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PATTERNS AND POLARIZATIONS OF SIMULATED CONFORMAL ARRAYS

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ABSTRACT

A conformal array on a surface of small curvature can be approximated by a number of planar arrays, several of which may be excited simultaneously so as to achieve a performance similar to that of a conformal array. Since the main beam of a planar array can be steered to any direction in visible space, several arrays, each oriented in a different direction, can be steered cooperatively to form a single beam in a desired direction. A general formulation of the radiated field of such a configuration of arrays is developed with the aid of formulas which relate the components into which a vector is resolved in one orthogonal coordinate system with those into which the same vector is resolved in a second orthogonal coordinate system. This formulation does not involve the integration of the current source but is solely dependent upon the knowledge of the far-field expressions of elementary radiators. By means of this formulation, it can be shown that within each array the conventional row and column phase setting can be used; each array, however, requires an additional phase shift to compensate for the phase difference caused by its position on the curved surface. As examples, the radiation patterns and polarizations of multiple arrays of short dipoles are studies with the aid of appropriate formulation. A comparison of the multiple planar array with the conventional conformal array is also presented.

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PATTERNS AND POLARIZATIONS OF SIMULATED CONFORMAL ARRAYS

INTRODUCTION

In recent years, considerable interest has been shown in conformal arrays. This type of array has a variety of potential uses. For example, on an airplane or a missile, due to the limitation of available space, it is often difficult to install a large conventional antenna. However, a conformal array can be fitted onto some surface area of the vehicle body in such a way that it does not interfere with the operation of the vehicle. Furthermore, because part of the vehicle body forms the ground plane of the antenna, the electrical interferance problem should be minimized. On the other hand, conformal arrays generally have several drawbacks. First, the phasing of such arrays is very difficult; except for a few particular cases, there is no available approach for phasing such arrays. Second, the switching of the beam of such arrays is exceedingly complicated. Moreover, the complexity of the switching network usually introduces very high power losses into the system and hence degrades its performance.

It is conceivable that, on a surface of small curvature, one may approximate this conformal array by a number of planar arrays, several of which may be excited simultaneously so as to achieve performance similar to that of a conformal array. Since the main beam of a planar array can be steered to any direction in real space, several planar arrays, each oriented in a different direction, can be steered cooperatively to form a single beam in a desired direction. With such an arrangement, the problem of array phasing is greatly simplified. Within each array, the conventional row and colum phase setting can be used, although each array requires an additional phase shift to compensate for the phase difference caused by its position on the curved surface. However, this correction is much simpler than that required for a conventional conformal array, in which each element requires this compensating phase setting. Furthermore, the switching is greatly simplified, as it involves only a few planar arrays, in contrast to the large number of elements among which radiating power must be switched in the usual conformal array.

The problem was formulated in an earlier report [1] for multiple planar arrays of vertical dipoles which might be used to approximate a conformal array of vertical dipoles on a cylindrical surface. It was shown that a function giving composite array patterns can be defined in such a way that the far field is the product of the functions of the element radiation pattern and the composite array pattern. Numerical examples for this kind of composite array were also presented, and properties of the far field were discussed. The analysis in the earlier report is strictly valid only when the element pattern of each planar array in the composite array is similarly polarized. The present report deals with the general case where the polarization of the far field of each planar array may be different. The radiation pattern and polarization characteristics of arrays of short dipoles are studied.

SIMULTANEOUSLY EXCITED PLANAR ARRAYS OF ANTENNAS

When the field of a system of simultaneously excited planar arrays of antennas is analyzed, it is convenient to write the field expressions in coordinate variables of different coordinate systems. Figures 1 and 2 illustrate these various coordinate systems. Figure 1

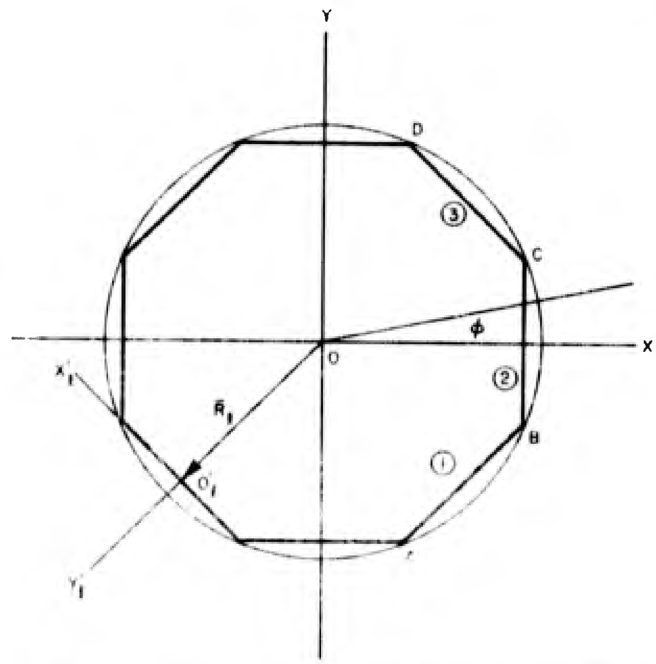


Fig. 1—Geometry of multiple planar arrays of antennas.

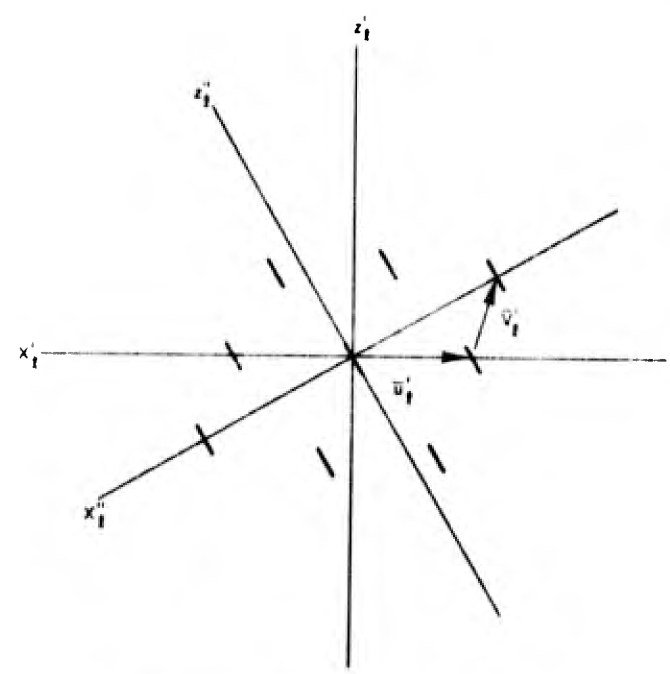


Fig. 2—A planar array of short dipoles.

shows the position of each planar array relative to the origin of the unprimed coordinate system. Figure 2 shows the lattice structure of the ℓth planar array and its radiators. One primed coordinate system is assigned to each planar array, e.g., $(x'_\ell, y'_\ell, z'_\ell)$ for the ℓth planar array. One double-primed coordinate system is also assigned to the radiators of a planar array, e.g., $(x''_\ell, y''_\ell, z''_\ell)$ for the radiators of the ℓth planar array. The primed coordinate systems are useful since the array pattern functions for the planar arrays are known in terms of the primed coordinate variables $(\theta'_\ell, \varphi'_\ell)$. The double-primed coordinate systems are useful since the element pattern of the radiators of the ℓth planar array may be known in a different coordinate system than the primed coordinates $(\theta'_\ell, \varphi'_\ell)$.

The far field of a system of simultaneously excited planar arrays of antennas may be written, omitting the time phase factor $e^{j\omega t}$, as

$$\mathbf{E} = \sum_{\ell=1}^{L} \mathbf{E}_\ell = \sum_{\ell=1}^{L} \left\{ \mathcal{E}_\ell(\theta''_\ell, \varphi''_\ell) \sum_{m,n} A_{\ell mn} e^{jk(\mathbf{R}_\ell + \mathbf{R}'_{\ell mn})\cdot\hat{R}} \right\}, \tag{1}$$

where

$$\mathbf{R}'_{\ell mn} = m\,\mathbf{u}'_\ell + n\,\mathbf{v}'_\ell. \tag{2}$$

In these expressions,

L = the number of planar arrays,

$\mathbf{E}_\ell$ = the electric field due to the ℓth planar array,

$\mathcal{E}_\ell(\theta''_\ell, \varphi''_\ell)$ = the element pattern of the ℓth planar array,

k = the free space wavenumber,

$A_{\ell mn}$ = the complex excitation coefficient of the radiating element at the point $\mathbf{R}_\ell + \mathbf{R}'_{mn}$, and

$\hat{R}$ = unit radial vector.

In spherical coordinates,

$$R = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta). \tag{3}$$

For maximum radiation in a certain direction (θ_o, φ_o), the conventional row and column phase setting can be used within each array to steer the beam in that direction. In addition to aligning the main beams spatially, one must also be sure that each E_ℓ is in time phase in the desired direction (θ_o, φ_o). This condition can be met if

$$A_{\ell mn} = \mathcal{A}_\ell\, \mathcal{A}_{nm} e^{-jk(\mathbf{R}_\ell + \mathbf{R}'_{\ell mn})\cdot\hat{R}_o}, \tag{4}$$

where

$$R_o = (\sin\theta_o\cos\varphi_o, \sin\theta_o\sin\varphi_o, \cos\theta_o), \tag{5}$$

and $\mathcal{A}_\ell$ and $\mathcal{A}_{mn}$ are constants to be specified soon. Substituting (4) in (1),

$$\mathbf{E} = \sum_{\ell=1}^{L} \mathcal{A}_\ell \mathcal{E}_\ell(\theta''_\ell, \varphi''_\ell) e^{jk\mathbf{R}_\ell \cdot (\hat{R}-\hat{R}_o)} \left[\sum_{mn} \mathcal{A}_{\ell mn} e^{jk\mathbf{R}'_{\ell mn}\cdot(\hat{R}-\hat{R}_o)}\right]$$

$$= \sum_{\ell=1}^{L} \mathcal{A}_\ell \mathcal{E}_\ell(\theta''_\ell, \varphi''_\ell) e^{jk\mathbf{R}_\ell \cdot (\hat{R}-\hat{R}_o)} f_\ell(\theta'_\ell, \varphi'_\ell), \tag{6}$$

where

$$f_\ell(\theta'_\ell, \varphi'_\ell) = \sum_{mn} \mathcal{A}_{\ell mn} e^{jk\mathbf{R}'_{\ell mn}\cdot(\hat{R}-\hat{R}_o)}. \tag{7}$$

Note that $f_\ell(\theta'_\ell, \varphi'_\ell)$ is simply the array factor for the ℓth planar array phased to have maximum radiation in the direction $\hat{R}_o$. The complex constants $\mathcal{A}_\ell$ may be set to certain convenient values that will cancel out any phase differences of the fields E_ℓ in the direction $\hat{R}_o$. For instance, $\mathcal{A}_\ell$ values may be set in the following way:

$$\mathcal{A}_\ell = (-) \text{ phase of } \left[\mathcal{E}_\ell(\theta''_\ell, \varphi''_\ell) f_\ell(\theta'_\ell, \varphi'_\ell)\right] \text{ in the direction } \hat{R}_o. \tag{8}$$

The constant $\mathcal{A}_{\ell mn}$ is simply the amplitude of the excitation coefficient of the element with the indices ℓ, m, n. Each element pattern $\mathcal{E}_\ell(\theta''_\ell, \varphi''_\ell)$ is assumed to be known in the form

$$\mathcal{E}_\ell(\theta''_\ell, \varphi''_\ell) = \mathcal{E}_{\theta''_\ell}(\theta''_\ell, \varphi''_\ell)\hat{\theta}''_\ell + \mathcal{E}_{\varphi''_\ell}(\theta''_\ell, \varphi''_\ell)\hat{\varphi}''_\ell. \tag{9}$$

Two problems will have to be solved before Eq. (6) can be used. First, one must decompose each set of unit vectors $(\hat{\theta}''_\ell, \hat{\varphi}''_\ell)$ in terms of the unit vectors $\hat{\theta}$ and $\hat{\varphi}$ in order to perform the vector addition of the $\mathcal{E}_\ell$'s. Second, the double-primed and primed coordinate variables θ''_ℓ, φ''_ℓ, θ'_ℓ, φ'_ℓ, etc, must be expressed as functions of the unprimed coordinate variables θ and φ.

TRANSFORMATIONS OF COORDINATE SYSTEMS

The problem of coordinate transformation is to express a vector function $\mathbf{V}$ known in the primed coordinate variables and unit vectors in terms of the unprimed coordinate variables and unit vectors. That is, $\mathbf{V}$ is known in the form

$$\mathbf{V} = V_{\theta'}(\theta', \varphi')\hat{\theta}' + V_{\varphi'}(\theta', \varphi')\hat{\varphi}', \tag{10}$$

where $V_{\theta'}$ and $V_{\varphi'}$ denote two functions of θ' and φ'.

We are interested in finding V in the form

$$\mathbf{V} = V_\theta(\theta, \varphi)\hat{\theta} + V_\varphi(\theta, \varphi)\hat{\varphi}, \tag{11}$$

where V_θ and V_φ denote two functions of θ and φ. Note that we have confined ourselves to the discussion of radiation fields. Thus, the vector has no radial component and is independent of the radius variable R.

In the following paragraphs, we will be dealing with a class of matrices known as real orthogonal matrices. These matrices transform a vector from one orthogonal coordinate system to a second orthogonal coordinate system in the three-dimensional Euclidean space. That is, if $(\hat{\alpha}, \hat{\beta}, \hat{\gamma})$ and $(\hat{\alpha}', \hat{\beta}', \hat{\gamma}')$ are the two sets of unit vectors of two orthogonal coordinate systems, then the components of a vector V in the two coordinate systems are related by an orthogonal matrix $|D|$:

$$|V| = |D| \cdot |V'|, \tag{12}$$

where $|V|$ and $|V'|$ are the column matrix representations of the vector V in the two coordinate systems,

$$|V| = \begin{vmatrix} V_\alpha \\ V_\beta \\ V_\gamma \end{vmatrix} \tag{13}$$

$$|V'| = \begin{vmatrix} V_{\alpha'} \\ V_{\beta'} \\ V_{\gamma'} \end{vmatrix} . \tag{14}$$

Real orthogonal matrices have two useful properties: first, the inverse $|D|^{-1}$ of a real orthogonal matrix $|D|$ is the transpose $|D|^T$ of $|D|$, or

$$|D|^{-1} = |D|^T; \tag{15}$$

second, the product of real orthogonal matrices is a real orthogonal matrix. These properties will be utilized in later discussions.

Coordinate transformations involve either a linear translation or a change of orientation of the coordinate system (Fig. 1). The only translations involved in the present problem are in moving the origins of the primed coordinate systems back to the common reference point, the origin of the unprimed coordinates. For the far field, the only effect of this translation is to introduce the phase factor $e^{jk\mathbf{R}_\ell \cdot (\hat{R}-\hat{R}_o)}$, which appears in Eq. (6). The functional dependence of a field vector V on the coordinate variables and unit vectors is not altered by coordinate translations. This fact is illustrated in Fig. 3, where, for clarity, the translation $\mathbf{R}'_\ell$ between the two coordinate systems is assumed to be in the xy plane. Let P be the field observation point; then it can be seen that if the field point P is truly at infinity, one would have

$$R' = R - \mathbf{R}'_{\ell} \cdot \hat{R}$$

$$\theta' = \theta \qquad (16)$$

$$\varphi' = \varphi$$

and

$$\hat{R}' = \hat{R}$$

$$\hat{\theta}' = \hat{\theta} \qquad (17)$$

$$\hat{\varphi}' = \hat{\varphi}.$$

The field vector **V** in terms of θ, φ, $\hat{\theta}$, $\hat{\varphi}$ can be obtained by substitution of Eqs. (16) and (17) in (10). Since **V** is independent of R', it is obvious that the functions V_θ and V_φ in Eq. (11) are identical to the functions $V_{\theta'}$ and $V_{\varphi'}$ in (10). For example, the radiation field of a short dipole lying along the z' axis and at the origin of the primed coordinate system in Fig. 3 is given by

$$\mathbf{E} = E_o(t, R') \sin \theta' \hat{\theta}' \qquad (18)$$

$$E_o(t, R') = \frac{j\omega I S e^{j\omega [t-(R'/C)]}}{4\pi \epsilon C^2 R'}\,, \qquad (19)$$

where I is the current and S is the length of the dipole.

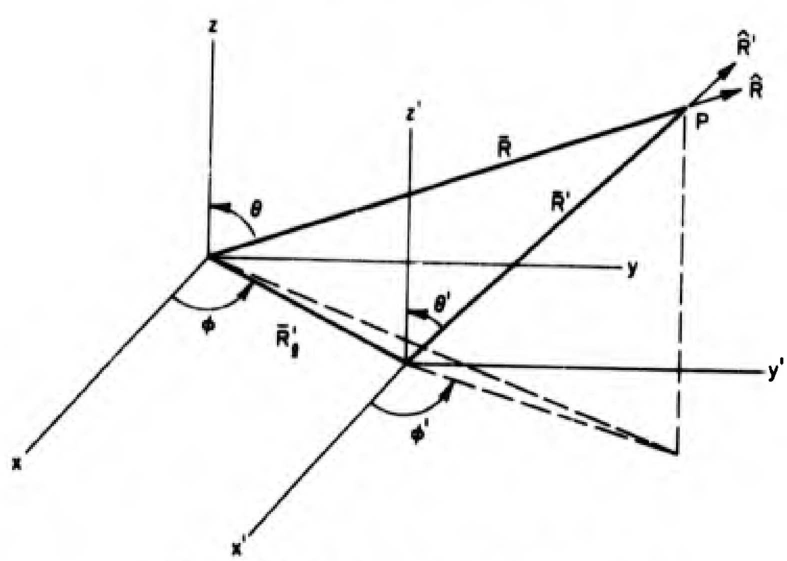


Fig. 3—Translation of a coordinate system.

Substituting Eqs. (16), and (17) in (18) and (19) and using the far-field approximation $R' = R$ for the denominator of (19),

$$\mathbf{E} = \left[E_o(t, R) \sin\theta\hat{\theta}\right] e^{jk\mathbf{R}\cdot\hat{R}}. \tag{20}$$

Note that the factor inside the bracket of Eq. (20) has a form identical to that of (18).

From the above discussion, we conclude that, with the inclusion of the exponential factor $e^{jk\mathbf{R}_\ell\cdot(\hat{R}-\hat{R}_o)}$, the translation $\mathbf{R}_\ell$ has no other effect on Eq. (6); this factor will be ignored in later discussions.

The second transformation is one in which the orientation of the coordinate system is changed. In classical mechanics problems the transformation formula has been set up using matrices. Three independent parameters are needed to specify the orientation of a rigid body. These are known as Eulerian angles. The transformation is described by the three angles, as explained in the following paragraphs.

The change of the orientation of the coordinate system is accomplished by three successive rotations about the three coordinate axes. These rotations are shown in Fig. 4. The first rotation is for an angle ξ_y about the y axis. The orthogonal matrix between the primed and the unprimed coordinate systems for this rotation is

$$|A| = \begin{vmatrix} \cos\xi_y & 0 & \sin\xi_y \\ 0 & 1 & 0 \\ -\sin\xi_y & 0 & \cos\xi_y \end{vmatrix}. \tag{21}$$

The second rotation is for an angle ξ_z about the z-axis. The orthogonal matrix for this rotation is

$$|B| = \begin{vmatrix} \cos\xi_z & -\sin\xi_z & 0 \\ \sin\xi_z & \cos\xi_z & 0 \\ 0 & 0 & 1 \end{vmatrix}. \tag{22}$$

The third rotation is for an angle ξ_x about the x-axis:

$$|C| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\xi_x & -\sin\xi_x \\ 0 & \sin\xi_x & \cos\xi_x \end{vmatrix}. \tag{23}$$

In all three rotations the angle of rotation is positive when the rotation is counterclockwise with respect to the axis of rotation. The overall transformation may be written as

$$|D| = |C|\,|B|\,|A|. \tag{24}$$

One should note here that the order of matrix multiplication is not commutative; thus, the sequence of these transformations is not interchangeable.

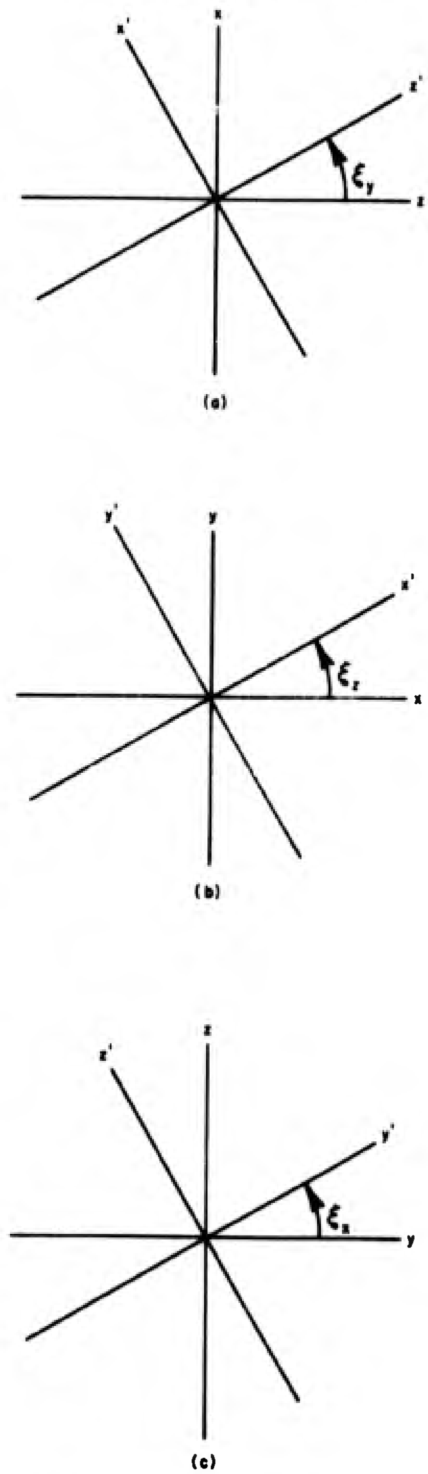


Fig. 4—Rotations of coordinate systems.

Note that matrices $|A|$, $|B|$, $|C|$, $|D|$ are real orthogonal. Thus,

$$|D|^{-1} = |D|^T. \tag{25}$$

Multiplying Eq. (12) by $|D|^{-1}$, one obtains

$$|V'| = |D|^{-1}\,|V|. \tag{26}$$

By the use of $|D|$ and $|D|^T$, a vector decomposed in one rectangular coordinate system may be decomposed again in a different rectangular coordinate system. To treat radiation fields, however, one would also have to deal with components in spherical coordinates. The spherical coordinate components and the rectangular coordinate components of a vector are also related by real orthogonal matrices, as follows:

$$\begin{vmatrix} V_r \\ V_\theta \\ V_\varphi \end{vmatrix} = |D_{RP}| \begin{vmatrix} V_x \\ V_y \\ V_z \end{vmatrix} \tag{27}$$

$$\begin{vmatrix} V_x \\ V_y \\ V_z \end{vmatrix} = |D_{PR}| \begin{vmatrix} V_r \\ V_\theta \\ V_\varphi \end{vmatrix} \tag{28}$$

where

$$|D_{PR}| = \begin{vmatrix} \sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{vmatrix} \tag{29}$$

and

$$|D_{RP}| = |D_{PR}|^{-1} = |D_{PR}|^T. \tag{30}$$

Substituting Eq. (27) and (28) in (12), and if **V** is the radiation field, also in (10), we obtain

$$\begin{vmatrix} V_R \\ V_\theta \\ V_\varphi \end{vmatrix} = |D_{RP}| \cdot |D| \cdot |D'_{PR}| \cdot \begin{vmatrix} 0 \\ V_{\theta'}(\theta', \varphi') \\ V_{\varphi'}(\theta', \varphi') \end{vmatrix}, \tag{31}$$

where $|D'_{PR}|$ is given by Eq. (29), with (θ', φ') replacing (θ, φ). It is used since the first transformation is from the primed polar coordinates to the primed rectangular coordinates. Equations (6) and (7) can now be rewritten in the matrix form with the aid of (31) and (12), respectively:

$$\begin{vmatrix} E_R(\theta,\varphi) \\ E_\theta(\theta,\varphi) \\ E_\varphi(\theta,\varphi)\end{vmatrix} = \sum_{\ell} \mathcal{A}_\ell e^{jk\mathbf{R}_\ell\cdot(\hat{R}-\hat{R}_o)} f_\ell(\theta'_\ell,\varphi'_\ell)|D_{RP}|\cdot|D_{2\ell}|\cdot|D''_{PR}|\cdot \begin{vmatrix}0\\ \mathcal{E}''_{\theta\ell}(\theta''_\ell,\varphi''_\ell)\\ \mathcal{E}''_{\varphi\ell}(\theta''_\ell,\varphi''_\ell)\end{vmatrix} \tag{32}$$

and

$$f_\ell(\theta_\ell,\varphi_\ell) = \sum_{mn} \mathcal{A}_{mn} e^{jk(|D_{1\ell}|\cdot|\mathbf{R}'_{\ell mn}|)\cdot(\hat{R}-\hat{R}_o)}, \tag{33}$$

where $|\mathbf{R}'_{\ell mn}|$ is the matrix of $\mathbf{R}'_{\ell mn}$ in $(x'_\ell, y'_\ell, z'_\ell)$ coordinates. The matrix $D_{2\ell}$ is the transformation matrix from $(x''_\ell, y''_\ell, z''_\ell)$ to (x, y, z), and $D_{1\ell}$ is the transformation matrix from $(x'_\ell, y'_\ell, z'_\ell)$ to (x, y, z). Equations (32) and (33) give the formal solution of the radiation field of multiple planar arrays, on the assumption that the double-primed and primed coordinate variables θ''_ℓ, φ''_ℓ, θ'_ℓ, φ'_ℓ, etc., are functions of the unprimed polar coordinate variables θ and φ.

Next, we consider relations between coordinate variables under coordinate transformations. The relations between rectangular coordinate variables are obtained by substituting the position vector $\mathbf{R}$ for the vector $\mathbf{V}$ in (26):

$$|R'| = |D|^{-1}\,|R|, \tag{34}$$

where

$$|R'| = \begin{vmatrix} x' \\ y' \\ z' \end{vmatrix} \tag{35}$$

and

$$|R| = \begin{vmatrix} x \\ y \\ z \end{vmatrix}. \tag{36}$$

The relations between polar coordinate variables are found by substituting

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{vmatrix} \tag{37}$$

and

$$\begin{vmatrix} x' \\ y' \\ z' \end{vmatrix} = \begin{vmatrix} \sin\theta' \cos\varphi' \\ \sin\theta' \sin\varphi' \\ \cos\theta' \end{vmatrix} \quad (38)$$

in Eq. (34) and solving for θ' and φ'.

By the use of the three Eulerian angles, the following can be shown:

$$\theta' = \cos^{-1} Z'(\theta, \varphi), \quad (39)$$

$$\varphi' = \tan^{-1} \frac{Y'(\theta, \varphi)}{X'(\theta, \varphi)}, \quad (40)$$

$$X'(\theta, \varphi) = \sin\theta \cos\phi \cos\xi_y \cos\xi_z + \sin\theta \sin\phi \sin\xi_z + \cos\theta \sin\xi_y \cos\xi_z, \quad (41)$$

$$Y'(\theta, \varphi) = -\sin\theta \cos\varphi (\cos\xi_y \sin\xi_z \cos\xi_x + \sin\xi_y \sin\xi_x)$$
$$+ \sin\theta \sin\varphi \cos\xi_z \cos\xi_x - \cos\theta (\sin\xi_y \sin\xi_z \cos\xi_x$$
$$- \cos\xi_y \sin\xi_x), \quad (42)$$

and

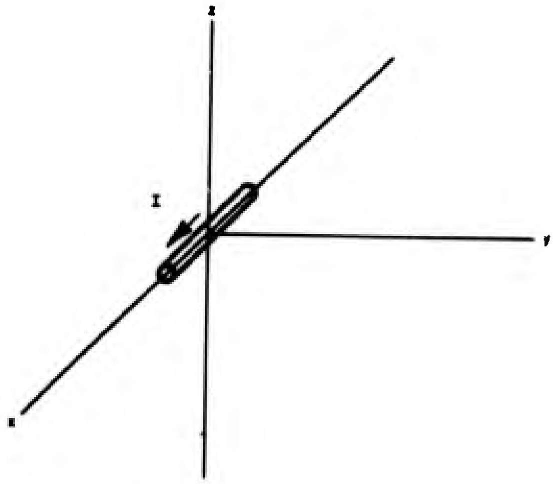
$$Z'(\theta, \varphi) = \sin\theta \cos\phi (\cos\xi_y \sin\xi_z \sin\xi_x - \sin\xi_y \cos\xi_x)$$
$$- \sin\theta \sin\phi \cos\xi_z \sin\xi_x + \cos\theta (\sin\xi_y \sin\xi_z \sin\xi_x$$
$$+ \cos\xi_y \cos\xi_x). \quad (43)$$

The ambiguity in the value of the arc tangent function in Eq. (40) is resolved by applying the same set of rules that one uses to determine the value of $\tan^{-1}(y/x)$, where x and y are the rectangular coordinate variables.

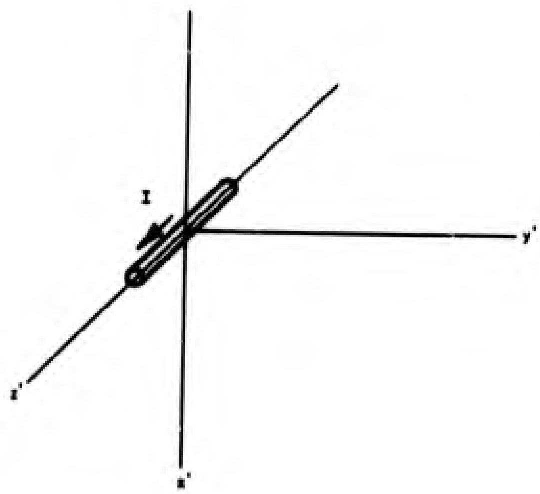
THE RADIATION FIELD OF A HORIZONTAL SHORT DIPOLE

In Fig. 5a, let z be the elevation axis; then the dipole lying along the x axis may be referred to as a horizontal dipole. The far field of a dipole is commonly expressed in the coordinate system shown in Fig. 5b and is given by Eqs. (18) and (19). The far field of the horizontal dipole may be obtained from Eq. (18) by the use of coordinate transformation formulas. The primed rectangular coordinate system in Fig. 5b is obtained from the unprimed rectangular coordinate system in Fig. 5a by a simple rotation of -90 degrees about the y axis. From Eq. (21), use of the Eulerian parameter ξ_y = 90 degrees, gives

$$|D| = |A| = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{vmatrix}. \quad (44)$$



(a)



(b)

Fig. 5—A short dipole in two coordinate systems.

The relations between the rectangular coordinate variables are from Eq. (34)

$$\left.\begin{aligned} x' &= -z, \\ y' &= y, \\ z' &= x. \end{aligned}\right\} \tag{45}$$

The relations between the polar coordinate variables can be obtained from Eq. (45):

$$\left.\begin{aligned}
\cos\theta' &= \frac{z'}{R'} = \frac{x}{R} = \sin\theta\cos\varphi \\
\sin\theta' &= \sqrt{1-\sin^2\theta\cos^2\varphi} \\
\cos\varphi' &= \frac{x'}{R'\sin\theta'} = \frac{-\cos\theta}{\sqrt{1-\sin^2\theta\cos^2\varphi}} \\
\sin\varphi' &= \frac{y'}{R'\sin\theta'} = \frac{\sin\theta\sin\varphi}{\sqrt{1-\sin^2\theta\cos^2\varphi}}\,.
\end{aligned}\right\} \tag{46}$$

The matrix form for E in the primed polar coordinates is

$$\begin{vmatrix} E_{r'} \\ E_{\theta'} \\ E_{\varphi'} \end{vmatrix} = \begin{vmatrix} 0 \\ E_o(t)\sin\theta' \\ 0 \end{vmatrix}. \tag{47}$$

Carrying out the successive matrix multiplication in Eq. (31) by means of (29), (30), (44), and (47), we get

$$\begin{vmatrix} E_r \\ E_\theta \\ E_\varphi \end{vmatrix} = \begin{vmatrix} -\sin\theta\cos\varphi\sin\theta' + \sin\theta\sin\varphi\cos\theta'\sin\varphi' - \cos\theta\cos\theta'\cos\varphi' \\ -\cos\theta\cos\varphi\sin\theta' + \cos\theta\sin\varphi\cos\theta'\sin\varphi' + \sin\theta\cos\theta'\cos\varphi' \\ \sin\varphi\sin\theta' + \cos\varphi\cos\theta'\sin\varphi' \end{vmatrix}. \tag{48}$$

If we now substitute Eqs. (46) in the above expression to get rid of the primed coordinate variables, we obtain

$$\begin{vmatrix} E_r \\ E_\theta \\ E_\varphi \end{vmatrix} = \begin{vmatrix} 0 \\ -E_o(t)\cos\theta\cos\varphi \\ E_o(t)\sin\varphi \end{vmatrix} \tag{49}$$

or

$$\mathbf{E} = E_o(t)\,(-\cos\theta\cos\varphi\hat{\theta} + \sin\varphi\hat{\varphi}). \tag{50}$$

PLANAR ARRAYS OF SHORT DIPOLES

In this section, we will consider using planar arrays of short dipoles to approximate a conformal array of short dipoles on a cylindrical surface. The geometry of the problem is shown in Fig. 1. The elements of the ℓth array are symmetrically placed about the reference point θ'_ℓ. The elements are short dipoles oriented in the direction either of the x'_ℓ axis or the z'_ℓ axis shown in Fig. 2. Use of the element pattern functions in Eqs. (50) and (18) shows that the double-primed coordinates are obviously the same and are the primed coordinates for both these cases. The Eulerian parameters between the (x, y, z) coordinates and the $(x'_\ell, y'_\ell, z'_\ell)$ coordinates are $\xi_{x\ell} = 0$, $\xi_{y\ell} = 0$, $\xi_{z\ell} = -[(\ell - 1)(2\pi/L) + (\pi/2)]$. Thus,

$$|D| = \begin{vmatrix} \cos \xi_{z\ell} & -\sin \xi_{z\ell} & 0 \\ \sin \xi_{z\ell} & \cos \xi_{z\ell} & 0 \\ 0 & 0 & 1 \end{vmatrix} . \tag{51}$$

It is easy to see that, for this simple rotation, the following relations hold for any far-field expressions:

$$\left\{ \begin{aligned} \theta'_\ell &= \theta \\ \varphi'_\ell &= \varphi - \xi_{z\ell} \end{aligned} \right\} \tag{52}$$

and

$$\left\{ \begin{aligned} \hat{\theta}'_\ell &= \hat{\theta} \\ \hat{\varphi}'_\ell &= \hat{\varphi} . \end{aligned} \right\} \tag{53}$$

Since the vectors $\hat{\theta}'_\ell$ and $\hat{\varphi}'_\ell$ are identical to $\hat{\theta}$ and $\hat{\varphi}$, one can use Eq. (6) in place of Eq. (32) for the vector addition process. Thus,

$$\mathbf{E} = \sum_\ell \mathcal{A}_\ell [\mathcal{E}_{\ell\theta}(\theta'_\ell, \varphi'_\ell)\hat{\theta}'_\ell + \mathcal{E}_{\ell\varphi}(\theta'_\ell, \varphi'_\ell)\hat{\varphi}'_\ell] e^{jk\mathbf{R}_\ell \cdot (\hat{R}-\hat{R}_o)} f_\ell(\theta'_\ell, \varphi'_\ell). \tag{54}$$

The excitation coefficients are assumed to have uniform magnitude of 1. The array pattern function $f_\ell(\theta, \varphi)$ is obtained from Eq. (34) as

$$f_\ell(\theta, \varphi) = \sum_{m=-M}^{M} \sum_{n=-N}^{N} e^{jk(|D||\mathbf{R}'_{\ell mn}|) \cdot |\hat{R}-\hat{R}_o|}, \tag{55}$$

where

$$|\mathbf{R}'_{\ell mn}| = \begin{vmatrix} m\, d_x \\ 0 \\ n\, d_z \end{vmatrix} . \tag{56}$$

By carrying out the matrix multiplication and rearranging terms, it can be shown that

$$f_\ell(\theta, \varphi) = f_{x\ell}(\theta, \varphi) f_{z\ell}(\theta, \varphi), \tag{57}$$

where

$$f_{x\ell}(\theta, \varphi) = 1 + 2 \sum_{m=1}^{M} \cos mkd_x [\sin\theta \cos(\varphi - \xi_{z\ell}) - \sin\theta_o \cos(\varphi_o - \xi_{z\ell})] \text{ and} \tag{58}$$

$$f_{z\ell}(\theta, \varphi) = 1 + 2 \sum_{n=1}^{N} \cos nkd_z (\cos\theta - \cos\theta_o). \tag{59}$$

By substituting Eqs. (18), (52), and (53) in (56), the radiation field for a system of planar arrays of vertical short dipoles is shown to be

$$\mathbf{E} = E_o(t) \sin\theta \left[\sum_{\ell=1}^{L} e^{jk\mathbf{R}_\ell \cdot (\hat{R} - \hat{R}_o)} f_\ell(\theta_\ell, \varphi_\ell) \; \hat{\theta} \right]. \tag{60}$$

By substituting (50), (52), and (53) in (54), the radiation field of planar arrays of horizontal dipoles is shown to be

$$\mathbf{E} = E_o(t) \left[\sum_{\ell} - \cos\theta \cos(\varphi - \xi_{z\ell}) e^{jk\mathbf{R}_\ell \cdot (\hat{R} - \hat{R}_o)} f_\ell(\theta, \varphi) \right] \hat{\theta}$$

$$+ E_o(t) \left[\sum \sin(\varphi - \xi_{z\ell}) e^{jkR_\ell \cdot (\hat{R} - \hat{R}_o)} f_\ell(\theta, \varphi) \right] \hat{\varphi}. \tag{61}$$

When both (55) and (56) were obtained, the complex constants $\mathcal{C}_\ell$ in (6) were set equal to 1.

NUMERICAL CALCULATIONS

For the multiple planar arrays of vertical dipoles, one can define a composite array function $A(\theta, \varphi)$ as follows:

$$A(\theta, \varphi) = \sum_{\ell=1}^{L} e^{jk\mathbf{R}_\ell \cdot (\hat{R} - \hat{R}_o)} f_\ell(\theta_\ell, \varphi_\ell). \tag{62}$$

The radiation field is then, from Eq. (60),

$$\mathbf{E} = \left[E_o(t) \sin\theta \hat{\theta} \right] A(\theta, \varphi). \tag{63}$$

This composite array function was calculated for different parameters in an earlier report [1].

In the case of multiple planar arrays of horizontal dipoles, the far field can not be factored into the product of an array function and an element pattern function, as can be seen from Eq. (61). This is characteristic of multiple planar arrays where the element polarization differs from one array to another. The radiation pattern and the polarization of the multiple planar arrays shown in Fig. 1 were calculated for both cases. Each planar array is assumed to be a linear array in the direction of the x'_ℓ axis. In one case, the elements of the arrays are short dipoles lying parallel to the z axis, and Eq. (62) was used to calculate the array pattern function of the composite array. In another case, the elements of the arrays are short dipoles lying parallel to the x'_ℓ axis of each array, and the far field components E_θ and E_φ were calculated using Eq. (61). Note that if there are more than two elements along the z'_ℓ axis, the only modification to the present calculation would be to multiply the radiation field by the factor $f_{z\ell}(\theta, \varphi)$ in Eq. (54). It is obvious that $f_{z\ell}(\theta, \varphi)$ is simply the array factor of a linear array in the direction of the z axis.

The following parameters were used:

Aperture length along the x'_ℓ axis = 10λ

Element spacing = 0.4λ

Number of elements of each array = 23

$(\theta_o, \varphi_o) = (90°, 0°)$

Note that there is no element at either end of each aperture. Figure 6 shows the array pattern function vs the angle φ in the xy plane for multiple arrays of vertical dipoles when the planar arrays 1, 2, and 3 in Fig. 1 are excited simultaneously. Figure 7a shows E_φ vs the angle φ in the xy plane when only the planar array No. 2 is active. Figure 7b shows the same when the three planar arrays 1, 2, and 3 are active. Note the improvement in the directivity of the composite array by having three active planar arrays as compared with just one active array. The half-power beamwidth is about 6° in Fig. 7a and about 2° in Fig. 7b. The more interesting comparison is between Fig. 7b and Fig. 7c, which shows E_φ vs φ in the xy plane for a conformal array of 76 equally spaced, horizontal, tangential, short dipoles on the arc ABCD (Fig. 1). In both Figs. 7b and 7c, the half-power beamwidth is 2 degrees and the sidelobe level is 13 dB. Patterns were also calculated for the conformal array and the multiple planar arrays for scanning angles φ_o = 10°, 20°, and 30° in the xy plane. In all instances, the beamwidth and the sidelobe level are the same as in Figs. 7b and 7c. It is, therefore, concluded that the performance of a conformal array on a cylindrical surface can be closely matched by a small number of multiple planar arrays having roughly the same total number of elements and occupying roughly the same space.

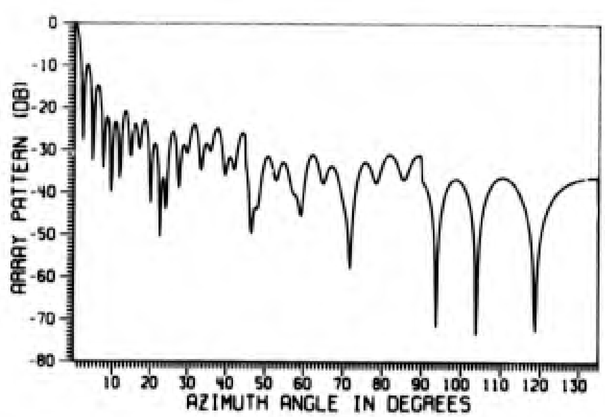


Fig. 6—Composite array pattern function for the multiple arrays of Fig. 1, vertical dipole case.

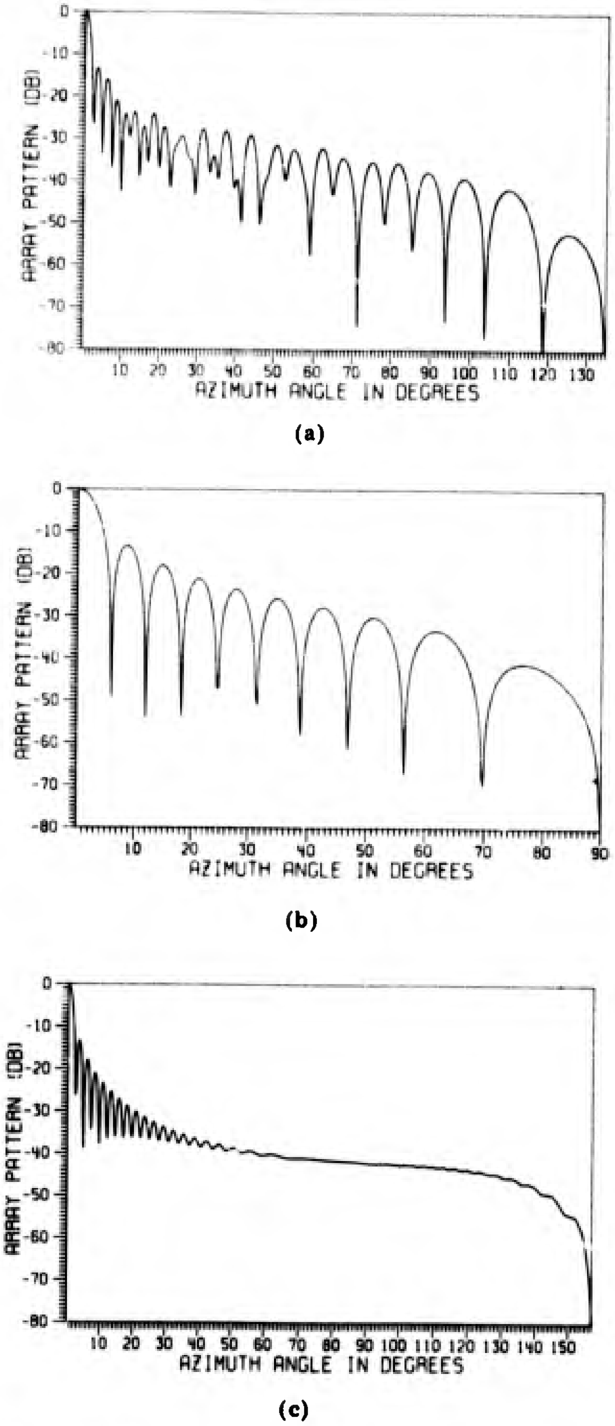


Fig. 7—Far-field patterns of the multiple arrays of Fig. 1, horizontal dipole case. (a) Planar array No. 2 is active. (b) Planar arrays 1, 2, and 3 are active. (c) A conformal array of 76 active elements on the arc ABCD.

So far, we have ignored the other component, E_θ. The reason is that $E_\theta = 0$ in the xy plane ($\theta = 90^\circ$), as can be seen from Eq. (61). The far field is thus horizontally polarized in the xy plane. The crosspolarized component E_θ becomes more important at large elevation angles (smaller θ). It can also be seen from Eq. (61) that E_θ is always in phase with E_φ if $f_\ell(\theta, \varphi)$ is real, or if each planar array is symmetrically excited relative to the center element of the array. The exact value of the crosspolarized field E_φ depends on θ, φ, θ_o, and φ_o for a given composite array and can be calculated from Eq. (61). For example, Fig. 8 shows the two components E_θ and E_φ on the conical surface $\theta = 80^\circ$ for the multiple planar arrays used in calculating the data shown in Fig. 7b.

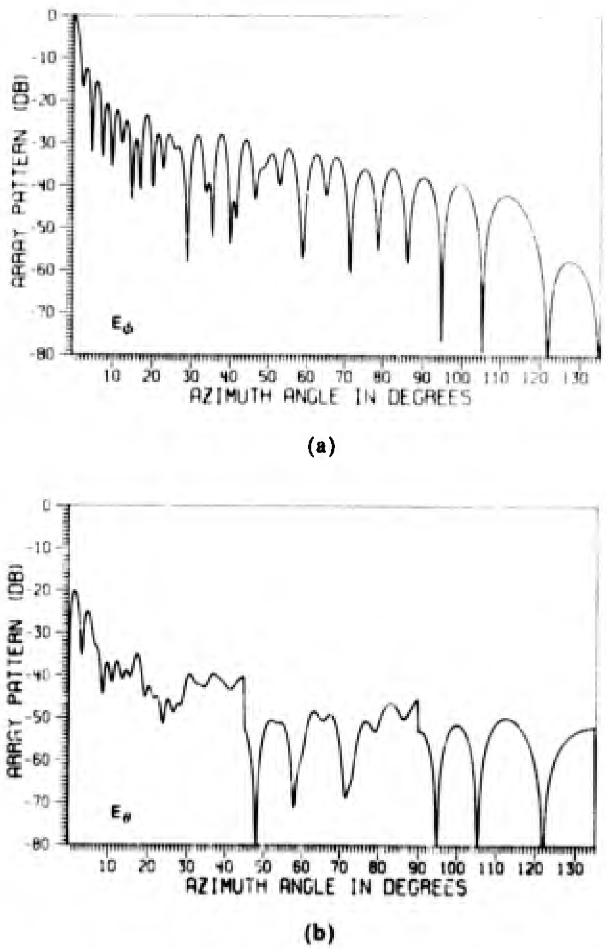


Fig. 8—Far-field patterns on the conical surface $\theta = 80^\circ$ for the multiple arrays in Fig. 1. Planar arrays 1, 2, and 3 are active. (a) E_φ. (b) E_θ.

CONCLUSIONS

It has been demonstrated that the radiation characteristics of a conformal array on a cylindrical surface can be closely matched by a number of planar arrays approximating the cylindrical surface. It is expected that the same technique will be applicable to many other types of conformal surfaces. The phase setting and switching problems of the multiple planar arrays are considerably simpler than those encountered with the conventional conformal array. As a result, the multiple planar arrays would have a less complicated switching network and lower power losses than a conventional conformal array. It is also worth noting that the present formulation, which is based on vector decomposition, also provides a very efficient numeric algorithm for calculating the far field of many complex radiating structures. By means of this approach, the structures are broken down into pieces and are treated as arrays of elementary radiators. The computation efficiency of the present approach results from making use of known pattern functions of elementary sources. In this way, the time-consuming numerical integrations and differentiations that one normally encounters in far-field calculations are greatly reduced.

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