DRAG REDUCTION BY POLYMER ADDITION

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13. ABSTRACT

In the present review of the subject special attention is paid to the changes in turbulence structure brought about by the additives, and an effort is made to understand these in the light of recent advances in the study of shear flow turbulence in ordinary fluids. Also a simplified theoretical model of shear flow wall turbulence is presented, with the aid of which an exploratory study of the influence of some non-Newtonian fluid properties is carried out. A particularly intriguing question is posed by the remarkable effectiveness of the extremely high-molecular-weight polymers with which only a few parts per million of weight of solvent is sufficient in some cases to lead to drag reductions of 50 per cent or more; how is it possible that such minute amounts can change the mechanical properties of the fluid enough to produce so dramatic effects? A general review of the very voluminous literature on the subject of drag reduction has not been attempted here (to date several hundred publications on the subject have appeared). Generally, the emphasis is put on the more recent contributions, particularly those having a bearing on turbulence structure, but some of the older material of interest for giving the background is also included.

1. Introduction

The reduction of flow friction drag through polymer addition, a phenom-
enon first subjected to systematic study by Toms (1948) (with whose
name it is commonly associated) together with Oldroyd (1948), but
of which the first documented discovery is due to Mysels (1949), has
been the subject of very extensive research during the last decade.
However, despite the combined efforts of specialists from many differ-
ent fields, no satisfactory explanation of the phenomenon has as yet
been found. Drag reduction has been observed with a large number of
additive-fluid combinations (a few of which are shown in figure 1),
the most effective ones being high-molecular-weight linear polymers
(such as polyethyleneoxide) in water. There are clear experimental
indications that drag reduction is associated with a modification of
the turbulence structure, and this is most remarkable in view of the
permanence and statistical stability exhibited by ordinary Newtonian
shear flow turbulence. Therefore, the phenomenon of drag reduction
becomes of interest not only because of the promising technical appli-
cations it offers but just as much for fundamental fluid mechanical
reasons. Through the unravelling of the mechanism underlying the phe-
nomenon one should not only improve the prospects of finding more
efficient and perhaps completely novel methods of turbulence control,
but should also hope to gain a substantially deepened understanding
of the fundamental nature of turbulence as well.

In the present review of the subject special attention is paid to the
changes in turbulence structure brought about by the additives, and
an effort is made to understand these in the light of recent advances
in the study of shear flow turbulence in ordinary fluids. Also, a
simplified theoretical model of shear flow wall turbulence is presented,

with the aid of which an exploratory study of the influence of some non-Newtonian fluid properties is carried out. A particularly intriguing question is posed by the remarkable effectiveness of the extremely high-molecular-weight polymers with which only a few parts per million of weight of solvent is sufficient in some cases to lead to drag reductions of 50 per cent or more; how is it possible that such minute amounts can change the mechanical properties of the fluid enough to produce so dramatic effects? A general review of the very voluminous literature on the subject of drag reduction has not been attempted here (to date several hundred publications on the subject have appeared). Generally, the emphasis is put on the more recent contributions, particularly those having a bearing on turbulence structure, but some of the older material of interest for giving the background is also included.

2. Data on drag reduction and mean velocity profiles

Most of the data on drag reduction published to date have been obtained in pipe flows following Toms (1948) original procedure. However, results for flat-plate boundary layers such as those by Kowalski (1967) and by Latto & Shen (1970) show the same qualitative and quantitative behaviour as for pipe flow as do also those obtained in the rotating coaxial cylinder experiments of Merrill et al. (1966). This is perhaps not surprising in view of the similar behaviour of all turbulent wall shear flows in Newtonian fluids. We shall therefore here consider only the pipe flow data.

Systematic studies of pipe flow (e.g., Metzner & Park, 1964; Savins, 1964; Elata & Tirosh, 1965; Fabula, 1965; Goren & Norbury, 1967; Seyer & Metzner, 1967; Virk et al., 1967 and Virk et al., 1970) have identified several drag reduction charcteristics common to dilute solutions of linear polymers such as polyethylene-oxide. (Other drag reducing agents such as complex soaps with micellar molecular structures produce quite different behaviour with onset of drag reduction occurring at very low shear rates, but these will not be discussed here.) At low flow rates such fluids obey the laminar friction law with a slight increase in drag due to the increased viscosity caused by the polymer. At higher velocities the flow will go through transition at a Reynolds number which - at least if the pipe inlet disturbances are moderately high - will not be appreciably different from the Newtonian value. Drag reduction sets in at a well defined flow rate and above this the frictional resistance depends on the polymer-solvent system, molecular weight and concentration. With increase in flow rate a maximum drag reduction independent of polymeric

molecular parameters may be reached. For still higher flow rates the drag reduction effect could eventually diminish due to polymer degradation. Pipe flow drag data typical of dilute polyethyleneoxide solutions are shown in figure 2 taken from Paterson & Abernathy (1970). The onset of drag reduction has been shown by Virk et al. (1967) to be approximately independent of pipe diameter and for a given solvent-polymer combination to depend only on the wall shear stress. Furthermore, by comparing results for polymers of different molecular weight, they were able to show that the onset data correlated with the formula

$$D_M u_T^* / \nu = 0.015 \pm 0.005 \qquad (1)$$

where D_M is the effective diameter of the randomly coiled macro-molecule at rest and u_T^* the value of the shear velocity $u_T = \sqrt{\tau_w / \rho}$ at onset. This result is quite surprising because it implies that drag reduction would start when the diameter of the molecule is about 10^{-3} times the size of the smallest dynamically significant eddy in the flow. Simplified theoretical models for the macromole-cule (Peterlin, 1966) show that a large deformation of the mole-cule - possibly leading to a complete extension of the random coil - would take place when the shortest time scale of motion would fall below twice the relaxation time of the molecule. However, the onset data correlate poorly on a time hypothesis. Virk (1971a) has also found that the onset of drag reduction in rough pipes occurs at the same shear stress as in smooth.

The existence of a maximum drag reduction asymptote has by now been well documented for most known solvent-polymer systems. Figure 3, reproduced from Virk et al. (1970), shows data presented in Prandtl's coordinates $1/\sqrt{f}$ versus $Re \, f^{1/2}$. It is convenient to present results for pipe flow in this manner, because in these

coordinates both the laminar drag law and the Prandtl - von Karman law for Newtonian turbulent pipe flow

$$1/\sqrt{f} = 4.0 \log_{10}(\text{Re} f^{1/2}) - 0.4 \qquad (2)$$

appear as straight lines in a semilogarithmic diagram. The drag asymptote was also shown to yield an approximately straight line of the form (Virk et al., 1970)

$$1/\sqrt{f} = 19.0 \log_{10}(\text{Re} f^{1/2}) - 32.4 \qquad (3)$$

In the regime below the maximum drag asymptote the friction factor varies both with polymer properties and flow variables.

Several attempts have been made to correlate friction data in this polymeric regime below the maximum drag asymptote and we have singled out the recent ones of Seyer & Metzner (1969b) and of Virk (1971b) as being representative of two different approaches, one (Seyer & Metzner's) based on measured viscoelastic properties and the other (Virk's) working directly with molecular parameters. These are best understood in light of the behaviour of the mean velocity profiles. Virk (1971b) has compared published experimental data on mean velocities, and in figure 4 we have reproduced these (figure 2 of Virk, 1971b) showing the measured velocities plotted in the usual wall variables $U^+ = U/u_\tau$, yu_τ/ν. From these one can distinguish three different regions which can be represented approximately by

$$(\text{I}) \qquad U^+ = y^+ \; ; \quad 0 \leq y^+ \leq y_v^+ \qquad (4)$$

$$(\text{II}) \qquad U^+ = A_2 \ln y^+ + B_2 \; ; \quad y_v^+ \leq y^+ \leq y_e^+ \qquad (5)$$

$$(\text{III}) \qquad U^+ = A_3 \ln y^+ + B_3 \; ; \quad y_e^+ \leq y^+ \leq R^+ \qquad (6)$$

respectively, where R^+ is the value of y^+ corresponding to the
The first region is the usual viscous sublayer

extending to about $y^+ = 10$. The second region, called the "elastic"
region, is best represented by the values $A_2 = 11.7$ and $B_2 = -17.0$.
In the third region U^+ shows the same logarithmic variation with y^+
as in the Newtonian case but the curve is displaced upwards. Thus,
$A_3 \simeq 2.5$, but B_3 is a function of molecular and flow parameter.
The mean velocity profile data hence indicate the existence of a
Newtonian "plug" in the center of the pipe with a mixing length
apparently unaffected by the polymers. At the condition of maximum
drag reduction the elastic region II is assumed to extend all the
way to the center of the pipe, and the Newtonian region III then
disappears. Virk (1971b) determined the coefficients in the region II
so as to make the profile give a drag variation consistent with the
maximum drag asymptote under this assumption. Using (4) - (6) one
can integrate the total volumetric flow rate and hence obtain a re-
lation between friction force and flow rate.

Seyer and Metzner (1969b) assumed the viscous wall layer to extend
all the way to the Newtonian region without an intermediate region.
Thus, setting

$$U^+ = A \ln y^+ + B(\theta^+) \tag{7}$$

where $\theta^+ = \theta u_\tau^2 / \nu$ and θ is the relaxation time of the fluid they
were able to correlate data for different polymer solutions. The
constant A is the same as in the Newtonian case. (Seyer and Metzner
took $A = 2.46$.) The nondimensional relaxation time θ^+ is the Deborah
number formed by the ratio of fluid relaxation time to natural vis-
cous time scale of the turbulence. The curve of the intercept func-
tion $B(\theta^+)$ obtained in this manner is reproduced in figure 5.
Their data indicate an asymptotic value of B of about 30 for large
relaxation times. Virk (1971b) made

a comparison of the Seyer - Metzner model with his newer one and found that the latter is superior in representing measure profiles at conditions of large drag reduction. The Seyer - Metzner correlation is also difficult to use for highly dilute solutions, for which no reliable methods of determining the relaxation time θ are as yet available (see further the following section).

Virk (1971b) tries to relate the intercept constant B_3 in the following manner

$$B_3 = B_n + \delta \ln (R^+/R^{+**})$$

where B_n is the value for a Newtonian flow, $B_n = 5.5$. The factor $\ln(R^+/R^{+**})$, where R^{+**} is the value of R^+ at onset, is assumed to represent an "excitation" factor which depends both on flow parameters and such polymetric parameters that may be associated with the excitation of the macromolecule. The function δ, which Virk termes a "slope increment factor" (since it gives the fractional increase due to the polymer in the slope of the curve of $1/\sqrt{f}$ versus $\log(Re \sqrt{f})$) is assumed to depend solely on molecular parameters. By examining a large body of data Virk (1971b) established that for each polymer-solvent system δ is approximately proportional to $c^{1/2}$, where c is the weight fraction of polymer additive. Setting

$$\delta = A_3(\bar{A}c/M)^{1/2}\pi \tag{8}$$

where $\bar{A}$ is Avogadros number, c the weight fraction and M the molecular weight, respectively, of the polymer (so that the quantity under square root represents the number of macromolecules per unit weight of solution) Virk finds that for all polymer species with the exception of one (guar gum) the function π (termed "intrinsic slope increment factor") can be fairly well

represented by a single curve with the number of macromolecular backbone chain links, N, as a parameter. His results are reproduced in figure 6 (Virk's figure 5). Furthermore, in a logarithmic diagram all the data with the exception of that for guar gum were found to cluster around a single straight line of slope $\frac{3}{2}$ indicating a simple power law of the form

$$\pi = KN^{3/2} \tag{9}$$

where K is a constant found to have the approximate value $K = (2.3 \pm 0.8) \times 10^{-14}$. From a theoretical reasoning based on the estimate of the turbulent strain energy stored in the flexible macromolecule, Virk comes to the conclusion that π should be proportional to b^3, where b is the bond length of the monomer making up the macromolecule. Guar gum with a bond length of $b \sim 20$ Å, as compared to the other polymers having $b \sim 4$ Å, should therefore be about $(20/4)^3$ as effective as the others, as indeed roughly indicated by the data for guar gum which give $K \simeq 280 \times 10^{-14}$.

3. Rheological properties

One of the most important and intricate questions in drag reduction
research has been to what specific bulk rheological property or
properties of the fluid the effect might be ascribed to. It is
usually stated (see, e.g., Metzner & Park, 1964) that drag reducing
fluids show viscoelastic behaviour, and the suggestion that visco-
elasticity is the cause of drag reduction has been almost uni-
versially accepted. Indeed, in larger concentrations the polymers
effective in reducing the drag produce normal stress differences
in pure shear of moderate magnitude, an effect usually associated
with viscoelasticity. Direct demonstration of reversible elastic
effects has also been given by Darby (1970), who with the aid of
a specially designed transient rheometer was able to measure
elastic rebound for fluids with polymer concentrations of 100
to 500 parts per million. However, such rheometric data are usu-
ally taken at low shear rates for which the normal stresses are
expected to be much lower than at the high shear velocities of
interest in drag reduction. Normal stress differences for drag
reducing fluids are therefore usually determined from the measure-
ment of the thrust of a capillary jet (see Metzner & Park, 1964),
a method which does not suffer from this shortcoming. This method
was the one used by Gadd (1966a,b) and by Oliver (1966), the
latter also reporting results for fairly low concentration levels
at which the method becomes unreliable, however. With the aid of
a specially designed concentric tube rheometer, Gadd (1966b), was
also able to detect a non-zero second normal stress difference
$(\tau_{22} - \tau_{33})$ for dilute (40 p.p.m.) polyethyleneoxide solution
but did not find any appreciable second normal stress difference
for other good drag reducing polymers such as guar gum and poly-

acrylamide.

It has also been found that the viscoelastic properties exhibited depend strongly on aging and degradation of the polymer solution, especially in the case of polyethyleneoxide. Brennen and Gadd (1967) reported a much diminished total head loss with age of solution for a pitot tube in a stream of polyethyleneoxide solution indicating a diminished first normal stress. They also stated that the second normal stress for a 70 p.p.m. solution vanished with aging. Several later investigations have noted the anomalous flow behaviour of fresh polyethyleneoxide solution; thus Kalashnikov and Kudin (1970) found that the Strouhal number for the Karman vortex sheet behind a circular cylinder was substantially reduced in a flow of fresh solution, whereas the same solution passed through a centrifugal pump gave a small increase in frequency as compared to water, as did a solution of guar gum. Despite such marked degradation effects a degraded solution was still able to produce a substantial drag reduction (although somewhat diminished as compared to that of a fresh solution). Such behaviour has led Barenblatt et al. (1965) to propose that the viscoelastic behaviour of a polyehtyleneoxide solution is a result of the formation of very large aggregates of macromolecules which in part disappear with aging. The larger aggregates will also be broken up by the shear field in the flow apparatus, so that large-scale motion such as vortex shedding would be affected by degradation but not the drag reduction.

Although standard rheometric measurements for very dilute polymer solutions (< 20 p.p.m.) usually give no clear indication of departure from a purely Newtonian liquid (Bryson et al., 1971), there

is other evidence that such fluids can show substantial
non-Newtonian behaviour. Metzner and Metzner (1970) measured the
stress levels developed in dilute solutions of polyacrylamide
issuing from small orifices at high velocities. In this flow field
the deformation is approximately that of pure rapid elongation.
The ratio of elongational to shear viscosity (which is equal to
three in a Newtonian fluid) determined in this way was found to
be exceedingly high; a conservative estimate of the deformation
rate level yielded values as high as 29,000. Possibly, the ob-
served tendency of polymer solutions to form long threads may be
related to such high values of bulk viscosity (Lumley, 1969).

In order to use laboratory rheological data in the equations of
motion for a continuum fluid one needs to incorporate them in a
constitutive model for the fluid. A large number of such models
have been proposed and studied in the rheological literature.
Spriggs et al. (1966) made an appraisal of twenty different models
for viscoelastic materials, and they concluded that in order to
reproduce all essential rheological properties a model with at
least four different constants was needed. In view of the uncer-
tainty as to how viscoelastic effects come into play in the
phenomenon of drag reduction, however, it would seem most approp-
riate to select for exploratory studies the simplest model incor-
porating viscoelasticity. Such a model is the convected Maxwell
one proposed by White and Metzner (1963), which in Cartesian
coordinates may be written

$$\tau_{ij} + \theta \frac{\delta \tau_{ij}}{\delta t} = 2\mu d_{ij} \, , \qquad (10)$$

where τ_{ij} is the deviatoric shear stress, θ a relaxation time,

$\delta/\delta t$ the Oldroyd time derivative

$$\frac{\delta \tau_{1j}}{\delta t} = \frac{\partial \tau_{1j}}{\partial t} + U_m \frac{\partial \tau_{1j}}{\partial x_m} - \tau_{mj} \frac{\partial U_1}{\partial x_m} - \tau_{im} \frac{\partial U_j}{\partial x_m} ,$$

and μ and d_{1j} have their usual meaning of shear viscosity and deformation rate tensor

$$d_{1j} = \frac{1}{2} \left(\frac{\partial U_1}{\partial x_j} + \frac{\partial U_j}{\partial x_1} \right) ,$$

respectively. The material coefficients θ and μ generally depend on the deformation rate. The constitutive relation (10) is a simplified version of Oldroyd's (1958) original one containing eight material constants. For exploratory purposes it is a great advantage to have a model with only one positive constant, namely θ, to describe departure from a Newtonian fluid. It has also some foundation in physical reality as Lumley (1971) has shown that a simple model for a dilute solution of polymer molecules employing a dumbell model for the molecules leads to a constitutive relation of the Oldroyd type (albeit with three instead of two material coefficients). It is also the relation usually employed to convert data on measured normal stress differences to relaxation times; application of (10) to simple steady shear flow, $U = (U_1(x_2),0,0)$, gives

$$\tau_{11} - \tau_{22} = 2\theta \tau_{12} \mu \Gamma , \tag{11}$$

where $\Gamma = \partial U_1/\partial x_2$. In this model the fluid relaxation time θ is thus proportional to the first normal stress difference. On the basis of (10) one can also demonstrate the possibility of high resistance due to stretching as observed in the experiments by Metzner and Metzner (1970) quoted above.

In view of the great difficulties in measuring departures from
Newtonian behaviour of very dilute drag reducing fluids, it would
be natural to turn to theory to see what guidelines it may offer.
In view of the large size of the macromolecules in comparison
with the solvent molecules involved a good mechanistic model for
a drag reducing fluid is to regard it as a suspension of small
particles in a continuum fluid. The rheology of suspensions is
a field of research which has undergone a very extensive develop-
ment since its inception by Albert Einstein in 1905 with his
famous analysis of the increase in fluid viscosity due to the
presence of rigid spheres. Good review articles on the subject
can be found in Vols. I and IV of the series "Rheology", edited
by Eirich (1956, 1966). In theoretical suspension mechanics one
can distinguish the following three separate stages in the ana-
lysis:

i) calculation of the influence of the local flow field on the
individual particle;

ii) determination of the effect of a statistical ensemble of
particles on the bulk rheological properties for the suspension; and

iii) analysis of the bulk motion of the suspension.

In i) fluid inertia can be ignored because of the small size of
the particle, but the calculation may nevertheless become very
complicated if the particle undergoes large deformations or is
subjected to hydrodynamic interference from neighboring particles,
situations that do arise for the fluid of interest here. Also,
the effect of Brownian motion may need to be incorporated in
determining the shape and average orientation of the particles.
The greatest complication is perhaps caused by the coupling

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between stages iii) and i); the bulk motion of the suspension is influenced by the shape and orientation of the particles, which in turn depend on the local flow field created. The problem of determining the shape of the macromolecule can also become quite difficult depending on the degree of sophistication with which the molecule is modelled. The linear so-called bead- and-spring models employed by Rouse (1953) and by Zimm (1956) have been employed to calculate the relaxation time of a dilute suspension of coiled macromolecules subjected to a small oscillatory shear (James & Acosta, 1970). Peterlin (1966) employing a linear dumbell model (i.e., two beads and one spring) demonstrated that the molecule can become infinitely extended in an irrotational straining field if the strain rate exceeds the inverse of twice the relaxation time of the molecule. In a pure shear, on the other hand, it will never become fully extended.

The infinite extension is of course a consequence of the assumption of spring linearity. With the aid of a more realistic nonlinear thread model, Hinch (1972) has shown recently that there is a critical strain rate above which the molecule becomes completely (at least 1/2) extended; below this the molecule is not extended at all. One would therefore expect that in a turbulent field of sufficient intensity a great proportion of the macromolecules is uncoiled. On the basis of Peterlin's (1966) result Dudley (1969) estimated that 70% of all molecules should be extended in a turbulent flow if the root mean-square strain rate exceeds the critical value of the inverse of twice the relaxation time.

Such simple rheological theories point to the important conclusion that the molecular conformation that may be very common in a turbulent flow might be an extended thread rather than a random coil, so that the rheological properties of the polymer solution in a turbulent flow therefore might be quite different from what they are in a laminar flow. As a theory realistically modelling coiling - uncoiling macromolecules in an unsteady flow field is likely to lead to unsurmountable mathematical difficulties, it would therefore be more useful for an exploratory study to search for a model which gives the maximum rheological effects of extended particles.

Such a model can in fact be found from the extensive recent studies of the rheological properties of suspensions of rigid elongated particles carried out by Batchelor (1970a,b, 1971) and his group at Cambridge University. The main concern in this group has been with highly elongated rigid particles. The problem of determining the particle motion then simplifies to that of determining the velocity and orientation giving zero net force and moment on the particle. (Inertia of the particle is neglected.) This was a problem considered by Jeffrey (1922), who determined the orbit of spheroids in a shearing motion and in an irrotational straining field. Leal & Hinch (1971) and Hinch & Leal (1972) have extended this analysis to incorporate Brownian motion and have shown that the orbits of highly elongated particles in shear flow tend to become such as to make them spend most of their time with their longest axes aligned with the direction of flow. Jeffrey (1922) also demonstrated that in an irrotational straining motion all the particles would end up in a direction with their longest axes aligned with the flow. Hence, there is a strong tendency for elongated particles to line up in the flow direction. If the par

ticles are very slender, their contribution to the stress in pure shear will be quite small so that the suspension will behave nearly like a Newtonian fluid. On the other hand, any straining perturbation in the mean flow direction would be strongly resisted. We can use such general results together with that for the particle stress contribution due to pure straining motion calculated by Batchelor (1971) to determine the perturbation stress τ_{11} arising from an unsteady twodimensioned perturbation with velocity components $u_1 = u$ and $u_2 = v$ of a parallel shear flow, a model problem that, as will be argued below, has some bearing on the dynamics of a turbulent shear flow.

Batchelor shows that the contribution $\Delta\tau_{11}$ to the stress in pure straining motion due to the presence of slender particles, all of length l and diameter d, is given approximately by

$$\Delta\tau_{11} = \mu\bar{B}\,\frac{\partial U_1}{\partial x_1}\ , \tag{12}$$

where

$$\bar{B} = \frac{4}{3}\,\varphi\, r^2/\ln(\pi/\varphi) = \frac{\pi}{3}\,nl^3/\ln(\pi/\varphi)\ , \tag{13}$$

φ is the volume fraction of particles, $r = l/d$ their aspect ratio, and n their number density. For a perturbation in the mean flow direction, the perturbation stress would therefore be obtained by the above formula with U_1 replaced by u. However, the perturbation velocity component v normal to the flow would in addition cause a misalignment of the fluid particles with the (small) angle

$$\epsilon = \int\frac{\partial v}{\partial x}\,Dt\ , \tag{14}$$

where the symbol Dt means that the integration should be carried out

following the particle. Since the particles are now tilted with respect
to the mean flow they will experience a velocity gradient along their
length proportional to $\varepsilon(dU/dy)$; hence the total first-order contri-
bution to the stress will be

$$\Delta \tau_{11} = \mu \bar{B} \left[\frac{\partial u}{\partial x} + \frac{dU}{dy} \int^{t} \frac{\partial v}{\partial x} Dt \right].$$ (15)

This result has been derived by Hinch[*], who also gave estimates of the
error arising because not all particles are aligned. We will in the
following refer to (15) as the Batchelor - Hinch model. Adding the
contribution due to the solvent itself, which is obtained from the
ordinary Newtonian formula, one will thus find the total perturbation
stress due to the perturbation velocity field. This result will be
used below with a simplified model for a turbulent shear flow exploring
its large- and small-scale stability properties. Batchelor (1971)
derived the formula (12) for rigid rods under the restriction that
the suspension is dilute in the volumetric sense so that the average
distance between the rods is large compared to their diameter but that
the rod length is much larger than the average distance between the
rods as illustrated in figure 7. Under such conditions, which would be
satisfied for the dilute polymer solutions of present interest if the
macromolecular coils were extended, the coefficient $\bar{B}$ can be very large,
indeed, leading to very large ratios of extensional viscosity to shear
viscosity of the order of 10^{3} or more, values that order-of-magnitude-
wise are consistent with the experimental results reported in the paper
of Metzner and Metzner (1970) quoted above. Where the model obviously
fails is in the assumption of rigid rods. A fluid filled with threads

E.J. Hinch, private communication.

from uncoiled macromolecules would be able to resist extension but not compression. A more realistic model for such a fluid would have to be nonlinear and quite complicated, however, and in view of the fact that some suspensions of elongated inelastic particles have shown drag reduction (Bobkowicz & Gauvin, 1965), an investigation based on the simple Batchelor - Hinch model (15) should be of some value for an exploratory study.

4. Stability and transition

In the mechanics of turbulence the phenomena of instability and transition of a laminar flow are believed to be of fundamental importance. Therefore, many theoretical and experimental investigations have been aimed at a study of non-Newtonian effects on instability and transition.

The theory of hydrodynamic stability is concerned with whether small disturbances in a steady laminar flow field will grow or decay. We shall here discuss only two-dimensional infinitesimal disturbances of a plane parallel shear flow.

$$U_1 = \left(U(x_2), 0, 0 \right) \tag{16}$$

$$(x_1 = x, \quad x_2 = y, \quad x_3 = z)$$

For such disturbances it is a straight-forward, if often somewhat tedious process to derive from the linearized momentum and the chosen constitutive relation a single differential equation for the stream function of the perturbation. Setting in the usual manner for the perturbation velocity components

$$u = (D\varphi)\, e^{i\alpha(x-ct)} \tag{17}$$

$$v = -i\alpha\varphi\, e^{i\alpha(x-ct)}$$

where α and c are the wave number and phase velocity, respectively, of the wave-like disturbance; $D = d/dy$; and the real part of $\varphi(y)\exp\left[i\alpha(x-ct) \right]$ is the perturbation stream function, one obtains for the two constitutive models selected the following modified Orr - Sommerfeld equations:

a) Convected Maxwell model:

$$(U-c)(D^2-\alpha^2)\varphi - \varphi D^2 U = \frac{1}{i\alpha S Re}\left[(D^2-\alpha^2)^2\varphi + \sum_{n=0}^{3} b_n D^2\varphi\right] \qquad (18)$$

where Re is the Reynolds number, and

$$S = 1 + i\alpha\theta(U-c)$$

$$b_0 = \underline{D^4 S} - \alpha^2(D^2 S)T - 2\alpha^2(DS)^2(1 + S^{-2})$$

$$- 4(DS)(D^3 S)S^{-1} - 3(D^2 S)^2 S^{-1} + 4(DS)^4 S^{-2}$$

$$- 6(DS)^2(D^2 S)TS^{-1}$$

$$b_1 = - 2\alpha^2(DS)T + 2D^3 ST + 4(DS)(D^2 S)T^2$$

$$- 4(DS)^3 TS^{-1}$$

$$b_2 = 3D^2 ST + 2(DS)^2 T$$

$$b_3 = 2DST$$

This equation was given by Porteous and Denn (1971), who also demon-
strated that for small fluid elasticity, i.e., small θ, only the
underlined term in the above expressions for b_0 should be retained.
The equation then becomes equivalent to the one for a second-order
fluid employed by Chun & Schwartz (1968) and others, an equation
also arrived at by Walters (1962) by considering a fluid of short
memory.

b) Batchelor - Hinch model:

$$(U-c)(D^2-\alpha^2)\varphi - \varphi D^2 U = \frac{1}{i\alpha Re}\left[(D^2-\alpha^2)^2\varphi \qquad (19)\right.$$

$$\left. - \alpha^2 B(U-c)D^2\left(\frac{\varphi}{U-c}\right)\right]$$

All quantities appearing in (18) and (19) have been nondimensionalized
in the usual manner for boundary layers employing shear layer thickness
δ and free stream velocity U_1 as reference length and velocity, respec-
tively, and δ/U_1 as reference time scale. The Batchelor parameter $\bar{B}$
is nondimensional as such and is independent of flow parameters.

Most stability results available to date have been based on the second
order fluid constitutive model. For plane Poiseulle flow Chan Man Fong
and Walters (1965) found that a small elasticity tended to lower the
critical Reynolds number. This destabilizing effect was confirmed by
Chun & Schwartz (1968) through direct numerical calculations of disturb-
ance amplification rates, which for moderate values of the elasticity
parameter were found to increase somewhat over those for a Newtonian
fluid. Further demonstrations of the destabilization due to elasticity
has also been given by Porteous and Denn (1971) who employed both (18)
and the second order fluid model and found very little difference
between the results for small values of the fluid elasticity. They also
carried out an analysis for finite amplitudes. No results obtained on
basis of the Batchelor - Hinch model (19) for aligned rigid rods have
as yet published, but our own preliminary calculations reported below
indicate that their effect should generally be stabilizing. In the
literature there have also appeared several analysis of stability of
Couette and Taylor - Couette flow of viscoelastic fluid, of which per-
haps the most ambitious one as far as the constitutive model employed
is that of Karlsson et al. (1971). Elasticity is found sometimes to act
stabilizing for a Taylor - Couette flow but it is difficult to judge
what possible implications such results may have for shear flow
turbulence.

Experiments directly aimed at studying the stability of drag reducing fluids have been carried out by Berman & Cooper (1972). They investigated the flow response due to the periodic motion of a thin sleeve. Although the shear rates were not high enough for viscoelastic effects to be expected, they found that the periodic disturbance generated by the sleeve persisted longer downstream with the dilute polymer solution than with the pure solvent.

The pipe flow transition experiments that have been reported in the literature to date give rather contradictory information regarding whether a drag reducing fluid will speed up or delay transition to turbulence. Virk et al. (1967) reported no substantial effect of polymer addition. Castro and Squire (1967) and White & McEligot (1970) found a slight increase in the transition Reynolds number, whereas Chung & Graebel (1972) obtained a large increase. Paterson and Abernathy (1972), on the other hand, observed a decrease in the transition Reynolds number of strongly drag-reducing solutions as compared to the pure solvent.

Possibly, these strongly differing results could in part have their explanation in the different methods of detecting transition used. However, it is also well known from pipe flow transition measurements in Newtonian fluids (Lindgren, 1957) that by reducing inlet disturbances one can prolong the laminar regime almost indefinitely. Therefore, one can expect that only experiments conducted at reasonably high inlet disturbance levels, such as those of Virk et al. (1967) will yield reproducible results for the lower transition Reynolds number. It is of interest that Paterson & Abernathy (1972), when using a squared-off pipe producing disturbed inlet flow, found little effect from the polymer on the transition Reynolds number, in agreement with the findings of Virk et al. (1967).

5. Turbulence structure

In recent years considerable advances have been made in the under-
standing of the dynamical processes taking place in a turbulent
Newtonian wall shear flow. Earlier measurements of the turbulent
field were focussed on determining space-time correlations of the
fluctuating quantities. From such long-term averages one could hope
to detect any persistent or recurring larger-scale flow features that
might be of dynamical significance. Measurements of correlations such
as those of Favre et al. (1967) and of Willmarth & Tu (1967) depict
the main structure as a decaying and convected pattern which propagates
with a velocity close to but not exactly equal to the local flow velo-
city. Corcos (1964) carried out a spectral analysis of the wall pressure
fluctuation correlations obtained by Willmarth & Wooldridge (1962) and
showed that the convection velocity had a uniquely defined value for
each frequency. This was interpreted as evidence for the presence
of waves in the turbulent boundary layer by Landahl (1967), who
showed theoretically that in a system admitting propagation of waves,
(a wave guide) the least damped wave mode should dominate in the
statistical average. Morrison & Kronauer (1969) detected such waves
from measurements of frequency-wave-number spectra of the fluctu-
ating velocities in the wall layer of a turbulent pipe flow and found
that the fluctuating velocity field is dominated by waves that are
highly swept. Hussain & Reynolds (1972) excited periodic disturbances
in a two-dimensional turbulent channel flow and measured their ampli-
tude distributions and propagation constants. They showed that the
effect of background turbulence must be included in the equations
modelling the disturbances (Reynolds & Hussain, 1972). An entirely
different approach in analysing correlation data was taken by Bakewell
& Lumley (1967), who decomposed the correlation functions in a set of

orthogonal eigenfunctions and demonstrated that the dominating large-
scale feature in the wall layer consists of a pair of randomly ocur-
ring contrarotating streamwise vortices separated in the spanwise
direction of a distance of about $\lambda_z^* = 100$ expressed in wall vari-
ables.

The conceptual two-layer model proposed by Townsend (1956), which
divides the turbulent boundary layer into an "active" part comprising
the region closest to the wall and a "passive" outer one, has been
a source of inspiration for many of the later invistigations. That
the energetics processes in the boundary layer are mainly confined
to the inner layer near the wall was given detailed confirmation by
Kline et al. (1967) and by Kim et al. (1971). Their visual observations
of a thick turbulent boundary layer in water with the aid of hydrogen
bubble tracers revealed that most of the turbulent energy and Reynolds
stress was produced during intermittently ocurring localized "bursts"
or turbulence. The burst is a violent instability process found to
occur primarily in the zone $0 < y^+ < 100$. The observations suggested
a local structure like a hair-pin snaped vortex lifting up from the
lower part of the boundary layer and being stretched by the mean shear
as it is transported downstream (see figure 8 reproduced from Kline
et al., 1967). The average transverse distance between such events
was found to be about $\lambda_z^+ \sim 100$ expressed in wall variables, strongly
suggesting a connection to the statistical picture of Bakewell and
Lumley (1967). Further visual observations by Corino and Brodkey
(1969) revealed two distinct violent processes; an "ejection" of low-
speed fluid away from the wall and a "sweep" of high-speed fluid to-
wards the wall.

Because of their strong intermittency the presence of turbulent bursts cannot normally be detected by means of ordinary statistical methods, and their more detailed quantitative study has necessitated the developments of new experimental techniques based on selective sampling of the signal during bursts. Although different criteria have been employed by different investigators as to what should signify the occurance of an "event", all are in substantial agreement as to the main picture. Willmarth & Lu (1972) discovered that 60% of the Reynolds stress is produced when the velocity in the sublayer is less than the mean. Blackwelder & Kaplan (1972) succeeded in determining the average instantaneous velocity profile just preceding burst for different locations of the triggering probe. In figure 9 is shown the instantaneous profile thus obtained before burst for a setting of the triggering probe at $y^+ = 15$. The large negative velocity perturbation up to 20% of the local mean and the strong inflexional character is apparent. The inflexional character of the velocity profile preceding burst has been observed in all investigations of the phenomenon and suggests a strong similarity with the final state of breakdown of a laminar boundary layer to turbulence (Klebanoff et al., 1962). Other selective sampling data have been presented by Wallace et al. (1972), who, extending the observations by Corino and Brodkey (1969), found that the ejection and sweep stages each contributed about seventy per cent of the mean stress for a total of 140 per cent. The excess was accounted for by "interactions" towards and away from the wall each producing a negative turbulent shear stress of about twenty per cent of the mean. In between ejections, sweeps and interactions the flow was found to be of essentially laminar character but unsteady.

Most of the experimental data on the turbulent structure of a non-Newtonian fluid reported in the literature so far have pertained to fluctuation intensities. From these it is immediately apparent that drag reduction is not accompanied by a corresponding decrease in fluctuation intensities. In fact, the measurements reported by Virk et al. (1967) showed actually an increase in the streamwise fluctuation velocity. Seyer and Metzner (1969b) on the other hand, using a hydrogen bubble technique, found a decrease. Development of laser anemometers in recent years has alleviated many of the special difficulties associated with measurements in polymer solutions employing traditional techniques. Measurements by Goldstein et al. (1969) gave very little difference in the u-component due to the presence of an additive. Rudd (1972) found a noticeable increase in the streamwise fluctuating component near the wall but hardly any change near the center of the pipe. (See figures 10 and 11 reproduced from Rudd, 1972.) The transverse (w-)component was decreased near the wall but increased above its value for the solvent at larger distances.

A few measurements of the spatial structure of non-Newtonian turbulences have been reported. For isotropic grid turbulence, Fabula (1966) could not detect any clear effect on the spectrum from polymer addition. Friehe & Schwartz (1970) obtained a decrease in the decay rate but no substantial change in the spectrum. Measurements in a flat plate boundary layer by Kowalski (1967) showed a substantial increase in the turbulence microscale near the wall with as much as by a factor of two. This is consistent with the observations from mean velocity measurements that the thickness of the viscous wall layer increases. On basis of data from flow in rough tubes Virk (1971a) concludes that the maximum increase in the viscous sublayer due to polymer additives is about 2.5 in qualitative agreement with Kowalski's result. Results

from correlation measurements have only recently become available.
Fortuna & Hanratty (1972) employing an electrochemical technique
determined the wall-stress fluctuation in pipe flow. They discovered
a strong increase in the transverse (spanwise) correlation scale
(see figure 12) as well as a much slower longitudinal (streamwise)
correlation decay (see figure 13) than in the Newtonian case. A re-
duction of the stress fluctuation amplitude for the same mean wall
stress was also observed; for the transverse component up to a factor
of about two, but less for the axial component. Spectra were also
determined but showed no dramatic differences from their Newtonian
counterparts when normalized with wall parameters. However, the low-
frequency part increased somewhat and the high frequency one decreased
with increasing drag reduction.

In summary, the qualitative picture that emerges from such experimental
results is that polymer addition causes an increase in the scale of
the smallest turbulents eddies near the wall accompanied by a reduction
of Reynolds stress and an associated lowering of the wall friction.
Turbulence is not supressed in the wall layer but actually enhanced.
The most spectacular change in the turbulence structure is in the
transverse scale, which can increase with up to a factor of five
(Fortuna and Hanratty, 1972). The reduction of streamwise decay of
the longitudinal correlation as compared to the solvent points to an
increase in the life-time for the big eddies.

6. Proposed mechanisms

A number of suggestions, all of a qualitative nature, have appeared
in the literature as to what mechanism might be the primary cause of
the drag reduction. The key element in all of these has been the
thickening of the wall layer, first observed by Elata et el. (1966).
The explanations differ widely, however, as to how a very small amount
of polymer addition could interfere with the turbulent mixing pro-
cesses so as to produce a substantial increase in the scale of the
smallest stress-producing eddies near the wall. There is even differing
opinions on such a fundamental question as whether an explanation on
a continuum basis is sufficient. Barenblatt et al. (1965 and 1969)
point to the observed appearance of large agglomerations of molecules
and suggest that these will form viscoelastic particles at least as
large as the Kolmogoroff scale of turbulence which - because they tend
to become effectively rigid at very high fluctuation frequencies - will
act to stabilize the small viscous eddies in the wall layer. Their
effect on the inertial large-scale motion, on the other hand, is to
lead to an increased intensity. The idea that entanglements of mole-
cules would resist the motion in the wall layer was also suggested by
Fabula et al. (1966) and discussed by Lumley (1967), but in his later
review article (Lumley, 1969) he comes cautiously to the
conclusion that agglomerations of molecules is not an important factor
in drag reduction. A continuum viewpoint was taken by Seyer & Metzner
(1969a), who based their reasoning on the model of large streamwise
eddies for the wall layer proposed by Bakewell & Lumley (1967) and
the convected Maxwell constitutive model for the fluid. According to
the latter, large values of normal stresses would be set up in a rapid
elongational flow, such as would be produced when the two Bakewell -
Lumley counterrotating vortices force the fluid up from the wall
between them. This motion would thus be strongly resisted. Pfenninger

(1967), following Theodorsen's (1955) idea that the turbulence could be modelled as a distribution of horseshoe vortices formed at the wall and lifting up from it as they are swept downstream, proposed that drag reduction was associated with the strong resistance to vortex stretching which experiments have shown to be exhibited by some (but not all) drag reducing fluids (see also Gadd, 1968). The horseshoe vortex concept has also been embraced by Black (1969), who however thinks that stretching of the more or less longitudinal vortices in the horseshoe "legs" is perhaps not as important as the direct stabilization of the wall layer. Influence on the energy budget of the turbulent fluctuations in a more unspecified manner has also been invoked. Astarita (1965) notes that the energy dissipation ought to decrease in a viscoelastic fluid since the dynamically process will become more elastic and hence more reversible at high frequencies. Thus, he argues, the drag must decrease in order to preserve the energy balance. Energy arguments were also used by Savins (1964) and are also implicit in Virk's (1971b) elastic sublayer model. Adsorption of a resilient layer of molecules on the wall layer has also been proposed as contributing to the thickened wall layer by Dryson et al. (1971), but no explanation has been suggested as to how this can change the turbulent processes.

7. Application of a new model

To make any quantitative or qualitative assessments of such proposed explanations of drag reduction is presently not possible because of the lack of an adequate theoretical model for the dynamics of shear flow turbulence. In view of the extreme complexity, randomness and nonlinearity of turbulence, the prospects for the discovery in the near future of a reasonably complete such model are very poor, indeed. However, in view of the very drastic effects produced by additives one could reasonably hope that even a fairly crude theoretical model - if retaining the essential dynamics of the turbulent processes - should be able to indicate the possibility of modifications of the turbulent field leading to drag reduction, at least qualitatively. We shall here present the first attempts at an extremely simplified such model which, hopefully, contains enough of the essential dynamical elements of the turbulence structure to make an exploratory study of the effect of additives possible.

The new model represents an outgrowth from ideas in classical hydrodynamic stability applied to a fully turbulent flow, but it also retains Townsend's (1956) basic concept of a two-layer structure. The scale of wall turbulence being roughly proportional to distance from the wall, one would expect that the significant stress and energy producing eddies in the inner "active" layer close to the wall are of much smaller scale than those in the "passive" outer layer. It may therefore be meaningful to think of the turbulence conceptually as consisting of a coupled motion at two disparate scales: a primary one of large scale (of, say, the order of the boundary layer thickness) and a much smaller secondary one (of the order of the viscous length scale ν/u_τ). The random fluctuating

field is superimposed on a mean velocity field, which to a good approximation can be regarded as a parallel shear flow. Accordingly, if the fluctuation velocities are small compared to the reference free stream speed so that a linearization is allowable, a simple model equation for the fluctuating velocity field (when Fourier transformed in x, z and t) would be of the Orr-Sommerfeld type, and a decomposition of the general unsteady motion in terms of the eigenmodes of the Orr-Sommerfeld problem should be the obvious way to proceed. However, nonlinearity would modify the picture in three important ways.

First, the primary wave field (belonging to the large-scale motion) would propagate through a flow filled with small-scale eddies, hence these will act effectively as scattering sources for the large-scale motion producing a gross effect perhaps much like that of an eddy viscosity (see Reynolds & Hussain, 1972, for efforts to model this effect). We shall here ignore this effect completely finding some justification in doing so in the calculation of Landahl (1967) which gave propagation and decay properties of the large-scale pressure-producing eddies in good qualitative agreement with observations without including eddy viscosity.

Secondly, nonlinearity is required to excite the primary waves since these are damped, as shown by Landahl (1967), and the most obvious excitation sources that have come to light so far are the turbulent bursts. As the recent experiments reviewed above indicate that these are highly localized in time and space, the excitation may be modelled as effectively point sources on the scale of the primary motion.

Thirdly, the small-scale secondary motion sees not only the parallel mean velocity field but also the primary fluctuating field, so that the secondary waves may be thought of as riding on the primary waves in very much the same way as capillary water waves on a large-scale gravity wave. On the scale of the secondary motion the mean plus the primary field will be a slowly varying one in space and time and may therefore locally be approximated as a parallel one.

For both the primary and the secondary motion we will thus have as a model equation one of the Orr-Sommerfeld type with the mean velocity replaced by mean plus primary in the latter. The secondary wave could thus be locally unstable, for example because of inflexion. Now, it has been shown recently with the aid of kinematic wave theory (Landahl, 1972) that there is a critical condition for a locally selfexcited secondary wave given by

$$c_g = c_o \qquad (22)$$

where c_g is the group velocity of the secondary wave and c_o the phase velocity of the primary wave on which it is riding, at which the secondary wave will grow to very large amplitudes. The reason for this is that the group of secondary waves at this condition experiences space-time focusing (i.e., neighboring groups of waves catch up with each others and hence "compress"), and that the wave group becomes trapped in a region of the primary wave (generally near its crest), where it can continue to amplify (see figure 14). It is therefore hypothesized that (22) should identify the condition at which the breakdown of the primary waves into strong secondary small-scale motion would take place. Application to the data from the transition experiments by Klebanoff et al. (1962) demonstrates that the condition is very closely satisfied at the appearance of the first "spikes" in the final stage of transition of a laminar

boundary layer, and that it gives good agreement with the measured
frequency of secondary motion. In view of the observed similarities
between breakdown in the final stage of transition and turbulent
bursts (as pointed out, for example, by Willmarth and Lu, 1972) we
shall hypothesize that the bursts are the results of the attaintment
of local criticality in the primary field, and that the subsequent
violent motion will cause excitation of new primary waves. (The
model is illustrated conceptually in figure 15.) Admittedly, the
model is highly speculative at this stage and contains many diffi-
cult details which are not yet worked out and which will require
a great deal more study, such as how randomness could be incorpo-
rated into the model, and how primary waves, which probably are
highly oblique, could interact so as to produce a critical condition.
Nevertheless, for the purpose of studying the effects of additives,
a great deal could perhaps be learned from the propagation charac-
teristics, in particular the stability characteristics, of primary
and secondary motion. Accordingly, some numerical stability calcu-
lations have been carried out for velocity profiles that may be
typical of the mean and instantaneous primary motion for both a
Newtonian situation and one with drag reduction. These calculations
were performed by F. Bark of the Royal Institute of Technology,
Stockholm, and a detailed account of them will be given later.
The three rheological models that were used were the ones described
above; the ordinary Newtonian one, the convected Maxwell one and
the Batchelor - Hinch model for a suspension of microscopical rigid
rods. Three different velocity profiles were investigated, two
qualitatively representative of the mean motion for a Newtonian
and a drag-reducing fluid in a boundary layer, and one broadly
similar to the inflexional profile appearing in the sublayer just
before breakdown. All three are illustrated in figure 16. The mean

profile for the Newtonian case was constructed from Reichhardt's representation for the velocity distribution in the constant-stress layer coupled with Coles' wake law profile for the outer portion. The drag-reduction velocity profile was obtained in essentially the same way, except that the viscous wall layer thickness parameter in Reichhardt's expression was increased to correspond to the maximum wall layer thickness possible according to the Seyer - Metzner (1969b) correlation. The same plate-length Reynolds number was employed as for the Newtonian profile. By comparisons of results for the two profiles with the same constitutive model one will thereby be able to isolate the effect on the stability characteristics of the change of profile from an ordinary turbulent one to a more laminar-looking one.

The velocity profile chosen to model the instantaneous wall layer breakdown profile was of the simple tanhyp-variety with the distance of the inflexion point to the wall adjusted so as to be roughly in the same ratio to the internal shear layer as in the experimental profile determined by Blackwelder and Kaplan (1972). This was selected in preference to a direct numerical fit to the measured profile in order to avoid undue numerical difficulties that would arise because of the fourth derivative of U appearing in the equation for the convected Maxwell model. Such simplifications should not be serious at the present stage, however, where only qualitative indications of a strong effect are being sought.

Results from the primary wave calculations are shown in figure 17. Following Landahl (1967) these were obtained in spatial form with the complex wave number $\alpha = \alpha_R + i\alpha_I$ determined as function of the real frequency ω (made dimensionless with the "outer" reference quantities, δ and U). The imaginary part, α_I, gives a measure of the

rate of decay with streamwise distance for a wave of given frequency.
The results obtained with the Maxwell fluid model show that for the
lower frequency range, $\omega < 1.5$, a moderate fluid elasticity (non-
dimensional relaxation time θ less than unity) causes very little
change in α_I relative to the Newtonian model for the same velocity
profile. There is an increase of α_I relative to the value for the
Newtonian profile which is entirely a profile effect. At higher
frequencies a destabilizing effect is seen which at $\omega = 3$ is
strong enough to lead to a complete loss of stability. Calculations
for larger values of θ (not shown here) give frequency regions
in which α_r becomes negative, i.e., the wave is unstable. It is
difficult to judge the significance of such results as the model
employed requires that all primary waves are stable, otherwise the
assumption of statistical stationarity has to be given up (Landahl,
1967).

With the Batchelor - Hinch constitutive model, on the other hand,
the decay rate α_I is increased over the value for the Newtonian
case by about a factor of 2.5. At a given non-dimensional frequency
based on outer variables one would thus expect to see a correspon-
dingly more rapid decay of the cross-power spectrum of the turbulent
fluctuations with streamwise distance. Such a result might seem to
be at variance with the experiments of Fortuna & Hanratty (1972),
which showed a much reduced decay of the streamwise correlation
of the wall stress fluctuations with streamwise distance as compared
to the Newtonian case for the same mean wall stress. However, since
the correlation is the inverse transform of the cross-power spectrum,
the reduction in the correlation decay rate may simply be due to a
shifting of the whole spectrum towards lower frequencies, as the
experiments indeed indicate will take place during drag reduction.

Results of the stability calculations for the secondary disturbance
with the model inflexional profile selected are presented in figure
18. In these calculations the fairly high Reynolds number of Re = 4600
was chosen in order to make the Newtonian stability characteristics
become essentially the inviscid ones. For a study of the shortest
scale in a Newtonian wall layer, perhaps a more realistic value of
the Reynolds number would have been less than 1000; however, in
order to bring out more clearly any stabilization potentialities
of the other rheological models, a high Reynolds number was chosen.
From the results presented in figure 17 it becomes clear that a
moderately high elasticity of the fluid $(\theta = O(1))$ has only a small
destabilizing influence. However, with the Batchelor - Hinch model
a strong stabilizing effect is demonstrated including the possi-
bility of a complete stabilization of the inflexional profile for
sufficiently high values of the parameter $\bar{B}$. In particular, it is
observed that the strongest stabilizing effect is found for the
largest wave numbers, so that the smallest scale of unstable motion
is increased. This would be expected to lead to an increase in the
size of the smallest stress producing eddies and a corresponding
increase in the wall layer thickness. The values of $\bar{B}$ used in the
calculations may perhaps seem high, but they are not unrealistic,
as they give a measure of the ratio of elongational to shear visco-
sity of the same order as found in the experiments by Metzner &
Metzner (1970). Furthermore, it is seen that the parameter enters
into the basic stability equation (22) in the ratio $\bar{B}/Re$, so that
for a lower Reynolds number the value of $\bar{B}$ required for stabilization
becomes proportionally lower.

The simplified turbulence model presented thus shows that fluid elasticity as modelled by the Maxwell constitutive equation causes destabilization of both primary and secondary motion. The Batchelor - Hinch constitutive model, on the other hand, is found to produce a strong stabilization of both primary and secondary motion. The stabilizing effect on the secondary motion is probably of most significance for the drag reduction phenomenon since the associated increase in scale of the unstable secondary disturbances could be expected to lead to a thickening of the wall layer.

8. Concluding remarks

The first basic and as yet unresolved question in the puzzling phenom-
enon of drag reduction due to polymer addition is whether a satis-
factory explanation based on continuum mechanics is possible or
whether one instead would have to invoke processes on the non-continuum
macromolecular level such as proposed by Barenblatt et al. (1965 & 1969).
From a heuristic point of view, a theory based on non-continuum con-
cepts will be quite difficult to quantify and the theoretician would
therefore naturally be quite reluctant to part with the continuum
approach. Although the initial efforts towards a simplified mechanistic
theory of wall turbulence presented here have not yet led to definite
conclusions in this regard, they have at least pointed to a possible
continuum mechanical effect due to the polymer additive which could
be highly effective even when the molecular size is much smaller than
the scale of fluctuating motion. The large frictional resistance to
streamwise straining perturbations caused by elongated particles
aligned in the mean flow direction was shown to have a strong stabili-
zing effect on inflexional instability. Since there is both experi-
mental and theoretical evidence that such instability is involved in
the creation of the stress producing turbulent bursts observed to
occur intermittently in the wall layer, such stabilization could be
expected to lead to a reduction of the turbulent stresses near the
wall and an associated thickening of the wall layer. Extended polymer
macromolecules mostly aligned in the mean flow direction of the tur-
bulent velocity field could be expected to act qualitatively like the
rigid rods employed in the simplified fluid model, at least during
the extensional phase of the secondary fluctuation cycle. That the
polymer molecules could have a rheological effect of this kind is
indicated indirectly by the very high values of the ratio of elonga-

tional to shear viscosity, of the order 10^4, measured by Metzner & Metzner (1970). Such high values are consistent with what Batchelor's (1971) simplified theory would give for rods having aspect ratios like those of the extended polymer molecules of typical drag reduction additives in dilute solutions.

As yet no direct experimental demonstration of the presence of fully extended macromolecular threads during drag reduction has been given. Complete extension would not be attainable in a laminar shear flow, but the fluctuating irrotational strains present in the turbulent field could possibly cause some coils to stay extended a large fraction of time. Intuitively, one would feel that the onset phenomenon is a manifestation of the extension of the macromolecular coils, although the observation (Virk et al. , 1967) that onset data best correlate with a length hypothesis based on the ratio between coil radius and the smallest scale of turbulence has not yet found a satisfactory theoretical explanation.

The calculations based on the Maxwell fluid model give quite clear indications of the effects of fluid elasticity on fluctuation field stability. Both the large-scale primary motion and the small-scale secondary one were found to be destabilized by moderate amounts of elasticity. With the model employed there is thus no indication that viscoelasticity as such could be the cause of drag reduction.

The explanation that drag reduction is associated with the high elongational viscosity produced by aligned uncoiled macromolecular threads is broadly in accord with Virk's (1971b) findings that the drag curve slope increment δ is approximately proportional to the square root of concentration and the three half power of the number of backbone links in the polymer. In the Batchelor - Kinch model the governing

parameter $\bar{B}$ is proportional to nl^3; (ignoring the logarithmic term) where n is the number density and l the length of the aligned particles. The experimentally determined variation with concentration, i.e., of n, to the one half power would thus indicate the effect to be proportional to $(nl^3)^{1/2}$, which thus yields a variation with the bond length b and number of backbone links N as $b^{3/2} N^{3/2}$. Thus, proportionality to $c^{1/2}$ would be consistent with the $N^{3/2}$ variation.[*] The two models differ as to the variation with molecular chain bond length, however. On the basis of an estimate of the turbulent strain energy stored in the elastic molecules, Virk (1971b) predicted a proportionality to b^3, in contrast to the $b^{3/2}$ dependence inherent in the $(nl^3)^{1/2}$ variation. The high drag reduction values exhibited by guar gum are more in line with the b^3 variation proposed by Virk than with the proportionality to $b^{3/2}$ tending to give support for Virk's hypothesis that elasticity is an important molecular property in drag reduction. In the Batchelor - Hinch model only the total length of the particle plays a role; its hydrodynamic effect extends over a sphere of diameter l and the quantity nl^3 thus measures the number of such influence spheres per unit volume. The high values of $\bar{B}$ that would be typical for dilute drag reducing fluids - of the order of 10^3 if the macromolecules were fully extended - indicate that the hydrodynamic interaction between the extended macromolecular threads should be very strong (i.e., their spheres of hydrodynamic influence would overlap strongly). In order for the solution to be dilute in the rheological sense so that the molecules take up only a small fraction of the available volume, they have to have extremely high length to diameter ratios of the order 10^3 or more.

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[*] This was pointed out to me by E.J. Hinch.

However, there is recent experimental evidence that it may not only be the length of the extended molecule that is of importance for drag reduction. Hand & Williams (1971) measured the drag reduction effectiveness of polyacrylic acid and of calf thymus DNA at various pH levels of the solution. They found that the effectiveness was considerably larger for pH ranges in which the molecules are believed to have a fairly compact helical structure. For pH ranges where the molecules are supposed to be extended and behave essentially as rigid rods, the drag reduction was found to be considerably lower. Results such as these are very difficult to explain on basis of any continuum model; but possibly entanglements and agglomerations could be an important factor.

It should be pointed out that stabilization of the instantaneous inflexional velocity profile before burst - as indicated by the calculations for a suspension of rod-like particles - is only one of several conceivable mechanisms that could produce a decrease in the rate or intensity of the bursts and thereby cause a lowering of turbulent stress. An alteration of propagation characteristics of the secondary wave group in relation to the primary disturbance field could presumably have such an effect according to the recently proposed model for breakdown (Landahl, 1972), but the model has not yet been sufficiently developed to allow any conclusions to be drawn regarding the possibility of an effect of this kind. Conceivably, the enhanced elongational viscosity could also have a strong influence on the structure of the longitudinal vortices appearing in the wall layer. In particular, a strong resistance to vortex stretching would be expected. However, a quantitative model to investigate this effect is likely to be quite difficult to set up.

As pointed out in the Introduction, perhaps the most interesting aspect of the research on the drag reduction effect of polymer additives is what new knowledge can be gained from it regarding the nature of turbulence. That one now has a very effective tool with which the turbulent processes can be manipulated should open up new possibilities for further experimental and theoretical research on the mechanics of turbulence. For example, such experiments as those by Wells & Spangler (1967) on the effectiveness of injection of polymer at the wall demonstrate quite unequivocally that wall shear flow turbulence is controlled by processes in a layer very close to the wall.

An important engineering motivation for turbulence research is to find new and effective methods for control of turbulence. From the research on polymer additives reviewed here it becomes apparent that to influence skin friction it is the very small-scale stress producing eddies near the wall which may need to be interfered with directly. This may put very difficult constraints on the dynamical response characteristics of any mechanical system employed for drag modification, but the search for efficient such systems should be an extremely interesting challenge from both a scientific and engineering point of view.

Acknowledgements

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ADDITIVE-FLUID COMBINATIONS SHOWING DRAG REDUCTION

ADDITIVE	FLUID
Polyethyleneoxide (PEO, Polyox)	water
Polyacrylamide (PAM, Separen)	"
Guar gum (GGM)	"
- - - - - - - - - - - - - - - -	
Polymethylmethacrylate (PMM)	monochlorobenzene (Toms, 1948)
- - - - - - - - - - - - - - - -	
Various soaps	water, toluene
- - - - - - - - - - - - - - - -	
Nylon fibres	water
	(Bobkowicz & Gauvin, 1965)

Figure 1.   Some additive-fluid systems exhibiting drag reduction
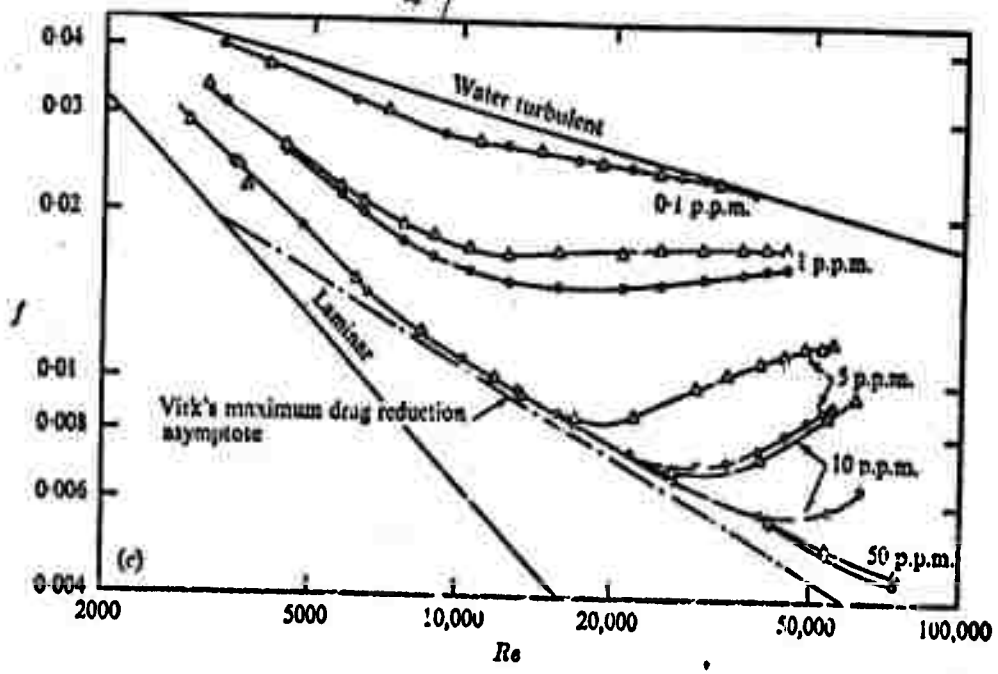
Figure 2. Pipe flow drag data for solutions of polyethyleneoxide
(from Paterson & Abernathy, 1970)
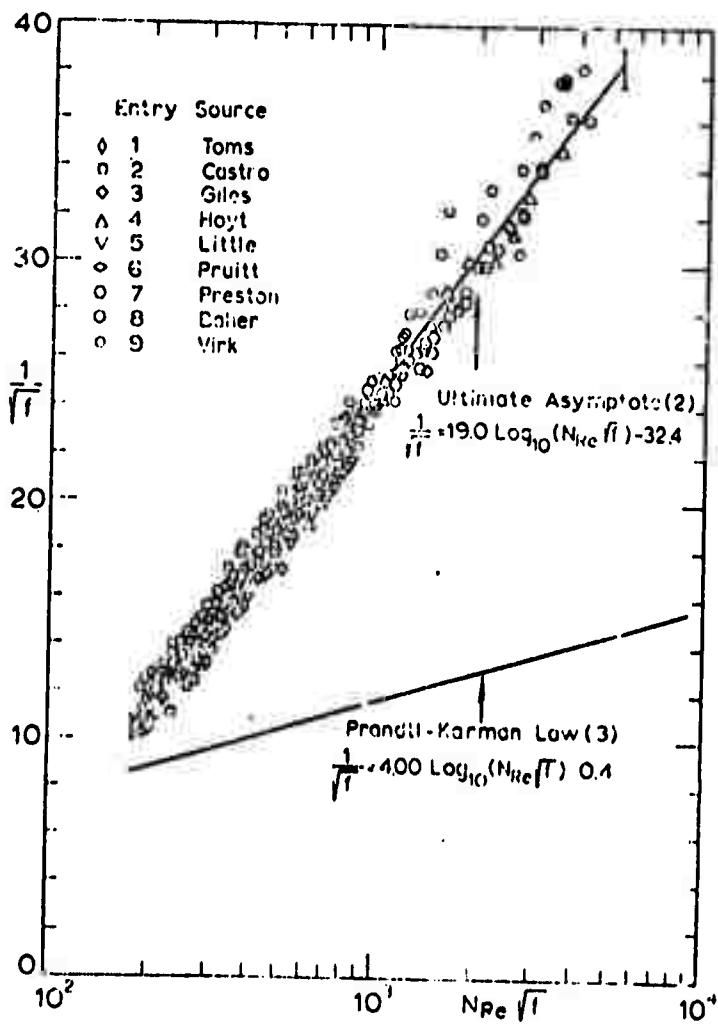
Figure 3. The maximum drag reduction asymptote
(from Virk et al., 1970)
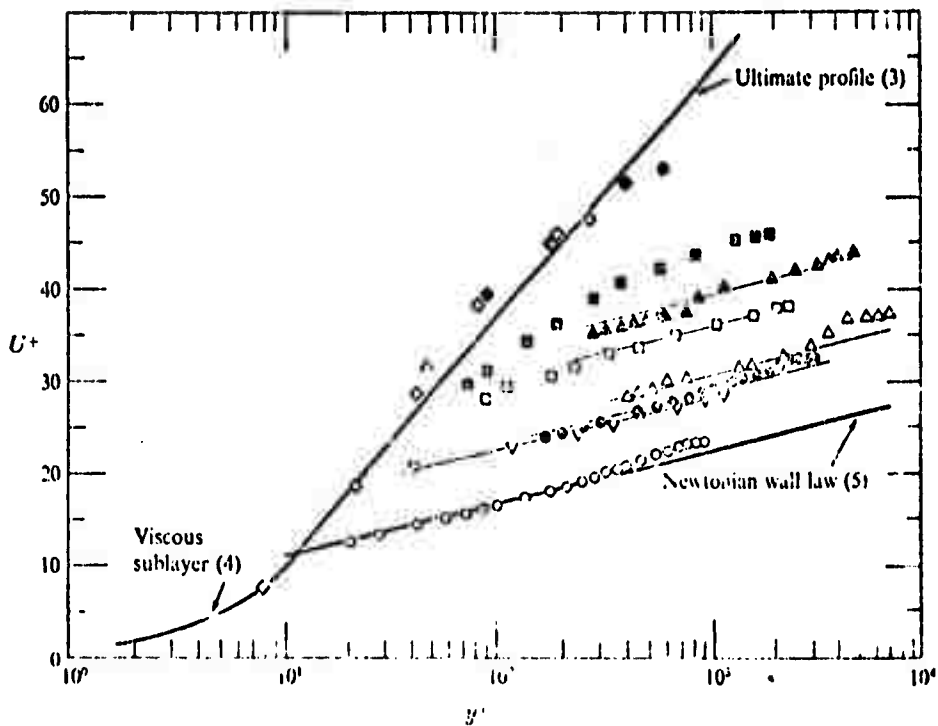
Figure 4. Experimental mean velocity profiles during drag reduction.
(from Virk, 1971b). △ ▲, Elata et al. (1966);
◻ ▨ Goren & Norbury (1967); ▽ Patterson & Florez (1969;
◇ ◆ Seyer & Metzner (1969b); ○ ◑ Virk et al. (1967)
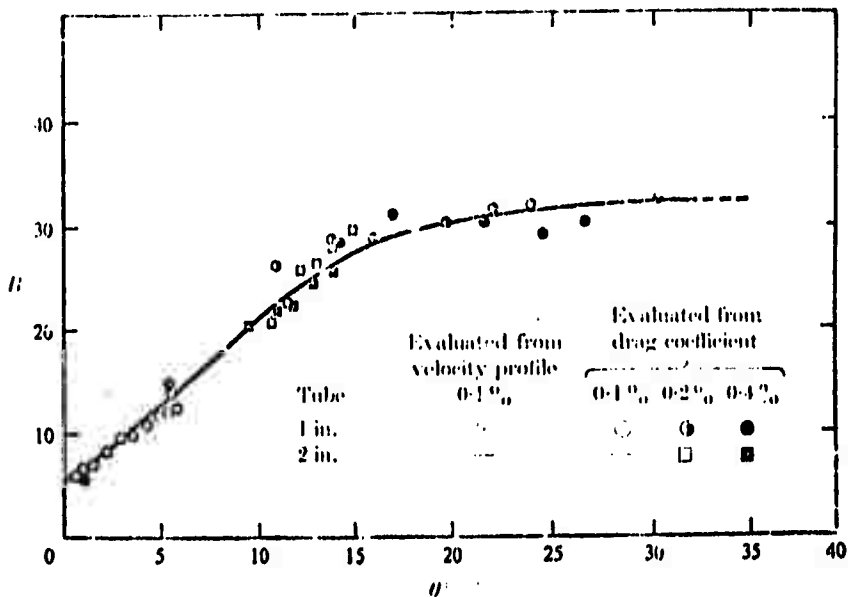
Figure 5. Logarithmic intercept function for drag reduction
velocity profiles (from Seyer & Metzner, 1969b)
Data obtained with polyacrylamide in water.
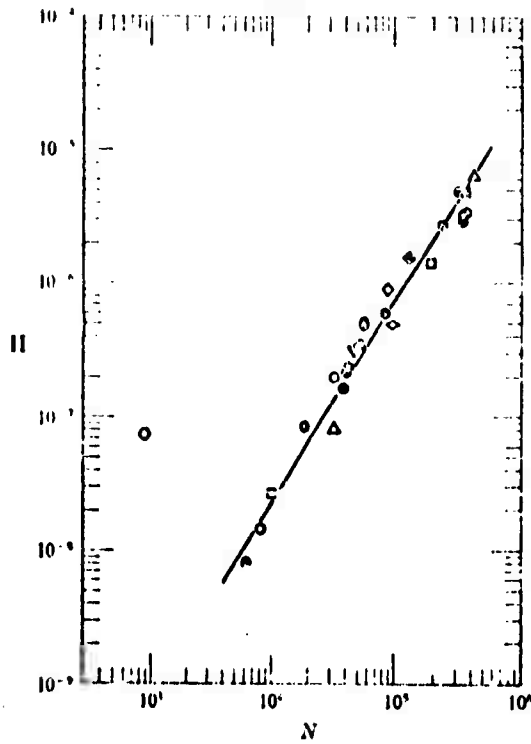
Figure 6. Intrinsic slope increment factor π as function of number of backbone links, N (from Virk, 1971b) $\circ$ guar gum, water; $\diamondsuit$ polyacrylamide, water; $\Leftrightarrow$ polycisisoprene, toluene; $\square$ polydimethylsiloxane, toluene; OOpolyethyleneoxide, water, benzene 0.6 m K_2SO_4; $\triangle$ polyisobutylene, cyclohexane; $\triangledown$ polymethylmethacrylate, chlorobenzene.
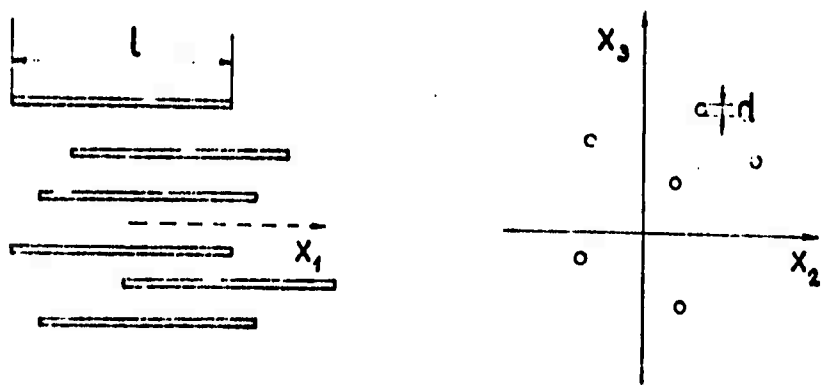
Figure 7. Batchelor's (1971) model of a suspension of aligned rigid rods.

Dynamically unstable
local shear layer

Lifted and stretched
vortex element

Secondary flow
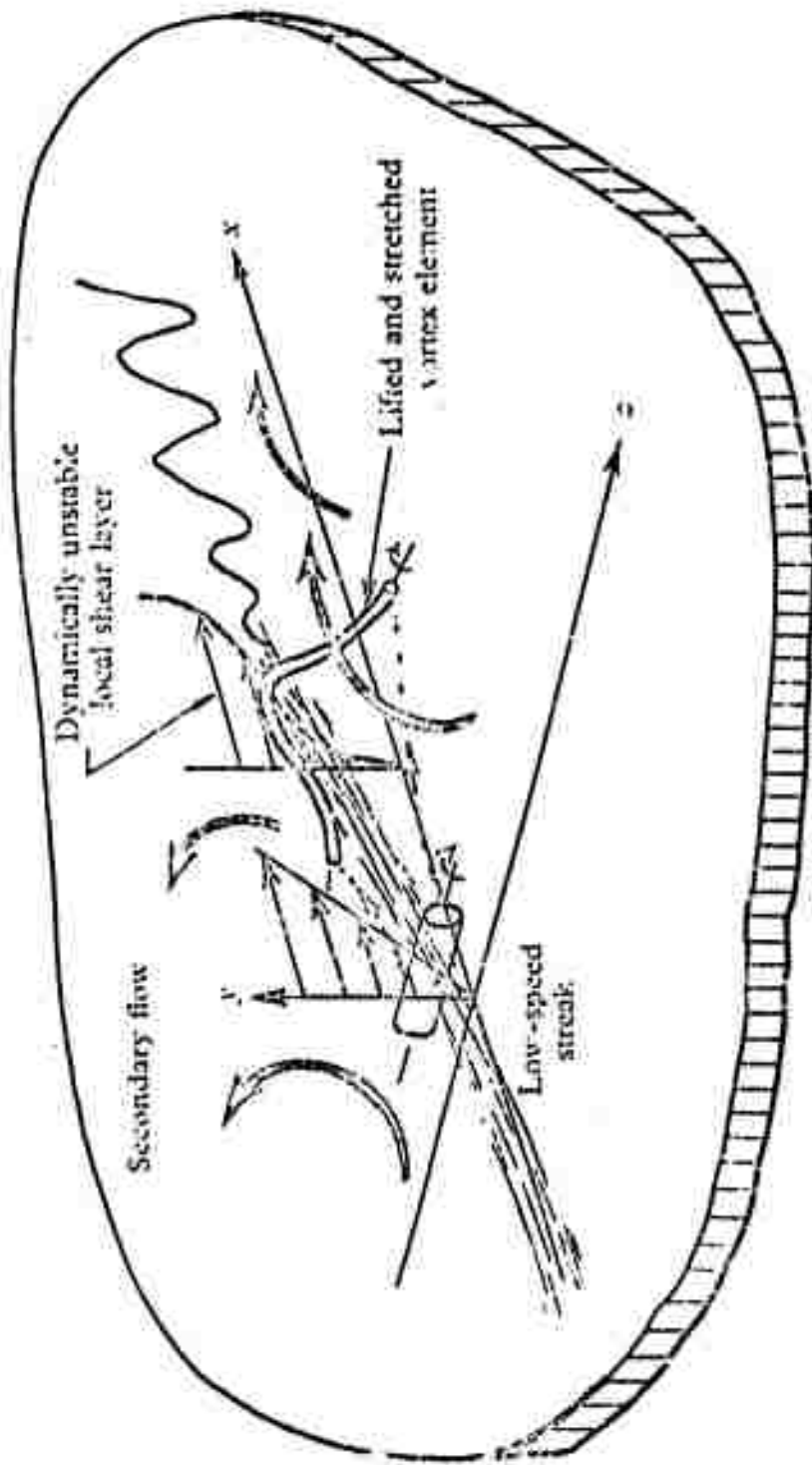
Low-speed
streak

Figure 8. Conceptual model of the structure of a low-speed streak breakup ("burst") in the wall region of a turbulent boundary layer (from Kline et al., 1967)
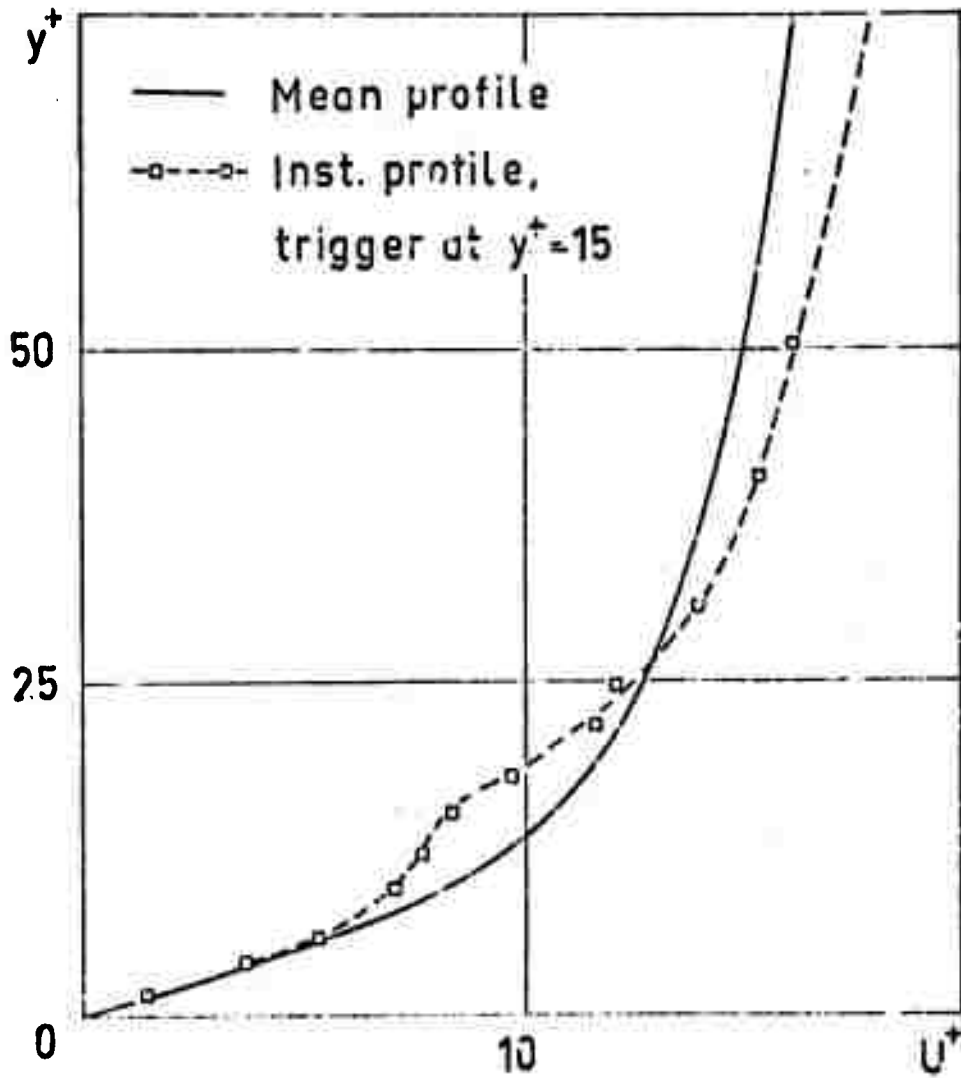
Figure 9. Instantaneous velocity distribution just preceding
           burst measured by Blackwelder & Kaplan (1972)
           Trigger set at y^+ = 15.
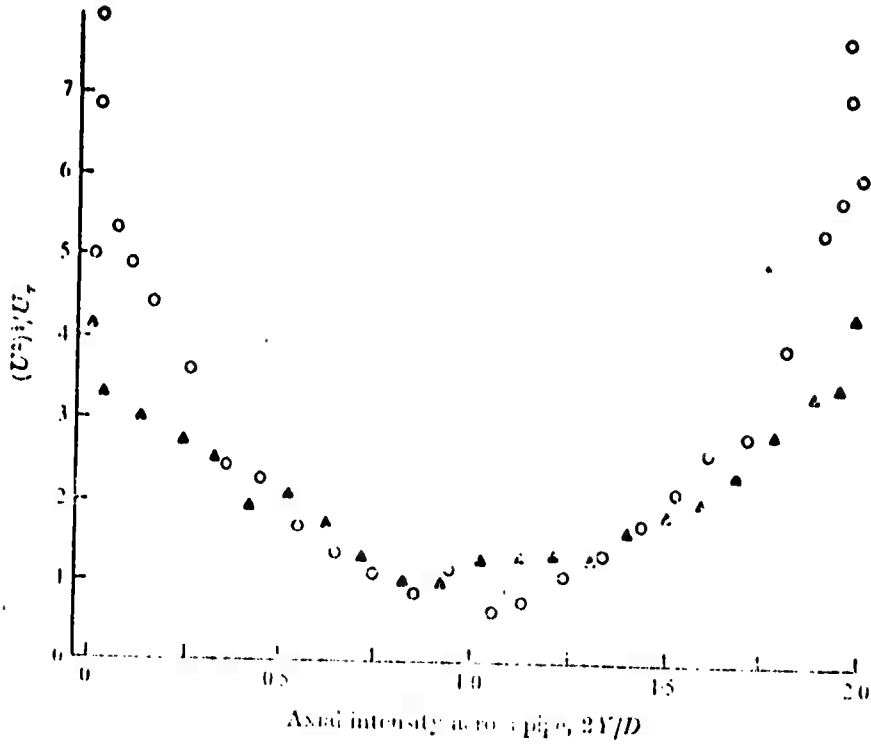
Figure 10.   Intensity of the streamwise turbulent velocities across pipe
(from Rudd, 1972). △ water, ○ 0.01% polyacrylamide.
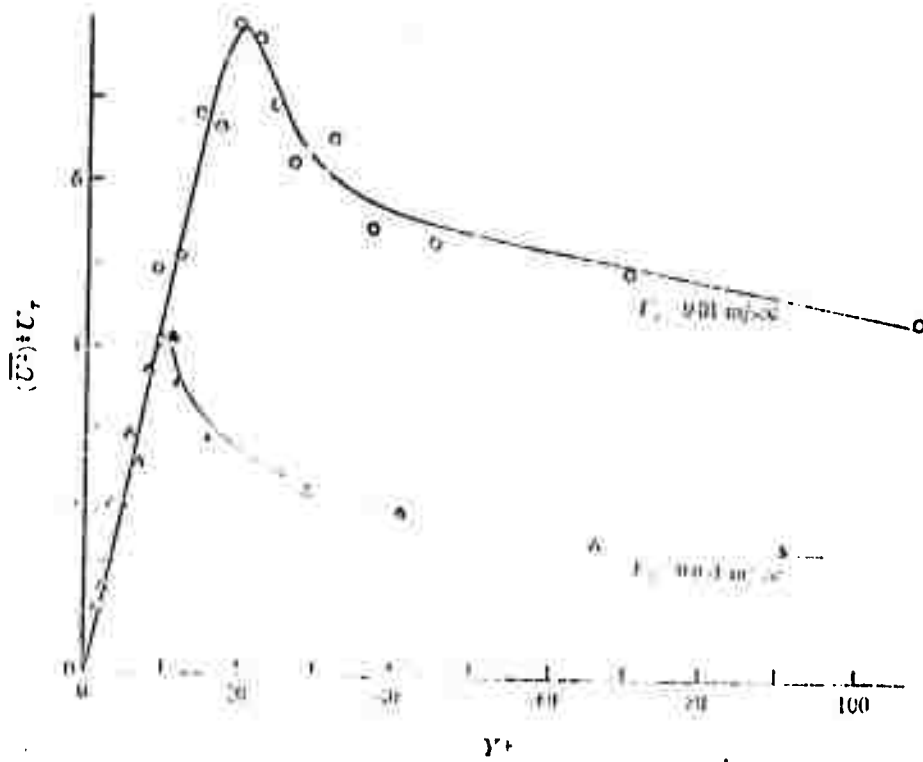
Figure 11.   Intensity of axial turbulent velocities in wall layer
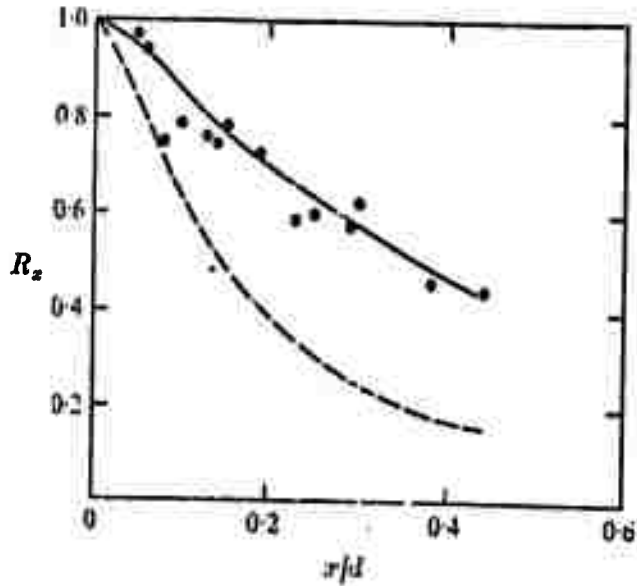
Figure 12.   Spatial correlation coefficient of streamwise fluctuating
wall stresses (from Fortuna & Hanratty, 1977)
--- water; -●-○- solution of polyacrylamid giving 65%
drag reduction.
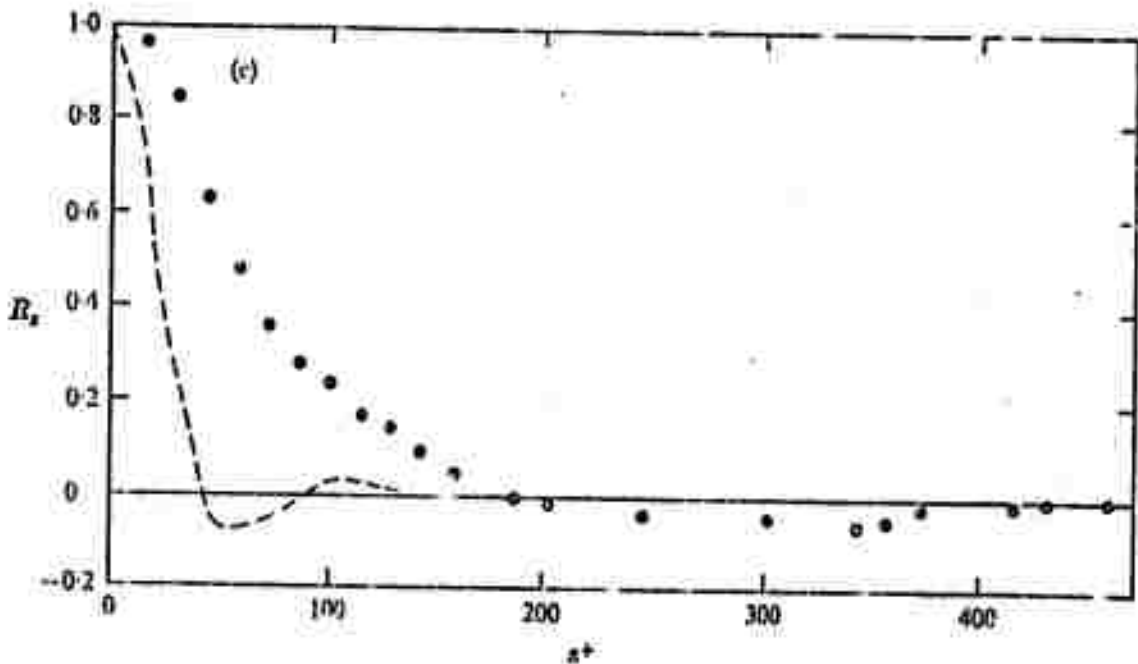
Figure 13.   Spatial correlation coefficient of transverse fluctuating
wall stresses (from Fortuna & Hanratty, 1972). --- water;
○ solution of polyacrylamide giving 49.3% drag reduction.
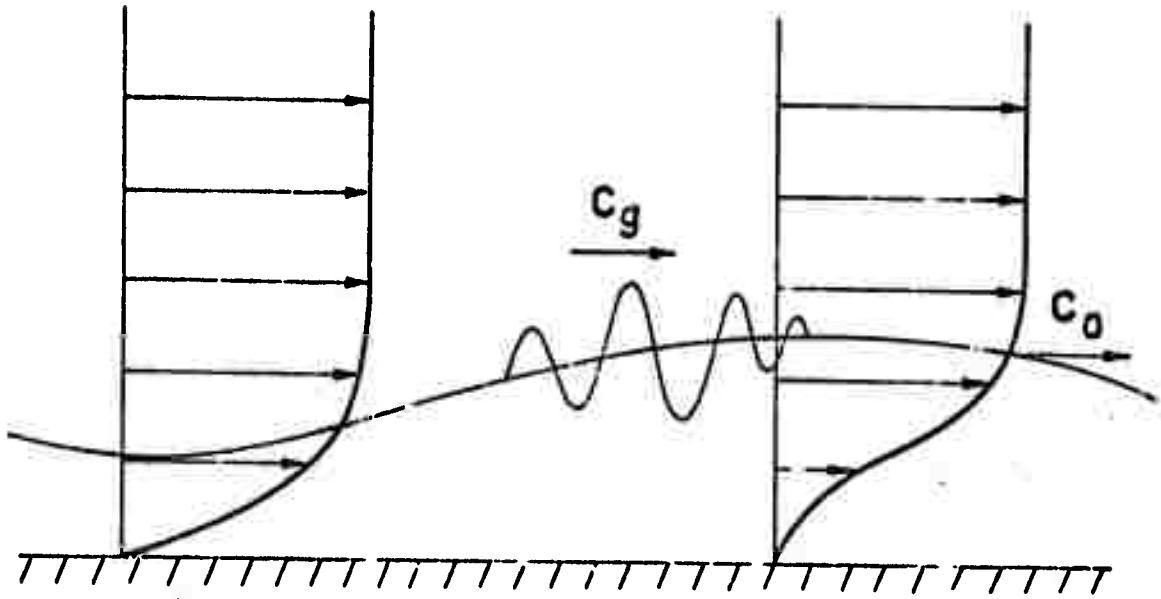
Figure 14. Illustration of critical condition for secondary small-scale wave riding on a primary large-scale one. Breakdown occurs when $c_g = c_0$ and $c_I > 0$.
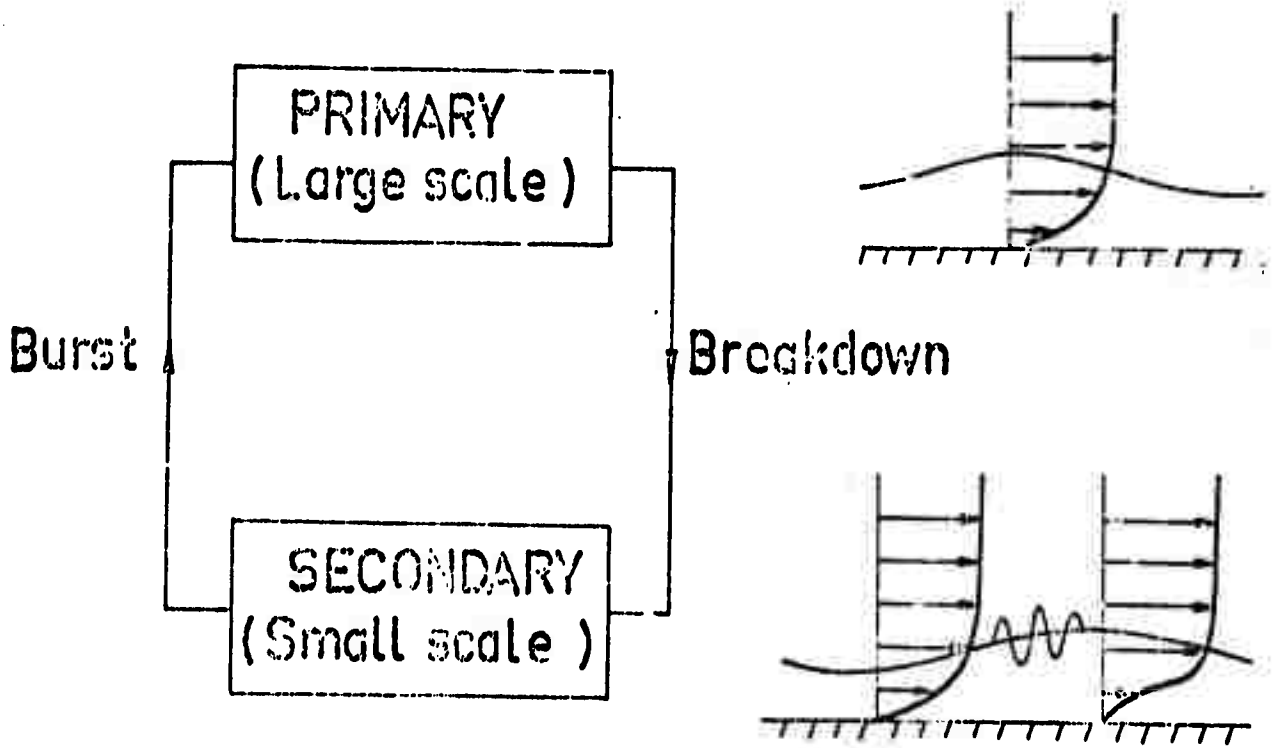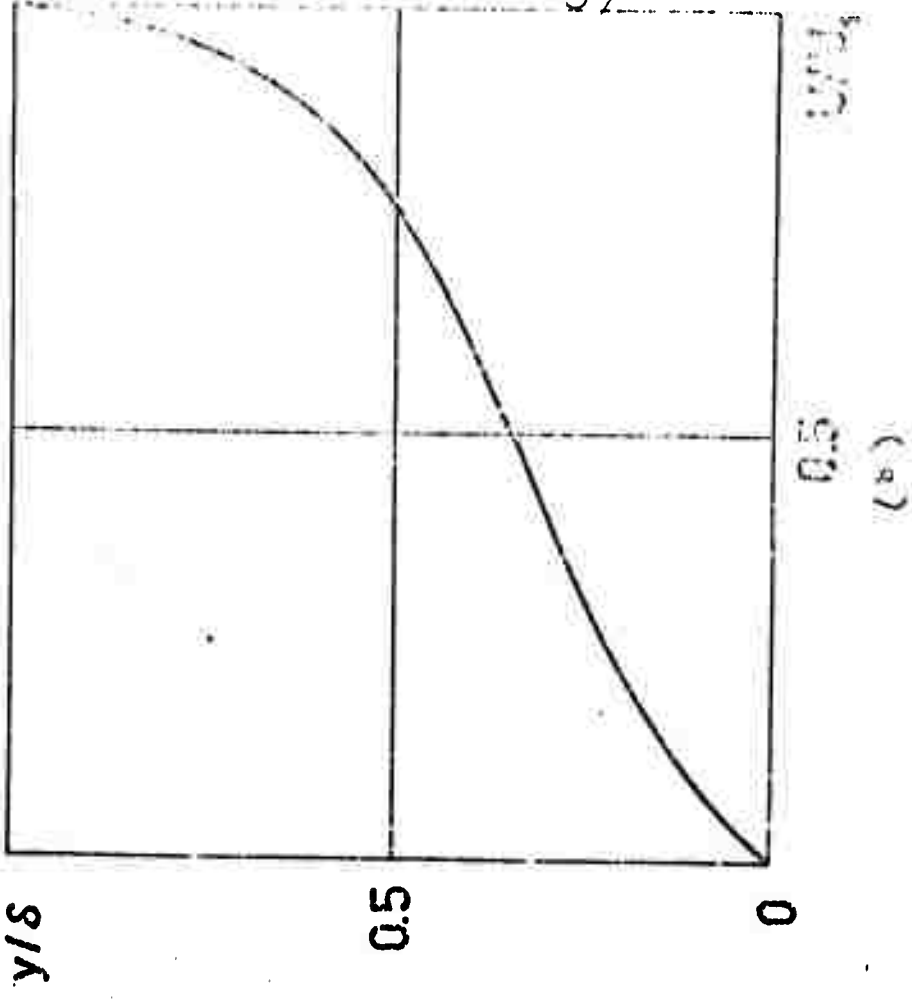

Figure 15. Two-scale model (conceptual) of the fluctuating velocity field in a turbulent boundary layer.

y/δ

0.5

0

U/U₁

DRAG REDUCTION
VELOCITY PROFILE
NEWTONIAN

0.5
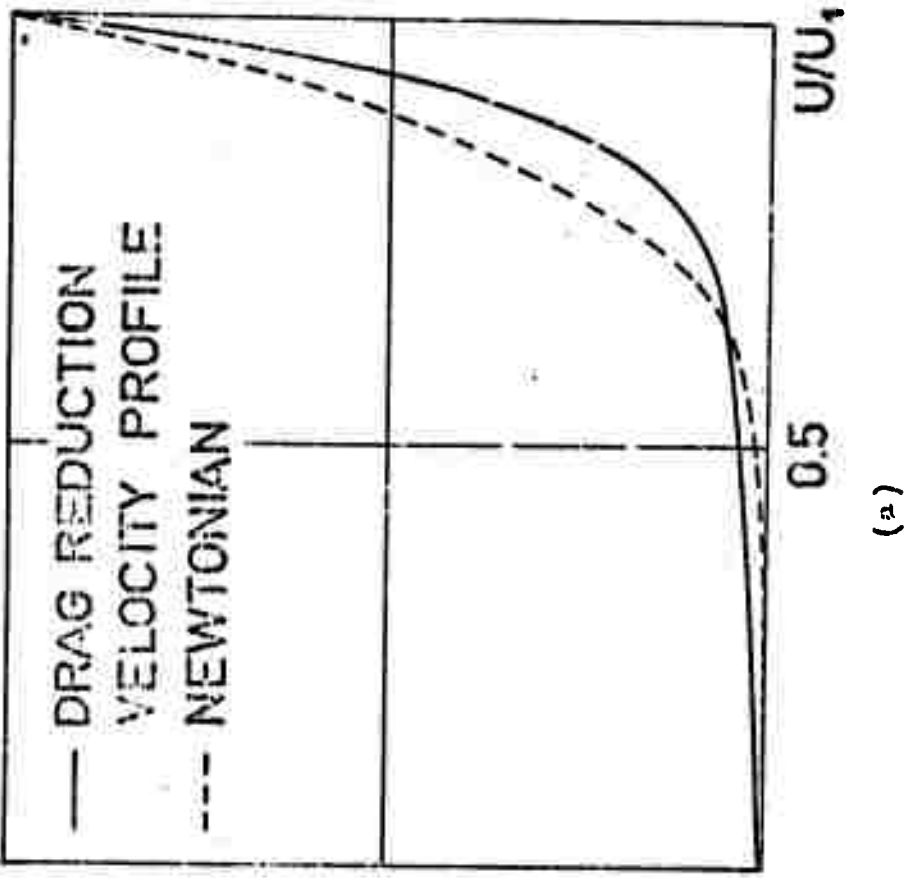
(a)

(b)

Figure 16, a and b. Model velocity profiles used in stability studies. (a) Turbulent mean profiles; -- Newtonian profile (1), — profile with drag reduction (2). (b) Instantaneous velocity profile assumed in study of secondary instability.
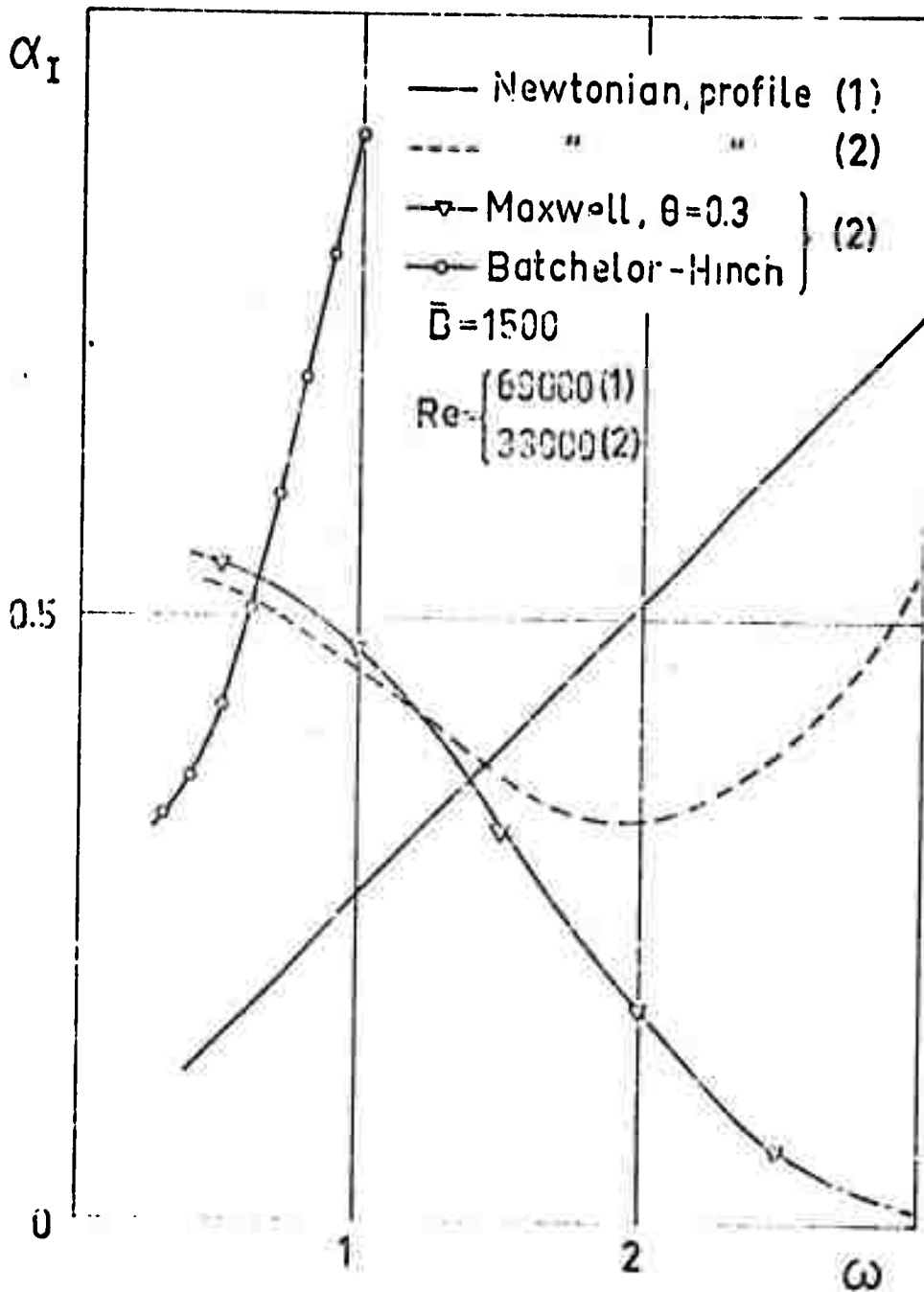
Figure 17. Decay rates for primary waves as function of frequency for the different models. All quantities nondimensionalized with outer variables, δ and U_1.
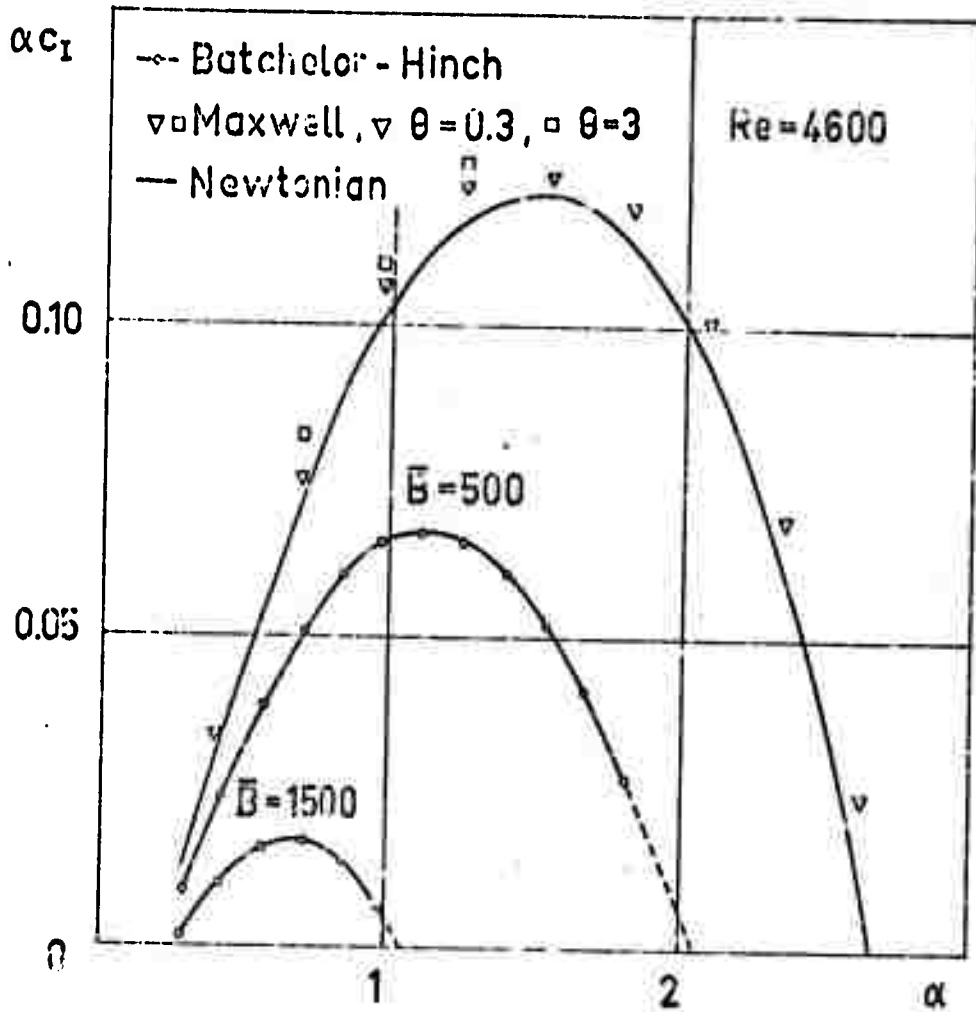
Figure 18. Stability characteristics of secondary disturbance for the different constitutive models. Quantities made nondimensional as in Figure 17.