2	2t-1
	dist. at origin	2t-1	2t-2	• • •	1	0
2^{nd} time -	time of origin.	0	1	• • •	2t-4	2t-3
	dist. of origin.	2t-3	2t-4	• • •	1	0
. . .	. . .					
$(t-1)^{th}$ time -	Time of origin.	0	1	2	3	
	dist. at origin.	3	2	1	0	
t^{th} time -	time of origin.	0	1			
	dist at origin.	1	0			

The grand sum is;

$$X_o^{d,1} = 1+4\sum_{j=1}^{2t-1}(2t-j)\ ,\quad X_o^{d,2} = 1+4\sum_{j=3}^{2t-1}(2t-j),\ldots,\quad X_o^{d,t} = 4\sum_{j=2t-1}^{2t-1}(2t-j)$$

since the number of messages originating at distance d is 4d as we have seen earlier. Therefore,

$$X_o^d(2t-1) = t+4\sum_{k=1}^{t}\ \sum_{j=2k-1}^{2t-1}(2t-j)$$

After summing we obtain:

$$X_o^d(2t-1) = \frac{t(4t+1)(2t+1)}{3},$$

By a similar analysis we can show that

$$X_o^d(2t) = \frac{(4t+3)(2t+1)(t+1)}{3},$$

Thus,

$$X_o(2t) = 4 \cdot X_o^d(2t-1) + 1 = \frac{4t(4t+1)(2t+1)}{3} + 1, \quad \text{or}$$

$$X_o(2t) = \frac{(4t+3)(8t^2+1)}{3}.$$

Similarly,

$$X_o(2t+1) = 4 \cdot X_o^d(2t) + 1$$

$$= \frac{4(4t+3)(2t+1)(t+1)}{3} + 1$$

$$= \frac{32t^3+72t^2+52t+15}{3}.$$

Hence:

$$N_o(2t) = \frac{\lambda(4t+3)(8t^2+1)}{3},$$

$$N_o(2t+1) = \frac{\lambda(32t^3+72t^2+52t+15)}{3},$$

$$N_o^1(2t) = \lambda(2t^2+2t+1) = \lambda(8t^2+4t+1).$$

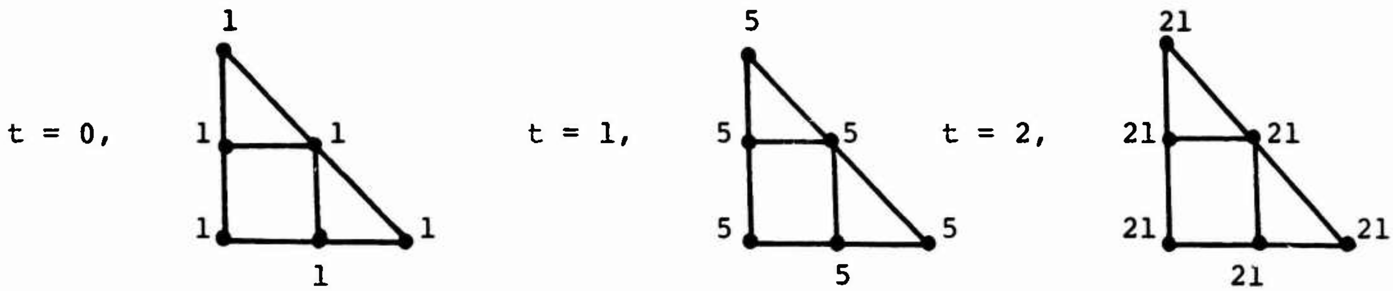
The efficiency of this mode is:

$$\text{Eff}(2t) = \frac{(4t+3)(8t^2+1)}{3(8t^2+4t+1)} \sim \frac{4t}{3}.$$

12.3 A Repeater Never Transmits The Same Message More Than Once Except Upon Initial Reception

This mode of operation implies infinite but not instantaneous memory in the repeater. We analyze the deterministic case with no loss of generality.

Pictorially the first few time points appear as follows;



Of the 21 messages received at t=2 there are 17 which are received for the first time. These can be enumerated by point and time of origin: 1 new one at (t=2, d=0), 4 at (t=1, d=1), 12 at (t=0, d=2). Thus there will be 69 messages received at each repeater at t=3.

In general let:

$X_o(t)$ be the number of messages received at the origin at time t, and $X_o^1(t)$ be the number of messages received at the origin for the first time at time t. Then,

$$X_o(t) = 4\, X_o^1(t-1) + 1 \, .$$

If a message was received for the first time at any repeater at time t-1 it originated at distance t-1 at time 0, or at d=t-2 at time 1 or, ..., at distance 0 at time t-1. Summing these by their appropriate multiplicities we obtain,

$$X_o^1(t-1) = 4\sum_{k=2}^{t}\sum_{j=0}^{t-k}\binom{t-k+1}{0}\binom{t-k+1}{j} + 1,$$

or,

$$X_o^1(t-1) = 2^{t+2}-4t-3 \qquad \text{and},$$

$$Xo(t) = 2^{t+4}-16t-11 .$$

Similarly;

$$No(t) = \lambda(2^{t+4}-16t-11) \quad \text{and as before}$$

$$N_o^1(t) = \lambda(2t^2+2t+1)$$

The efficiency of this mode is:

$$Eff = \frac{No(t)}{No^1(t)} = \frac{2^{t+4}-16t-11}{2t^2+2t+1} \sim \frac{2^{t+3}}{t^2} .$$

12.4 Mixture of Modes 12.2 and 12.3

In this mixed mode a repeater has infinite memory for comparison of messages and instant memory. The result (operational) of this mixed mode is that when multiple reception of the same message is received only one transmission is made. Furthermore if a message is received for the second time it is not transmitted at all. The result of this combined mode is to reduce the transmission in the instantaneous mode to only messages received for the first time.

Therefore:

$$X_o(t) = 4X_o^{',d}(t-1) + 1$$

where

$X_o^{',d}(t)$ is the number of distinct messages received at the origin for the first time at time t. This quantity has been previously computed to be,

$$X_o^{',d}(t-1) = 2(t-1)^2 + 2(t-1) + 1 = 2t^2 - 2t - 1.$$

It follows that,

$$X_o(t) = 8t^2-8t-3, \text{ and}$$

$$N_o(t) = \lambda(8t^2-8t-3).$$

The efficiency of this mixed mode is seen to be

$$\text{Eff}(t) = \frac{\lambda(8t^2-8t-3)}{\lambda(2t^2+2t+1)} \quad \frac{8t^2-8t-3}{2t^2+2t+1} \sim 4 \quad .$$

12.5 Mixture of Modes 12.1and 12.2

In this mixture of modes no message can be transmitted more than k-times and each repeater has instantaneous memory but not finite memory. This mode also provides an upper bound to the case where each repeater has zero capture and messages are dropped after k transmissions.

A little analysis will show that for $t \le k$ there is no change in the message flow from the instantaneous memory case. For $t \ge k$ the exact same analysis as in the instantaneous memory only case shows that the formulas are exactly those except that t should be replaced by k.

Therefore in this mixed mode, for memory of k.

$$X_o(2t) = \frac{(2k+3)(8k^2+1)}{3}, \quad t \ge [\tfrac{k}{2}]$$

$$= \frac{(2t+3)(8t^2+1)}{3}, \quad t \le [\tfrac{k}{2}]$$

$$X_o(2t+1) = \frac{32k^3+72k^2+52k+15}{3} \qquad t \ge k$$

$$= \frac{32t^3+72t^2+52t+15}{3} \qquad t \le k.$$

Furthermore,

$$N_o(2t) = X_o(2t),$$

$$N_o(2t+1) = X_o(2t+1).$$

We arrive at the same result with t replaced by k.

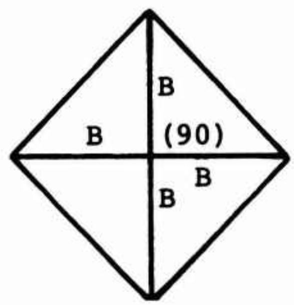
$$_I\mathrm{Eff}(2t) = \frac{(4k+3)(4k^2+1)}{3(8k^2+4k+1)} \sim \frac{4}{3}\,k.$$

In tabular form;

$_I$Eff	1.33	2.67	4.0	5.3	6.67	8.0
k	1	2	3	4	5	6

12.6 A Closed Boundary Model

In this part of the model it is assumed that each repeater is less than B units from the origin. As in earlier cases, the units are measured in steps. The region is now closed and appears as follows:



Furthermore, in the initial stages of this model, it is assumed that a single message originates at the origin at time t=0. All message flow is generated as a result of this single message. Unfortunately, this model causes a loss of symmetries which facilitated the ease of obtaining closed form solutions for message flow in the previous models. However, some closed form analysis can be obtained. In particular it is not difficult to obtain an algorithm which will supply a complete analysis of message flow at any point in time on any repeater or station.

Let $N^{d}_{B,j}(2t+d)$ be the number of messages received at a station or repeater which is d-units from the origin and j-units from the y-axis at time (2t+d) where d=0, 1, 2,...,B-1, j=0, 1, 2,...B.

The equations for $N^d_{B,j}(2t+d)$ are identical to the equations for the open boundary case when $0 \leq t \leq B-d$ since the closing of the boundary does not affect message flow at a repeater at distance B-d until t=B+d. Therefore;

$$N^d_{B,j}(2t+d) = \binom{2t+d}{t}\binom{2t+d}{t+j}; \quad t \leq \frac{B}{2}.$$

We can use this equation to compute message flow at all time points up to and including t=B. To compute message flow after time B, we can work backwards from the boundary and successively compute message flow at each station or repeater for <u>any</u> time point using the equation we will develop. First however, it is helpful to examine a particular case with a diagram and compare the closed boundary with the open boundary. We let B=4.

<u>Closed Boundary at B=4</u> <u>Open Boundary</u>

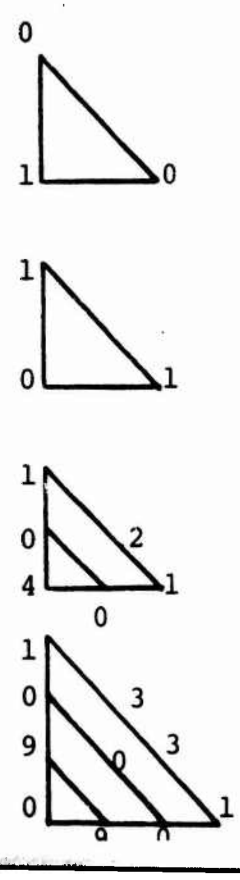
The closing of the boundary will not affect any repeaters or stations until t=6.

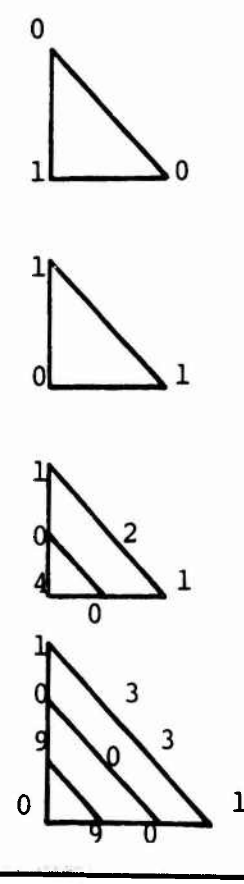
t=0

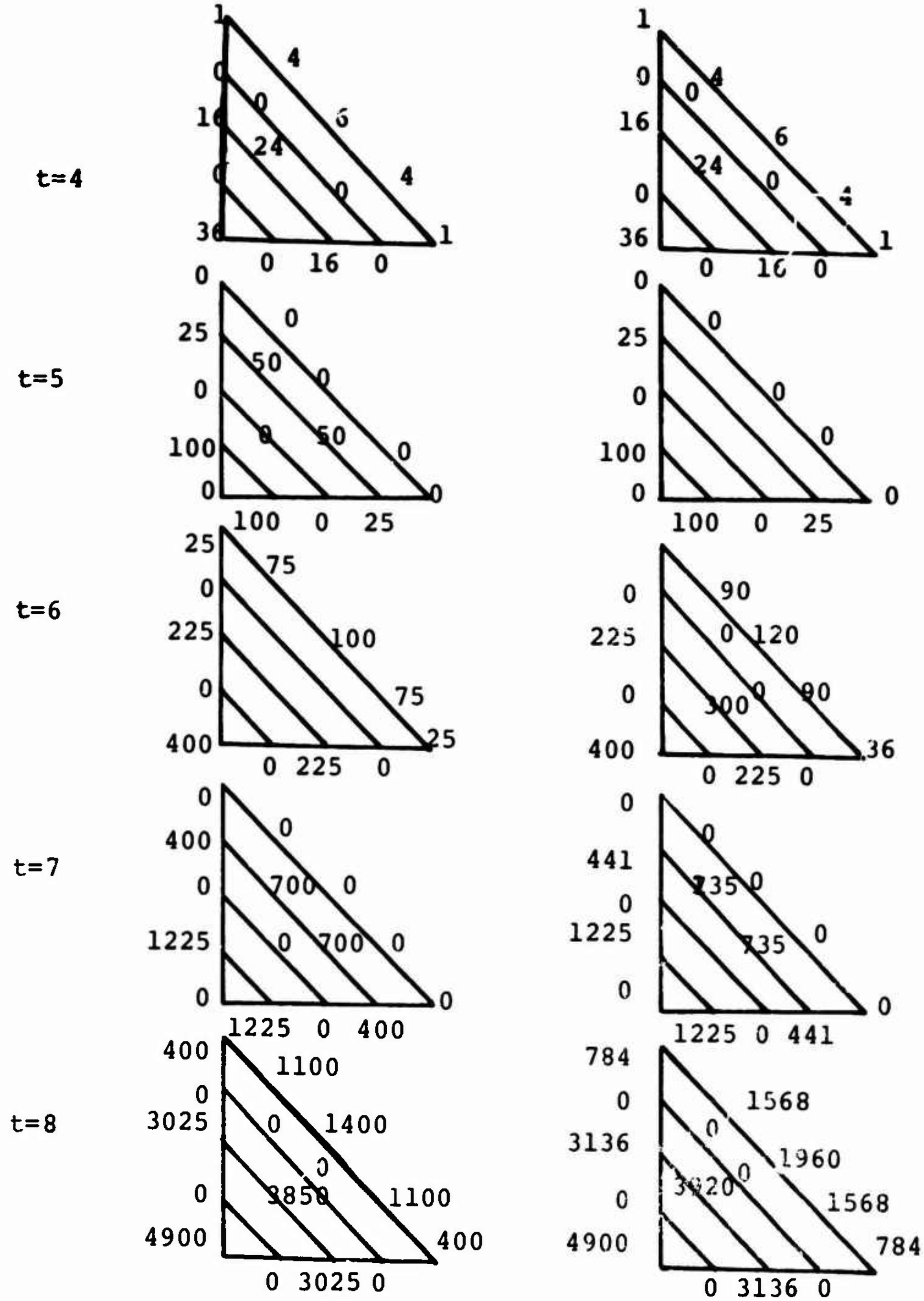
t=1

t=2

t=3







It is obvious how to continue for as long as one wishes to obtain the message flow. Since all message flow can be computed by the formulas of the preceding section for $t \leq B$, the message flow with closed boundary can be computed by backward iteration for all times larger than B. The initial conditions at time t=B are given by the following relations,

<u>At time t=B</u>

$$N^{d}_{B,j}(B) = \begin{cases} \binom{B}{k}\binom{B}{k+j} & \text{if } B=2k+d \\ 0 & \text{otherwise.} \end{cases}$$

since $N^{d}_{B,j}(B)$ is zero unless B and d are both odd or even numbers.

<u>At time t=B+1</u>

$N^{B}_{B,j}(B+1)=0$, i.e. no messages at the boundary,

$$N^{d}_{B,j}(B+1)=\begin{cases}\binom{B+1}{k}\binom{B+1}{k+j} & \text{if } B=2k+d-1,\\ 0 & \text{otherwise.}\end{cases}$$

<u>At time t=B+2</u>

On the boundary:

$$N^{B}_{B,0}(B+2) = N^{B-1}_{B,0}(B+1) = \binom{B+1}{1}^2 = \binom{B+1}{B}^2 = N^{B}_{B,B}(B+2).$$

$$N^{B}_{B,j}(B+2) = N^{B-1}_{B,j}(B+1) + N^{B-1}_{B,j-1}(B+1)$$
$$=\binom{B+1}{B}\binom{B+2}{j+1} = N^{B}_{B,B-j}(B+2)$$

for $j=1, 2,\ldots,B-1$.

for $d<B$

$$N^{d}_{B,j}(B+2) = \begin{cases}\binom{B+2}{k}\binom{B+2}{k+j} & \text{if } B=2k+d-2, \qquad j=0, 1, 2,\ldots,d.\\ 0 & \text{otherwise}\end{cases}$$

<u>At time t=B+3</u>

On the boundary

$$N^{B}_{B,j}(B+3) = 0: \quad j=0, 1, 2,\ldots,B.$$

<u>At distance d=B-1</u>

$$N^{B-1}_{B,0}(B+3) = N^{B}_{B,0}(B+2) + N^{B-2}_{B,0}(B+2) + 2N^{B}_{B,1}(B+2)$$
$$=\left[\binom{B+1}{B}+\binom{B+2}{B}\right]^2 = N^{B-1}_{B,B-1}(B+3).$$

$$N^{B-1}_{B,j}(B+3) = N^{B-2}_{B,j-1}(B+2)+N^{B}_{B,j+1}(B+2) + N^{B-2}_{B,j}(B+2)$$
$$+ N^{B}_{B,j}(B+2)$$
$$=\binom{B+3}{j+2}\left[\binom{B+1}{B} + \binom{B+2}{B}\right] = N^{B-1}_{B,B-1-j}(B+3).$$
$$j=1,\ 2,\ldots,B-2.$$

<u>At distance $d < B-1$</u>

$$N^{d}_{B,j}(B+3) = \begin{cases}\binom{B+3}{k}\binom{B+3}{k+j} & B=2k+d-3 \\ 0 & \text{otherwise}\end{cases} \qquad j=0,\ 1,\ldots,d.$$

<u>At time $t=B+4$</u>

On the boundary

$$N^{B}_{B,0}(B+4) = N^{B-1}_{B,0}(B+3) = \left[\binom{B+1}{B}+\binom{B+2}{B}\right]^2 = N^{B}_{B,B}(B+4).$$

$$N^{B}_{B,1}(B+4) = N^{B-1}_{B,0}(B+3) + N^{B-1}_{B,1}(B+3)$$

(Note that $N^{B}_{B,1}(B+4)$ must be computed separately since $N^{B-1}_{B,0}(B+3)$ and $N^{B-1}_{B,j}(B+3)$ for $j > 1$ do <u>not</u> share a common formula, i.e. are not special cases of a general formula.)

$$N^{B}_{B,1}(B+4) = N^{B}_{B,B-1}(B+4) = \left[\binom{B+1}{B}+\binom{B+2}{B}\right]\left[\binom{B+1}{B}+\binom{B+2}{B}+\binom{B+3}{B}\right]$$

$$N^{B}_{B,j}(B+4) = N^{B-1}_{B,j-1}(B+3) + N^{B-1}_{B,j}(B+3)$$

(Note the formula is the same for $j=1$ as above.)

$$= \left[\binom{B+1}{B}\binom{B+2}{B}\right]\left[\binom{B+4}{j+2}\right] = N^{B}_{B,B-j}(B+4)$$
$$j=2,\ldots,B-2.$$

<u>At distance $d=B-2$</u>

$$N^{B-2}_{B,0}(B+4) = N^{B-3}_{B,0}(B+3) + N^{B-1}_{B,0}(B+3) + 2\,N^{B-1}_{B,1}(B+3)$$
$$= \left[\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B}\right]^2 = N^{B-2}_{B,B-2}(B+4)$$

$$N^{B-2}_{B,j}(B+4) = N^{B-3}_{B,j-1}(B+3) + N^{B-1}_{B,j+1}(B+3) + N^{B-1}_{B,j}(B+3)$$

$$+ N^{B-3}_{B,j}(B+3).$$

$$= \binom{B+4}{j+3}\left[\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B}\right]$$

$$= N^{B-2}_{B,B-j-2}(B+4).$$

for $d < B-2$

$$N^{d}_{B,j}(B+4) = \begin{cases} \binom{B+4}{k}\binom{B+4}{k+j} & \text{if } B=2k+d-4 \\ 0 & \text{otherwise} \end{cases}.$$

<u>At time t=B+5</u>

<u>at distance d=B-1</u>

$$N^{B-1}_{B,0}(B+5) = N^{B-2}_{B,0}(B+4) + N^{B}_{B,0}(B+4) + 2\,N^{B}_{B,1}(B+4)$$

$$= \left[2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}\right]^2 = N^{B-1}_{B,B-1}(B+5)$$

$$N^{B-1}_{B,1}(B+5) = N^{B-2}_{B,0}(B+4) + N^{B}_{B,2}(B+4) + N^{B-2}_{B,1}(B+4)$$

$$+ N^{B}_{B,1}(B+4)$$

$$= \left[2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}\right]$$

$$\cdot \left[\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B} + \binom{B+4}{B}\right] = N^{B-1}_{B,B-1}(B+5).$$

$$N^{B-1}_{B,j}(B+5) = N^{B-2}_{B,j-1}(B+4) + N^{B}_{B,j+1}(B+4) + N^{B-2}_{B,j}(B+4)$$

$$+ N^{B}_{B,j}(B+4)$$

$$= \binom{B+5}{j+3}\left[2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}\right]$$

$$= N^{B-1}_{B,B-j-1}(B+5).$$

$$j=2,\ldots,B-3.$$

At distance B-3

$$N_{B,0}^{B-3}(B+5) = N_{B,0}^{B-4}(B+4) + N_{B,0}^{B-2}(B+4) + 2N_{B,1}^{B-2}(B+4)$$

$$= [\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B} + \binom{B+4}{B}]^2 = N_{B,B-3}^{B-3}(B+5).$$

$$N_{B,j}^{B-3}(B+5) = N_{B,j-1}^{B-4}(B+4) + N_{B,j+1}^{B-2}(B+4)$$

$$+N_{B,j}^{B-4}(B+4) + N_{B,j}^{B-2}(B+4)$$

$$= \binom{B+5}{j+4}[\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B} + \binom{B+4}{B}]$$

$$= N_{B,B-j-3}^{B-3}(B+5) \qquad j=1,\ 2,\ldots,B-4.$$

for $d < B-3$

$$N_{B,j}^{d}(B+5) = \begin{cases} \binom{B+5}{k}\binom{B+5}{k+j} & \text{if } B = 2k+d-5 \\ 0 & \text{otherwise.} \end{cases}$$

$$j=0,\ 1,\ 2,..,d.$$

We will continue for three more time points to get some idea as to how the solution behaves with time. We will omit the general equations and give only the results since it is clear that the expressions become too cumbersome for easy comprehension.

For time t=B+6

at the boundary

$$N_{B,0}^{B}(B+6) = [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}]^2 = N_{B,B}^{B}(B+6)$$

$$N_{B,1}^{B}(B+6) = [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}][3\binom{B+1}{B} + 3\binom{B+2}{B}+2\binom{B+3}{B}+\binom{B+4}{B}]$$

$$N_{B,2}^{B}(B+6) = [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}][\binom{B+1}{B} + \binom{B+2}{B} + \cdots + \binom{B+5}{B}]$$

$$N_{B,j}^{B}(B+6) = [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}][\binom{B+6}{J+3}]$$

$$j=3,\ 4,\ldots,B-3.$$

<u>At distance d=B-2</u>

$$N_{B,0}^{B-2}(B+6) = [3\binom{B+1}{B} + 3\binom{B+2}{B} + 2\binom{B+3}{B} + \binom{B+4}{B}]^2 = N_{B,B-2}^{B-2}(B+6)$$

$$N_{B,1}^{B-2}(B+6) = [3\binom{B+1}{B} + 3\binom{B+2}{B} + 2\binom{B+3}{B} + \binom{B+4}{B}]$$

$$\cdot [\binom{B+1}{B} + \binom{B+2}{B} + \cdots + \binom{B+5}{B}] = N_{B,B-3}^{B-2}(B+6)$$

$$N_{B,j}^{B-2}(B+6) = \binom{B+6}{j+4}\ [3\binom{B+1}{B} + 3\binom{B+2}{B} + 2\binom{B+3}{B} + \binom{B+4}{B}] = N_{B,B-2-j}^{B-2}(B+6)$$

$$j=2,\ldots,B-4.$$

<u>At distance d=B-4</u>

$$N_{B,0}^{B-4}(B+6) = [\binom{B+1}{B} + \binom{B+2}{B} + \cdots + \binom{B+5}{B}]^2 = N_{B,B-4}^{B-4}(B+6)$$

$$N_{B,j}^{B-4}(B+6) = \binom{B+6}{j+5}[\binom{B+1}{B} + \binom{B+2}{B} + \cdots + \binom{B+5}{B}]^2 = N_{B,B-4-j}^{B-4}(B+6)$$

$$j=1,\ 2,\ldots,B-5.$$

<u>For $d < B-4$</u>

$$N_{B,j}^{d}(B+6) = \begin{cases} \binom{B+6}{k}\binom{B+6}{k+j} & \text{if } B=2k+d-6 \\ 0 & \text{otherwise.} \end{cases}$$

<u>At time t=B+7</u>

<u>At distance d=B-1</u>

$$N_{B,0}^{B-1}(B+7) = [5\binom{B+1}{B} + 5\binom{B+2}{B} + 3\binom{B+3}{B} + \binom{B+4}{B}]^2 = N_{B,B-1}^{B-1}(B+7).$$

$$N^{B-1}_{B,1}(B+7) = [5\binom{B+1}{B} + 5\binom{B+2}{B} + 3\binom{B+3}{B} + \binom{B+4}{B}]$$

$$\cdot\ [4\binom{B+1}{B} + 4\binom{B+2}{B} + 3\binom{B+3}{B} + 2\binom{B+4}{B} + \binom{B+5}{B}]$$

$$= N^{B-1}_{B,B-2}(B+7).$$

$$N^{B-1}_{B,2}(B+7) = (\sum_{j=1}^{6}\binom{B+j}{B})\ [5\binom{B+1}{B} + 5\binom{B+2}{B} + 3\binom{B+3}{B} + \binom{B+4}{B}]$$

$$= N^{B-1}_{B,B-3}(B+7).$$

$$N^{B-1}_{B,j}(B+7) = \binom{B+7}{j+4}[5\binom{B+1}{B} + 5\binom{B+2}{B} + 3\binom{B+3}{B} + \binom{B+4}{B}].$$

$$j=3,\dots,B-4.$$

<u>At distance d=B-3</u>

$$N^{B-3}_{B,0}(B+7) = [4\binom{B+1}{B} + 4\binom{B+2}{B} + 3\binom{B+3}{B} + 2\binom{B+4}{B} + \binom{B+5}{B}]^2$$

$$N^{B-3}_{B,1}(B+7) = [\sum_{j=1}^{6}\binom{B+j}{B}][4\binom{B+1}{B}+4\binom{B+2}{B}+3\binom{B+3}{B}+2\binom{B+4}{B}+\binom{B+5}{B}]$$

$$N^{B-3}_{B,j}(B+7) = \binom{B+7}{j+5}[4\binom{B+1}{B}+4\binom{B+2}{B}+3\binom{B+3}{B}+2\binom{B+4}{B}+\binom{B+5}{B}]$$

$$j=2,\dots,B-5.$$

<u>At distance B-5</u>

$$N^{B-5}_{B,0}(B+7) = [\sum_{j=1}^{6}\binom{B+j}{j}]^2 = N^{B-5}_{B,B-5}(B+7)$$

$$N^{B-5}_{B,j}(B+7) = [\sum_{j=1}^{6}\binom{B+j}{j}]\binom{B+7}{j+6}. \qquad j=1,\ 2,\dots,B-6.$$

<u>At $d \angle B-5$</u>

$$N^{d}_{B,j}(B+7) = \begin{cases} \binom{B+7}{k}\binom{B+7}{k+j} & \text{if } B=2k+d-7 \\ 0 & \text{otherwise} \end{cases}$$

At time t=B+8

at the boundary

$$N^{B}_{B,0}(B+8) = N^{B}_{B,B}(B+8) = [5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]^2$$

$$N^{B}_{B,1}(B+8) = [5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]$$

$$\cdot[9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+2}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]$$

$$N^{B}_{B,2}(B+8) = [5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]$$

$$\cdot[5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]$$

$$N^{B}_{B,3}(B+8) = (\sum_{j=1}^{7}\binom{B+j}{B})[5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B} + \binom{B+4}{B}]$$

$$N^{B}_{B,j}(B+8) = \binom{B+8}{B}[5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]$$

$$j=4,\ldots,B-4.$$

At distance d=B-2

$$N^{B-2}_{B,0}(B+8) = [9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+3}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]^2$$

$$N^{B-2}_{B,1}(B+8) = [9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+3}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]$$

$$\cdot[5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]$$

$$N^{B-2}_{B,2}(B+8) = (\sum_{j=1}^{7}\binom{B+j}{B})[9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+3}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]$$

$$N^{B-2}_{B,j}(B+8) = \binom{B+8}{B}[9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+3}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]$$

At distance d=B-4

$$N_{B,0}^{B-4}(B+8) = [5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]^2$$

$$= N_{B,B-4}^{B-4}(B+8)$$

$$N_{B,1}^{B-4}(B+8) = [\sum_{k=1}^{7}\binom{B+k}{B}][5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]$$

$$N_{B,j}^{B-4}(B+8) = \binom{B+8}{j+6}[5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]$$

$$j=2,\ldots,B-6.$$

At distance d < B-4

$$N_j^d(B+8) = \begin{cases} \binom{B+8}{k}\binom{B+8}{k+j} & \text{if } B=2k+d-8 \\ 0 & \text{otherwise.} \end{cases}$$

This completes an analysis of the first eight time points past the time to the boundary. The expressions are quite unwieldy. However, the algorithm is clear and can compute message distribution throughout the grid at any time point. We write down the general set of equations and then indicate a "closed form" combinatorial method to make "sense" out of the unwieldy expressions.

The General Equations

The initial conditions for t=B are:

At time t=B: Initial Conditions

$$N_{B,j}^d(B) = \begin{cases} \binom{B}{k}\binom{B}{k+j} & \text{if } B=2k+d \\ 0 & \text{otherwise} \end{cases}$$

The General Equations

On the boundary $k \geq 1$

1. $N^{B}_{B,0}(B+2k) = N^{B}_{B,B}(B+2k) = N^{B-1}_{B,0}(B+2k-1)$.

2. $N^{B}_{B,j}(B+2k) = N^{B-1}_{B,j}(B+2k-1) + N^{B-1}_{B,j-1}(B+2k-1)$

$j=1, 2, \ldots, B-1$.

Note that:

$$N^{B}_{B,j}(B+2k) = N^{B}_{B,B-j}(B+2k), \qquad j=0,\ldots,B.$$

Along the axes $0 < d < B$, $k=1, 2, \ldots$.

$$\begin{aligned} N^{d}_{B,0}(B+2k) &= N^{d+1}_{B,0}(B+2k-1) + N^{d-1}_{B,0}(B+2k-1) \\ &+ 2N^{d+1}_{B,1}(B+2k-1) \\ &= N^{d}_{B,d}(B+2k) \end{aligned}$$

Off the axes $0 < d < B$, $0 < j < d$

$$\begin{aligned} N^{d}_{B,j}(B+2k) &= N^{d+1}_{B,j}(B+2k-1) + N^{d-1}_{B,j}(B+2k-1) \\ &+ N^{d-1}_{B,j-1}(B+2k-1) + N^{d+1}_{B,j+1}(B+2k-1). \end{aligned}$$

At the origin:

$$N^{0}_{B,0}(B+2k) = 4N^{1}_{B,1}(B+2k-1).$$

This completes the general equations.

A Conjecture on Solutions to the General Equation

An examination of the coefficients of the $\binom{B+k}{B}$ expressions in the first eight time points indicates that the coefficients are the same as the rows in the following infinite sequence of tableaus.

$\underline{T_1}$

	j=1	2	3	4	5	6
k=1	1					
k=2	1	1				
k=3	2	2	1			
k=4	5	5	3	1		
k=5	14	14	9	4	1	
k=6	42	42	28	14	5	1
	.	.	.	.	.	.
	.	.	.	.	.	.

$\underline{T_2}$

k \ j	1	2	3	4	5	6	7
1	0	1					
2	1	1	1				
3	3	3	2	1			
4	9	9	6	3	1		
5	28	28	19	10	4	1	
6	90	90	62	34	15	5	1
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.

$\underline{T_3}$

k=1	0	0	1					
2	1	1	1	1				
3	4	4	3	2	1			
4	14	14	10	6	3	1		
5	48	48	34	20	10	4	1	
6	165	165	116	69	35	15	5	1
	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.

$\underline{T_4}$

0	0	0	1					
1	1	1	1	1				
5	5	4	3	2	1			
20	20	15	10	6	3	1		
75	75	55	35	20	10	4	1	
275	275	200	125	70	35	15	5	1
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.

The tableaus through T_4 cover all the coefficients which arise up to and including $t=B+8$. It is possible to give a solution for $S_{kj}(T_\nu)$ which is the j^{th} element in the k^{th} row of the tableau T_ν, $\nu = 1, 2, \ldots$ However, it is perhaps not obvious how these tableaus are generated. The j^{th} term of the k^{th} row of any tableau is obtained by summing the last $(k+\nu-j)$ terms in the previous row. That is, the third term in the fourth row of T_2 is obtained by summing the last three terms of the third row. In mathematical form the equation for the tableaus are:

<u>Initial Condition</u> $$S_{1,j}(T_\nu) = \begin{cases} 0, j<\nu, \\ 1, j=\nu \\ 0, j>\nu. \end{cases}$$

$$S_{k,1}(T_\nu) = S_{k,2}(T_\nu) = \sum_{j=1}^{k+\nu-2} S_{k-1,j}(T_\nu).$$

$$S_{kj}(T_\nu) = \sum_{w=j-1}^{k+\nu-2} S_{k,w}(T_\nu), \qquad k > 1$$

It is clear that the between tableau equations are:

$$S_{k,j}(T_\nu) = S_{k,j}(T_{\nu+1}) - S_{k-1,j}(T_{\nu+2}), \quad k > 1, \quad \nu = 1, 2, \ldots$$

$$S_{k,j}(T_2) = S_{k+1,j}(T_1) - S_{k,j}(T_1).$$

Thus, to solve these equations it suffices to give a general formula for $S_{k,j}(T_1)$. The other terms can then be computed recursively. It is easy to check that the solution for T_1 is given by,

$$S_{k,j}(T_1) = \frac{j}{k}\binom{2k-j-1}{k-1}, \qquad j=1, 2, \ldots, k \quad k=1, 2, \ldots.$$

Then by recursion:

$$S_{k,j}(T_2) = \frac{j}{k+1}\binom{2k-j+1}{k} - \frac{j}{k}\binom{2k-j-1}{k-1}, \qquad k=1, 2, \ldots. \quad j=1, 2, \ldots, k+1.$$

$$S_{k,j}(T_3) = \frac{j}{k+2}\binom{2k-j+3}{k} + \frac{2j}{k+1}\binom{2k-j+1}{k}$$

$$\vdots$$

To apply these results we take a particular example to conjecture:

$$N^B_{B,0}(B+2k) = \left[\sum_{j=1}^{k} S_{k,j}(T_1)\binom{B+j}{B}\right]^2, \qquad k \geq 1$$

$$= \left[\sum_{j=1}^{k} \frac{j}{k}\binom{2k-j-1}{k-1}\binom{B+j}{B}\right]^2$$

This conjecture holds for time points k=1, 2, 3, 4.

$$N_{B,0}^{B-1}(B+2k+1) = [\sum_{j=1}^{k+1} S_{k+1,j}(T_1)\binom{B+j}{B}]^2$$

This conjecture holds for all k=1, 2, 3, and satisfies the general equations.

13 SUMMARY OF RESULTS OF THEORETICAL MODELS

Recall that;

$N_o(t)$ = expected number of messages received at the origin at time t.

$N'_o(t)$ = expected number of distinct messages received at the origin for the first time at time t.

$$Eff(t) = \frac{N_o(t)}{N'_o(t)}.$$

These symbols are in the model where there is Poisson input at each repeater at each time point. In all modes;

$$N'_o(t) = (2t^2+2t+1).$$

A. Deterministic The first model analyzed is the effect of a single message originally at the origin at time zero. No other messages ever enter the system. The propagation of this message can be measured by the following three quantities.

1) $B(t)$ - the number of repeaters which receive the message for the 1^{st} time at time t.

$$\boxed{B(t) = 4t \text{ for } t \geq 1, \quad B(0) = 1}$$

2) $A(t)$ = the number of repeaters which have received the message by time t,

$$\boxed{A(t) = 2t^2+2t+1.}$$

3) $N_j^d(t)$ = number of copies of the message received at time t at a repeater with coordinates (d,j).

$$N_j^d(t) = 0, \quad \text{if} \quad t < d$$

$$N_j^d(d) = \binom{d}{j} \quad j=0,\ 1,\ldots,d; \quad d=0,\ 1,\ldots\ldots,$$

$$N_j^d(d+2k+1) = 0 \quad k=0,\ 1,\ 2,\ldots \quad ,$$

$$N_j^d(d+2k) = \binom{d+2k}{k}\binom{d+2k}{k+j} \quad k=0,\ 1,\ldots \quad .$$

The quantity $N_j^d(d+2k)$ can be more generally interpreted as the number of copies of a message received at a repeater which is at distance d and horizontal distance j from the originating repeater, after d+2k time units.

B. Poisson Input Results

1. With no restrictions imposed on the operation of any repeater;

$$N_o(t) = \frac{\lambda}{3}(4^{t+1}-1)$$

$$N_o'(t) = \lambda(2t^2+2t+1)$$

$$\mathrm{Eff}(t) = \frac{4^{t+1}-1}{3(2t^2+2t+1)} \sim \frac{4^{t+1}}{6t^2}$$

2. No Message Can Be Transmitted More Than k-Times

$$N_o(t) = \frac{\lambda}{3}[4^{k+1}-1]$$

$$N_o'(t) = \lambda[2k^2+2k+1]$$

$$\mathrm{Eff}\ (t) = \frac{4^{k+1}-1}{3(2k^2+2k+1)}$$

Eff	1	1.60	3.40	8.3	22.4
k	1	2	3	4	5

3. <u>If the Same Message Arrives From Different Sources Only One Transmission is Made</u>

This mode of operation assumes that a repeater can compare all messages arriving at the same time and transmit only one copy of duplicate messages. It is inherently assumed in this mode that there is only instantaneous memory (which we may call 0-memory)

$$N_o(2t) = \lambda \frac{4t+3)(8t^2+1)}{3}$$

$$N_o'(2t) = \lambda(8t^2+4t+1)$$

$$\mathrm{IEff}(2t) = \frac{(4t+3)(8t^2+1)}{3(8t^2+4t+1)} \sim \frac{4t}{3}$$

This mode produces a substantial reduction in duplicate traffic.

4. <u>All Messages Are Transmitted in the Direction of a Fixed Ground Station</u>

In this mode it is assumed that repeater "knows" where the message should be received.

$$N_o(t) = \lambda\ 2^{t+3}+4t-7)$$

$$N_o'(t) = \lambda(2t^2+2t+1)$$

$$\mathrm{IEff}(t) = \frac{2^{t=3}+4t-7}{2t^2+2t+1} \sim \frac{2^{t+2}}{t^2}$$

This mode is better than no mode, but holds little promise by itself.

5. <u>A Repeater Never Transmits The Same Message More Than Once, Except Upon Initial Reception</u>

In this mode a repeater has "infinite" memory but <u>not</u> instantaneous memory.

$$N_o(t) = \lambda(2^{t+4}-16t-11)$$

$$N'_o(t) = \lambda(2t^2+2t+1)$$

$$IEff(t) = \frac{2^{t+4}-16t-11}{2t^2+2t+1} \sim \frac{2^{t+3}}{t^2}$$

It appears that infinite memory without instant memory does not "damp" the message flow significantly:in fact it is worse than "directionality" in the repeater.

6. <u>Infinite Memory Plus Instant Memory</u>

If we combine the two modes of operation

$$N_o(t) = \lambda(8t^2-8t-3)$$

$$N'_o(t) = \lambda(2t^2+2t+1)$$

$$IEff(t) = \frac{8t^2-8t-3}{2t^2+2t+1} \sim 4$$

The effect of combining the instant memory feature with the instantaneous memory feature is to reduce the message flow in the instantaneous mode by a factor of t/3 which is significant.

7. <u>Instant Memory Plus no Message Transmitted More Than k-Times</u>

In this mixed mode the analysis shows that all results in the instant memory case are the same except that the flow becomes independent of time after $t \geq k$. The inefficiency is

$$IEff(t) \sim \frac{4}{3}k$$

IEff	1.33	2.67	4	5.33	6.67	8
k	1	2	3	4	5	6

CHAPTER 12

TIME AND SPACE CAPTURE IN SPREAD SPECTPUM RANDOM ACCESS

I. INTRODUCTION AND SUMMARY

When a receiving station for packet data communication is being accessed in a random access mode, the use of spread spectrum communication offers the possibility of time capture. That is, a packet may be distinguished and received correctly even if contending packets overlap the transmission as long as the signal strength of the contending packets is not too great. Moreover, if multiple receivers are available at the receiving station, spread spectrum coding can be used to allow the reception of several distinct packets being transmitted with overlap on a single channel.

When the transmitters are widely distributed, geometric or power capture is possible [Roberts; 1972]. With or without spread spectrum, if a competing signal is much weaker (further away), than the desired signal, there is no interference. Both types of capture can give rise to performance superior to that predicted by a simple unslotted ALOHA model. On the other hand, power capture gives rise to bias against the more distant transmitters because the close in transmitters overpower the ones further away. Thus, the probability q(r) of a successful transmission at a distance r from the station will decrease as r increases and the number of retransmissions increase as well as delay.

The purpose of this chapter is to characterize the probability of successful transmission as a function of:

1. distance from the receiver, r,

2. the multiplicity, M, of receivers available at the receiving location,

3. the packet arrival rate,

4. the time bandwidth product K (extent of spectrum spreading),

5. the required signal to noise ratio SN of the receivers,

6. the exponent α in the inverse power law (s/r^{α}) assumed for the transmission power of a transmitter at distance r from the receiver.

The situation we consider is that of a circular field of transmitters of radius R accessing a receiving station in the center with M receivers where R is the range of the receivers. Arriving traffic is assumed to have a uniform rate per unit area in the field and the arrivals plus retransmissions are assumed to be Poisson distributed.

Particularly important for the design of a Packet Radio System is the analysis of multiple receiver stations. Adding extra receivers to the Packet Radio Station adds throughput and shortens delay. This is a very attractive approach because the modification is simple and is applied only at the station. No modifications are required to the terminals, the repeaters, or their geographical location. Finally, the enhancement is applied directly where it is needed at the station bottleneck where all the information flow of the system must eventually pass. The simulation results show that multiple receivers will in fact increase performance substantially; moreover, very few additional receivers are required to realize most of the advantage (usually only one additional is required). For addition of receivers to be effective, two prerequisites must be satisfied: (i) the traffic rate must be quite high; otherwise, the chance of several messages arriving simultaneously will be small and (ii) the spread spectrum factor K must be sufficiently large compared to the required signal to noise ratio SN; otherwise, even if there are sufficient receivers to receive a number of simultaneous messages the interference created by the messages will cause none of them to be received correctly.

The q(r) function derived here will also be very useful in the location of repeaters to provide coverage for terminals. The simplest models require that each possible terminal location be "covered" by some number of repeaters in order to guarantee reliable communication. This type of model assumes a binary characterization of transmitter-repeater communication: either communication is possible or it is not. The q(r) function could be used to reflect the fact that a far away receiver does not provide as good service as a nearby one.

In Section 2, the model and the assumptions it is based upon are introduced. In Section 3, analysis is used to predict the qualitative behavior of the q(r) function especially with respect to limiting values of parameters. It was found, for example, that to a user at far distances, $r \to R$, the system performs as in unslotted ALOHA. That is, a transmission arriving at time t_o gets through if and only if no other transmission starts in the interval $[t_o-T, t_o+T]$ where T is a packet transmission time. On the other hand, for $r \to o$ for the one receiver case performance approximates slotted ALOHA in the sense that only packets preceeding, $t_o - T \leq t \leq t_o$, can interfere. Explicit bounds on the benefits of adding multiple receivers are also given. In Section 4, simulation is used to verify and extend these results. Finally, the simulation program is documented in an Appendix.

The first analysis of the geometric effect of capture for a circular field of packet transmitters accessing a receiver in a random access mode was [Roberts; 1972]. Roberts made (among others) the following assumptions:

1. the probability q of a packet being received correctly for a given transmission or retransmission is:

(a) independent of distance r from the receivers,

(b) independent of whether the transmission is the first or a retransmission,

2. the probability of more than one contending packet is negligible,

3. noise is negligible compared to competing signal strengths.

Kleinrock and Lam [1972] improved on these results by relaxing assumption (1 b) to allow different q's for the initial transmission and for retransmissions. John Leung [1973] considered the same problem where the signal power, instead of being represented as a deterministic inverse square of distance, had a Rayleigh distribution to represent the effects of multipath. One consequence of this is that the receiver has unlimited range. Leung also essentially assumes (1) and (2) (see in particular, eq. 7, where G(r) is apparently assumed proportional to r; i.e., that the rate of arrivals plus retransmissions per unit area is independent of r.

Fralick [1972] also does an analysis of the field of terminals situation, extending Roberts' results for the use of spread slotted spectrum, including propagation delays. He assumes (1), (2), and (3).

Abramson [1973] considers a central receiver in an infinite field of transmitters and determines the "Sisyphus distance" R_s which is the radius beyond which the expected number of retransmissions is infinite. He assumes (2) and (3). Thus, the critical distance R_s is defined by competing signals while the R we use is defined by noise considerations. All the above models assumed one receiver, M=1.

The model for spread spectrum reception is similar to those described in [Kaiser; 1973, p. 2], [McGuire; 1973], and [Fralick; 1973a]. Fralick [1973a] considers several models of spread spectrum random access the one closest to ours is the one using "Synch Preamble." In particular, our lower bound for q(o) is based on his analysis of this case. Also, Fralick pointed out an error in normalization in Section 2 that appeared in an earlier version of this note. For simplicity, we assume that the surface wave devices for encoding the spread spectrum code are programable and that the chip code varies from bit to bit in a pseudo-random manner so that competing signals appear more nearly like noise; this, for example, avoids many of the difficulties pointed out by Fralick [1973]. Programmable surface wave devices are discussed in [Staples and Claiborne; 1973] and [LaRosa; 1973].

2. MODEL AND ASSUMPTIONS

A central receiving station is receiving messages from an infinite number of terminals within a radius R where R is defined to be the range of the station; that is, the distance at which a terminal can just be heard. The terminals send packets according to a Poisson distribution with arrival rate density ρ packets per unit time per unit area. The principal objective is to determine the grade of service for terminals as a function of distance from the station. The grade of service q(r) is defined to be the probability that a message sent by a terminal at radius r from the station will be received by the station on any one transmission (first and subsequent retransmissions are assumed to have the same probability of success [Kleinrock & Lam; 1972].)

Having q(r) we can also define G(r) = S(r)/q(r) to be the arrival plus retransmission rate density at radius r where S(r) is the arrival rate density at radius r given by $2\pi r\rho$.

For our purposes, the receiving station can be in one of M+1 states; o receivers busy, 1 receiver busy, ..., M receivers busy. We will denote these states as S_o, S_1, ..., S_M. In state $S_i (i<M)$, i receivers are busy and one of M-i remaining is awaiting a synchronization code from the beginning of some packet. If the receiver achieves synchronization with a new packet, the receiver is captured ($S_i \rightarrow S_i+1$) and is now busy (in a receiving state) and remains in that state for one packet transmission time, T. The new packet is correctly received if the total power of contending signals does not become too high during the transmission time. The receiver is busy for the full time T in either case. When all the receivers are busy, state S_M, all arriving packets are lost until one of the receivers becomes free. We specifically ignore the possibility of false alarms; that is, the receiver going from free to busy on the basis of noise. Thus, a packet arriving at time t is assumed to be received correctly by the station if the

following three conditions are satisfied:

1. a receiver is free at t.

2. the signal strength is sufficiently stronger than ambient noise and other signals arriving in (T-t,t) so that the receiver achieves synchronization.

3. signals arriving in (t, t+T) are not strong enough to drown out the signal after synchronization is achieved.

In our model of the receiver, we assume that a signal can be heard if the desired signal energy is sufficiently greater than the competing energy which consists of two elements: ambient noise and the other undesired signals. We are allowed to use spectrum spreading to increase discrimination between signal and the competing energy.

To be more specific, assume we divide each bit into a coded sequence of K chips. Further, suppose the receiver works by integrating the received signal correlated with the chip code. If the integral of the signal amplitude over one chip time is A, then, if there is correlation with the chip code, the received energy in one bit is $K^2A^2 = s/r^\alpha$ for a signal at distance r. If the signal is uncorrelated with the chip code, the received signal is the integral of a sum of K binomially distributed signals of amplitude A which for K reasonably large approaches a Gaussian distribution with mean o and variance $KA^2 = s/Kr^\alpha$. Thus, received signals in synchronization with the receiver deliver after correlation a factor of K more power than competing signals of the same strength which are uncorrelated with the receiver.

If we let N be the ambient noise per bit, the signal to noise energy ratio is:

$$\frac{\dfrac{s}{r_o^{\alpha}}}{\dfrac{1}{K}\left(\sum \dfrac{s}{r_i^{\alpha}}\right) + N} \geq SN \tag{1}$$

which we require to be no less than SN for reliable reception where r_o is the radius of the desired signal which is assumed to be in synch with the receiver's chip code and r_i, $i \neq o$ correspond to competing signals not in synch which effectively look like noise added to the ambient noise N.

The range R is determined by:

$$\frac{\dfrac{s}{R^{\alpha}}}{N} = SN, \text{ or}$$

$$R = \left(\frac{1}{N}\ \frac{s}{SN}\right)^{1/\alpha} \tag{2}$$

The parameters we wish to study are SN, α, and K; so, by judicious choice of units, we may normalize the units of power and distance, so that s = 1 and R = 1. This results in the decision criterion:

$$\frac{K\dfrac{1}{r_o^{\alpha}}}{\sum \dfrac{1}{r_i^{\alpha}} + \left(\dfrac{K}{SN}\right)} \geq SN \tag{3}$$

$$0 \leq r_i \leq 1 \qquad i = 0, 1, \ldots$$

See [Kaiser; 1973] for a similar model.

3. PRELIMINARY ANALYSIS

The exact analysis of the probability, $q(r)$, of a message getting through on any given transmission from a radius r seems most difficult; however, we can get quite a good qualitative idea of $q(r)$ by analysis. This analysis will be supplemented by simulation results in the next section.

Since the signal power is monotone decreasing with increasing radius and the message arrivals are independent, it is clear that $q(r)$ is monotone non-increasing. A message being transmitted, starting at time t from the critical radius $R = 1$, will be successful if, and only if, there is no other transmission in the interval $(t-T, t+T)$ since any additional signal will prevent reception. But this is exactly the situation for unslotted ALOHA. Thus, if G is the rate of message arrivals and retransmissions for the entire circular area of R radius from the receiver:

$$q(R) = e^{-2\,GT}. \tag{4}$$

Unfortunately, we do not yet know G. Clearly from (4) and the fact that $q(r)$ is monotone $G \leqq G_u$ where G_u is the arrival plus retransmission rate for an unslotted ALOHA system; i.e., G_u is the solution of

$$S = G_u\, e^{-2G_uT}$$

where $S = \pi R^2 \rho$ is the total arrival rate. We can generalize (4) somewhat. For, even if $r < R = 1$, if r is sufficiently large, any other transmission can prevent it from being received. Thus, suppose we are considering a packet from r_o contending with a packet arriving from $r_1 = R = 1$, the weakest possible. The message from r_o can be heard whenever,

$$\frac{K \frac{1}{r_o^{\alpha}}}{1 + \frac{K}{SN}} \geqq SN$$

or

$$r_o \leqq \left(\frac{1}{1 + \frac{SN}{K}} \right)^{1/\alpha} = \bar{r} ;$$

On the other hand, if $r_o \geq \bar{r}$, any other message is sufficient to block reception. Therefore, we have,

$$q(r) = e^{-2GT} \geqq e^{-2G_u T} \qquad \text{for } \bar{r} \leqq r \leqq 1. \tag{5}$$

As $r \to o$, the signal power becomes infinite, so that the probability of successful transmission depends only on whether a receiver is free or not. For the case where M=1 and with perfect time capture, Fralick [1973a] has shown that the probability the receiver is free is $\frac{1}{1+G}$. Since there isn't perfect time capture in our model, the probability the receiver is free is greater than that. So we have:

$$g(o) \geqq \frac{1}{1+G}.$$

Given there are one or more transmissions in (t-T, t) the probability the receiver is captured is greater than the probability that one specific transmission, say the first in the interval, captures the receiver which is, in turn, greater than the probability that that specific transmission is received which is S/G on the average.

Thus,

q(o) = 1 - (Prob. of transmission in (t-T, t) times the probability receiver is captured by one of the transmissions)

$$\leq 1 - (1 - e^{-GT}) \frac{S}{G}$$

yielding

$$\frac{1}{1+G} \leq q(o) \leq 1 - (1-e^{-GT}) \frac{S}{G} \qquad (6)$$

If we add additional receivers we get improved performance for small r, but not for $\bar{r} \leq r \leq 1$. In order for additional receivers to be helpful, there must be sufficient traffic to keep them busy but not so much that the interference caused by non-synchronized messages overwhelms the desired one. We first determine the maximum number of receivers which can all be correctly receiving at the same time. This clearly happens when all signals are at the same radius and that radius approaches zero. For n receivers at radius ε with $\varepsilon \to o$, (3) has the limit

$$\frac{K}{n} \geq SN$$

Thus, we see that

$$\text{Maximum number of receivers} \leq K/SN \qquad (7)$$

For example, if the spread spectrum factor K were 100 and the required signal to noise power SN were 20, then at most, five receivers could be simultaneously receiving correctly. This indicates that the number of repeaters which can profitably be added is small.

Another factor affecting the utilization of multiple receivers is the amount of traffic in the system. Suppose the gross traffic including retransmissions is a Poisson Process with arrival rate G per second. Then, the number of messages being received at a given time t_o is simply the messages which arrived in the interval (t_o-T, T) where T is the transmission time. The probability P(k) of k active messages is then given by the Poisson distribution:

$$P(GT;k) = \frac{(GT)^k e^{-GT}}{k!} \qquad k = 0, 1, \ldots \tag{8}$$

We define

$$Q(GT;k) = \sum_{i=o}^{k} P(GT;i)$$

For an M-receiver system, a new arrival at r=o will be received if less than M messages arrive in the previous T seconds (although sometimes a receiver will be free even if M or more arrived in the interval); thus,

$$q(o) \geqq Q(GT, M-1).$$

Similarly for $K \to \infty$ we have

$$q(r) \equiv \frac{S}{G} \geqq Q(GT, M-1).$$

We do not know G but we can define $\bar{G}$ to be the solution of

$$S = G\ Q(GT, M-1)$$

yielding the bound $\frac{S}{G} \geqq \frac{S}{\bar{G}}$ in the case $K \to \infty$ and $q(o) \geqq \frac{S}{\bar{G}}$ in general.

To get an upper bound, we consider a hypothetical system with perfect capture in which messages turn themselves off if they do not capture a receiver. In other words, a message is received correctly if and only if there are less than M receivers which are in the process of correctly receiving a message. For such a system,

$$q(r) \equiv \frac{S}{G} = Q\ (ST,\ M-1)$$

where we assume the packets received correctly follow a Poisson process with arrival rate S. Since messages do not turn themselves off, our performance is worse and for real systems we have,

$$q(r) \leqq Q(ST,\ M-1) \qquad (9)$$

For M=1 we obtain,

$$q(r) \leqq e^{-ST} \qquad (10)$$

Since a finite K will only make the system behave still worse, (9) and (10) also hold for finite K.

It is important to note that almost all these results depend on the processes involved being Poisson. There are three processes of interest the arrivals to the system, the starting times of first transmissions and retransmissions, and the starting times of successful transmissions. The first and third processes have arrival rates S in equilibrium while the second process has rate G. It is reasonable to assume the first process is Poisson, and by being sufficiently clever with the retransmission scheme used, the second process can probably be made Poisson to an arbitrarily close approximation, but there is no way the third process can be Poisson. For example, the probability of M+1 successful receptions starting in an interval of length T is zero for the third process and positive for a Poisson process. The Poisson approximation has been shown to be accurate for lower traffic rates and one receiver but for the high traffic rates examined here and with multiple receivers discrepancies can be expected.

For example, safer bounds for (9) and (10) respectively might be,

$$(9') \qquad q(r) \leqq \frac{Q(ST,\ M-1)}{Q(ST,\ M)}$$

and for M=1

$$(10') \qquad q(r) \leqq \frac{1}{1 + ST}$$

In the simulations of the next section only the second process is assumed Poisson.

Now let us summarize our knowledge. In general we have:

(i) $q(r_1) = q(r_2)$ for $o \leqq r_1 \leqq r_2 \leqq 1$

(ii) $e^{-2G_u T} \leqq e^{-2GT} \leqq q(r)$ for $o \leqq r \leqq 1$ where G_u satisfies

$S = G_u e^{-G_u}$

(iii) $q(r) = e^{-2GT}$ for $\bar{r} \leq r \leq 1$ where $\bar{r} = \left(\frac{1}{1 + \frac{SN}{K}}\right)^{1/\alpha}$

(iv) $q(r) \leqq Q(ST,\ M-1)$

(v) $q(o) \geqq \frac{S}{\bar{G}}$ where $\bar{G}$ satisfies $S = GQ(GT,\ M-1)$

(vi) $q(o) = \frac{1}{1 + G}$ if $K \to \infty$ and $q(o) \geq \frac{1}{1 + G}$ for finite K.

(vii) $q(o) \leqq 1 - (1-e^{-GT}) \frac{S}{G}$

Finally, the maximum number of receivers which can be correctly receiving messages at the same time is less than or equal to K/M.

4. A SIMULATION APPROACH

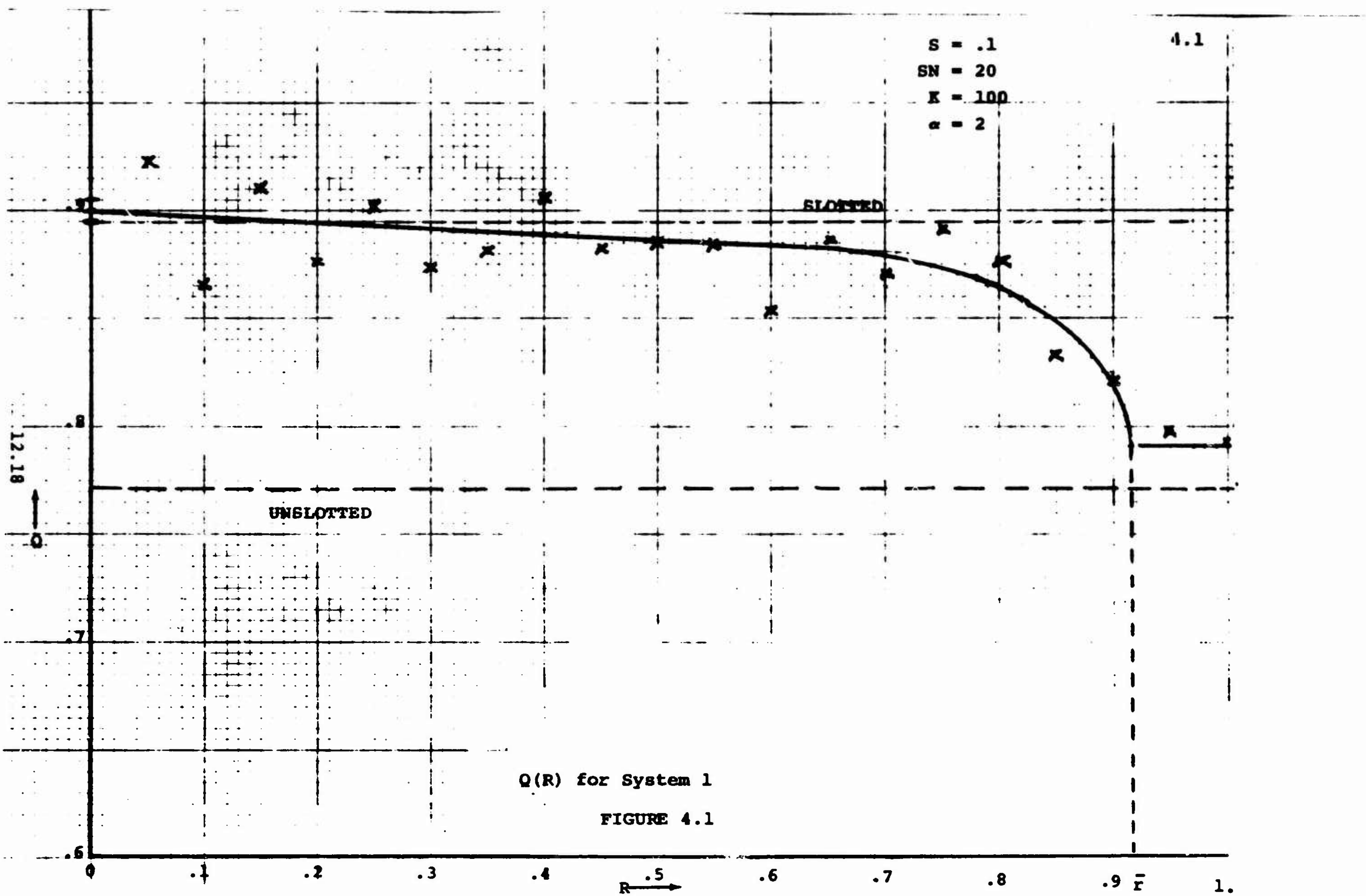
Simulation was resorted to in order to verify the relations derived in Section III and to estimate more detailed attributes of the system in particular to estimate G which is required for several of the estimates in Section III. Details of the simulation method are given in the Appendix. In general, the estimates of G were quite stable, with apparent accuracies of better than 1%. The functional form of q(r) was not as successfully simulated; however, the simulations were consistent with the approximations and bounds of Section III within sampling error.

The nominal system simulated had S = .1, SN = 20, K = 100, M = 1, and α = 2; T was everywhere taken to be 1. Table 1 summarizes the systems simulated. Figures 1 - 7 are q(r) curves for the first seven systems simulated. Figure 8 shows the change in q(r) obtained by adding a second receiver (Systems 2 and 10); changes for adding more than one receiver were negligible in all cases. Figure 9 illustrates the dependence of G on K for fixed S; while Figure 10 shows the relation between G and S for K = 100 and K = 1000.

SYSTEM NUMBER	ARRIVAL RATE S	SIGNAL TO NOISE SN	SPREAD SPECTRUM FACTOR K	POWER EXPONENT α	CRITICAL RADIUS $\bar{r}$	MEASURED RATE G	NUMBER OF RECEIVERS M
1	.1	20	100	2	.913	.1170	1
2	.2	20	100	2	.913	.2920	1
3	.1	10	100	2	.953	.1152	1
4	.1	20	10	2	.577	.1255	1
5	.1	20	1000	2	.990	.1126	1
6	.1	20	1	2	.223	.1290	1
7	.1	20	100	4	.955	.1166	1
8	.1	20	100	2	.913	.1087	2
9	.1	20	100	2	.913	.1085	3
10	.2	20	100	2	.913	.2414	2
11	.2	20	100	2	.913	.2397	3
12	.2	20	100	2	.913	.2397	5
13	.333	20	1000	2	.990	.5503	1
14	.333	20	1000	2	.990	.3718	2
15	.333	20	1000	2	.990	.3589	3

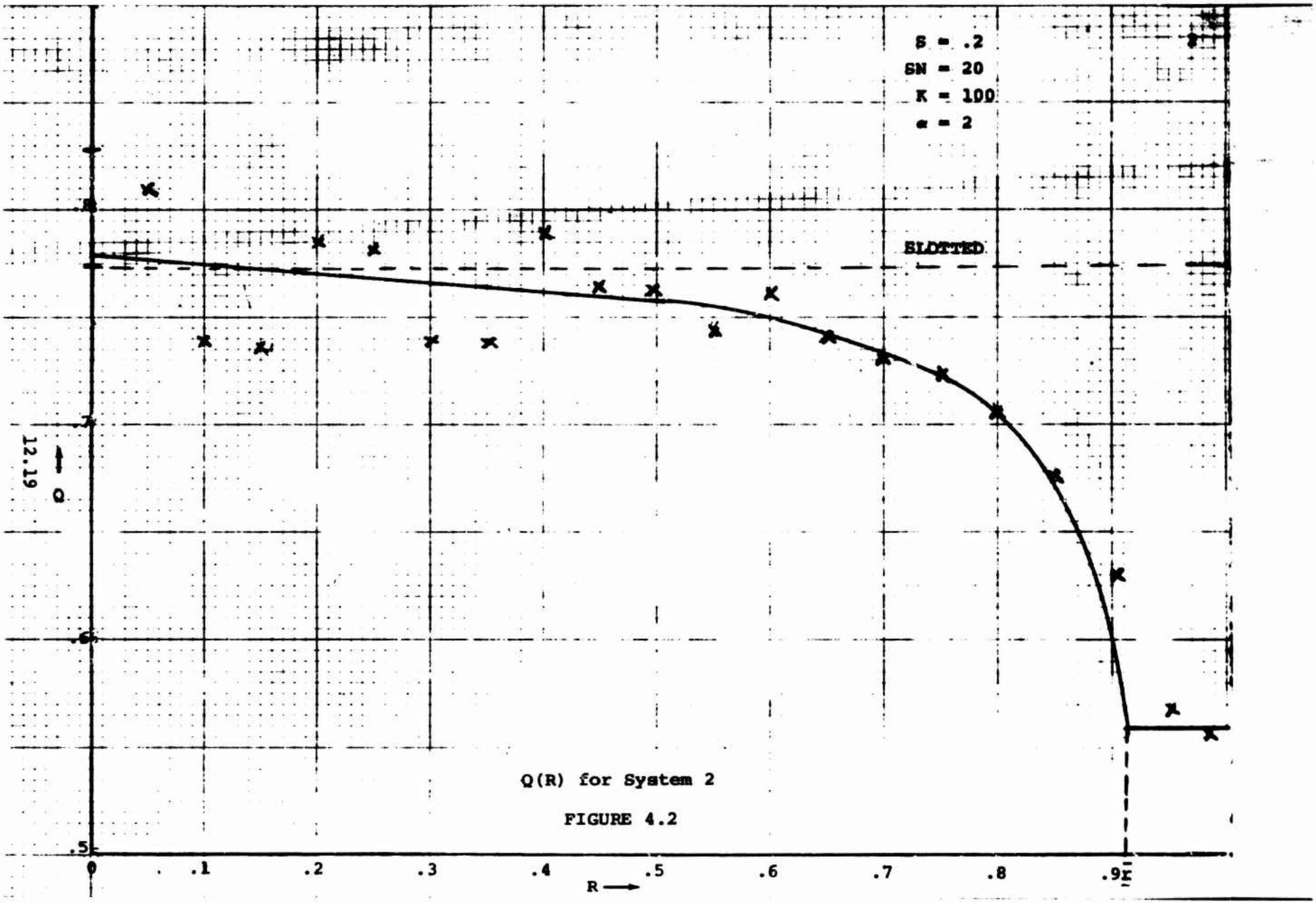
TABLE 1

Simulation Runs



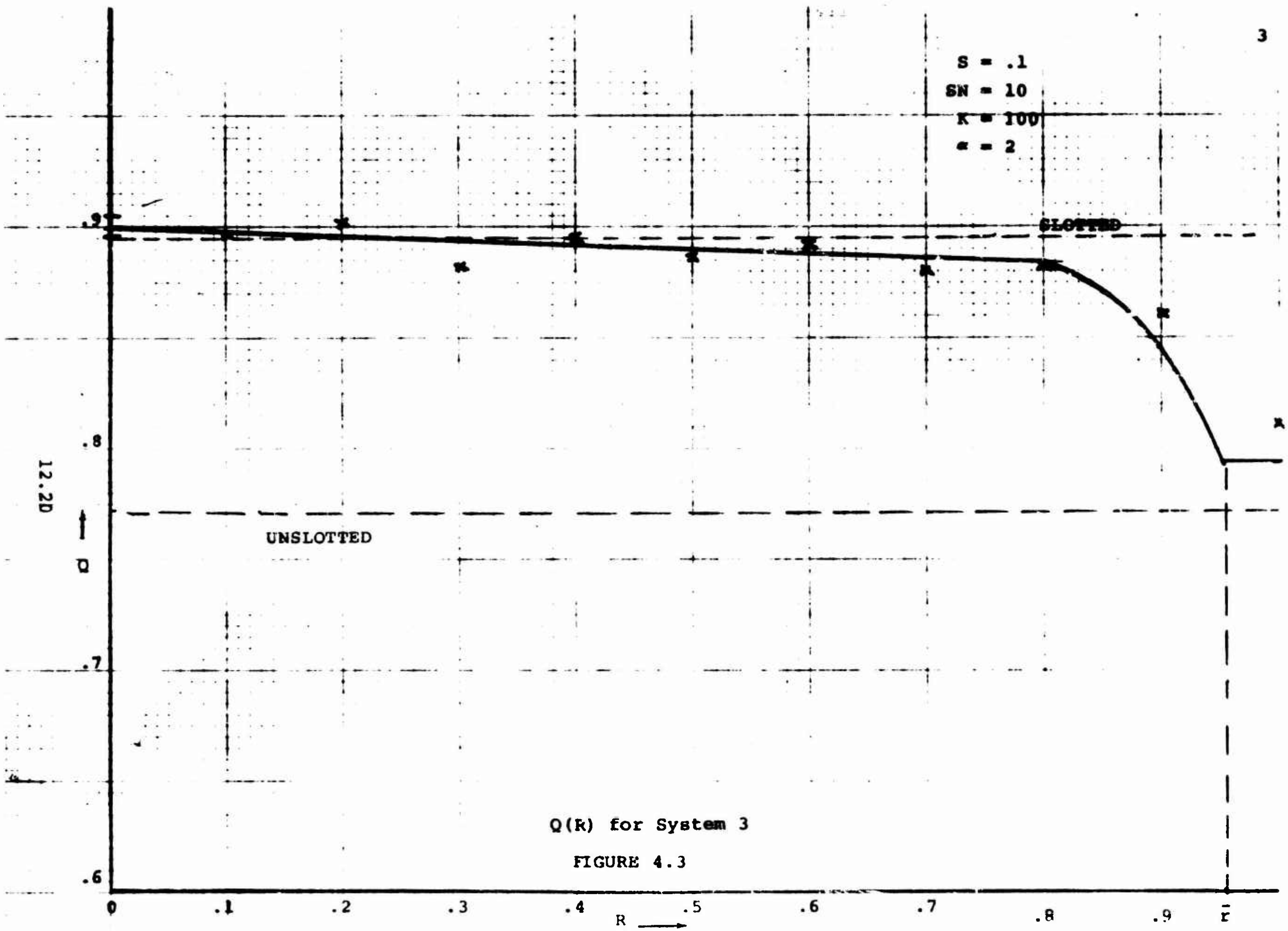
Q(R) for System 1

FIGURE 4.1



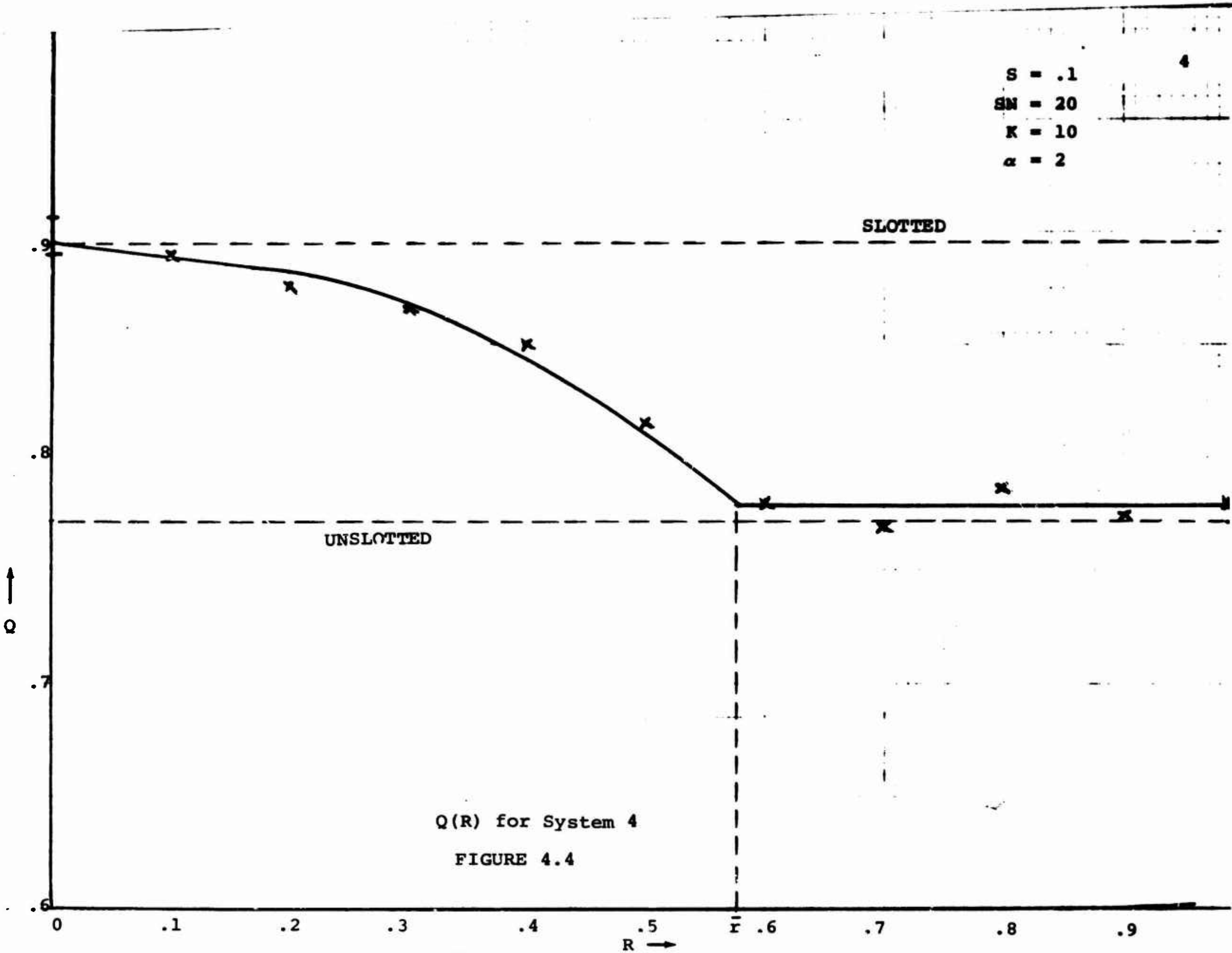
Q(R) for System 2

FIGURE 4.2



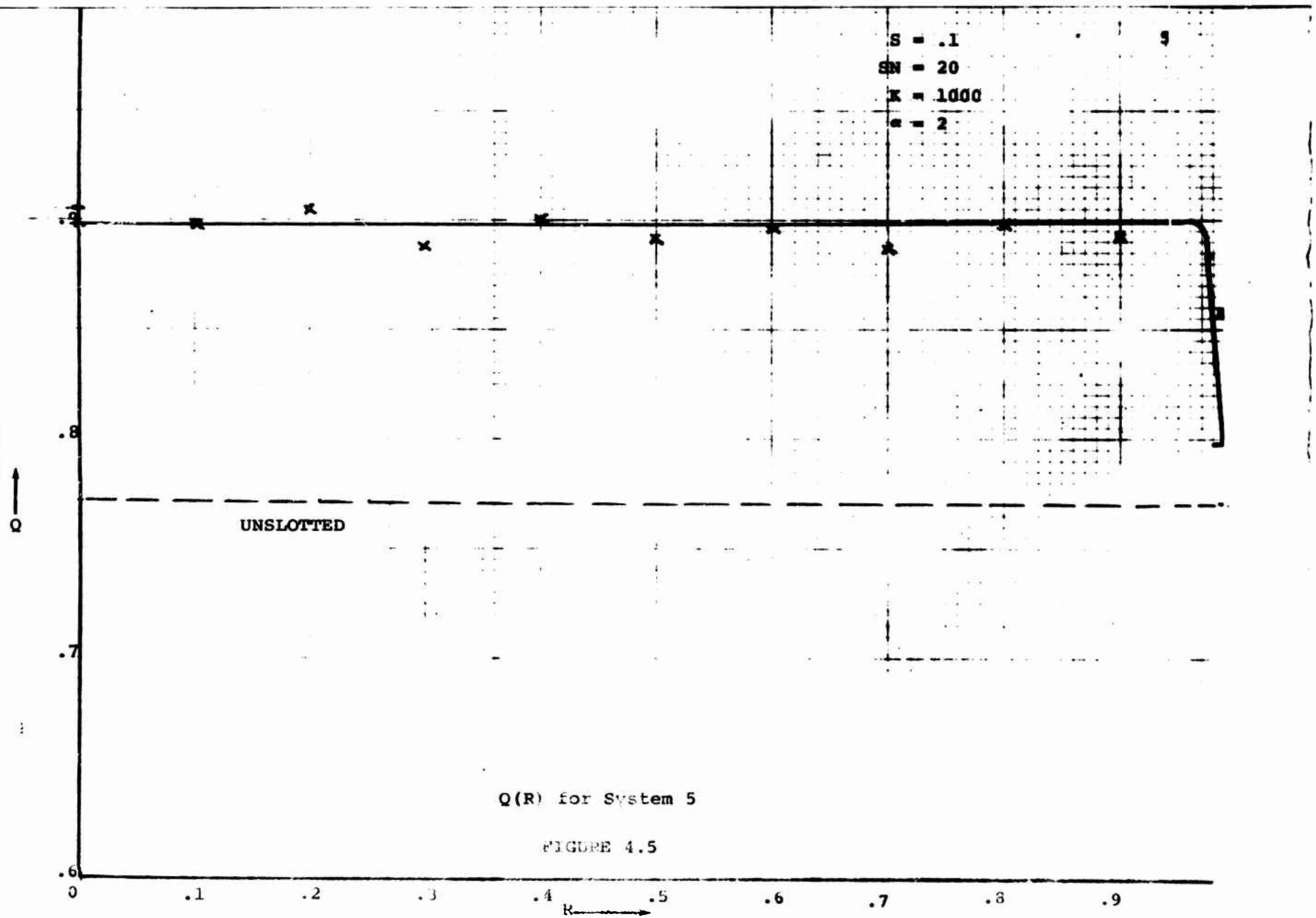
Q(R) for System 3

FIGURE 4.3



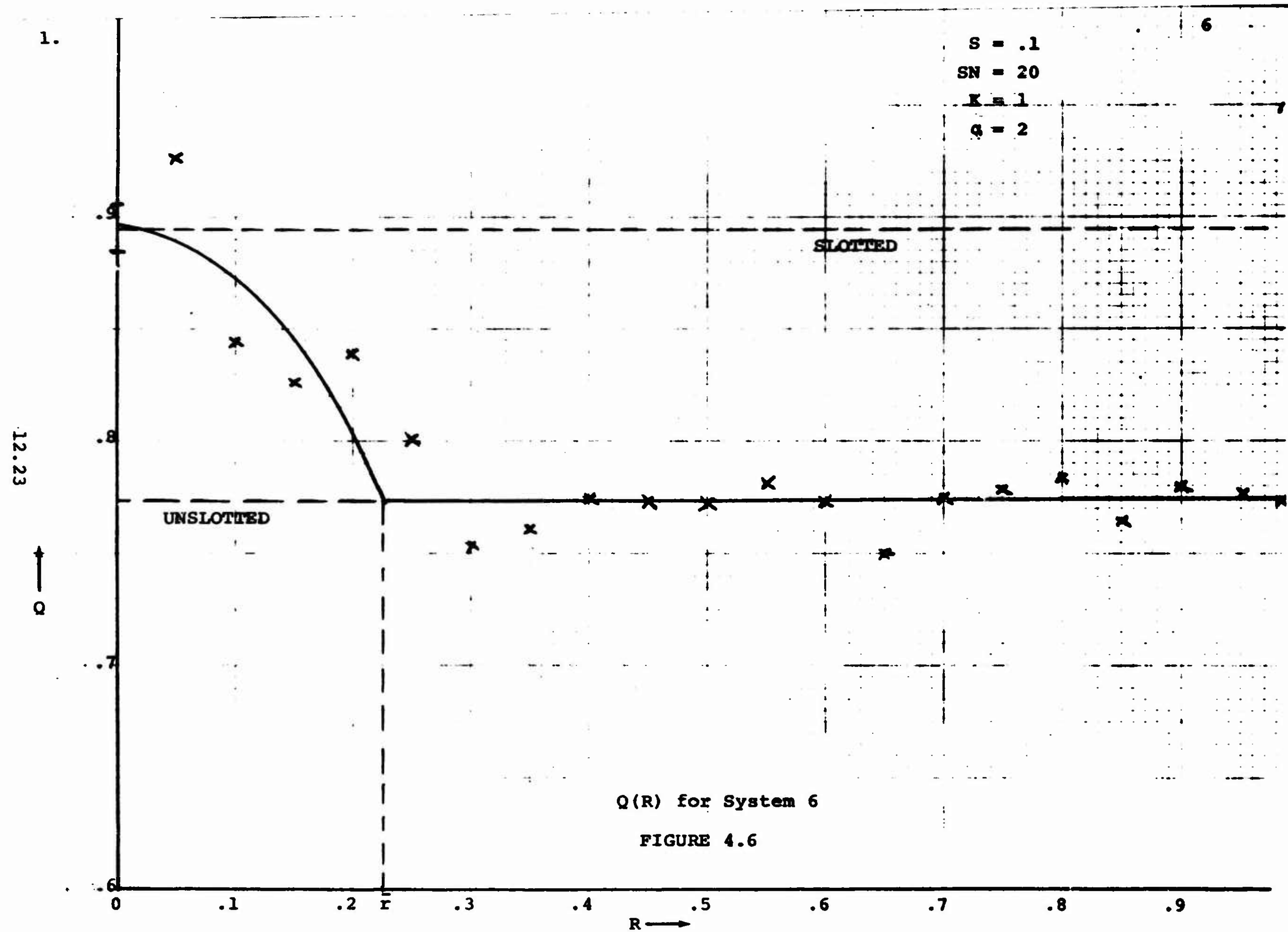
Q(R) for System 4

FIGURE 4.4



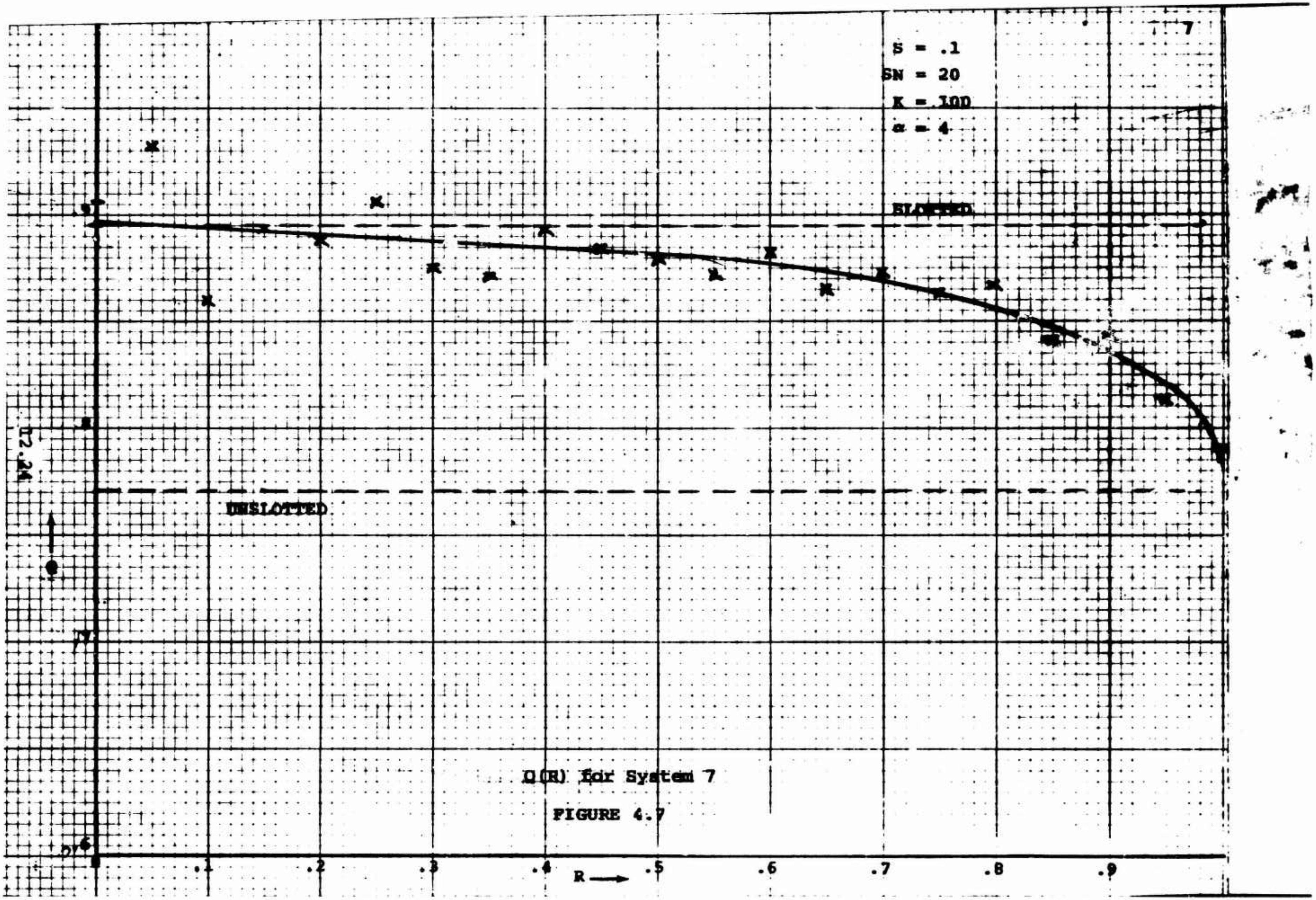
Q(R) for System 5

FIGURE 4.5



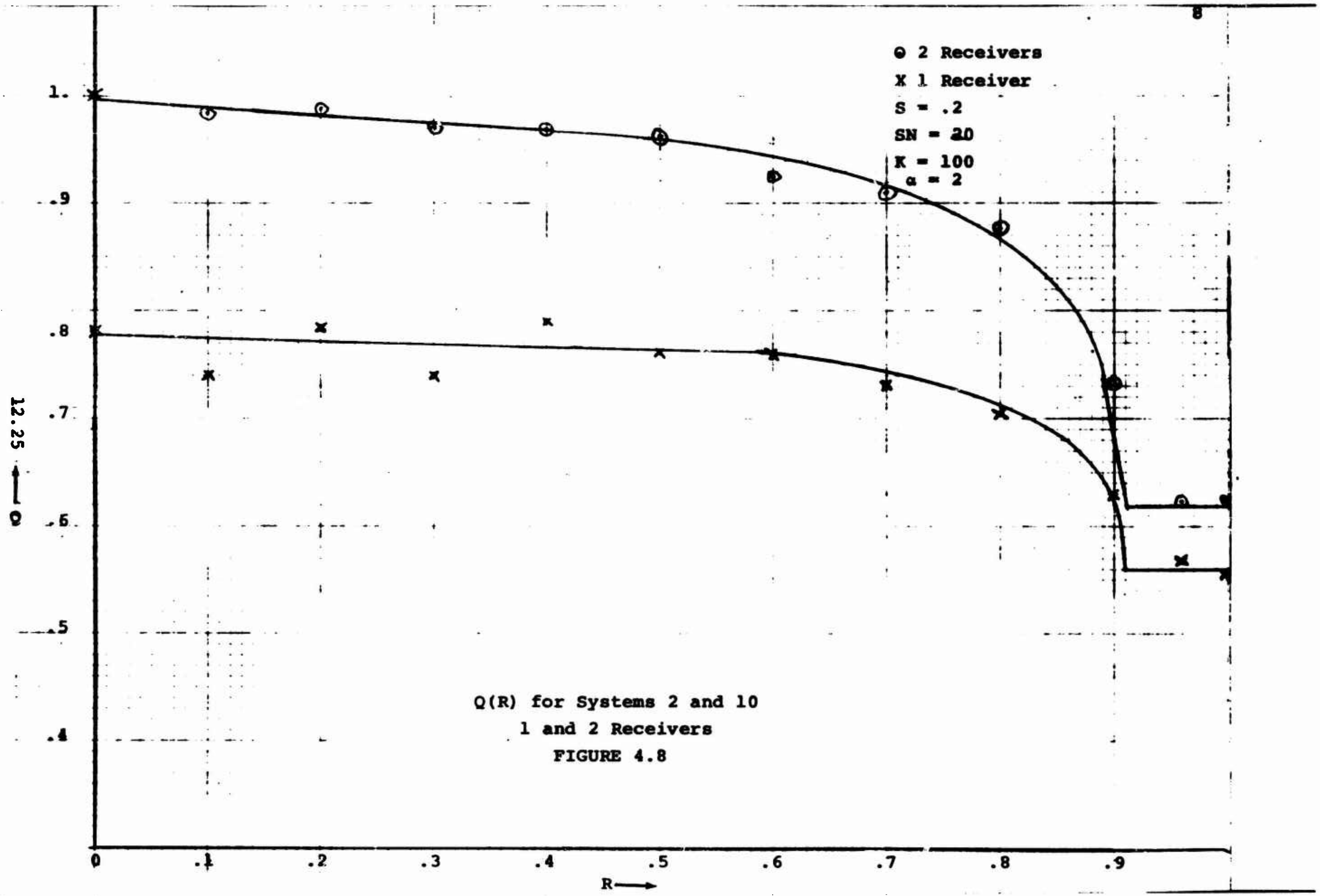
Q(R) for System 6

FIGURE 4.6



Q(R) for System 7

FIGURE 4.7



Q(R) for Systems 2 and 10
1 and 2 Receivers
FIGURE 4.8

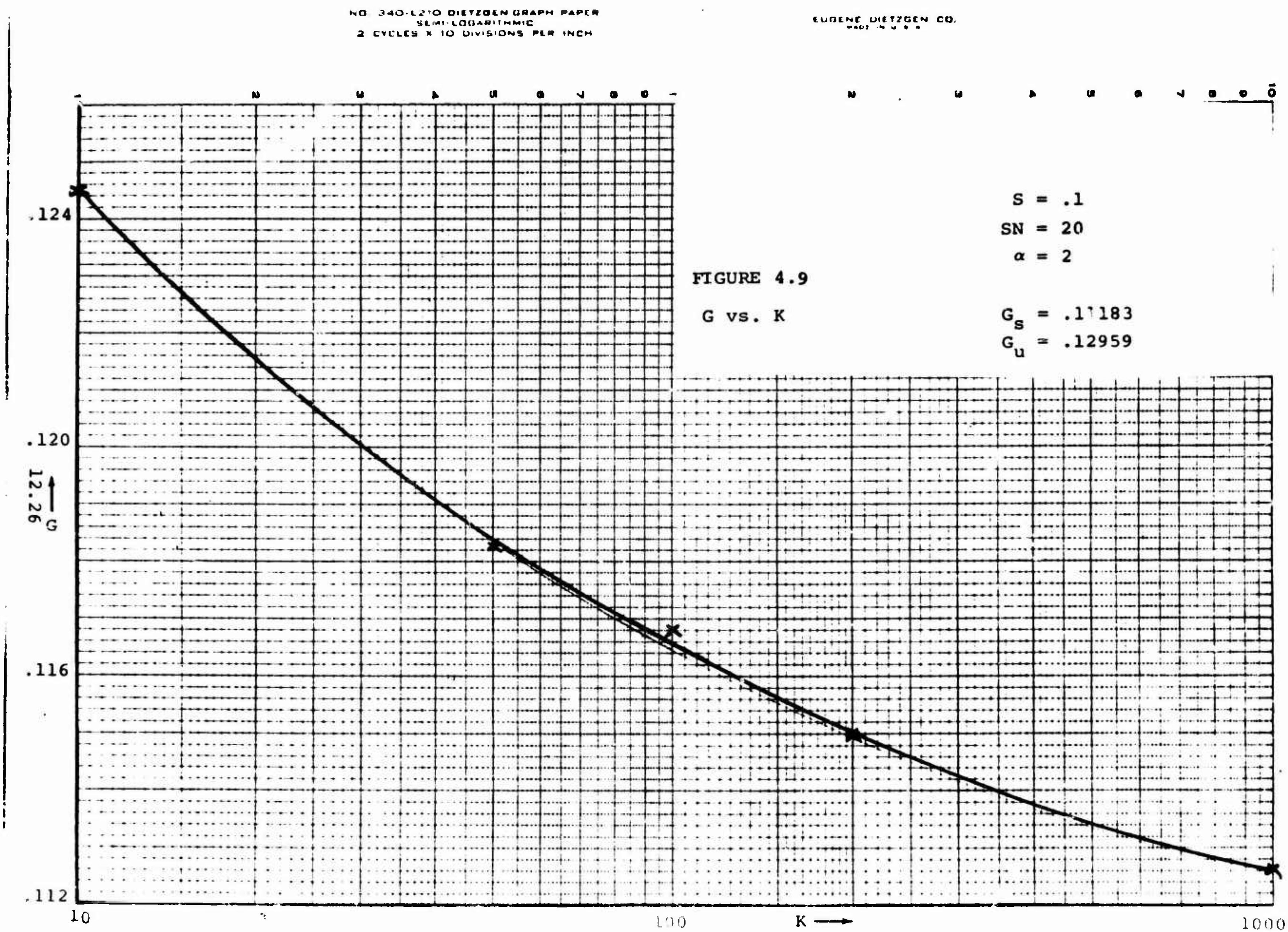


FIGURE 4.9

G vs. K

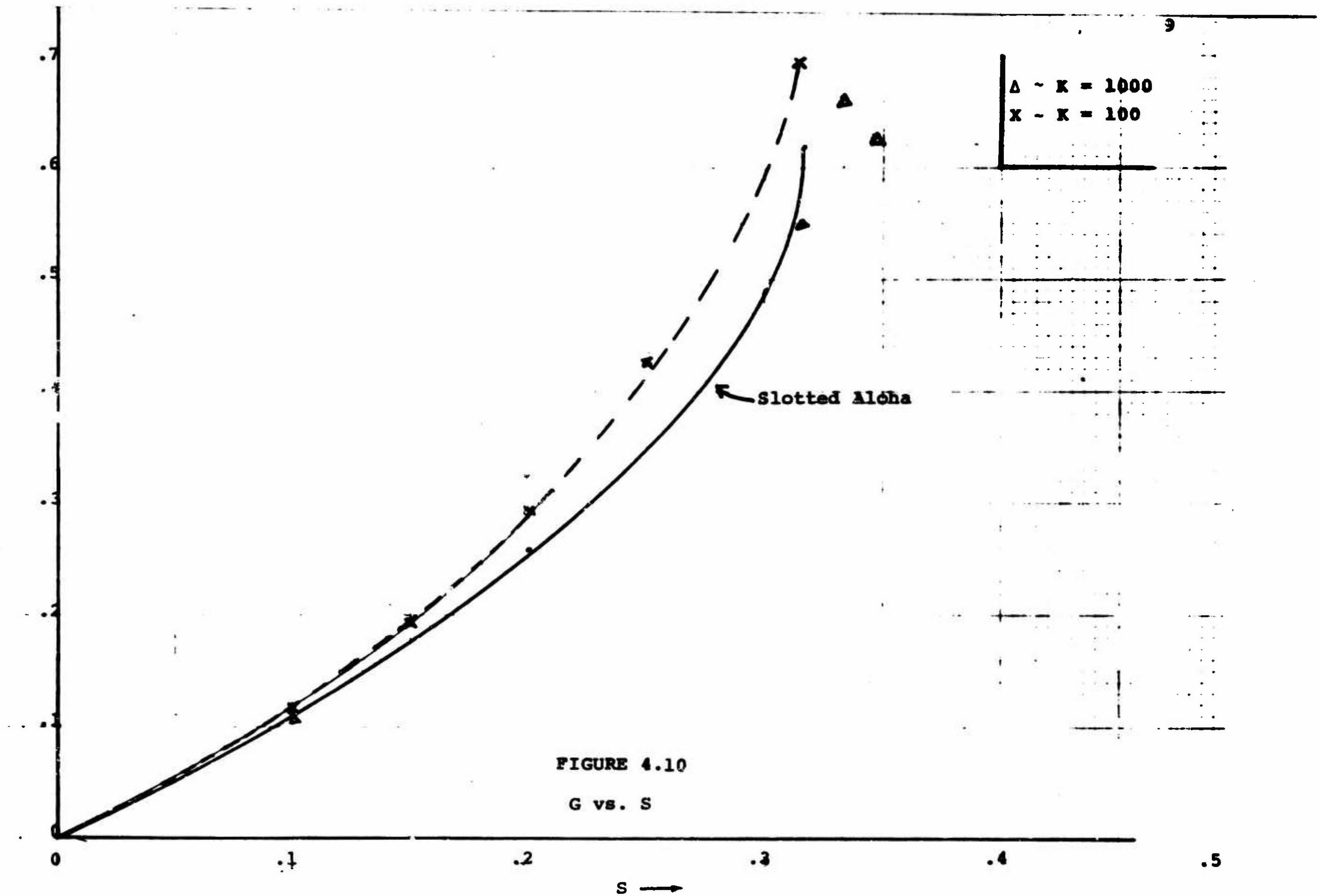


FIGURE 4.10

G vs. S

5. APPENDIX: METHOD OF SIMULATION

The radius from the station (R=o) to R=1 is divided into ND intervals. Associated with each interval, I, is a ring of width 1/ND and area 2 π I/ND. Thus, each ring is assigned a weight (proportional to radius) of arrivals in an array RDN, so that the arrival rate in the ring is RDN(I)* SIGMA; Q(I) is the current estimate of the probability a transmission from the I^{th} ring is successful. RDL(I) is defined proportional to RDN(I)/Q(I), so that the estimated arrival plus retransmission rate in the I^{th} interval is RDL(I) * GAMMA, where SIGMA~S and GAMMA~G.

Initially, we take GAMMA = SIGMA and RDL = RDN. The general step is to generate NS Poisson arrivals with arrival rate GAMMA. The transmissions are assigned radii according to the distribution RDL and the corresponding signal powers calculated in an array S. In an array ST, the instantaneous power is calculated for each time at which a transmission is initiated. Finally, in an array SMX the maximum power over the T seconds subsequent to each start of transmission is calculated.

To calculate the number of successful transmissions, the transmissions are considered in order. An indicator keeps track of how many receivers are busy. For a given transmission, the program first determines which interval from 1 to ND the transmission originates from; if it is in the I^{th} for example, then RBX(I) is bumped. Then the program makes the following tests to see if the transmission is successful:

1. Using the ST array, the program determines if the receiver can hear the beginning of the transmission. If so we go to Step 2. If not, we go on to the next transmission.

2. Is a receiver free? If one isn't, go on to the next transmission; if one is, mark the receiver busy for the next T seconds and go on to Step 3.

3. Can the entire transmission get through without being drowned out by subsequent competing transmissions? This is determined using the array SMX. If this test is passed, the transmission is assumed successful and a counter SBX(I) is bumped.

After all the transmissions are processed, and improved estimate for Q(I) given by

$$Q(I) = SBX(I)/RBX(I) \tag{A.1}$$

is obtained and this is repeated several times until the distribution settles down. It turns out that this process was very unstable, at least in the estimate for Q(I). The estimates for GAMMA were quite easy to obtain and reliable, but there were rather wide fluctuations in the Q(I) between iterations or when using different seeds for the random number generator. The reason is apparently because the power for originations near the origin become unboundedly large and hence have disproportionate influence, while the number of such events is quite small so the resulting variance is rather large. This was ameliorated to some extent by using exponential smoothing; that is, replacing (A.1) with

$Q(I) := RLX * Q(I) + (1-RLX)\ SBX(I)/RBX(I)$ where $0 \le RLX < 1$.

CHAPTER 13

PACKET DATA COMMUNICATIONS ON MATV AND CATV SYSTEMS: A FEASIBILITY STUDY

1. INTRODUCTION

In 1970, the ARPA network was still written about in terms of goals to be reached.

> "For many years, small groups of computers have been interconnected in various ways. Only recently, however, has the interaction of computers and communications become an important topic in its own right. In 1968, after considerable preliminary investigation and discussion, the Advanced Research Projects Agency of the Department of Defense (ARPA) embarked on the implementation of a new kind of nationwide computer interconnection known as the ARPA Network. This network will initially interconnect many dissimilar computers at ten ARPA-supported research centers with 50-kilobit common-carrier circuits. The network may be extended to include many other locations and circuits of higher bandwidth.
>
> The primary goal of the ARPA project is to permit persons and programs at one research center to access data and use interactively programs that exist and run in other computers of the network. This goal may represent a major step down the path taken by computer time-sharing, in the sense that the computer resources of the various research centers are thus pooled and directly accessible to the entire community of network participants." [F.E. Heart, et.al., 1970]

Now these goals have been completely achieved with even further developments largely solidified. For example, TIPS (Terminal Interface Processors) to connect terminals to the ARPANET are a reality as are VDH (Very Distant Host) connections. In fact, in 1972 the result of the ARPA network development led to the conclusive statement that its performance is superior to other existing methods. Extensions to personal hand held radio terminals, are in the offing using a random access mode as begun in the University of Hawaii. The merits of the ALOHA packet mode have been thoroughly investigated with a conclusion:

> "...packet technology is far superior to circuit technology, even on the simplest radio transmission level, so long as the ratio of peak bandwidth to average bandwidth is large. Most likely, the only feasible way to design a useful and economically attractive personal terminal is through some type of packet communication technology. Otherwise one is restricted to uselessly small numbers of terminals on one channel. This result may also apply to many other important developments, only to be discovered as the technology of packet communication is further developed." [Roberts, 72]

Both as an adjunct to the interactive packet radio mode and as an extension of the ARPANET, this report describes the use of MATV and CATV coaxial cable systems for local distribution of packet data.

We first explain the need for a local distribution medium such as an MATV and CATV System, to augment a packet radio system in urban and suburban areas. We next investigate the properties of the coaxial cable systems in order to evaluate their data handling capability. To demonstrate the validity of our conclusions, actual designs are discussed for the existing CATV system in the Metropolitan Boston area. Specifications are given for all required digital equipment, and finally, test procedures are given. The overall conclusion of the study is that MATV and CATV interactive packet systems would form an excellent local distribution medium.

2. THE IN-BUILDING PROBLEM

In urban areas two difficulties impede the reception of radio signals in buildings:

1. The attenuation of signals in passing through building walls; and
2. The reception of multipath signals due to reflections off buildings.

In a study of the in-building reception problem, C. H. Vandament of Collins Radio concludes,

> "...signal strength environment on a city street will probably need to be 25 to 30 db higher than that previously considered if direct radiation into buildings is to be considered (i.e. no building distribution system employing amplification). This is quite unattractive since this implies either excessive transmit power and/or quite close repeater spacing." [Vandament, 1973]

Both of these problems can be avoided while still retaining the main idea of using ALOHA random access multiplexing. Instead of operating in an over-the-air broadcast transmission mode, data can be sent over the existing wideband coaxial cable facilities of master antenna (MATV) and cable television (CATV) systems. Recent FCC rulings require that all new CATV systems have two way capability. Penetrations of 40-60% of U.S. homes is projected by the end of the decade [Sloan, 1971]. Moreover, most of the major new office buildings will have wide-band communication channels built in. For example, in the New York World Trade Center, there is a system of switched wideband communication channels. The user can select 60KHz audio channels or 15 MHz video channels and has an individual wideband cable connection to the central switch [Friedlander, 1972]. By 1980, "the wired city" will be close to reality.

Vandament, after considering radiation, telephone wires, power cables, and other special wiring schemes for the in-building problem, comes to a similar conclusion regarding the merits of coaxial cable transmission:

> "...Every building will require some analysis to determine which technique is required to deliver a useable signal to a terminal inside.

> If the building has windows and is relatively close to a system repeater, no special techniques will be required. At greater distances, a simple repeater with directional antennas focused on specific buildings will deliver the signals to interior users by radiation. Buildings which are effectively shielded must have a simple, dedicated repeater to receive a signal and pipe it into the building over conductors of one type or another...High grade coaxial cables solve that problem nicely, but this answer would probably be prohibitively expensive for the packet radio scenario of general distribution throughout every important building in the U.S. ...Where such cables exist, they do offer an attractive solution to the in-building distribution problem."
> [Vandament, 1973]

In the next five sections, we will show the technical merits of MATV systems for solution of the in-building problem and CATV systems for local distribution in high density suburban and urban areas. To do this, we first describe the properties of typical modem MATV and CATV systems in Sections 3 and 4 respectively.

3. A TYPICAL MATV SYSTEM

A master antenna television system as shown in Figure 3.1 serves a concentration of television sets such as in an apartment building, hotel, or motel. The main purpose of the MATV system, as

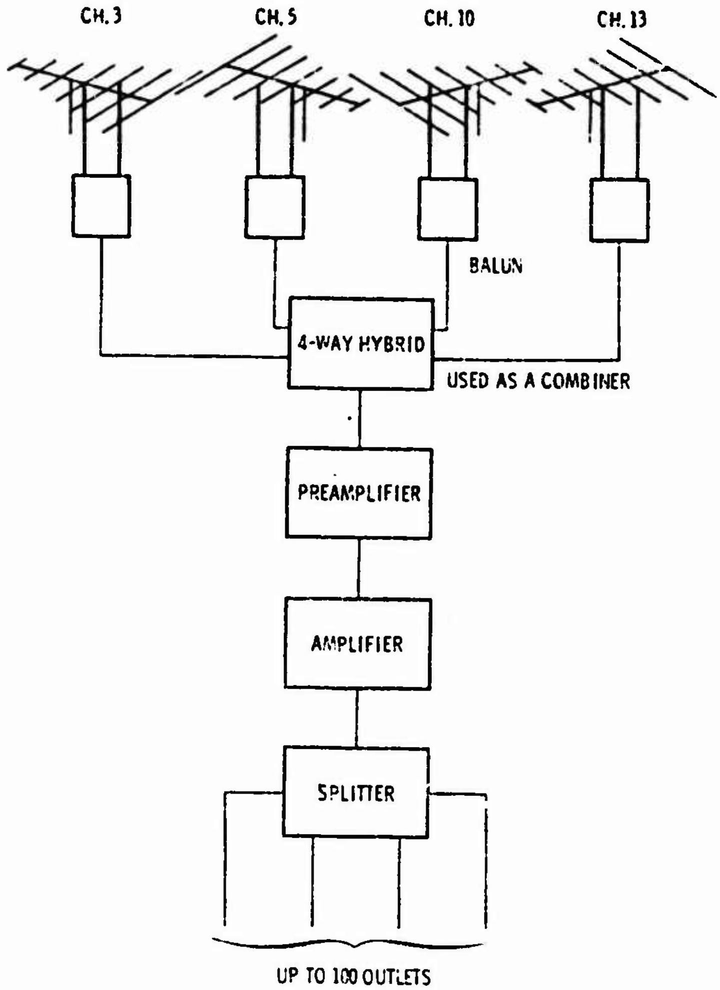


Figure 3.1 Typical Hotel or Apartment Building MATV System

shown in Figures 3.2, 3.3, 3.4, is to provide a usable signal to a large number of television sets fed by a local distribution network.

A number of television sets connected to the same antenna system without a signal amplifier would not provide any of the sets with a strong enough signal to produce good pictures. An all-channel

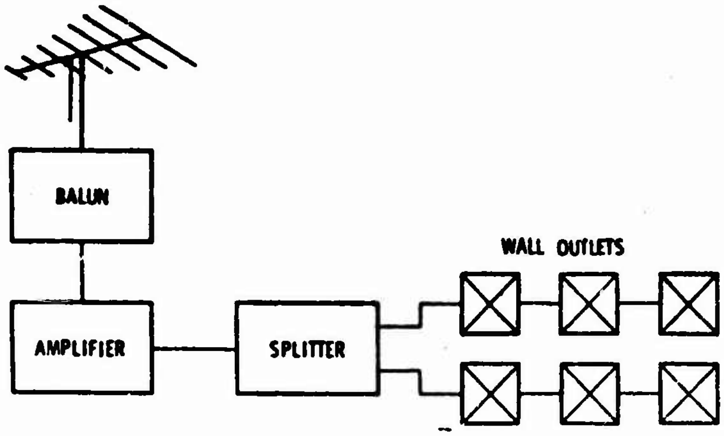


Figure 3.2 Typical Motel MATV System

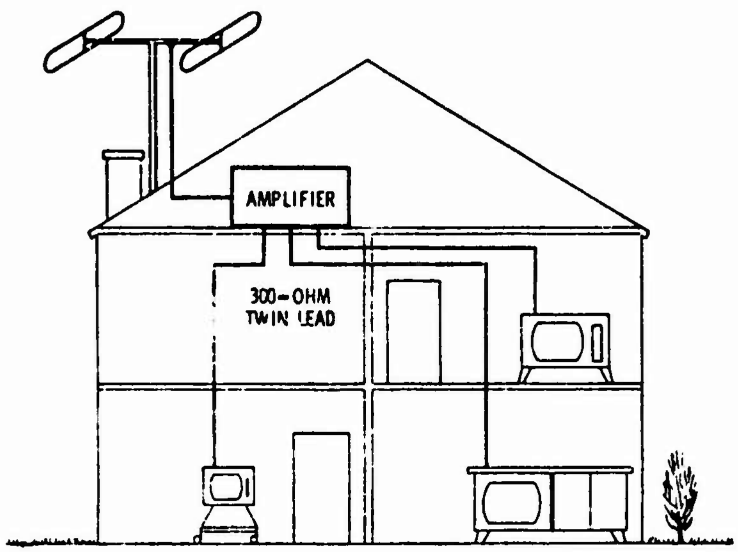


Figure 3.3 Typical Home MATV System

master antenna amplifier is connected to one antenna (sometimes more) which provides across-the-band amplification of all television signals in the VHF band and the f-m broadcast band.

Some MATV systems employ more than one amplifier. A separate single-channel amplifier may be used as shown in Figure 3.5, to provide greater amplification of a single channel. Generally, a separate antenna is used with a single-channel amplifier.

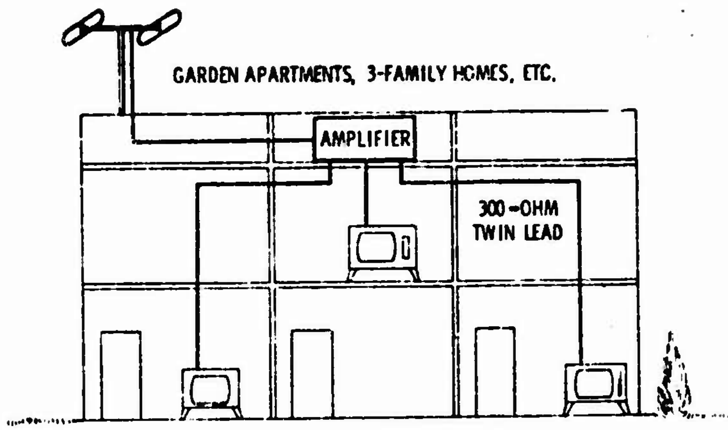


Figure 3.4 MATV System for Multiple Dwelling

For reception of UHF television stations, a UHF-to-VHF translator is required.

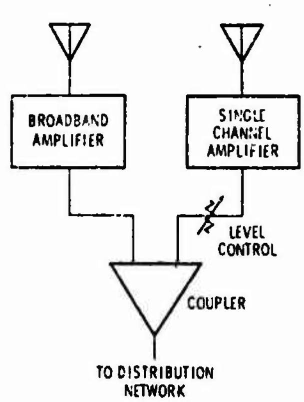


Figure 3.5

Use of Single Channel and Broadband Amplifiers for Receiving Distant Stations

For MATV systems with more than 100 outlets, further amplification may be required, and in a large office building cascades of two or three amplifiers might be expected. Nevertheless, compared to CATV systems - to be described in the next section - modern MATV systems are relatively uncomplicated media for data transmission. Signal to noise ratios of better than 43 db are reasonable; there are few environmental problems, since the system is indoors; there is minimal temperature variation; and amplifier cascades are low since the wide band capabilities of 0.5 inch coaxial cable are used. In a CATV system, the signal at the building is received from a cable distribution system rather than from antennas. However, the in-building part of the system is essentially unchanged from that used for an MATV system.

4. A TYPICAL CATV SYSTEM

CATV systems are an historical outgrowth of MATV and community antenna television systems. CATV systems perform roughly the same function for an entire town that a MATV system performs for a building, namely the distribution of TV singals to many terminals from a central reception area. Although the basic engineering strategies are the same for MATV and CATV systems, the CATV system is different in one crucial aspect; since it is larger, as many as thirty amplifiers may have to be cascaded to deliver a TV signal to the most distant terminal in the system. Therefore, the system design requirements are stringent; very high quality amplifiers and off the air reception equipment are essential. In order to motivate our proposal of data options and techniques for CATV systems, a detailed description of these systems is in order.

CATV systems in the U.S.A. are almost universally tree structured networks of coaxial cables installed for the distribution of broadcast type television signals from a central receiving station, called the "head end," to home type television receivers. Different television signals, which may be received at a central site or relayed over long distances by microwave systems, are processed at the head end and frequency division multiplexed onto coaxial cable for distribution. Coaxial cables now in use are universally 75 ohm inpedance types, usually of seamless aluminum sheathed construction, foam polyethylene dielectric, and solid copper or copper clad aluminum center conductor. The coaxial cables range in size from 0.75 inch outer diameter for "main trunk cables," through 0.5 inch size down to a 0.412 inch size for "local distribution." The service drop lines to the houses are usually flexible cables of about 0.25 inch diameter. The useful frequency range includes the VHF television band, 54-216 MHz, and broadband transistorized amplifiers are installed with equalizers to compensate for cable losses. Practical systems are aligned to be unity gain networks with amplifiers spaced about 20 db at the highest transmitted frequency.

Cable losses range from about 1 db/100' at 220 MHz for the 0.75 inch size cable to about 5 db/100' for the flexible service drop cables. Power division at multiple cable junctions and taps into subscribers' homes are accomplished through hybrid and directional couplers. All system components are carefully matched to 75 ohms to minimize internal signal reflections within the system.

System amplifiers are subject to rigorous linearity specifications. Amplifier overloads manifest themselves as cross-modulations between channels and as undesired second and third order intermodulation and harmonic products. Amplifier operating levels are bounded on the lower side by system signal to noise ratio objectives which are about 40 db over a 4 MHz band for reasonable acceptable performance as a television distribution system. A typical system will have amplifier inputs at about +10 dbmv and outputs at about +30 dbmv. System operating levels are controlled by automatic gain control circuits driven by pilot carriers and thermal compensation devices.

More recent cable systems have been built using a "hub" principal in which tree structured networks originate from a number of "hubs" throughout the community serviced (see Figure 4.1). The "hubs" may contain equipment for more elaborate control of signal levels and it may be possible to perform some special switching functions such as the interconnection of sub-trunks for special purposes.

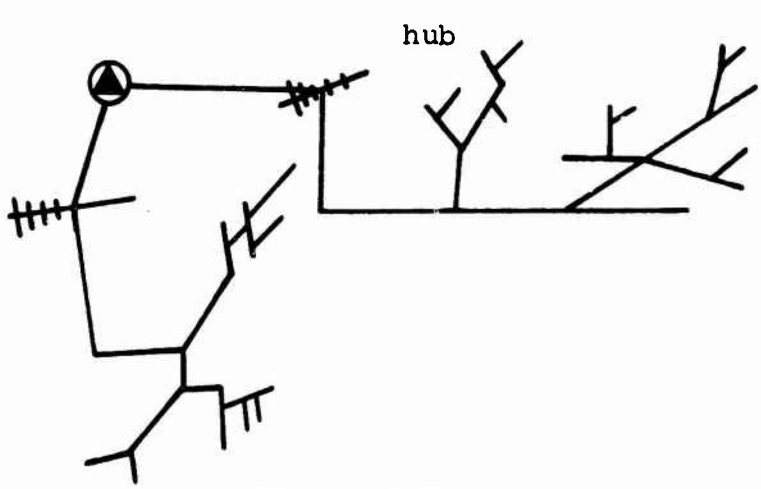


Figure 4.1 Hub System

Two-Way CATV Configurations:

FCC regulations now require that new CATV systems must have two-way capability. Practically speaking, this does not mean that all new systems are two-way systems, but rather that amplifier units are installed with forward amplifier modules in place and with distances between amplifiers constrained so that at some future date reverse amplifier modules can be installed for two-way operation. However, a number of actual fully two-way systems are presently being built and the number is increasing rapidly. Most present two-way systems use the configuration in Figure 4.2. Filters at each end of the station separate low (L) and high (H) frequencies and direct them to amplifiers usually referred to as "downstream" from the head end and "upstream" toward the head end.

A number of possible "two-way" configurations are shown in Figures 4.2, 4.3, and 4.4 [Jerrold, 1971]. The final choice between single cable/two-way, multi-cable two-way, and multi-cable without two-way filters will probably be made on the basis of marketing opportunities for special services.

There are no government regulations as to minimum specifications for a system. Hence, we will base our discussion on the characteristics of a representative two-way system, the Boston complex. This system is being built in about 10 stages, one of which is already installed and the final phase of which is to be completed within a year. At its completion, Boston will be one of the largest systems in existence in the U.S. The Boston system uses the "feederbacker" configuration shown in Figure 4.4 with the frequencies assigned to the upstream and downstream paths specifically indicated.

Data transmission on CATV systems will generally have to be fitted into space not being used to TV channels or for pilot frequencies. Hence, it is best to first describe the frequency allocation for video signals. TV channels are allocated six Megahertz

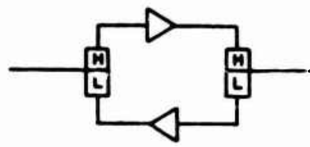


FIGURE 4.2

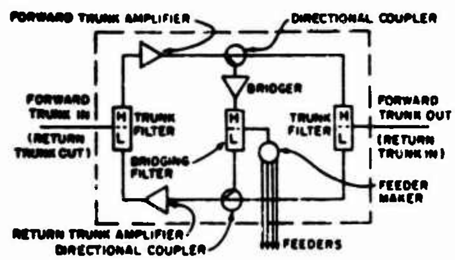


FIGURE 4.3 Two-way CATV Repeater (with feeders).

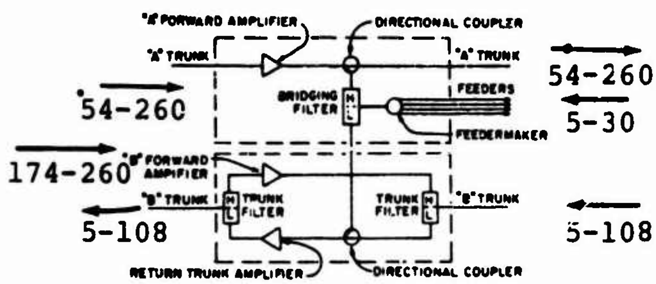


FIGURE 4.4 Dual Trunk/Single Feeder Station (Boston Configuration)

bandwiths. Broadcasted TV channels are in the Lo-VHF range 54 MHz-88MHz, the Hi-VHF range 174 MHz-216 MHz, and the UHF range 470 MHz-890MHz. The TV frequency allocations on cable are different. They partition the 54-300 MHzspectrum as follows:

Sub-VHF	5-54 MHz
Lo-VHF	54-88 MHz
Mid-band	88-174 MHz
Hi-VHF	174-216 MHz
Super-band	216-300 MHz

There is still discussion going on as to whether the mid-band should be used within a cable system because of danger of interference to aircraft navigation in case of signal leakage out of the cable. Data might be squeezed into bands not used by TV signals. Some space is available below Channel 2 at 48-54 MHz. The space between Channel 4 and Channel 5 (72-76 Mhz) might be used for low level data signals. High signal levels in this area could cause harmful picture interference on some TV sets.

A more likely situation is that two 6 MHz video channels - one upstream and one downstream - would be set aside for data transmission. The simple fact is that 21 to 30 channels are available on a single cable system and 42 to 60 channels on a dual cable system. These channels have not all yet been pre-empted by video transmission.

5. DATA OPTIONS ON MATV AND CATV SYSTEMS

Based on the details of MATV and CATV systems, we can demonstrate the merits of in-building and local distribution on MATV and CATV systems and describe detailed data options for experimentation and practical implementation.

Most two-way systems are being developed with an upstream channel designed to permit input from virtually any location in the network. The result is a large number of noise sources being fed upstream toward a common source. Individual cable television amplifiers usually have a noise figure of about 10db for a 6 MHz channel. Cascading amplifiers can increase effective system noise figure by 30db or more. Nevertheless, we shall see that system specifications on signal-to-noise ratio for CATV systems are stringent enough so that packets can be sent with existing analog repeaters, and no digital repeaters, such that bit rate error probabilities are negligible.

For example, in Boston the worst signal-to-thermal noise ratio is limited to 43db and the worst cross-modulation to signal ratio is limited to -47db. System operators may want to limit data channel carriers to a level of 10 to 20db below TV operating levels in order to minimize additional loading due to the data channel carriers [Switzer, 1972]. The cable operator, at least for the time being, is making his living by providing a maximum number of downstream television channels and will accept data channels only on a non-interfering basis.

Accepting these restrictions, in the worst case, we would be limited to 23db signal to thermal noise ratio and -27db cross-modulation to signal ratio. Let us consider both of these sets or restrictions to determine the resulting CATV system performance for random access packet transmission.

From the calculations in Appendix A, we have the error rates shown in Table 5.1. The calculations are performed for a FSK system with incoherent detection to determine a lower bound for system performance.

TABLE 5.1
ERROR RATES FOR FSK

System Label	Type of Specification	(N_c/S)	(S/N_r)	m	Pe
A	Boston Specs.	-47db	43db	2	$1/2\, e^{-5,148} \approx 1/2 \times 10^{-2,239}$
B	Boston Specs.	-47db	43db	4	$3/4\, e^{-571} \approx 3/4 \times 10^{-248}$
C	Boston Specs.	-47db	43db	8	$7/8\, e^{-104} \approx 7/8 \times 10^{-45}$
D	Boston Specs. - degraded by 20db	-27db	23db	2	$1/2\, e^{-51} \approx .5 \times 10^{-22}$

It is well known that for effecti signal-to-noise ratios above 20db there is a threshold effect for error probabilities. This is born out by the negligible error rates in Table 5.1. Even for the degraded specifications the error rate is low enough for the most stringent practical data requirements.

At a rate of 10^6 pulses/second the FSK signal will occupy the 6MHz bandwidth [Switzer, 1972] with negligible intermodulation into TV channels [Schwartz et al., 1966].

In Figure 5.1 we plot the maximum number of active terminals for the systems in Table 5.1 as calculated in Appendix B. The curves labeled A,B,C, and D correspond to the slotted systems in Table 5.1 labe B,C, and D. The lines labeled A',B',C', and D', are for t sponding unslotted systems.

We can now ine Figure 5.1 to determine system performance under some typical data transmission requirements. For a data rate of 40 bits/second per terminals, a single trunk can handle 900 terminals with a slotted ALOHA system (1 Megabit/sec) with an error rate of 10^{-22} at a signal to noise ratio degraded by 20db. The average number of TV sets per trunk in Boston is approximately 27,000. Hence, the simplest modulation scheme will handle one third of all terminals in Boston as active terminals. At 100Kbits/second, the system will handle 900 active terminals.

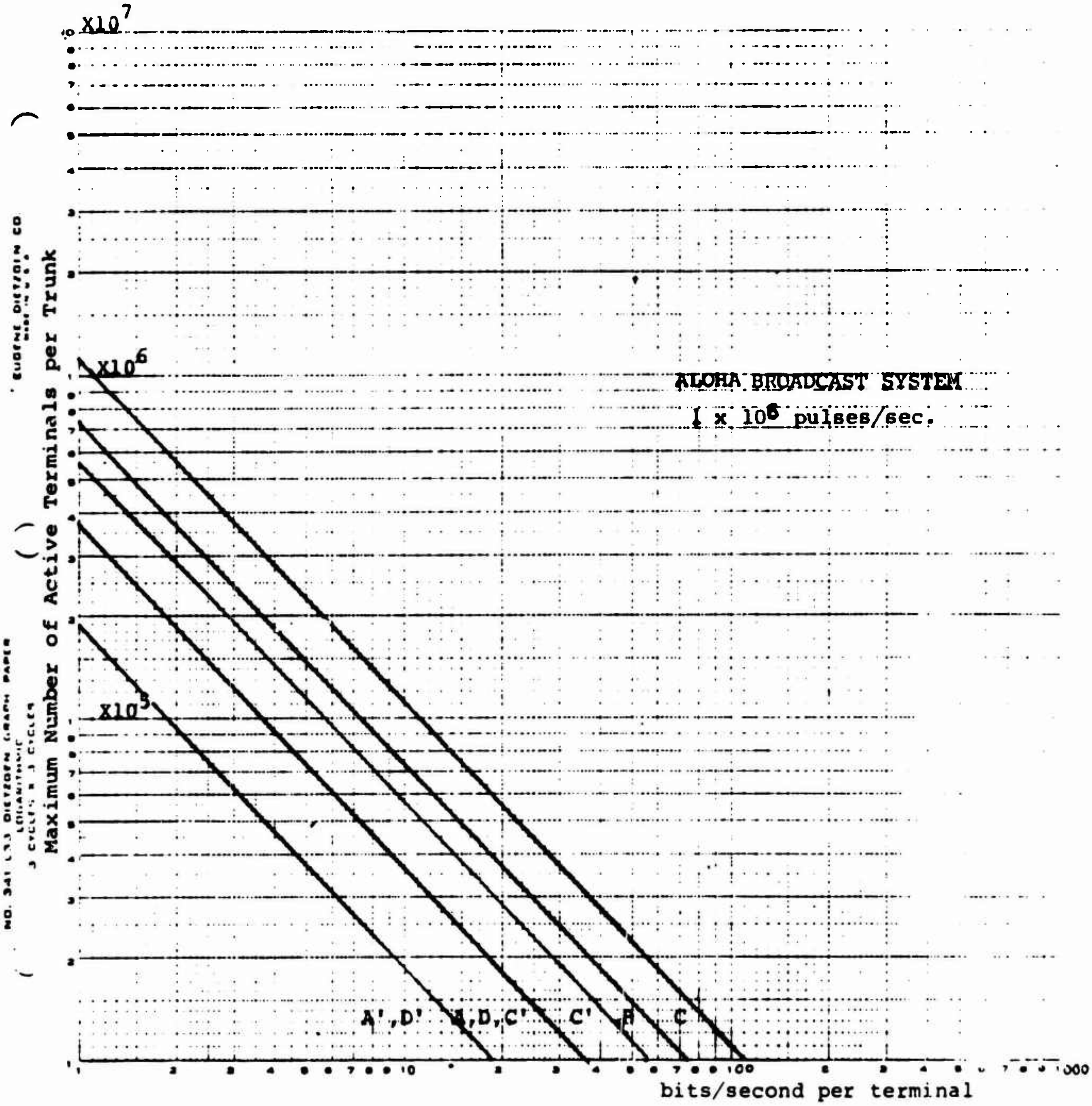


FIGURE 5.1
NUMBER OF TERMINALS PER TRUNK VS.BITS/SEC. PER TERMINAL

We will also consider the introduction of a number of basic data processing options into the CATV system in order to make our data transmission system adaptable to a variety of traffic requirements while satisfying the system constraints.

We will use the terminology of the cable TV industry in describing the direciton of signal flow. Signals traveling from the head end toward terminals will be said to be directed in the "forward" direction on a "forward" link and signals traveling from terminals toward the head end will be said to be directed in a "reverse" direction on a "reverse" link. A convenient synonym for "forward" will be "downstream", and for "reverse", will be "upstream". To install the device to be described in the forward and reverse channels simple diplex and triplex filters can be used. More detailed specifications for the filters and the data devices are given in Appendix E.

CARRIER FREQUENCY CONVERSION:

In the simplest version of a data system, two carrier frequencies are used; one for forward transmission from the head end to the terminals, and one for reverse transmission from the terminals to head end. Let us call these angular frequencies ω_f and ω_r respectively. The next simplest option is to use frequency converters at a small selected set of points in the system. In the forward direction, the converter converts from ω'_f to ω_f and in the reverse direction, it converts from ω_r to ω'_r. The net result is that the terminals still receive and transmit at the frequencies ω_f and ω_r. However, in the trunk between the converters and the head end, there are four frequencies in use, ω_r, ω'_r, ω_f and ω'_f so that in these trunks twice the traffic can be handled.

The converter and the other devices to be described in this section are shown schematically in Figure 5.2.

The advantages of this scheme are:

a) All terminals are identical.

b) The converters also act as digital repeaters to reshape signals.

c) The capacity of the system is increased since two channels are available in each direction.

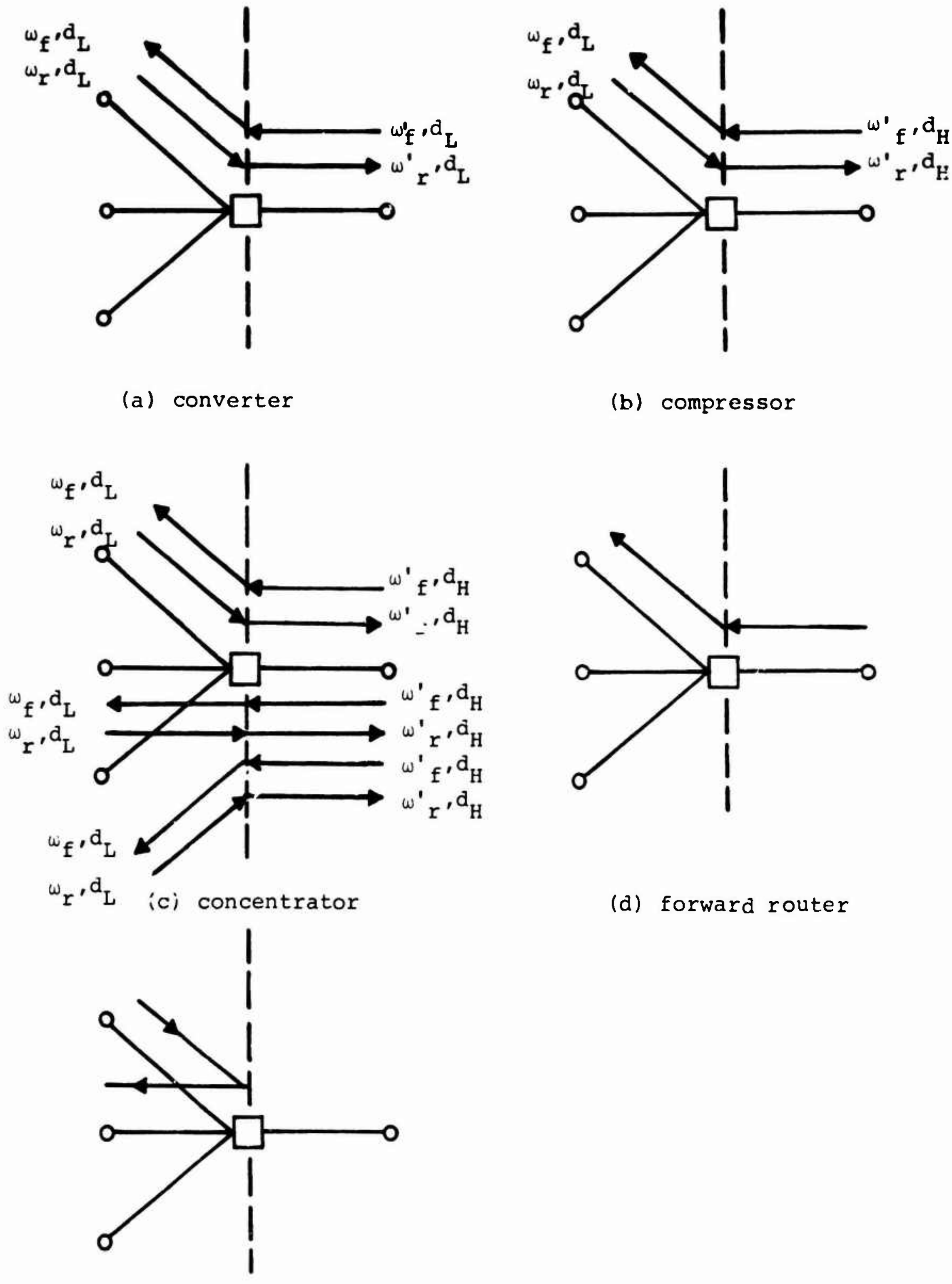


Figure 5.2
Device schematics; devices indicated by squares. Carrier frequencies ω_r, ω'_r, reverse; ω_f, ω'_f, forward. High data rate d_H; Low data rate, d_L.

ROUTING:

In the interactive packet system, there is no requirement for routing since the basic premise is that all receivers listen to all messages that reach them and merely select the ones addressed to them. Nevertheless, we will consider the addition of some primitive low cost routing schemes to study their effect on system capacity.

In the central transmission mode there is routing needed in the reverse direction since all messages reach the head end along the unique paths from the originating terminal. In the forward direction the signals at frequency ω'_f are blocked by filters at the converters and yields a simple form of routing.

In fact, at any section of trunk not requiring signals at ω'_f filters can be added to block ω'_f. Adding these filters does not increase system capacity but may be useful if the frequency ω'_f can be used for local signaling when not being used for data transmission.

At any junction containing a converter, digital routers must be added to send messages at ω_f down the trunk to which they are addressed rather than all trunks. Such a router may be added at any other point in the system as well. This is called "forward routing" and can increase system capacity. Forward routing requires a digital router which can read and interpret message addresses.

Let us now consider these system options in the presence of local traffic. The option of frequency conversion is unaffected and performs in exactly the same manner as in the central transmission mode. However, an extra routing option is available for local transmission. In particular, if two terminals are on the same trunk, then the message between them can be intercepted and routed at a routing station rather than travel all the way to the head end. Such routing is called "local routing". Local routing reduces the traffic on the main trunk.

COMPRESSION:

As the next more complicated option, the data rate as well as the carrier frequency is changed at a converter, i.e., a compressor can be used. The advantages of this arrangement are:

A. All terminals operate at low data rate.

B. On heavily used lines near the head end a higher data rate, say one megabit/second, can be used to increase the number of potential active users or decrease the delay.

C. Even though a section of the trunk can carry high data rate traffic at carrier frequencies ω'_r anand ω'_f other users can still use the system at the low data rates at ω_r and ω_f. Thus, the number of compressors required is small.

D. At the low data rate, signal distortion is kept to a minimum.

CONCENTRATION:

Finally, the compressor at junctions may be replaced by a concentrator. That is, messages arriving simultaneously on two or more links in the reverse direciton are buffered and sent out sequentially at the higher data rate. This essentially makes the system downstream from the concentrator appear to operate at the higher data rate and hence increases the system capacity even further.

With the use of converters, compressors , concentrators and routers, we have a highly flexible interactive packet data system which obviously meets most of the system requirements of capatibility with the CATV system, the packet radio system and the population. In particular, the system has the following characteristics:

A. The number of active users that may have access to the system can be readily controlled by varying the number of converters, compressors , concentrators and routers.

B. The transmission rates at every terminal are at the low data rate of 100 kilobits/second.

C. Terminal equipment is inexpensive because modulation takes place at the low data rate.

D. All terminals receive and transmit at the same frequencies and data rates.

FREQUENCY DIVISION MULTIPLEXING:

In case the data rate is limited by the head end mini-computer, an available option is to frequency division multiplex several 100Kbits/sec. channels, each of which is processed by a separate head end mini-computer.

The assignment of these options in an optimal fashion requires detailed expressions for the traffic in the links. Formulae for the traffic are given in Appendix C and are used to give an augmented design for the Boston system in Appendix D.

6. FUNCTION OF EXPERIMENTAL DIGITAL SYSTEM DESIGN

The previous considerations indicate the desirability of developing a system of MATV and/or CATV lines for inter- and intrabuilding communication to and from packet data terminals. A picture of the system is shown in Figure 6.1.

In order to communicate through the cable system, each terminal will be connected to an interface and modem. A minicomputer will be connected to the cable system at its head end by means of a similar modem and modem-to-computer interface. The minicomputer is used as a test device to:

A. Receive and generate basic traffic to establish the viability of the system configuration.

B. Provide message loading to determine limits of system performance.

C. Demonstrate the functional capabilities required for the head end processor.

D. Determine the performance required to communicate with external facilities.

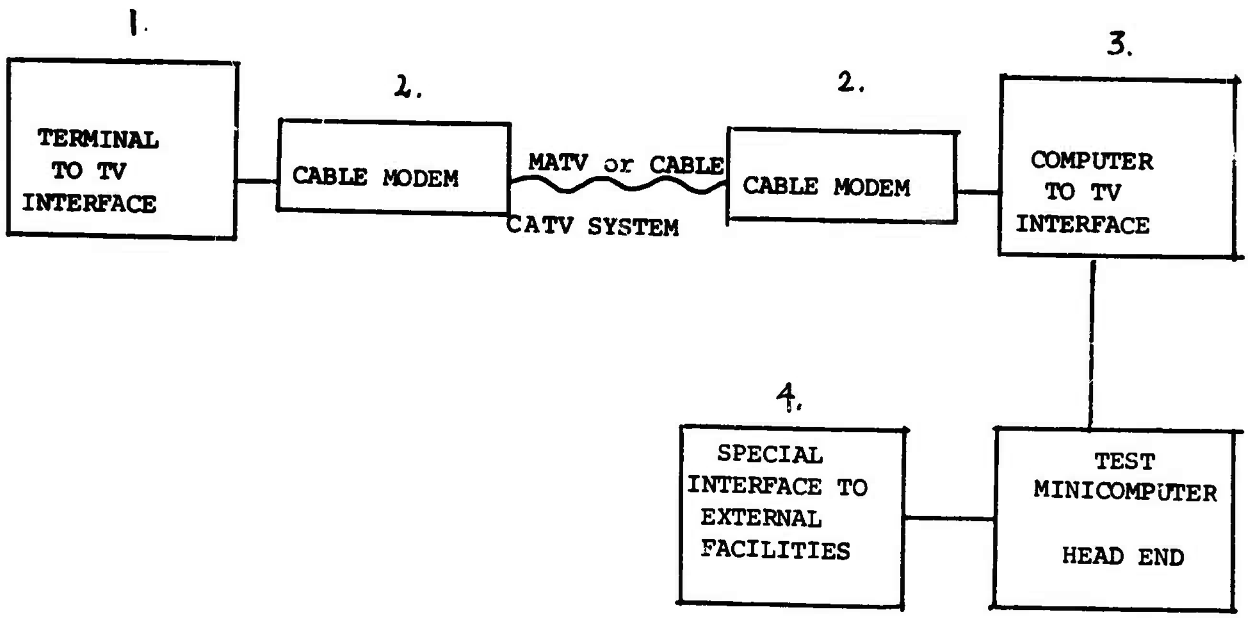


FIGURE 6.1 SYSTEM DIAGRAM

7. SYSTEM CONSTRAINTS

In specifying hardware, a dominant consideration is the fact that the interactive packet cable transmission system must interface with a CATV system and a packet radio transmission system, among others, and an existing population of unsophisticated users. Thus, any hardware innovation must satisfy the following three classes of system constraints.

Interface With CATV System:

Two Way Options:

The data transmission system must be readily adaptable to a wide variety of existing CATV system designs and two-way options.

Data Rates:

The data signals must not cause visible interference with video signals.

Installation:

If auxiliary data equipment is to be added to the CATV system, it must satisfy the following requirements:

- It can be installed with only minor changes in the CATV system.
- It need be installed in only a small number of locations.
- It can be installed rapidly in early hours of the morning to prevent interference with TV service.

Low Cost:

To maximize the marginal utility of data distribution over the CATV system, any equipment introduced must be inexpensive. Depending upon the data rates and bandwidths involved, the radio and CATV systems can be arranged to share some modules, at the baseband or IF frequency range.

Interface With Population:

Population Density Variations:

Standard transmission configuration options must be available for systems of various sizes, population densities and percent of active users. Because of the huge number of potential users, all terminal equipment must be simple and inexpensive.

Unsophisticated Users:

To minimize user interaction with the system operating mode, all terminal equipment must be the same for each location; it must use the same frequencies and data rates; and it must have no options for equipment modification by the user.

8. COMPONENTS OF EXPERIMENTAL SYSTEM

With the overall system plan described in Section 6 and the specifications described in Section 7, we can now develop a detailed description of each system component. These components are examined in depth since the system under consideration must be compatible with both CATV and data transmission technologies. Systems similar to the one proposed have never been designed or built. Hence, major system components must be designed to establish the difficulty of meeting technical requirements and to ultimately establish a cost for each component.

A future report will study in detail the cost/performance tradeoffs of this system versus other local transmission methods involving other technologies such as packet radio, polled multidrop lines, dial up, and other communication techniques.

8.1 MATV AND CATV SYSTEMS

The digital MATV and CATV study should be carried out in two phases. Phase one will develop data transmission techniques for a locally built MATV system. In the second phase, this technology will be transferred to an existing CATV system. The advantages of this mode of operation are the following:

- The MATV system will be local and hence there would be no initial problems of equipment transportation.
- There are no initial restrictions on TV interference or picture quality.
- There are no initial restrictions on carrier frequency or bandwidth.
- The initial system can be varied in structure and performance to approximate the degradation in any CATV system.
- A final advantage of performing the tests on high quality, well built MATV system is that it would completely prove out the viability of using MATV systems to solve the in-building distribution problem. Although MATV equipment is generally poorer quality than CATV systems, there should be only minor technical problems with the local MATV system since cascades are low and environmental conditions are good. The local system will also provide a standard to which older, poorly built MATV systems must be raised for data transmission capabilities.

The MATV system should meet the following specifications:

- The system will have a separate antenna and pre-amplifier for each channel, so the levels for each channel can be controlled independently.

- All available off-the-air channels will be received.
- The system will cover the full VHF and UHF range.
- The head end amplifiers will be driven at full output capability of about 55 dbmV in order to test significant noise and cross modulation figures.
- The system will be two-way so that at any tap a signal modulated by digital data can be inserted and received at any other tap. This will be done by feeding the signal back to the head end and redistributing it through an amplifier after conversion.
- The system will contain at least one extender amplifier on a leg of at least 1500 feet to simulate paths in a large building.
- The system will contain converters so that signals can be converted from a sub-VHF channel to a mid band channel.

A block diagram for the system is shown in Figure 8.1.1 and a bill of materials using Jerrold equipment is given in Table 8.1.1.

Once the equipment and techniques are perfected for the MATV data system, they must be tested and modified under environmental operating conditions on an actual CATV system. CATV systems use much higher quality equipment than do MATV systems, but have much longer cascades. Since the test CATV system should be an operating system, there will be certain restrictions on test operations.

The requirements on the data transmission are as follows:

- The data signal bandwidth be limited to an available 6 MHZ TV channel. The placement of the signal is still being investigated. If operation were to be at $\sim$70 MHZ, the 4 MHZ guard band between 70 and 74 MHZ could be used.

- The data signal must not yield visible interference with TV service. A reasonable guarantee is achieved if the data signal is kept 20 db below the video transmission.

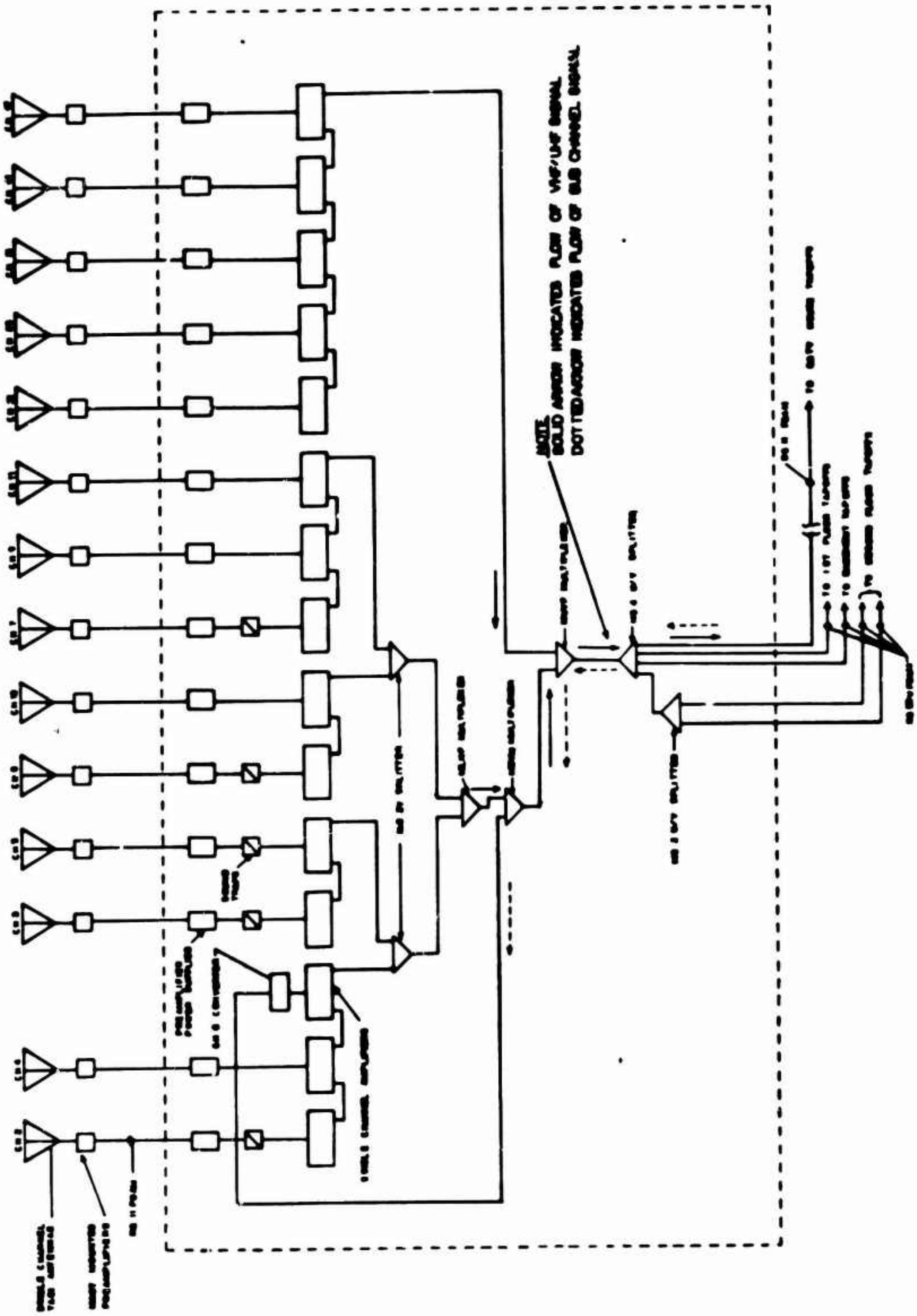


FIGURE 8.1.1
BLOCK DIAGRAM OF MATV SYSTEM

MATV SYSTEM BILL OF MATERIALS

QUANTITY	JERROLD CATALOG NUMBER	DESCRIPTION
4	J-Series	Lo band VHF antenna
5	J-Series	Hi band VHF antenna
5	J-275	UHF antenna
9	TPR	VHF preamplifiers
5	TPR (1443, 3968)	UHF preamplifier
2	PPS-8A	preamplifier power supply
10	THPM	VHF amplifier
5	UCA	UHF amplifier
3	TLB-2	trap filter
2	THB-2	trap filter
1	SCON-Sub-V	subchannel to VHF converter
2	1597	four-way splitter, UHF/VHF
2	1592B	two-way splitter, VHF
1	LHS-76	VHF multiplexer
1	FCO-320	UHF/VHF multiplexer
1	FCO47	subchannel - VHF, multiplexer
1	1596A	two-way splitter, VHF/UHF
38	UT-82	Subscriber taps
1	SPS-30/60A	power supply
1	--	rack and miscellaneous hardware
1	SLE-300-2W	CATV extender amplifier
4000 ft.	JA-500-cc-J	5 inch coaxial cable

TABLE 8.1.1

8.2 TERMINAL INTERFACE

A. The interface can use a non-slotted ALOHA random access mode. The first transmission, as well as subsequent retransmissions, will be randomized over time.

B. The interface will transmit and receive data to a suitable modem at a rate of 100K or 1M bits per second. Data will be supplied to and received from the modem in bit serial form. The interface will be able to provide crystal controlled clock signals to the modem or utilize modem provided clock signals along with modem data.

C. The interface will provide and receive data for the interactive terminal at the rate required by the particular terminal. The exact rate chosen will be selected from the rates given below by means of simple plug changes or terminal block jumper wiring at the interface unit. The data rates possible are:

110	bits per second
134.5	bits per second
150	bits per second
300	bits per second
600	bits per second
1200	bits per second
2400	bits per second

The terminal input and output rates shall be independently selectable. In addition, it should be possible to control the transfer of data to and from the terminal by means of an externally supplied clock signal(s) which is below 2400 bits per second.

D. The interface shall be capable of receiving or transmitting 5, 6, 7, and 8 bit characters. The number of bits per character shall be selectable independently for transmitter and receiver by means of plugs or jumper straps within the interface.

E. The interface shall be capable of transparently receiving or transmitting all bits included in each character, including a parity bit, independently for transmitter and receiver.

F. The following formats will be used for the TV/Terminal system:

1. Data Packets

HEADER	TEXT	CHECKSUM

2. Acknowledge Packets

HEADER	CHECKSUM

The format within the header will be as follows:

DEST. I.D.	SOURCE I.D.	CONTROL CODES	MESSAGE NUMBER	TERMINAL RCVR RATE
40 bits	40 bits	8 bits	4 bits	4 bits

The field sizes are chosen to allow expansion to a reasonable number of terminal interfaces on a given TV system, and to provide sufficient control codes for system operation.

There are three possible means of identifying source and destination in the TV/Terminal Interface.

A. "Log in" to the interface via the terminal and establish the ID codes which the interface then uses in each header.

B. Use short, hard wired (or switch selectable) interface ID codes for header information. The user of the terminal "logs in" to the head end to identify fully.

C. Use full switch selectable ID codes on the interface into which the user sets his ID number. A likely choice is a Social Security number of 9 4-bit digits. No further identification of the user is required to the head end.

The first alternative is unattractive because the implementation of a dialogue capability between the terminal and the interface will be more costly and complicated than is warranted. A hardware design which allows (C), also allows (B) as an alternate later on with only software changes while the reverse is not true. For this reason, the initial test interfaces will be built to allow insertion of a 36 bit (9 4-bit digits) ID code for source and destination. If at a later date it is desired to use a log in procedure at the head end, the switches will be used to set in a smaller interface ID code.

Since there are boundaries at 8 and 16 bits due to most mini-computers which might be used at the head end of the CATV/MATV systems, initial choice of 40 bit fields for both source and destination ID's is recommended. An 8 bit control code field should also be adequate. The control code field will contain information such as message ID and acknowledge bits. The total header length is then 96 bits or six 16 bit words.

G. The following description of the proposed operation of the transmitter section of the TV/Terminal interface assumes that the terminal has not been operated with the interface prior to this operation.

1. The switches, jumpers, and plugs on or in the interface should be set to conform to the parameters of the terminals. This includes input and output data rates, character size, and interfacing conventions such as EIA, current loop, etc.

2. The initial parameters which are required for operation with a given head end computer are set in via switches, plugs, or jumpers. These parameters include source and destination I.D. codes and special escape characters which, upon receipt by the interface, cause initiation of various functions (if any). As an example, the character sequence which, when received from the terminal, causes the interface to initiate message transmission is one such parameter.

3. The terminal should be connected to the interface and modem, and A.C. power should be applied to all units.

4. The terminal operator should compose the first message and enter it on the terminal keyboard. The interface will accept and buffer the message up to a maximum number of characters. For interactive terminals, the maximum number of characters will be 125, the maximum line length. Depending on the terminal type and mode of operation, the interface might provide data character echoing to indicate correct message receipt to the terminal operator.

5. Upon receipt of the escape character sequence from the terminal (line terminator such as carriage return, line feed, or a similar sequence), or upon filling of the interface buffer, a transmission would be initiated from the interface, and the message would be transmitted at 100K or 1M bits per second. Upon transmission of the message, a timer would be started to time out the reception of an acknowledgement from the message destination.

The timer will serve two functions: (a) time out the reception of an acknowledgement (of the transmitted message) from the head end for retransmission by the terminal interface, and (b) randomize the starting time for the retransmission. The fixed minimum delay before acknowledgement should be slightly greater than 14 milliseconds. The exact number will be chosen later and will reflect the ease of generation of the hardware,and be based on a multiple of an available clock rate.

The random interval of delay with respect to the starting time for the retransmission will vary from 0 to $\approx$ 5 full packet intervals (0 to $\approx$ 64 milliseconds) with an exact choice determined in the same manner as for the fixed interval. The maximum random interval will be divided into 1/2 millisecond sub-intervals as the various transmission starting times, corresponding to 128 possible starting delays. If the timer runs out without the reception of an acknowledgement, the message is retransmitted. The transmitter retransmits a message N times, where N is selected between 0 and 15 by means of jumpers or a switch in the interface. If, after N retransmissions, no acknowledgement has been received, an indicator will be activated to notify the terminal operator of failure to communicate with the head end computer.

There will be a "Packet Acknowledged" light on the interface which allows the terminal operator to know if the packet he last sent has been acknowledged and tells him when he can start to send the next packet. In the case of an unacknowledged packet, the interface will refuse any new data from the terminal, and that state may be cleared by pressing a "reset" button on the interface.

6. As soon as an acknowledgement is received, the local copy of the transmitted message may be released and a new message accepted from the terminal for transmission. With this type of operation, only one message may be handed at a given time by the interface. Once a given message has been accepted for transmission by an interface, the terminal user must wait until either the message is sent and acknowledged or the message is cleared by the interface reset button.

7. A preliminary flow chart for operation of the transmitter is shown in Figure 8.2.1.

H. The description of the proposed operation of the receiver section of the TV/Terminal interface assumes that the terminal has not been operated with the interface prior to this operation.

1. The parameters of the terminal should be set into the interface as in steps G1, G2, and G3 above.

2. Upon detection of modulation on the TV channel, the raw digital data will be passed through the receiver and bit and character synchronization will be obtained.

3. The header will be inspected to see for which terminal the message is destined. If the message is not for this terminal, it will be ignored.

4. If the message is destined for this terminal, the checksum will be checked to insure that the message is error free. If there are errors, the message will be ignored.

5. If the checksum is valid, the header will be inspected to see if the message indicates an acknowledgement or other control function. If so, appropriate action will be taken. For example, if the message is an acknowledgement, the transmitter will be notified to drop the copy of the last message sent and will be able to accept a new message.

6. The message text will be received and stored in a buffer in the interface, and if an acknowledgement is required, the interface will transmit one to the message

TV/TERMINAL INTERFACE

TRANSMITTER FLOW CHART

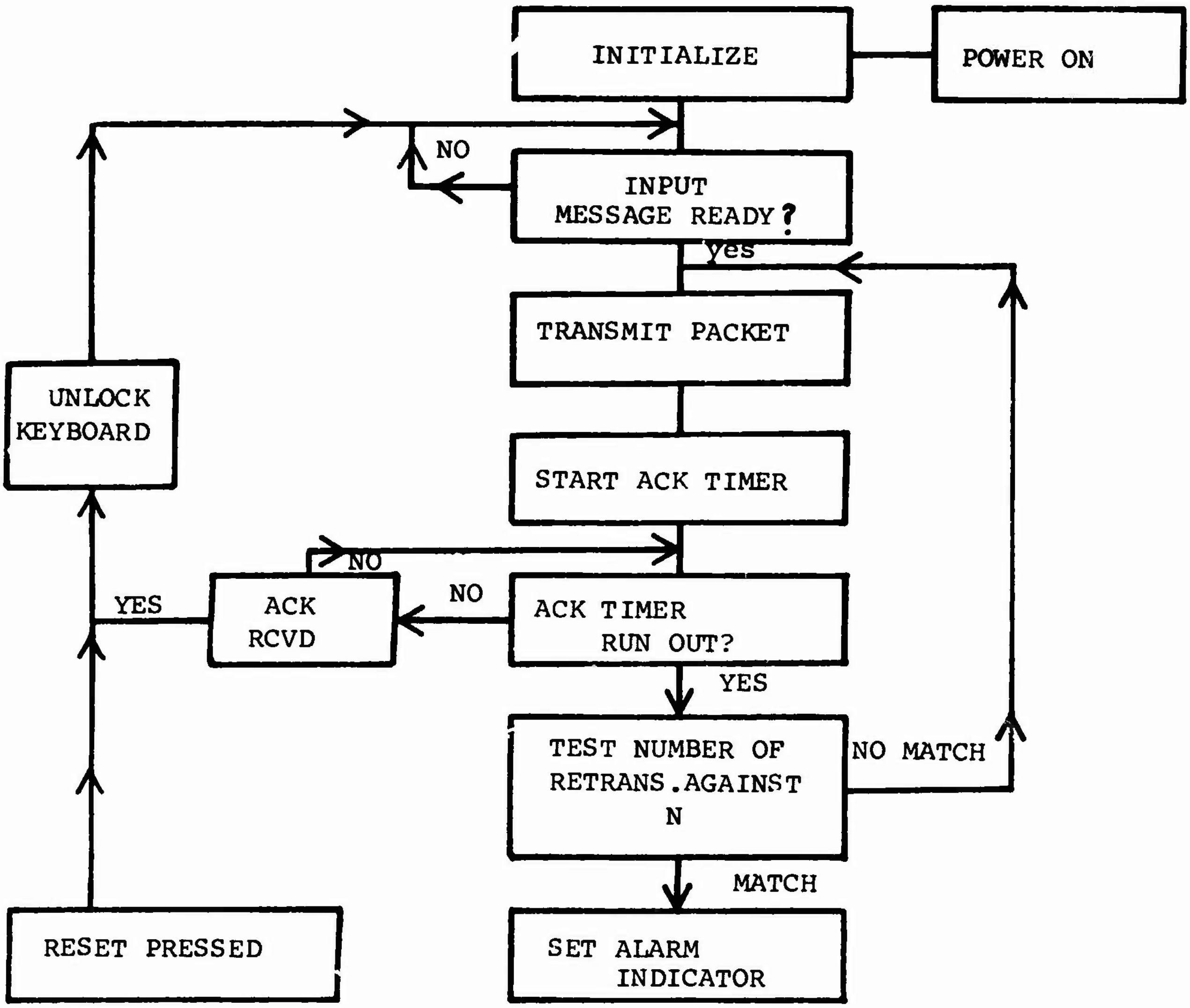


FIGURE 8.2.1

source. The interface will sense when the terminal is turned off, using the Data Terminal Ready (DTR) signal and will reject all packets received for that terminal during terminal power off conditions. The head end will not send messages to the same terminal interface without waiting a fixed "safety" period. The duration of this safety period will be determined at the head end by the specific terminal receiver data rate information which is a part of the header of all packets which originate from that terminal interface. The acknowledgement from the terminal interface will be sent to the head end simultaneously with the sending of the message from the interface to the terminal.

7. The number of the most recently received message will be stored in the interface logic to allow detection of duplicate messages.

8. A preliminary flow chart of the operation of the receiver is shown in Figure 8.2.2.

I. A Cyclic Redundant Code (CRC) checksum will be employed on the data for error control in the TV/Terminal system. The checksum for the message text will be chosen to provide adequate undetected error probability under channel conditions which are expected to be encountered in the TV system. The analysis and experience of the Packet Radio and ALOHA systems will be used to pick the best checksum. For hardware estimates, assume that the ARPANET type of 24 bit CRC will be used in the TV/Terminal interface.

TV/TERMINAL INTERFACE

RECEIVER FLOW CHART

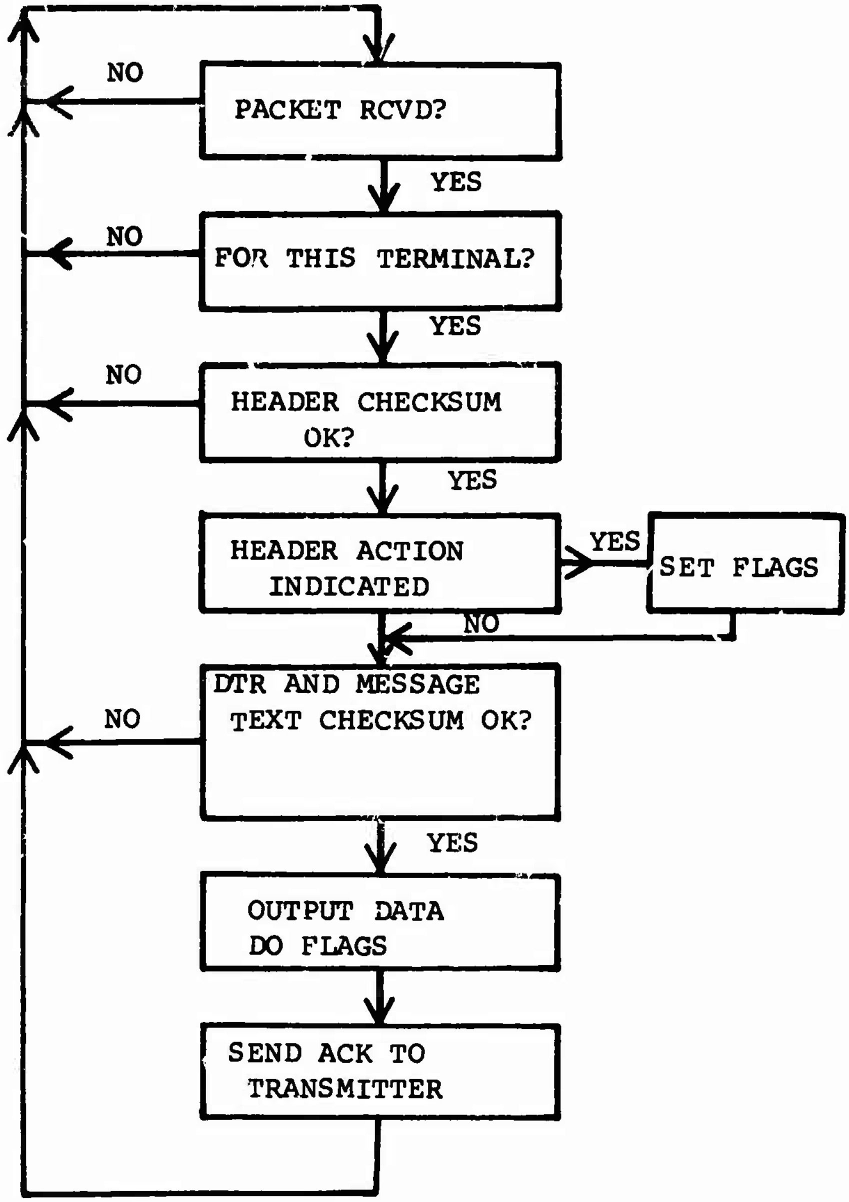


FIGURE 8.2.2

Electrical Specifications

A. The interface shall be capable of being configured to match a wide variety of terminal requirements. This flexibility shall be provided either by having a set of the various possible circuits required in each interface and selecting the proper form by jumpers of plugs, by having a set of sub-modules which can be interchanged in a given interface to adopt to a particular terminal's requirements, or, ideally (but possibly quite difficult), by having a single "universal" type of interface circuit which can be easily adapted to each terminal requirement. The types of requirements which can be expected are:

1. TTY (Current loop @ 20ma or 60ma)
2. EIA-RS-232 (Either full or partial depending on the terminal)
3. CCITT (Either full or partial depending on the terminal)
4. MIL STD 188B
5. Standard low level Signaling Interface
6. Specials

In all cases, the interface circuits should prevent or minimize damage to either the interface or to the terminal under conditions of short circuit, open circuit, or voltage or current transients. State-of-the-art isolation techniques will be used, including optically-coupled interface circuits and protective diodes on sensitive elements.

B. The interface will be designed to match the choice of modem. The same isolation and protective features used in the connection of the interface to the terminal will be included in the circuits which connect the interface to the modem.

C. The power supply and interface circuits will be housed together. Input power requirements are:

1. 110 volts A.C.
2. 60 Hz, single phase
3. Estimated input power required less than 100 watts

The power supplies will provide suitable voltages and currents for the operation of all circuits and modules within the interface. Standard modular off-the-shelf supplies will be used whenever possible. The interface will have a master power switch, power-on indicator, adequate protective fusing, transmission and reception status indicators, and a reset button.

D. The logical design of the interface will be realized using standard integrated circuits and components. The possible use of Large Scale Integration (LSI) logic modules, such as a microcomputers and Read Only Memory (ROM), will be investigated. The characteristics of available microcomputers will be carefully studied and a choice will be made based on the TV/Terminal interface requirements. At a later date, for reasons of improved microcomputer components or desired compatibility with other packet systems, a different microcomputer can be selected and programmed to perform the same functions without major hardware changes. The project will result in a reliable, cost effective interface with sufficient flexibility to allow reasonable system modifications during testing.

Mechanical Specifications

A. The interface will be packaged in an enclosure which is capable of standing alone. The estimated size of the interface is approximately 19" wide, 20" deep, and 5" high. The interface will have a 6' power cord and suitable connectors or terminal strips to match the connections to the terminals and modem with which the interface will operate. Sound construction practice will be used throughout with an emphasis on ease of assembly and maintenance along with low cost. The estimated weight of the interface is less than 30 pounds.

B. The front panel of the TV/Terminal interface will have a power switch and whatever other switches and visual indicators are required for operator assurance, such as power-on and incomplete transmission. All other switches, connectors, and seldom used controls will be mounted in a conveniently accessible, yet protected, fashion. Some connectors will be mounted on the rear panel of the interface, while other controls and terminals will be accessible only with a set of tools such as a screwdriver or key to open a service access panel or to remove the protective cover(s).

BLOCK DIAGRAM OF TERMINAL INTERFACE (FIGURE 8.2.3)

The cable modem data and control signals pass through appropriate level-conversion logic on entering the terminal interface. Modem control signals and status signals are set or sensed by the micro computer program via paths in the micro computer I/O multiplexer and controller. All incoming data are passed through a data examination register where special high speed character detection logic looks for synchronization characters and enables the checksum verification logic upon synchronization. All synchronized data are stored automatically in a high speed received packet buffer (First In, First Out - FIFO). Upon completion of reception of a packet, the micro computer is interrupted and begins to process the packet. The checksum is verified, and, if false, the buffer is cleared, and the sync logic is reinitialized. If the checksum is verified as true, the packet header is examined for destination, and if no match is detected, the receiver section is reinitialized and the buffer is cleared.

If a destination match is detected, the appropriate acknowledge and control information is stored in The Random Access Memory (RAM), and the data is enabled to pass on a byte basis to the Universal Asynchronous Receiver - Transmitter, (UAR-T), which converts the parallel data bytes to serial asynchronous data at the terminal speed. The data passes through appropriate terminal level conversion circuits before leaving the interface. The terminal control and status sense circuits are also connected to the micro computer and the I/O multiplexer. The micro computer can insert or delete characters in the actual data stream between the received packet buffer and the UAR-T, as for example in the case of an acknowledge for a previously transmitted message where no data would be passed to the terminal.

Data from the terminal is automatically stripped of start-stop bits by another UAR-T and stored in byte format in the transmit packet buffer. Upon detection of the terminal special end-of-line character or on filling up a packet, the micro computer is interrupted and attaches the header information to the data in the transmit packet buffer. At the appropriate randomized time, the data is gated to the output register and the checksum generation logic is enabled. The information about the message for retransmission purposes is stored in RAM and the message is recirculated in the transmit buffer at the same time it is sent to the modem. The micro computer controls the number and timing of retransmissions based on acknowledge information from received packets. The crystal clock provides timing pulses to all circuits as required.

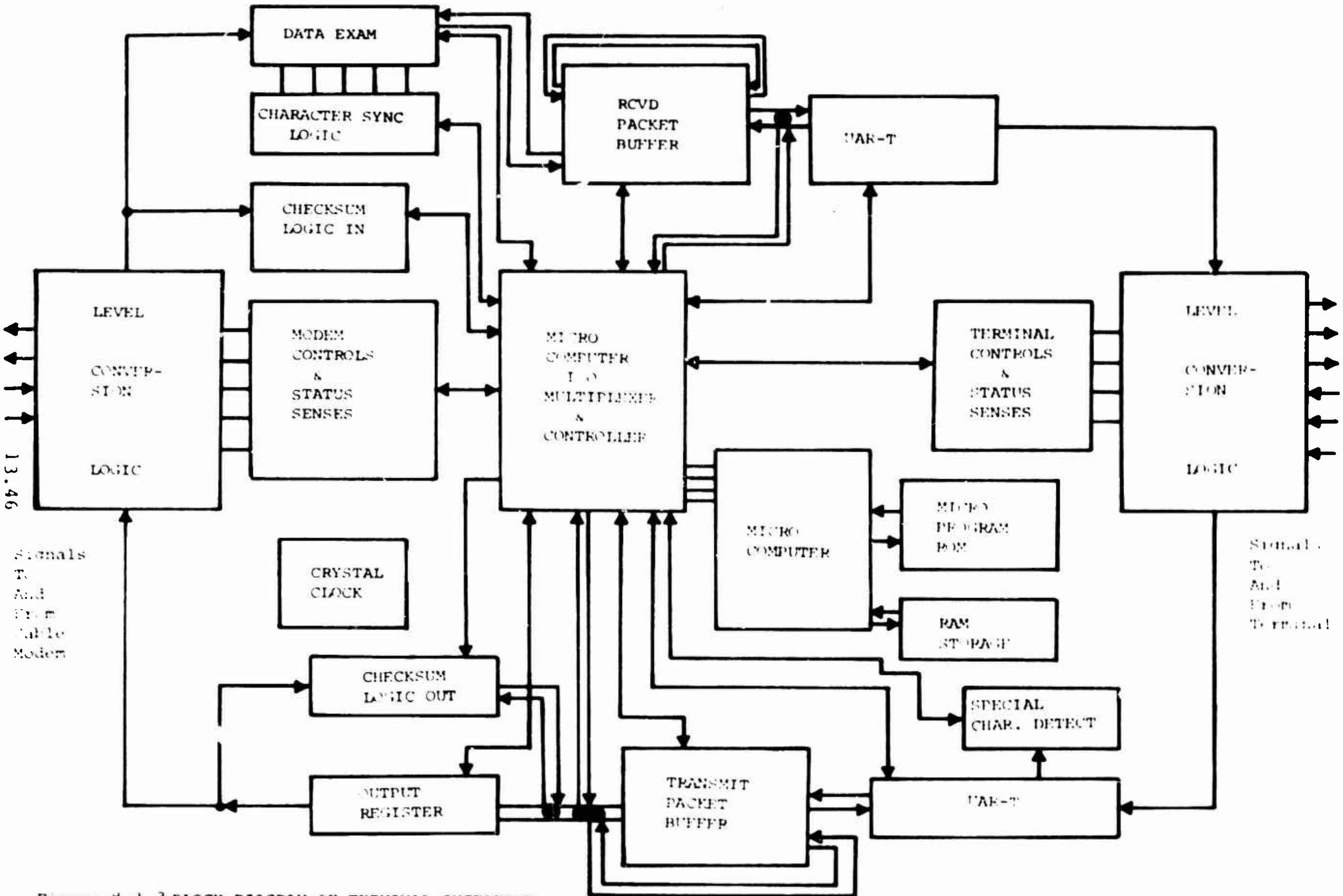


Figure 8.2.3 BLOCK DIAGRAM OF TERMINAL INTERFACE

8.3 MODEMS

For initial experiments a 2-phase differential phase shift keyed (DPSK), coherently detected signal with a 100 kilobit data rate is adequate. This selection of parameters enables the use of much off-the-shelf equipment while still giving the properties of a system meeting practical data requirements. For example, modems are commercially available to produce a baseband PSK binary data stream which can then be modulated onto a video carrier. These modems can produce a 0 dbmv signal on a 75 ohm line with good capture properties and excellent phase stability. The off-the-shelf modem will probably have poor acquisition time, on order of 100 milliseconds, which will limit the system to less than two hundred terminals in initial experiments.

Later experiments can be performed with 4 level DPSK and at a Megabit/sec data rate. However, the short acquisition times required for 1 Megabit/sec data rate would require special development of a Surface Acoustic Wave Device type detector [Matthaei, 1973]. The final selection of parameters for a practical data transmission system depends upon several variables whose values are uncertain and subject to change with improvements in technology:

1. The bandwidth of the head end minicomputer
2. The amount of core at the head end minicomputer
3. The bandwidth of the links to external test facilities.
4. The throughput at the terminals
5. The subscriber saturation level of CATV system

The specifications to be met by the final modems are given in Table 8.3.1.

MODEM SPECIFICATIONS

type of modulation	DPSK
data throughput	100 KB/sec. or 1MB/sec.
carrier frequency	5-240 MHZ
nominal signal level	input 10 dbmv,output 32 dbmv
impedance	75Ω
reflection coefficient	-23 db
noise figure	7.5 db
power hum	60 db below signal level

Table 8.3.1

Modems meeting these specifications are available with performance parameters and resulting system degradation indicated in Table 8.3.2, [Cuccia, 1973]. The signal degradation is within acceptable limits for error rates better than 10^{-20}.

MODULATOR

Carrier Instability in Carrier Source	1/2° RMS in a PLL with Bandwidth = 0.03% Bit Rate Bandwidth	0.05 db
Static Phase Error in QPSK Modulator	2°	0.10 db
Rise Time in Each Phase Change	1/4 Bit Period	0.40 db
Amplitude Unbalance in OPSK Modulator	0.2 db, 10°/db	0.10 db
Data Asymmetry from Data Source	±2%	0.05 db
Group Delay Distortion (Modulator)	25°	0.20 db
Clock Instability	1° RMS in the Bit Synchronizer PLL	0.10 db
	SOURCE TOTAL =	1.00 db

DEMODULATOR

Incidental FM from all Oscillators in RF Channel and Carrier Reconstruction	1/2° RMS in PLL with Bandwidth = 0.03% of Bit Rate Bandwidth	0.05 db
Static Phase Error in QPSK Demodulator	2°	0.10 db
Reference Phase Noise in Reference PLO	1°	0.10 db
Total Group Depay Distortion (Total Channel from Modulator)	20°	0.35 db
Timing Jitter in Matched Filter Sampler	4%	0.25 db
Timing Bias Error in Matched Filter Sampler	1.5%	0.15 db
DC Offsets-Total Receive	4%	0.10 db
Data Waveform/Matched Filter Detector Mismatch	--	0.60 db
	SOURCE TOTAL =	1.70 db

Table 8.3.2

8.4 TEST MINICOMPUTER

A. General

The minicomputer used as the test head end controller in the Terminal/MATV-CATV system will have the following characteristics:

A. 16 bit word length

B. ∿1 μ sec cycle time core memory

C. Memory size expandable to at least 32k words

In order to adequately test the Terminal/TV System, a variety of peripherals and interfaces provided by the mini computer vendor may be required. In addition, several custom interfaces may have to be developed for cases where standard interfaces are not available.

B. Data Rate Constraints

The data rates expected in the head end computer system are shown in Figure 8.4. The data to and from the cable modem will flow at the rates of either 10^5 or 10^6 bits per second, although the flow will not be continuous in general. At the rate of 10^6 bits/second, a 16 bit computer word will be assembled in the interface every 16 μ seconds. The transfers of these words into and out of the computer will require the availability of a high speed data channel, or alternately a Direct Memory Access (DMA) channel as a required option on the computer. Transfers to and from memory will be accomplished automatically without requiring program intervention.

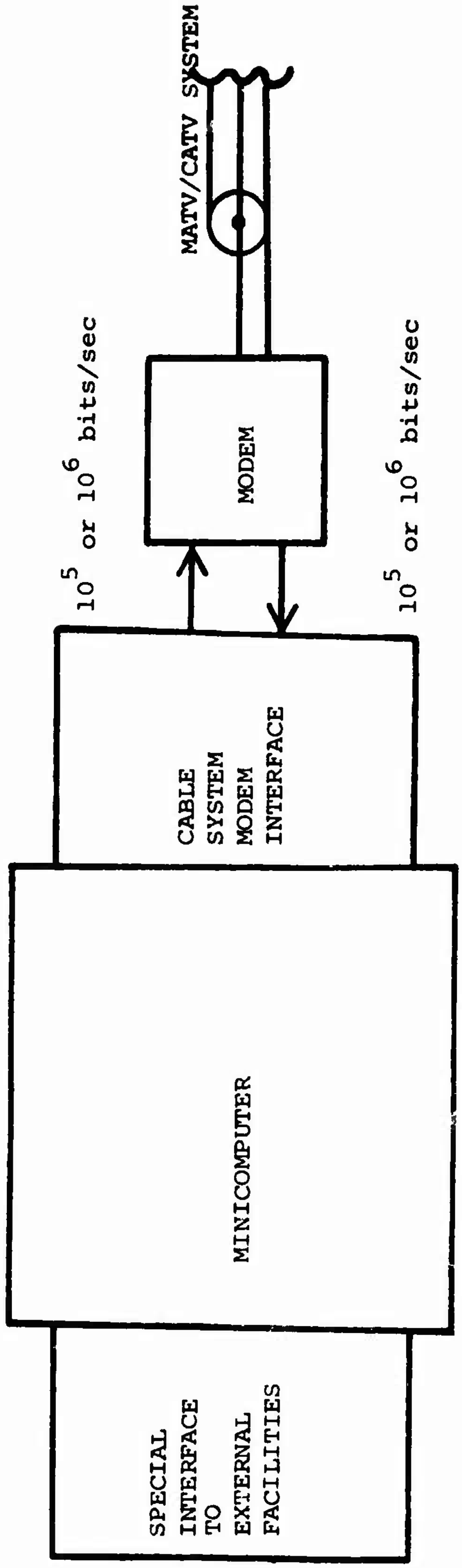


Figure 8.4.1 Head End Computer System Data Rates

C. Functional Description

The computer at the head end must implement the programs to test the following functions for the system:

- Buffering of packets to and from terminals.
- Flow control of cable system
- Implementation of network protocols.

The headend computer will provide sufficient buffer storage to match the difference between the peak to average data rates seen on the cable system and any external sources of data. Initially, storage for a total of the order of 100 packets will be provided.

The head end computer will provide the required cable system flow control by means of acknowledges for received packets. When traffic from the cable terminals begins to congest the system, the head end will effect the flow control by refusing to acknowledge packets which cannot be accepted. This will cause the terminal to retransmit messages, producing the same effect as errors on the cable system. The head end computer will provide the implementation of all protocols required to communicate on the system.

The following is a list of some of the major program pieces which must be implemented:

1. Cable modem interface handler
2. Special interface to external facilities handler
3. Monitor or supervisor for system
4. Cable flow control routine
5. Buffer allocation and management
6. Miscellaneous background tasks such as garbage collect, timeouts, accounting, etc.

8.5 MINICOMPUTER TO CATV INTERFACE

This specification describes the interface between the head end computer and the MATV and/or CATV system. A diagram of the system is shown in Figure 8.5.1.

FUNCTIONAL SPECIFICATIONS

A. The MATV-CATV computer interface will use a non-slotted ALOHA random access mode. The first transmission as well as subsequent retransmission of messages will be randomized over time.

B. The interface will be able to simultaneously and independently transmit to and receive data from a suitable modem at a rate of 100K or 1M bits per second. Data will be supplied to and received from the modem in bit serial form. The interface will be able to provide crystal controlled clock signals to the modem, and receive modem clock signals along with modem data.

C. The interface will be capable of receiving or transmitting 8 bit characters. The computer program shall be able to control the actual number of valid character bits in each received or transmitted character on a message by message basis.

D. The interface will be capable of transparently receiving or transmitting all bits included in each character, including a parity bit, independently for transmitter and receiver.

E. The following formats will be used for the TV/Terminal system:

(1) Data Packets

HEADER	TEXT	CHECKSUM

(2) Acknowledge Packets

HEADER	CHECKSUM

The format within the header will be as follows:

DEST. I.D.	SOURCE I.D.	CONTROL CODES	MESSAGE NUMBER	TERMINAL RCVR RATE
40 bits	40 bits	8 bits	4 bits	4 bits

COMPUTER TO TV/TERMINAL CABLE SYSTEM INTERFACE

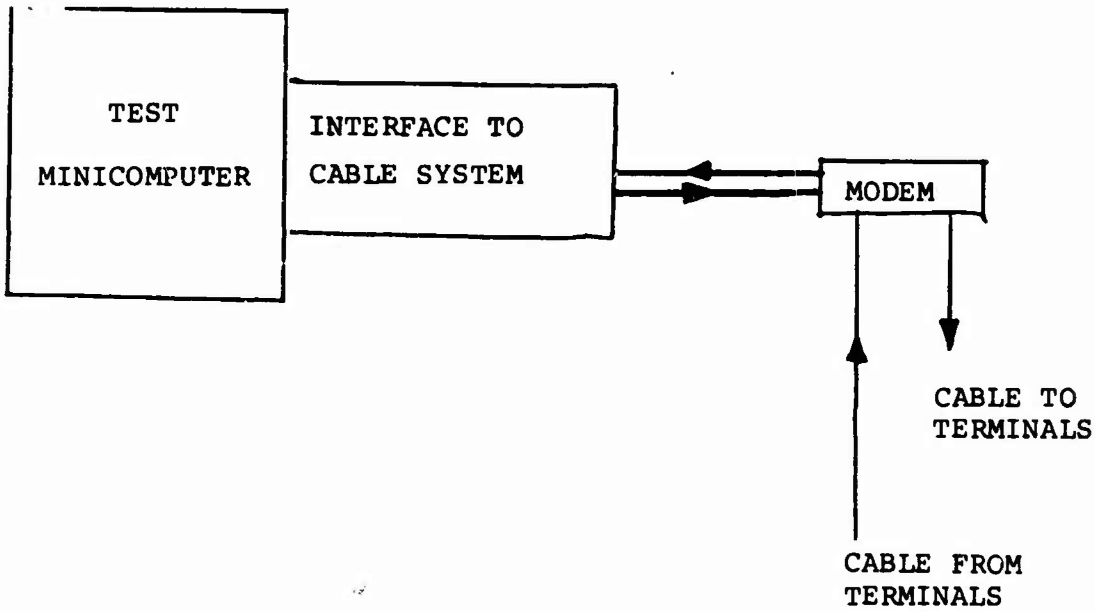


FIGURE 8.5.1

The field sizes are chosen to allow expansion to a reasonable number of terminal interfaces on a given TV system, and to provide sufficient control codes for system operation. The destination and source I.D. fields will each be 40 bits long. This length is the lowest 16 bit word boundary which will contain the temporary 36 bit source and destination identifier.

F. The transmitter section will operate in the following manner:

1. Messages within the computer which are to be transmitted will be placed in one of two queues, a high priority acknowledgement queue or a regular message queue.

2. All messages in the high priority queue will be transmitted before any regular messages are sent. The high priority queue will be serviced before each regular message transmission.

3. Copies of transmitted acknowledgements will not be kept, since acknowledgements are never retransmitted.

4. Copies of all transmitted regular messages will be kept in the computer until either receipt of an acknowledgement from the destination or the occurrence of a program-determined number of unsuccessful retransmissions.

5. The transmitter program will set up a parameter table for each terminal active on the cable system. This table will contain terminal-specific information, such as terminal output rate and character set; and will be used to format each transmitted message to match the characteristics of the destination terminal and to allow the spacing of messages for each specific terminal in time to match terminal output speed.

6. The transmitter program will operate in the computer in conjunction with other programs which will deal with the various functions required of the head end test mini computer system. There will be a monitor or executive program which will control the activities of

the cable system and external interface programs. This monitor will also control or provide various required console related activities and accounting functions.

7. Transfer of messages from the computer to the interface logic will be by Direct Memory Access (DMA) of the head end computer.

8. The transmitter interface hardware will calculate the message checksum for each message and append it to the message. In addition, the hardware will automatically perform such routine tasks as synchronization character generation, special escape character generation (for transparency), and others as required. When the interface hardware completes the transmission of each message, a program interrupt will be generated.

G. The receiver section will operate in the following manner:

1. Message transfers between the cable system interface and the computer memory will be by means of DMA.

2. The interface logic will compute the checksum of the received message and notify the computer by interrupt if an error has occurred. In this case, the computer program will discard the data for that message.

3. The interface logic will automatically acquire character synchronization and will strip synchronization and control characters from the input data stream.

4. The interface will allow the transparent reception of full binary text.

5. As each message is received, its checksum will be verified and a computer interrupt will be generated.

6. The receiver section of the computer program will, upon notification by a receiver interrupt, examine the new message and dispatch the message to the appropriate queue for further action. Sample actions are given below:

a. New message - determine destination queue and forward message, send acknowledgement to terminal, and check for retransmission.

b. Acknowledge - remove copy of acknowledged message from transmitter retransmission storage.

c. Duplicate - discard

7. The receiver section of the program will operate under the control of the computer monitor program and will interface to the transmitter and protocol programs.

ELECTRICAL SPECIFICATIONS

A. Modem Connection

The interface will be designed to match the choice of modem. Interface circuits should prevent or minimize damage to either the computer or the modem under conditions of short circuit, open circuit, or voltage or current transients. State-of-the-art isolation techniques will be used, including optically-coupled circuits and protective diodes on sensitive elements.

B. Computer Connection

The interface will be designed to connect to the Input/Output bus of the head end computer. The manufacturer's recommendations and specifications will be met in all cases to insure proper computer and interface operation.

C. Power

Power for the interface logic will be obtained from the computer power supply. Power for circuits beyond the isolation devices will be obtained by small modular power supplies which will be incorporated into the interface assembly.

D. Logical Design

The logical design of the interface will be realized using standard integrated circuits and components either of the types recommended by the minicomputer manufacturer or their equivalent. The project will result in a reliable cost effective interface with sufficient flexibility to allow reasonable system modifications during testing.

MECHANICAL SPECIFICATION

A. The interface will be packaged on circuit boards of the type used by the minicomputer for interface circuits. The circuit boards will plug into the I/O bus of the minicomputer in the enclosure provided by the minicomputer and will connect to the modem with a cable and connector. Estimated weight of the interface is less than 10 lbs. The prototype version of the interface will most likely use wire-wrapped sockets for the integrated circuits to facilitate design changes during testing.

B. There will be no operational controls associated with the interface other than program control. Test modes and switches for their control will be provided.

BLOCK DIAGRAM
OF MINICOMPUTER TO CATV INTERFACE

Figure 8.5.2

Assume that any frequency or phase acquisition is accomplished by the modem. The received bit stream is examined by the interface hardware for sync characters and synchronization is automatically accomplished without computer intervention. The interface hardware also handles DLE doubling and other special signaling conventions such as STX detection and ETX detection on the cable system automatically. As each character is received, it is packed into a minicomputer 16 bit word and when the 16 bits are ready, the word is transferred to core memory on a DMA cycle stealing basis. A computer interrupt is generated at the end of a packet or upon an error condition.

The transmit section of the interface is also automatic in that after initialization of the interface for a new packet, all communication discipline is accomplished automatically by the interface hardware, and words of data are simply requested by the interface via DMA from core, unpacked and serialized. The hardware also calculates and appends the checksum.

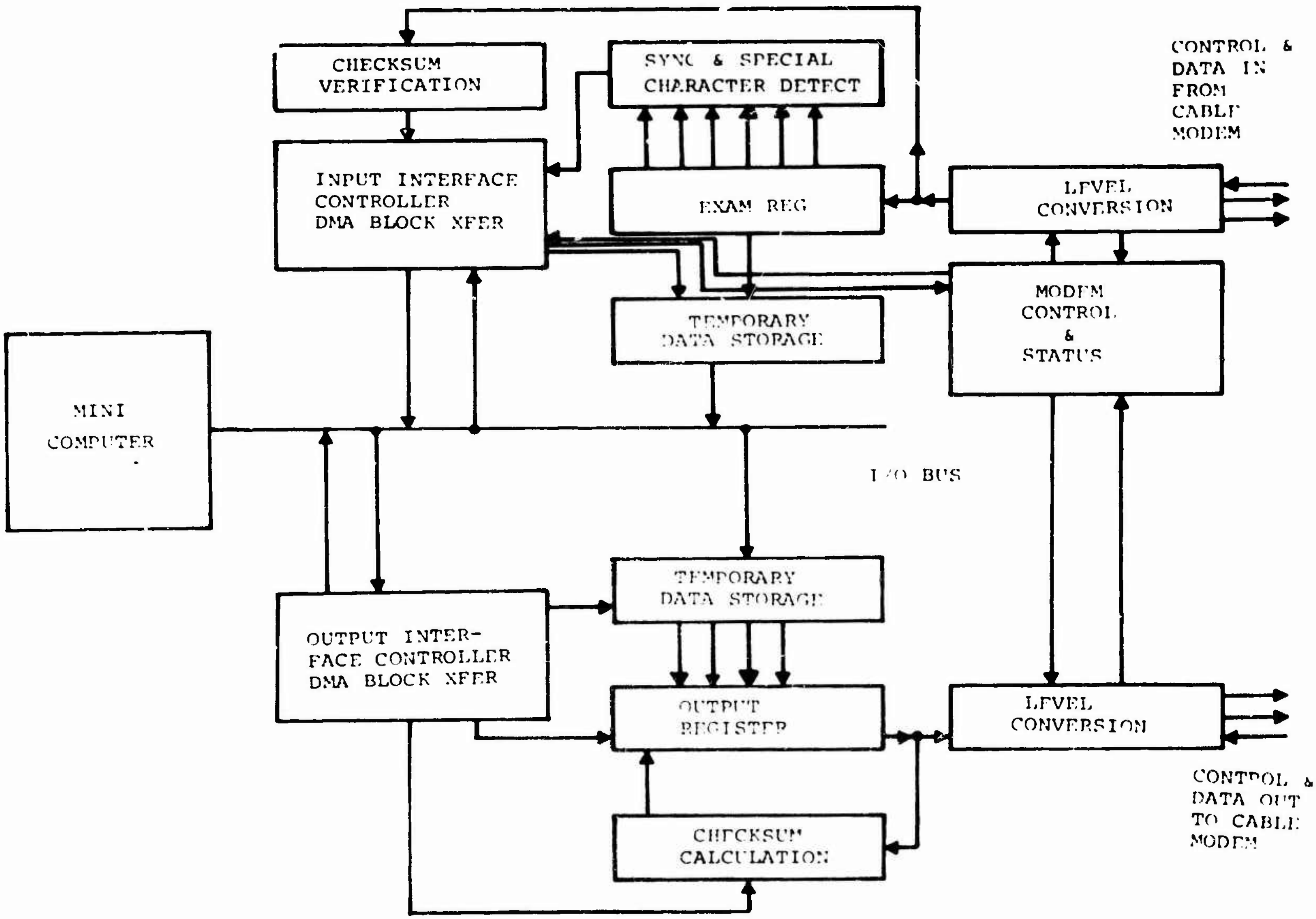


Figure 8.52 BLOCK DIAGRAM OF MINICOMPUTER TO CABLE SYSTEM INTERFACE

8.6 MINICOMPUTER TO EXTERNAL TEST FACILITIES INTERFACE

This specification describes the interface between the head end test minicomputer and a communications link to external test facilities.

Functional Specifications

A. The interface will communicate with the external test facilities over leased lines using commercially available modems, and will provide all of the control signals required to operate these modems.

B. The interface will connect to the I/O system of the selected minicomputer and will comply with all specifications set forth for such connections.

C. The interface will operate in full duplex mode at a rate of up to at least 4800 bits per second.

Electrical Specifications

A. The interface will be able to connect to a suitable data modem operating at 4800 bps full duplex. The connections will conform to EIA RS-232-C. The interface will incorporate adequate isolation features to prevent damage to itself or to the modem under conditions of short circuit or open circuit signal leads. In addition, sensitive interface elements will be protected from damage by voltage or current transients from external sources.

B. Power Supplies

The interface will be powered from the minicomputer logic power supplies, except for circuits which are to be isolated from the computer system. Any isolated sections of interface logic circuits will be powered by isolated power supplies with characteristics to match the degree of isolation desired. It is estimated that less than 50 watts of DC power will be required for the interface circuits.

C. Logic Realization

The logical design of the interface will be realized using standard integrated circuits and components. The minicomputer vendor's recommendations of logic elements and design techniques will be followed, especially those related to I/O bus loading, driving, and termination.

At the present time, it appears possible that a standard vendor Synchronous Line Controller will be adequate for the job at hand.

Mechanical Specifications

The interface will be packaged on plug-in circuit boards which conform to minicomputer vendor I/O board specifications. The interface will be housed in the minicomputer I/O enclosure. There will be no operational controls or indicators for the interface. However, test switches and indicators will be provided as required for troubleshooting. The modem will connect to the interface via a cable assembly which will run from the socket of the logic board to a suitable connector mounting panel.

9. SYSTEM TESTS

In addition to testing the various digital devices, extensive testing must be performed to demonstrate the viability of data transmission on MATV and CATV systems.

Data users may find cable system reliability quite poor when compared with the common carrier facilities that they are used to. One of the major problems with data transmission on MATV and CATV systems is that there is virtually no redundancy in these systems. Most present systems do not even have standby primary power. Alternate routings are not available in case of system "castrophe". There are no Government or industry minimal standards for acceptable performance; hence, performance will vary from system to system. Many old systems were built to extremely loose specifications on noise and cross-modulation and have serious reflection problems because of the use of unmatched subscriber taps. Fortunately, systems in large cities and new buildings are much newer and are required to meet more exacting standards.

Even with these systems, the construction norms are still those which satisty casual TV viewers, not data users. Thus, loose connections and cracked cable sheaths cause intermittant transmission conditions, and momentary "disconnects". These would cause only minor "flashes" on a TV picture but constitute major data dropouts in a high speed data circuit. Cable connections often loosen under the influence of vibration and the cold-flow property of aluminum. Strong RF sources, such as nearby mobile radio transmitters, amateur radio transmitters, and AM and short wave broadcasting stations, leak into both upstream and downstream cables and interfere with the low level television and data signals [Switzer, 1972]. Finally, systems are tested haphazardly, and hence, in some parts of a CATV system noise and cross-modulation levels may not meet written system specifications. The limiting factor in determining the performance of the system will not be Gaussian noise interference, but a number of practical factors which provide interference, generally categorized as "impulse noise". These factors are difficult to characterize and include phenomena such as loose connections, cracked cable sheaths, and R-F leaks. Hence, we must run tests on an actual CATV system to evaluate system performance.

Tests should be conducted in three phases. The first phase consists of the tests necessary to verify operation of the MATV system. The second phase should measure the performance of the MATV system under conditions of simultaneous video and data communications. Tests should be made to determine the interactions between the video and data signals. The third phase of testing should measure the characteristics of data transmission on the MATV system.

Verification of MATV System Operation

The testing of the completed MATV System involves a standard set of well documented procedures which fall into the following general categories:

1. Balancing the head end.
   - A. Check signals at antennas and align antennas
   - B. Adjust amplifier output levels, gains, AGC, and tilt.
   - C. Tune sound traps.
   - D. Record all settings and signal levels.
2. Testing all lines.
   - A. Verify proper installation of each line without short or open circuits.
   - B. Verify proper terminations of each line.
   - C. Verify proper signal reception at each tap.
3. Check picture quality.
   - A. Perform visual check of signals:
     - (1) "windshield wiper effect" caused by excessive cross modulation.
     - (2) "snow" caused by low signal to noise ratio

(3) "beats" caused by overloads, mistuned trap filters or improper sound carrier levels in adjacent channels.

(4) "ghosts" caused by improperly terminated lines or direct pickup of signals.

(5) "hum bars" caused by 60 cycle pickup.

B. Measure cross-modulation, second order distortion and carrier to noise ratio.

C. Compare pictures delivered throughout system with those obtained directly at the antennas.

These tests will not all be necessary for the CATV system if it is an already tested operational system. Some measurements should be made to record actual system parameters.

The next step is to establish the digital characteristics of the MATV or CATV system. The following tests should be run first on the MATV system, and second, on an actual CATV system. The UA-Columbia CATV system in Brookhaven, Long Island has given permission to run such tests. The tests should be run first with packets from a single terminal only and then with simulated background traffic from multiple terminals.

<u>Performance of MATV System with Digital and Video Signals</u>

9.1 Find data error rate as a function of the level of the data signal relative to combinations of the TV channel carrier levels on the cables with and without extender amplifiers (i.e. short term error rate).

9.2 Test visual interference into the television channels and the relation of this interference to bit patterns (i.e., random data or repeating pattern). Tests should be configured to maximize interference so that test can be considered worst case tests. Hopefully, interference will only be visible at levels far above that which will result in satisfactory error rates for the data system.

9.3 Based on short term data from tests under 9.1 above, set the operating level at some reasonable short term error rate (for example, 10^{-7}) and operate the system for at least a 1-to-2 week period of time and record the long term error statistics. After a period of test of 1-to-2 weeks, it may be desirable to reset the level and repeat these relatively long term tests. The purpose of this test is to determine whether error statistics vary significantly during the day or are related to other factors such as weather. If impulsive noise type interference is seen, then it would probably be wise to repeat the test at a number of different levels to determine to what extent the operating level affects error statistics. In order to minimize the data reduction and other labor needed for these long term tests, data will be recorded digitally in a manner that is adaptable to computer processing.

Performance of Digital Data Transmission System

The tests of the Digital Data Transmission System fall into two categories, initial simple tests to debug components of the system and verify system operation, and system performance tests. Testing of system components proceeds to a large degree as a part of component development. Listed below, roughly in order of complexity, are the tests which will be performed. These tests will be carried out under typical and worst case combinations of video signals on the cable systems.

1. Single Terminal With Data Path Through MATV System. For these tests, the terminal interface will be configured so that it can receive its own transmitted packets, and an upstream-to-downstream converter will provide a data loop at the head end of the MATV system (Figure 9.1). The tests will verify the operation of the interface on the cable system. In addition, several intermediate signal or data looping arrangements such as modem analog loop at baseband, digital loop at modem input, and digital loop at interface input and output will be tested to develop trouble diagnostic aids. Several tests will be performed to insure that different terminals located separately on the MATV system can communicate with each other.

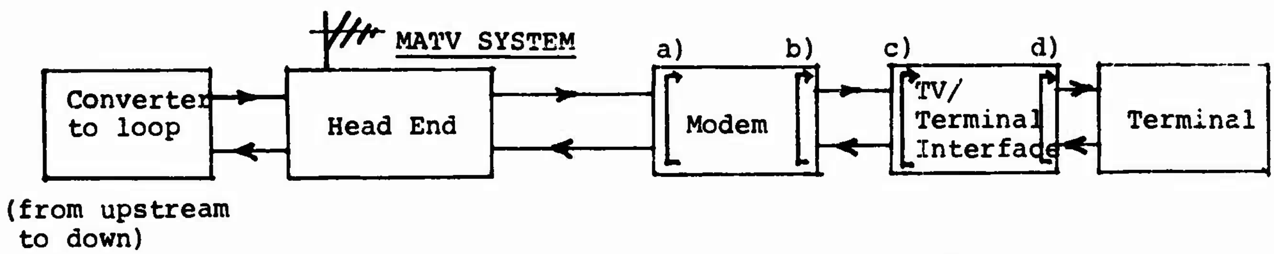
2. Mini Computer Tests With Data Path Through MATV System. The mini computer will be connected to the cable system configured for the data loop from downstream to upstream (Figure 9.2). It

will be tested to verify operation of the computer, the computer interfaces and modems by sending packets to itself. In addition, error rate measurements will be made using the computer to generate data and record the test results. Receiver acquisition time will also be measured.

3. Initial Data System Tests. The mini computer and a small number of TV/terminal interfaces will be connected to the MATV system (Figure 9.3). Operation of the system will be verified and human factors such as delay and convenience will be checked for acceptibility. Test messages will be passed between the terminals and the head end computer and vice versa.

4. Traffic Simulation. The mini computer and a number of TV/terminal interfaces will be connected to the MATV system (Figure 9.4). The mini computer will provide data to the terminal interfaces to simulate the effect of a large number of terminals using the system. The computer will be used to generate test messages and interfering traffic. System data throughput, transmission delay and number of retransmissions will be measured as a function of number of terminals active.

5. Operation of System With External Data Sources. The head end mini computer will be connected to external test facilities (Figure 9.3) and the performance of the entire data transmission system will be verified under typical user situations. These tests will involve users at individual terminals performing operations representative of actual terminal use.



Possible Diagnostic Loops

a) Modem analog loop at baseband

b) Digital Loop at modem input

c) Digital Loop at interface output

d) Digital loop at interface input

Figure 9.1 Test 1

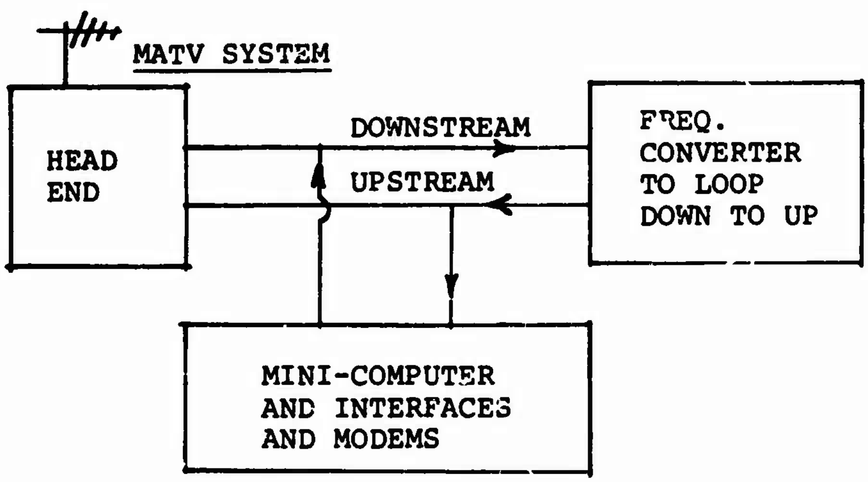


Figure 9.2 Test 2

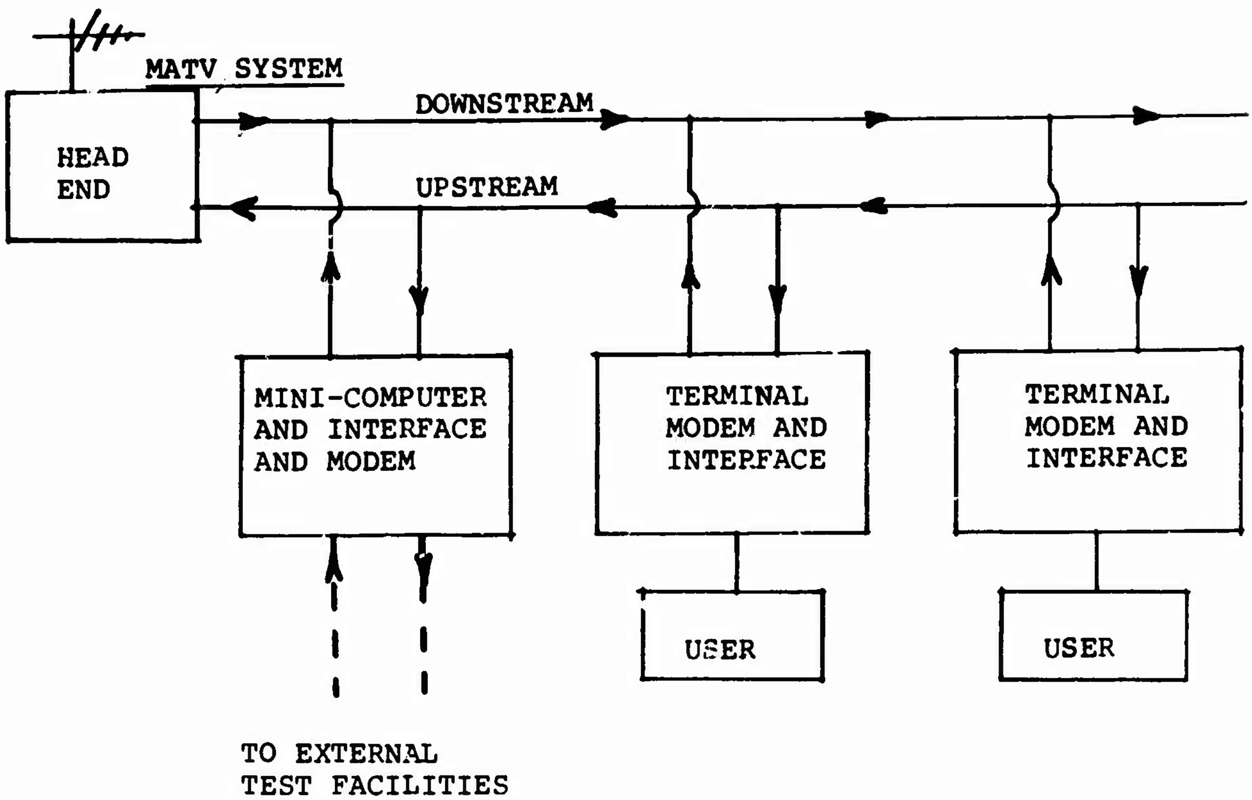


Figure 9.3 Test 3

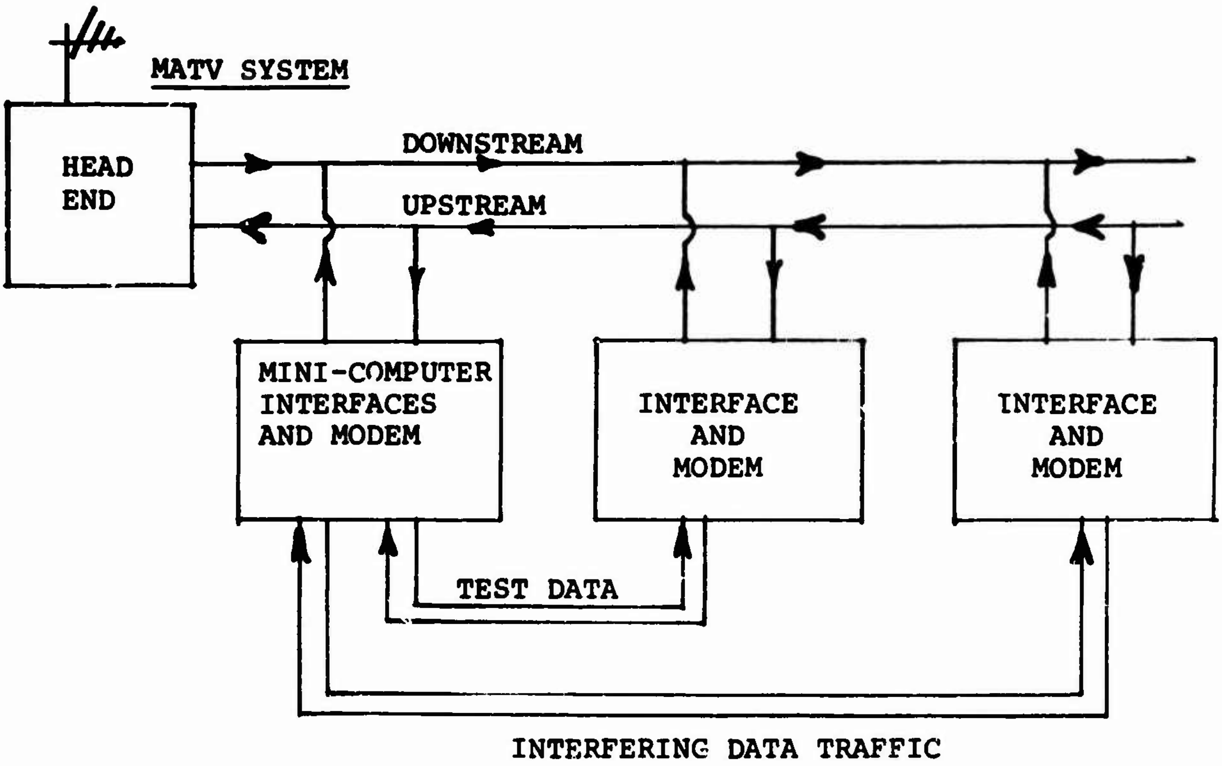


Figure 9.4 Test 4

10. CONCLUSION

An investigation of all aspects of the use of MATV and CATV systems for interactive packet transmission has led to the conclusion that the operation is feasible and appears to offer an excellent high bandwidth method for local distribution. The characteristics of the MATV and CATV systems are such that they would form a highly favorable environment for data transmission. Detailed specification and study of the required modems and digital equipment indicate that the devices can be built to meet reasonable component specifications and to meet overall system requirements.

It is also clear that the techniques and some of the components which are used to solve the local transmission problem mentioned above may be applied to solve communications problems in other areas, most notably for military applications.

I. Military Installations

A. Existing Wiring Systems

The techniques of converting terminal data to packets at the terminal and transmitting and receiving those packets on shared channels can be applied to existing wiring systems such as twisted pair networks. In the case of utilizing an existing wiring system, the performance of the entire terminal-computer system would depend on the bandwidth of the wire transmission medium and the distances involved. In the case of twisted pairs, for instance, modulation of terminal signals to VHF would be unnecessary and unworkable, but low-level standard signalling could be used on such a system. In certain cases where distance could be gained at a sacrifice in bandwidth, low frequency audio modems could be used and operation would be quite similar to operation of Remote Job Entry (RJE) terminals on a multi-drop telephone line.

One of the major differences between a cable system configuration and an existing twisted-pair network configuration is that the cable system generally tends to be a "tree" structure while the twisted pair network usually has a "star" structure, although "tree" structures are also used. This difference can easily be surmounted by connecting all wires of the star together at some point, even the center. The important point is that all signals from all terminals be available on the same channel (or pair). The trade-off in this case is that instead of having the available system bandwidth of the sum of the bandwidths of all lines, the effective bandwidth is reduced to that of a single line. This effect will have to be evaluated for each potential use.

B. New Installations

A detailed analysis of the requirements of each new installation will be essential to determine the choice of type of data transmission system to be implemented. Both coaxial cable and twisted pair systems have advantages and disadvantages, some of which are listed below.

1. Twisted Pair - Advantages

    a) Relatively low cost of wire

    b) Non-critical installation

    c) Terminal connection is simple, no special interface required

2. Twisted Pair - Disadvantages

    a) Low bandwidth, therefore expansion is limited

    b) High noise pickup

    c) Data easily eavesdropped

    d) In star networks, high cost of switch at star center

    e) Physically large cable bundles requiring large conduit space

    f) Limited signalling distance

3. Coaxial Cable - Advantages

    a) Extremely wide bandwidth - virtually unlimited expansion

    b) High noise immunity

    c) Relatively inexpensive head end switch

    d) Single cable requires little physical conduit space

    e) Longer distances allowed due to modulation

4. Coaxial Cable - Disadvantages

    a) Higher cable cost - system components cost

b) Installation relatively more difficult technically

c) Higher terminal equipment cost

In those situations where large numbers of terminals are to be used and easy expansion to larger numbers is important, the advantages of a coaxial cable system are probably enough greater than those of a twisted pair system to justify the choice of the coaxial system on that basis alone.

II. Tactical Radio and Satellite Communications

The solution of the local distribution problem will be based upon the use of an existing, or readily obtainable, channel for communication, namely a coaxial cable. However, the equipment which will be used in the solution will not, in general, depend on the channel used so long as the transmission properties or channel characteristics do not change significantly.

The cable represents an almost ideal channel in the sense that there is no multipath (although there are reflections from mismatches) and the cable is a much more controlled environment, from the standpoint of external moise sources for instance, than a radio channel. However, in the situations where an alternate channel has sufficiently good characteristics, it is possible to use the same approach and hardware (with minor modifications such as replacement of the modems with units capable of operating on the new channel frequencies or the addition of radio transmitters and receivers) on the new channel.

A point to point tactical radio system is one such example. The ALOHA system has proven the viability of the technique on a radio channel. It would thus be possible to use the equipment built for a cable system with an additional transmitter and receiver on a radio channel.

A similar channel which would be very close to a cable is satellite link to a tactical terminal. A tactical station with a small antenna, transmitter, and receiver could set up a channel good enough to communicate with a central computer.

The characteristics of the system which make it attractive for tactical command and control situations are the shared use of a limited resource, namely, spectrum. Several examples of such situations might be:

1. Army battlefield reporting, inputs to data bases, outputs from data bases.

2. Shipboard use between ships - allows several ships to share the data base of each.

Another advantage of the cable type of system is that data can be encrypted fairly easily between the terminal and interface. It is also possible to randomize frequencies of transmission and reception by periodically sending frequency shift packets which defines a new set of channel frequencies. The Positive acknowledgement scheme used in the TV/Terminal system is ideal for tactical Command and Control data transmissions in a jammed environment.

11. APPENDIX A: CALCULATION OF ERROR RATES FOR CATV SYSTEM

For the sake of illustration we will consider noncoherent frequency shift keying (FSK). The error rates for coherent detection or phase shift keying, of course, would be even lower [Schwartz et al., 1966].

Let S = Signal Power
N = Noise power
N_c = Cross modulation noise power
N_r = Thermal noise power
t = Average synchronization error time
T = Bit width time

Then the signal to noise ratio is:

$$\frac{S}{N} = \frac{Sq}{N_c + N_r} = \frac{q}{(N_c/S) + (N_r/S)}$$

where

$$q = (1 - \frac{2t}{T})^2$$

Let Pe be the bit rate error probability.
Let m be the number of keying frequencies in a multiple FSK System.
Then [Schwartz et al., 1966].

$$Pe = \frac{m-1}{m} e^{\frac{-(S/N)}{2(m-1)}2}$$

We assume that each packet carries its own synchronizing bit and hence there is no need to synchronize every terminal to a master clock. Therefore, temperature, pressure, and humidity variations which have approximately the same effects at all frequencies do not enter into the calcualtion of t. The group delay variation over a six Megahertz bandwidth is less than .2 μ seconds. [Rogeness, 1972] For a 1 Megabit pulse rate $T = 1$ μ second and $t = .2$ μ second. Hence $q = .36$. Using thes formulas and numbers we have the error probabilities in Table 5.1 for the Boston complex.

Consideration of reflections, intersymbol interference and 60 cycle hum also lead to the conclusion that MATV systems and CATV systems are excellent media for packet data transmission.

Intersymbol Interference

The signal levels in a CATV system are controlled via AGC and dual pilot carriers. Ripples are kept to less than 1 db over the whole frequency band. In any case, frequency shift keying is insensitive to small amplitude variations. The effect of group delay error has already been taken into account in the use of q in the formula for error probability. The remaining source of intersymbol interference is the reflection of pulses and the effect of the reflected pulses on the transmitted data.

There are three types of disturbances due to reflections. In each case we shall see that video restrictions are certainly stringent enough to avoid any difficulties for data transmission.

1) Periodic changes of minute magnitude uniformly distributed along the cable length, the magnitude of changes being essentially equal from period to period due to the nature of the manufacturing process, cause reflections which add in phase at certain frequencies. The signal strength relationship of the reflected wave to the incident wave is referred to as structural return loss (SRL). Typical values for the magnitude of SRL are better than -26db. [Olszewski and Lubars, 1970].

Assuming that the reflected signal is always of an opposite sign to the original signal, the signal level is degraded by at most S-a where a is the amplitude of the reflected signal. The signal-to-noise ratio becomes [BTL,1971]

$$\frac{S-a}{N} = \frac{S}{N}\left(1-\frac{a}{S}\right)$$

In other words $\frac{S}{N}$ is degraded by $\left(1-\frac{a}{S}\right)$

For a reflected signal of -26db $\left(1-\frac{a}{S}\right)$ is .9975. Hence the reflection problem is completely under control.

2) A localized change or changes on the cable cause echo phenomena. Low reflection coefficients of active and passive devices and the use of directional couplers at all subscriber taps ensure that the magnitudes of reflected pulses are in the "no ghost range" of Figure A1. [Rheinfelder, 1970; Shekel, 1962; Mertz, 1953]. These are translated into critical distances for different types of cable in Figure A2. Thus, for example, considering the reflection on .412 inch cable at Channel 13 the critical distance is about 250 feet and the ratio of the magnitude of the reflected signal to the magnitude of the original signal is -23db.

3) Randomly distributed changes of random magnitude which persist throughout the cable length cause reflections which do not add in phase. These can be taken into account in noise calculations and are usually negligible.

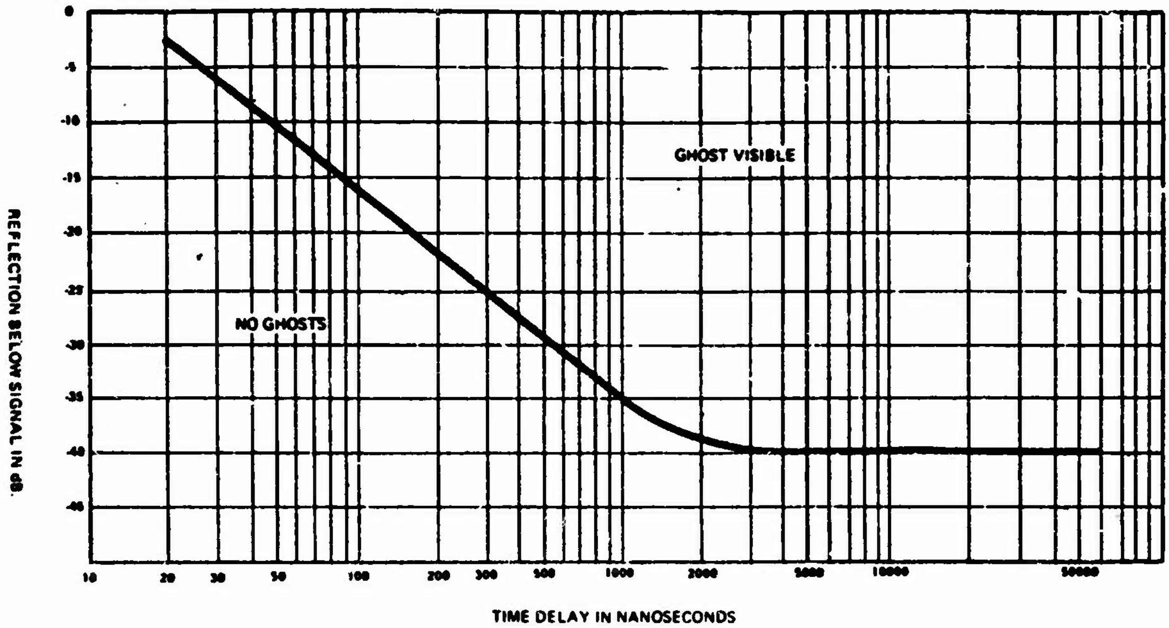


FIGURE A1 Curve showing perceptibility of ghosts

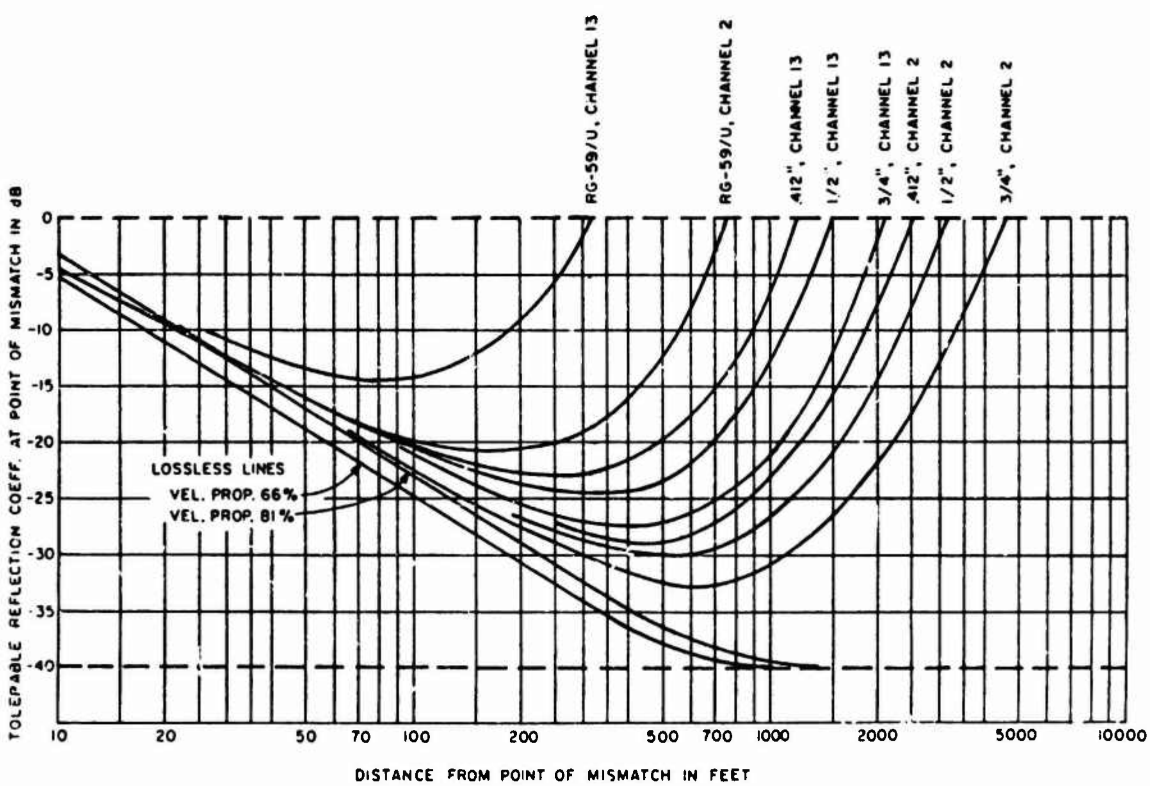


FIGURE A2 Graph for determination of critical cable lengths

Interference from Power Frequencies

Cable system amplifiers are powered by low voltage 60 Hz power through the co-axial cable. This power may be as high as 60 volts (RMS) and currents may run to 10 amperes (RMS) with peak currents even higher. There are significant harmonics of the power line frequencies present. Some amplifiers use switching mode power supplies with switching frequencies in the 10-20 KHz range. Hash from these switching regulators also finds its way into the cable. However, both the 60 cycle harmonics and hash limit only the area of very low frequencies which are generally avoided for data transmission anyway.

12. APPENDIX B: MAXIMUM NUMBER OF ACTIVE TERMINALS

In a CATV System there are separate channels allocated for upstream and downstream messages. Since the responses are longer and more frequent than the inquiries, they will limit the number of terminals.

For integers w, x, y, z, let

$450\ w$ = packet size, (no. of bits/packet)

$2x$ = rate of messages, (no. of responses/hr.)

$4y$ = message size, (number of packets/response)

$1 \times 10^{-6} z$ = pulse duration, seconds

At a 1 Megabit/sec. rate $z = 1$ and the pulse duration is 1 μsec on a binary system.

Note that the number of bits/second is

$$\frac{450w \cdot 2x \cdot 4y}{3600} = wxy \text{ which we define as } \Omega.$$

Then the number of packets/second for each terminal is

$$\lambda = \frac{8xy}{3600}$$

At a 3 Megabit/sec rate the pulse duration is .33 seconds. The corresponding average packet duration is

$$\tau = zw\ 450 \times 10^{-6} \text{ seconds}$$

The maximum number of active users per trunk in a system is given by [Abramson, 1970]

$$k_{max_u} = \frac{1}{2e\lambda\tau} \quad \text{for an unslotted system}$$

and

$$k_{max_s} = \frac{1}{e\lambda\tau} \quad \text{for a slotted system.}$$

In other words the bandwidth of an ALOHA type interactive channel is effectively reduced by a factor of 2e for an unslotted system and a factor of e for a slotted system.

Hence

$$k_{max_u} = \frac{3600 \times 10^6}{2e \cdot 3600\ wxyz}$$

$$= \frac{.184 \times 10^6}{\Omega z}$$

Similarly

$$k_{max_s} = \frac{.368 \times 10^6}{\Omega z}$$

13. APPENDIX C: FORMULAS FOR TRAFFIC IN LINKS

In order to precisely formulate the traffic requirements throughout the network, it is convenient to introduce some elementary descriptive terminology from graph theory.

We define a set of points called nodes to represent the head end, junction points, router, converter, concentrator, multiplexer and bridger locations. The nodes are represented by integers; 0 for the head end and, 1 to n for the remaining nodes, where the total number of nodes is n + 1. The transmission system including amplifiers and any single or dual cable joining two points corresponding to nodes a and b, is represented by an undirected arc [a,b]. The arcs and nodes together comprise a graph G = (N,A) where N is the set of nodes and A is the set of arcs. G is a tree, in our case, that is a graph joining all nodes and containing no "cycles".

Let $P_{i,j}$ be the "path" in G from node i to node j, that is the sequence of nodes and arcs listed in order in tracing a route from i to j. If node 0 is the head end, then path $P_{0,k}$ is a path from the head end to node k. If node i precedes node j in $P_{0,k}$ we say that node i is upstream of node j and node j is downstream of node i.

$$i = u(j)$$
$$j = d(i)$$

Let k=d(j), and i=d(k) or i=k then we write;

$$i = d(j,k)$$

$$D(j,k) = \{ i \mid i = d(j,k) \}$$

Let D(j) be the set of all downstream points of j.

$$D(j) = \{ i \mid j=d(i) \}$$

Let A x B be the cartesian product of the sets A and B, that is, $A \times B = \{(a,b) \mid a \in A, b \in B\}$

Let I(a,b) be the number of inquiries originating at node a addressed to node b.

Let R(a,b) be the number of responses originating at node a addressed to node b.

Then define:

$$I(A,B) = \sum_{(a,b) \in (A \times B)} I(a,b)$$

$$R(A,B) = \sum_{(a,b) \in (A \times B)} R(a,b)$$

$$M(A,B) = I(A,B) + R(A,B)$$

<u>EXAMPLE</u>:

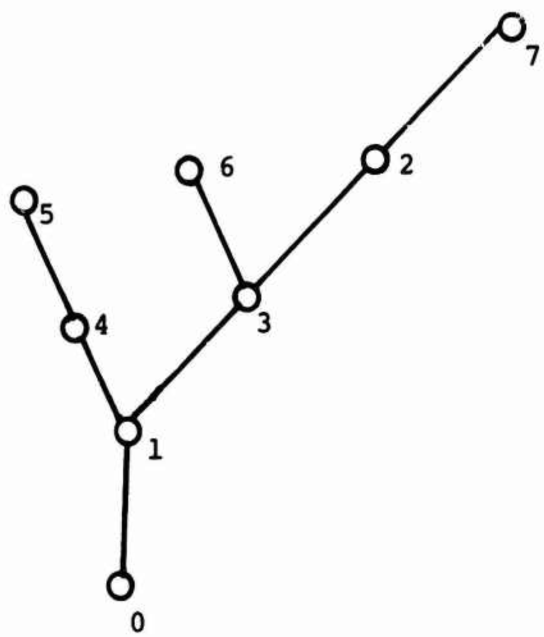


Figure C.1

$N = \{0, 1, 2, 3, 4, 5, 6, 7\}$

$2 = u(3)$

$2 = u(1)$

$2 = u(1,3)$

$D(1,3) = \{2, 6, 7\}$

$D(1) = \{2, 3, 4, 5, 6, 7\}$

For $A = \{4,5\}$ and $B = \{2,3\}$

$$M(A,B) = I(4,2) + I(4,3) + I(5,2) + I(5,3)$$
$$+ R(4,2) + R(5,3) + R(5,2) + R(5,3)$$

In this section we present formulae for the link traffic in terms of the location of concentrators, multiplexers, converters and routers. We first assume there are only converters and routers. We use the notation developed in Appendix C with intuitive explanations of the significance of the notation.

Let us consider the arbitrary junction in the network and designate it by the integer i. We are interested in the traffic in links downstream from i.

There are three cases to consider:

A. There is local routing at i.

B. There is forward routing at i.

C. There is no routing at i.

The notation used is defined precisely in the terminology of graph theory in Appendix C. A simple intuitive explanation is given in this section.

We assign the integer 0 to represent the head end and we assign a unique integer to every junction, bridger* multiplexer, concentrator and converter location in the system. Let N represent the integers corresponding to all these locations. We wish to consider the traffic in a specific link downstream from the integer i representing the junction under consideration. Let i' represent the integer corresponding to the first point downstream from i. We wish to know the traffic in the link between i and i', [i,i']. The traffic in [i,i'] depends upon the local routers downstream from i and the forward routers upstream from i. We therefore introduce some special terminology to represent these points. Trace any path from i in the downstream direction so the path contains i'. The first local router on this path (aside from i itself) is designated $t_{i,\ell}$ where ℓ is the number of the point at which the router is located. If there is no local router on the path, then the last point on the path is called $t_{i,\ell}$. Next, trace the path from i upstream to the head end; let r_i be the first forward router encountered. Let r'_i be the first point downstream of r_i on the path. As an example of this terminology, i, r_i, r'_i, $t_{i,1}$ and $t_{i,2}$ are shown in one case in Figure C2. Note,

*Bridgers are amplifiers which feed into feeder cable from which customer taps and drop lines emenate.

in particular, that even if point e is a local router, it is not labelled $t_{i,e}$ and even if point d is a forward router, it is not labelled r_i.

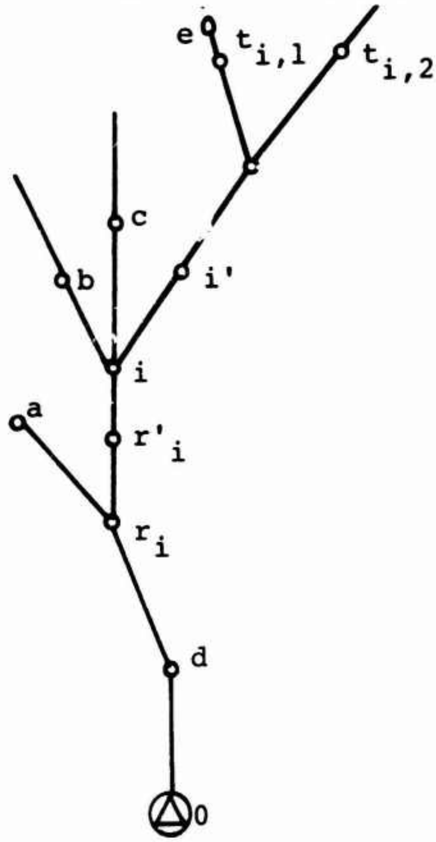


Figure C2

We next introduce some general terminology to be able to describe sets of points downstream from specified points along possibly specified paths. Let D(a,a') be all points in N downstream of a on the paths containing a'. Let D(a) be all points in N downstream from a. For sets of points A and B chosen from N, I(A x B) is the number of inquiries directed from terminals fed from bridgers in A to terminals fed by bridgers in B and R(A x B) the number of responses directed from points in A to points in B. The total number of messages is M(A,B) = I(A,B) + R(A,B). Thus, for example, M(N x D(i,i')) is the number of messages directed from any point to any point upstream of i on the route containing the router or terminating bridger i'. Finally, we assume:

N_0 = [0], where the head end is point 0,

$N_1 = N-N_0$, that is all points except the head end,

N_t = N, that is all points.

Then for y=0 the expressions below give the usage of link [i,i'] due to central traffic.

For y=1 the expressions below give the usage of link [i,i'] due to local traffic.

For y=t the expressions use the total traffic to and from all nodes N, including central traffic and local traffic.

We first give the expressions in the case in which there are no converters. Then all forward messages are at ω_f and all reverse messages are at ω_r.

Regardless of the routing at i the traffic in the reverse link [i,i'] is given by:

$$M(D(i,i') \times N_y)) - \sum_{\ell} M(D(t_{i,\ell}) \times D(t_{i,\ell})) \qquad \text{(C.1a)}$$

The expression $M(D(t_{i,\ell}) \times D(t_{i,\ell}))$ is the local traffic which is prevented by the router at $t_{i,\ell}$ from appearing upstream of $t_{i,\ell}$. The summation gives all such traffic from all local routers downstream of points along the path containing the link [i,i']. Since this summation appears repeatedly, we introduce an abbreviation for it, L(i,i') to indicate the local traffic downstream from i and i' which does not reach i. Rewriting (C.1a) we then have,

$$M(D(i,i') \times N_y)) - L(i,i') . \qquad \text{(C.1b)}$$

The traffic in the forward link [i,i'] is given for the three cases by the following expressions:

A. <u>Local</u> <u>Routing</u> <u>at</u> <u>i</u>:

$$M(N_y \times D(r_i,r'_i)) - L(i,i') \qquad \text{(C.2)}$$

B. <u>Forward Routing at i</u>:

$$M(N_y \times D(i,i')) - L(i,i') \qquad \text{(C.3)}$$

C. <u>No Routing at i</u>:

$$M(N_y \times D(r_i, r'_i)) - L(i,i') \qquad \text{(C.4)}$$

As an example of the use of the notation, for y=0 the expression in (C.1) indicates that the traffic in the reverse link is given by the inquiries from the terminals downstream from i on the routes contains i' directed to the head end plus the responses directed from the same terminals to the head end less the inquiries and responses directed from terminals downstream of $t_{i,\ell}$ to other terminals downstream ot $t_{i,\ell}$.

We now consider the addition of converters. For links which are downstream of converters, all messages are at ω_r and ω_f and the above formulae are unchanged. For the remaining links messages are carried at ω_r, ω'_r, ω_f and ω'_f and the formulae must be modified.

The expressions below give the traffic in link [i,i'] at the frequencies ω'_r and ω'_f. We introduce a terminology for concentrators similar to that used for local routers. Trace any path from i in the downstream direction so the path contains i'. The first concentrator is called $c_{i,\ell}$ if it is located at point ℓ .

Regardless of the routing at i, the traffic in the reverse link [i,i'] at ω'_r is:

$$\sum_a M(D(c_{i,a}) \times N_y)) - \Bigg[\sum_{\substack{t_{i,b} \text{ is downstream of} \\ \text{a concentrator } c_{i,c}}} M(D(t_{i,b})) \times D(t_{i,b})) + \sum_{\substack{c_{i,a} \text{ is downstream} \\ \text{of } t_{i,b}}} M(D(c_{i,a})) \times D(t_{i,b})) \Bigg] \qquad \text{(C.5a)}$$

The terms in brackets give the local traffic which does not reach the link $[i,i_j]$ because of a local router at $t_{i,b}$. The expression is more complicated than before, because the concentrator may be in two different positions with respect to the router. The first summation in brackets corresponds to the situation in Figure C3 (a) where the router is downstream of the concentrator. The second summation corresponds to the situation in Figure C3 (b) where the concentrator is downstream of the router.

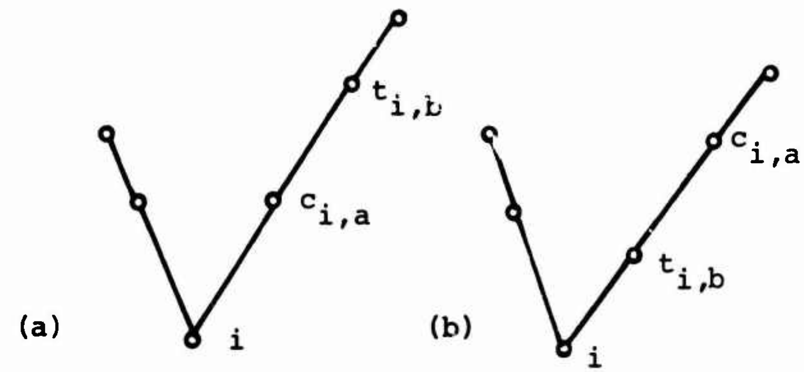


Figure C3

Since the term in brackets appears repeatedly, we introduce an abbreviation for it, $L_c(i,i')$ the local traffic with concentrators. Rewriting (C5a) we have,

$$\sum_a M(D(c_{i,a}) \times N_y)) - L_c(i,i') \qquad \text{(C.5b)}$$

The traffic at ω'_f in the forward link $[i,i']$ is given by the expressions below. In these expressions $A \cap B$ denotes the points common to A and B.

A. <u>Local Routing at i</u>:

$$\sum_a M(N_y \times D(c_{i,a}) \cap D(r,r'_i)) - L_c(i,i') \qquad \text{(C.6)}$$

B. Forward Routing at i:

$$\sum_a M(N_y \times D(c_{i,a}) \cap D(i,i')) - L_c(i,i') \qquad (C.7)$$

C. No Routing at i:

$$\sum_a M(N_y \times D(c_{i,a}) \cap D(r,r'_i)) - L_c(i,i') \qquad (C.8)$$

To obtain the traffic in these lines at ω_r and ω_f subtract the expressions in (C.6) - (C.8) from the corresponding expressions in (C.2) - (C.4).

The effect of multiplexing at converters is to increase the data rate at ω'_r and ω'_f. The effect of concentration at the converters is to isolate the links feeding upstream into the concentrator; that is each line operates separately at the low data rate, and their combined traffic is handled only at the higher data rate.

14. APPENDIX D: DESIGN OF BOSTON SYSTEM

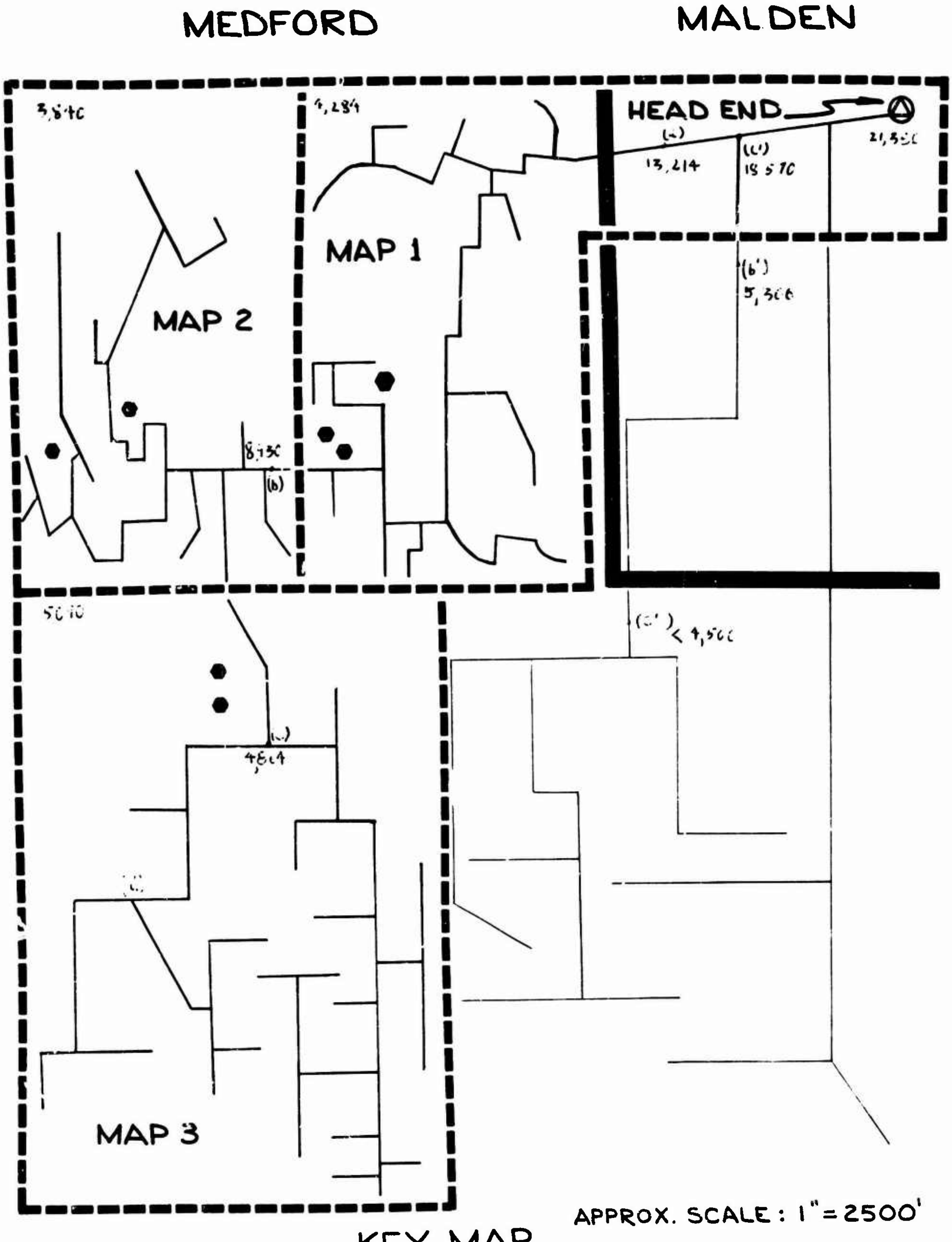
To determine the usefulness of the various options and devices we have considered, we will apply them to the design of an interactive packet data system for Medford, Massachusetts, a section of the Boston CATV complex. In Figure D.1.b a branch of the trunk is drawn for Medford, Massachusetts. The triangles represent bridger amplifiers. These amplifiers feed into feeder cable and extender amplifiers with customer taps and drop lines emanating from the feeder cable. In the design for Medford, the feederbacker arrangement is used so that the trunk lines are dual cable and the amplifiers are two-way units as shown in Figure 4.4. The hexagons represent local origination stations.

The feeder system emanating from a given bridger amplifier is called a cluster. The number next to each amplifier gives the number of terminals in the cluster associated with that amplifier. The average number of terminals per cluster is 137 with complete coverage of all homes.

We now consider the design of the Boston interactive packet cable system by focusing our attention on one trunk in the Medford area. We assume that the average traffic per terminal is 40 bits/sec. We conservatively assume that this is the rate for inquiries as well as responses. We assume that in the design the data from the terminals is 100 kilobits/sec; the upconverted rate is 1 Megabit/sec. In appendix B, we obtained the results in Table D.1 showing the number of active terminals that can be supported on a trunk at each data rate.

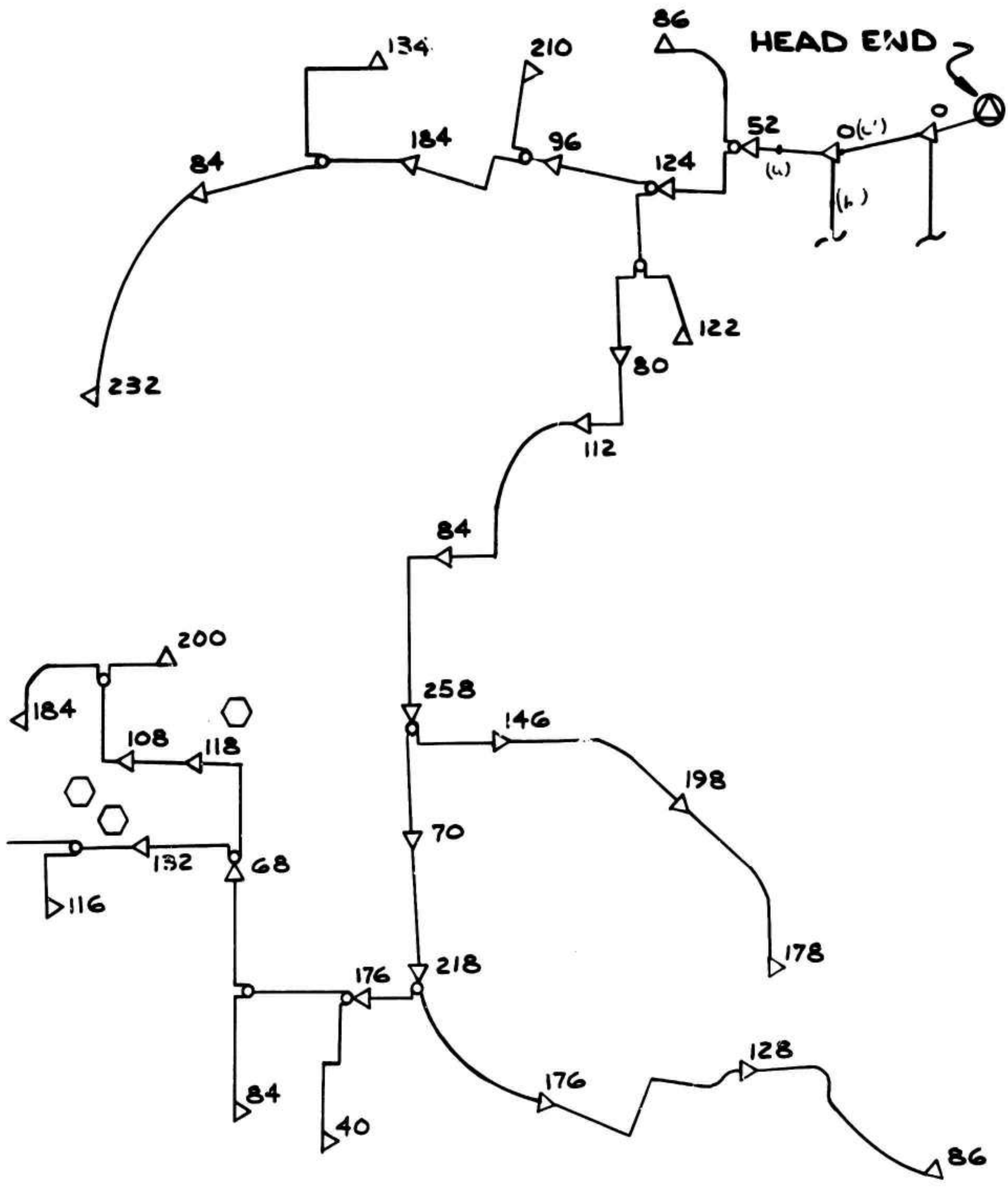
data rate \ type of system	slotted system	unslotted system
1 Megabit/sec	9,000	4,500
100Kilobit/sec	900	450

Table D.1 Number of Active Users Per Trunk



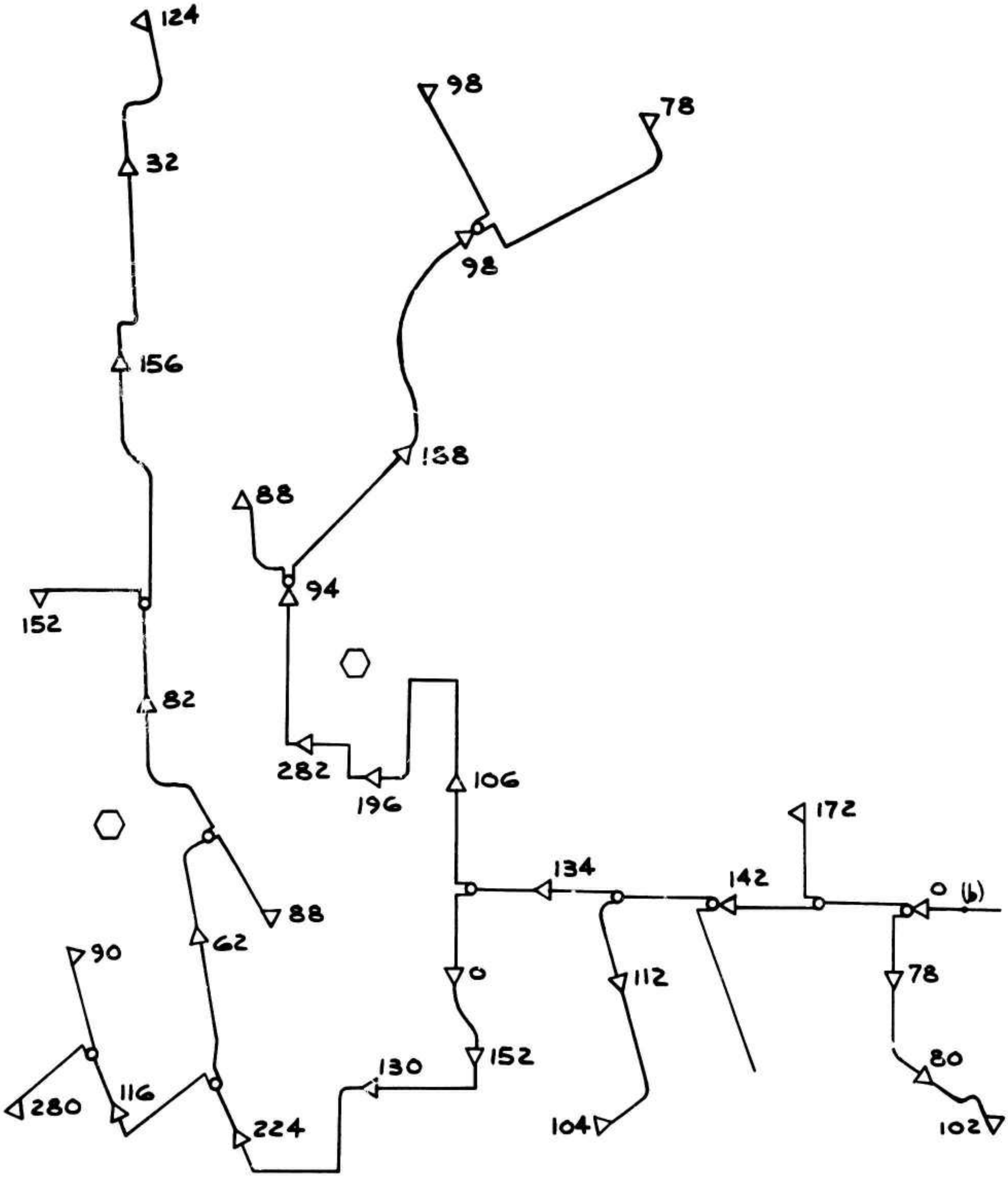
KEY MAP

Figure D.1a



MAP 1

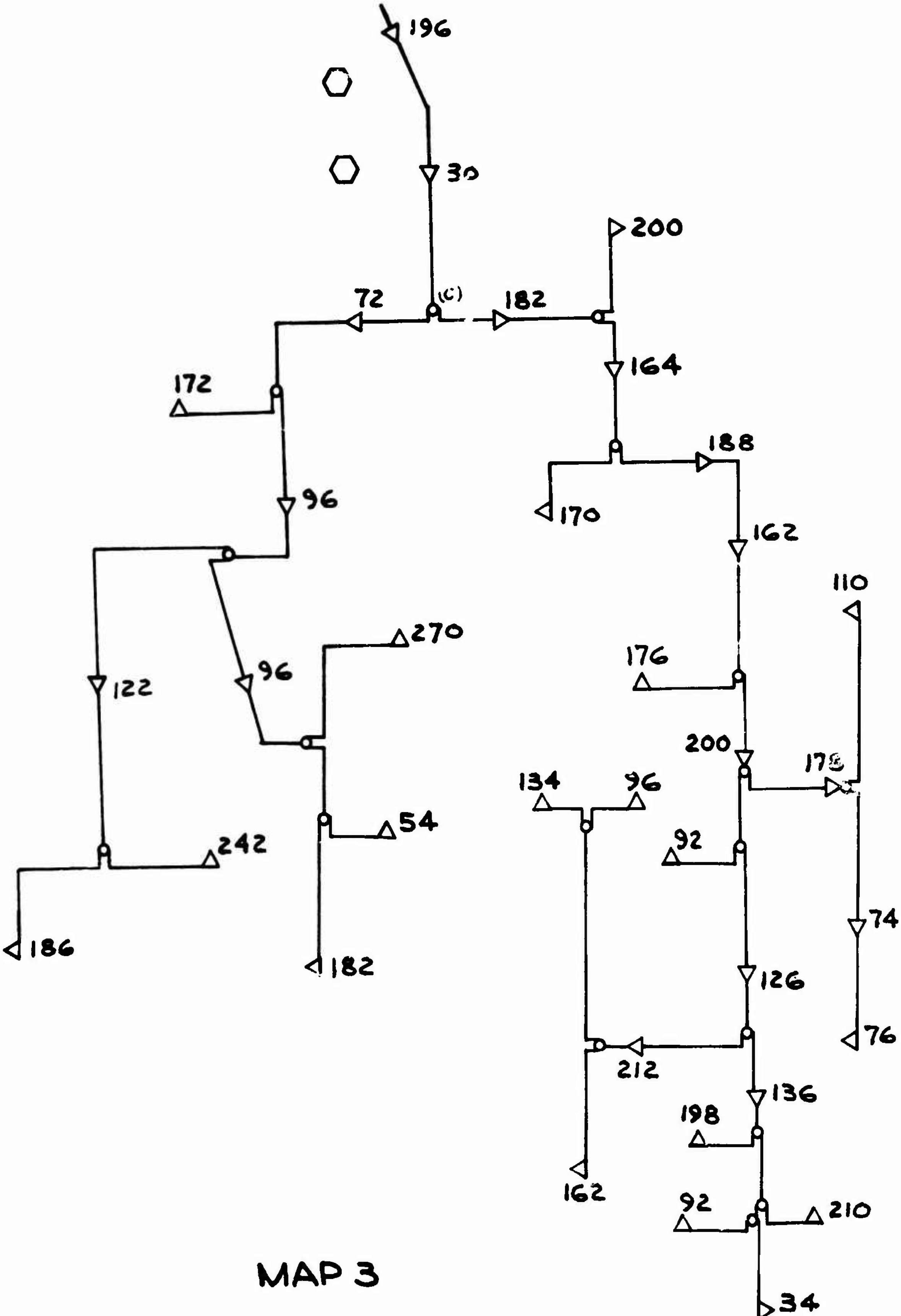
Figure D.1b



MAP 2

Figure D.1c

13.98



MAP 3

Figure D.1d

We first design the Medford trunk for the case in which there is no local traffic. We design the system for 1%, 3%, 10% and 15% active users.

Since we do not yet have a firm idea of relative costs of converters, concentrators, multiplexers and routers, we are in no sense optimizing the design. We are merely presenting feasible designs to demonstrate the wide range and flexibility achieved by combinations of a few devices. The designs are easily described by simply indicating the location of the various devices on the map in terms of an alphabetic label on the map. To aid in visualizing the design, the number in the rectangle beside the letter (on the key maps) indicates the population downstream from that point. The designs are as shown in Table D.2.

% active terminals	DESIGN Slotted	Unslotted
1%	no devices	no devices
3%	no devices	converter at (a)
10%	compressors at (b) & (b')	compressors at (c'') and concentrators at (b), (c), & (c')
15%	compressors at (c'') and concentrators at (b), (c) & (c')	

Table D.2 Feasible Designs for Central Transmission Mode

Suppose now that X% of the traffic from every terminal is local traffic whose destination is uniformly distributed throughout the network and suppose there are no routers, converters, compressors or concentrators. Then every local message must go from the originating terminal to the head end on reverse links and from the head end to the destination terminal on reverse links. Therefore, each local inquiry traverses both forward and reverse links instead of only reverse links as for central transmission. Similarly, each response occupies both forward and reverse links rather than only forward links as for central transmission. The net effect is that for X% local traffic, traffic in the branches is the same as if all the traffic were in the central transmission mode but the population were increased by X%. Hence, the previous design procedures are still applicable and the design already given can be used for local transmission superimposed on the central transmission provided the number of active users are appropriately adjusted. For example, the design for 10% active terminals in the central transmission mode can be used for 8% active terminals with 25% of the traffic on a local basis. In addition, as an example, if there were heavy traffic among the stations downstream of point (d), this might be handled by a local router at (d).

15. APPENDIX E: SPECIFICATIONS FOR FILTERS AND DATA AUGMENTATION DEVICES

BAND SEPARATION FILTERS:

The most common two-way CATV configuration at present is the simplest single-trunk subchannel split in Figure E1 . A much more sophisticated and flexible system is the dual trunk "feederbacker" arrangement in Figure E2 , which is the system used in Boston. For both these systems we must have an inexpensive, simple method of interfacing with a converter or router to be added after the construction of the CATV system has been completed. Furthermore, the interface must be readily adaptable to other possible two-way configurations.

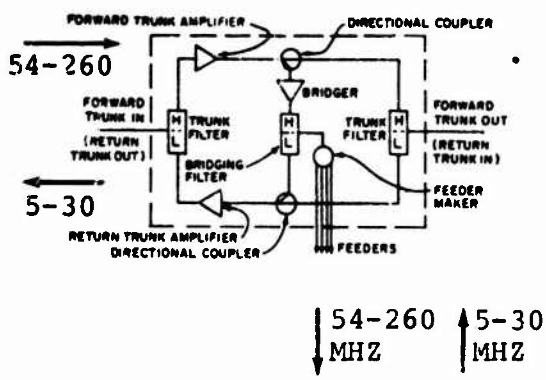


Figure E1 Simplest two-way configuration

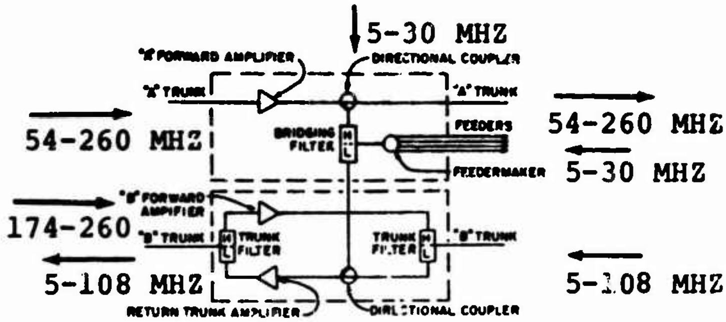


Figure E2 "Feederbacker" Configuration

The problem is readily solved by use of a pair of diplex filters as shown in Figure E3 or in some cases by a pair of triplex filters as shown in Figure E4. The units can be housed in casings a few inches in each dimension and are readily installed on line. The passbands of the filters can be set for the particular system transmission frequencies. The conditions for perfect band separation are that the passbands of the filter F_f, F_r and F_t do not overlap and that the passbands cover the full frequency range of the channel. Thus, for the two-way configuration in Figure 4.1, the triplex filter is used with the following passbands.

F_f passes the signals at carrier frequencies ω_f and ω'_f

F_r passes the signals at carrier frequencies ω_r and ω'_r

F_t passes all frequencies not passed by F_f and F_r.

F_f therefore isolates the forward channel for conversion or routing and F_r, the reverse channel.

For the feederbacker configuration in Figure E.2, the diplex filter is used for line A with F_f and F_t specified as follows:

F_f passes the signals at carrier frequencies ω_f and ω'_f

F_t passes all frequencies not passed by F_f.

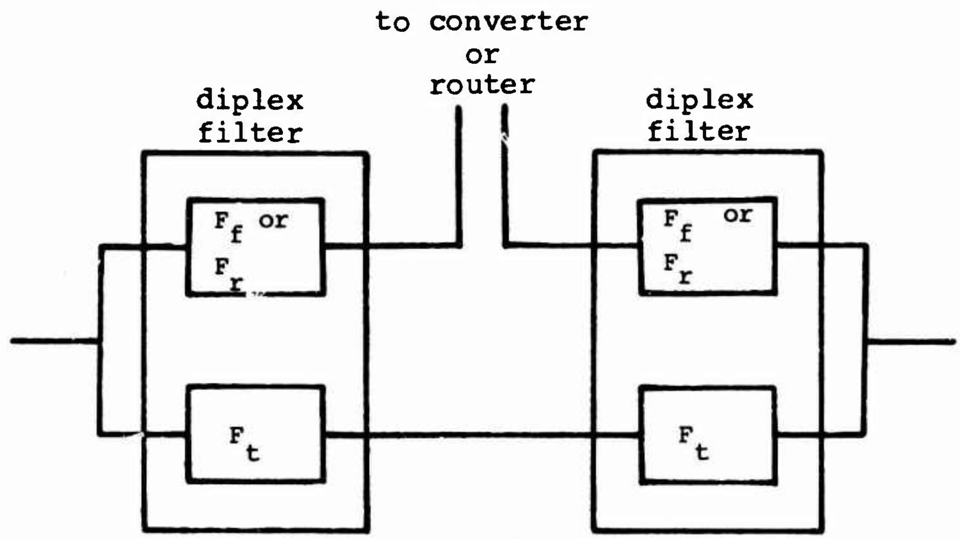


Figure E3 Diplex Band Separation Filter

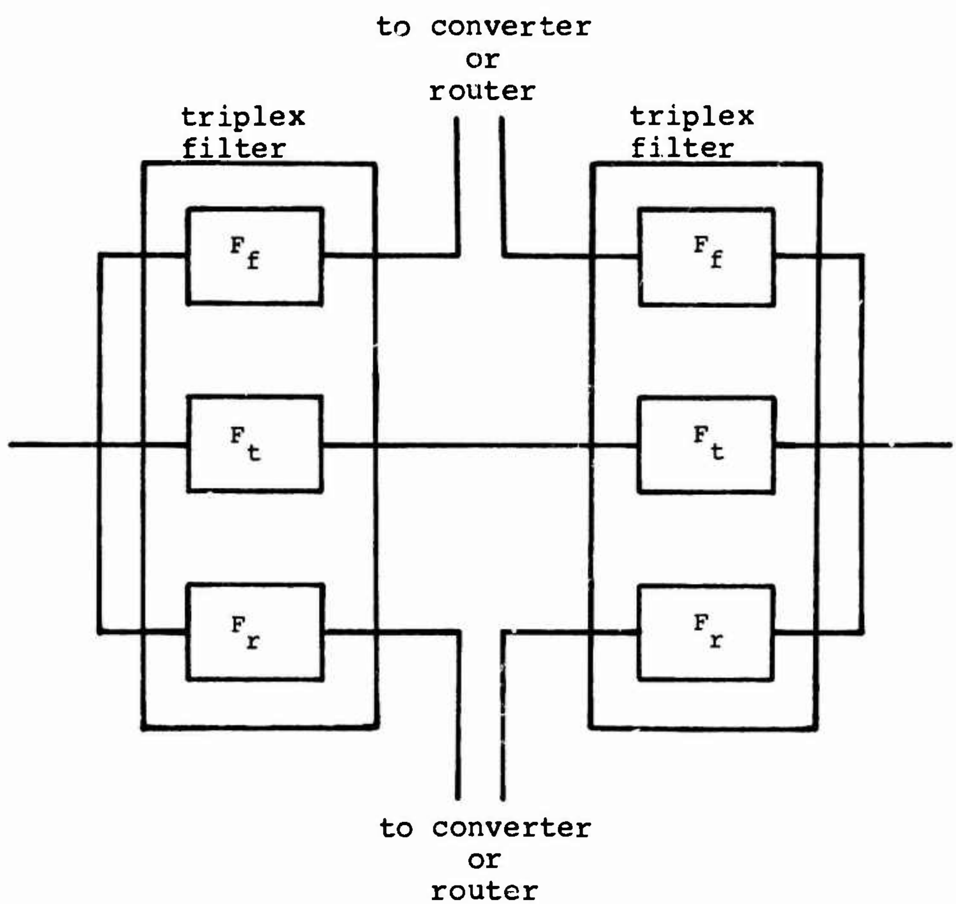


Figure E4 Triplex Band Separation Filter

For standard CATV circuits, each band separation unit adds a flat insertion loss of at most, 1.75 db. To make up this loss, the gain of the subsequent amplifier might be increased by 3.5 db. With most lines of equipment, this simply means reducing the value of the input attenuation pad by 3.5 db. However, if the attenuator is already set at too low a value, then the 3.5 db can be made up by increasing the size of the cable feeding into the band separation filter. For example, for a broad range of foam dielectric coaxial cables, the difference between the losses of .5 inch and .75 inch coaxial cable at channel 13 is about .4 db/100 ft. Thus, less than 900 feet of cable would have to be converted from .5 inch to .75 inch. This change would necessitate that at most ten subscriber taps be changed in value and that the equalizer for one amplifier be adjusted--all minor adjustments to an existing system.

The required specifications for the individual filters are consistent with typical parameters for CATV bandpass and notch filters. For example, the characteristics below are more than adequate for the data system and are derived from off-the-shelf CATV filters. [Jerrold Filter Specifications]

FILTER SPECIFICATIONS

Center Frequency	in the range 30-260 MHZ
Bandwidth	5.5 MHZ at .5 db points
Insertion Loss	1.25 at low VHF to 1.75 at high VHF
Impedance	75 Ω ,-23 db reflection coefficient
Passband Group Delay Variation	10 nanoseconds max
Passband Ripple	.01 db max
Stopband Attenuation	at least 30 db at .5 db bandedge plus 1 MHZ

Table E1

CONVERTERS:

The converter specifications are in Table E2.

function	convert carrier frequencies in VHF range
input impedence	75Ω ,-23 db reflection coefficient
output impedence	75Ω ,-23 db reflection coefficient
oscillator accuracy	.005%
conversion gain	5 db min
noise figure	10 db in analog mode
amplitude variation	± .75 db over 6MHZ Bandwidth

Table E2.

COMPRESSORS; CONCENTRATORS AND ROUTERS

The compressors, concentrators, and routers all operate digitally at the same frequencies and data rates and differ only in details and relative memory and logic requirements. The specifications for all three are given in Table E3.

	Compressor	Concentrator	Router
number of inputs	≤3	≤3	≤3
number of outputs	≤3	≤3	≤3
input data rate	100KB/sec	100KB/sec	100KB/sec or 1 Megabit/sec
output data rate	1 Megabit/sec	1 Megabit/sec	100KB/sec or 1 Megabit/sec
throughput (packets)	1 packet/11.5 msec	1 packet/~11.5msec	1 packet/11.5msec for 100 Kbit operation
packet length	1150 bits/packet	1150 bits/packet	1150 bits/packet
carrier frequency	VHF range	VHF range	VHF range
core memory requirement	two packets	six packets	two packets

Table E3

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