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SNAP LOADS IN LIFTING AND MOORING CABLE SYSTEMS INDUCED BY SURFACE WAVE CONDITIONS

NAVAL CIVIL ENGINEERING LABORATORIES

September 1973

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F. C. Liu

September 1973

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SNAP LOADS IN LIFTING AND MOORING CABLE SYSTEMS INDUCED BY SURFACE WAVE CONDITIONS

Technical Note N-1288

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by

F. C. Liu

ABSTRACT

The emplacement and recovery of large deep ocean cable systems containing in-line packages require a knowledge of the dynamic tension and motions of the system under the influence of surface ship motions, subsurface ocean currents and other external loads. A two-dimensional lumped mass model was developed to simulate simply connected cables and in-line packages. Cable tensions and mass poin position and velocity are determined by a finite difference method using a predictor - corrector technique. The resulting computer program is applicable to single payload lowering, cable laying, deep sea mooring and deployment of large undersea cable structures.



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INTRODUCTION

Ocean mooring and lifting are necessary surface support capabilities for the emplacement of seafloor structures and underwater cable systems. To proper 'v select mooring and lifting lines, predictions of line tensions throughout the various operational phases are required. The selection of undersized lines due to lack of dynamic stress information can result in catastrophic failures. Therefore, methods of analyzing cable systems are needed for proper design.

The maximum line tension may be considered as either the combination of dead weight and dynamic load or the snap load. Since the dead weight is a constant, it is easily determined. The maximum Jynamic load is defined as the peak of the tension increase or decrease from the static tension load. Dynamic loads are generated by the longitudinal and horizontal oscillations at the surface end of the line. The snap load is an impact load caused by the sudden retensioning of the line after a state of zero tension (Figure 1) which occurs when the cable syntem is subjected to surface motions of large amplitude and/or high frequency. Once the decrease in dynamic tension exceeds the magnitude of the static load, slack occurs in the line. A snap load develops in the line as the platform subsequently moves upward. The duration of the snap load is short but its amplitude may be many times greater than the maximum dynamic load, depending upon the properties of the line and the payload. It is therefore necessary to know the expected peak snap load in mooring or lifting operations in order to provide an adequate cable size.

While dynamic tension solutions for lifting and mooring systems have been sought by many investigators (references 1-7) the snap load problem has received relatively little attention. Goeller studied snap loads for a single degree of freedom system (reference 8). Wilkins developed an approximate solution for the problem of a short cable system (reference 9). Snap tensions caused by a free falling anchor have been investigated by Nath; good results were achieved using a lumped mass model and solving the problem with a predictor - corrector type numerical method (reference 10). However, methods for predicting snap loads in long cable systems are not available, nor is there any literature on snap loads in catenary mooring lines.

The present work involves the development of a computer method for the calculation of snap loads in lifting and mooring lines caused by oscillatory excitations at the surface (Figure 2). Exact solutions of the problem are not feasible because a cable cannot resist compression. Therefore, Hooke's law cannot be fulfilled when the line tension drops below zero. This nonlinear elastic property of the cable makes the equation of motion nonlinear. In this work, a lumped mass model is used and numerical solutions are sought. The equations of motion are written for each mass point and are solved by a computer subroutine which has proven to be effective in solving a set of nonlinear first order differential equations.

The end product of this work is a Fortran IV computer program called SNAPLG. This program can solve transient, dynamic and snapload responses to surface excitations of any composite underwater cable system in any current conditions with fixed or weighted lower end boundary conditions. The program is applicable to deep sea emplacements and retrievals, deep sea moorings and the construction of underwater cable systems. It is also valid for analyzing the free fall anchor dynamics. The limitations are planar motion and one continuous cable. Three dimensional motion and branched out legs can be included in the program with some modifications.

NUMERICAL SOLUTION

Lumped Mass Model

The cable system is divided into a number of short line segments. Each such segment is assumed to be small enough that it acts as a rigid body, whose centroid is called a mass point. Furthermore, all external forces act through this point. Adjacent mass points are connected by springs whose elastic property is equivalent to that of the line segment between the mass points. Dashpots are attached to mass points to simulate hydrodynamic damping provided by the fluid drag force along the line segment.

Any cable system suspended from the surface may be modeled as shown in Figure 3. The upper end represents the forced circular or ellipsoidal surface motions. The bottom end of the cable may be a heavy concentrated payload as in a lift system or an anchor fixed to the seafloor as in a mooring system.

The mathematical model was based on a single set of coordinates; + x is the horizontal distance to the right of the origin and + z is the vertical distance above the origin. The basic mass point geometrical relationaship is shown in Figure 4. The origin can be located anywhere desired. But a convenient point is at the lower end of the unstretched cable. The distance between mass points and the inclinations are expressed in terms of the x and z coordinates as follows:

$$\ell_{i} = \sqrt{(x_{i} - x_{i-1})^{2} + (z_{i} - z_{i-1})^{2}}$$

$$\ell_{i+1} = \sqrt{(x_{i+1} - x_{i})^{2} + (z_{i+1} - z_{i})^{2}}$$

$$\theta_{i} = \arctan \frac{z_{i+1} - z_{i}}{x_{i+1} - x_{i}}$$

$$\phi_{i} = \arctan \frac{z_{i} - z_{i-1}}{x_{i} - x_{i-1}}$$

$$\psi_{i} = \arctan \frac{z_{i+1} - z_{i-1}}{x_{i+1} - x_{i-1}}$$

Mathematical Formulation

The equation of motion for the ith mass is derived based on the principle of dynamic force equilibrium. Therefore, by resolving forces into the x and z directions, the equilibrium condition for the ith mass can be expressed as follows:

$$(I_{x})_{i} + (B_{x})_{i} + (D_{x})_{i} + (T_{x})_{i+1} + (T_{x})_{i} + F_{x} = 0$$
(1)
$$(I_{x})_{i} + (B_{x})_{i} + (D_{x})_{i} + (T_{x})_{i+1} + (T_{x})_{i} + F_{x} = 0$$

where the subscripts x and z denote direction of the force, the subscripts i and i+l identify the appropriate mass or line segment, and

I = inertial force

- B = body or gravitational force (weight)
- D = hydrodynamic drag force
- T = cable tension

and and graded at

F = imposed external force

The inertial force is the product of the virtual mass and the acceleration. For the i^{th} mass point the x and z components of this force are:

 $(\mathbf{I}_{\mathbf{x}})_{\mathbf{i}} = \mathbf{m}_{\mathbf{i}} \frac{d^{2}\mathbf{x}}{dt^{2}}$ $(\mathbf{I}_{\mathbf{z}})_{\mathbf{i}} = \mathbf{m}_{\mathbf{i}} \frac{d^{2}\mathbf{z}}{dt^{2}}$ (2)

where

 $m_{i} = \frac{1}{g} \left\{ 0.5 \xi_{i} \ell_{i}^{0} + 0.5 \xi_{i+1} \ell_{i+1}^{0} + 0.5 \alpha \left[(\xi_{i} - \xi_{i}^{*}) \ell_{1}^{0} + (\xi_{i+1} - \xi_{i+1}^{*}) \ell_{i+1}^{0} + W_{i} + \alpha^{*} (W_{i} - W_{i}^{*}) \right\}$

- g = acceleration due to gravity
- ξ = dry weight per unit length of cable
- ξ^* = submorged weight per unit length
- 2⁰ = length of segment at zero tension
 - mass coefficient of cable.
 - a dry weight of payload
- submerged weight of payload
- mass coefficient of payload

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The body force is simply the gravitational force or the submorged weight of the cable segment and the in-line mass at the ith mass point. Therefore, the x and z components are:

The current drag is calculated bases on the assumption that the vertical component of the current velocity is negligible and that the tangential drag along the cable is assumed to be negligible. The orientation of the payload is assumed constant with change of cable slope. The relative horizontal velocity of the mass point becomes:

$$P_{i} = \frac{dx_{i}}{dt} - Q_{i}$$
 (4)

There are three hydrodynamic damping forces: (1) the payload, (2) the normal cable drag and (3) the tangential cable drag.

$$(D_{x})_{i} = -(C_{x})_{i} P_{i} |P_{i}| - (C_{N})_{i} \sin \psi_{i} (V_{N})_{i} | (V_{N})_{i} | - (C_{t})_{i} \cos \psi_{i} (V_{t})_{i} | (V_{t})_{i} |$$

$$(D_{z})_{i} = -(C_{z})_{i} \frac{dz_{i}}{dt} | \frac{dz_{i}}{dt} | - (C_{N})_{i} \cos \psi_{i} (V_{N})_{i} | (V_{N})_{i} | - (C_{t})_{i} \sin \psi_{i} (V_{t})_{i} | (V_{t})_{i} |$$

$$(5)$$

where

$$(C_{N})_{i} = 0.5 (a_{N})_{i} v_{u} (A_{N})_{i}$$

$$(C_{N})_{i} = 0.25 (a_{N})_{i} v_{u} (A_{Z})_{i}$$

$$(C_{N})_{i} = 0.25 (a_{N})_{i} v_{u} = (d_{i} z_{i} + d_{i+1} z_{i+1})$$

$$(C_{L})_{i} = 0.25 (a_{L})_{i} v_{u} = (d_{i} z_{i} + d_{i+1} z_{i+1})$$

$$(V_{N})_{i} = P_{i} \sin z_{i} - \frac{dz_{i}}{dt} \cos z_{i}$$

$$(V_{L})_{i} = P_{i} \cos z_{i} - \frac{dz_{i}}{dt} \sin z_{i}$$

$$= drag coefficient of cable$$

a drag coefficient of payload
 a mass density of seawater
 A = payload drag area

d = cable diameter

Q = current velocity

The elastic forces are calculated based on Hooke's Law using the spring constant k_1 of the cable segment.

$$(T_{x})_{i} = -k_{i} (\ell_{i} - \ell_{i}^{0}) \cos \phi_{i}$$

for $(\ell_{i} - \ell_{i}^{0}) > 0$ (6)
 $(T_{z})_{i} = -k_{i} (\ell_{i} - \ell_{i}^{0}) \sin \phi_{i}$

and

$$r_i = 0 \text{ for } (l_i - l_i^0) \le 0$$

Other external force components $(F_x)_i$ and $(F_z)_i$ are to be defined as known quantities.

By substituting equations (2) through (6) into the corresponding portion of equation (1), the equation of motion of the ith mass point in the x and z directions can be obtained.

Boundary Conditions

The upper end boundary conditions may be expressed as follows:

vhore

x, z = steady state coordinates in x and z direction, respectively

x. r = peak upper and displacement in x and x direction, respectively

excitation frequency

 u_n * forced upper end velocity in x direction

With the proper combination of values of $r_{\rm g}$, $r_{\rm Z}$, and $u_{\rm h}$, surface excitations of several configurations are possible.

The lower end can be either free or fixed. Free end cable systems may have a heavy payload, a buoy or no payload at the free end. For free ends, the boundary conditions can be described by setting i = o in equations (1) through (6). The resulting equations are:

$$\frac{d^{2}x_{o}}{dt^{2}} = \frac{1}{m_{o}} \left[- (C_{x})_{o} P_{o} | P_{o} | - (C_{N})_{o} \sin \theta_{o} (V_{N})_{o} (V_{N})_{o} - (C_{t})_{o} \right]$$

$$cos \theta_{o} (V_{o})_{o} (V_{t})_{o} + k_{1} (\ell_{1} - \ell_{1}^{o}) \cos \theta_{o} + (F_{x})_{o} \right]$$

$$\frac{d^{2}z_{o}}{dt^{2}} = \frac{1}{m_{o}} \left[- (C_{z})_{o} \frac{dz_{o}}{dt} | \frac{dz_{o}}{dt} | - (C_{N})_{o} \cos \theta_{o} (V_{N})_{o} | (V_{N})_{o} | - (C_{t})_{o} \right]$$

$$sin \theta_{o} (V_{t})_{o} (V_{t})_{o} + k_{1} (\ell_{1} - \ell_{1}^{o}) \sin \theta_{c} - 0.5\xi_{1} \ell_{1} - W_{o} + (F_{z})_{o} \right]$$

where

0

$$(C_{N})_{o} = 0.25 (\beta_{N})_{o} \beta_{W} d_{1} x_{1}$$

$$(C_{t})_{o} = 0.25 (\beta_{t})_{o} \beta_{W} \pi d_{1} t_{1}$$

$$(C_{x})_{o} = 0.5 (\beta_{x})_{o} \beta_{W} (A_{x})_{o}$$

$$(C_{z})_{o} = 0.5 (\beta_{z})_{o} \beta_{W} (A_{z})_{o}$$

$$(V_{N})_{o} = P_{o} \sin \theta_{o} \frac{dz_{o}}{dt} \cos \theta_{o}$$

$$(V_{t})_{o} = P_{o} \cos \theta_{o} \frac{dz_{o}}{dt} \sin \theta_{o}$$

For a fixed lower end at the origin the boundary conditions are:

$$x_0 = 0$$

for all t's
$$z_0 = 0$$

Initial Conditions

Physically any selected initial condition will eventually lead to a steady harmonic response to the external excitation. The transient period depends on the frequency response of the system. An initial condition which describes a realistic initial state of the cable system requires little or no transient period. On the other hand, because a numerical solution technique is used, a poorly prepared initial condition can result in instability at the beginning of the calculation. Therefore, it is most desirable to determine first the static configuration of the cable system in question under static loads. Velocities of the mass points may be assigned if necessary to matcing the upper boundary condition. Analytical solutions are available for simple lift and mooring systems with uniform line properties but with no in-line concentrated masses and current drag forces. Computer programs can be used to calculate the static state configuration of complex cable system in a current field. The static configuration can then be input to Program SNAPLG as the initial conditions for dynamic analysis. Although causing a somewhat longer transient time a graphically determined static configuration is often adequate.

For a simple lift system, the static configuration is a vertically stretched cable. The elongation of the ith line segment due to static loadings of system dead weights and other static forces may be expressed as:

$$E_{i} = \frac{\sum_{i=1}^{1} \frac{1}{k_{i}} \left[(B_{z})_{i} + (F_{z})_{i} \right]$$

The coordinates of the ith mass point become:

$$\overline{z}_{i} = \overline{z}_{i-1} + \ell_{i}^{0} + \epsilon_{i} \qquad \overline{z}_{0} = -\sum_{0}^{1} \epsilon_{i}$$
$$\overline{x}_{i} = 0$$

For a catenary shaped cable system the static configuration is calculated based on inextensible catenary equations. Assuming that the upper end is subjected to a vertical load, V, and a horizontal load, H, and that the lower end is fixed, the tension at the upper end is:

$$T^2 = H^2 + v^2$$

Based on catenary equations:

and

$$\overline{z}_{n} - \overline{z}_{h} = \frac{H}{\xi} \left[\cosh^{-1} \frac{\xi^{-} (\overline{x}_{n} - \overline{x}_{h})}{H} - 1 \right] = \frac{T-H}{\xi}$$

where, \overline{x}_h and \overline{z}_h , are the coordinates of the lowest point on the catenary, where the slope is zero; \overline{x}_n and \overline{z}_n , are the coordinates of the top mass point.

$$\overline{z}_{h} = \overline{z}_{n} - \frac{H}{\xi} \left[\cosh^{-1} \frac{\xi^{2} (\overline{x}_{n} - \overline{x}_{h})}{H} - 1 \right]$$

and the lower end tension is:

$$T_o = -\overline{z}_h \xi^{\prime} + H$$

 $\overline{x}_{h} - \overline{x}_{h} = \frac{H}{\xi} \cosh^{-1} \frac{T}{H}$

therefore,

$$\mathbf{x}_{h} = \frac{-H}{\xi} \cosh^{-1} \left(\frac{T_{o}}{H} \right)$$

the total scope of the cable is calculated based on the third catenary equation:

$$s = \frac{H}{\xi} \left[\sinh \frac{\xi' \bar{x}}{H} - \sinh \left(\frac{-\xi' \bar{x}}{H} \right) \right]$$

The imaginary cable scope to the lowest point of the catenary is

$$s_0 = \sinh\left(\frac{-\xi'\vec{x}_h}{H}\right)$$

Let

$$=\frac{s}{n}$$

∆s

and

$$s_{i} = s_{i} + (i - 0.5) \Delta s$$
 for $i = 1, \cdots$

Using again the catenary equations:

$$\overline{\mathbf{x}}_{\mathbf{i}} = \frac{\mathrm{H}}{\xi} \sinh^{-1} \frac{\xi \, \mathbf{s}_{\mathbf{i}}}{\mathrm{H}} + \overline{\mathbf{x}}_{\mathrm{h}}$$
$$\overline{\mathbf{z}}_{\mathbf{i}} = \frac{\mathrm{H}}{\xi} \left(\cosh^{-1} \frac{\xi'(\overline{\mathbf{x}}_{\mathbf{i}} - \overline{\mathbf{x}}_{\mathrm{h}})}{\mathrm{H}} - 1\right) + \overline{z}_{\mathrm{h}}$$

for
$$i = 1, \cdots (n + 1)$$

Method of Solution

There are two differential equations for each mass point. For n mass points, the number of equations is 2n. These equations are second order second degree differential equations in terms of x and z. In order to solve these equations by the Adams numerical method, each equation is transformed into first order differential equations by substituting



As a result, there are a total of 4n first order parallel differential equations to be solved by numerical integration.

The fourth order Adams-Predictor-Corrector Method which is a method for solving a system of first-order ordinary differential equations (reference 12) is used in the present study. The Runge Kutta formula is used to start the integration and whenever the time step size is changed. The Adams-Bachforth and Adams-Moulton formulas are used as predictor and corrector respectively. The relative and absolute errors are computed and compared with the predetermined error bounds. If the relative and absolute errors are either too small or too large, the time step size will be changed automatically to reduce computing time and instability. It also has a provision for storing and printing the results at selected time increments (reference 13). A snap load test was developed to detect the buckling of the line segments and reassign the value of the segment spring constant to account for the lack of compressive stiffness. The test is made before each computation of the velocity and acceleration at each mass point. The line buckling is detected by comparing the instantaneous cable segment lengths with the unstretched lengths. If slackness is detected in a segment, the corresponding elastic spring constant k_1 is set to zero. When this segment becomes taut again the spring constant is reassigned its original value. This simulates the temporary disassociation of two adjacent mass points caused by a slack line segment. As these two points move far enough apart, the line segment regains its ability to resist tension. The basic set of differential equations has thus been altered to fulfill the condition of line buckling.

PROGRAM SNAPLG

Computer program SNAPLG consists of the main program and six subroutines. These subroutines are DIFEQ1, ADAMS, LOAD, RNGKTA, ERRTST and STATIC.

Program SNAPLG provides entrance for line and payload properties. It also calculates the initial configuration of the system if not given as input. Properties for each line segment to be input are segment length, cross-sectional area, diameter, dry and wet unit weights, and Young's modulus of elasticity. The properties for each in-line payload include the weight in air and in water, mass and drag coefficients, drag areas in vertical and horizontal directions. The upper boundary conditions are the vertical and horizontal components of line tension at the support. Surface displacement excitation is input in the form of amplitude and frequency. Also input may include the horizontal velocity of the surface ship. The depth of water must be given for catenary systems and the depth of suspension must be given for lift systems. The desired period of real time computation must be specified. The current velocity at each mass point may be included if required.

Based on the depth of suspension the unstretched position of each mass point is computed for a lift system. The positions will be used as the initial condition. For a catenary system, the initial positions can be either input or calculated by SUBROUTINE STATIC based on surface tension and line properties.

All constants used in the differential equations are computed here. Complete time domain solutions can be printed and plotted at the end of the SNAPLG. SUBROUTINE DIFGQ1 has two functions. First, it tests the slack condition in each line segment and corrects the spring constant when necessary. Secondly, it computes the top boundary conditions and calculates the acceleration and velocity of each mass point.

SUBROUTINE ADAMS' main function is to select the best time increment for the next round of calculations. It starts the calculations using the RUNGE-KUTTA method and continues the calculations with the ADAMS method. It also stores and outputs calculated information at desired time intervals.

SUBROUTINE LOAD is called by ADAMS at desired time intervals whenever results need to be output. The output includes the real time, position and velocity of each mass point and the line tension in each segment. This information is also stored in a matrix to be called out when needed at the end of SNAPLG.

SUBROUTINE RNGKTA is called by ADAMS to calculate the position of each mass point at the beginning of the calculation and after each output time increment.

LOGICAL FUNCTION ERRTST is also called by ADAMS. It determines whether the error between the predicted and corrected value is within the desired bound.

SUBROUTINE STATIC is called by PROGRAM SNAPLG to compute the static configuration of a catenary system based on the assumption that the line is not stretchable.

RESULTS

An example problem was analyzed by program SNAPLG. The cable was a l-inch wire rope mooring line suspended in 600 feet of water. The bottom end is fixed to the seafloor and the top end is subjected to vertical and horizontal sinusoidal excitations. Current loads were neglected. Part of the computer output is plotted in Figure 7. The snap loads are the sharp tension increase immediately following each occurrence of slack. The second peaks are the dynamic peaks. In this case, the value of peak snap loads is smaller than that of the dynamic loads, probably due to the large damping capability of the catenary. The snap loads are expected to be larger for larger surface excitations. The output is reasonably stable and consistent at the end of the second wave period. This rapid convergence characteristic means a savings in computer time.

A second example is the deployment of a buoy/anchor assembly. The 3800-pound anchor is suspended 1800 feet below a cylindrical buoy by a

1-inch Samson Cordage Power Braid rope. The buoy has a net buoyancy of 2500 pounds. The whole assembly is supported 50 feet under the platform by a 5/8-inch Samson Cordage Power Braid rope.

The platform is assumed to move ± 2.0 feet from its mean position at a period of 3.5 seconds. The result is plotted in Figure 8. The three curves represent the total tension in the three line segment ts. Segment 12 is the 5/8-inch rope above the buoy. A snap load occurs only in segment 12. The buoy acts as a filter to dampen out the shocks from the surface. The values of the snap load peaks are not high since the synthetic rope can absorb a great amount of shock energy.

The third example is a much more complex cable system construction problem. A mooring leg is being deployed in deep water. The leg consists of 4186 feet of 1/2-inch diameter 3 x 19 three-conductor torque balanced wire rope which weighs 0.49 lb/ft dry and 0.31 lb/ft wet. The cable is supported by a buoy weighing 525 lb dry and has a net buoyancy of 1625 bounds. Attached at the lower end is a 25,000 pound clump anchor weighing 20,000 pounds submerged. A crown line, 25,000 feet of 1-inch 3 x 19 three-conductor wire rope, is used to lift and position the anchor to the desired site. It weighs 1.75 lb/ft in air and 1.23 lb/ft in water. The upper end of the crown line is supported by a surface ship which is subjected to wave actions. In this example, the ship motion consists of a constant velocity of 3 knots pulling away from the buoy and a circular motion of 3 foot radius and 1.26 radians per second frequency in a vertical plane. A horizontal force of 810 pounds is exerted on the buoy by the balance of the cable system. The dynamic effect caused by the surfaced motion is assumed negligible at the buoy. Figure 9 is a schematic presentation of the cable system in the problem. Figures 10-17 show the tension history of each segment in the system. Although the displacements and velocities in each mass point are not shown here, they can be displayed in the same fashion as the tensions.

DISCUSSION

Program SNAPLG is a useful tool for designing hardware and planning deep ocean deployment and recovery. The snap load is generally affected by four major elements: Seastate, surface platform, cable and payload, or static tension. With these elements as input to the program, the occurrence of snap load may be predicted and the magnitude of the snap load caluclated. If based on the maximum tension, the cable is not safe, any of the four inputs may be modified for another trial. The iteration can continue until the most favorable combination of four elements has been reached. For a lift system, the maximum tension should be checked throughout the depth of the emplacement and recovery. Particularly, the depth where a prolonged payload suspension is anticipated. For a catenary mooring cable, the tension for a series of catenary configurations should be checked. The surface excitation is input as sinusoidal motions of a surface platform. By modifying the equations of motion, random motion may be input as a Fourier series. The near surface wave particle velocity loadings may also be included into the equation of motion without difficulty. This method may be useful in solving the cable strumming problem. The only difficulties will be the determination of the current induced lift force on a strumming cable and tee large number of mass points required.

Information needed on the line properties and the payload properties are standard. It is desirable to know the static configuration of a catenary. If this is not available, the top tension and the inclination should be given. An approximate static configuration can be obtained by a graphic method.

By using proper control parameters, the program will select the proper subroutines for the lift or moor calculations. Control wariables for the ADAMS subroutines include tee initial time step size, relative and absolute error bounds, length of real time calculation, and the time interval of printing output. The results of the computation are plotted on oscillographs for presentation and visualization. Movies can be made from these graphs to animate the dynamics of the cable system.

One advantage of this program is that the computer versus real time ratio is less than 2. This is based on a limited number of runs on a 5 segment cable system. A twenty-second real time computation can be made with a cost of no more than \$40.00 using a CDC 6600 computer.

This method can be extended to three dimensional cable systems with branched out legs and members. Three differential equations will be required to define the motion of the mass point. Consequently, longer computation time is expected. There is no forseen technical difficulty in achieving the three dimensional capability.

The accuracy of the solution depends on the mathematical model adopted for the problem. High precision may be achieved with a large number of lumped masses, small error bounds and small output time intervals. This program has not been validated by experimental data.

CONCLUSIONS

1. A computer program SNAPLG has been developed to solve the maximum peak tension in a lift line or in a mooring line subjected to surface excitations. The program is based upon a numerical method called the Fourth Order ADAMS Predictor-Corrector Method. Lumped mass models were used to simulate the cables. The program tests the line tension continuously to avoid negative values. Consequently, snap load conditions can be simulated and predicted. 2. The program provides a useful tool for the design of hardware and the emplacement and recovery of heavy payloads and deep mooring systems. Knowledge of the expected dynamic and snap loads makes possible the optimal design of the lift and mooring systems and the handling equipment.

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REFERENCES

1. Little, A. D., Inc., Report No. 1370863, Stress Analysis of Ship-Suspended Heavily Loaded Cables for Deep Underwater Emplacements, Bureau of Ships Contract NObsr-81564, August 1963.

2. Navel Civil Engineering Laboratory. Technical Report R-703, Dynamic Stress Response of Lifting Lines for Oceanic Operations, by C. L. Liu, Port Hueneme, CA, November 1970. (AD 715771)

3. Nath, J. H., and Flex, M. P., A Report on a Study of the Hull and Mooring Line Dynamics for the Convair Ocean Data Station Telemetering Buoy, General Dynamics Corporation, Engineering Science Report AN-1108, Convair Division of General Dynamics, San Diego, CA, December 1, 1967.

4. Reid, R. O., Dynamics of Deep-Sea Mooring Lines, Reference 68-11F, Department of Oceanography, Texas A&M University, College Station, Texas, July 1968.

5. Walton, T. S., and Polachek, H., Calculations of Non-Linear Transient Motion of Cables, David Taylor Model Basin, Applied Math. Lab Report No. 1279, July 1959.

6. Wilson, B. W., and Garbaccio, D. H., Dynamics of Ship Anchor Lines in Waves and Currents, Proceedings of the Conference on Civil Engineering in the Oceans, American Society of Civil Engineers, San Francisco, CA, September 1967.

7. Zajac, E. E., Dynamics and Kinematics of the Laying and Recovery of Submarine Cable, Bell Systems Technical Journal, Vol. 36, No. 5, pp. 1129-1207, September 1957.

8. Goeller, J. E., and Laura, P. A., A Theoretical and Experimental Investigation of Impact Loads in Stranded Steel Cables During Longitudinal Excitation, The Catholic University of America, The Institute of Ocean Science and Engineering, Report 70-2, Office of Naval Research Contract No. N00014-68-0001, April 1970.

9. Wilkins, G. A., RUWS Snap Loading, Unpublished Notes, May 1971.

10. Nath, J. H., Personal Communication, January 1972.

11. Harris, C. O., Mechanics for Engineers, The Ronald Press Co., New York, 1965, pp. 101.

12. Heldebrand, F. B., Introduction to Numerical Analysis, McGraw Hill, New York, 1956, Ch. 6.

13. User's Instructions for Computer Program ADAMS.

LIST OF SYMEOLS

٦ (

A = drag area of payload (f	ft")	
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B = gravitational force (1b)

D = hydrodynamic damping force (1b)

d = cable diameter (ft)

F = imposed external force (lb)

g = acceleration due to gravity (ft/sec²)

H = horizontal component of tension (1b)

I = inertial force (1b)

k = spring constant of cable segment (1b/ft)

 \pounds = length of cable segment (ft)

 \mathfrak{k}^{0} = length of cable segment at zero tension (ft)

m = total mass (slug)

P = relative velocity of mass point with respect to current (ft/sec)

Q = horizontal component of current (ft/sec)

r = forced peak upper end displacement (ft)

s * inextensible arc length along cable (ft)

T = cable tension, elastic spring force (1b)

t = time (sec)

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u = horizontal velocity, (ft/sec)

V = vertical component of tension (1b)

v = vertical velocity (ft/sec)

W # weight of payload (1b)

W' = submerged weight of payload (1b)

horizontal coordinate (ft) horizontal coordinate under steady state conditions (ft) x vertical coordinate (ft) 2 vertical coordinate under steady state conditions (ft) ž mass coefficient of cable mass coefficient of payload aʻ drag coefficient of cable R drag coefficient of payload unit weight of cable (lb/ft) submerged unit weight of cable (1b/ft) elongation of cable (ft) mass density of seawster (slug/ft³) 9 excitation frequency (rad/sec) t+L acute angle between horizontal and line through m_i and m_{i+1} θ, = acute angle between horizontal and line through $m_{i=1}$ and m_i acute angle between horizontal and line through m_{i-1} and m_{i+1} ý. **

Subscripts:

refers to location on catenary where slope is zero

- i = refers to i mass point or i th cable
- N = component normal to the cable
- n = upper end of cable
- o . lower end of cable

t = component tangential to the cable

- x = component in x direction
- z * component in z direction



Figure 1. Static load is the weight of the cable system. Dynamic load is additional load caused by the motion of the system. Snap load is usually the maximum possible load in the system. It is created by a sudden tensioning of a slack cable system.



subject to surface excitations.



















Figure 10. Tension history in line segment No. 4.



Figure 11. Tension history in line sugment No. 8.



Figure 12. Tension history in line segment No. 12.



Figure 13. Tension history in line segment No. 16.



Figure 14. Tension history in line segment No. 20.





Figure 16. Tension history in line segment No. 28.