AD-777 627

SCATTERING OF ELECTROMAGNETIC WAVES FROM THE TWO-DIMENSIONAL OCEAN SURFACE. I. LINEAR WAVES IN THE PRESENCE OF CURRENT

Kenneth M. Watson, et al

Physical Dynamics, Incorporated

Prepared for:

Advanced Research Projects Agency
Rome Air Development Center

January 1974

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER RADC-TR-74-42	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER AD 777 627
4. TITLE (and Subtitle) SCATTERING OF ELECTROMAGNETIC WAVES FROM THE TWO-DIMENSIONAL OCEAN SURFACE: I. LINEAR WAVES IN THE PRESENCE OF CURRENT		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER PD 73-050
7. AUTHOR(s) Kenneth M. Watson Bruce J. West		8. CONTRACT OR GRANT NUMBER(s) F30602-72-C-0494
9. PERFORMING ORGANIZATION NAME AND ADDRESS Physical Dynamics, Inc. P.O. Box 1069 Berkeley, CA 94701		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62301E 16490402
11. CONTROLLING OFFICE NAME AND ADDRESS ARPA/STO 1400 Wilson Blvd Arlington, VA 22209		12. REPORT DATE Jan 1974 13. NUMBER OF PAGES 26
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) RADC/OCSE ATTN: Leonard Strauss Griffiss AFB, NY 13441		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

See 16.

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Scattering theory
Wigner distribution function

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The scattering of electromagnetic waves from the ocean surface when a current is present is studied. The electromagnetic waves are singly scattered by two-dimensional linear surface gravity waves interacting with a surface current. The surface is described in terms of a Wigner function, i.e., the spectral density, which has finite spatial gradients. The effect of two dimensions on the spectral perturbation is explored in a preliminary calculation for a Phillip's ambient spectrum.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

PD-73-050

SCATTERING OF ELECTROMAGNETIC WAVES FROM THE TWO-DIMENSIONAL OCEAN SURFACE: I. LINEAR WAVES IN THE PRESENCE OF CURRENT

Kenneth M. Watson
Bruce J. West

Contractor: Physical Dynamics Incorporated
Contract Number: F30602-72-C-0494
Effective Date of Contract: 1 May 1972
Contract Expiration Date: 31 December 1974
Amount of Contract: $91,818.00
Program Code Number: 2E20

Principal Investigator: J. Alex Thomson
Phone: 415 848-3062

Project Engineer: Joseph J. Simons
Phone: 315 330-3055

Contract Engineer: Leonard Strauss
Phone: 315 330-3055

This research was supported by the Defense Advanced Research Projects Agency of the Department of Defense and was monitored by Leonard Strauss RADC (OCSE), GAFB, NY 13441 under contract F30602-72-C-0494.

PUBLICATION REVIEW

This technical report has been reviewed and is approved.

Leonard Strauss

RADC Project Engineer

ABSTRACT

The scattering of electromagnetic waves from the ocean surface when a current is present is studied. The electromagnetic waves are singly scattered by two-dimensional linear surface gravity waves interacting with a surface current. The surface is described in terms of a Wigner function, i.e., the spectral density, which has finite spatial gradients. The effect of two dimensions on the spectral perturbation is explored in a preliminary calculation for a Phillips' ambient spectrum.

TABLE OF CONTENTS

Preceding page blank

1. Introduction

In a previous paper we have reviewed the effect of surface currents in modulating the ocean surface wave spectrum[1]. In this work we have restricted ourselves to the linear approximation for the surface waves. The role of nonlinear surface wave-surface wave interactions has been studied in a preliminary way elsewhere[2], and a code has been developed to make a more thorough study of nonlinear interaction.

In this paper we use the rather simple results of III to survey qualitative features of electromagnetic wave scattering from the surface. A simple version of scattering theory will be used in which the electromagnetic wave is scattered only once. The more quantitative aspects of the scattering will be addressed in a later paper.

2. Description of the Scattering

Following Callen and Dashen[3] we consider the scattering of a scalar wave field ψ by ocean surface waves. They separated the ocean surface into a long and short wavelength part; the long wavelength part having a small curvature at the ocean surface. The scattering is performed by the short wavelength surface gravity waves riding on the longer waves. In the present analysis, the long wavelength waves become the surface current. This will be discussed in the next section. For scattering an incident plane wave of wavenumber $\underset{\sim}{k}$ to a final wavenumber $\underset{\sim}{k}'$, we have the scattered field in a simple single scattering approximation[3]

$$\psi_{sc} = R_o \int_{A_o} d^2x \, e^{i\underset{\sim}{x}\cdot\Delta\underset{\sim}{k}} \zeta(\underset{\sim}{x},t) \quad . \tag{2.1}$$

Here $\Delta\underset{\sim}{k} \equiv \underset{\sim}{k} - \underset{\sim}{k}'$ is the change of wavenumber vector, R_o is an amplitude factor, and the integral is taken over the illuminated area A_o of the ocean. The vector $\underset{\sim}{x}$ lies in the horizontal plane of the ocean surface when no waves are present and $\zeta(\underset{\sim}{x},t)$ is the vertical displacment of the surface at $\underset{\sim}{x}$ and time t due to wave motion.

We use the notation of III and write

$$\zeta(\underset{\sim}{x},t) = i\left[Z(\underset{\sim}{x},t) - Z^*(\underset{\sim}{x},t)\right]/2$$

$$= -\mathrm{Im}\left[Z(\underset{\sim}{x},t)\right] \tag{2.2}$$

where

$$Z(\underset{\sim}{x},t) = \sum_{\underset{\sim}{k}} e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}}\, a(\underset{\sim}{k}) \quad . \tag{2.3}$$

Equation (2.3) provides a Fourier representation of Z in an assumed large area A_o of the ocean and the Fourier coefficients $a(\underset{\sim}{k})$ are functions of time.

As is customary in changing from "box" or discrete to "continuum" normalization, we may replace the sum over discrete $\underset{\sim}{k}$ by an integral over continuous $\underset{\sim}{k}$ with the substitution

$$\sum_{\underset{\sim}{k}} \to \frac{A_o}{(2\pi)^2}\int d^2\, k \quad . \tag{2.4}$$

The power spectrum of $\zeta(\underset{\sim}{x},t)$ at a position $\underset{\sim}{x}$ is given by the Wigner[4] transformation

$$F(\underset{\sim}{x},\underset{\sim}{k}) = \tfrac{1}{2}\sum_{\underset{\sim}{\rho}} e^{i\underset{\sim}{\rho}\cdot\underset{\sim}{x}} \left\langle a\left(\underset{\sim}{k} + \frac{\underset{\sim}{\rho}}{2}\right) a^*\left(\underset{\sim}{k} - \frac{\underset{\sim}{\rho}}{2}\right)\right\rangle$$

$$= \frac{1}{2A_o}\int d^2 r\; e^{-i\underset{\sim}{r}\cdot\underset{\sim}{k}} \left\langle Z\left(\underset{\sim}{x} + \frac{\underset{\sim}{r}}{2}\right) Z^*\left(\underset{\sim}{x} - \frac{\underset{\sim}{r}}{2}\right)\right\rangle , \tag{2.5}$$

where $\langle\ldots\rangle$ represents an ensemble average over many observations of the sea state.

Now,

$$\frac{1}{A_o}\sum_{\underset{\sim}{k}} e^{i\underset{\sim}{k}\cdot(\underset{\sim}{x}-\underset{\sim}{y})} = \delta(\underset{\sim}{x}-\underset{\sim}{y}) \quad , \tag{2.6}$$

the Dirac δ-function, and

$$\frac{1}{A_o}\int d^2r\, e^{i(\underset{\sim}{k}-\underset{\sim}{p})\cdot\underset{\sim}{r}} = \delta_{\underset{\sim}{k},\underset{\sim}{p}} \quad , \tag{2.7}$$

the Kronecker δ-function. Thus, from (2.5) we obtain

$$\sum_{k} F(\underset{\sim}{x},\underset{\sim}{k}) = \tfrac{1}{2}\langle |Z(\underset{\sim}{x})|^2\rangle = \langle \zeta^2(\underset{\sim}{x})\rangle \quad , \tag{2.8}$$

since we anticipate that

$$\langle a(\underset{\sim}{k})\ a(\underset{\sim}{p})\rangle = \langle Z(\underset{\sim}{x})\ Z(\underset{\sim}{y})\rangle = 0 \quad , \tag{2.9}$$

i.e., the phase of the complex amplitude varies so rapidly in time that its net contribution to the statistical average vanishes.

If we wish to use continuum wavenumber variables, we refer to (2.4) and define the spectral function

$$\Psi(\underset{\sim}{x},\underset{\sim}{k}) = \frac{A_o}{(2\pi)^2} F(x,\underset{\sim}{k}) \quad , \tag{2.10}$$

with the normalization

$$\int d^2k\ \Psi(\underset{\sim}{x},\underset{\sim}{k}) = \langle \zeta^2(\underset{\sim}{x})\rangle \quad . \tag{2.11}$$

Now, we return to Eq. (2.1) and obtain for the mean scattered power the expression

$$P = |\psi_{sc}|^2$$

$$= |R_o|^2 \int_{A_o} d^2x d^2y \; e^{i\Delta\underset{\sim}{k}\cdot(\underset{\sim}{x}-\underset{\sim}{y})} \langle \zeta(\underset{\sim}{x},t)\, \zeta^*(\underset{\sim}{y},t)\rangle$$

$$= \frac{|R_o|^2}{4} \int_{A_o} d^2x d^2y \; e^{i\Delta\underset{\sim}{k}\cdot(\underset{\sim}{x}-\underset{\sim}{y})} \left[\langle Z(\underset{\sim}{x})\, Z^*(\underset{\sim}{y})\rangle + \langle Z^*(\underset{\sim}{x}) Z(\underset{\sim}{y})\rangle\right]$$

$$= \frac{|R_o|^2}{4} \int_{A_o} d^2v \quad d^2r \; e^{-i\underset{\sim}{r}\cdot\Delta\underset{\sim}{k}} \Big[\Big\langle Z\Big(\underset{\sim}{v}+\frac{\underset{\sim}{r}}{2}\Big)\, Z^*\Big(\underset{\sim}{v}-\frac{\underset{\sim}{r}}{2}\Big)\Big\rangle + \Big\langle Z\Big(\underset{\sim}{v}-\frac{\underset{\sim}{r}}{2}\Big)\, Z^*\Big(\underset{\sim}{v}+\frac{\underset{\sim}{r}}{2}\Big)\Big\rangle\Big]$$

$$= \frac{A|R_o|^2}{2} \int_{A_o} d^2x\left[F(\underset{\sim}{x},\Delta\underset{\sim}{k}) + F(\underset{\sim}{x}, -\Delta\underset{\sim}{k})\right]$$

$$= (2\pi)^2 \frac{|R_o|^2}{2} \int_{A_o} d^2x\left[\Psi(\underset{\sim}{x},\Delta\underset{\sim}{k}) + \Psi(\underset{\sim}{x}, -\Delta\underset{\sim}{k})\right] \quad . \qquad (2.12)$$

When the illuminated area A_o is sufficiently small that Ψ is nearly constant over that area, we have

$$P \cong (2\pi)^2 A_o\, |R_o|^2 \left[\Psi(\underset{\sim}{x}_o, \Delta\underset{\sim}{k}) + \Psi(\underset{\sim}{x}_o, -\Delta\underset{\sim}{k})\right] \qquad (2.13)$$

where $\underset{\sim}{x}_o$ lies within the illuminated area. To observe $\Psi(\underset{\sim}{x}_o, \Delta\underset{\sim}{k})$ or $\Psi(\underset{\sim}{x}_o, -\Delta\underset{\sim}{k})$ individually, it is necessary to measure the Doppler shift. For backscatter, for example, a downshift in frequency will be associated with $\Psi(\underset{\sim}{x}_o, \Delta\underset{\sim}{k})$ and a shift up for $\Psi(\underset{\sim}{x}_o, -\Delta\underset{\sim}{k})$.

It is seen from Eqs. (2.12) and (2.13) that the basic requirement for evaluating the scattered energy from the ocean surface requires the expressions for Z obtained in III to calculate Ψ.

3. The Wigner Power Spectrum of $Z(\underset{\sim}{x},t)$ in the Resonance Region

In III we assumed the interaction of the surface gravity waves with the prescribed surface current to be "turned on" at time t = 0. The expression obtained for Z was of the form

$$Z(\underset{\sim}{x},t) = \sum_{\underset{\sim}{k}} e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}} A(\underset{\sim}{k})\, G_{\underset{\sim}{k}}(\underset{\sim}{x},t) \quad , \tag{3.1}$$

where $G_{\underset{\sim}{k}}(\underset{\sim}{x},o) = 1$. The quantities $A(\underset{\sim}{k})$ thus correspond to the $a(\underset{\sim}{k})$ of Eq. (2.3) when there is no surface current. The "ambient" power spectrum, corresponding to the absence of a surface current, is then

$$F_a(\underset{\sim}{x},\underset{\sim}{k}) = \tfrac{1}{2} \sum_{\underset{\sim}{\rho}} e^{i\underset{\sim}{\rho}\cdot\underset{\sim}{x}} \left\langle A\left(\underset{\sim}{k}+\frac{\underset{\sim}{\rho}}{2}\right) A^*\left(\underset{\sim}{k}-\frac{\underset{\sim}{\rho}}{2}\right)\right\rangle \quad , \tag{3.2}$$

or

$$\Psi_a(\underset{\sim}{x},\underset{\sim}{k}) = \frac{A_o}{(2\pi)^2} F_a(\underset{\sim}{x},\underset{\sim}{k}) \quad . \tag{3.3}$$

When the ambient wave spectrum has a negligible spatial variation, we have

$$\langle A(\underset{\sim}{k})\, A^*(\underset{\sim}{\ell})\rangle = \langle |A(\underset{\sim}{k})|^2\rangle\, \delta_{\underset{\sim}{k},\underset{\sim}{\ell}}$$

and

$$F_a(\underset{\sim}{x},\underset{\sim}{k}) \equiv F_a(\underset{\sim}{k}) = \tfrac{1}{2}\langle|A(\underset{\sim}{k})|^2\rangle \quad . \tag{3.4}$$

Continuing to follow III, we suppose the surface current to be sinusoidal, with wavenumber K and phase velocity c_I , and to have the form

$$\underset{\sim}{U}(\xi) = \underset{\sim}{\hat{i}}\, U(\xi) = \underset{\sim}{\hat{i}}\, U_o \cos(K\xi) \quad , \tag{3.5}$$

where

$$\xi = x - c_I\, t \quad . \tag{3.6}$$

The modulation function $G_{\underset{\sim}{k}}$ was modelled for the "resonance region" in III as follows. First, the resonance wavenumber k_r is determined by the condition

$$c_I - c_g(k_r)\cos\theta_r = 0 \quad , \tag{3.7}$$

where $c_g(k_r) = (g/k_r)^{\frac{1}{2}}/2$ and $\cos\theta_r = \underset{\sim}{k}_r \cdot \underset{\sim}{\hat{i}}/k$. The resonance region was described as extending over a range

$$\Delta k = K\, S^{\frac{1}{2}} \quad ,$$

$$S = 2\left(\frac{k_r}{K}\right)^2 \frac{U_o}{c_I(3\cos^2\theta - 2)} \quad , \tag{3.8}$$

about k_r .

In this region we use Eq. (4.42) of III to write

$$G_{\underset{\sim}{k}} = \exp\left[-i\underset{\sim}{k}\cdot\underset{\sim}{U}(\xi)t\right] \hat{G} \quad ,$$

$$\hat{G} \cong \Bigg[1 + \left(\frac{U_o Kt}{2}\right) (1 + \tfrac{1}{2}\cos^2\theta) \sin(K\xi)$$

$$+ (U_o kt/2)^2 \cos(K\xi) \; (1 - t/\tau_p)^2/S$$

$$+ \left(\frac{t}{\tau_p}\right)^4 \cdot \frac{\sin\left[2S^{1/8} \sin(K\xi)\right]}{\sin(K\xi)}\Bigg] \quad , \qquad t < \tau_p \quad . \qquad (3.9)$$

and

$$\tau_p = \frac{\pi}{2} S^{\frac{1}{2}}/U_o k_r \quad . \qquad (3.10)$$

If we substitute (3.9) into (3.1) we obtain in a straightforward way

$$\left\langle Z\left(\underset{\sim}{x}+\frac{\underset{\sim}{r}}{2}\right) \; Z^*\left(\underset{\sim}{x}-\frac{\underset{\sim}{r}}{2}\right)\right\rangle \cong \sum_{\underset{\sim}{P}} \sum_{\underset{\sim}{\rho}} e^{i\underset{\sim}{\rho}\cdot\underset{\sim}{x}} \, e^{i\underset{\sim}{P}\cdot\underset{\sim}{r}}$$

$$\times \left\langle A\left(\underset{\sim}{P}+\frac{\underset{\sim}{\rho}}{2}\right) \; A^*\left(\underset{\sim}{P}-\frac{\underset{\sim}{\rho}}{2}\right)\right\rangle$$

$$\times \exp\left[2it \, \underset{\sim}{P}\cdot\underset{\sim}{U}_o \sin(K\xi) \; \sin\left(\underset{\sim}{K}\cdot\frac{\underset{\sim}{r}}{2}\right)\right]$$

$$\times \exp(-i\underset{\sim}{\rho}\cdot\underset{\sim}{U}t) \; |\hat{G}|^2 \quad , \qquad (3.11)$$

where $\underset{\sim}{K} = K\hat{\underset{\sim}{i}}$ and $\underset{\sim}{U}_o = U_o\hat{\underset{\sim}{i}}$. The relation

$$e^{iz\,\sin\theta} = \sum_{\nu=-\infty}^{\infty} e^{\nu i\theta}\, J_\nu(z) \quad ,$$

where $J_\nu(z)$ is a Bessel function of the first kind of order ν, in conjunction with Eq. (2.5) allows us to write the spectral density as

$$F(\underset{\sim}{x},\underset{\sim}{k}) = \sum_{\nu=-\infty}^{\infty} F_a\left((\underset{\sim}{x}-\underset{\sim}{U}t),\ \underset{\sim}{k}-\frac{\nu}{2}\underset{\sim}{K}\right)$$

$$\times\, J_\nu\left[2t\left(\underset{\sim}{k}-\frac{\nu}{2}\underset{\sim}{K}\right)\cdot\underset{\sim}{U}_o\ \sin(K\xi)\right]\,|\hat{G}|^2 \quad . \tag{3.12}$$

4. The Wigner Power Spectrum of $Z(\underset{\sim}{x},t)$ Outside the Resonance Region

We use the expressions (3.8) to specify the non-resonant region by the condition that

$$\frac{U_o}{(c_I - c_g \cos\theta)} << \left(\frac{U_o}{c_I}\right)^{\frac{1}{2}} \quad , \tag{4.1}$$

where now $c_g(k) = (g/k)^{\frac{1}{2}}/2$ and $\cos\theta = \underset{\sim}{k}\cdot\hat{i}/k$. Here we may use the WKB approximation to evaluate $G_{\underset{\sim}{k}}$ in Eq. (3.1). From Eqs. (5.4), (5.7), and (5.9) of III we obtain

$$G_{\underset{\sim}{k}} = \exp\left[\frac{i\underset{\sim}{k}\cdot\underset{\sim}{U}_o \sin(K\xi)}{K(c_I - c_g \cos\theta)}\right]\hat{G} \quad ,$$

$$\hat{G} = 1 + U\cos\theta\ (c_I - 2c_g\cos\theta + 2c_g \sin^2\theta)\Big/\left[4(c_I - c_g\cos\theta)^2\right] . \tag{4.2}$$

Evaluation of $F(\underset{\sim}{x},\underset{\sim}{k})$ now leads to the expression

$$F(\underset{\sim}{x},\underset{\sim}{k}) \cong \sum_{\nu=-\infty}^{\infty} F_a\left[\underset{\sim}{x} + \underset{\sim}{h}\sin(K\xi),\ \underset{\sim}{k} - \frac{\nu}{2}\underset{\sim}{K}\right]$$

$$\times\ J_\nu\left(\frac{2\left(\underset{\sim}{k}-\frac{\nu}{2}\underset{\sim}{K}\right)\cdot\underset{\sim}{U}}{K\left[c_I - c_g(|\underset{\sim}{k}-\frac{\nu}{2}\underset{\sim}{K}|)\cos\theta\right]}\right)|\hat{G}|^2 \quad . \tag{4.3}$$

Here we have evaluated $\hat{G}$ at the wavenumber $\underset{\sim}{k}$, a seemingly adequate approximation. The quantity $\underset{\sim}{h}$ in Eq. (4.3) is

$$\underset{\sim}{h} = \nabla_{\underset{\sim}{k}} \left[\frac{\underset{\sim}{k} \cdot \underset{\sim}{U}_o}{K(c_I - c_g \cos\theta)} \right]$$

$$= \frac{U_o}{K(c_I - c_g \cos\theta)^2} \left[c_I \underset{\sim}{\hat{i}} - \frac{3}{2} \left(\frac{\underset{\sim}{k}}{k} \right) c_g \cos^2\theta \right] \quad . \quad (4.4)$$

For wavelengths $2\pi/k$ much less than the wavelength at which resonance can occur, there is another contribution to (4.3) due to interaction with waves of wavenumber $\underset{\sim}{L}$ near resonance. In this case, Eq. (4.3) can be used, but with $\hat{G}$ given by Eq. (5.15) of III:

$$|\hat{G}|^2 \cong 1 + \frac{U \cos\theta}{2\, c_I} + \int d^2L\ \Psi_a(\underset{\sim}{L})\ \frac{L^2}{2}\ (\underset{\sim}{\hat{k}} \cdot \underset{\sim}{\hat{L}})\ \mathrm{Re}(G_{\underset{\sim}{L}} - 1)\ , \quad k >> L. \quad (4.5)$$

Here we have assumed that the ambient spectral function $\Psi_a(\underset{\sim}{L})$ is independent of $\underset{\sim}{x}$, as was done in III.

5. Simplification for Small Amplitude Current Effects

At this point it is convenient to make the transformation (2.12) to the continuum power spectra $\Psi(\underset{\sim}{x},\underset{\sim}{k})$ in Eqs. (3.12) and (4.3). We shall also assume that the ambient spectrum is independent of position, so

$$\Psi_a(\underset{\sim}{x},\underset{\sim}{k}) \equiv \Psi_a(\underset{\sim}{k}) \quad . \tag{5.1}$$

When $\frac{\nu}{2}\,|\underset{\sim}{K}| << |\underset{\sim}{k}|$ and the current is sufficiently weak, we can expand both the ambient spectra and Bessel functions in power of $\left(\frac{\nu K}{2}\right)$. On keeping only the lowest order and linear term, we can carry out the sum over ν in closed form with the relations[5]

$$\sum_{\nu=-\infty}^{\infty} J_\nu(a) = 1$$

$$\sum_{\nu=-\infty}^{\infty} \nu\, J_\nu(a) = a \quad .$$

For the case corresponding to Eq. (3.12) we obtain

$$\Psi(\underset{\sim}{x},\underset{\sim}{k}) = \Psi_a(\underset{\sim}{k})\left\{1 + \left|\frac{U_o K t}{2}\right| \cos^2\theta \sin(K\xi)\right.$$

$$+ \tfrac{1}{2}(U_o K t)^2 \cos(K\xi) \left|1 - \frac{t}{\tau_p}\right|^2 / S$$

$$\left. + 2\left(\frac{t}{\tau_p}\right)^4 \frac{\sin\left[2S^{1/8} \sin(K\xi)\right]}{\sin K\xi}\right\}$$

$$- \left[k \frac{\partial \Psi_a(\underset{\sim}{k})}{\partial k_x}\right] \left[tKU_o \sin(K\xi)\right] \quad . \tag{5.2}$$

For waves far from resonance, we make the corresponding approximation in Eq. (4.3) to obtain

$$\Psi(\underset{\sim}{x},\underset{\sim}{k}) = \Psi_a(\underset{\sim}{k}) \left\{1 + \frac{U}{2(c_I - c_g \cos\theta)^2} \left[c_I(\cos\theta - 2)\right.\right.$$

$$\left.\left. + 2c_g \cos\theta(1-\cos\theta) + c_g \cos^3\theta\right]\right\}$$

$$- \left[k \frac{\partial \Psi_a(\underset{\sim}{k})}{\partial k_x}\right]\left[\frac{U \cos\theta}{c_I - c_g \cos\theta}\right] \quad . \tag{5.3}$$

Here $c_g = (g/k)^{\frac{1}{2}}/2$ and $\cos\theta = \hat{\underset{\sim}{i}} \cdot \hat{\underset{\sim}{k}}$, as before.

Finally, for wavelengths very short compared to wavelengths near resonance, we use Eq. (4.5) to obtain

$$\Psi(\underset{\sim}{x},\underset{\sim}{k}) = \Psi_a(\underset{\sim}{k}) \left[1 + \frac{U}{2c_I}(\cos\theta - 2)\right.$$

$$\left. + \int d^2L \; \Psi_a(\underset{\sim}{L}) \; \frac{L^2}{2} \; (\hat{\underset{\sim}{k}} \cdot \hat{\underset{\sim}{L}}) \; \mathrm{Re}(G_L - 1)\right]$$

$$- \left[k \frac{\partial \Psi_a(\underset{\sim}{k})}{\partial k_x}\right] \left[\frac{U \cos\theta}{c_I}\right] \quad . \tag{5.4}$$

6. Application to Phillips' Spectrum

As an application, we choose for $\Psi_a(\underset{\sim}{k})$ the spectrum from Phillips[6]

$$\Psi_a(\underset{\sim}{k}) = \frac{B}{\pi} |\underset{\sim}{k}|^{-4}, \ \underset{\sim}{k} \text{ within } 90^\circ \text{ of wind direction,}$$

$$B = 0.4\times10^{-2} \quad . \tag{6.1}$$

Thus

$$k \frac{\partial\Psi_a(\underset{\sim}{k})}{\partial k_x} = -4 \cos\theta \ \Psi_a(\underset{\sim}{k}) \quad . \tag{6.2}$$

Our parameters are chosen to correspond to typical oceanographic conditions:

$$\begin{aligned} U_o &= 10^{-2} \text{ m/sec} \\ K &= 10^{-2} \text{ m}^{-1} \\ k_r &= 8.66 \text{ m}^{-1} \\ c_I &= 0.50 \text{ m/sec} \\ \theta &= 20^\circ \end{aligned} \tag{6.2}$$

Here k_r is the resonant wavenumber defined by $c_I = c_g \cos\theta$. The strength parameter S is

$$S = 4.609\times10^4 \tag{6.3}$$

and

$$\tau_r = \frac{2}{k_{res}\, U_o} = 23.09 \text{ sec} \quad . \tag{6.4}$$

Rosenbluth's "pile up time" is then

$$\tau_p = 3894 \text{ sec} \tag{6.5}$$

and the extent of the resonance region [Eq. (3.8)] is

$$\Delta k = KS^{1/2} = 2.147 \text{ m}^{-1} \quad . \tag{6.6}$$

For the spectrum (6.1) the term involving an integral over $\underset{\sim}{L}$ in Eq. (5.4) is negligible compared with the other two. In Eqs. (5.2), (5.3) and (5.4) the term involving $k\,\frac{\partial\Psi_a}{\partial k_x}$ appears to dominate [except for $t \gg \tau_r$ in Eq. (5.2)]. This was pointed out by Milder[7] in his discussion of spectral perturbations. For a spectrum of the form k^p the relative contribution of the first and last terms in Eq. (5.4) is approximately $\frac{1}{2p}$, which in two dimensions is .125.

For the parameters given by Eqs. (6.2) - (6.6), Eq. (5.7) becomes, for an adverse current,

$$\Psi(\underset{\sim}{x},\underset{\sim}{k}) = \Psi_a(\underset{\sim}{k}) \left\{ 1 - .01 \left(\frac{t}{\tau_r}\right) \sin K\xi - 4.34\times10^{-5} \left(\frac{t}{\tau_r}\right)^2 \left(1 - \frac{t}{\tau_p}\right)^2 \cos K\xi \right\} . \tag{6.7}$$

The term linear in t above results primarily from the spectral gradient term, i.e., 90% of its value.

To illustrate the spectral perturbation in the regions away from resonance, we graph Eq. (5.3) in Figure 1 excluding the interval Δk about k_r. It is evident that above the maxima of the surface current ($\xi = 0$), the longer waves are enhanced and the shorter waves suppressed. The magnitude of the perturbation is found to increase as one goes to smaller angles. At $\theta = 0°$, the perturbation extremes are in substantial agreement with previous calculations[8,9].

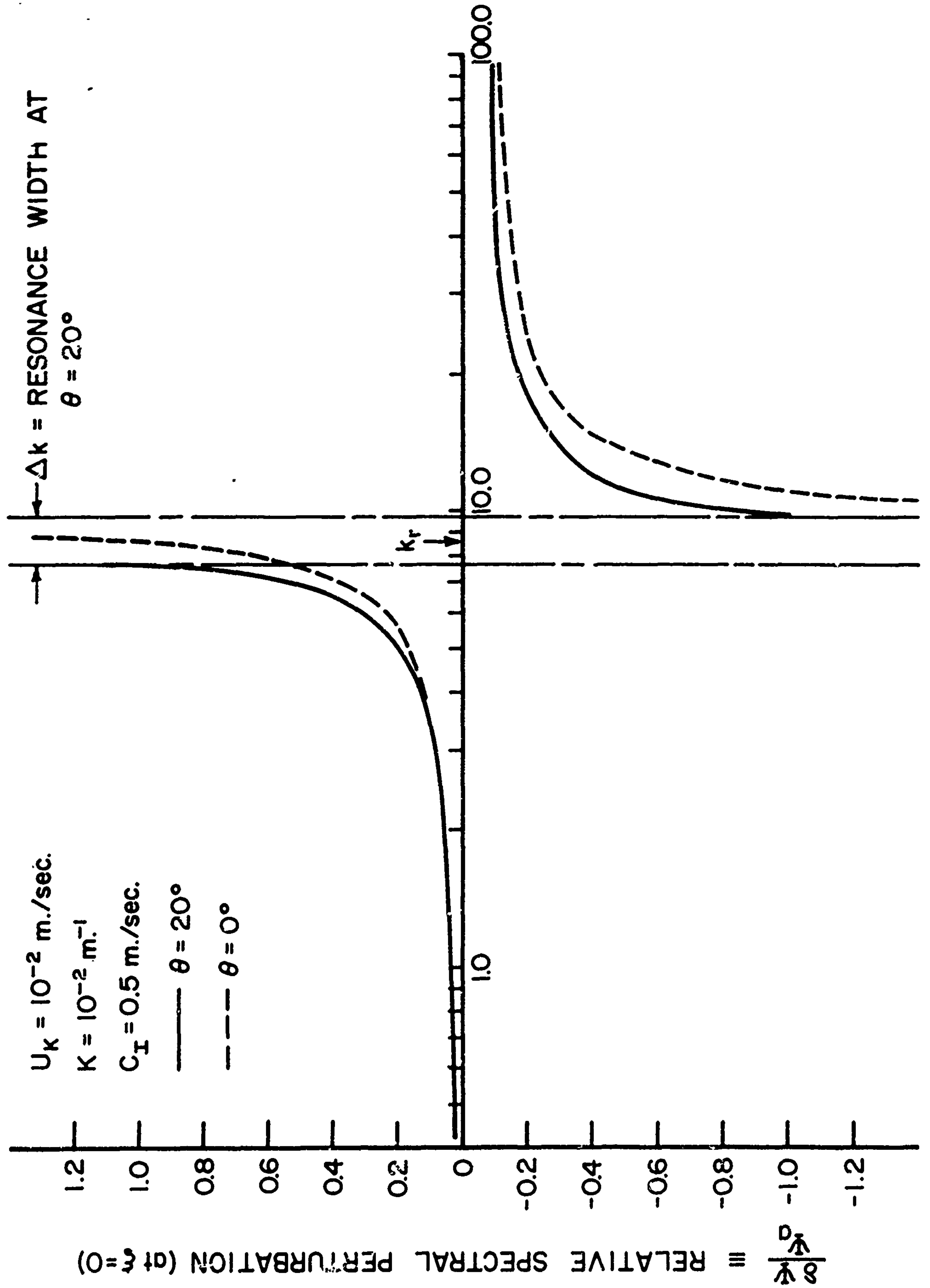
The width of the resonant region in Figure 1 is given by

$$\Delta k = KS^{\frac{1}{2}}$$

so that using Eq. (3.8) with the parameters we have been discussing,

$$\frac{\Delta k}{k_r} = \frac{.1155}{\sqrt{\cos^2\theta - .666}} \qquad (6.9)$$

which is plotted in Figure 2. We see from the slow rise of Eq. (6.9) from its minimum at $\theta = 0°$ that the resonance width is proportional to the resonance wavenumber for



Δk = RESONANCE WIDTH AT
θ = 20°
$U_K = 10^{-2}$ m./sec.
$K = 10^{-2}$ m.$^{-1}$
$C_I = 0.5$ m./sec.
θ = 20°
θ = 0°
k_r
1.0
10.0
100.0
1.2
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
$\frac{\delta \Psi}{\Psi_0}$ ≡ RELATIVE SPECTRAL PERTURBATION (at ξ=0)

Figure 1.

$|\theta| \lesssim 30°$ to within a factor of 2. In fact, it is only within a relatively narrow angular range about $\theta = \pm 35.26°$ that the two-dimensional effects become very strong.

Finally we assume $k >> k_{res}$ and write Eq. (5.4) as

$$\Psi(\underset{\sim}{x},\underset{\sim}{k}) \cong \Psi_a(\underset{\sim}{k}) \left[1 - 0.06 \ \cos(K\xi)\right] \quad . \qquad (6.10)$$

Here also, the $\partial\Psi_a/\partial k_x$ term provides the dominant contribution to the modulation. The coefficient (.065) in Eq. (6.10) is seen to be the asymptote of the relative perturbation for the short waves as seen in Figure 1.

In Figure 1, we see the effect of the dependence of the resonant wavenumber (k_r) on angle, and from Eq. (6.9) the dependence of the spectral interval on angle. The solid line is the spectral perturbation for $\theta = 20°$ and the dashed for $\theta = 0°$. The resonance interval $(k_r \pm \Delta k/2)$ is indicated for the $\theta = 20°$ case. The $0°$ perturbation is seen to be shifted to a higher wavenumber and the maximum perturbation exceeds the $20°$ case, although for $k \lesssim (k_r - \Delta k/2)_{\theta=20°}$ the $20°$ perturbation exceeds the $0°$ perturbation. The suppression of the shorter waves, however, is greater for all wavenumbers in the $0°$ case.

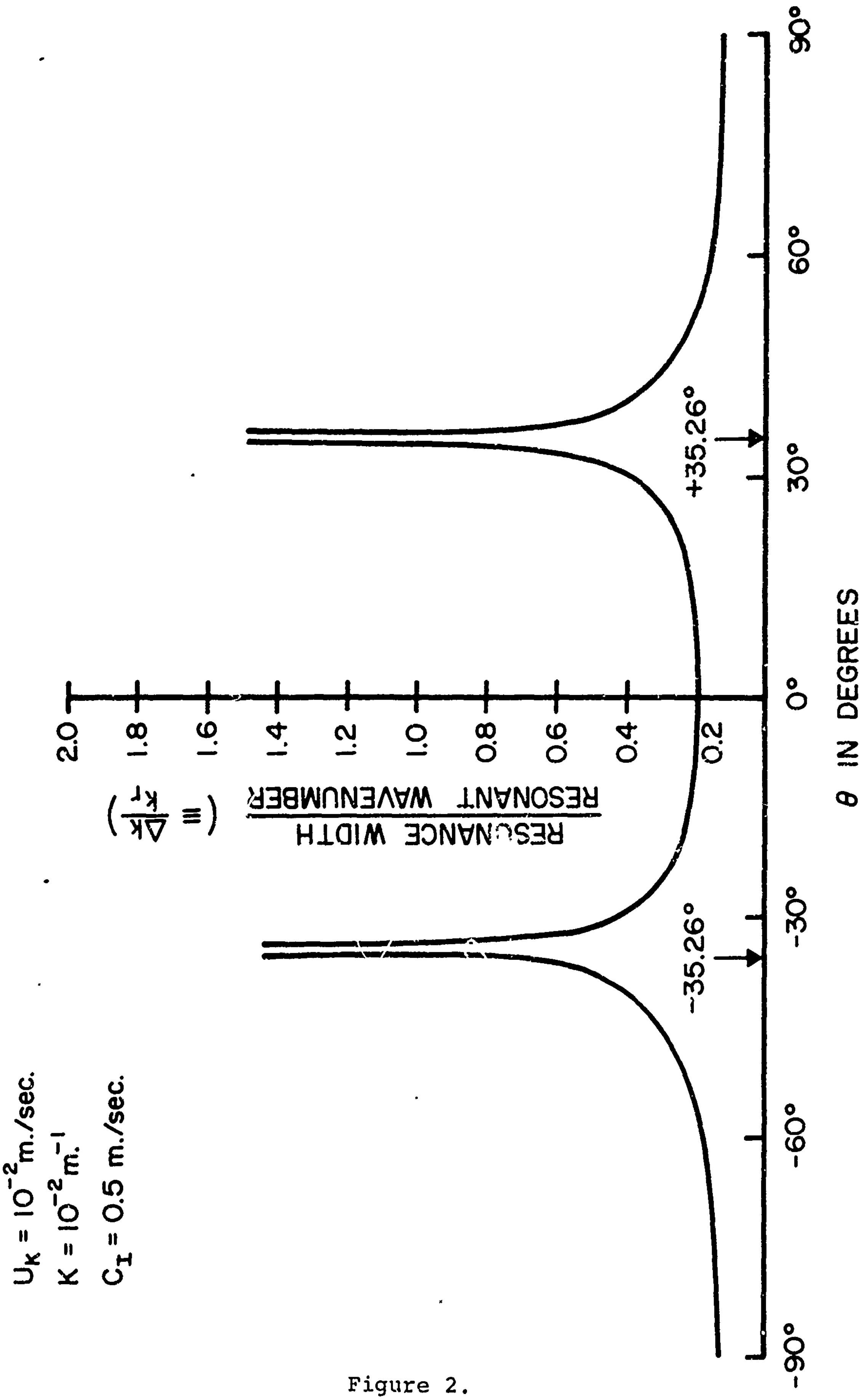

$U_k = 10^{-2}$ m./sec.
$K = 10^{-2}$ m.$^{-1}$
$C_I = 0.5$ m./sec.
RESONANCE WIDTH
RESONANT WAVENUMBER
$(\equiv \frac{\Delta k}{k_r})$
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
-90°
-60°
-30°
0°
30°
60°
90°
-35.26°
+35.26°
θ IN DEGREES

Figure 2.

References

1. K.M. Case, K. M. Watson, and B.J. West, "Energy Spectra of the Ocean Surface: III. Modulation by a Surface Current", PD 73-047. This paper will henceforth be referred to as "III".

2. K.M. Watson, B.J. West and B.I. Cohen, "Energy Spectra of the Ocean Surface: II. Interaction with Surface Current", PD 73-037.

3. C. Callen and R. Dashen, "Generation and Airborne Detection of Internal Waves from an Object Moving Through a Stratified Ocean, Vol. II: Supporting Analyses", Study S-334, Institute for Defense Analyses, April 1969.

4. E.P. Wigner, Phys. Rev. 40, 749 (1932).

5. A.D. Wheelon, Tables of Summable Series and Integrals Involving Bessel Functions, p. 49, Halden-Day, Inc., San Francisco, Ca. (1968).

6. O.M. Phillips, The Dynamics of the Upper Ocean, p. 111 Cambridge University Press (1969).

7. INTENTIONALLY DELETED,

8. J.A.L. Thomson and B.J. West, "Interaction of Non-Saturated Surface Gravity Waves with Internal Waves", PD 72-023 (1972).

9. B.J. West, J.A.L. Thomson and K.M. Watson, "Statistical Mechanics of Ocean Waves", AIAA Paper No. 74-65, to be published.

MISSION
of
Rome Air Development Center

RADC is the principal AFSC organization charged with planning and executing the USAF exploratory and advanced development programs for electromagnetic intelligence techniques, reliability and compatibility techniques for electronic systems, electromagnetic transmission and reception, ground based surveillance, ground communications, information displays and information processing. This Center provides technical or management assistance in support of studies, analyses, development planning activities, acquisition, test, evaluation, modification, and operation of aerospace systems and related equipment.