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MULTI-ATTRIBUTE UTILITY THEORY: MODELS AND ASSESSMENT PROCEDURES

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This article reviews multi-attribute utility theory from a measurement theoretic perspective. It describes and classifies decision situations according to three salient aspects of choice: uncertainty, time-variability, and multi-dimensionality. For each choice situation the main mathematical representations, their inter-relations and differences are discussed. Measurement theoretic tests are described which separate between multi-attribute utility models in riskless and risky time invariant choice situations. Assessment

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procedures are outlined to encode utility functions for the representations developed, and experimental applications of multi-attribute utility theory are briefly reviewed.

MULTI-ATTRIBUTE UTILITY THEORY: MODELS AND ASSESSMENT PROCEDURES[1]

Technical Report
5 November 1973

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Introduction

Multi-attribute utility theory (MAUT) combines a class of psychological measurement models and scaling procedures that can be applied to the evaluation of alternatives with multiple value relevant attributes. For example, MAUT can be used to analyze preferences between cars described by the attributes cost, comfort, prestige, and performance. MAUT may also be applied as a decision aiding technology for decomposing a complex evaluation task into a set of simpler subtasks. For example, the decision maker might be asked to assess the utility of each alternative with respect to each attribute and to assign importance weights to each attribute. Then an appropriate combination rule is used to aggregate utility across attributes.

Two major theoretical approaches to multi-attribute utility assessment have been developed. Both provide an axiomatic justification for the existence of a utility function over multi-attributed alternatives which decomposes into single attribute utility functions. The approaches to the representations, however, differ substantially. The theory of conjoint measurement (Krantz, 1964; Luce and Tukey, 1964; Krantz, Luce, Suppes, and Tversky, 1971) simultaneously constructs the overall and single attribute utility functions. In its additive form the conjoint measurement representation is given by

$$F(x_1,x_2,\ldots,x_i,\ldots,x_n) = \sum_{i=1}^{n} f_i(x_i) \qquad [1]$$

where x_i denotes the state of the outcome $\underline{x} = (x_1,x_2,\ldots,x_i,\ldots,x_n)$ in the i-th attribute, f_i is the utility function over the states of the i-th attribute, and F is the overall utility function. The conjoint measurement representation preserves the decision maker's preference ordering for riskless decisions, but it cannot necessarily be applied to decision under risk, where alternatives are not only multi-attributed but also uncertain.

Multi-attribute expected utility theory (Fishburn, 1965, 1970; Keeney, 1969, 1971, 1973; Raiffa, 1969), on the other hand, was explicitly designed for decisions under risk. The utility function U obtained with this approach not only preserves the decision maker's riskless preference order, but also may be used in expected utility computations to select among risky alternatives. For example, the additive expected utility representation is of the form

$$U(\underline{x}_1,\underline{x}_2,\ldots,\underline{x}_j,\ldots,\underline{x}_m) = \sum_{j=1}^{m} p_j \sum_{i=1}^{n} u_i(x_{ij}) \qquad [2]$$

where $\underline{\underline{x}} = (\underline{x}_1,\underline{x}_2,\ldots,\underline{x}_j,\ldots,\underline{x}_m)$ is a risky alternative for which the multi-attributed outcome $\underline{x}_j$ is received if event E_j occurs, p_j is the probability of this event, x_{ij} is the state of the i-th attribute of outcome $\underline{x}_j$, u_i is the utility function over the i-th attribute, and U is the expected utility for the risky alternative $\underline{\underline{x}}$.

The models above consider the problem of risk preferences and multi-attribute preferences. In most complex models of this sort, we would also want to reflect time preferences. No joint axiomatization of time, risk, and multi-attribute preferences is available at present, but Meyer (1969), Pollard (1969), and Fishburn (1970) axiomatized joint time and risk preferences. In the multi-attribute context one might want to consider a time stationary, additive expected utility model :

$$G(\underline{\underline{x}}_1,\underline{\underline{x}}_2,\ldots,\underline{\underline{x}}_k,\ldots,\underline{\underline{x}}_\ell) = \sum_{k=1}^{\ell} \lambda^{k-1} \sum_{j=1}^{m} p_j \sum_{i=1}^{n} g_i(x_{ijk}) \qquad [3]$$

where $\underline{\underline{x}}_k$ is an uncertain, multi-attributed alternative to be received at time k. λ is a factor which discounts the expected utility for the k-th time interval. The total discount in this particular model is an exponential function of the time index k.

Equations [1]-[3] are examples of highly structured decomposition models. The degree to which a model allows a decomposition of the evaluation of complex alternatives into independent evaluation aspects such as uncertainties, time-discounts, and single attribute utilities distinguishes between the models. How far one can go in decomposing the evaluation task depends on crucial independence assumptions which constitute the measurement theoretic basis for the models and justify their application in a particular choice situation.

MAUT is primarily concerned with the independence of attributes, which permits the evaluation of multi-attribute alternatives by breaking them down into singl attribute evaluations. This "riskless decomposition", as we shall call it, is, however, only a first step in MAUT. If alternatives become risky or time-variable, the decomposition over attributes is closely linked to the decomposition over uncertain events and time intervals. It should be k.p in mind that the kinds of MAUT representations and construction procedures are quite different if one considers time preferences, risk preferences, neit..., or both. MAUT is ultimately linked to expectation and time discounting models in their joint tization. This fact is expressed in [1]-[3] in the different single attribute utility function f_i, u_i, and g_i.

Another distinguishing factor of the model examples discussed so far is their algebraic (i.e. non-probabilistic) and compensatory nature. This is the class of models we will discuss in this paper. We will exclude some algebraic compensatory models such as the additive difference models (Beals, Krantz, and Tversky, 1968; Tversky, 1969), because they allow intransitive preferences. We will also exclude models of the conjunctive-disjunctive type like lexicographic models, satisficing models, or elimination by aspects models. These models are discussed in Fishburn (1970), Tversky (1972), and Fischer (1972a). Non-compensatory models may describe heuristic strategies applied by the decision maker in actual evaluation situations, but they seldom can be justified as models of rational choice behavior (Tversky, 1969).

Researchers concerned with the technological aspects of MAUT, notably Edwards (1971), typically worked with simple additive models like [1] or [2]. The argument for MAUT as a decision technology goes as follows. Since the evaluation of multi-attributed alternatives is often difficult, leading to inconsistent judgments and simplistic strategies, the choice problem is first structured by determining the basic dimensions of importance. Then the evaluation task is decomposed into the evaluation of each alternative with respect to each attribute, and the estimation of importance weights for the different attributes. Weights and single attribute utility functions are aggregated using a weighted additive model to generate an overall evaluation. Where the choice situation becomes more complex, as in [2] or [3], parameters like λ and p_j are assessed in addition to weights and utility functions.

These models have intuitive rational appeal and are robust against minor model violations. Additive models can approximate other models rather well, when utilities in single attributes are monotone functions of the attribute values. Arguments for the robustness of models like [1]-[3] can be found in Yntema and Torgerson (1961), Fischer (1972b), and v. Winterfeldt and Edwards (1972).

Note that a rigorous axiomatic test of the models is impossible in complex real choice situations because it would require judgments which the decision maker is unable to make, for example, ordering complex alternatives consistently. It is just this inability which leads to the application of MAUT as a decision aid. Consequently the applied work in MAUT has typically been more concerned with structuring the decision problem, assessing model parameters, and sensitivity analyses than with an axiomatic justification of the models used.

Recent summary papers by v. Winterfeldt (1971), Fischer (1972a), and MacCrimmon (1973), give an account of the theoretical and applied MAUT research which has been done since the pioneering work of Yntema and Torgerson (1961) and Shepard (1964). The present paper will try to fill in some gaps left by these articles. Stressing the modelling aspect of MAUT, we want to demonstrate that MAUT is much more than a simple additive algorithm. We will discuss a variety of additive and non-additive multi-attribute utility models, their inter-relations and their differences. Especially we want to stress the qualitative measurement theoretic assumptions on which these models rest. An understanding of these assumptions and their relations can assist a decision analyst in choosing an appropriate multi-attribute model for a particular choice problem.

The outline of this paper is as follows. First we will give a general classification scheme for choice situations and models which apply to them. Then we will discuss some special cases like the riskless and the risky time invariant multi-attribute situation, and we will describe a general analysis to test models for these cases. Some situations for which no model yet exists will be discussed and some structural relations among models for risk, time, and multi-attribute preferences will be sketched. Next, we give a summary of assessment procedures for the evaluation of multi-attributed alternatives, and finally, we briefly review the experimental applications of MAUT.

A Classification of Choice Situations and Models

To facilitate the discussion of choice situations and models, we will use the following conventions. An outcome is a (possibly multi-attributed) sure thing to be received at a specified time. A gamble is a distribution of outcomes over events. A consumption stream is a distribution of outcomes or gambles over time periods. As a general term for any choice entity like an outcome, a gamble, or a consumption stream, we will use the term alternative.

Decision problems are complicated by the multi-dimensionality of outcomes, by uncertainty, and by time variability. The presence or absence of these three aspects lead to a classification of choice situations into 2^3 cases, which are described in Table 1. The last column contains a brief

Insert Table 1 about here

description of the basic models which apply to these cases. Cases 1-4 are the crucial ones for the present analysis since they include multi-attribute preferences. Cases 5-8 are included for completeness and to demonstrate some interesting relations among models.

In each of the 8 choice situations, the basic alternative can be described as a vector or a matrix. In case 1, for example, alternatives are vectors of values in the single attributes, in case 3 they are vectors representing a consumption stream in which each outcome is itself a vector. Such a vectorial representation presupposes that the choice situation is already highly structured and that attribute states have been characterized numerically. We will not go into a discussion of the very important problem of arriving at a vectorial representation through structuring the decision problem. The reader is referred to Raiffa (1968, 1969) and Edwards (1971). Rather we will assume these vectors as a starting point.

Instead of a vector representation, we could have described alternatives by trees, as it is common practice in decision analysis. We use the vector notation, however, because it facilitates the explanation of the various tests and models we will discuss. We concede that vectorial representations are not always convenient, and sometimes they do not even comply with the rationale of a model (for example, Luce and Krantz's conditional expected utility model (1971)). Thus, our vector representation should be viewed as a convenient simplification for the sake of explanation and discussion.

Case 1 is characterized by time invariance, certainty, and multi-dimensionality of alternatives. To be classified into this category a decision situation must satisfy three criteria. First, the outcome to be received must have multiple value relevant attributes. Second, all outcomes must be received at the same time (not necessarily the present). And third, the outcome associated with each alternative must be known with certainty. Situations like this are extremely rare in real world decision problems. But if uncertainty and time are of little importance to the decision maker, or if their variability is but minor, one may want to consider case 1 as an idealized prototype.

In case 1 the basic choice alternative is a multi-attributed outcome $\underline{x}$ which can be represented as a vector of its components in the single attributes:

Attributes

$$A_1, A_2, A_3, \ldots, A_i, \ldots, A_n$$
$$\underline{x} = (x_1, x_2, x_3, \ldots, x_i, \ldots, x_n)$$

The weakest representation among the models for case 1 is a simple order preserving map F from vectors into the real numbers, where no decomposition of F is allowed. If F can be decomposed into single attribute functions f_i without the existence of a specified composition rule, we can apply trade-off models. The most structured models are the conjoint measurement models which specify the rule which combines the functions f_i. We will later discuss each of these classes of models in detail.

Case 2 in Table 1 describes a time invariant choice between gambles with multi-attributed outcomes. For example, alternative $\underline{\underline{x}}$ may be to immediately accept a job offer with a fair salary and location. Alternative $\underline{\underline{y}}$ might be to refuse this sure offer and wait to see if an application for a different job with higher salary and a good location will be accepted. A choice between $\underline{\underline{x}}$ and $\underline{\underline{y}}$ depends not only on attribute values, but also on the probability attached to the uncertain event, namely being accepted for the more desirable job.

A choice alternative in case 2 can be represented as a vector of multi-attributed outcomes $\underline{x}_j$, which are received if an event E_j occurs:

$$\begin{array}{c} \text{Events} \\ E_1,\ E_2,\ E_3,\ \ldots\ldots,\ E_j,\ \ldots\ldots,\ E_m \end{array}$$

$$\underline{\underline{x}} = (\underline{x}_1,\ \underline{x}_2,\ \underline{x}_3,\ \ldots\ldots,\ \underline{x}_j,\ \ldots\ldots,\ \underline{x}_m)$$

i.e. as a distribution of multi-attributed outcomes over events, which can also be written as a matrix:

$$\begin{array}{c} \text{Events} \end{array}$$

$$\underline{\underline{x}} = \begin{array}{c} \begin{array}{ccccc} E_1 & E_2 & \ldots\ldots & E_j & \ldots\ldots\ E_m \end{array} \\ \left[\begin{array}{ccccccc} x_{11} & x_{12} & \cdots\cdots & x_{1j} & \cdots\cdots & x_{1m} \\ x_{21} & x_{22} & \cdots\cdots & x_{2j} & \cdots\cdots & x_{2m} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{i1} & x_{i2} & \cdots\cdots & x_{ij} & \cdots\cdots & x_{im} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{n1} & x_{n2} & \cdots\cdots & x_{nj} & \cdots\cdots & x_{nm} \end{array}\right] \end{array} \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_n \end{array} \quad \text{Attributes}$$

Two types of basically different models represent case 2, both based on the expected utility hypothesis. The first assumes some riskless representation, as in case 1, and then monotonically transforms it to obtain a risky utility function U (see Boyd, 1970; Fischer, 1972a). The second assumes a decomposition of U directly into components similar to the decomposition of F in case 1. Our discussion of the models in case 2 will mainly be concerned with the latter case. Prominent representations are the additive and the multiplicative expected utility models (Pollak, 1967; Raiffa, 1969; Keeney, 1969, 1971, 1973; Fishburn, 1970).

Case 3 in Table 1 stands for a riskless choice between multi-attributed alternatives which may be received at different time intervals. To stay in our job-offer example, both jobs may be open for sure, but in case $\underline{\underline{x}}$ the job could begin in two weeks, whereas in case $\underline{\underline{y}}$ the job might begin in half a year.

The vectorial representation of alternatives in case 3 is very similar to case 2. Here the multi-attributed outcomes are distributed over different time intervals instead of events:

$$\begin{array}{c} \text{Times} \\ t_1,\ t_2,\ \ldots,\ t_k,\ \ldots,\ t_\ell \\ \underline{\underline{x}} = (\underline{x}_1,\ \underline{x}_2,\ \ldots,\ \underline{x}_k,\ \ldots,\ \underline{x}_\ell) \end{array} = \begin{array}{cc} & \text{Times} \\ & \begin{array}{ccccccc} t_1 & t_2 & \cdots & t_k & \cdots & t_\ell \end{array} \\ & \left[\begin{array}{ccccccc} x_{11} & x_{12} & \cdots & x_{1k} & \cdots & x_{1\ell} \\ x_{21} & x_{22} & \cdots & x_{2k} & \cdots & x_{2\ell} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ik} & \cdots & x_{i\ell} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nk} & \cdots & x_{n\ell} \end{array}\right] \end{array} \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_n \end{array} \ \text{Attributes}$$

We would like to offer a discussion of some models at this point, but we can't. In spite of the structural similarity with the uncertain, multi-attribute case, nobody has undertaken the task of modelling case 3 yet. From our later discussion of case 2, the approach to such a model should be obvious, and basically the same models and tests will apply for this case as for case 2.

Things are not much better in case 4, the most complex and probably also the most common decision situation; here the alternatives are consumption streams of gambles with multi-attributed outcomes. Putting our examples from cases 2 and 3 together, alternative $\underline{\underline{x}}$ may be to accept the first job offer right now, and get--in two weeks--a job with fair salary and location. Alternative $\underline{\underline{y}}$ is to refuse the sure job and see if the application for the job with the higher salary and location is accepted, then wait for half a year before starting the job.

An alternative $\underline{\underline{x}}$ in case 4 is represented by a vector of matrices or a matrix with vector elements:

$$\underline{\underline{x}} = \overset{\text{Events}}{\begin{array}{c} E_1, E_2, \ldots, E_j, \ldots E_m \\ (\underline{\underline{x}}_1, \underline{\underline{x}}_2, \ldots, \underline{\underline{x}}_j, \ldots, \underline{\underline{x}}_m) \end{array}} = \begin{array}{cc} \overset{\text{Events}}{\begin{array}{ccccc} E_1 & E_2 \; \cdots & E_j & \cdots & E_m \end{array}} & \\ \begin{bmatrix} \underline{x}_{11} & \underline{x}_{21} & \cdots & \underline{x}_{j1} & \cdots & \underline{x}_{m1} \\ \underline{x}_{12} & \underline{x}_{22} & \cdots & \underline{x}_{j2} & \cdots & \underline{x}_{m2} \\ \vdots & \vdots & & \vdots & & \vdots \\ \underline{x}_{1k} & \underline{x}_{2k} & \cdots & \underline{x}_{jk} & \cdots & \underline{x}_{mk} \\ \vdots & \vdots & & \vdots & & \vdots \\ \underline{x}_{1\ell} & \underline{x}_{2\ell} & \cdots & \underline{x}_{j\ell} & \cdots & \underline{x}_{m\ell} \end{bmatrix} & \begin{array}{c} t_1 \\ t_2 \\ \vdots \\ t_k \\ \vdots \\ t_\ell \end{array} \;\; \text{Times} \end{array}$$

where

$$\begin{array}{lcl} & & \textbf{Attributes} \\ & & A_1 \quad A_2 \quad \ldots \quad A_i \quad \ldots \quad A_n \\ \underline{x}_{jk} & = & (x_{1jk}, x_{2jk}, \ldots, x_{ijk}, \ldots, x_{njk}) \end{array}$$

No model is available at present for this most complex case. But the work of Meyer (1969) who modelled case 8--the case in which the outcomes in the above matrix are single valued--suggests a combination of the models for cases 2 and 3 to a joint axiomatization of time, risk, and multi-attribute preferences.

We conclude the discussion of the classification of choice situations and models with the cases 5-8, in which outcomes are not multi-attributed but rather single valued, as in the case of monetary outcomes. These cases have interesting structural relations with the cases 1-4.

Case 5 is the simplest situation. Here choices are made between single valued outcomes, which are typically received immediately and with no uncertainty. Choices between more or less profit, a higher or a lower production rate are examples. Models for this case include simple orders and difference structures (Krantz, Luce, Suppes, and Tversky, 1971). Case 5 is usually uninteresting for decision aiding since preferences are generally immediate and obvious--notably by monotonicity assumptions.

Case 6 has been discussed most extensively in the literature. Choices between uncertain single valued outcomes characterize this situation. Gambles for monetary outcomes are the typical examples, and they can be described, much like case 1, as a vector of single values, where values are distributed over events instead of attributes:

$$\begin{array}{ccc} & & \text{Events} \\ & & E_1 \quad E_2 \quad \ldots \; E_j \quad \ldots \; E_m \\ \underline{x} & = & (x_1, x_2, \ldots x_j, \ldots x_m) \end{array}$$

The most prominent models are the SEU and the EU models, discussed in Savage (1954), Fishburn (1970), and Luce and Krantz (1971). Decision analysis (Raiffa, 1968) rests heavily on the assumptions of these models.
Other models proposed are the minimax model, the minimax regret model (Savage, 1954; Luce and Raiffa, 1957) and portfolio theory (Coombs, 1972).

Case 7 describes situations in which single valued outcomes are received at different times with certainty. A realistic alternative may be a salary distribution over the next year. Again--as in cases 1 and 6--the representation of such an alternative is a vector of outcomes, this time distributed over time intervals:

$$\begin{array}{ccc} & & \text{Times} \\ & & t_1 \quad t_2 \quad \ldots \; t_k \quad \ldots \; t_\ell \\ \underline{x} & = & (x_1, x_2, \ldots, x_k, \ldots x_\ell) \end{array}$$

Models applicable here include a simple additive conjoint measurement model, additive models with variable discount rates, or models with constant discount rates. These are discussed in Koopmans (1960), Fishburn (1970), and Krantz, Luce, Suppes, and Tversky (1971).

The final case 8 is structurally very similar to our cases 2 and 3. Here the basic choice alternative is a distribution of single valued outcomes over events (i.e. gambles) which are to be received at different times. Investment plans are typically of that sort. The alternatives can be described as vectors of gambles or matrices:

$$
\underline{\underline{x}} = \begin{array}{c} \text{Events} \\ \begin{array}{ccccccccc} E_1 & E_2 & \ldots & E_j & \ldots & E_m \end{array} \\ (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_j, \ldots, \underline{x}_m) \end{array} = \begin{array}{c} \text{Events} \\ \begin{array}{cccccc} E_1 & E_2 & \ldots & E_j & \ldots & E_m \end{array} \\ \left[\begin{array}{cccccc} x_{11} & x_{21} & \ldots & x_{j1} & \ldots & x_{m1} \\ x_{12} & x_{22} & \ldots & x_{j2} & \ldots & x_{m2} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{1k} & x_{2k} & \ldots & x_{jk} & \ldots & x_{mk} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{1\ell} & x_{2\ell} & \ldots & x_{j\ell} & \ldots & x_{m\ell} \end{array}\right] \end{array} \begin{array}{c} t_1 \\ t_2 \\ \vdots \\ t_k \\ \vdots \\ t_\ell \end{array} \quad \begin{array}{c} \text{T} \\ \text{i} \\ \text{m} \\ \text{e} \\ \text{s} \end{array}
$$

Pollak (1967) and Meyer (1969) have modelled this case basically in the same fashion as Keeney (1971, 1973) modelled case 2, replacing multi-attribute outcomes by consumption streams (see also Pollard (1969); Fishburn (1970). Their more unique representations are achieved by assuming additional constraints like time stationarity.

MAUT Models and Their Axiomatic Foundation

In the following we will assume that a preliminary analysis has identified the choice situation which a decision maker faces and that the structuring process resulted in a description of alternatives as matrices or vectors, in the form described in the preceding section. This section will discuss the multi-attribute models in Table 1 in more detail. The basic axioms which determine the admissible degree of decomposition and thus separate these models are described in a way that provides a decision analyst with a systematic testing procedure to choose an evaluation model among those suitable for the choice situation.

A word of caution is appropriate before we go into more detail. Measurement theoretic tests can never guarantee that a model chosen is the correct one. Rather they provide a tool to elimiate models that are clearly wrong. The axioms we will discuss are typically necessary for a given representation. None of these axioms can be verified, since they generally apply to an infinite domain. We also cannot expect a decision maker to satisfy these axioms in a descriptive sense since he almost certainly will show inconsistencies and he may use non-compensatory simplistic strategies in complex choice situations (for a discussion of man's limited ability to handle complex choice situations see Slovic and Lichtenstein, 1971; and Slovic, 1972).

To illustrate this last point, consider the transitivity assumption, a cornerstone of almost all measurement theoretic models. Inconsistent judgments and imperfect discriminations will frequently cause intuitive judgments to violate the transitivity principle. Does it still make sense to talk about a measurement theoretic justification of these models ? We think so.

First, the basic axioms, like the independence assumptions, can be tested roughly by presenting the decision maker with "easy" choices. In fact, the analyst, who constructs these choices, wants to make it easy for the decision maker to violate model assumptions systematically, so that he can discover which assumptions are appropriate.

Second, model parameters can often be assessed on the basis of judgments about alternatives, that are easy to compare. The more structured a model, the easier such constructive procedures typically are.

If, therefore, in a subset of choice alternatives the decision maker satisfies transitivity and all other model assumptions, and if this subset is sufficiently rich to assess the basic model parameters, then one can have some faith in an extrapolation of the evaluation function constructed. One should be willing to follow the prescription of such a model as long as one believes that outside the "easy" subset failures of model assumptions in actual tests are not systematic, or are due to systematic applications of obviously unsatisfactory simplifying strategies.

In summary, our discussion of the axiomatic foundation of MAUT is designed to sharpen the decision analyst's eye for the places where things can go wrong with a model, and to enable him to ask sophisticated questions to discover those systematic violations which are intended and rationally justified by the decision maker.

Case 1: Riskless, time invariant, multi-attributed alternatives.

The tests which separate between the models in case 1 are summarized in Table 2 as a tree to which we will frequently refer.

Insert Table 2 about here

The first test, in the table labelled as 1-WCUI, checks to determine if any one attribute is Weakly Conditional Utility Independent of all others, i. e. if preferences for values in that attribute are independent of constant values in the other attributes. The term WCUI has been used by Raiffa (1969) for the single independence assumption in conjoint measurement theory (Krantz, Luce, Suppes, and Tversky, 1971). It is also sometimes called preferential independence (Keeney, 1973), monotonicity, or single cancellation. Formally, the test is of the following form : does there exist an i such that

Attributes		Attributes	
$A_1\ A_2 \cdots A_{i-1}\ A_i\ A_{i+1} \cdots A_n$		$A_1\ A_2 \cdots A_{i-1}\ A_i\ A_{i+1} \cdots A_n$	
$(a_1,a_2,\ldots,a_{i-1},x_i,a_{i+1},\ldots,a_n)$	$\not\succsim$	$(a_1,a_2,\ldots,a_{i-1},y_i,a_{i+1},\ldots,a_n)$	iff
$(b_1,b_2,\ldots,b_{i-1},x_i,b_{i+1},\ldots,b_n)$	$\not\succsim$	$(b_1,b_2,\ldots,b_{i-1},y_i,b_{i+1},\ldots,b_n)$	

for all x_i, y_i, a_j, b_j, $j \neq i$.

Here and in the following a and b stand for values which are constant across alternatives, x and y stand for variables. $\underline{x} \not\succsim \underline{y}$ means "$\underline{y}$ is not preferred to $\underline{x}$", $\doteq$ means $\not\succsim$ and $\not\precsim$. "iff" should be read as "if and only if".

As an illustration for 1-WCUI consider the attributes of a used car such as mileage, age, and body condition. Can you imagine that you would prefer a more expensive car over a less expensive one, if all other attribute values are equal ? If not, the attribute "price" is 1-WCUI. Now add the attributes "size of the car" and "power-steering". You may always prefer a larger car over a smaller car, if both have power-steering, but this preference can reverse if they don't, since in this case the larger car is more difficult to handle in many situations. Therefore size may not be WCUI of the rest of the attributes, when the attribute "power-steering" is included.

If 1-WCUI fails , i. e. if no attribute is WCUI of the rest, the only model applicable is a simple order model of the type

<u>Model 1.1</u>

$$\underline{x} \succsim \underline{y} \quad \text{iff} \quad F(\underline{x}) \geq F(\underline{y})$$

In this model, the choice is left to the decision maker's intuition, and it would be an appropriate representation for his judgmental process, if his choices are transitive. Model 1.1 is obviously of little use in decision analysis.

If 1-WCUI holds, we next test if n-WCUI holds, that is, if 1-WCUI holds for all n attributes. The size of an apartment, its location and its rent could be considered attributes which are n-WCUI. You typically prefer less rent, more room and a better location, no matter what the other attribute values are. If this test fails, we can apply model 1.2 which allows a partial decomposition of the function F in model 1.1 :

Model 1.2

$$\underline{x} \succsim \underline{y} \quad \text{iff } F[f(x_1,x_2,\ldots,x_{i-1}, x_{i+1},\ldots x_n);\ f_i(x_i)] \geq F[f(y_1,y_2,\ldots,y_{i-1}, y_{i+1},\ldots y_n);\ f_i(y_i)]$$

This model is of some help in decision analyses. For every multi-attribute outcome an outcome is found which is indifferent to it and has constant values in all dimensions except the dimension which is WCUI. Instead of comparing the original outcomes, one now compares the outcomes which are equated in all attributes except attribute i. If this attribute is, for example, monetary costs or payoffs, choices are immediately prescribed through the monotonicity of the the utility function over this attribute. Of course, the whole model stands and falls with the ability of the decision maker to make reasonable trade-offs. Otherwise the constructive procedure will fail. Often the trade-off procedure can be facilitated by a stepwise iteration in which one dimension is traded off after another to arrive at a standard outcome which varies only in one dimension.

If n-WCUI is satisfied, we next test more general independence conditions, called joint independencies. A set of attributes is said to be jointly independent of the rest if the preference order of alternatives which vary only in these attributes, remains invariant for any fixed levels of the remaining attributes. For example, attributes 1 and 2 are jointly independent of attributes 3 through n if

$$\begin{array}{lcc} & \text{Attributes} & & \text{Attributes} \\ & \underline{A_1\ A_2\ A_3\ A_4 \ldots A_i \ldots A_n} & & \underline{A_1\ A_2\ A_3\ A_4 \ldots A_i \ldots A_n} \\ & (x_1,x_2,a_3,a_4,\ldots,a_i,\ldots,a_n) & \gtrsim & (y_1,y_2,a_3,a_4,\ldots,a_i,\ldots,a_n) \\ \text{iff} & (x_1,x_2,b_3,b_4,\ldots,b_i,\ldots,b_n) & \gtrsim & (y_1,y_2,b_3,b_4,\ldots,b_i,\ldots,b_n) \end{array}$$

for all x_1,x_2,y_1,y_2,a_i,b_i, $i = 3,4,\ldots n$.

For example, when choosing among different job offers, your preferences over combinations of salary and staff benefits will most certainly be jointly independent of, say, the size of the town you would work in. The most interesting cases of violations of the joint independence conditions are those in which n-WCUI is satisfied. Those violations are typically subtle in nature and hard to find. Suppose you work in a large city and want to rent a house or an apartment. Your options have the following attribute values :

a) farm (F) - city apartment (A) ,

b) one hour car ride to work (1h) - 20 minutes car ride to work (20 min),

c) high speed transportation system near by (HST) - no high speed transportation system (NHST).

The authors gave the following rank orders of alternatives, depending on the presence or the absence of the high speed transit system :

1) (F, 20 min, HST)	1) (F, 20 min, NHST)
2) (F, 1h, HST)	2) (A, 20 min, NHST)
3) (A, 20 min, HST)	3) (F, 1h, NHST)
4) (A, 1h, HST)	4) (A, 1h, NHST).

Living on a farm in the country seemed to us very attractive, and the long car ride to work did not matter much with the convenience of the high speed transportation system. With no high speed transportation system the shorter ride from the apartment outweighed the benefits of living on the farm. This produced a switch in our preferences which violated the joint independence assumption. But the reader can also check that we always preferred the farm over the apartment and the shorter ride over the longer ride, when other values were held fixed. Since, in addition, we would always prefer HST over NHST, we satisfied n-WCUI.

If no joint independence condition is satisfied, we can, by virtue of n-WCUI, apply the following total decomposition model:

<u>Model 1.3</u>

$$\underline{x} \dot{\succcurlyeq} \underline{y} \quad \text{iff}$$

$$F[f_1(x_1),\ f_2(x_2),\ \ldots\ ,\ f_n(x_n)] \succcurlyeq F[f_1(y_1),\ f_2(y_2),\ \ldots\ ,\ f_n(y_n)]$$

This model is already of substantial help in decision analyses, since it allows us to construct independent utility functions in the single attributes and to think about trade-offs between some dimensions independent of the values of others. Even more helpful is an admissibility analysis which excludes outcomes which are dominated in all single dimension utilities. Once the inadmissible alternatives are eliminated, one can think about reasonable composition rules F, which model 1.3 does not specify. The application of different combination rules and a sensitivity analysis of model parameters can then determine the best or a few good outcomes.

Between models 1.2 and 1.3 there is a large class of models which allow a decomposition depending on which 1-WCUI conditions hold. The reader will readily be able to extrapolate to these models.

If all joint independencies can be justified or if critical tests of them do not fail, we can apply the most structured model in case 2, the additive conjoint measurement model 1.4:

Model 1.4

$$\underline{x} \gtrsim \underline{y} \text{ iff } F(\underline{x}) = \sum_{i=1}^{n} f_i(x_i) \geq \sum_{i=1}^{n} f_i(y_i) = F(\underline{y})$$

In this model single attribute utility functions are constructed and the sum of these functions represents the worth of each alternative.

Some other models can be expressed as special cases of model 1.4. The multiplicative representation

$$\underline{x} \gtrsim \underline{y} \text{ iff } F'(\underline{x}) = \prod_{i=1}^{n} f_i'(x_i) \geq \prod_{i=1}^{n} f_i'(y_i) = F'(\underline{y})$$

is an additive representation by

$$F(\underline{x}) = \log F'(\underline{x}) = \sum_{i=1}^{n} \log f_i'(x_i) = \sum_{i=1}^{n} f_i(x_i)$$

Similarily, the quasi-additive representation (here in a two attribute example)

$$\underline{x} \succsim \underline{y} \qquad \text{iff}$$

$$F''(\underline{x}) = f_1''(x_1) + f_2''(x_2) + k\, f_1''(x_1) \cdot f_2''(x_2) \geq$$
$$f_1''(y_1) + f_2''(y_2) + k\, f_1''(y_1) \cdot f_2''(y_2) = F''(\underline{y})$$

can be transformed into a multiplicative model. Let

$$F'(\underline{x}) = 1 + k\, F''(\underline{x}) = [1 + k\, f_1''(x_1)] \cdot [1 + k\, f_2''(x_2)]$$
$$f_i'(x_i) = 1 + k\, f_i''(x_i)$$

which gives us the multiplicative form

$$F'(\underline{x}) = f_1'(x_1) \cdot f_2'(x_2)$$

This multiplicative form can then be reduced to the additive representation by logarithmic transformations as above.

The reason that these representations are equivalent in a conjoint measurement sense is that the logarithmic transformations of F' and F'' are admissible , because they preserve the order of preferences, which is the basic property of F, F', and F''. This point is important, since herein lies a basic difference between additive riskless models and additive expected utility models.

As we discussed the riskless multi-attribute models, we proceeded from the most general and unstructured model 1.1 to the special and highly structured case 1.4 by adding assumptions and restrictions. Of course, the decision analyst may choose to apply the more general models, whenever a special case can be justified on the basis of qualitative tests.

For example, he may use the partial decomposition model 1.2, if an additive model is appropriate. Which model among those justified to choose is a practical problem. The more structrured models 1.3 and 1.4 usually make the task of assessing model parameters easier. However, if one believes that the decision maker can make reasonable trade-offs of the type discussed, applying model 1.2 is often more suitable, because trade-off models are more economical, if the number of alternatives is small. In addition, some decision makers are more familiar with the kinds of judgments required in those models, e.g. trade-offs into the dollar dimension.

We will not discuss other so called simple polynomials as combination rules in representations for multi-attribute choice situations, since this has been done in detail in the literature. The reader interested in such representations is referred to Krantz, Luce, Suppes, and Tversky (1971) and to Krantz and Tversky (1971).

Case 2 : Risky, time invariant, multi-attributed alternatives.

At first glance a possible approach to multi-attribute representations in the presence of uncertainty may be to first check which axioms for riskless choices are satisfied, then find a representation for choices under uncertainty and combine both models. For example, one might want to consider the additive model 1.4 with F and f_i as the multi-attribute riskless representation, and an expected utility model with a function U defined over multi-attributed alternatives.

Unfortunately, nothing guarantees that U will decompose additively if F does (in fact, this will be the case only if U is a linear function of F). Take, for example, $U(\underline{x}) = \log F(\underline{x})$. We would then obtain the following representation, which does not allow an additive decomposition of U into single attribute u_i's, although F is decomposed into f_i's:

$$\underline{\underline{x}} \succsim \underline{\underline{y}} \quad \text{iff} \quad \sum_{j=1}^{m} p_j \log \sum_{i=1}^{n} f_i(x_{ij}) \geqslant \sum_{j=1}^{m} p_j \log \sum_{i=1}^{n} f_i(y_{ij})$$

In other words, multi-attributed alternatives may have an additive representation under certainty and not under uncertainty. It can easily be seen by taking degenerate gambles that the reverse is not true. An additive representation under risk implies an additive representation under certainty.

At this point we distinguish between two approaches to modelling multi-attribute preferences under uncertainty. The first approach makes use of riskless representations and searches for a suitable transformation h which maps F into U. The second approach first constructs U and then adds assumptions which justify a decomposition of U into single attribute components.

Both approaches, however, assume the existence of U, i.e., a representation for choices among risky alternatives where multiattribute outcomes $\underline{x}$ are treated as primitives. The classical representation is, of course, that of expected utility theory, or subjective expected utility theory, with the representation

Model 2.1

$$\begin{array}{cccc} & \multicolumn{3}{c}{\text{Events}} \\ & \underline{E_1\ E_2\ E_3\ \ldots\ E_j\ \ldots\ E_m} & & \underline{E_1\ E_2\ E_3\ \ldots\ E_j\ \ldots\ E_m} \\ \underline{\underline{x}} = & (\underline{x}_1,\underline{x}_2,\underline{x}_3,\ldots,\underline{x}_j,\ldots,\underline{x}_m) & \succsim & (\underline{y}_1,\underline{y}_2,\underline{y}_3,\ldots,\underline{y}_j,\ldots,\underline{y}_m) = \underline{\underline{y}} \quad \text{iff} \end{array}$$

$$U(\underline{\underline{x}}) = \sum_{j=1}^{m} p_j\, U(\underline{x}_j) \geq \sum_{j=1}^{m} p_j\, U(\underline{y}_j) = U(\underline{\underline{y}})$$

The crucial assumption to be satisfied for an expected utility representation is the sure thing principle. One testable version of the sure thing principle says that preferences among gambles should not depend on the values of outcomes which are constant in a subset of events. In vector notation the sure thing principle asserts, for example, that the following is true (note the similarity with the joint independence assumption in case 1) :

$$\begin{array}{ccc} \text{Events} & & \text{Events} \\ \underline{E_1\ E_2\ E_3\ E_4 \ldots E_j \ldots E_m} & & \underline{E_1\ E_2\ E_3\ E_4 \ldots E_j \ldots E_m} \\ (\underline{x}_1,\underline{x}_2,\underline{a}_3,\underline{a}_4,\ldots,\underline{a}_j,\ldots,\underline{a}_m) & \gtrless & (\underline{y}_1,\underline{y}_2,\underline{a}_3,\underline{a}_4,\ldots,\underline{a}_j,\ldots,\underline{a}_m) \text{ iff} \\ (\underline{x}_1,\underline{x}_2,\underline{b}_3,\underline{b}_4,\ldots,\underline{b}_j,\ldots,\underline{b}_m) & \gtrless & (\underline{y}_1,\underline{y}_2,\underline{b}_3,\underline{b}_4,\ldots,\underline{b}_j,\ldots,\underline{b}_m) \end{array}$$

for all $\underline{x}_1,\underline{x}_2,\underline{y}_1,\underline{y}_2,\underline{a}_i,\underline{b}_i$, $i = 3,4,\ldots,m$.

In the above case the sure thing principle would hold for the events 3 to m. Theoretically, the sure thing principle would have to be tested for all subsets of events and for all combinations of outcomes. But the idea behind the axiom can be easily checked by constructing some critical examples.

Another critical assumption in an EU-representation is that no outcome should be infinitely desirable or undesirable. A stronger version of this assumption asserts that for all outcomes $\underline{x}$, $\underline{y}$, and $\underline{z}$ for which $\underline{x} \succsim \underline{y} \succsim \underline{z}$ events E and $\bar{E}$ (non E) exist such that

$$y \doteq (\overset{E}{\underline{x}}, \overset{\bar{E}}{\underline{z}})$$

This assumption can usually be checked by thought experiments. The reader interested in a discussion of the two basic axioms of an EU-representation is referred to Savage (1954), Luce and Raiffa (1957), and Ellsberg (1961) all of which describe illuminating counterexamples.

If either of the two EU-assumptions is violated, then decision theory can do little to assist the decision maker. Other models which apply to decision making under uncertainty are Coombs' portfolio theory and the minimax-models. Neither has been widely used for decision aiding and it can be argued that both lack the rational justification of the EU-model. Accordingly, we will base our subsequent discussion on the assumption that there exists a utility function U over multi-attributed alternatives which follows the EU-principle.

Table 3 describes in tree form the sequence of tests which separate between the different risky multi-attribute utility models. The test which

- - - - - - - - - - - - - -
Insert Table 3 about here
- - - - - - - - - - - - - -

separates the two modelling approaches we discussed earlier is called Strong Conditional Utility Independence (SCUI). This axiom is the probabilisitic equivalent of WCUI and joint independence. SCUI says that preferences among uncertain multi-attributed alternatives in which a subset of attributes has constant values across all outcomes should not depend upon the particular level at which these constant values are held fixed. In matrix notation, for example, attributes 1 and 2 would be SCUI of attributes 3 to n if

Events

$$\begin{array}{c} \begin{array}{ccccccc} E_1 & E_2 & \cdots & E_j & \cdots & E_m \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2m} \\ a_3 & a_3 & \cdots & a_3 & \cdots & a_3 \\ \vdots & \vdots & & \vdots & & \vdots \\ a_i & a_i & \cdots & a_i & \cdots & a_i \\ \vdots & \vdots & & \vdots & & \vdots \\ a_n & a_n & \cdots & a_n & \cdots & a_n \end{bmatrix} \end{array} \succsim \begin{array}{cc} \begin{array}{ccccccc} E_1 & E_2 & \cdots & E_j & \cdots & E_m \end{array} & \\ \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1j} & \cdots & y_{1m} \\ y_{21} & y_{22} & \cdots & y_{2j} & \cdots & y_{2m} \\ a_3 & a_3 & \cdots & a_3 & \cdots & a_3 \\ \vdots & \vdots & & \vdots & & \vdots \\ a_i & a_i & \cdots & a_i & \cdots & a_i \\ \vdots & \vdots & & \vdots & & \vdots \\ a_n & a_n & \cdots & a_n & \cdots & a_n \end{bmatrix} & \begin{matrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_i \\ \vdots \\ A_n \end{matrix} \end{array} \quad \begin{matrix} \text{A} \\ \text{t} \\ \text{t} \\ \text{r} \\ \text{i} \\ \text{b} \\ \text{u} \\ \text{t} \\ \text{e} \\ \text{s} \end{matrix}$$

iff

$$
\begin{array}{c}
\begin{array}{cccccc} E_1 & E_2 & \ldots & E_j & \ldots & E_m \end{array} \\
\begin{bmatrix}
x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1m} \\
x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2m} \\
b_3 & b_3 & \ldots & b_3 & \ldots & b_3 \\
\vdots & \vdots & & \vdots & & \vdots \\
b_i & b_i & \ldots & b_i & \ldots & b_i \\
\vdots & \vdots & & \vdots & & \vdots \\
b_n & b_n & \ldots & b_n & \ldots & b_n
\end{bmatrix}
\end{array}
\;\dot{\succsim}\;
\begin{array}{c}
\begin{array}{cccccc} E_1 & E_2 & \ldots & E_j & \ldots & E_m \end{array} \\
\begin{bmatrix}
y_{11} & y_{12} & \cdots & y_{1j} & \cdots & y_{1m} \\
y_{21} & y_{22} & \cdots & y_{2j} & \cdots & y_{2m} \\
b_3 & b_3 & \ldots & b_3 & \ldots & b_3 \\
\vdots & \vdots & & \vdots & & \vdots \\
b_i & b_i & \ldots & b_i & \ldots & b_i \\
\vdots & \vdots & & \vdots & & \vdots \\
b_n & b_n & \ldots & b_n & \ldots & b_n
\end{bmatrix}
\end{array}
\begin{array}{c}
\\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_i \\ \vdots \\ A_n
\end{array}
$$

for all x_{1j}, x_{2j}, y_{1j}, y_{2j}, a_i , b_i , $i = 3, 4, \ldots n$; $j = 1, 2, \ldots, m$

Since only outcome values in the first two attributes are uncertain, we dropped the second index in the a's and b's. The constants a_i and b_i can be thought of as sure things restricted to a subset of attributes. Note that WCUI and joint independencies are SCUI-conditions for degenerate gambles, i.e. for single events. Other useful formulations of SCUI can be found in Raiffa (1969) and Keeney (1969, 1973).

For a counter-example consider the following two gambles $\underline{\underline{x}}$ and $\underline{\underline{y}}$:

$$\underline{\underline{x}} = \begin{array}{cc} E_1 & E_2 \\ \left[\begin{array}{cc} \$\ 100 & -\ \$\ 50 \\ \text{Color TV set} & \text{Color TV set} \end{array}\right] \end{array} \quad \underline{\underline{y}} = \begin{array}{cc} E_1 & E_2 \\ \left[\begin{array}{cc} \$\ 15 & \$\ 15 \\ \text{Color TV set} & \text{Color TV set} \end{array}\right] \end{array}$$

where the events E_1 and E_2 are equally likely. You may prefer $\underline{\underline{x}}$ over $\underline{\underline{y}}$, since the possible loss of \$ 50 does not weigh too much with the color TV set as a sure thing, and since a shot at an additional \$ 100 seems more attractive than \$ 15 for sure. Now assume that both gambles had the same money amounts as outcomes but no color TV set. In this case you may choose $\underline{\underline{y}}$ over $\underline{\underline{x}}$ since the possible loss of \$ 50 appears more severe with no color TV set as a consolation. If you think that such a switch is reasonable, you violate SCUI. Although the riskless independence assumptions are clearly satisfied in this example, the violation of SCUI prevents the application of a simple risky decomposition model.

When SCUI is not satisfied, we consider the class of models in which a riskless decomposition is monotonically transformed to obtain a utility function U. So the left branch of table 3 goes through all the tests which we discussed in the preceding section. Depending on which riskless decomposition model holds, we can apply a partial decomposition expected utility model (2.2), a total decomposition expected utility model (2.4), or a riskless additive decomposition expected utility model (2.4) by transforming the riskless utility function F into an EU-representation U. Raiffa (1969), Boyd (1970), and Fischer (1972a) discuss these cases in greater detail. Methods for constructing the appropriate transformation h will be discussed in the assessment part of the present paper.

If SCUI holds, we next test a condition named <u>marginal equivalence</u> (Fishburn, 1965, 1970) or <u>marginality</u> (Raiffa, 1969). Marginality requires that risky multi-attributed alternatives are judged solely on the basis of the marginal probability distribution over the single attribute values. Specifically, marginality requires the following alternatives to be judged equivalent:

Events

$$
\begin{array}{c}
\begin{array}{ccccc} E_1 & E_2 & \ldots E_j & \ldots & E_m \end{array} \\
\left[\begin{array}{ccccc}
x_{11} & x_{12} \cdot & \cdot x_{ij} \cdot & \cdot & \cdot x_{1m} \\
\vdots & \vdots & \vdots & & \\
x_{i1} & x_{12} & x_{ij} \cdot & \cdot & \cdot x_{im} \\
\vdots & \vdots & \vdots & & \\
x_{n1} & x_{nj} & x_{nj} \cdot & \cdot & \cdot x_{nm}
\end{array}\right]
\end{array}
\doteq
\begin{array}{c}
\begin{array}{ccccc} E_1 & E_2 & \ldots E_j & \ldots & E_m \end{array} \\
\left[\begin{array}{ccccc}
x'_{11} & x'_{12} \cdot & \cdot x'_{ij} \cdot & \cdot & \cdot x'_{im} \\
\vdots & \vdots & \vdots & & \vdots \\
x'_{i1} & x'_{i2} \cdot & \cdot x'_{ij} \cdot & \cdot & \cdot x'_{im} \\
\vdots & \vdots & \vdots & & \vdots \\
x'_{ni} & x'_{n2} \cdot & \cdot x'_{nj} \cdot & \cdot & \cdot x'_{nm}
\end{array}\right]
\end{array}
\begin{array}{c} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_n \end{array}
\quad \text{Attributes}
$$

where all E_j are assumed to be equally likely and the right alternative is generated by permuting values x_{ij} within the rows of the left alternative.
It can easily be seen that under these two conditions the marginal probability of receiving any attribute value is the same for both alternatives.
Pollak (1967) and Fishburn (1965, 1970) state equivalent but formally different versions of the marginality assumption, assuming only two equally likely events. Which formulation to use is a matter of the practicability and the sensitivity of the tests.

As a counter-example for the marginality assumption, consider a commodity bundle with two attributes, a car (A_1) and a certain amount of money (A_2), as outcomes of a gamble. Marginality would require you to be indifferent between the following gambles $\underline{\underline{x}}$ and $\underline{y}$:

$$\underline{\underline{x}} = \begin{array}{cc} E_1 & E_2 \\ \left[\begin{array}{ll} 4000\ \$ & 0\ \$ \\ \text{a 1973} & \text{a 1961} \\ \text{Porsche} & \text{VW} \end{array}\right] \end{array} \qquad \underline{\underline{y}} = \begin{array}{cc} E_1 & E_2 \\ \left[\begin{array}{ll} 0\ \$ & 4000\ \$ \\ \text{a 1973} & \text{a 1961} \\ \text{Porsche} & \text{VW} \end{array}\right] \end{array}$$

whenever E_1 and E_2 are equally likely.

It does not take much sophisticated experimentation to see that most decision makers will prefer the right alternative, although the marginal distribution are the same in both cases. The most typical reason given for this preference is that in any event the right alternative gives you a fairly good outcome, while in the left alternative you take a chance at getting stuck with an old and rusty VW. Such a variance preference seems to be the most probable reason for violations of marginality.

If marginality is not satisfied, but SCUI is, we can represent the choice situation by the following multiplicative model:

Model 2.5

Events

$$\begin{array}{cc} \underline{E_1\ E_2\ E_3\ \dots\ E_j\ \dots\ E_m} & \underline{E_1\ E_2\ E_3\ \dots\ E_j\ \dots\ E_m} \\ (\underline{x}_1, \underline{x}_2, \underline{x}_3, \dots, \underline{x}_j, \dots, \underline{x}_m) \succsim & (\underline{y}_1, \underline{y}_2, \underline{y}_3, \dots, \underline{y}_j, \dots, \underline{y}_m) \end{array} \quad \text{iff}$$

$$\sum_{j=1}^{m} p_j \left[\prod_{i=1}^{n} u_i(x_{ij})\right] \geq \sum_{j=1}^{m} p_j \left[\prod_{i=1}^{n} u_i(x_{ij})\right]$$

This model is equivalent to the quasiadditive model, by

$$u_i(x_{ij}) = (1 + k_i u_i'(x_{ij})), \qquad k_i > 0.$$

If marginality is satisfied, an additive expected utility model is justified.

Model 2.6

Events

$$\begin{array}{ccc} \underline{E_1\ E_2\ \ldots\ E_j\ \ldots\ E_m} & & \underline{E_1\ E_2\ \ldots\ E_j\ \ldots\ E_m} \\ (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_j, \ldots, \underline{x}_m) & \dot{\succsim} & (\underline{y}_1, \underline{y}_2, \ldots, \underline{y}_j, \ldots, \underline{y}_m) \end{array} \qquad \text{iff}$$

$$\sum_{j=1}^{m} p_j \sum_{i=1}^{n} u_i(x_{ij}) \geq \sum_{j=1}^{m} p_j \sum_{i=1}^{n} u_i(y_{ij})$$

Model 2.5 and its quasiadditive equivalent cannot be treated as a special case of Model 2.6, as we did in the riskless context, since the log-transformation would destroy the interval scale properties of U, which exists by virtue of the EU-representation. Note that both 2.5 and 2.6 require that preferences be additive in a conjoint measurement sense. Thus,whenever the assumptions of these models are met, the analyst may also use model 2.4, in which a conjoint measurement representation F is transformed to reflect the decision maker's attitude towards risk. Of course, he could also apply the more general cases 2.2 and 2.3, when practical and feasible.

Cases 3 and 4 : Other multi-attribute models.

The choice situation described in cases 3 and 4 of table 1 have not been modelled yet, but the approach is straightforward.

Case 3 can be handled very similarly to case 2. First, one models time preferences over multi-attributed alternatives. Models for this case are available and coincide with the models for case 7. Such a model could be of the form

$$\begin{array}{ccc} \multicolumn{3}{c}{\text{Times}} \\ \underline{t_1\ t_2 \ldots t_k \ldots t_\ell} & & \underline{t_1\ t_2 \ldots t_k \ldots t_\ell} \\ (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_k, \ldots, \underline{x}_\ell) & \succsim & (\underline{y}_1, \underline{y}_2, \ldots, \underline{y}_k, \ldots, \underline{y}_\ell) \end{array} \qquad \text{iff}$$

$$\sum_{k=1}^{\ell} \lambda_k \, G(\underline{x}_k) \geq \sum_{k=1}^{\ell} \lambda_k \, G(\underline{y}_k)$$

or similar versions, where $\lambda_k = \lambda^{k-1}$. Here, of course, $\underline{x}_k$ is the multi-attributed alternative to be received at time k, λ_k is a discounting factor. Both representations are discussed in Fishburn (1970) and Krantz, Luce, Suppes and Tversky (1971).

Given a time preference utility function G, we can then ask what condition must be satisfied to allow a decomposition of G into single attribute components g_i. The main condition is again SCUI, i.e., its time - equivalent formulation. In fact, all conditions in Table 2 (except of marginality) can be applied directly to this situation by replacing events by time intervals. Marginality does not seem to have a direct time - equivalent formulation.

Case 4 in Table 1 describes the most complex choice situation. Conceivably a representation for the time variable, uncertain, multi-attribute choice can be constructed by a sequential application of the steps discussed in case 2. For example, one could start with a utility function U defined over risky consumption streams, then go through Table 2 to discover if U decomposes additively over time, and then repeat that analysis to see if the u_k's decompose additively or multiplicatively over attributes.

Some relations between multi-attribute and unidimensional models.

Unidimensional models like the ones for the choice situations 6-8 in Table 1 have some interesting structural relations to multi-attribute models. Cases 6 and 7 can be modelled basically in the same way as in case 1. In all three cases we are dealing with a function from vectors into the real numbers; what changes is the interpretation of vector elements. In case 1 an element x_i is a value of a multi-attributed outcome in attribute i, in case 6 it is the amount of a single commodity to be received if event E_i occurs, in case 7 it is the value of a single commodity to be received at time t_i. All three cases can be analyzed in a similar way as outlined in Table 2. In case 1, however, we were satisfied with a simple additive conjoint measurement representation. For cases 6 and 7 we would wish more unique representations, since we would expect some relation between utility functions u_i or g_i.

One additional condition we could test is whether a standard sequence in one time interval or conditional on one event is also a standard sequence in any other. This condition, introduced in an axiom system by Luce and Krantz (1971) will guarantee that the u_i's or the g_i's differ only in their units. For case 6 this would mean an expected utility representation. For case 7 it is an additive time preference model with a variable discounting factor.

Another condition, which results in an additive time preference structure with constant discounting rates as is a stationarity assumption discussed in Fishburn (1970) and Krantz, Luce, Suppes, and Tversky, (1971).

The two models in case 8, explicated by Meyer (1969) follow exactly the pattern of Table 2, i.e., they are additive or multiplicative expected utility representations where the decomposition is over time intervals instead of attributes. Meyer achieves his constant discount rate representation with an additional stationarity assumption.

So much for the models MAUT is concerned with and embedded in. One is tempted to ask : is it really that damaging if one decomposes the judgmental task more than the tests allow ? What would happen, for example, if an additive model is applied, although some independence assumptions are violated ?

First, the model parameters will depend on the subset of alternatives which were used to construct them. This is typically the point where the decision maker himself raises objections against the procedure. The judgments to elicit the parameters may not make any sense to him because of dependencies. Second, violations of the independence assumptions imply that at least in some cases the model prescriptions will be necessarily wrong. Which specific prescriptions are wrong will depend on the model parameters used. Both consequences suggest the usefulness of consistency checks while constructing model parameters as a non-qualitative complement to the previously discussed tests.

In the following section we will discuss the assessment procedures necessary to construct model parameters. The reader should bear in mind that these procedures only make sense if the model assumptions have been checked and accepted.

Assessment Procedures

The three most important constructive devices for assessing utility functions like F, G, U, f_i, g_i, and u_i are trade-offs, standard sequences, and Basic Reference Lottery Tickets (Brlts). Trade-offs are used in models 1.2, 1.3, 2.2, and 2.3. Standard sequences are constructed to assess single dimension utility functions f_i in the additive models 1.4 and 2.4. Brlts are the basic tools to construct the expected utility functions U and u_i in the models of case 2.

Trade-off procedures. Let us assume that we have to base our decision aid on model 1.2 of the preceding section, i.e., we found only one attribute i which is independent of all others. We can then construct for each multi-attributed outcome an equivalent outcome with specified constant values in all but the i-th attribute. In other words, we find for each $\underline{x}$ and $\underline{y}$

$$\underline{x} = (x_1,x_2,\ldots x_i,\ldots,x_n) \doteq (a_1,a_2,\ldots,x_i^1,\ldots,a_n)$$

$$\underline{y} = (y_1,y_2,\ldots y_i,\ldots,y_n) \doteq (a_1,a_2,\ldots,y_i^1,\ldots,a_n)$$

for some constant values a_j, $j \neq i$. Here and in the following the superscripts for the values of the trade-off attribute i mean that in the r-th trade-off step this attribute value is changed to, say, x_i^r. After this trade-off we determine the preference order for attribute i, i.e., we construct an order preserving utility function f_i. This can easily be done for commodities like money. Then the decision maker should prefer $\underline{x}$ to $\underline{y}$ if and only if

$$f_i(x_i^1) \gtrless f_i(y_i^1)$$

The trade-off procedure can be facilitated by decomposing it into n-1 steps as demonstrated in the following example :

$$\begin{aligned}
\underline{x} = (x_1, x_2, \ldots, x_i, \ldots, x_n) &\doteq \\
(a_1, x_2, \ldots, x_i^1, \ldots, x_n) &\doteq \\
(a_1, a_2, \ldots, x_i^2, \ldots, x_n) &\doteq \\
&\vdots \\
(a_1, a_2, \ldots, x_i^{n-1}, \ldots, a_n)&
\end{aligned}$$

Again $\underline{x}$ would be preferred to an alternative $\underline{y}$ if and only

$$f_i(x_i^{n-1}) \gtrless f_i(y_i^{n-1})$$

Standard sequences. Constructing utility functions with standard sequences is necessary in a conjoint measurement representation which yield F and f_i which are unique up to linear transformations. Fishburn (1967) describes this as the "saw-tooth" method. The procedure can best be demonstrated in two dimensions, i.e., we assume that multi-attributed outcomes are of the form (x_1, x_2).

We first pick an arbitrary zero point of the scale, usually the worst conceivable or actual combination of attribute values, say (x_{1*}, x_{2*}), and define

$$F(x_{1*}, x_{2*}) = 0$$

We also pick an arbitrary unit value on the first attribute, say x_1^1. We then construct a standard sequence on attribute 2, i.e., a sequence of x_2^i

which is equally spaced in utilities. In order to guarantee equally spaced utilities, we make every increase in attribute 2 equivalent to the standard increase in attribute 1 from x_{1*} to x_1^1 ; thus the standard sequence is defined by the following judgements :

$$(x_1^1, x_{2*}) \doteq (x_{1*}, x_2^1)$$

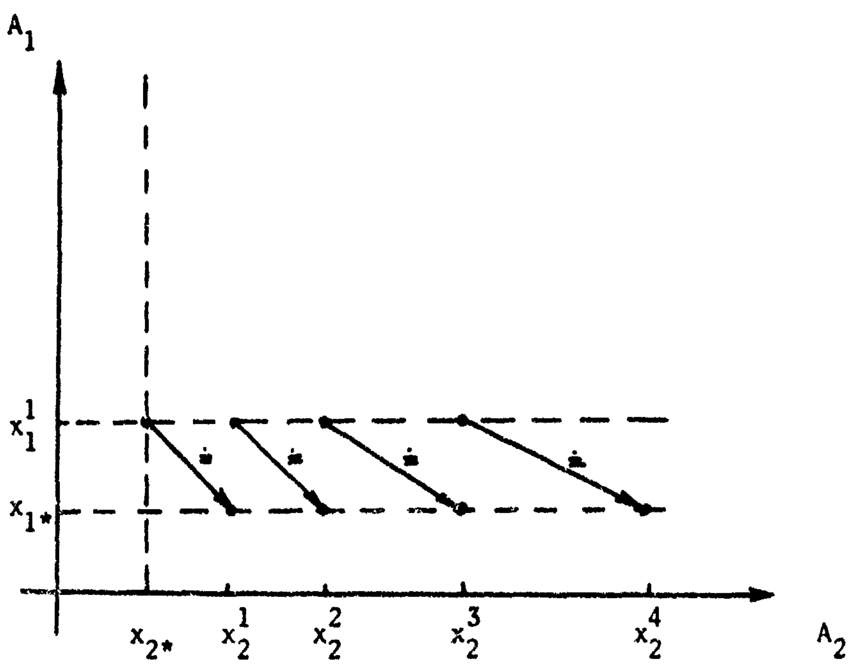
$$(x_1^1, x_2^1) \doteq (x_{1*}, x_2^2)$$

$$(x_1^1, x_2^2) \doteq (x_{1*}, x_2^3)$$

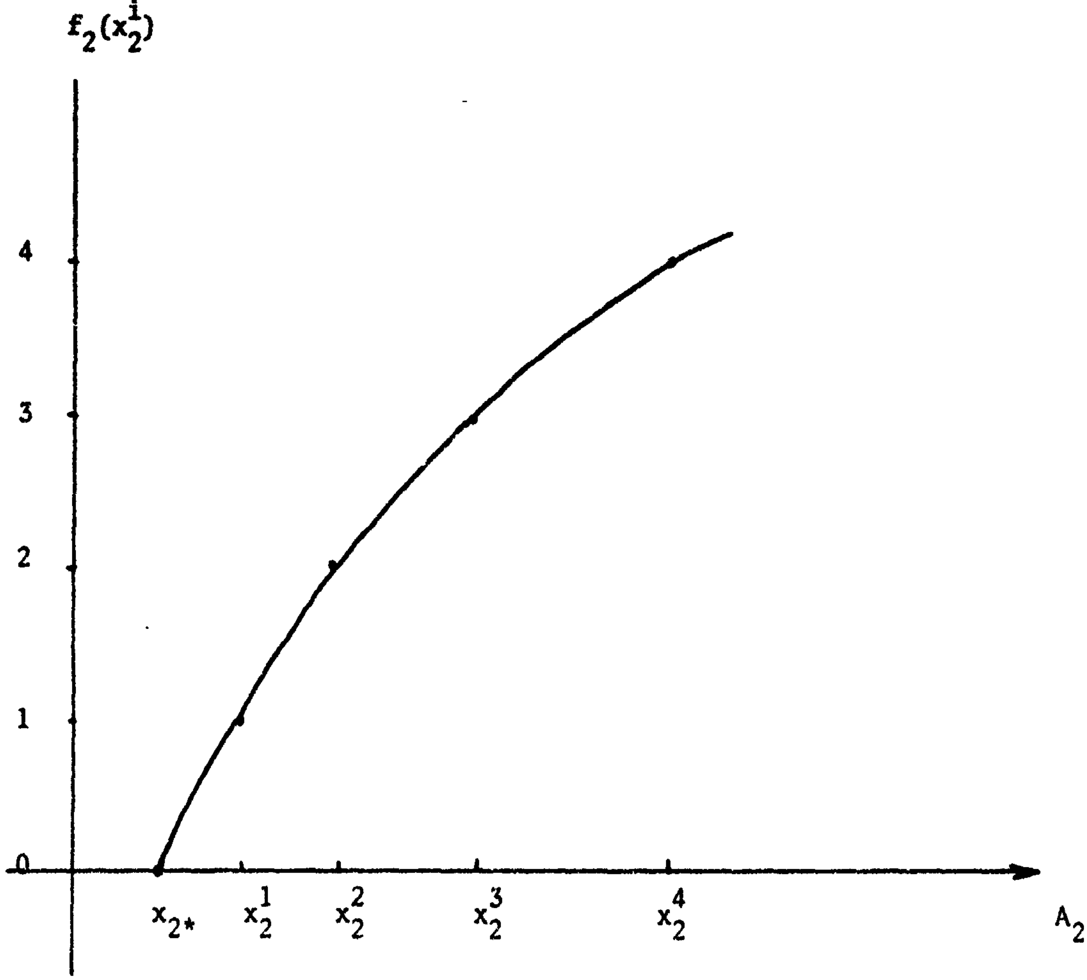
and, in general,

$$(x_1^1, x_2^{i-1}) \doteq (x_{1*}, x_2^i).$$

Graphically, this procedure can be represented as follows :



Since x_{2*}, x_2^1, x_2^2, ... , x_2^i, ... are equally spaced in utility, we can plot the utility function f_2 :



Similarily, we can use the interval x_{2*}, x_2^1 to lay off a standard sequence in attribute 1, thus constructing f_1. Note that f_1 and f_2 are automatically in the appropriate units by the choice of the unit in attribute 1, which determined the unit in attribute 2.

In some applications of MAUT weighting factors and utility functions are separated. From a conjoint measurement point of view such weights are vacuous. Separating the estimation of weights and utility functions may nevertheless be a helpful further decomposition of the assessment task.

To illustrate, suppose that in the example above we choose arbitrary units x_1^1 and x_2^1, i.e., it is not necessarily true that $(x_1^1,x_{2*}) \doteq (x_{1*},x_2^1)$. Then we are working not with the utility functions f_1 and f_2, which have a common unit of measure, but rather with f_1' and f_2', which are linearly related to f_1 and f_2, respectively. In other words

$$f_i = w_i\, f_i' + k_i \quad \text{for some } w_i > 0,\ k_i,\ i=1,2$$

Now take any two indifferent (but not equal) outcomes

$$(x_1,x_2) \doteq (y_1,y_2)$$

and we get by the additive conjoint measurement representation

$$w_1 f_1(x_1) + w_2 f_2(x_2) + k_1 + k_2 =$$
$$w_1 f_1(y_1) + w_2 f_2(y_2) + k_1 + k_2$$

$$\frac{w_1}{w_2} = \frac{f_2'(y_2) - f_2'(x_2)}{f_1'(x_1) - f_1'(y_1)}$$

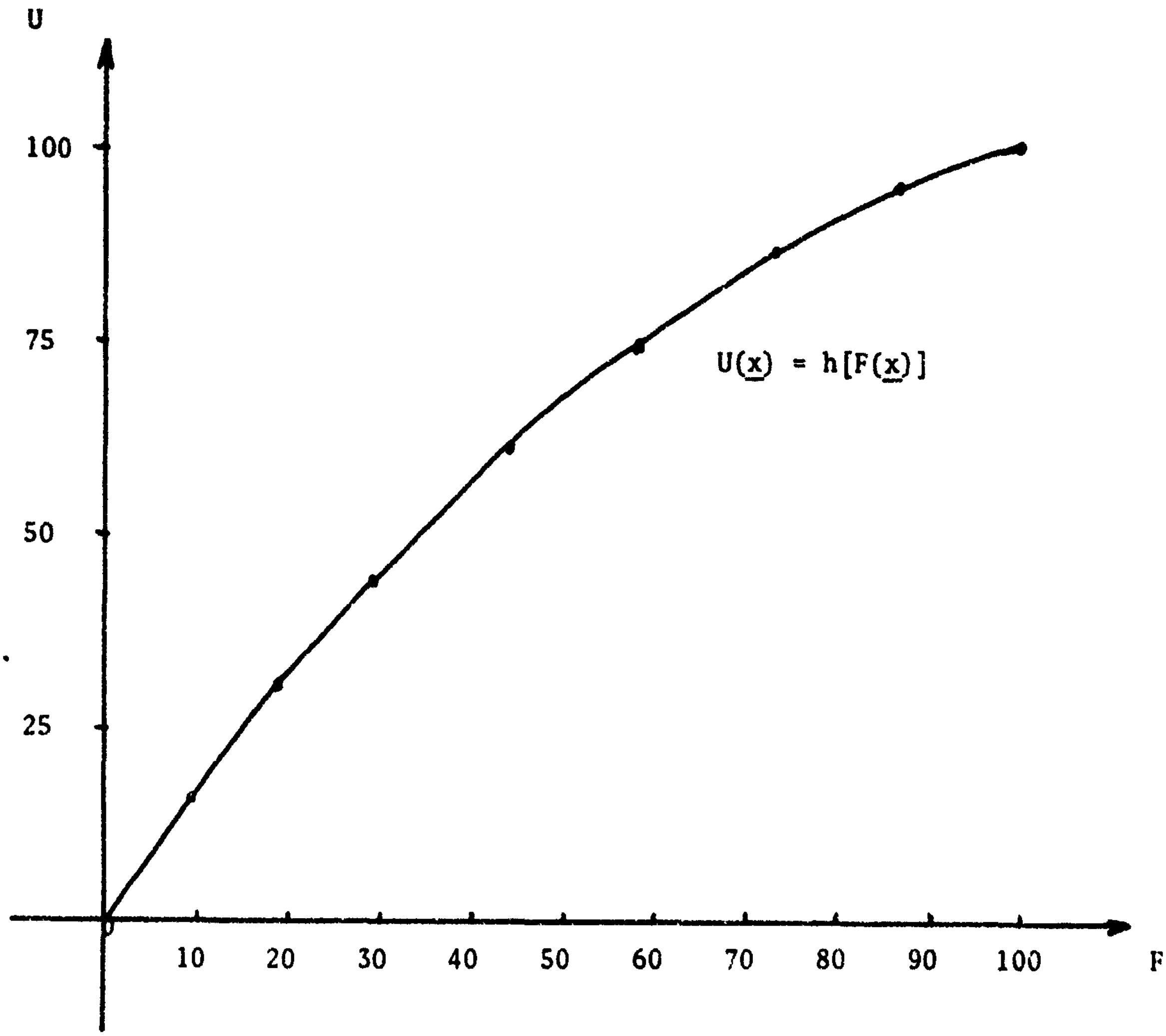
Any two weights which satisfy the above condition will do.

Basic reference lottery tickets.--In any expected utility model the utility of an alternative $\underline{x}$ is defined as the probability $U(\underline{x})$ which makes $\underline{x}$ as a sure thing indifferent to a gamble in which one receives an alternative $\underline{x}^*$ with probability $U(\underline{x})$ and an alternative $\underline{x}_*$ with probability $1-U(\underline{x}) \cdot \underline{x}^*$ is the outcome with the best conceivable values x_i in all attributes, $\underline{x}_*$ is the outcome, with the worst conceivable values x_{i*} in all attributes. Raiffa (1968) terms such lotteries brlts, and they are often denoted by $\{\underline{x}^*, U(\underline{x}), \underline{x}_*\}$.

We argued earlier that a riskless utility function could be transformed into a risky utility function by finding an appropriate monotone transform h over riskless functions. Brlts provide a means for finding this transform. For example, consider the trade-off procedures of models 2.2 and 2.3. These procedures trade-off all values into a standard attribute i, with all remaining attributes held fixed at standard levels a_j. Thus, to obtain a risky utility function U over the set of outcomes in question, we need only to assess a utility function over the standard trade-off attribute i. Consider the outcome which after trade-offs is equivalent to $(a_1,a_2,a_3,\ldots,x_i,\ldots,a_n)$. Then we define the utility of this outcome to be $u_i(x_i)$, where $u_i(x_i)$ is obtained from the brlt

$$(a_1,a_2,\ldots,x_i,\ldots,a_n) \doteq \{(a_1,a_2,\ldots,x_i^*,\ldots, a_n),\ u_i(x_i),$$
$$(a_1,a_2,\ldots,x_{i*},\ldots,a_n)\}.$$

Brlts may also be used to transform riskless additive conjoint measurement utility functions into risky (possibly non-additive) utility functions. First, we construct F and f_i's using the conjoint measurement procedure. Then we select several outcomes which cover the full range of F and which are easy to compare with the two extreme outcomes $\underline{x}^*$ and $\underline{x}_*$. We assign utilities U to these selected points using the brlt procedure. We next plot the U-values corresponding to the F values for the particular outcomes selected. By smoothing a curve through these points we may approximate the transform h which relates F to U. Thus, for any riskless outcome $\underline{x}$, we first find $f_i(x_i)$, then sum the f_i's to get $F(\underline{x})$, and then refer to a plot like the one below to find its associated $U(\underline{x}) = h\{F(\underline{x})\}$.



Brlts also provide the basic procedure for developing risky multi-attribute utility functions in which U and component u_i functions are obtained without reference to a riskless utility function F. In particular, we refer here to the multiplicative (2.5) and the additive (2.6) expected utility models developed by Raiffa (1969) and Keeney (1971, 1973). The component u_i functions are obtained by assessing brlts of the form

$$(a_1,a_2,\ldots,x_i,\ldots,a_n) \doteq \{(a_1,a_2,\ldots,x_i^*,\ldots,a_n),u_i(x_i),\\ (a_1,a_2,\ldots,x_{i*},\ldots,a_n)\}.$$

Such component functions necessarily range from 0 to 1, as does the overall utility function U when the brlt procedure is used. To match the u_i functions in units, weighting factors w_i in models 2.5 and 2.6 are obtained by assessing brlts of the form

$$(x_{1*},x_{2*},\ldots,x_i^*,\ldots,x_{n*}) \doteq \{\ \underline{x}^*,\ w_i,\ \underline{x}_*\ \}.$$

Assuming that SCUI is satisfied, U is additive if and only if these w_i sum to 1 (this is the non-qualitative equivalent to marginality). When they do, we obtain the representation

$$U(\underline{x}_j) = \sum_{i=1}^{n} w_i\, u_i(x_{ij}).$$

When the w_i do not sum to 1 we obtain the multiplicative form

$$1 + k\, U(\underline{x}_j) = \prod_{i=1}^{n} [1 + k\, w_i\, u_i(x_{ij})].$$

Here k is a constant reflecting the type and degree of non-additivity. It can be estimated from the w_i using a simple iterative procedure described in Keeney. Note that the above model formulations are equivalent to the formulations on pp. 32 . The only difference here is that the scaling constants w_i are made explicit.

Other scaling procedures. Fishburn (1967) describes 24 methods of estimating additive utilities, providing a good source for a wide variety of assessment techniques. Some of these methods, particularly those involving rating scales estimates of component functions and ratio estimates of importance weights, have been widely used in real world applications of MAUT (Edwards, 1971). It should be noted, however, that the trade-off, standard sequence, and brlt procedures are the only ones which are directly based on the models described. Other procedures may involve judgments, for which the models do not provide a basis. Nevertheless, there is reason to believe that other assessment techniques ought to provide excellent approximations to the methods explicitly justified by the axioms. This conclusion is based partly on mathematical arguments relating to the relative insensitivity of the models to minor errors in the parameters f_i, g_i, w_i etc., and in part on experimental studies which have indicated a high degree of convergence across scaling techniques (Fischer, 1972b).

Unfortunately, almost all validation studies have considered only riskless preferences. In addition, they have generally considered only simple additive rating scale methods for constructing a MAUT model. The approach adopted in these studies also has been relatively atheoretical. Rather than testing the assumptions required for additivity, experimenters have simply correlated a subject's wholistic responses with the utilities generated by his additive rating scale model. The results of these studies have generally been quite encouraging with correlations occasionally in the .70s, but typically in the high .80s or low .90s (Pollack, 1964; Yntema and Klem, 1965; Hoepfl and Huber, 1970; Huber, Daneshgar, and Ford; 1971, Pai, Gustafson, and Kiner, 1971; v. Winterfeldt , 1971; Fischer, 1972b). Fischer (1972b) also considered a simple additive trade-off procedure and obtained correlations in the mid .90s.

Experimental work in the area of risky multi-attribute decision making is almost non-existent. This seems to reflect the fact that most psychologists are unfamiliar with the multi-attribute utility approach to risky choices. Only two experimental applications of the risky MAUT measurement procedures have been reported. v.Winterfeldt (1971) studied students' preferences for apartments described by 14 attributes. Direct tests of independence assumptions where independence was doubtful revealed that the subjects' preferences were additive in a conjoint measurement sense, whereas in the same cases marginality was violated. Nevertheless, a correlational analysis indicated that the additive risky utility model (2.6) provided a fairly good approximation to the students' wholistic utility assessments. Fischer (1972b) studied preferences for risky job alternatives described by three attributes. Statistical analyses revealed that the wholistic preferences of 6 of the 10 subjects displayed small but significant departures from additivity.

Nevertheless, additive (2.6) and non-additive (2.4 and 2.5) risky utility models afforded essentially equal predictions of the wholistic utility judgements, with mean correlations in the mid .90s.

It seems apparent that much more research on risky and riskless multi-attribute decision making is required. The validity of the MAUT models must be established through direct tests of the measurement theoretic axioms. And the relative ability of the various MAUT models to approximate intuitive preferences or other appropriate validation criteria should be assessed in a wide variety of realistic contexts.

Statistical models of preferences for multi-attributed alternatives.

The experiments discussed above utilized a decomposition approach to multi-attribute utility assessment in which the decision maker makes assessments about the components of an alternative and a mathematical composition rule is used to aggregate information across components to assign an overall utility to an alternative. The testing procedure which we discussed provides a formal basis for selecting an appropriate composition rule. A number of investigators have adopted a different strategy in which subjects make only overall wholistic judgements about the utility of alternatives. Multivariate statistical procedures are then applied to obtain a multi-attribute utility function, and to identify a composition rule implied in the intuitive judgments. Because Slovic and Lichtenstein (1971) provide a recent and excellent review of this approach, we consider it only briefly here.

Like the MAUT validation studies discussed earlier, statistical modelling experiments have been primarily concerned with riskless choice. And like the MAUT studies, the goal of most statistical modelling studies has been pragmatic rather than theoretical -- experimenters have attempted to show that linear additive models can provide a good approximation to the decision maker's preferences. Typically these studies have attempted to represent these preferences with a linear regression model, using the multiple correlation coefficient as a measure of goodness of fit. In most cases the quality of the approximations provided by this approach has been quite good, with correlations ranging from the .70 to the .90s (Bowman,1963; Huber, Sahney, and Ford, 1969; Hoepfl and Huber, 1970; Huber, Daneshgar, and Ford, 1971; Dawes, 1970; Einhorn, 1971). Several researchers examined non linear regression models -- in which the independent variables were logarithmically transformed -- and multiplicative regression models -- in which both the independent and the dependent variables were logarithmically transformed (Huber,Sahney, and Ford, 1969; Huber, Daneshgar, and Ford, 1971; Einhorn, 1971). Although there were cases in which the data of individual subjects were better approximated by these more complex models, there are few instances in which the improvement in prediction was substantial. And in most cases simple linear additive models did just as well as or better than the more complex models.

Anderson (1970) has argued that regression procedures do not provide a sensitive test between models, and that analysis of variance procedures should be used instead. Sidowski and Anderson (1967) and Shanteau and Anderson (1969), for example, found significant and meaningful interactions in situations where additive models provided near perfect approximations to the data. Such findings are typical of analysis of variance studies. Although many subjects deviate significantly from additivity, additive models usually provide excellent approximations to the subjects' judgments (Slovic and Lichtenstein, 1971). It should be noted, however, that only one statistical modelling study (Fischer, 1972b) has dealt with risky decision making where the arguments against additivity are most compelling. Additional studies in risky real world environments are clearly required before any strong conclusions can be drawn.

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Table 1 : A CLASSIFICATION OF CHOICE SITUATIONS AND MODELS

case	the choice alternative is			models
	multi-attributed	uncertain	time-variable	
1	yes	no	no	1. simple order (model 1.1) 2. riskless trade-off models (1.2 and 1.3) 3. additive conjoint measurement (1.4)
2	yes	yes	no	1. simple expected utility model (2.1) 2. riskless decomposition - expected utility models (2.2-2.4) 3. multiplicative expected utility model (2.5) 4. additive expected utility model (2.6)
3	yes	no	yes	no model at present
4	yes	yes	yes	no model at present
5	no	no	no	1. simple order 2. difference structures
6	no	yes	no	1. EU and SEU models 2. minimax and minimax regret models 3. portfolio theory
7	no	no	yes	1. additive time preferences 2. additive time preferences with variable discounting rates 3. additive time preferences with constant discounting rates
8	no	yes	yes	1. additive time preferences - expected utility model (constant or variable discounting rates) 2. multiplicative time preferences - expected utility model (constant or variable discounting rates)

Table 2

TESTS OF MODELS FOR THE TIME-INVARIANT RISKLESS

MULTI-ATTRIBUTE CHOICE SITUATION

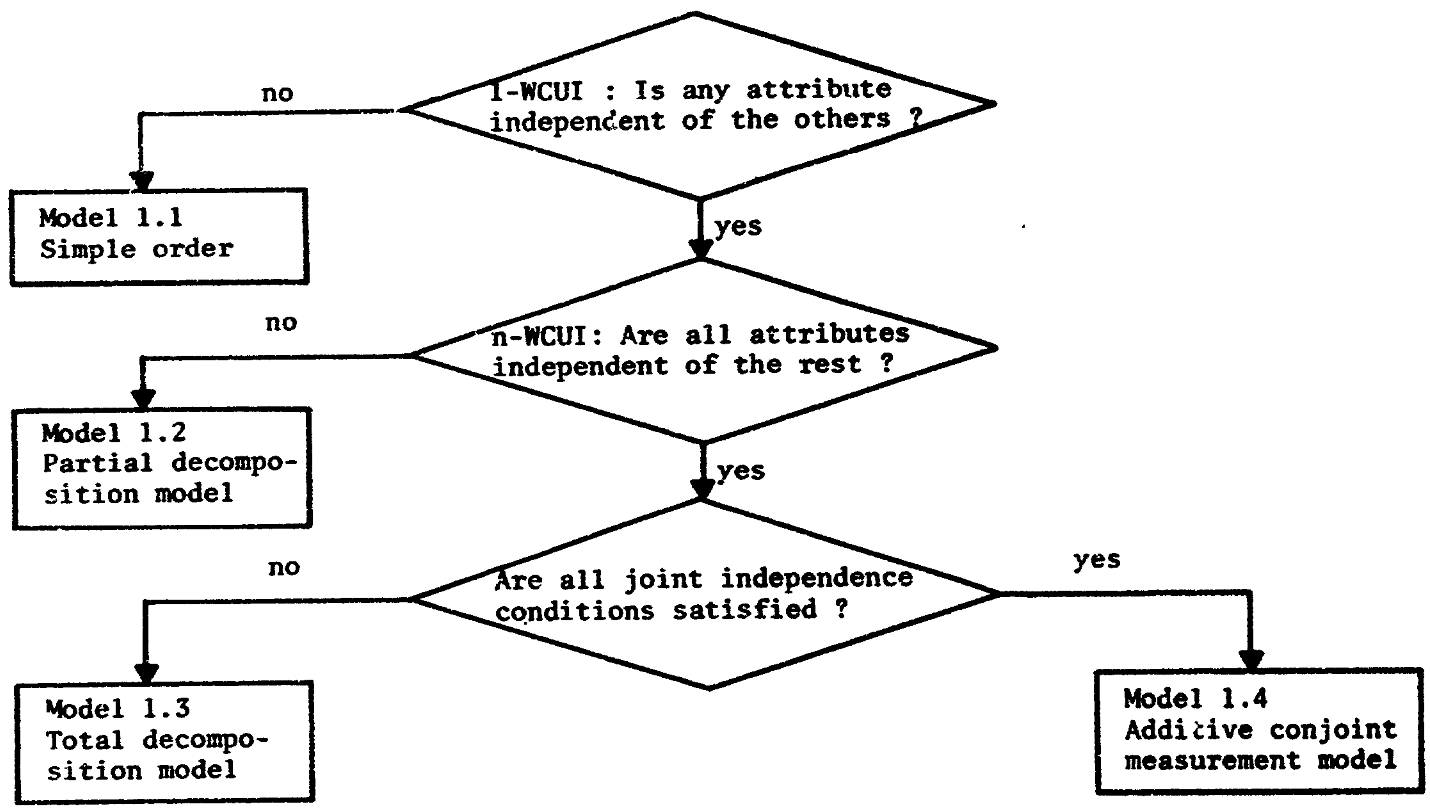

Table 3

TESTS OF MODELS FOR THE TIME-INVARIANT RISKY

MULTI-ATTRIBUTE CHOICE SITUATION