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A DESCRIPTION OF EXPECTED FAILURE RATES OF NEWLY ACQUIRED COMPONENTS PRIOR TO STEADY STATE

Richard F. Erickson, et al

Air Force Institute of Technology
Wright-Patterson Air Force Base

January 1974

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A study of the effects on the population failure rate of component failures during the period of transition to steady state is presented. Comparisons between the simulated failure rates and the system failure rate predicted under the present Air Force assumption of an underlying Poisson process are accomplished for various MTBF's and parameters of the Weibull distribution. The decreasing hazard, constant hazard, and increasing hazard functions are studied and presented.

This study was performed as a simulation of the transition period using the GASP II package developed by Pritsker and Kiviat. Such a simulation package greatly simplifies the required programs necessary to be written. The computer program used in this study is included as part of this paper.

The results of the study indicate there are significant differences between the present Air Force methods of predicting failure rates and the simulated failure rate distribution during the transition period. Plots of the simulated results compared to the present Air Force methods indicate these differences could reach 100% of the Air Force estimated failure rate when the Weibull shape parameter is greater than 3 and the components MTBF exceeds 1000 hours.

The study indicates that component failure rates, which can be described by a Weibull density function, will result in oscillations about the present Air Force computed rate at steady state.

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A DESCRIPTION OF EXPECTED FAILURE RATES OF NEWLY ACQUIRED COMPONENTS PRIOR TO STEADY STATE

A Thesis

Presented to the Faculty of the School of Systems and Logistics
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the Requirements for the
Degree of Master of Science in Logistics Management

By

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and

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CHAPTER I

INTRODUCTION

Structure

For the convenience of the reader, we have structured the following report so that Chapters III and IV are not required for an intuitive understanding of the background and results of our investigation. A reader with little time or desire for mathematical background material may wish to omit Chapters III and IV in the interest of brevity. In addition, we offer our definitions of the following commonly used terms.

Component--The general term for any part, subassembly, line replaceable unit, system, or piece of equipment whose failure and replacement must be considered in logistics plans.

Component Population--The total number of a component type in the Air Force operating inventory.

Failure--The opposite of "operating normally." Thus, for our purposes, a component has only two states of nature; either it is serviceable, or it has failed.

Failure Rate--The rate of failure in a population of identical components. This measures applies when components are repaired (to new condition) or replaced (by new components) and thus continue to operate. It is computed by dividing the total number of observed failures in a time period by the accumulated operating time during the period.

Mean Time Between Failures (MTBF)--The average operating time expected between failures in a population of identical components. This measure has meaning only when we are discussing a population where there is repair or replacement.

Hazard Rate--The instantaneous failure rate of the components under consideration; or, the expected failure rate during the next time interval.

Mean Time To Failure (MTTF)--The average operating time expected before failure of a component which is not repaired or replaced. This is simply the average time to failure of "n" units, i.e., the sum of "n" individual unit times to failure divided by "n" units.

Statement of the Problem

One of the basic factors associated with the logistical support of a weapons system, or a group of weapons systems, is the determination of future requirements. Specifically, we need to know how often parts and/or equipment will fail. In practice, the occurrence of a component failure is so basic to the support function that its importance is sometimes overlooked. Yet, the frequency and distribution of component failures determine how many spare parts are to be purchased, how many support manhours are to be scheduled, what base and depot support facilities are to be made available, and even how many aircraft are to be required to perform the mission.

In the established procedures for determining support requirements (5), the Air Force makes an implicit assumption as to the underlying distribution of failures. We assume each component population has a constant failure rate with respect to time. Under such an assumption, we ignore the reality of the situation where components actually deteriorate over time and that the probability of failure may increase as the components accrue larger and larger operating times. Thus, under such an assumption of a constant failure rate, we may unintentionally introduce serious errors into our logistics planning to meet future support requirements.

Objective

The objective of this research is to investigate, through digital computer simulation, the distribution of failure rates during the transition period from the initial acquisition of new weapons systems to the attainment of a "steady state" condition, resulting from the incremental introduction of a fleet of aircraft. For simplification we assume each aircraft consists of a self-contained, independent component whose probability of failure can be approximated by a Weibull distribution. Specifically, we were interested in the change of the component population's failure rate with respect to time. In reliability terms, we wanted to investigate the pooled output of N superpositioned renewal processes where $N(t) = f(t)$.[1]

Hypothesis

The fleet wide distribution of failures for any replaceable equipment/component is at all times a Poisson process; therefore, the assumption of a constant failure rate is always valid in estimating future expected failures.

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[1] We are interested in the system average failure rate at a point in time. This failure rate is the sum of the individual failure rates divided by the number of failures, i. e.,

$$N(t) = \frac{\sum_{i=1}^{n} N_i(t)}{n} = f(t)$$

CHAPTER II

BACKGROUND

The Air Force Method for Estimating Part Failures

If each part, subassembly, or line replaceable unit (LRU) in our supply inventory came with a "service life" stamped next to the part number, logistics plans for future requirements could be made by anyone with a desk calculator. Components could be replaced before failure, stocked exactly in the right amounts, and always reordered in the most economical quantities. Unfortunately, this is not the case, since individual component times to failure occur randomly and are not exactly predictable. To overcome this problem and prepare meaningful quantitative support plans, the Air Force has adopted a method for estimating the service time of components by averaging service times. This paper is concerned with the accuracy of such an estimation method during the initial provisioning process, and the early life of a weapon system.

The established Air Force procedures for predicting component failures are so basic that we seldom realize we are using a "method," and that there are requirements which must be met before applying it. The following is a rough description of present Air Force procedures (for mathematical derivations and references see Chapter III).

In order to sensibly deal with random failures, we first must make a few assumptions. One of the most helpful assumptions is to assume that while individual components fail at random, the total parent population is in a steady state condition. For our purposes, a component's supply system would be in steady state if the average Air Force-wide demand for the component remained constant year after year.[2] Where the parent population is in steady state, or sufficiently close so that we could assume steady state, then we can use the population's long run average demand as an estimator for a future period of time. For example, we might predict 1125 F-4 tire failures next year because we have averaged 1125 failures for the last five years. Such a quantitative measure belongs to a parent population in steady state and identifies the average number of failures experienced during the prior time period.

Next, we assume that component failures are distributed uniformly over the entire interval. If we have a homogeneous system which is not subject to large seasonal demands, then this assumption

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[2] We would not expect a particular F-4E aircraft to use exactly the same number of right main tires every year; however, if the yearly F-4 flying program did not change, we could expect the fleetwide demand for F-4 tires to be approximately constant.

will be valid.[3] We can now calculate an estimate of the population failure rate for any desired small time interval. Thus, if we averaged 1125 F-4 tire failures each year while logging 225,000 flying hours, we can assume we experience .005 failures per flying hour, or an average of 200 flying hours between failures. Since the component population is in steady state, is homogeneous, and composed of identical items, we can assume the above estimate applies uniformly to the individual components. Thus, each component's failure rate and time between failures would be identical to the population failure rate and time between failures. In our example, we would assume each F-4 tire had a failure rate of .005 failures per flying hour, or an average service life (MTBF) of 200 flying hours.

The procedure is complete; a quantitative measure has been determined which can be used in estimating future requirements. However, this method provides quantitative estimates only for the steady state conditions. Thus, future requirements may be estimated from current supply demand rates. As the component and its parent population age, these estimates can be refined as more data is collected.

Identical or Similar Components

When we begin the planning process for a new weapons system,

---

[3] Seasonal demands for items by their very nature do not exhibit a uniform distribution throughout the time interval. However, where the seasonal demands are relatively small in comparison to total demands, then such demands would not significantly affect this assumption.

we do not have to start over for every new component. Many items on a new aircraft are identical or quite similar to components used before. If we have collected data on similar systems, we may be able to accurately predict future requirements and provision for the new components using our past demand data. However, if the basic assumptions we made while applying the previous "method" are not valid for the new component population, sizable errors may be introduced into provisioning plans.

If the new aircraft are to be acquired in large numbers, say, 100, 500, or 700 in the total fleet, it is unlikely that all the aircraft will be placed into operation immediately. They will probably be introduced into the active inventory over a period of months, or even years. Does our first assumption of a steady state system still hold? If the components we are buying are not yet members of a steady state population, can we attribute to each of them the old system failure rate and mean time between failures? In some cases yes; in some cases no.

To illustrate this point, we will use a hypothetical situation. Consider a "specialist" shop in a maintenance squadron that supports ten aircraft. This shop repairs a photographic system that has two components: a power supply unit and a camera unit. Assume that the system has been in the Air Force inventory for a few years and that it is in steady state. The specialist repairman is new and eager. On his first day in the shop he erects a four-month wall chart for each component, complete with aircraft tail numbers and the date. For four months he religiously records each failure as a large "X" on the chart until, at the end of April, the charts appear as in Figure 1. The charts appear quite similar with five failures each month. Knowing that each aircraft flies 100 hours a month, our camera specialist calculates each component's failure rate

$$\frac{5 \text{ failures}}{1000 \text{ flying hours}} = .005 \text{ failures per hour}$$

and mean time between failures

$$\frac{1000 \text{ hours}}{5 \text{ failures}} = 200 \text{ hours.}$$

While the results from the charts appeared equal for the power supply units and the cameras, they disagreed with the repairman's intuition. He knew the components were entirely different; and therefore, he guessed that they should possess different failure patterns. The power supply unit was a solid state, fully electronic assembly that appeared to fail at random. Conversely, the camera unit was composed of small gears, bearings, and sprockets which became clogged with dust-like film emulsion and thus failed at regular intervals. To test his conjecture, the repairman went back to his charts and connected the "X"s for each aircraft with a solid line. The lines represent the operating time prior to failure for each individual component (Figure 2).

For the power supply components, the service time lines were

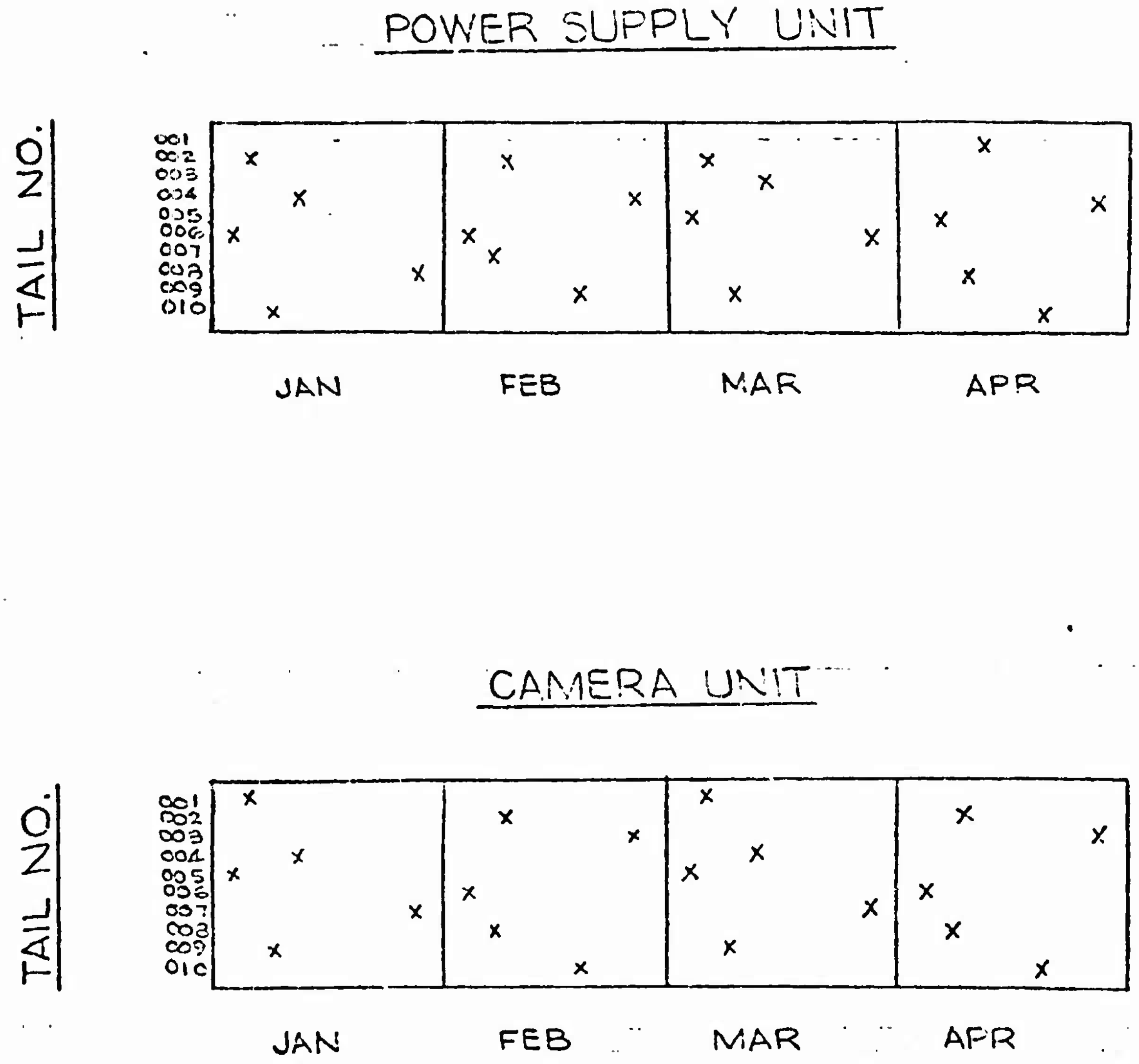
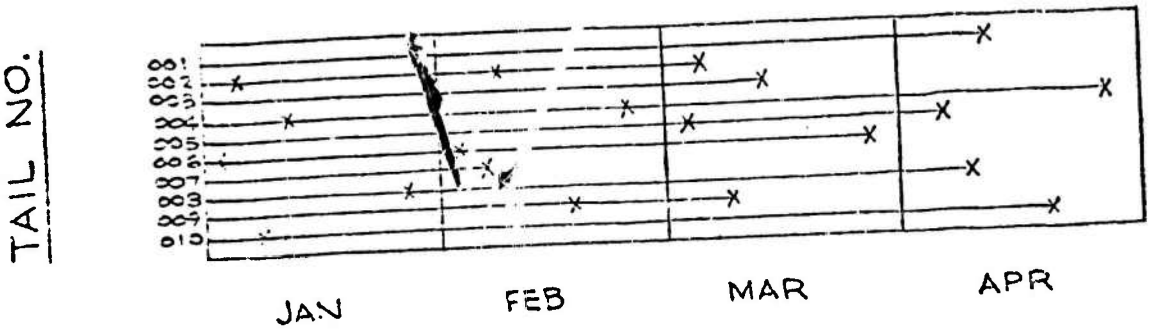


FIGURE 1

COMPONENT FAILURES - EXAMPLE
FAILURE DENOTED BY "X"
SYSTEM IN STEADY STATE

POWER SUPPLY UNIT



CAMERA UNIT

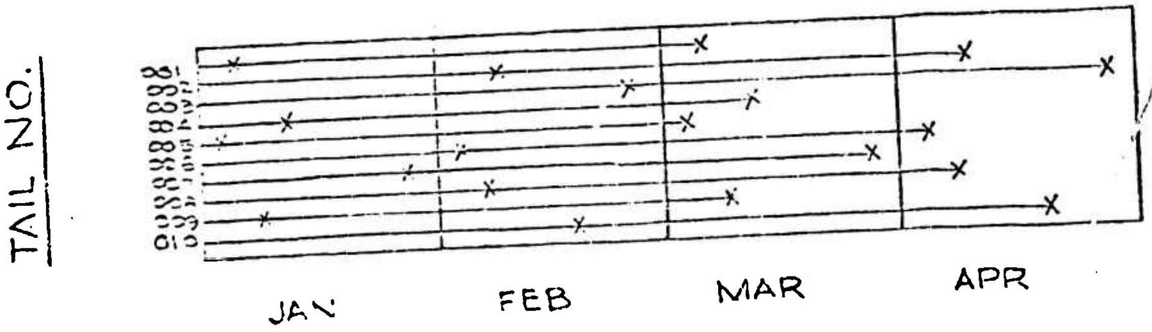


FIGURE 2

COMPONENT FAILURES - EXAMPLE
FAILURE DENOTED BY "X"
SYSTEM IN STEADY STATE

many different lengths; but, the average length was, as computed, 200 hours. The probability that they will fail in the next moment is not a function of operating age. Because of this property the length of service varies randomly from very short to very long. The service times for the camera units appear very consistent, varying only over a small range of times around 200 hours.

Now that we have accumulated failure data in our hypothetical shop, suppose we go one step further and attempt to provision for an entirely new camera system. Assume the squadron acquires ten aircraft from the production line equipped with the new camera system and expects to fly each aircraft an average of 100 hours a month beginning 1 January. We use our hypothetical (steady state) failure rates and stock parts/equipment based upon the prediction of five power supply and five camera failures each month. Figure 3 shows the hypothetical results experienced on the basis of our earlier assumptions.

The power components appear in steady state immediately, however, the camera components experience no failures for the first two months. Then they begin a two month failure cycle which continues until the small variations in service time smooth the distribution to the steady state condition shown in Figure 1. We have over stocked parts/equipment for the camera unit during the months of January and February since no failures were experienced. However, during the month of March, we discover that we do not have enough parts/

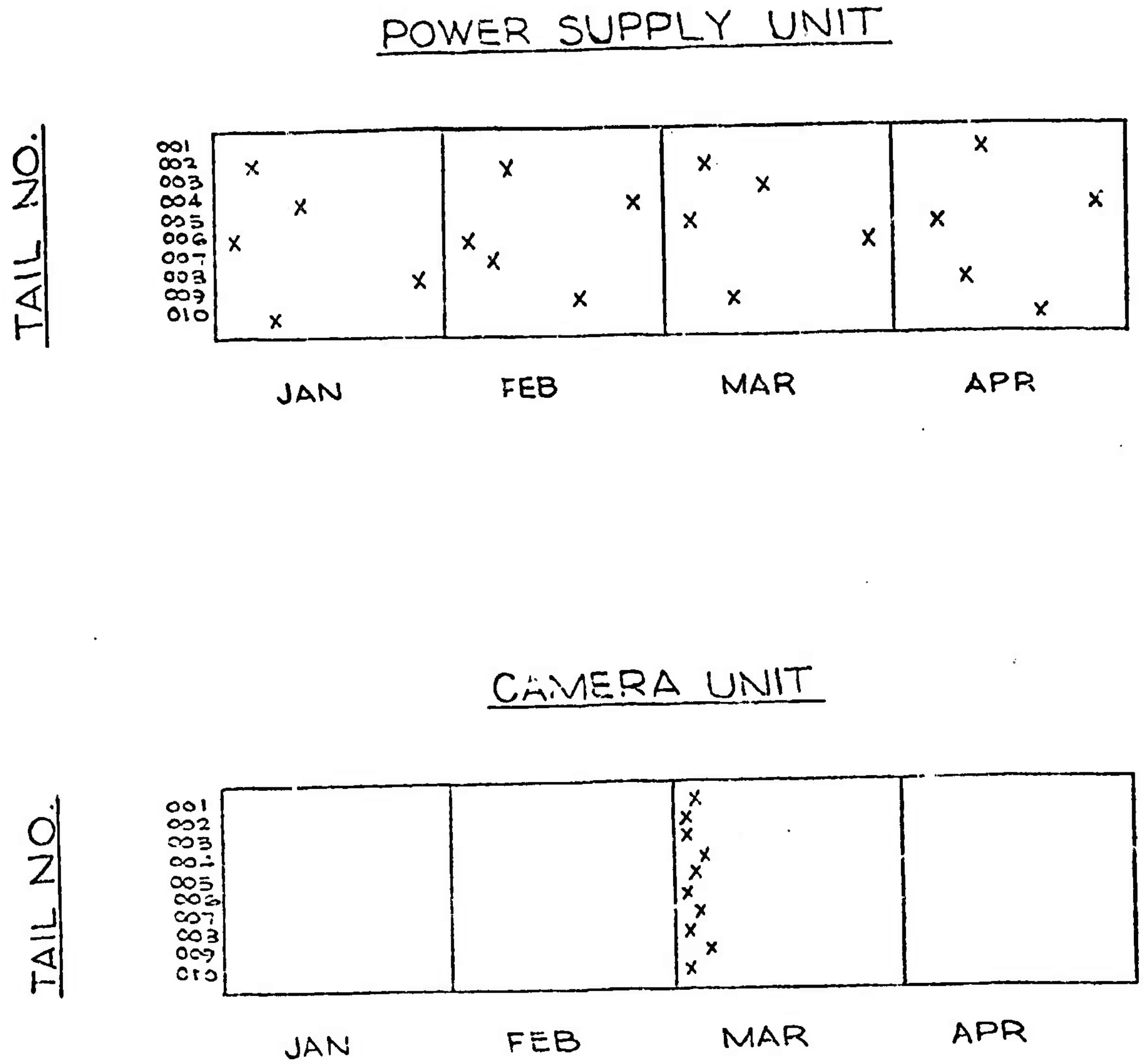


FIGURE 3

COMPONENT FAILURES - EXAMPLE
FAILURE DENOTED BY "X"
SYSTEM IN TRANSITION

equipment on hand to satisfy all failures and the mission is affected due to lack of parts. The stockage requirements for repair support of the camera system may be increased to insure sufficient spares are available to meet future requirements. When the system eventually reaches steady state, we find that we now have excess spares on hand to meet the system repair requirements; i.e., five failures per month.

Although the new power supply and camera units were similar to the old system, both had not achieved steady state. Therefore, assuming that our previous estimates of failure rates and service times were applicable to the new systems caused us to acquire erroneous stocks of parts/equipment for the camera unit.

Totally New or Dissimilar Components

When the Air Force procures a new weapons system, a large number of new items of supply are also received. Since the Air Force may not have prior supply experience for these new units, it must depend upon the contract engineering estimates for component failure rates and service times. A problem arises in that Air Force estimation methods may not be the same as those used by the contract engineers. To understand this problem, we need to define a few key terms.

If we are discussing a population of components that operate together as a system in steady state, and if each component is repaired or replaced upon failure, we use the term Mean Time Between

Failures (MTBF). (18:197) "MTBF may be determined by testing several equipments through several failure incidents and averaging the total operating time for the number of failures." (17:15-5) If we discuss the failure or service time of one component, we use its Mean Time To Failure (MTTF). We could life-test several of these units until they failed, and then average their service lives (time to failure) to find the unit's MTTF.

> It is often convenient to characterize a failure model or set of failure data by a single parameter. One generally uses the mean time to failure or the mean time between failures for this purpose.... Unfortunately these two quantities are sometimes wrongly thought of as equivalent, probably because for certain simple constant-hazard cases they are equal. In a single-parameter distribution, specification of the MTTF fixes the parameter. (18:197)

In Figures 1, 2, and 3 the power supply was of this simple type where the MTBF calculated by our steady state method was always equal to the individual unit's MTTF. This was not true of the camera unit.

When procuring a new system, we tell the contractor exactly what we want. We want a component that will have a given MTBF (calculated by our methods) regardless of when and how we use it.

> In many instances military contracts for complex systems express quantitative reliability requirements in terms of mean time between failures (MTBF). In fact, more than one MTBF may be specified. No matter how many MTBF's are specified, there should be a definition of what MTBF means. One of the pitfalls in current contracts is that a MTBF is required but what it stands for is not explained....
>
> 1. Is this MTBF the design, minimum acceptable hardware, or the operational requirement?
> 2. When is this MTBF to be attained?
> 3. What definition of failure is to be used in calculating MTBF?

> ....
> The second question above, regarding "when", is of paramount interest. The... MTBF may be attainable at some future date, but not in the first operational system. Usually, the best approach is to make this MTBF the average of systems, for instance, the first squadron or wing of delivered systems for a military contract.... (12:14-12)

The Air Force contracts for a given MTBF and the contractor designs for a steady state failure rate. But how long will it take before our actual failure rate reaches the steady state failure rate? The answer to this may be different for each component population. However, present Air Force initial provisioning procedures consider only the steady state MTBF. Air Force planners must also consider the transition period prior to steady state. To do this we must know something about the behavior of failure rates during the transition period.

Burn-in Phase

There is another problem associated with the present "method." Many complex electronic systems go through an infant mortality or "burn-in" phase. During this phase, many systems exhibit a decreasing failure rate with respect to time. Their initial failure rate is high, but after the first few repairs the system failure rate settles down to a constant low level. (17:15-14) This condition is often encountered in the real world, however we have not analyzed this type of phenomenon in this study. A long run average or an engineer's steady state estimate may suppress indications of a burn-in phase. Unless we are forewarned, the sudden appearance of a high failure rate may cause undue alarm during the first few operational months.

Summary

In this chapter we have attempted to show that the procedures used by the Air Force to estimate future logistics requirements have necessary underlying assumptions. The most basic of these is the assumption of a steady state condition. If a component has a constant failure rate, its component population will immediately appear in steady state. However, if a component has an increasing or a decreasing failure rate with respect to time, or if we receive a steady state engineering estimate, the component's population may pass through a transition period prior to reaching the steady state failure rate. The objective of this research is to investigate failure rates to be expected during this transition period.

CHAPTER III

MATHEMATICS

General

In published procedures for determining quantitative requirements for initial spare parts, the Air Force assumes that the occurrence of component failures can be approximated by a Poisson probability distribution. (5) While this assumption is not explicitly stated, it is implicit in Air Force tables which suggest that every spare part has a unique mean time between failure (MTBF) and a constant failure rate. A constant failure rate implies an exponential density function (18:185) and, therefore, that failures are generated by a Poisson process. (8:111)

In the following discussion we will show that there are two procedures which lead to a Poisson process. The first procedure requires each component to have a constant hazard function, but does <u>not</u> require steady state. The second procedure requires a steady state, but <u>not</u> a constant hazard function.

Constant Hazard Model

If we are discussing one component and we can assume its time to failure (TTF) follows an exponential distribution, then,

(1) $f(t) = \lambda e^{-\lambda t}, \ 0 < t \leq \infty$ (exponential density function)

(2) $F(t) = \int_0^t f(x)dx = 1 - e^{-\lambda t}, \ 0 < t \leq \infty$ (cumulative density function)

where λ is a constant and is defined as the hazard rate. (18:48) and t is operating time. We can also find the mean time to failure, MTTF, by

(3) $\text{MTTF} = E[\text{TTF}] = \int_0^\infty t \, f(t)dt = \frac{1}{\lambda}$ (7:165)

However, if we are discussing a population of identical independent components, we can discuss the fractional portion of the original population which fail prior to a time t by

(4) $P(x \leq t) = \int_0^t f(x)dx = \int_0^t e^{-\lambda x}dx = F(t), \ 0 < t \leq \infty.$

Which is identical to the cumulative function above, equation (3). Thus, 1 - F(t) represents the fraction of the population surviving to time t (7:164), defined as the reliability function as follows,

(5) $R(t) = 1 - F(t) = e^{-\lambda t}$ (Reliability function)

We can define the population hazard rate or the distribution hazard function as the rate at which the survivors fail. (7:165) Then,

(6) $$Z(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda, \quad 0 < t \leq \infty \text{(hazard function)}$$

which is constant for all values of t. For this distribution it has been proven that the reciprocal of the hazard function is the mean time between failures, (MTBF) (14:4-43), or,

(7) $$\text{MTBF} = \frac{1}{Z(t)} = \frac{1}{\lambda}$$

Thus, for the constant hazard model a component's hazard rate equals the population hazard rate and the MTTF = MTBF, and the MTBF is independent of the value of t.

The Renewal Model

If we are discussing the failure of one component, but, this time, we replace the failure with an identical new component (or an old component which has been repaired to new condition), we have a renewal process. (3) Furthermore, if we assume time to be component operating time and that we can replace a failed component instantaneously, the system's operating time becomes:

(8) $$T_{sys} = t_1 + t_2 + t_3 \ldots + t_n$$

Where t_n = the operating time for the n^{th} renewal. The density function

$$(9) \qquad f(T_{sys}) = f(t_1, t_2, t_3, \ldots, t_n)$$

may be found by the use of Laplace transforms, which simplify the function to

$$(10) \qquad f^*_{Tsys}(S) = f^*_1(S)\, f^*_2(S)\, f^*_3(S) \ldots f^*_n(S)$$

where $f^*_n(S)$ is the Laplace transform of $f_n(t)$ such that

$$(11) \qquad f^*_n(S) = \int_0^\infty e^{-st_n} f_n(t_n)dt$$

or, finally, where all units are identical:[4]

$$(12) \qquad f^*_{Tsys}(S) = [f^*(S)]^n \qquad (18{:}351\text{-}352)$$

Up to this point we have been discussing the renewal (or repair) of <u>one</u> component. A population of components, all operating, failing, and being instantaneously renewed, is referred to as a superposition of renewal processes. In order to analyze a pool of individual component processes, it is helpful to visualize each component's renewal process as an <u>independent distribution of failure times</u>, and then refer to the following basic reliability references.

---

[4] In the special case where the components are constant hazard we obtain:

$$f_{Tsys}(t) = \frac{\lambda(\lambda t)^{n-1}}{(n-1)!} e^{-\lambda t} \quad \text{(the special Erlangian function)}$$

Which is called the Poisson process.

> One of the first areas of reliability to be approached with any mathematical sophistication was the area of machine maintenance (Khintchine, 1932, and C. Palm, 1947). The techniques used to solve these problems grew out of the successful experiences of A. K. Erlang, C. Palm and others in solving telephone trunking problems. (2:1)

As Erlang and Palm demonstrated, the Poisson distribution was a very good approximation to the actual behavior of their "simple stream" of phone calls. (11:12) However, Palm, and later, Khintchine realized that groups of telephone calls did not always satisfy the strict assumptions of the Poisson distribution. This was especially true of the requirement that the distribution be memoryless or in their words, "without after-effects." (11:12) In the introduction to his mathematical proof of Palm's theorem, Khintchine wrote:

> Thus, although an investigator, when applying the theory to experimental data, usually has the task of explaining the causes of deviation of the actual processes from the course theoretically predicted for them, the situation is in fact the wrong way round--experimental data agree with the deductions from the constructed theory, as a rule, better than could have been expected on the principal considerations, and it is precisely this 'much better' agreement which requires explanation. Palm has made a noteworthy attempt to explain facts of this kind by the assumption that a given process represents a simple sum (superposition) of a large number of mutually independent processes of small intensity, in which each of these processes is stationary and orderly <u>and can be either with or without after-effects.</u> [author's underlining] In this it is shown that under broad assumptions the summary stream must be very near to a simple one. Such a statement of the problem is, on the face of it, very near to the real situation. So, if a large number of subscribers are attached to the given exchange, a general stream of calls is composed of streams (of a comparatively small intensity) arising from different subscribers, in which the component streams can in the first instance be regarded as stationary, orderly, and mutually independent. (11:49)

Khintchine then presents a chapter-long proof of Palm's proposal.

While this particular proof is too lengthy to reproduce in this report, two other authors have interpreted the meaning for us. In his text on renewal theory, Cox discusses Khintchine's proof in relation to the pooled output from superpositioned renewal processes.

> Khintchine...has proved that in the limit the numbers of renewals in non-overlapping intervals follow independent Poisson distributions, thus showing that in the limit the pooled output is a Poisson process. His proof does not require the component process to have identical distributions of failure time. (3:78)

And in his text on reliability, Shooman writes:

> Since we are now dealing with the superposition of sums of random variables, the arguments for using the central-limit theorem and obtaining limiting forms are even stronger. Specifically one can state that for K superposed (pooled) processes:
>
> 1. If each process is Poisson with parameter λ, the pooled process is Poisson with parameter $K\lambda$.
> 2. For arbitrary processes, if a large number is pooled, the output approaches a Poisson process.
> 3. As K becomes large, the time between renewals approaches an exponential distribution with parameter $K\lambda$... (18:356-357)

If each component of a population operates, fails, and is renewed; we have a population of independent renewal processes. If such a population were in the Air Force inventory, the Air Force would pool the outputs from these processes into a maintenance report. Khintchine's and Shooman's proofs show that this procedure would, in the limit, produce a Poisson process.

Summary

We have attempted to demonstrate in this chapter that there are two methods which produce a Poisson process. First, if a component has a constant failure rate, it can be described by the constant hazard model. This model is independent of time and number in the population. Second, if a component has an increasing or decreasing failure rate with time, the constant hazard model does not apply. However, the Poisson process still applies, at steady state, if the component is part of a pooled renewal process.

CHAPTER IV

MATHEMATICAL TOOLS

The Weibull Distribution

The Poisson and exponential distributions enjoyed wide acceptance in the 1950's due to their good results and appealing simplicity.

> A comprehensive analysis of failure distributions was done [in the early 1950's] at [the] RAND [Corporation] by D. J. Davis, who found that the exponential distribution. . . accurately described the failure characteristics of a wide variety of devices. As examples, he cited. . . commercial aircraft radio tubes, radar set components, tubes, resistors, . . . and combinations of various components. (9:209)

Many other investigators added items to the growing list. Yet, there were intuitive and theoretical arguments against the universal use of these distributions.

> The exponential law is reasonably appropriate where chance alone dictates failure occurrence. If it is known, for example, that failure is consistently due to deterioration, wear-out, degradation, fatigue, or any repetitive mechanism, almost certain nonexponentiality is implied. . . . To be truly random, failure cannot be due to design deficiencies or manufacturing errors. Being assignable as to cause, failure distributions associated with early life of a product usually are not exponential in that they are not random. (13:394)

We were cautioned--

> It is to be emphasized that considerable care is required in the selection of the appropriate underlying distribution for reliability testing. The validity of the test results depends to a large degree upon how well the selected probability distribution represents the actual distribution of the time to failure upon which observations are being made.... This point is emphasized because the widespread (perhaps indiscriminate) use of the exponential distribution as a model of failure patterns may lead one to believe that failure times in general may be adequately represented by such a distribution.... The use of other distributions introduced complications into the testing procedures; however, if one is to draw reasonable conclusions from the tests, he must be willing to face these other problems. (4:3-6)

Professor Weibull was willing to face the problems. He "... suggested it [the Weibull distribution] as the distribution for a variety of problems.... The agreement he found between his observations and those predicted with the fitted Weibull was very impressive...." (1:1)

The Weibull Mathematical functions are:

Weibull density function:

$$(13) \qquad f(t) = \frac{K}{\theta^k}(t-c)^{k-1}\,\mathrm{EXP}\left[-\left(\frac{t-c}{\theta}\right)^k\right], \; K>0, \theta>0, 0 < t \leq \infty.$$

Weibull cumulative function:

$$(14) \qquad F(t) = \int_0^t f(t)dt = 1 - \mathrm{EXP}\left[-\left(\frac{t-c}{\theta}\right)^k\right], \; K>0, \theta>0, 0<t \leq \infty.$$

Weibull reliability function:

$$(15) \qquad R(t) = \mathrm{EXP}\left[-\left(\frac{t-c}{\theta}\right)^k\right], \; K>0, \theta>0, 0<t \leq \infty.$$

Where C is the location parameter (for purposes of this research

$C = 0$ and time begins at the origin); K is the shape parameter; and, θ is the scale parameter.

> Since this distribution includes the. . . exponential as a special case, the Weibull is sometimes thought of as a generalization of the exponential distribution. Its use would then be considered by anyone who has been using the exponential distribution but felt a more flexible model was needed. Indeed, the popularity of the exponential distribution as the model for many experimenters in the fifties was challenged by the popularity of the Weibull in the sixties. (1:1ff)

The Weibull distribution, with its generality, has indeed given the model builder a powerful tool. By the appropriate choice of the parameter, K, we can change the distribution from the decreasing hazard model--a model sometimes used to show the "infant mortality" or break-in period, to the constant hazard exponential model, to the increasing hazard model--models characterized by the Rayleigh and normal distribution where a unit wears-out with time. By proper choice of the scaling factor θ, we can adjust the amplitude of the distribution much like we do when choosing the appropriate λ for the exponential. Therefore, by adjusting K and θ we may model a great number of hazard distributions. (18:190)

To properly define the failure rate characteristics of a system of equipment/components, it is necessary to describe the instantaneous failure rate function associated with the underlying failure generating distribution. This instantaneous failure rate function has been generally described by many authors as the hazard rate function, $Z(x)$, as follows:

(16) $Z(x) = \frac{f(x)}{R(x)}$ (10:2-5)

Where f(x) is the failure density function and R(x) is the reliability function.

The failure density function has been defined as:

(17) $f(x) = Z(x)\,\mathrm{EXP}\left[-\int_0^t Z(x)dx\right]$ (10:2-5)

Thus given an instantaneous failure rate function, we can easily obtain the failure density function by simple integration based upon the exponential law of growth or decay.

Many failure distributions can therefore be determined by integration of their characteristic instantaneous failure rate function. For example:

If: $Z(t) = \frac{1}{\theta}, \theta > 0$

Then: $f(t) = \frac{1}{\theta}\,\mathrm{EXP}\left[-\int_0^t \frac{1}{\theta}\,dx\right]$

$= \frac{1}{\theta}\,\mathrm{EXP}\left[-\frac{t}{\theta}\right]$

which is the well known exponential function.

Other instantaneous failure rate functions may be integrated to obtain the required failure density functions in the same manner.

The instantaneous failure rate function used in this simulation is as follows:

(18) $$Z(t) = \frac{K}{\theta^k} t^{k-1}, \theta > 0, K > 0, 0 < t \leq \infty.$$

Then: $$f(t) = Z(t) \text{ EXP} \left[- \int_0^t Z(t)dt \right]$$

(19) $$f(t) = \frac{K}{\theta^k} t^{k-1} \text{ EXP} \left[- \left(\frac{t}{\theta} \right)^k \right], \theta > 0, K > 0, 0 < t \leq \infty.$$

This is one form of the Weibull function and it is interesting to note that the exponential distribution is a special case of the Weibull distribution when K = 1. Other authors have indicated that when K is approximately 3, the Weibull approximates the shape of the Normal distribution. (19:21) Other failure distribution shapes can be approximated by the Weibull model. Decreasing failure rate models have a $0 < K < 1$ and increasing failure rate models have a $K > 1$.

The parameter θ of the Weibull model determines the variance or dispersion of the model. The variance of the Weibull model is given as:

(20) $$\text{Variance Weibull} = \theta^2 \left\{ \Gamma(1 + 2/k) - (\Gamma(1 + 1/k))^2 \right\}$$

Thus by changing the θ value we can set the variance of the failure density function and in conjunction with the K value approximate many failure distributions.

Mathematics of the Model

Using equations 16, 17, and 18 of the previous section, we can derive the following mathematical tools used in this study.

Hazard function:

(21) $$Z(t) = \frac{K}{\theta^k} t^{k-1}, \; K > 0, \theta > 0, \; 0 < t \leq \infty.$$

Failure density function:

(22) $$f(t) = Z(t) \; \text{EXP} \left[-\int_0^t Z(t) dt \right]$$

$$= \frac{K}{\theta^k} t^{k-1} \; \text{EXP} \left[-\left(\frac{t}{\theta} \right)^k \right], \; K > 0, \theta > 0, \; 0 < t \leq \infty.$$

Cumulative density function:

(23) $$F(t) = \int_0^t f(t) dt, \; 0 < t \leq \infty.$$

$$= 1 - \text{EXP} \left[-(t/\theta)^k \right], \; K > 0, \theta > 0, \; 0 < t \leq \infty.$$

Reliability function:

(24) $$R(t) = 1 - F(t)$$

$$= \text{EXP} \left[-(t/\theta)^k \right], \; K > 0, \theta > 0, \; 0 < t \leq \infty.$$

where K is the shape parameter and θ is the scale parameter. (15:94) From equation 3 and 22 we can then determine the expected value of the density function, MTTF, as follows:

$$\text{MTTF} = \int_0^\infty t\, f(t)dt$$

$$= \int_0^\infty t \left(\frac{K}{\theta^k}\, t^{k-1}\, \text{EXP}\left[-(t/\theta)^k \right] \right) dt$$

$$= \theta\Gamma(1 + 1/k),\ \theta > 0,\ K > 0$$

As shown earlier, this relationship yields the steady state failure rate for a pooled output of renewal processes, where each component's density function is described by a Weibull density function with parameters K and θ, and if the number of processes is large. Under such conditions, we can equate the MTTF of the individual components to the MTBF of the component population as follows:

$$\text{(25)} \qquad \text{MTBF} = \theta\Gamma(1 + 1/k)$$

We used equation 25 to compute our simulation model scale parameter, θ. When the shape parameters, $0.5 \leq K \leq 5.0$, were then chosen, we demonstrated that an appropriate choice of the scale parameter, θ, caused the pooled output of the renewal processes to approach the MTBF predicted by the constant hazard function of the exponential, i.e.,

$$\text{MTBF}_{\text{exp}} = \frac{1}{\text{Constant Failure Rate}}\ .$$

And therefore, at steady state, the MTTF for any individual component would equal the MTBF for the component population--a condition which exists only in a Poisson process. (18:197)

Random Numbers

To generate random numbers to compute individual times to failure for components, we used the inverse transform method as follows: (20:511)

$$v = 1 - \mathrm{EXP}\left[-(t/\theta)^k\right]$$

$$\mathrm{EXP}\left[-(t/\theta)^k\right] = 1 - v = u$$

$$-(t/\theta)^k = \ln u$$

$$t^k = \theta^k(-\ln u)$$

$$t = \theta\,(-\ln u)^{1/k}$$

where v is a uniformly distributed pseudo-random number, $0 \leq v \leq 1$, and thus, $(1 - v) = u$ is also a uniformly distributed pseudo-random number, $0 \leq u \leq 1$.

Therefore, by this inverse transform procedure, we can generate a stream of values for t which follow the Weibull distribution.

CHAPTER V

PROCEDURES

The Computer Model

We developed a computer model for this investigation to simulate the renewal processes underlying a portion of the Air Force maintenance system.

The time frame of our model covers approximately four years. We selected a 30 day month as our reporting interval so that our 4 year period contained 48 monthly reporting points. We assumed a 4 hour daily utilization rate per component and, thus, each component in the inventory operated 120 hours each month. The required inputs for each simulation were the total fleet size, the introduction rate, and the desired shape parameter.

The computer model introduced a percentage of the total fleet into our inventory at the beginning of each month. As each unit entered, a service time was selected from a random Weibull distribution and then added to the present computer time to form the unit's future failure time. The model then stored the unit with its failure time in a future events file. When the computer clock reached a scheduled failure time, the model removed the associated unit from the events file; recorded the failure; selected a new future failure time; and refiled the unit and time in the events file. Once started, this cycle continued independently for each unit in the inventory.

At the end of each computer month, the model stored the total number of failures which occurred in a memory array. We repeated each 4 year simulation 40 times for each set of input parameters. After the 40th run, the model calculated an average number of failures for each monthly period, and then divided this value by each month's operating time. The results were 48 average monthly failure rates. For a more detailed description of the simulation model see Appendix A.

Scope

General. --The research for this paper was limited to a literature search and digital computer simulation techniques. We have not attempted to evaluate our simulated data against actual failure data. However, we have attempted to select our model parameters so that they would present a reasonable approximation to actual conditions.

Fleet Size. --The total size of a fleet of aircraft may depend on many variables, e.g., planned mission, projected costs, actual cost, adaptability to alternate missions, world conditions, etc. Therefore, we set a range of fleet sizes, 20 to 1000 aircraft, and tested increments of this range in our preliminary simulations. Our preliminary results indicated that the total number in the fleet had little effect on our prime performance variable, failure rate, as long as a constant percentage of that total were introduced each month. For this reason we chose a fleet size, 100 aircraft, that was well within the feasible range, and one that would simplify later extrapolation.

Introduction Rate. --As with fleet size, the introduction rate of a fleet of aircraft is subject to political and economic pressures and, therefore, highly variable. In addition, this rate is limited by the physical problems of "tooling up" for production. Therefore, we selected as an average figure the monthly introduction rate at 5%. We used percentage values, as stated before, to simplify extrapolation to actual introduction phases.

Average Operating Period. --The number of hours an aircraft flies each day again depends on the type of aircraft and the mission. However, when we are discussing components on an aircraft, the problem becomes more complex. A C-5A may use a system for the full mission, a B-52 may use the same type system only on take off and landing, and an F-106 may use the same system for special missions only. In our model, all component failures are a function of operating time; thus, all results are directly related to the utilization rate. For this reason we chose the "ball park" value of four hours per unit per day for our utilization rate.

Mean Time Between Failures. --We selected the values for this variable to cover a feasible range for Air Force repair or replacement parts. We conjectured that a major system with less than 50 hours MTBF would be unacceptable for reliability reasons, and that a system with failure intervals greater than 2000 hours (nearly 17 months in our model) would cause few logistics problems. We then investigated the values of 50, 100, 500, 1000, and 2000 hours to cover our assumed range.

Shape Parameters. --We investigated the effects of this parameter through the values of 0.5, 1.0, 1.5, 2.0, 3.0, 4.0, and 5.0; and we conjecture that the properties of our performance variable, failure rate, are continuous within this range. These values take us from a rapidly decreasing failure rate (infant mortality phase) through a constant failure rate (random failures) to a rapidly increasing failure rate (wear-out phase).

Scale Parameters. --The scale parameter, θ, was determined as previously indicated from equation 25. Equation 25 allowed us to transform the MTBF desired or indicated to an appropriate scale parameter of our simulation model. Thus, we were able to track the experienced failure rates through the transition period until the component population steady state failure rate was achieved.

CHAPTER VI

SIMULATION RESULTS

General

The simulation results have been divided into decreasing hazard rate, constant hazard rate and increasing hazard rate categories to more easily discuss and analyze the simulation output. The discussion and analysis will be based upon frequency plots of the output data. The actual tabulated output data is included in Appendix B for readers wishing to perform a more extensive analysis than presented in this study.

As indicated earlier, our objective is to investigate the distribution of failures per hour during the transition period from initial acquisition of weapons systems to the attainment of steady state; and we want to compare such failure rates to the constant failure rate predictions presently in use by the Air Force. (5) Thus, our main interest is the magnitude of the difference between the simulation results and the Air Force prediction methods.

Definitions

The analysis has been performed in terms of the average absolute deviation between the present prediction methods and the simulation results prior to the achievement of the system steady state condition. For purposes of analysis, system steady state is "broadly" defined as relatively minor absolute deviations of the simulation results from the current predicted constant failure rate.

The following definitions are necessary to the analysis.

Constant Hazard Rate (CHR)--the constant failure rate predicted by present Air Force methods

$$\mathrm{CHR} = \frac{1}{\mathrm{MTBF}} .$$

Constant Failure Rate--the condition of the Weibull failure generating function where the shape parameter, K, is equal to 1.

Decreasing Failure Rate--the condition of the Weibull failure generating function where the shape parameter, K, is $0 < K < 1$.

Increasing Failure Rate--the condition of the Weibull failure generating function where the shape parameter, K, is greater than 1.

Mean Time Between Failure (MTBF)--the time between failures predicted by the present Air Force method

$$\mathrm{MTBF} = \frac{1}{\text{Constant Failure Rate}} .$$

Period Failure Rate (PFR)--the monthly simulation produced failure rate.

Per Cent Average Absolute Deviation (AD)--the per cent of average absolute deviation between the simulation results and the CHR computed as follows:

$$AD = \frac{\sum_{i=1}^{48} |PFR_i - CHR|}{48\ (CHR)} \times 100\% .$$

Per Cent Maximum Absolute Deviation (MD)--the per cent of maximum absolute deviation between the simulation results and the CHR computed as follows:

$$MD = \frac{MAX|PFR_i - CHR|}{CHR} \times 100\%, \quad i = 1, 2, 3, \ldots, 48.$$

Per Cent In Range (IR)--the number of individual PFR's within the range CHR ± AD computed as follows:

$$IR = \frac{\sum_{i=1}^{48} Oi}{48} \times 100\%$$

where:

$$Oi = \begin{cases} 1, & \text{when } (CHR - AD) \leq PFR_i \leq (CHR + AD) \\ 0, & \text{otherwise} \end{cases}$$

The Decreasing Failure Rate Case

The Decreasing Failure Rate Case is defined as the condition where the hazard function of the components is inversely related to time, i.e., the function decreases over time. In this case, the simulation program assigned to each component of the population such a decreasing failure rate, and we were interested in observing what effect such a failure rate would have on the total population failure rate.

Figures 6, 7, and 8 of Appendix B are plots of the system failure rates produced by the simulation program. (The simulation output may be found in Appendix B of this study.) The plots of the system failure rates indicate relatively large failure rates during the initial months of introduction. However, these large failure rates rapidly reduced in magnitude in a relatively short period of time. The rate of change of the system failure rate is negative and decreases at a decreasing rate eventually approaching the CHR asymptotically from above.

Each plot indicates that deviations from the CHR may be 10% or higher for the first 24-28 months of operation. Such deviations appear to be directly related to the size of the MTBF as can be seen from the plots. Figure 6 indicates that with a MTBF of 500 hours, the system of components enters steady state (our broad definition) in approximately 36 months. However Figures7 and 8 with MTBF's of 1000 hours and 2000 hours are not to be considered in steady state throughout the entire simulation period of 48 months.

The simulation results indicate that components of a given population characterized by a decreasing hazard function affect the population failure rate distribution. Much higher population rates than predicted by the present Air Force methods should be experienced during the initial months of the transition period to steady state. It appears that the population failure rate will eventually achieve steady state conditions, however, the transition period may be quite lengthy, and for large MTBF's may not actually be attainable. Table 1 indicates the simulated maximum % deviations (MD) and the average % deviations (AD) from such a CHR prediction method.

TABLE 1

THE DECREASING FAILURE RATE CASE

MTBF	Shape Parameter	MD	AD	% OBS. (1/MTBF) (1±AD)
500	.5	272.9%	36.57%	68.75%
1000	.5	412.5%	59.89%	60.41%
2000	.5	591.6%	95.58%	62.50%

Under such conditions, the Air Force planners may decide to approximate the expected failure rate by averaging several period failure rates. However, since the population failure rate distribution is continuously decreasing, such a procedure would result in the acquisition of replacement/spare parts in quantities larger than actually necessary. If the MTBF supplied by the contractor is used as the basis for logistical support planning, the simulation results indicate that fewer items would be planned for than would actually be required.

The Constant Failure Rate Case

The Constant Failure Rate Case is defined as the condition where the hazard rate function of the components is a constant over time. The simulation program assigned to each component of the population such a failure rate. Again, the effect of such a component characteristic on the total population failure rate is our main interest.

Figures 9, 10, 11, 12, and 13 of Appendix B are plots of the system failure rates produced by the simulation program. All plots indicate that the system failure rates immediately tend to oscillate around the CHR predicted by the present Air Force method. A transition period does not appear to be present and according to our "broad" definition of steady state, we conjecture the system of components to be immediately in steady state. The oscillations around the predicted CHR tend to increase proportionally with increases in the MTBF used. Table 2 indicates the simulated maximum % deviations (MD) and the average % deviations (AD) from the CHR.

TABLE 2

THE CONSTANT FAILURE RATE CASE

MTBF	Shape Parameter	MD	AD	% OBS (1/MTBF) (1±AD)
50	1.0	8.33%	1.78%	56.25%
100	1.0	10.83%	2.06%	62.50%
500	1.0	10.95%	3.14%	64.58%
1000	1.0	17.40%	4.79%	58.33%
2000	1.0	20.80%	5.48%	60.41%

Under such conditions it appears that the present Air Force procedures for predicting expected failures for new weapons systems would produce "satisfactory" results since the system would immediately be in a steady state condition.

The Increasing Failure Rate Case

The Increasing Failure Rate Case is defined as the condition where the hazard function of the components is directly proportional to time, i. e., the function increases over time. As with the other cases, we are still interested in the effect such component failure rates have upon the population failure rate. We conjecture that component failures which possess this characteristic are more representative of the real world situation than either the decreasing or constant failure rate cases. Intuitively, it is more reasonable to expect larger probabilities of failure as items age than either constant or decreasing probabilities of failure with increasing age.

Figures 14 through 36 of Appendix B are plots of the simulation results for varying MTBF's and shape parameters for this case. (The simulation data may be found in Appendix B of this study.) The plots of the simulated failure rates have been categorized into groups on the basis of MTBF's and will be discussed in this order.

MTBF = 50.--For shape parameters, $2 \leq k \leq 5$, the plots indicate a minimum of 5% deviation between the predicted failure rate and the simulation results for the first few months during transition to steady state. The magnitude of these deviations appears to increase with increases in the shape parameter. Such a condition is to be expected with the Weibull distribution, since shape parameters less than, or greater than approximately 3 generate skewed distributions, right and left respectively, with a symetrical distribution occurring when the shape parameter equals 3. The length of the transition period appears to be in the 16-24 month range for MTBF's of 50, and the variation around the predicted CHR during steady state is small. The rate of change in the population failure rate distribution is positive and approaches the predicted CHR asymptotically from below. See Figures 14 through 17, Appendix B.

MTBF = 100.--Again the same general observations can be made of these plots. The rate of change of the failure rate is positive, increasing at a decreasing rate and approaching the predicted CHR asymptotically from below. The system failure rate appears to achieve steady state in about 20 months. The simulated results deviate from the predicted CHR by at least 10% during the first few months of the transition period. The maximum deviation of the simulated failure rate from the predicted CHR is approximately 31% during the first month. See Figures 18 through 21, Appendix B.

MTBF = 500. --The same general observations made above are again observed. However, the maximum deviation of the simulated failure rates from the predicted CHR has increased considerably. For shape parameters of 4 and 5 the maximum deviation is 100%. The system appears to achieve steady state in approximately 24-28 months. The rate of change in the failure rate also appears to have decreased with increasing scale parameters. See Figures 22 through 26, Appendix B.

MTBF = 1000. The same general observations made above also apply here. Steady state appears to be achieved in approximately 24-28 months, however, the relative deviations around the predicted CHR at steady state has increased. The rate of change of the failure rate has again decreased. See Figures 27 through 31, Appendix B.

MTBF = 2000. The same general observations are applicable to this group also. There is a marked difference in the rate of change of the system failure rate. Steady state appears to be reached in the 26-28 month period. The relative fluctuations in the failure rate have also increased markedly. During the first 12 months, the minimum deviation between the simulation results and the predicted CHR is at least 40%. See Figures 32 through 36, Appendix B.

In all of the plots, there appears to be a change to the shape of the failure distribution during the transition period resulting from increasing MTBF's. The relative fluctuation of simulated failure rates also appears to increase as the MTBF is increased. All the failure distributions approach the predicted CHR asymptotically from below. Tables 3 and 4 summarize the results of the simulation for the parameters and ranges tested. Table 3 indicates that for increases in MTBF for a given shape parameter, the MD and AD (as defined before)

TABLE 3

INCREASING FAILURE RATE CONDITION WITH CONSTANT SHAPE

MTBF	Shape Parameter	MD	AD	% OBS (1/MTBF) (1±AD)
50	2.0	12.71%	1.43%	77.08%
100	2.0	25.42%	2.73%	75.00%
500	2.0	77.10%	10.63%	70.80%
1000	2.0	85.40%	18.35%	60.41%
2000	2.0	100.00%	28.74%	60.41%
50	3.0	15.83%	1.54%	79.16%
100	3.0	31.67%	2.94%	79.16%
500	3.0	93.75%	13.96%	72.91%
1000	3.0	100.00%	21.71%	62.50%
2000	3.0	100.00%	34.46%	62.50%
50	4.0	16.66%	1.56%	75.00%
100	4.0	31.67%	2.94%	72.91%
500	4.0	100.00%	14.47%	70.83%
1000	4.0	100.00%	23.09%	64.58%
2000	4.0	100.00%	36.44%	62.50%
50	5.0	17.5%	1.58%	77.08%
100	5.0	30.00%	2.99%	72.91%
500	5.0	100.00%	14.51%	72.91%
1000	5.0	100.00%	24.06%	64.58%
2000	5.0	100.00%	37.70%	62.50%

have increased. Table 4 indicates that for increases in the shape parameter for a given MTBF that the MD and AD have increased. Thus for large MTBF's or large shape parameters, we can expect larger deviations from the predicted CHR.

TABLE 4

INCREASING FAILURE RATE CASE WITH CONSTANT MTBF

MTBF	Shape Parameter	MD	AD	% OBS (1/MTBF) (1±AD)
50	2.0	12.71%	1.43%	77.08%
50	3.0	15.83%	1.54%	79.16%
50	4.0	16.66%	1.56%	75.00%
50	5.0	17.50%	1.58%	77.08%
100	2.0	25.42%	2.74%	75.00%
100	3.0	31.67%	2.95%	79.16%
100	4.0	31.67%	2.94%	72.91%
100	5.0	30.00%	2.99%	72.91%
500	2.0	77.10%	10.63%	70.80%
500	3.0	93.75%	13.96%	72.91%
500	4.0	100.00%	14.47%	70.83%
500	5.0	100.00%	14.51%	72.91%
1000	2.0	85.40%	18.35%	60.41%
1000	3.0	100.00%	21.71%	62.50%
1000	4.0	100.00%	23.09%	64.58%
1000	5.0	100.00%	24.06%	64.58%
2000	2.0	100.00%	28.74%	60.41%
2000	3.0	100.00%	34.46%	62.50%
2000	4.0	100.00%	36.44%	62.50%
2000	5.0	100.00%	37.70%	62.50%

Standardized Table for Steady State

The following is the development of a standard table, Table 5, and curve, Graph 1, which was used to estimate the required MTTF or scale parameter, θ, of the failure generating function for this simulation. Once the shape and scale parameters have been estimated for a given equipment/component failure distribution, then the MTTF can be estimated by using the table. For example

The time to failure P. D. F. is:

$$f(t) = \frac{K}{\theta^k} t^{k-1} e^{-\left(\frac{t}{\theta}\right)^k}, \quad K > 0, \theta > 0, 0 < t \leq \infty .$$

$$E(t) = \text{MTTF} = \int_0^\infty t \, f(t) \, dt$$

$$= \int_0^\infty t \left\{ \frac{K}{\theta^k} \quad t^{k-1} \quad e^{-\left(\frac{t}{\theta}\right)^k} \right\} dt$$

$$= \theta \cdot \Gamma \, (1 + 1/K)$$

$$\text{Mean Failures Per Hr} = \frac{1}{E(t)} = \frac{1}{\theta \cdot \Gamma(1+1/K)} = \frac{1}{\text{MTTF}}$$

From the above, given the MFPH and K, the scale parameter can be estimated from:

$$\theta = \frac{1}{\text{MFPH} \cdot \Gamma(1+1/K)} = \frac{\text{MTTF}}{\Gamma(1+1/K)}$$

Thus, by estimating the shape parameter, K, and determining one of the other two parameters θ or MFPH, we can derive an estimate for the remaining variable when the failure distribution is of the Weibull form:

$$f(t) = \frac{K}{\theta^k} \quad t^{k-1} \quad e^{-\left(\frac{t}{\theta}\right)^k}$$

The table is constructed with $\theta = 1$ and the K parameter is varied from 0 to 10 in increments of .25. All that is necessary to determine the MFPH or θ value is to obtain the tabulated mean value,

TABLE 5

STANDARDIZED TABLE TO DETERMINE THE STEADY STATE MFPH OF COMPONENTS WHICH HAVE A WEIBULL TIME TO FAILURE DISTRIBUTION

STANDARDIZED TABLE SHAPE	EXPECTED MEAN
0.250	0.04166667
0.500	0.50000000
0.750	0.83988503
1.000	1.00000000
1.250	1.07367142
1.500	1.10773224
1.750	1.12281597
2.000	1.12837909
2.250	1.12901184
2.500	1.12706082
2.750	1.12377663
3.000	1.11984669
3.250	1.11565629
3.500	1.11142324
3.750	1.10726975
4.000	1.10326244
4.250	1.09943514
4.500	1.09580223
4.750	1.09236656
5.000	1.08912425
5.250	1.08606765
5.500	1.08318715
5.750	1.08047223
6.000	1.07791221
6.250	1.07549660
6.500	1.07321534
6.750	1.07105894
7.000	1.06901857
7.250	1.06708595
7.500	1.06525347
7.750	1.06351410
8.000	1.06186141
8.250	1.06028937
8.500	1.05879261
8.750	1.05736604
9.000	1.05600511
9.250	1.05470550
9.500	1.05346334

TABLE 5 (Continued)

SHAPE	EXPECTED MEAN
9.750	1.05227505
10.000	1.05113725

given the estimated K parameter and then divide the value by the predetermined scale parameter, or MFPH respectively.

Examples:

(1) Given $K = 3.5$

and, $\theta = 950$.

Obtaining from the table, MEAN = 1.11142324

the $\text{MFPH} = \frac{\text{MEAN}}{950} = .0011699$

(2) Given $K = 2$

and, $\text{MTTF} = 1500$

or, $\text{MFPH} = (1500)^{-1}$

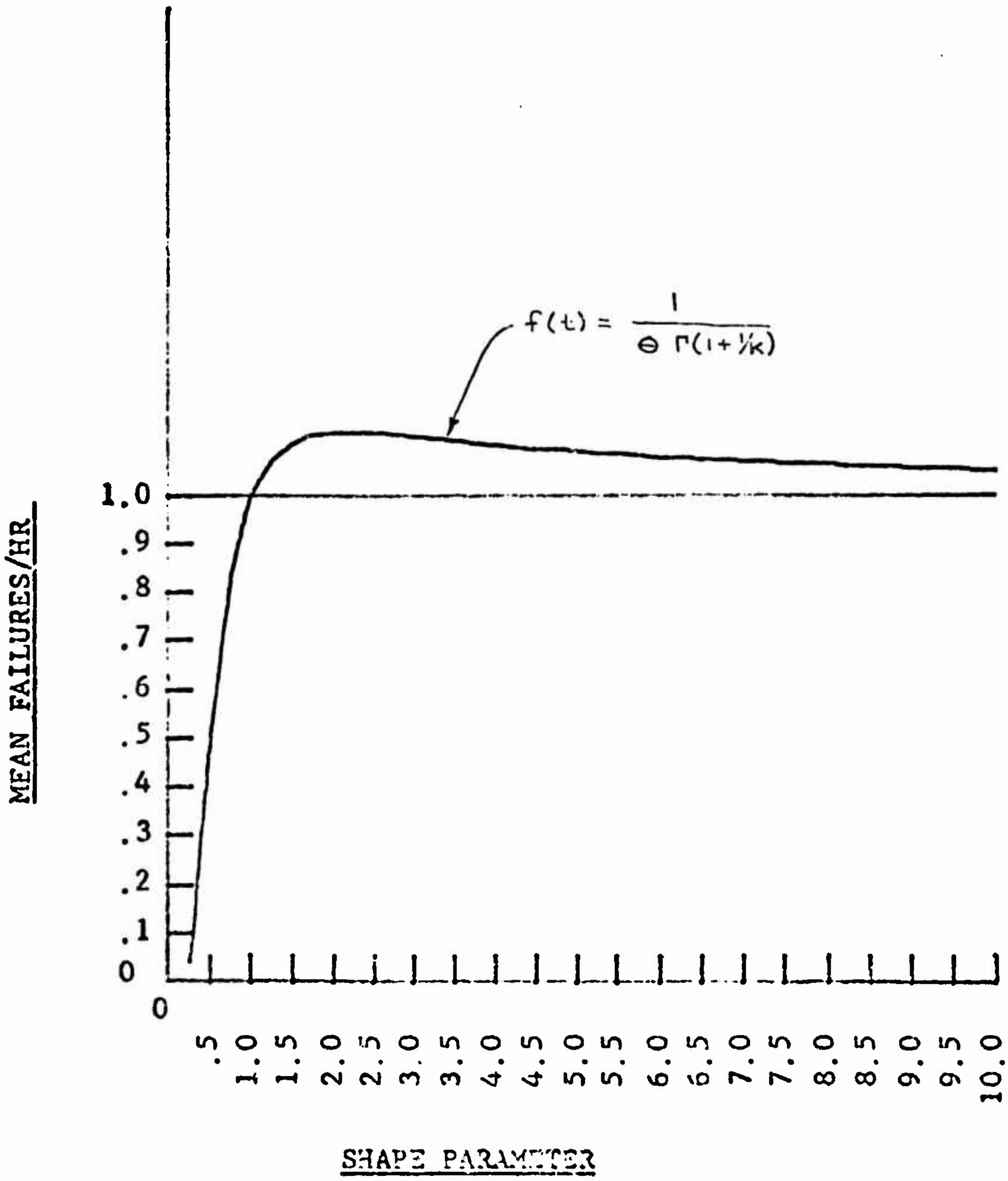
Obtaining from the table, MEAN = 1.12837909

$$\theta = \frac{\text{MEAN}}{(1500)^{-1}} = 1692.5685$$

In the case of the constant failure rate, the shape parameter, k, for the Weibull cumulative density function is 1 and the scale parameter, θ, is directly proportional to the MTBF as follows:

$$\text{MTTF} = \int_0^\infty t\, f(t)\, dt$$

$$= \int_0^\infty t\, \frac{K}{\theta^k}\, t^{k-1}\, e^{-\left(\frac{t}{\theta}\right)^{k}} dt, \quad K > 0,\ \theta > 0$$



GRAPH 1

STANDARD CURVE FOR MFPH

$= \theta\Gamma\ (1 + 1/K)$

With K = 1, MTTF becomes:

$\text{MTTF} = \theta\Gamma\ (1 + 1/1)$

or

$\text{MTTF} = \theta\Gamma(2)$

But $\Gamma(2)$ is equal to 1, therefore

$\text{MTTF} = \theta \cdot 1 = \theta$

and,

$$\text{MFPH} = \frac{1}{\text{MTBF}} = \frac{1}{\theta}$$

Thus, when the underlying failure distribution follows the exponential distribution, we can equate the scale parameter to the reciprocal of the MFPH, or equate the scale parameter directly to the MTTF. Under these conditions, we can predict the steady state MFPH by computing the reciprocal of the MTTF or of θ.

Summary

Of the three cases investigated, only the constant failure rate case appeared to be in steady state immediately. The decreasing (DFR) and increasing (IFR) failure rate cases result in significant deviations from the failure rates predicted by the present Air Force methods which may lead to supply difficulties in supporting weapons systems during the transition period.

The transition period prior to the attainment of steady state conditions for the DFR and IFR exceeds 12 months for the MTBF's and parameters tested and may actually be longer than 24 months for large MTBF's. Systems in transition tend to approach steady state more rapidly for lower MTBF values, and the random relative fluctuations appear to be less erratic than those for larger MTBF values.

The plots of the system failure rates indicate that upon attainment of steady state, the expected future failure rates for the system can be approximated or predicted by the present Air Force methods quite satisfactorily. In the limit, the population failure distributions appear to exhibit the random oscillations around the steady state failure rate characteristic of the constant failure rate case.

CHAPTER VII

CONCLUSION

Our original hypothesis that the fleetwide distributions of failures for any replaceable equipment/component is at all times a Poisson process and that the assumption of a constant failure rate is always valid in the estimation of future expected failures is not supported by the simulation results and we reject the hypothesis.

The simulation results using the Weibull failure generating function as the underlying failure distribution for the population of equipments/components indicate that a deviation between the simulated failures and those failures predicted by a Poisson process is to be expected when aircraft components are introduced incrementally into the Air Force inventory. The transition period between the initial level of failures and the steady state level of failures is related to the MTTF of the introduced components and the characteristic shape parameter of the underlying failure distribution.

Since the shape parameter of the Weibull failure distribution determines the general "shape" of the distribution, we can approximate the exponential distribution and the normal distributions which are the basic statistical forms assumed to describe the failure characteristics of components. As the shape parameter is increased above the value 1, the Weibull tends to approach a normal distribution. The larger the K parameter becomes the smaller the deviation from the predicted failure rate becomes, until in the limit, i. e., $K \to \infty$, the distribution has no deviation and the failures, MFPH, can be approximated by $\frac{1}{\text{MTTF}}$. This can be seen from the following:

$$\text{MFPH} = \frac{1}{\text{MTTF}} = \frac{1}{\theta\Gamma(1 + 1/k)}$$

$$\lim_{K \to \infty} \Gamma(1 + 1/K) = \Gamma(1) = 1$$

therefore,

$$\lim_{K \to \infty} \text{MFPH} = \lim_{K \to \infty} \frac{1}{\theta\Gamma(1 + 1/K)} = \frac{1}{\theta} = \frac{1}{\text{MTTF}}$$

As can be seen from the Standardized Table (Table 5) and graph (Graph 1) the maximum average deviation from the predicted exponential MFPH is the range $2. \leq K \leq 2.25$. The maximum deviation occurs at approximately $K = 2.17$. Such a K value is quite possible in actual experience with equipment/component failure rates, since such a K value places the underlying failure distribution "between" the exponential, i. e., $K = 1$, and an approximation to the shape of the normal distribution, i. e., $K = 3$. Thus, when the Air Force acquires equipments/components incrementally and the actual underlying failure distribution is of the increasing failure rate type, we should not be surprised to observe increasing failure rates during the transition period to steady state.

When we have equipments/components which are characterized by decreasing failure rates, the Weibull failure generating function has a shape parameter $0 < K < 1$. Such a case has been explored in this simulation and the results have been indicated in the Analysis Section of this report as the Decreasing Failure Rate Case. Simulation results indicate that a shape parameter value of .5 causes the experienced failure rate to approach the predicted failure rate asymptotically from above. The deviation from the predicted failure rate and the simulated failure rate increasing directly with the MTBF. In such a situation, we should expect to see larger initial failure rates which decrease throughout the introduction period and eventually settling down when steady state is reached.

In the case of constant failure rates, i.e., a shape parameter $K = 1$, the simulation results indicate that failures will be randomly distributed about the predicted failure rate increasing in absolute deviation from the predicted failure rate as the MTBF increases. We should not be surprised nor concerned with apparent increases or decreases in the number of failures observed when the components have an underlying exponential failure distribution, since such deviations from the expected or predicted failure rate are due to random fluctuations.

The simulation results also indicate that after steady state has been attained for a system of equipments/components the observed distribution of failures resembles the random fluctuations of the constant failure rate case. This is the result proved by Khintchine as previously indicated in Chapter III. Thus, it appears that upon the attainment of steady state, the expected failures can be estimated by assuming a Poisson process, regardless of the underlying failure generating distribution; and, in steady state the MTBF should equal the MTTF.

APPENDIX A

COMPUTER SIMULATION PROGRAM

The program developed for this research is shown in Figure 4 and a simplified logic chart is shown in Figure 5. This program is a FORTRAN language program using the GASP2B simulation package developed by A. Alan B. Pritsker and Philip J. Kiviat. (16) The use of GASP2B simulation package greatly simplifies the programming required to be written for this study.

Basically the program is composed of a Main Program and six Subroutines as shown in Figure 4. The Main Program is used to initialize non-GASP variables and to call the GASP2B program package. When the GASP package is called, the internal GASP executive routines control all further simulation events until simulation termination is reached.

The GASP executive routines call Subroutine EVNTS which in turn selects the appropriate INTRO, FAILR, REPORT, or ENDSM subroutines based upon the currently processed event.

The INTRO subroutine is primarily used to introduce the aircraft into the inventory incrementally until the total fleet size has been attained. Each unit introduced incrementally to the system is assigned a time to failure (service time) and internally filed in a future events file.

The FAILR subroutine is called by the GASP routine whenever a failure event occurs. To simulate the instantaneous repair/replacement of failed components, this subroutine immediately computes a new time to failure and then refiles this new time to failure in the future events file. Statistics are collected on the service time of each failure.

The REPORT subroutine schedules the next monthly reporting, and records the total period failures and service times which have been experienced. A check is made against the total simulation time for each run and the simulation run is terminated if required.

The ENDSM subroutine sets specific GASP variables for the termination of the current simulation run. A check is made to determine if the total number of simulation runs has been completed and the OUTPUT subroutine is called as required.

The OUTPUT subroutine has been written to provide the total simulation output when the total number of simulation runs has been completed. In this research, a simulation run is approximately 4 years long and each simulation run is performed 40 times. The OUTPUT subroutine then takes an average of all statistics collected over the 40 runs before the output is printed. In this manner, we minimize the effect of extreme values in our output data. The printed output format and data are shown in the simulation output in Appendix B.

```
C MAIN PROGRAM HANDLE WEIBULL SIMULATION
 DIMENSION NSET(10,120)
C    *****DIMENSION AND COMMON CARDS FOR GASP2B**************
 COMMON SEED
 DOUBLE PRECISION SEED
 COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,NOQ,
&NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,TBEG,TFIN,
&MXX,NPRCT,NEP,NPROJ,MON,NDAY,NYR,JCLR
 COMMON ATRIB(10),ENQ(20),INN(20),JCELS(10,22),KRANK(20),MAXNQ(20),
&MFE(20),MLC(20),MLE(20),NCELS(10),NQ(20),PARAM(20,4),QTIME(20),
&SSUMA(20,5),SUMA(20,5),NAME(6),VNQ(20),FAIL(48),CTIME(48),FPH(48)
 COMMON FPH2(48)
 COMMON XNEFFIT,XINRATE,XNINTRD,THEDA,AK,KPIPERD,SIMTIME,L,ANTOTAL
&,ALASTPD
 I=1.0
 ANTOTAL=0.0
 ALASTPD=0.0
 DO 1 I=1,48
 FAIL(I)=0.0
 FPH2(I)=0.0
 CTIME(I)=0.0
 1 FPH(I)=0.0
 CALL GASP2B(NSET)
 STOP
 END
```

Figure 4. --Computer Program

```
10-11-75    16.825    SUBROUTINE EVENTS HAMRIC WEIBULL ***

  C **** SUBROUTINE EVENTS HAMRIC WEIBULL ***
   SUBROUTINE EVNTS(IX,NSET)
   DIMENSION NSET(10,1)
  C   *****DIMENSION AND COMMON CARDS FOR GASP20************
   COMMON SEED
   DOUBLE PRECISION SEED
   COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,NOQ,
  $NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,TBEG,TFIN,
  &MXX,NPRNT,NEP,NPROJ,MON,NDAY,NYR,JCLR
   COMMON ATRIB(10),ENQ(20),INN(20),JCELS(10,22),KRANK(20),MAXNO(20),
  &MFE(20),MLC(20),MLE(20),NCELS(10),NO(20),PARAM(20,4),QTIME(20),
  $SSUMA(20,5),SUMA(20,5),NAME(6),VNO(20),FAIL(48),CTIME(48),FPH(48)
   COMMON FPH2(48)
   COMMON XNFLEET,XINRATE,XNINTRO,THEDA,AK,RPIPERD,SIMTIME,.,ANTOTAL
  &,ALASTPD
   GO TO(1,2,3,4),IX
   1 CALL INTRO(NSET)
   RETURN
   2 CALL FAILR(NSET)
   RETURN
   3 CALL REPORT(NSET)
   RETURN
   4 CALL FUDSH(NSET)
   RETURN
   END
```

Figure 4 continued

```
C ***** SUBROUTINE INTRO-HAMRIC WEIBULL **
 SUBROUTINE INTRO(NSET)
 DIMENSION NSET(10,1)
C    *****DIMENSION AND COMMON CARDS FOR GASP2D************
 COMMON SEED
 DOUBLE PRECISION SEED
 COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,NOQ,
&NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,TBEG,TFIN,
&MXX,NPRNT,NEP,NPROJ,NOU,NDAY,NYR,JCLR
 COMMON ATRIB(10),ENQ(20),INN(20),JCELS(10,22),KRANK(20),MAXNQ(20),
&MFE(20),MLE(20),MCE(20),NCELS(10),NQ(20),PARAM(20,4),QTIME(20),
&SSUMA(20,5),SUMA(20,5),NAME(6),VNQ(20),FAIL(48),CTIME(48),FPH(48)
 COMMON FPH2(48)
 COMMON XNFLEET,XINRATE,XNINTRO,THEDA,AK,RPIPERD,SIMTIME,I,ANTOTAL
&,ALASTPD
 XNFLEET=PARAM(1,1)
 XINRATE=PARAM(1,2)
 XNINTRO=PARAM(1,3)
 RPIPERD=PARAM(1,4)
 THEDA=PARAM(2,1)
 AK=PARAM(2,2)
 SIMTIME=PARAM(2,3)
 CALL TMST(ANTOTAL,TNOW,1,NSET)
 ANTOTAL=ANTOTAL+XNINTRO
 IF(XNFLEET-ANTOTAL)1,1,2
 2 ATRIB(1)=TNOW+XINRATE
 ATRIB(2)=1.0
 CALL FILEM(1,NSET)
 1 NTRO=XNINTRO
 DO 10 I=1,NTRO
 Y=1.0/AK
 ATRIB(1)= TNOW +(THEDA*(EXP(Y*ALOG(-ALOG(DRAND(SEED))))))
 ATRIB(2)=2.0
 CALL FILEM(1,NSET)
 SERVIM= ATRIB(1)-TNOW
 10 CALL HISTO(SERVIM,100.0,100.0,1)
 RETURN
 END
```

Figure 4 continued

```
C ****SUBROUTINE FAILURE HANDIC WEIBULL**
 SUBROUTINE FAILR(NSET)
 DIMENSION NSET(10,1)
C   *****DIMENSION AND COMMON CARDS FOR GASP2B************
 COMMON SEED
 DOUBLE PRECISION SEED
 COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,NOQ,
&NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,TBEG,TFIN,
&MXX,NPRNT,JTRY,NPROJ,MON,NDAY,NYR,JCLR
 COMMON ATRIB(10),ENQ(20),INN(20),JCELS(10,22),KRANK(20),MAXNQ(20),
&MFE(20),MLC(20),MLE(20),NCELS(10),NQ(20),PARAM(20,4),QTIME(20),
&SSUMA(20,5),SUMA(20,5),NAME(6),VNQ(20),FAIL(48),CTIME(48),FPH(48)
 COMMON FPH2(48)
 COMMON XNEFFCT,XTHRATE,XNINTRO,THEDA,AK,RPTPERD,SIMTIME,L,ANTOTAL
&,ALASTFD
 Y=1.0/AK
 ATRIB(1)=TNOW+(THEDA*(EXP(Y*ALOG(-ALOG(DRAND(SEED))))))
 ATRIB(2)=2.0
 CALL FILEM(1,NSET)
 SERVTM=ATRIB(1)-TNOW
 CALL HISTO(SERVTM,100.0,100.0,1)
 CALL COLCT(1.0,1,NSET)
 RETURN
 END
```

Figure 4 continued

```
C ****SUBROUTINE REPORT HAMRIC WEIBULL **
 SUBROUTINE REPORT(NSET)
 DIMENSION NSET(10,1)
C  *****DIMENSION AND COMMON CARDS FOR GASP2B************
 COMMON SEED
 DOUBLE PRECISION SEED
 COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,NOQ,
&NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,TBEG,TFIN,
&MXX,NPRNT,NEP,NPROJ,MON,NDAY,NYR,JCLR
 COMMON ATRIB(10),ENQ(20),INN(20),JCELS(10,22),KRANK(20),MAXNQ(20),
&MFE(20),MLC(20),MLE(20),NCELS(10),NQ(20),PARAM(20,4),QTIME(20),
&SSUMA(20,5),SUMA(20,5),NAME(6),VNQ(20),FAIL(48),CTIME(48),FPH(48)
 COMMON FPH2(48)
 COMMON XNFIFTI,XINRATE,XNINTRO,THETA,AK,RPIPERD,SIMTIME,L,ANTOTAL
&,ALASTPD
 ATRIB(1)=TNOW + RPIPERD
 ATRIB(2)=3.0
 CALL FILEM (1,NSET)
 CALL TMST(ANTOTAL,TNOW,1,NSET)
 CTIME(L)=CTIME(L)+ (SSUMA(1,2)-ALASTPD)
 FPH(L)=FPH(L)+(SUMA(1,1)/(SSUMA(1,2)-ALASTPD))
 FPH2(L)=FPH2(L)+(FPH(L)**2)
 FAIL(L)=FAIL(L)+SUMA(1,1)
 ALASTPD=SSUMA(1,2)
 L=L+1.
 DO 1 J=1,5
 1 SUMA(1,J)=0
 IF(SIMTIME.GE.L) GO TO 2
 ATRIB(1)=TNOW
 ATRIB(2)=4.0
 CALL FILEM(1,NSET)
 2 RETURN
 END
```

Figure 4 continued

```
C **** SUBROUTINE FOR OF SIM HAMRIC WEIBULL **
 SUBROUTINE ENDSM(NSET)
 DIMENSION NSET(10,1)
C    ****DIMENSION AND COMMON CARDS FOR GASP2B************
 COMMON SEED
 DOUBLE PRECISION SEED
 COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,NOQ,
&NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,TBEG,TFIN,
&MXX,NPRNT,NCR,NPROJ,MON,NDAY,NYR,JCLR
 COMMON ATRIB(10),ENQ(20),INN(20),JCELS(10,22),KRANK(20),MAXNQ(20),
&MFE(20),MLC(20),MLE(20),NCELLS(10),NQ(20),PARAM(20,4),QTIME(20),
&SSUMA(20,5),SUMA(20,5),NAME(6),VNQ(20),TAIL(48),CTIME(48),FPH(48)
 COMMON FPH,(48)
 COMMON XOFFLET,XINRATE,XNIOTRO,THEDA,AK,RPIPERU,SINTIME,L,ANTOTAL
&,ALASTFD
 MSTOP=-1
 IF(NRUN-40)1,2,2
 1 NORPT=1
 L=1.0
 ANTOTAL=0.0
 ALASTFD = 0.0
 RETURN
 2 NORPT=0
 RETURN
 END
```

Figure 4 continued

```
 SUBROUTINE OTPUT(NSET)
 DIMENSION NSET(10,1)
C ****DIMENSION AND COMMON CARDS FOR GASP2B************
 COMMON QSET
 DOUBLE PRECISION QSET
 COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,NOQ,
&NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,TBEG,TFIN,
&MXX,NPRNT,NEP,NPROJ,MON,NDAY,NYR,JCLR
 COMMON ATRIB(10),ENQ(20),INN(20),JCELS(10,22),XRANK(20),MAXNQ(20),
&MFE(20),MLC(20),MLE(20),NCELS(10),NQ(20),PARAM(20,4),QTIME(20),
&SSUMA(20,5),SUMA(20,5),NAME(6),VNQ(20),FAIL(48),CTIME(48),FPH(48)
 COMMON FPH2(48)
 COMMON XNFLEET,XIBRATE,XNINTRO,THEDA,AK,RPIPERD,SIMTIME,L,ANIDIAL
&,XLASTPD
 PRINT 99
 99 FORMAT(1H1,//,27X,"AVG",8X,"FLY",6X,"FAIL PER")
 PRINT 104
 104 FORMAT(" ",15X,"PERIOD",4X,"FAIL",7X,"HOURS",5X,"FLY HOUR",
&6X,"STD.DEV.")
 PRINT 98
 98 FORMAT(1H )
 DO 1 IJ=1,13
 AVFAIL=FAIL(IJ)/40.0
 AVCTME=CTIME(IJ)/40.0
 AVFPH=FPH(IJ)/40.0
 AFPH2=FPH2(IJ)/40.0
 STD=AFPH2-(AVFPH**2)
 PRINT 101,IJ,AVFAIL,AVCTME,AVFPH,STD
 101 FORMAT(" ",16X,I3,F10.1,3X,F10.1,5X,F8.6,5X,F8.6)
 1 CONTINUE
 PRINT 102,XNFLEET,XIBRATE
 102 FORMAT(///,15X,"NUMBER IN FLEET =",F6.1,5X,"INTRU INTERVAL =",
&F6.1)
 PRINT 103,RPTPERD,THEDA,AK
 103 FORMAT(" ","REPORT INTERVAL =",F6.1,5X,"THETA =",F6.1,
&3X,"SHAPE =",F3.1)
 PRINT 107,SIMTIME,XNINTRO
 107 FORMAT(15X,"SIM PERIODS =",F5.1,3X,"NUMBER INTRO / PERIOD =",
&F5.1)
 RETURN
 END
```

Figure 4 continued

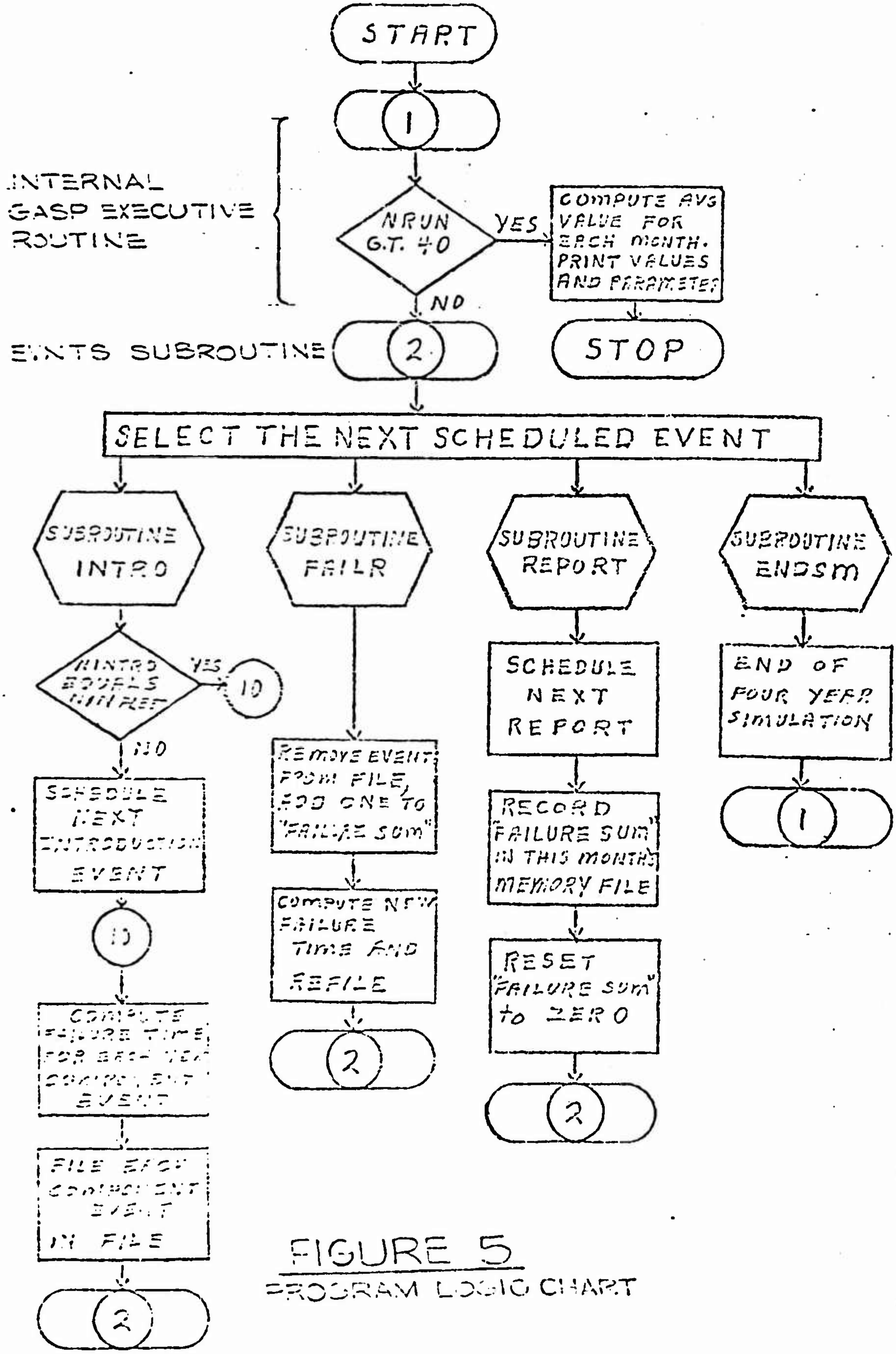


FIGURE 5

PROGRAM LOGIC CHART

APPENDIX B

SIMULATION OUTPUT DATA

This appendix contains the final simulation output listings and plots for the various parameters investigated in this study. Each listing contains the average number of failures per period, the period operating hours, the average number of failures per operating hour per period and the period standard deviation of failures per operating hour. The legend at the bottom of each listing describes the conditions for which the data was generated. Definitions of these conditions are as follows:

NUMBER IN FLEET--the maximum size of the fleet to be in the inventory.

INTRO INTERVAL--the time interval between increments of the fleet in hours. (120 hrs. equals 1 Report Interval).

REPORT INTERVAL--the time length of each reporting period in hours.

SCALE--a parameter of the Weibull distribution determined by the equation

$$\theta = \frac{\text{MTBF}}{\Gamma(1+1/k)} .$$

SHAPE--the other parameter, K, of the Weibull distribution tested in this study.

MTBF--the Mean Time Between Failures under the assumption of an exponential time to failure distribution.

NUMBER INTRO/PERIOD--the number of items introduced to the simulation until the total fleet size is attained.

The failures per ops hour have been plotted for each listing and are included in Chapter VI of this report.

The listings are presented as follows:

TABLE NO.	MTBF	SHAPE
B-1	500	.5
B-2	1000	.5
B-3	2000	.5
B-4	50	1.0
B-5	100	1.0
B-6	500	1.0
B-7	1000	1.0
B-8	2000	1.0
B-9	50	2.0
B-10	50	3.0
B-11	50	4.0
B-12	50	5.0
B-13	100	2.0
B-14	100	3.0
B-15	100	4.0
B-16	100	5.0
B-17	500	1.5
B-18	500	2.0
B-19	500	3.0
B-20	500	4.0
B-21	500	5.0
B-22	1000	1.5
B-23	1000	2.0

TABLE NO.	MTBF	SHAPE
B-24	1000	3.0
B-25	1000	4.0
B-26	1000	5.0
B-27	2000	1.5
B-28	2000	2.0
B-29	2000	3.0
B-30	2000	4.0
B-31	2000	5.0

TABLE B-1

```
              AVG          OPS          FAIL PER
PERIOD        FAIL         HOURS        OPS HOUR           STD.DEV
     1         4,5          600,0       0,007458          0,026953
     2         7,2         1200,0       0,005979          0,018714
     3         9,5         1800,0       0,005250          0,012758
     4         9,6         2400,0       0,004000          0,008490
     5        11,2         3000,0       0,003725          0,008012
     6        12,9         3600,0       0,003583          0,006766
     7        14,6         4200,0       0,003518          0,006663
     8        15,5         4800,0       0,003219          0,005650
     9        17,2         5400,0       0,003185          0,005657
    10        18,6         6000,0       0,003096          0,005493
    11        19,8         6600,0       0,003004          0,004910
    12        22,9         7200,0       0,003184          0,005430
    13        24,7         7800,0       0,003160          0,005543
    14        22,4         8400,0       0,002670          0,003885
    15        25,5         9000,0       0,002833          0,004234
    16        25,4         9600,0       0,002646          0,004064
    17        27,2        10200,0       0,002662          0,004018
    18        29,7        10799,9       0,002750          0,004139
    19        30,0        11399,9       0,002536          0,003729
    20        31,4        11999,9       0,002617          0,003852
    21        29,2        12000,0       0,002431          0,003145
    22        26,0        12000,0       0,002165          0,002679
    23        27,8        12000,0       0,002317          0,002978
    24        26,5        12000,0       0,002208          0,002678
    25        25,8        12000,0       0,002154          0,002470
    26        26,8        12000,0       0,002238          0,002715
    27        26,5        12000,0       0,002206          0,002856
    28        26,9        12000,0       0,002240          0,002827
    29        26,2        12000,0       0,002183          0,002485
    30        27,5        12000,0       0,002290          0,002846
    31        26,1        12000,0       0,002177          0,002578
    32        25,5        12000,0       0,002129          0,002520
    33        26,5        12000,0       0,002204          0,002655
    34        26,5        12000,0       0,002210          0,002651
    35        25,5        12000,0       0,002123          0,002264
    36        25,2        12000,0       0,002102          0,002426
    37        24,3        12000,0       0,002021          0,0022[illegible]
    38        25,2        12000,0       0,002096          0,002446
    39        24,1        12000,0       0,002010          0,002250
    40        23,4        12000,0       0,001948          0,002139
    41        24,0        12000,0       0,002002          0,002235
    42        25,0        12000,0       0,002081          0,002461
    43        24,6        12000,0       0,002052          0,0024[illegible]
    44        23,7        12000,0       0,001973          0,002095
    45        22,7        12000,0       0,001894          0,0020[illegible]
    46        24,7        12000,0       0,002054          0,002377
    47        24,5        12000,0       0,002044          0,002340
    48        24,1        12000,0       0,002006          0,002229

NUMBER IN FLEET = 100,0      INTPO INTERVAL = 120,0
REPORT INTERVAL = 120,0      SCALE = 250,0   SHAPE =0,50
MTBF = 500,0   NUMBER INTPO / PERIOD =  5,0
```

TABLE B- 2

```
                AVG          OPS         FAIL PER
 PERIOD         FAIL        HOURS        OPS HOUR        STD,DEV
    1            3,1         600,0       0,005125        0,014348
    2            4,4        1200,0       0,003708        0,006944
    3            6,3        1800,0       0,003514        0,005805
    4            6,9        2400,0       0,002865        0,004052
    5            7,5        3000,0       0,002492        0,003549
    6            7,8        3600,0       0,002167        0,002503
    7            9,4        4200,0       0,002232        0,002806
    8           10,1        4800,0       0,002109        0,002464
    9            9,4        5400,0       0,001741        0,001572
   10           10,5        6000,0       0,001604        0,001669
   11           12,4        6600,0       0,001883        0,001849
   12           12,5        7200,0       0,001740        0,001744
   13           13,5        7800,0       0,001740        0,001789
   14           14,4        8400,0       0,001717        0,001534
   15           16,0        9000,0       0,001783        0,001710
   16           17,5        9600,0       0,001620        0,001876
   17           16,8       10200,0       0,001645        0,001482
   18           17,8       10799,9       0,001646        0,001537
   19           18,0       11399,9       0,001579        0,001365
   20           19,3       11999,9       0,001608        0,001479
   21           15,0       12000,0       0,001250        0,000889
   22           16,8       12000,0       0,001400        0,001080
   23           15,0       12000,0       0,001252        0,000854
   24           14,8       12000,0       0,001235        0,000833
   25           14,9       12000,0       0,001240        0,000841
   26           13,6       12000,0       0,001133        0,000714
   27           13,8       12000,0       0,001148        0,000696
   28           14,3       12000,0       0,001194        0,000769
   29           13,6       12000,0       0,001131        0,000707
   30           13,8       12000,0       0,001152        0,000734
   31           15,0       12000,0       0,001252        0,000804
   32           12,9       12000,0       0,001077        0,000658
   33           12,3       12000,0       0,001021        0,000600
   34           14,0       12000,0       0,001167        0,000781
   35           13,1       12000,0       0,001094        0,000653
   36           14,4       12000,0       0,001198        0,000788
   37           12,5       12000,0       0,001042        0,000596
   38           13,1       12000,0       0,001094        0,000684
   39           12,5       12000,0       0,001040        0,000554
   40           12,9       12000,0       0,001071        0,000625
   41           12,5       12000,0       0,001044        0,000588
   42           13,1       12000,0       0,001096        0,000598
   43           13,3       12000,0       0,001108        0,000572
   44           13,1       12000,0       0,001095        0,000745
   45           13,5       12000,0       0,001123        0,000708
   46           13,1       12000,0       0,001090        0,000663
   47           12,7       12000,0       0,001056        0,000549
   48           12,3       12000,0       0,001029        0,000557

 NUMBER IN FLEET = 100,0      INTRO INTERVAL = 120,0
 REPORT INTERVAL = 120,0      SCALE = 500,0     SHAPE =0,50
 MTBF =1000,0    NUMBER INTRO / PERIOD =  5,0
```

TABLE B-3

```
                AVG          OPS        FAIL PER
PERIOD          FAIL        HOURS       OPS HOUR          STD.DEV
   1             2.1         600.0      0.003458          0.006739
   2             2.9        1200.0      0.002438          0.003095
   3             3.8        1800.0      0.002097          0.001987
   4             4.9        2400.0      0.002021          0.001831
   5             5.5        3000.0      0.001825          0.001879
   6             4.9        3600.0      0.001375          0.001038
   7             5.9        4200.0      0.001417          0.001096
   8             6.4        4800.0      0.001339          0.001063
   9             7.5        5400.0       .001389          0.001040
  10             7.7        6000.0      0.001288          0.000942
  11             7.0        6600.0      0.001061          0.000535
  12             7.5        7200.0      0.001038          0.000646
  13             7.5        7800.0      0.000962          0.000489
  14             8.9        8400.0      0.001060          0.000556
  15             9.3        9000.0      0.001031          0.000613
  16             9.8        9600.0      0.001023          0.000534
  17             9.9       10200.0      0.000968          0.000492
  18            10.8       10799.9      0.000998          0.000508
  19            11.8       11399.9      0.001033          0.000573
  20            12.5       11999.9      0.001038          0.000626
  21             9.9       12000.0      0.000823          0.000382
  22             9.8       12000.0      0.000815          0.000373
  23             9.0       12000.0      0.000752          0.000315
  24             9.0       12000.0      0.000748          0.000325
  25             7.8       12000.0      0.000652          0.000249
  26             9.2       12000.0      0.000767          0.000310
  27             8.8       12000.0      0.000735          0.000320
  28             9.0       12000.0      0.000754          0.000304
  29             7.2       12000.0      0.000602          0.000223
  30             7.6       12000.0      0.000637          0.000226
  31             7.7       12000.0      0.000644          0.000218
  32             7.8       12000.0      0.000650          0.000253
  33             8.0       12000.0      0.000665          0.000252
  34             7.6       12000.0      0.000629          0.000212
  35             7.1       12000.0      0.000594          0.000191
  36             7.6       12000.0      0.000629          0.000221
  37             7.5       12000.0      0.000623          0.000220
  38             7.5       12000.0      0.000625          0.000195
  39             7.4       12000.0      0.000617          0.000221
  40             7.0       12000.0      0.000581          0.000190
  41             7.1       12000.0      0.000588          0.000178
  42             6.6       12000.0      0.000548          0.000158
  43             7.1       12000.0      0.000596          0.000199
  44             6.6       12000.0      0.000552          0.000161
  45             7.2       12000.0      0.000598          0.000187
  46             6.9       12000.0      0.000577          0.000194
  47             6.1       12000.0      0.000505          0.000137
  48             6.9       12000.0      0.000577          0.000179

NUMBER IN FLEET = 100.0      INTRO INTERVAL = 120.0
REPORT INTERVAL = 120.0      SCALE =1000.0   SHAPE =0.50
MTBF =2000.0   NUMBER INTRO / PERIOD = 5.0
```

```
                AVG          OPS       FAIL PER
PERIOD          FAIL         HOURS     OPS HOUR        STD.DEV
    1           13.0         600.0     0.021667        0.244795
    2           25.0        1200.0     0.020875        0.227015
    3           34.6        1800.0     0.019222        0.205380
    4           48.4        2400.0     0.020188        0.221661
    5           57.8        3000.0     0.019283        0.201221
    6           69.8        3600.0     0.019389        0.208494
    7           82.6        4200.0     0.019673        0.216171
    8           95.7        4800.0     0.019932        0.222196
    9          107.9        5400.0     0.019977        0.216236
   10          120.4        6000.0     0.020075        0.222688
   11          134.7        6600.0     0.020439        0.226803
   12          143.4        7200.0     0.019913        0.219229
   13          154.1        7800.0     0.019755        0.218330
   14          173.6        8400.0     0.020673        0.235528
   15          183.6        9000.0     0.020403        0.231060
   16          196.2        9600.0     0.020435        0.226982
   17          207.3       10200.0     0.020324        0.223536
   18          222.7       10799.9     0.020618        0.233327
   19          225.7       11399.9     0.019796        0.216921
   20          247.3       11999.9     0.020611        0.233564
   21          238.        12000.0     0.019860        0.216788
   22          237         12000.0     0.019810        0.214436
   23          238.7       12000.0     0.019890        0.219047
   24          241.0       12000.0     0.020083        0.223878
   25          237.9       12000.0     0.019821        0.218736
   26          234.6       12000.0     0.019548        0.215024
   27          245.7       12000.0     0.020475        0.233321
   28          247.1       12000.0     0.020596        0.233856
   29          245.3       12000.0     0.020444        0.229164
   30          240.7       12000.0     0.020058        0.221789
   31          245.9       12000.0     0.020490        0.231067
   32          242.6       12000.0     0.020219        0.225421
   33          242.1       12000.0     0.020177        0.226727
   34          241.4       12000.0     0.020119        0.221331
   35          236.1       12000.0     0.019675        0.215930
   36          235.5       12000.0     0.019625        0.211694
   37          236.0       12000.0     0.019669        0.213825
   38          238.0       12000.0     0.019829        0.218186
   39          242.9       12000.0     0.020240        0.225078
   40          245.0       12000.0     0.020421        0.227945
   41          235.6       12000.0     0.019639        0.214527
   42          231.9       12000.0     0.019321        0.206287
   43          239.3       12000.0     0.019944        0.217497
   44          245.0       12000.0     0.020421        0.226055
   45          236.7       12000.0     0.019725        0.217558
   46          242.8       12000.0     0.020233        0.225827
   47          241.3       12000.0     0.020106        0.221023
   48          239.4       12000.0     0.019945        0.219304

NUMBER IN FLEET = 100.0      INTRO INTERVAL = 12 .0
REPORT INTERVAL = 120.0      SCALE =  50.0   SHAPE =1.00
MTBF =  50.0   NUMBER INTRO / PERIOD =  5.0
```

TABLE B- 4

TABLE B- 5

```
                AVG            OPS         FAIL PER
PERIOD          FAIL          HOURS        OPS HOUR           STD.DEV
   1             6.5           600.0       0.011083          0.064460
   2            12.9          1200.0       0.010708          0.056275
   3            18.1          1800.0       0.010070          0.055324
   4            23.2          2400.0       0.009646          0.051510
   5            29.8          3000.0       0.009933          0.054463
   6            35.7          3600.0       0.009917          0.054594
   7            41.3          4200.0       0.009827          0.051356
   8            49.0          4800.0       0.010208          0.057469
   9            49.5          5400.0       0.009162          0.045966
  10            59.8          6000.0       0.009963          0.056754
  11            65.0          6600.0       0.009849          0.054562
  12            72.1          7200.0       0.010010          0.054408
  13            78.1          7800.0       0.010010          0.055395
  14            84.0          8400.0       0.010003          0.055628
  15            91.5          9000.0       0.010167          0.055525
  16            97.7          9600.0       0.010180          0.056901
  17           103.3         10200.0       0.010130          0.056830
  18           103.8         10799.9       0.009607          0.051694
  19           115.7         11399.9       0.010151          0.057671
  20           124.3         11999.9       0.010358          0.058873
  21           121.9         12000.0       0.010163          0.057388
  22           123.0         12000.0       0.010250          0.057078
  23           123.2         12000.0       0.010269          0.057637
  24           121.7         12000.0       0.010144          0.055546
  25           121.6         12000.0       0.010131          0.054864
  26           124.6         12000.0       0.010385          0.060052
  27           120.0         12000.0       0.010002          0.054844
  28           121.7         12000.0       0.010140          0.057201
  29           120.6         12000.0       0.010052          0.055584
  30           124.2         12000.0       0.010348          0.059103
  31           119.1         12000.0       0.009929          0.054085
  32           119.8         12000.0       0.009983          0.054559
  33           119.9         12000.0       0.009990          0.055097
  34           119.4         12000.0       0.009946          0.054316
  35           117.2         12000.0       0.009765          0.051733
  36           118.8         12000.0       0.009900          0.055065
  37           122.6         12000.0       0.010221          0.057604
  38           118.9         12000.0       0.009904          0.054236
  39           119.8         12000.0       0.009985          0.055599
  40           117.1         12000.0       0.009763          0.052513
  41           112.5         12000.0       0.009377          0.054683
  42           119.1         12000.0       0.009927          0.055543
  43           118.3         12000.0       0.009854          0.051617
  44           124.1         12000.0       0.010340          0.059083
  45           126.3         12000.0       0.010527          0.062465
  46           120.9         12000.0       0.010079          0.055933
  47           122.8         12000.0       0.010235          0.058353
  48           123.1         12000.0       0.010256          0.057484

NUMBER IN FLEET = 100.0        INTRO INTERVAL = 120.0
 REPORT INTERVAL = 120.0       SCALE = 100.0    SHAPE =1.00
MTBF = 100.0   NUMBER INTRO / PERIOD =   5.0
```

TABLE B-6

PERIOD	AVG FAIL	OPS HOURS	FAIL PER OPS HOUR	STD.DEV
1	1.2	600.0	0.001958	0.002259
2	2.6	1200.0	0.002167	0.002464
3	3.9	1800.0	0.002139	0.002171
4	5.3	2400.0	0.002219	0.002433
5	6.5	3000.0	0.002175	0.002435
6	7.4	3600.0	0.002069	0.002270
7	8.1	4200.0	0.001929	0.002026
8	10.0	4800.0	0.002094	0.002519
9	11.1	5400.0	0.002056	0.002386
10	11.5	6000.0	0.001917	0.001982
11	12.5	6600.0	0.001898	0.002004
12	13.1	7200.0	0.001826	0.001827
13	15.8	7800.0	0.002026	0.002246
14	16.4	8400.0	0.001949	0.002165
15	18.0	9000.0	0.002003	0.002097
16	20.2	9600.0	0.002099	0.002347
17	21.0	10200.0	0.002061	0.002395
18	22.0	10799.9	0.002039	0.002290
19	22.2	11399.9	0.001945	0.002002
20	23.2	11999.9	0.001935	0.002138
21	22.5	12000.0	0.001879	0.002000
22	22.9	12000.0	0.001908	0.002020
23	23.9	12000.0	0.001992	0.002137
24	24.0	12000.0	0.002002	0.002279
25	23.3	12000.0	0.001944	0.002126
26	23.9	12000.0	0.001994	0.002138
27	23.3	12000.0	0.001946	0.002064
28	24.2	12000.0	0.002015	0.002225
29	23.7	12000.0	0.001971	0.002112
30	24.0	12000.0	0.002002	0.002229
31	22.8	12000.0	0.001900	0.002040
32	24.7	12000.0	0.002060	0.002309
33	23.5	12000.0	0.001962	0.002302
34	24.5	12000.0	0.002046	0.002305
35	23.5	12000.0	0.001954	0.002113
36	24.2	12000.0	0.002017	0.002269
37	23.8	12000.0	0.002150	0.002370
38	24.5	12000.0	0.002042	0.002312
39	24.6	12000.0	0.002065	0.002314
40	23.4	12000.0	0.001948	0.002096
41	24.9	12000.0	0.002077	0.002366
42	24.1	12000.0	0.002008	0.002117
43	23.8	12000.0	0.001983	0.002145
44	24.0	12000.0	0.002000	0.002148
45	24.5	12000.0	0.002037	0.002327
46	23.5	12000.0	0.001957	0.002173
47	24.5	12000.0	0.002040	0.002305
48	24.2	12000.0	0.002012	0.002218

NUMBER IN FLEET = 100.0   INTRO INTERVAL = 120.0
REPORT INTERVAL = 120.0   SCALE = 500.0   SHAPE =1.00
MTBF = 500.0   NUMBER INTRO / PERIOD = 5.0

TABLE B-7

```
              AVG        OPS        FAIL PER
PERIOD        FAIL       HOURS      OPS HOUR          STD.DEV
   1          0,6        600,0      0,001000          0,000547
   2          1,4       1200,0      0,001146          0,000750
   3          1,9       1800,0      0,001042          0,000560
   4          2,7       2400,0      0,001135          0,000582
   5          2,7       3000,0      0,000900          0,000373
   6          4,2       3600,0      0,001174          0,000744
   7          4,3       4200,0      0,001012          0,000506
   8          5,1       4800,0      0,001052          0,000673
   9          5,7       5400,0      0,001056          0,000641
  10          6,2       6000,0      0,001033          0,000575
  11          7,2       6600,0      0,001087          0,000634
  12          7,3       7200,0      0.001010          0,000572
  13          7,7       7800,0      0,000987          0,000542
  14          7,7       8400,0      0.000914          0,000427
  15          7,6       9000,0      0,000850          0,0004[illegible]8
  16          9,1       9600,0      0.000951          0,000475
  17         10,5      10200,0      0,001025          0,000595
  18         10,5      10799,9      0.000975          0,000525
  19         11,0      11399,9      0,000963          0,000494
  20         11,9      11999,9      0.000994          0,000527
  21         12,0      12000,0      0,000996          0,000537
  22         13,5      12000,0      0.001125          0,000685
  23         12,0      12000,0      0,000996          0,000556
  24         12,5      12000,0      0.001044          0,000608
  25         11,6      12000,0      0,000969          0,000542
  26         13,0      12000,0      0.001081          0,000628
  27         11,8      12000,0      0,000985          0,000523
  28         11.4      12000,0      0,000948          0,000475
  29         11,9      12000,0      0,000994          0,000554
  30         11,7      12000,0      0,000977          0,000540
  31         11.2      12000,0      0,000935          0,000509
  32         11,3      12000,0      0,000940          0,000483
  33         11,3      12000,0      0,000940          0,000480
  34         11,1      12000,0      0,000925          0.000502
  35         12,6      12000,0      0,001054          0,000641
  36         12,1      12000,0      0.001006          0,000544
  37         12,3      12000,0      0.001023          0.000578
  38         12,0      12000,0      0,001002          0.000554
  39         11,4      12000,0      0,000950          0,0005[illegible]0
  40         11.5      12000,0      0,000954          0,000515
  41         11,6      12000,0      0.000963          0,000497
  42         12,0      12000,0      0.001000          0.000539
  43         11,9      12000,0      0.000992          0,000525
  44         11,3      12000,0      0.000936          0.000486
  45         [illegible],5      12000,0      0,00[illegible]738          0,000593
  46         12,0      12000,0      0,001004          0.000563
  47         11,1      12000,0      0.000925          0.000475
  48         12,4      12000,0      0,001035          0,000550

NUMBER IN FLEET = 100,0      INTRO INTERVAL = 120,0
REPORT INTERVAL = 120,0      SCALE =1000,0    SHAPE =1,00
MTBF =1000,0   NUMBER INTRO / PERIOD =  5,0
```

TABLE B- 8

PERIOD	AVG FAIL	OPS HOURS	FAIL PER OPS HOUR	STD.DEV
1	.4	600,0	0,000583	0,000194
2	0,6	1200,0	0,000500	0,000125
3	0,9	1800,0	0,000500	0,000148
4	1,5	2400,0	0,000604	0,000169
5	1,7	3000,0	0,000550	0,000126
6	1,8	3600,0	0,000507	0,000116
7	2,1	4200,0	0,000500	0,000132
8	2,4	4800,0	0,000510	0,000139
9	2,9	5400,0	0,000528	0,000160
10	2,9	6000,0	0,000488	0,000124
11	3,3	6600,0	0,000500	0,000149
12	4,3	7200,0	0,000594	0,000206
13	4,1	7800,0	0,000529	0,000150
14	4,9	8400,0	0,000583	0,000192
15	4,6	9000,0	0,000508	0,000135
16	4,5	9600,0	0,000482	0,000130
17	5,4	10200,0	0,000532	0,000151
18	5.1	10799,9	0,000477	0,000126
19	5,3	11399,9	0,000463	0,000115
20	5,5	11999,9	0,000456	0,000105
21	5,4	12000,0	0,000450	0,000113
22	5.4	12000,0	0,000446	0,000112
23	6,0	12000,0	0,000502	0,000139
24	5,4	12000,0	0,000446	0,000105
25	5,7	12000,0	0,000477	0,000118
26	6,3	12000,0	0,000523	0,000138
27	6,0	12000,0	0,000502	0,000133
28	5,8	12000,0	0,000481	0,000120
29	5,9	12000,0	0,000492	0,000138
30	6,2	12000,0	0,000519	0,000146
31	6,2	12000,0	0,000517	0,000154
32	6,0	12000,0	0,000500	0,000135
33	6,2	12000,0	0,000515	0,000140
34	6,3	12000,0	0,000525	0,000150
35	6,0	12000,0	0,000502	0,000138
36	6,5	12000,0	0,000540	0,000175
37	6,3	12000,0	0,000525	0,000155
38	6,5	12000,0	0,000554	0,000169
39	5,9	12000,0	0,000490	0,000128
40	6,1	12000,0	0,000506	0,000158
41	6,0	12000,0	0,000498	0,000131
42	6,0	12000,0	0,000498	0,000132
43	6,9	12000,0	0,000573	0,000171
44	5,3	12000,0	0,000444	0,000133
45	5,6	12000,0	0,000471	0,000126
46	5.8	12000,0	0,000483	0,000123
47	6,1	12000,0	0,000506	0,000126
48	5,6	12000,0	0,000471	0,000121

```
NUMBER IN FLEET = 100,0      INTRO INTERVAL = 120,0
REPORT INTERVAL = 120,0      SCALE =2000,0    SHAPE =1,00
MTBF =2000,0   NUMBER INFO / PERIOD =  5,0
```

TABLE B-

```
               AVG         OPS       FAIL PER
PERIOD        FAIL        HOURS      OPS HOUR        STD.DEV
     1        10,5        600,0      0,017458        0,165011
     2        22,8       1200,0      0,019021        0,188774
     3        33,5       1800,0      0,018611        0,193972
     4        47,1       2400,0      0,019604        0,212547
     5        56,6       3000,0      0,018875        0,195905
     6        69,8       3600,0      0,019389        0,205766
     7        81,3       4200,0      0,019345        0,208306
     8        95,1       4800,0      0,019807        0,218622
     9       105,9       5400,0      0,019602        0,210512
    10       118,1       6000,0      0,019679        0,214462
    11       132,2       6600,0      0,020030        0,219911
    12       143,2       7200,0      0,019885        0,216582
    13       152,8       7800,0      0,019590        0,214023
    14       169,0       8400,0      0,020113        0,222405
    15       179,4       9000,0      0,019936        0,221164
    16       191,3       9600,0      0,019930        0,218879
    17       204,6      10200,0      0,020064        0,220596
    18       216,6      10799,9      0,020058        0,220934
    19       226,6      11399,9      0,019875        0,217891
    20       239,4      11999,9      0,019952        0,219556
    21       239,0      12000,0      0,019912        0,218956
    22       239,8      12000,0      0,019983        0,220382
    23       238,1      12000,0      0,019844        0,218390
    24       241,5      12000,0      0,020123        0,223539
    25       238,9      12000,0      0,019903        0,219446
    26       236,5      12000,0      0,019706        0,216265
    27       242,7      12000,0      0,020227        0,226430
    28       242,3      12000,0      0,020192        0,226937
    29       241,6      12000,0      0,020133        0,223789
    30       243,0      12000,0      0,020248        0,224845
    31       240,4      12000,0      0,020031        0,222207
    32       243,3      12000,0      0,020275        0,227731
    33       241,8      12000,0      0,020145        0,225780
    34       240,8      12000,0      0,020063        0,221999
    35       237,5      12000,0      0,019790        0,216635
    36       237,4      12000,0      0,019779        0,215101
    37       236,7      12000,0      0,019727        0,215706
    38       240,1      12000,0      0,020006        0,221237
    39       239,8      12000,0      0,019981        0,219930
    40       240,7      12000,0      0,020056        0,222005
    41       240,0      12000,0      0,020000        0,220062
    42       236,6      12000,0      0,019721        0,216458
    43       239,2      12000,0      0,019931        0,219560
    44       244,3      12000,0      0,020358        0,225699
    45       238,0      12000,0      0,019837        0,217822
    46       240,5      12000,0      0,020044        0,221420
    47       235,5      12000,0      0,019585        0,218412
    48       242,3      12000,0      0,020192        0,224737

NUMBER IN FLEET = 100.0      INTRO INTERVAL = 120,0
REPORT INTERVAL = 120,0      SCALE =  56,4   SHAPE =2,00
MTBF =  50,0   NUMBER INTRO / PERIOD =  5,0
```

TABLE B-10

```
            AVG          OPS        FAIL PER
PERIOD      FAIL        HOURS       OPS HOUR       STD.DEV
   1        10.1        600.0      0.016833       0.155651
   2        22.4       1200.0      0.018646       0.185025
   3        32.8       1800.0      0.018298       0.185432
   4        46.6       2400.0      0.019427       0.208211
   5        57.3       3000.0      0.019100       0.209975
   6        69.3       3600.0      0.019243       0.203652
   7        81.8       4200.0      0.019476       0.210895
   8        93.9       4800.0      0.019552       0.211547
   9       105.9       5400.0      0.019611       0.211821
  10       118.4       6000.0      0.019733       0.214629
  11       131.5       6600.0      0.019932       0.214565
  12       142.0       7200.0      0.019715       0.213366
  13       152.9       7800.0      0.019599       0.212570
  14       167.3       8400.0      0.019914       0.219609
  15       178.8       9000.0      0.019867       0.218224
  16       190.2       9600.0      0.019813       0.216093
  17       203.4      10200.0      0.019939       0.217753
  18       216.3      10799.9      0.020023       0.221443
  19       224.8      11399.9      0.019717       0.214370
  20       239.4      11999.9      0.019950       0.219393
  21       237.8      12000.0      0.019812       0.217062
  22       239.9      12000.0      0.019990       0.221455
  23       240.6      12000.0      0.020048       0.222070
  24       241.6      12000.0      0.020138       0.223140
  25       237.2      12000.0      0.019765       0.216828
  26       238.5      12000.0      0.019873       0.218773
  27       241.7      12000.0      0.020140       0.223868
  28       241.0      12000.0      0.020083       0.223732
  29       241.0      12000.0      0.020083       0.223861
  30       241.5      12000.0      0.020127       0.223420
  31       241.3      12000.0      0.020106       0.223401
  32       243.0      12000.0      0.020243       0.226354
  33       239.6      12000.0      0.019967       0.221997
  34       241.1      12000.0      0.020092       0.222179
  35       238.0      12000.0      0.019838       0.217861
  36       237.9      12000.0      0.019821       0.216712
  37       238.9      12000.0      0.019906       0.219327
  38       239.7      12000.0      0.019977       0.220876
  39       239.1      12000.0      0.019925       0.219316
  40       240.5      12000.0      0.020054       0.221212
  41       240.6      12000.0      0.020054       0.221900
  42       237.5      12000.0      0.019788       0.216571
  43       239.5      12000.0      0.019956       0.219897
  44       243.1      12000.0      0.020263       0.225909
  45       237.8      12000.0      0.019813       0.216990
  46       240.2      12000.0      0.020015       0.221604
  47       24[illegible]      12000.0      0.020029       0.221419
  48       240.5      12000.0      0.020050       0.221223

NUMBER IN FLEET = 100.0       INTRO INTERVAL = 120.0
REPORT INTERVAL = 120.0       SCALE =  56.0    SHAPE =3.00
MTBF =  50.0   NUMBER INTRO / PERIOD =  5.0
```

TABLE B-11

```
              AVG           OPS        FAIL PER
PERIOD        FAIL         HOURS       OPS HOUR          STD.DEV
   1          16.0          600.0      0.016667         0.151928
   2          22.3         1200.0      0.018563         0.185382
   3          32.8         1800.0      0.018236         0.186163
   4          46.1         2400.0      0.019188         0.201955
   5          57.5         3000.0      0.019192         0.203409
   6          69.2         3600.0      0.019229         0.203806
   7          81.9         4200.0      0.019512         0.211115
   8          93.4         4800.0      0.019453         0.210389
   9         105.4         5400.0      0.019520         0.209845
  10         118.2         6000.0      0.019700         0.214449
  11         131.5         6600.0      0.019936         0.217981
  12         141.4         7200.0      0.019639         0.212071
  13         153.2         7800.0      0.019641         0.213288
  14         167.0         8400.0      0.019881         0.219036
  15         178.4         9000.0      0.019820         0.216608
  16         190.2         9600.0      0.019810         0.215882
  17         203.2        10200.0      0.019919         0.218550
  18         214.7        10799.9      0.019880         0.217546
  19         224.5        11399.9      0.019693         0.214323
  20         238.9        11999.9      0.019908         0.218676
  21         239.0        12000.0      0.019921         0.219319
  22         239.9        12000.0      0.019992         0.221675
  23         239.7        12000.0      0.019975         0.220249
  24         241.4        12000.0      0.020113         0.222586
  25         238.1        12000.0      0.019842         0.218676
  26         238.4        12000.0      0.019865         0.218097
  27         241.6        12000.0      0.020131         0.223740
  28         240.6        12000.0      0.020048         0.223210
  29         241.0        12000.0      0.020087         0.223178
  30         241.3        12000.0      0.020106         0.223153
  31         240.4        12000.0      0.020031         0.222437
  32         242.1        12000.0      0.020175         0.223801
  33         239.9        12000.0      0.019990         0.222457
  34         241.2        12000.0      0.020098         0.222899
  35         239.2        12000.0      0.019935         0.219364
  36         236.6        12000.0      0.019721         0.214833
  37         240.5        12000.0      0.020042         0.222018
  38         239.1        12000.0      0.019925         0.220744
  39         239.2        12000.0      0.019935         0.219281
  40         241.0        12000.0      0.020083         0.222186
  41         240.0        12000.0      0.020000         0.219835
  42         238.2        12000.0      0.019848         0.218126
  43         239.7        12000.0      0.019977         0.220427
  44         242.4        12000.0      0.020200         0.224497
  45         238.4        12000.0      0.019863         0.216160
  46         239.9        12000.1      0.019996         0.221549
  47         240.3        12000.0      0.020027         0.220979
  48         240.6        12000.0      0.020052         0.221940

NUMBER IN FLEET = 100.0       INTRO INTERVAL = 120.0
REPORT INTERVAL = 120.0       SCALE =  55.2   SHAPE =4.00
MTBF =  50.0   NUMBER INTRO / PERIOD =  5.0
```

TABLE B-12

```
               AVG          OPS          FAIL PER
PERIOD         FAIL         HOURS        OPS HOUR         STD.DEV
   1            9.9          600.0       0.016500        0.149551
   2           22.3         1200.0       0.018625        0.187072
   3           32.6         1800.0       0.018195        0.184488
   4           46.0         2400.0       0.019156        0.201517
   5           57.7         3000.0       0.019242        0.204961
   6           69.0         3600.0       0.019160        0.202984
   7           82.2         4200.0       0.019571        0.211688
   8           93.0         4800.0       0.019375        0.209249
   9          105.6         5400.0       0.019565        0.209706
  10          118.4         6000.0       0.019725        0.215396
  11          130.9         6600.0       0.019837        0.215536
  12          141.5         7200.0       0.019653        0.213251
  13          153.1         7800.0       0.019631        0.212681
  14          166.2         8400.0       0.019783        0.216602
  15          178.1         9000.0       0.019795        0.216252
  16          189.8         9600.0       0.019768        0.215315
  17          203.6        10200.0       0.019961        0.219767
  18          213.9        10799.9       0.019803        0.216045
  19          225.2        11399.9       0.019752        0.215721
  20          238.6        11999.9       0.019888        0.218102
  21          238.4        12000.0       0.019865        0.217909
  22          239.7        12000.0       0.019975        0.220819
  23          240.6        12000.0       0.020048        0.222200
  24          241.5        12000.0       0.020129        0.223251
  25          238.1        12000.0       0.019844        0.218973
  26          238.8        12000.0       0.019902        0.217375
  27          240.7        12000.0       0.020060        0.222943
  28          240.9        12000.0       0.020077        0.222797
  29          241.1        12000.0       0.020092        0.223919
  30          240.1        12000.0       0.020013        0.221973
  31          240.4        12000.0       0.020029        0.222270
  32          241.8        12000.0       0.020152        0.223624
  33          239.3        12000.0       0.019940        0.220892
  34          241.8        12000.0       0.020152        0.224379
  35          239.1        12000.0       0.019927        0.218944
  36          237.2        12000.0       0.019769        0.216102
  37          240.1        12000.0       0.020008        0.221162
  38          240.0        12000.0       0.019998        0.221850
  39          239.4        12000.0       0.019952        0.220249
  40          240.3        12000.0       0.020069        0.221128
  41          239.7        12000.0       0.019975        0.219916
  42          239.1        12000.0       0.019925        0.219993
  43          238.8        12000.0       0.019902        0.219920
  44          242.0        12000.0       0.020167        0.224191
  45          238.7        12000.0       0.019894        0.218231
  46          240.3        12000.0       0.020021        0.221927
  47          240.1        12000.0       0.020013        0.221065
  48          240.7        12000.0       0.020060        0.222118

NUMBER IN FLEET = 100.0       INTRO INTERVAL = 120.0
REPORT INTERVAL = 120.0       SCALE =  54.5    SHAPE =5.00
MTBF     50.0   NUMBER INTRO / PERIOD =  5.0
```

```
                 AVG           OPS          FAIL PER
PERIOD          FAIL          HOURS         OPS HOUR          STD.DEV
   1             4.5           600.0        0.007456          0.039403
   2            10.6          1200.0        0.008854          0.041436
   3            16.3          1800.0        0.009069          0.042967
   4            21.9          2400.0        0.009125          0.045916
   5            28.7          3000.0        0.009558          0.050650
   6            33.7          3600.0        0.009354          0.047943
   7            38.7          4200.0        0.009220          0.047265
   8            47.2          4800.0        0.009839          0.052399
   9            50.7          5400.0        0.009394          0.048685
  10            58.1          6000.0        0.009683          0.052825
  11            64.1          6600.0        0.009708          0.052118
  12            70.2          7200.0        0.009750          0.052576
  13            77.0          7800.0        0.009872          0.053157
  14            81.5          8400.0        0.009699          0.052302
  15            89.8          9000.0        0.009975          0.054359
  16            94.9          9600.0        0.009891          0.053735
  17           100.8         10200.0        0.009882          0.053696
  18           105.2         10799.9        0.009738          0.052579
  19           112.4         11399.9        0.009858          0.054609
  20           120.7         11999.9        0.010056          0.054772
  21           120.1         12000.0        0.010008          0.056180
  22           120.8         12000.0        0.010069          0.055464
  23           120.9         12000.0        0.010075          0.   5916
  24           121.9         12000.0        0.010163          0.056211
  25           120.8         12000.0        0.010065          0.055859
  26           121.9         12000.0        0.010160          0.056318
  27           121.4         12000.0        0.010117          0.056246
  28           119.0         12000.0        0.009917          0.054338
  29           119.2         12000.0        0.009931          0.054772
  30           123.4         12000.0        0.010281          0.057361
  31           119.9         12000.0        0.009988          0.055577
  32           118.6         12000.0        0.009881          0.054097
  33           119.1         12000.0        0.009921          0.054427
  34           121.4         12000.0        0.010117          0.055964
  35           117.4         12000.0        0.009779          0.052322
  36           120.8         12000.0        0.010069          0.057384
  37           119.5         12000.0        0.009958          0.053841
  38           120.6         12000.0        0.010050          0.055690
  39           120.0         12000.0        0.010000          0.056293
  40           117.8         12000.0        0.009812          0.052734
  41           120.3         12000.0        0.010021          0.055163
  42           117.5         12000.0        0.009790          0.053799
  43           119.1         12000.0        0.009925          0.055359
  44           122.9         12000.0        0.010246          0.058371
  45           121.3         12000.0        0.010110          0.056570
  46           122.2         12000.0        0.010183          0.057497
  47           120.6         12000.0        0.010053          0.055573
  48           121.6         12000.0        0.010133          0.057565

NUMBER IN FLEET = 100.0       INTRO INTERVAL = 120.0
REPORT INTERVAL = 120.0       SCALE = 112.8    SHAPE =2.00
MTBF = 100.0    NUMBER INTRO / PERIOD =  5.0
```

TABLE B-13

TABLE B-14

```
                AVG          OPS         FAIL PER
PERIOD          FAIL         HOURS       OPS HOUR          STD.DEV
   1             4.1          600.0      0.006833          0.024860
   2            10.2         1200.0      0.008521          0.039618
   3            15.8         1800.0      0.008764          0.040143
   4            21.5         2400.0      0.008979          0.045444
   5            28.5         3000.0      0.009508          0.050326
   6            33.5         3600.0      0.009313          0.047444
   7            38.5         4200.0      0.009173          0.046687
   8            46.3         4800.0      0.009646          0.050990
   9            50.7         5400.0      0.009394          0.048937
  10            59.2         6000.0      0.009858          0.053714
  11            62.8         6600.0      0.009515          0.050647
  12            69.9         7200.0      0.009708          0.051781
  13            76.1         7800.0      0.009760          0.052077
  14            81.8         8400.0      0.009735          0.052661
  15            88.9         9000.0      0.009878          0.053206
  16            95.0         9600.0      0.009896          0.054186
  17            99.6        10200.0      0.009760          0.052663
  18           106.1        10799.9      0.009824          0.052677
  19           111.6        11399.9      0.009785          0.053496
  20           119.6        11999.9      0.009965          0.054875
  21           120.3        12000.0      0.010023          0.055304
  22           119.9        12000.0      0.009996          0.054746
  23           120.6        12000.0      0.010054          0.055400
  24           120.9        12000.0      0.010073          0.055698
  25           121.1        12000.0      0.010096          0.056465
  26           120.7        12000.0      0.010058          0.055214
  27           120.5        12000.0      0.010040          0.055637
  28           120.1        12000.0      0.010006          0.055272
  29           118.9        12000.0      0.009906          0.054255
  30           122.4        12000.0      0.010196          0.057096
  31           119.5        12000.0      0.009960          0.055567
  32           119.1        12000.0      0.009925          0.053939
  33           119.7        12000.0      0.009977          0.055400
  34           120.6        12000.0      0.010048          0.055416
  35           119.1        12000.0      0.009929          0.054023
  36           118.8        12000.0      0.009900          0.054997
  37           120.2        12000.0      0.010017          0.054887
  38           121.0        12000.0      0.010085          0.056197
  39           119.7        12000.0      0.009973          0.055386
  40           117.9        12000.0      0.009829          0.053476
  41           121.2        12000.0      0.010098          0.056041
  42           117.9        12000.0      0.009823          0.053914
  43           120.0        12000.0      0.010000          0.055648
  44           121.3        12000.0      0.010104          0.056659
  45           120.9        12000.0      0.010077          0.056348
  46           121.1        12000.0      0.010092          0.056096
  47           120.1        12000.0      0.010008          0.055854
  48           121.4        12000.0      0.010113          0.056492

NUMBER IN FLEET = 100.0        INTRO INTERVAL = 120.0
REPORT INTERVAL = 120.0        SCALE = 112.0    SHAPE =3.00
MTBF    100.0    NUMBER INTRO / PERIOD =   5.0
```

TABLE B-15

```
             AVG          OPS          FAIL PER
PERIOD       FAIL         HOURS        OPS HOUR        STD.DEV
   1          4.1          600.0       0.006833        0.025360
   2         10.1         1200.0       0.008438        0.036684
   3         15.7         1800.0       0.008736        0.040886
   4         21.2         2400.0       0.008813        0.043550
   5         28.9         3000.0       0.009625        0.050890
   6         33.0         3600.0       0.009174        0.046718
   7         38.8         4200.0       0.009232        0.047329
   8         45.8         4800.0       0.009542        0.049428
   9         50.7         5400.0       0.009384        0.048883
  10         59.2         6000.0       0.009858        0.053374
  11         63.4         6600.0       0.009610        0.051738
  12         69.3         7200.0       0.009629        0.051617
  13         75.7         7800.0       0.009705        0.051327
  14         82.1         8400.0       0.009774        0.052837
  15         87.1         9000.0       0.009683        0.051476
  16         95.1         9600.0       0.009904        0.054037
  17        100.1        10200.0       0.009814        0.053291
  18        106.2        10799.9       0.009831        0.053028
  19        111.8        11399.9       0.009835        0.053363
  20        118.7        11999.9       0.009892        0.054044
  21        120.2        12000.0       0.010019        0.055593
  22        120.1        12000.0       0.010004        0.055019
  23        119.9        12000.0       0.009992        0.054727
  24        120.8        12000.0       0.010065        0.055598
  25        120.5        12000.0       0.010042         .056143
  26        120.4        12000.0       0.010035        0.055307
  27        120.3        12000.0       0.010023        0.055315
  28        120.1        12000.0       0.010013        0.055132
  29        119.5        12000.0       0.009958         .054880
  30        121.3        12000.0       0.010108        0.056310
  31        119.8        12000.0       0.009983        0.055515
  32        119.9        12000.0       0.009987        0.054631
  33        119.7        12000.0       0.009973        0.055296
  34        119.9        12000.0       0.009988        0.054992
  35        120.0        12000.0       0.009998        0.055008
  36        118.6        12000.0       0.009887        0.054698
  37        120.8        12000.0       0.010065        0.055593
  38        119.9        12000.0       0.009992        0.054885
  39        119.0        12000.0       0.009915        0.054873
  40        119.0        12000.0       0.009915        0.054427
  41        121.6        12000.0       0.010131        0.056346
  42        118.2        12000.0       0.009850        0.054185
  43        120.4        12000.0       0.010029        0.055438
  44        119.8        12000.0       0.009987        0.055601
  45        121.7        12000.0       0.010144        0.057266
  46        119.9        12000.0       0.009993        0.054934
  47        120.0        12000.0       0.010002        0.055398
  48        121.3        12000.0       0.010108        0.056574

NUMBER IN FLEET = 100.0      INTRO INTERVAL = 120.0
REPAIR INTERVAL = 120.0      SCALE = 110.3    SHAPE = 4.00
MTTF = 100.0    NUMBER INTRO / PERIOD =  5.0
```

TABLE B-16

```
                AVG          OPS        FAIL PER
PERIOD          FAIL        HOURS       OPS HOUR        STD.DEV
   1             4.2        600.0       0.007000       0.026585
   2            10.1       1200.0       0.008396       0.039027
   3            15.7       1800.0       0.008736       0.041246
   4            21.3       2400.0       0.008875       0.043193
   5            28.2       3000.0       0.009408       0.046873
   6            33.3       3600.0       0.009264       0.047860
   7            39.0       4200.0       0.009286       0.047713
   8            45.2       4800.0       0.009411       0.048508
   9            51.0       5400.0       0.009440       0.048986
  10            58.8       6000.0       0.009800       0.052777
  11            63.9       6600.0       0.009682       0.053031
  12            69.1       7200.0       0.009597       0.050825
  13            75.6       7800.0       0.009692       0.051574
  14            82.5       8400.0       0.009824       0.052886
  15            86.8       9000.0       0.009639       0.051005
  16            94.8       9600.0       0.009870       0.053708
  17            99.3      10200.0       0.009738       0.052810
  18           106.8      10799.9       0.009887       0.053536
  19           111.5      11399.9       0.009783       0.053045
  20           118.2      11999.9       0.009850       0.053726
  21           119.9      12000.0       0.009992       0.055489
  22           119.7      12000.0       0.009975       0.054387
  23           120.1      12000.0       0.010013       0.055174
  24           121.0      12000.0       0.010083       0.055899
  25           120.0      12000.0       0.009998       0.055479
  26           120.5      12000.0       0.010044       0.055652
  27           119.9      12000.0       0.009994       0.054609
  28           119.8      12000.0       0.009979       0.055165
  29           120.6      12000.0       0.010050       0.055573
  30           120.5      12000.0       0.010048       0.055469
  31           120.1      12000.0       0.010013       0.056092
  32           118.9      12000.0       0.009906       0.053936
  33           121.1      12000.0       0.010096       0.056227
  34           119.1      12000.0       0.009921       0.054230
  35           120.1      12000.0       0.010006       0.055348
  36           119.0      12000.0       0.009917       0.054739
  37           121.3      12000.0       0.010108       0.056218
  38           119.7      12000.0       0.009973       0.054855
  39           118.7      12000.0       0.009892       0.054296
  40           119.4      12000.0       0.009946       0.054739
  41           121.1      12000.0       0.010092       0.055758
  42           118.8      12000.0       0.009898       0.054586
  43           120.3      12000.0       0.010021       0.055714
  44           120.2      12000.0       0.010017       0.055532
  45           121.5      12000.0       0.010123       0.055936
  46           119.6      12000.0       0.009965       0.054918
  47           119.4      12000.0       0.009946       0.054793
  48           121.4      12000.0       0.010113       0.056326

NUMBER IN FLEET = 100.0        INTRO INTERVAL = 120.0
REPORT INTERVAL = 120.0        SCALE = 108.9    SHAPE =5.00
MTBF = 100.0   NUMBER INTRO / PERIOD =  5.0
```

TABLE B-17

```
            AVG        OPS         FAIL PER
PERIOD      FAIL       HOURS       OPS HOUR        STD.DEV
   1         0,5       600,0       0,000833       0,000429
   2         1,5      1200,0       0,001271       0,000858
   3         2,3      1800,0       0,001306       0,000890
   4         3,8      2400,0       0,001563       0,001244
   5         5,3      3000,0       0,001750       0,001483
   6         6,1      3600,0       0,001708       0,001567
   7         7,2      4200,0       0,001720       0,001558
   8         8,0      4800,0       0,001677       0,001592
   9         9,3      5400,0       0,001727       0,001754
  10        11,0      6000,0       0,001829       0,001930
  11        11,6      6600,0       0,001765       0,001661
  12        12,3      7200,0       0,001705       0,001588
  13        13,7      7800,0       0,001753       0,001695
  14        14,8      8400,0       0,001765       0,001686
  15        17,1      9000,0       0,001900       0,002011
  16        17,9      9600,0       0,001862       0,001928
  17        19,5     10200,0       0,001907       0,001899
  18        21,0     10799,9       0,001947       0,002026
  19        21,5     11399,9       0,001890       0,002041
  20        22,8     11999,9       0,001902       0,001958
  21        21,9     12000,0       0,001823       0,001816
  22        22,1     12000,0       0,001844       0,001943
  23        23,2     12000,0       0,001935       0,002129
  24        24,1     12000,0       0,002010       0,002248
  25        24,5     12000,0       0,002044       0,002347
  26        24,2     12000,0       0,002015       0,002267
  27        23,3     12000,0       0,001940       0,002152
  28        24,6     12000,0       0,002048       0,002221
  29        24,1     12000,0       0,002008       0,002129
  30        23,7     12000,0       0,001977       0,002183
  31        24,3     12000,0       0,002027       0,002370
  32        23,6     12000,0       0,001967       0,002167
  33        25,0     12000,0       0,002085       0,002359
  34        23,1     12000,0       0,001927       0,002104
  35        23,8     12000,0       0,001981       0,002193
  36        24,0     12000,0       0,002000       0,002159
  37        23,9     12000,0       0,001995       0,002191
  38        25,5     12000,0       0,002125       0,002356
  39        25,2     12000,0       0,002098       0,002369
  40        23,3     12000,0       0,001940       0,002028
  41        24,0     12000,0       0,002004       0,002248
  42        23,8     12000,0       0,001983       0,002220
  43        25,1     12000,0       0,002090       0,002337
  44        23,5     12000,0       0,001958       0,002120
  45        24,2     12000,0       0,002019       0,002253
  46        24,2     12000,0       0,002012       0,002226
  47        24,5     12000,0       0,002044       0,002334
  48        24,1     12000,0       0,002010       0,002282

NUMBER IN FLEET = 100,0     INTRO INTERVAL = 120,0
REPORT INTERVAL = 120,0     SCALE = 553,9    SHAPE =1,50
MTBF = 500,0   NUMBER INTRO / PERIOD =  5,0
```

TABLE B-18

```
               AVG          OPS        FAIL PER
 PERIOD        FAIL        HOURS       OPS HOUR        STD.DEV
    1           0.3        600.0       0.000458        0.000097
    2           0.8       1200.0       0.000667        0.000260
    3           1.9       1800.0       0.001056        0.000599
    4           3.2       2400.0       0.001344        0.000967
    5           4.3       3000.0       0.001442        0.000963
    6           5.6       3600.0       0.001549        0.001281
    7           7.0       4200.0       0.001673        0.001431
    8           7.7       4800.0       0.001609        0.001512
    9           8.4       5400.0       0.001556        0.001349
   10          10.7       6000.0       0.001788        0.001814
   11          11.5       6600.0       0.001735        0.001668
   12          12.6       7200.0       0.001747        0.001678
   13          12.7       7800.0       0.001625        0.001436
   14          14.3       8400.0       0.001705        0.001592
   15          16.4       9000.0       0.001819        0.001876
   16          18.2       9600.0       0.001898        0.002019
   17          18.0      10200.0       0.001767        0.001615
   18          19.9      10799.9       0.001843        0.001805
   19          20.8      11399.9       0.001822        0.001878
   20          23.4      11999.9       0.001948        0.002106
   21          22.3      12000.0       0.001862        0.001866
   22          22.6      12000.0       0.001885        0.002060
   23          23.0      12000.0       0.001912        0.002027
   24          25.5      12000.0       0.002127        0.002582
   25          23.2      12000.0       0.001935        0.002062
   26          24.0      12000.0       0.001995        0.002182
   27          24.4      12000.0       0.002033        0.002321
   28          23.9      12000.0       0.001994        0.002132
   29          24.8      12000.0       0.002067        0.002388
   30          23.7      12000.0       0.001973        0.002140
   31          23.5      12000.0       0.001960        0.002127
   32          24.5      12000.0       0.002040        0.002285
   33          23.7      12000.0       0.001975        0.002107
   34          24.3      12000.0       0.002025        0.002255
   35          23.4      12000.0       0.001950        0.002189
   36          24.2      12000.0       0.002017        0.002205
   37          23.8      12000.0       0.001979        0.002139
   38          25.1      12000.0       0.002090        0.002390
   39          25.0      12000.0       0.002081        0.002299
   40          23.8      12000.0       0.001987        0.002143
   41          23.6      12000.0       0.001965        0.002180
   42          24.5      12000.0       0.002069        0.002378
   43          24.2      12000.0       0.002012        0.002283
   44          24.0      12000.0       0.001995        0.002114
   45          23.9      12000.0       0.001992        0.002195
   46          24.0      12000.0       0.001998        0.002159
   47          24.8      12000.0       0.002062        0.002367
   48          23.5      12000.0       0.001981        0.002178

NUMBER IN FLEET = 100.0       INTRO INTERVAL = 120.0
REPORT INTERVAL = 120.0       SCALE = 564.2    SHAPE =2.00
MTBF = 500.0   NUMBER INTRO / PERIOD =  5.0
```

PERIOD	AVG FAIL	OPS HOURS	FAIL PER OPS HOUR	STD.DEV
1	0,1	600,0	0,000125	0,000035
2	0,4	1200,0	0,000333	0,000079
3	1,3	1800,0	0,000722	0,000257
4	2,2	2400,0	0,000917	0,000439
5	4,5	3000,0	0,001500	0,001206
6	5,3	3600,0	0,001479	0,001123
7	6,3	4200,0	0,001494	0,001188
8	7,2	4800,0	0,001500	0,001213
9	8,6	5400,0	0,001588	0,001439
10	9,6	6000,0	0,001608	0,001531
11	11,6	6600,0	0,001758	0,001736
12	12,4	7200,0	0,001715	0,001602
13	13,0	7800,0	0,001670	0,001565
14	14,0	8400,0	0,001673	0,001466
15	15,7	9000,0	0,001742	0,001729
16	17,0	9600,0	0,001768	0,001748
17	17,6	10200,0	0,001723	0,001630
18	19,2	10799,9	0,001773	0,001643
19	22,3	11399,9	0,001952	0,002096
20	21,9	11999,9	0,001827	0,001909
21	22,7	12000,0	0,001894	0,001930
22	22,8	12000,0	0,001900	0,001995
23	24,3	12000,0	0,002023	0,002323
24	24,7	12000,0	0,002054	0,002353
25	23,4	12000,0	0,001950	0,002159
26	23,9	12000,0	0,001994	0,002152
27	24,0	12000,0	0,002000	0,002182
28	24,1	12000,0	0,002010	0,002325
29	24,0	12000,0	0,001998	0,002086
30	24,8	12000,0	0,002065	0,002400
31	23,9	12000,0	0,001992	0,002224
32	23,5	12000,0	0,001956	0,002063
33	24,3	12000,0	0,002027	0,002297
34	24,5	12000,0	0,002038	0,002302
35	23,5	12000,0	0,001960	0,002213
36	23,7	12000,0	0,001975	0,002140
37	24,5	12000,0	0,002044	0,002271
38	23,7	12000,0	0,001971	0,002153
39	24,6	12000,0	0,002050	0,002275
40	24,5	12000,0	0,002038	0,002201
41	24,0	12000,0	0,002000	0,002225
42	24,8	12000,0	0,002069	0,002350
43	23,7	12000,0	0,001975	0,002174
44	24,7	12000,0	0,002054	0,002354
45	23,8	12000,0	0,001983	0,002088
46	23,2	12000,0	0,001933	0,002142
47	25,0	12000,0	0,002081	0,002376
48	23,4	12000,0	0,001948	0,002133

NUMBER IN FLEET = 100,0 INTRO INTERVAL = 120,0
REPORT INTERVAL = 120,0 SCALE = 559,9 SHAPE =3,00
MTBF = 500,0 NUMBER INTRO / PERIOD = 5,0

TABLE B-19

TABLE B-20

```
            AVG          OPS       FAIL PER
PERIOD      FAIL        HOURS      OPS HOUR        STD,DEV
   1        0,          600,0      0,              0,
   2        0,3        1200,0      0,000229        0,000028
   3        0,7        1800,0      0,000403        0,000097
   4        2,4        2400,0      0,000990        0,000505
   5        4,1        3000,0      0,001383        0,001015
   6        5,4        3600,0      0,001507        0,001208
   7        5,7        4200,0      0,001357        0,000989
   8        7,1        4800,0      0,001469        0,001144
   9        8,8        5400,0      0,001620        0,001494
  10        9,5        6000,0      0,001508        0,001516
  11       11,0        6600,0      0,001670        0,001530
  12       13,0        7200,0      0,001799        0,001757
  13       12,2        7800,0      0,001561        0,001332
  14       14,3        8400,0      0,001705        0,001586
  15       15,6        9000,0      0,001736        0,001693
  16       17,0        9600,0      0,001766        0,001703
  17       17,3       10200,0      0,001696        0,001669
  18       19,3       10799,9      0,001785        0,001690
  19       21,7       11399,9      0,001904        0,001940
  20       22,0       11999,9      0,001835        0,001921
  21       22,4       12000,0      0,001867        0,001916
  22       22,9       12000,0      0,001906        0,001943
  23       24,8       12000,0      0,002065        0,002431
  24       24,5       12000,0      0,002046        0,002345
  25       24,4       12000,0      0,002031        0,002322
  26       22,8       12000,0      0,001898        0,001977
  27       24,0       12000,0      0,001998        0,002185
  28       24,8       12000,0      0,002065        0,002363
  29       24,3       12000,0      0,002023        0,002190
  30       23,3       12000,0      0,001944        0,002173
  31       24,8       12000,0      0,002065        0,002340
  32       23,3       12000,0      0,001946        0,002003
  33       24,6       12000,0      0,002050        0,002276
  34       23,2       12000,0      0,001935        0,002100
  35       25,5       12000,0      0,002127        0.002607
  36       22,3       12000,0      0,001855        0,001912
  37       25,3       12000,0      0,002110        0,002395
  38       22,8       12000,0      0,001900        0,001929
  39       25,0       12000,0      0,002079        0,002436
  40       24,5       12000,0      0,002037        0,002237
  41       23,8       12000,0      0,001961        0,002133
  42       25,9       12000,0      0,002156        0,002535
  43       23,1       12000,0      0,001925        0,002066
  44       23,8       12000,0      0,001987        0,002225
  45       23,5       12000,0      0,001913        0,002044
  46       24,3       12000,0      0,002029        0,002344
  47       23,4       12000,0      0,001948        0,002080
  48       24,1       12000,0      0,002008        0,002333

NUMBER IN FLEET = 100,0        INTRO INTERVAL = 120,0
REPORT INTERVAL = 120,0        SCALE = 551,6    SHAPE =4,00
MTBF = 500,0   NUMBER INTRO / PERIOD =  5,0
```

TABLE B-21

PERIOD	AVG FAIL	OPS HOURS	FAIL PER OPS HOUR	STD.DEV
1	0.	600,0	0.	0.
2	0,2	1200,0	0,000188	0,000016
3	0,5	1800,0	0,000264	0,000045
4	2,1	2400,0	0,000854	0,000395
5	4,5	3000,0	0,001500	0,001193
6	5,2	3600,0	0,001444	0,001118
7	5,8	4200,0	0,001381	0,001015
8	6,9	4800,0	0,001427	0,001092
9	8,5	5400,0	0,001588	0,001398
10	9,5	6000,0	0,001608	0,001527
11	10,6	6600,0	0,001614	0,001481
12	12,8	7200,0	0,001774	0,001713
13	12,7	7800,0	0,001628	0,001437
14	14,8	8400,0	0,001759	0,001719
15	15,0	9000,0	0,001661	0,001522
16	16,5	9600,0	0,001716	0,001645
17	18,0	10200,0	0,001760	0,001747
18	19,4	10799,9	0,001794	0,001716
19	21,0	11399,9	0,001844	0,001838
20	21,8	11999,9	0,001817	0,001855
21	22,5	12000,0	0,001875	0,001935
22	23,1	12000,0	0,001923	0,001993
23	25,3	12000,0	0,002108	0,002521
24	24,6	12000,0	0,002052	0,002364
25	23,2	12000,0	0,001933	0,002102
26	22,8	12000,0	0,001904	0,002017
27	24,6	12000,0	0,002052	0,002298
28	24,6	12000,0	0,002050	0,002329
29	24,5	12000,0	0,002037	0,002266
30	23,5	12000,0	0,001960	0,002143
31	24,3	12000,0	0,002029	0,002278
32	23,7	12000,0	0,001977	0,002145
33	24,0	12000,0	0,001998	0,002157
34	24,2	12000,0	0,002019	0,002209
35	24,5	12000,0	0,002044	0,002453
36	22,7	12000,0	0,001888	0,001953
37	24,8	12000,0	0,002067	0,002258
38	23,7	12000,0	0,001971	0,002114
39	24,5	12000,0	0,002042	0,002363
40	24,0	12000,0	0,002000	0,002158
41	24,3	12000,0	0,002027	0,0022 6
42	25,4	12000,0	0,002115	0,002443
43	23,5	12000,0	0,001958	0,002157
44	23,1	12000,0	0,001927	0,002091
45	24,6	12000,0	0,002050	0,002241
46	24,7	12000,0	0,002058	0,002340
47	23,2	12000,0	0,001935	0,002055
48	23,3	12000,0	0,001944	0,002219

NUMBER IN FLEET = 100,0   INTRO INTERVAL = 120,0
ABSORB INTERVAL = 120,0   SCALE = 544,6   SHAPE =5,00
MTBF = 500,0   NUMBER INTRO / PERIOD = 5,0

TABLE B-22

PERIOD	AVG FAIL	OPS HOURS	FAIL PER OPS HOUR	STD.DEV
1	0.3	600.0	0.000417	0.000084
2	0.5	1200.0	0.000375	0.000098
3	0.9	1800.0	0.000514	0.000123
4	1.5	2400.0	0.000635	0.000221
5	1.9	3000.0	0.000625	0.000193
6	2.7	3600.0	0.000743	0.000283
7	2.7	4200.0	0.000643	0.000195
8	3.9	4800.0	0.000802	0.000330
9	4.4	5400.0	0.000810	0.000360
10	4.9	6000.0	0.000808	0.000374
11	5.3	6600.0	0.000799	0.000374
12	6.1	7200.0	0.000851	0.000363
13	7.2	7800.0	0.000920	0.000480
14	7.0	8400.0	0.000836	0.000388
15	7.0	9000.0	0.000778	0.000347
16	7.4	9600.0	0.000768	0.000316
17	8.8	10200.0	0.000863	0.000416
18	8.8	10799.9	0.000813	0.000360
19	9.8	11399.9	0.000857	0.000411
20	10.4	11999.9	0.000863	0.000417
21	11.3	12000.0	0.000940	0.000464
22	11.4	12000.0	0.000948	0.000439
23	11.5	12000.0	0.000960	0.000525
24	12.0	12000.0	0.000996	0.000503
25	12.6	12000.0	0.001052	0.000606
26	11.6	12000.0	0.000969	0.000536
27	12.2	12000.0	0.001019	0.000567
28	12.6	12000.0	0.001054	0.000605
29	12.3	12000.0	0.001021	0.000588
30	12.4	12000.0	0.001031	0.000578
31	10.6	12000.0	0.000885	0.000408
32	11.0	12000.0	0.000912	0.000464
33	11.3	12000.0	0.000940	0.000506
34	12.1	12000.0	0.001006	0.000595
35	11.3	12000.0	0.000942	0.000487
36	11.3	12000.0	0.000940	0.000495
37	11.9	12000.0	0.000990	0.000557
38	12.4	12000.0	0.001031	0.000583
39	11.9	12000.0	0.000990	0.000541
40	12.6	12000.0	0.001052	0.000531
41	11.8	12000.0	0.000981	0.000521
42	12.0	12000.0	0.000993	0.000559
43	12.5	12000.0	0.001044	0.000611
44	11.9	12000.0	0.000988	0.000557
45	11.0	12000.0	0.000913	0.000427
46	12.2	12000.0	0.001019	0.000565
47	12.4	12000.0	0.001035	0.000590
48	12.4	12000.0	0.001033	0.000585

NUMBER IN FLEET = 100.0   INTRO INTERVAL = 120.0

REPORT INTERVAL = 120.0   SCALE =1107.7   SHAPE =1.50

MTBF =1000.0   NUMBER INTRO / PERIOD = 5.0

TABLE B-23

```
                 AVG        OPS       FAIL PER
PERIOD          FAIL       HOURS      OPS HOUR        STD.DEV
   1             0.2       600.0      0.000250        0.000033
   2             0.2      1200.0      0.000146        0.000010
   3             0.5      1800.0      0.000264        0.000048
   4             0.9      2400.0      0.000354        0.000061
   5             1.6      3000.0      0.000533        0.000152
   6             1.6      3600.0      0.000444        0.000098
   7             2.8      4200.0      0.000667        0.000222
   8             2.7      4800.0      0.000557        0.000153
   9             3.6      5400.0      0.000671        0.000238
  10             4.8      6000.0      0.000792        0.000350
  11             5.1      6600.0      0.000769        0.000322
  12             5.3      7200.0      0.000740        0.000311
  13             5.8      7800.0      0.000744        0.000301
  14             6.9      8400.0      0.000815        0.000355
  15             7.3      9000.0      0.000808        0.000390
  16             7.6      9600.0      0.000794        0.000364
  17             8.0     10200.0      0.000782        0.000333
  18             8.6     10799.9      0.000794        0.000342
  19             8.8     11399.9      0.000770        0.000314
  20            10.0     11999.9      0.000829        0.000395
  21            10.5     12000.0      0.000877        0.000441
  22            11.5     12000.0      0.000956        0.000471
  23            11.2     12000.0      0.000931        0.000481
  24            11.6     12000.0      0.000971        0.000513
  25            12.6     12000.0      0.001050        0.000586
  26            12.1     12000.0      0.001013        0.000558
  27            13.0     12000.0      0.001083        0.000616
  28            11.9     12000.0      0.000990        0.000566
  29            11.8     12000.0      0.000985        0.000554
  30            12.8     12000.0      0.001067        0.000620
  31            11.1     12000.0      0.000925        0.000470
  32            11.8     12000.0      0.000985        0.000503
  33            10.8     12000.0      0.000900        0.000455
  34            11.5     12000.0      0.000956        0.000520
  35            12.5     12000.0      0.001040        0.000620
  36            11.1     12000.0      0.000927        0.000487
  37            12.5     12000.0      0.001044        0.000636
  38            12.3     12000.0      0.001027        0.000603
  39            11.5     12000.0      0.000958        0.000515
  40            11.8     12000.0      0.000983        0.000527
  41            12.3     12000.0      0.001025        0.000539
  42            12.6     12000.0      0.001050        0.000627
  43            11.4     12000.0      0.000948        0.000491
  44            12.8     12000.0      0.001063        0.000635
  45            11.7     12000.0      0.000975        0.000550
  46            11.7     12000.0      0.000973        0.000533
  47            11.5     12000.0      0.000954        0.000454
  48            12.8     12000.0      0.001063        0.000628

NUMBER IN FLEET = 100.0      INTRO INTERVAL = 120.0
REPORT INTERVAL = 120.0      SCALE =1128.4    SHAPE =2.00
NTEP =1000.0   NUMBER INTRO / PERIOD =  5.0
```

TABLE B-24

```
                AVG          OPS        FAIL PER
PERIOD          FAIL         HOURS      OPS HOUR       STD.DEV
   1            0.           600.0      0.             0.
   2            0.1         1200.0      0.000063       0.000001
   3            0.2         1800.0      0.000111       0.000008
   4            0.4         2400.0      0.000156       0.000018
   5            0.7         3000.0      0.000225       0.000022
   6            1.3         3600.0      0.000361       0.000076
   7            1.9         4200.0      0.000458       0.000099
   8            2.5         4800.0      0.000526       0.000141
   9            3.2         5400.0      0.000597       0.000183
  10            4.0         6000.0      0.000663       0.000234
  11            4.8         6600.0      0.000720       0.000282
  12            5.3         7200.0      0.000736       0.000275
  13            5.6         7800.0      0.000721       0.000314
  14            6.0         8400.0      0.000711       0.000292
  15            7.2         9000.0      0.000803       0.000366
  16            7.4         9600.0      0.000768       0.000340
  17            8.0        10200.0      0.000782       0.000332
  18            8.2        10799.9      0.000759       0.000313
  19            9.0        11399.9      0.000789       0.000348
  20           10.1        11999.9      0.000844       0.000389
  21            8.8        12000.0      0.000735       0.000295
  22           10.9        12000.0      0.000910       0.000460
  23           12.3        12000.0      0.001021       0.000568
  24           12.4        12000.0      0.001033       0.000621
  25           11.8        12000.0      0.000981       0.000512
  26           12.1        12000.0      0.001006       0.000540
  27           11.8        12000.0      0.000985       0.000520
  28           12.7        12000.0      0.001058       0.000617
  29           12.6        12000.0      0.001050       0.000596
  30           11.5        12000.0      0.000958       0.000521
  31           13.0        12000.0      0.001085       0.000685
  32           11.5        12000.0      0.000965       0.000500
  33           11.9        12000.0      0.000990       0.000535
  34           11.0        12000.0      0.000915       0.000457
  35           11.1        12000.0      0.000923       0.000468
  36           12.1        12000.0      0.001013       0.000573
  37           12.8        12000.0      0.001067       0.000665
  38           11.8        12000.0      0.000981       0.000540
  39           12.2        12000.0      0.001019       0.000574
  40           11.6        12000.0      0.000967       0.000521
  41           12.5        12000.0      0.001038       0.000597
  42           11.9        12000.0      0.000994       0.000567
  43           11.9        12000.0      0.000986       0.000527
  44           12.3        12000.0      0.001023       0.000572
  45           11.5        12000.0      0.000960       0.000493
  46           11.9        12000.0      0.000994       0.000554
  47           12.5        12000.0      0.001038       0.000525
  48           12.1        12000.0      0.001013       0.000516

NUMBER IN FLEET = 100.0       INTRO INTERVAL = 120.0
REPORT INTERVAL = 120.0       SCALE =1119.6    SHAPE =3.00
MTBF =1000.0    NUMBER INTRO / PERIOD =  5.0
```

PERIOD	AVG FAIL	OPS HOURS	FAIL PER OPS HOUR	STD.DEV
1	0.	600,0	0.	0.
2	0.	1200,0	0.	0.
3	0,2	1800,0	0,000097	0,000036
4	0,1	2400,0	0,000052	0,000031
5	0,4	3000,0	0,000133	0,000014
6	0,8	3600,0	0,000229	0,000021
7	1,6	4200,0	0,000369	0,000022
8	2,3	4800,0	0,000484	0,000119
9	3,3	5400,0	0,000602	0,000176
10	4,2	6000,0	0,000704	0,000269
11	4,5	6600,0	0,000686	0,000250
12	4,8	7200,0	0,000663	0,000242
13	5,7	7800,0	0,000734	0,000286
14	6,2	8400,0	0,000738	0,000339
15	6,0	9000,0	0,000669	0,000264
16	7,9	9600,0	0,000818	0,000372
17	8,1	10200,0	0,000799	0,000343
18	8,2	10799,9	0,000759	0,000319
19	9,1	11399,9	0,000803	0,000354
20	9,1	11999,9	0,000760	0,000316
21	9,8	12000,0	0,000815	0,000354
22	10,6	12000,0	0,000883	0,000429
23	11,8	12000,0	0,000983	0,000555
24	12,5	12000,0	0,001044	0,000600
25	11,7	12000,0	0,000977	0,000539
26	11,6	12000,0	0,000965	0,000512
27	13,0	12000,0	0,001081	0,000592
28	12,8	12000,0	0,001065	0,000636
29	12,5	12000,0	0,001038	0,000598
30	12,5	12000,0	0,001044	0,000596
31	11,5	12000,0	0,000963	0,000537
32	11,7	12000,0	0,000975	0,000513
33	11,1	12000,0	0,000927	0,000462
34	11,4	12000,0	0,000952	0,000494
35	11,7	12000,0	0,000975	0,000524
36	12,5	12000,0	0,001040	0,000628
37	12,5	12000,0	0,001052	0,000633
38	11,7	12000,0	0,000973	0,000552
39	12,4	12000,0	0,001031	0,000552
40	12,0	12000,0	0,001002	0,000572
41	12,0	12000,0	0,000996	0,000554
42	11,8	12000,0	0,000979	0,000522
43	11,6	12000,0	0,000969	0,000511
44	11,9	12000,0	0,000994	0,000547
45	12,0	12000,0	0,000996	0,000554
46	11,9	12000,0	0,000994	0,000563
47	13,1	12000,0	0,001096	0,000636
48	12,1	12000,0	0,001006	0,000553

NUMBER IN FLEET = 100.0   INTRO INTERVAL = 120,0
REPORT INTERVAL = 120,0   SCALE =1103,3   SHAPE =4,00
MTBF =1000,0   NUMBER INTRO / PERIOD = 5,0

TABLE B-25

TABLE B-26

PERIOD	AVG FAIL	OPS HOURS	FAIL PER OPS HOUR	STD,DEV
1	0,	600,0	0,	0,
2	0,	1200,0	0,	0,
3	0,0	1800,0	0,000014	0.000000
4	0,2	2400,0	0,000094	0,000005
5	0,1	3000,0	0,000042	0,000001
6	0,6	3600,0	0,000167	0,000018
7	1,4	4200,0	0,000339	0.000055
8	2,0	4800,0	0,000411	0,000095
9	3.5	5400,0	0,000644	0,000209
10	4,1	6000,0	0,000666	0,000246
11	4,9	6600,0	0,000742	0,000289
12	4,4	7200,0	0,000611	0,000212
13	5,8	7800,0	0,000737	0,000298
14	5,9	8400,0	0,000702	0,000280
15	6,3	9000,0	0,000703	0,000303
16	7,2	9600,0	0,000753	0,000311
17	8,0	10200,0	0,000787	0,000339
18	8,6	10799,9	0,000794	0,000353
19	8,5	11399,9	0,000750	0,000304
20	9,9	11999,9	0,000827	0,000371
21	9,5	12000,0	0,000790	0,000331
22	11,2	12000,0	0,000933	0,000486
23	10,6	12000,0	0,000883	0,000457
24	12,5	12000,0	0,001040	0,000594
25	12,0	12000,0	0,000998	0,000569
26	12,3	12000,0	0,001027	0,000563
27	13,1	12000,0	0,001094	0,000636
28	12,9	12000,0	0,001075	0,000625
29	12,5	12000,0	0,001040	0.000565
30	11,3	12000,0	0,000944	0,000511
31	11,6	12000,0	0,000969	0,000528
32	11,8	12000,0	0,000983	0,000542
33	10,5	12000,0	0,000881	0,000424
34	12,4	12000,0	0,001029	0,000562
35	12,0	12000,0	0,000996	0,000551
36	12,2	12000,0	0,001017	0,000594
37	12,4	12000,0	0,001031	0,000566
38	13,0	12000,0	0,001079	0,000563
39	12,1	12000,0	0.001005	0,000564
40	11,3	12000,0	0,000942	0,000490
41	11,9	12000,0	0,000994	0,000557
42	11,7	12000,0	0,000975	0,000530
43	11,9	12000,0	0,000988	0,000533
44	11,7	12000,0	0,000973	0,000516
45	12,9	12000,0	0,001077	0,000548
46	11,5	12000,0	0,000953	0,000506
47	12,8	12000,0	0,001065	0,000613
48	12,0	12000,0	0,001000	0,000555

```
NUMBER IN FLEET = 100,0       INTRO INTERVAL = 120,0
REPORT INTERVAL = 120,0       SCALE =1089,1    SHAPE =5,00
MTBF =1000,0   NUMBER INTRO / PERIOD =  5,0
```

TABLE 3-27

```
               AVG          OPS          FAIL PER
PERIOD         FAIL         HOURS        OPS HOUR         STD.DEV
   1           0.2           600.0       0.000250         0.000033
   2           0.2          1200.0       0.000125         0.000028
   3           0.4          1800.0       0.000194         0.000026
   4           0.4          2400.0       0.000167         0.000014
   5           0.8          3000.0       0.000258         0.000035
   6           1.0          3600.0       0.000264         0.000036
   7           1.3          4200.0       0.000310         0.000048
   8           1.3          4800.0       0.000261         0.000037
   9           1.5          5400.0       0.000287         0.000037
  10           1.6          6000.0       0.000271         0.000042
  11           2.6          6600.0       0.000398         0.000082
  12           2.1          7200.0       0.000295         0.000046
  13           2.8          7800.0       0.000362         0.000087
  14           3.4          8400.0       0.000405         0.000081
  15           3.5          9000.0       0.000392         0.000088
  16           3.3          9600.0       0.000349         0.000074
  17           4.5         10200.0       0.000449         0.000133
  18           3.9         10799.9       0.000366         0.000073
  19           4.4         11399.9       0.000386         0.000086
  20           4.8         11999.9       0.000398         0.000066
  21           4.7         12000.0       0.000394         0.000087
  22           5.1         12000.0       0.000429         0.000102
  23           5.3         12000.0       0.000444         0.000105
  24           4.6         12000.0       0.000381         0.000078
  25           4.6         12000.0       0.000388         0.000076
  26           4.8         12000.0       0.000398         0.000093
  27           5.4         12000.0       0.000446         0.000117
  28           5.6         12000.0       0.000465         0.000113
  29           6.0         12000.0       0.000500         0.000129
  30           6.3         12000.0       0.000527         0.000158
  31           5.7         12000.0       0.000477         0.000126
  32           4.9         12000.0       0.000412         0.000095
  33           6.5         12000.0       0.000544         0.000161
  34           5.8         12000.0       0.000483         0.000125
  35           6.0         12000.0       0.000498         0.000137
  36           5.8         12000.0       0.000485         0.000126
  37           6.4         12000.0       0.000529         0.000163
  38           6.2         12000.0       0.000515         0.000134
  39           5.6         12000.0       0.000465         0.000115
  40           7.0         12000.0       0.000585         0.000135
  41           6.1         12000.0       0.000510         0.000138
  42           5.3         12000.0       0.000479         0.000125
  43           6.1         12000.0       0.000504         0.000154
  44           6.7         12000.0       0.000560         0.000159
  45           5.9         12000.0       0.000490         0.000146
  46           6.1         12000.0       0.000513         0.000134
  47           6.0         12000.0       0.000509         0.000146
  48           6.4         12000.0       0.000529         0.000153

NUMBER IN FLEET = 100.0     INTRO INTERVAL = 120.0
REPORT INTERVAL = 120.0     SCALE =2215.5   SHAPE =1.50
MTBF =2000.0   NUMBER INTRO / PERIOD =  5.0
```

```
                  AVG          OPS        FAIL PER
   PERIOD         FAIL        HOURS       OPS HOUR           STD.DEV
      1           0.           600.0      0.                 0.
      2           0.2         1200.0      0.000146           0.000013
      3           0.1         1800.0      0.000069           0.000002
      4           0.2         2400.0      0.000063           0.000004
      5           0.4         3000.0      0.000117           0.000006
      6           0.5         3600.0      0.000146           0.000009
      7           0.6         4200.0      0.000149           0.000012
      8           1.0         4800.0      0.000208           0.000023
      9           1.2         5400.0      0.000218           0.000020
     10           1.3         6000.0      0.000221           0.000026
     11           1.6         6600.0      0.000235           0.000025
     12           1.5         7200.0      0.000208           0.000026
     13           2.5         7800.0      0.000317           0.000049
     14           2.1         8400.0      0.000250           0.000036
     15           2.8         9000.0      0.000314           0.000062
     16           3.6         9600.0      0.000370           0.000069
     17           3.2        10200.0      0.000316           0.000054
     18           3.8        10799.9      0.000352           0.000078
     19           3.9        11399.9      0.000340           0.000062
     20           4.8        11999.9      0.000400           0.000086
     21           4.1        12000.0      0.000346           0.000067
     22           5.0        12000.0      0.000415           0.000087
     23           4.6        12000.0      0.000385           0.000086
     24           4.8        12000.0      0.000400           0.000092
     25           5.4        12000.0      0.000452           0.000116
     26           5.0        12000.0      0.000417           0.000094
     27           4.9        12000.0      0.000406           0.000087
     28           5.5        12000.0      0.000456           0.000118
     29           5.1        12000.0      0.000429           0.000112
     30           5.9        12000.0      0.000494           0.000132
     31           5.7        12000.0      0.000475           0.000136
     32           6.8        12000.0      0.000565           0.000168
     33           5.7        12000.0      0.000473           0.000122
     34           6.1        12000.0      0.000506           0.000145
     35           6.2        12000.0      0.000519           0.000143
     36           5.9        12000.0      0.000492           0.000135
     37           6.1        12000.0      0.000510           0.000148
     38           5.9        12000.0      0.000492           0.000129
     39           6.3        12000.0      0.000521           0.000136
     40           6.3        12000.0      0.000521           0.000157
     41           6.1        12000.0      0.000508           0.000137
     42           5.2        12000.0      0.000433           0.000092
     43           6.9        12000.0      0.000573           0.000179
     44           6.7        12000.0      0.000560           0.000170
     45           5.9        12000.0      0.000488           0.000129
     46           6.4        12000.0      0.000531           0.000153
     47           5.5        12000.0      0.000455           0.000120
     48           6.4        12000.0      0.000535           0.000168

   NUMBER IN FLEET = 100.0      INTRO INTERVAL = 120.0
   REPORT INTERVAL = 120.0      SCALE =2256.8    SHAPE =2.00
   MTBF =2000.0   NUMBER INTRO / PERIOD =  5.0
```

TABLE B-28

TABLE B-29

```
              AVG        OPS      FAIL PER
 PERIOD      FAIL       HOURS     OPS HOUR         STD.DEV
    1        0.          600.0    0.               0.
    2        0.         1200.0    0.               0.
    3        0.0        1800.0    0.000014         0.000000
    4        0.1        2400.0    0.000031         0.000000
    5        0.2        3000.0    0.000067         0.000003
    6        0.1        3600.0    0.000021         0.000000
    7        0.2        4200.0    0.000042         0.000001
    8        0.5        4800.0    0.000094         0.000005
    9        0.5        5400.0    0.000097         0.000004
   10        0.9        6000.0    0.000154         0.000014
   11        1.0        6600.0    0.000144         0.000010
   12        1.3        7200.0    0.000181         0.000017
   13        1.4        7800.0    0.000179         0.000012
   14        1.8        8400.0    0.000208         0.000025
   15        2.0        9000.0    0.000219         0.000023
   16        2.5        9600.0    0.000260         0.000035
   17        3.4       10200.0    0.000331         0.000058
   18        3.3       10799.9    0.000308         0.000051
   19        3.4       11399.9    0.000303         0.000053
   20        4.2       11999.9    0.000346         0.000058
   21        4.4       12000.0    0.000363         0.000073
   22        4.4       12000.0    0.000369         0.000065
   23        4.6       12000.0    0.000387         0.000084
   24        5.1       12000.0    0.000427         0.000106
   25        5.1       12000.0    0.000429         0.000116
   26        5.6       12000.0    0.000469         0.000121
   27        5.1       12000.0    0.000429         0.000099
   28        5.5       12000.0    0.000456         0.000124
   29        5.7       12000.0    0.000477         0.000118
   30        6.6       12000.0    0.000550         0.000156
   31        6.1       12000.0    0.000508         0.000149
   32        5.1       12000.0    0.000429         0.000105
   33        5.5       12000.0    0.000460         0.000116
   34        6.5       12000.0    0.000544         0.000163
   35        6.7       12000.0    0.000558         0.000157
   36        6.6       12000.0    0.000550         0.000171
   37        5.8       12000.0    0.000483         0.000135
   38        6.6       12000.0    0.000546         0.000153
   39        6.3       12000.0    0.000527         0.000159
   40        5.8       12000.0    0.000481         0.000132
   41        6.2       12000.0    0.000519         0.000151
   42        5.4       12000.0    0.000454         0.000107
   43        5.9       12000.0    0.000496         0.000124
   44        5.9       12000.0    0.000486         0.000124
   45        5.6       12000.0    0.000465         0.000128
   46        6.0       12000.0    0.000498         0.000137
   47        6.4       12000.0    0.000529         0.000150
   48        6.3       12000.0    0.000523         0.000139

NUMBER IN FLEET = 100.0      INFO INTERVAL = 120.0
REPORT INTERVAL = 120.0      SCALE =2239.7    SHAPE =3.00
MTBF =2000.0  NUMBER INFO / PERIOD =  5.0
```

```
                    AVG         OPS        FAIL PER
   PERIOD          FAIL        HOURS      OPS HOUR          STD.DEV
      1             0.          600.0      0.                0.
      2             0.         1200.0      0.                0.
      3             0.         1800.0      0.                0.
      4             0.         2400.0      0.                0.
      5             0.1        3000.0      0.000017          0.000000
      6             0.1        3600.0      0.000035          0.000001
      7             0.2        4200.0      0.000036          0.000001
      8             0.1        4800.0      0.000010          0.000000
      9             0.3        5400.0      0.000051          0.000002
     10             0.5        6000.0      0.000075          0.000002
     11             0.7        6600.0      0.000098          0.000005
     12             0.9        7200.0      0.000128          0.000009
     13             1.3        7800.0      0.000173          0.000016
     14             1.1        8400.0      0.000134          0.000009
     15             2.2        9000.0      0.000247          0.000027
     16             1.9        9600.0      0.000201          0.000022
     17             2.9       10200.0      0.000284          0.000043
     18             3.0       10799.9      0.000280          0.000042
     19             4.2       11399.9      0.000368          0.000078
     20             3.9       11999.9      0.000327          0.000059
     21             4.4       12000.0      0.000365          0.000077
     22             4.1       12000.0      0.000346          0.000065
     23             5.1       12000.0      0.000427          0.000091
     24             4.8       12000.0      0.000402          0.000102
     25             5.5       12000.0      0.000456          0.000118
     26             5.6       12000.0      0.000467          0.000136
     27             5.6       12000.0      0.000469          0.000121
     28             5.3       12000.0      0.000438          0.000106
     29             6.1       12000.0      0.000506          0.000128
     30             5.6       12000.0      0.000465          0.000121
     31             6.5       12000.0      0.000542          0.000168
     32             6.2       12000.0      0.000517          0.000133
     33             5.5       12000.0      0.000458          0.000118
     34             6.3       12000.0      0.000521          0.000146
     35             6.4       12000.0      0.000533          0.000155
     36             6.9       12000.0      0.000579          0.000189
     37             6.6       12000.0      0.000546          0.000175
     38             5.9       12000.0      0.000496          0.000137
     39             6.1       12000.0      0.000504          0.000135
     40             6.0       12000.0      0.000500          0.000152
     41             5.6       12000.0      0.000465          0.000120
     42             5.1       12000.0      0.000425          0.000089
     43             5.5       12000.0      0.000455          0.000115
     44             6.0       12000.0      0.000500          0.000131
     45             6.1       12000.0      0.000512          0.000138
     46             5.9       12000.0      0.000488          0.000133
     47             6.6       12000.0      0.000546          0.000153
     48             6.3       12000.0      0.000527          0.000127

 NUMBER IN FLEET = 100.0       INTRO INTERVAL = 120.0
 REPORT INTERVAL = 120.0       SCALE =2206.5    SHAPE =4.00
 MTBF =2000.0   NUMBER INTRO / PERIOD =  5.0
```

TABLE B-30

TABLE B-31

PERIOD	AVG FAIL	OPS HOURS	FAIL PER OPS HOUR	STD.DEV
1	0.	600.0	0.	0.
2	0.	1200.0	0.	0.
3	0.	1800.0	0.	0.
4	0.	2400.0	0.	0.
5	0.	3000.0	0.	0.
6	0.1	3600.0	0.000014	0.000020
7	0.1	4200.0	0.000012	0.000010
8	0.2	4800.0	0.000036	0.000031
9	0.1	5400.0	0.000019	0.000010
10	0.1	6000.0	0.000021	0.000000
11	0.5	6600.0	0.000072	0.000033
12	0.6	7200.0	0.000083	0.000033
13	1.0	7800.0	0.000122	0.000058
14	1.7	8400.0	0.000196	0.000019
15	1.3	9000.0	0.000144	0.000011
16	2.3	9600.0	0.000245	0.000028
17	2.3	10200.0	0.000223	0.000027
18	3.2	10799.9	0.000294	0.000047
19	4.2	11399.9	0.000366	0.000074
20	4.1	11999.9	0.000342	0.000064
21	4.4	12000.0	0.000365	0.000074
22	4.7	12000.0	0.000390	0.000090
23	5.0	12000.0	0.000415	0.000097
24	4.8	12000.0	0.000402	0.000094
25	5.3	12000.0	0.000442	0.000115
26	5.6	12000.0	0.000465	0.000118
27	5.4	12000.0	0.000448	0.000115
28	6.0	12000.0	0.000498	0.000139
29	5.4	12000.0	0.000448	0.000178
30	6.4	12000.0	0.000538	0.000169
31	5.9	12000.0	0.000490	0.000130
32	6.1	12000.0	0.000508	0.000125
33	6.1	12000.0	0.000513	0.000147
34	6.3	12000.0	0.000527	0.000147
35	6.9	12000.0	0.000573	0.000181
36	6.4	12000.0	0.000529	0.000161
37	6.6	12000.0	0.000550	0.000168
38	6.3	12000.0	0.000521	0.000156
39	5.6	12000.0	0.000465	0.000135
40	5.5	12000.0	0.000460	0.000116
41	5.3	12000.0	0.000442	0.000111
42	5.6	12000.0	0.000471	0.000123
43	5.3	12000.0	0.000438	0.000097
44	6.4	12000.0	0.000531	0.000167
45	5.6	12000.0	0.000471	0.000119
46	6.1	12000.0	0.000513	0.000134
47	6.3	12000.0	0.000523	0.000143
48	6.4	12000.0	0.000536	0.000150

```
NUMBER IN FLEET = 100.0      INTRO INTERVAL = 120.0
PRIORT INTERVAL = 120.0      SCALE =2178.2    SHAPE =5.00
MTBF =2000.0   NUMBER INTRO / PERIOD =  5.0
```

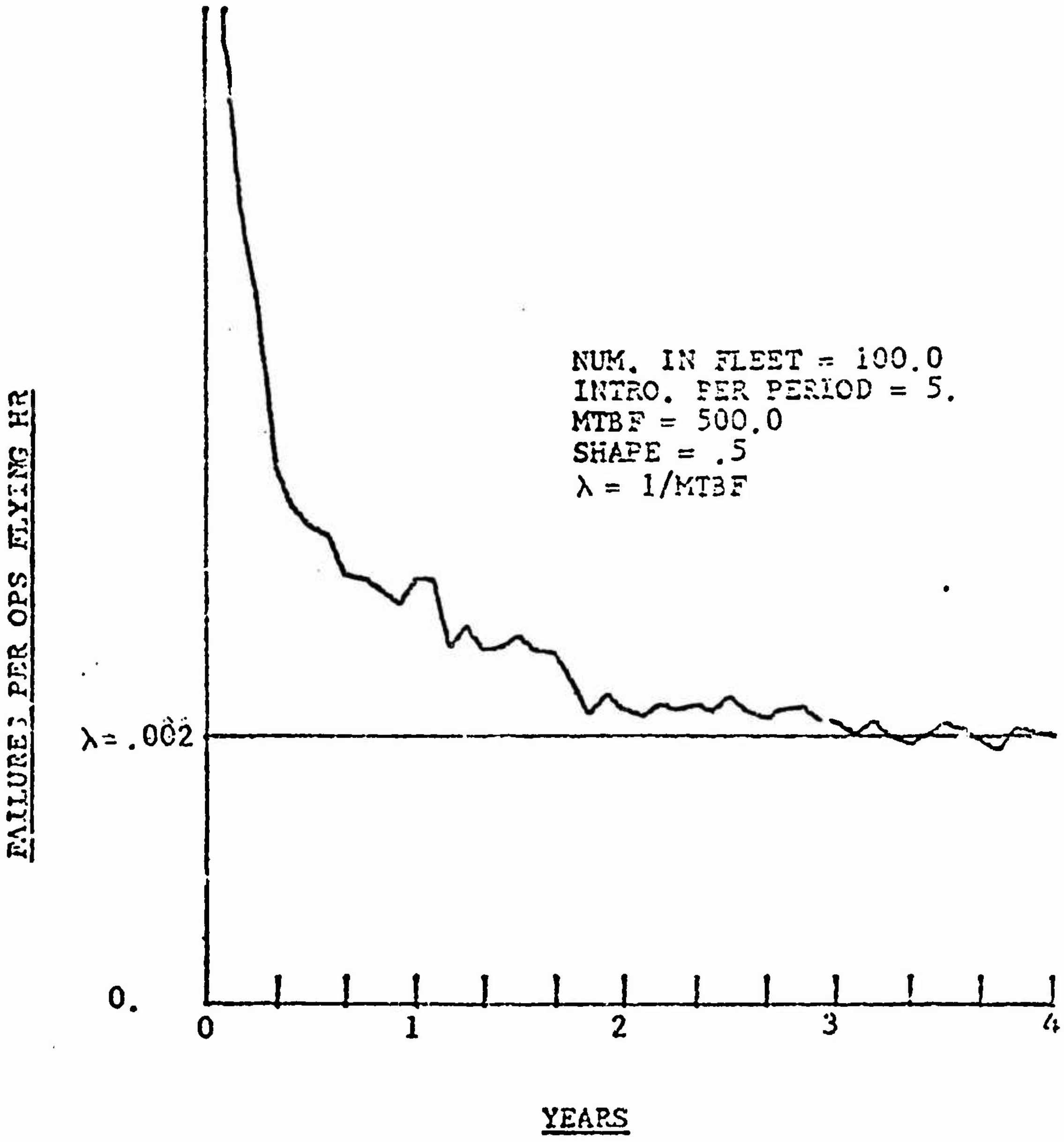


Figure 6

Decreasing Failure Rate

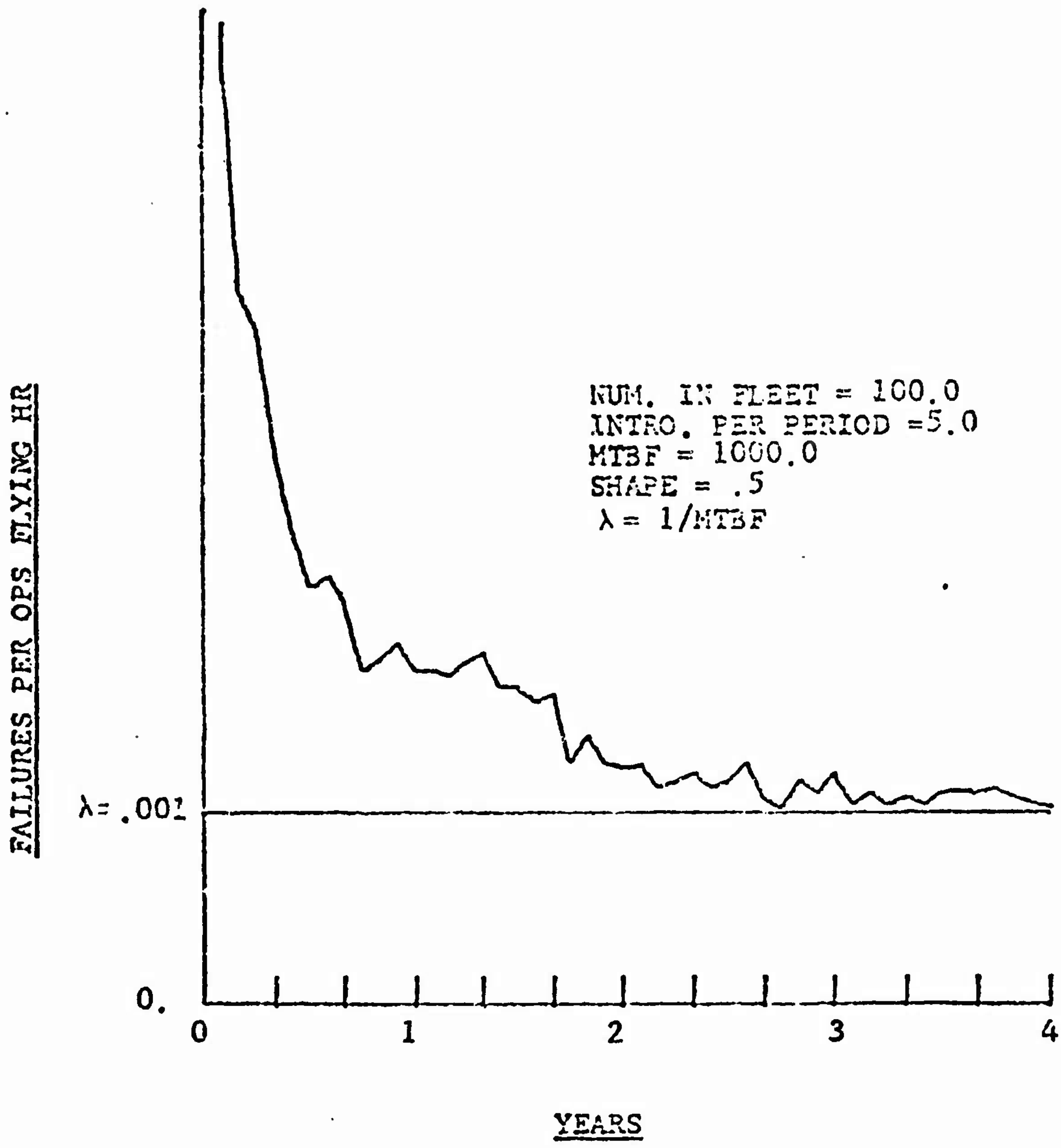


Figure 7

Decreasing Failure Rate

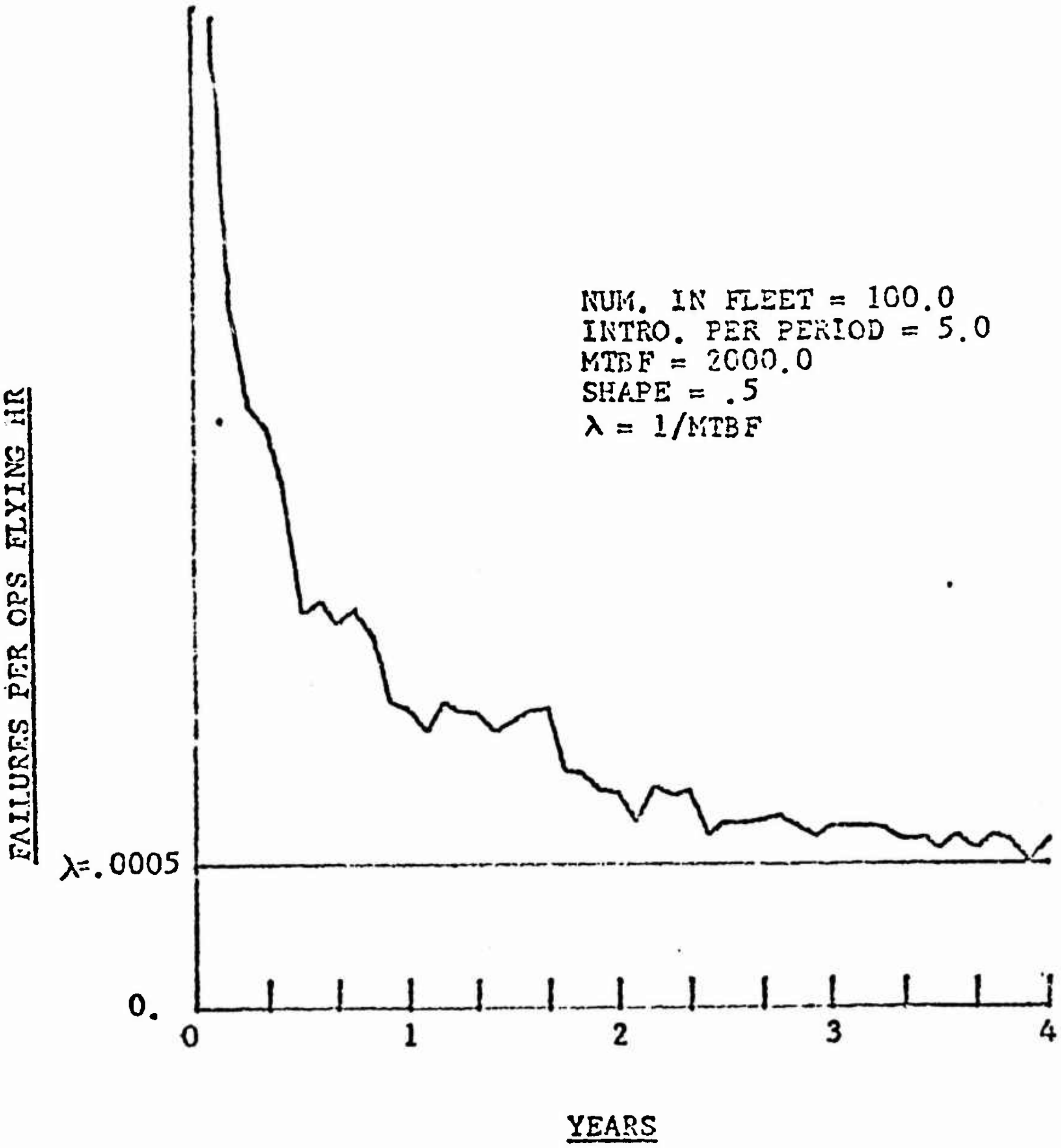


Figure 8

Decreasing Failure Rate

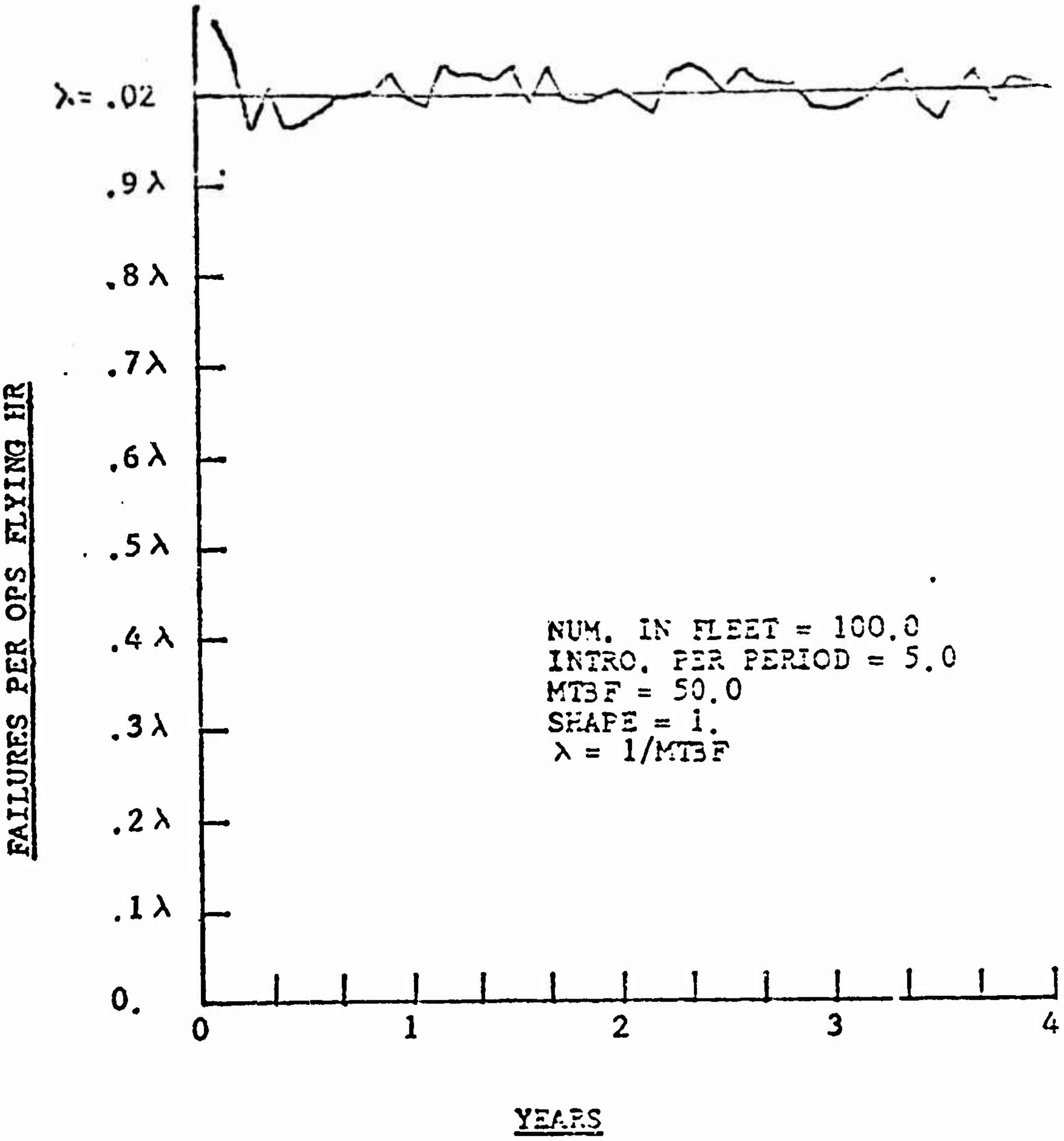


Figure 9

Constant Failure Rate

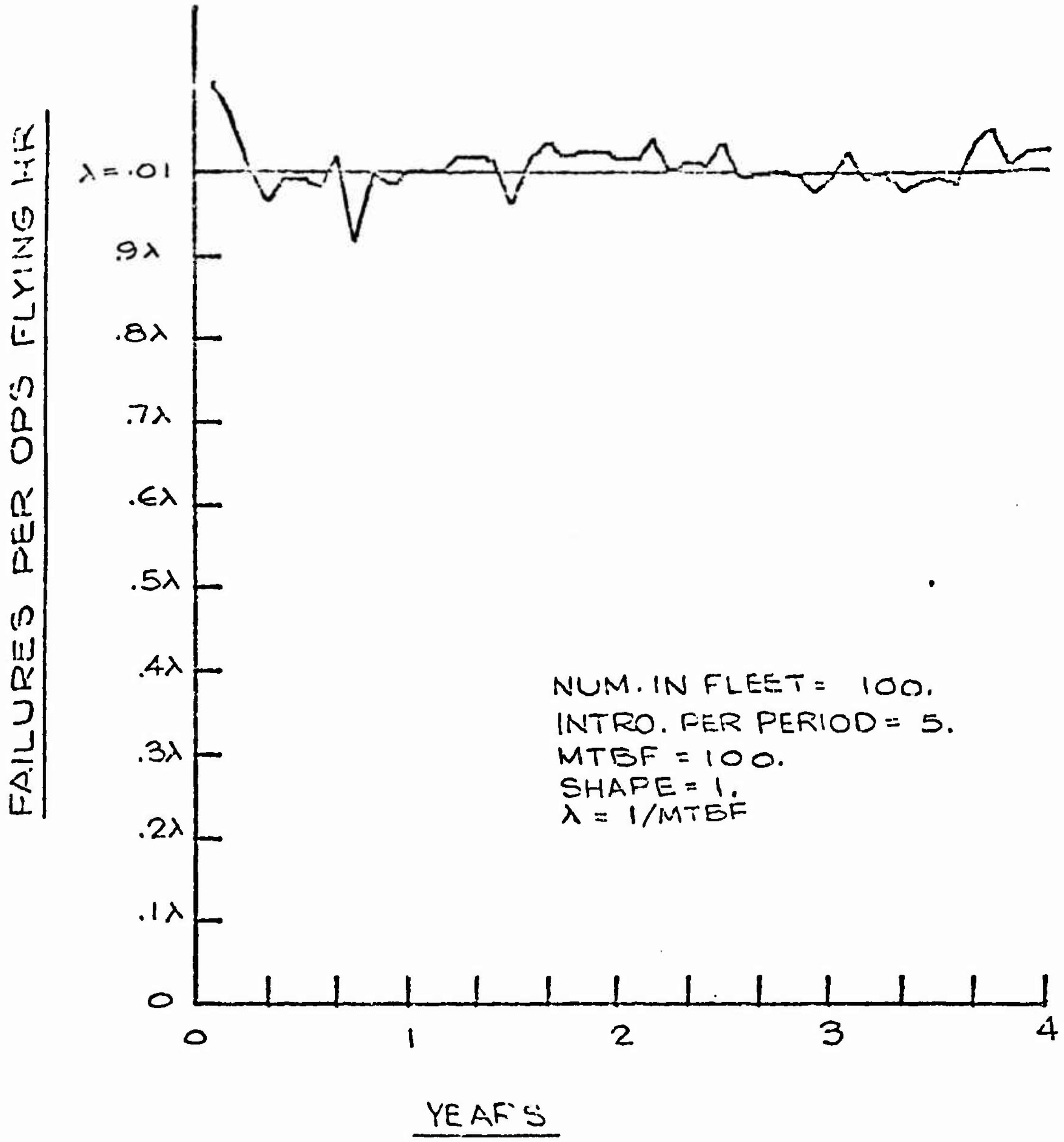


Figure 10

Constant Failure Rate

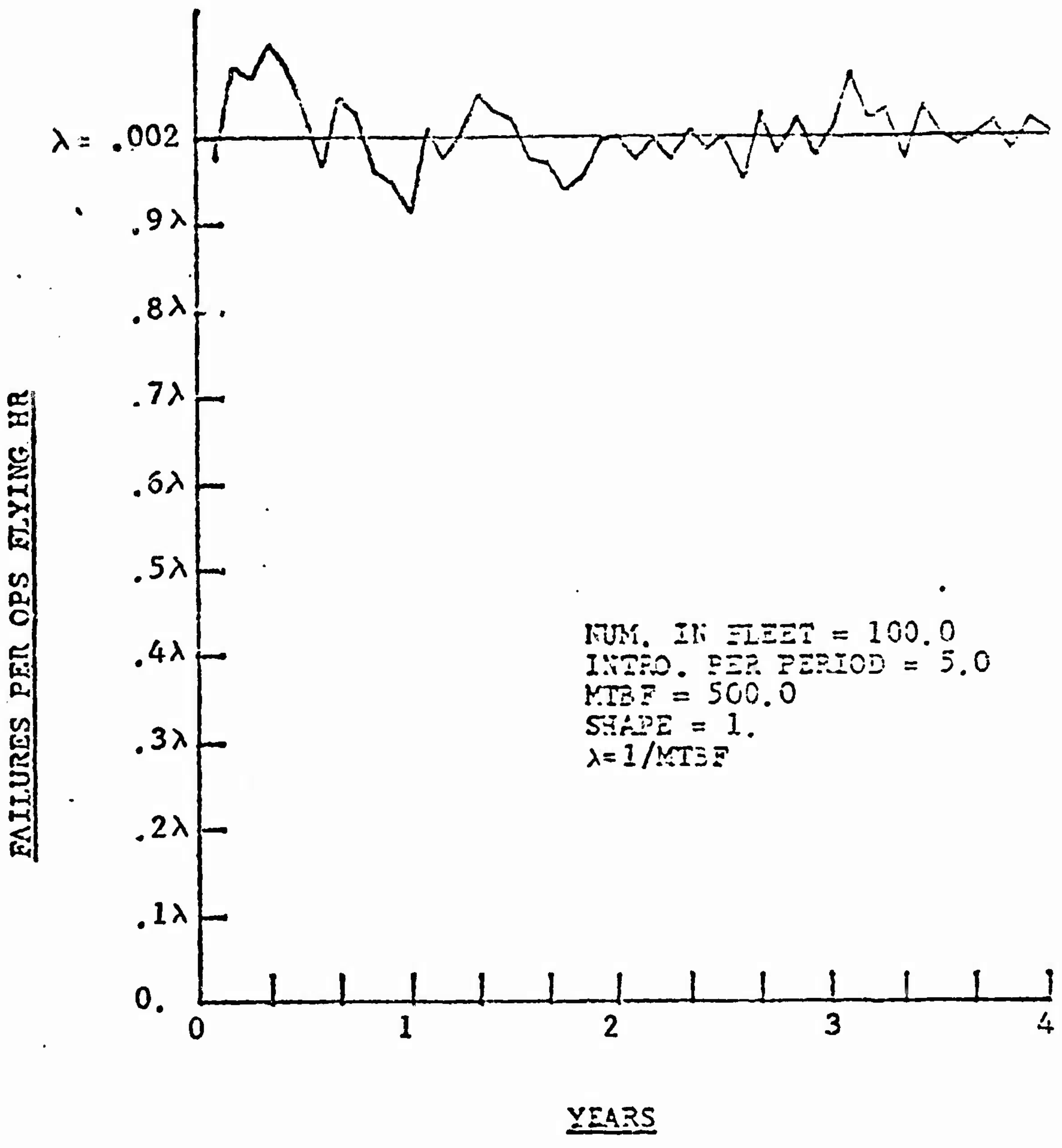
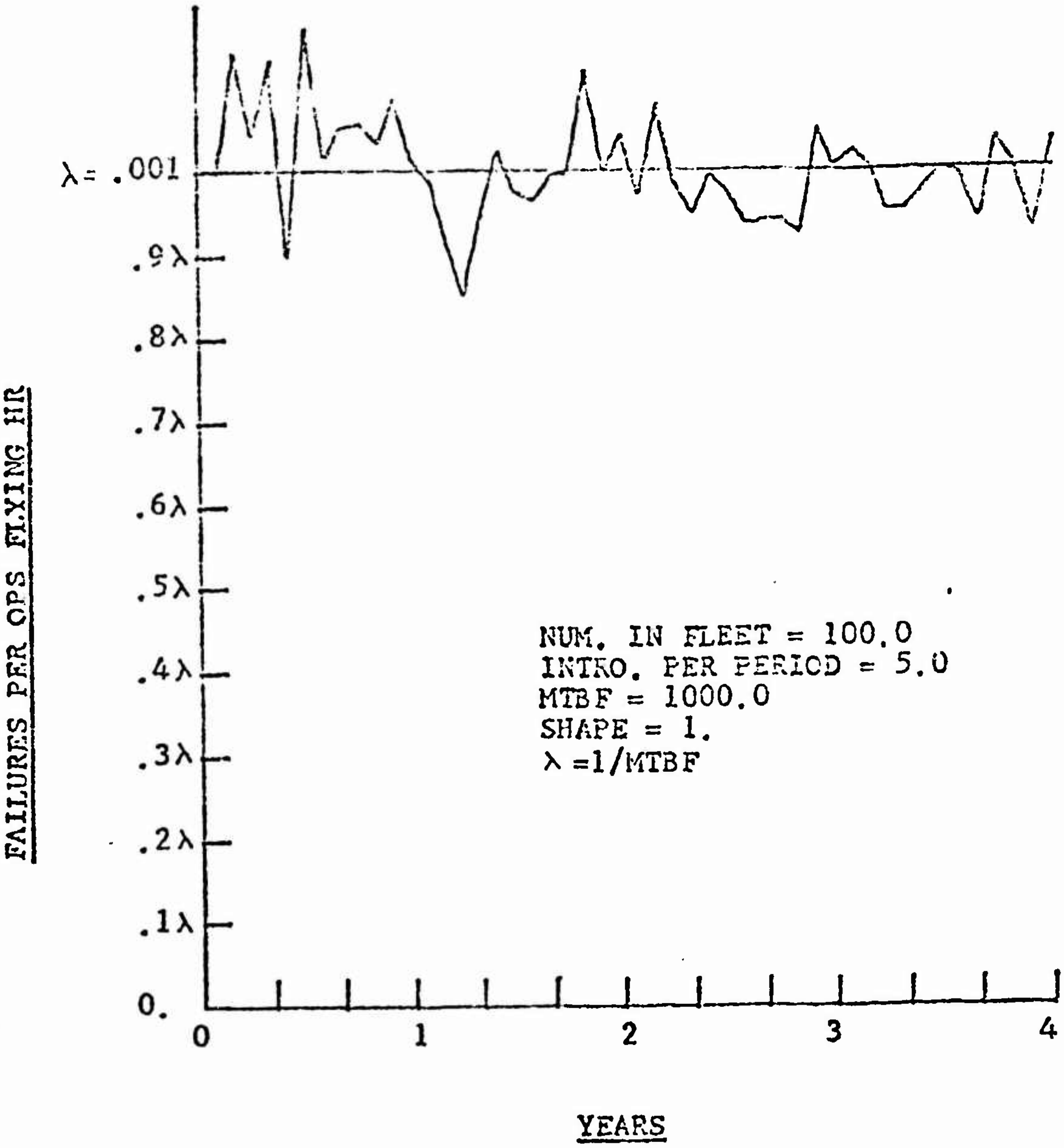


Figure 11

Constant Failure Rate



NUM. IN FLEET = 100.0
INTRO. PER PERIOD = 5.0
MTBF = 1000.0
SHAPE = 1.
λ =1/MTBF

Figure 12

Constant Failure Rate

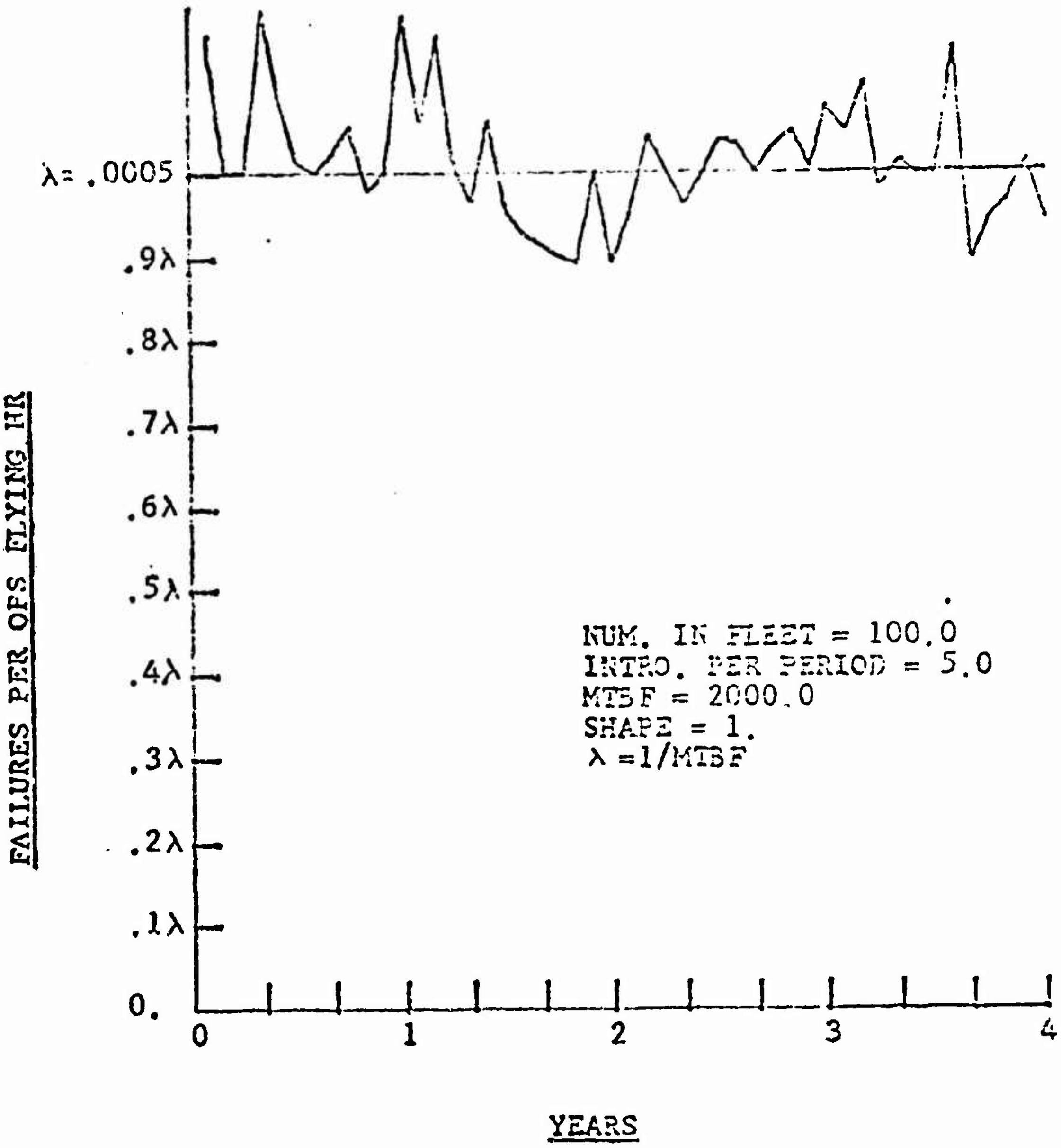


Figure 13

Constant Failure Rate

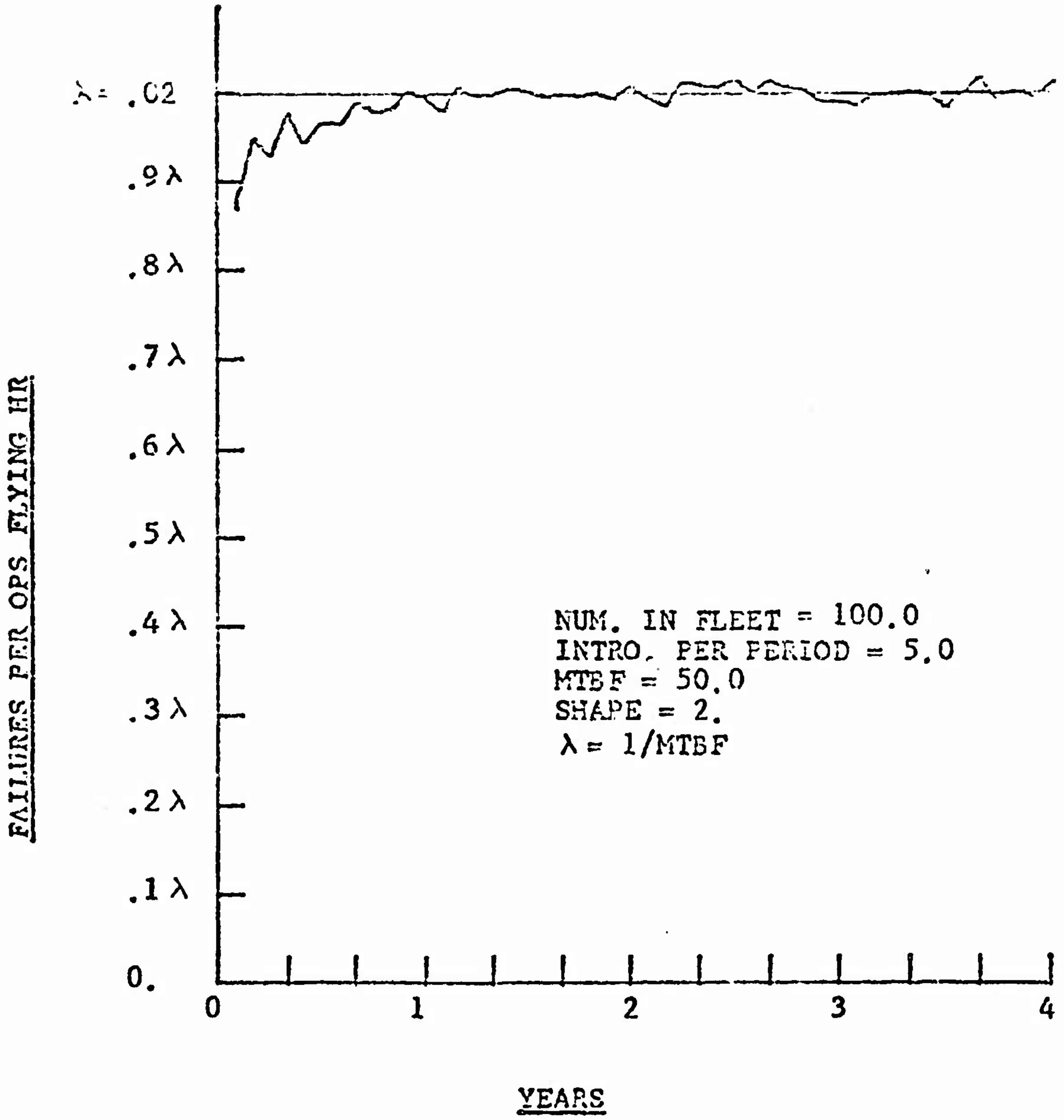


Figure 14

Increasing Failure Rate

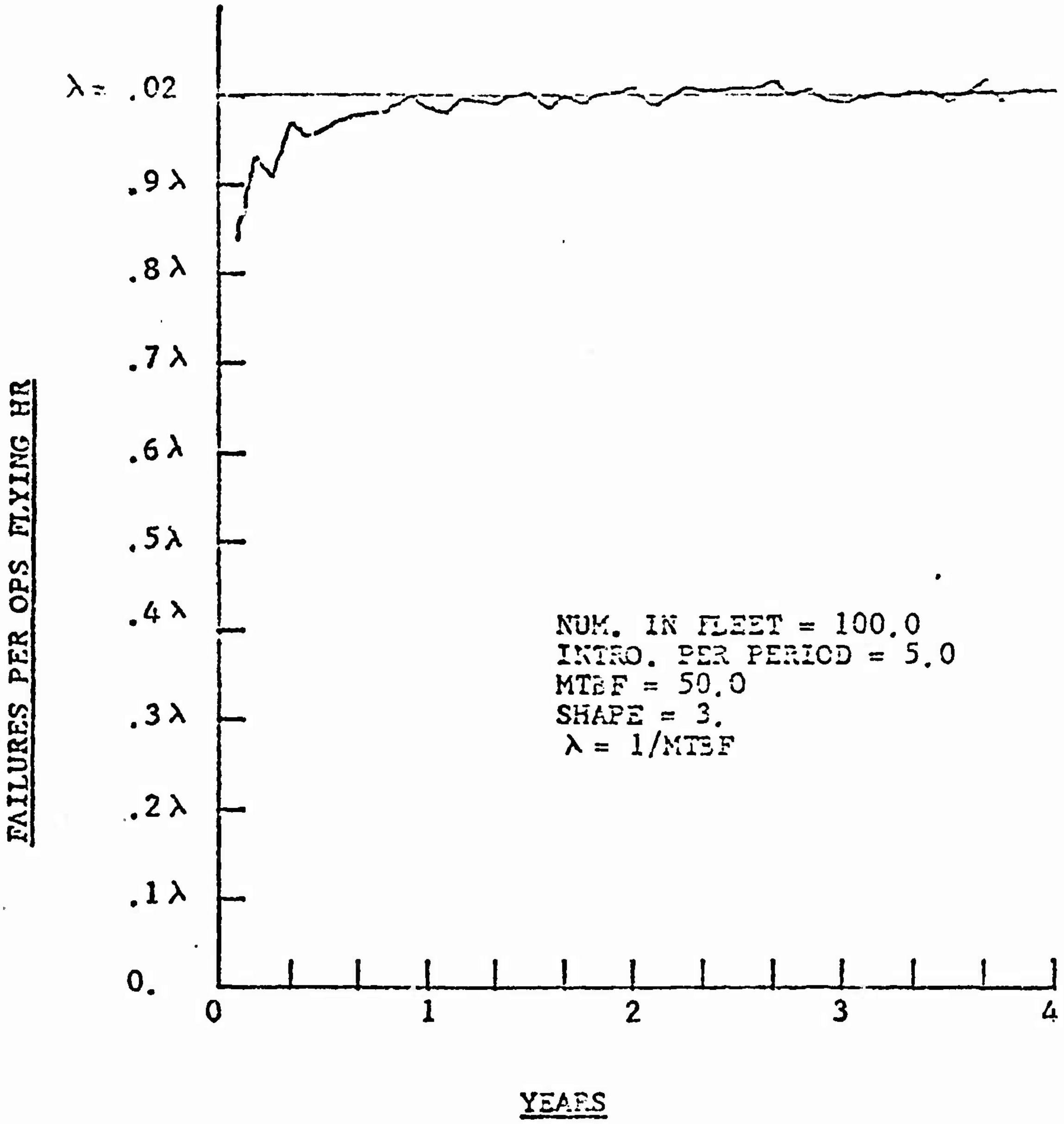


Figure 15

Increasing Failure Rate

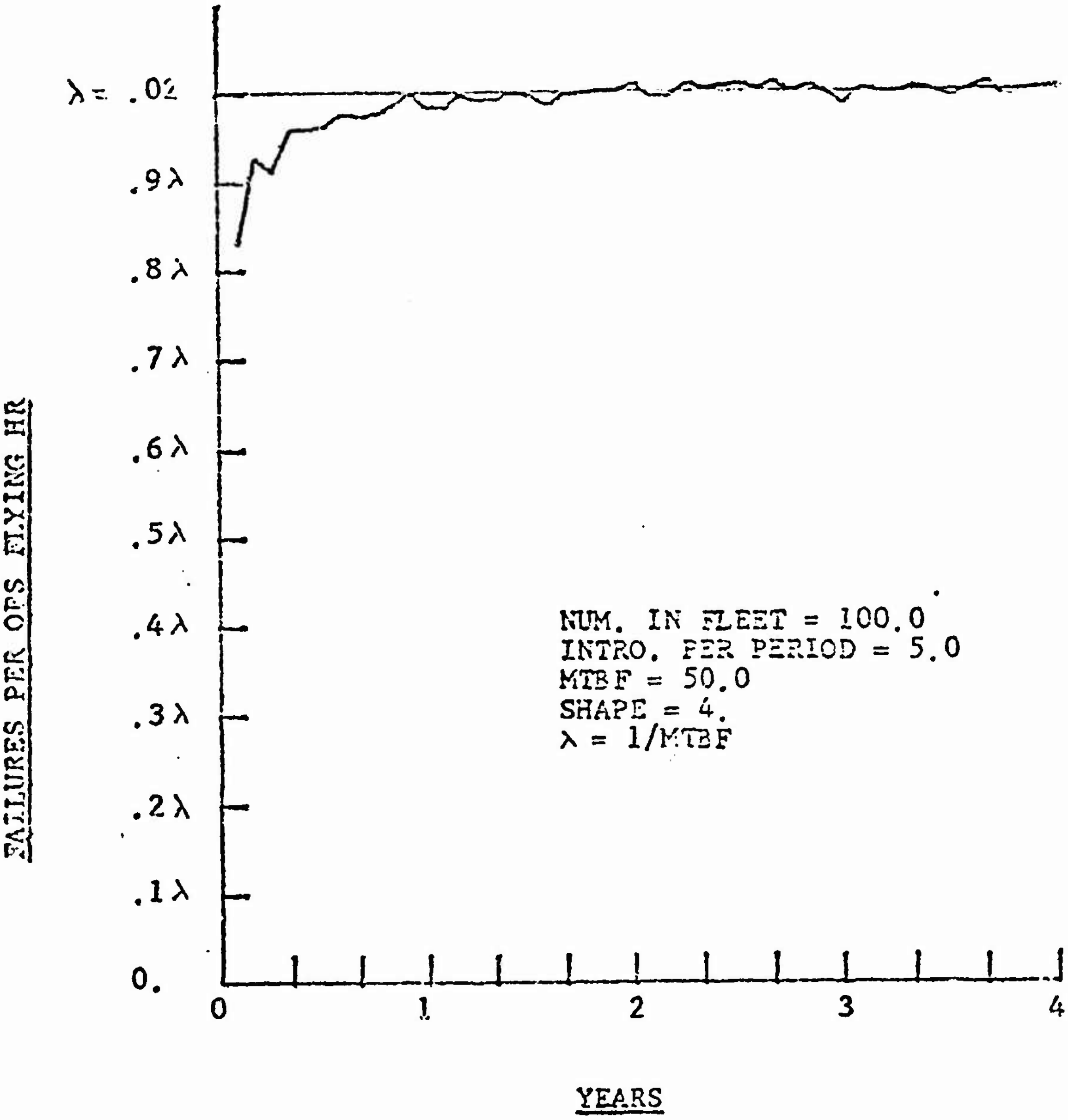


Figure 16

Increasing Failure Rate

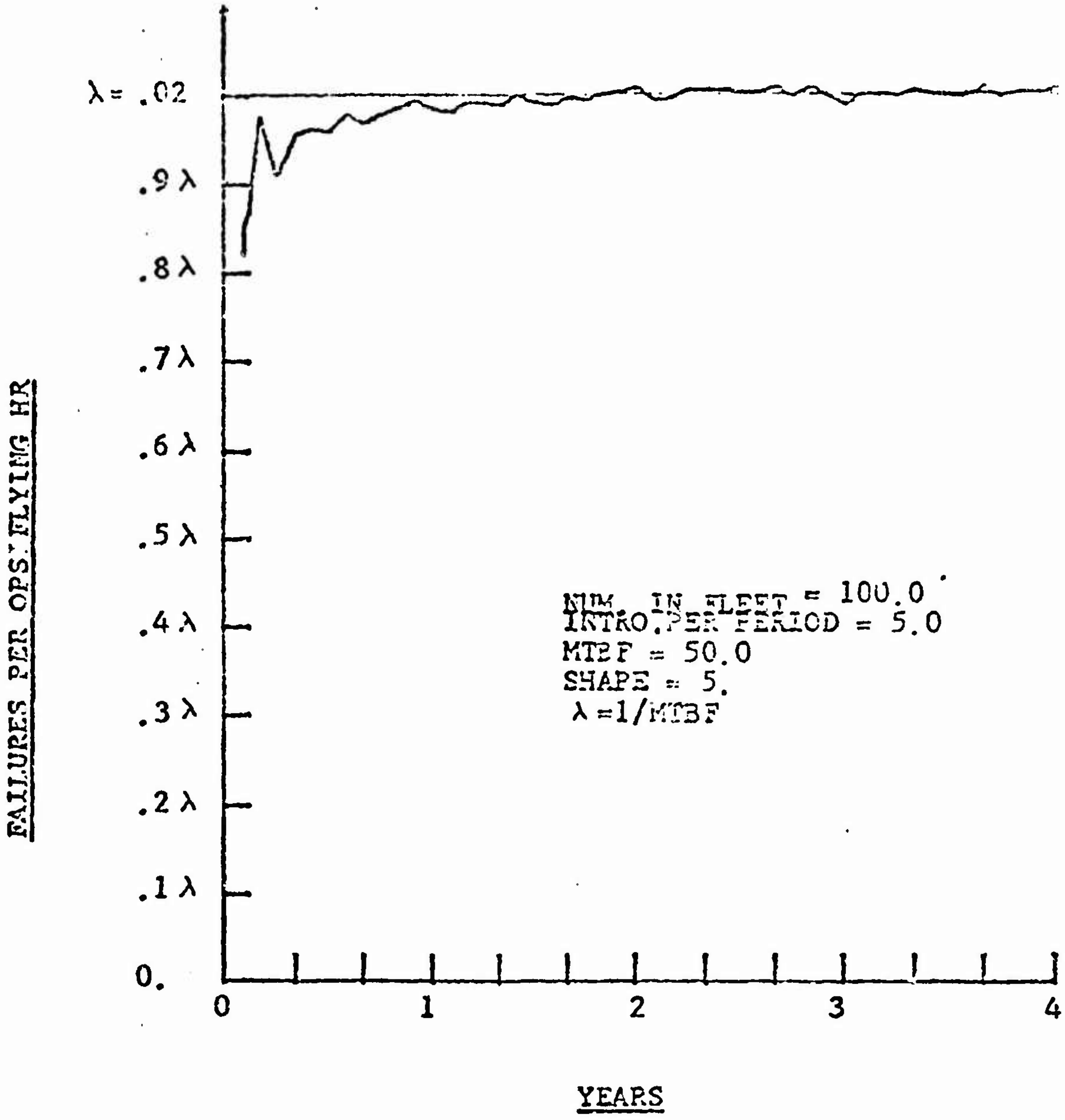


Figure 17

Increasing Failure Rate

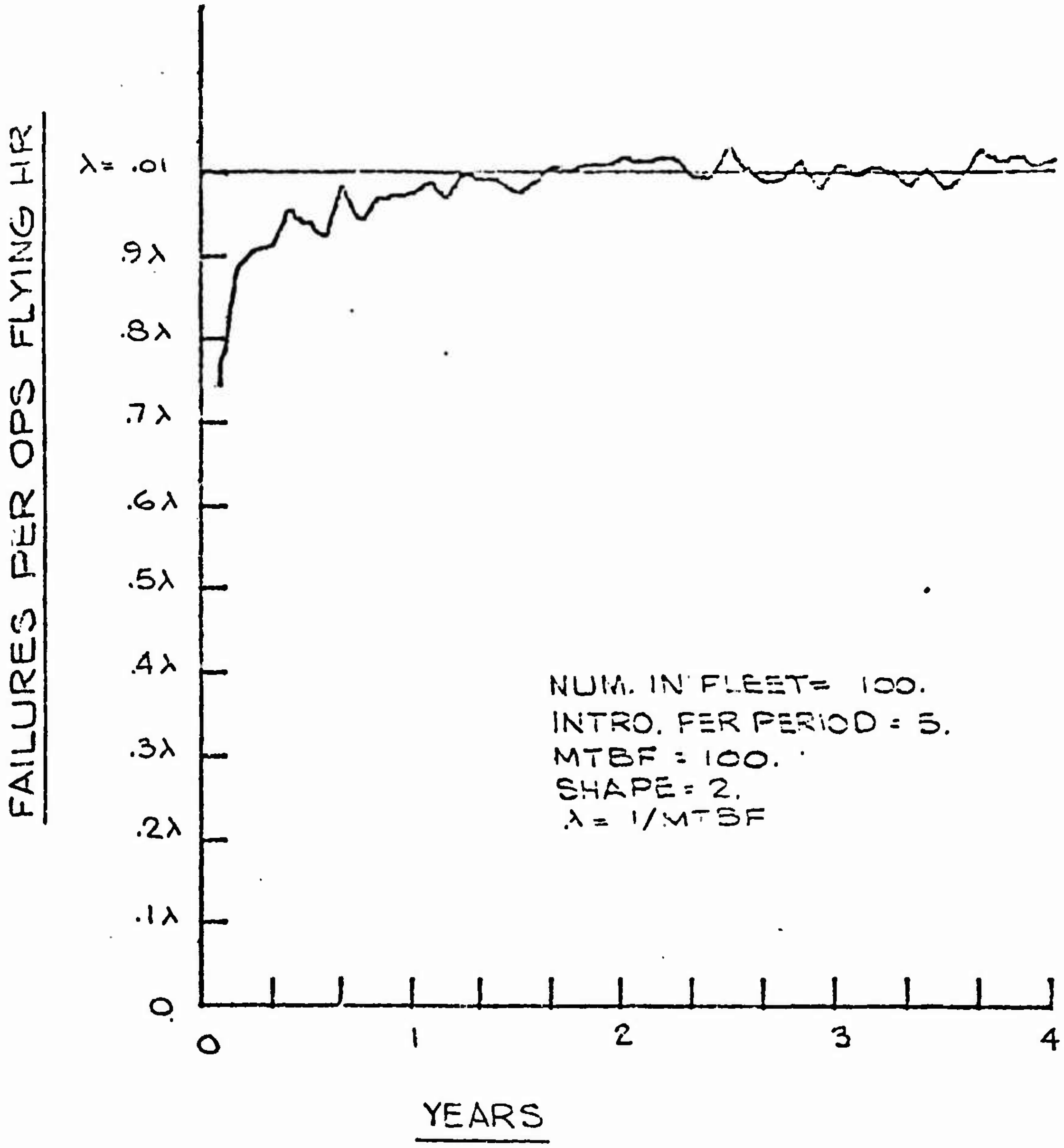


Figure 18

Increasing Failure Rate

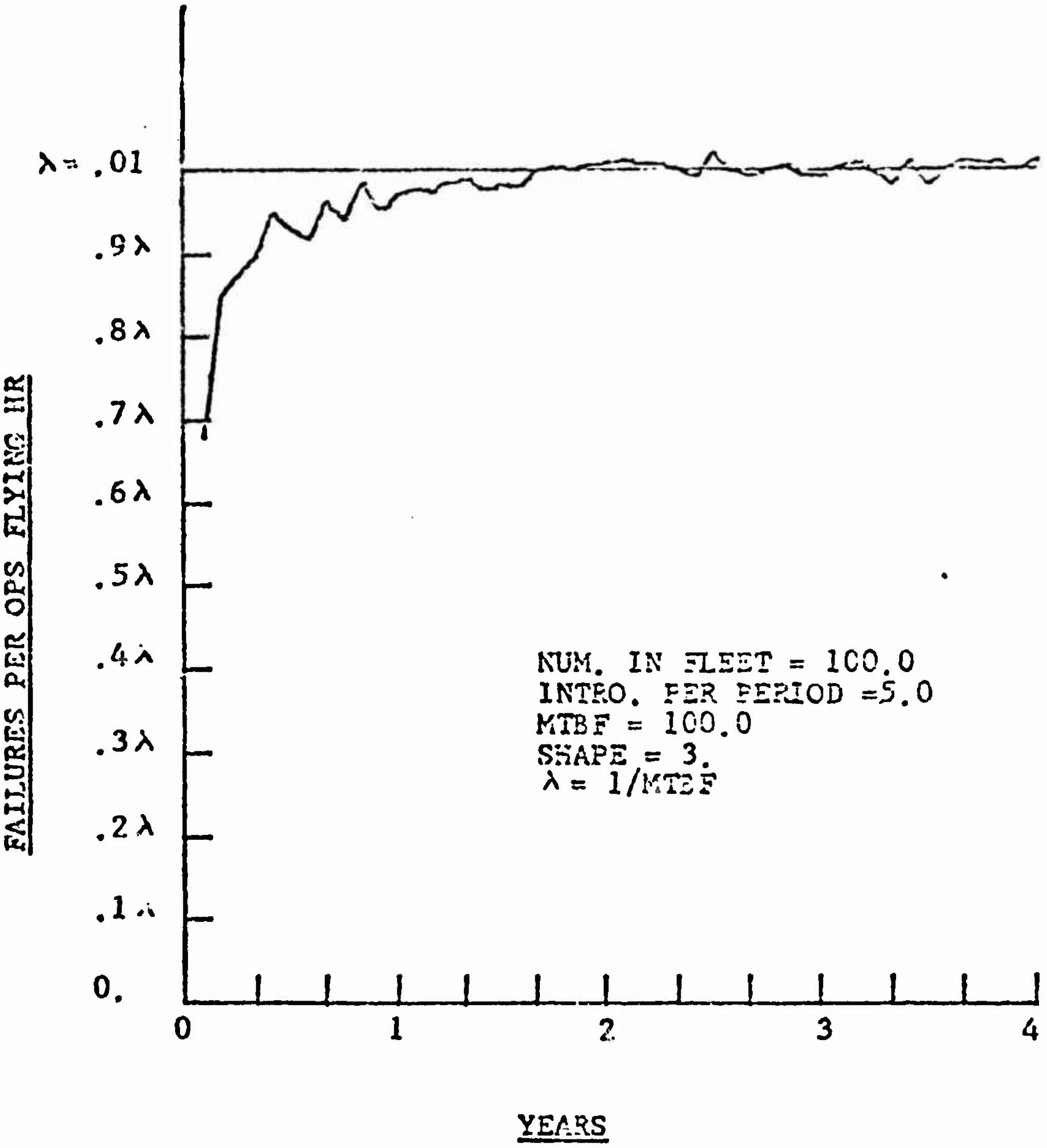


Figure 19

Increasing Failure Rate

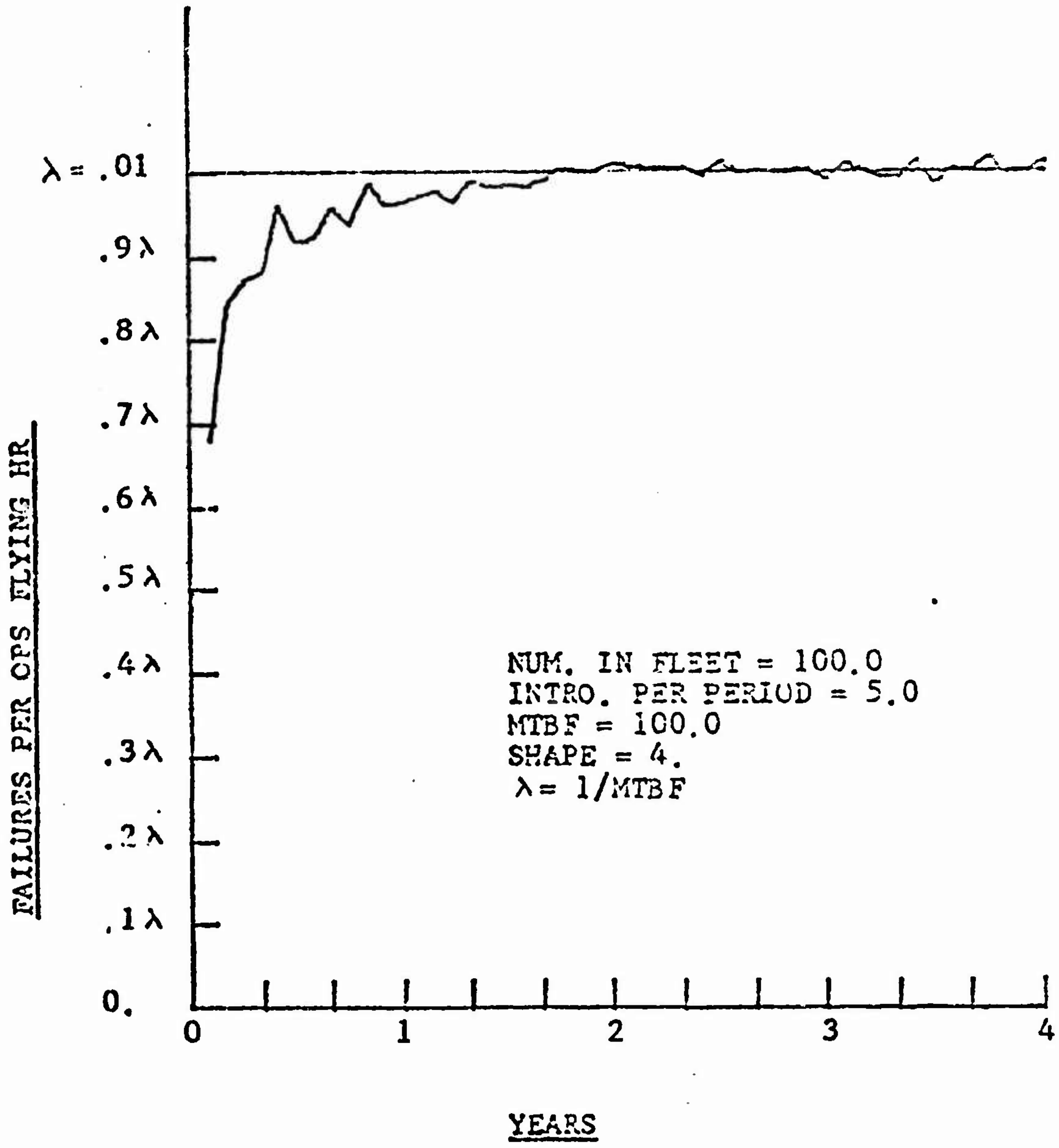


Figure 20

Increasing Failure Rate

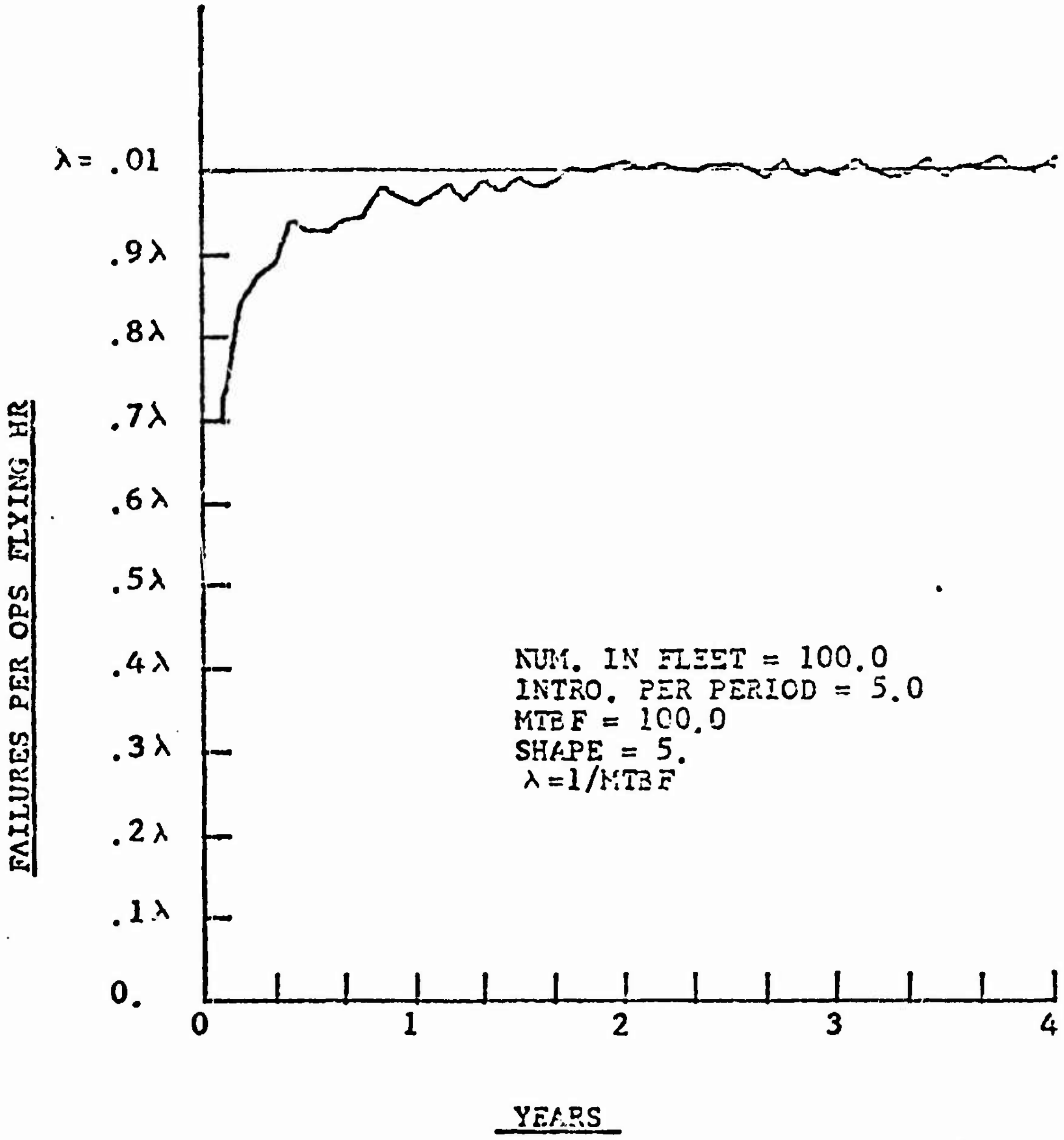


Figure 21

Increasing Failure Rate

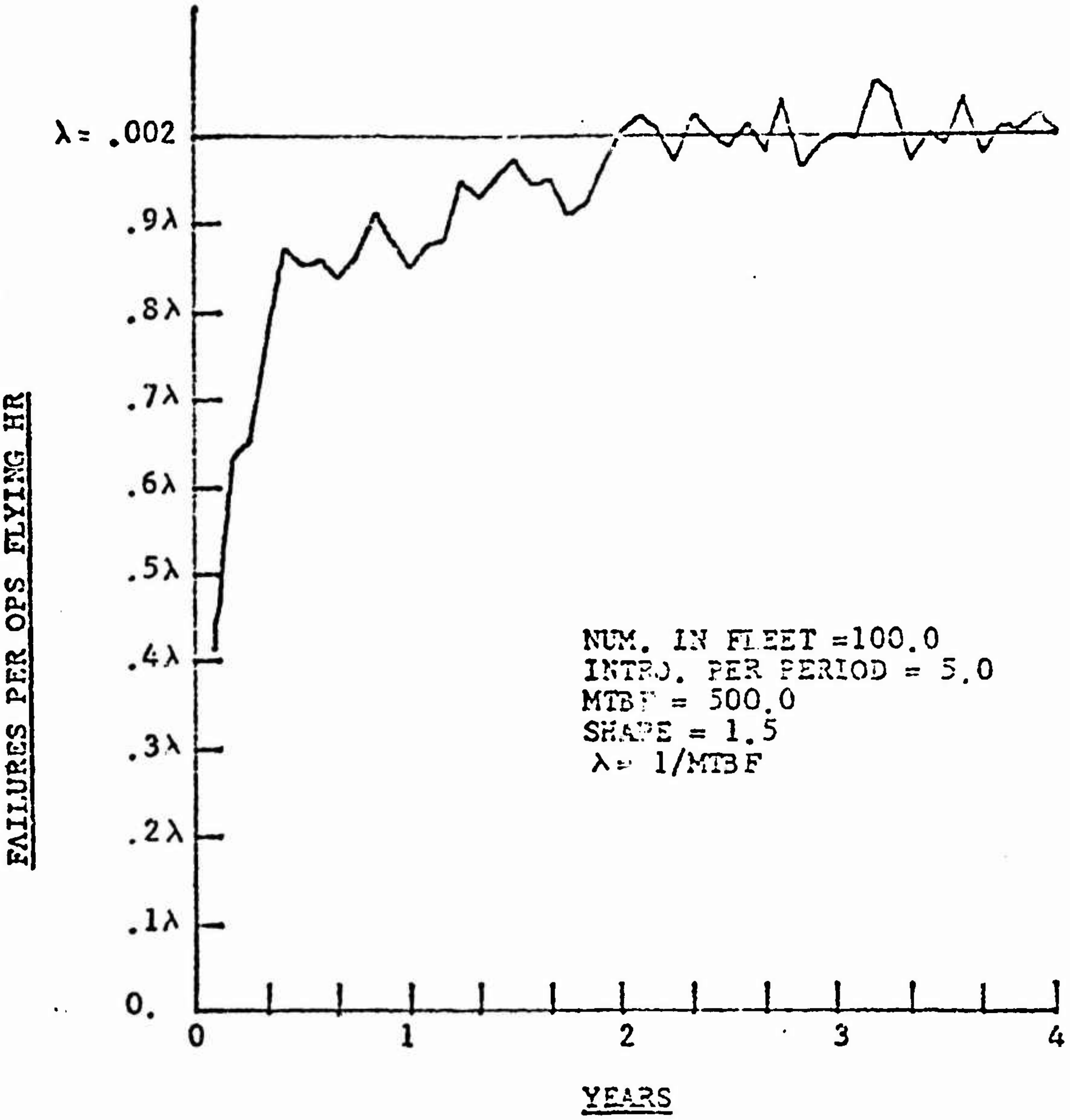


Figure 22

Increasing Failure Rate

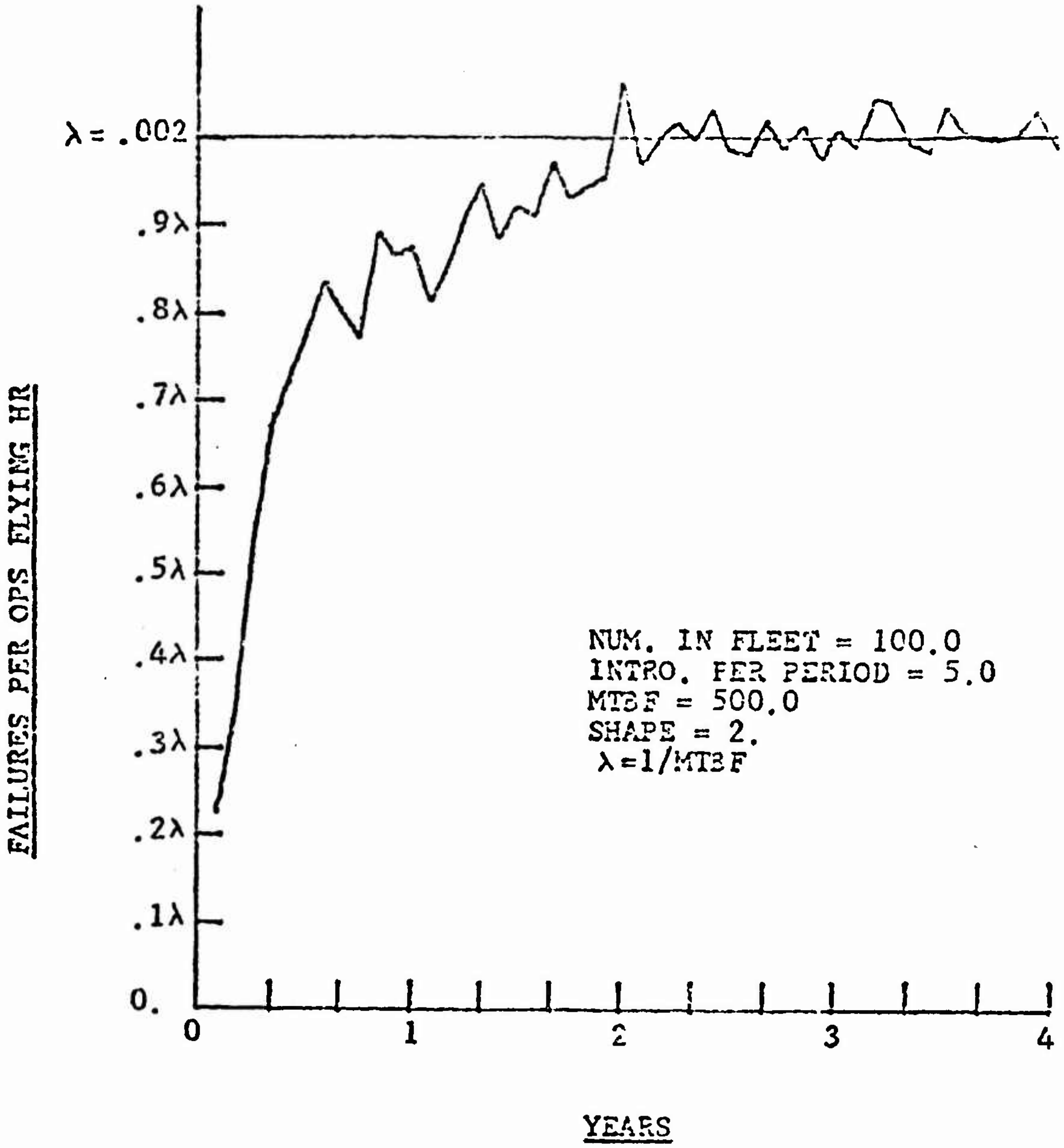


Figure 23

Increasing Failure Rate

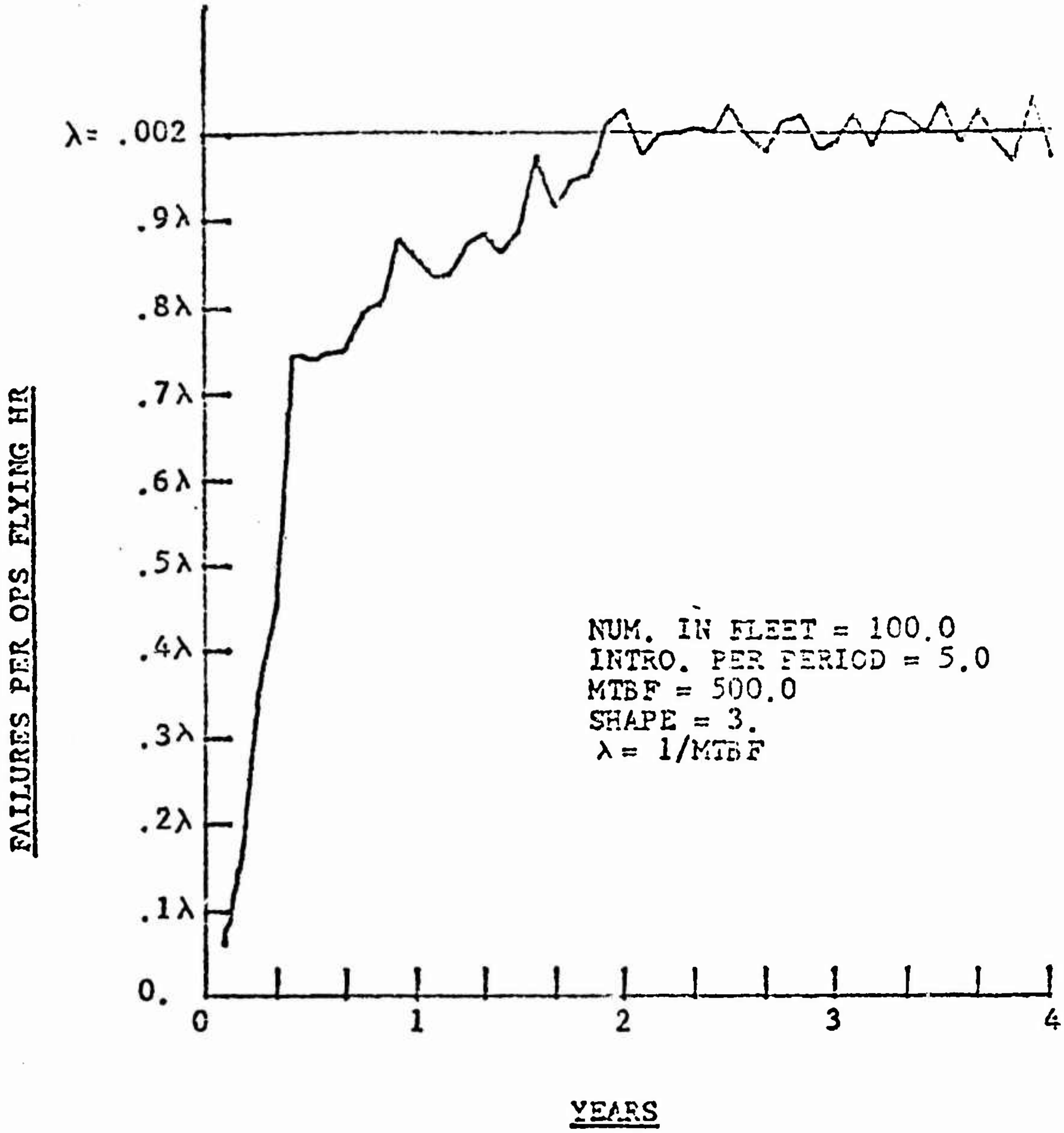


Figure 24

Increasing Failure Rate

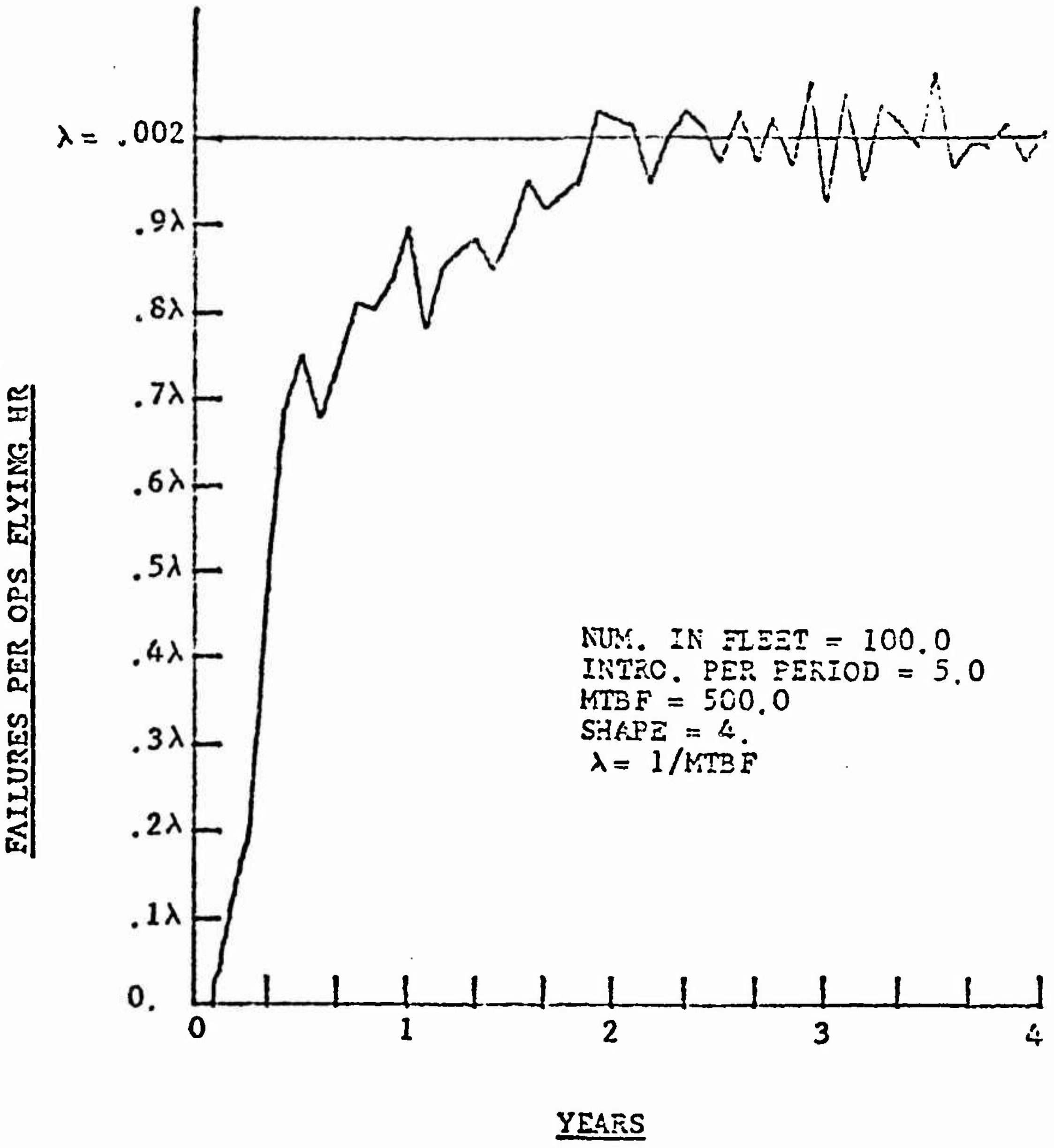


Figure 25

Increasing Failure Rate

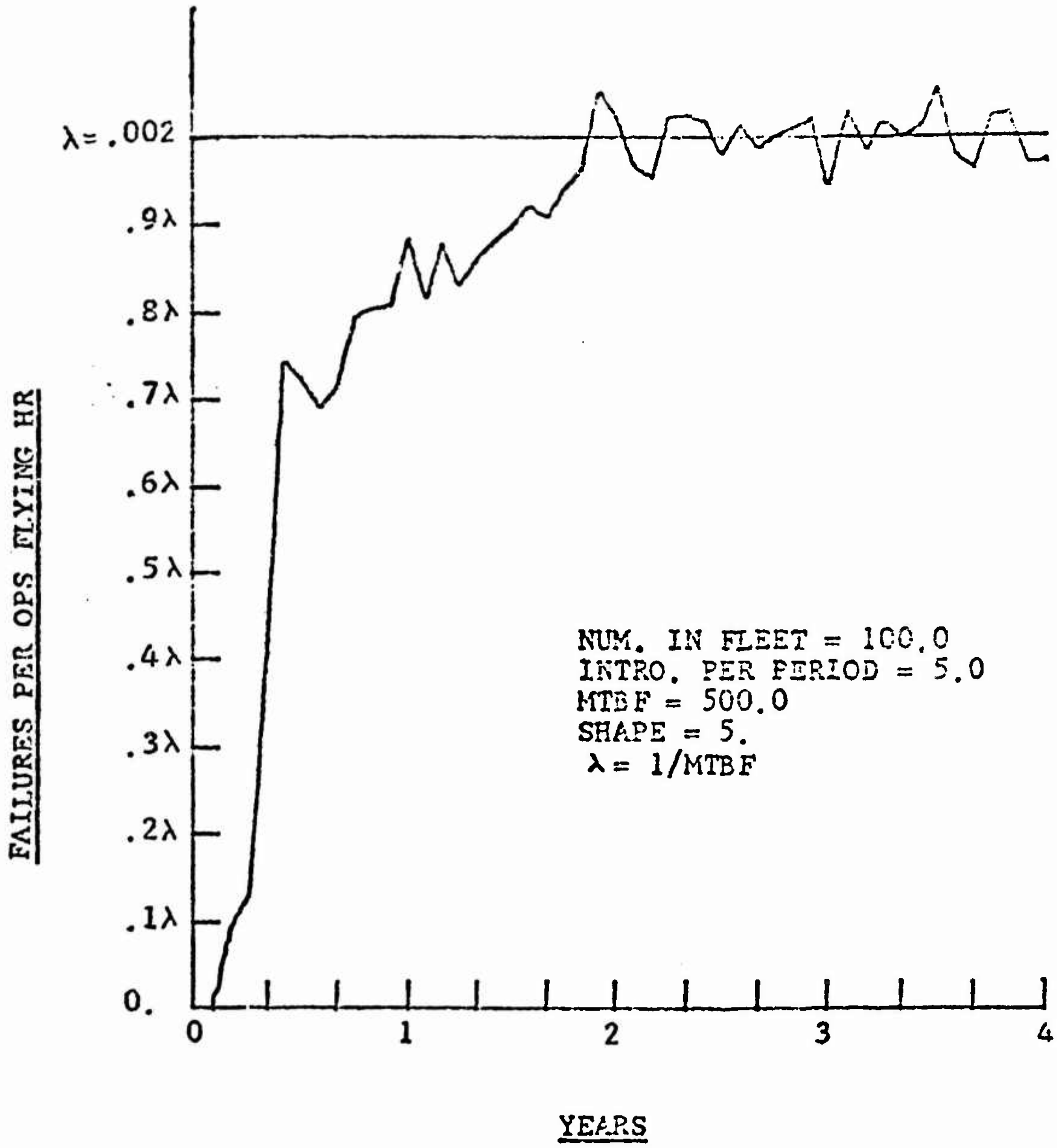


Figure 26

Increasing Failure Rate

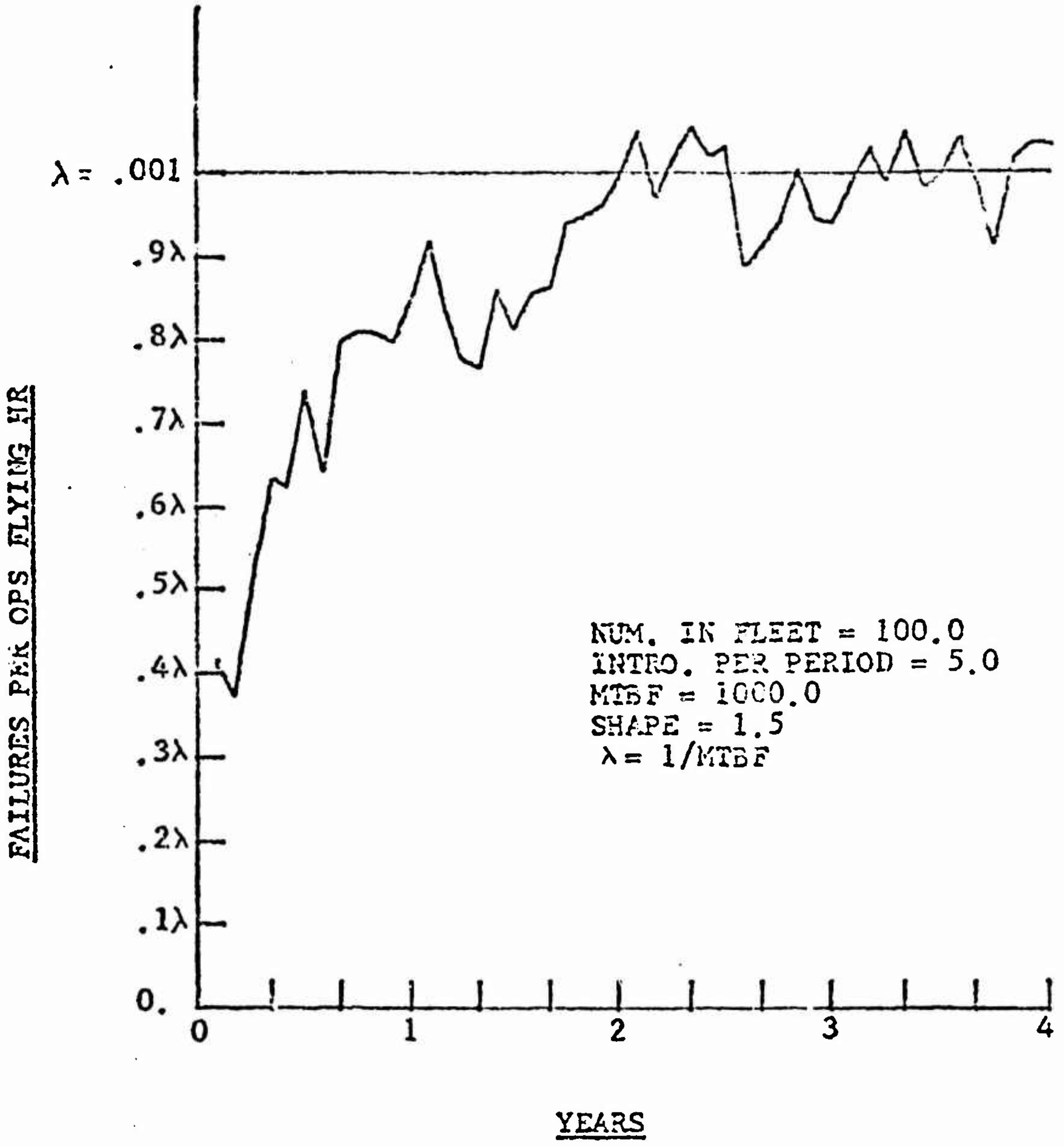


Figure 27

Increasing Failure Rate

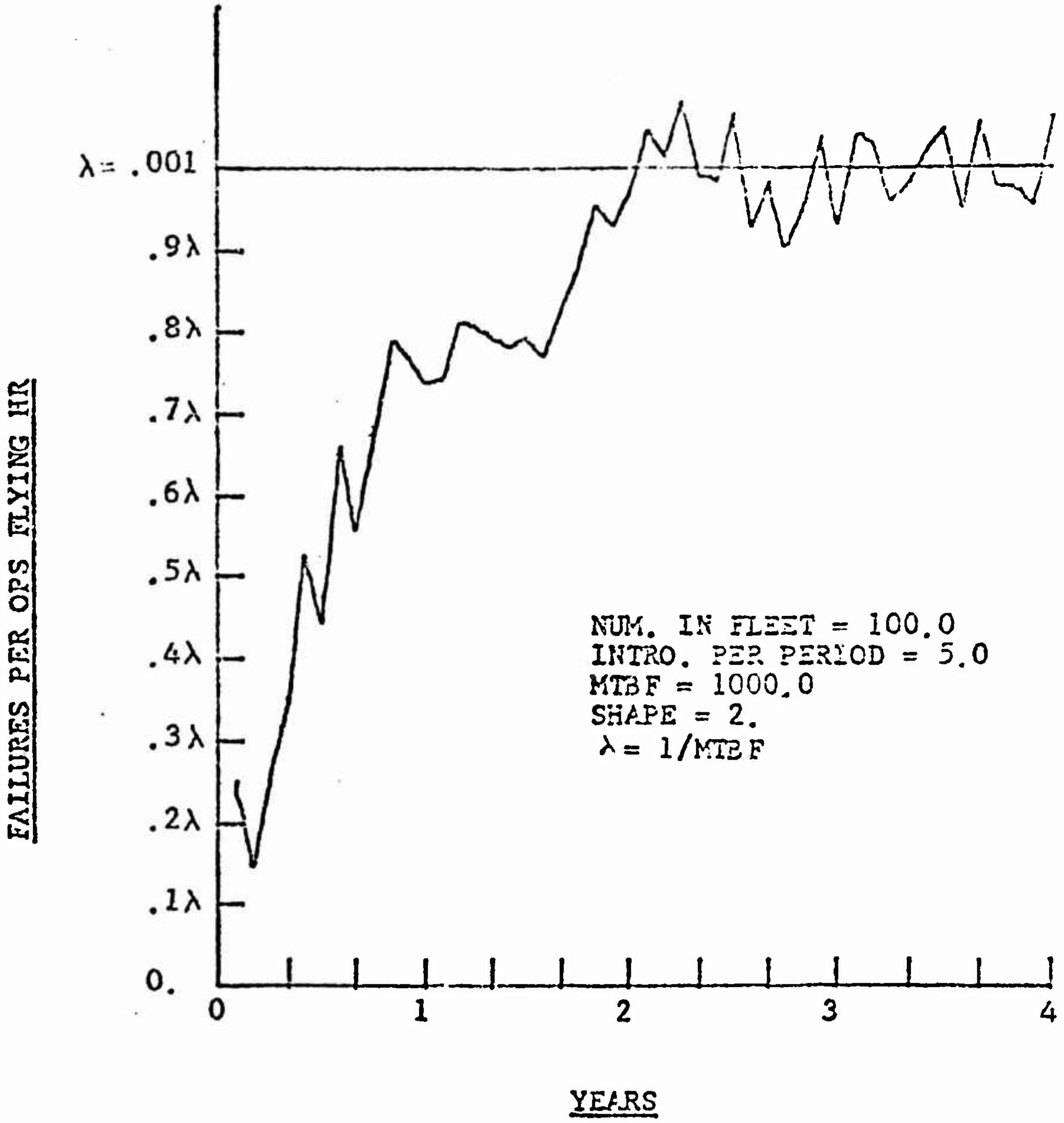


Figure 28

Increasing Failure Rate

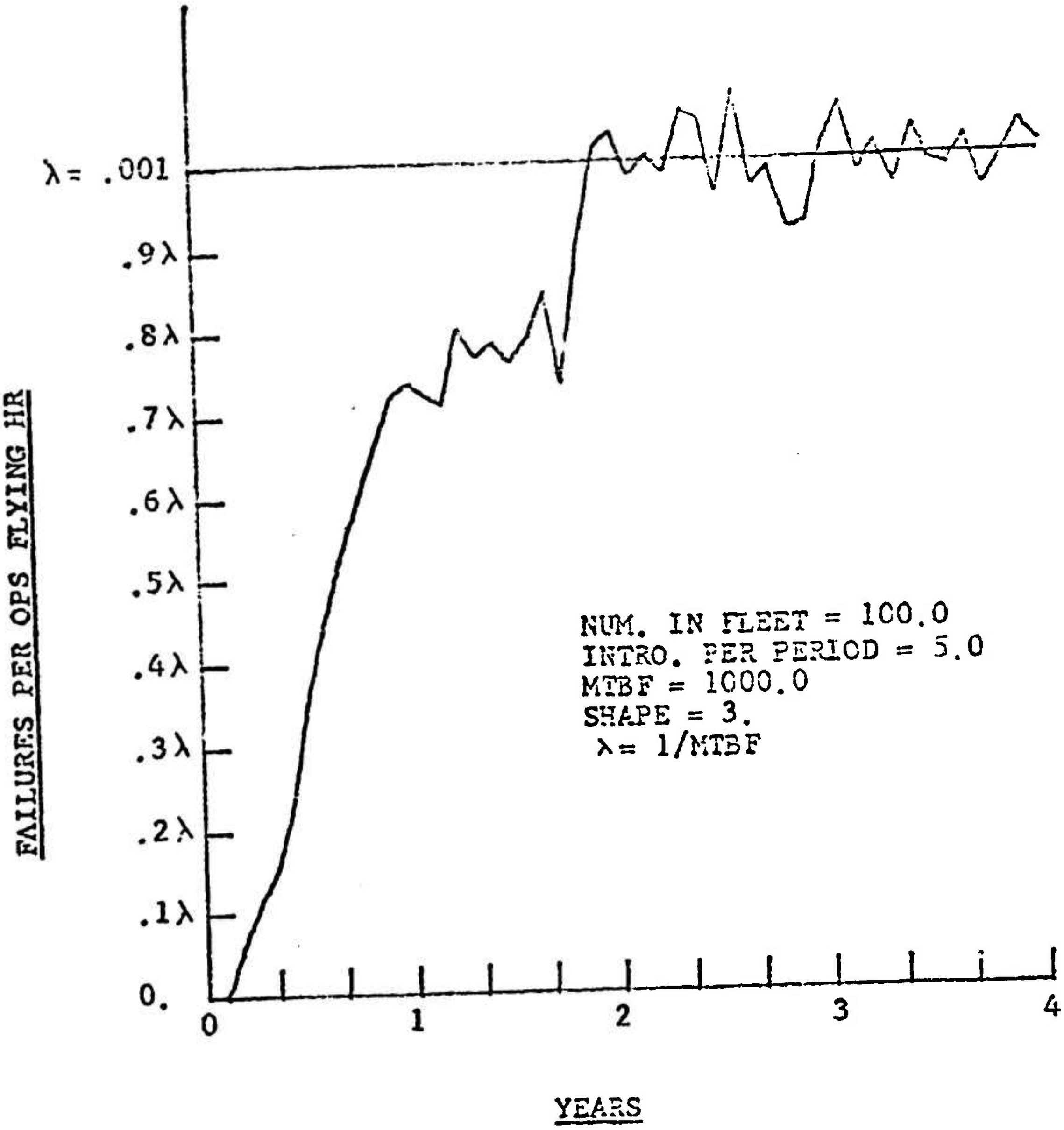


Figure 29

Increasing Failure Rate

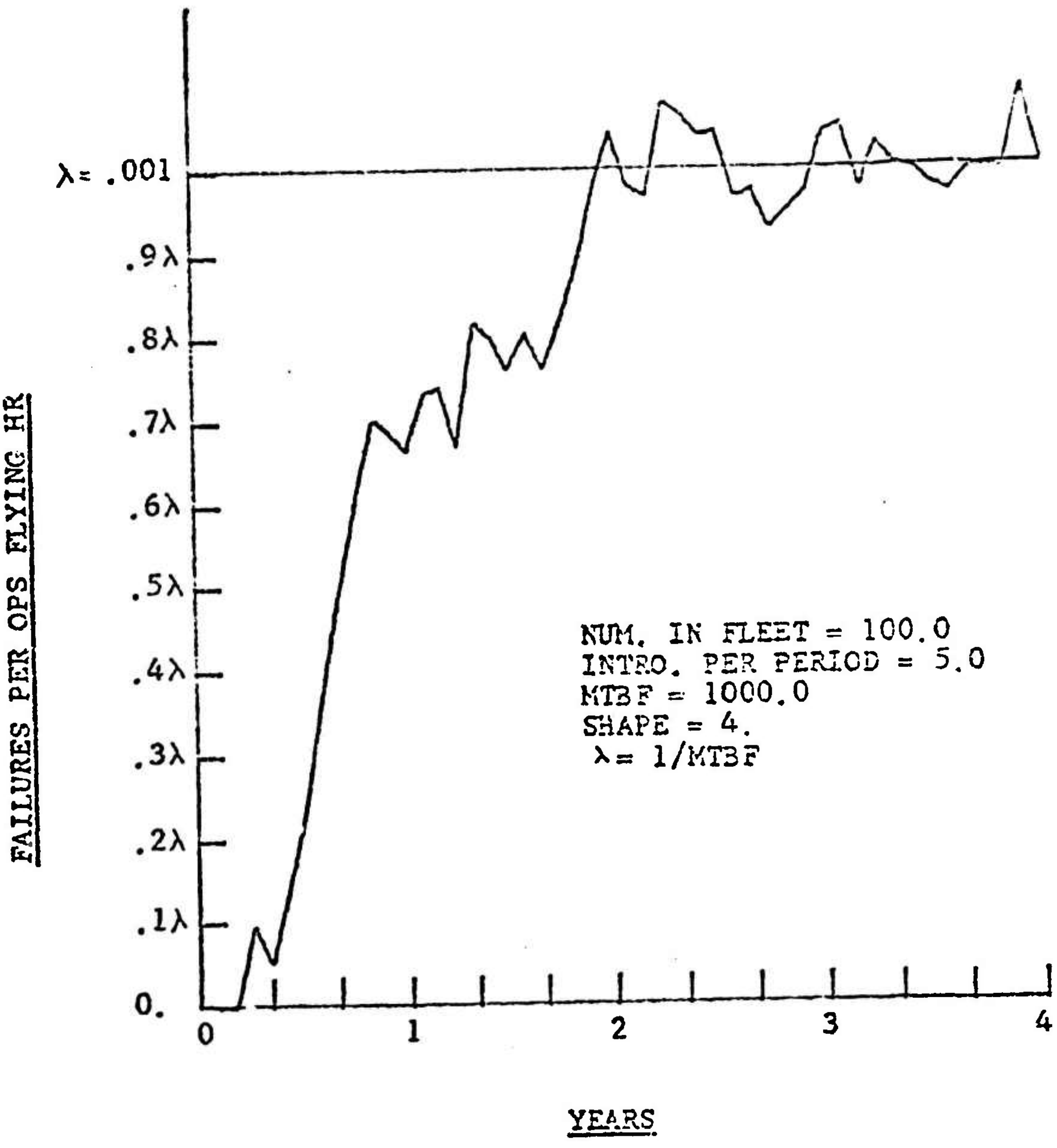


Figure 30

Increasing Failure Rate

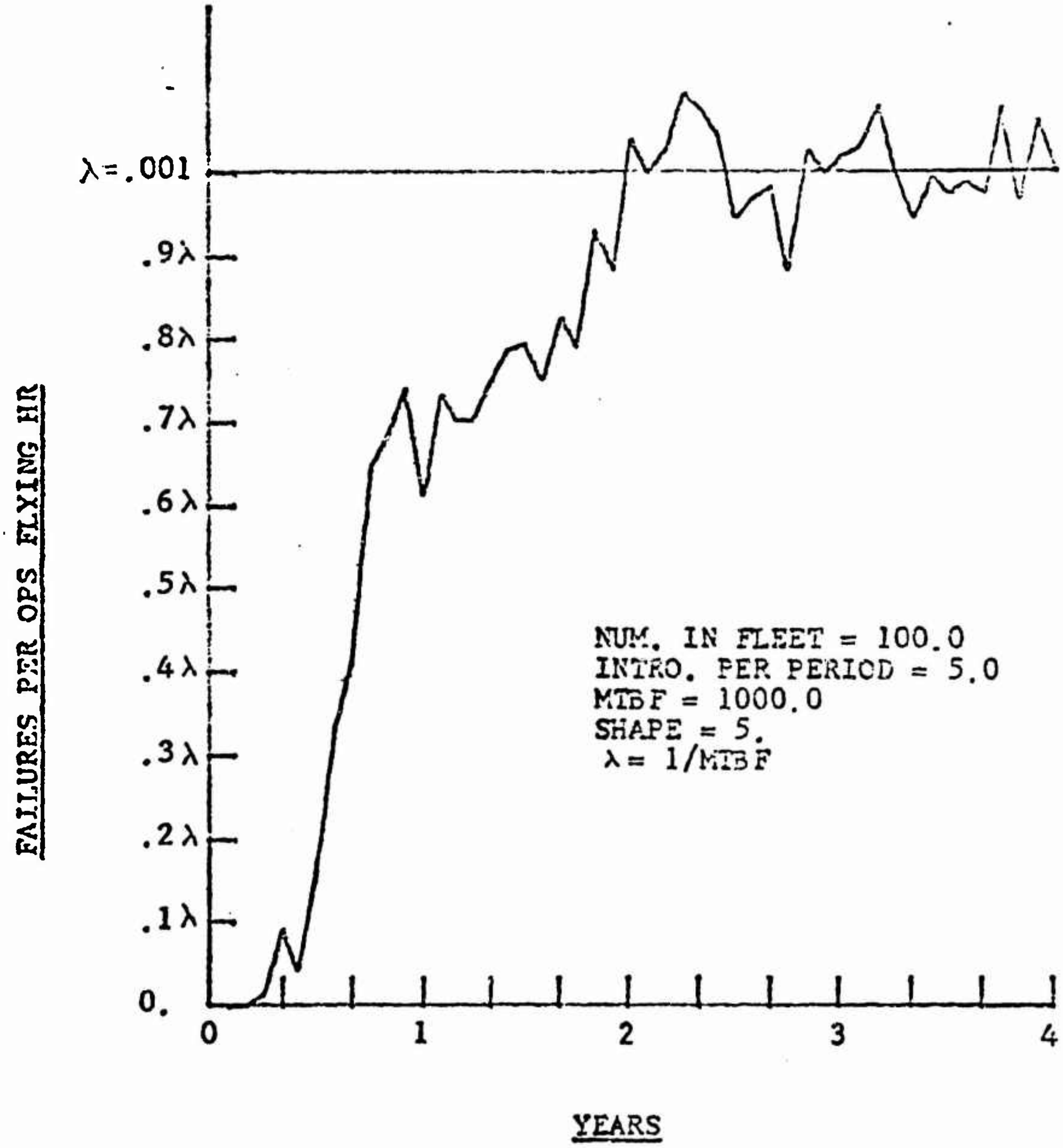


Figure 31

Increasing Failure Rate

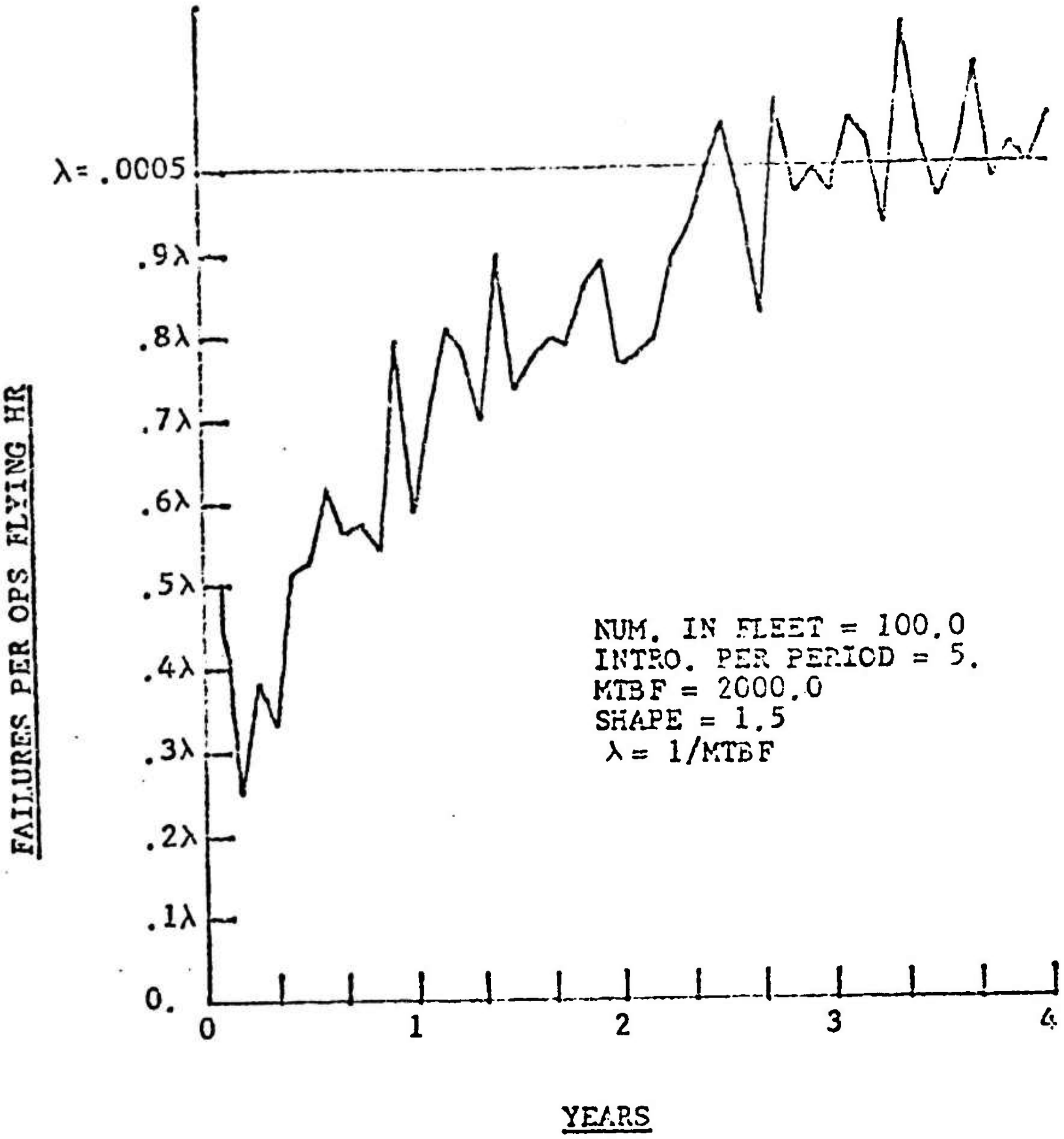


Figure 32

Increasing Failure Rate

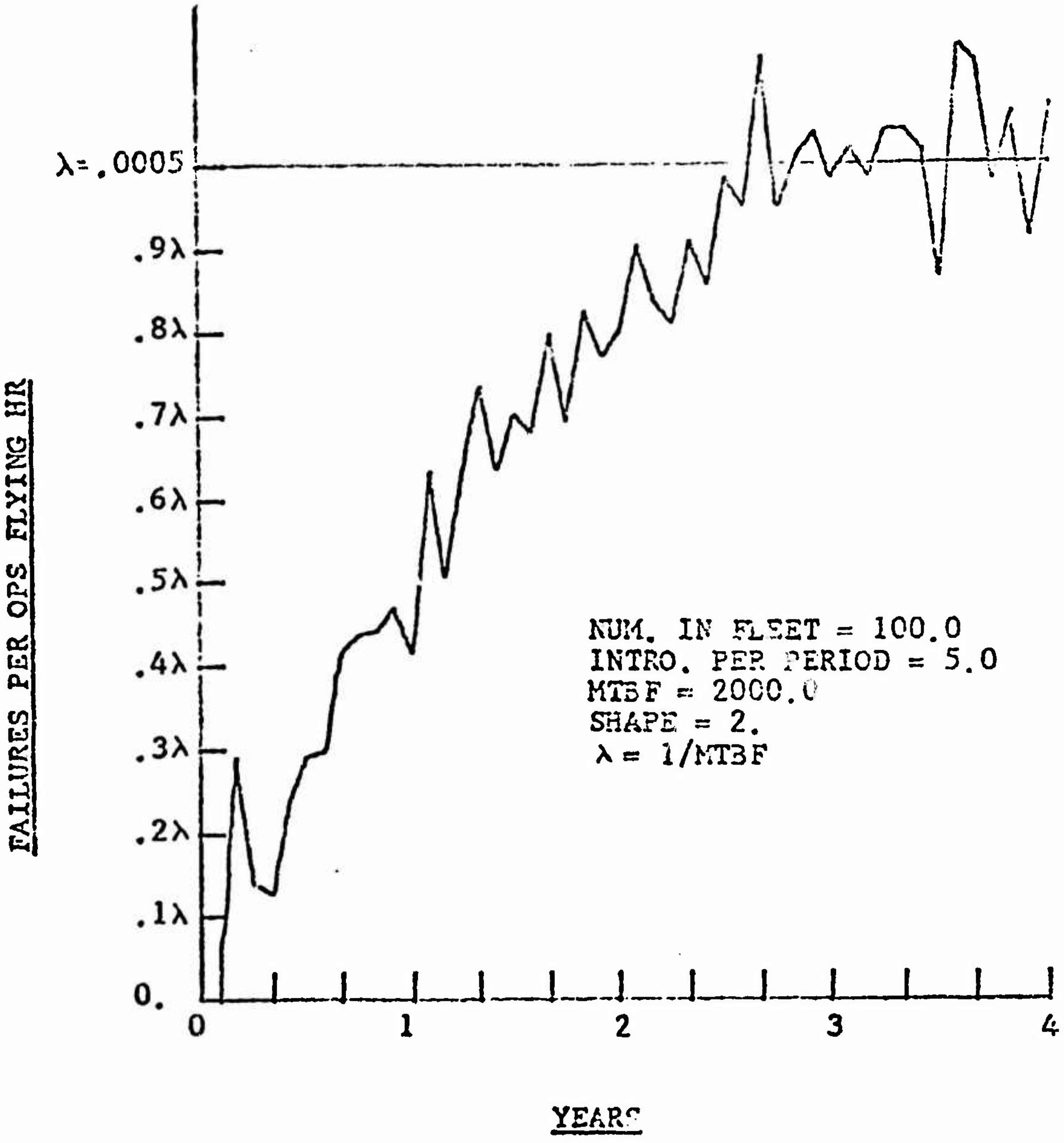


Figure 33

Increasing Failure Rate

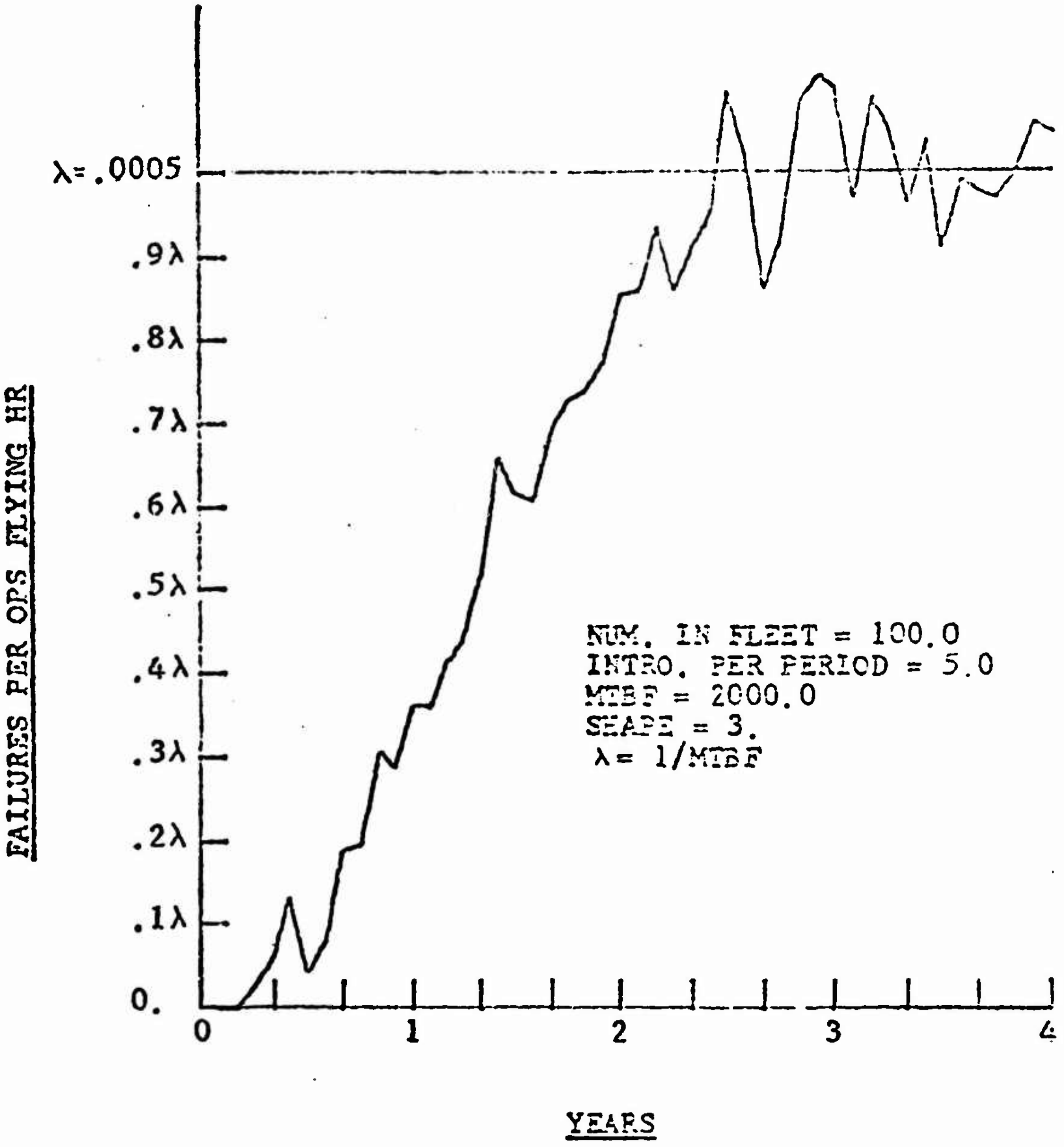


Figure 34

Increasing Failure Rate

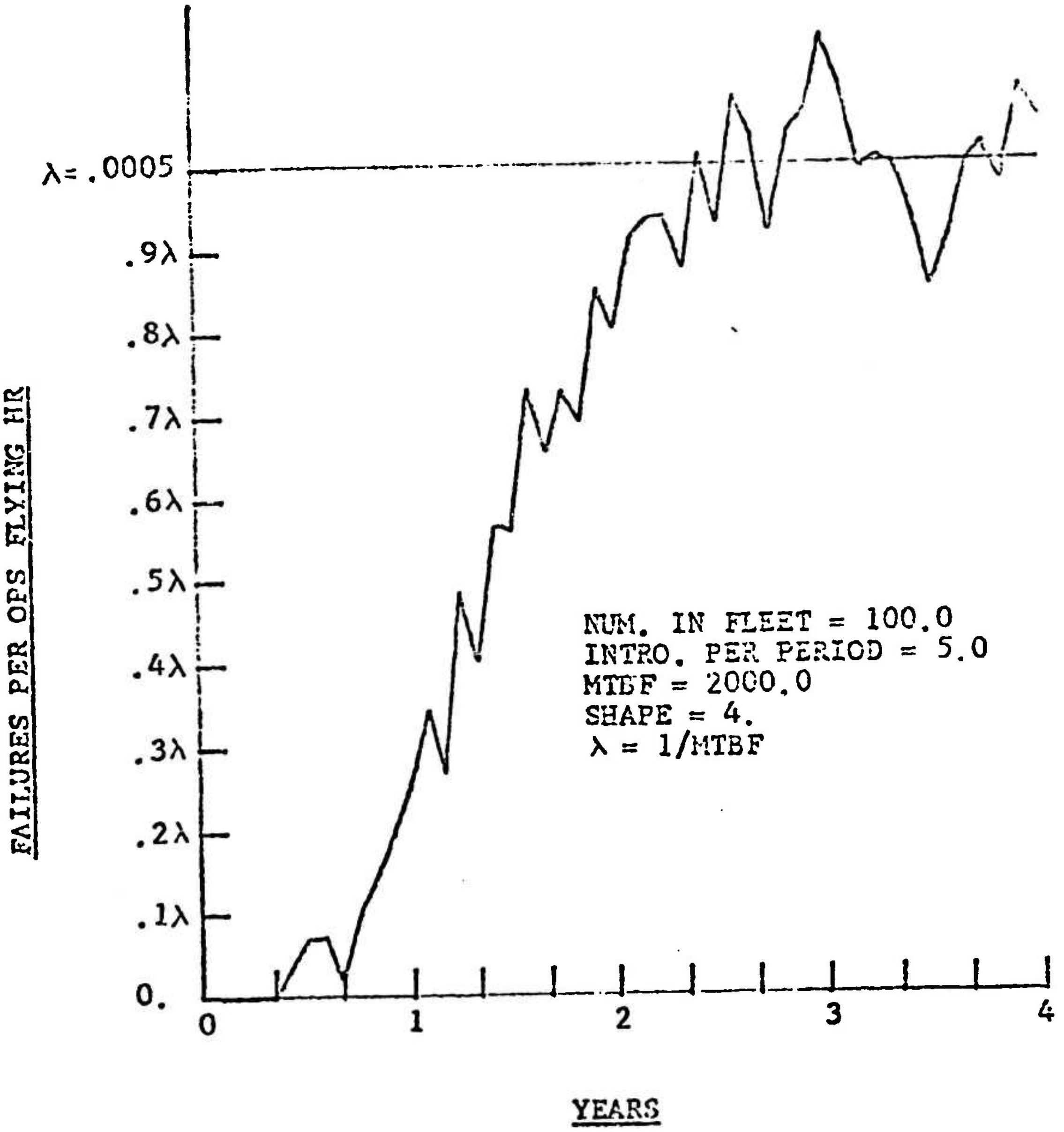


Figure 35

Increasing Failure Rate

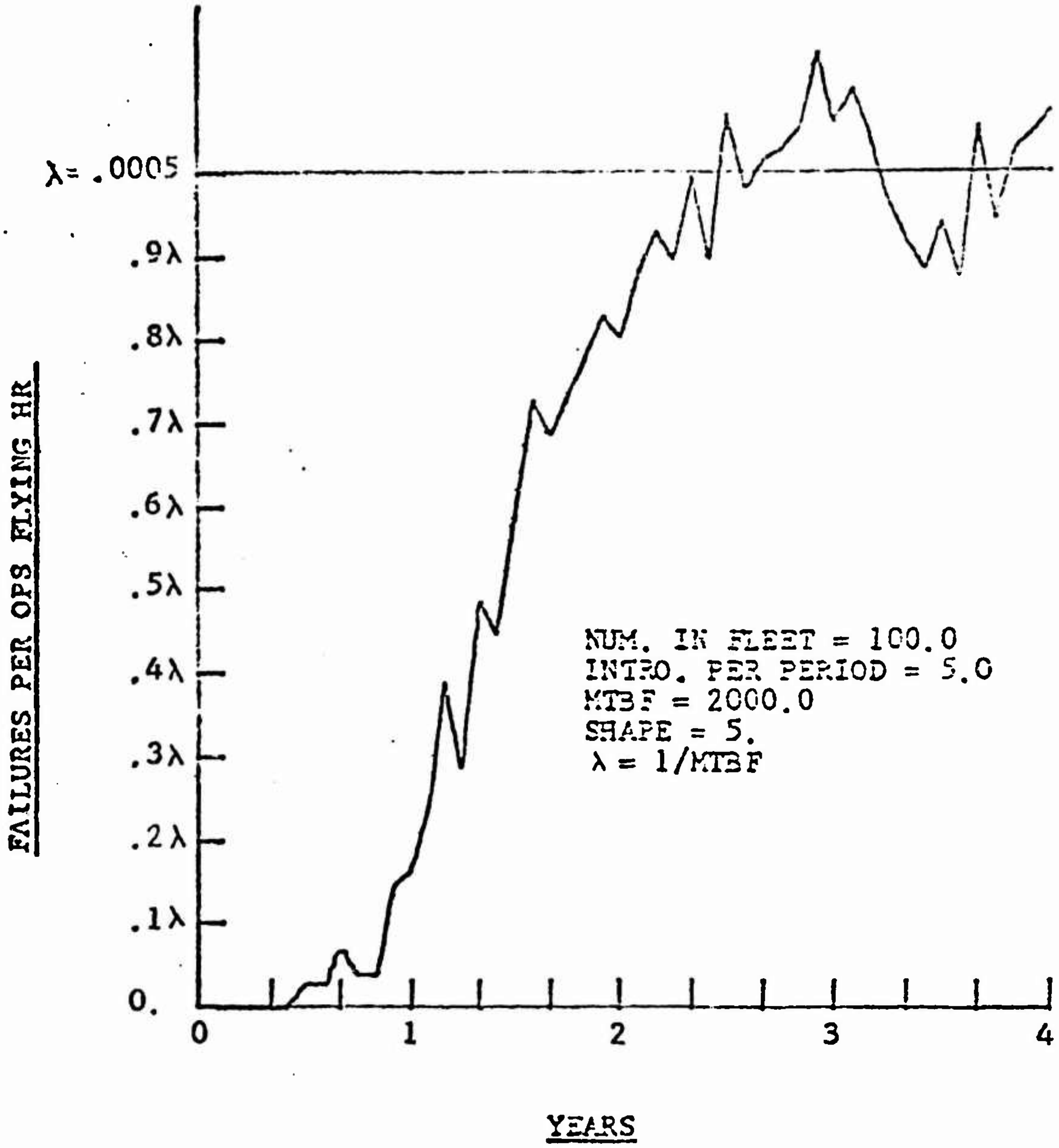


Figure 36

Increasing Failure Rate

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