

**Special Programs for Analysis of Radiation by Wire
Antennas,**

SYRACUSE UNIV NY

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BY WIRE ANTENNAS

by

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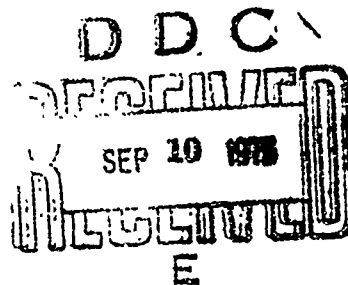
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BY WIRE ANTENNAS

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ABSTRACT

Two user-oriented computer programs are presented and described. The first is suitable for handling efficiently typical analysis and design problems involving linear arrays of parallel thin-wire antennas. The second is designed to enable efficient analysis of radiation from vertical wire antennas over systems of radial ground wires. Examples are given to illustrate various applications of both programs. Special attention is devoted to use of the first program together with a standard optimization procedure to design linear arrays of wire elements with unequal spacings and/or unequal wire lengths.

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CONTENTS

	PAGE
ABSTRACT-----	i
ACKNOWLEDGEMENTS-----	ii
CONTENTS-----	iii
1. INTRODUCTION-----	1
2. THE ANALYSIS PROGRAM	
2.1 Summary of the Method-----	2
2.2 Program Description-----	5
2.3 Comments on the Number of Expansion Functions-----	8
3. OPTIMIZATION PROBLEMS	
3.1 Introduction-----	15
3.2 Design of Unequally Spaced Arrays-----	16
3.3 Use of Unequal Wire Lengths-----	17
3.4 Adjustments of Both Lengths and Spacings-----	24
4. A SPECIAL PROGRAM FOR HANDLING CERTAIN WIRE GROUND SYSTEMS	
4.1 Introduction-----	28
4.2 Data Input-----	30
4.3 Program Description-----	31
4.4 Examples-----	32
5. CONCLUSION-----	36
6. REFERENCES-----	36
7. APPENDICES	
7.1 Appendix A - Evaluation of the Elements of the Generalized Impedance Matrix [Z]-----	38
7.2 Appendix B - Program for Linear Arrays of Parallel Wires-----	41
7.3 Appendix C - Program for Wire Over Radial Wire System-----	56

1. INTRODUCTION

This report has three objectives. The first is to present and describe a user-oriented computer program suitable for handling typical radiation problems involving linear arrays of parallel thin-wire antennas. This program assumes that the wires are all of the same radius, although they may be of different lengths. It is also assumed the wires are centered with feed points all in the same plane and that the wires are lossless with no externally applied loading. The program is based on an analysis procedure suggested by Harrington and represents an application of the method of moments. [1,2] Within Harrington's general formalism the subsectional piecewise sinusoidal functions suggested by Richmond [3] are used for both expansion and weighting resulting in a Galerkin solution to the analysis problem. The corresponding computer program presented here appears to have an advantage with respect to convergence over previous programs, [4,5] that were based on other sets of expansion and weighting functions. This appears especially true for radiation problems involving unloaded thin wires near resonant lengths. The program is described in Chapter 2 of this report and is characterized by very simple data input requirements. The program listing is included in Appendix B.

A second objective of this report is to demonstrate use of the program described above together with an appropriate optimization procedure to treat some typical optimization and design problems of interest. These include design of unequally spaced arrays to provide equal sidelobe patterns, selection of wire lengths and interelement spacings to optimize array directivity, and reduction of pattern sidelobes through adjustments of wire lengths. Results for several typical problems are included in Chapter 3 along with a discussion of the optimization procedure used.

The third objective of this report is to present a modified version of a user-oriented computer program that was presented in an earlier Scientific Report [4] together with some results of its application. This program is

another example of use of the Galerkin procedure in that subsectional piecewise linear (triangle) functions are used for both expansion and weighting. The initial program was written specifically to analyze radiation and scattering by complicated configurations of arbitrarily bent and interconnected thin wires. As yet, however, data have only been presented for certain plane-cross scattering problems [5,6] and for certain radiation problems involving rectangular and circular wire loops, double-V antennas, and symmetrical-T antennas. [5,7] The modified program presented here is useful for handling certain situations involving special symmetry such as vertical wire antennas over radial wire (counterpoise) systems. As mentioned earlier, examples of its use are included. The program is described in Chapter 4, and the listing is presented in Appendix C.

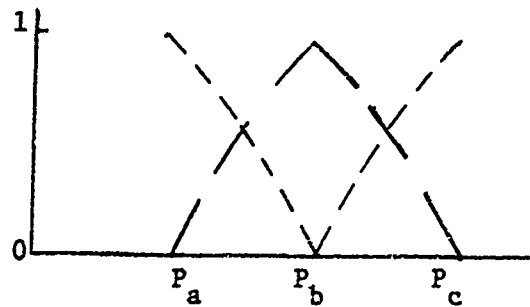
2. THE ANALYSIS PROGRAM

2.1 Summary of the Method

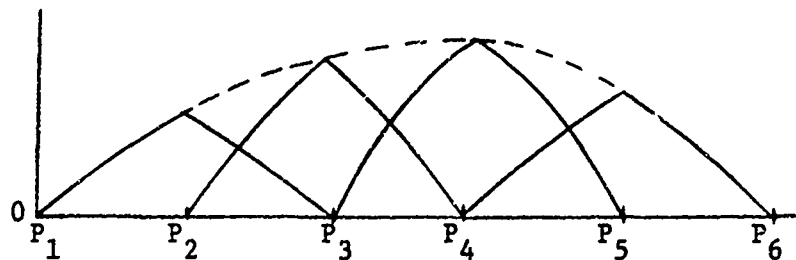
The details of Harrington's general method of analysis as applied to thin-wire antennas are readily available elsewhere. [1,2] It is assumed here the wires are thin and that current flows only in the axial direction of each. Current and charge densities are approximated by filaments of current and charge on the wire axes. The boundary condition regarding the tangential component of the electric field at the wire surfaces is satisfied (approximately) by requiring that the axial component vanish at the surface of each wire. In the subsectional approach used here each wire is thought of as divided into a number of short segments or subsections connected together. The integrodifferential equation characterizing the analysis problem is then reduced to a matrix equation of the form

$$[V] = [Z][I] \quad (1)$$

by expanding each wire current in a sequence of subsectional expansion functions where each is nonzero only over a small portion of a wire; that is, over a small number of subsections. There are many useful choices of sets of expansion functions, and many have been investigated in detail [1,2]. For the program presented here the piecewise sinusoidal functions suggested by Richmond are used, and these are depicted in Fig. 1. [3]



(a) Sinusoidal function



(b) Piecewise sinusoidal approximation

Fig. 1 - Subsectional bases and functional approximation.

In order to compute the elements of the matrices in (1) by the method of moments it is necessary to define a set of testing or weighting functions in addition to the current expansion functions used. If the same functions are used for both expansion and testing then the result is known as a Galerkin solution to the analysis problem. This is the procedure used here.

With reference to Fig. 1 the n th current expansion function can be written as (the wires of the array are all assumed to be z -directed)

$$\begin{aligned} \vec{I}_n(z) &= u_z \frac{\sin k(z-z_{n-1})}{\sin k(z_n-z_{n-1})} & z_{n-1} \leq z < z_n \\ &= u_z \frac{\sin k(z_{n+1}-z)}{\sin k(z_{n+1}-z_n)} & z_n \leq z < z_{n+1} \end{aligned} \quad (2)$$

where \hat{u}_z is the obvious unit vector and $k = 2\pi/\lambda$. The field corresponding to this single current function is given by [8]

$$E_\rho = \frac{j30}{\rho} \left[\frac{(z-z_{n-1})e^{-jkR_1}}{R_1 \sin k(z_n - z_{n-1})} - \frac{(z-z_n)e^{-jkR_2} \sin k(z_{n+1} - z_{n-1})}{R_2 \sin k(z_n - z_{n-1}) \sin k(z_{n+1} - z_n)} + \frac{(z-z_{n+1})e^{-jkR_3}}{R_3 \sin k(z_{n+1} - z_n)} \right] \quad (3)$$

$$E_z = j30 \left[-\frac{e^{-jkR_1}}{R_1 \sin k(z_n - z_{n-1})} + \frac{e^{-jkR_2} \sin k(z_{n+1} - z_{n-1})}{R_2 \sin k(z_n - z_{n-1}) \sin k(z_{n+1} - z_n)} - \frac{e^{-jkR_3}}{R_3 \sin k(z_{n+1} - z_n)} \right] \quad (4)$$

where E_ρ and E_z are, of course, the field components normal to and parallel to the direction of the current element respectively. If I_n is the complex amplitude of the nth current expansion function, then the total current of the array can be written as

$$\vec{I}(z) = \sum_n I_n \vec{I}_n(z) \quad (5)$$

and the total field is an obvious corresponding generalization of (3) and (4) using (5).

Formulas for the elements of the matrices in (1) can be found easily from the general theory. [1,2,5,9] The matrix [I] is called the current matrix and is simply a column matrix of dimension equal to the total number of current expansion functions used. Each element equals the complex amplitude of the corresponding current expansion function; that is, the nth element is I_n . If the total number of expansion functions used is denoted by M then [Z] in (1) is a square matrix of dimension M known as

the generalized impedance matrix. If \vec{E}_m is the vector field produced by the m th expansion function $\vec{I}_m(z)$ then a general formula for the typical element of $[Z]$ denoted by Z_{nm} is

$$Z_{nm} = - \int_{z_{n-1}}^{z_n} \frac{\sin k(z-z_{n-1})}{\sin k(z_n-z_{n-1})} \hat{u}_z \cdot \vec{E}_m dz + \int_{z_n}^{z_{n+1}} \frac{\sin k(z_{n+1}-z)}{\sin k(z_{n+1}-z_n)} \hat{u}_z \cdot \vec{E}_m dz \quad (6)$$

Detailed formulas for the real and imaginary parts of Z_{nm} are obtained by using (4) in (6) and are given in Appendix A of this report in terms of sine and cosine integrals. The formulas are valid if all segment lengths $(z_j - z_{j-1})$ are equal along any one wire.

The matrix $[V]$ in (1) is a column matrix of dimension M , and its elements are related to the excitation of the array. There is an element of $[V]$ that corresponds to each expansion function. However, it is assumed here that excitation voltages will be applied only at the center points of the array wires. Hence, the only nonzero elements of $[V]$ are those corresponding to current expansion functions that are centered at the midpoints or feedpoints of the array wires. These nonzero elements are numerically equal to the complex excitation voltages that are applied. This result is derived directly from the general formula for $[V]$ by using impulsive excitation functions at the points described.

Once the elements of $[Z]$ and $[V]$ have been computed the current distribution is found from (1) by simply inverting $[Z]$. Input impedances and field patterns of interest are then determined using straightforward and well-known matrix manipulations, completing the analysis problem.

2.2 Program Description

In the program printout of Appendix B the main program is listed first followed by the various subroutines. The example included in the Appendix is for analysis of a linear array of eight half-wave wires that are half-wavelength

spaced and all centered with real unit voltages. Required input data included in the main program are listed as follows:

- a) The fifth statement is

$$AK = TP * (\text{wire radius}) \quad (TP = 2\pi)$$

which inputs the wire radius in wavelengths. (Recall that all array wires are assumed to have the same radius.) The wire radius used in the example included in Appendix B is 0.007022λ .

b) The total number of wires making up the array is provided with the sixth statement of the main program. Of course, for the example considered this statement is simply $N=8$.

c) The eighth statement is the first of $(2N-1)$ consecutive data statements providing information on the geometry of the array. The first N of these are typified by $Y(j) = (\text{half-length of the } j\text{th wire in wavelengths}) * TP$ where TP is simply 2π as determined in statement four. The half-lengths of the wires included in the example of Appendix B are all 0.25λ . The next $(N-1)$ data statements indicate the positions of the array wires relative to the first. An example of these is

$$Y(k) = (\text{distance of the } k\text{th wire from the first in wavelengths}) * TP$$

Note that the wires in the array of the example are all spaced one half-wavelength apart.

d) The purpose of the DO LOOP formed by statements 23-25 of the main program is to provide information concerning excitation of the array. As mentioned earlier it is assumed the wires are all centered so that $[V]$ is a column matrix with each nonzero element equalling the complex excitation in volts applied at the midpoint of the corresponding wire. In the illustrative example of the Appendix all wires are centered with real unit voltages.

e) Part of the computed output is the field pattern of the array taken in the principal H-plane. This pattern is computed over 180° of the azimuth angle centered on the broadside direction. Values are calculated at intervals

determined by the 26th statement of the main program. The statement $MQ=3$ shown in the sample printout means that intervals of three degrees are desired. If it is only necessary to compute the pattern at five degree intervals of the azimuth angle then, of course, this statement should read $MQ=5$.

f) The only other information required is concerned with the number of current expansion functions that should be used to represent the current for each array wire. This decision is made by way of statement 40 included in Subroutine ASCTFD. For analysis problems where an accurate representation of the current is needed to determine correctly input impedances and other quantities of interest the statement

$$MM = \text{INT}(Y(I) - 0.571) + 3$$

is recommended, where $Y(I)$ is defined in paragraph (c) above. The total number of expansion functions for the I th wire is $[2(MM)-1]$ so that the relationship between this number and the wire length L is approximately

$$0 < L < \frac{\lambda}{2} \dots 2(MM)-1 = 5$$

$$\frac{\lambda}{2} < L < 0.82\lambda \dots 2(MM)-1 = 7$$

$$0.82\lambda < L < 1.14\lambda \dots 2(MM)-1 = 9$$

and so on.

This seems to be a reasonable choice based on the discussion of convergence included in Section 2.3 of this report. If, on the other hand, the primary concern of the user is with pattern characteristics as is the case with the optimization problems discussed in Chapter 3 then fewer current expansion functions are needed for each wire and the statement

$$MM = \text{INT}(Y(I) - 0.571) + 1$$

can be used. With this choice the numbers of expansion functions used for the intervals defined above are 1,3,5,... . Obviously the program user can make either of these choices or an independent one, depending on the particular kinds of results sought.

Normal printed output of the program includes all input data, the current distribution as expressed by the real and imaginary parts of the complex amplitude of each current expansion function, input impedances corresponding to each feed point, and the normalized electric-field pattern in the principal H-plane together with the normalization constant.

Program operation proceeds roughly as follows: The generalized impedance matrix $[Z]$ is computed using instructions 53-97 with the obvious symmetry about the feed points taken into account in order to reduce overall computation time and storage requirements. This matrix is inverted using Subroutine CSMIN, a complex inversion routine authored at the University of Illinois. The current matrix $[I]$ is determined by forming the matrix product

$$[I] = [Z]^{-1} [V] \quad (7)$$

in Subroutine MULTPY. Finally, the electric-field pattern is computed for the principal H-plane using instructions 101-120, and the input impedances corresponding to feed points are calculated using the DO LOOP of statements 146-152. The FUNCTIONS ZMN and ZM1 compute mutual terms of $[Z]$ when the segment lengths of the corresponding wires are unequal and equal respectively. This is done with the aid of Subroutine SICI made available through the IBM Scientific Subroutine Package for the purpose of computing the required sine and cosine integrals $S_1(x)$ and $C_1(x)$. The program described here was developed with an IBM 370/155 computer and storage-time requirements are quoted with the various examples as they are presented in later sections of this report.

2.3 Comments on the Number of Expansion Functions

The number of current expansion functions that should be used in any given situation has been an open subject of discussion for some time. It is clear that requirements vary considerably in this respect depending on the kinds of results sought. For example, for some pattern synthesis and optimization problems it may be possible to use as few as say five expansion functions per wavelength, while in some other situations where careful analyses of current distributions are required it is advisable to use considerably more,

say 10-15 expansion functions per wavelength. This, of course, is the reason for incorporating the special instruction discussed in Section 2f above.

Figures 2-5 illustrate the kinds of results that are obtained as the numbers of expansion functions are varied for certain, single-wire problems. First, consider a half-wave centered wire of radius 0.007022λ . The input admittance is calculated for this problem using different numbers of current expansion functions and for two different choices of subsectional expansion functions. One of these choices involves the piecewise sinusoidal functions used here while the other involves the piecewise linear (triangle) functions used for the analysis programs developed earlier to handle arbitrary configurations of bent and interconnected wires. [4] Computed results are shown in Fig. 2 along with the corresponding experimental data reported by Mack. [10] Similar results are shown in Figs. 3-5 for longer wires. These and other available data indicate that the piecewise sinusoidal functions offer somewhat faster convergence than the functions used with the arbitrary wire program and also that at least 9 or 10 expansion functions are needed per wavelength to describe accurately the current distributions encountered in the kinds of problems considered here. As indicated by Fig. 2 an even larger number may be advisable for wires near resonant lengths.

Figure 6 provides computed information on the self and mutual admittances of two, half-wavelength, centered parallel wires that are spaced various distances apart and are both of radius 0.007022λ . As before, data are recorded as functions of the number of expansion functions used for each wire and also versus the separation distance d , for both the piecewise sinusoidal and the piecewise linear cases. Once again, Mack's corresponding experimental data are included. As before, the data indicate that the piecewise sinusoidal functions offer relatively rapid convergence and that approximately 10 expansion functions are needed per wavelength of wire to obtain good results for the current distributions and input admittances.

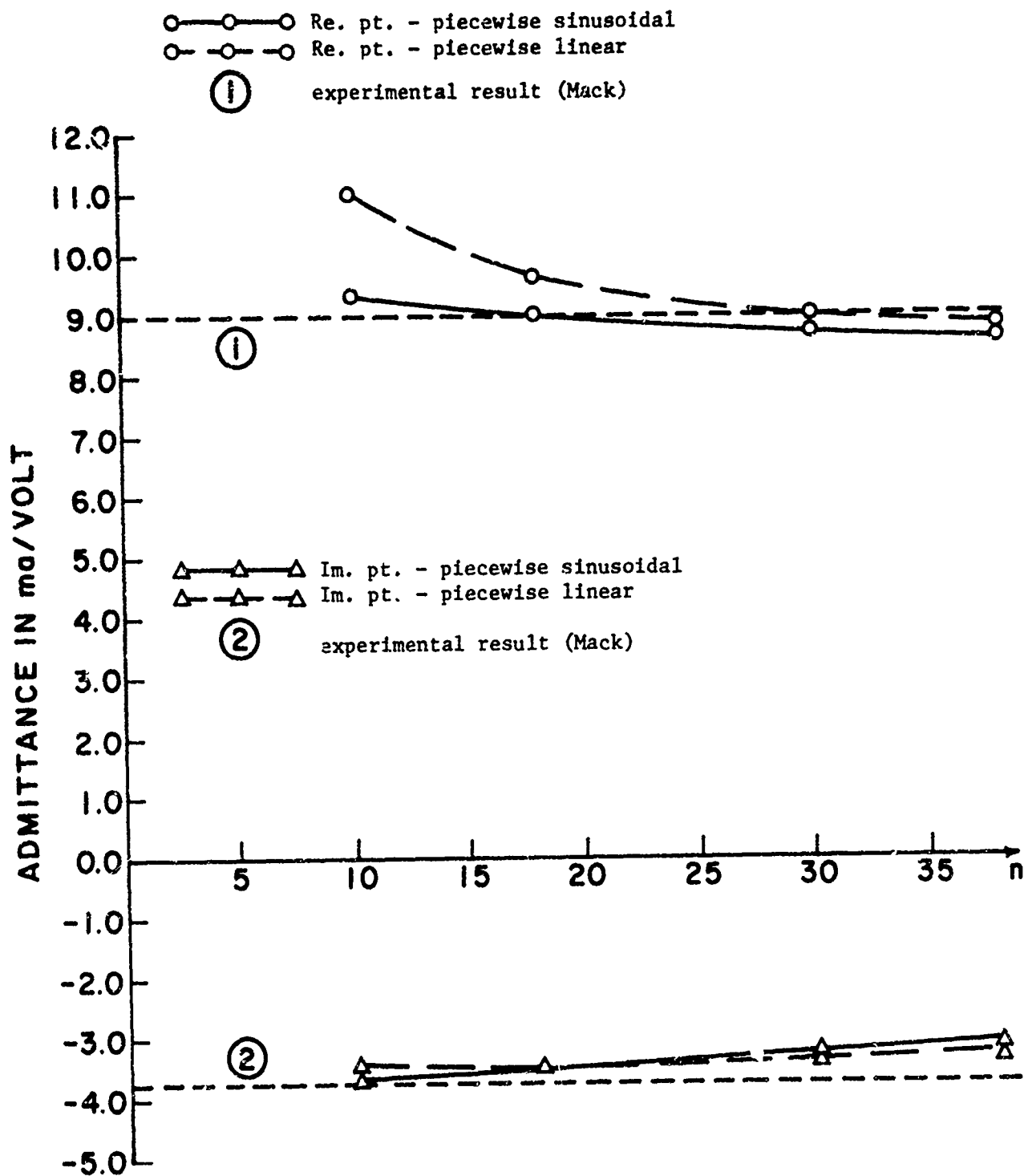


Fig. 2 - Input admittance of a centered linear antenna of length 0.5λ and radius 0.007022λ plotted vs. n , the number of subsectional expansion functions used per wavelength.

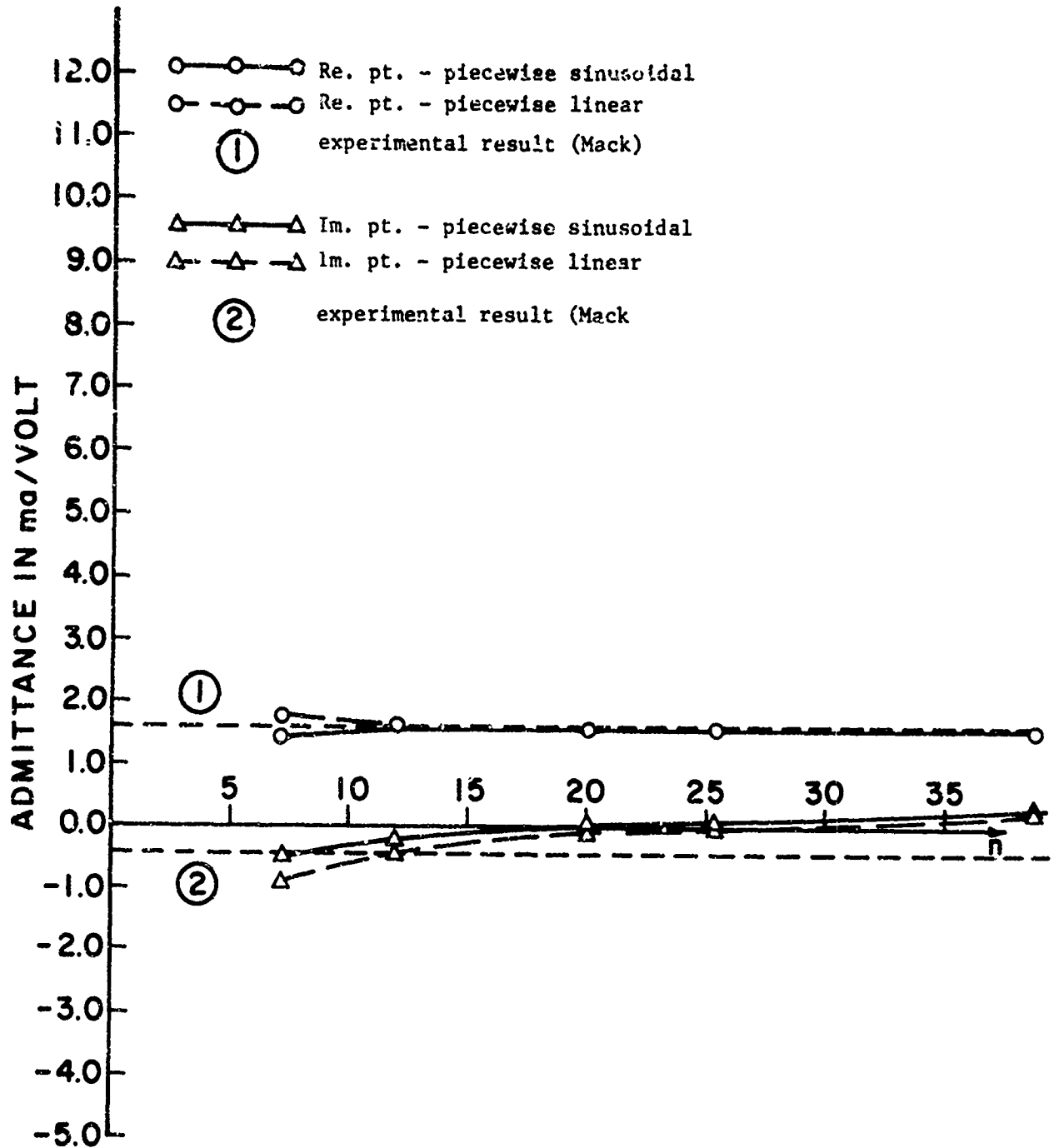


Fig. 3 - Input admittance of a centered linear antenna of length 0.75λ and radius 0.007022λ plotted vs. n , the number of subsectional expansion functions used per wavelength.

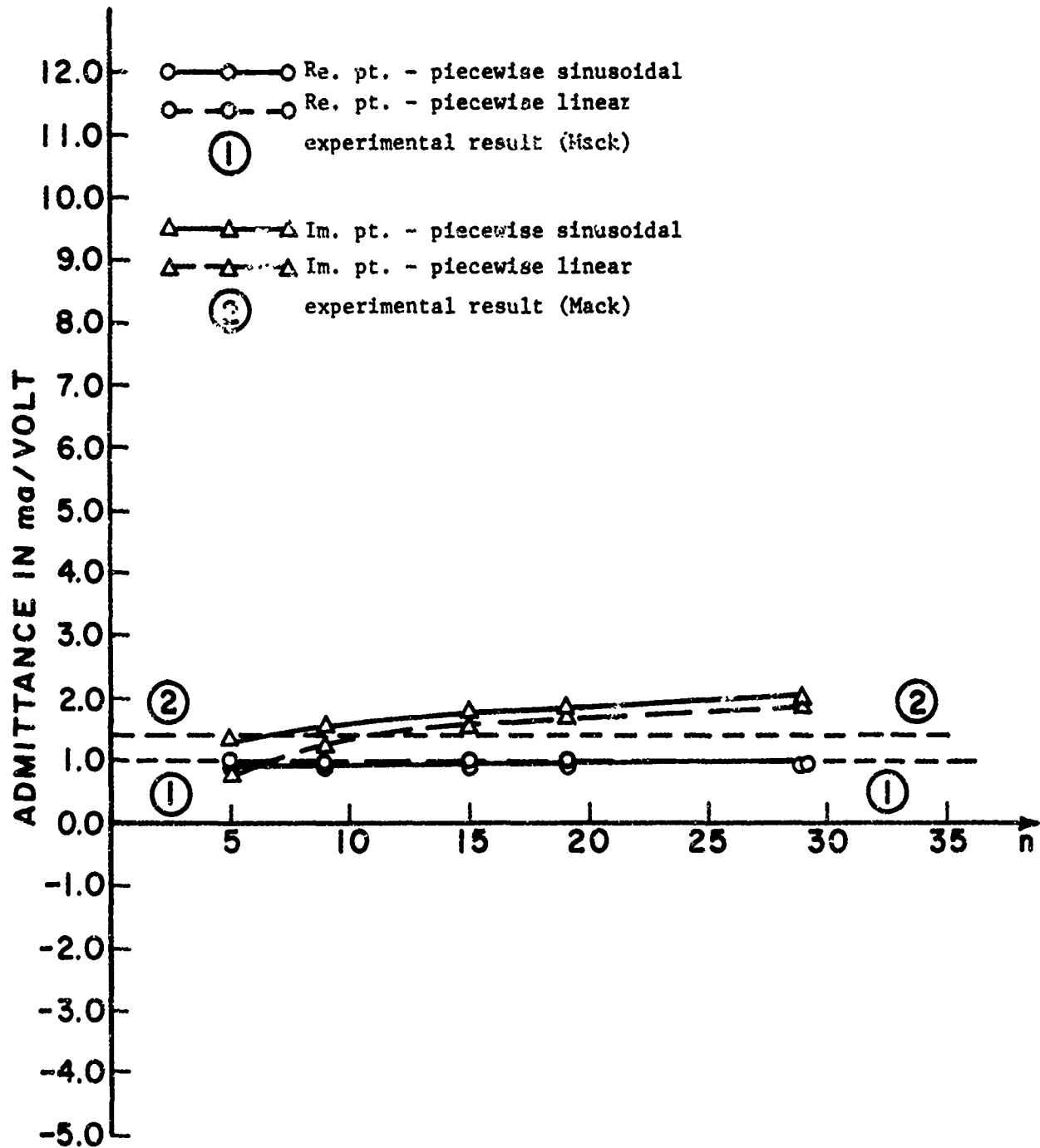


Fig. 4 - Input admittance of a centered linear antenna of length λ and radius 0.007022λ plotted vs. n , the number of subsectional expansion functions used for wavelength.

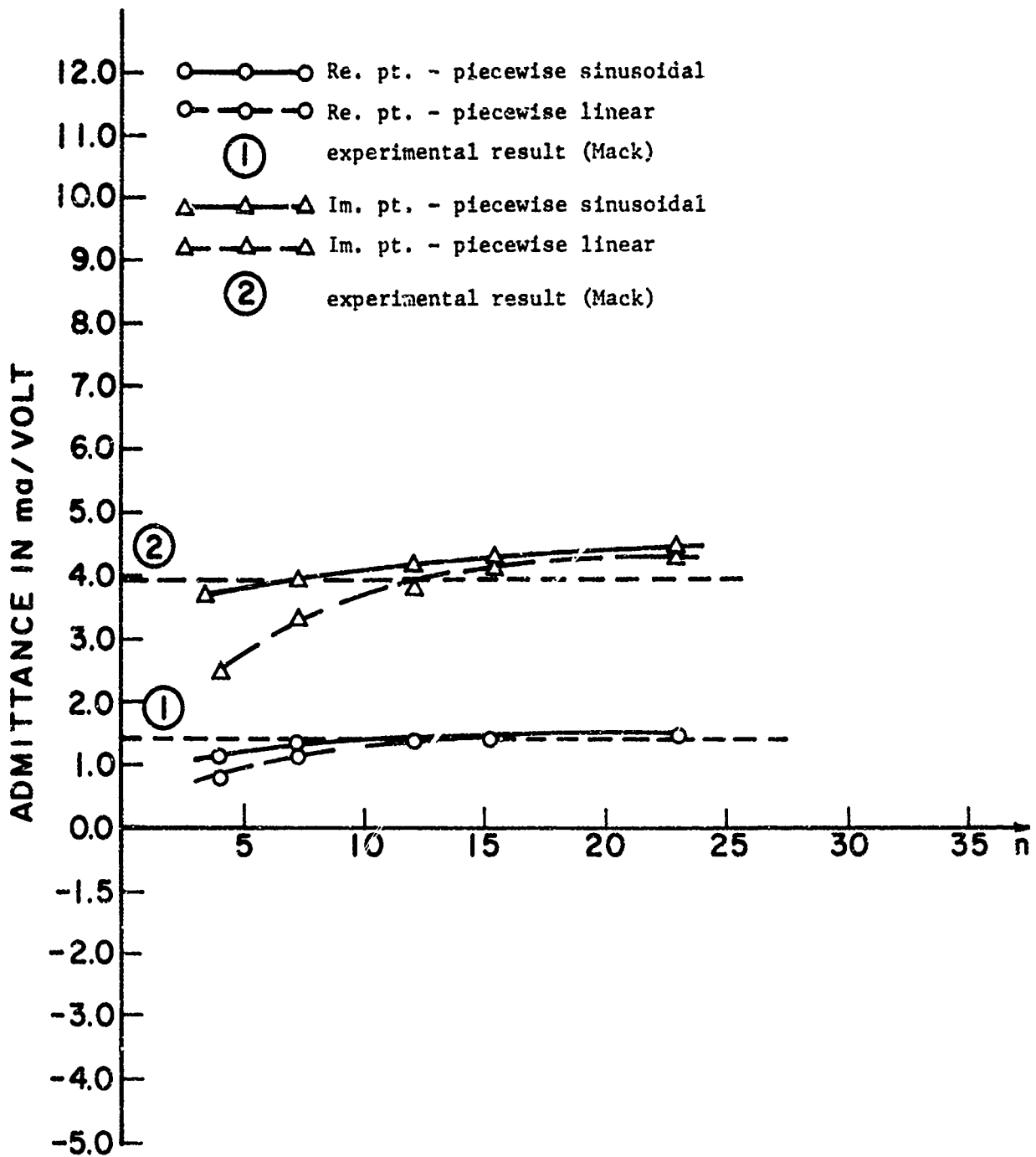


Fig. 5 - Input impedance of a centered linear antenna of length 1.25λ and radius 0.007022λ plotted vs. n , the number of subsectional expansion functions used per wavelength.

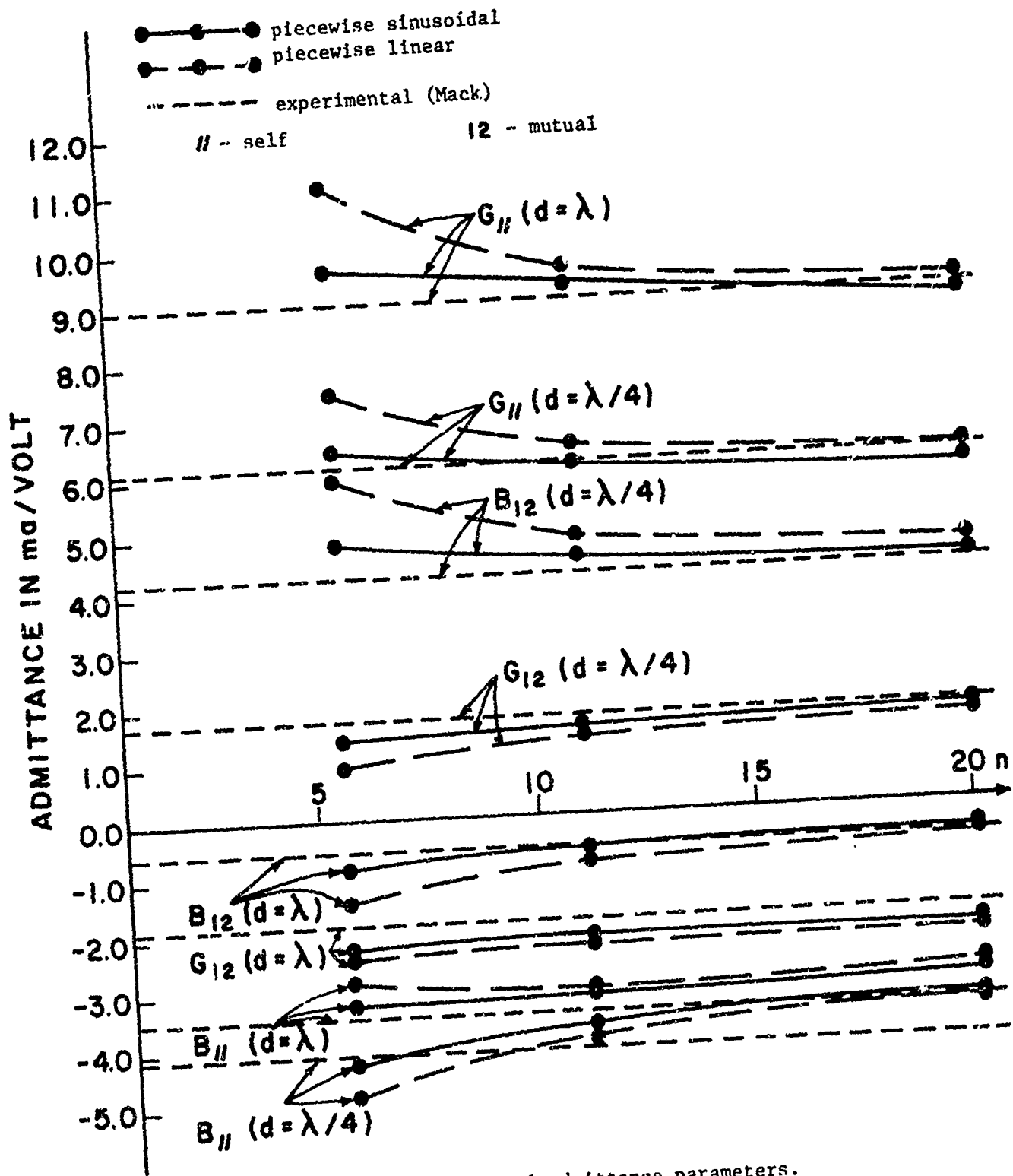


Fig. 6 - Convergence of terminal admittance parameters.

The somewhat faster convergence that characterizes the piecewise sinusoidal current approximation for problems involving thin wires is due in part to the inherent shape of the individual expansion functions used and also in part to the fact that the integrations required to obtain the elements of $[Z]$ can be performed analytically. This is in contrast to certain programs based on the piecewise linear current approximation that incorporate a substantial number of approximate numerical integrations. The piecewise linear data of Figs. 2-6 were derived using one of these. It is interesting to note that convergence is significantly improved even in the piecewise linear case if the integrations required to obtain $[Z]$ are performed analytically [11].

Another advantage of the formulation used here is that there is no tendency towards matrix instability as the number of expansion functions is increased, a difficulty encountered with some other methods available for handling thin-wire problems.

3. OPTIMIZATION PROBLEMS

3.1 Introduction

In this chapter an optimization procedure is used together with the analysis program described in the previous chapter to treat certain optimization and design problems involving arrays of parallel thin-wire antennas. As mentioned earlier these problems include design of unequally spaced arrays to provide equal-sidelobe patterns, reduction of pattern sidelobes through adjustments of wire lengths, and selection of wire lengths and spacings to optimize array directivity. The optimization procedure used is due to Rosenbrock [12] and represents a modified version of the method of steepest descent. This method does not require calculation of any derivatives and is particularly well-suited to automatic calculation by digital computer. The method has been used by other antenna engineers in treating challenging optimization problems of practical interest.

3.2 Design of Unequally Spaced Arrays

The analysis program presented in the previous chapter has been used with the optimization method of Rosenbrock [12] to develop a design technique for linear arrays of unequally spaced, parallel-wire antennas. As before the wires are assumed to be centered and all of the same radius. In this case it is also assumed that each wire is excited with a real unit voltage, and that the interelement spacings are symmetrical about the array center.

During the course of this investigation several error criteria were tried in association with the optimization procedure used. Most of these resulted in relatively poor or slow convergence to acceptable solutions, and quite often an acceptable solution was never reached within the reasonable time constraints placed on the computer. Of course, the error criterion is the quantity to be minimized as the optimization procedure progresses and the most successful of those tried is given by

$$\epsilon = \sum_{i=2}^p \left| |T(i)|^2 - |T(1)|^2 \right| \quad (8)$$

where $T(i)$ is the peak of the i th sidelobe with $T(1)$ being the peak of the sidelobe closest to the main beam. (Note that the resulting broadside pattern is symmetrical about the main beam since the excitations are uniform and the spacings are symmetrical about the array center.) The limit p is simply the number of sidelobes to be controlled on either side of the main beam. The starting point for the iterative procedure is the pattern of an equally spaced uniform array with a given number of elements. The result is an equally excited, unequally spaced array with nearly equal sidelobes. Of course the design parameters are the interelement spacings, although the spacing with respect to the array center of the outermost symmetrical pair of elements is kept fixed at its initial value so that the beamwidth will remain essentially unchanged from its value at the start.

As an example consider an 8-element linear array of centered, half-wave, parallel wires that are half-wavelength spaced and all of radius

0.007022λ . With equal in-phase excitations the program of the previous chapter can be used easily to obtain the current distributions, input impedances, and the H-plane pattern of Fig. 7. The program can then be used in conjunction with the optimization procedure and error criterion given by (8) to obtain a new pattern with reduced and nearly equal sidelobes using only the interelement spacings as design parameters. The positions of the elements of the outermost pair are kept fixed to provide roughly the beamwidth of the uniform array and the remaining three interelement spacings are adjusted to achieve nearly equal sidelobes at a reduced level. The resulting pattern is also shown in Fig. 7 along with the initial and final spacing configurations. Note that because of symmetry the spacings are shown for only half the array and the patterns are shown for only one quadrant.

Figures 8-10 show results of applying the same procedures to linear arrays of 9, 12, and 18 parallel, half-wave wires respectively. It is noted that there is no substantial beamwidth deterioration even though the sidelobes have been reduced in overall level. These computations were all performed using an IBM 370/155 computer with total times of 25, 36, 70, and 185 seconds required for obtaining the results shown in Figs. 7-10, respectively.

Finally, it should be noted that a different error criterion was used earlier in conjunction with an alternative analysis procedure and an optimization method due to Fletcher and Powell [13] to treat the same problem discussed above [14]. However, the rate of convergence proved to be a serious problem in that case and the present choices appear to be much more satisfactory for this particular problem.

3.3 Use of Unequal Wires Lengths

A second problem treated successfully using the optimization method of the previous section involves use of array wire lengths rather than the interelement spacings as design parameters. The objective is to reduce the sidelobes of an equally excited, equally spaced array of (initially) equal-length

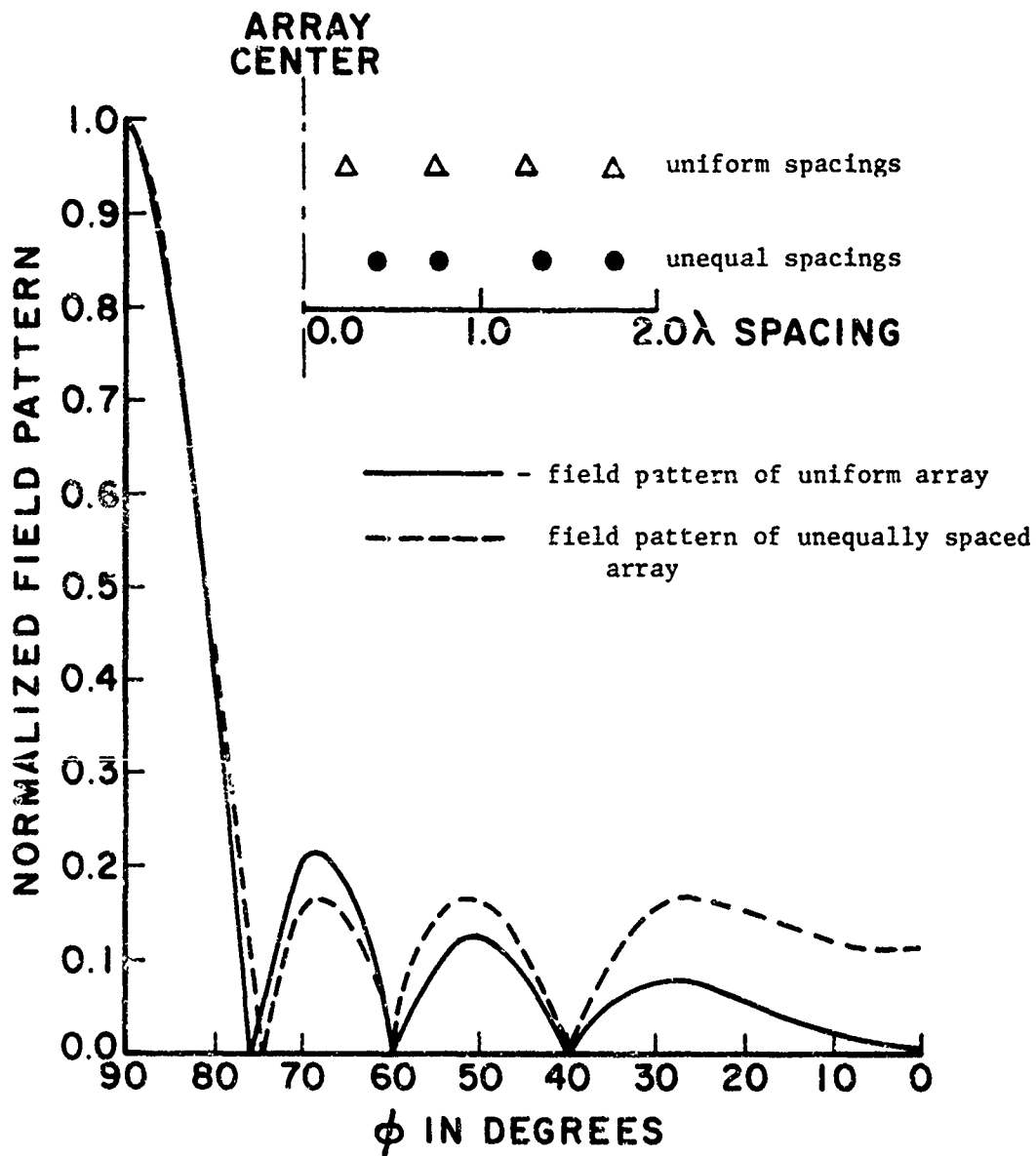


Fig. 7 - Field patterns of an 8-element uniform array along with the pattern of a similar array with unequal spacings designed for equal sidelobes.

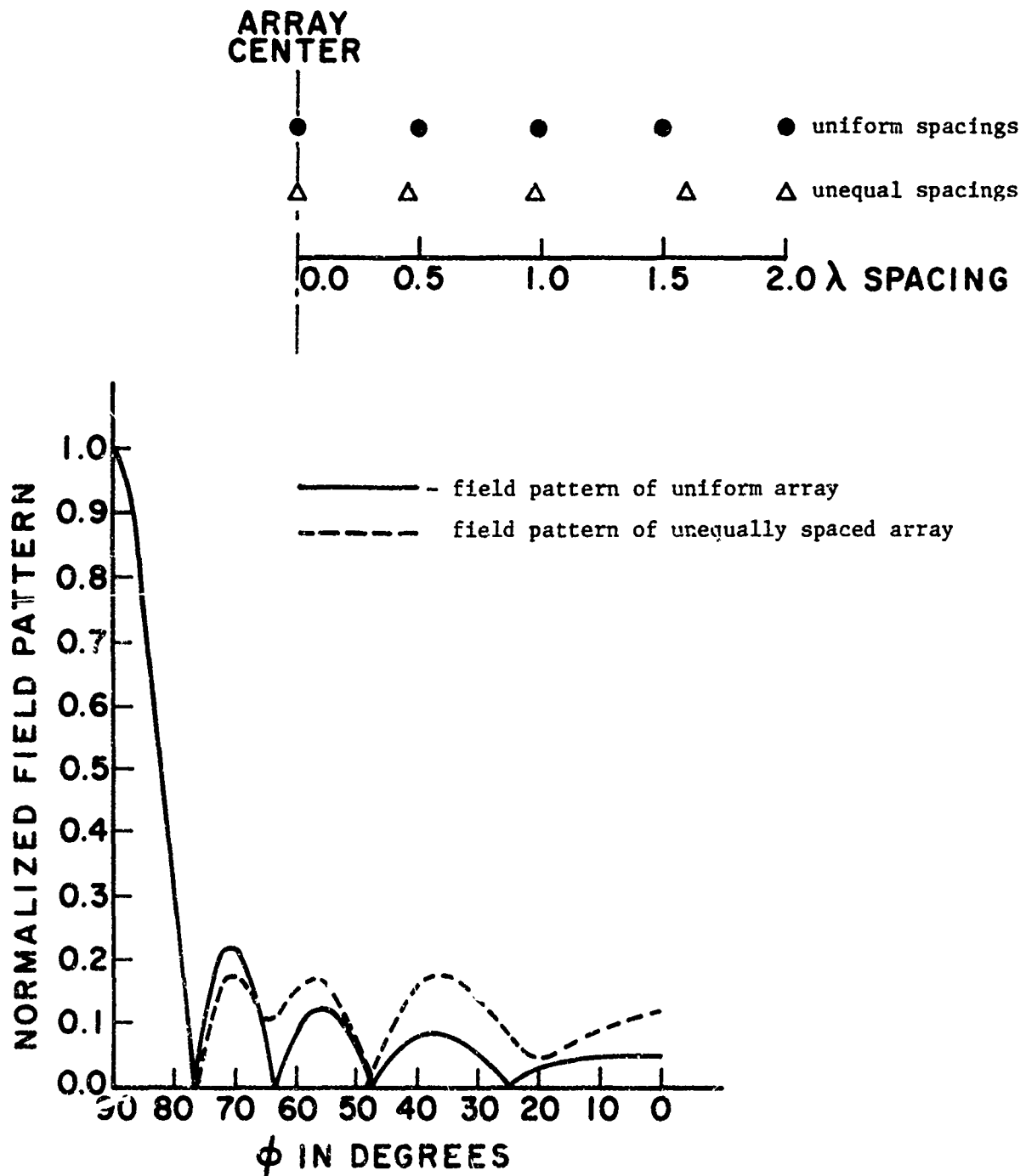


Fig. 8 - Field pattern of a 9-element uniform array along with the pattern of a similar array with unequal spacings designed for equal sidelobes.

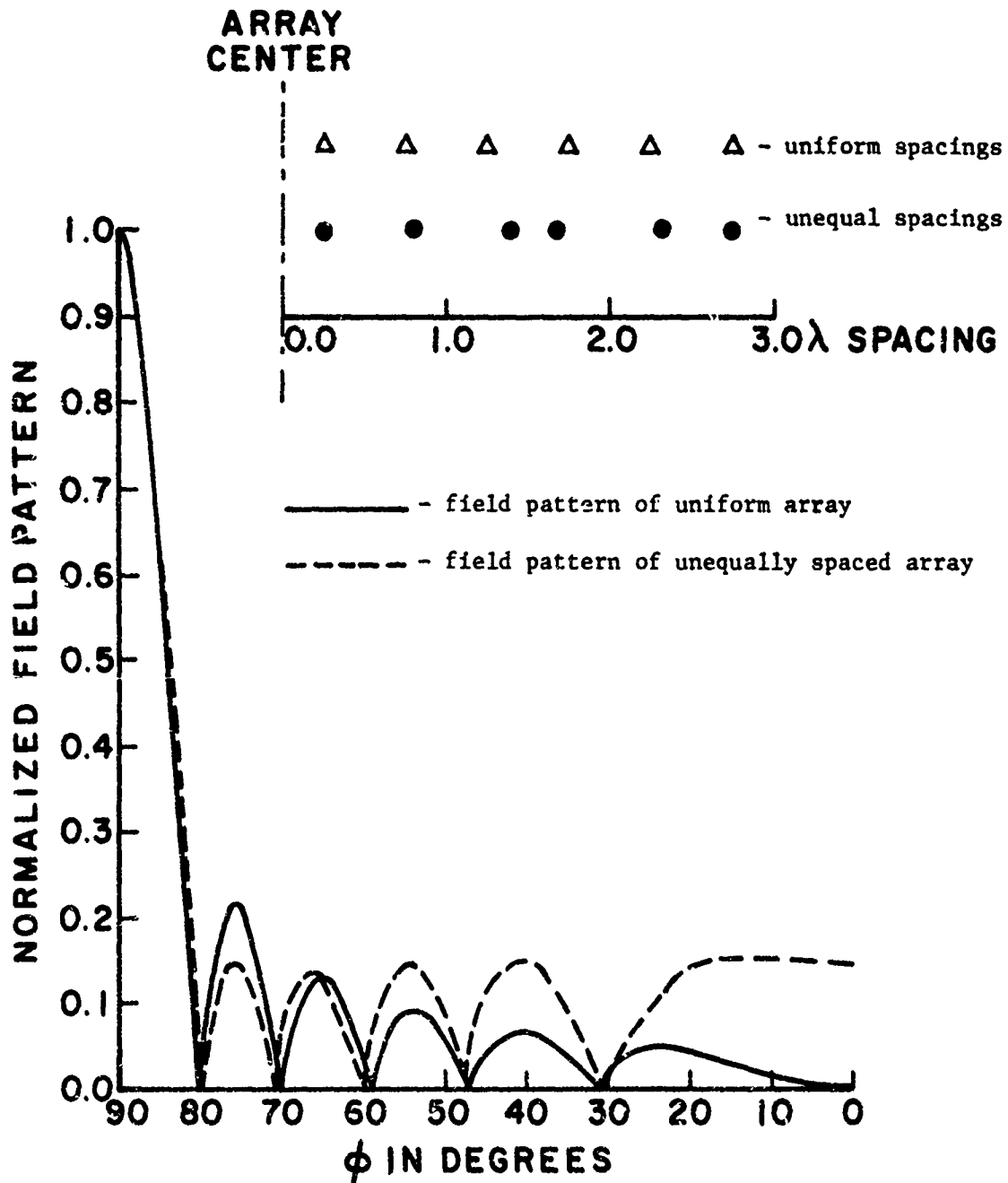


Fig. 9 - Field pattern of a 12-element uniform array along with the pattern of a similar array with unequal spacings designed for equal sidelobes.

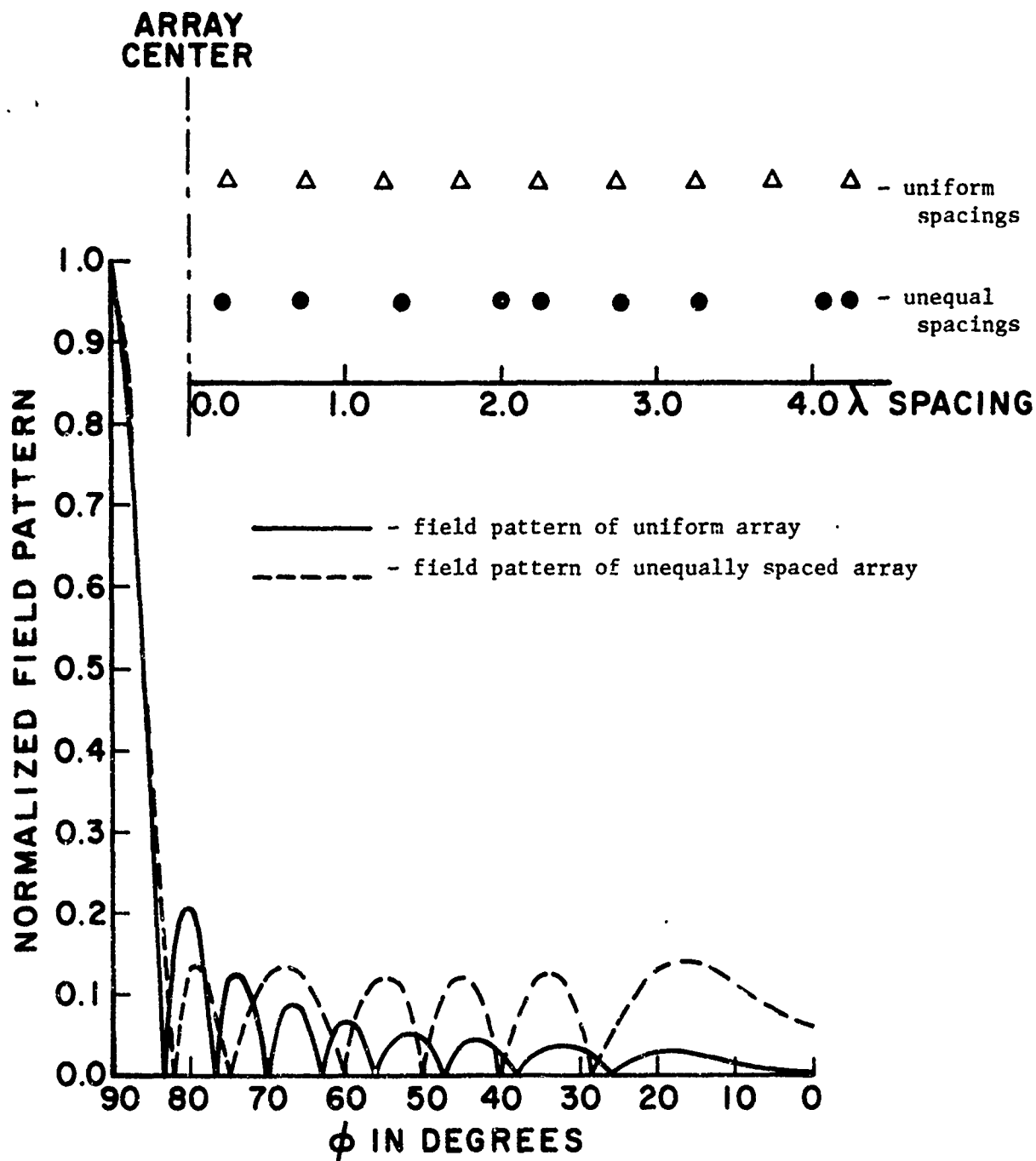


Fig. 10 - Field pattern of an 18-element uniform array along with the pattern of a similar array with unequal spacings designed for equal sidelobes.

wires by adjusting the wire lengths. As before, it is assumed here that the resulting array remains symmetrical about its center and also with respect to the plane containing the feed points. The wires are again assumed to be centered and all of the same radius.

In this case the most success was achieved using an error criterion given by

$$\begin{aligned} \epsilon_1 &= \sum_{i=1}^n \left| |T(i)|^2 - |T_0|^2 \right| \quad \text{for } |T(i)| > |T_0| \\ &= 0 \quad \text{for } |T(i)| \leq |T_0| \end{aligned} \quad (9)$$

where in this case $T(i)$ is the value of the field pattern in the i th specified direction and n is the number of observations calculated (e.g. one every two or three degrees) over the sidelobe region of the pattern. The quantity $|T_0|^2$ is simply the square of the desired sidelobe level. The starting point for the iterative procedure is the pattern of an equally spaced uniform array with a given number of equal-length wires. The result is an equally excited, equally spaced array of wires of different lengths with sidelobes near the desired level.

As an example consider a 12-element array of centered, parallel wires that are half-wavelength spaced, uniformly excited, and initially all one half-wavelength long. The wires are all of radius 0.007022λ , and the initial H-plane pattern is shown in Fig. 9. The analysis program of the previous chapter can be used together with the optimization procedure and error criterion given by (9) to obtain the new pattern of Fig. 11 by adjusting only the wire lengths. The desired level in this case was $+0.1$ corresponding to $|T_0|^2 = 0.01$. This corresponds to a desired sidelobe level of -20 dB. Figure 11 also includes the final wire lengths which are assumed symmetric about the array center.

Results for this example are quite close to those desired as indicated by Fig. 11. However, the procedure is time-consuming with convergence much

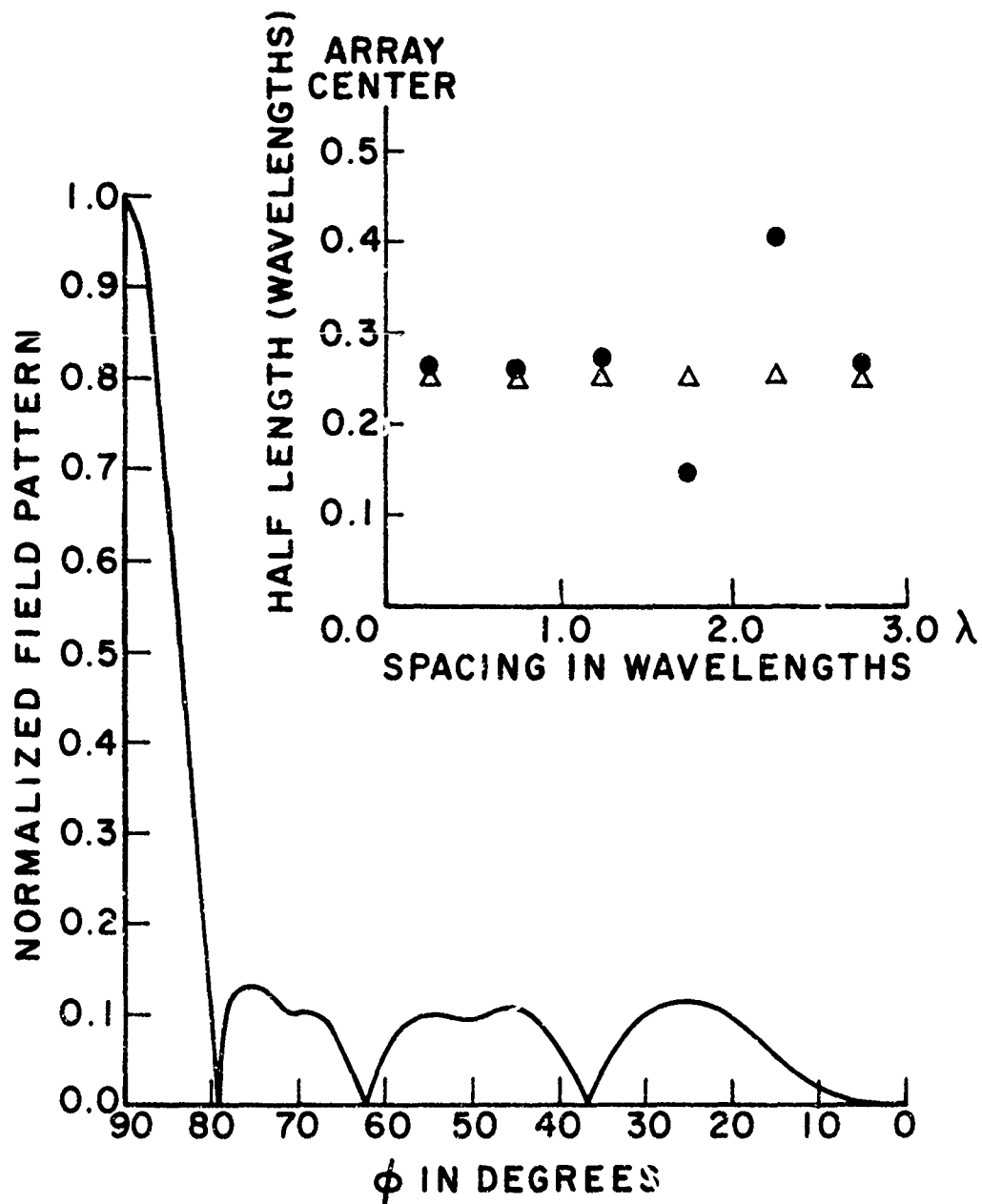


Fig. 11 - Pattern of a 12-element array with sidelobe minimized with respect to wire lengths.

slower than in the examples of the previous section. A total of about nine minutes was required to obtain the data corresponding to Fig. 11 using an IBM 370/155 computer.

3.4 Adjustments of Both Lengths and Spacings

An obvious extension of the work for the last two sections involve adjustments of both wire lengths and interelement spacings simultaneously to achieve reduced sidelobes. Figure 12 shows the final result of minimizing the error criterion given by (9) for an 8-element array of parallel wires. The starting point for the iterative procedure was the initial pattern of Fig. 7 corresponding to a uniform 8-element array of half-wave wires that are half-wavelength spaced, and the final result is that of Fig. 12 while Fig. 13 indicates the corresponding array configuration. Symmetry about the array center and about the plane containing the feed points is again assumed and the wire radii are all 0.007022λ . The result is quite good although convergence was considerably slower than in the case involving just the spacings. The results of Fig. 12 required about 11 minutes using an IBM 370/155 computer.

As a final illustration of the use of the analysis program described in Chapter 2 together with Rosenbrock's optimization procedure consider the design of a typical Yagi antenna. Here, lengths and spacings are to be determined for an array of parallel wires (with symmetry assumed only about the center-points of the array wires) such as to maximize the endfire gain G_0 defined as

$$G_0 = \frac{4\pi \text{ (radiation intensity for the endfire direction)}}{\text{power input to the array}} \quad (10)$$

Formulas for computing the gain using matrices available from the analysis program are well-known [1,2]. These can be used with the iterative optimization procedure to optimize a Yagi antenna with respect to its wire lengths and spacings. The starting point can be an initial state corresponding to some classical or other improved design procedure. Figure 14 shows results after three iterations for Yagi antennas numbering 3-6 elements where

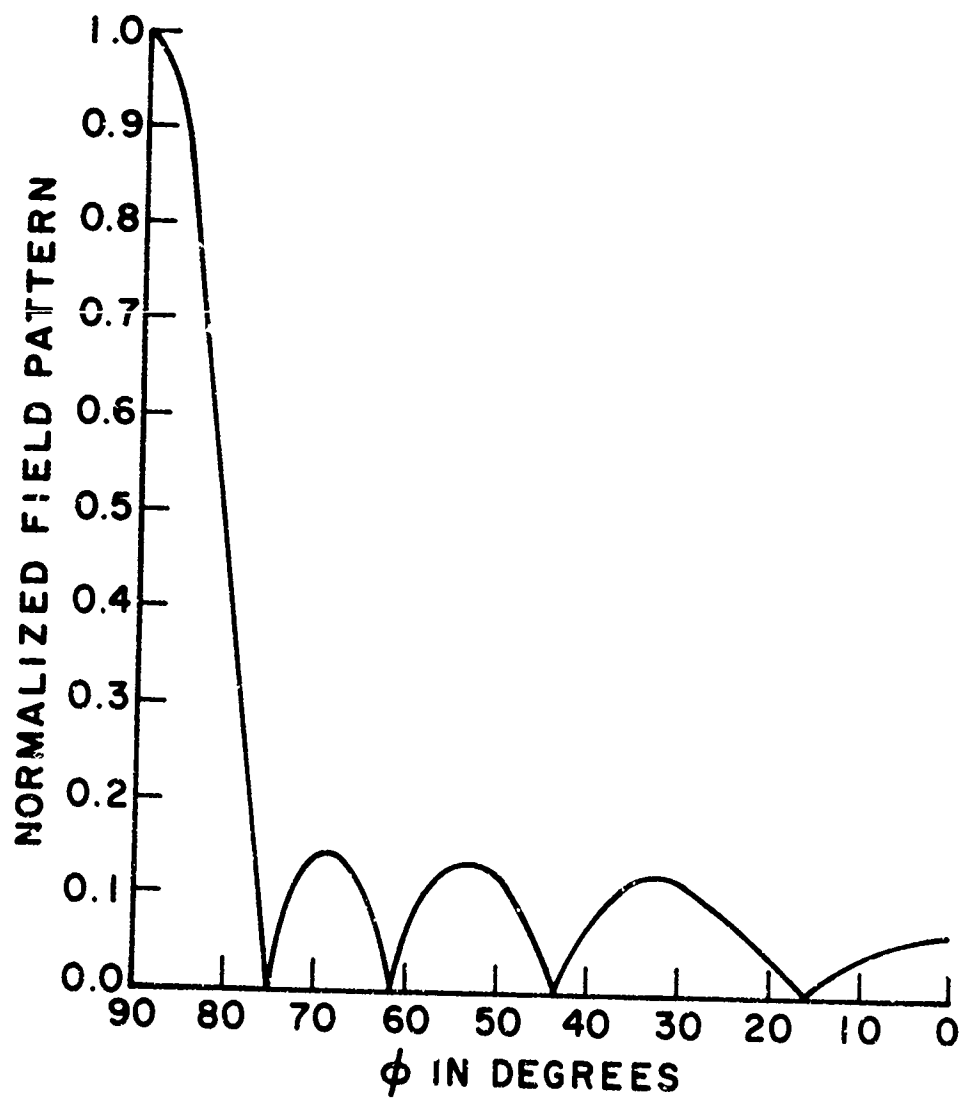


Fig. 12 - Pattern of an 8-element array with sidelobe level minimized with respect to both lengths and spacings.

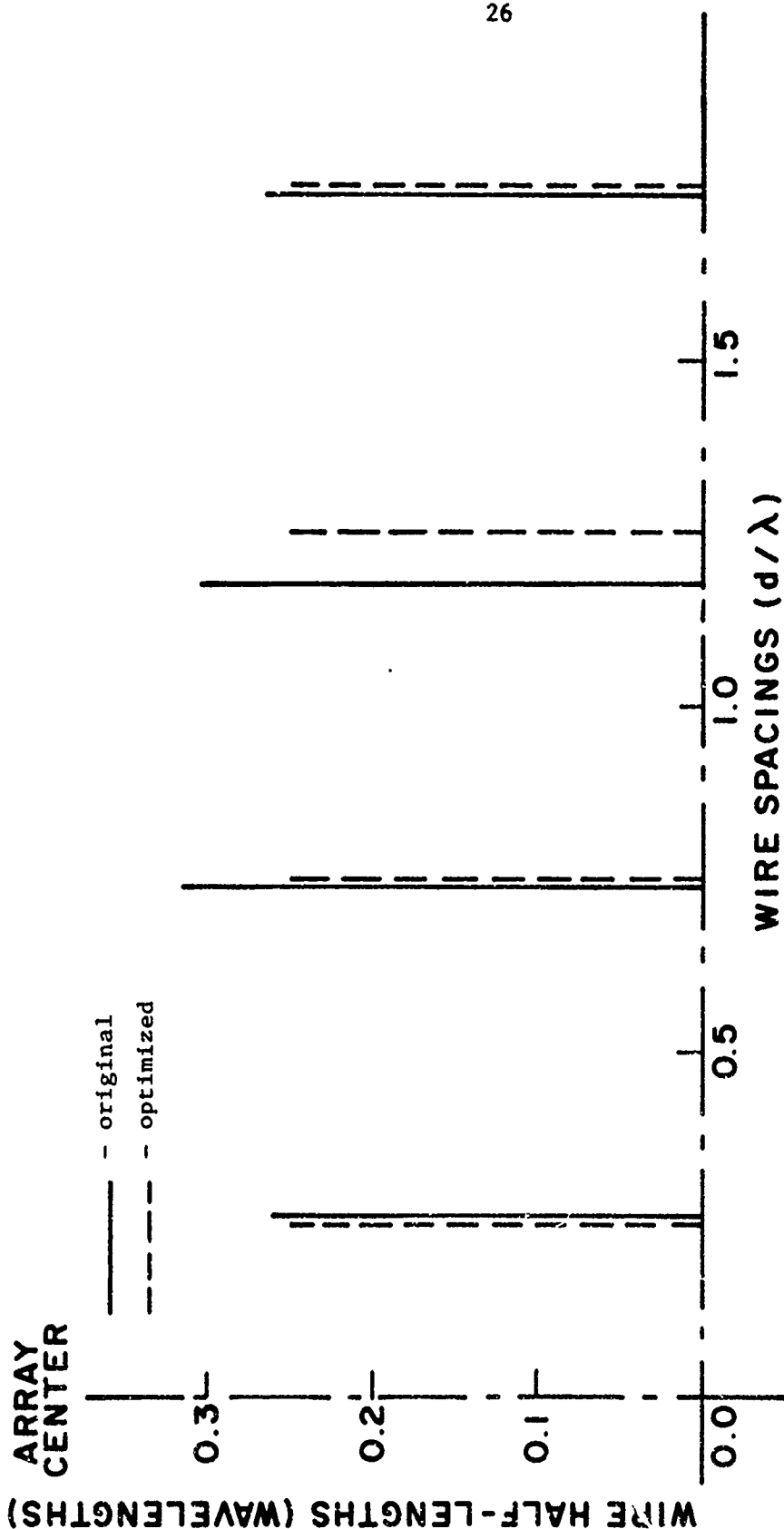


Fig. 13 - Array configuration corresponding to the pattern of Fig. 12.

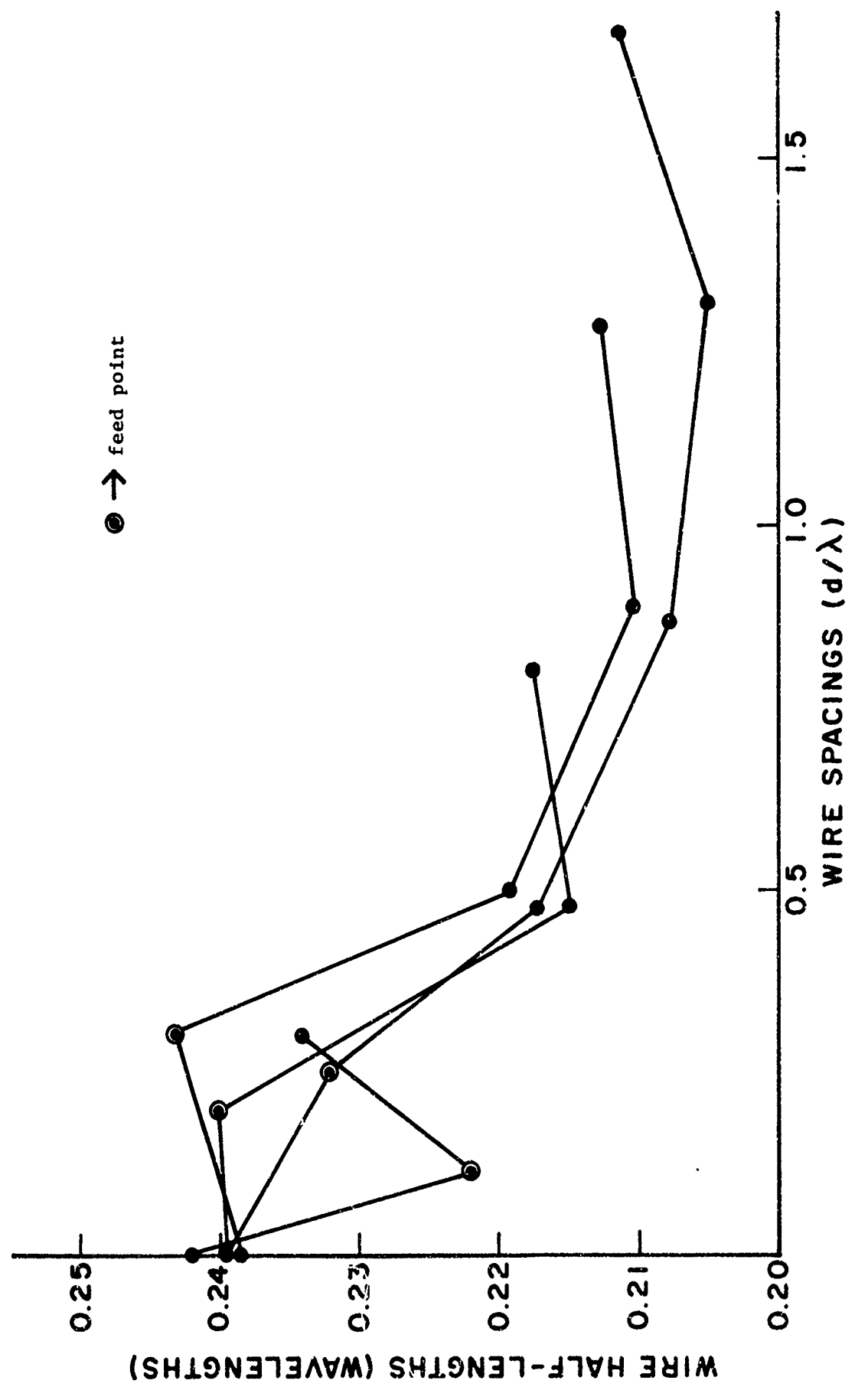


Fig. 14 - Gain maximization of Yagi arrays of 3-6 elements. Wire lengths are 0.01λ .

in each case the initial state (already the result of some gain-maximizing design procedure) was at least slightly improved upon. For the 3-element case the initial state was the result of a classical design procedure [15] and in the other cases the initial status were due to another analysis and optimization program based on matrix methods [16]. The time required to optimize the 6-wire Yagi antenna was 2 minutes on the IBM 370/155 computer. Improvements in gain were

- a) from 5.9 to 9.1 for 3-element case, starting from an antenna designed by a standard technique [15]
- b) from 14.2 to 14.5 for 4-element case, starting from an antenna optimized by matrix methods [16]
- c) from 18.9 to 19.4 for 5-element case, starting from an antenna optimized by matrix methods [16]
- d) from 24.4 to 24.9 for 6-element case, starting from an antenna optimized by matrix methods [16].

The examples presented in this chapter are by no means exhaustive and serve simply to illustrate possible uses of the analysis program described in Chapter 2 and presented in Appendix B. The examples do point out, however, that this particular analysis program can be used in solving quite difficult problems without requiring unreasonable computer time as is often the case.

4. A SPECIAL PROGRAM FOR HANDLING CERTAIN WIRE GROUND SYSTEMS

4.1 Introduction

This chapter describes a special user-oriented computer program that was designed specifically to handle certain problems involving vertical wire antennas over radial ground systems. The program, presented in Appendix C of this report, is capable of treating fairly general radiation problems for wire configurations such as depicted in Fig. 16. The program is a specialization of the general analysis program presented in an earlier Scientific

Report [4] that is capable of handling radiation problems involving arbitrary configurations of thin wires including junctions. By incorporating available symmetries the program presented here offers considerable advantage over the arbitrary wire program with regard to both speed of computation and simplicity.

The typical problem geometry is assumed to consist of a z-directed center wire plus a number NB of branches as in Fig. 15. The branches are

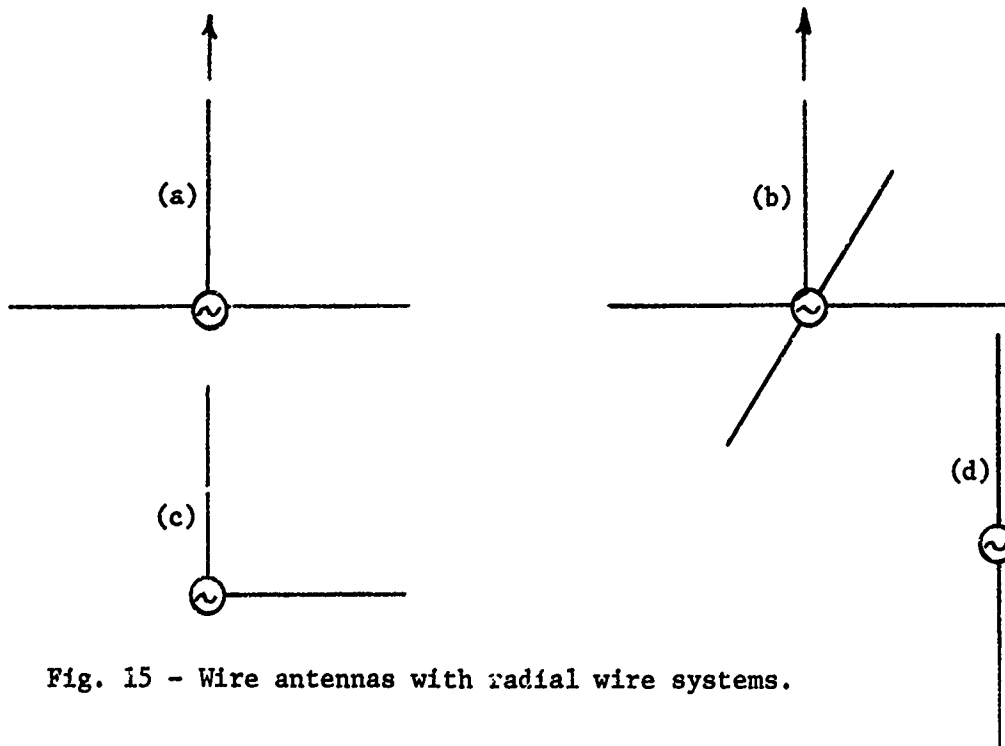


Fig. 15 - Wire antennas with radial wire systems.

assumed to be all the same length and all at the same angle with the center wire. The branches are uniformly distributed about the center wire, so that they are situated at regular intervals of $2\pi/NB$ radians.

As in the previous program all wires are assumed here to be perfect conductors. Each wire length L and radius a are such that $L/a \gg 1$ and $a \ll \lambda$. Within Harrington's general procedure the so-called triangle functions are used for subsectional current expansion functions resulting in a piecewise linear approximation to the current of each wire. Triangle functions are also used for testing or weighting functions resulting in a Galerkin solution to the analysis problem which, as mentioned earlier, is

characterized by relatively fast convergence. The wires are assumed to be unloaded and the possible locations of applied excitations are restricted to points along the z-directed wire (see Fig. 15) which correspond to peaks of triangle current expansion functions. Most of the necessary details of the program were presented earlier with the arbitrary wire programs [4]. The only details included here are those that differ significantly from the previous work.

4.2 Data Input

All input data are provided through the main program. The first data statement reads in quantities NB, NEB, NEC, NR, BK, BLENTH, CLENTH, and ANGLE where

NB = the number of branches in the wire configuration not including the center wire. For example, in Fig. 15a NB = 2 while in Fig. 15b NB = 4.

NEB = the number of triangle expansion functions chosen for each branch. (All branches have the same length. The center wire is not included among the branches.)

NEC = the number of triangle expansion functions chosen for the center wire. (It should be noted with regard to both NEC and NEB that for accurate descriptions of current distributions it is advisable to use at least 12 triangle current expansion functions per wavelength of wire. On the other hand a smaller number may be used if only far-field patterns are of interest.)

BK = the factor 2π divided by the wavelength in meters.

BLENTH = the length in meters of each branch. (As mentioned earlier, all branches have the same length.)

CLENTH = the length in meters of the center wire.

ANGLE = the angle between the branches and the positive z-axis. (All branches are assumed to be at the same angle with the center wire.) This angle is expressed in degrees so that in Fig. 15a-c ANGLE = 90 and for Fig. 15d ANGLE = 180.

The second data statement provides the wire radius in meters. (For the example included with the program of Appendix C the wire radius is given as 10^{-4} meters.) The third reads in a number NF which equals the number of independent feed voltages to be applied to the center wire. As mentioned earlier these feed voltages must be applied at wire positions corresponding to peaks of triangle current expansion functions so that NF must not exceed the number of triangles assigned to the center wire. For radiation problems it is assumed $NF \geq 1$, and DO LOOP 44 executes a total of NF times with an integer J1 and a complex number V read in using a single additional data card with each execution. J1 is the number of the particular triangle whose peak marks the desired position of the first excitation voltage and V is the desired value of that excitation in volts. Thus, following the data statement providing NF, the next NF data cards read in feed positions (corresponding to the numbers of specific triangles) and excitation voltages for each of the feed points along the center wire.

Far-field patterns are calculated and printed out using DO LOOP 17. Here, of course, it is necessary to provide the polar and azimuth angles of observation desired. In the example of the program included in Appendix C the instructions

```
DO 17 IPH = 1,2
PHI = (IPH-1)* 90
DO 17 ITH = 1,181,5
THE = ITH-1
```

cause field patterns to be computed in five degree steps of the polar angle θ for two different choices of azimuth angle $\phi = 0^\circ, 90^\circ$.

4.2 Program Description

Details of the problem geometry are calculated from the input data using subroutine GOMTRY and the generalized impedance matrix [Z] is computed using subroutine ZBRCH. [Z] is a square matrix of dimension equal to the number of independent expansion functions used, and its elements are functions only of the problem geometry. This matrix is inverted using sub-

routine LINEQ and the current matrix $[I]$ is calculated by using this inverted matrix in (1) and DO LOOP 9. Finally, the field patterns specified are computed and printed out by way of DO LOOP 17 as mentioned earlier. In passing it should be pointed out that in the program presented in Appendix C the matrix $[Z]^{-1}$ occupies the same storage locations previously held by $[Z]$. Hence, the term $[Z]$ denotes the generalized impedance matrix before inversion, and the generalized admittance matrix $[Z]^{-1}$ after inversion. It should also be noted that $[Z]$ is manipulated in this program as a column matrix of N^2 complex numbers where N is the number of independent expansion functions used.

4.3 Examples

To illustrate use of this special purpose program consider the wire configurations shown in Fig. 15. For the antenna of Fig. 15a suppose the length of the center wire and the lengths of the branches are all $\lambda/4$.

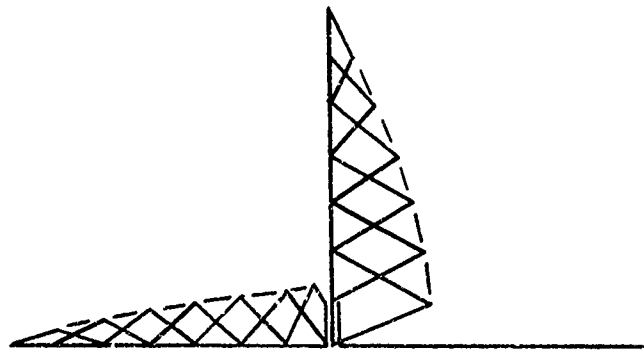


Fig. 16 - Junction treated as overlapping open-ended wires.

Suppose further that the wavelength is given to be one meter. Thus, $BK = 2\pi$, $CLENTH = 0.25$ and $BLENTH = 0.25$. In accordance with previous discussions of the treatment of wire junctions [5] the antennas of Fig. 15 can be treated as systems of overlapping open-ended wires as shown in Fig. 16, corresponding to

the antenna of Fig. 15a. As indicated the antenna can be analyzed conveniently using six triangle current expansion functions for the center wire and seven for each branch, so that $NEB = 7$ and $NEC = 6$. Obviously, the branches are normal to the center wire in this case so that $ANGLE = 90$. The radius of each wire is 0.007λ so for the second data statement $RAD = 0.007$. Finally, the antenna is assumed to be base fed with a real unit excitation voltage. Hence, $NF = 1$ and since the base corresponds to the peak of the seventh triangle the fourth data statement provides $J1 = 7$ (locating the feed point) and an excitation given by $V = 1.0 + j0.0$ volts.

If the wire lengths and radii are the same as in Fig. 15a then the antennas of Figs. 15b-d can be handled with very simple changes in the input data. For example, if the antenna of Fig. 15b has a center wire and four branches, all of length $\lambda/4$, and if all remaining data are the same as before the only change required in the input is $NB = 4$. The vertical wire is always taken to be z-directed and the projection of the first branch on the x-y plane is always assumed to be x-directed, with the projections of the remaining branches distributed at equal-angle intervals. Thus, in Fig. 15b the first branch assumes the positive x-direction, the second the negative y-direction, and so on. Next, for the L-shaped antenna of Fig. 15c, it is obvious that the only change in input data required is $NB = 1$ if all other characteristics are the same as in the first example. Of course, in this case the only branch is +x-directed. Finally, the straight $\lambda/2$ -wire of Fig. 15d can be treated as a $\lambda/4$ z-directed wire with a single branch of length $\lambda/4$ at an angle of 180° with the +z-axis. Here, input data differing from the specifications of Fig. 15a are $NB = 1$ and $ANGLE = 180$.

Current distributions for the antennas of Fig. 15 are plotted in Fig. 17. Input-output data, current distribution, and sample field patterns for an additional example are included with the program in Appendix C.

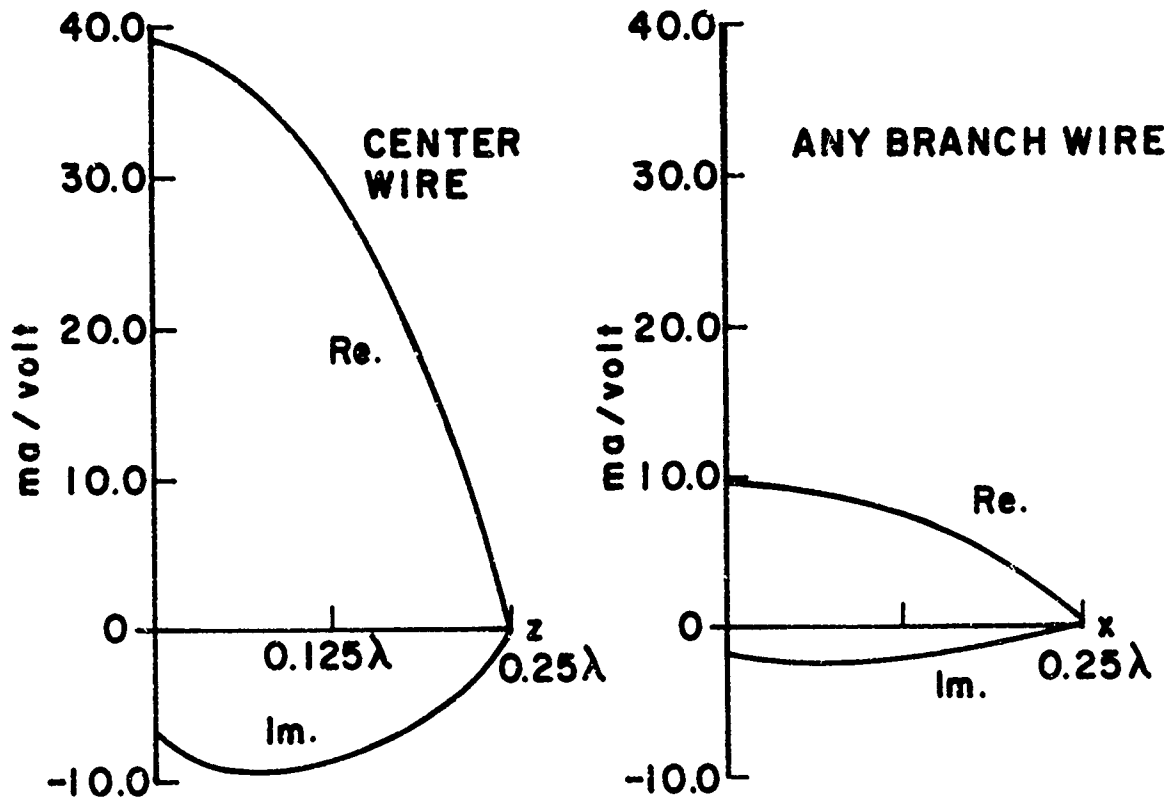


Fig. 17b - Current for Fig. 15b.

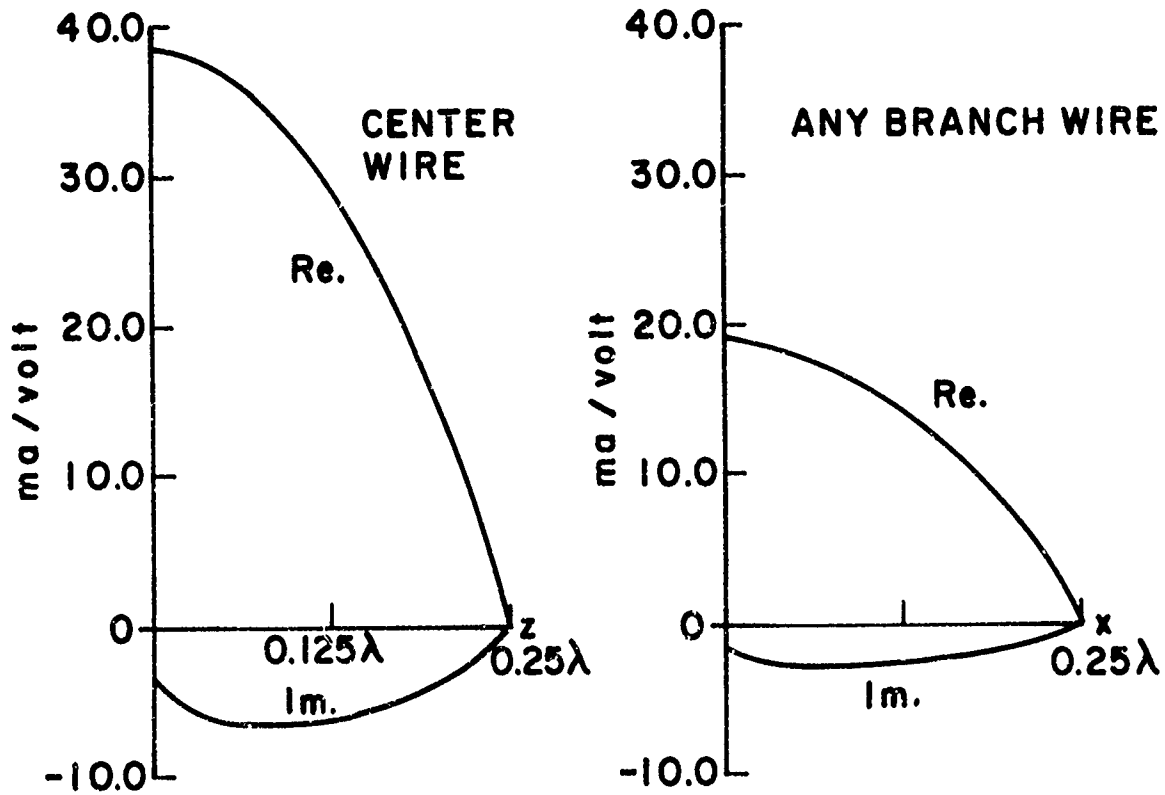


Fig. 17a - Current for Fig. 15a.

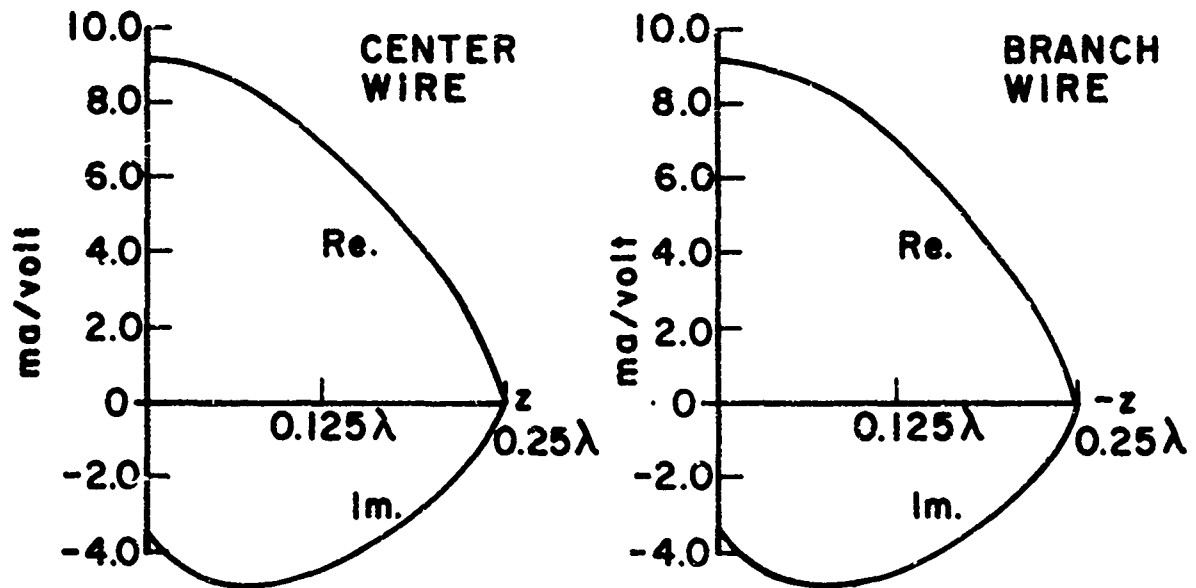


Fig. 17d - Current for Fig. 15d.

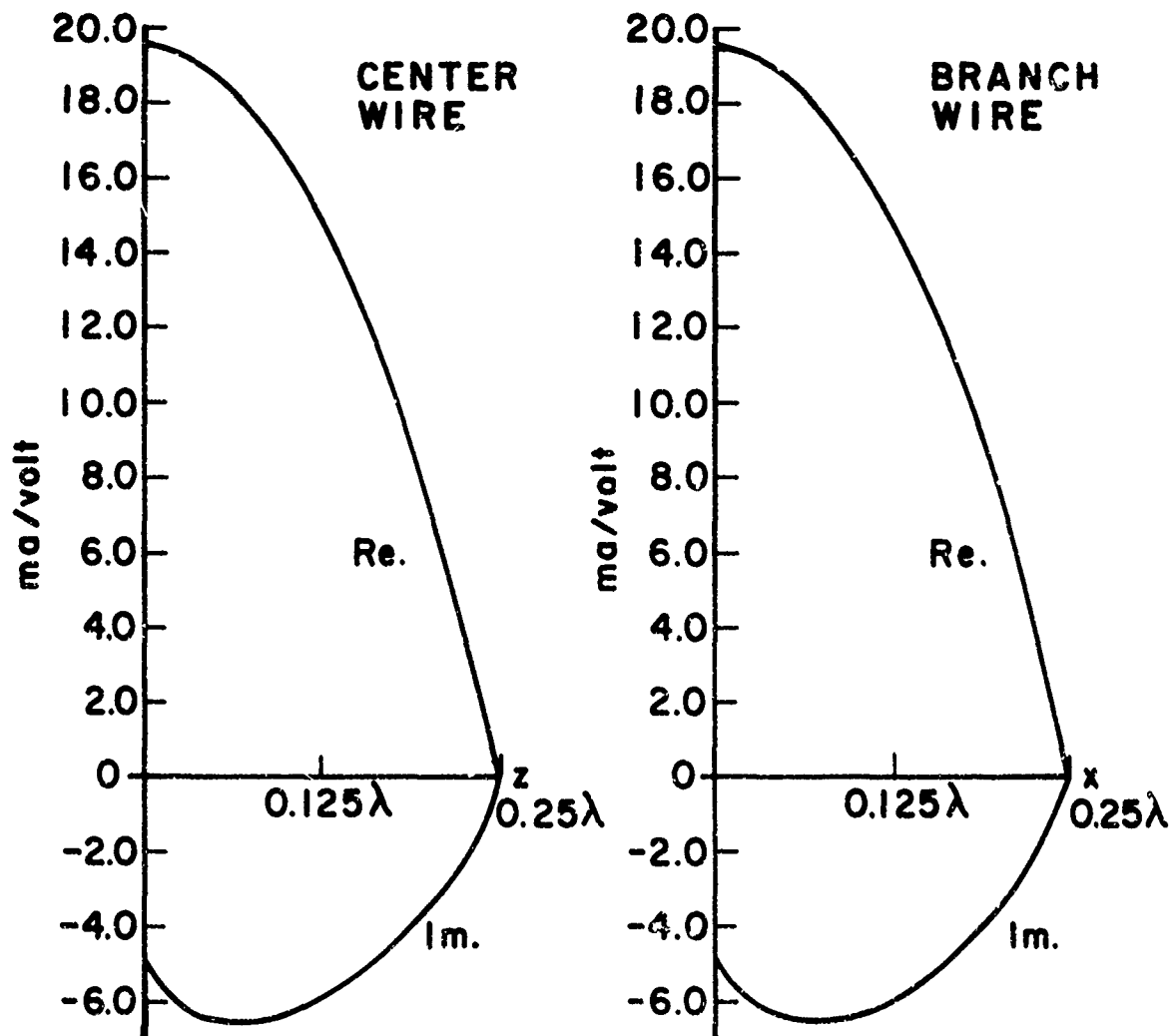


Fig. 17c - Current for Fig. 15c.

5. CONCLUSION

Two user-oriented computer programs have been presented and described for handling radiation problems involving certain special configurations of thin-wire antennas. Both of these stem from the method of moments with the first employing piecewise sinusoidal current expansion functions and the second a piecewise linear current approximation. The first treats radiation by arrays of thin, parallel, centered wires where the feed points all lie in the same plane. The wires are assumed to be lossless and all of the same radius. To illustrate use of the program several array design problems were treated. These include design of unequally spaced arrays to provide equal-sidelobe patterns, selection of wire lengths and spacings to optimize directivity, and reduction of pattern sidelobes through adjustments of wire lengths.

The second program presented here treats efficiently certain special situations involving vertical wire antennas over radial wire systems. Here again the wires are assumed lossless and all of the same radius, and feed points are restricted to the vertical wire. Several examples were included to illustrate use of this sound program as well.

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APPENDIX A

EVALUATION OF THE ELEMENTS OF THE GENERALIZED IMPEDANCE MATRIX [Z]

For an array of straight, z-directed wires the elements of [Z] are given by (6) and can be evaluated as follows:

$$\begin{aligned}
 Z_{nm} &= - \int_n \vec{I}_n \cdot \vec{E}_m \cdot dz \\
 &= - \left[\int_{z_{n-1}}^{z_n} \frac{\sin k(z-z_{n-1})}{\sin(k\Delta z_n)} + \int_{z_n}^{z_{n+1}} \frac{\sin k(z_{n+1}-z)}{\sin(k\Delta z_n)} \right] \times \frac{j30}{\sin(k\Delta z_m)} \\
 &\quad \left[\frac{-jkR_1}{R_1} - 2 \cos(k\Delta z_m) \frac{-jkR_2}{R_2} + \frac{-jkR_3}{R_3} \right] dz \tag{A1}
 \end{aligned}$$

where

$$R_1 = \sqrt{a^2 + (z-z_{m-1})^2}$$

$$R_2 = \sqrt{a^2 + (z-z_m)^2}$$

$$R_3 = \sqrt{a^2 + (z-z_{m+1})^2}$$

$$\Delta z_n = z_n - z_{n-1} = z_{n+1} - z_n$$

$$\Delta z_m = z_m - z_{m-1} = z_{m+1} - z_m$$

The computed elements are: $Z_{nm} = R_{nm} + jX_{nm}$ where,

$$\begin{aligned}
R_{nm} = & \frac{15}{\sin(k\Delta z_m) \sin(k\Delta z_n)} \times \\
& [\cos k(z_{m-1} - z_{n-1}) \{Ci(v_0) + Ci(u_0) - Ci(u_1) - Ci(v_1)\} \\
& + \sin k(z_{m-1} - z_{n-1}) \{Si(v_0) - Si(u_0) + Si(u_1) - Si(v_1)\} \\
& + \cos k(z_{m+1} - z_{n-1}) \{Ci(v_4) + Ci(u_4) - Ci(u_5) - Ci(v_5)\} \\
& + \sin k(z_{m+1} - z_{n-1}) \{Si(v_4) - Si(u_4) + Si(u_5) - Si(v_5)\} \\
& - 2 \cos(k\Delta z_m) \cos k(z_m - z_{n-1}) \{Ci(v_2) + Ci(u_2) - Ci(u_3) - Ci(v_3)\} \\
& - 2 \cos(k\Delta z_m) \cos k(z_m - z_{n-1}) \{Si(v_2) - Si(u_2) + Si(u_3) - Si(v_3)\} \\
& + \cos k(z_{m-1} - z_{n+1}) \{Ci(v_6) - Ci(v_1) + Ci(u_6) - Ci(u_1)\} \\
& + \sin k(z_{m-1} - z_{n+1}) \{Si(v_6) - Si(u_6) + Si(u_1) - Si(v_1)\} \\
& + \cos k(z_{m+1} - z_{n+1}) \{Ci(v_8) - Ci(v_5) - Ci(u_5) + Ci(u_8)\} \\
& + \sin k(z_{m+1} - z_{n+1}) \{Si(v_8) - Si(u_8) + Si(u_5) - Si(v_5)\} \\
& - 2 \cos(k\Delta z_m) \cos k(z_m - z_{n+1}) \{Ci(v_7) - Ci(v_3) - Ci(u_3) + Ci(u_7)\} \\
& - 2 \cos(k\Delta z_m) \sin k(z_m - z_{n+1}) \{- Si(u_7) + Si(v_7) \\
& \quad + Si(u_3) - Si(v_3)\}] \tag{A2}
\end{aligned}$$

```

A(3*(I-1)+J)=ZN+7M
5 7M=7M+DZM
4 ZN=ZN+DZN
6 DO 3 I=1,9
  IF (A(I) .LT. 0.0) GO TO 21
  U(I)=SQRT(RR+A(I)*A(I))+A(I)
  V(I)=RR/U(I)
  GO TO 22
21 V(I)=SQRT(RR+A(I)*A(I))-A(I)
  U(I)=RR/V(I)
22 CALL SICI(SI,CI,U(I))
  SU(I)=SI
  CU(I)=CI
  CALL SICI(SI,CI,V(I))
  SV(I)=SI
  CV(I)=CI
  SP(I)=SU(I)+SV(I)
  SN(I)=SU(I)-SV(I)
  CP(I)=CU(I)+CV(I)
  CN(I)=CU(I)-CV(I)
2 CONTINUE
  C(1)=COS(A(1))
  C(2)=COS(A(2))
  C(3)=COS(A(3))

  C(7)=COS(A(7))
  C(8)=COS(A(8))
  C(9)=COS(A(9))
  S(1)=SIN(A(1))
  S(2)=SIN(A(2))
  S(3)=SIN(A(3))
  S(7)=SIN(A(7))
  S(8)=SIN(A(8))
  S(9)=SIN(A(9))
  PL=C(1)*(CP(1)-CP(4))-S(1)*(SN(4)-SN(1))+C(3)*(CP(3)-CP(6))-S(3)*
  *(SN(6)-SN(3))+C(7)*(CP(7)-CP(4))-S(7)*(SN(4)-SN(7))+C(9)*(CP(9)
  *-CP(6))-S(9)*(SN(6)-SN(9))-CC*(C(2)*(CP(2)-CP(5))-S(2)*(SN(5)
  *-SN(2))+C(8)*(CP(8)-CP(5))-S(8)*(SN(5)-SN(8)))
  AG=C(1)*(SP(4)-SP(1))-S(1)*(CN(4)-CN(1))+C(3)*(SP(6)-SP(3))-S(3)*
  *(CN(6)-CN(3))+C(7)*(SP(4)-SP(7))-S(7)*(CN(4)-CN(7))+C(9)*(SP(6)-
  *-SP(9))-S(9)*(CN(6)-CN(9))-CC*(C(2)*(SP(5)-SP(2))-S(2)*(CN(5)-CN(
  *2))+C(8)*(SP(5)-SP(8))-S(8)*(CN(5)-CN(8)))
  7MN=CMPLX(RL,AG)*15./(SIN(DZN)*SIN(DZM))+7MN
  IF(N.EQ.2 .OR. N.EQ.3) RETURN
  GO TO 10
END

```

APPENDIX B

PROGRAM FOR LINEAR ARRAYS OF PARALLEL WIRES

This program is suitable for analyzing radiation by linear arrays of parallel thin-wire antennas. The wires can be of unequal lengths although all wire radii are assumed to be the same. Symmetry is assumed only about the plane containing the midpoints of the wires. The wires are assumed to be lossless with no externally applied loading. The sample output corresponds to analysis of an 8-element array of half-wave wires that are half-wavelength spaced and all of radius 0.01λ . Five expansion functions are used for approximating the current of each wire. (Because of symmetry only three of these are independent.) Each wire is fed with a real unit excitation and the principal H-plane pattern is computed in steps of three degrees.

The program is described in Chapter 2. The main program is presented first followed by the subroutines.

```

C
C
C   ***   MAIN PROGRAM   ***
C
C   DIMENSION   Y(20),VV(10)
C   COMMON      AK,N,NL,N7,NV,VV,MQ
C   COMPLEX     CMPLX,CEXP,VV
C   TP=6.2831853
C
C   AK = RADIUS * (2*PI)
C
C   AK=0.01*TP
C
C   N = TOTAL NO. OF ELEMENTS IN THE LINEAR ARRAY
C
C   N=8
C
C   NV = TOTAL NO. OF VARIABLES ( WIRE LENGTHS & INDEPENDENT ARRAY SPACINGS )
C
C   NV=15
C
C   FIRST FEW ELEMENTS OF Y(I) DENOTE THE LENGTHS OF THE ELEMENTS AND
C   THE REST DENOTE THE DISTANCES OF THE ELEMENTS FROM THE FIRST, WHICH
C   IS CONSIDERED AS THE ORIGIN OF THE ARRAY.

```


C
C *** NOTE *** ALL DIMENSIONS OF Y(I) ARE METERS/WAVELENGTH * (2*PI)
C
C

Y(1)=0.25*TP
Y(2)=0.25*TP
Y(3)=0.25*TP
Y(4)=0.25*TP
Y(5)=0.25*TP
Y(6)=0.25*TP
Y(7)=0.25*TP
Y(8)=0.25*TP
Y(9)=0.50*TP
Y(10)=1.0*TP
Y(11)=1.5*TP
Y(12)=2.0*TP
Y(13)=2.5*TP
Y(14)=3.0*TP
Y(15)=3.5*TP

C
C VV(I) SPECIFIES THE FEED VOLTAGES
C

DO 3 I=1,N
VV(I)=CMPLX(1.0,0.0)
CONTINUE

C
C PRINCIPAL H - PLANE PATTERN IS COMPUTED IN STEPS OF MQ DEGREES

MQ=3

C
C ***** END OF DATA *****
C

NZ=N+1
NL=N-1
CALL ASCTFD(Y)
STOP
END

SUBROUTINE ASCTFD(Y)
COMPLEX Z,V,SS,DETERM,CEXP,CMPLX,TERM,AZ,ZMN,YJ,7M1,VV
COMMON AK,N,NL,NZ,NV,VV,MO
DIMENSION M(15),YZ(10),NZ(15),7(50,50),V(50,1),SS(50,1),CS(15)
DIMENSION TE(200),PH(200),Y(20),VV(10)
BIG=0.0
M(1)=0
DO 2 I=1,N

C
C ***** NOTE *****
C
C FOR OPTIMIZATION PROBLEMS MM=INT(Y(I)-0.571)+1
C
C FOR ANALYSIS PROBLEMS MM=INT(Y(I)-0.571)+3
C

```

C
C
MM=INT(V(I)-0.571)+3
M(I+1)=M(I)+MM
2  DZ(I)=V(I)/FLDZ(MM)
   MN=M/N+1
   DO 64 I=1,MM
69  V(I,1)=COMPLX(0.0,0.0)
   DO 4 J=1,N
   M22=M(I)+1
   V(M22,1)=VV(J)
4  CONTINUE
   YZ(1)=0.0
   DO 3 J=1,N1
3  Y7(J+1)=Y(N+J)
   DO 86 L=1,N
   ZN=0.0
   ML1=M(L)+1
   ML2=M(L+1)
   DO 81 I=ML1,ML2
   ZM=0.0
   DO 82 J=ML1,ML2
   Z(I,J)=ZM1(ZM,ZN,DZ(L),AK)
R2  ZM=ZM+I*Z(L)
R1  ZN=ZN+DZ(L)
R6  CONTINUE
   DO 74 K=1,N1
   K1=K+1
   ZN=-DZ(K)
   MK1=M(K)+1
   MK2=M(K+1)
   DO 71 I=MK1,MK2
   DO 73 L=K1,N
   ZM=-DZ(I)
   ML1=M(L)+1
   ML2=M(L+1)
   DO 72 J=ML1,ML2
   SD=Y7(L)-YZ(K)
   Z(I,J)=ZMN(ZM,ZN,DZ(L),DZ(K),SD)
72  ZM=ZM+DZ(L)
73  CONTINUE
71  ZN=ZN+DZ(K)
74  CONTINUE
   DO 64 K=2,N
   K1=K-1
   ZN=-DZ(K)
   MN1=M(K)+1
   MN2=M(K+1)
   DO 61 I=MN1,MN2
   DO 63 I=1,K1
   ZM=-DZ(I)
   ML1=M(L)+1
   ML2=M(L+1)
   DO 62 J=ML1,ML2
   SD=YZ(K)-Y7(L)

```

```

Z(I,J)=ZMN(7M,7N,DZ(L),DZ(K),S)
62 ZM=7M+DZ(L)
63 CONTINUE
61 7N=ZN+DZ(K)
64 CONTINUE
CALL CSMIN(Z,MN,DETERM)
CALL MULTPY(MN,MN,1,Z,V,SS)
LL=0
DO 11 LK=1,181,MQ
PC=0.0174583*FLOAT(LK-1)
PC=COS(PC)
A7=CMPLX(0.0,0.0)
DO 44 J=1,N
YJ=CMPLX(0.0,PC*YZ(J))
TERM=CEXP(YJ)
CS(J)=TAN(0.5*DZ(J))
MC=M(J)+1
MR=M(J+1)
MA=MC+1
A7=A7+SS(MC,1)*TERM*CS(J)
IF(MA.GT.MR) GO TO 44
DO 45 I=MA,MR
45 AZ=AZ+SS(I,1)*2.0*TERM*CS(J)
44 CONTINUE
LL=LL+1
TE(LL)=CABS(AZ)
IF(TE(LL).GT.RIG) RIG=TE(LL)
PH(LL)=ATAN2(AIMAG(AZ),REAL(AZ))/0.0174533
11 CONTINUE
DO 43 I=1,NV
Y(I)=Y(I)/6.2931853
43 CONTINUE
WRITE(3,111) (Y(I), I=1,N)
WRITE(3,112) (Y(I), I=N7,NV)
112 FORMAT('0', ' SPACING= ',10F10.5///)
111 FORMAT('0', ' LENGTH= ',10F10.5///)
WRITE(3,892)
892 FORMAT('0', ' CURRENT DISTRIBUTION ON THE ELEMENTS ')
DO 49 K=1,N
MK1=M(K)+1
MK2=M(K+1)
WRITE(3,895)
WRITE(3,898)K
898 FORMAT('0', ' CURRENT ON ',15,5X,'ELEMENT'//)
DO 50 NZ=MK1,MK2
WRITE(3,999) SS(NZ,1)
999 FORMAT(' ',2F20.6)
50 CONTINUE
49 CONTINUE
WRITE(3,895)
895 FORMAT('0'///)
WRITE(3,893)
893 FORMAT('0', ' INPUT IMPEDANCES AT THE FEED POINTS'///)
DO 51 J=1,N
M22=M(J)+1

```

```

      IF(CABS(V(M22,1)).EQ.0.0) GO TO 51
      A7=V(M22,1)/SS(M22,1)
      WRITE(3,894) J,AZ
894  FORMAT('0', 'INPUT IMPEDANCE OF', I5.5X, 'ELEMENT =', 2F20.6//)
51  CONTINUE
      WRITE(3,891)
891  FORMAT('1', 'NORMALIZED E- FIELD PATTERN '//)
      LM=0
      DO 95 L=1,LI
      TF(L)=TF(L)/RIG
      WRITE(3,842) LM,TF(L),PH(L)
842  FORMAT(' ', I5.2F40.6)
      LM=LM+MC
95  CONTINUE
      WRITE(3,890) RIG
890  FORMAT('0', 'NORMALIZATION CONSTANT =', E15.5//)
      RETURN
      END

```

FUNCTION ZMN(ZM1,7N1,DZM,DZN,R)

C
C
C
C
C
C
C
C

COMPUTES MUTUAL IMPEDANCE BETWEEN TWO ELEMENTS OF LENGTHS DZM AND DZN
EACH OF RADIUS R HAVING STARTING POINTS AT ZM1 AND ZN1
*** NOTE *** ALL DIMENSIONS ARE (*2*PI)

```

      COMPLEX ZMN,CMPLX
      DIMENSION A(9),U(9),V(9),SU(9),SV(9),CU(9),CV(9),C(9),S(9),SP(9),
      SN(9),CN(9),CP(9)
      N=1
      IF(ZM1.EQ.-DZM) N=2
      CC=2.0*CCOS(DZM)
      KK=R*P
      ZN=7N1
      DO 1 I=1,3
      ZM=ZM1
      DO 2 J=1,3
      A(3*(I-1)+J)=ZN-ZM
2  ZM=ZM+DZM
1  ZN=ZN+DZN
      ZMN=CMPLX(0.0,0.0)
      GO TO 6
10  ZN=7N1
      N=3
      DO 4 I=1,3
      ZM=ZM1
      DO 5 J=1,3

```

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```

A(3*(I-1)+J)=ZN+ZM
5  ZM=ZM+DZM
4  ZN=ZN+DZN
6  DO 3 I=1,9
   IF (A(I) .LT. 0.0) GO TO 21
   U(I)=SQRT(RR+A(I)*A(I))+A(I)
   V(I)=RR/U(I)
   GO TO 22
21  V(I)=SQRT(RR+A(I)*A(I))-A(I)
   U(I)=RR/V(I)
22  CALL SIC(SI,CI,U(I))
   SU(I)=SI
   CU(I)=CI
   CALL SCI(SI,CI,V(I))
   SV(I)=SI
   CV(I)=CI
   SP(I)=SU(I)+SV(I)
   SN(I)=SU(I)-SV(I)
   CP(I)=CU(I)+CV(I)
   CN(I)=CU(I)-CV(I)
2  CONTINUE
   C(1)=COS(A(1))
   C(2)=COS(A(2))
   C(3)=COS(A(3))

   C(7)=COS(A(7))
   C(8)=COS(A(8))
   C(9)=COS(A(9))
   S(1)=SIN(A(1))
   S(2)=SIN(A(2))
   S(3)=SIN(A(3))
   S(7)=SIN(A(7))
   S(8)=SIN(A(8))
   S(9)=SIN(A(9))
   RL=C(1)*(CP(1)-CP(4))-S(1)*(SN(4)-SN(1))+C(3)*(CP(3)-CP(6))-S(3)*
$(SN(6)-SN(3))+C(7)*(CP(7)-CP(4))-S(7)*(SN(4)-SN(7))+C(9)*(CP(9)
$-CP(6))-S(9)*(SN(6)-SN(9))-CC*(C(2)*(CP(2)-CP(5))-S(2)*(SN(5)
$ -SN(2))+C(8)*(CP(8)-CP(5))-S(8)*(SN(5)-SN(8)))
   AG=C(1)*(SP(4)-SP(1))-S(1)*(CN(4)-CN(1))+C(3)*(SP(6)-SP(3))-S(3)*
$(CN(6)-CN(3))+C(7)*(SP(4)-SP(7))-S(7)*(CN(4)-CN(7))+C(9)*(SP(6)-
$SP(9))-S(9)*(CN(6)-CN(9))-CC*(C(2)*(SP(5)-SP(2))-S(2)*(CN(5)-CN(
$2))+C(8)*(SP(5)-SP(8))-S(8)*(CN(5)-CN(8)))
   ZMN=CMPLX(RL,AG)*15./(SIN(DZN)*SIN(DZM))+ZMN
   IF(N.EQ.2 .OR. N.EQ.3) RETURN
   GO TO 10
END

```

FUNCTION ZM1(ZM,ZN,DK,AK)

COMPUTES MUTUAL IMPEDANCE BETWEEN TWO ELEMENTS OF RADIUS AK AND
LENGTH DK HAVING STARTING POINTS AT ZM AND ZN.

*** NOTE *** ALL DIMENSIONS ARE (*2*PI)

```

COMPLEX CMPLX,ZM1
CDK=COS(DK)
SDK=15.7/(1.-CDK*CDK)
DSQ=AK*AK
D4=4.*CDK
DD4=2.+D4*CDK
N=1
IF(ZM.EQ.C.C) N=0
A=ZN-ZM
R=A-DK
C=A+DK
D=R-DK
F=C+DK
CA=COS(A)
SA=SIN(A)
CR=COS(R)
SR=SIN(R)
CC=COS(C)
SC=SIN(C)
CD=COS(D)
SD=SIN(D)
CF=COS(F)
SF=SIN(F)
U0=SQRT(DSQ+A*A)+A
V0=DSQ/U0
U1=SQRT(DSQ+C*C)+C
V1=DSQ/U1
U2=SQRT(DSQ+R*R)+R
V2=DSQ/U2
U4=SQRT(DSQ+D*D)+D
V4=DSQ/U4
U6=SQRT(DSQ+F*F)+F
V6=DSQ/U6
CALL SICI(SI,CI,U0)
SI0=SI
CI0=CI
CALL SICI(SI,CI,V0)
SV0=SI
CV0=CI
CALL SICI(SI,CI,U1)
SI1=SI
CI1=CI
CALL SICI(SI,CI,V1)
SV1=SI
CV1=CI

```

```

CALL      SICI(SI,CI,U2)
SU2=SI
CU2=CI
CALL      SICI(SI,CI,V2)
SV2=SI
CV2=CI
CALL      SICI(SI,CI,U4)
SU4=SI
CU4=CI
CALL      SICI(SI,CI,V4)
SV4=SI
CV4=CI
CALL      SICI(SI,CI,U6)
SU6=SI
CU6=CI
CALL      SICI(SI,CI,V6)
SV6=SI
CV6=CI
RL=DD4*(CA*(CU2+CV2)+(SU2-SV2)*SA)-D4*(CC*(CU1+CV1)+CR*(CU2+CV2)+
+SC*(SU1-SV1)+SR*(SU2-SV2))+CD*(CU4+CV4)+SD*(SU4-SV4)+CE*(CU6+CV6)
+SF*(SU6-SV6)
AG=SE*(CU4-CV6)-CF*(SU6+SV6)+SD*(CU4-CV4)-CD*(SU4+SV4)-D4*(SC*
+(CU1-CV1)+SB*(CU2-CV2)-CC*(SU1+SV1)-CR*(SU2+SV2))+DD4*(SA*(CU2-
+CV2)-CA*(SU2+SV2))
ZM1=CMPLX(RL,AG)*SDK
IF(N.EQ.0) RETURN
A=7N+ZM
R=A+DK
C=A+DK
D=R+DK
E=C+DK
CA=COS(A)
SA=SIN(A)
CR=COS(R)
SR=SIN(R)
CC=COS(C)
SC=SIN(C)
CD=COS(D)
SD=SIN(D)
CE=COS(E)
SE=SIN(E)
V0=SQRT(DSQ+D*D)+D
U0=DSQ/V0
V1=SQRT(DSQ+B*B)+B
U1=DSQ/V1
V2=SQRT(DSQ+A*A)+A
U2=DSQ/V2
V4=SQRT(DSQ+C*C)+C
U4=DSQ/V4
V6=SQRT(DSQ+E*E)+E
U6=DSQ/V6
CALL      SICI(SI,CI,U0)
SU0=SI
CU0=CI

```

```

CALL SICI(SI,CI,VO)
SVO=SI
CVO=C I
CALL SICI(SI,CI,U1)
SU1=SI
CU1=CI
CALL SICI(SI,CI,V1)
SV1=SI
CV1=C I
CALL SICI(SI,CI,U2)
SU2=SI
CU2=C I
CALL SICI(SI,CI,V2)
SV2=SI
CV2=C I
CALL SICI(SI,CI,U4)
SU4=SI
CU4=C I
CALL SICI(SI,CI,V4)
SV4=SI
CV4=C I
CALL SICI(SI,CI,U6)
SU6=SI
CU6=C I
CALL SICI(SI,CI,V6)
SV6=SI
CV6=C I
RL=DD4*(CA*(CU7+CV7)-SA*(SU2-SV2))-D4*(CC*(CU4+CV4)+CH*(CU1+CV1)-
+SC*(SU4-SV4)-SR*(SU1-SV1))+CD*(CU0+CVO)-SD*(SU0-SV0)+CE*(CU6+CV6)
+SF*(SU6-SV6)
AG=-SF*(CU6-CV6)-CE*(SU6+SV6)-SD*(CU0-CVO)-CD*(SU0+SV0)+D4*(SC*(
+CU4-CV4)+SR*(CU1-CV1)+CR*(SU1+SV1)+CC*(SU4+SV4))+DD4*(-CA*(SU2+
+SV2)-SA*(CU2-CV2))
7M1=CMPLX(RL,AG)*SDK+7M1
RETURN
END

```

SUBROUTINE SICI(SI,CI,X)

COMPUTES SINE AND COSINE INTEGRALS

```

C
C
C
7=ARS(X)
IF(7-4.)1,1,4
1 Y=(4.-7)*(4.+7)
3 SI=X*(((1.753141E-9*Y+1.564988E-7)*Y+1.374168E-5)*Y+6.939889E-4)
1*Y+1.964882E-2)*Y+4.395509E-1)
CI=((5.772156E-1+ALOG(Z))/Z-Z*(((1.386985E-10*Y+1.584996E-8)*Y
+1.725752E-6)*Y+1.185999E-4)*Y+4.990920E-3)*Y+1.315308E-1)*7
4 RETURN
4 SI=SIN(Z)
Y=COS(Z)
Z=4./7
U=(((1.048069E-3*Z-2.279143E-2)*Z+5.515070E-2)*Z-7.261642E-2)

```



```

1*Z+4.987716E-2)*Z-3.332519E-3)*Z-2.314617E-2)*Z-1.134958E-5)*Z
2+4.250011E-2)*Z+7.583989E-10
V=((((( ((( (-5.108699E-3*Z+2.819179E-2)*Z-6.537283E-2)*Z
1+7.902034E-2)*Z-4.400416E-2)*Z-7.945556E-3)*Z+2.601293E-2)*Z
2-3.764000E-4)*Z-3.122418E-2)*Z-6.646441E-7)*Z+2.500000E-1
C1=7*(S1*V-Y*U)
S1=-Z*(S1*U+Y*V)      +1.570796
RETURN
END

```

```
SUBROUTINE CSMIN(A,N,DETERM)
```

```
C
C
C
```

```
INVERTS A GIVEN SQUARE MATRIX
```

```

COMPLEX A,PIVOT,AMAX,T,SWAP,DETERM,U,CMLX,CONJ
DIMENSION IPIVOT(50),INDEX(50,2),A(50,50),PIVOT(50)
DETERM=CMPLX(1.0,0.0)
DO 20 J=1,N
20 IPIVOT(J)=0
DO 600 I=1,N
AMAX=CMPLX(0.0,0.0)
DO 105 J=1,N
60 IF(IPIVOT(J)-1) 60,105,60
DO 100 K=1,N
80 IF(IPIVOT(K)-1) 80,100,740
TEMP=AMAX*CONJ(AMAX)-A(J,K)*CONJ(A(J,K))
85 IF(TEMP)85,85,100
IROW=J
ICOLUM=K
AMAX=A(J,K)
100 CONTINUE
105 CONTINUE
IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
IF(IROW-ICOLUM) 140,260,140
140 DETERM=-DETERM
DO 200 L=1,N
SWAP=A(IROW,L)
A(IROW,L)=A(ICOLUM,L)
200 A(ICOLUM,L)=SWAP
260 INDEX(I,1)=IROW
INDEX(I,2)=ICOLUM
PIVOT(I)=A(ICOLUM,ICOLUM)
DETERM=DETERM*PIVOT(I)
TEMP=PIVOT(I)*CONJ(PIVOT(I))
IF(TEMP) 330,720,330
330 A(ICOLUM,ICOLUM)=CMPLX(1.0,0.0)
DO 350 L=1,N
U=PIVOT(I)
350 A(ICOLUM,L)=A(ICOLUM,L)/U
380 DO 550 LI=1,N
IF(LI-ICOLUM)400,550,400

```

```

400 T=A(L1,ICOLUM)
    A(L1,ICOLUM)=CMPLX(0.0,0.0)
    DO 450 I=1,N
    U=A(ICOLUM,L)
450 A(L1,I)=A(L1,I)-U*T
550 CONTINUE
600 CONTINUE
    DO 710 I=1,N
    L=N+1-I
    IF (INDEX(L,1)=INDEX(L,2)) 630,710,630
630 JROW=INDEX(L,1)
    JCOLUM=INDEX(L,2)
    DO 705 K=1,N

    SWAP=A(K,JROW)
    A(K,JROW)=A(K,JCOLUM)
    A(K,JCOLUM)=SWAP
705 CONTINUE
710 CONTINUE
    RETURN
720 WRITE(3,730)
730 FORMAT('C',5X,' MATRIX IS SINGULAR '//)
740 RETURN
    END

```

```

FUNCTION CONJ(Z7)
COMPLEX CMPLX,Z7,CONJ
CONJ=CMPLX(REAL(Z7),-AIMAG(Z7))
RETURN
END

```

```

SUBROUTINE MULTPY(L,M,N,A,B,C)
C
C MULTIPLIES TWO MATRICES
C
COMPLEX A(50,50),B(50,1),C(50,1)
DO 1 I=1,L
DO 1 K=1,N
C(I,K)=0.0
DO 1 J=1,M
1 C(I,K)=C(I,K)+A(I,J)*B(J,K)
RETURN
END

```

LENGTH= 0.25000 0.25000 0.25000 0.25000 0.25000 0.25000 0.25000 0.25000
 SPACING= 0.50000 1.00000 1.50000 2.00000 2.50000 3.00000 3.50000 3.50000

CURRENT DISTRIBUTION ON THE ELEMENTS

CURRENT ON	1	ELEMENT	CURRENT ON	4	ELEMENT
0.135590E-01		-0.727654E-03	0.168413E-01		0.107999E-02
0.120027E-01		-0.296037E-02	0.149134E-01		-0.135921E-02
0.792061E-02		-0.246207E-02	0.985340E-02		-0.140274E-02

CURRENT ON	2	ELEMENT	CURRENT ON	5	ELEMENT
0.174993E-01		0.107519E-02	0.168413E-01		0.109001E-02
0.154946E-01		-0.136203E-02	0.149134E-01		-0.135910E-02
0.102342E-01		-0.140119E-02	0.985340E-02		-0.140265E-02

CURRENT ON	3	ELEMENT	CURRENT ON	6	ELEMENT
0.165198E-01		0.110923E-02	0.165198E-01		0.110928E-02
0.146296E-01		-0.133398E-02	0.146296E-01		-0.133391E-02
0.966777E-02		-0.138788E-02	0.966777E-02		-0.138792E-02

CURRENT IN	7	ELEMENT	CURRENT IN	8	ELEMENT
0.174992F-C1		0.107520F-C2	0.135590F-C1		-0.727606E-C3
0.154944F-01		-0.136201F-02	0.120028F-01		-0.296031E-02
0.102342F-01		-0.140117E-02	0.792063F-02		-0.246204E-C2

INPUT IMPEDANCES AT THE FEED POINTS

INPUT IMPEDANCE OF	1	ELEMENT =	0.735400E 02	0.394658E 01
INPUT IMPEDANCE OF	2	ELEMENT =	0.569303F 02	-0.349788E 01
INPUT IMPEDANCE OF	3	ELEMENT =	0.602618E 02	-0.404629E 01
INPUT IMPEDANCE OF	4	ELEMENT =	0.591347F 02	-0.379179E 01
INPUT IMPEDANCE OF	5	ELEMENT =	0.591345F 02	-0.379221E 01
INPUT IMPEDANCE OF	6	ELEMENT =	0.602617E 02	-0.404646F 01
INPUT IMPEDANCE OF	7	ELEMENT =	0.569303F 02	-0.349793E 01
INPUT IMPEDANCE OF	8	ELEMENT =	0.735398F 02	0.394630E 01

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NORMALIZED F- FIELD PATTERN

0	0.000001	53.712891
3	0.001453	66.059052
6	0.005793	63.462265
9	0.012975	59.139740
12	0.022916	53.071182
15	0.034996	45.249466
18	0.048338	35.656738
21	0.061772	24.258026
24	0.073137	10.986127
27	0.079783	-4.302690
30	0.078781	-21.962402
33	0.067565	-43.012985
36	0.045304	-71.680725
39	0.020482	-141.776978
42	0.042617	118.120651
45	0.083445	80.080215
48	0.115672	50.298401
51	0.126299	21.032181
54	0.105935	-10.648551
57	0.053941	-54.991577
60	0.044821	161.263107
63	0.129797	106.062408
66	0.198734	70.245300
69	0.214862	36.055420
72	0.151932	-0.682838
75	0.030137	-121.462845
78	0.236566	131.869217
81	0.503893	95.018326
84	0.755990	60.819748
87	0.935622	27.297760
90	1.000000	-5.863531
93	0.933421	-38.657883
96	0.752121	-70.907059
99	0.499261	-102.105713
102	0.232193	-130.024353
105	0.029792	-81.258011
108	0.153752	-29.985626
111	0.215196	-55.546463
114	0.197901	-82.309097
117	0.128347	-105.952866
120	0.043571	-107.648117
123	0.055012	-20.748306
126	0.106574	-31.308990
129	0.126362	-52.028732
132	0.115269	-72.925568
135	0.082777	-90.934418
138	0.041938	-97.939575
141	0.020670	-39.324295
144	0.045762	-10.939811
147	0.067850	-19.835052

150	0.078877	-33.298767
153	0.079726	-47.126251
156	0.072973	-60.234039
159	0.061552	-72.199875
162	0.048101	-82.808685
165	0.034663	-91.933441
168	0.022626	-99.493362

171	0.012829	-105.420761
174	0.005694	-109.682693
177	0.001400	-112.246414
180	0.000001	-96.129089

NORMALIZATION CONSTANT = 0.13669E 00

APPENDIX C

PROGRAM FOR WIRE OVER RADIAL WIRE SYSTEM

This program is suitable for analyzing radiation by wires over radial wire systems as exemplified by Fig. 15. The wires are assumed lossless with no externally applied loading and excitations are restricted to center-wire positions corresponding to peaks of triangle current expansion functions. The first triangle begins at the junction and extends towards the end of the center wire. The junction itself corresponds to the peak of the first triangle of the branch wire. The sample output is for a $\lambda/4$ center wire with two branches, each of length $\lambda/8$. The branches are normal to the center wire as in Fig. 15a. The center wire has six triangle expansion functions while each branch supports four. The wavelength is one meter and each wire is of radius 10^{-4} meter. The configuration has a single unit excitation and that is located right at the junction itself. Since the junction marks the position of the first branch triangle which is the seventh triangle the feed point is assigned the number seven. Patterns are computed for the $\phi = 0^\circ, 90^\circ$ planes. x,y,z-coordinates of axial points defining the problem geometry are labeled PX, PY, PZ, respectively in the sample output.

Subroutines are listed first, followed by the main program and typical output. The program is described in Chapter 4. Further information on computational procedures is available elsewhere [4].

```

SUBROUTINE LINEQ (N,Z)
COMPLEX Z( 400),STOR,ST0,ST,S
DIMENSION LA(50)
DO 20 I=1,N
LA(I)=I
20 CONTINUE
M1=0
DO 18 M=1,N
K=M
DO 2 I=M,N
K1=M1+I
K2=M1+K
IF (CABS(Z(K1))-CABS(Z(K2))) 2,2,6
6 K=I
2 CONTINUE
LS=LA(M)
LA(M)=LA(K)
LA(K)=LS

```

```

K2=M1+K
STOR=Z(K2)
J1=0
DO 7 J=1,N
K1=J1+K
K2=J1+M
STO=Z(K1)
Z(K1)=Z(K2)
Z(K2)=STO/STOR
J1=J1+N
7 CONTINUE
K1=M1+M
Z(K1)=1./STOR
DO 11 I=1,N
IF(I-M) 12,11,12
12 K1=M1+I
ST=Z(K1)
Z(K1)=0.
J1=0
DO 10 J=1,N
K2=J1+M
K1=J1+I
Z(K1)=Z(K1)-Z(K2)*ST
J1=J1+N
10 CONTINUE
11 CONTINUE
M1=M1+N
18 CONTINUE
J1=0
DO 9 J=1,N
IF (J-LA(J)) 14,8,14
14 LAJ=LA(J)
J2=(LAJ-1)*N
31 DO 13 I=1,N
K2=J2+I
K1=J1+I
S=Z(K2)
Z(K2)=Z(K1)
Z(K1)=S
13 CONTINUE
LA(J)=LA(LAJ)
LA(LAJ)=LAJ
IF (J-LA(J)) 14,8,14
8 J1=J1+N
9 CONTINUE
RETURN
END

```

C

```

SUBROUTINE ROW(N,TH,PH,E)
COMPLEX C(20),E(2),U,U1,U2,U3,U4,U5
COMMON XX(50),XY(50),XZ(50),TX(50),TY(50),TZ(50),AL(50)
COMMON T(100),TP(100),RAD2(10),L(10),LL(10),LR(10),BK
COMMON /COA/ C
DIMENSION BKR( 50),DT( 50),DP( 50)
U=(0.,1.)
ETA=376.7307

```



```

CT=COS(TH)
ST=SIN(TH)
CP=COS(PH)
SP=SIN(PH)
S1=CT*CP
S2=CT*SP
BK1=BK*ST*CP
BK2=BK*ST*SP
BK3=BK*CT
N2=1
N3=-2
DO 1 NS=1,N
IF (L(N2)-NS) 2,3,2
3 KK=1
N3=N3+2
N2=N2+1
GO TO 4
2 KK=3
4 DO 5 K=KK,4
N7=N3+K
BKR(N7)=XX(N7)*BK1+XY(N7)*BK2+XZ(N7)*BK3
DT(N7)=TX(N7)*S1+TY(N7)*S2-TZ(N7)*ST
DP(N7)=-TX(N7)*SP+TY(N7)*CP
5 CONTINUE
N3=N3+2
1 CONTINUE
N2=1
N3=-2
U3=0.
U4=0.
DO 6 NS=1,N
IF (L(N2)-NS) 7,8,7
8 N3=N3+2
N2=N2+1
7 J1=(NS-1)*4
U1=0.
U2=0.
DO 9 JS=1,4
J2=J1+JS
J3=N3+JS
S1=BKR(J3)
U5=T(J2)*(COS(S1)+U*SIN(S1))
U1=U1+U5*DT(J3)
U2=U2+U5*DP(J3)
9 CONTINUE
U3=U3+U1*C(NS)
U4=U4+U2*C(NS)
N3=N3+2
6 CONTINUE
S1=.0745774*ETA*BK
E(1)=-U*S1*U3
E(2)=-U*S1*U4
RETURN
END

```

SUBROUTINE GOMTRY(NBRCH,NEC,NEB,N1,N,CLENTH,BLENTH,THETA)

THIS SUBROUTINE GENERATING PARAMETERS OF A UMBRELLA SHAPE WIRE
STRUCTURE (ANTENNA OR SCATTERER)

I/P NBRCH= NO. OF BRANCHES
N1=TOTAL NO. OF SEGMENTS
N=TOTAL NUMBER OF EXPANSION FUNCTIONS
NEC=NO. OF EXPANSION FUNCTION OVER CENTRAL WIRE
NEB=NO. OF EXPANSION FUNCTION OVER EACH BRANCH
CLENTH=LENGTH OF CENTRAL WIRE MAY BE NEGATIVE TO REPRESENT THE CASE WITH NEGATIVE-DIRECTED CENTRAL WIRE
BLENTH=LENGTH OF EACH BRANCH (IN METER)
THETA=ANGLE BETWEEN EACH BRANCH AND THE POSITIVE Z-AXIS

O/P XX(I) = I'TH COMPONENTS OF THE COORDINATES OF THE CENTER
OF THE I'TH SEGMENT
XY(I) IS THE Y COMPONENTS
XZ IS THE Z COMPONENTS
AL(I) IS THE I'TH SEGMENT LENGTH
TX IS THE PROJECTION OF AL TO X-AXIS
TY IS THE Y COMPONENTS
TZ IS THE Z COMPONENT

COMMON XX(50),XY(50),XZ(50),TX(50),TY(50),TZ(50),AL(50)

COMMON T(100),TP(100),RAD2(10),L(10),LL(10),LR(10)

DIMENSION PX(50),PY(50),PZ(50)

DOUBLE PRECISION DP1,DP2,DP3

NPR=2*NEB+3

NPC=2*NEC+3

DP1=NPC-1

DP2=CLENTH

DP2=DP2/DP1

L(1)=1

LL(1)=1

L(2)=1+NEC

LL(2)=1+NPC

DO 10 I=1,NPC

S1=I-1

PX(I)=0.

PY(I)=0.

PZ(I)=S1*DP2

10 CONTINUE

DP1=THETA

DP2=.0174533

TH=DP1*DP2

DP1=BLENTH

DP3=2*NEB

DP3=DP1/DP3

D3=DP3*COS(TH)

D1=DP3*SIN(TH)

J1=NPC+1

DO 20 I=1,NBRCH

BRCH=NBRCH

X=I-1

```

DP1=360.*X/BRCH
PH=DP1*DP2
D2=D1*SIN(PH)
D4=D1*COS(PH)
LL(I+2)=LL(I+1)+NPR
L(I+2)=L(I+1)+NER
DO 40 J=1,3
PX(J1)=0.
PY(J1)=0.
PZ(J1)=PZ(4-J)
J1=J1+1
40 CONTINUE
NP1=2*NER
DO 30 J=1,NP1
S1=J.
PX(J1)=D4*S1
PY(J1)=D2*S1
PZ(J1)=D3*S1
J1=J1+1
30 CONTINUE
20 CONTINUE
L(NBRCH+2)=200
LL(NBRCH+2)=200
NP=NPC+NBRCH*NPR
WRITE (3,301)(PX(I),I=1,NP)
WRITE (3,302)(PY(I),I=1,NP)
WRITE (3,303)(PZ(I),I=1,NP)
301 FORMAT('OPX'/(1X, 7E11.4))
302 FORMAT('OPY'/(1X, 7E11.4))
303 FORMAT('OPZ'/(1X, 7E11.4))
J1=1
J2=2
N1=0
DO 2 J=1,NP
IF (LL(J1)-J) 3,4,3
4 J2=J2-1
L(J1)=J2
J1=J1+1
GO TO 2
3 N1=N1+1
J3=J-1
IF ((N1/2*2-N1).EQ.0) J2=J2+1
XX(N1)=.5*(PX(J)+PX(J3))
XY(N1)=.5*(PY(J)+PY(J3))
XZ(N1)=.5*(PZ(J)+PZ(J3))
S1=PX(J)-PX(J3)
S2=PY(J)-PY(J3)
S3=PZ(J)-PZ(J3)
S4=SQRT(S1*S1+S2*S2+S3*S3)
TX(N1)=S1/S4
TY(N1)=S2/S4
TZ(N1)=S3/S4
AL(N1)=S4

```

```

2 CONTINUE
L(J1)=J2
N=J2-2
J1=1
J2=-2
DO 11 J=1,N
IF(L(J1)-J) 13,14,13
14 J2=J2+2
J1=J1+1
13 J3=(J-1)*4
J4=J3+1
J5=J4+1
J6=J5+1
J7=J6+1
K4=J2+1
K5=K4+1
K6=K5+1
K7=K6+1
S1=AL(K4)+AL(K5)
S2=AL(K6)+AL(K7)
T(J4)=AL(K4)*.5*AL(K4)/S1
T(J5)=AL(K5)*(AL(K4)+.5*AL(K5))/S1
T(J6)=AL(K6)*(AL(K7)+.5*AL(K6))/S2
T(J7)=AL(K7)*.5*AL(K7)/S2
TP(J4)=AL(K4)/S1
TP(J5)=AL(K5)/S1
TP(J6)=-AL(K6)/S2
TP(J7)=-AL(K7)/S2
J2=J2+2
11 CONTINUE
RETURN
END

```

SUBROUTINE ZBRCH(NBRCH,N1,N,NEC,NEB,Y)

C
C
C
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C

THIS ROUTINE COMPUTE THE GENERALIZED IMPEDANCE MATRIX OF UMBRELLA
SHAPE OF WIRE STRUCTURE

I/P NBRCH= NO. OF BRANCHES
N1=TOTAL NO. OF SEGMENTS
N=TOTAL NUMBER OF EXPANSION FUNCTIONS
NEC=NO. OF EXPANSION FUNCTION OVER CENTRAL WIRE
NEB=NO. OF EXPANSION FUNCTION OVER EACH BRANCH
XX,XY,XZ,AL, TX, TY, TZ ARE GEOMETRY PARAMETERS
RAD=RADIUS OF EACH WIRE

O/P Y IS THE GENERALIZED IMPEDANCE MATRIX COMPUTED

COMPLEX Z(400),Y(400),PSI(200),U,U1,U2,U3,U4,U5,U6
COMMON XX(50),XY(50),XZ(50),TX(50),TY(50),TZ(50),AL(50)
COMMON T(100),TP(100),RAD2(10),L(10),LL(10),LR(10),BK
DIMENSION DC(200)
U=(0.,1.)
PI=3.141593
FTA=376.7307

```

C1=.125/PI
C2=.25/PI
U3=U*RK*ETA
U4=-U/RK*ETA
BK2=BK*RK/2.
BK3=BK2*BK/3.
N9=0
N2=1
N0=1
N3=-2
M1=NEC+NFR
M2=2*M1+4.
M4=NEC
M3=2*NEC+2
DO 10 NS=1,M1
IF(NS.GT.NEC) M3=N1-((NBRCH-1)/2)*(2*NEB+2)
IF(NS.GT.NEC) M4=NEC+((NBRCH+2)/2)*NEB
IF(L(N2)-NS) 12,11,12
11 KK=1
N3=N3+2
N2=N2+1
GO TO 13
12 KK=3
DO 14 NF=1,M3
N4=NF+M3
N5=N4+M3
N6=N5+M3
DC(NF)=DC(N5)
DC(N4)=DC(N6)
PSI(NF)=PSI(N5)
PSI(N4)=PSI(N6)
14 CONTINUE
13 IF (N3+1-LR(N0)) 5,6,5
6 AA=RAD2(N0)
N0=N0+1
5 CONTINUE
DO 15 K=KK,4
N7=N3+K
K1=(K-1)*M3
DO 16 NF=1,M3
N8=NF+K1
S1=XX(N7)-XX(NF)
S2=XY(N7)-XY(NF)
S3=XZ(N7)-XZ(NF)
R2=S1*S1+S2*S2+S3*S3+AA
R=SQRT(R2)
RT=ABS(S1*TX(N7)+S2*TY(N7)+S3*TZ(N7))
RT2=RT*RT
RH=(R2-RT2)
ALP=.5*AL(N7)
AR=ALP/R
S1=BK*R
U2=COS(S1)-U*SIN(S1)
IF(AR-.1) 22,22,21
21 U2=U2*C1/ALP
S1=RT-ALP

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```

S2=RT+ALP
S3=SQRT(S1*S1+RH)
S4=SQRT(S2*S2+RH)
IF(S1) 18,18,14
18 AI1=ALOG((S2+S4)*(-S1+S3)/RH)
GO TO 20
19 AI1=ALOG((S2+S4)/(S1+S3))
20 AI2=AL(N7)
AI3=(S2*S4-S1*S3+RH*AI1)/2.
AI4=AI2*(RH+ALP*ALP/3.+RT2)
S3=AI1*R
S1=AI1-BK2*(AI3-R*(2.*AI2-S3))
S2=-BK*(AI2-S3)+BK3*(AI4-3.*AI3*R+R2*(3.*AI2-S3))
GO TO 28
22 U2=U2*C2/R
BA=BK*ALP
BA2=BA*BA
AR2=AR*AR
AR3=AR2*AR
ZR=RT/R
ZR2=ZR*ZR
ZR3=ZR2*ZR
ZR4=ZR3*ZR
H1=(3.-30.*ZR2+35.*ZR4)*AR3/40.
A1=AR*(-1.+3.*ZR2)/6.+(3.-30.*ZR2+35.*ZR4)*AR3/40.
A0=1.+AR*A1
A2=-ZR2/6.-AR2*(1.-12.*ZR2+15.*ZR4)/40.
A3=AR*(3.*ZR2-5.*ZR4)/60.
A4=ZR4/120.
S1=A0+BA2*(A2+BA2*A4)
S2=BA*(A1+BA2*A3)
28 PSI(N8)=U2*(S1+U*S2)
DC(N8)=TX(NF)*TX(N7)+TY(NF)*TY(N7)+TZ(NF)*TZ(N7)
16 CONTINUE
15 CONTINUE
N3=N3+2
J3=(N3-1)*4
J7=-2
J9=1
DO 25 NF=1,M4
J1=(NF-1)*4
IF(L(J9)-NF) 26,27,26
27 J9=J9+1
J7=J7+2
26 N9=N9+1
U5=0.
U6=0.
J5=0
DO 23 JS=1,4
J4=J3+JS
J8=J5+J7
DO 24 JF=1,4
J6=J8+JF
J2=J1+JF
U5=T(J2)*T(J4)*DC(J6)*PSI(J6)+U5

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```

U6=TP(J2)*TP(J4)*PSI(J6)+U6
24 CONTINUE
J5=J5+M3
23 CONTINUE
Z(N9)=U5*U3+U6*U4
J7=J7+2
25 CONTINUE
10 CONTINUE
J1=0
J2=0
M3=NEC*NFC
DO 30 I=1,NEC
J3=M3+I
DO 31 J=1,NEC

J2=J2+1
J1=J1+1
Y(J1)=Z(J2)
31 CONTINUE
S1=NBRCH
DO 32 J=1,NER
J1=J1+1
Y(J1)=Z(J3)*S1
J3=J3+M4
32 CONTINUE
30 CONTINUE
J2=M3
DO 35 I=1,NER
DO 36 J=1,NEC
J2=J2+1
J1=J1+1
Y(J1)=Z(J2)
36 CONTINUE
DO 37 J=1,NER
J1=J1+1
J2=J2+1
Y(J1)=Z(J2)
IF(NBRCH.EQ.1) GO TO 37
KK=(NBRCH+2)/2
DO 41 K=2,KK
KKK=(NBRCH/2)*2-NBRCH
IF((K.EQ.KK).AND.(O.EQ.KKK)) GO TO 42
J3=J2+(K-1)*NER
Y(J1)=Y(J1)+Z(J3)*2
GO TO 41
42 J3=J2+(K-1)*NER
Y(J1)=Y(J1)+Z(J3)
41 CONTINUE
37 CONTINUE
J2=J2+(NBRCH/2)*NER
35 CONTINUE
RETURN
END

```

C
C

```

      MAIN PROGRAM          MMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMM
COMPLEX 7(400),U(20),C(20),E(3),EI(2),UV(4),U1,V
COMMON XX(50),XY(50),XZ(50),TX(50),TY(50),TZ(50),AL(50)
COMMON T(100),TP(100),RAD2(10),L(10),LL(10),LR(10),BK
COMMON /COA/ C
DIMENSION IFP(20),RAD(10)
1 READ(1,101,END=500) NR,NEB,NEC,BK,BLENT,CELENT,ANGLE
101 FORMAT(3I3,3E14.7,F6.2)
WRITE (3,301)NR,NEB,NEC,BK
301 FORMAT('0 NB NEB NEC    BK'/1X,I3,2I4,E14.7)
WRITE (3,302) BLENT,CELENT,ANGLE
302 FORMAT('0LENGTH OF EACH BRANCH '/1X,E14.7// ' LENGTH OF CENTER WI
1RE'/1X,F14.7//' ANGLE'/1X,F7.2)
NR=1
LR(1)=1
LR(NR+1)=200
READ(1,109) RAD(1)
109 FORMAT(E14.7)
WRITE (3,305) RAD(1)
305 FORMAT('ORAD'/1X,E14.7)
DO 46 I=1,NR
RAD2(I)=RAD(I)**2
46 CONTINUE
CALL GOMTRY(NR,NEC,NEB,N1,N,CELENT,BLENT,ANGLE)
CALL ZBRCH(NR,N1,N,NEC,NEB,Z)
NE=NFC+NFB
NE2=NE*NE
CALL LINEQ (NE,Z)
DO 45 I=1,N
U(I)=0.
45 CONTINUE
READ (1,107) NF
107 FORMAT(20I3)
WRITE (3,303)NF
303 FORMAT ('0NF'/I3/'0FP          VOLTAGE')
DO 44 I=1,NF
READ (1,119) J1,V
119 FORMAT(I3,2E14.7)
WRITE (3,120) J1,V
120 FORMAT(1X,I3,2E14.7)
U(J1)=V
IFP(I)=J1
44 CONTINUE
WRITE (3,111)
111 FORMAT ('0CURRENT'/I   I   REAL   IMAG   MAGNITUDE
1   PHASE')
J3=0
J1=0
5 J1=J1+1
U1=0.
J2=0
6 J2=J2+1
J3=J3+1
U1=U1+Z(J3)*U(J2)
IF (J2-NE)6,7,7
7 C(J1)=U1
IF (J1-NE)5,8,8
8 CONTINUE

```



```

DO 9 I=1,NE
U1=C(I)
CM=CABS(U1)
IF (CM) 11,12,11
11 CP=ATAN2(AIMAG(U1),REAL(U1))*57.2858
GO TO 10
12 CP=0.
10 CONTINUE
WRITE (3,112) I,C(I),CM,CP
112 FORMAT (I5,3E14.6,F10.3)
9 CONTINUE
IF(NR.F0.1) GO TO 42
J1=NER+NEC
DO 41 I=2,NR
J2=NER
DO 41 J=1,NER
J1=J1+1
J2=J2+1
C(J1)=C(J2)
41 CONTINUE
42 CONTINUE
WRITE (3,113)
113 FORMAT ('OFIELD PATTERN!/' K=1 FOR THETA COMPONENT K=2 FOR PHI

1 COMPONENT!/'O THETA PHI K REAL IMAG MAGNITUDE
2E PHASE')
DO 17 IPH=1,2
PHI=(IPH-1)*90
PH=PHI*.0174533
DO 17 ITH=1,181,5
TH=ITH-1
TH=THE*.0174533
CALL ROW (N,TH,PH,E)
DO 16 K=1,2
U1=E(K)
GM=CABS(U1)
IF (GM)13,13,14
14 EP=ATAN2 (AIMAG(U1),REAL(U1))
GO TO 15
13 EP=0.
15 GP=EP*57.2858
WRITE (3,114) THE,PHI,K,U1,GM,GP
114 FORMAT(F6.0,F5.0,I3,3E14.7,F10.3)
16 CONTINUE
17 CONTINUE
GO TO 1
500 STOP
END

/*
//GD.SYSIN DD *
2 4 6+0.6283185E+01+0.1250000E+00+0.2500000E+00+90.00
+0.1000000E-03
1
7+0.1000000E+01+0.0000000E+00

```

NR NEB NFC BK
2 4 6 0.6283184E 01

LENGTH OF EACH BRANCH
0.1250000E 00

LENGTH OF CENTER WIRE
0.2500000E 00

ANGLE
90.00

RAD
0.9999999E-04

PX

0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.1563E-01	0.3125E-01	0.4688E-01
0.6250E-01	0.7813E-01	0.9375E-01	0.1094E 00	0.1250E 00	0.0	0.0
0.0	-0.1563E-01	-0.3125E-01	-0.4688E-01	-0.6250E-01	-0.7813E-01	-0.9375E-01
-0.1094E 00	-0.1250E 00					

PY

0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	-0.5089E-08	-0.1018E-07	-0.1527E-07	-0.2036E-07	-0.2544E-07	-0.3053E-07
-0.3562E-07	-0.4071E-07					

PZ

0.0	0.1786E-01	0.3571E-01	0.5357E-01	0.7143E-01	0.8929E-01	0.1071E 00
0.1250E 00	0.1429E 00	0.1607E 00	0.1786E 00	0.1964E 00	0.2143E 00	0.2321E 00
0.2500E 00	0.3571E-01	0.1786E-01	0.0	0.4906E-08	0.9812E-08	0.1472E-07
0.1962E-07	0.2453E-07	0.2944E-07	0.3434E-07	0.3925E-07	0.3571E-01	0.1786E-01
0.0	0.4906E-08	0.9812E-08	0.1472E-07	0.1962E-07	0.2453E-07	0.2944E-07
0.3434E-07	0.3925E-07					

NF
1

FP VOLTAGE
7 0.1000000E 01 0.0

CURRENT

i	REAL	IMAG	MAGNITUDE	PHASE
1	-0.511069E-03	-0.476440E-02	0.479223E-02	-96.105
2	-0.508337E-03	-0.437440E-02	0.440384E-02	-96.612
3	-0.471473E-03	-0.380923E-02	0.383829E-02	-97.039
4	-0.400856E-03	-0.307407E-02	0.310009E-02	-97.412
5	-0.298497E-03	-0.218866E-02	0.220892E-02	-97.749
6	-0.167006E-03	-0.117641E-02	0.118820E-02	-98.063
7	0.241828E-03	0.253185E-02	0.254337E-02	84.529
8	0.195378E-03	0.199148E-02	0.200105E-02	84.382
9	0.138629E-03	0.139732E-02	0.140418E-02	84.319
10	0.745126E-04	0.744863E-03	0.748581E-03	84.273

FIELD PATTERN

K=1 FOR THETA COMPONENT K=2 FOR PHI COMPONENT

THETA	PHI	K	REAL	IMAG	MAGNITUDE	PHASE
0.	0.	1	0.1852448E-01	-0.2404486E-02	0.1867987E-01	-7.394
0.	0.	2	-0.4318437E-08	0.2318754E-09	0.4324658E-08	176.896
5.	0.	1	0.2811516E-01	0.4627321E-02	0.2849341E-01	9.345
5.	0.	2	-0.4283006E-08	0.5410978E-09	0.4317048E-08	172.770
10.	0.	1	0.3760987E-01	0.1145996E-01	0.3931709E-01	16.943
10.	0.	2	-0.4232792E-08	0.8457961E-09	0.4316465E-08	168.671
15.	0.	1	0.4702598E-01	0.1788098E-01	0.5031075E-01	20.815
15.	0.	2	-0.4168804E-08	0.1143016E-08	0.4322661E-08	164.639
20.	0.	1	0.5636786E-01	0.2368479E-01	0.6114167E-01	22.787
20.	0.	2	-0.4092520E-08	0.1429995E-08	0.4335156E-08	160.712
25.	0.	1	0.6562126E-01	0.2867664E-01	0.7161349E-01	23.601
25.	0.	2	-0.4005798E-08	0.1704222E-08	0.4353250E-08	156.926
30.	0.	1	0.7474923E-01	0.3267739E-01	0.8157974E-01	23.609
30.	0.	2	-0.3910849E-08	0.1963477E-08	0.4376066E-08	153.314
35.	0.	1	0.8368844E-01	0.3552905E-01	0.9091789E-01	22.999
35.	0.	2	-0.3810115E-08	0.2205868E-08	0.4402590E-08	149.905
40.	0.	1	0.9234822E-01	0.3710096E-01	0.9952223E-01	21.884
40.	0.	2	-0.3706218E-08	0.2429835E-08	0.4431719E-08	146.725
45.	0.	1	0.1006104E 00	0.3729643E-01	0.1073008E 00	20.336
45.	0.	2	-0.3601869E-08	0.2634155E-08	0.4462311E-08	143.796
50.	0.	1	0.1083333E 00	0.3605935E-01	0.1141769E 00	18.407
50.	0.	2	-0.3499781E-08	0.2817917E-08	0.4493231E-08	141.135
55.	0.	1	0.1153556E 00	0.3338032E-01	0.1200880E 00	16.136
55.	0.	2	-0.3402594E-08	0.2980492E-08	0.4523379E-08	138.759
60.	0.	1	0.1215046E 00	0.2930132E-01	0.1249877E 00	13.556
60.	0.	2	-0.3312790E-08	0.3121491E-08	0.4551733E-08	136.679
65.	0.	1	0.1266061E 00	0.2391834E-01	0.1288456E 00	10.696
65.	0.	2	-0.3232655E-08	0.3240725E-08	0.4577373E-08	134.905
70.	0.	1	0.1304952E 00	0.1738180E-01	0.1316477E 00	7.586
70.	0.	2	-0.3164200E-08	0.3338133E-08	0.4599485E-08	133.444
75.	0.	1	0.1330277E 00	0.9892862E-02	0.1333951E 00	4.252
75.	0.	2	-0.3109131E-08	0.3413756E-08	0.4617405E-08	132.303
80.	0.	1	0.1340917E 00	0.1697588E-02	0.1341023E 00	0.725
80.	0.	2	-0.3068799E-08	0.3467675E-08	0.4630579E-08	131.485
85.	0.	1	0.1336166E 00	-0.6923538E-02	0.1337957E 00	-2.966
85.	0.	2	-0.3044198E-08	0.3499985E-08	0.4638643E-08	130.993
90.	0.	1	0.1315810E 00	-0.1566721E-01	0.1325104E 00	-6.789
90.	0.	2	-0.3035932E-08	0.3510747E-08	0.4641358E-08	130.829
95.	0.	1	0.1280149E 00	-0.2422226E-01	0.1302862E 00	-10.713
95.	0.	2	-0.3044198E-08	0.3499985E-08	0.4638643E-08	130.993
100.	0.	1	0.1230005E 00	-0.3228397E-01	0.1271666E 00	-14.704
100.	0.	2	-0.3068799E-08	0.3467677E-08	0.4630582E-08	131.485
105.	0.	1	0.1166678E 00	-0.3956900E-01	0.1231953E 00	-18.732
105.	0.	2	-0.3109131E-08	0.3413756E-08	0.4617405E-08	132.303
110.	0.	1	0.1091857E 00	-0.4582763E-01	0.1184132E 00	-22.765
110.	0.	2	-0.3164200E-08	0.3338133E-08	0.4599485E-08	133.444
115.	0.	1	0.1007528E 00	-0.5085400E-01	0.1128594E 00	-26.777
115.	0.	2	-0.3232657E-08	0.3240725E-08	0.4577373E-08	134.905
120.	0.	1	0.9158385E-01	-0.5449336E-01	0.1065698E 00	-30.748
120.	0.	2	-0.3312790E-08	0.3121491E-08	0.4551733E-08	136.679
125.	0.	1	0.8189815E-01	-0.5664512E-01	0.9957898E-01	-34.664
125.	0.	2	-0.3402594E-08	0.2980492E-08	0.4523379E-08	138.759

170.	0.	1-0.2737458E-02-0.1548368E-01	0.1572380E-01	-100.009
170.	0.	2-0.4232792E-08 0.8457470E-09	0.4316465E-08	168.671
175.	0.	1-0.1072789E-01-0.6631736E-02	0.1261220E-01	-148.251
175.	0.	2-0.4283006E-08 0.5410976E-09	0.4317048E-08	172.770
180.	0.	1-0.1852450E-01 0.2404514E-02	0.1867989E-01	172.574
180.	0.	2-0.4318437E-08 0.2318762E-09	0.4324658E-08	176.896
0.	90.	1 0.1497844E-08-0.5230831E-09	0.1586554E-08	-19.247
0.	90.	2-0.1852448E-01 0.2404486E-02	0.1867987E-01	172.574
5.	90.	1 0.9491581E-02 0.5323514E-02	0.1088255E-01	29.282
5.	90.	2-0.1852448E-01 0.2404486E-02	0.1867987E-01	172.574
10.	90.	1 0.1901302E-01 0.1048635E-01	0.2171309E-01	28.873
10.	90.	2-0.1852448E-01 0.2404485E-02	0.1867987E-01	172.574
15.	90.	1 0.2858743E-01 0.1532894E-01	0.3243791E-01	28.196
15.	90.	2-0.1852448E-01 0.2404482E-02	0.1867987E-01	172.574
20.	90.	1 0.3822480E-01 0.1969456E-01	0.4300012E-01	27.254
20.	90.	2-0.1852448E-01 0.2404482E-02	0.1867987E-01	172.574
25.	90.	1 0.4791601E-01 0.2343156E-01	0.5333837E-01	26.055
25.	90.	2-0.1852448E-01 0.2404481E-02	0.1867987E-01	172.574
30.	90.	1 0.5762853E-01 0.2639677E-01	0.6338638E-01	24.606
30.	90.	2-0.1852448E-01 0.2404481E-02	0.1867987E-01	172.574
35.	90.	1 0.6730169E-01 0.2845993E-01	0.7307178E-01	22.918
35.	90.	2-0.1852448E-01 0.2404480E-02	0.1867987E-01	172.574
40.	90.	1 0.7684582E-01 0.2950929E-01	0.8231694E-01	21.003
40.	90.	2-0.1852448E-01 0.2404480E-02	0.1867987E-01	172.574
45.	90.	1 0.8614147E-01 0.2945792E-01	0.9103906E-01	18.876
45.	90.	2-0.1852448E-01 0.2404479E-02	0.1867987E-01	172.574
50.	90.	1 0.9504282E-01 0.2825044E-01	0.9915251E-01	16.551
50.	90.	2-0.1852448E-01 0.2404479E-02	0.1867987E-01	172.574
55.	90.	1 0.1033820E 00 0.2586960E-01	0.1065695E 00	14.046
55.	90.	2-0.1852448E-01 0.2404476E-02	0.1867988E-01	172.574
60.	90.	1 0.1109769E 00 0.2234155E-01	0.1132034E 00	11.380
60.	90.	2-0.1852448E-01 0.2404476E-02	0.1867988E-01	172.574

150.	90.	1 0.4938317E-01-0.3884482E-01	0.6283003E-01	-38.182
150.	90.	2-0.1852448E-01 0.2404480E-02	0.1867988E-01	172.574
155.	90.	1 0.4069822E-01-0.3371320E-01	0.5284812E-01	-39.630
155.	90.	2-0.1852448E-01 0.2404480E-02	0.1867988E-01	172.574
160.	90.	1 0.3222238E-01-0.2784979E-01	0.4258981E-01	-40.830
160.	90.	2-0.1852448E-01 0.2404480E-02	0.1867988E-01	172.574
165.	90.	1 0.2395230E-01-0.2139971E-01	0.3211947E-01	-41.771
165.	90.	2-0.1852448E-01 0.2404481E-02	0.1867988E-01	172.574
170.	90.	1 0.1585945E-01-0.1451008E-01	0.2149569E-01	-42.449
170.	90.	2-0.1852448E-01 0.2404481E-02	0.1867988E-01	172.574
175.	90.	1 0.7895697E-02-0.7327948E-02	0.1077222E-01	-42.857
175.	90.	2-0.1852448E-01 0.2404481E-02	0.1867988E-01	172.574
180.	90.	1-0.3092213E-07 0.2796228E-07	0.4169014E-07	137.854
180.	90.	2-0.1852448E-01 0.2404484E-02	0.1867988E-01	172.574