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MISS DISTANCE MEASUREMENT USING ONE CAMERA

by

A. T. Smith

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SUMMARY

Data from one high frame speed camera on a target can give missile trajectory and miss distance. This Report gives a theoretical solution and indicates how it can be modified to process practical data in a computer. The sources of error and their magnitudes are given.

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1 INTRODUCTION

Whenever a missile is fired at a target it is important during the final stage of attack, to obtain relative trajectory and miss distance information. At Aberporth these are either aerial or sea targets and the attacking missile is normally sighted by one or more high speed cameras. These cameras are free running and run nominally at one hundred frames per second. Time information from the Range Central Timing Unit is recorded on the edge of the camera film, so that an accurate time can be given to each frame.

Only two types of aerial targets at present carry a camera system, these are aircraft targets and towed targets (infra-red or radar sources towed by an aircraft). Normally the missile is photographed by two camera packs (each pack consisting of two cameras) one on each wing tip of the target aircraft. The data obtained from these cameras can then be interpolated to corresponding times, to allow a simple two camera solution[1] using the wing span of the aircraft as a baseline. It has been found from experience that the extrapolation of camera data seldom gives satisfactory results.

For aircraft targets and 13 metres towed targets the camera packs consist of two WRETAR Mark 3 cameras[2]. These cameras have a field of view of 186° and are mounted to give a complete spherical optical coverage. At least three occasions arise on these targets when a single camera solution must give trajectory data. These occur when

(a) a faulty camera fails to record either pictures or the timing information

(b) a camera pack is removed to allow additional equipment on the aircraft, (e.g. a telemetry pack)

(c) the time interval common to both cameras for interpolating data is too short to allow a satisfactory two camera solution over the intercept period. Although the camera packs have full spherical coverage, on some flight paths the missile is obscured by the aircraft, for part of the intercept, from one or both cameras. This situation can be made worse by certain events (e.g. flash) which results in the loss of further data. Quite often in these conditions one or both cameras will give sufficient data for a single camera solution, given an independent velocity estimate.

The single camera solution is considered too inaccurate for use on the longer range, 30 metres and 61 metres, towed targets. The greater ranges involved, of the missile from the camera, (see section 5) and the probable inaccuracies in the knowledge of the tow position are the main reasons for this decision.

The Range's sea targets were basically designed for a single camera solution as the fields of view of the target cameras do not overlap to any great extent, except on the largest targets. The cameras used are GW1 Mark 1A and GW2 Mark 2, and these have a field of view of 120°. This means that most missile trials against sea targets require a single camera solution.

Until recently a manual single camera solution described in a previous Report[3] has been used. This manual solution has now been superseded by a computer solution. This Report describes the theoretical solution of the problem and how this solution is modified for computer use in the practical case.

2 THEORETICAL SOLUTION

To obtain a theoretical solution for trajectory data using a single camera, certain assumptions have to be made about the trajectory during the period of time covered by the solution. The trajectory in question is the relative trajectory of the missile with respect to (a) the camera origin and (b) the camera axes system. These assumptions are

(1) the missile flies in a straight line

(2) the velocity V of the missile is a known constant.

In practice these assumptions imply that the target (and hence camera) is also moving in a straight line at constant velocity and that the attitude of the target is steady.

Since basically any camera system supplies direction cosine vectors of a trajectory with respect to the camera, the following theory is based on direction cosine vectors. For the theoretical solution, it will be assumed that there are no errors in the direction cosine vectors. The effect of these and other practical errors will be discussed briefly in a later section.

The missile flies in a straight line and therefore, provided the camera does not lie in the path of the missile the missile trajectory and the camera origin define a unique plane in space. In addition all rays from the camera to the missile and hence all direction cosine vectors of the missile with respect to the camera, lie in this plane. The missile also flies at a constant velocity both spatially and with respect to camera axes, and therefore the distance flown between two given time points depends only on the time difference and not on absolute time.

Let $\underline{P}_i = (x_i, y_i, z_i)$ be the position vector of the missile with respect to the camera, at the time t_i. Then a constant velocity implies that

$$D_i \propto t_{i+1} - t_i \quad ,$$

i.e. $\dfrac{D_i}{t_{i+1} - t_i}$ = constant (= velocity of the missile)

where $D_i = |\underline{P}_{i+1} - \underline{P}_i|$

$$= \sqrt{\left\{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 + (z_{i+1} - z_i)^2\right\}}$$

see Fig.1.

2.1 Calculation of missile direction

Consider any line parallel to the missile trajectory which also lies in the camera-trajectory plane. The length of a section of this line, formed by two rays from the camera to the missile trajectory, depends only on the time difference between these two rays. This follows because the missile trajectory also has this property. Therefore the direction of the missile can be found from just direction cosine vectors. This is achieved by finding a line in the camera-trajectory plane with the following property:-

The direction cosine vectors from the camera should cut the line into sections whose length depend only on the time difference between the successive direction cosine vectors.

With reference to Fig.2 let $\underline{d}_i$ be the direction cosine vector of the missile at time t_i. Let ℓ_i be the length of the section of a line formed by the successive direction cosine vectors $\underline{d}_{i+1}$ and $\underline{d}_i$. Then

$$\frac{\ell_i}{t_{i+1} - t_i} = \text{constant for a line parallel to the missile trajectory} \quad .$$

A line with this property can be obtained by using the following procedure:-

Define a new system of axes X', Y', Z' such that

(a) Z' is in the direction of a vector perpendicular to the camera-trajectory plane,

(b) Y' is in the direction of the first direction cosine vector $\underline{d}_1$,

(c) X' is in the direction of a vector perpendicular to both the Y' and Z' axes, and

(d) X', Y' and Z' form a right handed triad.

Both the new X' and Y' axes lie in the camera-trajectory plane and hence form a base for the plane. Any direction cosine vector in this plane can now be described by the single variable

$$S_i = \frac{\underline{y} \cdot \underline{d}_i}{\underline{x} \cdot \underline{d}_i} \text{ (see Fig.3)}$$

where $\underline{x}$ and $\underline{y}$ are unit vectors along the axes X' and Y' respectively. Note that $y = S_i x$ is a line in the plane (X', Y') which lies in the same direction as $\underline{d}_i$ and that $S_1 = \infty$.

Since any line with the relevant property is required, a line of the form

$$y = Mx + 1$$

is calculated. This line intersects

$$y = S_i x$$

when

$$S_i x = Mx + 1 ,$$

i.e.

$$x = \frac{1}{S_i - M} \quad \text{and} \quad y = \frac{M}{S_i - M} + 1 .$$

In theory three direction cosine vectors are now required. For convenience let these be the first three vectors. The line $y = Mx + 1$ intersects the lines $y = S_1x$, $y = S_2x$ and $y = S_3x$ at the points

$$(x_1, y_1) = (0, 1)$$

$$(x_2, y_2) = \left(\frac{1}{S_2 - M}, \frac{M}{S_2 - M} + 1\right)$$

and

$$(x_3, y_3) = \left(\frac{1}{S_3 - M}, \frac{M}{S_3 - M} + 1\right) \quad \text{respectively.}$$

See Fig.4. Then

$$\ell_1 = \sqrt{\left\{\left(\frac{1}{S_2 - M}\right)^2 + \left(\frac{M}{S_2 - M}\right)^2\right\}}$$

and

$$\ell_2 = \sqrt{\left\{\left(\frac{1}{S_3 - M} - \frac{1}{S_2 - M}\right)^2 + \left(\frac{M}{S_3 - M} - \frac{M}{S_2 - M}\right)^2\right\}}$$

$$= \sqrt{(1 + M^2)}\left|\frac{1}{S_3 - M} - \frac{1}{S_2 - M}\right| .$$

For the line $y = Mx + 1$ to have the required property

$$\frac{\ell_1}{t_2 - t_1} = \frac{\ell_2}{t_3 - t_2}$$

where t_1, t_2 and t_3 are the times of the first three direction cosine vectors.

Therefore $$\frac{1}{(S_2 - M)(t_2 - t_1)} = \frac{1}{t_3 - t_2}\left(\frac{1}{S_3 - M} - \frac{1}{S_2 - M}\right)$$

therefore $$\frac{1}{S_2 - M}\left(\frac{t_3 - t_1}{t_2 - t_1}\right) = \frac{1}{S_3 - M}$$

therefore $$M\left(1 - \frac{t_2 - t_1}{t_3 - t_1}\right) = S_3 - S_2\,\frac{t_2 - t_1}{t_3 - t_1} .$$

Hence

$$M = \frac{S_3(t_3 - t_1) - S_2(t_2 - t_1)}{t_3 - t_2} \; .$$

Note that the x and y co-ordinates of the missile in the (X', Y') plane are both linear in time t. It is therefore possible to find a pair of equations of the form

$$y = m(t - t_1) + 1$$

and

$$x = n(t - t_1) \; ,$$

instead of the single equation $y = Mx + 1$. The auxiliary equation $y = \frac{m}{n} x + 1$ then automatically has the required property.

Therefore $$n(t_2 - t_1) = x_2 = \frac{1}{S_2 - \frac{m}{n}} \; ,$$

and

$$n(t_3 - t_1) = x_3 = \frac{1}{S_3 - \frac{m}{n}} \; .$$

Therefore $$S_2 - S_3 = \frac{1}{n}\left\{\frac{1}{t_2 - t_1} - \frac{1}{t_3 - t_1}\right\} \; ,$$

therefore $$n = \frac{t_3 - t_2}{(S_2 - S_3)(t_2 - t_1)(t_3 - t_1)}$$

and

$$m = \frac{S_3(t_3 - t_1) - S_2(t_2 - t_1)}{(S_2 - S_3)(t_2 - t_1)(t_3 - t_1)} \; .$$

In the computer program solution of the problem, the solution is simplified by calculating m and n.

2.2 Calculation of missile trajectory

Now the trajectory of the missile has an equation of the form

$$y = \frac{m}{n} x + r \; ,$$

in the (X', Y') plane and the co-ordinates have equations of the form

$$y = m(t - t_1)r + r$$

$$x = n(t - t_1)r$$

where m and n have already been calculated. Now in one second the missile would have moved a distance of V units, assuming that time is measured in seconds.

Therefore $$r\sqrt{(m^2 + n^2)} = V ,$$

therefore $$r = \frac{V}{\sqrt{(m^2 + n^2)}} .$$

Therefore the x co-ordinate of the missile, in the (X', Y') plane is

$$\frac{nV}{\sqrt{(m^2 + n^2)}} (t - t_1)$$

at the time t. The corresponding y co-ordinate is

$$\frac{mV}{\sqrt{(m^2 + n^2)}} (t - t_1) + \frac{V}{\sqrt{(m^2 + n^2)}} .$$

The position vector of the missile at the time t in camera axis and origin is therefore

$$\frac{V}{\sqrt{(m^2 + n^2)}} \left[n(t - t_1)\underline{x} + \{m(t - t_1) + 1\}\underline{y}\right] .$$

This now completes the theoretical solution since it is possible to calculate the missile position at any given time. The calculation of the time and range of nearest miss, and other useful parameters is given in section 4.

3 PRACTICAL SOLUTION

Considering a missile travelling at 700 m/s, with a limit of 25 g on the lateral acceleration, it can be shown that there exists a straight line from which the missile cannot deviate by more than 16 cm, over a period of 1/10 s (a likely maximum time for a missile to be in view). Therefore in practice, a missile will not deviate much from the assumed model, at least while passing the camera. This model can therefore be used as an approximation, to obtain a trajectory. The solution requires the velocities of the missile and the target, which must be supplied from external sources. Due to errors introduced by the camera system and the error of applying the model to the missile, the solution needs to be modified in the practical case.

The following information will be assumed for the solution of a practical situation:

(a) At least three direction cosine vectors of the missile with respect to a camera and their corresponding times, i.e.

$$(t_i, \underline{d}_i) \qquad \text{for } i = 1,2,\ldots,r\ (r \geqslant 3)$$

where $\underline{d}_i = (\ell_i, m_i, n_i)$, and

(b) the relative velocity V of the missile or at least an estimate for it.

3.1 Calculation of the camera-trajectory plane

The first stage of the computer solution is the calculation of the camera-trajectory plane. Since the direction cosine vectors will not lie perfectly in a common plane, the 'best' plane passing through them must be defined and then calculated. Before continuing it must be noted that any plane passing through the origin can be defined by a single unit vector (L, M, N) . This vector is perpendicular to any vector lying in that plane. If (L, M, N) satisfies this condition then so does $(-L, -M, -N)$, therefore it will be assumed that $N \geqslant 0$. With reference to Fig.5 define θ_i by

$$\begin{aligned} L\ell_i + Mm_i + Nn_i &= \cos\left(\frac{\pi}{2} - \theta_i\right) \\ &= \sin\theta_i \quad . \end{aligned}$$

The 'best' plane is defined as that plane which minimizes

$$\begin{aligned} S(L,M,N) &= \sum_{i=1}^{r} \theta_i^2 \\ &= \sum_{i=1}^{r} \left\{\sin^{-1}\left[L\ell_i + Mm_i + Nn_i\right]\right\}^2 \quad . \end{aligned}$$

Now for small θ , $\sin\theta \approx \theta \approx \sin^{-1}\theta$. Therefore the problem is simplified by finding the vector (L, M, N) which minimizes

$$S(L,M,N) = \sum_{i=1}^{r} \left\{L\ell_i + Mm_i + Nn_i\right\}^2$$

Now since $L^2 + M^2 + N^2 = 1$ and N is assumed non-negative the problem is to find, say L and M , such that S(L,M) is a minimum, where

$$S(L,M) = \sum_{i=1}^{r} \left\{L\ell_i + Mm_i + \sqrt{\left[1 - L^2 - M^2\right]}n_i\right\}^2$$

S(L, M) has a minimum when

$$\frac{\partial S}{\partial L} = \frac{\partial S}{\partial M} = 0 \ ,$$

i.e.

$$\sum_{i=1}^{r} \left\{L\ell_i + Mm_i + Nn_i\right\} * \left\{\ell_i - \frac{n_i L}{N}\right\} = 0$$

and

$$\sum_{i=1}^{r} \left\{L\ell_i + Mm_i + Nn_i\right\} * \left\{m_i - \frac{n_i M}{N}\right\} = 0$$

where $N = \sqrt{\{1 - L^2 - M^2\}}$,

i.e.

$$L\left\{\sum_{i=1}^{r} \ell_i^2 - \sum_{i=1}^{r} n_i^2\right\} + M\sum_{i=1}^{r} \ell_i m_i + \frac{(N^2 - L^2)}{N}\sum_{i=1}^{r} n_i \ell_i - \frac{ML}{N}\sum_{i=1}^{r} m_i n_i = 0$$

and

$$M\left\{\sum_{i=1}^{r} m_i^2 - \sum_{i=1}^{r} n_i^2\right\} + L\sum_{i=1}^{r} \ell_i m_i + \frac{(N^2 - M^2)}{N}\sum_{i=1}^{r} n_i m_i - \frac{ML}{N}\sum_{i=1}^{r} \ell_i n_i = 0 \ .$$

The root of the above two non-linear equations gives the required L and M . The problem of finding the best plane has now reduced to solving these two non-linear equations. Since $\Sigma\ell_i^2 - \Sigma n_i^2$, $\Sigma\ell_i m_i$, $\Sigma n_i \ell_i$, $\Sigma m_i n_i$ and $\Sigma m_i^2 - \Sigma n_i^2$ are constants they will be replaced by a, b, c, d and e respectively.

Let

$$f_1(M,L) = La + Mb + \frac{(N^2 - L^2)c}{N} - \frac{ML_d}{N} = 0$$

and

$$f_2(M,L) = Me + Lb + \frac{(N^2 - M^2)d}{N} - \frac{ML_c}{N} = 0 \quad .$$

The root of this pair of non-linear equations is found by using the Newton-Raphson technique. The details of this technique are given in Appendix A. Let (M, L) be the root required and $(\tilde{M}, \tilde{L})$ be an approximation to this root, then

$$M \approx \tilde{M} - \frac{\beta\phi - \alpha\lambda}{\beta\gamma - \alpha^2}$$

and

$$L \approx \tilde{L} - \frac{\gamma\lambda - \alpha\phi}{\beta\gamma - \alpha^2} \quad ,$$

where $\alpha = \dfrac{\partial f_1(\tilde{M},\tilde{L})}{\partial M} = \dfrac{\partial f_2(\tilde{M},\tilde{L})}{\partial L}$;

$\beta = \dfrac{\partial f_1(\tilde{M},\tilde{L})}{\partial L}$;

$\gamma = \dfrac{\partial f_2(\tilde{M},\tilde{L})}{\partial M}$;

$\lambda = f_1(\tilde{M},\tilde{L})$ and

$\phi = f_2(\tilde{M},\tilde{L})$.

$$\frac{\partial f_1(M,L)}{\partial M} = b + \frac{1}{N^3}\left\{(M^2 - 1)Mc + (L^2 - 1)Ld\right\}$$

$$\frac{\partial f_1(M,L)}{\partial L} = a + \frac{1}{N^3}\left\{(3M^2 + 2L^2 - 3)Lc + (M^2 - 1)Md\right\}$$

$$\frac{\partial f_2(M,L)}{\partial M} = e + \frac{1}{N^3}\left\{(3L^2 + 2M^2 - 3)Md + (L^2 - 1)Lc\right\}$$

$$\frac{\partial f_2(M,L)}{\partial L} = b + \frac{1}{N^3}\left\{(M^2 - 1)Mc + (L^2 - 1)Ld\right\} .$$

The Newton-Raphson technique gives rise to an iterative process for finding the root (L, M) . An initial estimate for the root can be obtained by considering the plane formed by two of the original direction cosine vectors. Using the first and last direction cosine vectors, the initial estimate for L and M is given by

$$(L, M, N) = \frac{\underline{d}_1 \wedge \underline{d}_r}{|\underline{d}_1 \wedge \underline{d}_r|} \qquad \text{if} \quad \ell_1 m_r - m_1 \ell_r \geqslant 0$$

$$= \frac{\underline{d}_r \wedge \underline{d}_1}{|\underline{d}_r \wedge \underline{d}_1|} \qquad \text{if} \quad \ell_1 m_r - m_1 \ell_r < 0$$

where $\underline{d}_i = (\ell_i, m_i, n_i)$

and $\underline{d}_1 \wedge \underline{d}_r = (m_1 n_r - n_1 m_r, n_1 \ell_r - n_r \ell_1, \ell_1 m_r - m_1 \ell_r)$.

Note both functions $f_1(M,L)$ and $f_2(M,L)$ and their derivatives have poles at $N = 0$, (i.e. $L^2 + M^2 = 1$) . Therefore if N is small or zero, the iteration may not converge. These poles have no physical significance and are a consequence of the axes system used. Therefore if N is small or zero a change to another system of axes X', Y', Z' , where N' is not small, allows the vector (L', M', N') to be calculated. This by-passes the poles and the problem of convergence. Once the vector (L', M', N') has been calculated a rotation back to the original system of axes gives (L, M, N) .

At this stage the 'best' plane nearly containing the direction cosine vectors $\underline{d}_i$ has been calculated. Now in general these direction cosine vectors will not lie in this plane, therefore these vectors must be projected onto the plane. Let $\underline{P} = (L, M, N)$, then the projection of $\underline{d}_i$ onto the camera-trajectory plane is

$$\underline{d}_i - (\underline{P} \cdot \underline{d}_i)\underline{P} .$$

This vector will not in general be a unit vector, therefore it should be normalized before continuing. Let $\underline{e}_i$ be this modified direction cosine vector, then

$$\underline{e}_i = \frac{\underline{d}_i - (\underline{P} \cdot \underline{d}_i)\underline{P}}{|\underline{d}_i - (\underline{P} \cdot \underline{d}_i)\underline{P}|} .$$

3.2 Calculation of missile direction

Define a new system of axes X', Y' and Z'

where X' is in the direction of $\underline{e}_1 \wedge \underline{P} = \underline{Q}$ say

Y' is in the direction of $\underline{e}_1$

and Z' is in the direction of $\underline{P} = (L, M, N)$.

$$\underline{Q} = (g_1N - h_1M,\ h_1L - Nf_1,\ f_1M - g_1L) ,$$

where $\underline{e}_i = (f_i, g_i, h_i)$.

This gives a right handed system of axes $\underline{Q}$, $\underline{e}_1$ and $\underline{P}$. Using the vectors $\underline{Q}$ and $\underline{e}_1$ which both lie in the camera-trajectory plane, define the variable S at the time t

$$S_i = \frac{\underline{e}_1 \cdot \underline{e}_i}{\underline{Q} \cdot \underline{e}_i} .$$

Note $S_1 = \infty$.

At this stage, the best pair of equations of the form

$$y = m(t - t_1) + 1 \qquad \text{and} \qquad x = n(t - t_1)$$

to satisfy the assumed model, have to be defined and then calculated. The curve $y = mx/n + 1$ intersects the lines $y = S_i x$ when

$$x = \frac{n}{nS_i - m} \qquad \text{and} \qquad y = \frac{m}{nS_i - m} + 1 .$$

In theory these co-ordinates should be

$$x = n(t_i - t_1) \qquad \text{and} \qquad y = m(t_i - t_1) + 1 .$$

Therefore the best values of m and n are defined as those which minimize

$$A(m,n) = \sum_{i=1}^{r} \left\{ \left(m(t_i - t_1) - \frac{m}{nS_i - m} \right)^2 + \left(n(t_i - t_1) - \frac{n}{nS_i - m} \right)^2 \right\}$$

$$= (m^2 + n^2) \sum_{i=2}^{r} \left(t_i - t_1 - \frac{1}{nS_i - m} \right)^2 .$$

$A(m,n)$ has a minimum when $\frac{\partial A}{\partial n} = \frac{\partial A}{\partial m} = 0$, i.e.

$$m \sum_{i=2}^{r} \left(t_i - t_1 - \frac{1}{nS_i - m} \right)^2 - (m^2 + n^2) \sum_{i=2}^{r} \left(t_i - t_1 - \frac{1}{nS_i - m} \right) \frac{1}{(nS_i - m)^2} = 0 .$$

and

$$n \sum_{i=2}^{r} \left(t_i - t_1 - \frac{1}{nS_i - m} \right)^2 + (m^2 + n^2) \sum_{i=2}^{r} \left(t_i - t_1 - \frac{1}{nS_i - m} \right) \frac{S_i}{(nS_i - m)^2} = 0 .$$

The root of this pair of non-linear equations gives the best values of m and n . Again the Newton-Raphson technique is used to find the root.

Let

$g_1(m,n)$

$$= m \sum_{i=2}^{r} \left(t_i - t_1 - \frac{1}{nS_i - m} \right)^2 - (m^2 + n^2) \sum_{i=2}^{r} \left(t_i - t_1 - \frac{1}{nS_i - m} \right) \frac{1}{(nS_i - m)^2}$$

and

$g_2(m,n)$

$$= n \sum_{i=2}^{r} \left(t_i - t_1 - \frac{1}{nS_i - m} \right)^2 + (m^2 + n^2) \sum_{i=2}^{r} \left(t_i - t_1 - \frac{1}{nS_i - m} \right) \frac{S_i}{(nS_i - m)^2} .$$

Let (m, n) be the root and $(\tilde{m}, \tilde{n})$ be an approximation to the root, then

$$m \approx \tilde{m} - \frac{\alpha\phi - \gamma\lambda}{\alpha^2 - \beta\gamma} \quad \text{and} \quad n \approx \tilde{n} - \frac{\alpha\lambda - \beta\phi}{\alpha^2 - \beta\gamma} ,$$

where $\alpha = \dfrac{\partial g_2(\tilde{m},\tilde{n})}{\partial m} = \dfrac{\partial g_1(\tilde{m},\tilde{n})}{\partial n}$;

$\beta = \dfrac{\partial g_1(m,n)}{\partial m}$;

$\gamma = \dfrac{\partial g_2(m,n)}{\partial n}$

$\lambda = g_1(\tilde{m},\tilde{n})$ and

$\phi = g_2(\tilde{m},\tilde{n})$.

$$\frac{\partial g_1(m,n)}{\partial m}$$

$$= \sum_{i=2}^{r}\left(t_i - t_1 - \frac{1}{nS_i - m}\right)^2 - 4m\sum_{i=2}^{r}\left(t_i - t_1 - \frac{1}{nS_i - m}\right)\frac{1}{(nS_i - m)^2}$$

$$+ (m^2 + n^2)\sum_{i=2}^{r}\frac{1}{(nS_i - m)^4} - 2(m^2 + n^2)\sum_{i=2}^{r}\left(t_i - t_1 - \frac{1}{nS_i - m}\right)\frac{1}{(nS_i - m)^3}$$

$$\frac{\partial g_1(m,n)}{\partial n}$$

$$= 2m\sum_{i=2}^{r}\left(t_i - t_1 - \frac{1}{nS_i - m}\right)\frac{S_i}{(nS_i - m)^2} - 2n\sum_{i=2}^{r}\left(t_i - t_1 - \frac{1}{nS_i - m}\right)\frac{1}{(nS_i - m)^2}$$

$$+ 2(m^2 + n^2)\sum_{i=2}^{r}\left(t_i - t_1 - \frac{1}{nS_i - m}\right)\frac{S_i}{(nS_i - m)^3} - (m^2 + n^2)\sum_{i=2}^{r}\frac{S_i}{(nS_i - m)^4}$$

$$\frac{\partial g_2(m,n)}{\partial m} = \frac{\partial g_1(m,n)}{\partial n}$$

$$\frac{\partial g_2(m,n)}{\partial n}$$

$$= \sum_{i=2}^{r}\left(t_i - t_1 - \frac{1}{nS_i - m}\right)^2 + 4n\sum_{i=2}^{r}\left(t_i - t_1 - \frac{1}{nS_i - m}\right)\frac{S_i}{(nS_i - m)^2}$$

$$- 2(m^2 + n^2)\sum_{i=2}^{r}\left(t_i - t_1 - \frac{1}{nS_i - m}\right)\frac{S_i^2}{(nS_i - m)^3} + (m^2 + n^2)\sum_{i=2}^{r}\frac{S_i^2}{(nS_i - m)^4}$$

An initial estimate for this iteration can be obtained using the data of three time points. For example, t_1, t_2 and t_3, then

$$\tilde{n} = \frac{(t_3 - t_2)}{(S_2 - S_3)(t_2 - t_1)(t_3 - t_1)}$$

and

$$\tilde{m} = \frac{S_3(t_3 - t_1) - S_2(t_2 - t_1)}{(S_2 - S_3)(t_2 - t_1)(t_3 - t_1)}$$

3.3 Calculation of missile trajectory

The missile trajectory is now calculated as in the theoretical solution. The position vector of the missile at the time t, in camera axes and with respect to the camera origin is therefore

$$\frac{V}{\sqrt{(m^2 + n^2)}}\left[n(t - t_1)\underline{Q} + \left\{m(t - t_1) + 1\right\}\underline{e}_1\right] .$$

The target origin and axes system is in general different to that of the camera, therefore to complete the solution the trajectory has to be converted to this new origin and axes system. The actual target origin is normally a nominal position on the target, for example the centre of gravity of the aircraft.

This solution has been written into a computer program subroutine, a listing and short description is given in Appendix B.

4 MISS DISTANCE PARAMETERS

This section gives the mathematics for the calculation of the normal parameters required at the time of nearest miss. The time of nearest miss is the time at which the range of the missile from the target is a minimum. The parameters required at nearest miss are

(a) Time t_{miss}

(b) Range R_{miss}

(c) Co-ordinates $(X_{miss}, Y_{miss}, Z_{miss})$

(d) Direction cosine vector of the trajectory (ℓ, m, n) and

(e) Velocity, assumed supplied from other sources.

Let the position of the missile, with respect to the target, at time t, be (x_1, y_1, z_1) and at time t_2 be (x_2, y_2, z_2). Then the position of the missile at time t is

$$(x_0 + Lt, y_0 + Mt, z_0 + Nt)$$

where $$x_0 = x_1 + \frac{t_1}{t_2 - t_1}(x_1 - x_2) = \frac{t_2 x_1 - t_1 x_2}{t_2 - t_1}$$

$$y_0 = y_1 + \frac{t_1}{t_2 - t_1}(y_1 - y_2) = \frac{t_2 y_1 - t_1 y_2}{t_2 - t_1}$$

$$z_0 = z_1 + \frac{t_1}{t_2 - t_1}(z_1 - z_2) = \frac{t_2 z_1 - t_1 z_2}{t_2 - t_1}$$

$$L = \frac{x_2 - x_1}{t_2 - t_1}, \quad M = \frac{y_2 - y_1}{t_2 - t_1} \quad \text{and} \quad N = \frac{z_2 - z_1}{t_2 - t_1}.$$

L, M and N are velocity components.

The velocity of the missile is

$$V = \sqrt{\left\{L^2 + M^2 + N^2\right\}} = \frac{1}{t_2 - t_1}\sqrt{\left\{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right\}}$$

and the direction cosine vector of the missile trajectory is given by

$$(\ell, m, n) = \frac{1}{V}(L,M,N)$$

$$= \frac{1}{\sqrt{\left\{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right\}}}(x_2 - x_1, y_2 - y_1, z_2 - z_1).$$

Now the range of the missile from the target is given by

$$R = \sqrt{\left\{(x_0 + Lt)^2 + (y_0 + Mt)^2 + (z_0 + Nt)^2\right\}} \quad .$$

This has a minimum when

$$\frac{dR}{dt} = 0 \ , \qquad \left(\text{or } \frac{d(R^2)}{dt} = 0\right)$$

i.e. when

$$L(x_0 + Lt) + M(y_0 + Mt) + N(z_0 + Nt) = 0$$

i.e.

$$\begin{aligned} x_0L + y_0M + z_0N &= -(L^2 + M^2 + N^2)t \\ &= -V^2t \quad . \end{aligned}$$

Therefore the time of nearest miss is

$$t_{miss} = -\frac{(x_0L + y_0M + z_0N)}{V^2} = -\frac{(x_0\ell + y_0m + z_0n)}{V}$$

The range at t_{miss} is

$$R_{miss} = \sqrt{\left\{(x_0 + Lt_{miss})^2 + (y_0 + Mt_{miss})^2 + (z_0 + Nt_{miss})^2\right\}}$$

and the position co-ordinates are

$$\begin{aligned} X_{miss} &= x_0 + Lt_{miss} \\ Y_{miss} &= y_0 + Mt_{miss} \\ Z_{miss} &= z_0 + Nt_{miss} \quad . \end{aligned}$$

5 SOURCES OF ERROR

Throughout this section, let T be the length of time covered by the single camera solution and during this period of time let

(a) R be the maximum range of the missile from the camera, and

(b) V be the average missile velocity.

There are three sources of error for the single camera solution, in a practical situation, these are:-

(a) Reading errors

Both time and angular data, have to be read from films. This can only be done to a finite accuracy which depends on the camera system used.

The timing information on the Range's films can be read to an accuracy of 0.1 milliseconds, therefore the maximum percentage error in the calculated trajectory data, due to a timing error is

$$2 \times \frac{0.0001}{T} \times 100\% = \frac{0.02}{T}\% \qquad \text{(T in seconds)}$$

Let α be the accuracy of the angular data read from the film in radians, then the maximum percentage error is $100\alpha\%$ which is probably pessimistic since the least squares technique used should reduce this error.

(b) The error in applying the model, required by the single camera solution, to an actual trajectory

The missile can fail to satisfy the model for at least three reasons:-

(i) a lateral acceleration, of say 'a' m/s^2

(ii) a longitudinal acceleration, of say 'ℓ' m/s^2, and

(iii) a rotating camera axes system, say the axes system is rotating at a rate of 'b' rad/s.

(N.B. The lateral and longitudinal accelerations are in general the combined effects of both the missile and the camera moving.)

It can easily be shown that the maximum deviations from the model are

(1) $$\frac{1}{2}\left(\frac{1}{2}a\left(\frac{T}{2}\right)^2\right) = \frac{aT^2}{16}$$ metres, due to the lateral acceleration

(2) $$\frac{1}{2}\ell\left(\frac{T}{2}\right)^2 = \frac{\ell T^2}{8}$$ metres, due to the longitudinal acceleration

and

(3) $$\frac{bV}{2}\left(\frac{T}{2}\right)^2 = \frac{bVT^2}{8}$$ metres, due to the rotating axes system.

(c) The error in the velocity estimate

A 5% error in the velocity will produce a 5% error in the calculated trajectory data. (It should be noted, that all the percentage errors quoted above, apply only when the trajectory origin is identical to the camera origin.)

Given the characteristics (actual or theoretical) of the missile, target and camera system (e.g. velocity, lateral accelerations, etc.), it is therefore possible to calculate the maximum error to be expected from using a single camera solution. For example, consider the representative figures for a missile fired against an aerial target, typical maximum values would be

$$R = 35 \text{ m}$$
$$a = 150 \text{ m/s}^2 \ (\approx 15 \text{ g})$$
$$\ell = 50 \text{ m/s}^2 \ (\approx 5 \text{ g})$$
$$b = 10^\circ/\text{s}$$
$$\alpha = \tfrac{1}{2}^\circ$$
$$V = 700 \text{ m/s with a 5\% error.}$$

Then the maximum compounded error in position, assuming a typical time T of 0.05 s, is approximately 2.27 m, of which 1.75 m is due to the velocity error. It should be noted that this compounded error has its maximum value at the extreme range from the camera and will be considerably less nearer the camera.

The main source of error for the single camera solution, in the example quoted, is the velocity estimate. This is considered to be the situation for most single camera solutions calculated at Aberporth. The reason being that the tracking radars usually fail to give trajectory data and hence velocity data during the period of the intercept.

When the trajectory data is calculated with respect to an origin and axes system, other than that of the camera (e.g. if the target aircraft is towing a decoy target behind it), additional errors can easily be introduced. These errors are obviously dependent on the accuracy of the trajectory origin and axes system with respect to those of the camera, but must be smaller for origins nearer the camera.

6 CONCLUSION

The single camera solution puts fairly restrictive conditions on the trajectory of the missile. These conditions limit the situations in which a single camera solution can be used. This is in contrast with solutions obtained from two or more cameras. For example, the single camera solution is unsuitable for obtaining trajectory data during the launch phase of a missile. This is because the missile is undergoing substantial longitudinal acceleration at relatively low speed, during this period.

Another restriction on the single camera solution is the fact that the velocity of the missile relative to the camera must be supplied from an external source.

Nevertheless, taking these restrictions into account the single camera solution can be very useful in obtaining trajectory data. This is particularly so during the attack phase of a missile, when the missile trajectory normally approximates to the model required by the single camera solution. The solution is not specifically restricted to aerial or sea targets and will apply to any single camera situation where the necessary extra information (timing, missile velocity, etc.) is available.

Appendix A

DETAILS OF THE 'NEWTON-RAPHSON' TECHNIQUE

Given here are the details for finding the root of a pair of non-linear equations, using the Newton-Raphson technique.

Let the pair of non-linear equations be

$$f_1(M,L) = 0$$

and

$$f_2(M,L) = 0 \ .$$

Let (M, L) be the root and $(\tilde{M}, \tilde{L})$ be an approximation to this root, then

$$f_1(M,L) \approx f_1(\tilde{M},\tilde{L}) - \delta M \frac{\partial f_1(\tilde{M},\tilde{L})}{\partial M} - \delta L \frac{\partial f_1(\tilde{M},\tilde{L})}{\partial L}$$

and

$$f_2(M,L) \approx f_2(\tilde{M},\tilde{L}) - \delta M \frac{\partial f_2(\tilde{M},\tilde{L})}{\partial M} - \delta L \frac{\partial f_2(\tilde{M},\tilde{L})}{\partial L}$$

where $\delta M = \tilde{M} - M$ and $\delta L = \tilde{L} - L$.

Since

$$f_1(M,L) = f_2(M,L) = 0$$

it follows that

$$f_1(\tilde{M},\tilde{L}) \approx \delta M \frac{\partial f_1(\tilde{M},\tilde{L})}{\partial M} + \delta L \frac{\partial f_1(\tilde{M},\tilde{L})}{\partial L}$$

and

$$f_2(\tilde{M},\tilde{L}) \approx \delta M \frac{\partial f_2(\tilde{M},\tilde{L})}{\partial M} + \delta L \frac{\partial f_2(\tilde{M},\tilde{L})}{\partial L} \ .$$

Let

$$\frac{\partial f_1(\tilde{M},\tilde{L})}{\partial M} = \alpha \quad ; \quad \frac{\partial f_1(\tilde{M},\tilde{L})}{\partial L} = \beta \quad ; \quad \frac{\partial f_2(\tilde{M},\tilde{L})}{\partial M} = \gamma$$

$$\frac{\partial f_2(\tilde{M},\tilde{L})}{\partial L} = \omega \quad ; \quad f_1(\tilde{M},\tilde{L}) = \lambda \quad \text{and} \quad f_2(\tilde{M},\tilde{L}) = \phi \ ,$$

then

$$\lambda \approx \alpha\delta M + \beta\delta L$$

and

$$\phi \approx \gamma\delta M + \omega\delta L$$

Therefore

$$\delta M \approx \frac{\beta\phi - \omega\lambda}{\beta\gamma - \omega\alpha}$$

and

$$\delta L \approx \frac{\gamma\lambda - \alpha\phi}{\beta\gamma - \omega\alpha}$$

therefore

$$M \approx \tilde{M} - \delta M$$

and

$$L \approx \tilde{L} - \delta L \quad .$$

This gives rise to an iterative process for calculating the root (M, L) .

Appendix B

SHORT DESCRIPTION AND LISTING OF THE SINGLE CAMERA SOLUTION SUBROUTINE SINCA

The subroutine SINCA performs the single camera solution described in Section 3. Starting with a velocity estimate and at least three direction cosine vectors and their corresponding times, the subroutine calculates trajectory data and miss distance information with respect to any given origin and axes system. Trajectory data is automatically produced at the times of the direction cosine vectors and the time of nearest miss. Since trajectory data is often required at special times, e.g. at fuse triggering time, there is an option to supply data at other given times. The subroutine is called

SINCA (T, RL, RM, RN, IN, JN, X, V, RMISS, D)

T, RL, TM, RN, X, RMISS and D are arrays dimensioned as follows: T(25), RL(25), RM(25), RN(25), X(3), RMISS(8) and D(3,3).

INPUT DATA

IN : number of direction cosine vectors (3 $\leqslant$ IN $\leqslant$ 24)

JN : number of special time points (IN+JN $\leqslant$ 25)

T(I) : (a) time of Ith direction cosine vector I = 1, IN

(b) time of a special time point I = IN+1, IN+JN

RL(I) : l component of Ith direction cosine vector I = 1, IN

RM(I) : m component of Ith direction cosine vector I = 1, IN

RN(I) : n component of Ith direction cosine vector I = 1, IN

X : (x, y, z) co-ordinates of the target, with respect to the axes system of the direction cosine vectors and the camera origin.

D : rotation matrix

V : velocity estimate

OUTPUT DATA

IN : as input

JN : as input

V : as input

X : as input

D : as input

T(I) : as input I = 1, IN+JN

RL(I) : x co-ordinate of trajectory at time T(I), I = 1, IN+JN

RM(I) : y co-ordinate of trajectory at time T(I), I = 1, IN+JN

RN(I) : z co-ordinate of trajectory at time T(I), I = 1, IN+JN

RMISS : miss distance parameters, as follows

RMISS(1), time of nearest miss
RMISS(2), x co-ordinate of trajectory at time of nearest miss
RMISS(3), y co-ordinate of trajectory at time of nearest miss
RMISS(4), z co-ordinate of trajectory at time of nearest miss
RMISS(5), range at time of nearest miss
RMISS(6), l component of trajectory direction cosine vector
RMISS(7), m component of trajectory direction cosine vector
RMISS(8), n component of trajectory direction cosine vector

```
// JOB

LOG DRIVE   CART SPEC   CART AVAIL  PHY DRIVE
  0000        0006        0006        0000

V2 M09   ACTUAL 16K  CONFIG 16K

// FOR
*LIST ALL
*ONE WORD INTEGERS
*EXTENDED PRECISION
*NAME SINCA
** SINGLE CAMERA SOLUTION  SUBROUTINE
```

SINGLE CAMERA SOLUTION SUBROUTINE

```
      SUBROUTINE SINCA(T,RL,RM,RN,IN,JN,X,V,RMISS,D)
      DIMENSION T(25),RL(25),RM(25),RN(25),X(3),RMISS(8),D(3,3),A(9),
     1S(25)
C     CALCULATE INITIAL VALUES FOR (L,M,N),ROTATING AXES IF NECESSARY
      RLL=RM(1)*RN(IN)-RN(1)*RM(IN)
      RMM=RN(1)*RL(IN)-RL(1)*RN(IN)
      RNN=RL(1)*RM(IN)-RM(1)*RL(IN)
      RR=SQRT(RLL*RLL+RMM*RMM+RNN*RNN)
      RLL=RLL/RR
      RMM=RMM/RR
      RNN=RNN/RR
      K2=1
      J=1
      IF(ABS(RNN)-0.1)100,100,107
  100 IF(ABS(RMM)-0.1)104,104,101
  101 KK=2
  102 DO 103 I=1,IN
      RR=RN(I)
      RN(I)=RM(I)*J
  103 RM(I)=-RR*J
      RR=RNN
      RNN=RMM*J
      RMM=-RR*J
      GO TO (108,109),K2
  104 KK=3
  105 DO 106 I=1,IN
      RR=RN(I)
      RN(I)=RL(I)*J
  106 RL(I)=-RR*J
      RR=RNN
      RNN=RLL*J
      RLL=-RR*J
      GO TO (108,109),K2
  107 KK=1
  108 IF(RNN)10,11,11
   10 RLL=-RLL
      RMM=-RMM
      RNN=-RNN
C     CALCULATE COEFFICIENTS FOR THE FIRST ITERATION,TO FIND (L,M)
   11 DO 12 I=1,6
   12 A(I)=0.0
      DO 13 I=1,IN
      A(1)=A(1)+RL(I)*RL(I)
      A(2)=A(2)+RM(I)*RM(I)
      A(3)=A(3)+RN(I)*RN(I)
      A(4)=A(4)+RL(I)*RM(I)
      A(5)=A(5)+RN(I)*RL(I)
   13 A(6)=A(6)+RM(I)*RN(I)
      A(1)=A(1)-A(3)
      A(2)=A(2)-A(3)
C     PERFORM FIRST ITERATION,AND IMPROVE ESTIMATE FOR (L,M)
      DO 14 I=1,30
      RLL2=RLL*RLL
      RMM2=RMM*RMM
      RNN2=RNN*RNN
      RR=1.0/RNN
      F1=RLL*A(1)+RMM*A(4)+RR*((RNN2-RLL2)*A(5)-RMM*RLL*A(6))
      F2=RMM*A(2)+RLL*A(4)+RR*((RNN2-RMM2)*A(6)-RMM*RLL*A(5))
      RR=RR/RNN2
      F3=A(4)+RR*((RMM2-1)*RMM*A(5)+(RLL2-1)*RLL*A(6))
      F4=A(1)+RR*((-3*RNN2-RLL2)*RLL*A(5)+(RMM2-1)*RMM*A(6))
```

SINGLE CAMERA SOLUTION SUBROUTINE

```
      F5=A(2)+RR*((-3*RNN2-RMM2)*RMM*A(6)+(RLL2-1)*RLL*A(5))
      RR=F4*F5-F3*F3
      DM=(F4*F2-F3*F1)/RR
      DL=(F1*F5-F3*F2)/RR
      RMM=RMM-DM
      RLL=RLL-DL
      RNN=SQRT(1.0-RMM*RMM-RLL*RLL)
      RR=SQRT(DM*DM+DL*DL)
      IF(RR-0.0000001)15,15,14
   14 CONTINUE
   15 J=-1
      K2=2
C     ROTATE BACK TO ORIGINAL AXES SYSTEM IF NECESSARY
      GO TO (109,102,105),KK
C     CALCULATE MODIFIED DIRECTION COSINE VECTORS AND THE VECTOR Q
  109 DO 16 I=1,IN
      RR=RLL*RL(I)+RMM*RM(I)+RNN*RN(I)
      RL(I)=RL(I)-RR*RLL
      RM(I)=RM(I)-RR*RMM
      RN(I)=RN(I)-RR*RNN
      RR=SQRT(RL(I)*RL(I)+RM(I)*RM(I)+RN(I)*RN(I))
      RL(I)=RL(I)/RR
      RM(I)=RM(I)/RR
   16 RN(I)=RN(I)/RR
      RL(25)=RM(1)*RNN-RN(1)*RMM
      RM(25)=RN(1)*RLL-RL(1)*RNN
      RN(25)=RL(1)*RMM-RM(1)*RLL
C     CALCULATE THE VARIABLES S(I)
      DO 17 I=2,IN
      RR=RL(1)*RL(I)+RM(1)*RM(I)+RN(1)*RN(I)
      F4=RL(25)*RL(I)+RM(25)*RM(I)+RN(25)*RN(I)
   17 S(I)=RR/F4
      I=(IN-1)/2
      DM=1.0/(T(IN)-T(1))
      DL=1.0/(T(I+1)-T(1))
      ANN=(DL-DM)/(S(I+1)-S(IN))
      AMM=ANN*S(IN)-DM
C     PERFORM SECOND ITERATION AND IMPROVE ESTIMATE FOR (M,N)
      DO 20 J=1,30
      DO 18 I=1,9
   18 A(I)=0.0
      DO 19 I=2,IN
      DL=1.0/(ANN*S(I)-AMM)
      DM=T(I)-T(1)-DL
      RMM2=DM*DM
      RLL2=DL*DL
      A(1)=A(1)+RMM2
      A(2)=A(2)-DM*RLL2
      A(3)=A(3)+S(I)*RLL2*DM
      A(4)=A(4)+RLL2*RLL2
      A(5)=A(5)-2*DM*RLL2*DL
      A(6)=A(6)+S(I)*RLL2*RLL2
      A(7)=A(7)+2*S(I)*DM*RLL2*DL
      A(8)=A(8)-2*S(I)*S(I)*DL*DM*RLL2
   19 A(9)=A(9)+S(I)*S(I)*RLL2*RLL2
      RLL=AMM*AMM+ANN*ANN
      F1=AMM*A(1)+RLL*A(2)
      F2=ANN*A(1)+RLL*A(3)
      F3=A(1)+4*AMM*A(2)+RLL*A(4)+RLL*A(5)
      F4=2*(AMM*A(3)+ANN*A(2))+RLL*(A(7)-A(6))
```

```
      F5=A(1)+4*ANN*A(3)+RLL*(A(8)+A(9))
      RR=1.0/(F4*F4-F5*F3)
      DM=(F4*F2-F5*F1)*RR
      DL=(F4*F1-F3*F2)*RR
      AMM=AMM-DM
      ANN=ANN-DL
      RR=SQRT(DM*DM+DL*DL)
      IF(RR-0.0000001)21,21,20
   20 CONTINUE
C     CALCULATE TRAJECTORY DATA AND MISS DISTANCE PARAMETERS
   21 DL=SQRT(AMM*AMM+ANN*ANN)
      RR=V/DL
      DL=AMM*(T(IN)-T(1))+1.0
      DM=ANN*(T(IN)-T(1))
      RL(IN)=RL(25)*DM+RL(1)*DL
      RM(IN)=RM(25)*DM+RM(1)*DL
      RN(IN)=RN(25)*DM+RN(1)*DL
      RL(1)=RR*RL(1)-X(1)
      RM(1)=RR*RM(1)-X(2)
      RN(1)=RR*RN(1)-X(3)
      RL(IN)=RR*RL(IN)-X(1)
      RM(IN)=RR*RM(IN)-X(2)
      RN(IN)=RR*RN(IN)-X(3)
      DO 800 I=1,3
      A(I)=RL(1)*D(I,1)+RM(1)*D(I,2)+RN(1)*D(I,3)
  800 A(I+3)=RL(IN)*D(I,1)+RM(IN)*D(I,2)+RN(IN)*D(I,3)
      F1=1.0/(T(IN)-T(1))
      RLL2=(A(4)-A(1))*F1
      RMM2=(A(5)-A(2))*F1
      RNN2=(A(6)-A(3))*F1
      J=IN+JN
      DO 25 I=1,J
      RR=T(I)-T(1)
      RL(I)=A(1)+RR*RLL2
      RM(I)=A(2)+RR*RMM2
   25 RN(I)=A(3)+RR*RNN2
      RR=-(RL(1)*RLL2+RM(1)*RMM2+RN(1)*RNN2)/(V*V)
      RMISS(1)=RR+T(1)
      RMISS(2)=RL(1)+RR*RLL2
      RMISS(3)=RM(1)+RR*RMM2
      RMISS(4)=RN(1)+RR*RNN2
      RMISS(5)=SQRT(RMISS(2)*RMISS(2)+RMISS(3)*RMISS(3)+RMISS(4)*RMISS(4
     1))
      RMISS(6)=RLL2/V
      RMISS(7)=RMM2/V
      RMISS(8)=RNN2/V
      RETURN
      END
VARIABLE ALLOCATIONS
     A(R )=0018-0000      S(R )=0063-001B   RLL(R )=0066        RMM(R )=0069        RNN(R )=006C        RR(R )=006F
  RLL2(R )=0072        RMM2(R )=0075       RNN2(R )=0078         F1(R )=007B         F2(R )=007E        F3(R )=0081
    F4(R )=0084          F5(R )=0087         DM(R )=008A         DL(R )=008D        ANN(R )=0090       AMM(R )=0093
    K2(I )=00A2           J(I )=00A3         KK(I )=00A4          I(I )=00A5

STATEMENT ALLOCATIONS
 100  =0203  101  =020B  102  =020F  103  =0225  104  =0259  105  =025D  106  =0273  107  =02A7  108  =02AB  10   =02B0
 11   =02BF  12   =02C3  13   =0318  14   =0494  15   =049D  109  =04AD  16   =053F  17   =05D3  18   =062F  19   =06DB
 20   =07E3  21   =07EC  800  =08E5  25   =0977

FEATURES SUPPORTED
```

SINGLE CAMERA SOLUTION SUBROUTINE

```
ONE WORD INTEGERS
EXTENDED PRECISION

CALLED SUBPROGRAMS
 ESQRT   EABS    EADD    EADDX   ESUB    ESUBX   EMPY    EMPYX   EDIV    ELD     ELDX    ESTO    ESTOX   ESBR    ESBRX
 EDVR    FLOAT   SUBSC   SNR     SUBIN

REAL CONSTANTS
  .100000000E 00=00B4     .000000000E 00=00B7     .100000000E 01=00BA     .100000000E-06=00BD

INTEGER CONSTANTS
     1=00C0     2=00C1     3=00C2     6=00C3     30=00C4     9=00C5     4=00C6

CORE REQUIREMENTS FOR SINCA
 COMMON       0  VARIABLES     180  PROGRAM    2408

RELATIVE ENTRY POINT ADDRESS IS 00C7 (HEX)

END OF COMPILATION
```

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2	F. Spencer	Airborne and missile borne cameras developed by Weapons Research Establishment. WRE Tech. Note ISD 78 (September 1966)
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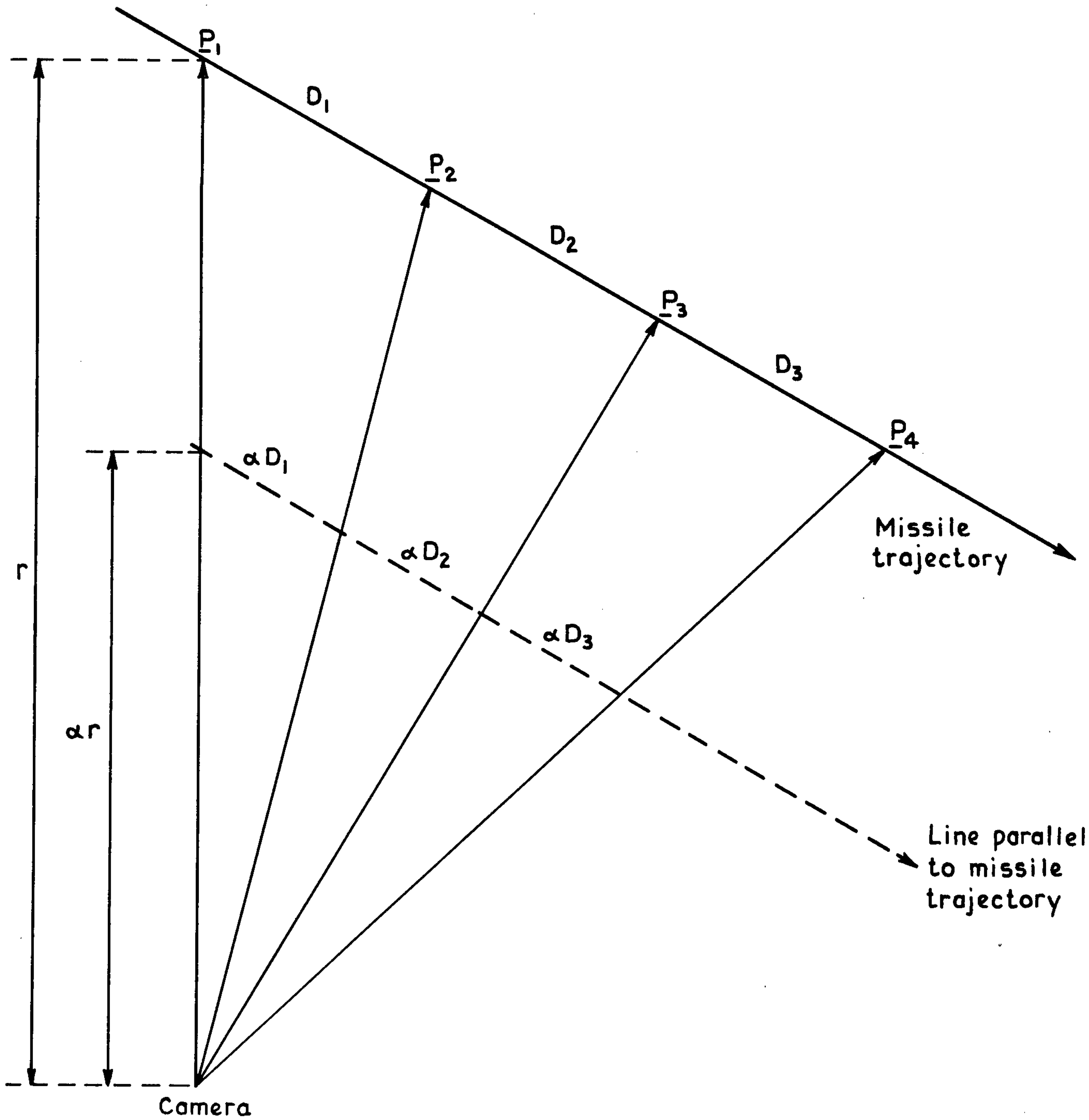
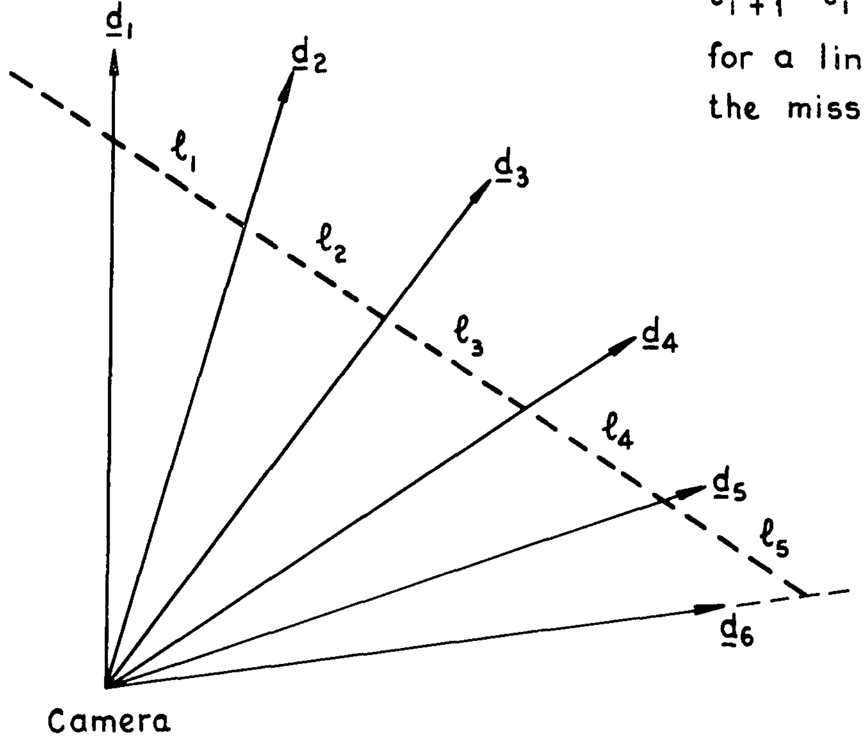


Fig.1 Camera – trajectory plane

Fig. 2 & 3



$$\frac{\ell_i}{t_{i+1} - t_i} = \text{constant}$$

for a line parallel to the missile trajectory

Fig. 2

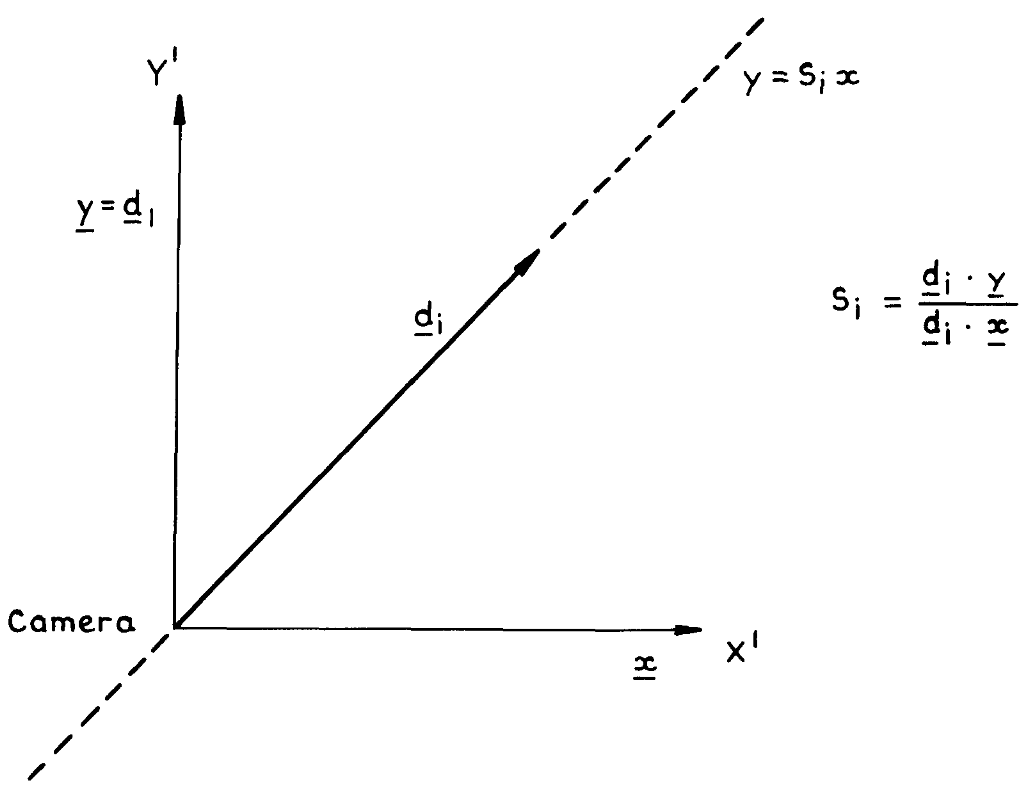


Fig. 3

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Fig.4 & 5

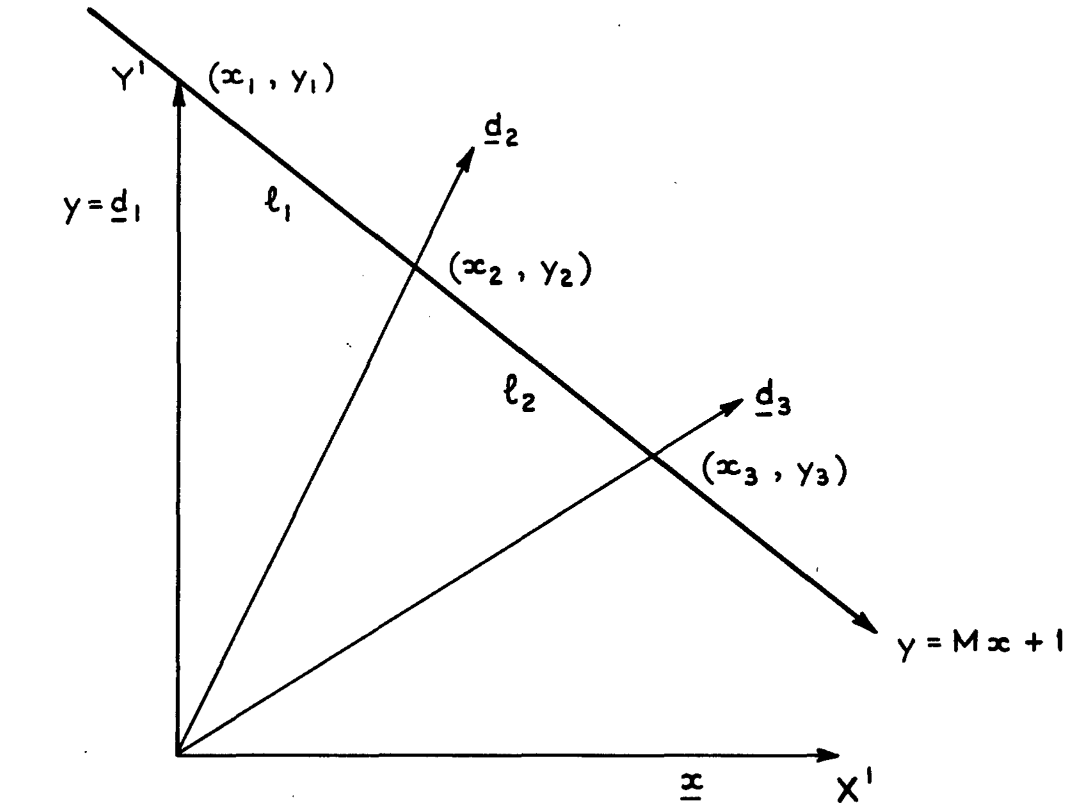

Fig. 4

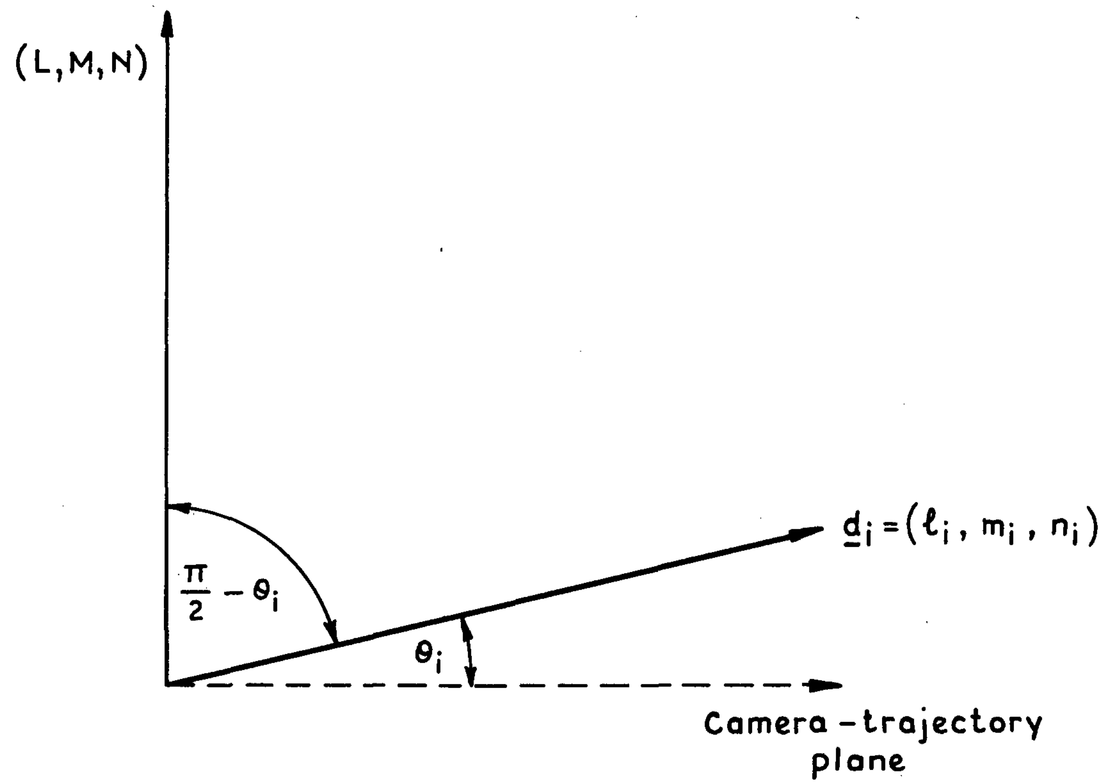

Fig. 5

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