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RADIAL PRESSURE ON A GUN LAUNCHED MOTOR  
CASE DUE TO SLUMPING PROPELLANT

David Salinas, et al

Naval Postgraduate School  
Monterey, California

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## NOTATION

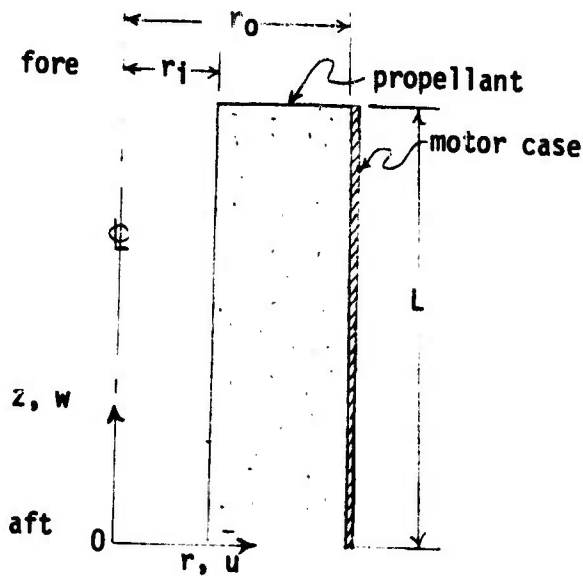
$a_m$	maximum rigid body acceleration of the projectile
A, B	material constants, eq. (8)
$C_i$	constants due to body forces
e	dilatation
$e_{rr}, e_{zz}, e_{\theta\theta}$	normal strain components
E	Young's modulus of elasticity
$f_z$	body force in the z direction
$F_{ij}$	coefficients of eqs. (21) from Rayleigh-Ritz method
G	shear modulus
L	length of propellant
$r, r_i, r_o$	radius, inner radius, outer radius
T	total potential energy
u	displacement along r axis
U	elastic strain energy
V	potential energy of external forces
w	displacement along z axis
$\alpha, \beta, \gamma, \delta, \eta, \lambda, \xi, \zeta$	Rayleigh Ritz undetermined coefficients
$\gamma_{rz}$	shear strain component
$\rho$	weight per unit volume
$\sigma_{zz}, \sigma_{rr}, \sigma_{\theta\theta}$	normal stress components
$\tau_{rz}$	shear stress component

## INTRODUCTION

The launch phase of a gun-launched rocket is associated with high acceleration loading of up to 10,000 g. As a result of the high acceleration the motor case portion of the rocket, which contains the propellant, is subjected to axial and lateral loads. The lateral loads are associated with the deformation of the contained propellant. This "slumping propellant" behavior is the concern of the present investigation.

## ANALYSIS

The propellant-motor case system may be idealized as a two-dimensional system because of axial symmetry. With cylindrical coordinates,  $r$  and  $z$ , a typical section is shown below.



$u$  = displacement in the  $r$  direction

$w$  = displacement in the  $z$  direction

$r_i$  = inside radius of propellant

$r_o$  = outside radius of propellant

$L$  = length of propellant

The particular problem of the unbonded propellant is considered here. The stiff motor case shell is assumed to completely restrain axial displacement at the aft end ( $r, 0$ ) as well as radial displacement along the outer radius

$(r_o, z)$  with the resultant boundary conditions,

$$w(r, 0) = 0 \quad \tau_{rz}(r, 0) = 0 \quad (1a)$$

$$u(r_o, z) = 0 \quad \tau_{rz}(r_o, z) = 0 \quad (1b)$$

The remaining sides  $(r, L)$  and  $(r_i, z)$  are stress free, that is

$$\sigma_z(r, L) = 0 \quad \tau_{rz}(r, L) = 0 \quad (1c)$$

$$\sigma_r(r_i, z) = 0 \quad \tau_{rz}(r_i, z) = 0 \quad (1d)$$

The acceleration of the propellant subjects every particle to the body force

$$f_z = -\rho a_m \quad (2)$$

where  $\rho$  is the weight/unit volume, and  $a_m$  is the maximum acceleration in g's.

A solution to the problem is obtained by the Rayleigh-Ritz method. According to this method, an approximate solution satisfying the displacement boundary conditions is formed. The unknown coefficients of the solution are determined by minimizing the total potential energy of the system.

Here the displacement fields

$$\begin{aligned} u(r,z) &= (L-z)(\alpha + \beta r + \gamma r^2 + \delta r^3) = (L-z)f(r) \\ w(r,z) &= z(L - \frac{z}{2})(\eta + \lambda r + \xi r^2 + \zeta r^3) = z(L - \frac{z}{2})g(r) \end{aligned} \quad (3)$$

are assumed. The linear  $z$  term in the  $u$  displacement and the quadratic  $z$  term in the  $w$  displacement were chosen to satisfy the following conditions:

i) Results from the Rohm and Haas finite element solution,

ref. 1, Appendix B, for the accelerated, unbonded propellant

show  $\sigma_r$  and  $\sigma_z$  to be linear functions of  $z$ . Note that Equations (3) give this condition, i.e.

$$\begin{aligned} \sigma_r &= \frac{E}{1-\nu^2} \left[ \frac{\partial u}{\partial r} + \nu \left( \frac{u}{r} + \frac{\partial w}{\partial z} \right) \right] = \frac{E}{1-\nu^2} \left[ \frac{\partial f}{\partial r} + \nu \left\{ \frac{f(r)}{r} + g(r) \right\} \right] (L-z) \\ \sigma_z &= \frac{E}{1-\nu^2} \left[ \frac{\partial w}{\partial z} + \nu \left( \frac{\partial u}{\partial r} + \frac{u}{r} \right) \right] = \frac{E}{1-\nu^2} \left[ g + \nu \left\{ \frac{\partial f}{\partial r} + \frac{f(r)}{r} \right\} \right] (L-z) \end{aligned}$$

ii) The propellant is assumed to be nearly incompressible, and thus

linearly varying compression along the  $z$  coordinate causes the propellant to move towards the centerline since it is restrained by the case from moving outward, i.e.  $u \leq 0$  for all  $z$ , and  $u = 0$  for  $z = L$ .

This assumed displacement field is not valid for the bonded propellant.

The equation for  $w$  satisfies the displacement boundary condition  $w(r,0) = 0$ .

The displacement boundary condition,  $u(r_0,z) = 0$  is satisfied by setting

$$\alpha = -\beta r_0 - \gamma r_0^2 - \delta r_0^3 \quad (4)$$

The total potential energy of the system,  $T$ , resulting from the displacement fields given by equations (3) must be formed. Denoting the elastic strain energy as  $U$  and the potential energy of external forces as  $V$ , we have

$$T = U + V \quad (5)$$



Determination of the elastic strain energy  $U$  proceeds as follows.

According to the strain-displacement relations

$$\begin{aligned}
 e_r &= \frac{\partial u}{\partial r} = (L - z)(\beta + 2\gamma r + 3\delta r^2) \\
 e_\theta &= \frac{u}{r} = (L - z)\left(\frac{\alpha}{r} + \beta + \gamma r + \delta r^2\right) \\
 e_z &= \frac{\partial w}{\partial z} = (L - z)(\eta + \lambda r + \xi r^2 + \zeta r^3) \\
 \gamma_{rz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} = -(\alpha + \beta r + \gamma r^2 + \delta r^3) \\
 &\quad + z\left(\frac{L}{2} - z\right)(\lambda + 2\xi r + 3\zeta r^2)
 \end{aligned} \tag{6}$$

and the stress-strain relations,

$$\begin{aligned}
 \sigma_r &= Ae_r + B(e_z + e_\theta) \\
 \sigma_\theta &= Ae_\theta + B(e_r + e_z) \\
 \sigma_z &= Ae_z + B(e_r + e_\theta) \\
 \tau_{rz} &= G\gamma_{rz}
 \end{aligned} \tag{7}$$

where

$$\begin{aligned}
 A &= \frac{(1 - \nu) E}{(1 + \nu)(1 - 2\nu)} \\
 B &= \frac{\nu E}{(1 + \nu)(1 - 2\nu)}
 \end{aligned} \tag{8}$$

and

$$G = \frac{E}{2(1 + \nu)} \tag{9}$$

Substituting equations (6) into (7) yields stress-displacement equations. The elastic strain energy of the system is given by

$$U = \frac{1}{2} \iiint \sigma_{ij} e_{ij} dV = \frac{1}{2} \iiint (\sigma_r e_r + \sigma_\theta e_\theta + \sigma_z e_z + \tau_{rz} \gamma_{rz}) dV \quad (10)$$

where the integration is over the volume of the propellant. The independence with the  $\theta$  coordinate gives

$$U = 2\pi \cdot \frac{1}{2} \int_{r_0}^{r_i} \int_0^L (\sigma_r e_r + \sigma_\theta e_\theta + \sigma_z e_z + \tau_{rz} \gamma_{rz}) r dr dz \quad (11)$$

For convenience, let

$$U_a = \pi \iint \sigma_r e_r r dr dz$$

$$U_b = \pi \iint \sigma_\theta e_\theta r dr dz \quad (12)$$

$$U_c = \pi \iint \sigma_z e_z r dr dz$$

$$U_d = \pi \iint \tau_{rz} \gamma_{rz} r dr dz$$

then

$$U = U_a + U_b + U_c + U_d \quad (13)$$

Substituting the stress-displacement, and strain displacement relations into equation (12), and integrating, we obtain

$$\begin{aligned}
U_a = \frac{\pi L^3}{3} & \left[ A \left\{ \frac{1}{2} \beta^2 \lambda^2 + \frac{1}{2} \gamma^2 \lambda^4 + \frac{3}{2} \delta^2 \lambda^6 + \frac{4}{3} \beta \gamma \lambda^3 + \frac{9}{4} \beta \delta \lambda^4 \right. \right. \\
& \left. \left. + \frac{12}{5} \gamma \delta \lambda^5 \right\} \right. \\
& + B \left\{ \alpha \beta \lambda + \frac{\lambda^2}{2} (\beta \eta + \beta^2 + 2\gamma \alpha) + \frac{\lambda^3}{3} (3\beta \gamma \right. \\
& \left. + 2\gamma \eta + 3\delta \alpha + \beta \lambda) + \frac{\lambda^4}{4} (4\beta \delta + 2\gamma^2 + 3\delta \eta \right. \\
& \left. + 2\gamma \lambda + \beta \xi) + \frac{\lambda^5}{5} (5\gamma \delta + 3\delta \lambda + \beta \zeta \right. \\
& \left. + 2\gamma \xi) + \frac{\lambda^6}{6} (3\delta^2 + 3\delta \xi + 2\gamma \zeta) + \frac{3}{7} \delta \zeta \lambda^7 \right\} \Big]_{n_i}^{n_o} \quad (14)
\end{aligned}$$

$$\begin{aligned}
U_b = \frac{\pi L^3}{3} & \left[ A \left\{ \alpha^2 \lambda \eta \lambda + 2\alpha \beta \lambda + \frac{\lambda^2}{2} (\beta^2 + 2\alpha \gamma) \right. \right. \\
& \left. \left. + \frac{\lambda^3}{3} (2\alpha \delta + 2\beta \gamma) + \frac{\lambda^4}{4} (\gamma^2 + 2\beta \delta) \right. \right. \\
& \left. \left. + \frac{2}{5} \gamma \delta \lambda^5 + \frac{1}{6} \delta^2 \lambda^6 \right\} \right. \\
& + B \left\{ \lambda (\alpha \eta + \alpha \beta) + \frac{\lambda^2}{2} (2\alpha \gamma + \beta \eta + \beta^2 + \alpha \lambda) \right. \\
& \left. + \frac{\lambda^3}{3} (3\alpha \delta + 3\beta \gamma + \gamma \eta + \beta \lambda + \alpha \xi) \right. \\
& \left. + \frac{\lambda^4}{4} (4\beta \delta + 2\gamma^2 + \delta \eta + \gamma \lambda + \beta \xi + \alpha \zeta) \right. \\
& \left. + \frac{\lambda^5}{5} (5\gamma \delta + \delta \lambda + \gamma \xi + \beta \zeta) + \frac{\lambda^6}{6} (3\delta^2 + \delta \xi + \gamma \zeta) \right. \\
& \left. + \delta \zeta \frac{\lambda^7}{7} \right\} \Big]_{n_i}^{n_o} \quad (15)
\end{aligned}$$

$$\begin{aligned}
U_c = \frac{\pi L^3}{3} & \left[ A \left\{ \frac{\lambda^2}{2} \eta^2 + \frac{2}{3} \eta \lambda \lambda^3 + \frac{\lambda^4}{4} (\lambda^2 + 2\eta \xi) + \frac{\lambda^5}{5} (2\lambda \xi + 2\eta \zeta) \right. \right. \\
& \left. \left. + \frac{\lambda^6}{6} (\xi^2 + 2\lambda \zeta) + \frac{3}{7} \xi \zeta \lambda^7 + \zeta^2 \frac{\lambda^8}{8} \right\} \right. \\
& + B \left\{ \alpha \eta \lambda + \frac{\lambda^2}{2} (2\beta \eta + \alpha \lambda) + \frac{\lambda^3}{3} (3\gamma \eta + 2\beta \lambda + \alpha \xi) \right. \\
& \left. + \frac{\lambda^4}{4} (4\delta \eta + 3\gamma \lambda + 2\beta \xi + \alpha \zeta) + \frac{\lambda^5}{5} (4\delta \lambda + 3\gamma \xi \right. \\
& \left. + 2\beta \zeta) + \frac{\lambda^6}{6} (4\delta \xi + 3\gamma \zeta) + \frac{4}{7} \delta \zeta \lambda^7 \right\} \Big]_{n_i}^{n_o} \quad (16)
\end{aligned}$$

$$\begin{aligned}
U_d = \pi G \left[ L \left\{ \frac{1}{2} \alpha^2 \lambda^2 + \frac{2}{3} \alpha \beta \lambda^3 + \frac{\lambda^4}{4} (\beta^2 + 2\alpha\gamma) + \frac{\lambda^5}{5} (2\alpha\delta \right. \right. \\
\left. \left. + 2\gamma\beta) + \frac{1}{6} \beta\delta \lambda^6 + \frac{1}{7} \gamma\delta \lambda^7 \right\} \right. \\
- \frac{L^3}{12} \left\{ \frac{1}{2} \alpha \lambda \lambda^2 + \frac{\lambda^3}{3} (2\alpha\xi + \beta\lambda) + \frac{\lambda^4}{4} (3\alpha\zeta + 2\beta\xi \right. \\
\left. + \gamma\lambda) + \frac{\lambda^5}{5} (3\beta\zeta + 2\gamma\xi + \delta\lambda) + \frac{\lambda^6}{6} (3\gamma\zeta \right. \\
\left. + 2\xi\delta) + \frac{3}{7} \delta\zeta \lambda^7 \right\} \\
\left. + \frac{L^5}{30} \left\{ \frac{1}{2} \lambda^2 \lambda^2 + \frac{4}{3} \lambda\xi \lambda^3 + \lambda^4 (\xi^2 + \frac{3}{2} \lambda\zeta) + \frac{12}{5} \xi\zeta \lambda^5 \right. \right. \\
\left. \left. + \frac{3}{2} \zeta^2 \lambda^6 \right\} \right]_{\lambda_i}^{\lambda_0} \quad (17)
\end{aligned}$$

The potential energy of external forces  $V$ , associated with the inertia body force  $f_z$  given by equation (2) is,

$$V = - \iiint_{r_i}^{r_0} f_z \cdot w \, dV = - 2\pi \int_0^L \int_{r_i}^{r_0} \rho a_m \cdot w \cdot r \, dr \, dz \quad (18)$$

Substituting the expression for  $w$  from equation (3) into equation (18) yields

$$V = - \frac{\pi L^3}{3} \rho a_m \cdot \left[ \eta r^2 + \frac{2}{3} \lambda r^3 + \frac{1}{2} \xi r^4 + \frac{2}{5} \zeta r^5 \right]_{r_i}^{r_0} \quad (19)$$

The total potential energy of the system is obtained as

$$T = U_a + U_b + U_c + U_d + V \quad (20)$$

The theorem of minimum potential energy states that equilibrium is associated with a minimum of  $T$ . Here  $T$  is a function of the independent parameters  $\beta, \gamma, \delta, \eta, \epsilon$  and  $\zeta$ . The minimum of  $T$  is obtained by taking the partial derivative of  $T$  with respect to each of these seven coefficients. This yields the system of 7 linear algebraic equations in 7 unknowns,

$$\begin{bmatrix} F_{11} & F_{12} & \dots & \dots & \dots & \dots & F_{17} \\ F_{21} & F_{22} & \dots & \dots & \dots & \dots & F_{27} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ F_{71} & F_{72} & \dots & \dots & \dots & \dots & F_{77} \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ \delta \\ \eta \\ \lambda \\ \epsilon \\ \zeta \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ C_7 \end{bmatrix} \quad (21)$$

The explicit expressions for the  $F_{ij}$  and  $C_i$  ( $i, j = 1, 7$ ) coefficients are

$$F(1,1) = A \left\{ -2\lambda_0^2 - 2\lambda_i^2 + 4\lambda_0\lambda_i + 2\lambda_0^2 \ln \frac{\lambda_0}{\lambda_i} \right\} + B \left\{ -2\lambda_0^2 - 2\lambda_i^2 \right\} + 3G \left\{ \frac{1}{6}\lambda_0^4 - \lambda_0^2\lambda_i^2 + \frac{4}{3}\lambda_0\lambda_i^3 - \frac{1}{2}\lambda_i^4 \right\} L^2$$

$$F(1,2) = A \left\{ -\lambda_0^3 - 2\lambda_i^3 + 2\lambda_0^2\lambda_i + \lambda_0\lambda_i^2 + 2\lambda_0^3 \ln \frac{\lambda_0}{\lambda_i} \right\} + B \left\{ -2\lambda_0^3 + 2\lambda_0\lambda_i^2 + 2\lambda_0^2\lambda_i - 2\lambda_i^3 \right\} + 3G \left\{ \frac{7}{30}\lambda_0^5 - \lambda_0^3\lambda_i^2 + \frac{2}{3}\lambda_0^2\lambda_i^3 + \frac{1}{2}\lambda_0\lambda_i^4 - \frac{2}{5}\lambda_i^5 \right\}$$

$$F(2,1) = F(1,2)$$

$$\begin{aligned}
 F(1,3) = F(3,1) &= A \left\{ \frac{1}{12} \lambda_0^4 - \frac{11}{4} \lambda_i^4 + 2\lambda_0^3 \lambda_i + \frac{2}{3} \lambda_0 \lambda_i^3 + 2\lambda_0^4 \ln \frac{\lambda_0}{\lambda_i} \right\} \\
 &+ B \left\{ -2\lambda_0^4 + 2\lambda_0 \lambda_i^3 - 2\lambda_i^4 + 2\lambda_0^3 \lambda_i \right\} \\
 &+ 3G \sqrt{\lambda_i} \left\{ \frac{4}{15} \lambda_0^6 - \lambda_0^4 \lambda_i^2 + \frac{2}{3} \lambda_0^3 \lambda_i^3 + \frac{2}{5} \lambda_0 \lambda_i^5 - \frac{1}{3} \lambda_i^6 \right\}
 \end{aligned}$$

$$F(1,4) = F(4,1) = B \left\{ -2\lambda_i^2 + 2\lambda_0 \lambda_i \right\}$$

$$\begin{aligned}
 F(1,5) = F(5,1) &= B \left\{ \frac{4}{3} (\lambda_0^3 - \lambda_i^3) - \lambda_0^3 + \lambda_0 \lambda_i^2 \right\} \\
 &- 2G \left\{ -\frac{1}{6} \lambda_0^3 + \frac{1}{2} \lambda_0 \lambda_i^2 - \frac{1}{3} \lambda_i^3 \right\}
 \end{aligned}$$

$$\begin{aligned}
 F(1,6) = F(6,1) &= B \left\{ \frac{1}{3} \lambda_0^4 + \frac{2}{3} \lambda_0 \lambda_i^3 - \lambda_i^4 \right\} \\
 &- 2G \left\{ -\frac{1}{6} \lambda_0^4 + \frac{2}{3} \lambda_0 \lambda_i^3 - \frac{1}{2} \lambda_i^4 \right\}
 \end{aligned}$$

$$\begin{aligned}
 F(1,7) = F(7,1) &= B \left\{ \frac{3}{10} \lambda_0^5 + \frac{1}{2} \lambda_0 \lambda_i^4 - \frac{4}{5} \lambda_i^5 \right\} \\
 &- 2G \left\{ -\frac{3}{20} \lambda_0^5 + \frac{3}{4} \lambda_0 \lambda_i^4 - \frac{3}{5} \lambda_i^5 \right\}
 \end{aligned}$$

$$\begin{aligned}
 F(2,2) &= A \left\{ -\frac{1}{2} \lambda_0^4 - \frac{3}{2} \lambda_i^4 + 2\lambda_0^2 \lambda_i^2 + 2\lambda_0^4 \ln \frac{\lambda_0}{\lambda_i} \right\} \\
 &+ B \left\{ -2\lambda_0^4 + 4\lambda_0^2 \lambda_i^2 - 2\lambda_i^4 \right\} \\
 &+ 3G \sqrt{\lambda_i} \left\{ \frac{1}{3} \lambda_0^6 - \lambda_0^4 \lambda_i^2 + \lambda_0^2 \lambda_i^4 - \frac{1}{3} \lambda_i^6 \right\}
 \end{aligned}$$

$$\begin{aligned}
 F(2,3) = F(3,2) &= A \left\{ \frac{17}{15} \lambda_0^5 - \frac{14}{5} \lambda_i^5 + \frac{2}{3} \lambda_0^2 \lambda_i^3 + \lambda_0^3 \lambda_i^2 + 2\lambda_0^5 \ln \frac{\lambda_0}{\lambda_i} \right\} \\
 &+ B \left\{ -2\lambda_0^5 + 2\lambda_0^3 \lambda_i^2 + 2\lambda_0^2 \lambda_i^3 - 2\lambda_i^5 \right\} \\
 &+ \frac{3G}{\lambda_i} \left\{ \frac{27}{70} \lambda_0^7 - \lambda_0^5 \lambda_i^2 + \frac{1}{2} \lambda_0^3 \lambda_i^4 + \frac{2}{5} \lambda_0^2 \lambda_i^5 - \frac{3}{7} \lambda_i^7 \right\}
 \end{aligned}$$

$$F(2,4) = F(4,2) = B \left\{ -2\lambda_i^3 + 2\lambda_0^2 \lambda_i \right\}$$

$$\begin{aligned}
 F(2,5) = F(5,2) &= B \left\{ \frac{3}{2} (\lambda_0^4 - \lambda_i^4) - \lambda_0^4 + \lambda_0^2 \lambda_i^2 \right\} \\
 &- 2G \left\{ -\frac{1}{4} \lambda_0^4 + \frac{1}{2} \lambda_0^2 \lambda_i^2 - \frac{1}{4} \lambda_i^4 \right\}
 \end{aligned}$$

$$F(2,6) = F(6,2) = B \left\{ \frac{8}{15} \lambda_0^5 + \frac{2}{3} \lambda_0^2 \lambda_i^3 - \frac{6}{5} \lambda_i^5 \right\} \\ - 2G \left\{ -\frac{4}{15} \lambda_0^5 + \frac{2}{3} \lambda_0^2 \lambda_i^3 - \frac{2}{5} \lambda_i^5 \right\}$$

$$F(2,7) = F(7,2) = B \left\{ \frac{1}{2} \lambda_0^6 + \frac{1}{2} \lambda_0^2 \lambda_i^4 - \lambda_i^6 \right\} \\ - 2G \left\{ -\frac{1}{4} \lambda_0^6 + \frac{3}{4} \lambda_0^2 \lambda_i^4 - \frac{1}{12} \lambda_i^6 \right\}$$

$$F(3,3) = A \left\{ 2\lambda_0^6 - \frac{10}{3} \lambda_i^6 + \frac{4}{3} \lambda_0^3 \lambda_i^3 + 2\lambda_0^6 \ln \frac{\lambda_0}{\lambda_i} \right\} \\ + B \left\{ -2\lambda_0^6 - 2\lambda_i^6 + 4\lambda_0^3 \lambda_i^3 \right\} \\ + \frac{3G}{12} \left\{ \frac{1}{20} \lambda_0^8 - \lambda_0^6 \lambda_i^2 + \frac{4}{5} \lambda_0^3 \lambda_i^5 - \frac{1}{4} \lambda_i^8 \right\}$$

$$F(3,4) = F(4,3) = B \left\{ -2\lambda_i^4 + 2\lambda_0^3 \lambda_i \right\}$$

$$F(3,5) = F(5,3) = B \left\{ \frac{6}{5} (\lambda_0^5 - \lambda_i^5) - \lambda_0^5 + \lambda_0^3 \lambda_i^2 \right\} \\ - 2G \left\{ -\frac{3}{10} \lambda_0^5 + \frac{1}{2} \lambda_0^3 \lambda_i^2 - \frac{1}{5} \lambda_i^5 \right\}$$

$$F(3,6) = F(6,3) = B \left\{ \frac{2}{3} \lambda_0^6 + \frac{2}{3} \lambda_0^3 \lambda_i^3 - \frac{4}{3} \lambda_i^6 \right\} \\ - 2G \left\{ -\frac{1}{3} \lambda_0^6 + \frac{2}{3} \lambda_0^3 \lambda_i^3 - \frac{1}{3} \lambda_i^6 \right\}$$

$$F(3,7) = F(7,3) = B \left\{ \frac{9}{14} \lambda_0^7 + \frac{1}{2} \lambda_0^3 \lambda_i^4 - \frac{6}{7} \lambda_i^7 \right\} \\ - 2G \left\{ -\frac{9}{28} \lambda_0^7 + \frac{3}{4} \lambda_0^3 \lambda_i^4 - \frac{3}{7} \lambda_i^7 \right\}$$

$$F(4,4) = A \left\{ \lambda_0^2 - \lambda_i^2 \right\}$$

$$F(4,5) = F(5,4) = A \cdot \left\{ \frac{2}{3} (\lambda_0^3 - \lambda_i^3) \right\}$$

$$F(4,6) = F(6,4) = A \cdot \left\{ \frac{1}{2} (\lambda_0^4 - \lambda_i^4) \right\}$$

$$F(4,7) = F(7,4) = A \left\{ \frac{1}{2} (\lambda_0^4 - \lambda_i^4) \right\}$$

$$F(5,5) = A \left\{ \frac{1}{2} (\lambda_0^4 - \lambda_i^4) \right\} + GL^2 \left\{ \frac{2}{5} (\lambda_0^2 - \lambda_i^2) \right\}$$

$$F(5,6) = F(6,5) = A \left\{ \frac{2}{5} (\lambda_0^5 - \lambda_i^5) \right\} + GL^2 \left\{ \frac{8}{15} (\lambda_0^3 - \lambda_i^3) \right\}$$

$$F(5,7) = F(7,5) = A \left\{ \frac{1}{3} (\lambda_0^6 - \lambda_i^6) \right\} + GL^2 \left\{ \frac{3}{5} (\lambda_0^4 - \lambda_i^4) \right\}$$

$$F(6,6) = A \left\{ \frac{1}{3} (\lambda_0^6 - \lambda_i^6) \right\} + GL^2 \left\{ \frac{4}{5} (\lambda_0^4 - \lambda_i^4) \right\}$$

$$F(6,7) = F(7,6) = A \left\{ \frac{2}{7} (\lambda_0^7 - \lambda_i^7) \right\} + GL^2 \left\{ \frac{24}{25} (\lambda_0^5 - \lambda_i^5) \right\}$$

$$F(7,7) = A \left\{ \frac{1}{4} (\lambda_0^8 - \lambda_i^8) \right\} + GL^2 \left\{ \frac{6}{5} (\lambda_0^6 - \lambda_i^6) \right\}$$

$$C_1 = C_2 = C_3 = 0$$

$$C_4 = -\rho a_m (\lambda_0^2 - \lambda_i^2) \quad C_5 = -2\rho a_m (\lambda_0^3 - \lambda_i^3) / 3.$$

$$C_6 = -\rho a_m (\lambda_0^4 - \lambda_i^4) / 2. \quad C_7 = -2\rho a_m (\lambda_0^5 - \lambda_i^5) / 5.$$

A computer program was used to obtain the solution of the problem. A large number of problems were solved for various values of the physical parameters,  $r_i$ ,  $r_0$ ,  $L$  and  $a_m$ , and the propellant properties  $\nu$  and  $E$ . A listing of the program is given in Appendix A.

#### DISCUSSION OF RESULTS

It has been noted by other investigators that numerical difficulties arise in the analysis of propellants when Poisson's ratio approaches 0.5



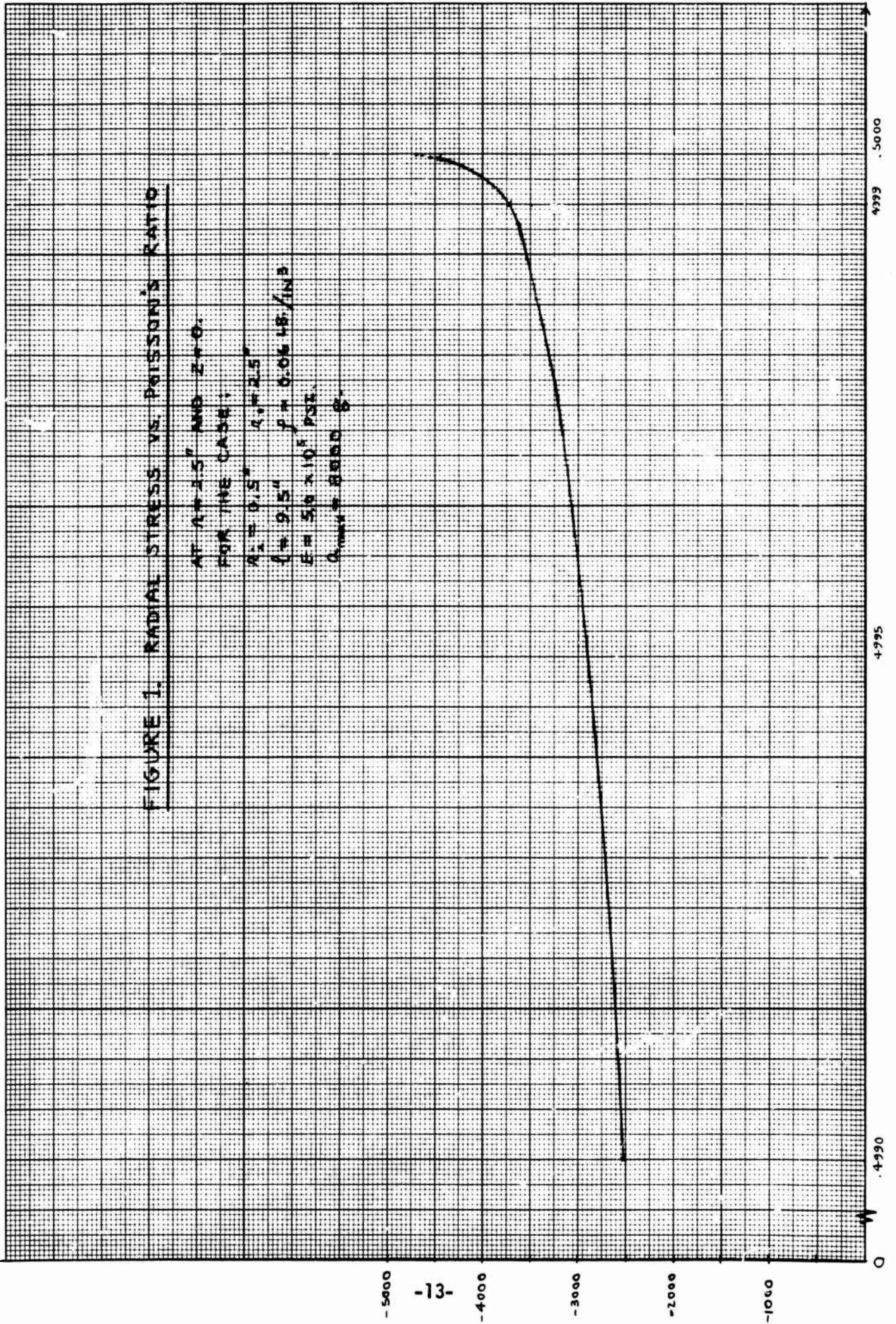
(ref. 2). To circumvent this difficulty Hermann (ref. 3) gives a special variational theorem for nearly incompressible materials. In the present investigation the numerical problem was overcome by using a CDC 6600 computer with double precision. This results in approximately 32 significant digits, sufficient for Poisson's ratio to 0.4999999.

A study was conducted on the effect of Poisson's ratio on the propellant stress for a propellant with 1.5 inch outer radius, 0.5 inch inner radius, 9.5 inch length, 500,000 psi Young's modulus and  $a_m$  equal to 8000 g. Poisson's ratio was varied between 0.45 and 0.4999999. The computer program results for  $\sigma_r$  at the propellant-case interface at the base are given in Table 1.

Table 1

$\nu$	$\sigma_r$ (psi)	$\Delta\nu$	$\Delta\sigma$	$e$
.45	-1589			$-2.0 \times 10^{-3}$
.48	-1691	.03	-112	$-8.9 \times 10^{-4}$
.49	-1771	.01	- 80	$-2.1 \times 10^{-4}$
.499	-2513	.004	-616	$-2.8 \times 10^{-5}$
.4999	-3744	.0009	-1231	$-4.0 \times 10^{-6}$
.49999	-4469	.0009	-725	$-4.8 \times 10^{-7}$
.499999	-4613	.00009	-144	$-5.0 \times 10^{-8}$
.4999999	-4630	.000009	- 27	$-5.0 \times 10^{-9}$

$\sigma_r$  (psi)



-5000  
-4000  
-3000  
-2000  
-1000  
0

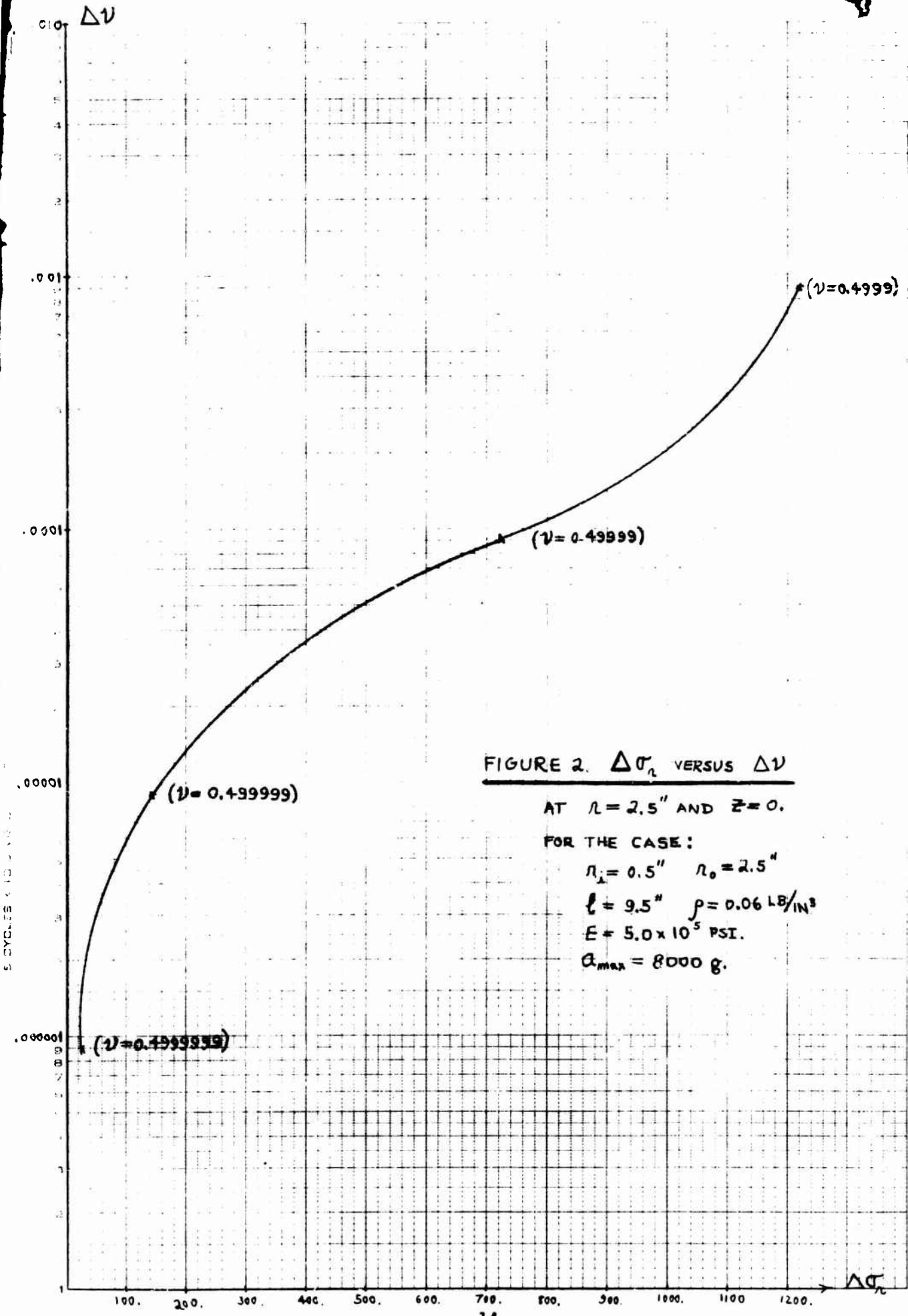


FIGURE 2.  $\Delta \sigma_r$  VERSUS  $\Delta \nu$

AT  $r = 2.5''$  AND  $z = 0$ .

FOR THE CASE:

$r_i = 0.5''$     $r_o = 2.5''$

$l = 9.5''$     $\rho = 0.06 \text{ LB/IN}^3$

$E = 5.0 \times 10^5 \text{ PSI}$ .

$\alpha_{max} = 8000 \text{ g}$ .



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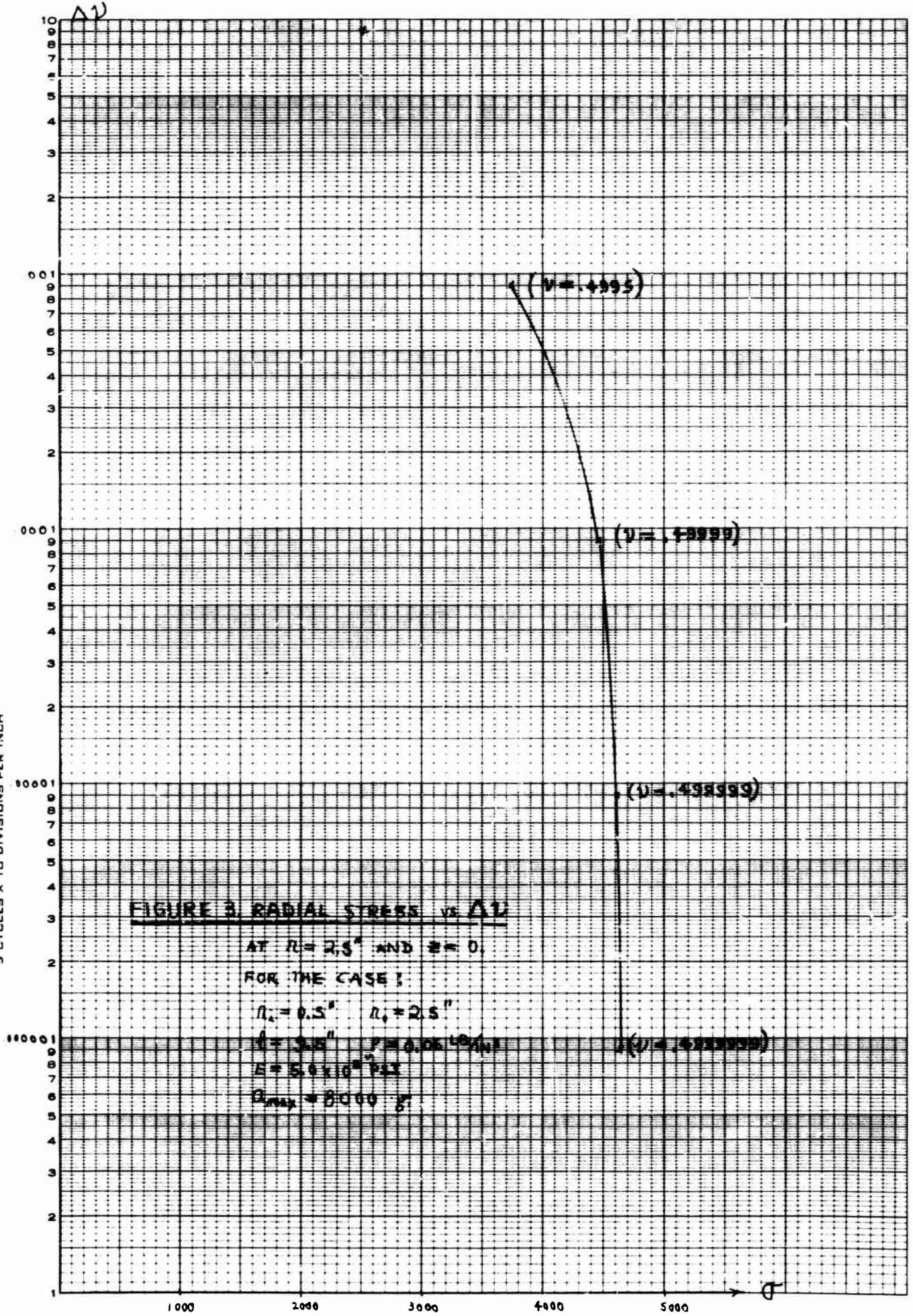


FIGURE 3. RADIAL STRESS vs Δv

AT  $r = 2.5''$  AND  $z = 0$ .

FOR THE CASE:

$r_1 = 0.5''$      $r_2 = 2.5''$

$\sigma = 3000''$      $\nu = 0.4999999$

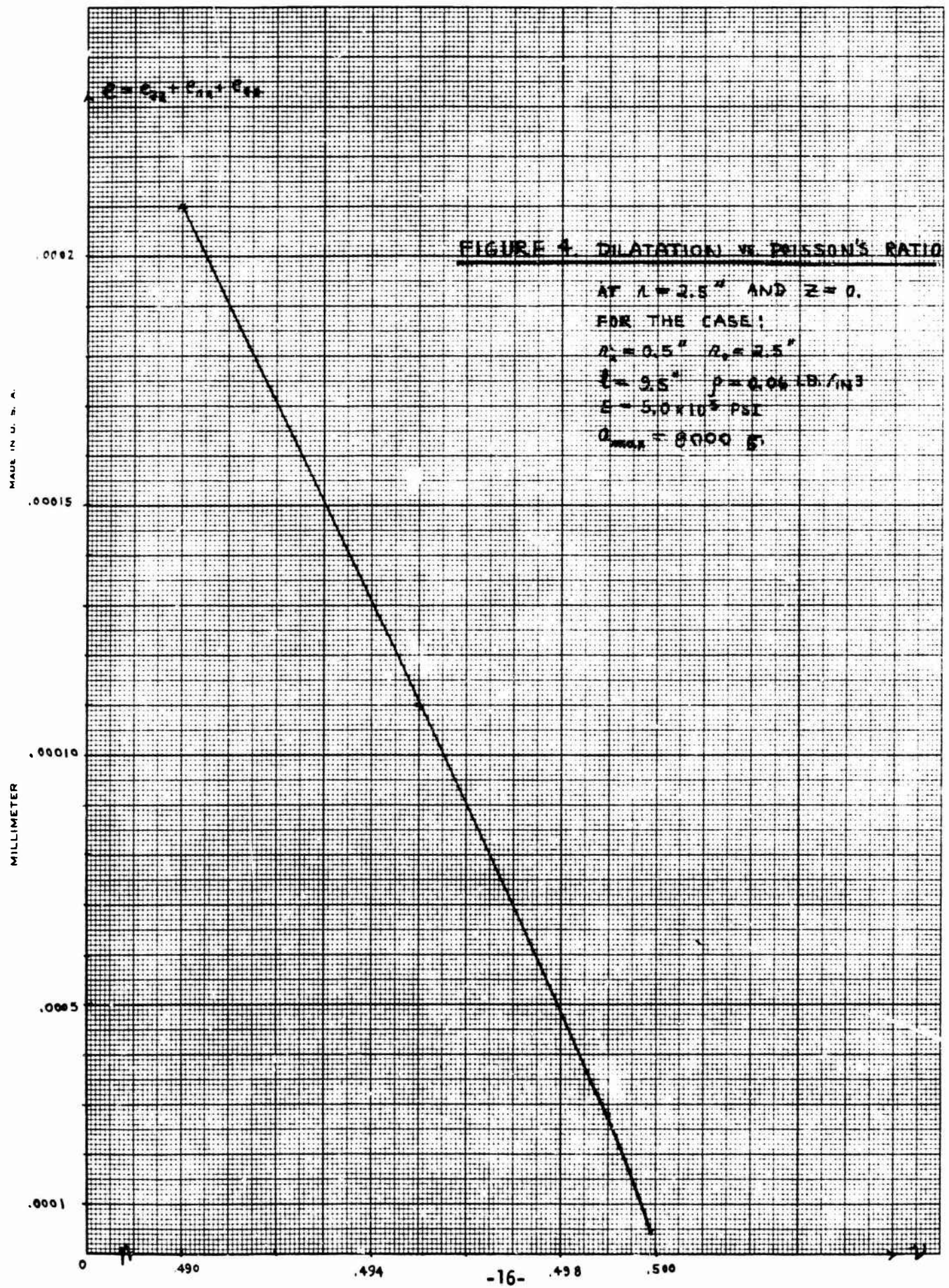
$E = 5.0 \times 10^{10}$  PSI

$\sigma_{max} = 8000$  PSI

$$\epsilon = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$$

FIGURE 4. DILATATION VS. POISSON'S RATIO

AT  $r = 2.5''$  AND  $z = 0$ .  
 FOR THE CASE:  
 $r_1 = 0.5''$   $r_2 = 2.5''$   
 $t = 9.5''$   $\rho = 0.06 \text{ LB./IN}^3$   
 $E = 5.0 \times 10^5 \text{ PSI}$   
 $Q_{max} = 8000 \text{ LB}$



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Figures 1 through 3 are curves showing these results; the first figure being a direct plot of stress  $\sigma_r$  versus Poisson's ratio  $\nu$ . Figures 2 and 3 show more clearly that the stress approaches a finite value of -4650 psi as Poisson's ratio approaches 0.5. In Figure 4 the dilatation  $e = e_\theta + e_r + e_z$ , at  $r = r_0$  and  $z = 0$  is plotted against Poisson's ratio. The resulting curve verifies that the incompressibility condition  $e$  approaches zero as  $\nu$  approaches 1/2 is satisfied.

In addition to the study on the effect of Poisson's ratio on propellant behavior, analyses were made to determine the effect of other parameters to behavior. The results obtained show that the radial stress varies linearly with respect to acceleration, radii, length and Young's modulus of elasticity.

The results of this analysis were compared with two other sources; a Rohm and Haas finite element analysis (ref. 3), and an exact solution for an incompressible material with zero interior radius. The present formulation resulted in a 20 percent higher radial stress than the finite element program, and 2 percent higher than the exact solution, at the propellant-case interface at the base.

#### CONCLUSIONS

A computer program was developed for the determination of the stress distribution due to a slumping propellant contained within a launched rigid motor case. The program was initially incorporated into the SATANS finite difference program for the calculation of the elastic static buckling load of the motor case subjected to high acceleration loading. The stiffening of the motor case, which is predicted by theory, was obtained in the SATANS analysis; a favorable structural response.



An unfavorable effect on structural integrity is associated with the additional contribution to yield from the propellant radial stress. For a motor case of 2.5 inch outer radius, zero inner radius, 1/8 inch thickness, 20 inch length, and a maximum acceleration of 8,000 g, the radial stress of approximately 10,000 psi gives a hoop stress of 100,000 psi. The contribution to yield of this stress component is of significant magnitude. That is, the propellant has a beneficial buckling stiffening effect but an adverse effect on yield.

The present analysis is based on the linear elastic theory of solids and, therefore, the results are not valid beyond the initial yield point of the propellant. Because it is difficult to obtain the material properties of propellants (Young's modulus, Poisson's ratio, and initial yield), definite assertions regarding the structural integrity of the propellant are not given here; however, the stress results obtained in some of the analyses show that additional study regarding this concern may be warranted.

## REFERENCES

1. Ball, Robert E., and Salinas, David, "Analysis of a Three Inch Gun Launched Finned Motor Case," Naval Postgraduate School, NPS-57Bp-72011A, January 1972.
2. Thacker, J. H., "Deformation of Case-Bonded Propellants Under Axial Acceleration," 20th Meeting Bulletin, JANAF-ARPA-NASA Panel on Physical Properties of Solid Propellants, I, Nov. 1961, 81-90.
3. Hermann, L. R., "Elasticity Equations for Incompressible and Nearly Incompressible Materials by a Variational Theorem," AIAA Journal, Vol. 3, No. 11, Oct. 1965, 1896-1900.



THIS PROGRAM CALCULATES THE STRESSES IN A SLUMPING PROPELLANT  
DUE TO ACCELERATION LOADING

IMPLICIT REAL\*8(A-H,O-Z)  
DIMENSION F(7,7),S(7),Y(7),T(7,7),FT(7,7),SIGXX(15),R(15)

READ(5,5) NLOOP,NR  
FCRMAT(2,110)

WRITE(6,6) NLOOP  
READ(5,8) (R(K),K=1,NR)

FCRMAT(8,10.5) (R(K),K=1,NR)

WRITE(6,9) (R(K),K=1,NR)  
FCRMAT(15,8,10.5)  
FCRMAT(14,10.5),NUMBER OF PROBLEMS IS',I10)

DC 200 NL=1,NLOOP  
WRITE(6,7) NL  
FCRMAT(14,10.5),PROBLEM ',I10)

READ(5,10) E,ZNU,ZL,AMAX,RO,RI,RHC  
FCRMAT(7,10.5)

WRITE(6,15) E,ZNU,ZL,AMAX,RO,RI,RHC  
FCRMAT(14,10.5),YOUNGS MODULUS=',E17.8,POISSONS RATIO=',E17.8

1, //5X, PROPELLANT LENGTH=',E17.8, //5X, MAX. ACCEL.=',E17.8, //5X  
2, C, OUTER RADIUS=',E17.8, //5X, INNER RADIUS=',E17.8, //5X, SPECIFIC  
3, DENSITY=',E17.8)

A=(1.-ZNU)\*E/((1.+ZNU)\*(1.-2.\*ZNU))  
B=ZNU\*E/((1.+ZNU)\*(1.-2.\*ZNU))  
G=E/(2.\*(1.+ZNU))

WRITE(6,30) A,B,G  
FCRMAT(2X,'A=',E17.8,10X,'B=',E17.8,10X,'C=',E17.8)

ZL2=ZL\*\*2  
RCCVRI=RO/RI

RC2=RO\*\*2  
RC3=RO\*\*RO2  
RC4=RO\*\*RO3  
RC5=RO\*\*RO4  
RC6=RO\*\*RO5  
RC7=RO\*\*RO6  
RC8=RO\*\*RO7

RI2=RI\*\*2  
RI3=RI\*\*RI2  
RI4=RI\*\*RI3  
RI5=RI\*\*RI4

RI6=RI\*RI5  
RI7=RI\*RI6  
RI8=RI\*RI7

C

RHS4=-RHO\*AMAX\*(RC2-RI2)  
RHS5=-2.\*RHC\*AMAX\*(RO3-RI3)/3.  
RHS6=-RHO\*AMAX\*(RO4-RI4)/2.  
RHS7=-2.\*RHC\*AMAX\*(RO5-RI5)/5.  
WRITE(6,20) RHS4,RHS5,RHS6,RHS7  
20 FCRMAT(4E20.8)

C

F(1,1)=\* (-2.\*RO2-2.\*RI2+4.\*RC\*RI+2.\*RO2\*DLOG(ROOVRT))  
1+B\*(-2.\*RO2+4.\*RO\*RI-2.\*RI2)+3.\*G\*(RO4/3.-RC2\*RI2+4.\*RO\*RI3/3.  
2-RI4/2.)/ZL2  
F(1,2)=A\*(-RO3-2.\*RI3+2.\*RO2\*RI+RC\*RI2+2.\*RC3\*DLOG(RCOVRT))  
1+B\*(-2.\*RO3+2.\*RI3+2.\*RO2\*RI+2.\*RI5/5.)/ZL2  
2+2.\*RO2\*RI3/3.+2.\*RI4/4.+2.\*RO3\*RI+2.\*RC\*RI3/3.+2.\*RO4\*DLOG(  
1RCOVRT))  
2+3.\*G\*(4.\*RO6/15.-RO4\*RI2+2.\*RO3\*RI3/3.+2.\*RO\*RI5/5.-RI6/3.)/ZL2  
F(1,4)=8\*(-2.\*RI2+2.\*RO\*RI)  
F(2,2)=A\*(-RO4/2.-3.\*RI4/2.+2.\*RO2\*RI2+2.\*RC4\*DLOG(ROOVRT))  
1+B\*(-2.\*RO4+4.\*RO2\*RI2-2.\*RI4,  
2+3.\*G\*(RO6/3.-RO4\*RI2+RO2\*RI4-RI6/3.)/ZL2  
1(ROOVRT))  
2+3.\*G\*(17.\*RO5/15.-14.\*RO3\*RI3/3.+RO3\*RI2+2.\*RO5\*DLOG  
2+3.\*G\*(27.\*RO7/70.-RO5\*RI2+RO3\*RI4/2.+2.\*FC2\*RI5/5.-2.\*RI7/7.)/ZL2  
F(2,4)=8\*(-2.\*RI3+2.\*RO2\*RI)  
F(3,3)=A\*(2.\*RO6-10.\*RI6/3.+4.\*RO3\*RI3/3.+2.\*RO6\*DLOG(ROOVRT))  
1+B\*(-2.\*RO6-2.\*RI6+4.\*RO3\*RI3)  
2+3.\*G\*(RO8/20.-RO6\*RI2+4.\*RO3\*RI5/5.-RI8/4.)/ZL2  
F(3,4)=8\*(-2.\*RI4+2.\*RO3\*RI)  
F(4,4)=A\*(RO2-RI2)  
F(1,5)=8\*(4.\*(-RO3-RI3)/3.-RC3+RC\*RI2)  
F(1,6)=-2.\*G\*(-RO3/6.+RO\*RI2/2.-RI3/3.)\*F(1,5)  
F(1,6)=-2.\*G\*(RO4/3.+2.\*RO\*RI3/3.-RI4)-2.\*G\*(-RC4/6.+2.\*RO\*RI3/3.-  
1RI4/2.)  
F(2,5)=8\*(3.\*(RO4-RI4)/2.-RC4+RC2\*RI2)  
F(2,5)=F(2,5)-2.\*G\*(-RO4/4.+RO2\*RI2/2.-RI4/4.)  
F(2,6)=8\*(8.\*RO5/15.+2.\*RO2\*RI3/3.-6.\*RI5/5.)-2.\*G\*(-4.\*RC5/15.+2.  
1\*RC2\*RI3/3.-2.\*RI5/5.)  
F(3,5)=8\*(8.\*(RO5-RI5)/5.-RO5+RO3\*RI2)  
F(3,5)=F(3,5)-2.\*G\*(-3.\*RO5/10.+RO3\*RI2/2.-RI5/5.)  
F(3,6)=8\*(2.\*RO6/3.+2.\*RO3\*RI3/3.-4.\*RI6/3.)  
1(-RO6/3.+2.\*RO3\*RI3/3.)  
F(4,6)=A\*(RO3-RI3)/3.  
F(4,6)=A\*(RO4-RI4)/2.  
F(5,5)=A\*(RO4-RI4)/2.

```

F(5,5)=F(5,5)+2.*G*(ZL**2)*(R02-RI2)/5.
F(5,6)=2.*A*(R05-RI5)/5.+8.*G*(ZL**2)*(RC3-RI3)/15.
F(6,6)=A*(R06-RI6)/3.+4.*G*(ZL**2)*(R04-RI4)/5.
F(1,7)=B*(R05/10.+R0*RI4/2.-4.*RI5/5.)
1-2.*G*(-3.*R05/20.+3.*R0*RI4/4.-3.*RI5/5.)
1F(2,7)=B*(R06/2.+R02*RI4/2.-RI6)-2.*G*(-R06/4.+3.*R02*RI4/4.-RI6/
12(3,7)=B*(9.*R07/14.+R03*RI4/2.-8.*RI7/7.)
1-2.*G*(-9.*R07/28.+3.*R03*RI4/4.-3.*RI7/7.)
1F(4,7)=2.*A*(R05-RI5)/5.
1F(5,7)=A*(R06-RI6)/3.+3.*G*ZL2*(RC4-RI4)/5.
1F(6,7)=2.*A*(R07-RI7)/7.+24.*G*ZL2*(R05-RI5)/25.
1F(7,7)=A*(R08-RI8)/4.+6.*G*ZL2*(RC6-RI6)/5.
F(7,1)=F(1,7)
F(7,2)=F(2,7)
F(7,3)=F(3,7)
F(7,4)=F(4,7)
F(7,5)=F(5,7)
F(7,6)=F(6,7)
F(2,1)=F(1,2)
F(3,1)=F(1,3)
F(3,2)=F(1,3)
F(3,3)=F(1,4)
F(4,1)=F(1,4)
F(4,2)=F(2,4)
F(4,3)=F(2,4)
F(4,4)=F(2,5)
F(5,1)=F(1,5)
F(5,2)=F(2,5)
F(5,3)=F(2,5)
F(5,4)=F(2,5)
F(5,5)=F(1,6)
F(6,1)=F(1,6)
F(6,2)=F(2,6)
F(6,3)=F(2,6)
F(6,4)=F(2,6)
F(6,5)=F(2,6)
F(6,6)=F(2,6)
F(6,7)=F(2,6)
WRITE(6,25)((F(K,J),J=1,7),K=1,7)
FCRMMAT(4E20,8)
DC 80 I=1,7
DC 80 J=1,7
T(I,J)=F(I,J)
CONTINUE
C CALL SYINV(F,7)
C WRITE(6,25)((F(K,J),J=1,7),K=1,7)
DC 90 I=1,7
DC 90 J=1,7

```

```

FT(I,J)=0.
CC 85 K=1,7
FT(I,J)=FT(I,J)+T(I,K)*F(K,J)
CC CONTINUE
85 CC WRITE(6,25) ((FT(L,M),M=1,7),L=1,7)
Y(1)=0.
Y(2)=0.
Y(3)=0.
Y(4)=RHS4
Y(5)=RHS5
Y(6)=RHS6
Y(7)=RHS7
C
DC 100 I=1,7
S(I)=0.
DO 100 J=1,7
S(I)=S(I)+F(I,J)*Y(J)
100 CC CONTINUE
CC WRITE(6,25) (S(K),K=1,7)
C
STRESSES AT X=0, R=RC AND TAU AT X=0, R=RI
C
ALPHA=-S(1)*RO-S(2)*RC2-S(3)*RO3
WRITE(6,105) ALPHA
FORMAT(5X,'ALPHA=',E20.8)
105
107 FORMAT(1H 2X,'STRAINS AND STRESSES')
ER=+L*(S(1)+2.*S(2)*RO+3.*S(3)*RO2)
ETHETA=ZL*(ALPHA/RC+S(1)+S(2)*RO+S(3)*RO2)
EX=ZL*(S(4)+S(5)*RO+S(6)*RO2+S(7)*RO3)
GAMRX=-L*(ALPHA+S(1)*RI+S(2)*RI2+S(3)*RI3)
WRITE(6,26) ER,ETHETA,EX,GAMRX
C
SIGR=A*ER+9*(EX+ETHETA)
SIGO=A*ETHETA+8*(EX+ER)
SIGX=A*EX+B*(ER+ETHETA)
TAU=G*GAMRX
WRITE(6,26) SIGR,SIGO,SIGX,TAU
C
RE=1.49999
RE2=RE**2
ERE=ZL*(S(1)+2.*S(2)*RE+3.*S(3)*RE)
ETHE=ZL*(ALPHA/RE+S(1)+S(2)*RE+S(3)*RE**2)
EXRE=ZL*(S(4)+S(5)*RE+S(6)*RE2+S(7)*RE**2)
SIGRE=A*ERE+B*(EXE+ETHE)
WRITE(6,888) SIGRE
888 FORMAT(5X,'RADIAL STRESS AT R=1.49999=',E20.8)

```

```

RC=(RO+RI)/2.
RC2=RC**2*(S(1)+2.*S(2)*RC+3.*S(3)*RC2)/2.
ERC=ZL*(S(ALPHA/RC+S(1)+S(2)*RC+S(3)*RC2)/2.
ETHC=ZL*(S(4)+S(5)*RC+S(6)*RC2+S(7)*RC*RC2)/2.
EXZ=ZL*(A*ERC+B*(ETHC+EXC)
SIGRC=A*ERC+B*(ETHC+EXC)
WRITE(6,889) SIGRC STRESS AT CENTER=',E20.8)
FCRMA(5X,'RADIAL STRESS AT CENTER=',E20.8)
889 FCRMA(5X,'RADIAL STRESS AT CENTER=',E20.8)
ZXR=1.499
ZXR=.001**2
ZR2=ZL-ZX)**2*(S(1)+2.*S(2)*ZR+3.*S(3)*ZR2)
ZERZ=(ZL-ZX)*(ALPHA/ZR+S(1)+S(2)*ZR+S(3)*ZR2)
ETHZ=(ZL-ZX)*(S(4)+S(5)*ZR+S(6)*ZR2+S(7)*ZR*ZR2)
EXZ=(ZL-ZX)*B*(ETHZ+EXZ)
SIGRZ=A*ERZ+B*(ETHZ+EXZ)
WRITE(6,901) ZR,ZX,SIGRZ
901 FCRMA(5X,'AT R=',F10.5,'AND X=',F10.5,'THE RADIAL STRESS =',E20
1.8)
C
DC 150 K=1,NR
SIGXX(K)=ZL*(A*(S(4)+S(5)*R(K))+B*(ALPHA/R(K)+2.*S(1)
+3.*S(2)*R(K)+4.*S(3)*(R(K)**2)))
+3.*S(2)*R(K)+4.*S(3)*(R(K)**2)+S(7)*(R(K)**3))
15C CCNTINUE
WRITE(6,151) (SIGXX(K),K=1,NR)
SIGXAV=0.
DO 160 K=1,NR
SIGXAV=SIGXAV+SIGXX(K)
CCNTINUE
SIGXAV=SIGXAV/NR
160 WRITE(6,161) SIGXAV
FORMAT(5X,'AVERAGE AXIAL STRESS=',E17.8)
161 FCRMA(5X,6E17.8)
C
FD=ZL*(A*S(4)*(RO2-RI2)/2.+B*(ALPHA*(RO-RI)+S(1)*(RO2-RI2)
+S(2)*(RO3-RI3)+S(3)*(RO4-RI4))+A*S(5)*(RC3-RI3)/3.)
FC=FD+A*ZL*S(6)*(RO4-RI4)/4.+A*ZL*S(7)*(RC5-RI5)/5.
FU=ZL*ARC*AMAX*(FU)*100.
PCTDIF=(1.+FD/FU)*100.
WRITE(6,205) FD,FU,PCTDIF
FORMAT(5X,'FD=',E17.8,'FU=',E17.8,'PCTDIF=',E17.8)
205 FCRMA(5X,'FD=',E17.8,'FU=',E17.8,'PCTDIF=',E17.8)
C
DC 300 I=1,11
ZX=ZL*(I-1)/10
ZER=(ZL-ZX)*(S(1)+2.*S(2)*RO+3.*S(3)*RO2)
ET=(ZL-ZX)*(ALPHA/RC+S(1)+S(2)*RO+S(3)*RO2)
EX=A*(ZL-ZX)*(S(4)+S(5)*RO+S(6)*RC2+S(7)*RO3)
SR=A*ER+B*(ET+EX)

```

```

C 301 WRITE(6,301) ZX,ER,ET,EX,SR,ER=,E17.8,5X,ETHETA=,E17.8,5X,EX,EX=,
FORMAT(5X,AT X=,F10.5,5X,ER=,E17.8,5X,ETHETA=,E17.8,5X,EX,EX=,
1,E17.8,5X,SR=,E17.8)
U=(ZL-ZX)*(ALPHA+S(1)*RI+S(2)*RI2+S(3)*RI3)
W=ZX*(ZL-ZX/2.1*(S(4)+S(5)*RI+S(6)*RI2+S(7)*RI3)
WRITE(6,302) ZX,U,SR
FCRMMAT(5X,AT X=,F10.5,5X,U=,E17.8,5X,h=,E17.8,5X,SR=,E17
302 1.8)
300 CCNTINUE
200 STCP
C
END

```