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COMPARISON OF SOME THEORETICAL NOISE MODELS WITH THE NORSOR MICROSEISMIC NOISE FIELD

Eivind Rygg

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Comparison of some Theoretical Noise Models with the NORSAR Microseismic Noise Field.

Eivind Rygg

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ABSTRACT

The effectiveness of large arrays in mapping the noise field is well known. This paper describes an attempt to estimate the noise fields by using a small number of sensors. The noise fields are defined by their power densities in the frequency-wavenumber space and their validity will be judged by comparing coherence estimates of real data with coherence computations on the basis of the models. The real data-base have been recordings from Oyer array - the first large installation in the NORSAR area.
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INTRODUCTION

During the last few years methods for mapping the noise fields have been presented in literature and results from different sites have been presented in numerous reports.

A commonly used and excellent way of presenting the results is by displaying the power density as a function of the frequency and the wavenumber, thus giving the distribution in azimuth and velocity of the noise fields. However, to map the noise fields this way one must have access to data from large and properly spaced arrays and of course the results are valid only at or near the array sites.

Now the number of large aperture arrays are not very large, and the number of sample points in space are usually limited. The normal case will be recordings from one or a few (2-3) sensors at each site. In this paper we have investigated the possibility of estimating the noise field in frequency – wavenumber space by using the experimental data from only a small number of sensors. The procedure has been to design noise models with specific power distributions and to check the models by coherence computations.
THE NOISE MODEL

Before defining the noise model, let us point out a few requirements that must be met: First of all the model must be simple enough to be mathematically formulated and to allow the calculation of the parameter of interest. Secondly it should not deviate too much from the noise fields as mapped by using arrays in the same area, and thirdly it must explain certain peculiar observations such as variation of coherence with direction (Rygg and al. 1969). With these restrictions in mind we define the model as follows: The theoretical noise field consists of a number of plane, uncorrelated wavetrains approaching from all azimuths and distributed over a certain velocity range. (Fig. 1). Each wavetrain is assumed to have a flat spectrum inside the frequency band of interest for our computations (white noise), and the power density is distributed with varying strength along the periphery.

The reason for choosing this model instead of disc noise sources or a combination of disc noise and fixed velocity arc noise is that we have experienced that this model is a good approximation to the experimentally estimated noise field (Bungum and al. 1971). In the model
we have also allowed for some power variation associated with varying velocity, thus taking into account energy connected with different modes of propagation. On evaluating the theoretical coherence function for this model we follow the lines of Murdock and Pfluke (1970): The periphery is divided into K discrete directions. From each direction we assume that L discrete wavetrains propagate with different velocities, carrying different amounts of power. The total number of wavetrains reaching a sensor is then K·L. The output in the transform domain is:

\[ S(f) = \sum_{k=1}^{K} \sum_{l=1}^{L} A_{k,l}(f) \cdot H_{k,l}(f) \]

Here \( A_{k,l} \) is the Fourier transform at a spatial reference point of the wavetrain with direction index k and velocity index l. \( H_{k,l}(f) \) is a transfer function expressing the effect of the medium from the reference point to the sensor. (We assume the instruments to be identical).

The cross-spectrum between two sensors (1 and 2) is then:

\[ P_{12}(f) = S_1(f) \cdot S_2(f) = \]
\[
\sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{m=1}^{K} \sum_{n=1}^{L} \overline{A_{k,l}(f)} \cdot A_{m,n}(f) \cdot H_{1,k,l}(f) \cdot H_{2,m,n}(f)
\]

The bars represent complex conjugates. Now, since the wavetrains are mutually uncorrelated:

\[
E\left(\overline{A_{k,l}(f)} \cdot A_{m,n}(f)\right) = P_{k,l}^{\text{(auto)-power}}(f), \text{ when } m \neq k \text{ and } n \neq l
\]

= 0 otherwise.

Here, \(P_{k,l}^{\text{(auto)-power}}(f)\) is the (auto)-power spectrum of the wavetrain coming from the \(k\)'th direction and propagating with the velocity which is tied to velocity index \(l\).

If we neglect attenuation and dispersion across the site, the transfer functions \(H_{k,l}(f)\) represent merely phase delays. Thus, if \(t_{1,k,l}\) is the time required for a specific wavetrain to pass from the reference point to sensor 1, the associated transfer function can be written:

\[
H_{1,k,l}(f) = e^{-j\omega t_{1,k,l}} \quad (\omega = 2\pi f)
\]

Then we have:
$$H_{1,k,l}(f) \cdot H_{2,k,l}(f) = e^{i\omega t_{1,k,l}} \cdot e^{-i\omega t_{2,k,l}} \cdot e^{-i\omega (t_{2,k,l} - t_{1,k,l})}$$

$$P_{12}(f) = \sum_{k=1}^{K} \sum_{l=1}^{L} P_{k,l}(f) e^{-i\omega \Delta t_{k,l}}$$

$$P_{21}(f) = \sum_{k=1}^{K} \sum_{l=1}^{L} P_{k,l}(f) e^{i\omega \Delta t_{k,l}}$$

Here $\Delta t_{k,l} = t_{2,k,l} - t_{1,k,l}$ is the time required for the wavefront with the direction index $k$ and velocity index $l$ to pass from sensor 1 to sensor 2. The expression for the coherence is

$$\gamma = \frac{P_{12}(f) P_{21}(f)}{(P_{11}(f) P_{22}(f))^{1/2}}$$

and according to this formula and the foregoing the coherence estimate will be
\[
\left\{ \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{m=1}^{K} \sum_{n=1}^{L} \frac{P_{Wk,l}(f) P_{Wm,n}(f) \cos \omega (\Delta t_{k,l} - \Delta t_{m,n})}{\sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{m=1}^{K} \sum_{n=1}^{L} P_{Wk,l}(f) P_{Wm,n}(f)} \right\}^{1/2}
\]

**DATA AND RESULTS**

In the following we shall compare some of the theoretically computed coherence curves with coherence estimates of real data collected at Oyer subarray (Fig. 2). The theoretical, continuous noise field has been approximated by 35 discrete azimuth directions and 7 individual wavetrains associated with each direction. The power densities of the wavetrains decrease exponentially from a maximum value at a velocity of 3.3 km/s. The total velocity span is 3.0–3.6 km/s. (Fig. 1). The cross and auto spectral power estimates of the real data have been obtained by averaging over 50 nonoverlapping blocks, each of 51.2 sec. Thus each estimate covers 42 2/3 min of recording starting at the times given on the figures. The sampling interval was 0.05 sec.
Fig. 3 shows a weather map for May 18, 1968. Meteorologically this is a very quiet day and by experience we do not expect a very anisotropic noise field due to Atlantic or coastal sources under such conditions. An assumed power distribution at Oyer is shown on top of Fig. 4. Even if we assume that the noise field has a maximum in one direction, there is no reason to believe that there is a minimum in the opposite direction. Therefore we propose isotropic condition around the opposite half periphery. As we see there is a very good fit between the experimental data for 1A1 - 10Y and the theoretical curve calculated for a sensor combination pointing towards the maximum noise power. The result is a support for the noise model proposed and if we use this model it should be located with its maximum in a north-west direction.

In the following we present examples of a more anisotropic model and for comparison we have selected coherence estimates made on a day with dominating Atlantic and coastal noise sources (Fig. 5). In the upper part of Fig. 6 is given a proposed power distribution for the weather situation shown in Fig. 5. The coherence curves of Figs. 5 and 7 demonstrate the general increase in
coherence compared to the more isotropic situation. This is particularly pronounced when the sensor pair is oriented towards the maximum noise power. If we look at a sensor combination abreast of the maximum noise concentration (Fig. 7), the coherence drops rapidly from a high start value.

One may object that the fit between the real estimates and the theoretical curves in Figs. 6 and 7 is not very good. It should be pointed out that there has been no attempt made to get a better fit, for instance by varying details in the parameters. This has several reasons: The noise fields are estimated the indirect way - through the coherence - and for computational reasons the power fields were defined using assumptions which are not valid in general (e.g. whiteness). Furthermore, finer details in the noise field structure can not be explored when one is using only two or three space sampling points.
CONCLUSION

In this paper we have used a most commonly measured parameter - the coherence - to estimate the noise power distribution. The procedure applied only allowed the corroboration of noise models, which were crude approximations to the actual noise fields.

In the way suggested the noise field can be roughly estimated even if the number of space sampling points is small (2-3) and one gets a direct measure of the noise anisotropy.
REFERENCES


Fig. 1. The theoretical Noise Model.
Fig. 2. Oyer Subarray.
Fig. 3. Weather Map May 19, 1968.
Fig. 4. Comparison between theoretical coherence curves (solid lines) and a real coherence estimate (dotted line). The theoretical curves have been calculated using the azimuthal power distribution shown on top. 0 refers to the curve calculated for a sensor combination pointing towards the maximum noise power, while 90 refers to a sensor combination at 90 degrees to this direction. In both cases the sensor separation was 3 km.
Fig. 5. Weather Map March, 28, 1968.
Fig. 6. A real coherence estimate (1A1 - 1F4), and a theoretical curve (solid line). The theoretical curve gives the coherence between two sensors 2.3 km apart and whose connection line is inclined 25 degrees relative to the maximum noise power direction.
Fig. 7. The coherence estimate between 1F1 and 1F4 compared with a theoretical curve for two sensors 2.8 km apart and making an angle of 80 degrees with the direction of maximum noise power. The power distribution is the same as in Fig. 6.