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A NEW TEST OPERATOR, VJ, BASED ON CLASS FREQUENCIES

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FOREWORD

The research work reported herein was conducted by Prof. Dr. Waloddi Weibull, Chemin Fontanettaz 15, 1012 Lausanne, Switzerland under USAF Contract No. F44620-72-C-0028. This contract, which was initiated under Project No. 7451, "Metallic Materials", Task No. 735106, "Behavior of Metals", was administered by the European Office, Office of Aerospace Research. The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio under the direction of Mr. W. J. Trapp, AFML/LL.

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This technical report has been reviewed and is approved.

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ABSTRACT

The test statistic X^2 of the Chi-square test may, if applied to a proper sample, be used for selecting distribution functions. When examining its use for this purpose its decision power was found to be very small due to a kind of pooling, an inherent property of its definition. In order to eliminate this pooling, a new test statistic, denoted by VJ, was introduced. It is defined by the number v_i of sample elements which fall within each of r properly[1] defined classes into which the space of the variable x has been divided. In fact X^2 may be regarded as a statistic obtained from VJ by a pooling procedure. For this reason VJ was expected to have a much larger decision power than X^2 as was verified by the example that the decision power for a specified case being 6.6% for X^2 was raised to 69.1% for VJ.

The properties of VJ have been thoroughly examined. In particular the class limits yielding the largest decision power have been determined with the result that, in some cases, the decision power was found to be somewhat larger than anyone so far attained.

The statistic VJ can also be used for stating whether a hypothetical distribution is acceptable or not and also for selecting the most probable one within a set of such distributions. Necessary tables for the practical use have been prepared.

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I

INTRODUCTION

Let the space of the variable x with the distribution function $F(x)$ be divided into r parts (classes) without common points. The corresponding values of the given probability function will be denoted by $p_1, p_2, \ldots, p_r$ $(\Sigma p_i = 1)$ The test statistic X^2 of the classical Chi-square test of goodness-of-fit is defined by

$$X^2 = \sum_1^r (v_i^2/N.p_i) - N \tag{1}$$

where v_i are the observed and $N.p_i = v_{oi}$ the expected class frequencies. For the particular case, that all the expected frequencies are equal to $v_o = N/r$, equ.(1) takes the form

$$X^2 = \sum_1^r v_i^2/v_o - N \tag{2}$$

It is evident that X^2 is, in this case, invariant under permutations of the order of the observed frequencies v_i. For example, if $N = 8$, $r = 2$, $v_o = 4$, then $X^2 = 8.0$ as well for $v_1 = 0$, $v_2 = 8$ as for $v_1 = 8$, $v_2 = 0$.

The test statistic X^2, which is a variable of the discrete type, has, in this example, the total mass of its distribution concentrated in the five mass points: 0,0.5,2.0,4.5 and 8.0. If we, alternatively, introduce a new test statistic VJ, defined by the possible pairs of observed class frequencies, which may be regarded as the coordinates of the mass points, that is

$$VJ = (v_1, v_2) \tag{3}$$

then the total mass of its distribution is concentrated in the nine points: (0,8),(1,7),...,(8,0). Thus it can be said that X^2 is derived from VJ by a procedure which is equivalent to the pooling of classes of a continuous test statistic. It is easily proved that a pooling procedure can never increase the decision power of a test operator (Cf.Reference [1]), but may, in some cases, catastrophically reduce it, as will be illustrated in the sequel. A more general definition of VJ, than that given by equ.(3), will now be presented.

II

SIGNIFICATION OF THE SYMBOL VJ

The symbol VJ will be used to denote as well a test operator as a test statistic. The test operator VJ is defined by the procedure of dividing the space of the variable x into an arbitrary number r of classes with the upper limits $c_1, c_2, \ldots, c_r$ which may be selected without regard to any distribution function. Such an operator may be denoted by VJA. Alternatively, the division of the space can be specified by the class probabilities p_i, in which case it may be preferable to take all the probabilities equal to

$$p_i = 1/r \tag{4}$$

The corresponding class limits for any hypothetical distribution function $F(x)$ are then given by

$$c_i = F^{-1}(i/r) - F^{-1}((i-1)/r) \tag{5}$$

If $F(x)$ is the distribution function of the population R, the operator may be denoted by VJR. The result of its acting upon random samples from the population S may be denoted by VJRS.

From any given sample $x_1, x_2, \ldots, x_N$ the number of elements falling within the i:th class, that is, the observed frequency v_i, is determined. The value of the test statistic VJ, thus obtained, will be denoted by

$$VJ = (v_1, v_2, \ldots, v_r) \quad (\Sigma v_i = N) \tag{6}$$

where the r values v_i may be regarded as the coordinates of the mass points of the distribution of the statistic VJ.

III

FUNDAMENTAL PROPERTIES OF THE STATISTIC VJ

The statistic VJ is a variable of the discrete type. The total mass of its distribution is concentrated in a finite number K of mass points, where

$$K = (N + r - 1)!/N!(r-1)! \tag{7}$$

In the particular case of pseudo-standardized samples, defined by the sample elements

$$t_i = (x_i - x_1)/(x_N - x_1) \tag{8}$$

the number of non-fixed elements of the sample is reduced to $(N-2)$, since $t_o = 0$; $t_N = 1$. Thus the number of mass points becomes

$$K_t = (N + r - 3)!/(N - 2)!(r - 1)! \tag{9}$$

Some values of K_t are presented in Table 1.

These many mass points may be brought together into groups, called types of VJ. Each type consists of all possible permutations of a given set of values of v_i. The number of different types are listed in Table 2 for some values of N and r.

As an illustration, the types and the corresponding numbers of permutations are given for $N = 10$, $r = 4$ and $r = 8$, pseudo-standardized samples, in Table 3.

The number of permutations NP is given by the formula

$$NP = (N-2)!/n_1!n_2!n_3! \tag{10}$$

where n_1 = the number of frequencies v_1, n_2 that of frequencies v_2, and so on. Hence

$$\Sigma n_i v_i = N - 2 \tag{11}$$

Also the values of X^2, corresponding to each type of VJ are included in Table 3.

The statistical properties of VJ are completely described by its distribution, that is, the set of probability masses p_v of each of the K or K_t mass points. This distribution can be computed, for any given situation, by means of Monte-Carlo studies, as will be described in the following.

Even with modern computers the number K_t will be too large for large enough samples, and some kind of pooling may be required.

From Table 3 different stages of pooling may be distinguished. The total number of mass points K_t may be grouped according to:

1. The types of VJ
2. The values of X^2
3. The number NE of empty classes

with a reduction of the number of mass points as indicated below for $N = 10$.

Stage of pooling	Nr. of mass points	
	$r = 4$	$r = 8$
VJ, ungrouped	165	6435
Types of VJ	15	22
X^2	14	18
NE	4	8

The reduction of the decision power due to the poolings will be illustrated below.

The test operators VJ may, just like other operators, be made to act upon three different types of samples, transformed by use of the following formulas:

1. $u_i = x_i - x_1$ producing location invariance

2. $s_i = x_i / x_N$ producing scale invariance (12)

3. $t_i = (x_i - x_1)/(x_N - x_1)$ producing both scale and location invariance

If the distribution function, including its shape parameter, if any, is known, then the sample $[u_i]$ depends only on the scale parameter β, and the sample $[s_i]$ only on the parameter quotient μ/β. In any case, the sample $[t_i]$ depends only on the shape parameter α.

These three types of operations may be used for two different purposes

1) to decide, with known decision power, between two different values of β, μ/β, or α, and

2) to test whether an assumed value of these parameters is acceptable on the basis of a preassigned level of significance.

The present report will deal only with the shape operators.

IV
THE SHAPE PARAMETER AND ITS DECISION POWER

The most important property of an operator, its decision power DP, is determined by comparing the distributions of the actual test statistics. It is evident that these distributions depend not only on N and r but also on the class limits selected. Thus, the fundamental problem will be to determine that set of class limits which will yield the highest decision power.

To this purpose the computer programs 1/72, 2/72, and 3/72 have been written by Göran W.Weibull. A large number of random samples, usually 10,000, of a given size N and belonging to a normal population or Weibull distribution with given shape parameter α are generated and transformed into t_i-samples. For an arbitrary set of class limits t_c, the frequency functions of the test statistic $VJ = (v_1, v_2)$ (Program 1/72) $VJ = (v_1, v_2, v_3)$ (Program 2/72) and $VJ = (v_1, v_2, v_3, v_4)$ (Program 3/72) are determined. From the obtained frequency dbns, the decision powers have been computed for various combinations of normal and Weibull distributions and class divisions. The results for the dbns: 1.0, 0.1, 0 and $N = 6, 10, 20$, $r = 2, 3, 4$ are listed in Table 4, and $N = 6, 10, 20$; $r = 4$ and $\alpha = 1.0, 0.9, 0.7, 0.5, 0.3, 0.1$, and 0 (normal dbn) in Table 5, where also the estimation powers EP (Cf.Ref[1]) are given.

The EP of the most powerful shape operator, so far obtained, denoted by $T(i,j,k)$, is also presented for $N = 20$ and is found to be somewhat lower than that of VJ.

V

THE USE OF VJ FOR TESTING THE ACCEPTABILITY OF AN ASSUMED DISTRIBUTION FUNCTION

Suppose, for illustration purpose, that we will test whether a given sample of size $N = 10$ is drawn from a normal population or not by use of the shape operator VJ specified by the class limits $t_c = 0, 0.250, 0.375, 0.500, 1$.

After having transformed the sample into a t_i-sample by use of equ.(8), the number of elements v_i within each of the four classes are determined. This procedure yields a test point

$$VJ = (v_1, v_2, v_3, v_4) \quad (\Sigma v_i = 8)$$

The question now arises whether this test value is acceptable or not. To this purpose the distribution of VJ is computed by use of Program 3/72 which generates 10,000 random normal samples and counts the number v_v corresponding to each of the 165 mass points. If the number corresponding to the actual test point is zero or very small, the hypothesis of normality will be rejected.

The critical value of v_v can be put in relation to a preassigned level of significance e by use of a statistic called <u>the test level</u> and denoted by TL (Cf.Ref.[1]). It is thus defined: Let n_i be the number of mass points which have a $v_v = i$ $(i = 1, 2, 3, \ldots)$, then

$$TL = \Sigma(i.n_i)/\Sigma v_v \tag{13}$$

that is, TL is equal to that relative number of mass points which have $v_v \leq i$. In this way there will be assigned a definite value of TL to each mass point. The computation of the TL-distribution is included as a part of Program 3/72.

The criterion of rejection now becomes: The hypothesis that the sample belongs to an assumed population is rejected if

$$TL \leq e$$

Some values of TL are presented in Table 6,7 and 8,9,10 for $N = 6$, 10 and 20, respectively, and for various hypothetical distributions.

By use of these tables it is quite simple to decide whether an assumed distribution function is acceptable or not and also which of various dbn functions is the most probable one. Suppose we have obtained a test value

$$VJ_{test} = (0,1,2,5)$$

using the class limits indicated in Table 7, then four of the hypothetical dbns are acceptable, but the normal one is the most probable. Three of the dbns are rejected on the basis of a 5% level of significance.

A test value $VJ = (0,1,6,1)$ motivates the rejection of all normal and Weibull distributions, which will be taken as a strong indication that the examined sample is not a simple but a composed one.

So far, the operator VJ has been specified by a set of class limits t_c selected without regard to the corresponding class probabilities p_i. This way of proceeding requires individual TL-tables for each hypothetical function.

It is, however, possible to produce a TL-table of more general applicability by selecting the class limits according to a given set of class probabilities, preferably all of them equal to $p_i = 1/r$. It seems plausible that we then will have the correct individual TL-functions by using the $(r+1)$ class limits

$$t_c = F^{-1}(c/r) \quad (c = 0,1,\ldots,r) \tag{15}$$

This statement seems self-evident when the operator is acting upon a sample, where the non-ordered elements are independent of each other, but not at all, when it is acting upon a t-sample, where the elements within each sample form an individual entirety.

The important hypothesis that an operator VJ, defined by a given set of class probabilities (instead of a set of class limits t_c), yields a distribution which can be used for any distribution function, provided that the class limits are selected according to equ.(15), has been proved by use of three different tests in the following way:

Program 3/72 was applied to 3x10,000 random t-samples of size $N-2=8$, drawn from the normal, the Weibull, $\alpha = 1.0$, and the Weibull, $\alpha = 0.1$, populations. The expected frequencies v_v of the 3x165 mass points of VJ were determined for the class limits $t_c = 0, 0.303, 0.500, 0.697, 1.0$; $t_c = 0, 0.090, 0.227, 0.445, 1.0$; and $t_c = 0, 0.380, 0.585, 0.763, 1.0$, respectively. These class limits yield class probabilities with insignificant deviations from $p_i = 25\%$.

The hypothesis that these three sets of obtained frequencies v_v belong to the same population was tested by use of three tests fully described by Dixon & Massey [2]:
a) the sign test b) the number-of-runs test, and
c) the rank-sum test, applied to fourteen of the fifteen types of VJ. The type VJ = (2,2,2,2) was excluded as having only one pair of elements. The results are presented in Table 11 with the following comments.

The sign test is based on the signs of the differences between the corresponding values of v_v. If some of the differences are zero, they will be excluded and the sample size reduced. The number of positive and negative differences are listed in the first column of the sign test. The sample size is equal to their sum. If r denotes the number of times the less frequent sign occurs, Table A-10a of Ref.[2] gives the critical value of r for 1, 5, 10, and 25 per cent levels of significance. From this table the values listed in the second column of the sign test have been taken. For example, for one plus sign and eleven minus signs, the size = 12, r = 1. A value of r less than or equal to 1 is rejected on the 1% level of significance.

Of the 28 tested samples there are

1	where the sample size is too small for decision							
1	where	the	hypothesis	is	rejected	on	the	1% level
3	"	"	"	"	"	"	"	5% "
1	"	"	"	"	"	"	"	10% "
22	"	"	"	"	"	"	"	≤25% "

There is no reason to reject the hypothesis on the whole.

The number-of-runs test is used for testing the random arrangement of the plus and the minus signs. The total numbers of runs observed are listed in the first column of the nr-of-runs test, and in the second column the chance of obtaining a value less than or equal to these numbers, taken from Table A -11 of Ref.[2]. On the whole, rejection of the hypothesis seems unmotivated.

The rank-sum test provides another means of testing the hypothesis. The two samples are arranged in order of size and rank scores are assigned to the individual observations. The sums of the ranks of each sample are denoted by T_1 and T_2. Table A-20 of Ref.[2] has been used for obtaining the rejection percentages, of which there is only one small value (2.5%). This test is strongly in favour of accepting the hypothesis.

On this condition, the three sets of v_v, based on 30,000 random samples, can be pooled. The corresponding TL-values have been computed and are presented in Table 12, which is applicable to any hypothetical distribution, if the test value VJ have been determined by use of the class limits given by equ.(15). The limits for $r = 4$ and $r = 8$ are identical with the percentiles listed in Table 13.

A graphical representation of these percentiles as functions of the shape parameter α of the Weibull distribution is shown in Fig.1. This graph is quite useful for a determination of the test value of VJ. Since the given sample has been transformed into a t-sample, the order statistics t_i may be plotted on a separate paper using the scale of t_p. By shifting this plotting horizontally, the number of sample elements within each of the classes are easily counted for any wanted α.

It is of interest to note that the percentiles of the normal distribution, which are plotted against $\alpha = 0$, coincide very closely with those of $\alpha = 0.28$. Hence, there will be no possibility of deciding between these two distributions by means of the operator VJ.

VI

THE EFFECT OF POOLING ON THE DECISION POWER OF VJ

In the table on page 4, three different stages of pooling the probability masses of the statistic VJ and the corresponding reduction of the number of its mass points have been indicated. It is expected that these considerable reductions will strongly affect the decision power.

For illustration purpose, the shape operator VJ, pseudo-standardized variables, $N = 10$, $t_c = 0, 0.250, 0.375, 0.500, 1.0$, will be taken as an example.

From the tables of the expected frequencies v_γ, corresponding to the 165 mass points and computed by use of Program 3/72, the expected values of v_γ, corresponding to each of the mass points of the reduced sets, are obtained by summing the v_γ-values within each group. In this way the required probability distributions are easily computed, and from them the decision powers. The results are listed in Table 14, which demonstrates the catastrophic effect of the poolings on the decision powers, and, in particular, the weak decision power of the test statistic X^2 of the Chi-square test of goodness-of-fit. This result implies that this test will, except for very large samples, frequently fail to reject false hypothetical distributions.

REFERENCES

1. Weibull, W., "Outline of a theory of powerful selection of distribution functions", AFML-TR-71-52, March 1971.

2. Dixon, W.J., "Introduction to Statistical Analysis". Mc Graw-Hill Book Comp., Inc., New York 1957

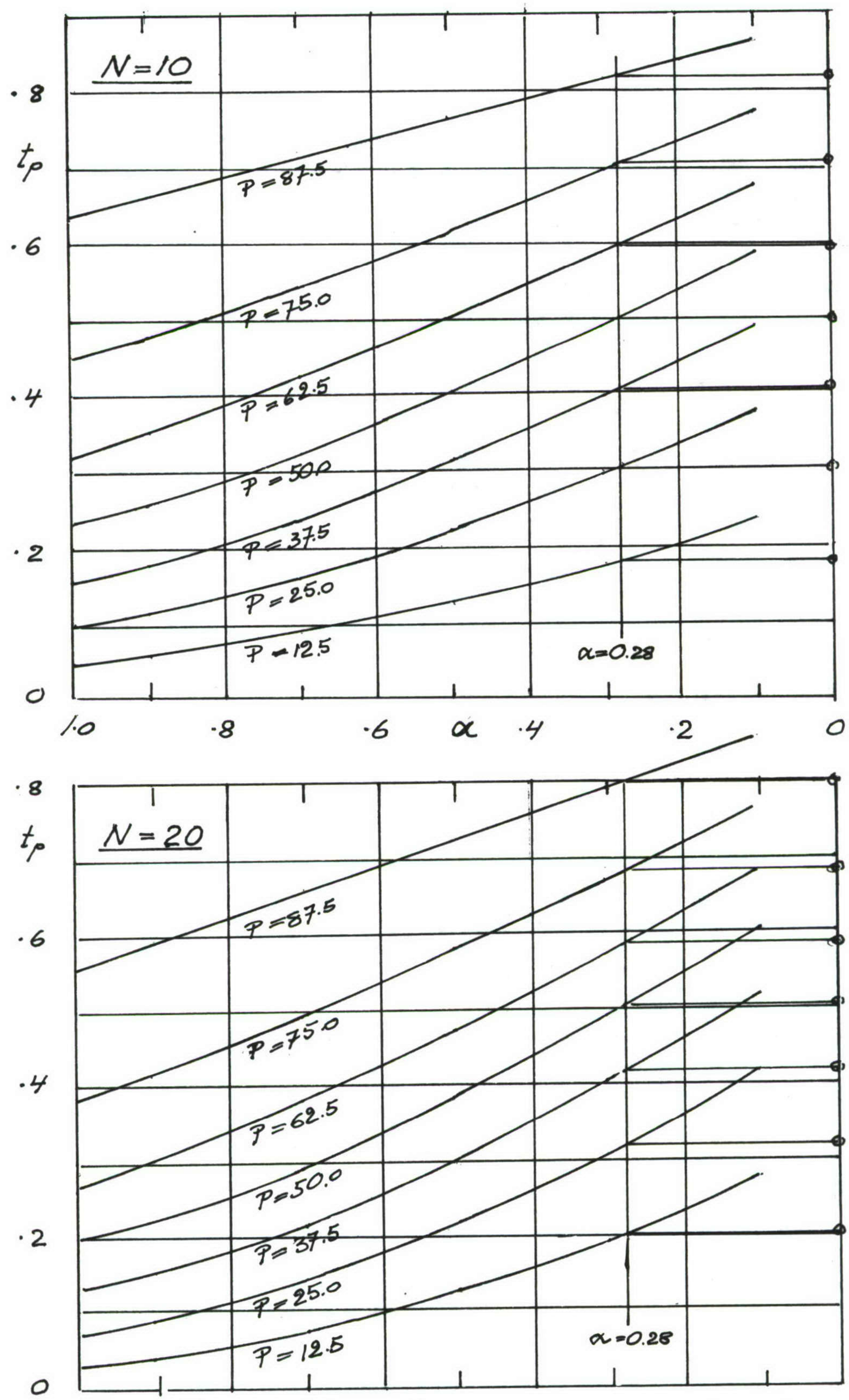

Fig. 1. Percentiles of pseudo-standardized variables for various dbn. functions

Table I. Number of mass points K_t for some N and r

N-2 \ r	2	3	4	5	6	7	8
2	3	-	-	-	-	-	-
3	4	10	-	-	-	-	-
4	5	15	35	-	-	-	-
5	6	21	56	126	-	-	-
6	7	28	84	210	462	-	-
7	8	36	120	330	792	1716	-
8	9	45	165	495	1287	3003	6435
9	10	55	220	715	2002	5005	
10	11	66	286	1001	3003	8008	
11	12	78	364	1365	4368		
12	13	91	455	1820	6188		
13	14	105	560	2380			
18	19	190	1330	7315			
23	24	300	2600				
28	29	435	4495				
33	34	595					
38	39	780					
43	44	990					
48	49	1225					

Table II. Number of different types of VJ for some N and r, pseudo-standardized samples

N-2 \ r	2	3	4	5	6	7	8	9	10
2	2	-	-	-	-	-	-	-	-
3	2	3	-	-	-	-	-	-	-
4	3	4	5	-	-	-	-	-	-
5	3	5	6	7	-	-	-	-	-
6	4	7	9	10	11	-	-	-	-
7	4	8	11	13	14	15	-	-	-
8	5	10	15	18	20	21	22	-	-
9	5	12	18	23	26	28	29	30	-
10	6	14	22	29	34	37	39	40	41

Table III. Different types of VJ for N = 10, r = 4 and 8, pseudo-standardized samples

r = 4				
	Type of VJ	Nr. of permutations		X^2
1	0008	4	4	24
2	0017	12		17
3	0026	12		12
4	0035	12		9
5	0044	6	42	8
6	0116	12		11
7	0125	24		7
8	0134	24		5
9	0224	12		4
10	0233	12	84	3
11	1115	4		6
12	1124	12		3
13	1133	6		2
14	1223	12		1
15	2222	1	35	0

Total 165

r = 8				
	Type of VJ	Nr. of permutations		X^2
1	0000.0008	8	8	56
2	0000.0017	56		42
3	0000.0026	56		32
4	0000.0035	56		26
5	0000.0044	28	196	24
6	0000.0116	168		30
7	0000.0125	336		22
8	0000.0134	336		18
9	0000.0224	168		16
10	0000.0233	168	1176	12
11	0000.1115	280		20
12	0000.1124	840		14
13	0000.1133	420		12
14	0000.1223	840		10
15	0000.2222	70	2450	8
16	0001.1114	280		12
17	0001.1123	1120		8
18	0001.1222	560	1960	6
19	0011.1113	168		6
20	0011.1122	420	588	4
21	0111.1112	56	56	2
22	1111.1111	1	1	0

Total 6435

Table IV. Decision power DP of the shape operator VJ, pseudo-standardized variables

Class limits	.125	.250	.375	.500	.625	.750	.875
	Program 1/72: N = 6; r = 2						
1.0 vs 0.1	46.0	50.1	51.8	50.5	41.9	35.3	22.0
1.0 vs 0	41.2	43.9	40.9	40.0	30.1	26.3	15.4
0.1 vs 0	4.9	9.1	10.8	13.0	11.8	9.0	6.9
	Program 1/72: N = 10 ; r = 2						
1.0 vs 0.1	65.6	73.9	74.2	69.8	61.0	50.0	34.7
1.0 vs 0	60.8	64.0	60.9	55.1	46.3	32.6	31.7
0.1 vs 0	8.2	14.3	18.0	20.6	20.3	17.5	11.6
	Program 1/72: N = 20 ; r = 2						
1.0 vs 0.1	91.1	94.2	93.9	90.7	84.0	70.8	46.7
1.0 vs 0	85.2	88.4	85.0	77.8	65.0	49.3	31.5
0.1 vs 0	14.3	23.7	31.1	35.9	35.4	29.5	19.4

Class limits	.125 .250	.250 .375	.375 .500	.500 .625	.625 .750	.750 .875
	Program 2/72: N = 6 ; r = 3					
1.0 vs 0.1	50.1	53.6	51.8	50.5	41.9	35.3
1.0 vs 0	43.9	45.5	42.8	40.0	30.1	26.3
0.1 vs 0	9.1	10.8	13.1	13.1	11.8	9.3
	Program 2/72: N = 10 ; r = 3					
1.0 vs 0.1	73.9	75.6	74.5	69.9	61.2	50.0
1.0 vs 0	59.8	65.4	61.2	53.1	46.3	34.0
0.1 vs 0	14.9	18.0	20.7	21.5	20.6	17.5
	Program 2/72: N = 20 ; r = 3					
1.0 vs 0.1	94.6	95.1	93.6	90.7	84.3	71.1
1.0 vs 0	89.3	88.6	85.2	77.8	65.4	49.7
0.1 vs 0	23.7	31.3	37.0	37.1	36.0	30.6

Class limits	.125 .250 .375	.250 .375 .500	.375 .500 .625	.500 .625 .750	.625 .750 .875
	Program 3/72: N = 6 ; r = 4				
1.0 vs 0.1	55.1	55.2	52.5	50.5	41.9
1.0 vs 0	47.0	45.8	42.4	40.0	30.2
0.1 vs 0	11.3	13.1	13.5	13.2	11.8
	Program 3/72: N = 10 ; r = 4				
1.0 vs 0.1	77.2	77.2	74.7	70.5	61.6
1.0 vs 0	68.3	69.1	62.5	56.5	46.4
0.1 vs 0	19.6	22.0	23.0	22.7	20.9
	Program 3/72: N = 20 ; r = 4				
1.0 vs 0.1	95.8	95.6	94.2	91.4	84.6
1.0 vs 0	90.7	89.1	85.6	78.4	65.4
0.1 vs 0	32.9	37.4	38.9	39.0	37.3

Table V. Decision power DP of the shape operator VJ, pseudo-standardized variables

$N = 6$; $r = 4$; $t_c = 0$; 0.250 ; 0.375 ; 0.500 ; 1.000

α \ α	1.0	0.9	0.7	0.5	0.3	0.1	normal
1.0	-	6.1	18.3	31.6	44.4	55.2	45.8
0.9	6.1	-	11.1	26.0	39.7	50.7	41.1
0.7	18.3	11.1	-	14.2	29.0	41.6	30.8
0.5	31.6	26.0	14.2	-	15.6	29.1	17.6
0.3	44.4	39.7	29.0	15.6	-	15.2	3.0
0.1	55.2	50.7	41.6	29.1	15.2	-	13.1
normal	45.8	41.1	30.8	17.6	3.0	13.1	-
EP	61.0	57.3	63.2	74.5	77.0	76.0	-

$N = 10$; $r = 4$; $t_c = 0$; 0.250 ; 0.375 ; 0.500 ; 1.000

α \ α	1.0	0.9	0.7	0.5	0.3	0.1	normal
1.0	-	9.5	28.5	47.8	65.2	77.2	69.1
0.9	9.5	-	20.5	40.9	59.6	73.8	61.2
0.7	28.5	20.5	-	23.6	45.2	62.3	46.9
0.5	47.8	40.9	23.6	-	26.1	47.1	27.4
0.3	65.2	59.6	45.2	26.1	-	24.1	7.0
0.1	77.2	73.8	62.3	47.1	24.1	-	22.0
normal	69.1	61.2	46.9	27.4	7.0	22.0	-
EP	95.0	100.0	110.2	124.2	125.5	120.5	-

$N = 20$; $r = 4$; $t_c = 0$; 0.125 ; 0.250 ; 0.375 ; 1.000

α \ α	1.0	0.9	0.7	0.5	0.3	0.1	normal
1.0	-	19.8	49.3	75.9	89.8	95.8	90.7
0.9	19.8	-	37.2	67.4	85.3	93.4	86.8
0.7	49.3	37.2	-	40.5	71.6	85.2	72.1
0.5	75.9	67.4	40.5	-	41.4	68.0	46.2
0.3	89.8	85.3	71.6	41.4	-	38.1	13.9
0.1	95.8	93.4	85.2	68.0	38.1	-	32.9
normal	90.7	86.8	72.1	46.2	13.9	32.9	-
EP	198.0	190.0	194.2	204.8	198.8	190.5	-
EP(T)	173.7	175.9	188.3	202.2	198.7	188.7	

Table VI. Test levels of the mass points VJ, pseudo-standard. variables, $N = 6$, $t_c = 0, 0.250, 0.375, 0.500, 1.000$

Mass points		Hypothetical distributions						
No.	VJ	$\alpha = 1.0$	0.9	0.7	0.5	0.3	0.1	normal
1	0004	4.0	5.8	13.2	34.2	74.3	100.0	81.8
2	0013	9.4	9.8	25.4	42.3	82.0	84.9	90.5
3	0022	4.0	3.0	8.2	19.4	27.1	24.1	30.1
4	0031	0.9	1.1	1.2	3.2	3.3	4.3	5.2
5	0040	0.0	0.0	0.1	0.1	0.2	0.1	0.1
6	0103	8.2	11.3	22.7	46.9	59.4	62.2	58.2
7	0112	12.4	16.2	31.0	51.6	52.3	38.5	51.0
8	0121	2.6	3.9	8.2	15.2	13.5	9.4	13.0
9	0130	0.1	0.2	0.3	0.4	0.6	0.4	0.6
10	0202	4.9	7.0	15.0	21.8	23.5	15.3	22.8
11	0211	5.8	5.8	4.0	9.8	11.6	7.0	11.0
12	0220	0.5	0.8	1.6	1.0	1.2	1.0	1.2
13	0301	1.3	1.5	2.1	2.5	2.6	1.2	2.3
14	0310	0.3	0.6	0.5	0.7	0.3	0.2	0.3
15	0400	0.0	0.1	0.1	0.2	0.1	0.0	0.0
16	1003	25.7	29.1	50.8	83.8	100.0	73.2	100.0
17	1012	35.1	42.8	63.5	91.8	90.7	53.3	73.6
18	1021	16.0	18.3	28.2	27.4	20.8	13.0	19.9
19	1030	0.7	0.4	0.8	1.4	1.6	0.3	1.5
20	1102	45.5	60.6	92.0	100.0	74.3	45.4	65.4
21	1111	40.2	53.9	57.0	63.3	40.6	28.1	39.4
22	1120	7.0	8.2	9.7	8.2	8.4	3.8	6.1
23	1201	22.3	25.0	37.4	30.7	18.1	11.0	17.3
24	1210	10.8	12.8	13.2	6.8	7.4	2.8	8.0
25	1300	2.6	2.1	2.8	1.8	0.9	0.6	1.2
26	2002	73.0	81.8	84.2	76.4	46.4	32.9	45.0
27	2011	58.1	67.3	76.7	57.1	35.4	17.8	34.3
28	2020	14.1	14.4	5.4	5.5	5.7	2.0	3.0
29	2101	90.3	100.0	100.0	69.5	31.2	20.5	26.3
30	2110	35.1	37.8	41.3	24.5	10.0	5.2	9.4
31	2200	19.1	21.6	20.0	11.6	4.9	2.3	4.4
32	3001	100.0	100.0	70.0	37.9	15.6	6.5	15.0
33	3010	51.0	33.2	33.8	13.3	6.6	3.2	7.0
34	3100	80.7	74.0	45.4	17.2	4.1	2.0	3.7
35	4000	65.4	48.2	17.5	4.2	2.0	0.7	1.9

Table VII. Test levels of the mass points VJ, pseudo-standard. variables, $N = 10, t_c = 0, 0.250, 0.375, 0.500, 1.000$

Mass points		Hypothetical distributions						
No.	VJ	$\alpha = 1.0$	0.9	0.7	0.5	0.3	0.1	normal
1	0008	0.1	0.1	0.5	3.6	19.9	94.0	35.6
2	0017	0.1	0.6	1.2	10.8	57.9	100.0	73.2
3	0026	0.0	0.5	3.1	7.6	47.8	57.9	59.4
4	0035	0.2	0.3	2.0	5.5	19.0	28.1	24.6
5	0044	0.2	0.3	0.9	1.6	6.6	7.3	10.1
6	0053	0.0	0.0	0.0	0.8	1.1	1.4	2.9
7	0062	0.0	0.0	0.0	0.3	0.1	0.0	0.5
8	0071	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0080	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0107	0.1	0.3	1.2	9.8	38.9	67.8	50.3
11	0116	0.5	1.5	4.2	28.8	78.1	79.6	92.5
12	0125	0.9	0.8	3.4	26.3	59.9	49.0	86.3
13	0134	0.9	1.0	3.8	15.7	32.6	19.4	25.5
14	0143	0.1	0.6	0.5	2.8	7.5	6.3	8.2
15	0152	0.0	0.0	0.1	0.6	0.4	0.8	2.6
16	0161	0.0	0.0	0.0	0.1	0.1	0.0	0.1
17	0170	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0206	0.4	0.3	3.1	9.1	30.4	37.9	37.0
19	0215	0.2	2.8	9.6	28.8	56.1	39.9	51.9
20	0224	1.6	1.0	5.1	26.3	33.8	25.4	33.2
21	0233	0.5	0.5	2.6	10.8	11.2	8.9	15.4
22	0242	0.0	0.3	0.3	0.8	2.7	1.6	6.3
23	0251	0.0	0.0	0.1	0.5	0.2	0.1	0.3
24	0260	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	0305	0.2	0.3	1.8	6.6	16.0	11.4	14.9
26	0314	1.1	2.7	3.8	16.2	21.6	12.6	18.8
27	0323	1.9	1.0	3.2	11.6	12.2	6.3	13.8
28	0332	0.0	0.5	0.9	2.4	5.7	2.2	6.3
29	0341	0.1	0.0	0.1	1.2	1.6	0.0	1.4
30	0350	0.0	0.0	0.0	0.0	0.0	0.0	0.0
31	0404	0.1	0.1	0.9	2.1	4.1	1.5	3.6
32	0413	0.4	0.8	1.2	2.4	4.7	2.7	7.2
33	0422	0.1	0.3	1.5	2.8	2.8	0.6	3.1
34	0431	0.0	0.1	0.2	1.2	1.1	0.0	0.3
35	0440	0.0	0.0	0.0	0.1	0.0	0.0	0.0
36	0503	0.0	0.0	0.2	0.3	0.4	0.2	1.2
37	0512	0.0	0.3	0.6	0.5	0.1	0.2	0.5
38	0521	0.0	0.0	0.2	0.1	0.4	0.2	0.1
39	0530	0.0	0.0	0.0	0.0	0.2	0.0	0.0
40	0602	0.0	0.0	0.0	0.0	0.0	0.0	0.3
41	0611	0.0	0.0	0.1	0.1	0.0	0.0	0.1
42	0620	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TableVII (Continued)

Mass points		Hypothetical distributions						
No.	VJ	α = 1.0	0.9	0.7	0.5	0.3	0.1	normal
43	0701	0.0	0.0	0.0	0.0	0.0	0.0	0.0
44	0710	0.0	0.0	0.0	0.0	0.0	0.0	0.0
45	0800	0.0	0.0	0.0	0.0	0.0	0.0	0.0
46	1007	0.9	1.4	6.4	19.2	46.2	75.1	38.4
47	1016	1.9	2.7	9.2	44.2	86.3	84.4	83.5
48	1025	2.6	4.6	10.8	40.6	73.1	61.0	68.3
49	1034	1.6	2.7	8.5	19.8	37.6	26.7	30.8
50	1043	0.9	0.8	1.8	6.1	14.8	6.3	10.9
51	1052	0.0	0.1	0.3	1.5	1.8	1.2	1.8
52	1061	0.0	0.0	0.1	0.1	0.8	0.0	0.7
53	1070	0.0	0.0	0.0	0.0	0.0	0.0	0.0
54	1106	1.1	3.6	9.2	32.4	68.4	71.4	75.7
55	1115	7.2	10.4	40.1	84.3	100.0	89.2	100.0
56	1124	7.5	9.4	30.4	77.2	92.4	64.1	89.4
57	1133	3.2	6.6	15.9	44.2	40.3	17.4	48.7
58	1142	0.2	1.4	3.1	15.7	8.9	7.0	12.8
59	1151	0.0	0.0	0.6	0.5	0.8	0.4	0.8
60	1160	0.0	0.0	0.0	0.0	0.0	0.0	0.0
61	1205	3.8	5.1	12.9	37.4	54.3	34.2	47.1
62	1214	7.9	14.2	41.3	74.9	83.5	42.0	80.8
63	1223	6.1	7.4	28.5	65.0	64.1	22.8	55.3
64	1232	1.9	4.6	16.5	21.8	20.7	8.5	18.1
65	1241	0.9	0.5	1.8	3.6	3.8	2.7	4.3
66	1250	0.0	0.0	0.0	0.3	0.1	0.0	0.0
67	1304	3.2	4.2	15.4	23.3	23.3	13.8	21.2
68	1313	6.5	7.8	25.8	55.5	29.4	16.5	34.4
69	1322	3.4	4.2	13.4	33.3	16.6	5.4	16.6
70	1331	0.5	1.7	5.4	5.5	5.4	1.2	4.8
71	1340	0.0	0.0	0.2	0.2	0.2	0.0	0.5
72	1403	2.6	3.4	4.7	7.6	4.5	2.4	0.3
73	1412	1.4	3.8	8.5	15.7	7.5	3.9	6.8
74	1421	1.1	1.4	4.2	4.7	2.5	1.2	2.3
75	1430	0.4	0.5	0.5	0.5	0.1	0.2	0.5
76	1502	1.2	0.6	1.2	1.5	1.1	0.5	0.7
77	1511	0.4	1.4	2.6	1.9	0.8	0.2	0.7
78	1520	0.0	0.0	0.3	0.1	0.0	0.1	0.3
79	1601	0.1	0.3	0.2	0.2	1.1	0.0	0.3
80	1610	0.0	0.1	0.1	0.2	0.0	0.0	0.5
81	1700	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table VII (Continued)

Mass points		Hypothetical distributions						
No.	VJ	α = 1.0	0.9	0.7	0.5	0.3	0.1	normal
82	2006	3.2	3.4	7.4	24.0	44.7	44.1	45.6
83	2015	7.2	11.4	37.7	72.8	89.2	52.0	66.0
84	2024	9.6	13.1	33.5	63.4	62.0	37.9	57.3
85	2033	4.1	6.3	17.1	30.6	26.3	15.1	22.8
86	2042	0.9	2.1	6.4	12.8	5.2	3.4	5.5
87	2051	0.1	0.5	0.9	1.2	0.4	0.5	0.3
88	2060	0.0	0.0	0.0	0.0	0.1	0.0	0.0
89	2105	13.9	22.4	51.0	70.8	70.7	46.4	61.4
90	2114	20.6	36.7	87.3	100.0	95.9	54.9	96.1
91	2123	21.7	31.5	75.5	89.7	80.8	32.5	78.2
92	2132	13.2	15.5	33.5	55.5	24.4	12.6	24.6
93	2141	2.3	1.7	6.4	8.5	4.1	1.2	5.2
94	2150	0.0	0.5	0.3	0.1	0.1	0.0	1.2
95	2204	12.6	18.6	45.2	51.1	54.3	21.6	53.6
96	2213	25.0	47.1	89.8	92.8	68.4	29.5	70.7
97	2222	16.1	25.5	67.2	68.8	41.8	14.4	41.2
98	2231	4.6	9.9	14.3	16.8	12.7	4.7	8.9
99	2240	0.4	0.3	0.9	1.2	0.2	0.0	1.2
100	2303	10.5	17.0	29.5	34.3	17.4	6.6	20.4
101	2312	15.3	23.4	49.5	51.1	26.3	8.1	22.0
102	2321	10.0	12.0	21.1	19.2	8.5	1.8	9.3
103	2330	2.0	1.5	2.3	4.0	0.8	0.1	1.2
104	2402	5.5	8.6	11.6	15.7	3.6	1.2	3.6
105	2411	4.9	7.4	10.8	13.3	6.6	1.2	3.9
106	2420	1.6	1.4	2.3	1.2	1.4	0.1	1.2
107	2501	2.2	2.1	1.5	1.9	1.1	0.2	0.7
108	2510	0.5	0.6	1.5	0.5	0.0	0.0	0.3
109	2600	0.1	0.1	0.3	0.1	0.0	0.0	0.0
110	3005	12.1	14.8	23.4	36.3	29.4	18.3	29.6
111	3014	23.8	30.2	54.2	66.8	50.9	31.0	44.1
112	3023	18.7	27.8	49.5	63.4	35.0	13.8	32.0
113	3032	10.9	13.1	19.0	24.8	14.8	5.4	10.5
114	3041	1.4	2.1	6.4	4.0	2.5	0.2	1.8
115	3050	0.0	0.1	0.6	0.0	0.8	0.0	0.1
116	3104	31.8	45.5	61.3	63.4	49.4	20.4	42.7
117	3113	62.1	73.9	100.0	96.3	75.6	24.0	63.6
118	3122	46.7	68.8	82.4	86.8	43.2	15.1	39.8
119	3131	11.5	17.0	24.2	20.5	10.2	3.5	8.9
120	3140	0.4	2.3	1.8	1.9	0.4	0.5	1.2
121	3203	37.8	52.2	71.2	48.2	27.3	11.4	26.4
122	3212	69.8	82.8	96.6	81.9	36.3	10.3	28.5
123	3221	30.4	40.9	57.7	45.5	9.8	3.4	16.0
124	3230	3.2	4.8	12.9	8.5	3.6	0.8	2.0

Table VII (Continued)

Mass points		Hypothetical distributions						
No.	VJ	α = 1.0	0.9	0.7	0.5	0.3	0.1	normal
125	3302	27.6	39.5	43.8	35.3	12.2	2.2	14.3
126	3311	26.2	34.1	59.5	39.5	8.5	2.4	7.5
127	3320	4.3	6.C	8.5	5.5	1.1	0.6	2.1
128	3401	8.3	10.9	10.0	8.5	3.6	0.6	1.8
129	3410	3.8	3.0	5.1	3.6	1.4	0.1	2.0
130	3500	0.9	1.0	2.0	0.3	0.2	0.1	0.0
131	4004	19.6	21.4	26.6	28.8	15.4	8.1	12.3
132	4013	41.1	56.0	63.2	58.5	18.2	9.8	17.4
133	4022	39.4	26.6	46.6	38.4	10.7	4.9	11.8
134	4031	8.7	8.6	14.3	9.8	3.6	0.4	1.8
135	4040	0.9	2.3	0.5	0.3	0.8	0.2	0.0
136	4103	59.6	71.2	69.2	57.0	22.5	9.3	19.6
137	4112	91.7	100.0	93.2	79.5	31.5	7.3	27.4
138	4121	48.6	62.1	73.3	41.8	9.3	4.7	13.8
139	4130	9.6	9.0	6.8	4.5	1.4	0.0	1.8
140	4202	75.4	79.7	77.7	46.8	13.2	3.9	11.8
141	4211	81.4	89.2	84.8	55.5	13.7	1.4	9.7
142	4220	17.8	17.8	19.0	12.8	2.5	0.5	3.2
143	4301	34.7	45.5	36.6	21.2	2.5	0.7	3.9
144	4310	14.6	19.6	25.8	6.6	1.8	0.2	1.2
145	4400	6.1	5.6	2.6	2.8	0.4	0.1	0.5
146	5003	33.2	38.1	31.4	18.0	6.8	2.2	5.2
147	5012	67.2	66.5	52.6	22.6	5.0	1.5	6.6
148	5021	36.3	29.0	21.8	11.6	4.5	1.2	2.7
149	5030	5.2	5.4	4.7	1.5	0.8	0.2	0.8
150	5102	87.9	92.4	67.2	30.6	6.6	1.8	4.5
151	5111	100.0	96.1	80.0	31.4	8.5	1.4	8.2
152	5120	30.4	34.1	22.6	9.1	2.8	0.0	1.3
153	5201	84.5	76.7	55.9	17.4	3.6	0.5	4.3
154	5210	50.7	54.0	38.9	15.7	1.6	0.1	2.3
155	5300	22.7	20.5	14.8	3.0	0.8	0.0	0.3
156	6002	52.8	50.5	27.5	7.6	1.9	0.8	2.7
157	6011	64.6	58.0	34.5	6.1	1.5	0.1	2.9
158	6020	16.9	13.6	12.0	3.6	0.1	0.0	0.1
159	6101	95.8	86.0	42.6	12.0	1.4	0.7	1.4
160	6110	72.6	64.3	35.6	10.1	1.4	0.2	0.7
161	6200	57.3	48.8	21.1	4.2	0.8	0.1	0.1
162	7001	55.0	42.4	19.0	2.1	0.4	0.1	0.5
163	7010	44.8	36.7	11.6	0.8	0.1	0.1	0.0
164	7100	78.3	60.0	19.6	1.5	0.2	0.0	0.3
165	8000	43.0	24.4	7.4	0.8	0.0	0.0	0.1

Table IIX. Test levels of the mass points VJ, pseudo-standardized Weibull samples, $\alpha = 1.0$, $N = 20$, $t_c = 0, .125, .250, .375, 1.0$

Mass points not appearing in this table have a TL ≤ 1.4%

v_1	v_2 \ v_3	0	1	2	3	4	5	6	7	8	9
0	4	-	-	-	-	-	2.8	-	-	-	-
0	5	-	-	-	-	2.8	-	-	-	-	-
0	6	-	-	2.8	-	-	-	-	-	-	-
1	2	-	-	-	2.8	2.8	2.8	-	-	-	-
1	3	-	-	2.8	6.2	6.2	4.2	4.2	-	-	-
1	4	-	-	4.2	2.8	2.8	-	-	-	-	-
1	5	-	-	6.2	2.8	2.8	6.2	-	-	-	-
1	6	-	-	-	6.2	2.8	4.2	2.8	-	-	-
1	7	-	-	-	-	4.2	-	-	-	-	-
1	8	-	-	-	2.8	-	-	-	-	-	-
2	1	-	-	-	-	-	4.2	2.8	-	-	-
2	2	-	-	4.2	9.9	12.4	6.2	2.8	-	-	-
2	3	-	8.1	12.4	23.2	18.8	21.5	8.1	2.8	-	-
2	4	-	-	9.9	17.1	23.2	13.7	2.8	2.8	-	-
2	5	-	9.9	2.8	28.7	12.4	9.9	2.8	2.8	-	-
2	6	2.8	9.9	9.9	8.1	13.7	2.8	6.2	-	-	-
2	7	-	-	12.4	11.2	12.4	8.1	2.8	-	-	-
2	8	-	-	6.2	8.1	-	4.2	-	-	-	-
2	11	-	-	2.8	-	-	-	-	-	-	-
3	0	-	-	-	-	2.8	-	-	-	-	-
3	1	-	-	-	-	6.2	2.8	6.2	4.2	-	-
3	2	-	4.2	11.2	9.9	21.5	17.1	9.9	4.2	-	-
3	3	2.8	12.4	23.2	36.7	27.4	37.7	21.5	8.1	-	-
3	4	2.8	17.1	37.7	49.8	49.8	32.3	17.1	4.2	-	-
3	5	-	21.5	41.5	75.2	33.2	28.7	8.1	8.1	-	-
3	6	2.8	11.2	28.7	36.7	33.2	13.7	12.4	-	-	-
3	7	4.2	11.2	18.8	32.3	13.7	6.2	-	-	-	-
3	8	-	11.2	21.5	18.8	11.2	-	-	-	-	-
3	9	4.2	4.2	6.2	4.2	4.2	-	-	-	-	-
4	1	-	-	6.2	21.5	8.1	12.4	6.2	2.8	-	-
4	2	-	6.2	31.0	29.6	31.0	21.5	9.9	4.2	2.8	4.2
4	3	11.2	14.7	38.8	65.0	53.6	41.5	28.7	9.9	2.8	-
4	4	9.9	27.4	55.2	62.3	65.0	43.0	25.6	2.8	-	-
4	5	6.2	34.6	73.5	69.4	60.5	25.6	21.5	2.8	-	-
4	6	4.2	44.5	53.6	53.6	39.6	18.8	6.2	-	-	-
4	7	4.2	18.8	55.2	27.4	23.2	-	-	-	-	-
4	8	6.2	12.4	24.0	14.7	13.7	-	-	-	-	-
4	9	2.8	6.2	9.9	8.1	2.8	-	-	-	-	-
4	10	-	-	4.2	-	-	-	-	-	-	-

Table IIX (Continued)

v_1	v_2 \ v_3	0	1	2	3	4	5	6	7	8
5	0	-	-	-	4.2	2.8	4.2	2.8	-	-
5	1	-	4.2	9.9	27.4	18.8	17.1	8.1	6.2	-
5	2	-	18.8	41.5	45.7	44.5	41.5	14.7	2.8	-
5	3	6.2	31.0	65.0	80.3	77.9	57.2	17.1	6.2	-
5	4	17.1	46.4	84.9	91.7	84.9	47.7	24.0	6.2	-
5	5	27.4	63.2	76.8	93.3	71.5	32.3	17.1	-	-
5	6	21.5	39.6	73.5	49.8	36.7	18.8	6.2	-	-
5	7	4.2	34.6	47.7	44.5	21.5	6.2	-	-	-
5	8	9.9	23.2	21.5	13.7	6.2	-	-	-	-
5	9	4.2	4.2	8.1	6.2	-	-	-	-	-
5	10	-	2.8	-	-	-	-	-	-	-
6	0	-	-	4.2	2.8	-	-	-	-	-
6	1	-	4.2	23.2	33.2	17.1	27.4	4.2	4.2	-
6	2	6.2	28.7	51.3	53.6	60.5	33.2	21.5	8.1	-
6	3	8.1	46.4	75.2	90.9	79.1	51.3	13.7	4.2	-
6	4	11.2	62.3	96.1	96.1	85.6	37.7	12.4	2.8	-
6	5	21.5	73.5	92.5	87.8	71.5	32.3	4.2	-	-
6	6	12.4	71.5	87.8	76.8	43.0	8.1	-	-	-
6	7	21.5	39.6	62.3	43.0	21.5	-	-	-	-
6	8	8.1	11.2	34.6	11.2	9.9	-	-	-	-
6	9	-	11.2	9.9	-	-	-	-	-	-
6	10	-	2.8	-	-	-	-	-	-	-
7	0	-	-	-	2.8	8.1	-	2.8	-	-
7	1	6.2	8.1	12.4	31.0	25.6	25.6	9.9	-	2.8
7	2	6.2	17.1	58.4	66.9	49.8	41.5	17.1	2.8	-
7	3	13.7	53.6	89.4	90.9	58.4	46.7	14.7	-	-
7	4	31.0	69.4	96.1	100.0	76.8	45.7	8.1	-	-
7	5	27.4	67.4	99.0	99.0	60.5	8.1	2.8	-	-
7	6	36.7	59.2	71.5	71.5	27.4	11.2	-	-	-
7	7	21.5	41.5	44.5	23.2	2.8	-	-	-	-
7	8	9.9	18.8	13.7	8.1	-	-	-	-	-
7	9	6.2	6.2	4.2	-	-	-	-	-	-
8	0	-	-	-	6.2	2.8	-	-	-	-
8	1	-	14.7	32.3	17.1	23.2	17.1	6.2	-	-
8	2	8.1	29.6	51.3	53.6	37.7	25.6	8.1	-	-
8	3	9.9	62.3	77.9	83.6	80.3	34.6	17.1	-	-
8	4	21.5	66.0	97.0	89.4	51.3	24.0	-	-	-
8	5	17.1	73.5	82.9	74.1	36.7	2.8	-	-	-
8	6	17.1	57.2	66.9	36.7	6.2	-	-	-	-
8	7	8.1	36.7	29.6	12.4	-	-	-	-	-
8	8	6.2	14.7	-	-	-	-	-	-	-
8	9	4.2	-	-	-	-	-	-	-	-
9	0	-	-	2.8	4.2	4.2	-	-	-	-
9	1	-	13.7	24.0	34.6	25.6	11.2	-	-	-
9	2	4.2	28.7	57.2	59.2	31.0	17.1	2.8	-	-

Table IIX (Continued)

v_1	v_2 \ v_3	0	1	2	3	4	5	6
9	3	18.8	66.0	81.6	81.0	34.6	17.1	-
9	4	27.4	69.4	87.8	79.1	25.6	6.2	-
9	5	23.2	55.2	82.9	55.2	11.2	-	-
9	6	17.1	43.0	48.4	9.9	-	-	-
9	7	18.8	25.6	13.7	-	-	-	-
9	8	4.2	6.2	-	-	-	-	-
10	0	-	-	4.2	2.8	-	-	-
10	1	4.2	8.1	21.5	17.1	14.7	11.2	-
10	2	9.9	27.4	36.7	38.8	13.7	6.2	-
10	3	8.1	47.7	65.0	45.7	21.5	6.2	-
10	4	17.1	58.4	69.4	38.8	8.1	-	-
10	5	29.6	63.2	57.2	8.1	-	-	-
10	6	14.7	25.6	8.1	-	-	-	-
10	7	11.2	8.1	-	-	-	-	-
11	0	-	-	-	4.2	-	-	-
11	1	-	8.1	12.4	11.2	8.1	2.8	-
11	2	6.2	25.6	41.5	31.0	9.9	-	-
11	3	6.2	38.8	48.4	29.6	13.7	-	-
11	4	21.5	57.2	43.0	14.7	-	-	-
11	5	13.7	36.7	18.8	-	-	-	-
11	6	21.5	18.8	-	-	-	-	-
11	7	6.2	-	-	-	-	-	-
12	0	-	-	-	6.2	-	-	-
12	1	2.8	6.2	9.9	9.9	6.2	2.8	-
12	2	2.8	27.4	24.0	17.1	2.8	-	-
12	3	9.9	18.8	44.5	11.2	-	-	-
12	4	23.2	32.3	23.2	-	-	-	-
12	5	8.1	9.5	-	-	-	-	-
12	6	2.8	-	-	-	-	-	-
13	0	-	-	-	2.8	-	-	-
13	1	2.8	4.2	6.2	6.2	2.8	-	-
13	2	8.1	9.9	6.2	8.1	-	-	-
13	3	12.4	14.7	11.2	-	-	-	-
13	4	8.1	28.7	-	-	-	-	-
13	5	4.2	-	-	-	-	-	-
14	0	-	-	-	4.2	-	-	-
14	1	-	-	9.9	-	-	-	-
14	2	9.9	4.2	-	-	-	-	-
14	3	9.9	-	-	-	-	-	-
14	4	6.2	-	-	-	-	-	-
15	1	-	2.8	4.2	-	-	-	-
15	2	2.8	8.1	-	-	-	-	-
15	3	4.2	-	-	-	-	-	-
16	1	-	2.8	-	-	-	-	-
16	2	2.8	-	-	-	-	-	-

Table IX. Test levels of the mass points VJ, pseudo-standardized normal samples, $N = 20$, $t_c = 0, .125, .250, .375, 1.0$

Mass points not appearing is this table have a TL<1%

v_1	v_2 \ v_3	0	1	2	3	4	5	6	7	8	9
0	0	52.9	94.6	90.0	76.0	58.3	35.7	20.1	5.7	1.6	-
0	1	56.1	97.0	100.0	92.2	74.3	49.9	21.7	9.0	4.6	-
0	2	41.7	68.3	85.6	87.7	65.6	41.7	15.6	9.9	4.6	-
0	3	15.6	46.3	55.0	60.6	41.7	31.7	17.8	4.0	4.6	1.6
0	4	7.8	33.4	32.8	30.7	25.0	13.2	4.6	1.6	2.4	-
0	5	4.0	13.8	13.8	13.2	10.6	8.1	3.2	-	-	-
0	6	2.4	5.7	6.3	-	3.2	-	1.6	-	-	-
0	7	-	2.4	3.2	-	-	-	-	-	-	-
0	8	-	-	-	1.6	-	-	-	-	-	-
1	0	28.4	49.0	64.3	54.0	38.3	28.4	13.2	5.7	3.2	2.4
1	1	27.5	71.2	83.6	77.8	66.9	43.9	22.0	7.8	4.6	-
1	2	35.7	63.0	81.5	79.6	72.7	37.6	21.0	7.8	6.3	1.6
1	3	21.0	47.2	69.8	59.5	44.7	23.5	14.9	9.9	1.6	-
1	4	9.9	32.8	37.0	35.7	25.8	13.2	7.8	3.2	-	-
1	5	4.0	13.8	22.8	16.8	15.6	6.3	4.0	-	-	-
1	6	1.6	5.7	13.2	8.1	3.2	3.2	-	-	-	-
1	7	-	2.4	4.0	4.0	-	-	-	-	-	-
1	9	-	-	-	-	1.6	-	-	-	-	-
2	0	7.8	24.6	36.3	26.2	24.6	10.3	11.0	2.4	1.6	-
2	1	14.4	46.3	50.9	61.8	43.2	20.1	16.1	7.8	3.2	-
2	2	20.1	39.6	51.9	57.2	39.6	22.8	16.1	7.8	2.4	-
2	3	14.9	29.3	42.4	48.0	31.2	14.4	9.0	4.0	-	-
2	4	6.3	21.0	30.7	28.8	17.8	9.9	8.1	1.6	-	-
2	5	2.4	7.8	10.3	13.2	11.3	5.7	3.2	-	1.6	-
2	6	2.4	3.2	9.9	7.8	4.0	1.6	-	-	-	-
2	7	-	-	3.2	-	-	-	-	-	-	-
2	8	-	-	-	1.6	-	-	-	-	-	-

Table IX (Continued)

v_1	v_2 \ v_3	0	1	2	3	4	5	6	7	8
3	0	3.2	11.3	13.2	18.7	9.9	4.0	4.0	-	-
3	1	5.0	16.8	25.8	27.5	20.1	13.2	7.8	3.2	-
3	2	10.3	21.7	29.8	33.9	27.5	17.8	4.6	1.6	-
3	3	9.0	16.8	24.6	23.5	18.9	7.8	5.0	-	-
3	4	1.6	5.7	17.8	13.2	13.2	7.8	4.0	-	-
3	5	5.0	7.8	2.4	4.0	3.2	1.6	-	-	-
3	6	-	1.6	1.6	4.0	1.6	1.6	-	-	-
3	7	-	-	-	1.6	-	-	-	-	-
3	8	-	1.6	-	-	-	-	-	-	-
4	0	-	4.0	3.2	2.4	3.2	5.0	2.4	-	-
4	1	-	5.0	18.7	9.0	9.0	6.3	-	-	-
4	2	2.4	11.0	18.7	14.4	5.7	7.8	-	1.6	-
4	3	1.6	4.6	10.5	7.8	5.7	6.3	1.6	1.6	1.6
4	4	-	9.0	9.9	9.0	3.2	-	-	-	-
4	5	1.6	3.2	2.4	3.2	-	-	-	-	-
4	6	-	-	1.6	2.4	-	-	-	-	-
5	0	-	-	3.2	-	-	-	-	-	-
5	1	-	2.4	2.4	5.7	4.0	1.6	-	-	-
5	2	2.4	-	4.6	4.6	4.6	-	-	-	-
5	3	-	2.4	6.3	2.4	-	-	-	-	-
5	4	-	-	4.0	2.4	1.6	-	-	-	-
5	5	-	-	-	2.4	-	-	-	-	-
6	1	-	-	2.4	-	-	-	-	-	-
6	2	-	1.6	1.6	2.4	-	-	-	-	-
6	3	-	1.6	-	-	-	-	-	-	-
7	1	-	-	-	2.4	-	-	-	-	-
7	2	-	-	-	-	-	-	-	-	-
7	3	-	1.6	-	-	-	-	-	-	-

Table X. Test levels of the mass points VJ, pseudo-standardized Weibull samples, $\alpha = 0.1$, $N = 20$, $t_c = 0; .125; .250; .375; 1.0$

Mass points not appearing in this table have a TL 1%

v_1	v_2 \ v_3	0	1	2	3	4	5	6	7	8
0	0	93.3	100.0	81.9	53.7	30.7	14.7	4.4	-	-
0	1	72.9	87.2	77.1	58.5	37.1	16.3	4.8	2.0	-
0	2	41.2	63.9	55.9	44.3	21.0	14.4	3.7	1.6	-
0	3	18.1	34.8	31.7	21.8	11.8	5.1	2.0	-	-
0	4	7.8	12.7	14.4	8.5	6.3	3.4	1.6	1.0	-
0	5	2.0	4.4	4.8	3.4	-	1.0	-	-	-
0	6	-	1.0	1.6	-	1.0	-	-	-	-
1	0	45.9	69.6	51.5	39.8	23.2	8.8	3.4	2.0	1.0
1	1	38.4	66.7	61.2	42.7	28.0	15.4	7.3	1.0	-
1	2	22.5	47.5	49.3	35.9	19.8	10.4	2.0	-	-
1	3	10.4	25.6	28.9	20.4	11.2	5.6	1.6	-	-
1	4	5.6	12.7	12.7	7.3	3.4	1.0	-	-	-
1	5	2.0	4.4	4.4	2.3	-	1.0	-	-	-
1	6	-	1.0	1.0	-	-	-	-	-	-
2	0	17.8	29.8	24.8	18.6	13.4	5.1	2.8	-	-
2	1	17.6	33.7	32.6	24.8	16.7	8.0	3.4	2.0	-
2	2	11.8	27.2	26.3	19.2	13.4	3.4	2.8	1.6	-
2	3	6.3	15.0	15.8	10.4	10.4	3.4	2.8	-	-
2	4	2.3	7.3	7.8	5.6	4.4	2.0	-	-	-
2	5	-	3.7	2.3	2.3	-	-	-	-	-
2	6	-	1.0	1.6	1.0	-	-	-	-	-
3	0	4.4	7.4	7.3	7.3	3.7	1.6	-	-	-
3	1	6.3	10.4	11.2	11.2	6.3	1.0	-	-	-
3	2	4.4	9.0	14.4	8.8	7.3	1.6	-	-	-
3	3	2.3	4.8	8.8	5.8	3.4	1.6	1.6	-	-
3	4	1.0	2.8	4.4	1.6	-	-	-	-	-
3	5	1.0	1.0	-	-	-	-	-	-	-
4	0	1.6	1.6	2.8	1.0	2.0	-	-	-	-
4	1	1.6	2.8	5.6	3.7	2.0	1.6	-	-	-
4	2	2.0	5.1	4.8	2.3	2.8	1.6	-	-	-
4	3	-	2.0	1.6	2.8	-	-	-	-	-
4	4	-	-	1.6	-	-	-	-	-	-
5	0	1.0	-	-	-	-	-	-	-	-
5	1	-	-	1.6	-	-	-	-	-	-
5	2	-	-	1.6	-	-	-	-	-	-
6	0	-	-	-	1.0	-	-	-	-	-

Table XI. Tests of the hypothesis that the distributions of VJR and VJS are identical for equal class probabilities

a) R = pseudo-standardized exponential population
S = -"- -"- normal -"-
$N = 10$, $r = 4$, $p_i = 1/r = 25\%$

Types of VJ	Nr. of pairs	Sign test Nr. of +	Sign test Nr. of −	Sign test Reject. %	Nr-of-runs test Nr. of runs	Nr-of-runs test Reject. %	Rank-sum test T_1	Rank-sum test T_2	Rank-sum test Reject. %
0008	4	1	1	-	2	-	18.0	18.0	55.7
0017	12	4	6	> 25	7	88.1	136.0	164.0	21.8
0026	12	1	11	= 1	3	-	129.5	170.5	12.3
0035	12	4	5	> 25	5	50.0	144.5	155.5	38.7
0044	6	0	6	= 10	0	-	31.0	47.0	12.0
0116	12	2	10	= 5	3	18.2	139.0	161.0	27.3
0125	24	13	11	> 25	9	7.5	604.5	571.5	37.1
0134	24	10	14	> 25	8	3.7	582.0	594.0	45.9
0224	12	6	6	> 25	5	17.5	149.0	151.0	48.8
0233	12	3	8	= 25	5	53.3	140.0	160.0	29.2
1115	4	2	2	> 25	4	> 90.0	15.5	20.5	29.3
1124	12	7	5	> 25	6	42.4	148.5	151.5	47.6
1133	6	4	2	> 25	3	40.0	34.0	34.0	24.2
1223	12	9	3	= 25	4	20.0	184.5	115.5	2.5

b) R = pseudo-standardized Weibull, $\alpha = 1.0$, population
S = -"- -"- Weibull, $\alpha = 0.1$, -"-
$N = 10$, $r = 4$, $p_i = 1/r = 25\%$

Types of VJ	Nr. of pairs	Sign test Nr. of +	Sign test Nr. of −	Sign test Reject. %	Nr-of-runs test Nr. of runs	Nr-of-runs test Reject. %	Rank-sum test T_1	Rank-sum test T_2	Rank-sum test Reject. %
0008	4	0	3	= 25	0	-	15.5	20.5	29.3
0017	12	1	8	= 5	3	-	127.5	172.5	10.0
0026	12	3	6	> 25	3	10.7	144.0	156.0	37.5
0035	12	4	8	> 25	7	78.8	143.0	157.0	35.5
0044	6	4	2	> 25	4	80.0	40.5	37.5	43.9
0116	12	5	7	> 25	6	42.4	133.0	167.0	17.0
0125	24	12	12	> 25	9	7.2	583.5	582.5	46.0
0134	24	12	12	> 25	13	50.0	582.0	594.0	45.9
0224	12	7	5	> 25	4	7.6	152.5	147.5	45.4
0233	12	7	5	> 25	7	65.2	154.0	146.0	42.1
1115	4	3	1	> 25	2	-	20.5	15.5	29.3
1124	12	5	7	> 25	8	85.4	147.0	153.0	44.3
1133	6	4	2	> 25	3	40.0	42.0	36.0	35.0
1223	12	10	2	= 5	2	3.0	170.0	130.0	13.1

Table XII. Test levels of the mass points VJ, pseudo-standardized variables, $N = 10$, $r = 4$, $p_i = 1/r = 25\%$

Mass points No	VJ	TL %	Mass points No	VJ	TL %	Mass points No	VJ	TL %	Mass points No	VJ	TL %
1	0008	0.2	43	0701	0.0	84	2024	29.0	125	3302	35.3
2	0017	2.1	44	0710	0.1	85	2033	36.1	126	3311	71.8
3	0026	7.4	45	0800	0.0	86	2042	25.5	127	3320	54.9
4	0035	11.8	46	1007	0.3	87	2051	6.3	128	3401	15.8
5	0044	9.8	47	1016	4.4	88	2060	0.0	129	3410	34.5
6	0053	3.3	48	1025	14.3	89	2105	10.4	130	3500	6.1
7	0062	0.4	49	1034	23.8	90	2114	51.3	131	4004	0.7
8	0071	0.2	50	1043	20.7	91	2123	79.5	132	4013	7.4
9	0080	0.0	51	1052	6.6	92	2132	85.6	133	4022	19.6
10	0107	0.8	52	1061	1.8	93	2141	47.8	134	4031	13.3
11	0116	9.5	53	1070	0.0	94	2150	3.7	135	4040	2.1
12	0125	33.0	54	1106	4.0	95	2204	20.7	136	4103	11.4
13	0134	40.1	55	1115	2.1	96	2213	73.7	137	4112	53.7
14	0143	31.0	56	1124	65.4	97	2222	100.0	138	4121	57.3
15	0152	13.7	57	1133	70.0	98	2231	87.8	139	4130	24.9
16	0161	2.5	58	1142	57.3	99	2240	22.7	140	4202	23.2
17	0170	0.0	59	1151	18.6	100	2303	27.8	141	4211	68.3
18	0206	2.6	60	1160	2.3	101	2312	83.5	142	4220	41.3
19	0215	26.6	61	1205	8.0	102	2321	90.0	143	4301	28.4
20	0224	44.7	62	1214	61.2	103	2330	43.6	144	4310	46.7
21	0233	58.6	63	1223	97.0	104	2402	19.6	145	4400	6.8
22	0242	37.0	64	1232	94.5	105	2411	52.5	146	5003	1.3
23	0251	11.1	65	1241	49.0	106	2420	38.8	147	5012	5.8
24	0260	0.4	66	1250	5.1	107	2501	8.5	148	5021	5.3
25	0305	1.3	67	1304	14.1	108	2510	16.2	149	5030	3.2
26	0314	21.1	68	1313	62.6	109	2600	2.1	150	5102	4.2
27	0323	42.6	69	1322	83.5	110	3005	1.4	151	5111	34.5
28	0332	37.9	70	1331	68.3	111	3014	9.5	152	5120	16.7
29	0341	14.8	71	1340	17.6	112	3023	27.8	153	5201	14.9
30	0350	3.0	72	1403	10.8	113	3032	30.3	154	5210	33.0
31	0404	2.8	73	1412	39.7	114	3041	12.9	155	5300	12.2
32	0413	14.5	74	1421	64.0	115	3050	1.0	156	6002	1.3
33	0422	29.6	75	1430	24.9	116	3104	10.4	157	6011	1.7
34	0431	22.1	76	1502	7.7	117	3113	50.2	158	6020	2.5
35	0440	2.9	77	1511	21.6	118	3122	75.6	159	6101	3.8
36	0503	0.7	78	1520	8.2	119	3131	59.9	160	6110	8.8
37	0512	4.7	79	1601	1.7	120	3140	15.3	161	6200	5.1
38	0521	5.6	80	1610	3.5	121	3203	26.6	162	7001	0.5
39	0530	1.0	81	1700	0.1	122	3212	77.6	163	7010	0.3
40	0602	0.2	82	2006	0.6	123	3221	92.2	164	7100	1.7
41	0611	2.3	83	2015	12.9	124	3230	45.7	165	8000	0.1
42	0620	1.0	–	–	–	–	–	–	–	–	–

Table XIII. Percentiles of pseudo-standardized variables for N = 10, N = 20, and various distribution functions

N = 10

P \ α	1.0	0.9	0.7	0.5	0.3	0.1	Normal dbn
0.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000
12.5	.0435	.0542	.0824	.1220	.1725	.2323	.1810
25.0	.0938	.1123	.1585	.2189	.2933	.3780	.3035
37.5	.1542	.1786	.2373	.3104	.3957	.4904	.4055
50.0	.2280	.2570	.3233	.4025	.4913	.5853	.5000
62.5	.3197	.3511	.4212	.4998	.5857	.6732	.5945
75.0	.4461	.4772	.5443	.6160	.6900	.7635	.6965
87.5	.6377	.6625	.7126	.7640	.8151	.8635	.8190
100.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

N = 20

P \ α	1.0	0.9	0.7	0.5	0.3	0.1	Normal dbn
0.0	.000	.000	.000	.000	.000	.000	.000
12.5	.030	.040	.079	.130	.192	.278	.205
25.0	.075	.095	.145	.217	.310	.416	.322
37.5	.130	.155	.215	.300	.405	.518	.416
50.0	.194	.220	.294	.384	.492	.605	.502
62.5	.268	.304	.382	.475	.577	.680	.585
75.0	.382	.418	.492	.580	.671	.762	.678
87.5	.560	.590	.660	.722	.790	.857	.795
100.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table XI The effect of pooling on the decision power of VJ

Stage of pooling	Nr. of mass points	Decision power in %		
		1 vs 0.1	1 vs 0	0.1 vs 0
VJ, ungrouped	165	77.2	69.1	22.0
Types of VJ	15	18.4	6.6	21.6
X^2	14	18.4	6.6	21.6
NE	4	13.0	5.6	13.6

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13. ABSTRACT

The test statistic X^2 of the Chi-square test may, if applied to a proper sample, be used for selecting distribution functions. When examining its use for this purpose its decision power was found to be very small due to a kind of pooling, an inherent property of its definition. In order to eliminate this pooling, a new test statistic, denoted by VJ, was introduced. It is defined by the number v_i of sample elements which fall within each of r properly defined classes into which the space of the variable x has been divided. In fact X^2 may be regarded as a statistic obtained from VJ by a pooling procedure. For this reason VJ was expected to have a much larger decision power than X^2 as was verified by the example that the decision power for a specified case being 6.6% for X^2 was raised to 69.1% for VJ.

The properties of VJ have been thoroughly exmanined. In particular the class limits yielding the largest decision power have been determined with the result that, in some cases, the decision power was found to be somewhat larger than anyone sofar attained.

The statistic VJ can also be used for stating whether a hypothetical distribution is acceptable or not and also for selecting the most propable one within a set of such distributions. Necessary tables for the practical use have been prepared.

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