AD-762 148

CHAPMAN-KIRK REDUCTION OF FREE-FLIGHT RANGE DATA TO OBTAIN NONLINEAR AERO-DYNAMIC COEFFICIENTS

Robert H. Whyte, et al

Ballistic Research Laboratories Aberdeen Proving Ground, Maryland

May 1973

**DISTRIBUTED BY:** 

National Technical Information Service U. S. DEPARTMENT OF COMMERCE 5285 Port Royal Road, Springfield Va. 22151

# BBR BL

# MEMORANDUM REPORT NO. 2298

# CHAPMAN-KIRK REDUCTION OF FRFE-FLIGHT RANGE DATA TO OBTAIN NONLINEAR AERODYNAMIC COEFFICIENTS

by

Robert H. Whyte Angela Jeung James W. Bradley



AD

May 1973

Approved for public release; distribution unlimited.

Peproduced by NATION AL TECHNICAL INFORMATION SERVICE US Department of Commerce US Department of Commerce

USA BALLISTIC RESEARCH LABORATORIES ABERDEEN PROVING GROUND, MARYLAND

58

Destroy this report when it is no longer needed. Do not return it to the originator.

Secondary distribution of this report by originating or sponsoring activity is prohibited.

Additional copies of this report may be obtained from the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia 22151.



The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized doluments.

DOCUMENT			
(Security classification of title hads of shrings tand	CONTROL DATA - R	& D	averall report is classified.
ORIGINATING ACTIVITY (Corporate author)	meaning minutation must be	20. REPORT S	ECURITY CLASSIFICATION
U. S. Army Ballistic Research Laborat	tories	UU	nclassified
Aberdeen Proving Ground, Maryland 2	1005	25. GROUP	
REPORT TITLE		_ <u></u>	
CHAPMAN-KIRK REDUCTION OF FREE-FLIGHT COEFFICIENTS	T RANGE DATA TO O	BTAIN NONL	INEAR AERODYNAMIC
DESCRIPTIVE NOTES (Type of report and inclusive dates)			
AUTHOR(S) (First name, middle initial, last name)			
Robert H. Whyte			
Angela Jeung			
James W. Bradley	Te. TOTAL NO. ,C	F PAGES	75. NO. OF REFS
MAY 1973	17.58		19
. CONTRACT OR GRANT NO	Se. ORIGINATOR	S REPORT NUM	BER(\$)
B. PROJECT NO. RDT&E 1T061102A33D	BRL MEMO	ORANDUM REI	PORT NO. 2298
ε.	9b. OTHER REPO this report)	RT NO(\$) (Any o	ther numbers that may be assigned
Approved for public release; distribut	ion unlimited.		
1 SUPPLEMENTARY NOTES	12 SPONSORING	MILITARY ACTI	
	U. S. An Washingt	on, D. C.	20315
ABSTRACT			
The Chapman-Kirk technique for obtain equations was applied to time, positi trajectories of twelve rounds fired is represent four spin-stabilized project out extension and the M329A1E1. The Standard, linear reduction techniques by the General Electric Company under GE results shows good agreement in the technique has the advantage - particut analysis breaks down - that it can do the nonlinear coefficients present in	ning the paramete ion and orientation in the BRL Transo ctile types: the rounds were previs; the Chapman-Ki r contract to BRL he linear coeffic alarly for large g etermine from a sing the equations or	rs in a sys on measuren nic Range. M71, the H iously reduction rk reduction . A compa- ients. The yaw rounds ingle traje f motion.	stem of differential ments taken along the The twelve rounds M329A1 with and with- aced at BRL by on das carried out rison of the BRL and e Chapman-Kirk , where the linear ectory the values of
The Chapman-Kirk technique for obtain equations was applied to time, positi trajectories of twelve rounds fired is represent four spin-stabilized project out extension and the M329A1E1. The Standard, linear reduction techniques by the General Electric Company under GE results shows good agreement in the technique has the advantage - particu- analysis breaks down - that it can do the nonlinear coefficients present in <b>D</b> . 1473	hing the paramete ion and orientation in the BRL Transon ctile types: the rounds were prevents; the Chapman-Kin r contract to BRL he linear coefficularly for large petermine from a sing the equations of the equations of	rs in a sym on measurem nic Range. M71, the liously reduction rk reduction. A comparise ients. The yaw rounds ingle trajo f motion.	stem of differential ments taken along the The twelve rounds M329Al with and with- aced at BRL by on das carried out rison of the BRL and e Chapman-Kirk , where the linear ectory the values of

# BALLISTIC RESEARCH LABORATORIES

MEMORANDUM REPORT NO. 2298

RHWhyte/AJeung/JWBradley/ds Aberdeen Proving Ground, Md. May 1973

# CHAPMAN-KIRK REDUCTION OF FREE-FLIGHT RANGE DATA TO OBTAIN NONLINEAR AERODYNAMIC COEFFICIENTS

# ABSTRACT

The Chapman-Kirk technique for obtaining the parameters in a system of differential equations was applied to time, position and orientation measurements taken along the trajectories of twelve rounds fired in the BRL Transonic Range. The twelve rounds represent four spin-stabilized projectile types: the M71, the M329Al with and without extension and the M329AlEl. The rounds were previously reduced at BRL by standard, linear reduction techniques; the Chapman-Kirk reduction was carried out by the General Electric Company under contract to BRL. A comparison of the BRL and GE results shows good agreement in the linear coefficients. The Chapman-Kirk technique has the advantage - particularly for large yaw rounds, where the linear analysis breaks down - that it can determine from a single trajectory the values of the nonlinear coefficients present in the equations of motion.

Preceding page blank

3

# TABLE OF CONTENTS

		Page
	ABSTRACT	3
	LIST OF TABLES	7
	LIST OF SYMBOLS	9
Ι.	INTRODUCT ION	15
11.	THE CHAPMAN-KIRK TECHNIQUE	17
III.	INPUT DATA, COORDINATE SYSTEMS AND YAW VARIABLES	23
IV.	THE EQUATIONS OF MOTION	27
	A. The Drag Equation	27
	B. The Roll Equation	31
	C. The Yaw Equations	34
	D. The CG Equations	38
ν.	RESULTS	42
	A. Background	42
	B. Drag Results	43
	C. Yaw Results	44
	D. CG Results	45
VI.	CONCLUSIONS AND RECOMMENDATIONS	45
	FIGURES	47
	REFERENCES	59
	DISTRIBUTION LIST	61

5

Preceding page blank

R

.

.

# LIST OF TABLES

Table		Page
Ι.	Physical Parameters and Environment	. 51
11.	Drag Reduction Results	. 52
III.	M71 Yaw Reduction Results	. 53
IV.	M329A1 (w/ext) Yaw Reduction Results	. 54
v.	M329Al (wo/ext) Yaw Reduction Results	. 56
VI.	M329A1E1 Yaw Reduction Results	. 57
VII.	M329A1E1 CG Reduction Results	. 58

ないとう

# LIST OF SYMBOLS

¥	vector of the N23 parameters $a_n$ to be determined
A1	$\frac{\pi \rho d^2}{8m} [L^{-1}]$
A 2	$\frac{\pi \rho d^3}{8 I_y} [L^{-2}]$
A 3	$\frac{\pi \rho d^4}{8 I_x} C_{\ell p} [L^{-1}]$
a n	the n-th unknown constant parameter to be determined $(n = 1, 2,, N23)$
<sup>a</sup> R	$\tan^{-1}(\dot{y}/\dot{x})$ , see Fig. 2c
a, a, a 1 2 3	coefficients in an expansion of time as a cubic in distance, eq. (36)
Ē	vector from the range system origin in the direction of the positive X - axis, see Fig. 2a
ţ	Coriolis acceleration, eq. (64a,b)
C <sub>A</sub>	$A_1 \overline{C}_D V_0 [T^{-1}], eq. (38)$
с <sub>D</sub>	drag coefficient, $\frac{ \text{drag force} }{m A_1 V^2}$ , eq. (30)
C <sub>Dj</sub>	coefficient of $\delta^j$ (j = 0, 2) in the expansion of $C_D$
C <sub>DA</sub>	axial drag coefficient, $(1 - \delta^2) C_{D}$
C <sub>DAj</sub>	coefficient of $\delta^j$ (j = 0, 2) in the expansion of $C_{DA}$
C <sub>DR</sub>	BRL range value of C <sub>D</sub> , eq. (36)
¯c <sub>D</sub>	average value of C <sub>D</sub> , eq. (35)
с <sub>е</sub> р	roll damping moment coefficient, $\pm \frac{ r_0   \text{ damping moment} }{ m   d^2 A_v V p}$
	eq. (47)

Preceding page blank

Magnus moment coefficient,  $\pm \frac{|Magnus moment|}{m d^2 A V p \delta}$ , eq. (51) С<sub>М</sub>ра coefficient of  $\delta^{j}$  (j = 0, 1, 2, 4, 6, 8) in the expansion <sup>C</sup>M paj of C<sub>M</sub> damping moment coefficient,  $\pm \frac{|\text{damping moment}|}{m d^2 A V | \dot{\theta} + i \dot{\psi} \cos \theta|}$ , с<sub>м</sub>а eq. (51) coefficient of  $\delta^j$  (j = 0, 2) in the expansion of  $C_{M_a}$ C<sub>M</sub>qj static moment coefficient,  $\pm \frac{|\text{static moment}|}{\text{m d A} \quad V^2 \quad \delta}$ , eq. (51) С<sub>Ма</sub> C<sub>M</sub>aj coefficient of  $\delta^{j}$  (j = 0, 2, 4) in the expansion of  $C_{M}$ Magnus force coefficient,  $\pm \frac{|Magnus force|}{m d A}$ , eqs. (59, 66) С<sub>N</sub>ра coefficient of  $\delta^{j}$  (j = 0, 2) in the expansion of  $C_{N_{DQ}}$ C<sub>N</sub>paj normal force coefficient,  $\pm \frac{|\text{normal force}|}{m A_v^2 \delta}$ , eqs. (59, 66) C<sub>Nα</sub> coefficient of  $\delta^{j}$  (j = 0, 2) in the expansion of  $C_{N_{a}}$ C<sub>N</sub>αj č, centripetal acceleration, eq. (63),  $[LT^{-2}]$ С<sub>үра</sub>  $2~C_{\ensuremath{N_{\text{pa}}}}$  , the Magnus force coefficient of Reference 14 N1 x N4 matrix of measurements d<sub>im</sub> D đ drag force, eq. (30),  $[MLT^{-2}]$ d reference diameter, [L] the measured value of  $y_i$  at  $x = x_m$ dim

E <sub>R</sub>	$\sin^{-1}$ (z/V), see Fig. 2c
ŧ	sum of the aerodynamic forces acting on the projectile, [MLT <sup>-2</sup> ]
F <sub>X</sub> ,F <sub>Y</sub> ,F <sub>Z</sub>	range components of F
F <sub>1</sub> , F, F <sub>1</sub>	fixed-plane components of F
f <sub>i</sub> ()	single-valued elementary function
ธ	gravitational force, eq. (21), [MLT <sup>-2</sup> ]
g	magnitude of the gravitational acceleration, $[LT^{-2}]$
I <sub>x</sub> ,I <sub>y</sub>	axial and transverse moments of inertia, $[ML^2]$
t	angular momentum vector, $[ML^2T^{-1}]$
L,L,L 1 2 3	fixed-plane components of $\vec{L}$ , eq. (42)
พี	sum of the aerodynamic moments acting on the projectile, $[ML^2T^{-2}]$
M , M , M 1 2 3	fixed-plane components of $\vec{M}$ , eq. (43)
m	mass, [M]
N1	the number of measured dependent variables in a system of equations
N2	the total number of dependent variables in a system of differential equations
N3	the number of unknown parameters appearing explicitly in a system of differential equations
N4	the number of measurements taken on each of the N1 depen- dent variables
N23	N2 + N3, the number of unknown constants in a system of equations
n	rifling, [cal/rev], Table I

SCHERMEN

1000

PE	probable error
P <sub>in</sub>	$\frac{\partial y_i}{\partial a_n}$ , the influence (or sensitivity) coefficients, eq. (3)
p	the X -component of the angular velocity of the missile- l fixed system with respect to the range system, eqs. (40- 41), [rad/sec]
Ŕ	position vector of the missile's CG, [L]
r	arclength along the trajectory of the missile's CG, [M]
t	time
t*	value of time at $x = x^*$
u,u,u 1 2 3	fixed-plane components of $\vec{V}$ , [LT <sup>-1</sup> ]
₹	velocity vector, [LT <sup>-1</sup> ]
v	magnitude of $\vec{V}$ , [LT <sup>-1</sup> ]
ž	vector of the N4 values $\mathbf{x}_{m}$ of the independent variable at which measurements are taken
XYZ	the axes of a range (earth-fixed) coordinate system; the X-axis is directed down-range along the intersection of a horizontal plane with the vertical plane contain- ing the gun; the Y-axis lies in this horizontal plane, directed to the left of an observer facing downrange; the Z-axis is directed upward.
X X X 1 2 3	the axes of a fixed-plane coordinate system; the X -axis
	lies along the missile's longitudinal axis, directed from the CG (origin) to the nose; the X -axis is con-
	strained to lie in the horizontal plane, directed to the left of an observer facing in the direction of the positive X -axis; the X -axis is directed upward. 1 $3$
X X' X' 1 2 3	the axes of the fixed-plane system of References 1 and 2, where $X'_2 = -X_2$ , $X'_3 = -X_3$ .

x	the independent variable of a system of equations
x,y,2	range components of R
x,y,z	range components of $\vec{V}$
x <sub>m</sub>	the value of the independent variable x at which the m-th measurement is taken, $m = 1, 2, N4$
<b>x*</b>	the mid-range value of the down-range coordinate x
у <sub>ј</sub>	the dependent variables of a system of equations
<sup>a</sup> nk	the coefficients of the increments $\Delta r_k$ in the differential corrections normal equations, see eqs. (6-7)
<sup>ã</sup> nk	the (n, k)-th element of the inverse of matrix $(\alpha_{nk})$
œŢ	the yaw angle, the total angle of attack, measured (in the X X X system) from the velocity vector to the X -axis. 1 2 3
β <sub>n</sub>	the terms on the right-hand side of the differential correc- tions normal equations, see eqs. (6, 8)
γ <sub>kn</sub>	a possible replacement for $\alpha_{nk}$ , suggested by Marquardt, eq. (15)
<sup>∆a</sup> k	the increments by which the parameters are changed in the differential corrections process
δ	sine of the yaw angle, eq. (17)
ε	the sum of the squares of the residuals in a least squares fit, eq. (2)
θ	an Euler angle; see Fig. 2a
θ <sub>A</sub>	azimuth of the line-of-fire, that is, the angle measured clockwise from North to the down-range axis (192° for the BRL Transonic Range)
θ <sub>L</sub>	latitude of the range, considered positive in the Northern hemisphere (39° 26' at BRL)
θ <sub>m</sub>	the missile's angle of yaw; see Fig. 2a and eq. (26)

.

λ	a constant in Marquardt's scheme, eq. (15)
ξ <sub>2</sub> ,ξ <sub>3</sub>	$-u_2/V$ , $-u_3/V$
ρ	air density, considered constant, [ML <sup>-3</sup> ]
τ <sub>ω</sub> Ε	the earth's angular velocity vector, eq. (64a)
ω <sub>E</sub>	magnitude of $\vec{\omega}_{E}$ , 0.00007292 rad/sec
<sup>*</sup> ωS <sub>1</sub> (S <sub>2</sub> )	(where S and S can be FP, R or R') angular velocity of the S coordinate system with respect to the S coor- dinate system

Subscripts:

( ) <sub>FP</sub>	components in a fixed-plane $(X X X)$ system
() <sub>R</sub>	components in a range (XYZ) system
() <sub>R</sub> /	components in an intermediate $(X'Y'Z')$ system
$\frac{d^2 \vec{R}}{dt^2} \Big _{I}$	second derivative of $\vec{R}$ in an inertial system
() <sub>o</sub>	initial condition, taken here to be the value at the first data station

## I. INTRODUCTION

In free flight studies, the main task is to determine from observations of a missile's motion the values of the aerodynamic coefficients appearing in the differential equations describing that motion. For normal enclosed-range firing conditions (symmetric shell, nearly horizontal flight, small yaw, constant or only slowly varying spin, etc.), we can approximate the solution to the differential equations quite adequately by convenient closed-form expressions. These expressions involve certain constants directly related to the aerodynamic coefficients. The values of these constants are determined by a least squares fit of the observed data to the closed-form expressions.

For the past two decades, the general technique of fitting observed data to convenient closed-form expressions has been the heart of freeflight, enclosed-range data reduction<sup>1,2\*</sup>. Highly successful results have been obtained for a variety of missile shapes and sizes. Over the Years, the technique has been gradually refined and extended to cover many types of force and moment nonlinearities. Unfortunately, such extension often requires that a number of rounds be fired at different Mach numbers, yaw levels, and so on, to obtain a single set of coefficients. The process can be costly in dollars and time. Moreover, the technique can be stretched only so far; an occasional round has defied analysis by conventional procedures.

It is relatively easy in these troublesome nonlinear situations to specify - by experience and by cunning - the sort of nonlinear terms that must appear in the differential equations of motion in order to produce the observed behavior. It is much more difficult to find convenient pseudo-solutions that will (1) represent the motion adequately and (2) contain constants that can be easily related to the coefficients of the differential equations. What we need in these situations is a method that doesn't require any knowledge or assumptions on our part regarding the form of the solution to the differential equation.

The problem is essentially one of parameter optimization. We are given

a. the form of a set of differential equations involving unknown constant parameters (coefficients and initial conditions);

b. a set of first estimates for the parameters;

c. a set of discrete measurements on one or more of the dependent variables;

d. a criterion function of the parameters (sey, the sum of the squares of the residuals for a given fit).

\*References are listed on page 59.

The problem is to devise a routine for adjusting the parameters sutomatically so as to minimize the criterion function.

The problem has received much attention, particularly in recent years with the proliferation of high-speed computers, and many ingenious schemes for optimizing the parameters have been proposed. For continuous rather than discrete input data, Meissinger<sup>3,4</sup> approached the criterion function minimum by a path of approximately steepest descent, using an analog computer. On the other hand, the techniques devised by Goodman<sup>5-7</sup> and by the team of Chapman and Kirk<sup>8</sup> to handle discrete data are better suited to the digital computer. In this paper, we will be concerned primarily with the Chapman-Kirk technique. The Goodman and Chapman-Kirk methods are quite similar, although Chapman and Kirk were unaware of Goodman's work when they presented their own results at the AIAA Seventh Aerospace Sciences Meeting, New York, 1969. From the standpoint of the aerodynamicist, Chapman and Kirk's contribution was to apply the process successfully to representative aerodynamic cases and, perhaps more important, to present their results at the meeting and later in a journal<sup>8</sup> where it came to aerodynamicists' attention. (It is unfortunate but often true that when a pertinent article such as Goodman's appears in a mathematical journal, the aerodynamicist either overlooks it or fails to recognize its applicability to his work.)

Applications<sup>9-13</sup> of the Chapman-Kirk technique are growing more and more sophisticated. The present report documents one such application carried out by the Armament Department, General Electric Company, Burlington, Vermont, for the U. S. Army Aberdeen Research and Development Center (ARDC), Aberdeen Proving Ground, Maryland, under Government Contract No. DAAD05-71-C-0265 during the period 4 February to 20 April 1971. The results of this work have also been issued as a General Electric report<sup>14</sup>.

The raw data for this study consisted of time, position and orientation measurements at discrete points along the trajectory for twelve rounds fired in the Transonic Free Flight Range<sup>15</sup>, Ballistic Research Laboratories (BRL). The twelve rounds consisted of four spin-stabilized projectile types (see Table I and Figure 1). Each of the rounds had been previously reduced<sup>11,16</sup> by the usual range technique (with varying degrees of success) and each was hand-picked for the present assignment.

Some of the twelve rounds could be fitted by assuming relatively simple force and moment systems; these rounds were chosen to enable the Chapman-Kirk technique to get a foot in the door. Other rounds of the twelve were oddities that had already annoyed and frustrated a team of data analysts; these rounds were chosen to give the Chapman-Kirk technique a good work-out. The primary purpose of the present study was to see how well the Chapman-Kirk technique could determine the values of the nonlinear coefficients present in the force and moment expressions.

# II. THE CHAPMAN-KIRK TECHNIQUE

Suppose for the moment that we have a system defined by elementary equations\*. Let

- N1 = the number of measured dependent variables
- N23 = the number of unknown constants in the system
- N4 = the number of measurements taken on each of the N1 dependent variables

where N4 is greater than N23. (The notation here is not entirely capricious. An N2 and N3, lying between N1 and N4, will be introduced later and N23 will be the sum of N2 and N3.) Assume that we can write  $\gamma$ ur system of equations in the form

$$y_i = f_i(x, a_1, a_2, \dots, a_{N23}), i = 1, 2, \dots, N1$$
 (1)

where  $y_i$  is the i-th dependent variable (i = 1, 2, ... N1)

f; is a single-valued elementary function

x is the independent variable

 $a_n$  is the n-th unknown constant parameter (n = 1, 2, . . ., N23)

We are given a set of measurements, which we represent by the N1 x N4 matrix  $D = (d_{im})$ , and a vector  $\vec{X} = (x_m)$  of the corresponding N4 values of the independent variable. That is,

ď im	Ξ	the measured value of $y_i$ at $x_m$
i	Ð,	1, 2,, N1
m	÷:	1, 2,, N4

For any parameter vector  $\vec{A} = (a_n)$ , we can obtain from (1) the corresponding solution  $y_i(x_m)$  at each point  $x_m$ . The problem is to determine the value of  $\vec{A}$  that minimizes  $\varepsilon$ , the sum of the squares of the residuals:

<sup>\*</sup>By an elementary equation, we mean an equation involving only elementary functions and a finite number of arithmetical operations. Specifically, we are excluding differential equations.

$$\epsilon = \sum_{m=1}^{N4} \sum_{i=1}^{N1} [d_{im} - y_i (x_m)]^2$$
 (2)

(For convenience, we omit weighting factors in (2) and the succeeding discussion; such factors might be needed, for example, to insure that the terms in (2) are dimensionally equal.) Now  $\varepsilon$  will be at a minimum only when its partial derivatives with respect to each of the parameters is zero. We introduce a convenient notation for these partial derivatives:

$$P_{in} = \frac{\partial y_i}{\partial a_n}$$
(3)  
 $i = 1, 2, ..., N1$   
 $n = 1, 2, ..., N23$ 

Because they reflect the influence of each parameter change, the P<sub>ir</sub> are sometimes called "influence" (or "sensitivity") coefficients. For a minimum  $\varepsilon$ , we must have

$$\frac{\partial}{\partial a_n} = -2 \sum_{m=1}^{N4} \sum_{i=1}^{N1} \left[ d_{im} - y_i(x_m) \right] P_{in} = 0$$
(4)

Equations (4) constitute a set of N23 equations in the N23 unknown parameters; the values of the parameters satisfying (4) are the desired optimum values.

If the functions f, are linear in the parameters a\_:

$$f_i = a_i \phi_{i_1}(x) + a_i \phi_{i_2}(x) + ... + a_{N23} \phi_{i_{N23}}(x)$$

then

$$P_{in} = \phi_{in}(x)$$

and set (4) is also linear in the parameters; hence (4) is easily solvable (in theory). If the functions  $f_i$  are <u>nonlinear</u> in the parameters. Sowever, then solving (4) can be quite difficult. The usual way out f this difficulty is to approximate the variables  $y_i$  by their linearly truncated Taylor expansions about a given set of values for the parameters:

$$y_i \simeq \hat{y}_i + \sum_{k=1}^{N23} \hat{P}_{ik} \Delta a_k$$
 (5)

where the circumflex (^) denotes evaluation at the given parameter values and where  $\Delta a_k$  is an indicated change in the given value of  $a_k$ .

If (5) is substituted in (4), and if the  $\rm P_{in}$  in (4) are replaced by  $\hat{\rm P}_{in}$ , we have

$$\sum_{m=1}^{N4} \sum_{i=1}^{N1} \left[ d_{im} - \hat{y}_{i} - \sum_{k=1}^{N23} \hat{p}_{ik} \Delta a_{k} \right] \hat{p}_{in} = 0$$

or

$$\sum_{k=1}^{N23} \alpha_{nk} \Delta a_k = \beta_n, n = 1, 2, ..., N23$$
(6)

where

$$\alpha_{nk} = \sum_{m=1}^{N4} \sum_{i=1}^{N1} \hat{P}_{in}(x_m) \hat{P}_{ik}(x_m)$$
(7)

$$\beta_n = \sum_{m=1}^{N4} \sum_{i=1}^{N1} \left[ d_{im} - \hat{y}_i(x_m) \right] \hat{P}_{in}(x_m)$$
 (8)

Equation (6) represents a set of N23 linear equations in the N23 increments  $\Delta a_n$ . Hence we can solve (6) for the increments:

$$\Delta a_{n} = \sum_{k=1}^{N23} \tilde{\alpha}_{nk} \beta_{k}, n = 1, 2, ..., N23 \qquad (9)$$

where

 $a_{nk}$  = the (n, k)-th element of the inverse of matrix  $(a_{nk})$ cofactor of element  $a_{nk}$ 

determinant of matrix 
$$(\alpha_{nk})$$

The new value of  $a_n$  is then obtained by adding  $\Delta a_n$  to the old value.

If the initial estimates of the parameters are "close enough" to the "true" values, (5) will be an adequate approximation and the new Values of the parameters will produce a smaller criterion function  $\varepsilon$ . The process can then be repeated as many times as necessary until some convergence criterion is satisfied. The probable error of the fit at the end of any iteration is given by

$$PE = 0.6745 \left[ \frac{\varepsilon}{N1 \times N4 - N23} \right]^{\frac{1}{2}}$$
(10)

and an estimate of the probable error in  $a_n$  is given by

$$E(a_n) = (\tilde{\alpha}_{nn})^{\frac{1}{2}} \cdot PE$$
(11)

This, in brief, is the well-known process of "differential corrections," as applied to elementary equations that are nonlinear in the parameters.

For a system of ordinary differential equations, the situation is naturally more complicated. Assume that the given system of differential equations has been reduced to a system of N2 first order, possibly nonlinear equations\*:

$$\frac{dy_{j}}{dx} = f_{j} (x, y_{1}, y_{2}, \dots, y_{N2}, a_{1}, a_{2}, \dots, a_{N3})$$

$$y_{j} (x_{0}) = a_{N3 + j}$$

$$j = 1, 2, \dots, N2$$

$$(12)$$

<sup>\*</sup>It is not necessary when carrying out the Chapman-Kirk technique to reduce the given system to first order equations; this was done here merely to aid the exposition.

where

- N2 = the total number of dependent variables in the system
- N3 = the number of unknown parameters appearing explicitly in the differential equations

and where N1 and N4 are defined as before. Note that we have juggled the notation so that while only N3 parameters appear explicitly in (12), there are still N23 (= N2 + N3) parameters to be determined:

 $a_1, a_2, \ldots, a_{N3}$  (the N3 explicit parameters)

 $a_{N3+1}, a_{N3+2}, \ldots, a_{1,23}$  (the N2 unknown initial conditions)

where

$$1 \le N1 \le N2 \le N23 \le N4$$
 (13)

The same differential correction technique that was applied to (1) can be applied to (12). Equations (6-11), which depend only on the definition (2) of the criterion function  $\varepsilon$ , are still valid. However, a new difficulty arises when the given set of equations are ordinary differential equations: how do we evaluate the dependent variables and their partial derivatives appearing in (7) and (8)? For the case of non-differential equations, this was no problem; we were presumably given explicit expressions for each of the variables and could easily write down expressions for the required partials. For a given set of differential equations, however, we will not, in general, know the form of the solution. Values of the dependent variables can be obtained by some numerical integration scheme, but part of the problem remains: how do we obtain the required values of the partial derivatives?

Chapman and Kirk tried various schemes for evaluating these partial derivatives and finally settled on the method\* of "parametric differentiation." This method consists of formally differentiating the given set (12) of differential equations with respect to each of the N23 constants to be determined. We have

$$\frac{\partial}{\partial \mathbf{a}_n} \left( \frac{dy_j}{dx} \right) = \frac{\partial \mathbf{f}_j}{\partial \mathbf{a}_n}$$

<sup>\*</sup>The method is not new. It was used, for example, by the previously cited Meissinger and Goodman (and apparently by Knalder<sup>17</sup>, with whose work we are unfamiliar but who is referenced in References 10 and 12).

or, assuming that the  $y_j$  are continuous in x and  $a_n$ , so that the order of differentiation can be interchanged,

$$\frac{d P_{jn}}{dx} = \frac{\partial f_j}{\partial a_n} \qquad j = 1, 2, \dots, N2 \qquad (14a)$$

$$n = 1, 2, \dots, N23$$

where

$$P_{jn}(x_0) = 1$$
 (if  $n - j = N3$ )  
= 0 (otherwise) (14b)

The somewhat strange-looking initial conditions (14b) merely reflect the fact that  $P_{jn}$  is initially 1 if and only if a represents the initial value of  $y_j$ . By (12), this occurs if and only if  $a_n = a_{N3} + j$ ; hence if and only if n = N3 + j.

What we have done above is to derive an auxiliary set of N2 x N3 equations (14) whose solutions are the partials  $P_{jn}$ , some of which (those for  $j \le N1$ ) are needed in solving (6). For a given set of estimates of the parameters and initial conditions, we can integrate by some numerical scheme both the original set of equations (12) and the auxiliary set (14). (The original set may or may not be linear; the auxiliary set will always be linear.) The numerical integration yields the values of the dependent variables and the influence coefficients required to solve (6) for the parameter changes.

Except for this more laborious way of determining  $y_i(x_m)$  and  $P_{in}(x_m)$ , the procedure for determining the unknown constants of a set of differential equations is the same as for non-differential equations.

As a final aside, we note a possible future improvement. In the N23-dimensional parameter space, the truncated Taylor series technique proceeds from a given point (whose coordinates are the given estimates of the N23 constants) in the direction of the vector  $\Delta A = (\Delta a_n)$ 

obtained by solving (6). The method of steepest descent, on the other hand, proceeds from the given point in the direction of the negative gradient of  $\varepsilon$ . Marquardt<sup>18,19</sup> points out that these two directions are nearly perpendicular, while the optimum direction lies somewhere in between. To proceed in approximately the optimum direction, he suggests replacing  $\alpha_{kn}$  in (6) by

$$\gamma_{kn} = \begin{cases} (1 + \lambda) \alpha_{kn} & \text{for } k = n \\ \alpha_{kn} & \text{for } k \neq n \end{cases}$$
(15)

where  $\lambda$  is a fudge factor constant whose value should be changed from one iteration to the next according to a few simple rules (the rules are listed - in slightly different form - in References 18 and 19). Using Marquardt's magic  $\lambda$ , the Chapman-Kirk process often converges for initial guesses far outside the previous region of convergence. The Marquard algorithm was pointed out to us by Chapman himself, who has used it (subsequent to the work reported on in Reference 8) with great success in hitherto intractable cases. The algorithm was not used in the investigation covered by this report because we were not aware of it at the time.

#### III. INPUT DATA, COORDINATE SYSTEMS AND YAW VARIABLES

A missile fired in the BRL Transonic Range is observed at twentyfive spark-photography stations distributed along a 680-foot portion of the trajectory. The observed data at each station consists of

a. the elapsed time t, reckoned from the instant the spark at the first station was triggered by the passing missile. The time error in a properly functioning timer is estimated to be no more than one microsecond. Only about two-thirds of the stations are instrumented at present to furnish timing data.

b.  $(x,y,z)_R$ : the position vector of the missile's CG in a <u>range</u> (earth-fixed) coordinate system XYZ. The X-axis is directed down-range along the intersection of a horizontal plane with the vertical plane containing the gun; the Y-axis lies in this horizontal plane, directed to the left of an observer facing downrange; the Z-axis is directed upward. The error in any position measurement should be no greater than 0.003 meter.

c.  $(0, \xi_2, \xi_3)_{FP}$ : the yaw vector in a <u>fixed-plane</u> coordinate system X X X. The X -axis lies along the missile's longitudinal axis, 1 + 2 + 3 + 1directed from the CG (the origin of the fixed-plane system) to the nose; the X -axis is constrained to lie in the horizontal plane, directed to the left of an observer facing in the direction of the positive X -axis; the X -axis is directed according to the right-hand rule (that is, upward). If u ,u ,u are the velocity components in the fixed-plane system, then

$$\xi_2 = -\frac{u^2}{V}, \quad \xi_3 = -\frac{u^3}{V}$$
 (16)

(17)

where V is the magnitude of the velocity vector. The minus signs in (16) appear because in range studies the yaw angle is measured from the velocity vector to the X -axis\*. The magnitude  $\delta$  of the yaw is given by

 $\delta = \left[\xi_{2}^{2} + \xi_{3}^{2}\right]^{\frac{1}{2}} = \frac{\left[u_{2}^{2} + u_{3}^{2}\right]^{\frac{1}{2}}}{V} = \sin \alpha_{T}$ 

where  $\alpha_T$  is the yaw angle, the so-called total angle of attack. The angular measurements that yield the yaw components are usually accurate to within 0.002 radian.

Although differential equations describing the yawing motion can be written in terms of  $\xi_{2}$  and  $\xi_{3}$ , we found it more convenient to work with the related Euler angles  $\psi$  and  $\theta$ . These angles appear in the transformation matrix that converts from range to fixed-plane\_coordinates. To derive this matrix, assume the existence of a vector B extending from the range system origin in the direction of the positive X\_-axis

(see Figure 2a). Rotate the range system XYZ about the Z-axis by the angle  $\psi$  so that X' (the rotated X-axis) coincides with the projection of the vector B on the XY-plane. The magnitude of  $\psi$  is not to exceed 180°; if this requires a counterclockwise rotation about the Z-axis (as in Figure 2a),  $\psi$  is considered positive and if clockwise, then  $\psi$  is considered negative. Then we have

\*In References 1 and 2, a fixed-plane system X X' X' is used, where the X' - and X' - axes have opposite directions to the X - and X - axes, respectively. In this X X' X' system, the yaw angle is measured from the  $X - \frac{123}{123}$  axis to the velocity vector. If  $u_1 u'_1 u'_1$  are the velocity components and  $0, \xi'_1, \xi'_1$  are the yaw components in the  $X X' X'_1$  system, then

$$\xi'_2 = \frac{u'_2}{V} = -\frac{u_2}{V} = \xi_2$$

and similarly,  $\xi'_{3} = \xi_{3}$ . That is, the yaw components in the two fixedplane systems are identical.

$$\begin{pmatrix} \mathbf{x}' \\ \mathbf{y}' \\ \mathbf{z}' \end{pmatrix}_{\mathbf{R}'} = \begin{pmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \\ \mathbf{R} \end{cases}$$
(18)

Next, rotate the intermediate system X'Y'Z' about the Y'-axis so that X" (the rotated X-axis) coincides with vector B. Let  $\theta$  denote the angle from Z' to B, where  $|\theta| \leq 90^{\circ}$ . If the rotation is counterclockwise about the Y'-axis,  $\theta$  is considered positive; if the rotation is clock-wise (as in Figure 2a),  $\theta$  is negative. Then we have

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix}_{\text{FP}} = \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}_{\text{R}'}$$
(19)

This final system X"Y"Z" has the orientation of the fixed-plane system X X X. Thus the transformation matrix from range to fixed-plane co-1 2 3 ordinates is the product of the  $\theta$ -matrix and the  $\psi$ -matrix:

$$\begin{pmatrix} x_{1} \\ x_{2} \\ x_{3} \\ FP \end{pmatrix} = \begin{pmatrix} \cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\ -\sin \psi & \cos \psi & 0 \\ \sin \theta \cos \psi & \sin \theta \sin \psi & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{R}$$
(20)

For an assumed flat, nonrotating earth, the gravitational force  $\vec{G}$  has the form

 $\vec{G} = (0, 0, -mg)_{D}$  (21a)

where the gravitational acceleration g is constant for range firings. Substituting the right-hand side of (21a) in (20), we see that the fixed-plane components of G depend on the Euler angle  $\theta$ :

 $\vec{G} = (mg \sin \theta, 0, - mg \cos \theta)_{FD}$  (21b)

Note that by the definitions of the two Euler angles, the angular velocity of the intermediate R'(X'Y'Z') system with respect to the range system is

$$\vec{\omega}_{R'(R)} = (0, \dot{\theta}, \dot{\psi})_{R'}$$

Substituting this vector in (19), we obtain the angular velocity of the fixed-plane system with respect to the range system

$$\vec{\psi}_{FP(R)} = (-\dot{\psi}\sin\theta, \dot{\theta}, \dot{\psi}\cos\theta)_{FP}$$
(22)

The above discussion gives us the physical interpretation of  $\psi$  and  $\theta$ , but in order to work with these angles, we needed explicit equations relating  $\psi$  and  $\theta$  to the given yaw components  $\xi$  and  $\xi$ . By some elementary but cumbersome vector analysis, it can be shown that the desired relations are

$$\psi = \sin^{-1} \left( \frac{\sin \psi_{\rm M}}{\cos E_{\rm R}} \right) + a_{\rm R}$$
 (23)

$$\theta = \theta_{M} - \sin^{-1} \left( \frac{\sin E_{R}}{\cos \psi_{M}} \right)$$
(24)

where  $\psi_{M}$  and  $\theta_{M}$  are the missile's pitch and yaw angles, respectively (see Figure 2b):

$$\psi_{\mathsf{M}} = \sin^{-1} \left( -\frac{u}{v} \right) = \sin^{-1} \left( \xi_{2} \right) \tag{25}$$

$$\theta_{M} = \tan^{-1} \left( \frac{u_{3}}{u_{1}} \right) = -\sin^{-1} \left( \frac{\xi_{3}}{\cos \psi_{M}} \right)$$
(26)

where  $a_R$  and  $E_R$  are the azimuth and elevation angles, respectively, of the velocity vector in the range system (see Figure 2c):

$$a_{R} = \tan^{-1} \left( \frac{\dot{y}}{\dot{x}} \right)$$
 (27)

$$E_{R} = \sin^{-1} \left(\frac{\dot{z}}{\nabla}\right)$$
 (28)

where x, y, z are the velocity components in the range system.

For the present analysis of range firings, we made the simplifying assumption that the angles  $a_R$  and  $E_R$  could be ignored. Letting  $a_p = E_p = 0$ , equations (23) and (24) reduce to

$$\psi = \psi_{M} \tag{23a}$$

$$\theta = \theta_{M}$$
(24a)

By equations (17), (25) and (26), the magnitude  $\delta$  of the yaw can be expressed in terms of the pitch and yaw angles:

$$\delta = (\sin^2 \psi_{\rm M} + \cos^2 \psi_{\rm M} \sin^2 \theta_{\rm M})^{\frac{5}{2}}$$
(29)

## IV. THE EQUATIONS OF MOTION

In this section, the working forms of the equations of motion are derived (albeit briefly) from the basic expressions for Newton's Second Law. By writing out this derivation, we can point out where and what kinds of assumptions and simplifications were made and thus facilitate future changes.

The interdependence of the various equations and of the three distinct reductions (drag, yaw and CG) are indicated in Figure 3.

# A. The Drag Equation

The classic drag equation has the form

$$\vec{D} + \vec{G} = m \frac{d^2 \vec{R}}{dt^2} \bigg|_{I} [MLT^{-2}]$$
(30)

where

$$\vec{D}$$
 = drag force  
= - (m A C<sub>D</sub> V)  $\vec{V}$ 

$$A_1 = \frac{\pi \rho d^2}{8m} [L^{-1}]$$

 $\vec{G}$  = gravitational force

# $\vec{R}$ = position vector of the missile's CG

and where the subscript I denotes vector differentiation in an inertial system. For enclosed-range studies, the density  $\rho$  and hence A are known constants. The drag coefficient  $C_D$  is in general a function of Mach number and of  $\delta^2$ . For each of the twelve Transonic Range rounds studied here, however, we could ignore the Mach number variation over the observed trajectory and assume a linear\* dependence of  $C_D$  on  $\delta^2$ :

$$C_{\rm D} = C_{\rm D_0} + C_{\rm D_2} \delta^2$$
 (31)

We seek the X-component of (30) in the range system. For the purpose of performing the drag reduction, we can assume that the range system is an inertial system and hence ignore the centripetal and Coriolis accelerations that arise in any earth-fixed system. Then the X-component of (30) can be written as

$$\begin{array}{c} \ddot{x} = -A_1 C_D V \dot{x} \\ = -A_1 C_D V^2 \end{array}$$

$$(32)$$

where

$$C_{DX} = \left(\frac{\dot{x}}{V}\right) C_{D} = C_{D} \cos E_{R} \cos a_{R}$$

## = down-range drag coefficient

For the present Transonic Range studies, (32) has the disadvantage that the independent variable t - which should be known exactly at each point - is obtained only at about two-thirds of the spark stations (and obtained with sufficient accuracy at a considerably smaller fraction). On the other hand, the down-range coordinate x of the missile's CG is usually known very accurately at each station. Thus a reasonable course of action is to convert the independent variable in (32) from t to x. We have

<sup>\*</sup>Provision was made in the coding to hardle  $C_D$  as a quadratic in  $\delta^2$ , but the higher-order term was never needed.

$$\frac{dt}{dx} = \frac{1}{\dot{x}}$$

$$\frac{d\dot{x}}{dx} = -A_1 C_D V = -A_1 \left(\frac{C_D^2}{C_D x}\right) \dot{x}$$
(33)

The presence in (33) of the velocity V is a nuisance for as yet we have no equation for generating V. For the nearly horizontal flights encountered in ballistic ranges, the distinction between V and

 $\dot{x}$  (= V cos  $E_R$  cos  $a_R$ ) and between  $C_D$  and  $C_{DX}$  can be ignored. Thus we can write (33) in the final form:

$$\frac{dt}{dx} = \frac{1}{V}$$

$$\frac{dV}{dx} = -\left(C_{D_0} + C_{D_2} \delta^2\right) A_1 V$$
(34)

The drag reduction - that is, the application of the Chapman-Kirk technique to (34) - is normally done before the yaw reduction and so at this stage we know the values of  $\delta^2$  only at certain discrete points (namely, at each spark station). From these scattered values, we obtained additional input values of  $\delta^2$  at selected values of x by interpolation. (After the yaw reduction has been performed, the drag reduction could be re-done, using the fitted values of  $\delta^2$  rather than the raw plus interpolated-raw values. For the present study, this wasn't necessary.)

In addition to the yaw data, the required input for a given round consisted of the measured times t, the measured x values and initial estimates of the two explicit parameters  $\begin{pmatrix} C_{D_o}, C_{D_o} \\ 0 & 2 \end{pmatrix}$  and the two initial conditions  $(t_o, V_o)$ , where we defined initial conditions as the conditions at the first spark station. The output consisted of

a. the least squares values of the explicit parameters and initial conditions;

b. appropriate error estimates in the above;

c. a reliable correlation of time with distance, so that in the remaining equations of motion, time could be used as the independent variable. Of course, it was not absolutely necessary to revert to a time base. For theoretical work, the arclength r along the trajectory of the missile's CC (or a nondimensional length r/d) is often a more convenient independent variable than either time or down-range distance. The equations of motion (in particular, the yaw equations) assume

simpler forms when r is the independent variable. Since r = V, our

approximation  $x \cong V$  is equivalent to ignoring the distinction between x and r. We could, then, have written all the equations with respect to this convenient length variable which we measure as x but are free to interpret as r. One reason we didn't is that the present study, while interesting in itself, is also regarded as preliminary, gettingour-feet-wet training for more ambitious studies that will require a time-based set of equations. By working with time-based equations in the present study, we could save considerable coding effort. Comprehensive computer programs, capable of handling the larger problems, were able to handle the current problem as a special case.

One additional bit of information can be gleaned from our reduction: a representative  $C_D$  value for each round, obtained by replacing  $\delta^2$  in (31) by its mean value:

$$\overline{C}_{D} = C_{D_{0}} + C_{D_{2}} \overline{\delta^{2}}$$
(35)

Although a value for  $\overline{\delta^2}$  was available for each round from the BRL yaw reduction, the quantities  $\overline{\alpha_T} = \sin^{-1}(\sqrt[4]{\overline{\delta^2}})$  listed in Tables II - VI were obtained by a slightly different averaging process than used by BRL. Thus these listed values differ slightly (by less than 4%) from the BRL values. Each representative drag coefficient  $\overline{C_D}$  can be compared with the range value,  $C_{DR}$ , obtained at BRL by fitting time as a cubic in down-range distance x:

$$t = t^{*} + a_{1} (x - x^{*}) + a_{2} (x - x^{*})^{2} + a_{3} (x - x^{*})^{3}$$

$$C_{DR} = \frac{2 a_{2}}{A_{1} a_{1}}$$

$$= \text{ range value of } C_{D} \text{ at } [(t, x, V) = (t^{*}, x^{*}, 1/a_{1})]$$
(36)

where x\* is the given mid-range value of x.

As we will see, the variable V appears in the yaw and CG equations and the question arises: how should we generate it there? We could, of course, store the large number of discrete velocity values available when we integrate the drag equation (34) numerically. This would be both tedious and unnecessary. Instead, we made a reasonable assumption: for purposes of performing the yaw reduction\* on the given Transonic Range rounds, V can be represented adequately over the observed trajectory by a known quadratic in time:

$$V = V_{0} + V_{1} (t - t_{0}) + V_{2} (t - t_{0})^{2}$$
(37)

Values for V and V in (37) can be obtained in various ways. For example, if we replace  $C_{D}$  with  $\overline{C}_{D}$  in the drag equation, the solution can be written at once:

$$V = V_{o} \exp \left[ -A_{1} \overline{C}_{D} (x - x_{o}) \right]$$
  
=  $\frac{V_{o}}{1 + C_{A} (t - t_{o})}$  (38)

where

$$C_{A} = A_{1} \overline{C}_{D} V_{0} [T^{-1}]$$

with the truncated series expansion:

$$V = V_{o} [1 - C_{A} (t - t_{o}) + C_{A}^{2} (t - t_{o})^{2}]$$
(37a)

# B. The Roll Equation

$$\dot{\widetilde{\omega}}_{MF(FP)} = (\dot{\phi}, 0, 0)_{FP}$$
(39)

<sup>\*</sup>The assumption is not necessary for the CG equations where, as we shall see, V can be generated handily.

where  $\phi$  is the roll angle (the angle between the X X axes and the X X  $_{23}^{23}$  axes). By (22) and (39), we can write an expression for the angular velocity of the missile-fixed system with respect to the range system:

$$\dot{\vec{\omega}}_{MF}(R) = \vec{\omega}_{MF}(FP) + \vec{\omega}_{FP}(R)$$
  
= (p,  $\dot{\theta}$ ,  $\dot{\psi} \cos \theta$ )<sub>FP</sub> (40)

7

where

$$\mathbf{p} = \mathbf{\phi} - \mathbf{\psi} \sin \theta \tag{41}$$

The angular momentum of a missile with rotational symmetry is then

$$\vec{L} = (L_1, L_2, L_3)_{FP} = (I_x p, I_y \dot{\theta}, I_y \dot{\psi} \cos \theta)_{FP} [ML^2T^{-1}]$$
(42)

The sum of the moments acting on the missile is equal to the time derivative of the angular momentum:

$$\vec{M} = (M_1, M_2, M_3)_{FP} = \frac{d\vec{L}}{dt} \Big|_{I} \qquad [ML^2T^{-2}]$$
$$= [(\vec{L}_1, \vec{L}_2, \vec{L}_3) + \vec{\omega}_{FP(R)} \times \vec{L}]_{FP} \qquad (43)$$

where again the range system is considered an inertial system. Hence by (22) and (42),

$$M_{1} = I_{x} \dot{p}$$
(44)

$$M_{2} = I_{y} \ddot{\theta} + (I_{x} p + I_{y} \dot{\psi} \sin \theta) \dot{\psi} \cos \theta \qquad (45)$$

$$M_{3} = I_{y} \ddot{\psi} \cos \theta - (I_{x} p + 2 I_{y} \dot{\psi} \sin \theta) \dot{\theta}$$
(46)

Equation (44) is the roll equation; (45) and (46) constitute the yaw equations.

The roll equation does not depend on the other two moment equations and hence can be solved separately. We define the axial moment as

$$M_{1} = \left(\rho \frac{V^{2}}{2}\right) \left(\frac{\pi d^{2}}{4}\right) (d) \left(\frac{pd}{V}\right) C_{\ell p} [ML^{2}T^{-2}]$$
(47)

where  $C_{\substack{\mu \\ p}}$  is the roll damping moment coefficient\*. Then the roll equation (44) becomes

 $\dot{p} = A_{3} p V [T^{-2}]$  (48)

where

$$A_{3} = \left(\frac{\pi \rho d^{4}}{8 I_{x}}\right) C_{\ell p} \qquad [L^{-1}]$$

For the present study, A can be considered constant. If we were given sufficient spin data (obtained, say, by measuring the position on each spark photograph of two distinguishable pins placed in the base of the missile), we could obtain the "best" values of  $p_0$  and A by the Chapman-Kirk technique or by a fit of the data to the solution of (48):

 $p = p_0 \exp [A_3 (x - x_0)]$  (49)

In the present study, however, such spin data was unavailable. Yet the roll rate p was needed in the yaw equations. We resolved this problem by assuming that p is a known linear function of time:

 $p = p_0 [1 + A_3 V_0 (t - t_0)]$  (50)

where

$$p_o = 2 \pi V_o/(nd)$$
, rad/sec

n = rifling, cal/rev (see Table I)

and where A was evaluated by assuming  $C_{l} = -.013$  for all twelve p rounds.

\*In some texts (e.g., in References 13 and 14), the combination pd/(2V) is preferred to pd/V and in those texts the C must be interpreted

accordingly:

$$\begin{pmatrix} C_{\ell} \\ p \end{pmatrix}_{\frac{pd}{2V}} = 2 \begin{pmatrix} C_{\ell} \\ p \end{pmatrix}_{\frac{pd}{V}}$$

## C. The Yaw Equations

The yaw equations describe the wobbling motion of the missile's longitudinal axis  $\vec{A}$  about the tangent to the trajectory, that is, about the velocity vector  $\vec{V}$ . Any two variables sufficient to orient  $\vec{V}$  with respect to  $\vec{A}$  can serve as the dependent variables. In the usual BRL Transonic Range reduction, the dependent variables are the yaw components  $\xi_{2}$  and  $\xi_{3}$ ; here we use the Euler angles  $\psi$  and  $\theta$  and the governing equations (45) and (46). We define the cross-moments acting on the missile as follows\*:

$$M_{2} + i M_{3} = \left(\rho \frac{V^{2}}{2}\right) \left(\frac{\pi d^{2}}{4}\right) (d) \left\{ C_{M_{q}} \left[ \frac{(\dot{\theta} + i \psi \cos \theta) d}{V} \right] + \left[ \left(\frac{pd}{V}\right) C_{M_{p\alpha}} - i C_{M_{\alpha}} \right] \left(\frac{u_{2} + i u}{V}\right) \right\} [ML^{2}T^{-2}]$$
(51)

where\*\*

$$C_{M_{\alpha}} = C_{M_{\alpha0}} + C_{M_{\alpha2}} \delta^{2} + C_{M_{\alpha4}} \delta^{4}$$

$$C_{M_{p\alpha}} = C_{M_{p\alpha0}} + C_{M_{p\alpha2}} \delta^{2} + C_{M_{p\alpha4}} \delta^{4}$$

$$C_{M_{q}} = C_{M_{q0}} + C_{M_{q2}} \delta^{2}$$
(52)

\*Provision was made in the computer program to cope with an asymmetrical missile by including some trim terms not shown in (51) above. These terms were not used in the present study.

\*\*Note that C in (51) is multiplied by the nondimensional spin pd/V, so that the remarks on C in a previous footwite apply to C as well. That is,

 $\begin{pmatrix} C_{M} \\ p\alpha \end{pmatrix}_{\frac{pd}{2V}} = 2 \begin{pmatrix} C_{M} \\ p\alpha \end{pmatrix}_{\frac{pd}{V}}$ 

Likewise,  $C_{M_q}$  as defined here is half the  $C_{M_q}$  of References 13 and 14.

Substituting (51) in (45) and (46), we obtain the final form of the yaw equations:

$$\ddot{\theta} = A_2 \left[ (V u_3) C_{M_{\alpha}} + (u_2 p d) C_{M_{p\alpha}} + (d V \dot{\theta}) C_{M_{q}} \right]$$

$$- \left[ (I_x/I_y) p + \dot{\psi} \sin \theta \right] \dot{\psi} \cos \theta \qquad (53)$$

$$\ddot{\psi} = \left\{ A_2 \left[ - (V u_2) C_{M_{\alpha}} + (u_3 p d) C_{M_{p\alpha}} + (d V \dot{\psi} \cos \theta) C_{M_{q}} \right]$$

$$+ \left[ (I_x/I_y) p + 2 \dot{\psi} \sin \theta \right] \dot{\theta} \right\} / \cos \theta \qquad (54)$$

where A is a known constant:

$$A_{2} = \frac{\pi \rho d^{3}}{8 I_{y}} [L^{-2}]$$

Equations (53) and (54) involve four initial conditions, the eight aerodynamic parameters indicated in (52) and (possibly) one physical parameter,  $I_x/I_y$ . Thus a maximum of thirteen constants could be determined for each round by an application of the Chapman-Kirk technique to (53) and (54). However, for any computer run, any one or more of the eight aerodynamic constants in (52) could be treated as a known constant rather than as a parameter to be determined. For example,  $C_M$  was  $q_2$ 

fixed at zero (that is, ignored) in all the present computer runs.

Note that in addition to  $\theta$ ,  $\psi$  and their derivatives, (53) and (54) involve five dependent variables:

V, p, 
$$\delta^2$$
, u and u  $2^3$ 

We have already given equations adequate for simulating V and p as functions of time, namely, (37) and (50), respectively. The squared yaw can be obtained by (17):

$$\delta^2 = \left(\frac{u^2}{2} + \frac{u^2}{3}\right) / V^2$$
 (17a)

provided we know u and u. Thus the ability to solve (53) and (54) hinges now on our ability to generate u and u. We proceed now to derive the required equations. Let

$$\vec{F} \equiv (F_1, F_2, F_3)_{FP}$$

# = the aerodynamic force vector

and assume that the only forces acting on the missile are the aerodynamic and gravitational forces. Then the force equation is

$$\vec{F} + \vec{G} = m \left[ \dot{\vec{u}}_1, \dot{\vec{u}}_2, \dot{\vec{u}}_3 \right] + \vec{\omega}_{FP(R)} \times \vec{V}_{FP}$$
(55)

or, by (21b) and (22),

$$F_{1} + mg \sin \theta = m \left( \dot{u}_{1} - \dot{u}_{2} \dot{\psi} \cos \theta + u_{3} \dot{\theta} \right)$$
(56)

$$F_{2} = m (\dot{u}_{2} + u_{3} \dot{\psi} \sin \theta + u_{1} \dot{\psi} \cos \theta)$$
 (57)

$$F_{3} - mg\cos\theta = m(\dot{u}_{3} - u_{1}\theta - u_{2}\dot{\psi}\sin\theta)$$
 (58)

We assume that the fixed-plane components of the aerodynamic force are given by

$$F_{1} = -mA_{1}C_{D}Vu_{1}$$

$$F_{2} + iF_{3} = -mA_{1}\left(C_{N_{\alpha}}V + iC_{N_{p\alpha}}pd\right)(u_{2} + iu_{3})$$
(59)

where for the present purposes,  $C_{N_{\alpha}}$  and  $C_{N_{p\alpha}}$  are considered known constants\*. Then equations (57) and (58) can be written as:

\*As with 
$$C_{\mu_{p}}, C_{M_{p\alpha}}$$
 and  $C_{M_{q}}, we have
$$\begin{pmatrix} C_{N_{p\alpha}} \end{pmatrix}_{pd} = 2 \begin{pmatrix} C_{N_{p\alpha}} \end{pmatrix}_{pd}$$$ 

$$\dot{u}_{2} = -A_{1} \left[ (V u_{2}) C_{N_{\alpha}} - (p d u_{3}) C_{N_{p\alpha}} \right]$$

$$- (u_{1} \cos \theta + u_{3} \sin \theta) \dot{\psi} \qquad (60)$$

$$\dot{u}_{3} = -A_{1} \left[ (V u_{3}) C_{N_{\alpha}} + (p d u_{2}) C_{N_{p\alpha}} \right]$$

$$+ u_{1} \dot{\theta} + u_{2} \dot{\psi} \sin \theta - g \cos \theta \qquad (61)$$

where

$$u_1 = (V^2 - u_2^2 - u_3^2)^{\frac{1}{2}} = (1 - \delta^2) V$$

Thus we generate u and u by assigning values to  $C_{N\alpha}$ ,  $C_{N\alpha}$ , u and u  $_{2}$   $_{30}^{20}$  and then solving (60) and (61) simultaneously with the yaw equations (53) and (54).

An alternative method for generating u and u follows at once from assumptions (23a) and (24a) that  $\psi = \psi_M$  and  $\theta = \theta_M$ . By (16), (25) and (26) we have

 $u_{2} = -V \sin \psi$  $u_{3} = V \cos \psi \sin \theta$ 

These expressions eliminate the need for (60) and (61). The only drawback is that the resulting computer program couldn't be used in future cases where assumptions (23a) and (24a) are invalid.

While we didn't use this simplification, we did ease the labor of computation by modifying the <u>auxiliary</u> set of yaw equations produced by parametric differentiation. That is, we replaced the functions  $f_j$  in equation (14) by approximating expressions. The validity of this procedure depends, of course, on the adequacy of the approximations.

One final remark: a by-product of the yaw reduction is  $\delta^2$  as a function of time. As mentioned in section IV A, a second drag reduction could be performed with this computed  $\delta^2$  as input. If the output of the second drag reduction differs significantly from the

original drag output, a second yaw reduction based on the new drag data should then be run. For the present study, none of this was necessary.

# D. The CG Equations

The motion of the missile's CG along the trajectory is defined by the force equation:

$$\vec{F} + \vec{G} = m \left. \frac{d^2 R}{dt^2} \right|_{I}$$
(62)

We are interested in the range components of (62). Whereas in (30) and (55) we could treat the range system as an inertial system, here such simplification is not as defensible. For the range (earth-fixed) system, the acceleration is given by

$$\frac{d^2 \vec{R}}{dt^2} \bigg|_{T} = [(\vec{x}, y, z) + \vec{C} + \vec{C}_{T}]_{R}$$
(63)

where  $\vec{C}_T$  is the centripetal acceleration, which we promptly ignore, and where  $\vec{C}$  is the Coriolis acceleration, which we retain:

where

$$\vec{\omega}_E$$
 = the earth's angular velocity vector  
=  $\omega_E (\cos \theta_L \cos \theta_A, \cos \theta_L \sin \theta_A, \sin \theta_L)_R$   
 $\omega_E$  = 2  $\pi$  radians/sidereal day  
 $\simeq 0.00007292$  rad/sec

and where  $\theta_A$  and  $\theta_L$  are known constants defined in the List of Symbols. If we make the approximation

$$\vec{V} = (\dot{x}, \dot{y}, \dot{z})_{R} \cong (V, 0, 0)_{R}$$

then the Coriolis acceleration is given by

$$\vec{C} = 2 \omega_E V (0, \sin \theta_L, -\cos \theta_L \sin \theta_A)_R$$
(64b)

Defining

$$\vec{F} = (F_{\chi}, F_{\gamma}, F_{Z})_{R} = (F_{1}, F_{2}, F_{3})_{FP}$$

and using (21a), (63) and (64b), we can write (62) in the form

$$\ddot{x} = F_{\chi}/m$$

$$\ddot{y} = F_{\gamma}/m - (2 \omega_E \sin \theta_L) V$$

$$\ddot{z} = F_{\chi}/m + (2 \omega_E \cos \theta_L \sin \theta_A) V - g$$
(65)

To obtain expressions is:  $F_X$ ,  $F_Y$  and  $F_Z$ , we assume that the fixed-plane components of  $\vec{F}$  have the form already indicated in (59):

$$F_{1} = -mA_{1}C_{D}Vu_{1}$$
  
= -mA\_{1}C\_{DA}V^{2}  
$$F_{2} + iF_{3} = -mA_{1}\left(C_{N_{\alpha}}V + iC_{N_{p\alpha}}pd\right)(u_{2} + iu_{3})$$
(66)

where

$$C_{DA} = \left(\frac{u}{V}\right) C_{D} = (1 - \delta^2) C_{D}$$

= axial drag coefficient

For the yaw reduction,  $C_{N_{\alpha}}$  and  $C_{N_{p\alpha}}$  were considered known constants and  $F_{1}$  wasn't needed; here we assume

$$C_{DA} = C_{DAO} + C_{DA_2} \delta^2$$

$$C_{N_{\alpha}} = C_{N_{\alpha O}} + C_{N_{\alpha 2}} \delta^2$$

$$C_{N_{p\alpha}} = C_{N_{p\alpha O}} + C_{N_{p\alpha 2}} \delta^2$$

AND ADDRESS TO THE REAL OF SHE SHERE ADDRESS

where the six coefficients on the right-hand side are unknown constants to be determined by the Chapman-Kirk technique\*. The range components of the aerodynamic force are obtained by multiplying the fixed-plane components by the transpose of the transformation matrix given in equation (20):

$$F_{\chi} = F_{1} \cos \theta \cos \psi - F_{2} \sin \psi + F_{3} \sin \theta \cos \psi$$

$$F_{\gamma} = F_{1} \cos \theta \sin \psi + F_{2} \cos \psi + F_{3} \sin \theta \sin \psi$$

$$F_{z} = -F_{1} \sin \theta + F_{2} \cos \theta$$
(67)

Substituting (66) and (67) in (65) we obtain the final form of the CG equations:

$$\ddot{\mathbf{x}} = -\mathbf{A}_{1} \left\{ \begin{pmatrix} \mathbf{V}^{2} \ \mathbf{C}_{DA} \end{pmatrix} \cos \theta \cos \psi \\ - \begin{bmatrix} (\mathbf{V} \ \mathbf{u}_{2}) \ \mathbf{C}_{N_{\alpha}} - (\mathbf{pd} \ \mathbf{u}_{3}) \ \mathbf{C}_{N_{p\alpha}} \end{bmatrix} \sin \psi \\ + \begin{bmatrix} (\mathbf{V} \ \mathbf{u}_{3}) \ \mathbf{C}_{N_{\alpha}} + (\mathbf{pd} \ \mathbf{u}_{2}) \ \mathbf{C}_{N_{p\alpha}} \end{bmatrix} \sin \theta \cos \psi \right\}$$
(68)

\*Since  $C_{DA} = (1 - \delta^2) C_D = C_{DO} + (C_{D_2} - C_{DO}) \delta^2 - C_{D_2} \delta^4$ , where  $C_{DO}$ and  $C_{D_2}$  are well-determined from the drag reduction, we could, as an alternative, consider  $C_{DA}$  a known function of  $\delta^2$ , thus reducing to four the number of aerodynamic coefficients to be determined by the CG equations.

$$\ddot{y} = -A_{1} \left\{ \begin{pmatrix} V^{2} C_{DA} \end{pmatrix} \cos \theta \sin \psi \\ + \left[ (V u_{2}) C_{N_{\alpha}} - (pd u_{3}) C_{N_{p\alpha}} \right] \cos \psi \\ + \left[ (V u_{3}) C_{N_{\alpha}} + (pd u_{2}) C_{N_{p\alpha}} \right] \sin \theta \sin \psi \right\}$$

$$- (2 \omega_{E} \sin \theta_{L}) V \qquad (69)$$

$$\ddot{z} = A_{1} \left\{ \begin{pmatrix} V^{2} C_{DA} \end{pmatrix} \sin \theta \\ - \left[ (V u_{3}) C_{N_{\alpha}} + (pd u_{2}) C_{N_{p\alpha}} \right] \cos \theta \right\}$$

$$+ (2 \omega_{E} \cos \theta_{L} \sin \theta_{A}) V - g \qquad (70)$$

The right-hand sides of equations (68 - 70) involve six dependent variables:

V, p, u, u, 
$$\theta$$
 and  $\psi$ .

The first of these can be generated internally:

$$V = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}}$$

The other five must be brought in from outside. For the spin p, we used the linear expression given in (50). The variables u, u,  $\theta$  and  $\frac{2}{3}$  were obtained from the yaw reduction, as described in the previous subsection.

The aerodynamic coefficients and initial conditions of equations (68 - 70) were adjusted by the Chapman-Kirk technique until the solution of the equations was a least squares fit to the measured  $(x,y,z)_R$  values of the CG.

#### V. RESULTS

The results obtained in the present study can be interpreted better in the light of a little background on the four shell types.

#### A. Background

The 90mm M71 HE shell was designed to be fired from a 1/32 twist gun at Mach 2.4; at lower muzzle velocities, the shell was known to perform unsatisfactorily. The two rounds studied here (round 2-4203 at 7° average yaw and round 2-4204 at 5°) were fired at about Mach 0.93. At that muzzle velocity and a 1/32 twist, the gyroscopic stability factor s was slightly less than unity at launch, so that the shell was gyroscopically unstable when it emerged from the tube. However, the situation (like the shell) took some turns for the better. As the shell travelled downrange, (a) the velocity decay exceeded the spin decay and (b) the static moment coefficient decreased with decreasing Mach number. As a result, s increased, eventually crossing the value 1.0, so that

the shell became and remained gyroscopically stable.

The M71 rounds were fired and initially reduced in 1956, but the values obtained for some of the coefficients were known to be nonsense. The photographic plates were reread several times in the next four years in a largely unsuccessful attempt to improve the linear theory fit. One positive result of all these readings is that the probable errors of the fit in the present study are smaller for the two M71 rounds than for any of the other rounds.

Before applying the Chapman-Kirk technique to the two M71 rounds, we simulated their motion on an analog computer, using partially linearized equations of motion. This side investigation served two purposes: it satisfied our curiosity as to the behavior of a shell flying along the borderline of gyroscopic stability - instability and it provided accurate estimates of the initial conditions required by the Chapman-Kirk technique. For the other three shell types, however, we were satisfied to use a simple digital subprogram to obtain the needed first estimates.

The M329A1 shell without extension was a 4.2-inch (107mm) spinstabilized mortar shell to which a subcaliber cylindrica after-body a boom - had been attached at the base (see Figure 1). This boom, about 0.35 caliber in diameter and 0.7 caliber long, was used to carry the ignition and propelling charge and to provide needed volume. The boom served no aerodynamic purpose and it was hoped at the time that the boom would have negligible aerodynamic effect on the shell's performance. Careful experiments<sup>11</sup> proved otherwise. The M329A1 shell with extension carried, as its name might imply, an extension to the original boom, bringing the total boom length to 1.35 calibers (see Figure 1). Tests<sup>11</sup> comparing the M329A1 with and without the extension clearly indicated that the added length affected the aerodynamics, decreasing the stability and increasing the drag and the shell's tendency to fly erratically now and then.

The M329A1E1 shell differed from the two previous types in that it had a longer ogive, a shorter body, a boattail rather than a square base and a shorter over-all length (see Figure 1). The boom was about 0.31 caliber in diameter and 0.75 caliber long.

# B. Drag Results

A comparison of the BRL drag results with those of the GE Chapman-Kirk approach is given in Table II. The last column lists the BRL values of  $C_{D_0}$  and  $C_{D_2}$  for each of the four projectile types, obtained by a least squares fit of the data for each round of the given type to

the equation

$$C_{DR} = C_{D_0} + C_{D_2} (\overline{\delta^2})_{BRL}$$
(71)

Although additional rounds of each of the four shell types were available for fitting BRL data to equation (71), it was considered a fairer comparison to work solely with the twelve given rounds. As a result, the fits for the M71 and M329A1E1 - where there are only two rounds each - are exact and unreliable. To make matters worse, the yaw levels of the two M71 rounds are roughly the same, so that we can expect the slope to be poorly determined. In fact, only for type M329A1 with extension, where there are five rounds (albeit only three yaw levels), can we regard the results of the fit with any semblance of confidence. In spite of all this, the BRL fitted values of  $C_{\rm D}$  and  $C_{\rm D}$ 

Chapman-Kirk values are not wildly dissimilar - they are, so to speak, in the same ball park - and perhaps that is all we can expect from such a meager number of rounds.

One encouraging feature of the GE data is that the values of  $C_{D_0}$ and  $C_D$  obtained are approximately the same for each round of a given type. This seems a necessary if not sufficient condition for trusting the results. For each round, we can also compare  $\overline{C}_D$  (from equation 35) with the BRL range value  $C_{DR}$  shown in the next to last column of Table II. Here we find that the agreement is quite good when the yaw is small and poorer when the yaw is large.

#### C. Yaw Results

The yaw results are shown in Tables III - VI. The BRL values of  $C_{M}$ ,  $C_{M}$  and  $C_{M}$  shown for each round are - for the small yaw rounds -  $\alpha$  q  $p\alpha$ the values obtained by the standard BRL epicycle yaw reduction<sup>1,2</sup>. For the large yaw rounds (8615, 8618, 8621 and 8721), the BRL reduction values were corrected to compensate for the fact that certain geometrical terms in the yaw reduction - terms which are only significant for large yaw angles - are ignored in the BRL reduction.

Again it should be noted that all the coefficients  $C_M and C_M q p\alpha$ shown in these tables, both BRL and GE values, are defined (see equation 51) as half the coefficients designated by the same symbols in, for example, References 9, 13 and 14. This disparity is unfortunate but necessary if the present paper is to be consistent with other BRL reports.

For most of the rounds, more than one GE case per round is shown in the tables. These cases differ in the selection of which parameters were fixed (indicated by an asterisk) and which were allowed to seek out their optimum values. As might be expected, the more parameters fitted, the smaller (with a few exceptions) was the probable error of the fit. For three rounds (8713, 8716 and 8981), GE assumed in one case per round that all the moment coefficients were constant. This affords a direct comparison with the BRL values.

The last row in Tables III - VI lists the four coefficients obtained by fitting the BRL data for each round of the given type to the equations

$$\begin{pmatrix} C_{M_{\alpha}} \\ BRL \end{pmatrix}_{BRL} = C_{M_{\alpha0}} + C_{M_{\alpha2}} \delta_{e}^{2}$$

$$\begin{pmatrix} C_{M_{p\alpha}} \\ p\alpha \end{pmatrix}_{BRL} = C_{M_{p\alpha0}} + C_{M_{p\alpha2}} \delta_{e}^{2}$$

$$(72)$$

where the constant  $\delta_e^2$  is an effective squared yaw, obtained as a byproduct of the BRL yaw reduction. The previous remarks on the fit to Equation (71) apply with equal force to Equation (72): the results of the fit are not too trustworthy but are least suspect for the M329A1 with extension (Table IV).

Round 8618 and to a lesser extent round 8621 (Table IV) revealed a very strong Magnus moment nonlinearity. In fact, by a separate computer curve fit program, we determined that the expansion

$$C_{M} = C_{M} + C_{M} \delta^{2} + C_{M} \delta^{4}$$

$$+ C_{M} \delta^{6} + C_{M} \delta^{8}$$
(73)

was needed in these two rounds to obtain a reasonable fit to the given data. Accordingly, our main program was modified to allow for the last terms in (73) - terms that are missing in equation (52). Unfortunately, we were unable to determine values for the two new coefficients; the process diverged on every attempt. That is, it was not possible with the present technique and/or the available data to determine more than three terms of the Magnus moment coefficient expansion.

This situation should be investigated further. A first step would be to analyze by the present technique the output of a six-degrees-offreedom program that simulates the motion of a projectile with a highly nonlinear Magnus moment. In this way, the quantity and the quality of the experimental data needed to obtain a satisfactory fit for this type of nonlinearity could be determined.

D. CG Results

The CG equations (68-70) were applied only to the two M329A1E1 rounds (see Table VII). The values obtained for the normal force coefficient are in good agreement with each other and with the BRL values. The Magnus force coefficient (which is less by that ubiquitous factor of two than the coefficient labelled  $C_{\gamma}$  in Reference 14) is  $p_{\alpha}$ 

not very well determined. The BRL values of the axial drag coefficient shown in Table VII were obtained by the approximation

$$\left(C_{DA}\right)_{BRL} = (1 - \overline{\delta^2}) C_{DR}$$

using the  $C_{DR}$  values in Table III.

#### VI. CONCLUSIONS AND RECOMMENDATIONS

We have shown that for twelve problem rounds the Chapman-Kirk technique could be applied satisfactorily to free-flight, enclosedrange data. However, before we can safely apply the technique on a steady, production basis to high-yaw rounds, additional theoretical study is needed. A systematic investigation should be made, whereby a variety of trajectories are computer-generated, suitable noise is introduced and the resultant data fed to the Chapman-Kirk system. Some force and moment coefficients are more easily determined than others; for example, those depending mainly on the frequencies of the motion are more easily pinpointed than those related to the damping. For a given force or moment expansion in  $\delta^2$ , the coefficients naturally become less reliable as the order of the term increases and shortly a limit is reached; when any term of order higher than this limit is included in the analysis, the process fails to converge (or, worse yet, converges to wrong answers). This limit is a function of the number and accuracy of the observations, so that any improvement in these factors (up to some point of diminishing return) would be helpful.

It might be possible to determine high order terms more accurately for a given set of data if the lowest order terms are fixed at good values. In particular, low yaw rounds (say,  $\overline{\alpha_T} < 3^\circ$ ) should establish adequate zero-yaw coefficients for use with the high yaw rounds. In general, the fewer the number of coefficients to be determined, the less likely that computational instabilities will arise.









# FIGURE I SIMPLIFIED SKETCH OF THE FOUR PROJECTILE TYPES





FIGURE 3 SIMPLIFIED BLOCK DIAGRAM OF THE DATA REDUCTION PROCEDURE

Environment
and
Parameters
Physical
Ι.
Table

.

				PHYSICA	L PARAMETE	RS		ENVIR	ONMENT
Type	BRL RD No. 2-	[W]	۳ [KG]	Ix [KG-M <sup>2</sup> ]	I y [KG-M <sup>2</sup> ]	CG (cal. from nose)	n=rifling [cal/rev]	[KG/M <sup>3</sup> ]	Vel. Sound [M/Sec]
M71	4203 4204	.08926 .08926	10.614 10.623	.011720	.11067 .11067	2.974 2.962	32 32	1.1770 1.1722	348.1 348.7
M329A1 w/ext	8615 8617 8618 8618 8621 8704	.10643 .10645 .10638 .10643 .10643	11.898 11.898 11.902 11.902 11.898	.018227 .018215 .018165 .018161 .018781	.23735 .23984 .23755 .23718 .23818 .24871	3.029 3.034 3.040 3.032 2.989	18 18 18 18 18	1.2043 1.1856 1.1825 1.1870 1.2046	345.4 347.9 348.2 347.0 344.7
M329Al wo/ext	8713 8716 8721	.10643 .10643 .10643	11.793 11.793 11.793	.018248 .018673 .018673	.22698 .23774 .23774	2.997 2.951 2.961	18 18 18	1.1800 1.1814 1.1981	345.0 344.9 344.5
M329A1 E1	8981 8988	.10640 .10640	10.006 10.002	.015080 .015080	.13025 .13025	2.487 2.487	20 20	1.1540 1.1527	350.6 351.2

Preceding page blank

51

Table II. Drag Reduction Results

Ser.m.

.

	L	( <sup>c</sup> <sup>0</sup> <sup>, c</sup> <sup>0</sup> )	(Eq 71)	105	2.53)			.190.	3.59)	_				(.223,	2.79)			(.115, 2.48)	
	BR	CDR	(Eq 36)	.219	.204	.372	.237	.389		.815	. 238		.323	.231	479		.174	.129	
	ļ	ם. م	(Eq 35)	.221	.199	. 392	.240	.412	t t	.779	.241		.328	.232	522	440.	.180	.129	
	t	<u>م</u> `	(PE)	4.19	5.27 (.23)	3.00	3.25*	3.20*		3.16 (06)	3.15*		3.40	(.20) 3.31	(2.28) 3 00	(.005)	2.0	(<<.) 2.0*	
RESULTS	τ	وں	(PE)	.162	.160 (.002)	.207	(.0003) .212	(.0006) .204	(.0008)	(110)	.207	(.0005)	.220	(.006) .209	(.015) 215*		.129	.117	(.0004)
<b>NG REDUCTION</b>	STATION 1	V <sub>o</sub> , m/sec	(PE, m/sec)	325.5	327.6 (.006)	221.53	(.0037) 219.54	(.0064) 214.57	(6200.)	214.25 (20192)	216.83	(.0055)	216.62	213.80	(.03) 213,84	(900.)	192.05	225.66	(.0061)
GE DRA	VALUES AT S	t <sub>o</sub> , sec	(PE, µ-sec)	01932	01921 (2.2)	01838	(3.4) 02615	(6.2) 01759	(8.4)	.00006	02750	(10)	02716	(3./) 02767	(1.6) - 02780	(4.9)	02988	02408	(4.2)
	DE cf rit	μ-sec	(No. Sta.)	2.0	2.8 (13)	3.9	(14)	(10) 6.6	(11)	10.4	7.2	(14)	3.4	11.3	(12) 6.8	(12)	4.3	(c1) (c1)	(14)
		BRL RD 2-	(ar_1, deg)	4203 (6 8)	4204 (4.9)	8615	(14.2) 8617	(5.3) 8618	(14.6)	0021 (24.6)	8704	(5.9)	8713	(10.2) 8716	(4.8) 8721	(18.3)	8981	(۲۰۶) لایا 88	(4.5)
		L	Type	M71			M329A1 w/ext						1.400.234	wo/ext			LVOCZM	E1	

\*Held Constant

		C <sub>M</sub>	C <sub>M</sub>	C <sub>M</sub>	C.	<sup>C</sup> M	1/1
БКL КU 2-( )	Nethod (PE of Fit)	α0 (PE)	α2 (PE)	q (PE)	рао (РЕ)	Pa2 (PE)	- x' _ y (PE)
2067	BRL	4.21		- 26.1	1.66		.1059*
(6.8°) (24 Sta)	GE (- 03 )	4.62	- 4.8	- 22	1.36		.1059*
	GE GE ( 096°)	4.61	- 4.7	- 8.8 (1.4.2)	0.21	20.6	.1059*
	GE (.082°)	4.38 (.03)	- 1.12 (.56)	- 8.4 (4.1)	0.20 0.34)	(1.0) 19.9 (6.0)	.1041 (.0003)
1001	BRL	4.23		- 44.2	2.52		.1059*
(4.9°)	GE (112°)	4.58	- 5.0+	- 28	3.20		.1059*
	() GE ()	4.62	- 8.3	- 5.9 (1 )	-0.43	99.4	.1059*
	GE (.064°)	4.37 (.03)	(.40) - 2.41 (.90)	- 6.8 (3.6)	-0.52 -0.52 (.29)	(9.2) 115.5 (8.2)	.1039 (.0003)
Combined	BRL Eq. (72)	4.25	- 0.83		3.23	- 35.6	
*Held Cons	stant						

Table III. M71 Yaw Reduction Results

T.

•

53

	22.0 (5.28)	- 100 (48) - 95 (48)	17.8 (5.4) 53.0 - 200 (13.7) (80) 59.5 - 258 (11.4) (66)	(continued)
C <sub>M</sub> pa1 (PE)			11.5 (2.96)	
C <sub>M</sub> pao (PE)	1.15 - 1.12 (.57)	.083 1.37 (.60) 1.35 (.54)	$\begin{array}{c} 1.09\\ 1.23\\ (.06)\\83\\ (.62)\\ - 2.83\\ (1.03)\\ - 2.53\\ (.77)\\ - 2.64\\ (.63)\end{array}$	
С <sub>М</sub> (PE)	- 12.7 - 0.35 (3.25)	- 3.2 - 5.5 - 6.1 (1.2)	- 11.6 - 12.6 (.89) - 1.22 (3.43) 2.47 (3.92) (3.34) 3.66 (2.68)	
C <sub>M</sub> α4 (PE)			66.4 (12.2)	
GM ∞2 (PE)	- 9.5 (.77)	- 24.7 (7.1)	- 8.9 - 9.0 (.93) - 9.0 (.77) - 8.4 (.73) - 7.9 (.73) - 17.8 (1.9)	
C <sub>M</sub> α0 (PE)	3.18 4.46 (.08)	4.32 4.31 (.009) 4.64 (.09)	3.15 4.40 (.11) 4.43 (.09) 4.36 (.08) 4.30 (.08) 4.68 (.08) 4.68	
Method (PE of Fit)	BRL (.227°) GE (.122°)	BRL (.135°) GE (.098°) GE (.093°)	BRL (.251°) GE (.162°) GE (.129°) GE (.125°) GE (.115°) GE (.097°)	
BRL RD 2-( )	8615 (14.2°)	8617 (5.3°)	8618 (14.6°)	

Table IV. M329Al (w/ext) Yaw Reduction Results

54

Table IV. M329Al (w/ext) Yaw Reduction Results (continued)

BRL RD 2-()	(PE of Fit)	C <sub>M</sub> αο (PE)	C <sub>M</sub> α2 (PE)	$C_{M_{\alpha_4}}^{C_M}$ (PE)	С <sub>М</sub> (РЕ)	С <sub>М</sub> рао (PE)	C <sub>M</sub> Pα1 (PE)	С <sub>М</sub> ра <sub>2</sub> (РЕ)	См Рац (РЕ)
8621 (24.6°)	BRL (1.40°) GE (.275°) GE (.238°)	1.75 3.85* 3.85 (.18)	- 6.1 (.38 - 6.7 (1.5)	8.6 (1.5) 11.3 (3.6)	- 25.4 - 5.75* 0.9 (2.9)	2.61 - 0.7 (.42) - 2.4 (.87)		20.2 (5.0) 33.4 (7.8)	- 29.6 (13.9) - 63.1 (20.3)
8704 (5.9°)	BRL (.135°) GE (.097°)	4.16 4.32 (.06)	- 16.1 (4.0)		- 5.85 - 5.75 (.87)	0.10 0.92 (.36)		- 55 (24.3)	
Combined	BKL Eq. (72)	4.11	- 6.0			0.192		6.09	

\*H. Id Constant

BRL RD 2-( )	Method (PE of Fit)	MαO (PE)	M (PE)	له (PE)	رM (PE)	CM Pao (PE)	См Раг (РЕ)	μ Paų (PE)
8713 10.2°)	BRL (.11°) GE (.096°) GE (.:34°) GE (.093°)	3.44 3.38 3.28 3.28 (.04) 3.28 (.04)	2.21 (.83) 2.22 (.83)		- 4.23 - 4.35 - 4.15 - 4.15 - 4.25 - 4.25	.065 .085 .085 .03) .05 .03) .08 (.03)	2.78 (4.81)	
8716 (4.8°)	BRL (.07°) GE (.071°) GE (.070°) GE (.069°)	3.49 3.28 (.044) 3.32 (.006) 3.32 (.006)	4.52 (4.42)		- 4.08 - 3.58 (.70) - 3.63 - 3.45 (.73)	058 08 075 075 .195 (.03) (.24)	- 27.3 (24.7)	
8721 18.3°)	BRL (.33°) GE (.135°) GE (.110°) GE (.103°)	2.83 3.09 3.30* 3.30* 3.67 (.08)	67 (.28) - 4.1 (.27) - 8.6 (1.0)	13.6 (1.7) 27.8 (3.6)	- 5.54 - 2.55 - 3.75* - 3.85 - 3.85 (1.1)	.36 55 .025* .24) .025* .2	6.25 6.25 - 2.0 (1.1) - 2.8 (6.2)	29.8 (7.4) :1.8 (22.2)
ombined Held Con	BRL Eq. (72) stant	3.59	- 4.54			075	2.69	

Table V. M329A.I (wo/ext) Yaw Reduction Results

56

Table VI. M329A1E1 Yaw Reduction Results

.

Areas.

BRL RU 2-( )	Method (PE of fit)	C <sub>M</sub> αο (PE)	$C_{M_{\alpha_2}}^{\alpha_2}$ (PE)	հ հ (PE)	С <sub>М</sub> (РЕ)	$C_{M}$ (PE) (PE)	С <sub>М</sub> рац (PE)
8981 (9.2°)	BRL (.104°) GE (.100°) GE (.099°) GE (.092°) GE (.087°)	2.975 2.988 (.004) 3.07 (.04) 3.12* 3.06 (.04)	$\begin{array}{c} & \cdot \cdot \cdot 4 \\ & \cdot \cdot 1 \\ & \cdot 1 \\ & \cdot 2 \cdot 0 \\ & \cdot 1 \\ & \cdot 1$	- 2.82 - 2.86 (.31) - 2.73 (.47) - 2.31 - 1.24 (.74)	180 175 05 .5* 1.8 (.70)	- 3.58 - 40.3 - 114 (41)	511 511 (183) 1406 (515)
8988 (4.5°)	BRL (.093°) GE (.105°) GE (.105°)	3.143 3.145 3.014 (.00)	15.5 (8.3)	- 3.06 - 2.84 - 2.76 (.75)	.14 .405 .376 (.44)	- 29.9 - 27.2 (52)	
Combined	BRL (Eq. 72)	3.20	- 6.79		.250	- 12.9	

\*Held Constant

Table VII. M329AIE1 CG Reduction Results

1.08 (.54) 1.10 (.58) 1.08\* .295 (1.53) 1.08\* C<sub>DA</sub> (PE) (.0005) (.013) .143 (.012) .117\* (.009) .120 (:0005) (.0005) C<sub>DA</sub> (PE) .117\* .117 .170 .122 .128 14.7 (2.4) - 34.8 (12) - 29.0 (11) (123)  $C_{N_{p\alpha_2}}^{C_N}$ 228 .0435 (.075) .0585 (.072) 1.27 1.04 (.17) .555 - 2.655 (1.14) (.37) .0566 CN Pao (PE) - .555 \*S. .64 2.88 (2.2) C<sub>N</sub> (PE) (.068) 1.444 (.013) 1.402 (.011) 1.364 (.012) 1.412 (.026) 1.315 (010) (.026) (.033) CN (PE) 1.400 1.402 1.401 1.403 1.396 Method (PE of Fit) ا (.0023) GE 1 (.0019) GE GE (.0021) GE (.0020) GE (.0017) GE BRL (.0019) GE GE (.0023) GE (.0023) GE (.0022) (.0017) BRL BRL RD 2-( ) 8988 (4.5°) 8981 (9.2°)

\*Held Constant

#### REFERENCES

- 1. C. H. Murphy, "Data Reduction for the Free Flight Spark Ranges," Ballistic Research Laboratories Report No. 900, 1954, AD 35833.
- 2. C. H. Murphy, "Free Flight Motion of Symmetric Missiles," Ballistic Research Laboratories Report 1216, 1963, AD 442757.
- 3. H. F. Meissinger, "The Use of Parameter Influence Coefficients in Computer Analysis of Dynamic Systems," Proc. Western Joint Computer Conference, San Francisco, California, May 1960, pp. 181-192.
- 4. G. A. Bekey and W. J. Karplus, Hybrid Computation, John Wiley & Sons, Ind., New York, 1968.
- T. R. Goodman, "System Identification and Prediction An Algorithm Using a Newtonian Iteration Procedure," *Quarterly of Applied Mathematics*, 24, No. 3, 1966, pp. 249-255.
- T. K. Goodman and T. P. Sargent, "A Method for Identifying Nonlinear Systems with Applications to Vehicle Dynamics and Chemical Kinetics," Oceanics, Inc., Technical Report No. 71-83, 1971.
- 7. J. W. Bradley, "CHLOE: a FORTRAN Subroutine for Fitting Ordinary Differential Equations to Observed Data," Ballistic Research Laboratories Memorandum Report 2184, 1972, AD 743878.
- G. T. Chapman and D. B. Kirk, "A New Method for Extracting Aerodynamic Coefficients from Free-Flight Data," *AIAA Journal*, <u>8</u>, No. 4, 1970, pp. 753-758.
- 9. R. H. Whyte and J. G. Beliveau, "Non-Linear Free Flight Aerodynamic Reduction Using Angle of Attack or Angular Rate Data," General Electric Report No. GE T1SR69APB6, Armament Department, Burlington, Vermont, 1969.
- D. R. Cormier, "Numerical Integration Method for Computing Stability Coefficients," Department of Aerospace and Mechanical Engineering, University of Notre Dame, Notre Dame, Indiana, 1969.
- L. C. MacAllister, V. Oskay, B. J. Reiter, R. Klein and W. Gazdayka, "The Effect of a Subcaliber Cylindrical After-Body on the Behavior of Spin-Stabilized Projectiles," AIAA Paper No. 70-558, AIAA Atmosphere Flight Mechanics Conference, Tullahoma, Tennessee, 1970.
- 12. T. E. Bullock, M. H. Clarkson and D. C. Daniel, "Preliminary Report On Extracting Aerodynamic Coefficients From Dynamic Data," University of Florida Technical Report AFATL-TR-72-52 under contract with the Air Force Armament Laboratory (DLGC), Eglin Air Force Base, Florida, 1972.

#### REFERENCES (condinued)

- R. H. Whyte and W. H. Mermagen, "A Method for Obtaining Aerodynamic Coefficients from Yawsonde and Radar Data," AIAA Paper No. 72-978, AIAA 2nd Atmospheric Flight Mechanics Conference, Palo Alto, California, 1972.
- R. H. Whyte and A. Jeung, "Aerodynamic Reduction of Free Flight Transonic Range Data Utilizing Numerical Integration," General Electric Report No. GE 71APB514, Armament Department, Burlington, Vermont, 1971.
- 15. W. K. Rogers, Jr., "The Transonic Free Flight Range," Ballistic Research Laboratories Report 1044, 1958, AD 200177.
- E. D. Boyer, "Aerodynamic Properties of the 90mm HE M-71 Shell," Ballistic Research Laboratories Memorandum Report 1475, 1963, AD 411804.
- 17. C. E. Knadler, Jr., "A Method for Determining the Parameters of Ordinary Differential Equations," NOL TR 68-1972, 1968.
- D. W. Marquardt, "An Algorithm for Least-Squares Estimation of Nonlinear Parameters," J. Soc. Indust. Appl. Math., <u>11</u>, No. 2, June 1963, pp. 431-441.
- 19. P. R. Bevington, Data Reduction and Error Analysis for the Physical Sciences, McGraw-Hill Book Co., New York, 1969.