AD-773 744

NOISE CANCELLATION IN THE PRESENCE OF CORRELATED SIGNAL AND NOISE

Norman L. Owsley

Naval Underwater Systems Center
New London, Connecticut

11 January 1974

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NUSC Technical Report 4639

AD773744

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PREFACE

This report was prepared under NUSC Project No. A-354-00, Principal Investigator, Dr. N. L. Owsley (Code TD111), and Navy Subproj and Task No. SF 11 121 110-15806, Program Manager, D. Hoffman, NAVSHIPS Code PMS-302-413. Additional support was provided under Project No. A-678-43, Principal Investigator, D. Gelfond (Code SA24), and Navy Subproject and Task No. X24X2-63794, Program Manager, CAPT V. Anderson (Code PME-124).

The Technical Reviewer for this report was Dr. D. M. Viccione (TC).

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New London, Connecticut 06320

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TR 4639	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) NOISE CANCELLATION IN THE PRESENCE OF CORRELATED SIGNAL AND NOISE		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Norman L. Owsley		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Underwater Systems Center New London Laboratory New London, Connecticut 06320		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS A-654-00 SF 11 121 110-15806
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Ship Systems Command (PMS-302) Washington, D. C. 20360		12. REPORT DATE 11 January 1974 13. NUMBER OF PAGES 18
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Acoustic Signal Processing
Multichannel Signal Processing
Noise Cancellation
Correlated Signal and Noise

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A model for a multichannel array signal processor that performs a noise-cancellation function is specified. The performance of the processor is examined in terms of the output signal-to-total-background-noise ratio. The parameters of interest are the correlation between signal and noise, the number of noise sampling channels, the relative gain of the noise in the signal-plus-noise versus the noise-only channels, and the array input signal-to-noise and interference-to-

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noise ratios. An important practical implication of the results presented is that, in systems such as adaptive beamformers for arrays, system performance can be seriously degraded if the signal and noise waveforms are correlated over finite time intervals.

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INTRODUCTION

Noise cancellation using the "estimate-subtract" approach is fundamental to the operation of numerous so-called optimum filter signal processors.[1] In particular, adaptive implementations of a noise-cancellation capability in a multichannel system form the basis for the optimum/adaptive processing of data outputs from an array of sensors as typified by beamforming applications.[2-5] Typically, in evaluating the performance of a sensor array processor with noise cancellation, one assumes that the signal is uncorrelated with the noise.[6,7] In fact, the signal can be highly correlated with the noise over a time interval comparable to the response time-constant implicit in an adaptive system that is first required to "learn" about the time-varying noise before it can cancel it.

This report considers the effects of signal and noise correlation (noise estimator characteristics and signal- (interference-) to-noise ratios) on the performance of an array signal processor with noise cancellation implemented by means of a noise estimation-subtraction scheme.

THE ESTIMATOR-SUBTRACTOR NOISE-CANCELLATION PROCESSOR

Consider the general K-channel noise-cancellation system shown in figure 1. The filter weights $\{w_k / k = 1, 2, \ldots, K\}$ can be expressed in vector form as

$$\underline{W}^H = \left[w_1^* \; w_2^* \ldots w_K^*\right], \tag{1}$$

where H denotes the matrix complex conjugate transpose, and * indicates the complex conjugate of a complex scalar. Similarly, the K noise sample channels, referred to as the auxiliary channels, can be expressed in vector notation as

$$\underline{X}^T = \left[x_1(t) \; x_2(t) \ldots x_K(t)\right] \tag{2}$$

$$= \left[\alpha_1 \; \alpha_2 \ldots \alpha_K\right] i(t) + \left[n_1(t) \; n_2(t) \ldots n_K(t)\right] \tag{3}$$

$$= \underline{A}^T i(t) + \underline{N}^T, \tag{4}$$

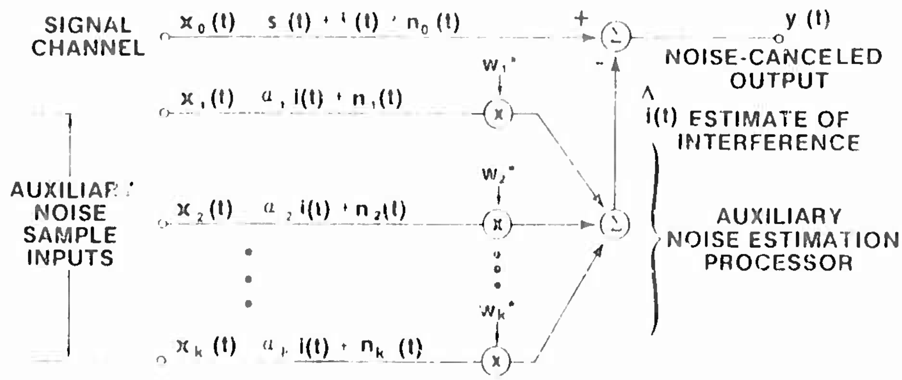

Figure 1. A K-Channel Estimator-Subtractor Noise-Cancellation Device

where T denotes the matrix transpose. The waveform i(t) is the noise component, or "interference," that is to be estimated, then subtracted from the signal channel. Also, $\{n_k(t)/k = 0,1,2,\ldots,K\}$ are the noise components, which are uncorrelated from channel to channel. The coefficients $\{\alpha_k/k = 1,2,\ldots,K\}$ represent the interference amplitude and phase difference factors between the signal channel and auxiliary channels. Notice that the signal s(t) does not appear in any of the auxiliary inputs. This type of noise canceler is a "best-case" configuration, in the sense that specifying the filter weight vector $\underline{W}$ to minimize the signal channel interference is not constrained by the problem of signal suppression resulting from the presence of signal in the auxiliary channels. Thus, any signal suppression that does occur is a result of the similarity (or correlation) of the interference component i(t) of the noise with the signal s(t).

An estimator-subtractor noise-cancellation processor works by selecting the filter weight vector $\underline{W}$ so as to minimize the mean-squared error (MSE), i.e., noise canceled output power

$$E\left\{|y(t)|^2\right\} = E\left\{|x_0(t) - \hat{i}(t)|^2\right\} \tag{5}$$

with respect to $\underline{W}$. In equation (5), $E\{\ \}$ is the statistical expectation operation and $\hat{i}(t)$ is a minimum MSE estimate of the interference component generated at the output of the auxiliary array. Since $\hat{i}(t)$ can not contain a signal component, the process of minimizing equation (5) would work primarily on reducing the level of interference i(t) in the signal channel.

Expressing the MSE noise-canceled output power in terms of the filter vector $\underline{W}$ gives

$$E\left\{\left|y(t)\right|^2\right\} = E\left\{\left|s(t) + i(t) + n_o(t) - \underline{W}^H \underline{X}\right|^2\right\} \tag{6}$$

$$= S + I + N + 2\mathrm{Re}\left\{P_{si}^*\right\} - 2\mathrm{Re}\left\{\underline{W}^H \underline{A}(P_{si}^* + I)\right\} + \underline{W}^H (I\underline{A}\underline{A}^H + \tilde{R})\, \underline{W}, \tag{7}$$

where $\mathrm{Re}\{z\}$ denotes the real part of a complex argument z, and

$$S = E\left\{\left|s(t)\right|^2\right\} \tag{8}$$

$$I = E\left\{\left|i(t)\right|^2\right\} \tag{9}$$

$$N = E\left\{\left|n_o(t)\right|^2\right\} \tag{10}$$

$$\tilde{R} = E\left\{\underline{N}\underline{N}^H\right\} \tag{11}$$

$$P_{si} = E\left\{s(t)\, i^*(t)\right\} \tag{12}$$

$$= (SI)^{1/2} f^*, \tag{13}$$

where f is the normalized value of the signal and interference correlation. Notice that P_{si} is assumed to be separable in terms of the expected power levels S and I for the signal and interference. The noise-canceled output power is minimized with respect to $\underline{W}$ for $\underline{W} = \underline{W}_0$ such that, if $\left[I\underline{A}\underline{A}^H + \tilde{R}\right]^{-1}$ exists, then

$$\underline{W}_o = \left[(SI)^{1/2} f^* + I\right]\left[I\underline{A}\underline{A}^H + \tilde{R}\right]^{-1} \underline{A}. \tag{14}$$

The minimum value of power at the noise subtractor output is therefore

$$E\left\{\left|y(t)\right|^2\right\}\Big|_{\underline{W} = \underline{W}_o} = S + I + N + 2(SI)^{1/2}\, \mathrm{Re}\{f\} - \left|(SI)^{1/2} f + I\right|^2 \underline{A}^H\left[I\underline{A}\underline{A}^H + \tilde{R}\right]^{-1} \underline{A}. \tag{15}$$

If $\tilde{Q}$ and $\tilde{P}$ are Hermitian, then the identity

$$\left[\tilde{M}^H \tilde{Q}^{-1} \tilde{M} + \tilde{P}^{-1}\right]^{-1} = \tilde{P} - \tilde{P}\tilde{M}^H (\tilde{M}\tilde{P}\tilde{M}^H + \tilde{Q})^{-1} \tilde{M}\tilde{P} \tag{16}$$

can be used, and equation (15) can be rewritten as

$$E\{|y(t)|^2\}\Big|_{\underline{W}=\underline{W}_o} = S + I + N + 2(SI)^{1/2}\,\mathrm{Re}\{f\} - |(SI)^{1/2}\,f + I|^2\,\frac{\underline{A}^H \underset{\sim}{R}^{-1} \underline{A}}{1 + I\underline{A}^H \underset{\sim}{R}^{-1} \underline{A}} . \tag{17}$$

If it is assumed that the level of uncorrelated noise in the auxiliary array noise reference channels is given by

$$E\{n_i(t)\, n_j^*(t)\} = \begin{cases} \gamma N, & i = j \\ 0, & i \neq j \end{cases} , \tag{18}$$

then

$$\underset{\sim}{R} = \gamma N \underset{\sim}{I}, \tag{19}$$

where $\underset{\sim}{I}$ is a K-by-K identity matrix. Using equation (19) in equation (17) yields

$$E\{|y(t)|^2\}\Big|_{\underline{W}=\underline{W}_o} = S\left[\frac{1 + (1 - |f|^2)(I/\gamma N)|\underline{A}|^2}{1 + (I/\gamma N)|\underline{A}|^2}\right] + N + \left[\frac{I + 2(SI)^{1/2}\,\mathrm{Re}\{f\}}{1 + (I/\gamma N)|\underline{A}|^2}\right]. \tag{20}$$

The performance of the noise-cancellation processor can be expressed in terms of the subtractor output signal-to-total-background-noise ratio S/B. This performance metric is obtained directly from equation (20) as

$$S/B = \frac{S/N}{1 + \dfrac{(I/N) + 2\left[(S/N)(I/N)\right]^{1/2}\,\mathrm{Re}\{f\}}{1 + (I/\gamma N)|\underline{A}|^2}}\left[\frac{1 + (1 - |f|^2)(I/\gamma N)|\underline{A}|^2}{1 + (I/\gamma N)|\underline{A}|^2}\right]. \tag{21}$$

AN EXAMPLE

Consider the specific case where the signal, interference, and uncorrelated noise are given, respectively, by

$$s(t) = (S)^{1/2} e^{j[\omega t + \phi_s(t)]} \tag{22}$$

$$i(t) = (I)^{1/2} e^{j[\omega t + \phi_i(t)]} \tag{23}$$

$$n_\ell(t) = \text{narrowband Gaussian noise with power } N \quad (\ell = 0, 1, \ldots, N). \tag{24}$$

In an actual implementation of the noise-cancellation processor, only a time-limited estimate of the correlation P_{si} (see equation (12)) is available, namely,

$$\hat{P}_{si} = (1/T) \int_{-T/2}^{T/2} s(t)\, i^*(t)\, dt \tag{25}$$

$$= (SI)^{1/2} f(T), \tag{26}$$

where $f(T)$ is the normalized signal-interference correlation

$$f(T) = (1/T) \int_{-T/2}^{T/2} e^{j[\phi_s(t) - \phi_i(t)]}\, dt. \tag{27}$$

For this example, let $\underline{A}$ be of the form

$$\underline{A} = a\underline{U}, \tag{28}$$

where a is a real-valued scale factor and $\underline{U}$ is a K-dimensional vector with elements of the form $e^{j\theta}$. Furthermore, let the uncorrelated noise level in the auxiliary channels be the same as the level in the signal-plus-noise channel, i.e., $\gamma = 1$. With the above simplifications, equation (21) becomes

$$S/B = \frac{S/N}{1 + \dfrac{(I/N) + 2[(S/N)(I/N)]^{1/2}\, \mathrm{Re}\{f(T)\}}{1 + (I/N)\, a^2 K}} \left\{ \frac{1 + [1 - |f(T)|^2](I/N)\, a^2 K}{1 + (I/N)\, a^2 K} \right\}. \tag{29}$$

Several limiting cases of equation (29) are of interest. First, in the limit as normalized correlation $f(T)$ approaches zero,

$$\lim_{f(T)\to 0} (S/B) = \frac{S/N}{1 + \dfrac{I/N}{1 + (I/N)\, a^2 K}} . \tag{30}$$

For $f(T) = 0$, perfect cancellation $(S/B = S/N)$ of the interfering noise component is approached as the auxiliary array gain factor a^2K becomes large. If only I/N is large and $a^2K \geq 1$, then the performance is 3 dB worse than perfect for $a^2K = 1$, and is essentially optimum for $a^2K \geq 10$. A second limiting case of interest occurs as either I/N or the auxiliary array gain factor become large. For a given normalized signal-interference correlation $f(T)$, the noise-canceled output signal-to-background-noise ratio S/B becomes

$$\lim_{I/N,\, a^2K \to \infty} (S/B) = \frac{S/N}{1 + (1/a^2K)\left[1 + 2(S/I)^{1/2}\, \mathrm{Re}\{f(T)\}\right]} \left[1 - |f(T)|^2\right] . \tag{31}$$

As the normalized signal correlation approaches unity, the signal-to-background ratio approaches zero, regardless of either the auxiliary array gain factor or the ratio S/I. Finally, for perfectly correlated signal and interference, $f(T) = 1$, and equation (29) provides the general expression of S/B as

$$\lim_{f(T)\to 1} (S/B) = \frac{S/N}{1 + (1 + a^2K)(I/N) + 2\left[(I/N)(S/N)\right]^{1/2}} . \tag{32}$$

This result indicates that, with a small S/N (even with unity signal-interference normalized correlation), complete signal suppression will be approached only as $a^2K(I/N) \to \infty$.

Figure 2 shows the noise-canceled output signal-to-background ratio S/B versus the normalized signal-interference correlation. These curves were computed using equation (29). They illustrate the effects of $f(T)$, S/N, I/N, and a^2K on S/B. The input signal-to-background-noise ratio $S/(N+I)$ is indicated to show either the relative improvement or degradation of performance resulting from implementing noise cancellation. The distance above the $S/(N+I)$

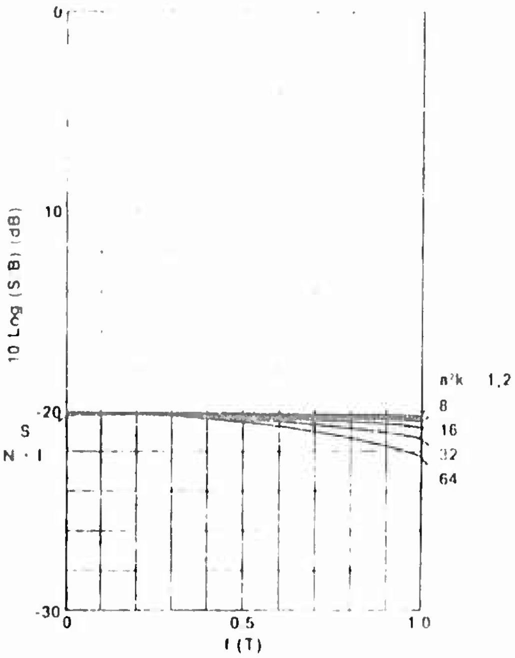

Figure 2A. S/N = -20 dB,
I/N = -20 dB

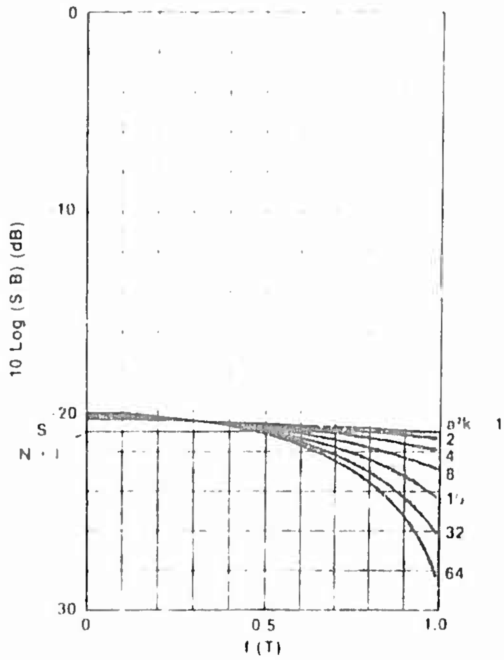

Figure 2B. S/N = -20 dB,
I/N = -10 dB

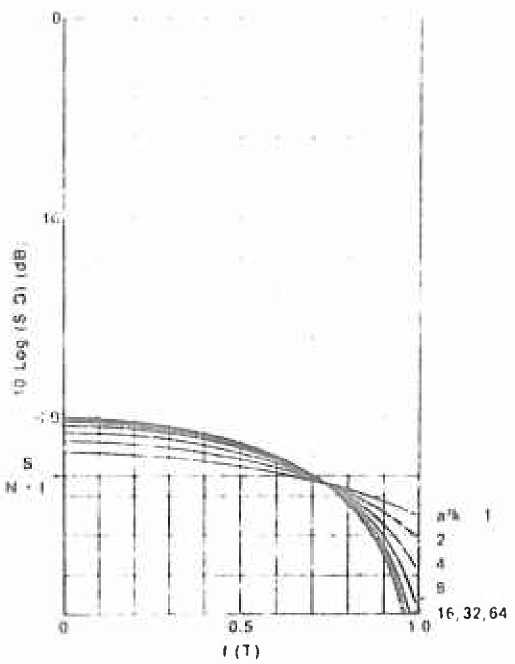

Figure 2C. S/N = -20 dB,
I/N = 0 dB

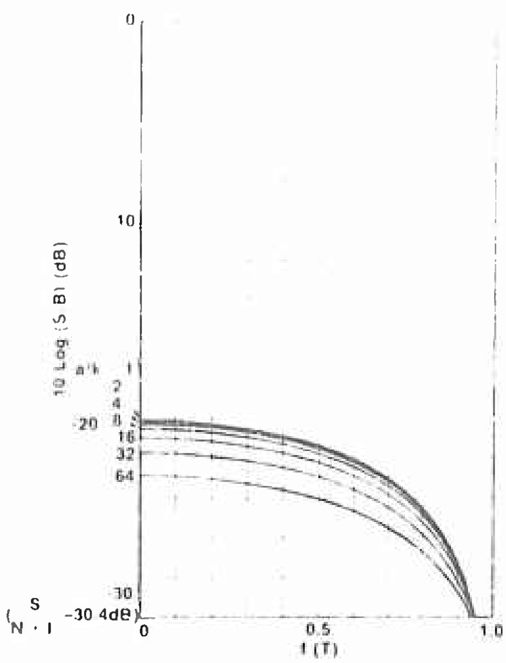

Figure 2D. S/N = -20 dB,
I/N = 10 dB

Figure 2. Output Signal-to-Total-Background-Noise Ratio versus Normalized Correlation Between Signal and Interference

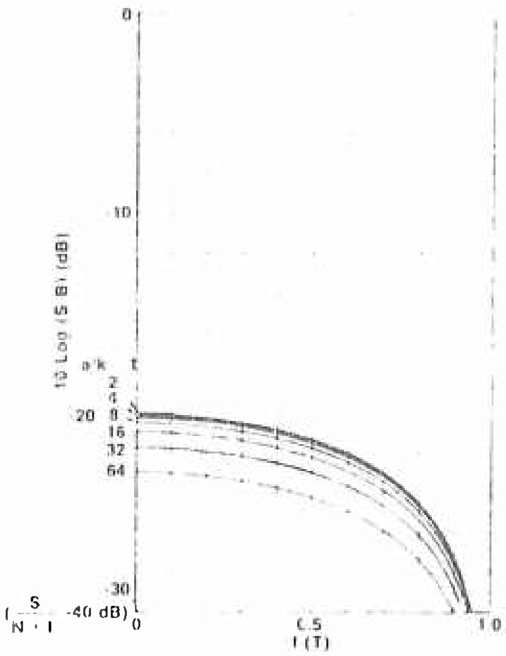

Figure 2E. S/N = -20 dB, I/N = 20 dB

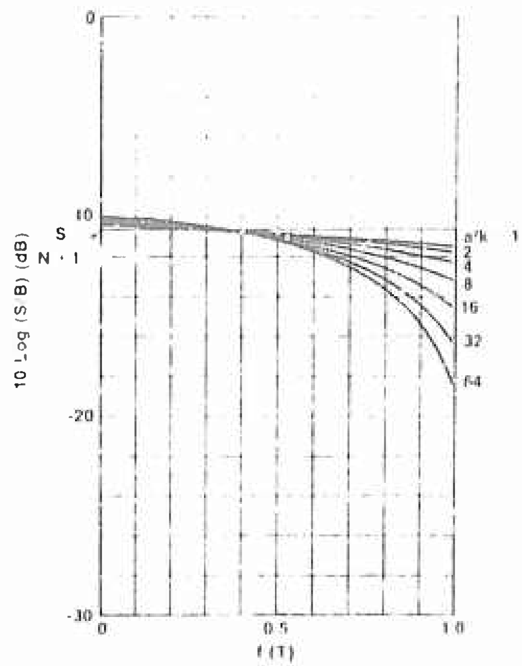

Figure 2F. S/N = -10 dB, I/N = -10 dB

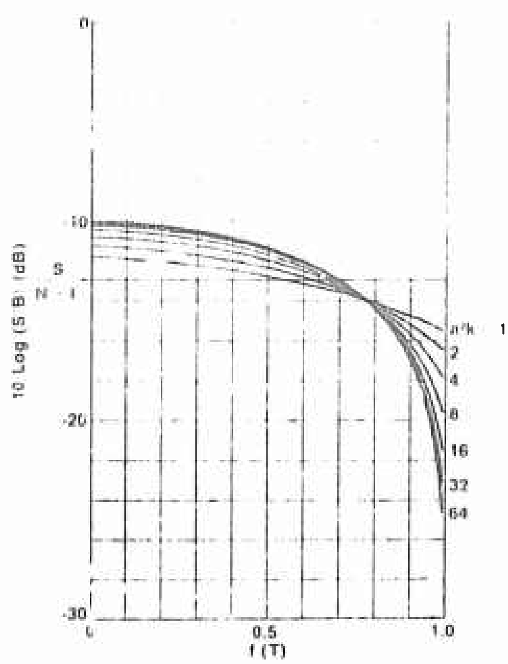

Figure 2G. S/N = -10 dB, I/N = 0 dB

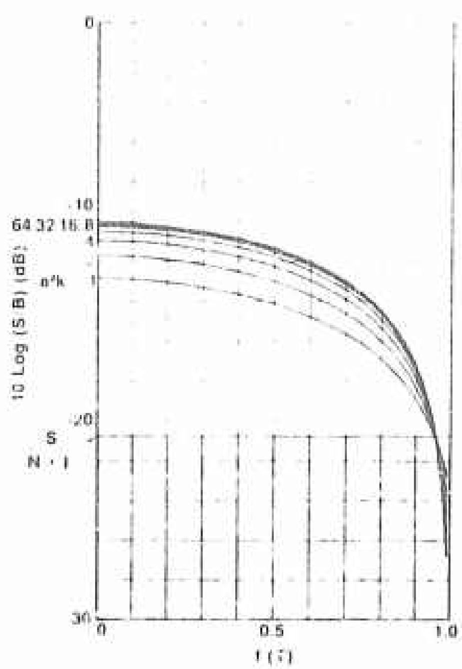

Figure 2H. S/N = -10 dB, I/N = 10 dB

Figure 2 (Cont'd). Output Signal-to-Total-Background-Noise Ratio versus Normalized Correlation Between Signal and Interference

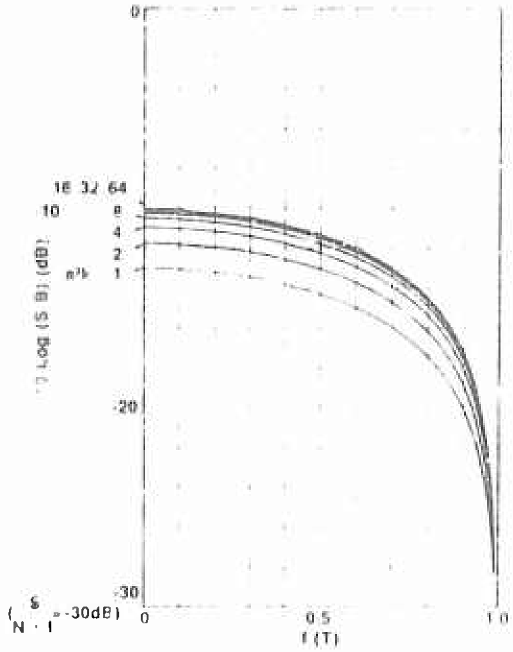

Figure 2I. S/N = -10 dB,
I/N = 20 dB

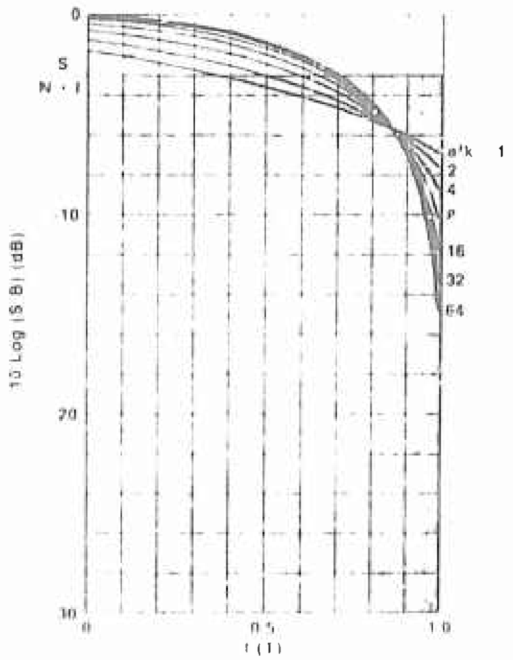

Figure 2J. S/N = 0 dB,
I/N = 0 dB

Figure 2 (Cont'd). Output Signal-to-Total-Background-Noise Ratio versus Normalized Correlation Between Signal and Interference

line to the curves is the degree of improvement; the distance below, in the shaded area, is the degree of degradation. As a specific example, consider the case in which the signal and interference are sinusoidal at frequencies ω and $(\omega + 2\pi\Delta)$, respectively. For this example,

$$s(t) = (S)^{1/2}\, e^{j\omega t} \tag{33}$$

$$i(t) = (I)^{1/2}\, e^{j(\omega + 2\pi\Delta)t} \tag{34}$$

$$f(T) = \frac{\sin \pi\Delta T}{\pi\Delta T}\,. \tag{35}$$

When equation (35) is used in equation (29), the noise-canceled output signal-to-total-background-noise ratio S/B is found and plotted in figure 3 for several values of S/N and I/N as a function of ΔT. Again, the input signal-to-total-background-noise ratio $S/(N+I)$ before cancellation is indicated to emphasize relative performance.

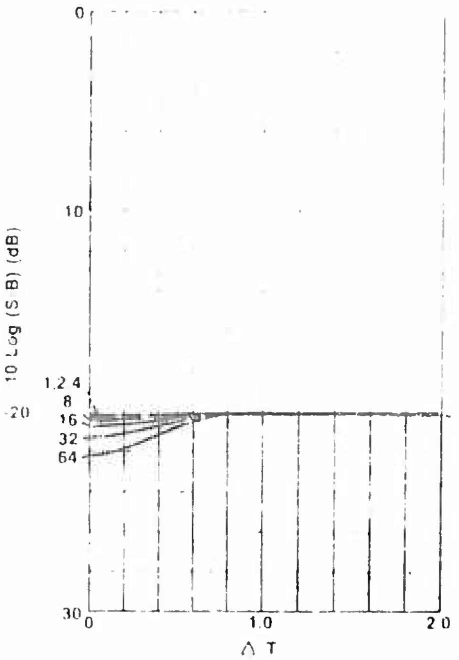

Figure 3A. S/N = −20 dB, I/N = −20 dB

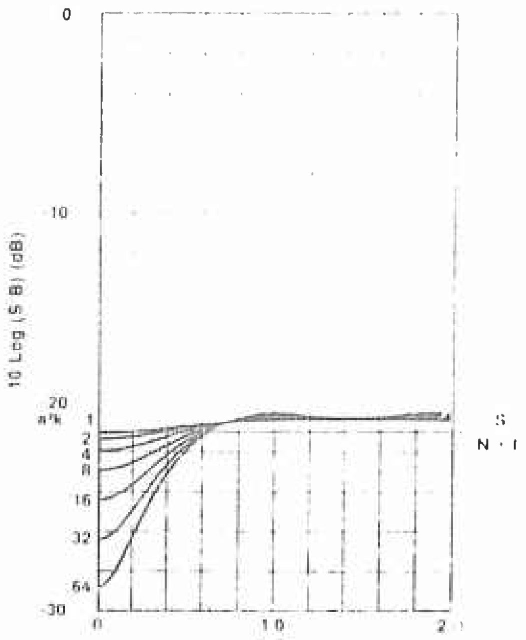

Figure 3B. S/N = −20 dB, I/N = −10 dB

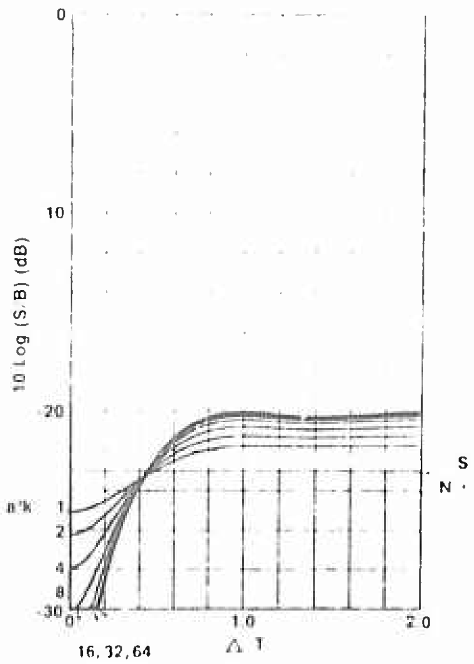

Figure 3C. S/N = −20 dB, I/N = 0 dB

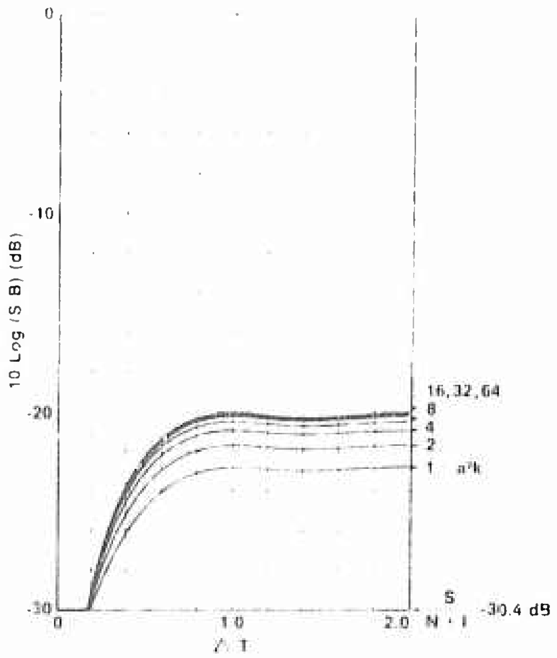

Figure 3D. S/N = −20 dB, I/N = 10 dB

Figure 3. Output Signal-to-Total-Background-Noise Ratio versus Integration Time-Bandwidth Product

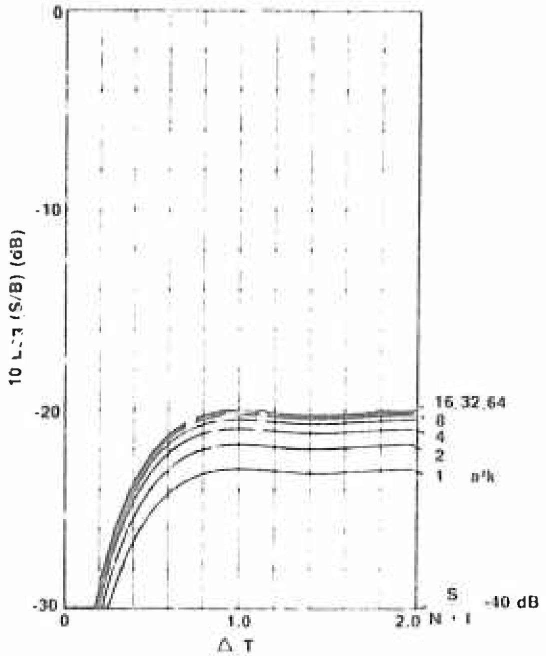

Figure 3E. S/N = -20 dB, I/N = 20 dB

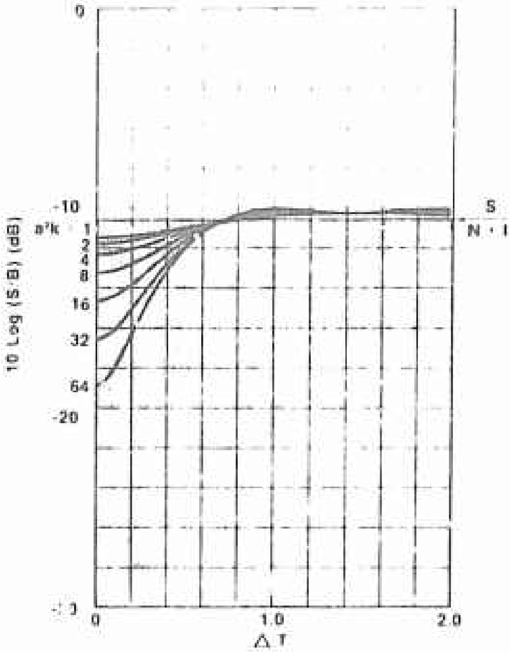

Figure 3F. S/N = -10 dB, I/N = -10 dB

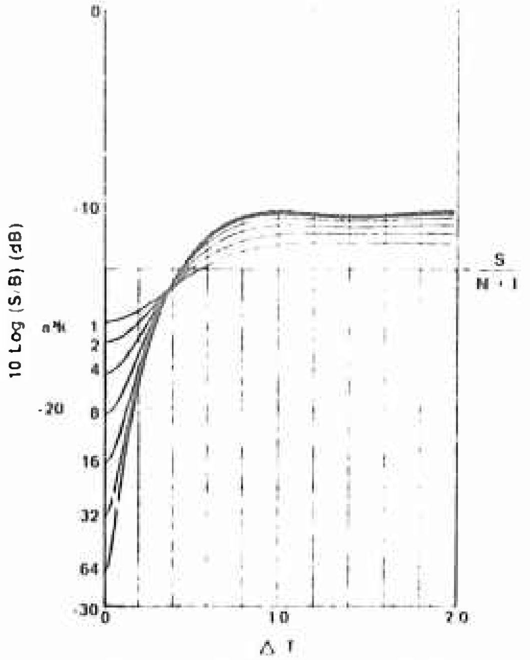

Figure 3G. S/N = -10 dB, I/N = 0 dB

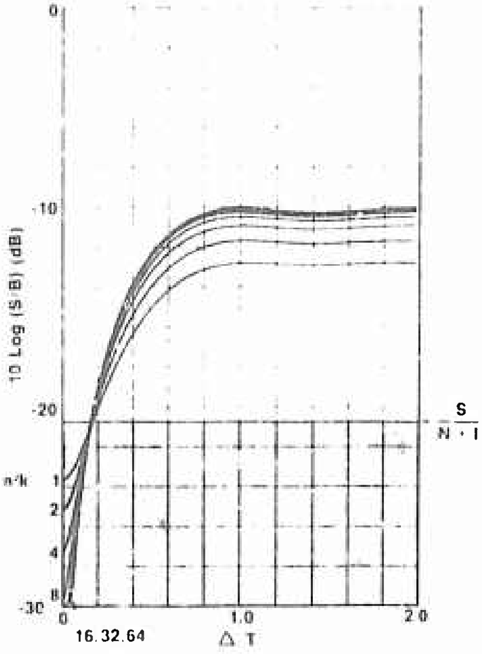

Figure 3H. S/N = -10 dB, I/N = 10 dB

Figure 3 (Cont'd). Output Signal-to-Total-Background-Noise Ratio versus Integration Time-Bandwidth Product

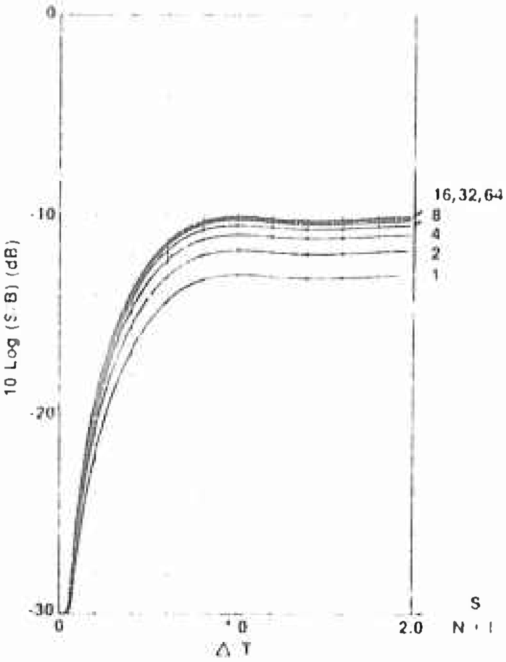

Figure 3I. S/N = -10 dB, I/N = 20 dB

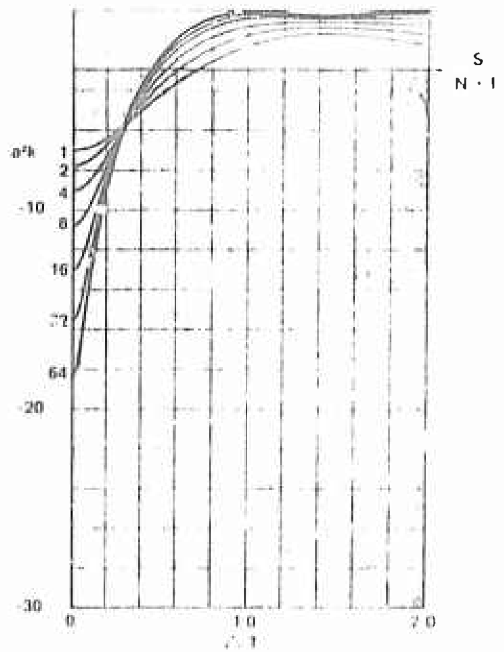

Figure 3J. S/N = 0 dB, I/N = 0 dB

Figure 3 (Cont'd). Output Signal-to-Total-Background-Noise Ratio versus Integration Time-Bandwidth Product

DISCUSSION

The assumption that signal and noise are uncorrelated is common in the theory of optimum filtering. In practice, the finite correlation times used to adaptively realize the so-called optimum filter for noise cancellation can lead to signal and noise correlation. This correlation degrades system performance.

To investigate this degradation, this report has derived a general expression for the signal-to-total-background-noise ratio at the output of a K-channel noise cancellation device. The primary results are as follows:

- An adaptively realized optimum noise-cancellation device can perform worse than a system without cancellation if the signal and interference are correlated, particularly for the small interference-to-noise ratios;
- The normalized signal and interfering noise correlation must exceed 0.5 (see figure 2) for serious signal suppression to occur as a result of implementing an interfering noise-cancellation device;

- If the signal and interference are single frequency with separation Δ Hz and if the adaptive noise-cancellation correlator has an effective integration time of T seconds, then the product ΔT must exceed unity to prevent signal suppression resulting from signal and interference correlation.

Consider some of the numerical results presented in figure 3. Recall that these results were obtained with an auxiliary array consisting of K channels, each giving a sample of a single-frequency interfering noise component in the presence of uncorrelated noise. Thus, the effectiveness of the interference-cancellation capability quite naturally is a function of the auxiliary array (interference) gain factor a^2K, where a is the attenuation factor of the interference in an auxiliary array channel relative to the signal channel. As a^2K becomes large, the interference waveform can be estimated better, simply because of array gain considerations in the presence of uncorrelated noise. However, this "clean" interference estimate can be detrimental when the correlation time-bandwidth product ΔT is less than unity. This is because a better-quality interference estimate allows more signal cancellation (suppression) when the signal and interference are correlated. On the other hand, as the interference-to-noise ratio I/N becomes large and significant interfering noise cancellation is possible, then some signal suppression can be tolerated, provided the background noise is canceled more than the signal is suppressed. (See figure 3H, for example.) Complete interfering noise cancellation without signal suppression is possible only when either the auxiliary array gain factor a^2K or the ratio I/N is large and the signal-interference correlation vanishes, i.e., $f(T) = 0$.

In contrast to the example given previously, suppose that all of the terms in the auxiliary array gain vector $\underline{A}$ are not of equal magnitude. For example, let N of the auxiliary array gain factors have magnitude Ga and the remaining (K - N) factors have magnitude a. Accordingly, there results

$$|\underline{A}|^2 = NG^2a^2 + (K - N)a^2 \tag{36}$$

$$= K_{eff}\, a^2 , \tag{37}$$

where $K_{eff} = (NG^2 + K - N)$ is an effective value of the parameter K which is referenced in equation (29). In other words, the results obtained using constant auxiliary array gain factors can be used for the variable gains case, provided an effective auxiliary array size K_{eff} can be computed as illustrated above. In particular, consider the case of interest for equation (36), wherein G = K and

$N = 1$. If the high interference-to-noise case is assumed, equation (31) gives $K_{eff} = K^2 + (K - 1) \approx K^2$ for large K. Accordingly,

$$S/B = \frac{S/N}{1 + (1/a^2K^2)\left[1 + (S/I)\right]^{1/2} \mathrm{Re}\{f(T)\}} \left[1 - |f(T)|^2\right] . \qquad (38)$$

Thus, for a given value of $f(T)$, the processor performance improves (i.e., $S/B \to S/N$), at a rate determined by K^2 rather than K. This can be noted by comparing equations (31) and (38).

SUMMARY

Signal and noise correlation can be a major source of degradation in a noise-cancellation system that has a finite response time adaptive realization. For given signal and interference noise ratios, the signal suppression effects are strictly a function of the interference gain factor for the noise estimation array and of the product between the signal-interference bandwidth and the convergence time-constant for an adaptively realized noise-cancellation system.

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