VISCOUS EFFECTS IN LOW-DENSITY NOZZLE FLOWS

David L. Whitfield
ARO, Inc.

June 1973

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FOREWORD

The work reported herein was conducted by the Arnold Engineering Development Center (AEDC) under the sponsorship of the Air Force Rocket Propulsion Laboratory (AFRPL), Air Force Systems Command (AFSC), under Program Element 62302F. This work was monitored by Captain Sam Thompson of AFRPL.

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This technical report has been reviewed and is approved.

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ABSTRACT

Viscous effects in low-density nozzle flows were investigated numerically, and comparisons were made with experimental data. The numerical method of Patankar and Spalding was modified to solve the internal laminar boundary-layer equations for two-dimensional flow or axisymmetric flow with or without transverse curvature. A listing is given of the computer code. Boundary-layer displacement thicknesses for typical nozzle geometries and flow conditions are presented. Solutions were obtained for specific conditions corresponding to experimental data. The result is a relatively fast, simple to use numerical procedure, which is shown to give results in good agreement with experimental data.
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NOMENCLATURE

A , Cross-sectional area of nozzle

\( c_p \) Specific heat at constant pressure

\( c_v \) Specific heat at constant volume

\( d^* \) Nozzle throat diameter

\( H \) Local total enthalpy

\( \bar{H} \) \( H/H_0 \)

\( h \) Heat-transfer coefficient

\( k \) Thermal conductivity

\( M \) Mach number

\( m \) Mass flux

\( \text{Pr} \) Prandtl number, \( \mu c_p/k \)

\( p \) Pressure

\( p_p \) Measured pitot pressure

\( \bar{p} \) \( (\gamma - 1)p/(\gamma p_0) \)

\( q_w \) Nozzle wall heat-transfer rate

\( R \) Longitudinal radius of curvature of converging portion of nozzle

\( \bar{R} \) \( R/r^* \)

\( \text{Re}_{O,r^*} \) Nozzle reservoir Reynolds number, \( \rho_0(2H_0)^{1/2} r^*/\mu_0 \)
Defined by Eq. (4)

\( r/r^* \)

Nozzle throat radius

\( T \)

Temperature

\( T_{aw} \)

Adiabatic wall temperature

\( \frac{T}{T_0} \)

\( u \)

Velocity component in x direction

\( \frac{u}{(H_0)^{1/2}} \)

\( v \)

Velocity component in y direction

\( \frac{v}{(H_0)^{1/2}} \)

\( x \)

Coordinate along nozzle wall

\( \frac{x}{r^*} \)

\( y \)

Coordinate normal to nozzle wall

\( \frac{y}{r^*} \)

\( z \)

Coordinate along nozzle axis referenced to the throat

\( \frac{z}{r^*} \)

\( \alpha \)

Nozzle wall angle

\( \frac{c_p}{c_v} \)

\( \delta \)

Boundary-layer thickness defined as the value of \( y \) where \( u/u_E = 0.99 \)

\( \delta^* \)

Boundary-layer displacement thickness

\( \xi \)

Exponent in power law viscosity, \( \mu \sim T^\xi \)

\( \eta \)

\( \frac{y}{y_E} \)

\( \mu \)

Viscosity

\( \frac{\mu}{\mu_0} \)

\( \nu \)

Denotes planar or axisymmetric flow in Eq. (23)

\( \xi \)

Transformed x coordinate, Eq. (11)

\( \rho \)

Mass density

\( \frac{\rho}{\rho_0} \)
\( \psi \) Stream function

\( \omega \) Transformed stream function, Eq. (12)

**SUPERSCRIPT**

Condition immediately downstream of a normal shock

**SUBSCRIPTS**

- \( C_L \) Nozzle centerline
- \( E \) Outer edge of boundary layer
- \( e \) Nozzle exit
- \( I \) Inner edge of boundary layer
- \( o \) Reservoir (total) conditions
- \( w \) Nozzle wall
SECTION I
INTRODUCTION

Investigations of viscous effects in low-density gas flows in two-dimensional and axisymmetric channels and nozzles have been conducted during the past few years in support of the design of low-density wind tunnel facilities and small microthrust rockets used for spacecraft attitude control. In addition to these areas of interest, there are two other areas where attention to viscous effects is required. One is that of internal boundary-layer scaling. When the nozzle and/or plume flow of a rocket engine is to be investigated in a wind tunnel or space chamber, it is usually necessary to significantly reduce the size of the nozzle used. To achieve adequate simulation, the model nozzle viscous effects must appropriately simulate those of the actual rocket. A specific example of this problem is found in the laboratory study of rocket exhaust plumes interacting with the free-stream (Ref. 1). The other area of interest is associated with the study of gas dynamic and chemical lasers (Ref. 2). The total laser power output is influenced by the static gas pressure in the optical cavity just downstream of the nozzle exit. Since the nozzles used are small and since they operate at relatively high total temperatures, the nozzle viscous effects significantly influence the nozzle static pressure distribution. Although numerous comparisons with experimental data from low-density wind tunnel nozzles will be presented to check the accuracy of the results, the present investigation was motivated by the current interest in rocket nozzle scaling and the viscous effects on the operation of gas dynamic and chemical laser systems.

Some previous investigations of the design and analysis of low-density wind tunnel nozzles are given in Refs. 3 through 5. The method of Potter and Carden (Ref. 3) was developed to design nozzles for particular test section flow conditions. It is based on an integral technique which uses the similar solutions of Cohen and Reshotko (Ref. 6). Although the work of Potter and Carden (Ref. 3) has proved successful for the design of nozzles to produce desired flow conditions, it is not directly applicable to the analysis of specified nozzle geometry. Also, non-similar solutions presented in Ref. 4 indicate that similarity does not exist in nozzle flows, particularly near the throat, and as a consequence, inaccuracies may result from the use of similar solutions for relatively short nozzles.
The method presented in Refs. 4 and 5 solves the non-similar laminar boundary-layer equations with or without first-order transverse curvature (referred to as second-order in Refs. 4 and 7), with or without velocity slip and temperature jump boundary conditions, and it has been shown to be accurate. However, the method has certain disadvantages: (1) the numerical integration scheme is that of Jaffe, Lind, and Smith (Ref. 7) which requires a relatively large amount of computer time, (2) the computer program is large and not necessarily simple to use, (3) the transformation variables are not amenable to internal flow problems, and (4) large flow expansions frequently require interpolating the solutions, changing the numerical step size across the nozzle, and resuming the calculations. These disadvantages discourage the use of the method described in Refs. 4 and 5.

The methods described in Refs. 3 through 5 assume that the nozzle flow consists of a viscous region and an inviscid (core) region. Some previous investigations which are not restricted to such flows, but permit viscous effects across the entire channel, are described in Refs. 8 through 10. These investigations are more suitable for the study of flow in microthrust rockets where the flow may be fully viscous. However, these methods also have certain disadvantages for the present application. The works of Adams (Ref. 8) and Williams (Ref. 9) are based on similar solutions, the conditions for which are not likely to be satisfied by a large class of practical problems. The work of Rae (Ref. 10) is a significant contribution to the study of low-density nozzle flows, and this method will be discussed further in this report. Numerical results from Rae's method will be compared with results from Ref. 5, results obtained in the present investigation, and experimental data. Rae's method was shown to give good results for fully viscous flows (Ref. 10); however, it will be shown herein to be less accurate than the method of Refs. 4 and 5 or the present method for the flow regime of interest in this investigation.

The objectives of the present investigation are: (1) to provide results for estimating the viscous effects in low-density converging-diverging nozzle flows based on certain flow parameters and nozzle geometries, and (2) to provide a fast, simple to use method for calculating viscous effects in low-density nozzle flows. To meet these objectives, the numerical integration scheme of Patankar and Spalding (Ref. 11) was used. In fact, the program used for this work was taken directly from Ref. 11 and then suitably modified for internal, low-density flows. The resulting program is similar to that of Mayne (Ref. 2) except for the numerical scheme used near the wall. Also, the
The present work is based on a set of dimensionless equations which reduces the amount of input and provides more convenient solutions in the sense that they are applicable to flows which satisfy certain parameters rather than specific inputs of pressure, temperature, etc.

The following section describes the governing equations and boundary conditions, the variables used to nondimensionalize the equations, and the transformed equations. Section III describes briefly the numerical solution as well as some of the computational difficulties which have been encountered with this program. Section IV is devoted to numerical results and Section V to comparisons of the present results with previous theoretical investigations and experimental data. Some conclusions are given in Section VI.

SECTION II
BASIC EQUATIONS

The governing system of equations and the boundary conditions are presented in this section. Certain dimensionless and transformation variables are introduced and used to transform the governing equations for convenience of the numerical solution. Also, the boundary-layer displacement thickness is derived in the transformed plane. Although the program will solve the two-dimensional equations or axisymmetric equations with or without transverse curvature, only the equations appropriate to the latter with the transverse curvature terms retained are considered here in detail because they are more general. The two-dimensional equations can be obtained from the equations considered by setting \( r = 1 \), and the axisymmetric equations without transverse curvature can be obtained by setting \( r = r_w \).

2.1 GOVERNING EQUATIONS

The governing system of equations is taken as that obtained by Probstein and Elliott (Ref. 12). The equations in curvilinear coordinates are:

Continuity Equation

\[
\frac{\partial (p vr)}{\partial x} + \frac{\partial (p uv)}{\partial y} = 0 \tag{1}
\]
Momentum Equation

\[ \rho u \frac{\partial u}{\partial x} + \rho \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{1}{\tau} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) \]  

Total Energy Equation

\[ \rho u \frac{\partial II}{\partial x} + \rho \nu \frac{\partial II}{\partial y} = \frac{1}{\tau} \frac{\partial}{\partial y} \left( \mu \frac{\partial II}{\partial y} + \mu \left( 1 - \frac{1}{\tau^2} \right) \frac{\partial u}{\partial y} \right) \]  

The total energy equation is obtained by multiplying the momentum equation by \( u \) and adding the result to the static energy equation. The coordinate system is defined in Fig. 1 (Appendix I) with the \( r(x, y) \) term defined for internal flow as

\[ r(x, y) = r_w(x) - \gamma \cos \alpha \]  

Probstein and Elliott (Ref. 12) obtained Eqs. (1) through (3) by an order of magnitude analysis of the general forms of the continuity equation, Navier-Stokes momentum equations, and energy equation. The assumptions made in the analysis were that the ratio of the boundary-layer thickness to the longitudinal radius of curvature of the body surface was small compared to unity, and the ratio of the boundary-layer thickness to the nozzle radius was on the order of unity. Therefore, Eqs. (1) through (3) are valid for nozzles which have a longitudinal radius of curvature much larger than the nozzle radius. This stipulation is normally satisfied by axisymmetric convergent-divergent nozzles used for low-density wind tunnels, rockets, and gas dynamic and chemical laser systems.

The axisymmetric boundary-layer equations without transverse curvature terms correspond to those which can be obtained from Eqs. (1) through (3) by replacing \( r(x, y) \) with \( r_w(x) \) as stated above. Because \( r_w(x) \) is a function of \( x \) only, it can be eliminated from Eqs. (2) and (3), and therefore appears only in the continuity equation. The resulting set of equations can be used to describe internal or external boundary layers. It was shown in Ref. 5, by solutions with and without the transverse curvature terms, that the effect of transverse curvature is important for \( \delta / r_w(x) \sim 0(1) \).
Implicit in Eq. (2) by the use of the total derivative of $p$ with respect to $x$ is the $y$ component of the momentum equation as given by

$$-\frac{\partial p}{\partial y} = 0$$  \hspace{1cm} (5)

The validity of this equation is sometimes questioned for thick laminar boundary layers. However, the analysis of Probstein and Elliott (Ref. 12) indicates that this equation is consistent with the other equations in the governing set. Equation (5) was used in Refs. 4 and 5, and good agreement between calculated and measured boundary-layer profiles was obtained for flow conditions where 99 percent of the cross-sectional area of a nozzle was boundary layer.

For the boundary conditions at the edge of the boundary layer, it was assumed that an isentropic core flow exists along the nozzle centerline, from which the flow properties at the edge of the boundary layer can be calculated. The boundary conditions at the nozzle wall were taken as zero velocity and a prescribed wall temperature distribution or a prescribed wall heat-transfer distribution. Solutions were presented in Ref. 5 with and without velocity slip and temperature jump boundary conditions. For the conditions investigated in Ref. 5, it was found that the nozzle flow merged (that is, the boundary layer completely filled the nozzle) before velocity slip and temperature jump became significant. Therefore, no-slip wall boundary conditions were assumed adequate for this investigation.

The system of equations is completed by using the equation of state, $p = \rho RT$, and expressing the viscosity as some function of $T$, $\mu = \mu(T)$. The governing equations are next made dimensionless.

### 2.2 NONDIMENSIONALIZED EQUATIONS

Dimensionless variables are used to nondimensionalize the governing equations as follows:

\begin{align*}
\bar{x} &= \frac{x}{r^*} \quad (6a) \\
\bar{\rho} &= \frac{\rho}{\rho_o} \quad (6d) \\
\bar{p} &= \frac{p}{(\gamma - 1)\rho_o} \quad (6g) \\
\bar{y} &= \frac{y}{r^*} \quad (6b) \\
\bar{\Pi} &= \frac{\Pi}{\Pi_o} \quad (6e) \\
\bar{u} &= \frac{u}{(\Pi_o)^{1/2}} \quad (6h) \\
\bar{r} &= \frac{r}{r^*} \quad (6c) \\
\bar{\mu} &= \frac{\mu}{\mu_o} \quad (6f) \\
\bar{v} &= \frac{v}{(\Pi_o)^{1/2}} \quad (6i)
\end{align*}
Using these equations in Eqs. (1) through (3) gives for the continuity, momentum, and total energy equation, respectively,

\[
\frac{\partial(p \bar{u})}{\partial x} + \frac{\partial(p \bar{v})}{\partial y} = 0
\]

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = -\frac{\partial \bar{p}}{\partial x} - \frac{(2)^{\frac{1}{2}}}{Re_{e_0,r^*}} \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{y}} \left( \frac{\bar{f}}{\mu} \frac{\partial \bar{u}}{\partial \bar{y}} \right)
\]

and

\[
\frac{\partial \bar{H}}{\partial x} + \frac{\partial \bar{H}}{\partial y} = \frac{(2)^{\frac{1}{2}}}{Re_{e_0,r^*}} \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{y}} \left( \frac{\bar{f}}{\mu} \frac{\partial \bar{u}}{\partial \bar{y}} + \bar{f} \left( 1 - \frac{1}{Pr} \right) \frac{\partial \bar{u}}{\partial \bar{y}} \right)
\]

where

\[
Re_{e_0,r^*} = \frac{\rho_o (2H_o)^{\frac{1}{2}} r^*}{\mu_o}
\]

A few comments are in order concerning the choice of the dimensionless variables. Note that \((H_0)^{1/2}\) was used to normalize the velocity components \(u\) and \(v\) instead of the more commonly used maximum velocity \((2H_0)^{1/2}\). The motivation for this was to recover the same form of the boundary-layer equations in dimensionless variables as in physical plane variables. If \((2H_0)^{1/2}\) were used in Eqs. (6h) and (6i) in place of \((H_0)^{1/2}\), then the term \(\mu \left[ 1 - (1/Pr) \right] \frac{\partial u}{\partial y}\) in the right hand side of Eq. (9) would be \(2\mu \left[ 1 - (1/Pr) \right] \frac{\partial u}{\partial y}\), which would not be the same as in Eq. (3). The maximum velocity \((2H_0)^{1/2}\) was used in the definition of \(Re_{e_0,r^*}\) for convenience. This introduces the term \((2)^{-1/2}\) as a coefficient of \(Re_{e_0,r^*}\), but since \(Re_{e_0,r^*}\) is a constant for each solution, the constant \((2)^{-1/2}\) causes no inconveniences in the numerical solutions. Note that by defining a new viscosity, \(\mu = (2)^{1/2} \mu/Re_{e_0,r^*}\), Eqs. (7) through (9) are identical to Eqs. (1) through (3). This, in fact, was done in the numerical computations.

For a given set of initial conditions, boundary conditions, \(\gamma, Pr,\) nozzle geometry, etc., the solution to Eqs. (7) through (9) depends only on \(Re_{e_0,r^*}\) if a power law viscosity variation with temperature is assumed, i.e., \(\mu = T_0^{\xi} = (T/T_0)^{\xi}\). However, if, for example, Sutherland's viscosity law is assumed, then the solutions will also depend on the absolute value of temperature. Because the calculation of
viscosity is carried out in a separate subroutine in the computer program, it is convenient to use any viscosity law desired. Solutions will be presented for both Sutherland's law and power law variations of viscosity.

2.3 TRANSFORMED EQUATIONS

Equations (7) through (9) were transformed from the dimensionless physical \( \bar{x} - \bar{y} \) plane to the \( \xi - \omega \) plane by the transformation variables

\[ \xi(\bar{x}) = \bar{x} \]  

(11)

and

\[ \omega(\bar{x},\bar{y}) = \frac{\psi(\bar{x},\bar{y}) - \psi_I(\bar{x})}{\psi_E(\bar{x}) - \psi_I(\bar{x})} \]  

(12)

where \( \psi(\bar{x},\bar{y}) \) is the stream function which identically satisfies the continuity equation, Eq. (7), i.e.,

\[ \frac{\partial \psi}{\partial \bar{x}} = -\bar{p} \bar{u} \bar{r} \]  

(13)

and

\[ \frac{\partial \psi}{\partial \bar{y}} = \bar{p} \bar{u} \bar{r} \]  

(14)

The transformation used is that due to von Mises with \( \omega \) introduced to restrict the integration across the boundary layer from zero to unity. The subscripts I and E denote the inner and outer edge of the boundary layer, respectively, and \( \psi_I \) and \( \psi_E \) are functions of \( \bar{x} \) only. The operators \( \frac{\partial}{\partial \bar{x}} \) and \( \frac{\partial}{\partial \bar{y}} \) are given by

\[ \frac{\partial}{\partial \bar{x}} = \frac{\partial}{\partial \bar{x}} + \frac{\partial \omega}{\partial \bar{x}} \frac{\partial}{\partial \bar{\omega}} = \frac{\partial}{\partial \bar{x}} \left( \frac{\partial \omega}{\partial \bar{\psi}} \frac{\partial \bar{\psi}}{\partial \bar{x}} + \frac{\partial \omega}{\partial \bar{\psi}_I} \frac{\partial \bar{\psi}_I}{\partial \bar{x}} + \frac{\partial \omega}{\partial \bar{\psi}_E} \frac{\partial \bar{\psi}_E}{\partial \bar{x}} \right) \frac{\partial}{\partial \bar{\omega}} \]  

(15)

and

\[ \frac{\partial}{\partial \bar{y}} = \frac{\partial \omega}{\partial \bar{y}} \frac{\partial}{\partial \bar{\omega}} = \frac{\partial \omega}{\partial \bar{\psi}} \frac{\partial \bar{\psi}}{\partial \bar{\omega}} \]  

(16)
Using Eqs. (12) through (14) in Eqs. (15) and (16) and applying them to Eqs. (8) and (9) give for the momentum and total energy equations

\[
\frac{\partial \tilde{u}}{\partial x} + \frac{\tilde{r}_1 \tilde{m}_1 + \omega (\tilde{r}_E \tilde{m}_E - \tilde{r}_1 \tilde{m}_1)}{\psi_E - \psi_1} \frac{\partial \tilde{u}}{\partial \omega} = \frac{(2)^{1/2}}{\Pi_{o,r}^*} \frac{\partial}{\partial \omega} \left[ \frac{\tilde{r} - \tilde{r} \bar{u} \bar{u}}{(\psi_E - \psi_1)^2 \Pr} \frac{\partial \tilde{u}}{\partial \omega} \right] - \frac{1}{\rho \bar{u}} \frac{d \tilde{p}}{d x} \tag{17}
\]

and

\[
\frac{\partial \tilde{n}}{\partial x} + \frac{\tilde{r}_1 \tilde{m}_1 + \omega (\tilde{r}_E \tilde{m}_E - \tilde{r}_1 \tilde{m}_1)}{\psi_E - \psi_1} \frac{\partial \tilde{n}}{\partial \omega} = \frac{(2)^{1/2}}{\Pi_{o,r}^*} \left\{ \frac{\partial}{\partial \omega} \left[ \frac{\tilde{r} - \tilde{r} \bar{u} \bar{u}}{(\psi_E - \psi_1)^2 \Pr} \frac{\partial \tilde{n}}{\partial \omega} \right] \right. \\
+ \left. \frac{\partial}{\partial \omega} \left[ \frac{\tilde{r} - \tilde{r} \bar{u} \bar{u}}{(\psi_E - \psi_1)^2 (1 - \frac{1}{\Pr})} \frac{d (\tilde{u}^2 / 2)}{d \omega} \right] \right\} \tag{18}
\]

where

\[
\frac{d \psi_1}{d x} = - \tilde{r}_1 \tilde{m}_1 \tag{19}
\]

and

\[
\frac{d \psi_E}{d x} = - \tilde{r}_E \tilde{m}_E \tag{20}
\]

and where \(\xi(\overline{x})\) has been replaced by \(\overline{x}\). Except for the constant \((2)^{1/2}/Re_{o,r}^*\), Eqs. (17) and (18) are identical to the momentum and total energy equations for external flow which are solved numerically by the method of Patankar and Spalding (Ref. 11). The calculation of the mass transfer fluxes \(\tilde{m}_1\) and \(\tilde{m}_E\) will be discussed in Section III.

The boundary-layer displacement thickness was used to calculate an effective nozzle geometry which in turn was used to calculate an axial pressure distribution. The displacement thickness, \(\delta^*\), which takes into account transverse curvature is expressed by Probstein and Elliott (Ref. 12) as

\[
\int_0^{\delta^*} 2 \pi r \bar{u}_E \bar{u}_E dy = \int_0^{\gamma_E} 2 \pi r (\bar{u}_E \bar{u}_E - \bar{u} \bar{u}) dy \tag{21}
\]
Solving Eq. (21) for $\delta^*$ gives

$$
\frac{\delta^*}{r^*} = \frac{r_w - \left[ \frac{r_w^2 - 2 \cos \alpha \left( \frac{r_w}{\bar{r}_w} - \frac{\bar{r}_E}{\bar{r}_w} \right) \cos \alpha - \frac{\psi_E - \psi_I}{\bar{r}_E u_E} \right]}{\cos \alpha}
$$

A quadratic equation for $\delta^*$ must be solved to obtain Eq. (22) from Eq. (21). The positive sign was chosen so that $\delta^*/r^* < r_w/\cos \alpha$. The displacement thicknesses for axisymmetric flows without transverse curvature and planar flows can be obtained by using $(r_w)^\nu$ in place of $r$ in Eq. (21). The displacement thicknesses are then given by

$$
\frac{\delta^*}{r^*} = \bar{r}_E - \frac{(\psi_E - \psi_I)}{(r_w)^\nu \bar{r}_E u_E}
$$

where $\nu = 0$ for planar flows and $\nu = 1$ for axisymmetric flows without transverse curvature.

### SECTION III

**NUMERICAL SOLUTION**

The numerical solution of Eqs. (17) and (18) is discussed in this section. The basic scheme is briefly described and a listing of the computer code is presented. A description of the necessary inputs to the program is given, and some computational difficulties which have been encountered with the program are discussed.

#### 3.1 BASIC SCHEME

As previously stated, the basic numerical integration scheme of Patankar and Spalding (Ref. 11) was suitably modified and used to solve the governing internal flow equations pertinent to the present investigation. Because the flow regime of the present investigation was entirely laminar, some subroutines of the computer code listed in Ref. 11 which were associated with turbulent flow were removed for the present code. Also, most, but not all, statements pertaining to turbulent flow were removed. The computer code as used for the present investigation is
listed in Appendix III. Essentially this same program was used on two different computers at AEDC, a Scientific Data Systems (SDS) 9300 and an International Business Machines (IBM) 370/155. The code in Appendix III is the one used on the IBM 370/155.

The basic numerical scheme used by Patankar and Spalding is discussed in detail in Ref. 11. One of the primary features of this finite difference technique is that the set of linear algebraic equations which must be solved has only three unknowns in each equation, and this set can be solved by simple successive substitution (Gauss reduction, or elimination, Ref. 13) rather than by matrix inversion. This technique provides a saving in computational time. For example, the time required to solve the algebraic equations by elimination (at a fixed x location) is proportional to the number of unknowns, whereas the time required to solve the equations by matrix inversion is proportional to the square of the number of unknowns (Ref. 13). On the IBM 370/155, a solution at a fixed x location required approximately 0.6 sec using 200 grid points across the boundary layer (u direction) in single precision. The corresponding time required on the SDS 9300 was approximately 10 sec. The IBM 370/155 carried 8 digits in single precision, whereas the SDS 9300 carried 12.

The general method of solution consisted of matching the inviscid and viscous flow regions in the nozzle by iterating on the axial pressure distribution. An initial guess of the axial pressure distribution throughout the nozzle is made, and a solution is obtained. The displacement thickness calculated in this solution is used to obtain an effective nozzle geometry which in turn is used to calculate a new pressure distribution from one-dimensional, perfect gas, expansion theory. A typical iteration and convergence process is illustrated in Fig. 2. The use of one-dimensional perfect gas expansion theory as opposed to a method-of-characteristics solution, for example, seems justified on the basis of the agreement with experimental data in Section V. The same iteration process as used here and some suggestions for choosing the initial pressure distribution are discussed further by Whitfield and Lewis (Ref. 5).

The symbols and subroutines used in the Patankar and Spalding code are clearly defined and discussed in Ref. 11. Therefore, the remainder of this section is directed toward the modifications and additions which have been made to the code of Ref. 11.
3.2 INPUT CONDITIONS

The input requirements to this program are particularly simple. The input was modified somewhat from that of the original code (Ref. 11), and the input variables are described below in the order they are read in the present code (Appendix III).

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>KRAD</td>
<td>This input permits the treatment of plane flows and axisymmetric flows with first-order transverse curvature. Also, although not pointed out in Ref. 11, axisymmetric flows without transverse curvature can be treated by setting KRAD = 0. Plane flows can be treated by setting KRAD = 0 and $r_w(x) \equiv 1$ (or a constant). Axisymmetric flows including first-order transverse curvature are treated by setting KRAD = 1 (at least not zero) and using the actual geometry $r_w(x)$.</td>
</tr>
<tr>
<td>IDIMEN</td>
<td>If the nozzle considered is two-dimensional, set IDIMEN = 0. This sets $r_w(x) \equiv 1$ in subroutine RAD. If the nozzle considered is axisymmetric set IDIMEN = 1.</td>
</tr>
<tr>
<td>NEQ</td>
<td>This is the number of partial differential equations to be solved. The code of Ref. 11 also includes the solution to the equation for the conservation of chemical species. However, this equation was never considered in this investigation and only the momentum and total energy equations were used, in which case NEQ = 2. The chemical species equation is, however, retained in the present code.</td>
</tr>
<tr>
<td>KEX</td>
<td>This input specifies the type of E boundary. It can be either 1, 2, or 3, according to whether the E boundary is a wall, free boundary, or a symmetry line, respectively. However, in the present investigation KEX was always 2, i.e., E was a free boundary, and certain modifications must be made to the present code if anything other than KEX = 2 is used.</td>
</tr>
</tbody>
</table>
KIN

This is similar to KEX except KIN specified the type of I boundary. For the present code KIN = 1 must be used; otherwise certain modifications must be made.

IHEAT

This is used in subroutine FBC and indicates a wall temperature boundary condition if IHEAT = 1 and a wall heat-transfer rate if IHEAT ≠ 1. In the present code, the only wall heat flux considered was zero, i.e., an adiabatic wall. If a heat flux other than zero is prescribed, then a few statements in subroutine SLIP must be modified (see Ref. 11). The only wall temperature distribution considered in the present work was $T_w/T_0 = \text{constant}$. However, subroutine FBC is easily modified to accommodate any desired variation of $T_w$.

N

This is the number of strips across the boundary layer, i.e., in the $ω$ direction. It must always be at least three less than the dimensions of the arrays of the variables across the layer, e.g., the maximum N which can be used with the dimensions of the arrays in the present code is 197.

REORS

Reservoir Reynolds number, $Re_0, r^*$.

ZETA

Exponent in the power law variation of viscosity with temperature, $μ \sim T^γ$.

PR(1)

Prandtl number, $Pr$.

GAM

Ratio of specific heats, $γ$.

ALPHA

Nozzle wall half-angle, $α$.

XR

Longitudinal radius of curvature of the nozzle upstream of the throat, $R$.

XL

Termination condition for the computations, maximum value of $\bar{x}$.

USUP

This input controls the location of the E boundary. It is associated with the entrainment rate and it will be discussed in the following subsection. USUP was varied from 0.99 to 0.999, and was usually 0.995.
YSTART

Initial velocity and total enthalpy profiles were calculated from the expressions

\[ \frac{u}{u_E} = 2n - \eta^2 \]  
\[ \frac{H}{H_0} = H_w/H_0 + \left[ 1 - \left( \frac{H_w}{H_0} \right) \right] \frac{u}{u_E} \]

where \( \eta = \bar{y}/\bar{y}_E \) and \( \bar{y}_E = Y\text{START} \). Suggestions for choosing YSTART will be given in the following subsection. A typical value of YSTART is 0.5.

TWT∅

Ratio of wall to total temperature, \( T_w/T_0 \)

XSTEP

Specification of the integration step-size in the \( \bar{x} \) direction in terms of local wall radius, e.g., step size, \( DX \), is given by the product of XSTEP times \( \bar{r}_w \). A typical value of XSTEP is 0.05.

LMAX

Number of \( \bar{x} \) locations where an input pressure is specified.

XX(L)

Array of \( \bar{x} \) locations where an input pressure is specified. Array goes from 1 to LMAX.

POP(L)

Input pressure for corresponding \( \bar{x} \) location, XX(L). Array goes from 1 to LMAX.

The last three inputs, LMAX, XX(L), and POP(L), are read-in in subroutine PRE, the other inputs are read-in in subroutine BEGIN.

Because of the nature of the process involved in iterating on the axial pressure distribution, it is important to input a smooth initial pressure distribution. For the present computations, the initial pressure distribution was calculated using the actual geometry at and upstream of the throat and some effective inviscid nozzle wall downstream of the throat. (Actually, the slope of the assumed inviscid nozzle wall just downstream of the throat was matched to the nozzle wall half-angle to produce a smooth wall in order to have a smooth pressure distribution.) The simple program used to calculate the initial pressure distribution for the present computations is included in Appendix IV for convenience of users where such an approximation is adequate. The particular version of the code presented in Appendix IV uses \( \delta^* \sim x^{3/2} \) for axisymmetric nozzles and \( \delta^* \sim x \) for two-dimensional nozzles. Although \( \delta^* \sim x^{3/2} \) was not used to calculate the initial pressure distribution for all axisymmetric nozzle computations presented herein, it appears to provide a reasonable approximation to the variation of \( \delta^* \) in axisymmetric nozzles.
Inasmuch as several wind tunnel and laser nozzles have conical sections downstream of the throat and constant longitudinal radius of curvature for the upstream converging portions, this general nozzle geometry (which can be sufficiently described by the inputs ALPHA and XR) was considered in this investigation. However, subroutine RAD can be easily modified to include any geometry, such as for example contoured nozzles which were analyzed in Ref. 5 using the same governing equations as used here.

3.3 SOME POSSIBLE COMPUTATIONAL DIFFICULTIES

The mass-transfer rate, or entrainment rate, across the E boundary essentially governs the location of the edge of the boundary layer. This technique of locating the edge of the boundary layer has certain advantages in analyzing low-density nozzle flows; however, it might also cause some difficulties if not handled correctly. The entrainment rate was calculated in the present investigation by evaluating the momentum equation, Eq. (17), along a constant \( \omega \) line, denoted as \( \omega_B \), near the E boundary. This technique is discussed in Ref. 11. The scheme requires the specification of the velocity along \( \omega = \omega_B \) (where \( \omega_B \) was taken as 0.9) at the next downstream station. This velocity is denoted as \( \overline{u}_B \) and it is suggested in Ref. 11 that it be taken as \( \overline{u}_B = 0.99 \overline{u}_E \), where \( \overline{u}_E \) is the velocity at the edge of the boundary layer at the next downstream station. (Note that \( \overline{u}_E \) can be calculated from the Euler equation since \( d\overline{p}/d\overline{x} \) is presumed known for the particular iteration.) It was found in the present work, however, that more flexibility could be obtained with the program if \( \overline{u}_B \) was taken as \( \overline{u}_B = (USUP)\overline{u}_E \) and USUP was input for each solution. The quantity USUP provides a means of suppressing the outer edge of the boundary layer which is advantageous in treating flows which are nearly merged. For example, during the process of iterating on the axial pressure distribution, it was observed (Fig. 2, and also Ref. 5) that \( \delta^* \) resulting from the first two iterations usually provided upper and lower bounds on the final converged \( \delta^* \). Therefore, if the flow is sufficiently rarefied, the pressure distribution resulting from the thinnest \( \delta^* \) may be such that the following iteration would predict a merged flow when in fact the flow is not merged and could be calculated if a better guess for the initial pressure distribution could be made. In some cases the calculation of merged flow in the iteration process can be avoided by using a small value of USUP (a value of 0.99 is herein regarded as small and 0.999 is regarded as large) to suppress the edge of the boundary layer and prevent an indication of merging. The suppression seems to
apply only to the outer edge of the layer and the calculated profiles over most of the layer remain essentially unchanged. For this reason, the converged $\delta^*$, or pressure distribution, obtained using a small USUP usually differs by a small amount from that obtained using a larger USUP, say 0.995. After convergence is obtained using a small value of USUP a final solution can be obtained using a larger value. The value of 0.99 for $\overline{\rho}/\overline{\mu}$ as suggested in Ref. 11 seems to excessively suppress the boundary layer for the present internal flow calculations. For most solutions reported herein, USUP was 0.995.

The step-size along the $\bar{x}$ component was taken for most of the solutions as 5 or 7.5 percent of the local wall radius. This step-size was sufficiently small for most problems. However, if calculations of properties in the nozzle throat region are of particular interest, such as the nozzle wall heat-transfer rate, a smaller step-size may be desirable.

In some applications where nozzle exit properties are of particular interest, it may be desirable to conserve computer time and use a relatively large $\bar{x}$ component step-size. For such problems some difficulties might be encountered in starting the solutions. Consider a finite-difference form of Eqs. (19) and (20)

$$
(\psi_E - \psi_I) \overline{\psi} = (\psi_E - \psi_I) U + (\overline{\rho}_E \overline{\mu}_E - \overline{\rho}_I \overline{\mu}_I) U (\overline{x}_D - \overline{x}_U)
$$

(24)

If Re$_{\infty}$, $r^*$ and/or YSTART is small, then $(\psi_E - \psi_I) U$ calculated from the initial profile will be small. Depending on the initial profiles and flow conditions, $\overline{\rho}_E$ may be positive for the first few stations and therefore $(\psi_E - \psi_I) \overline{\psi}$ could be less than $(\psi_E - \psi_I) U$ for these first few stations and might even become negative. To circumvent this difficulty one could reduce the step-size ($\overline{x}_D - \overline{x}_U$). However, if this is not desirable in view of computational time requirements, another approach is to increase YSTART in order to increase $(\psi_E - \psi_I) U$ at the first station. The first few station solutions would consequently not be as accurate as solutions obtained by using a small $\overline{x}$ step-size. Therefore, although the solutions are started at the beginning of the converging portion of the nozzle upstream of the throat, this technique of increasing YSTART should be used with caution if accurate solutions (particularly properties which depend on derivatives near the wall such as heat-transfer rate) in the throat region are required. As stated previously, a typical value of YSTART was 0.5. The largest value of YSTART used to obtain solutions was unity, but this depends on XR.
Heat transfer and viscous effects change the effective nozzle throat from the location corresponding to the actual geometric nozzle throat. The subroutine NEWPP0 searches for the minimum effective nozzle radius and uses it for the throat in calculating a one-dimensional pressure distribution for the next iteration. However, if a sufficient number of solutions near the throat are not taken, then the true minimum area and its location used in calculating a new pressure distribution may not be accurately approximated. Then, the resulting $dp/d\bar{x}$ in the throat region for the following iteration may not be smooth. In this case, the error in $dp/d\bar{x}$ for each successive iteration would become worse. This is especially a problem in solutions for flows which have adiabatic or relatively hot nozzle walls. It is suggested that if the initial guess of the $\delta^*$ distribution is not a particularly good one, e.g., if the calculated $\delta^*$ at the exit is not within 20 to 30 percent of the assumed $\delta^*$, then an improved pressure distribution should be calculated by the program in Appendix IV (or some similar method). This ensures a smooth pressure distribution, and since convergence is, in general, much faster in the throat region than further downstream where the relative displacement thicknesses are larger, few, if any, extra iterations are required.

Although areas have been pointed out where computational difficulties have been encountered with this program, it should also be pointed out that this numerical scheme is actually rather rugged, as for example compared to the method of Ref. 5. It is in general not sensitive to input conditions and seldom "blows up."

SECTION IV
NUMERICAL RESULTS

Solutions are presented in this section for typical nozzle configurations which provide some indication of the effect of $Re_{\infty}$, $r^*$, $\gamma$, $Pr$, $\delta$, $T_w/T_o$, $\alpha$, transverse curvature, and two-dimensional versus axisymmetric flows. Table I (Appendix II) summarizes the conditions of the solutions presented in this section.

Of particular interest in nozzle flows is the displacement thickness, $\delta^*$. By using $\delta^*$ and the nozzle geometry, an effective inviscid nozzle radius can be determined from which Mach number and other flow properties outside the boundary layer can be estimated. The displacement thicknesses for Conditions 1 to 8 of Table I are presented in Figs. 3 to 10.
Solutions were started at the beginning of the converging portion of the nozzles where \( z/r^* = -3 \). However, to conserve computing time, extremely small step-sizes were not utilized for all the solutions in this region (as discussed in Section III), and results are not presented in Figs. 3 to 10 for \( z/r^* < -2 \). Some solutions were repeated, however, with smaller step-sizes with no appreciable changes in the results presented.

At least for some values of \( \text{Re}_0 r^* \) in Figs. 3 to 7 and 9 to 10, negative displacement thicknesses were calculated in the throat region. This is due to the relatively cool wall increasing the gas density and hence the local mass flux near the wall. The adiabatic wall results presented in Fig. 8 do not indicate negative \( \delta^* \) for the same flow conditions. Similar results concerning the calculation of negative \( \delta^* \) have been reported previously, e.g., Potter and Carden (Ref. 3) and Whitfield and Lewis (Refs. 4 and 5).

Some indication of the effect of using various gases in a fixed nozzle geometry with fixed flow conditions is provided in Figs. 3 to 5. The specific heat ratio, Prandtl number, and power law viscosity variation with temperature of Figs. 3 to 5 closely approximate the properties of carbon dioxide (CO\(_2\)), helium (He), and nitrogen (N\(_2\)), respectively. For example, by considering the nozzle exit displacement thickness for \( \text{Re}_0 r^* = 10^4 \), one observes that, for the indicated flow conditions, \( \delta^* \) using He is about 200 percent of that when using CO\(_2\), and \( \delta^* \) using N\(_2\) is about 150 percent of that when using CO\(_2\).

Results are presented in Figs. 5 and 6 for identical conditions except for the nozzle wall half-angle. Figure 5 has a wall half-angle of 10-deg, and Fig. 6 has a half-angle of 15 deg. For the same geometric area ratio, the expansion process is more rapid for the \( \alpha = 15 \) deg nozzle than the \( \alpha = 10 \) deg. Also, the nozzle wall length for the same area ratio is longer for the 10-deg nozzle than for the 15-deg nozzle. The result is to produce a larger exit \( \delta^* \) for the 10-deg nozzle than for the 15-deg nozzle for the same geometric area ratio. For \( \text{Re}_0 r^* = 3 \times 10^3 \), the 10-deg nozzle exit \( \delta^* \) is about 18 percent larger than that of the 15-deg nozzle.

It should be pointed out that although \( \delta^* \) is relatively small because of the cool walls, \( \delta \) is not necessarily small. For example, in Fig. 5 for \( \text{Re}_0 r^* = 10^3 \), the flow merges (i.e., the boundary layer completely fills the nozzle) at a point where \( \delta/r^* \approx 3 \). For these conditions, \( \delta^*/\delta \approx 0(1/10) \). The ratio \( \delta^*/\delta \) tends to increase with wall temperature.
In general, the wall temperature seems to have a stronger effect on $\delta^*$ than on $\delta$ (Ref. 4). In Figs. 5, 7, and 8 the effect of nozzle wall temperature was investigated, with other conditions held constant, by considering $T_w/T_0 = 1/10$, $T_w/T_0 = 1/3$, and an adiabatic wall. For $Re_o r^* = 10^4$, the ratio $\delta^*/\delta$ at the nozzle exit was 0.30, 0.48, and 0.68, respectively.

The effect of Prandtl number was investigated by repeating the conditions of Fig. 5 but with $Pr = 1$ in place of $5/7$. The results are presented in Fig. 9. The displacement thickness was found to increase for $Pr = 1$ by approximately 20 percent at the nozzle exit.

The conditions of Fig. 5 were also repeated using $\xi = 1$ in place of $\xi = 2/3$ to investigate the effect of the power law variation of viscosity with temperature. For $\xi = 1$ the displacement thickness was found to decrease by 20 to 25 percent below that for $\xi = 2/3$ (see Figs. 5 and 10). Note that, since $\bar{\mu} = T^{\xi}$ and $T \leq 1$, then the flow is more viscous for $\xi = 2/3$ than for $\xi = 1$.

Velocity and temperature profiles calculated with and without transverse curvature terms are presented in Fig. 11 for Conditions 3 and 9 (Table I) with $Re_o r^* = 3 \times 10^3$. Neglecting transverse curvature decreases the boundary-layer thickness for internal flow. The effect of transverse curvature is negligible for thin boundary layers, but may be significant for thick boundary layers. Further results and discussion concerning the effects of transverse curvature are reported in Ref. 5.

The displacement thicknesses of an axisymmetric and two-dimensional nozzle are presented in Fig. 12. For each nozzle, $A_e/A^* = 5$, and therefore, the two-dimensional nozzle is considerably longer since $\alpha = 10$ deg for each. The result is to produce a significantly larger displacement thickness for the two-dimensional nozzle than for the axisymmetric nozzle.

SECTION V
COMPARISONS WITH EXPERIMENTAL DATA AND OTHER THEORETICAL RESULTS

Curves of $Re_o r^*(p_o r^*)$ versus total temperature for various gases are presented in Fig. 13 for the convenience of determining $Re_o r^*$. The viscosities used for all gases in Fig. 13 except carbon dioxide (CO$_2$) are
those given by Svehla (Ref. 14). The viscosity of CO₂ was taken from Table 8.4-2 of Hirschfelder, Curtiss, and Bird (Ref. 15).

Highly viscous nozzle flows are usually associated with nozzles of small physical size. The spatial resolution of experimental measurements in such nozzles is obviously restricted. However, highly viscous flows can also be produced in large nozzles if sufficiently large volume flows can be pumped at low pressures, thereby permitting more detailed investigations of the boundary layer. Such investigations are possible in the Aerospace Research Chamber 10V in the VKF at AEDC. The present and some previous calculation methods will be compared with experimental data taken in Chamber 10V using a nominal Mach three nozzle, denoted M3 nozzle. This is a 10-deg half-angle conical nozzle with d* = 27 cm, dₑ = 76.2 cm, and R = 11.2 cm. The walls of this nozzle were cooled with liquid nitrogen to maintain a constant nozzle wall temperature of about 85°K.

Calculated and measured pitot pressure profiles at the nozzle throat are presented in Fig. 14. Nitrogen was the test gas used for these and all other experimental data presented in this report. The present results are in good agreement with the results using the method of Ref. 5. The calculated profiles are in relatively good agreement with the measured profile for these conditions.

Calculated and measured pitot pressure profiles at the nozzle exit are presented in Fig. 15. This calculation was performed using ζ = 2/3, whereas some other results presented herein, e.g., Fig. 14, were obtained using Sutherland's viscosity law for nitrogen. However, over the temperature range of the experimental data taken in Chamber 10V, there was negligible difference in the results using either viscosity variation with temperature.

Present calculations are compared to calculations by the method of Rae (Ref. 10) and experimental data in Figs. 16 and 17. The present results are in relatively good agreement with the experimental data, but Rae's method is found to overestimate the size of the viscous region for these conditions. Results were also presented in Ref. 5 for the conditions of Fig. 17. The calculated profile from Ref. 5 is almost identical with the present result in Fig. 17 (see Ref. 5) and is not presented.

Pitot pressure profiles at about 7 throat radii downstream of the M3 nozzle throat are presented in Fig. 18 for Re₀, r* = 1170. A relative minimum exists at the nozzle centerline in the experimentally
measured profile, and slight "humps" or relative maximums exist near the edge of the boundary layer. It should be pointed out that the existence of humps does not necessarily imply that the flow is non-isentropic. Imagine a radial pitot pressure profile in an inviscid contoured supersonic nozzle which is sufficiently far downstream of the throat to pass through a portion of the uniform parallel flow near the centerline. Such a pitot profile would have lower values near the nozzle centerline than near the wall because of the larger Mach numbers near the centerline. However, the matching of such an inviscid flow with a realistic viscous flow requires the pitot pressure near the wall to approach the local static pressure which is less than the centerline pitot pressure. Therefore, it is not difficult to conceive of humps in such a radial pitot profile because of the matching of inviscid and viscous flow. This argument is based on flow in a contoured nozzle. It is applied to the present case because the displacement thickness effectively contours the nozzle. A more accurate approach of investigating such flows would be to remove the present assumption of one-dimensional inviscid flow outside the boundary layer and obtain more accurate solutions, such as, inviscid method-of-characteristics solutions. However, the pitot pressure profile is relatively well predicted in Fig. 18, and for this work, the assumption of one-dimensional inviscid flow outside the boundary layer is considered acceptable.

Just as radial pitot profiles with humps do not necessarily imply that the flow is non-isentropic, a flat radial profile does not necessarily imply that the flow is isentropic. Consider the pitot pressure data in Fig. 19. Both the calculated and measured profiles are relatively flat for $y/r_w$ larger than about 0.7. However, in this case the pitot profile is not a good measure of the extent of the nozzle wall viscous effects as shown in Fig. 20. From Fig. 20, the boundary-layer thickness as estimated from the pitot profile is about 55 percent of the nozzle radius, whereas it is calculated to be actually over 80 percent. The pitot profile for the case in Fig. 20 implies that 20 percent of the cross-sectional area of the nozzle at this point is core flow, whereas actually less than 4 percent is core flow. The reason a pitot profile might lead one astray is associated with the temperature or thermal boundary layer since pitot pressure depends, among other things, on $u/(T)^{1/2}$. The velocity variation is usually well behaved and fairly accurately predicted by simple analytical expressions based on boundary-layer thickness (Ref. 4); however, this is not the case with the thermal boundary layer. The temperature variation depends not only on local wall and edge values and gradients but also on the upstream conditions.
Therefore, some consideration should be given to the temperature variation in order to place limits-of-confidence on the use of pitot pressure as an indication of the nozzle wall viscous effects.

Carden (Ref. 16) measured local heat-transfer coefficients in an axisymmetric nozzle for \( \text{Re}_0 r^* = 5 \times 10^3 \). The experimental data were compared with calculated heat-transfer coefficients in Refs. 5 and 16. However, Whitfield made a mistake in Ref. 5 and presented solutions for \( r^* = 0.262 \text{ cm} \) instead of \( d^* = 0.262 \text{ cm} (0.103 \text{ in.}) \) which corresponds to the actual nozzle throat dimension. Because of this error, the calculated heat-transfer coefficients (Ref. 5) from the iterated solutions were significantly below the experimental data of Carden (Ref. 16). Heat-transfer coefficients were calculated using the present method but with the inappropriate throat dimension of \( r^* = 0.262 \text{ cm} \), and good agreement was obtained with the calculated results of Ref. 5. Results were also obtained with the present method using the proper throat dimension of \( d^* = 0.262 \text{ cm} \), and good agreement with the experimental data was obtained as shown in Fig. 21. Also, in Fig. 21 are results from two of the methods Carden (Ref. 16) used for calculating the heat-transfer coefficient and one solution from Ref. 5. Although the result from Ref. 5 which is presented in Fig. 21 was not iterated to include the higher-order displacement effect, the result was obtained using the pressure distribution corresponding to the proper nozzle geometry and, therefore, is included in Fig. 21. This result from Ref. 5 is in relatively good agreement with the present result. All calculation methods underestimate the most upstream uncorrected experimental data point in Fig. 21. However, Carden points out that radiation from the arc used to heat the gas could increase the total heat-transfer rate to this portion of the nozzle. No corrections for radiation heat transfer were made to the experimental data.

It might be pointed out that the heat-transfer rate printed out in the present program, denoted as \( \text{AJI}(1) \), is equal to \( q_w/(\rho_o H_o^3/2) \). Although made dimensionless, the numerical scheme used in Ref. 11 to calculate \( \text{AJI}(1) \) is retained in the present code in subroutine WALL.

SECTION VI

CONCLUSIONS

The present results were shown to be in good agreement with results from the method of Ref. 5. Although it was noted during the present investigation that more axial stations were required using the
present program than that of Ref. 5 to obtain the same degree of accuracy (presumably this is due to the transformation variables used in Ref. 5, which are advantageous for thin boundary layers but disadvantageous for thick internal boundary layers), the present program is much smaller, faster and easier to use. Comparisons with results from Rae's method (Ref. 10) and experimental data indicated the present method to be more accurate than Rae's for the conditions considered. It should be remembered, however, that the present work is not applicable to merged flows without modification, whereas Rae's method is more applicable to merged flows.

Consistent agreement was obtained between the present results and experimental data. Except for the heat-transfer data, the experimental data were taken in a relatively large nozzle where the boundary layer could be accurately probed. Similar agreement was obtained in Ref. 2 where extensive data were obtained in small nozzles, \( r^* \sim 0 \ (0.1 \text{ cm}) \). The numerical method used in Ref. 2 was developed by Mayne (Ref. 2), as stated previously, and differs from the original Patankar and Spalding method (Ref. 11) primarily in the numerical scheme near the wall.

The present method has not been modified for use in nozzle design; however, it is a straightforward matter to make such a modification (see Ref. 5). To design a nozzle, a new nozzle geometry, \( r_w(x) \), is calculated from the displacement thickness from each iteration, and the new geometry is used for the next iteration while the desired axial pressure distribution is maintained for each iteration. It was found in Ref. 5 that convergence was, in general, more rapid in iterating on the nozzle geometry than on the axial pressure distribution.

REFERENCES


APPENDIXES

I. ILLUSTRATIONS
II. TABLE
III. BOUNDARY-LAYER COMPUTER CODE
IV. INITIAL PRESSURE DISTRIBUTION COMPUTER CODE
Fig. 1 Definition of Coordinate System
Condition No. 5 (Table I)

\[
\begin{align*}
\gamma &= 7/5 \\
Pr &= 5/7 \\
\zeta &= 2/3 \\
T_w/T_o &= 1/3 \\
\alpha &= 10 \text{ deg} \\
A/A^* &= 25 \\
R &= 3 \\
Re_{o,r^*} &= 5 \times 10^3
\end{align*}
\]

Fig. 2 Displacement Thickness of Successive Iterations for Condition No. 5 (Table I)
Condition No. 1 (Table I)

\[ \gamma = \frac{9}{7} \]
\[ Pr = \frac{3}{4} \]
\[ \zeta = \frac{3}{4} \]
\[ \frac{T_w}{T_o} = 1/10 \]
\[ \alpha = 10 \text{ deg} \]
\[ A/A^* = 25 \]
\[ \bar{R} = 3 \]

Fig. 3 Displacement Thickness for Condition No 1 (Table I)
Condition No. 2 (Table I)

\begin{align*}
\gamma &= 5/3 \\
Pr &= 2/3 \\
\zeta &= 5/7 \\
\frac{T_w}{T_o} &= 1/10 \\
\alpha &= 10 \ deg \\
\frac{A/A^*}{R} &= 25 \\
\overline{R} &= 3
\end{align*}

Fig. 4 Displacement Thickness for Condition No. 2 (Table I)
Condition No. 3 (Table I)

\[ \gamma = \frac{7}{5} \]
\[ \Pr = \frac{5}{7} \]
\[ \zeta = \frac{2}{3} \]
\[ \frac{T_w}{T_o} = \frac{1}{10} \]
\[ \alpha = 10 \text{ deg} \]
\[ \frac{A}{A^*} = 25 \]
\[ \overline{R} = 3 \]

Fig. 5 Displacement Thickness for Condition No. 3 (Table I)
Condition No. 4 (Table I)

\[ \gamma = \frac{7}{5} \]

\[ \text{Pr} = \frac{5}{7} \]

\[ \zeta = \frac{2}{3} \]

\[ \frac{T_w}{T_o} = \frac{1}{10} \]

\[ \alpha = 15 \text{ deg} \]

\[ \frac{A}{A^*} = 25 \]

\[ \bar{R} = 3 \]
Condition No. 5 (Table I)

\[ \gamma = \frac{7}{5} \]
\[ \text{Pr} = \frac{5}{7} \]
\[ \zeta = \frac{2}{3} \]
\[ \frac{T_w}{T_o} = \frac{1}{3} \]
\[ \alpha = 10 \text{ deg} \]
\[ \frac{A}{A^*} = 25 \]
\[ \text{Re} = 3 \]

Fig. 7 Displacement Thickness for Condition No. 5 (Table I)
Condition No. 6 (Table I)
\[
\begin{align*}
\gamma &= 7/5 \\
\Pr &= 5/7 \\
\zeta &= 2/3 \\
\text{Re}_{o,r^*} &= \text{Adiabatic} \\
\alpha &= 10 \text{ deg} \\
A/A^* &= 25 \\
\bar{R} &= 3
\end{align*}
\]
Condition No. 7 (Table I)

\[ \gamma = 7/5 \]
\[ Pr = 1 \]
\[ \zeta = 2/3 \]
\[ T_w/T_o = 1/10 \]
\[ \alpha = 10 \text{ deg} \]
\[ A/A^* = 25 \]
\[ R = 3 \]

Fig. 9 Displacement Thickness for Condition No. 7 (Table I)
Condition No. 8 (Table I)

\[ \gamma = \frac{7}{5} \]
\[ \text{Pr} = \frac{5}{7} \]
\[ \zeta = 1 \]
\[ \frac{T_w}{T_o} = \frac{1}{10} \]
\[ \alpha = 10 \text{ deg} \]
\[ \frac{A}{A^*} = 25 \]
\[ N = 3 \]

Fig. 10 Displacement Thickness for Condition No. 8 (Table I)
Fig. 11 Nozzle Exit Velocity and Temperature Profiles
for Condition Nos. 3 and 9 (Table I)

\[ \text{Re}_{\text{o}, r^*} = 3 \times 10^3 \]
Fig. 12  Displacement Thicknesses of an Axisymmetric and Two-Dimensional Nozzle with Equal Area Ratios

Re_{o,r^*} = 3 \times 10^3
Fig. 13 Reservoir Reynolds Number as a Function of $p_0$, $T_0$, and $r^*$ for Various Gases
Fig. 14 Calculated and Measured Pitot Pressure Profiles at the M3 Nozzle Throat for Re₉ = 900

<table>
<thead>
<tr>
<th>Sym</th>
<th>p₀, Torr</th>
<th>T₀, °K</th>
<th>Tₘ, °K</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>○</td>
<td>0.10</td>
<td>267</td>
<td>85</td>
<td>Experimental Data</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>300</td>
<td>100</td>
<td>Calculated (Ref. 5)</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>300</td>
<td>100</td>
<td>Calculated (Present)</td>
</tr>
</tbody>
</table>
Fig. 15 Calculated and Measured Pitot Pressure Profiles at the M3 Nozzle Exit for $Re_{0,r^*} = 1800$

- Present Calculation
  $Re_{0,r^*} = 1800$
  $T_w/T_o = 1/3$

- Experimental Data
  $p_o = 0.20$ torr
  $T_o = 295^\circ K$
  $T_w = 89^\circ K$
  $r^* = 13.5$ cm
Table:

<table>
<thead>
<tr>
<th>Sym</th>
<th>p_o, torr</th>
<th>T_o, °K</th>
<th>T_w, °K</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>0.10</td>
<td>295</td>
<td>89</td>
<td>Experimental Data</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>295</td>
<td>100</td>
<td>Calculated</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>300</td>
<td>100</td>
<td>Calculated</td>
</tr>
</tbody>
</table>

Fig. 16 Calculated and Measured Pitot Pressure Profiles at the M3 Nozzle Exit for Re_{x,r} = 900
Fig. 17 Calculated and Measured Pitot Pressure Profiles at the M3 Nozzle Exit for $Re_{e,r} = 4500$
Fig. 18 Calculated and Measured Pitot Pressure Profiles Downstream of the M3 Nozzle Throat for \( \text{Re}_{o, r*} = 1170 \)

- \( \gamma = 7/5 \)
- \( P_r = 5/7 \)
- \( \zeta = 2/3 \)

<table>
<thead>
<tr>
<th>Sym</th>
<th>( p_o ), torr</th>
<th>( T_o ), K</th>
<th>( T_w ), K</th>
<th>( r_w ), cm</th>
<th>( z ), cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>0.359</td>
<td>717</td>
<td>85</td>
<td>30.5</td>
<td>96.5</td>
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<tr>
<td></td>
<td>0.360</td>
<td>717</td>
<td>89</td>
<td>30.2</td>
<td>95.3</td>
</tr>
</tbody>
</table>

Source: Experimental Data, Present Calculation
Fig. 19  Calculated and Measured Pitot Pressure Profiles Downstream of the M3 Nozzle Throat for $Re_o, r_0 = 330$
Fig. 20 Calculated Velocity, Temperature, and Pitot Pressure Profiles Downstream of the M3 Nozzle Throat for $Re_{o,*} = 330$

$p_o = 0.100$ torr
$T_o = 717°K$
$T_w = 89°K$
$r_w = 29.0$ cm
$r^* = 13.5$ cm
$z = 88.6$ cm
$\alpha = 10$ deg
<table>
<thead>
<tr>
<th>Curve</th>
<th>Numerical Method</th>
<th>Higher-Order Effects</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>———-</td>
<td>Beckwith and Cohen</td>
<td>None</td>
<td>Ref. 16</td>
</tr>
<tr>
<td></td>
<td>(Local Similarities, Ref. 17)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>———-</td>
<td>Incremental Flat Plate (Ref. 18)</td>
<td>None</td>
<td>Ref. 16</td>
</tr>
<tr>
<td>———-</td>
<td>Jaffe, Lind, and Smith (Nonsimilar, Ref. 7)</td>
<td>None</td>
<td>Ref. 5</td>
</tr>
<tr>
<td>———-</td>
<td>Patankar and Spalding (Nonsimilar, Ref. 11)</td>
<td>Transverse Curvature</td>
<td>Present</td>
</tr>
</tbody>
</table>

![Graph](image)

**Fig. 21** Calculated and Measured Heat-Transfer Coefficients in a Low-Density Nozzle

\[ h = \frac{q_w}{T_{aw} - T_w} \text{ Btu ft}^{-2} \text{ hr}^{-1} \text{ F}^{-1} \]

- ○ Experimental Data (Ref. 16)
- □ Experimental Data Corrected for Axial Conduction (Ref. 16)

\[ d^* = 0.103 \text{ in.} \]
<table>
<thead>
<tr>
<th>Condition No.</th>
<th>$\gamma$</th>
<th>Pr</th>
<th>$\xi$</th>
<th>$T_w/T_0$</th>
<th>$\alpha$</th>
<th>$A/A^*$</th>
<th>$R$</th>
<th>$Re_{o,r}^*$</th>
<th>Comment</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>9/7</td>
<td>3/4</td>
<td>3/4</td>
<td>1/10</td>
<td>10</td>
<td>25</td>
<td>3</td>
<td>$3 \times 10^3, 10^4, 10^5$</td>
<td>Axisymmetric Nozzle with Transverse Curvature</td>
</tr>
<tr>
<td>2</td>
<td>5/3</td>
<td>2/3</td>
<td>5/7</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td>$5 \times 10^3, 10^4, 2 \times 10^4, 5 \times 10^4, 10^5$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7/5</td>
<td>5/7</td>
<td>2/3</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td>$10^3, 3 \times 10^3, 10^4, 3 \times 10^4, 10^5$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15</td>
<td>$3 \times 10^3, 10^4, 10^5$</td>
<td></td>
</tr>
<tr>
<td>5</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>1/3</td>
<td>$5 \times 10^3, 10^4, 3 \times 10^4, 10^5$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$10^4, 10^5$</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/10</td>
<td>$3 \times 10^3, 10^4, 3 \times 10^4, 10^5$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$10^3, 10^4, 10^5$</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$3 \times 10^3$</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>2/3</td>
<td>$3 \times 10^3$</td>
<td></td>
</tr>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>5</td>
<td>9</td>
<td>Two-Dimensional Nozzle</td>
</tr>
</tbody>
</table>
APPENDIX III
BOUNDARY-LAYER COMPUTER CODE

<table>
<thead>
<tr>
<th>FORTRAN IV G LEVEL</th>
<th>MAIN</th>
<th>DATE</th>
<th>22/52/18</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>MAIN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0001</td>
<td>CUMON /GEN/ PEI, AME, OME, DPDX, PREF (2), PR (2), P (2), DEN, AMU, XU, XD, XP</td>
<td>A 2</td>
<td></td>
</tr>
<tr>
<td>1XL, DX, INTG, CSALPHA, ALPHA, XR, REORS, GAM, ZETA, PP0, TWD0, START, USUP, IDI</td>
<td>A 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2ME, IHEAT, I, TD, XSTEP /N, NP1, NP2, NP3, NEQ, NPH, KEX, KIN, KASE, KRAO /W</td>
<td>A 4</td>
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<td></td>
</tr>
<tr>
<td>3EX, GAM (2), TAU (2), XJ (2), AMU (2), INI (2), Q (2), W (2), U (2), V (2)</td>
<td>A 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0012</td>
<td>COMMON /L/ AK, ALMG</td>
<td>A 10</td>
<td></td>
</tr>
<tr>
<td>0007</td>
<td>CALL BEGIN</td>
<td>A 13</td>
<td></td>
</tr>
<tr>
<td>0009</td>
<td>CALL PRE (XU, OPDX)</td>
<td>A 14</td>
<td></td>
</tr>
<tr>
<td>0010</td>
<td>CALL ENTRN</td>
<td>A 15</td>
<td></td>
</tr>
<tr>
<td>0013</td>
<td>IF (XG.TG.XL) GO TO 13</td>
<td>A 16</td>
<td></td>
</tr>
<tr>
<td>0014</td>
<td>CONTINUE</td>
<td>A 17</td>
<td></td>
</tr>
<tr>
<td>0014</td>
<td>INTG = INTG + 1</td>
<td>A 18</td>
<td></td>
</tr>
<tr>
<td>0016</td>
<td>CHOICE OF FORWARD STEP</td>
<td>A 19</td>
<td></td>
</tr>
<tr>
<td>0017</td>
<td>X = X + DX</td>
<td>A 20</td>
<td></td>
</tr>
<tr>
<td>0018</td>
<td>IF (+NDT (XG.LT.XTH0AT.AND.XQ.GT.XTH0AT)) GO TO 4</td>
<td>A 21</td>
<td></td>
</tr>
<tr>
<td>0019</td>
<td>X = X + DX</td>
<td>A 22</td>
<td></td>
</tr>
<tr>
<td>0020</td>
<td>DX = DX - XU</td>
<td>A 23</td>
<td></td>
</tr>
<tr>
<td>0021</td>
<td>ITH0AT = INTG + 1</td>
<td>A 24</td>
<td></td>
</tr>
<tr>
<td>0022</td>
<td>CONTINUE</td>
<td>A 25</td>
<td></td>
</tr>
<tr>
<td>0023</td>
<td>IF (XG.LE.XL) GO TO 5</td>
<td>A 26</td>
<td></td>
</tr>
<tr>
<td>0024</td>
<td>X = X + DX</td>
<td>A 27</td>
<td></td>
</tr>
<tr>
<td>0025</td>
<td>DX = DX - XU</td>
<td>A 28</td>
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</tr>
<tr>
<td>0026</td>
<td>CONTINUE</td>
<td>A 29</td>
<td></td>
</tr>
<tr>
<td>0027</td>
<td>CALL PRE (XG, OPDX)</td>
<td>A 30</td>
<td></td>
</tr>
<tr>
<td>0028</td>
<td>CALL ENTRN</td>
<td>A 31</td>
<td></td>
</tr>
<tr>
<td>0029</td>
<td>CALL PRE (XG, OPDX)</td>
<td>A 32</td>
<td></td>
</tr>
<tr>
<td>0030</td>
<td>IF (KASE.EQ.2) GO TO 6</td>
<td>A 33</td>
<td></td>
</tr>
<tr>
<td>0031</td>
<td>IF (KIN, EQ.1) CALL MASS (XG, XD, AMU)</td>
<td>A 34</td>
<td></td>
</tr>
<tr>
<td>0032</td>
<td>CALL WALL</td>
<td>A 35</td>
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</tr>
<tr>
<td>0033</td>
<td>CALL OUTPUT</td>
<td>A 36</td>
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</tr>
<tr>
<td>0034</td>
<td>CALL PRE (XG, OPDX)</td>
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</tr>
<tr>
<td>0036</td>
<td>CALL GCOEFF</td>
<td>A 38</td>
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</tr>
<tr>
<td>0037</td>
<td>MODIFIED FOLLOWING STATEMENT FOR INTERNAL CORE FLOW</td>
<td>A 39</td>
<td></td>
</tr>
<tr>
<td>0038</td>
<td>IF (KEX.EQ.2) U(NP3) = SORT (U1) * (KX - XU) / (RHO (1))</td>
<td>A 40</td>
<td></td>
</tr>
<tr>
<td>0039</td>
<td>CALL SOLVE (AU, BU, CV, U, NP3)</td>
<td>A 41</td>
<td></td>
</tr>
<tr>
<td>0040</td>
<td>IF (KIN.EQ.3) GO TO 7</td>
<td>A 42</td>
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</tr>
<tr>
<td>0041</td>
<td>U(1) = U(2)</td>
<td>A 43</td>
<td></td>
</tr>
<tr>
<td>0042</td>
<td>IF (KRAD.EQ.0) U(1) = 0.75 * U(2) + 0.25 * U(3)</td>
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<tr>
<td>F77TRAN IV G LEVEL</td>
<td>20</td>
<td>MAIN</td>
<td>DATE = 72286</td>
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<td>---------------------</td>
<td>----</td>
<td>------</td>
<td>--------------</td>
</tr>
<tr>
<td>0043 7</td>
<td>IF (KEX.EQ.3) U(NP3) = .75<em>U(NP2) + .25</em>U(NP1)</td>
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<td></td>
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<tr>
<td>0044</td>
<td>IF (NEU.EQ.1) GO TO 14</td>
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</tr>
<tr>
<td>0045</td>
<td>DO 13 J = 1, NPH</td>
<td>A 60</td>
<td></td>
</tr>
<tr>
<td>0046</td>
<td>DO 8 I = 2, N P 2</td>
<td>A 61</td>
<td></td>
</tr>
<tr>
<td>0047 8</td>
<td>A(I) = A(J, I)</td>
<td>A 62</td>
<td></td>
</tr>
<tr>
<td>0048</td>
<td>B(J, I)</td>
<td>A 63</td>
<td></td>
</tr>
<tr>
<td>0049</td>
<td>C(J, I)</td>
<td>A 64</td>
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</tr>
<tr>
<td>0050</td>
<td>DO 9 I = 1, N P 3</td>
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</tr>
<tr>
<td>0051 9</td>
<td>SC(I) = F(J, I)</td>
<td>A 66</td>
<td></td>
</tr>
<tr>
<td>0052</td>
<td>CALL SOLVE (AU, BU, CU, SC, NP 3)</td>
<td>A 67</td>
<td></td>
</tr>
<tr>
<td>0053</td>
<td>DO 10 I = 1, N P 3</td>
<td>A 68</td>
<td></td>
</tr>
<tr>
<td>0054 10</td>
<td>F(J, I) = SC(I)</td>
<td>A 69</td>
<td></td>
</tr>
<tr>
<td>0055</td>
<td>IF (KASE.EQ.2) GO TO 11</td>
<td>A 70</td>
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<tr>
<td>0056</td>
<td>C SETTING UP WALL VALUES OF F</td>
<td>A 71</td>
<td></td>
</tr>
<tr>
<td>0057</td>
<td>IF (KIN.EQ.1.AND.IND(J)*EQ.2) F(J, I) = (1.*BETA+GAMA(J))*F(J, I)</td>
<td>A 72</td>
<td></td>
</tr>
<tr>
<td>0058</td>
<td>C SETTING UP SYMMETRY-LINE VALUES OF F</td>
<td>A 73</td>
<td></td>
</tr>
<tr>
<td>0059</td>
<td>IF (KIN.EQ.3) GO TO 12</td>
<td>A 74</td>
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<td>0060</td>
<td>F(J, I) = F(J, 2)</td>
<td>A 75</td>
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<tr>
<td>0061</td>
<td>IF (KRED.EQ.0) F(J, I) = 75<em>F(J, 2) + 25</em>F(J, 3)</td>
<td>A 76</td>
<td></td>
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<tr>
<td>0062</td>
<td>IF (KEX.EQ.3) F(J, NP3) = 75<em>F(J, NP2) + 25</em>F(J, NP1)</td>
<td>A 77</td>
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</tr>
<tr>
<td>0063</td>
<td>CONTINUE</td>
<td>A 78</td>
<td></td>
</tr>
<tr>
<td>0064 14</td>
<td>XU = XU</td>
<td>A 79</td>
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<tr>
<td>0065</td>
<td>RPS(INTG) = R(1)</td>
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<td>COSAL(INTG) = CSALFA</td>
<td>A 81</td>
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<tr>
<td>0067</td>
<td>XU = X D</td>
<td>A 82</td>
<td></td>
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<tr>
<td>0068</td>
<td>THE TERMINATION CONDITION</td>
<td>A 83</td>
<td></td>
</tr>
<tr>
<td>0069</td>
<td>IF (XU.LT.XL) GO TO 2</td>
<td>A 84</td>
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<tr>
<td>0070</td>
<td>CALL NEHPO</td>
<td>A 85</td>
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<td>0071</td>
<td>STOP</td>
<td>A 86</td>
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<tr>
<td>0072</td>
<td>END</td>
<td>A 87</td>
<td></td>
</tr>
</tbody>
</table>

C

THE TERMINATION CONDITION

A 88
SUBROUTINE NEWPPO
COMMON /GEN/ PEI,AM1,AME,DPDL,PREF12,PR(21,P12),DEN,AMU,XU,DX,XP,
1LX,DX,INTG,CALFA,ALPHA,XX,REOR,GM,GETA,PPD,TWTO,YSTART,USUP,IDI
2MENV,1HEAT,2,USTEP/E,OSTAR(300),XRS(300),WNRS(300),COSAL(300)
DIMENSION FM(3001),POP(3001),RIRSTVO(300)
IF (IDMEN.EQ.0) RETURN

C1=G-1/2, C2=(G-1)/2, C3=(0+1.1/2.)*C2, C4=C1**C3
C5=C4*C2*C3**2
DO 11 I=1,INTG
RIPSII)-RWRS(II-COSALM)*DSTAR(I)
DO 14 I=1,INTG
IF (RIKS(I).LT.RI8.IL8.)*1)> GO TO 3
CONTINUE
RTH0AT-RWRS(II-COSALM)*DSTAR(I)
DO 14 I=1,INTG
A=(RTH0AT/RI5II)**2
IF (A-1.O) 4,4,6
FH1=0.5
FM»FM1-(A**B**(C3**1.1-C4*FM1*0>/(C5*FH1**2-C4*BI
IF (ABS(FM-FH1)<.OO0Ö) 13,12,12
FM1=FM1-C2
IF (FM-FH1<.OO0Ö) 13,12,12
FH1=FM
FM1=FM1-0.1
CONTINUE
FM1=FM1-C2
GO TO 13
CONTINUE
FM1=FM1+0.1
GO TO 5
CONTINUE
FM1=FM1+0.1
GO TO 5
CONTINUE
FM1=FM1-0.1
GO TO 5
CONTINUE
FM1=FM1+0.1
CONTINUE
WRITE (16,151)
WRITE (16,152)
WRITE (16,153)
WRITE (15,154)
RETURN
15 FORMAT(1H1,2X,'THE DISPLACEMENT THICKNESS FROM THIS ITERATION AND '
1X,'THE PRESSURE DISTRIBUTION FOR THE NEXT ITERATION FOLLOWS','/
28X,FM1,11X,PR/P01,11X,DS1,1X,DX,12X,WN,12X,INTG'/)
END
FORTRAN IV G LEVEL 20 BEGIN DATE = 72286 22/52/18

0001 SUBROUTINE BEGIN

0002 COMMON /GEN/, PEI, AMI, AME, DPOX, PREF(2), PR(2), P(2), DEN, AMU, XU, XD, XP, C

0003 XL, DX, INTC, CSALFA, ALPHA, XR, REGRS, GAM, ETA, PPO, TWTO, YSTART; USUP, IDI C

0004 ZMEN, IHET, Z, TU, XSTEP /N, NP1, NP2, NP3, NEQ, NPH, KEX, KIN, KASE, KRAD, B/B C

0005 ETA, GAMAI2), TAU1, TAU2, AJE(2), AJE2), IND(2), INDI(2), INDI(2), INDI(2), INDI(2), INDI(2) C


0007 C PROBLEM SPECIFICATION

0008 READ 15.281 KRAD, DP1, DPM, NEQ, KEX, KIN, IHET, N C

0009 READ (5, 29) REGRS, ETA, PR(1), GAM, ALPHA, XR, XSUP, YST, XST, IT C

0010 PROBLEM SPECIFICATION

0011 READ 15.281 KEX, NEQ, KIN, IHET, N C

0012 READ 15.291 REGRS, ETA, PR(1), GAM, ALPHA, XR, XSUP, YST, XST, IT C

0013 IC INIT. EDGE OF BOUNDARY LAYER IS YSTART C

0014 IC APPROXIMATE CALCULATION OF UFDGE FROM ONE DIMENSION FLOW RELATIONS

0015 UUMXX = SQRT(2/(1 + XR1)) C

0016 UEDGE = UUMXX * SQRT(2.1) C

0017 IF (KIN .EQ. 1.0) KASE = 1 C

0018 KASE = 2 C

0019 IF (KRAD .NE. 0) GO TO 5 C

0020 U(2) = U(3) / (1.0 + ETA * ETA) C

0021 Y(2) = Y(3) / (1.0 + ETA * ETA) C

0022 Y(2) = Y(3) / (1.0 + ETA * ETA) C

0023 Y(2) = Y(3) / (1.0 + ETA * ETA) C

0024 CONTINUE C

0025 C INITIAL VELOCITY PROFILE

0026 Y(1) = 0.0 C

0027 U(1) = 0.0 C

0028 XNP2 = NP2 C

0029 DELY = YSTART/XNP2 C

0030 GO TO 2, 2 C

0031 ETA = Y(1) / DELY C

0032 GO TO 2, 2 C

0033 U(1) = (1.0 - ETA) * (U(1) - ETA) C

0034 U(2) = (1.0 - ETA) * (U(2) - ETA) C

0035 U(2) = (1.0 - ETA) * (U(2) - ETA) C

0036 GO TO 6 C

0037 IF (KRAD .NE. 0) GO TO 5 C

0038 U(2) = (1.0 - ETA) * (U(2) - ETA) C

0039 Y(2) = Y(3) / (1.0 + ETA * ETA) C

0040 GO TO 6 C

0041 U(3) = Y(3) / (1.0 + ETA * ETA) C

0042 Y(2) = Y(3) / (1.0 + ETA * ETA) C

0043 GO TO 7, 7 C

0044 Y(2) = Y(3) / (1.0 + ETA * ETA) C

0045 Y(2) = Y(3) / (1.0 + ETA * ETA) C

0046 Y(2) = Y(3) / (1.0 + ETA * ETA) C

52
AEDC-TR-73-52

BEGIN

DATE = 7/28/62

22/52/18

GO TO 10

C 57

GO TO 11

C 58

GO TO 12

C 59

GO TO 13

C 60

GO TO 14

C 61

GO TO 15

C 62

GO TO 16

C 63

GO TO 17

C 64

GO TO 18

C 65

GO TO 19

C 66

GO TO 20

C 67

GO TO 21

C 68

GO TO 22

C 69

GO TO 23

C 70

GO TO 24

C 71

GO TO 25

C 72

GO TO 26

C 73

GO TO 27

C 74

GO TO 28

C 75

GO TO 29

C 76

GO TO 30

C 77

GO TO 31

C 78

GO TO 32

C 79

GO TO 33

C 80

GO TO 34

C 81

GO TO 35

C 82

GO TO 36

C 83

GO TO 37

C 84

GO TO 38

C 85

GO TO 39

C 86

GO TO 40

C 87

GO TO 41

C 88

GO TO 42

C 89

GO TO 43

C 90

GO TO 44

C 91

GO TO 45

C 92

GO TO 46

C 93

GO TO 47

C 94

GO TO 48

C 95

GO TO 49

C 96

GO TO 50

C 97

GO TO 51

C 98

GO TO 52

C 99

GO TO 53

C 100

GO TO 54

C 101

GO TO 55

C 102

GO TO 56

C 103

GO TO 57

C 104

GO TO 58

C 105

GO TO 59

C 106

GO TO 60

C 107

GO TO 61

C 108

GO TO 62

C 109

GO TO 63

C 110

GO TO 64

C 111

GO TO 65

C 112
AEDC-TR-73-52

**FuTRAN IV G LEVEL 20**

**BEGIN**

**DATE = 72286 22/52/18**

```
0095      DM(1)=0.
0096      DO 25 I=3,NP2
0097      25   DM(I)=DM(I-1)+.5*(RHO(I)*U(I)*R(I)+RHO(I-1)*U(I-1)*R(I-1))+Y(I)*Y(I)-V

0098      PEI=DM(NP2)
0099      DO 26 I=3,NP1
0100      26   DM(I)=DM(1)/PEI
0101      OMIN(NP3)=1.
0102      IF INEQ.EQ.1 RETURN
0103      DO 27 J=1,NPH
0104      27   IF (KEQ.EQ.1) INDE(J)=1
0105      IF (KEQ.EQ.1) INDI(J)=1
0106      CONTINUE
0107      RETURN
0108      FORMAT(611,3)
0109      FORMAT(8E10.0)
0110      END
0111
```

END
```
<table>
<thead>
<tr>
<th>FORTRAN IV LEVEL 20</th>
<th>COEFF</th>
<th>DATE</th>
<th>2d/52/18</th>
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<tbody>
<tr>
<td>0001</td>
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<tr>
<td>0002</td>
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<tr>
<td>COMMON GEN/PE1,AM1,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1XL,OX,INTG,CALIFA,ALPHA,XP,REUK,GAY,ZETA,PPDO,TWTO,YSTART,USUP,IDI</td>
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<tr>
<td>0003</td>
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<tr>
<td>COMMON L/ALPHA,AK,ALMG</td>
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<tr>
<td>0004</td>
<td></td>
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<tr>
<td>DIMENSION G1(200),G2(200),G3(200),D1(2,200),S1(200),S2(200),S3(200)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
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<tr>
<td>CALCULATION OF SMALL C'S</td>
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<tr>
<td>0005</td>
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<tr>
<td>DO 1 I=2,NP1</td>
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<tr>
<td>0006</td>
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<tr>
<td>RA=5*(R(I+1)+R(I))</td>
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<tr>
<td>0007</td>
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<tr>
<td>KH=5*(RH0(I+1)+RH0(I))</td>
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<td>0008</td>
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<tr>
<td>UM=5*(U(I)+U(I))</td>
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<tr>
<td>0009</td>
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<tr>
<td>CALL VEFF (I,I+1,FMU)</td>
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<tr>
<td>010</td>
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<tr>
<td>SC(I)=RA<em>K</em>L<em>HUM</em>EMU/(PE1*PE1)</td>
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<td>011</td>
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<tr>
<td>THE CONVECTION TERM</td>
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<td>012</td>
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<tr>
<td>S=5*(R(I)+R(I-1))*AM1/PEI</td>
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<td>013</td>
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<td>DX=DX-DU</td>
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<td>014</td>
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<td>DO 4 I=3,NP1</td>
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<tr>
<td>015</td>
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<tr>
<td>Q=Q+UM(I-1)-OM(I-1)</td>
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<tr>
<td>016</td>
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<tr>
<td>P2=2.5/0X</td>
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<tr>
<td>017</td>
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<tr>
<td>P3=P2/OUM</td>
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<tr>
<td>018</td>
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<tr>
<td>P=UM(I)+OM(I+1)-P3</td>
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<tr>
<td>019</td>
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<tr>
<td>P3=OM(I)+OM(I-1)-P3</td>
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<tr>
<td>020</td>
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<tr>
<td>P2=2.5*P2</td>
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<tr>
<td>021</td>
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<tr>
<td>Q=SA/OUM</td>
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<tr>
<td>022</td>
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<tr>
<td>R2=SA+25</td>
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<tr>
<td>023</td>
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<tr>
<td>R3=P2/OUM</td>
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<td>024</td>
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<tr>
<td>R1=UM(I+1)+3.0*OM(I+1)*R3</td>
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<td>025</td>
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<tr>
<td>R3=OM(I+1)+3.0*OM(I+1)*R3</td>
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<td>026</td>
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<tr>
<td>G(I)I=PI*QVR</td>
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<td>027</td>
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<tr>
<td>G2(I)=P2*H2</td>
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<td>028</td>
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<tr>
<td>G3(I)=P3*JH3</td>
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<td>029</td>
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<tr>
<td>CUI(I)=P(I)+U(I+1)-P2<em>U(I)-P3</em>U(I-1)</td>
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<td>030</td>
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<tr>
<td>THE DIFFUSION TERM</td>
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<td>031</td>
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<tr>
<td>BUI(I)=SC(I-1)+AU(I)+OM(I)+OM(I-1)</td>
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<tr>
<td>032</td>
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<tr>
<td>AU(I)=SC(I)+AU(I)+OM(I)+OM(I-1)</td>
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<td>033</td>
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<tr>
<td>IF (NEQ.EQ.11) GO TO 3</td>
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<tr>
<td>034</td>
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<tr>
<td>DO 2 J=1,NPH</td>
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<td>035</td>
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<tr>
<td>C(J,I)=P(F(J,1)+2<em>F(J,1)+P3</em>F(J,1)-1)</td>
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<tr>
<td>036</td>
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<tr>
<td>CALL SOURCE (J,I,CS*DIJI))</td>
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<tr>
<td>037</td>
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<tr>
<td>C(J,I)=C(J,I)+CS*F(J,1)+D(J,I)</td>
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<tr>
<td>038</td>
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<tr>
<td>A(J,I)=AU(I)+PREF(I)</td>
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<tr>
<td>039</td>
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<tr>
<td>B(J,I)=BU(I)+PREF(I)</td>
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<tr>
<td>040</td>
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<td>CONTINUE</td>
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<tr>
<td>041</td>
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<tr>
<td>SOURCE TERM FOR VELOCITY EQUATION</td>
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<tr>
<td>042</td>
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<tr>
<td>S2(I)=P2*S(I)(1)+RHO(I)*U(I)</td>
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<tr>
<td>043</td>
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<tr>
<td>S3(I)=P3*S(11)(1)+KOH(I)-11-U(I-1)</td>
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<tr>
<td>044</td>
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<tr>
<td>S1(I)=P1*S(I)(1)+RHO(I+1)*U(I+1)</td>
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<tr>
<td>045</td>
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<tr>
<td>CUI=CU(I)-2.0*(S(I)+S2(I)+S3(I))</td>
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<tr>
<td>046</td>
<td></td>
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<tr>
<td>S1(I)=S1(I)+U(I+1)</td>
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<table>
<thead>
<tr>
<th>FORTRAN IV G LEVEL</th>
<th>20</th>
<th>COEFF</th>
<th>DATE = T2286</th>
<th>22/52/18</th>
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<tr>
<td>0047</td>
<td></td>
<td>S2(I) = S2(1)/U(I)</td>
<td>D 57</td>
<td></td>
</tr>
<tr>
<td>0048</td>
<td></td>
<td>S3(I) = S3(1)/U(I-1)</td>
<td>D 58</td>
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</tr>
<tr>
<td>0049</td>
<td></td>
<td>CONTINUE</td>
<td>D 59</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>COEFFICIENTS IN THE FINAL FORM</td>
<td>D 60</td>
<td></td>
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<tr>
<td>0050</td>
<td>5</td>
<td>I = 1, NPL</td>
<td>D 61</td>
<td></td>
</tr>
<tr>
<td>0051</td>
<td></td>
<td>RL = 1/(G2(I) + A(I) + B(I) - S2(I))</td>
<td>D 62</td>
<td></td>
</tr>
<tr>
<td>0052</td>
<td></td>
<td>A(I) = (A(I) + S1(I) - G1(I)) * RL</td>
<td>D 63</td>
<td></td>
</tr>
<tr>
<td>0053</td>
<td></td>
<td>B(I) = (A(I) + S3(I) - G3(I)) * RL</td>
<td>D 64</td>
<td></td>
</tr>
<tr>
<td>0054</td>
<td></td>
<td>C(I) = C(I) * RL</td>
<td>D 65</td>
<td></td>
</tr>
<tr>
<td>0055</td>
<td></td>
<td>IF (NEW &lt; EQ. 1) GO TO 7</td>
<td>D 66</td>
<td></td>
</tr>
<tr>
<td>0056</td>
<td></td>
<td>DO 6 J = 1, NPH</td>
<td>D 67</td>
<td></td>
</tr>
<tr>
<td>0057</td>
<td></td>
<td>DO 6 I = 1, NPL</td>
<td>D 68</td>
<td></td>
</tr>
<tr>
<td>0058</td>
<td></td>
<td>RL = 1/(G2(I) + A(J, I) + B(J, I) - D(J, I))</td>
<td>D 69</td>
<td></td>
</tr>
<tr>
<td>0059</td>
<td></td>
<td>A(J, I) = (A(J, I) - G1(I)) * RL</td>
<td>D 70</td>
<td></td>
</tr>
<tr>
<td>0060</td>
<td></td>
<td>B(J, I) = (B(J, I) - G3(I)) * RL</td>
<td>D 71</td>
<td></td>
</tr>
<tr>
<td>0061</td>
<td></td>
<td>C(J, I) = C(J, I) * RL</td>
<td>D 72</td>
<td></td>
</tr>
<tr>
<td>0062</td>
<td></td>
<td>CALL SLIP</td>
<td>D 73</td>
<td></td>
</tr>
<tr>
<td>0063</td>
<td></td>
<td>RETURN</td>
<td>D 74</td>
<td></td>
</tr>
<tr>
<td>0064</td>
<td></td>
<td>END</td>
<td>D 75</td>
<td></td>
</tr>
</tbody>
</table>
001 SUBROUTINE READY
002 COMMON /GEN/ PE1,AM1,AM6,OPDX,PREF2,P1,P(2),P(2),DEN,AMU,XU,XP,
1X,D/L,INTG,CSALFA,ALPHA,XR,REDH,GM,ETA,PPD,INTD,YSTART,USUP,IDI
2MEN,HEAT,T,T,XSTEP/V/U(200),F1(2),F2(200),R2(200),R2(200),OM(200),R(2)
300I/1,NP1,NP2,NP3,PH,NNH,NNP,NNP,NNP,KIN,KASE,KRAD/6/BETA,GAMA/2/,TAU,
4TAUE,AJ1(2),AJE1(2),IND1(2),IND2(2)
003 CALL DENSTY
004 CALL RAD (XU,R(1),CSALFA)
005 GO TO (1,2,3), KIN
006 1 Y(2)=1/(1+BETA)*OM(3)+2/(1+BETA)*OM(1)+U(2) U(1)
007 GO TO 4
008 2 Y(2)=1/(1+BETA)*OM(3)+2/(1+BETA)*OM(1)+U(2)+U(1)
009 GO TO 4
010 3 Y(2)=1/(1+BETA)*OM(3)+2/(1+BETA)*OM(1)+U(2)
011 GO TO 9
012 4 Y(NP2)=Y(NP1)+2/(1+BETA)*OM(NP1)+U(NP1)+U(NP2)
013 GO TO 9
014 5 Y(NP2)=Y(NP1)+2/(1+BETA)*OM(NP1)+U(NP1)+U(NP2)
015 GO TO 9
016 6 Y(NP3)=Y(NP2)+2/(1+BETA)*OM(NP2)+U(NP2)
017 GO TO 9
018 7 Y(NP3)=Y(NP2)+2/(1+BETA)*OM(NP2)+U(NP2)
019 GO TO 9
020 8 Y(NP3)=Y(NP2)+2/(1+BETA)*OM(NP2)+U(NP2)
021 IF (CSALFA.EQ.0) OR (KRED.GT.0) GO TO 11
022 IF (XXI.LT.0) GO TO 14
023 XXI=R(11)-P(11)-Y(11)*CSALFA
024 IF (XXI.LT.0) GO TO 14
025 IF (XXI.LT.0) GO TO 14
026 IF (V(2).LT.0) IF (CSALFA.LT.0) RETURN
027 GO TO 13
028 11 DO 13 I=2,NP3
029 12 Y(11)=PE1*Y(11)
030 13 Y(2)=2*(Y(11)-Y(11))
031 Y(NP2)=2.*YNP2-Y(NP1)
032 IF (V(2).LT.0) GO TO 14
033 DO 14 I=2,NP3
034 IF (KRED.GT.0) R(11)=Y(11)*CSALFA
035 IF (KRED.GT.0) R(11)=Y(11)*CSALFA
036 IF (V(2).LT.0) X0=2.*X0
037 IF (V(2).LT.0) X0=2.*X0
038 RETURN
039 END
SUBROUTINE DENSTV

COMMON /GEN/ PE, AI, AM, DPDX, PREF(2), PR(2), P2(2), DEN, AMU, XU, XG, XP,
1XL, DX, INTG, CSALFA, ALPHA, XR, RECRS, GAM, ZETA, PPO, TWU, YSTART, USUP, IDI
2FN, IHEAT, Z, TO, XSTEP/1/1(200), F(2, 200), R(200), RHO(200), T(200), TM(200), V(200)
3001/1/N, NP1, NP2, NP3, NEG, NPH, KEK, KIN, KASE, KRAD

RN3=F11, NP3=U(NP3)**2/2.

DO 3 NP3=1,NP3
3 EU=NP3**1/(GAM-1.*)

RETURN

END

THE FOLLOWING AME IS FOR LAMINAR FLOW

IF (INTG, NE, 1) GO TO 5

IF (M11, GT, 0.9)  N9=1
IF (M11, LT, 0.9)  I=NP3

CONTINUE

DOM=OM( V)$$+3M(N9-1)

CONTINUE

U9=U(h9)-0UM*5/0UM*1(U<N9l-U(N9-1))

TERM=1=(1MN9)-U(N9-1))/Lt'.1

TERMA = G'5*(U(N9)-U9)-G6*U9-UIN9-1)

TERMA=TERMA/TERMB

U9W=US*0.5*SORTT(2.)*SORTT*PPD**((GAM-1.*)/GAMI)

USUP IS READ-IN IN BEGIN, IT SUPPRESSES THE BL.

RETURN

END

RETURN
SUBROUTINE FBC (X, J, IND, AJFS)

COMMON /GFN/ PFI, AMI, AME, DPDX, PREF(2), PR(2), P(2), DEN, AMU, XU, XD, XP,
1XL, DX, INTG, CSALFA, ALPHA, XR, RKDS, GAM, ZETA, PPO, TTO, YSTART, USUP, ID1
2MEN, IHEAT, IT, TU, XSTEP

* TW IS PRESCRIBED IF IHEAT = 1 -- QODY IS PRESCRIBED IF NOT 1

IND = 1
AJFS = TTO
IF (IHEAT .EQ. 1) GO TO 1
IND = 2
AJFS = 0.0
1 CONTINUE
RETURN
END

SUBROUTINE MASS (XU, XD, AM)

APPLICABLE TO AN IMPERMEABLE-WALL SITUATION

AM = 0.0
RETURN
END
**FURTRAN IV G LEVEL 20**

### OUTPUT

**DATE** = 72286

22/52/18

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
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<tbody>
<tr>
<td>0001</td>
<td>SUBROUTINE OUTPUT</td>
</tr>
<tr>
<td>0002</td>
<td>COMMON /GEN/ PE1,A1,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XP,</td>
</tr>
<tr>
<td>0003</td>
<td>1XL,DX,INTG,CSALFA,ALPHA,XX,REDS,GAM,ZETA,PPU,TNTU,START,USUP,IDI</td>
</tr>
<tr>
<td>0004</td>
<td>2MEN,IHEAT,Z,TO,XSTEP/ /U(200),E(2),R(2),RHO(200),OM(200),X(2)</td>
</tr>
<tr>
<td>0005</td>
<td>300)C/ SC(200),AU(200),R(200),CIU(200),AI(2),R(2),C(2),C(200)/D</td>
</tr>
<tr>
<td>0006</td>
<td>4/Y(200),URK(200),HR(200),XK(200),PITD(200),TEMP(200)/ES</td>
</tr>
<tr>
<td>0007</td>
<td>5AR(300),XRS(300),RWS(300),CSAL(300)/N,NP1,NP2,NP3,NEQ,NPH,KEX</td>
</tr>
<tr>
<td>0008</td>
<td>6,KIN,KASE,KRAD/BETA,GAMMA/2,TAU,TAU/AJ(2),AJE(2),INDI(2),IMDE</td>
</tr>
<tr>
<td>0009</td>
<td>7(2)</td>
</tr>
<tr>
<td>0010</td>
<td>IF (INTG-NE.1) GO TO 1</td>
</tr>
<tr>
<td>0011</td>
<td>ALPHA**80./3'.</td>
</tr>
<tr>
<td>0012</td>
<td>KPITE(6,7)</td>
</tr>
<tr>
<td>0013</td>
<td>WRITE (6,6)</td>
</tr>
<tr>
<td>0014</td>
<td>IF (KRP,Eq.0) DSTAK(PIGI=Y(NP3)-PE1/R(1)*RHO(NP3)*U(NP3))</td>
</tr>
<tr>
<td>0015</td>
<td>IF (KRP,Eq.0) GO TO 2</td>
</tr>
<tr>
<td>0016</td>
<td>DSTAK(PIGI=SORR(1)**2-2.<em>CSALFA</em>(R(1)**2-0.5<em>CSALFA</em>(NP3)**2-PE1/RMCINP3)*U(NP3))</td>
</tr>
<tr>
<td>0017</td>
<td>IF (CSALFA-NE.0) DSTARIHINTG/DSTARIHINTO/CSALFA</td>
</tr>
<tr>
<td>0018</td>
<td>IF (FLOATINTG.INTG-1)/(FLOATINTG.INTG-1/2)) RETURN</td>
</tr>
<tr>
<td>0019</td>
<td>IF (XMU.NE.0) LM2=SORT(2.I)</td>
</tr>
<tr>
<td>0020</td>
<td>WRITE (6,8)</td>
</tr>
<tr>
<td>0021</td>
<td>WRJTE (6,9)</td>
</tr>
<tr>
<td>0022</td>
<td>WRITE (16,121)</td>
</tr>
<tr>
<td>0023</td>
<td>WRITE (16,131)</td>
</tr>
<tr>
<td>0024</td>
<td>WRITE (16,101)</td>
</tr>
<tr>
<td>0025</td>
<td>DO 3</td>
</tr>
<tr>
<td>0026</td>
<td>DO 4</td>
</tr>
<tr>
<td>0027</td>
<td>DO 5</td>
</tr>
<tr>
<td>0028</td>
<td>DO 6</td>
</tr>
<tr>
<td>0029</td>
<td>DO 7</td>
</tr>
<tr>
<td>0030</td>
<td>DO 8</td>
</tr>
<tr>
<td>0031</td>
<td>WRITE (6,8)</td>
</tr>
<tr>
<td>0032</td>
<td>WRITE (6,9)</td>
</tr>
<tr>
<td>0033</td>
<td>WRITE (6,9)</td>
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<tr>
<td>0034</td>
<td>WRITE (6,9)</td>
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<td>WRITE (6,9)</td>
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<tr>
<td>0037</td>
<td>WRITE (6,9)</td>
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<tr>
<td>0038</td>
<td>WRITE (6,9)</td>
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<tr>
<td>0039</td>
<td>WRITE (6,9)</td>
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<tr>
<td>0040</td>
<td>WRITE (6,9)</td>
</tr>
<tr>
<td>0041</td>
<td>WRITE (6,9)</td>
</tr>
<tr>
<td>0042</td>
<td>WRITE (6,9)</td>
</tr>
</tbody>
</table>

FORTRAN IV G LEVEL 20 OUTPUT DATE = 72286 22/52/18

0043 FORMAT(24H1THE VALUES OF OMEGA ARE/1P10E11.4) J 57
0044 FORMAT(1L4X,1NTG*9X,XU*11X,2*9X,REURS*7X,GAMMA*8X,PR* J 58
2*9X,TAUX*5X,TW*9X,ALPHA*8X,XR*10X,7X*5X,PR3*9X,D5* J 59
2*TAP*8X,PE1*9X,AME*8X,DPDX*6X,DX*8X,COSALF*7X,US*US*8X J 60

0045 FORMAT(IX,Y*9X,H/H0,9X,'U/UE',RX,'U/UM',9X,'R',10X,'RHO',10X) J 61
0046 FORMAT(1P10E12.5,) J 62
0047 FORMAT(1P10E12.5,) J 63
0048 FORMAT(1P10E12.4) J 64
0049 FORMAT(1P10E12.4,) J 65
0050 FORMAT(1P10E12.4,) J 66
0051 END

FORTRAN IV G LEVEL 20 PRE DATE = 72286 22/52/18

0001 SUBROUTINE PRE (X,DPOXX) K 1
0002 COMMON /GEM/ PEI,AM1,AME,DPOXI,PREF2,PZ,PZ,DEN,AMU,XU,XD,XP,K 2
1*XU,DX,INTG,CSalfa,ALPHA,XR,REURS,GAM,ETE,PO,TND,TSHOT,USUP,IDI,K 3
0003 DIMENSION XX(300), POP(300) K 5
0004 IF (INTG.NE.0) GO TO 1 K 6
0005 READ (5,3) LMAX K 7
0006 READ (5,4) (XXI,LPOP(L),L,LMAX) K 8
0007 WRITE (6,5) K 9
0008 WRITE (6,6) (XXI,LPOP(L),L,LMAX) K 10
0009 1 CONTINUE K 11
0010 L=1 K 12
0011 2 CONTINUE K 13
0012 L=L+1 K 14
0013 IF (XX(L).LT.X) GO TO 2 K 15
0014 DPOXX=(PDP(L)-POPOP(L-1))/(XX(L-XX(L-1))) K 16
0015 POP=POP(L-1)+DPOXX*(XX(L-1)) K 17
0016 DPOXX=DPOXX K 18
0017 DPOXX=DPOXX*(GAM+1)/GAM K 19
0018 RETURN K 20
0019 C CONTINUE K 21
0020 3 FORMAT(13) K 22
0021 4 FORMAT(2EI2.0) K 23
0022 5 FORMAT(1HE/9X,'X',12X,'P/P0',/) K 24
0023 6 FORMAT(1PE12.5,16I) K 25
0024 END K 26

END
SUBROUTINE RAD (X,YR,CALPHA)

COMMON /GEN/ PEI,AMI,AMC,DPDX,PREF(2),PH(2),P(2),DEN,AMU,XU,XD,XP,
1XL,DX,INTG,CSALFA,ALPHA,XR,REORS,GAM,ZETA,PPU,TIME,YSTART,USUP,IDI
300/I/N,P1P2,NP3,NEQ,NPH,KX,KIN,KASE,KRAAD

C APPLICABLE TO NOZZLES WITH CONSTANT LONGITUDINAL RADIUS OF
C CURVATURE OF THE CONVERGING SECTION- CONSTANT WALL HALF ANGLE OF
C DIVerging SECTION- AND WALL SLOPES MATCHED DOWNSTREAM OF THE THROAT

0003 IF (INTGNE.O) GO TO 1

0004 PI2=3.141592652

0005 CSALF=COS(ALPHA)

0006 SINALF=SIN(ALPHA)

0008 ZWIG*X=(1.+SINALF)

0009 WIG=X*(PI2+ALPHA)

0010 RIG=1.*X*K(1.-CSALF)

0011 CONTINUE

0012 IF (XGE.ZWIG) GO TO 2

0013 X(1)=1.*XR*(1.-SIN(X/XR))

0014 CSALFA=SIN(X/XR)

0015 CALPHA=CSALFA

0016 K=K(1)

0017 ZZ=X*K(1.-COS(X/XR))

0018 GO TO 3

0019 CONTINUE

0020 P(1)=(X-ZWIG)*SINALF*RIG

0021 CSALFA=COSALF

0022 CALPHA=CSALFA

0023 P=K(1)

0024 ZZ=(X-ZWIG)*CSALFA*ZWIG

0025 CONTINUE

0026 I=IODEN,EO)R]=1.

0027 ZZ=ZZ-XR

0028 Z=ZZ

0029 RETURN

0030 END
<table>
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<tr>
<th>FORTRAN IV G LEVEL</th>
<th>SLIP</th>
<th>DATE = 72286</th>
<th>22/52/18</th>
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<tr>
<td>0001</td>
<td>SUBROUTINE SLIP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0002</td>
<td>COMMON /GEN, PFI, AMI, AME, DPDX, PREF(2), PR(2), P(2), DEN, AMU, XU, XO, XP,</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1XL, DX, NTG, CSGALF, ALPHA, XR, KEPS, GAM, ZETA, PO, TMTO, YSTART, USUP, IDI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0003</td>
<td>COMMON /L, AK, AMG/C, STC(200), AMU(200), BJU(200), CUB(200), AQ(2, 200), B(2,</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2001), C(2, 200)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0004</td>
<td>SLIP COEFFICIENTS NEAR THE E BOUNDARY FOR VELOCITY EQUATION</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0005</td>
<td>C(U(NP2)) = 0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0006</td>
<td>GO TO (1, 2, 3), KIN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0007</td>
<td>BU(NP2) = 0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0008</td>
<td>AUK2 = 1./(1 + 2 * BETA)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0009</td>
<td>GO TO 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0010</td>
<td>SQ = B(4) * U(NP2) * U(NP2) * U(NP2) * U(NP2) * U(NP2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0011</td>
<td>B(2) = 6. * (U(NP2) + U(NP2)) * (U(NP2) + U(NP2)) * (U(NP2) + U(NP2))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0012</td>
<td>AUK1 = 1. - U(NP2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0013</td>
<td>GO TO 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0014</td>
<td>BU(NP2) = 0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0015</td>
<td>CALL VEFF(B(2), B(2))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0016</td>
<td>AK1 = 1./DPDX/(RHO(1) * U(1))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0017</td>
<td>AK2 = -U(1) * AK1 * DPDX/(RHO(1) * U(1))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0018</td>
<td>AJ = RHO(1) * U(1) * B(2) * Y(2) * Y(3) * Y(3) * Y(3) * Y(3) * Y(3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0019</td>
<td>IF (KRAD.EQ.0) GO TO 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0020</td>
<td>AUK1 = 1./ (2 + 2 * A)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0021</td>
<td>C(U(NP2)) = 1. - 5 * AJ = AK1 * AUK1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0022</td>
<td>GO TO 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0023</td>
<td>B(2) = 0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0024</td>
<td>C(U(NP2)) = 1./ (2 + 2 * A)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0025</td>
<td>C(U(NP2)) = C(U(NP2)) + 4 * A * AK2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0026</td>
<td>GO TO 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0027</td>
<td>B(2) = 0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0028</td>
<td>CALL VEFF(NP1, NP2, ENU)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0029</td>
<td>B(2) = U(NP2) * B(2) + U(NP2) * B(2) + U(NP2) * B(2) + U(NP2) * B(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0030</td>
<td>SQ = B(4) * U(NP2) * U(NP2) * U(NP2) * U(NP2) * U(NP2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0031</td>
<td>B(2) = 6. * (U(NP2) + U(NP2)) * (U(NP2) + U(NP2)) * (U(NP2) + U(NP2)) * (U(NP2) + U(NP2))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0032</td>
<td>B(2) = C(U(NP2))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0033</td>
<td>GO TO 9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0034</td>
<td>A(U(NP2)) = 0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0035</td>
<td>CALL VEFF(NP1, NP2, ENU)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0036</td>
<td>B(2) = U(NP2) * B(2) + U(NP2) * B(2) + U(NP2) * B(2) + U(NP2) * B(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0037</td>
<td>B(2) = U(NP2) * B(2) + U(NP2) * B(2) + U(NP2) * B(2) + U(NP2) * B(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0038</td>
<td>B(2) = U(NP2) * B(2) + U(NP2) * B(2) + U(NP2) * B(2) + U(NP2) * B(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0039</td>
<td>IF (INEQ.EQ.1) RETURN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0040</td>
<td>CALL FBC(XO, J, INDI(J), INDI)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0041</td>
<td>IF (INDI(J, EQ.1)) GO TO 1.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

63
SLIP
DATE = 72286
22/52/18

0049 AJ(J,J)=Q1
0050 AJ(J,J)=1.
0051 B(J,J)=0.
0052 1}=1.*BETA*PREF(J)*AJ(J)/(AK*AK*BETA*(1.+BETA)*1.+BE)
0053 GO TO 15
0054 11 FIJ,J,1)=Q1
0055 AJ(J,J)=1.*(BETA-GAMA(J))/(1.+BETA +GAMA(J))
0056 B(J,J)=1.-AJ(J,J)
0057 GO TO 15
0058 12 A(J,J)=U(1)*U(3)*8.*U(1)/U(2)+U(3)*8.*U(1)
0059 B(J,J)=0.+(1.+BETA-GAMA(J))/(1.+PREF(J))
0060 A(J,J)=B(J,J)+GF/(1.+B(J,J)+GF)
0061 B(J,J)=1.-AJ(J,J)
0062 GO TO 15
0063 CALL SOURCE(J,J,CS,DS)
0064 AK1=0./OX-OS
0065 AK2=AK1*(1.+CS)
0066 AJF=AJ*PREF(J)
0067 IF (KRAO.EQ.0) GO TO 14
0068 A(J,J)=Z./(2.+AJF*AK1)
0069 C(J,J)=AJF*AK2/AJF*AK1
0070 GO TO 15
0071 14 C(J,J)=1./(2.+3.*AJF*AK1)
0072 C(J,J)=C(J,J)*(2.+AJF*AK1)
0073 C(J,J)=C(J,J)*4.*AJF*AK2
0074 SLIP COEFFICIENTS NEAR THE E BOUNDARY FOR OTHER EQUATIONS
0075 GO TO (15,16,17, KEX)
0076 CALL FAC (XD,J,INDE(J),Q)
0077 IF (INDE(J).EQ.1) GO TO 17
0078 A(J,J)=QE
0079 B(J,NP2)=1.
0080 CALL SOURCE(J,J,CS,DS)
0081 C(J,NP2)=B(J,NP2)
0082 GO TO 20
0083 17 FIJ,NP3)=QE
0084 B(J,NP2)=1.*BETA-GAMA(J)/(1.+BETA +GAMA(J))
0085 A(J,NP2)=1.-B(J,NP2)
0086 GO TO 20
0087 18 B(J,NP2)=U(NP1)+U(NP1)*8.*U(NP1)/U(NP1)+U(NP1)+8.*U(NP3))
0088 GF=(1.-PREF(J))/(1.+PREF(J))
0089 B(J,NP2)=B(J,NP2)+GF/(1.+B(J,NP2)+GF)
0090 A(J,NP2)=1.-B(J,NP2)
0091 GO TO 20
0092 19 B(J,NP2)=0.
0093 CALL SOURCE(J,NP3,CS,DS)
0094 RK1=0./OX-OS
0095 SK2=SK1*F(J,NP3)-CS
0096 B(JF)=B(JF)*PREF(J)
0097 C(J,NP2)=1./(2.+3.*BJF*HK1)
0098 B(J,NP2)=C(J,NP2)*BJF*BJK1
0099 C(J,NP2)=C(J,NP2)*4.*BJF*BJK2
0100 CONTINUE
0101 RETURN
SUBROUTINE 'SOLVE' (A, 0, C, F, NP3)

C     THIS SOLVES EQUATIONS OF THE FORM
C     F(I) = A(I)*F(I+1) + B(I)*F(I-1) + C(I)

FOR I = 2, NP2

DIMENSION A(NP3), B(NP3), C(NP3)

BI2 = BI2 + FCT2

DO 1 I = 2, NP2

F(I) = A(I)*F(I-1) + BI2

1 RETURN

END

SUBROUTINE 'VEFF' (I, IP1, EMU)

COMMON /GEN/ PE1, AM1, AME, ODPX, PHEF(2), PR(2), P2, DEN, AMU, AX, XD, XP

1 AL = INT(J*X) + CSALFA + ALPHA + XR*REORS + GAM + ZETA*PPO + TWOT + YSTAW + USUP + UI

DO 2 J = 1, IP1

2 TI = TI + F(J)*A(I)*F(J)*F(J)/2.

RETURN

END

SUBROUTINE 'SOURCE' (J, CS, DS)

COMMON /GEN/ PEI, AMI, AME, ODPX, PHEF(2), PR(2), P2, DEN, AMU, AX, XD, XP

1 AL = INT(J*X) + CSALFA + ALPHA + XR*REORS + GAM + ZETA*PPO + TWOT + YSTAW + USUP + UI

DO 2 J = 1, IP1

2 TI = TI + F(J)*A(I)*F(J)*F(J)/2.

RETURN

END
FUNCTION VISCOS (I)  
COMMON /GEN/ PE1, AM1, AME, DPDX, PREF(2), PR(2), P(2), DEN, AMU, XU, XD, XP,  
IXL, DX, INTG, CSALFA, ALPHA, XR, REKS, GAM, ZETA, PPO, TINTO, YSTART, USUP, IDI  
2MEI, HFRAT, Z, TO, XSTEP, V/U(200), F(2), Z(200), R(200), RH(200), DM(200), Y(2)  
3001/I/N/PH1/NPH2/NPH3/NEQ/NPH, KEX, KIN, KASE, KRAD  
RETURN  
END  

SUBROUTINE WALL  
COMMON /GFN/ PE1, AM1, AME, DPDX, PREF(2), PR(2), P(2), DEN, AMU, XU, XD, XP,  
IXL, DX, INTG, CSALFA, ALPHA, XR, REKS, GAM, ZETA, PPO, TINTO, YSTART, USUP, IDI  
2MEI, HFRAT, Z, TO, XSTEP, V/U(200), F(2), Z(200), R(200), RH(200), DM(200), Y(2)  
3001/I/N/PH1/NPH2/NPH3/NEQ/NPH, KEX, KIN, KASE, KRAD  
RETURN  
END  

C** C** CALCULATION OF BETA FOR THE I BOUNDARY  
Y1=5*(Y(2)+Y(3))  
C    C FOR LAMINAR FLOW AND AM=0 (NEED DIFFERENT EXPRESSION IF F=0)  
S=1./RE-FP/2.  
TAU=6.*PH=U*U  
C    C FOR THE I BOUNDARY  
IF (NEQ.EQ.1) RETURN  
OD 2 J=1,NPH  
C FOR LAMINAR FLOW AND LIMITING ZERO AM  
SF=1./(PREF(J)*RE)  
GAM(J)=RE*PR(J)*(SF*AM)  
C    C FOR LAMINAR FLOW AND LIMITING ZERO AM  
IF (IND(J).EQ.1) AJI=SF*PH=U*(2.*F(2,J)+F(2,J))  
CONTINUE  
RETURN  
END
APPENDIX IV
INITIAL PRESSURE DISTRIBUTION COMPUTER CODE

11 C PROGRAM FOR INITIAL PRESSURE DISTRIBUTION
21 READ (10,14) GAMMA,XH,XSTEP,AACT,AEFF,IDIMEN
31 C GAMMA = RATIO OF SPECIFIC HEATS
41 C ALPHA IS DIVERGING NOZZLE WALL HALF ANGLE
51 C XH IS LONGITUDINAL RADIUS OF CURVATURE OF CONVERGING SECTION
61 C XSTEP IS STEP SIZE ALONG X COORDINATE (PERCENT OF LOCAL WALL RAD)
71 C AACT IS THE ACTUAL NOZZLE AREA RATIO
81 C AEFF IS THE EFFECTIVE NOZZLE AREA RATIO
91 C IDIMEN FOR TWO-DIMENSIONAL NOZZLE AND 1 FOR AXI-SYMMETRIC NOZZLE
101 C XWIG,ZWIG, AND RWIN ARE THE COORDINATES OF THE POINT WHERE THE
111 C NOZZLE WALL SLOPES ARE MATCHED JUST DOWNSTREAM OF THE THROAT
121 ALPH=ALPHA*3.14159/180
131 VUM=0
141 PI2=3.14159/2
151 COSALF=COS(ALPHA)
161 SINALF=SIN(ALPHA)
171 XWIG=XR*T2
181 XR=WIG=(PI2+ALPHA)
191 RWIN=1*XR*(1+COSALF)
201 XTHBAT=XR*PI2
211 REND=SQR(D(AACT))
221 IF (IDIMEN.EQ.0) REND=AACT
231 XEND=XWIG*(XSTEP-HWIN)/SINALF
241 IF (IDIMEN.EQ.0) CONST=AACT
251 IF (IDIMEN.EQ.0) CONST=AAC/(/COSALF*(XEND-XWIG)**1.5)
261 ALPH=ALPHA*180/3.14159
271 IF (IDIMEN.EQ.0) WRITE(108,18)
281 IF (IDIMEN.EQ.1) WRITE(108,19)
291 WRITE (108,15) ALPH,XR,XEND,XSTEP,AACT,AEFF
301 CI=(6+1)/2
311 C2=(6+1)/2
321 C3=CI/(U=1)
331 C4=C1+C3
341 C5=C4+C2+C3
351 FM=0.5
361 X=0.0
371 R=0.0
381 DS=0.0
391 CONTINUE
401 X=X+XSTEP
411 IF (X-XWIG)**2.2<=4
421 CONTINUE
431 2 CONTINUE
441 IF (.NOT.(X=LT.XTHBAT.AND.X.GT.XTHBAT)) GO TO 3
451 X=XTHBAT
461 3 CONTINUE
471 Z=X*PI2*COSALF(X/XR)
481 R+X*R**2+2=SINF(Y/XR)
491 R=INV/R
501 COSA=SINF(X/XH)
511 GO TO 5
521 4 Z=(X-XWIG)*COSALF+ZWIG
531 R=(X-XWIG)*SINALF+ZWIG
541 COSA=COSALF

67
55: DS=CONST*(X*HIG)**14
56: IF (IDIMNEW>0) DS=CONST*(X*HIG)
57: RINV=DS=CUSALF
58: 5 A=(1/RINV)**(1.IDIMN)
59: IF (A=1.0) 7,8,9
60: 6 A=1.0
61: FM=1+0
62: FM=1+0
63: IF (X=PI2*XH) 11,11,9
64: B=(1+C2+C2)**2
65: FM=FM1=(A***(C3+1))-C4*FM1)/(1+C5+FM1+C4*C4)
66: IF (ABS(FM-FM1)<0.00001) 12,11,11
67: CONTINUE
68: IF (FM=1.+20) 10,11,11
69: FM=1+20
70: CONTINUE
71: FM=FM
72: FM=FM
73: GO TO 8
74: CONTINUE
75: WRITE (105,16) FM,PAP,Z*X,DS,XP,IINV,NUM
76: WRITE (105,17) FM,PAP,Z*X,DS,XP,IINV,NUM
77: IF (X=XEND) 1,1,13
78: CONTINUE
79: STOP
80: 13
81: STOP
82: 12
83: 11
84: 10
85: 9
86: 8
87: 7
88: 6
89: 5
90: 4
91: 3
92: 2
93: 1
94: ALPHA = 2*F7.3/10X; 
SR = 2*F7.3/10X; 
XEND = 2*F7.3/10X; 
XSTEP = 2*F7.3/10X; 
XSTEPS = 2*F7.3/10X; 
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**Viscous Effects in Low-Density Nozzle Flows**

**March 9 to November 30, 1972—Final Report**

**David L. Whitfield, ARO, Inc.**

Viscous effects in low-density nozzle flows were investigated numerically, and comparisons were made with experimental data. The numerical method of Patankar and Spalding was modified to solve the internal laminar boundary-layer equations for two-dimensional flow or axisymmetric flow with or without transverse curvature. A listing is given of the computer code. Boundary-layer displacement thicknesses for typical nozzle geometries and flow conditions are presented. Solutions were obtained for specific conditions corresponding to experimental data. The result is a relatively fast, simple to use numerical procedure, which is shown to give results in good agreement with experimental data.
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