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THEORETICAL DEVELOPMENT FOR LOW DISTORTION BROADBANDING OF SEISMIC DATA

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13. ABSTRACT

This report presents the results of a theoretical study of non-linearities in seismometers. The effects of non-linear elements in the mechanical and electrical parts of the seismometer system were modeled and investigated with particular attention to the transfer of noise power between different regions of the seismic noise spectrum.

The seismometer was modeled as a simple second order mechanical system followed by a transducer and an electronic amplifier. The qualitative effects of non-linearities in each of these elements were investigated.

A numerical simulation demonstrated the quantitative effects of various non-linearities on the signal-to-noise ratio observed in the low-noise region of the seismic spectrum. Small non-linearities were shown to cause serious loss of signal-to-noise ratio.

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1.0 INTRODUCTION AND SUMMARY

This report presents the results of a theoretical study of nonlinearities in seismometers. The effects of nonlinear elements in the mechanical and electrical parts of the seismometer system have been modeled and investigated, with particular attention to the transfer of noise power between different regions of the seismic noise spectrum.

The study was motivated by the following considerations. A number of investigators have studied the spectral characteristics of microseismic noise, which is considered to be the principal limiting factor in the accurate estimation of the waveforms generated by seismic events of interest. There is general agreement that this noise has a spectral peak at roughly a seven second period, while at both longer and shorter periods, the spectrum has a fairly low noise level. The effects of the noise peak are customarily minimized by narrowband filtering of the seismometer output above or below the noisy region to give short or long period signals. If nonlinearities exist before the filter, however, one of the possible effects is the transfer of energy from the spectral components in the noisy region to low-noise regions of the spectrum, thus causing a loss of signal-to-noise ratio in the useful bands. It was hypothesized that if this were the case, then a possible solution would be to use seismometers whose mechanical systems would be sufficiently narrowband that they would reject the noise peak.

In pursuing the investigation, work has been concentrated in two areas. First, in order to investigate the effects of nonlinearities by numerical simulation, it was necessary to model the seismometer-electronic amplifier combination. This model was based on a simple second order mechanical system followed by a transducer and an electronic filter amplifier. The qualitative effects of nonlinearities in the seismometer spring or damper, the transducer, and the amplifier were investigated. This work is described in Section 2.

It was demonstrated quantitatively that a nonlinearity would indeed have the effect of decreasing the signal-to-noise ratio in the low noise portion of the seismic spectrum. This was shown by a numerical analysis which simulated the effect of a polynomial nonlinearity on typical seismic noise waveforms. The simulation work included investigations of how the signal-to-noise ratio reduction was affected by 1) the mechanical parameters of the seismometer and 2) whether the nonlinearity was located in the mechanical or electronic parts of the seismometer-amplifier combination. Section 3 gives the results and discussion.

It was found that the literature on seismometers which was available to us contained little information on nonlinearities which was directly applicable to this research. For this reason the nonlinearities which we investigated and the results which we obtained can not be associated with any particular seismometer. We did not include complicated forms of nonlinearity such as mechanical hystereses or stiction. Rather, we used small distortions which could be described by a Taylor series. This should provide a conservative estimate of the severity of the effects of nonlinearities. The results of this work then should be considered as tentative until further research provides more detailed information about the nonlinearities of actual seismometers.

We were more concerned with degradation of the signal-to-noise ratio due to nonlinearities than with amplitude or shape distortions of signals. A nonlinearity can have a negligible effect on a signal while still causing appreciable transfer of microseismic noise energy between spectrum bands, with accompanying degradation of the signal-to-noise ratio.

The designer of a seismometer is restricted in his choice of basic parameters. The second order nature of a mechanical seismometer leaves the designer essentially free to choose only the resonant frequency f_o and the damping factor Q. No control of the mechanical gain can be exercised independently of f_o and Q. This limitation has significant consequences in the consideration of nonlinear effects.

If a nonlinearity occurs in the mechanical elements of the seismometer, including the pick-off transducer, then one might expect its effect to be minimized by reducing the excursion of the pendulum, i.e. reducing the gain. For a given Q, this can only be accomplished by increasing f_o. But if one attempts to reduce the effect of a nonlinearity by designing a narrowband, low-frequency, seismometer, then the resonant peak, f_o, must necessarily be placed in the low frequency region. A low value of f_o, however, implies a large pendulum excursion, but a lower relative gain in the noisy region of the spectrum above f_o. Whether the net effect will be a reduction or an increase in the nonlinear distortion cannot be readily deduced from qualitative analysis.

If a nonlinearity exists in the electronic amplifier, then it is reasonable to assume that the nonlinear distortion will be reduced by reducing the amplifier gain.

The basic results of the study can be summarized as follows.

1. Small nonlinearities in a seismometer can cause serious loss of signal-to-noise ratio in both the short period and long period spectral regions by transfer of noise energy from the high-intensity spectral peaks at intermediate frequencies.

2. For certain mechanical nonlinearities, the distortion may be reduced by increasing the resonant frequency while the effect of reducing the resonant frequency below the noise peak is not clear.

3. The firm recommendation of design procedures to reduce the deleterious effects of nonlinearities depends upon a better understanding of nonlinear effects. It is recommended that this be based on analysis of experimental seismometer signals. Experimental data is needed to show how large the nonlinearities will be in normal operating conditions and whether their principal source is the seismometer or its associated electronics.

2.0 SEISMOMETER SYSTEM MODEL

A seismometer system can be considered to consist of three subsystems connected in series, any one of which could have a non-linear response.

The three subsystems are: 1) The mechanical seismometer, including the inertial mass, springs, and dampers. 2) The output transducer which senses the motion or position of the inertial mass and produces an electrical output signal. 3) Amplifiers, filters, analog-digital converters, tape recorders, etc. leading to the final recorded form. The input signal is the acceleration of the earth's surface, while the output signal from the seismometer is the displacement of the inertial mass. The output of the transducer is generally proportional to either the displacement or velocity of the inertial mass.

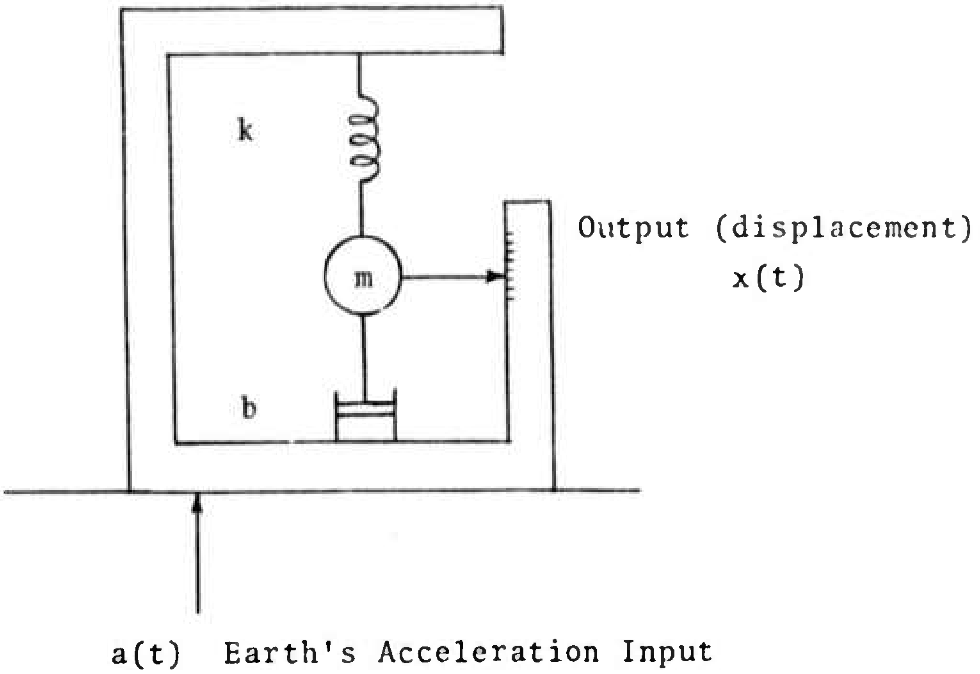
In this section we first consider briefly the equation of motion of a simplified second order model of a seismometer in order to examine the two parameters at the disposal of the designer, and to determine how these parameters effect the frequency response in the linear case. The equations of motion are then investigated for nonlinearities in the mechanical and electrical elements.

2.1 Simplified Model

A simplified seismometer model is illustrated in Figure 2-1. It consists of an inertial mass, m, a spring k, velocity damping b, a case or frame and a pointer. This model is not intended to reflect the details of construction of a particular seismometer, but merely to make clear that any single degree of freedom seismometer must be functionally equivalent to this simple model.

The actual spring in a physical seismometer may be a coiled spring, a u-shaped piece of spring steel, or a twisted quartz or metal fiber. All these, and/or combinations are represented by a single effective spring, with spring constant, k, in Figure 2-1. Similarly, various systems of velocity dependent damping are represented by the damping constant b. The pointer P, measures the displacement of the center of mass, x, of the effective suspended mass, m, relative to the frame, F.

The exact equivalent model of physical seismometers will of course differ from this model, since physical seismometers are three dimensional objects and therefore will have at least three degrees of freedom. However, the goal of any seismometer design would be to uncouple these extra degrees of freedom from the motion along the sensitive direction. Typical designs also place spurious vibrational modes above the seismic band of interest.



Basic Seismometer Model

Figure 2-1

From the forces acting on the mass the equation of mass motion may be written as

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \ddot{y} \qquad (2\text{-}1)$$

where y is the displacement of earth and frame relative to the inertial reference. This equation may be rewritten

$$\ddot{x} + \frac{\omega_o}{Q}\dot{x} + \omega_o^2 x = \ddot{y} \qquad (2\text{-}2)$$

where $\omega_o^2 = \frac{k}{m}$ is the undamped natural frequency and $Q = \frac{m\omega_o}{b}$ is the reciprocal damping factor.

From this equation it is clear that the seismometer is completely specified by ω_o and Q. In terms of modifying the performance of the seismometer, the designer is limited to the response characteristics of a damped second-order system. It should be noted that, unlike the case of an electronic amplifier, there is no arbitrary control of the system gain independent of ω_o and Q.

The actual gain versus frequency characteristics of a second order system is found by Laplace transform of equation 2-2, which gives

$$\left[s^2 + \frac{\omega_o}{Q} s + \omega_o^2\right] x(s) = -s^2 Y(s) \equiv A(s),$$

where A(s) is the input acceleration. The frequency response for both acceleration and displacement inputs can be written by letting $s = i\omega$ as

$$G_{da}(\omega) = \frac{|X(i\omega)|}{|A(i\omega)|} = \left[\left(\omega_o^2 - \omega^2\right)^2 + \left(\frac{\omega\omega_o}{Q}\right)^2\right]^{-1/2} \qquad (2\text{-}3)$$

and

$$G_{dd}(\omega) = \frac{|X(i\omega)|}{|Y(i\omega)|} = \omega^2 G_{da}(\omega). \qquad (2\text{-}4)$$

Typical plots of these transfer functions are given in Figures 2-2 and 2-3, showing the variation with the design parameters ω_o and Q. In particular, it is noted that seismometers with high natural frequencies have lower mechanical gain than those with low natural frequencies.

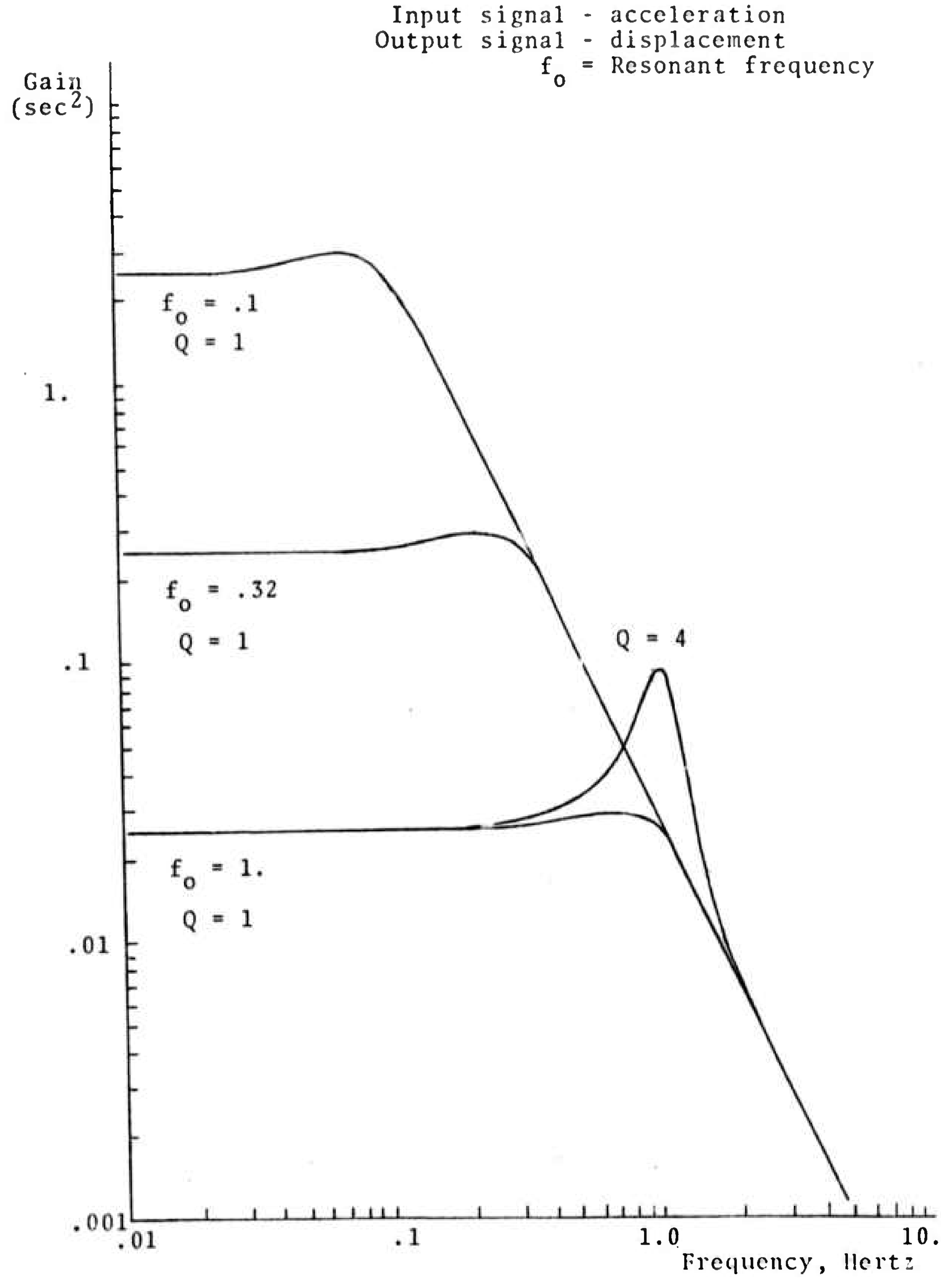
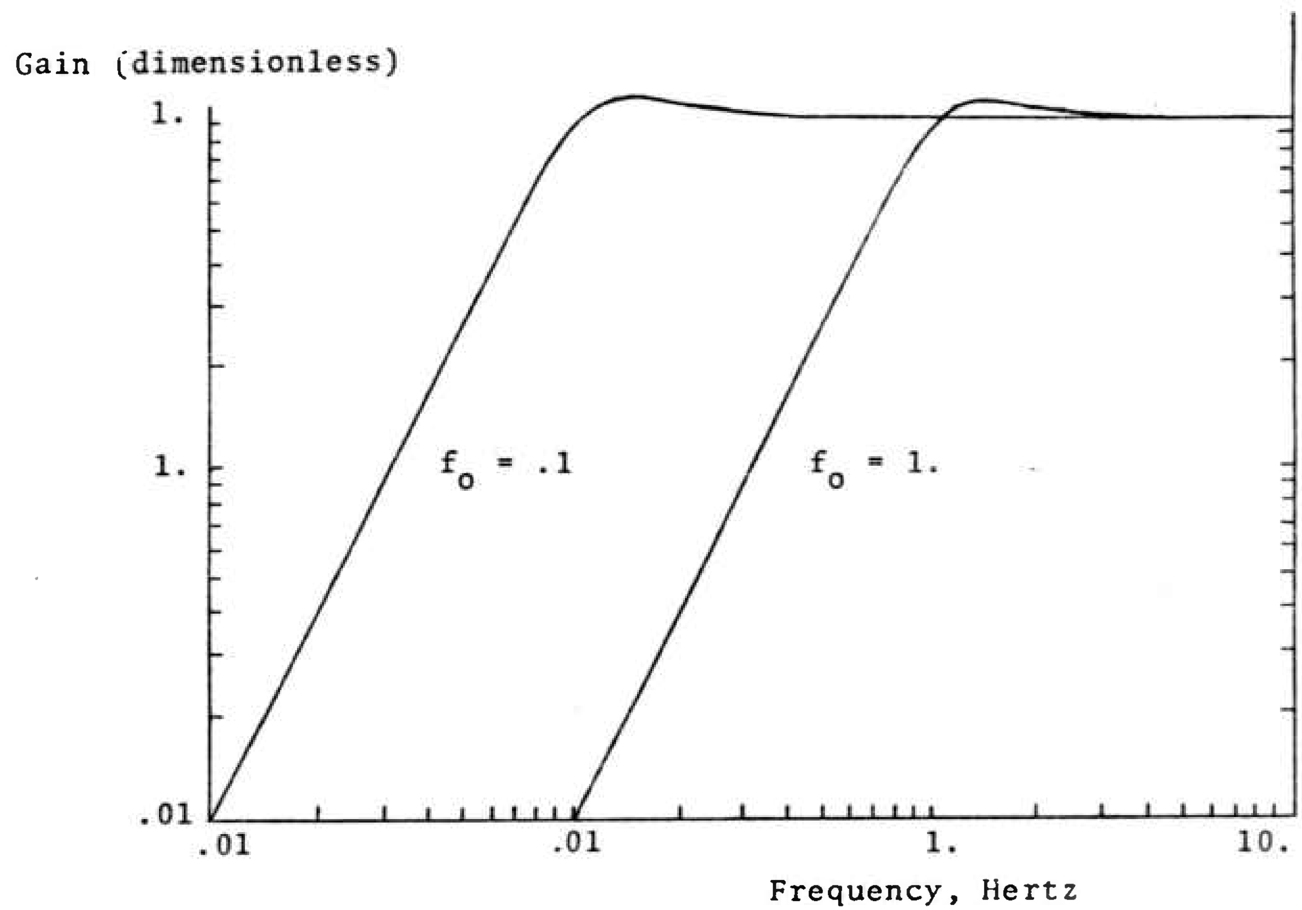


Figure 2-2 Seismometer Frequency Response



Input - Displacement

Output - Displacement

f_o = Resonant Frequency

Seismometer Mechanical Gain

Figure 2-3

2.2 Nonlinear Models

The addition of nonlinear elements into the seismometer models can lead to two possible models - one containing a nonlinear feedback element, and one containing a series nonlinearity. The nonlinear feedback model results from nonlinear mechanical elements in the seismometer itself (either spring or damper), while nonlinearities in the transducer or electronics can only be represented by series nonlinearities. In this section we illustrate the feedback nonlinearity by considering the equation of motion for a seismometer containing a nonlinear spring, and investigating its equivalent frequency response. Series nonlinearities are then discussed.

2.2.1 Nonlinear spring, feedback model

A nonlinear spring will in general have a restoring force of polynomial type. This leads to an equation of the form

$$m\ddot{x} + b\dot{x} + k_1 x + k_2 x^2 + k_3 x^3 = m\ddot{y}, \tag{2-5}$$

where nonlinear terms above the third order are neglected. In terms of ω_o and Q this can be written

$$\ddot{x} + \frac{\omega_o}{Q}\dot{x} + \omega_o^2 x = \ddot{y} - N(x), \tag{2-6}$$

where $$N(x) = \frac{\omega_o^2}{k_1}\left[k_2 x^2 + k_3 x^3\right].$$

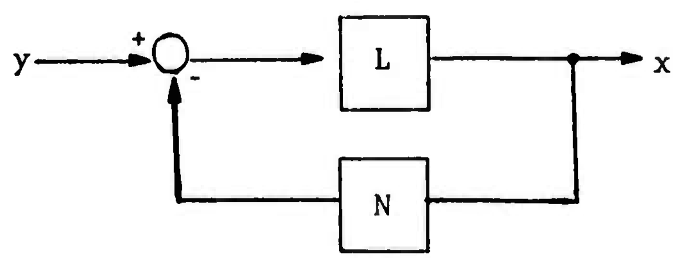
In the absence of nonlinearities, the seismometer output can be represented as a linear transformation L on the input, that is,

$$x = L(\ddot{y})$$

Analogously, the output in the nonlinear case can be considered to be the same linear operator acting on the right hand side of equation (2-6). Then we have

$$x = L[\ddot{y} - N(x)],$$

which must be represented by the nonlinear feedback circuit



We have computed the principal frequency response for a cubic spring non-linearity, using the perturbation theory techniques described in references 6 and 7. The differential equation of motion for the mechanical seismometer with a sinusoidal input is

$$\ddot{x} + \frac{\omega_o}{Q}\dot{x} + \omega_o^2 x + \frac{k_3 \omega_o}{k_1} x^3 = A \sin \omega t \quad .$$

This can be converted into a normalized form by the change of variables:

$$x' = x/x_2, \quad \text{where } x_2 = (k_1/k_3)^{1/2}$$

$$t' = \omega_o t$$

$$E = \frac{A}{\omega_o^2 x_2} \quad , \qquad f' = f/f_o$$

yielding

$$\frac{d^2x'}{dt'^2} + \frac{1}{Q}\frac{dx'}{dt'} + x' + x'^3 = E \sin f't'$$

In other words, length is measured in units of the length x_2, time in multiples of the time it takes a natural oscillation of frequency $f_o = \omega_o/2\pi$, to complete one radian $t_o = 1/\omega_o$. For an input acceleration of amplitude A, the output displacement amplitude is A/ω_o^2 for off resonance low frequencies:

then this quantity is measured in units of x_2. Therefore, the magnitude of the input signal E is normalized to the off-resonance output signal for a unit input. For weak non-linearities, the situation of interest, E is less than one, E << 1.

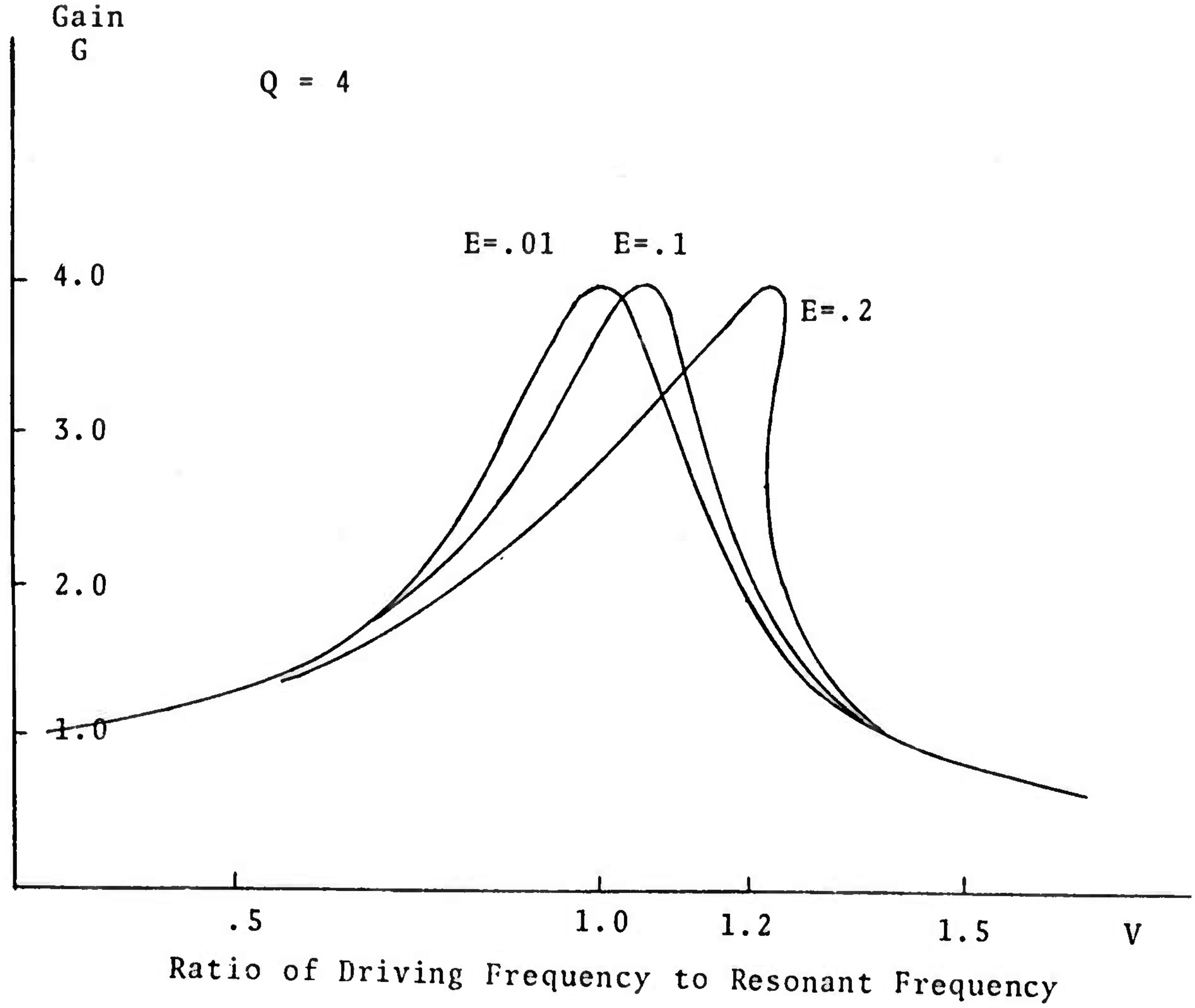
In this system of units, the gain G, as a function of frequency, ratio f' and input signal magnitude E is given by:

$$f' = \sqrt{\left(1 + \frac{3}{8}(GE)^2\right)^2 \pm \sqrt{\frac{1}{G^2} - \frac{1}{Q^2}}}$$

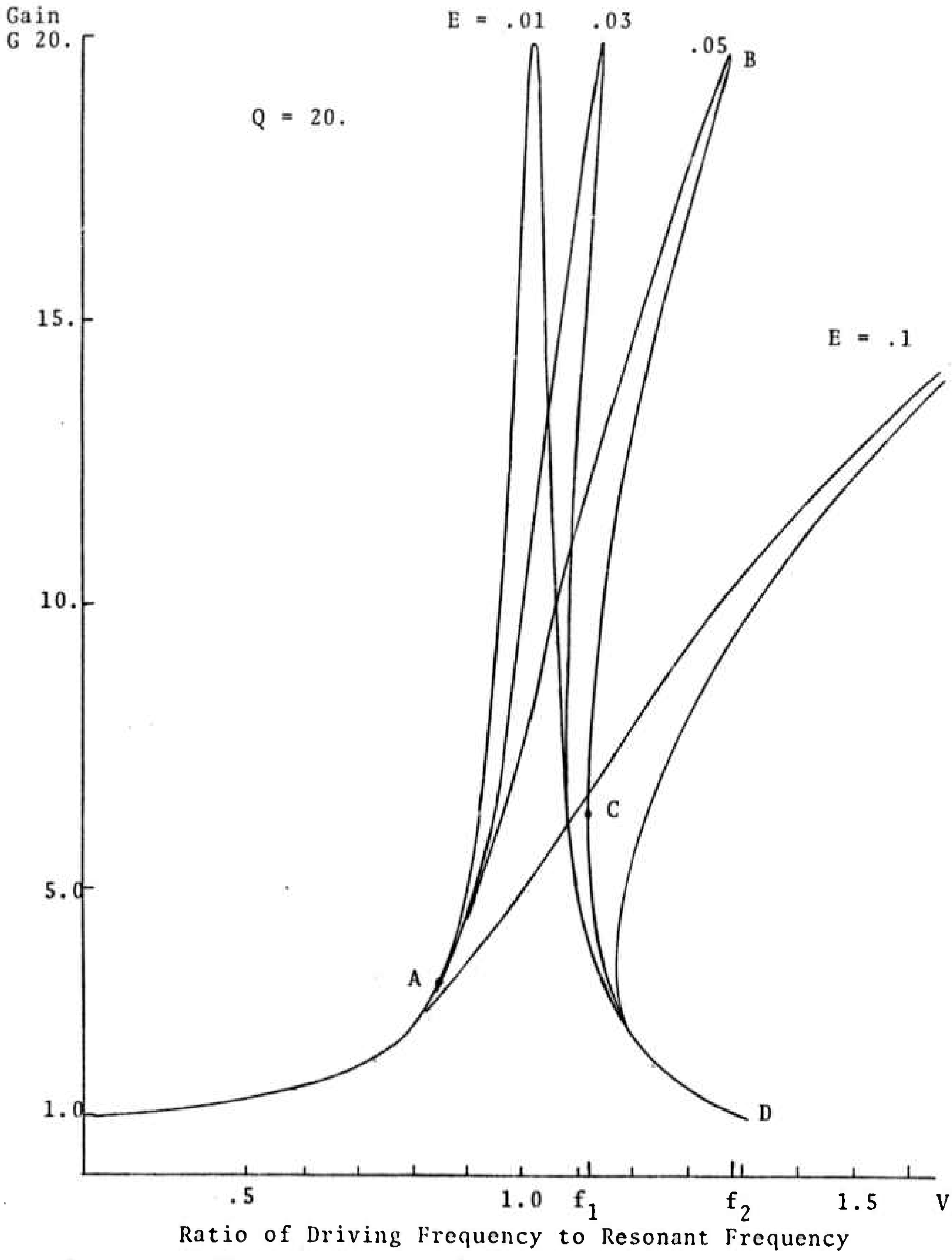
Figures 2-4, 2-5 and 2-6 show typical plots of G versus f' for various Q values. These curves show that neglecting harmonic generation, a small non-linearity can severely distort the frequency response of a high-Q system. The distortion caused by non-linearity must be added to the recognized reasons (primarily stability) for avoiding high Q mechanical systems.

For some curves, one observes that there are three possible gains at some frequencies. We can interpret this by referring to Figure 2-5. The actual gain at frequencies f_1 to f_2 depends on the initial conditions. If the frequency of a low frequency signal is slowly increased, then its gain will be specified by the section A - B of the gain curve; while if the frequency of a high frequency signal is slowly decreased, its gain will follow section C - D of the curve. At frequencies f_1 and f_2, the gain will jump. Gains along curve D - B are unstable.

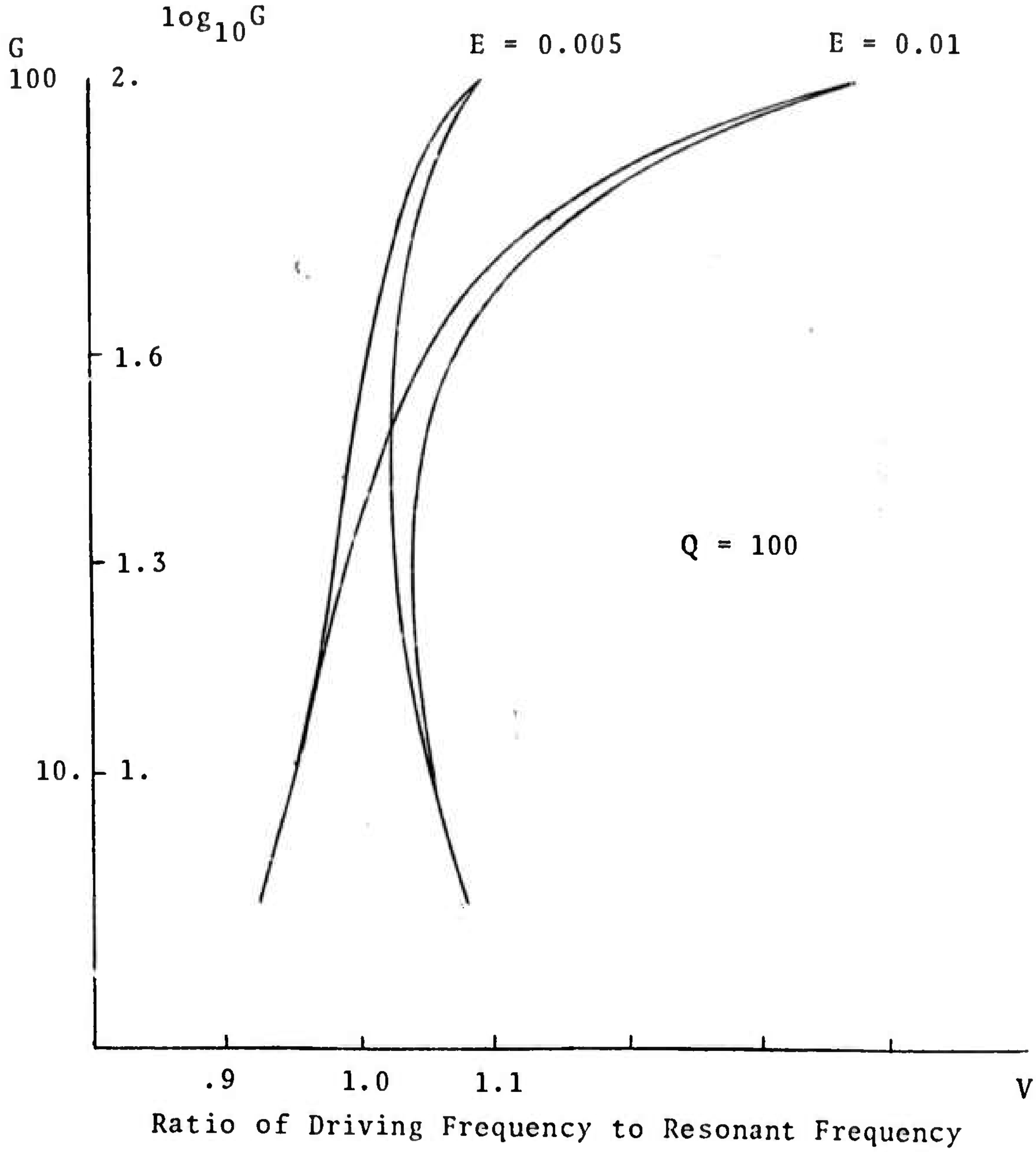
These results indicate that for a given non-linearity, a low-Q seismometer will suffer less distortion than a high-Q seismometer. Since the greatest distortion is near and above the resonant frequency, the resonant frequency should be above the frequencies of interest.



Frequency Response with a Cubin Non-Linearity, Q=4
Figure 2-4



Frequency REsponse with a Cubin Non-Linearity, Q=20
Figure 2-5

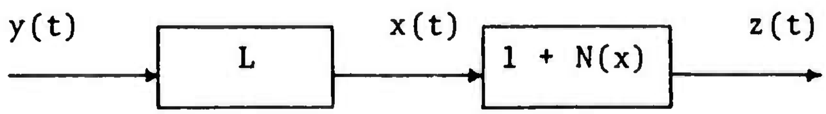


Frequency Response with a Cubic Non-Linearity, Q=100

Figure 2-6

In this model L is the linear transformation of the seismometer with no nonlinearity, while N contains the nonlinear operations of the nonlinear element. A nonlinearity in the damper leads to an equivalent representation. This feedback equivalent circuit cannot be reduced to a series nonlinearity such as that which represents a nonlinearity in either the output transducer or the amplifier.

For a series nonlinear system



,

the principal frequency response of the output z(t) will be almost the same as the principal frequency response of the linear output x(t), if the non-linearity N(x) is resonably small.

However, for a non-linear feedback system, the principal frequency response of the output x(t) may be considerably different from the linear frequency response, particularly near the resonant frequency for high-Q systems.

2.2.2 Transducer and Amplifier Non-linearities. (Series model)

In addition to the mechanical elements of the seismometer itself, non-linearities in the output electro-mechanical transducer and in the seismometer amplifier can also contribute to non-linear distortion of the output signal.

The transducer converts either the mass displacement or the mass velocity, depending on its type, to an electrical signal. In the linear case the transducer output x_t will therefore be either

$$x_t = Ax$$

or

$$x_t = A\dot{x} \quad .$$

Non-linearities in the transducer will introduce higher order terms into the output, resulting in expression of the form

$$x_t = A_1x + A_2x^2 + A_3x^3 + \cdots$$

or its equivalent in terms of velocity.

The seismometer amplifier may also introduce non-linear terms in its action on the transducer output x_t to produce the final observable signal x_a. The general expression is

$$x_a = C_1x_t + C_2x_t^2 + C_3x_t^3 + \cdots$$

In contrast to the feedback model required to represent non-linear mechanical elements, non-linearities in either the transducer or amplifier are represented by series non-linear elements:

$$x \longrightarrow \boxed{A + N_1} \xrightarrow{x_t} \boxed{C + N_2} \xrightarrow{x_a}$$

where A and C are the linear terms and N_1 and N_2 contain the higher order effects.

2.3 Discussion and Summary

In this section we have identified two distinct types of non-linearities - those associated with the mechanical elements of the seismometer, which require representation by a non-linear feedback element within the seismometer itself, and those which are associated with the transducer or amplifier and can be represented by series non-linear elements.

In addition to distinguishing "mechanical" and "electrical" non-linearities by their equivalent representation, it is reasonable to postulate that the errors incurred in these two cases will be differently affected by the signal amplitude observed at the mechanical elements.

In both cases, the output signal x_a can be represented as a linear function of the acceleration input plus an error term caused by the non-linearity.

$$x_a = L(\ddot{y}) + e$$

The fractional error incurred is

$$e_f = \frac{e}{L(\ddot{y})} \quad .$$

If the non-linearity occurs only in the mechanical elements, say in the spring, then its effects can be reduced by reducing the mass excusion, hence the spring extension. We assume in this case that the true situation can be approximated by allowing the fractional error to be directly proportional to signal amplitude.

In the case of the "electrical" non-linearity, the situation differs in the following way. The amplifier is used to bring the signal up to a useable level. A fixed change in mechanical gain resulting in a change in input signal level will probably be compensated by a change in amplifier gain so as to maintain roughly the same signal level. Since amplifier non-linearities are generally dependent on output signal level, this situation is approximated by assuming the fractional error to be constant, and independent of input signal level.

These two cases will be investigated numerically in Section 3 to determine their relative effects in reducing signal to noise ratio by non-linear transfer of energy between frequency bands. It should be emphasized that the above assumption regarding the action of mechanical and electrical nonlinearities represents an approximation to the true situation made for convenience of numerical simulation. The actual effects require study of actual seismometer waveforms.

3.0 NUMERICAL EFFECTS OF SMALL NON-LINEARITIES

The simulation work began with a brief investigation of the effects of non-linearities on the signal-to-noise ratio in low-noise regions of a typical seismic noise spectrum. This initial work was done without considering the frequency response of the seismometer, that is, the seismometer was modeled as an all-pass network.

This work was followed by an investigation of the signal-to-noise ratio reduction caused by non-linearities of the two particular types discussed in the preceding section. This work included the determination of the effects of the seismometer parameters f_o and Q.

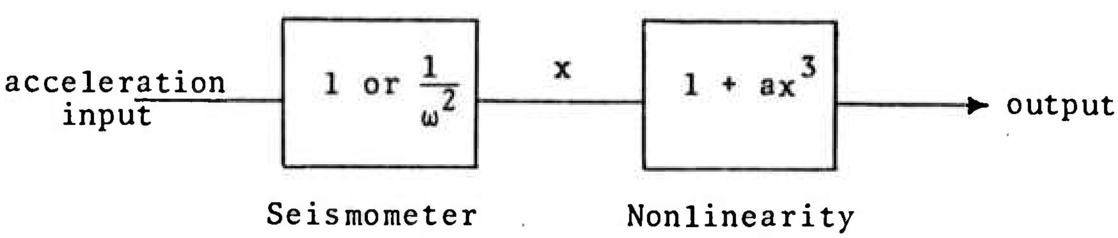
The non-linearities investigated have been consistently small - of the order of magnitude that may introduce one or two percent harmonic distortion in a pure sinusoid. It is assumed that large non-linearities can be easily avoided by careful design procedures. In spite of this restriction to small non-linearities, large degradation of signal-to-noise ratio were observed.

3.1 Non-Linearity Effects on Microseismic Noise

A model for microseismic displacement noise covering the period range from 0.1 to greater than 100 seconds was synthesized in consultation with ARPA technical representatives using measurement data primarily from Reference 4. This displacement noise spectrum is shown in Figure 3-1. A program was devised for generating a noise time signal which exhibits the spectral properties shown in Figure 3-1 and covering the range from .002 Hz to 8 Hz (.125 to 500 seconds period). These noise time signals were then processed through a cubic non-linearity of the form $y = x + ax^3$ and comparisons were made between input and output spectra.

An estimate for the corresponding acceleration spectrum was obtained by doubly differentiating with respect to time the displacement curve. Results of this process are shown in Figure 3-2.

The model can be depicted as shown



In this model the signal x is the seismometer transducer output, and the seismometer itself is simplified to be either all-pass or a perfect double integrator. Thus in the following discussions the output x is proportional either to mass acceleration or to mass displacement. The choice is made by choosing either the earth noise acceleration spectrum or displacement spectrum (respectively) used as the input.

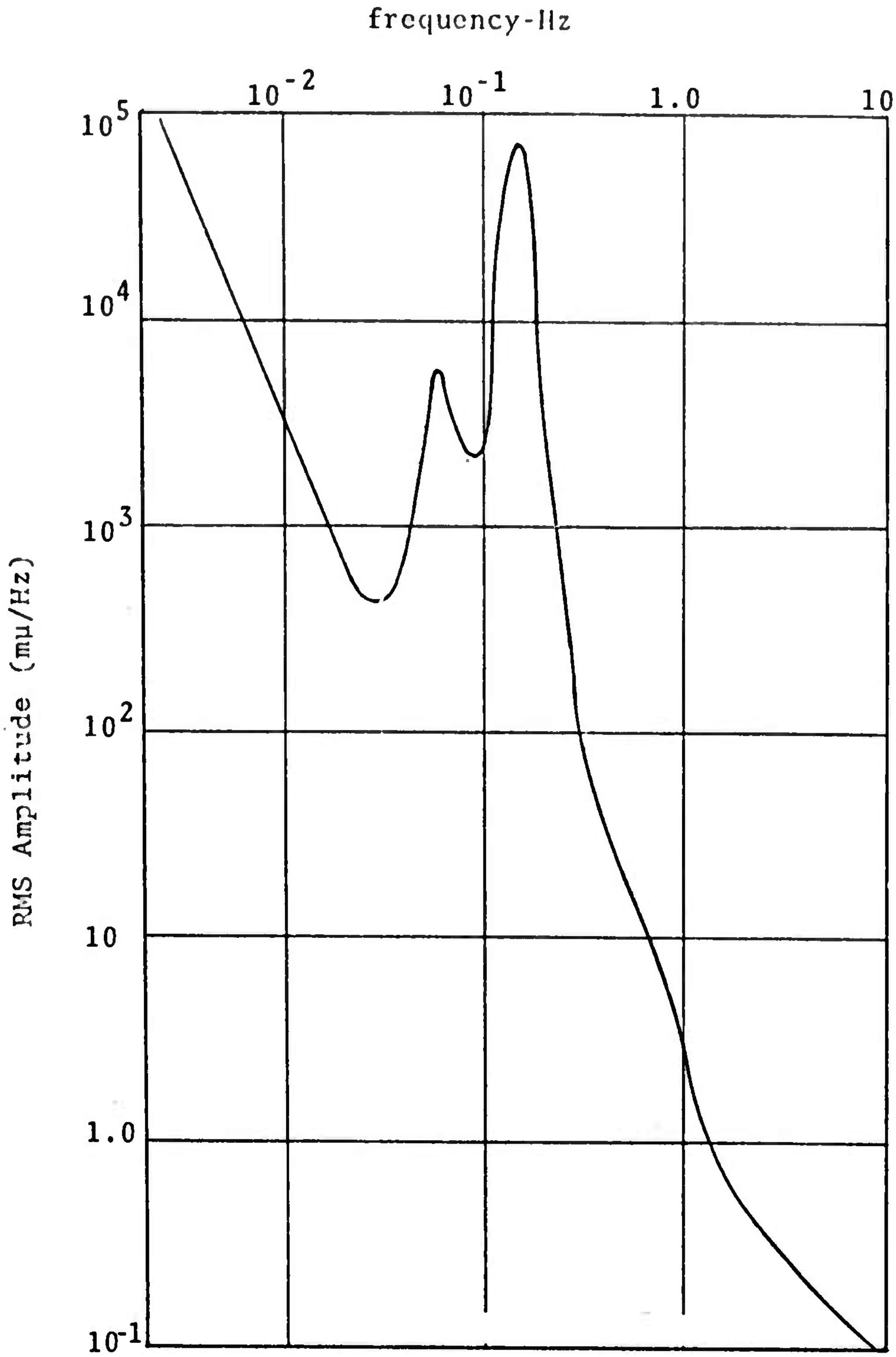


Figure 3-1. Displacement Micro-seismic Noise Spectrum

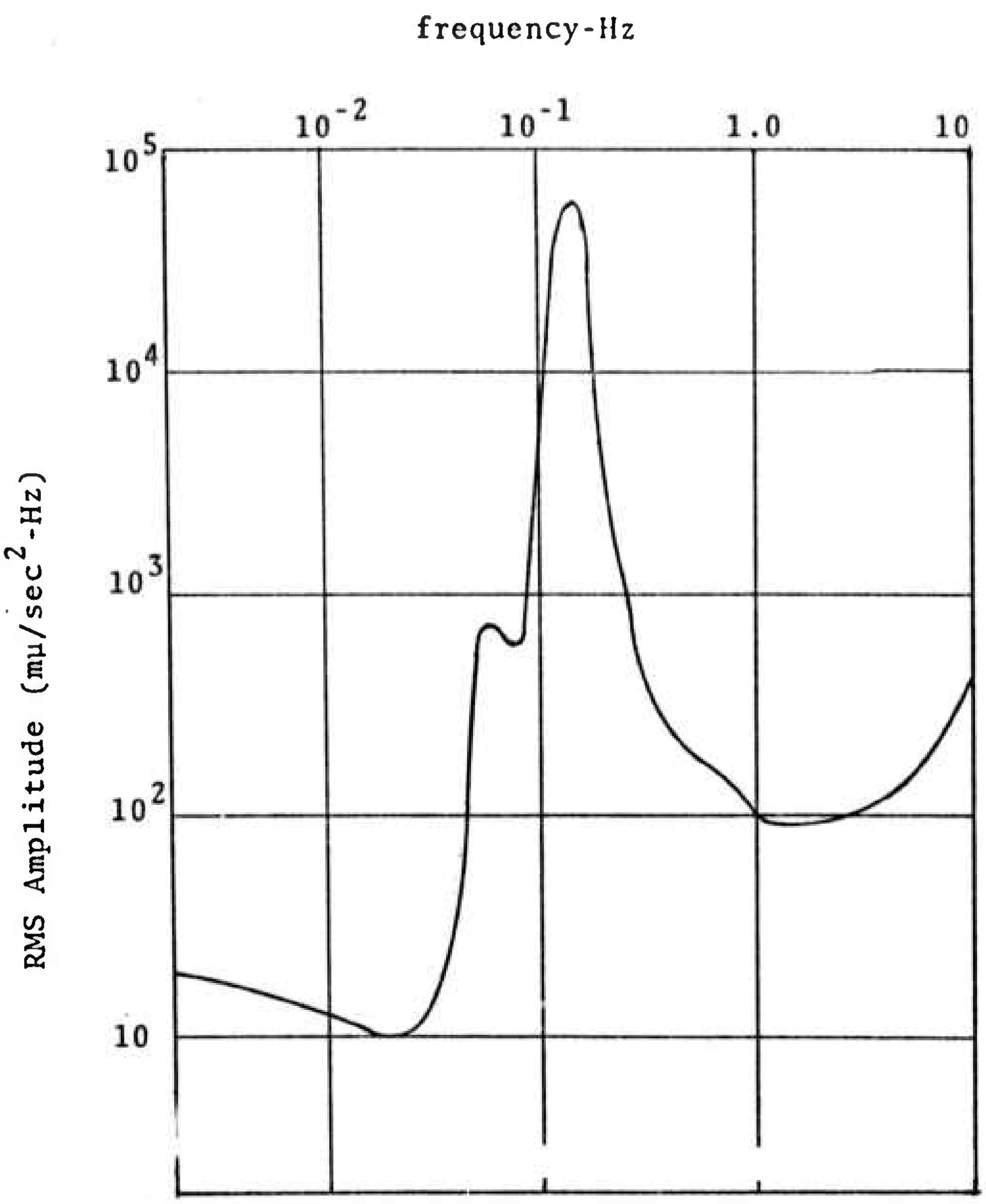


Figure 3-2. Acceleration Micro-seismic Noise Spectrum

The percentage distortion is based upon signal (microseismic noise) level. In a microseismic storm, for example, one would expect the non-linearity to have greater effect because of the higher signal levels. This effect was removed by adjusting the parameter "a" so that the distortion is related to the input RMS values. A P% distortion means that $1+ax^3 = 1+P\times10^{-2}$ when x is set equal to its RMS value.

Results of application of the non-linearity model to displacement noise are shown in Figure 3-3. The input spectrum is shown as a solid line and output spectra are shown for values of distortion (at rms) of 0.1%, 0.3%, 1.0% and 3%. As would be expected the high energy microseismic peak at approximately 6-seconds when passed thru the non-linearity model causes significant increases in noise energy at the third harmonic level (approximately 0.4 Hz) and at the low (difference) frequencies. A significant increase in a portion of the 20-40 second band is observed. Peak differences as high as 33 db at 0.4 Hz are observed with peak differences in the LP band at approximately 12 db.

The results of Figure 3-3 indicate the expected output spectrum for a non-linearity acting on the earth displacement if the input spectrum is truly representative of earth displacement noise. It is emphasized that this input spectrum is based on measurement data and is in fact a seismometer output spectrum upon which non-linearities (if present) have <u>already</u> acted.

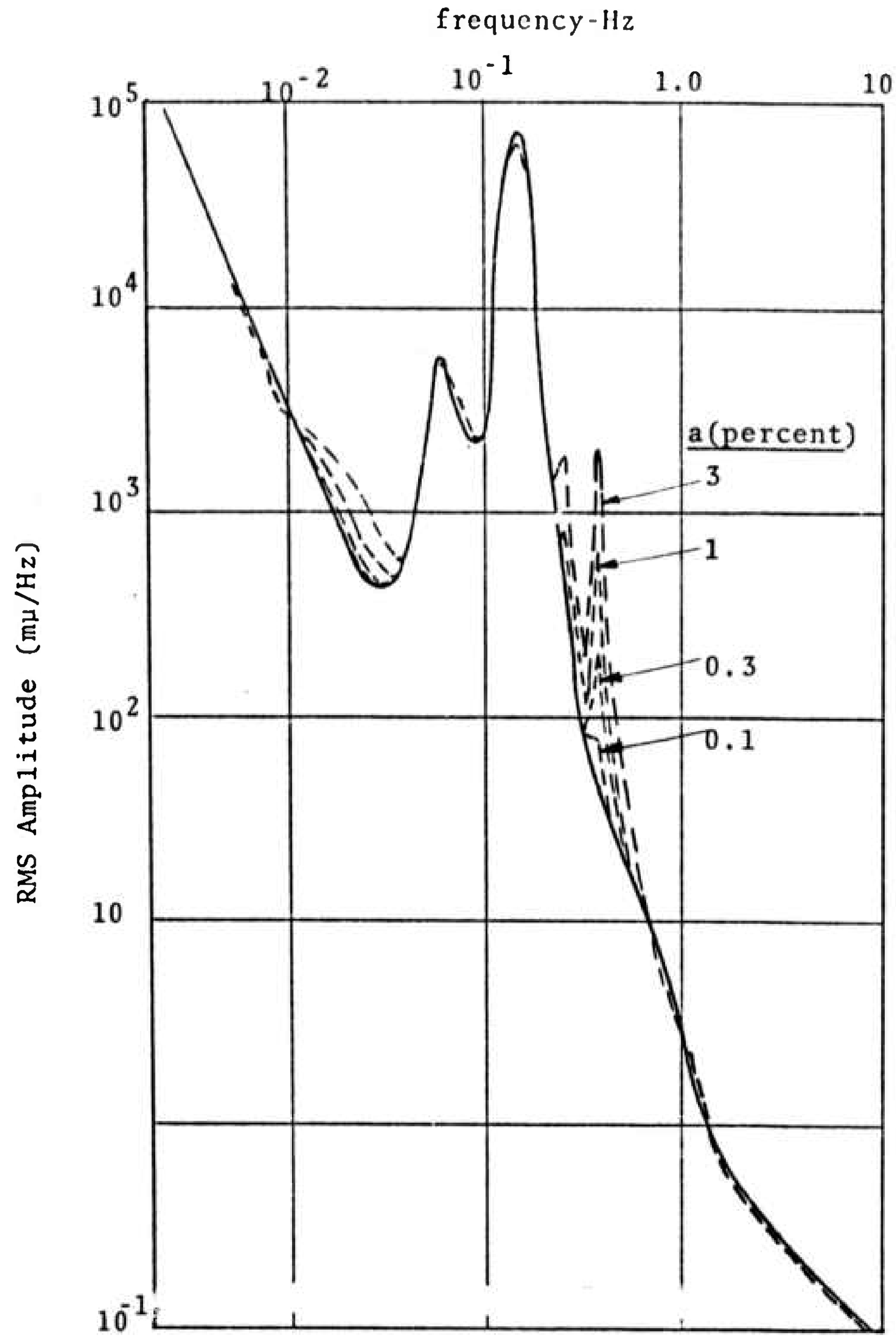


Figure 3-3. Effect of Non-Linearity Model on Displacement Noise Spectrum

Figure 3-4 shows results of the non-linearity model acting on the acceleration microseismic noise. The result of the non-linearity is to fill in the notch at 18 seconds and to increase energy in the low frequencies and near the third harmonic. The increase in low frequency energy for the acceleration model is more pronounced than for the displacement model because of the relative levels of low and high frequency energy at the input. That is, for the acceleration noise model, significant energy in the range from 1 to 8 Hz is also pumped down into the LP band.

Peak spectral noise increase occurs in the LP band and ranges from about 1 db for a = 0.1% to about 21 db for a = 3%. Thus, for an instrument with non-linearities acting on the acceleration input to the instrument, the non-linearity levels examined would cause significant degradation of the system performance.

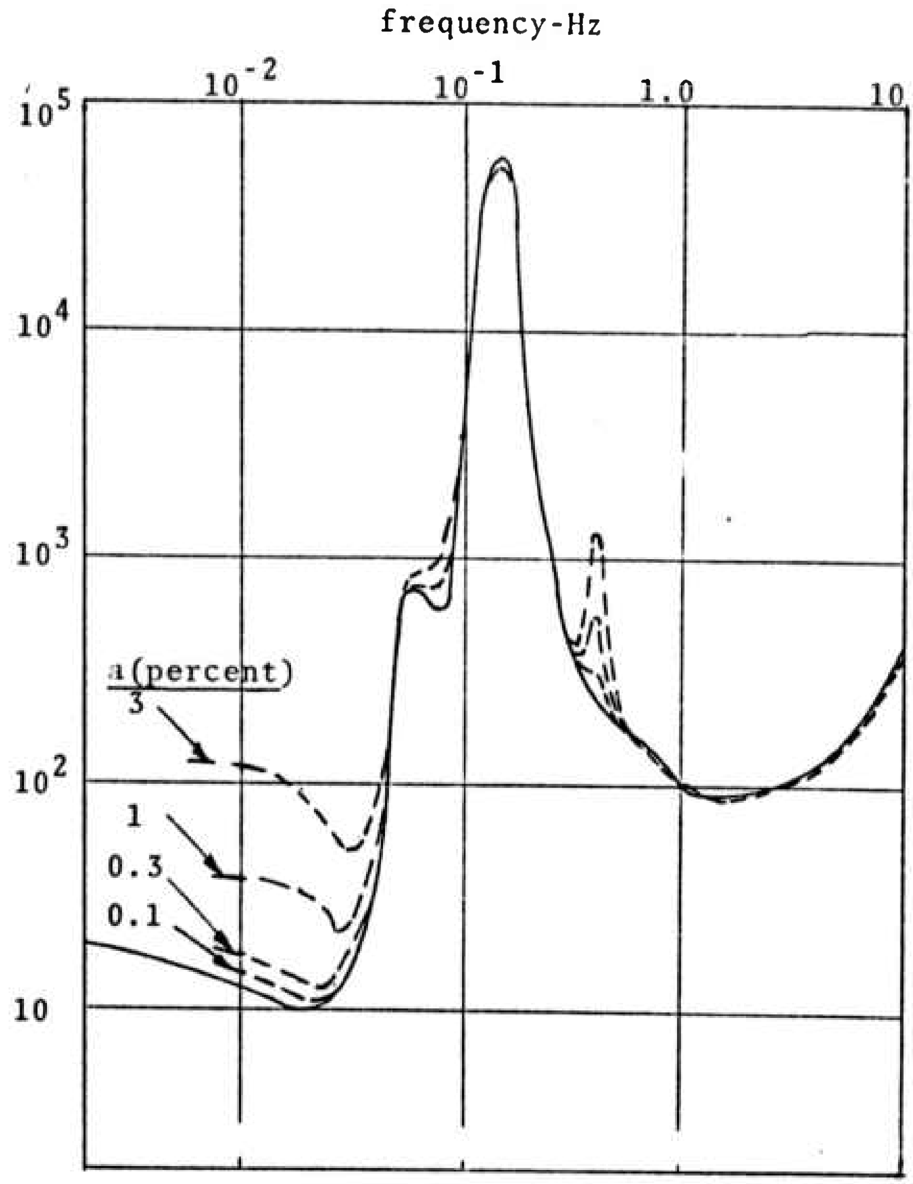


Figure 3-4. Effect of Nonlinearity Model on Acceleration Noise Spectrum

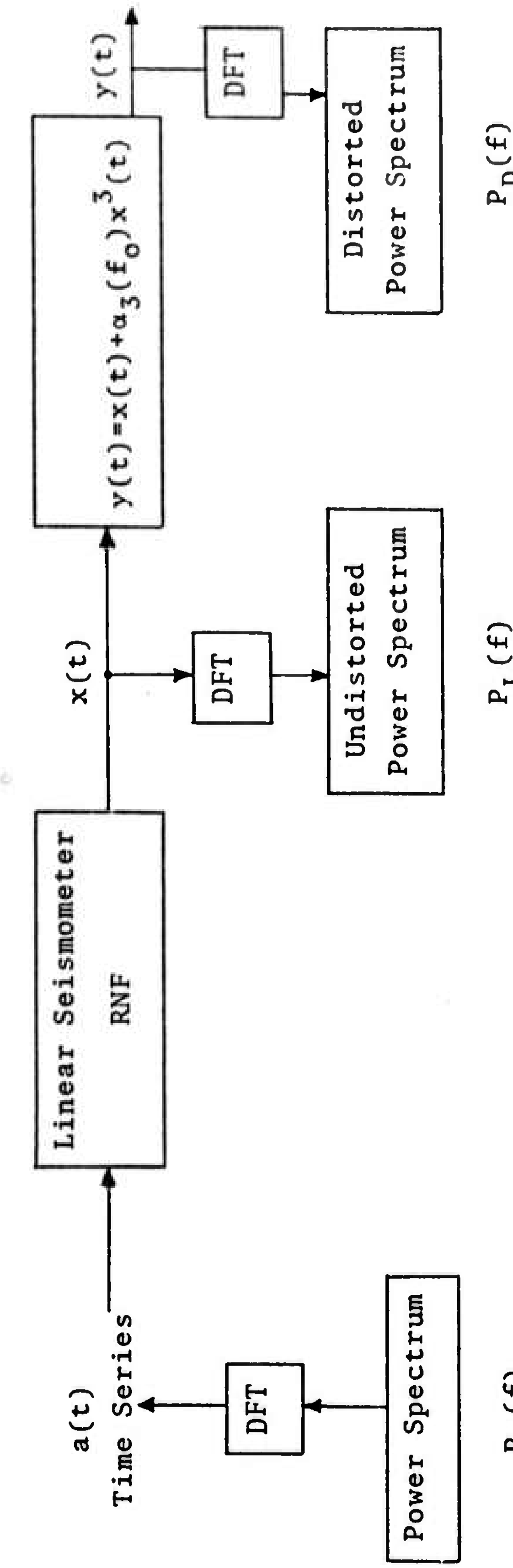
3.2 Effects of Seismometer Frequency Response

In section 3.1 the seismometer was assumed to be all-pass in the frequency band of interest. This is approximated in practice when f_o>1 Hz and Q is low. It can be hypothesized that one could decouple the LP bands from the microseismic band by choosing low f_o or/and high Q design parameters. In this section we show that this hypothesis may be true but that a simple conclusion in this regard is impossible.

Our simulation scheme is outlined in Figure 3-5. For this work we continue to utilize the series type non-linearity. The basic system consists of a linear seismometer followed by a polynomial non-linearity. The linear seismometer is specified by its resonant frequency f_o and Q and has the gain characteristics described in Section 2. The input signal is a time series having the same power spectrum, P_N, as the earth noise model shown in Figure 3-2. After the linear seismometer, we take the power spectrum, P_L. Then the time series is passed through the polynomial (cubic) non-linearity, characterized by x_3, and the power spectrum P_D, of the distorted data is taken. The difference between P_D and P_L represents the distortion caused by the non-linearity. We then compare the low frequency end of P_L and P_D by examining

$$\Delta(f) = \frac{P_D(f) - P_L(f)}{P_L(f)} \qquad \text{for } f \simeq .03 \text{ Hertz}$$

as we systematically vary f_o, Q and α_3.



Numerical Simulation of a Non-Linear Seismometer System

Figure 3-5

The input signal is a random noise process, therefore, there are variations in the realizations of the power spectra. To smooth out these variations the spectra were smoothed and averaged to a resolution of .01 Hz.

In Table 3-1 we present the results of the calculations of $\Delta(f)$ in terms of db.

$$\Delta(f)(db) = 10 \log_{10}\Delta(f)$$

in the leftmost columns of the table. The righthand columns give the r.m.s. magnitude of the seismometer output signal and the non-linearity parameter. The seismometer output signal is taken to be the inertial mass displacement, measured in millimicrons, mμ, responding to seismic noise. Instead of giving the value of α_3 we give $x_2 = (1/\alpha)^{1/2}$ which is the equivalent output displacement, in mμ's, at which the seismometer is highly non-linear.

The first half of the table shows that for "electronics non-linearity" (i.e. one for which fractional error is proportional to signal amplitude) the low frequency seismometer experiences less increase in low frequency noise, $\Delta(f)$, than the high frequency seismometer. In fact, the noise increase for $f_o = .02$, was unobservable for the non-linearity chosen. The relative noise increase is less for small f_o, since the low frequency background noise at the output of the seismometer is much larger, relative to the 7 second seismic peak, than it is for high f_o.

"ELECTRONICS NON-LINEARITY"

α_3 set for 2% non-linearity at each f_o and Q

$\alpha_3 = (\frac{1}{x_2})^2$ Q = 1

F_o	Q = 1. Q = 1	3.	10.	RMS mμ	x_2 mμ
.02	0. db	0.	4	8837.	62490
.05	3. db	7	12	5617.	39722
.1	12.	16	17	5435.	38433
1.0	15.	16	15	57.	405

"MECHANICAL NON-LINEARITY"

α_3 constant for all f_o.

F_o	Q=1.0	mμ RMS	mμ x_2
.02	65 db	8837	(405)
.1	87 db	5435	405
1.0	16	57	405

Increase in Low-Frequency Noise in db, $\Delta(f)$

Table 3-1

The second half of the table shows that for the "mechanical non-linearity" (one for which fractional error is independent of signal amplitude), the high frequency seismometer has the least low frequency noise increase $\Delta(f)$. This is because the r.m.s. magnitude of the seismometer output signal decreases rapidly at high resonant frequencies, f_o.

These two cases show that there is no unique optimum seismometer design that will be most effective in mitigating the effects of any non-linearities that may be present.

This conclusion is independent of the particular type of non-linearity used, even though the actual noise increases may be different. For example, if we have a quadratic non-linearity where α_3 is chosen to give a 0.2% non-linearity, then the noise increase will be 40 db for f_o - 1.0, Q = 1, [and 6 db for f_o = .02, Q = 1], as opposed to 16 db for a 2% cubic non-linearity. However, the overall trends are similar: for a fixed non-linearity, the noise increase $\Delta(f)$ is smaller for large f_o, while for a constant percentage α_2, the noise increase is smaller for small f_o.

4.0 CONCLUSIONS AND RECOMMENDATIONS

This study has considered two questions. First, if small nonlinearities exist in seismometer elements, what is their effect on the output signal? Second, what precautions, if any, can be taken in the basic design of seismometer systems to minimize the undesirable effects of nonlinearities.

We have made more progress toward answering the first question than the second. The work accomplished in modeling seismometers and simulating the effects of nonlinearities on typical seismic noise spectra has shown conclusively that even small nonlinearities in a simple linear system can cause reduction of the signal-to-noise ratio in quiet parts of the seismic spectrum of up to several orders of magnitude. Since it is in just those quiet spectral regions that attempts are made to estimate small signals, this result indicates that removal of such nonlinearities could do a great deal toward the improvement of signal estimation.

In considering the second question, we recognized that the art of seismometer design is an advanced one, and that the recommendation of major design modifications or radically different construction techniques would at this stage of our work be premature. We have restricted our modeling to the basic second order mechanics in order to gain insight into how variations of the fundamental parameters might affect nonlinear behavior. We have concluded that for some mechanical nonlinearities, an increase in natural frequency, with its attendent reduction of gain and extension of flat frequency response, will tend to reduce signal distortion. In addition, we have shown that gross distortions can occur near resonance in high-Q mechanical systems, which argues for keeping the resonant peak low in amplitude and above the frequencies of interest, as is the custom in practice.

The major question that remains unanswered by this study is a fundamental one. Do nonlinearities of the type and magnitude considered indeed exist in seismometers? The answer to this question cannot be approached by simulation or models, nor can the non-existence of nonlinearities be assured by refined and advanced design and construction techniques.

Since nonlinearities are an unwanted and unintended element in a physical system, their presence can only be confirmed or disproved by experiments performed on actual seismometers. Such experiments have as yet not been performed in a controlled way, although some results have been observed which show that identical seismometers rigidly mounted on the same platform produce output signals which are noticeably dissimilar. This is just the effect that would be expected from small nonlinearities acting in quiet spectral bands.

The detection and analysis of nonlinear effects in actual seismometer signals is not a straightforward problem, primarily because the seismometer input is unobservable. There are, however, signal processing techniques which provide a promising analytical approach to detecting nonlinearities by observing only the seismometer output. In view of the dividends that can be achieved in improved signal-to-noise ratio by the detection and removal of small nonlinearities, it is recommended that the investigation be continued in this direction.

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