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THE EFFECTS OF TARGET LOCATION UNCER-
TAINTY UPON WEAPON SYSTEM EVALUATION

Terry L. DePhillips

Naval Postgraduate School
Monterey, California

March 1973

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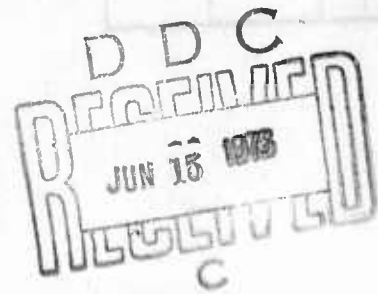
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NAVAL POSTGRADUATE SCHOOL

Monterey, California



THESIS

THE EFFECTS OF TARGET LOCATION UNCERTAINTY
UPON WEAPON SYSTEM EVALUATION

by

Terry Lee DePhillips

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March 1973

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The Effects of Target Location Uncertainty
Upon Weapon System Evaluation

by

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Submitted in partial fulfillment of the
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ABSTRACT

This thesis examines the influence of target location error upon small arms weapons system evaluation. The adequacy of the diffuse target approximation is examined by comparison with tabulated results for a salvo of N-rounds of small arms fire.

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TABLE OF SYMBOLS AND ABBREVIATIONS

<u>SYMBOL</u>	<u>DEFINITION</u>
a	Lethal radius of fragmentation projectile
$f(X_B, Y_B; \bar{x}_b, \bar{y}_b)$	Probability density function of ballistic error
$f(X_A, Y_A; x_a, y_a)$	Probability density function of aiming error
$f(X_L, Y_L; x_e, y_e)$	Probability density function of target location error
$\ell(X_I, Y_I; X_B, Y_B)$	Lethality function
N	Number of projectiles
$P_K(N)$	Salvo kill probability

I. INTRODUCTION

A. BACKGROUND

The trend in small arms development has been away from cumbersome, large caliber, slow firing weapons to light-weight, small caliber, rapid firing weapons. Sir Basil Liddell Hart [18] stresses the roles played by deception and mobility in all warfare, both ancient and modern. The decisive roles played by deception and mobility have made target detection (location) an extremely important variable to consider when evaluating and hence selecting a small arms weapon system.

In this thesis the influence of target location error upon several common measures of effectiveness (MOE's) used in small arms weapon system evaluation is examined. The adequacy of the diffuse target approximation [7] is also examined by comparison with tabulated results for salvo-firing of N-rounds of small arms ammunition against a square target.

B. MEASURES OF EFFECTIVENESS (MOE)

In view of the large number of small arms weapon systems available to the armed services today, an acceptable and practical set of criteria for comparing the overall combat effectiveness of these various systems is of great importance. The following measures were approached in this thesis: (1) hit probability, (2) lethality, (3) kill probability, and

(4) rate of fire. These measures were selected because of their wide usage and ease of representation by analytical models. References 3 and 13 are recommended to those interested in additional measures sometimes used. These references also contain a short discussion of the problems encountered when trying to decide which measures are important.

C. PURPOSE

The purpose of this thesis is to develop a means of effectively using the measures listed earlier when comparing various small arms weapon systems.

D. APPROACH

The approach taken in this thesis is to incorporate target location error into the various analytical expressions used with the measures listed earlier. The effects of location error upon these measures is then examined to obtain meaningful answers to the following questions:

1. When is it desirable to sacrifice accuracy for an increase in the rate of fire?
2. When is it desirable to use ball ammunition, shotgun ammunition, or grenades?
3. Which system best incapacitates the target?
4. Which systems provide the best suppressive fires?

Target location error was selected to permit examination of the survivability of the individual foot soldier under both static and dynamic conditions. What type weapon system

is most desirable when very little is known about the true target's location? What good is accuracy if a soldier doesn't know where to fire his rounds or is so pinned down by the enemy's superior fire power he doesn't have time to get a fix on the target?

D. THE FOOT SOLDIER SYSTEM

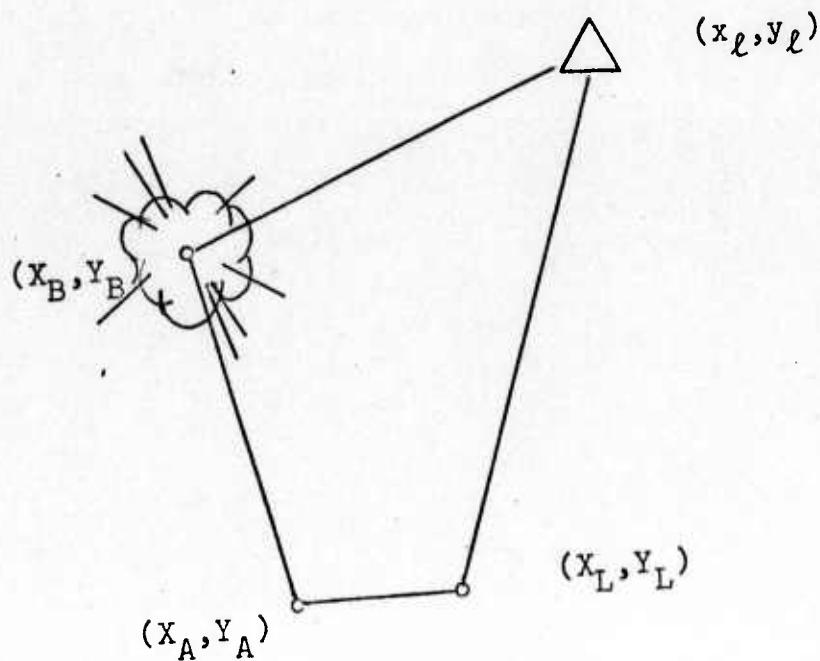
The individual foot soldier is required to both acquire and engage, with small arms fire, enemy ground targets. Under target acquisition he is responsible for target detection (the determination of the existence or presence of a target), target identification (the determination of the nature and composition of the target), and target location (the determination of the coordinates of the target). Of course, during the engagement phase he must successfully neutralize the target.

If these combined processes could be carried out with zero error, every target would be precisely located and then destroyed. This, of course, is not the case as errors are present in both processes (see Figure 1, page 9). The soldier makes both a target location error and an aiming error. There are also errors in the trajectory of the projectile due to such variables as barrel wear, temperature, humidity, and wind.

E. SYSTEM ERRORS

This section is an adaption to small arms fire of errors considered by C.H. Hess [19] for artillery fire.





Definition

target location error
 aiming error
 ballistic error
 lethality

Distribution

$f(X_L, Y_L; x_l, y_l)$
 $f(X_A, Y_Z; x_a, y_a)$
 $f(X_B, Y_B; x_b, y_b)$
 $l(X_I, Y_I; X_B, Y_B)$

Figure 1. System Errors

1. Target Location Error

Targets to be engaged by small arms fire are usually located by the foot soldier on the ground. These targets are imprecisely located due to errors in target acquisition and to the fact that the target is usually a fleeting target. The target's suspected location (X_L, Y_L) is distributed with respect to the true target location (x_ℓ, y_ℓ) according to some probability density function, $f(X_L, Y_L; x_\ell, y_\ell)$.

2. Aiming Error

Because of errors in firing techniques and corrections for other variables, an aiming error results in a separation of the desired aim point (x_a, y_a) and the actual aim point (X_A, Y_A) . The actual aim point is distributed with respect to the desired aim point according to some probability density function, $f(X_A, Y_A; x_a, y_a)$. If there is no mean aiming error, then the expected aim point coincides with the expected target location (X_L, Y_L) .

3. Ballistic Error

The impact of N rounds is distributed about the mean center of impact according to some probability density function, $f(X_B, Y_B; \bar{x}_b, \bar{y}_b)$.

4. Lethality

The amount of damage done to a target by N -rounds is a function of the type target, the type ammunition fired, and the distance from the center of impact to the target. The lethality functions used in this thesis were selected

based on this knowledge and in all cases were treated as known analytic functions.

II. ONE DIMENSION

A. INTRODUCTION

The problem addressed is that of maximizing the probability of hitting and thus killing a sharply defined one dimensional target. The target was assumed to occupy a sharply defined portion of a one dimensional coordinate system. A round impacting within the target area produces the full effects of a hit, while a round impacting outside the target has little or no effect on the target.

The one dimension model is presented to convey the essential features of the models used and to provide a solid foundation for the more involved and complicated models used later in this thesis.

B. KNOWN TARGET LOCATION

The single shot hit and kill probabilities were calculated for fire delivered against a stationary target by assuming that the distribution of the impact points was known or could be obtained from experimental data. The single shot hit probability, P_{SSH} , is obtained by use of Equation (1) while Equation (2) is used to obtain the single shot kill probability, P_{SSK} .

$$P_{SSH} = \int_{-L}^L f_{X_I}(x)dx \quad (1)$$

$$P_{SSK} = \int \ell(X_I) f_{X_I}(x)dx \quad (2)$$

For both calculations, $f_{X_I}(x)$ is the density of the impact points while $\ell(X_I)$ is an appropriate lethality function. The lethality function was used to account for the lethality of the various types of small arms ammunition available.

1. Impact Point

The distribution of the impact points from the center of the target is, in many cases, normal and was assumed such for this thesis. The Central Limit Theorem provides a theoretical basis for this empirical fact, since many factors together cause a soldier to miss the intended aim point. The density used was thus

$$f_{X_I}(x) = \frac{1}{(2\pi)^{1/2}\sigma_I} \exp^{-1/2 \left\{ \frac{x-\mu_I}{\sigma_I} \right\}^2},$$

where μ_I is the expected impact point and σ_I is the measure of dispersion about μ_I .

2. Lethality Functions

The lethality function, defined as the Prob [kill target | round impacts at x] is introduced to provide a means of calculating P_{SSK} 's for all types of small arms ammunition available, both fragmenting and non-fragmenting.

For non-fragmenting ammunition the zero-one lethality function, denoted by $\ell_o(x)$, is used to reflect the fact that a hit is required in order to obtain a kill. Thus, for a one dimension target of dimension $2L$, the lethality function is

$$\ell_c(x) = \begin{cases} 1; & \text{if } x \in [-L, L] \\ 0; & \text{otherwise} . \end{cases}$$

Fragmenting ammunition, grenades and the like, does not necessarily need to impact on the target to kill it.

Fragmenting ammunition can kill a fragment sensitive target by throwing shrapnel on the target. The damage done by the fragmentation type round depends on the target location and the point of impact of the round. Figure 2, page 15, from BRL report 1544 displays, rather vividly, why the negative exponential function was chosen for this thesis. The lethality function is therefore

$$\ell_E(x) = \exp^{-\frac{1}{2}\left\{\frac{x}{a}\right\}^2}$$

where a is a shape parameter derived from a fit to experimental lethality data which has been independently determined from fragmentation tests or other techniques and is called the lethal radius. The variable x is the distance from the center of impact to the target center.

3. Non-Fragmenting Model

Under the above assumptions the single shot kill probability is determined by evaluation of the integral

$$P_{SSK} = \int_{-\infty}^{\infty} \ell_c(x) \frac{1}{(2\pi)^{\frac{1}{2}}\sigma_I} \exp^{-\frac{1}{2}\left\{\frac{x-\mu_I}{\sigma_I}\right\}^2} dx. \quad (3)$$

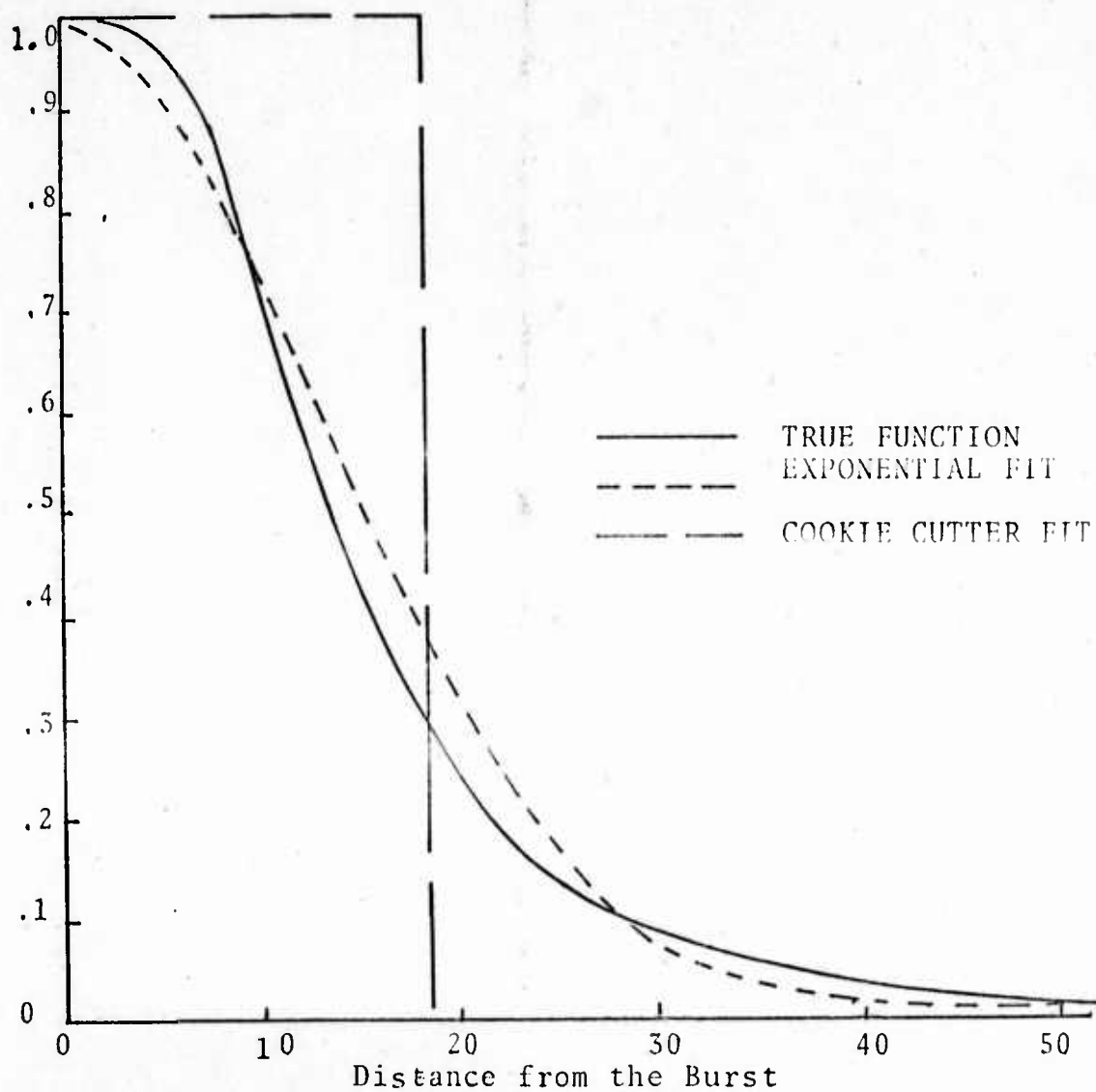


Figure 2. Round Lethality Function

Use of the zero-one lethality function reduces this evaluation to the interval $[-L, L]$ so that

$$P_{SSK} = \int_{-L}^L (1) \frac{1}{(2\pi)^{1/2} \sigma_I} \exp^{-\frac{1}{2} \left\{ \frac{x - \mu_I}{\sigma_I} \right\}^2} dx, \quad (4)$$

finally

$$P_{SSK} = \phi\left(\frac{L - \mu_I}{\sigma_I}\right) - \phi\left(\frac{-L - \mu_I}{\sigma_I}\right), \quad (5)$$

where

$$\phi(\mu_I) = \int_{-\infty}^{\mu_I} \frac{1}{(2\pi)^{1/2}} e^{-t^2/2} dt$$

is the cumulative distribution function for the standardized normal distribution.

4. Fragmenting Model

For this thesis all ammunition, other than ball ammunition, is considered fragmenting ammunition. Therefore,

$$P_{SSK} = \int_{-\infty}^{\infty} \ell_E(x) f_{X_I}(x) dx, \quad (6)$$

where

$$\ell_E(x) = \text{prob} [\text{kill target} | \text{round impacts at } x].$$

The single shot kill probability, P_{SSK} , is found to be

$$P_{SSK} = \frac{a}{(a^2 + \sigma_I^2)^{1/2}} \exp^{-\frac{1}{2} \left\{ \frac{\mu_I^2}{a^2 + \sigma_I^2} \right\}} \quad (7)$$

5. Maximizing P_{SSK}

An examination of the conditions under which P_{SSK} is maximized leads to an examination of the partial derivatives; $\frac{\partial P_{SSK}}{\partial \mu}$ and $\frac{\partial P_{SSK}}{\partial \sigma}$ [17]. The results of this examination are twofold:

1. P_{SSK} is always decreased when the expected impact point is moved away from the target center.
2. P_{SSK} can be increased when $\mu > L$, i.e. when the expected impact point is off the target. It is not unreasonable to postulate that there is no net effect on the expected impact point from ballistic sources. Therefore the condition that $\mu > L$ must be due to bias in either target location, aim point selection or both. Hence, P_{SSK} can be improved by increasing the dispersion of rounds about the expected impact point when these biases are present.

C. UNKNOWN TARGET LOCATION

The results of the last section indicate that to effectively evaluate weapon systems, a model which includes target location error is needed. Comparing weapon systems based on their performance against stationary, clearly visible targets is somewhat less than satisfactory, for, except in rare instances, targets do not remain motionless and in clear view.

1. Target Location Error

The results of the last section merely reflect the fact that in war, targets are seldom located where they are thought to be located. The suspected location is distributed with respect to the true target center according to some probability distribution. For the one dimension case, the target location error is assumed to be distributed normal with mean μ_ℓ and variance σ_ℓ^2 .

2. Ballistic Error

The impact of rounds about the target's suspected location is also assumed to be normally distributed so that

$$f_{X_B}(x_b) = \frac{1}{(2\pi)^{1/2}\sigma_b} \exp \left\{ -\frac{1}{2} \left(\frac{x_b - x_L}{\sigma_b} \right)^2 \right\},$$

where the ballistic dispersion σ_b is assumed to be composed of terminal ballistics dispersion only. The dispersion due to recoil was assumed to be zero.

3. Lethality Function

The zero-one lethality function is used for the non-fragmenting case while the negative exponential is assumed for the fragmenting case.

4. Analytical Model

The single shot hit and kill probabilities discussed earlier, although important, are not the entire story. The probability of hitting and/or killing a target when more than one round is fired is now examined by use of a salvo-fire model.

In a salvo-fire model it is assumed that (1) the aim point is constant for all N-rounds and (2) the N-rounds are fired simultaneously [17]. This model describes numerous tactical situations among which are (1) a soldier sequentially firing N identical rounds at the same aim point (suspected target location) and (2) a squad of t - men firing N/t identical rounds simultaneously at the same aim point from approximately the same location.

a. Non-fragmenting Case

The basic model and assumptions for salvo-firing of N-rounds of non-fragmenting ammunition when all rounds are sequentially fired at the same aim point are: (1) a soldier engages a target at location x_ℓ , (2) he aims his rifle at X_L where $x_\ell \neq X_L$ due to target location error, (3) he fires N-rounds at the suspected target location with the i^{th} round impacting at x_{bi} , and (4) the impact of the rounds about the fixed aim point are statistically independent. Using a rectangular coordinate system (see Fig. 3) with the target reference point, x_1 , located at 0, the assumed aim point is X_L the expected target location.

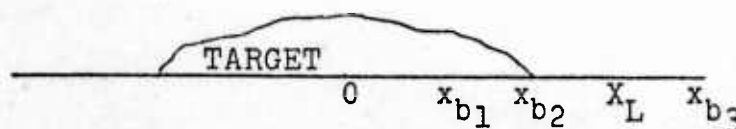


Figure 3. 1D Salvo Model

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The probability of hitting the target at least once with a salvo of N-rounds is calculated in the following manner:

STEP 1. Compute the single shot conditional probability of hitting the target given it is thought to be at X_L by

$$P_{SSH/X_L} = \int \frac{1}{(2\pi)^{1/2} \sigma_b} \exp \left\{ -\frac{1}{2} \left(\frac{x_b - x_L}{\sigma_b} \right)^2 \right\} dx_b \quad (8)$$

STEP 2. Compute the conditional probability of hitting the target at least once with a salvo of N-rounds given it is thought to be at X_L by

$$P_{H/X_L}^{(N)} = 1 - (1 - P_{SSH/X_L})^N. \quad (9)$$

STEP 3. Compute the unconditional probability of hitting the target at least once with a salvo of N-rounds by

$$P_H(N) = 1 - \int_{-\infty}^{\infty} (1 - P_{SSH/X_L})^N f_{X_L}(x_L) dx_L. \quad (10)$$

The probability of killing the target with a salvo of N-rounds is equal to $P_H(N)$, since the zero-one lethality function is used. The final equation becomes

$$P_K(N) = P_H(N) = 1 - \int_{-L}^L \frac{1}{2\pi\sigma_b} \exp \left\{ -\frac{1}{2} \left(\frac{x_b - x_L}{\sigma_b} \right)^2 \right\} dx_b \Big)^N \cdot \frac{1}{(2\pi)^{\frac{1}{2}} \sigma_\ell} \exp \left\{ -\frac{1}{2} \left(\frac{x_L - \mu_\ell}{\sigma_\ell} \right)^2 \right\} dx_\ell \quad (11)$$

for which no closed form solution has been found.

b: Fragmenting Case

The basic model and assumptions for salvo-firing of N-rounds of fragmenting ammunition when all rounds are sequentially fired at the same aim point are identical to the four of the non-fragmenting case with the additional assumption that (5) cumulative damage is negligible.

The probability of killing the target with a salvo of N-rounds is determined as follows:

STEP 1. Compute the single shot conditional probability of killing the target given the aim point is x_L by,

$$P_{SSK/x_L} = \int_{-\infty}^{\infty} \frac{1}{(2\pi)^{\frac{1}{2}} \sigma_b} \exp \left\{ -\frac{1}{2} \left(\frac{x_b - x_L}{\sigma_b} \right)^2 \right\} \exp \left\{ -\frac{1}{2} \left(\frac{x_b}{a} \right)^2 \right\} dx_b \quad (12)$$

thus

$$P_{SSK/x_L} = \frac{a}{(a^2 + \sigma_b^2)^{\frac{1}{2}}} \exp \left\{ -\frac{1}{2} \left(\frac{x_L^2}{a^2 + \sigma_b^2} \right) \right\} \quad (13)$$

STEP 2. Compute the conditional probability of killing the target with a salvo of N-rounds given the target is though to be at X_L by.

$$P_{K/X_L}(N) = 1 - (1 - P_{SSK/X_L})^N. \quad (14)$$

STEP 3. Compute the unconditional probability of killing the target with a salvo of N-rounds by,

$$P_K(N) = \int_{-\infty}^{\infty} f_{x_\ell}(x_L) P_{K/X_L}(N) dx, \quad (15)$$

thus

$$P_K(N) = \sum_{K=1}^N \binom{N}{K} (-1)^K \int_{-\infty}^{\infty} f_{x_\ell}(x_L) (P_{SSK/X_L})^K dx_\ell. \quad (16)$$

Under the above assumptions this equation reduces to

$$P_K(N) = a \sum_{K=1}^N \binom{N}{K} \left\{ \frac{-a}{(a^2 + \sigma_b^2)^{1/2}} \right\}^{K-1} \frac{\exp \left(\frac{-K\mu_a^2}{2(a^2 + \sigma_b^2 + K\sigma_\ell^2)} \right)}{(a^2 + \sigma_b^2 + K\sigma_\ell^2)^{1/2}} \quad (17)$$

When it is assumed that the location error is symmetric about the true target location, i.e. $E(X_L) = x_\ell = \mu_a = 0$, then the above closed form equation reduces to

$$P_K(N) = \sum_{K=1}^N \binom{N}{K} \left\{ \frac{-a}{(a^2 + \sigma_b^2)^{1/2}} \right\}^{K-1} \frac{a}{(a^2 + \sigma_b^2 + K\sigma_\ell^2)^{1/2}} \quad (18)$$

III. TWO DIMENSION

A. INTRODUCTION

The addition of a second dimension brought the model much closer to reality and provided a method of comparison with tabulated data for square targets [7]. Both range and deflection error were taken into account by use of bivariate error distributions.

B. TARGET LOCATION ERROR

The target location error was assumed to be uncorrelated bivariate circular normal so that,

$$f_{X_L, Y_L}(x_\ell, y_\ell) = \frac{1}{2\pi\sigma_\ell^2} \exp^{-\frac{1}{2} \frac{(X_L - x_\ell)^2 + (Y_L - y_\ell)^2}{\sigma_\ell^2}}$$

C. BALLISTIC ERROR

The ballistic error was also assumed to be uncorrelated bivariate circular normal so that,

$$f_{X_B, Y_B}(x_b, y_b) = \frac{1}{2\pi\sigma_b^2} \exp^{-\frac{1}{2} \frac{(x_\ell - x_b)^2 + (y_\ell - y_b)^2}{\sigma_b^2}}$$

D. AIMING ERROR

The aiming error was assumed to be uncorrelated bivariate circular normal with mean (μ_x, μ_y) and variance $\sigma_a^2 = \sigma_x^2 = \sigma_y^2$.

E. LETHALITY

1. Non-Fragmenting

$$\ell_c(x_b, y_b) = \begin{cases} 1 & ; \text{ if round impacts on the target} \\ 0 & ; \text{ otherwise} \end{cases}$$

where the target is assumed to be of area A.

2. Fragmenting

$$\ell_E(x_b, y_b) = \exp^{-\frac{1}{2} \frac{(x_L - x_b)^2 + (y_L - y_b)^2}{a^2}}$$

F. ANALYTICAL MODEL

1. Non-Fragmenting Case

Using the "diffuse" target approximation proposed by J. Von Neumann [4], and as discussed in Appendix A, the conditional probability of killing the target with one round, given the target location is (x_L, y_L) and aim at (μ_x, μ_y) is found to be

$$P_{SSK/X_L, Y_L}(\mu_x, \mu_y) = \frac{W^2}{W^2 + \sigma_b^2} \exp^{-\frac{1}{2} \frac{(\mu_x - x_L)^2 + (\mu_y - y_L)^2}{W^2 + \sigma_b^2}}$$

where the parameter W must be adjusted as discussed in Appendix A.

The probability of killing the target with N-rounds when the i^{th} round is aimed at (μ_{x_i}, μ_{y_i}) is obtained from solving the integral

$$P_K(\mu_{x_1}, \mu_{y_1}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X_L, Y_L} \left(1 - \frac{N}{1} (1 - P_{SSK/X_L, Y_L}(\mu_{x_1}, \mu_{y_1}))\right) dx_L, dy_L$$

where the origin of the (x,y) coordinate system is the center of the true target's location.

For salvo-fire, i.e. all N-rounds are aimed at the same aim point, (μ_X, μ_Y) , it follows that

$$P_K(N) = P_K(\mu_X, \mu_Y) = 1 - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X_L, Y_L} (1 - P_{SSK X_L, Y_L}(\mu_X, \mu_Y))^N dx_L, dy_L$$

now

$$\begin{aligned} & (1 - P_{SSK X_L, Y_L}(\mu_X, \mu_Y))^N \\ &= \left[1 - \frac{W^2}{W^2 + \sigma_b^2} \exp^{-\frac{1}{2} \frac{(\mu_X - x_L)^2 + (\mu_Y - y_L)^2}{W^2 + \sigma_b^2}} \right]^N \\ &= \sum_{K=0}^N \binom{N}{K} \left\{ \frac{-W^2}{W^2 + \sigma_b^2} \right\}^K \exp^{-\frac{K}{2} \frac{(\mu_X - x_L)^2 + (\mu_Y - y_L)^2}{W^2 + \sigma_b^2}} \end{aligned}$$

thus

$$P_K(N) = W^2 \sum_{K=1}^N \binom{N}{K} \left[\frac{-W^2}{W^2 + \sigma_b^2} \right]^{K-1} \frac{\exp \frac{\frac{K}{2} \frac{(\mu_x - x_L)^2 + (\mu_y - y_L)^2}{W^2 + \sigma_b^2} + K(\sigma_L^2 + \sigma_a^2)}{W^2 + \sigma_b^2 + K(\sigma_L^2 + \sigma_a^2)}}{W^2 + \sigma_b^2 + K(\sigma_L^2 + \sigma_a^2)}$$

The assumption that the constant aim point, (μ_x, μ_y) , selected by the soldier is also the expected target location, (x_L, y_L) , reduces the above formula to

$$P_K(N) = \sum_{K=1}^N \binom{N}{K} \left[\frac{-W^2}{W^2 + \sigma_b^2} \right]^{K-1} \frac{W^2}{W^2 + \sigma_b^2 + K(\sigma_a^2 + \sigma_L^2)} \quad (23)$$

Substituting the adjusted value of the parameter W (see Appendix A) into the above formula reduces the formula to

$$P_K(N) = \sum_{K=1}^N \binom{N}{K} \left[\frac{-4L^2/2\pi}{4L^2/2\pi + \sigma_b^2} \right]^{K-1} \frac{4L^2/2\pi}{4L^2/2\pi + \sigma_b^2 + K(\sigma_a^2 + \sigma_L^2)} \quad (24)$$

for a square target of sides $2L$. The computer program used to calculate $P_K(N)$ for $N=5,10$ (both with and without the adjustment on the parameter W) is given in Appendix B.

An indication of the nature of the diffuse target approximation was obtained by comparing the computer calculated $P_K(N)$ values with the exact values for a square target of sides $2L$. The latter values are tabulated in Reference 7. Figures 4

and 5 graphically depict this comparison for $N = 5, 10$, and W adjusted as per Appendix A. Table 1 is given to reflect the improvement of the approximation after adjustment of the parameter W .

The ratios $\frac{[2(\sigma_a^2 + \sigma_b^2)]^{\frac{1}{2}}}{L}$ and $\frac{\sigma_b (2)^{\frac{1}{2}}}{L}$ used in Figures

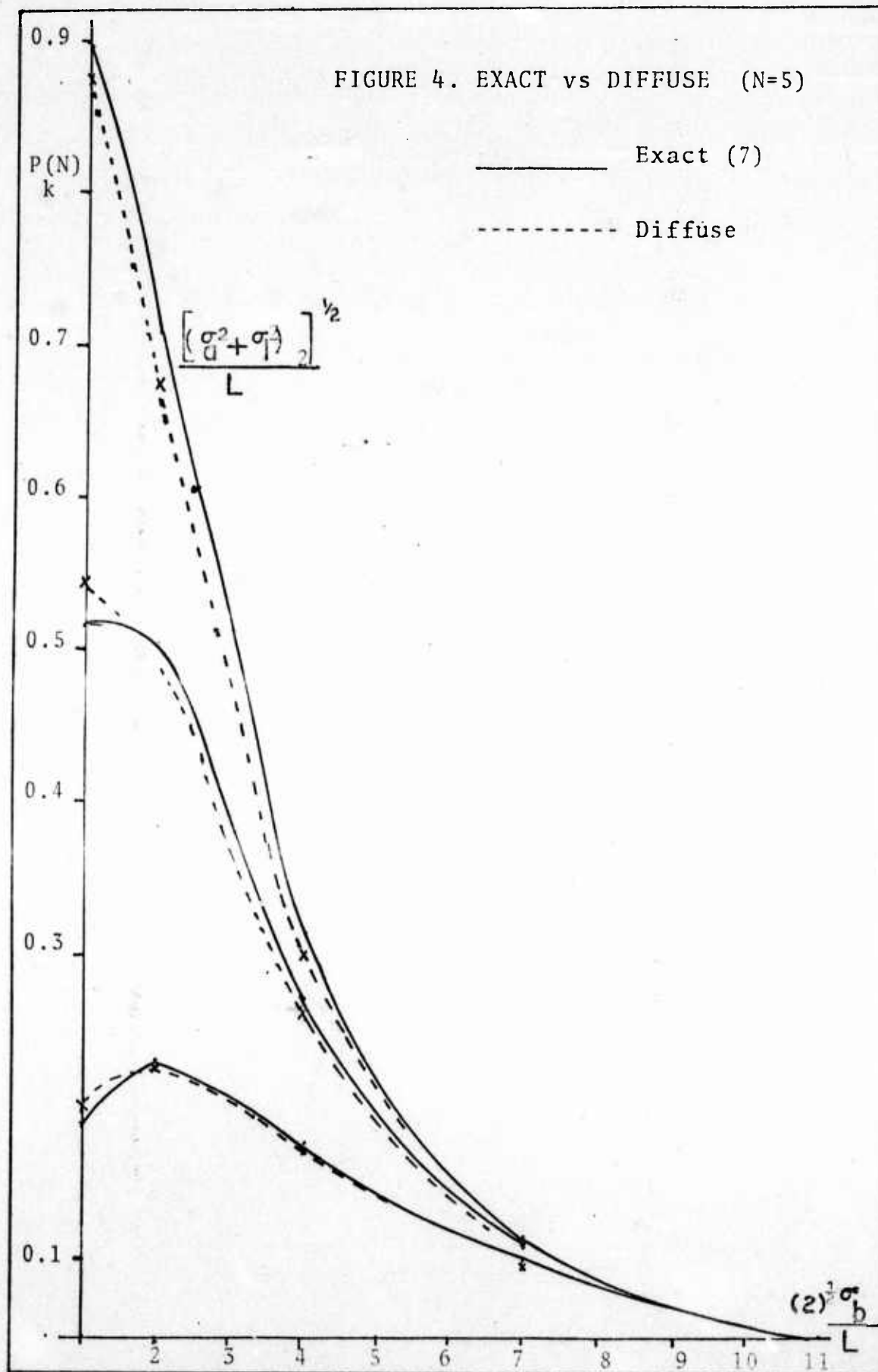
4 and 5 were formed in order that a direct comparison could be made with the tabulated results of Reference 7. These ratios were formed by multiplying through equation (24) by $((2)^{\frac{1}{2}}/L)^2$ to obtain

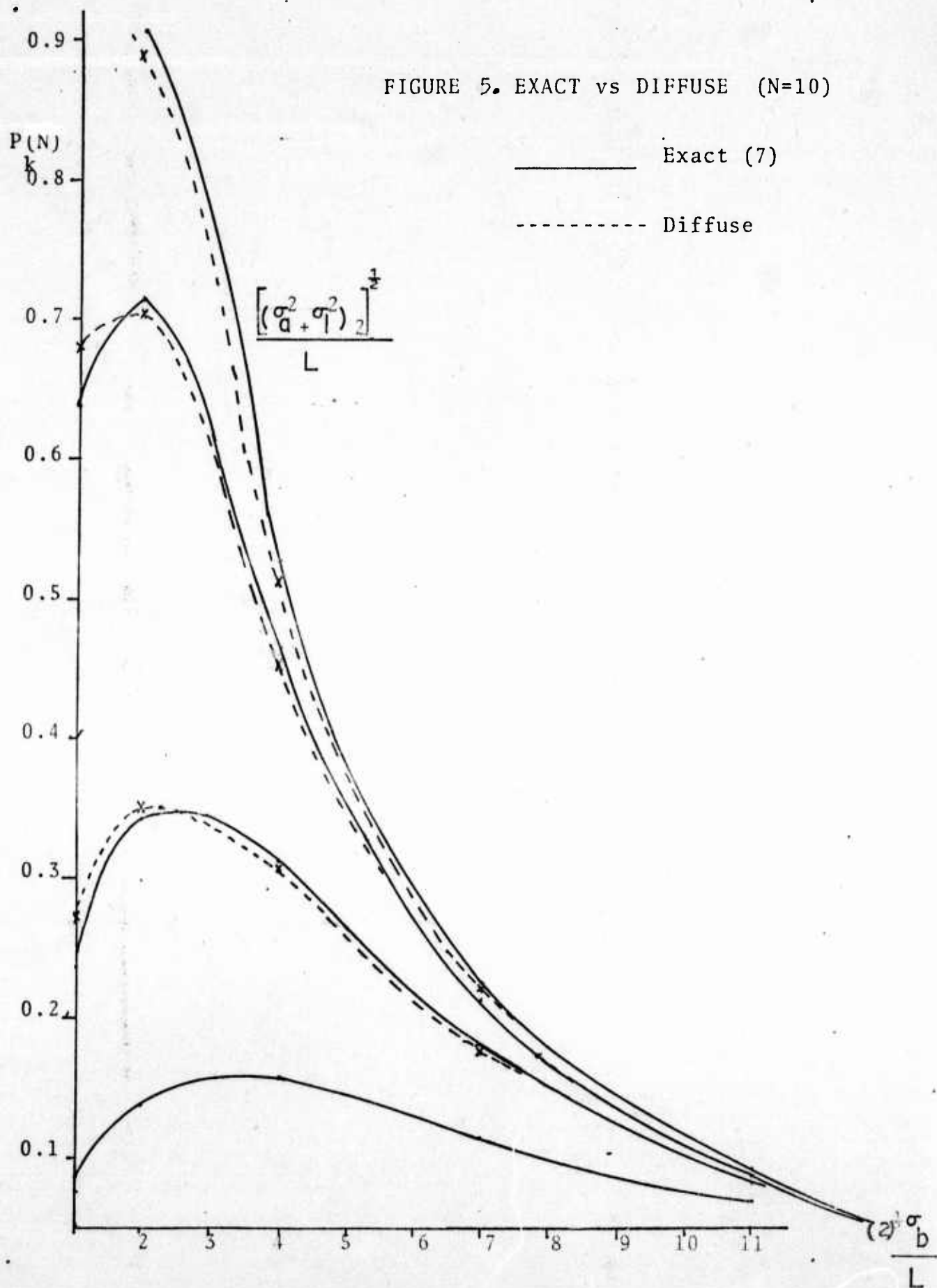
$$P_K(N) = \sum_{K=1}^N \binom{N}{K} \left[\frac{-4/\pi}{4/\pi + \left[\frac{(2)^{\frac{1}{2}} \sigma_b}{L} \right]^2} \right]^{K-1} \frac{4/\pi}{4/\pi + \left[\frac{(2)^{\frac{1}{2}} \sigma_b}{L} \right]^2 + K \frac{(2)^{\frac{1}{2}} \sigma_a^2 + \sigma_b^2}{L}} \quad (25)$$

The presence of the $(2)^{\frac{1}{2}}$ factor in the multiplier, $(2)^{\frac{1}{2}}/L)^2$, was found to be necessary in order to obtain probabilities close to those tabulated in Reference 7 and noted by H. J. Helgert in Reference 6. Although there was no a priori reason to include the $(2)^{\frac{1}{2}}$ factor in these ratios, calculations which did not include it in the ratios resulted in salvo kill probabilities far below those of Reference 7.

2. Fragmenting Case

The use of the negative exponential lethality function for fragmentation ammunition resulted in the final formula





NO ADJUSTMENT			ADJUSTED			TABULATED
RATA	RATB	$P_K(N)$	RATA	RATB	$P_K(N)$	$P_K(N)$
σ_b/L	$\frac{(\sigma_a^2 + \sigma_b^2)^{1/2}}{L}$		$(2)^{1/2} \sigma_b/L$	$\frac{[2(\sigma_a^2 + \sigma_b^2)]^{1/2}}{L}$		
1	1	0.8715	1	1	0.9631	0.9611
	2	0.6902		2	.8872	.9804
	4	0.3072		4	.5138	.5257
	7	0.1187		7	.2222	.2245
	11	0.0506		11	.0985	.0989
	16	0.0244		16	.0482	.0481
2	1	0.5055	2	1	.6796	.6337
	2	0.4974		2	.7059	.7139
	4	0.2674		4	.4567	.4659
	7	0.1124		7	.2111	.2131
	11	0.0494		11	.0963	.0966
	16	0.0241		16	.0476	.0476
4	1	0.1774	4	1	.2732	.2359
	2	0.2239		2	.3510	.3443
	4	0.1740		4	.3083	.3120
	7	0.0925		7	.1754	.1767
	11	0.0452		11	.0882	.0885
	16	0.0230		16	.0456	.0456
7	1	0.0633	7	1	.1017	.0854
	2	0.0882		2	.1441	.1389
	4	0.0880		4	.1600	.1606
	7	0.0620		7	.1189	.1195
	11	0.0365		11	.0716	.0718
	16	0.0206		16	.0407	.0407
11	1	0.0263	11	1	.0429	.0357
	2	0.0379		2	.0628	.0602
	4	0.0422		4	.0776	.0776
	7	0.0360		7	.0695	.0697
	11	0.0258		11	.0507	.0508
	16	0.0167		16	.0330	.0330
16	1	0.0125	16	1	.0205	.0170
	2	0.0183		2	.0305	.0292
	4	0.0213		4	.0395	.0394
	7	0.0201		7	.0391	.0391
	11	0.0166		11	.0327	.0327
	16	0.0123		16	.0244	.0244

TABLE I
SALVO KILL PROBABILITIES FOR N=10

$$P_K(N) = a^2 \sum_{K=1}^N \binom{N}{K} \left[\frac{-a^2}{a^2 + \sigma_b^2} \right]^{K-1} \frac{\exp \left[-\frac{K}{2} \frac{(\mu_x - x_L)^2 + (\mu_y - y_L)^2}{a^2 + \sigma_b^2 + K(\sigma_a^2 + \sigma_l^2)} \right]}{a^2 + \sigma_b^2 + K(\sigma_a^2 + \sigma_l^2)}, \quad (26)$$

where a is the lethal radius of the round. The similarity between this final equation and that of the non-fragmenting case were expected. The diffuse target approximation used a negative exponential function similar to the lethality function used in this case. The parameter a in this case is a function of both the type ammunition used and the type target engaged, while the parameter W in the diffuse target approximation is a function of the target and not of the type ammunition fired.

IV. OPTIMAL AIMING

When firing at a target whose true location is unknown, but a "guess" can be made as to the suspected target location the question of where to aim arises. For the case when two rounds are to be fired and assuming that (1) the origin of the coordinate system is at the center of the suspected target location and (2) that the range error is of greater importance than deflection error, the optimal two aim points are μ_{y_1} , and μ_{y_2} where $\mu_{y_1} = -\mu_{y_2}$, as shown by Figure 6.

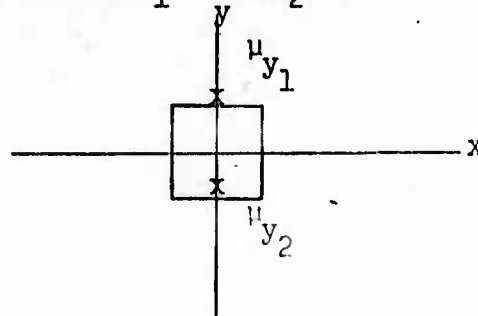


Figure 6

The probability of killing the target is

$$P_K(\mu, -\mu) = \frac{2W^2}{W^2 + \sigma_\ell^2 + \sigma_a^2 + \sigma_b^2} \exp^{-\frac{1}{2} \frac{\mu^2}{W^2 + \sigma_a^2 + \sigma_\ell^2 + \sigma_b^2}} - \frac{W^3}{(W^2 + \sigma_b^2)[2(\sigma_a^2 + \sigma_\ell^2) + W^2 + \sigma_b^2]} \exp^{-\frac{\mu^2}{W^2 + \sigma_b^2}}, \quad (27)$$

where $\mu_{y_1} = -\mu_{y_2} = \mu$.

For the case when three rounds are to be fired the optimal aim points are μ_{y_1} , 0, μ_{y_2} as depicted in Figure 7.

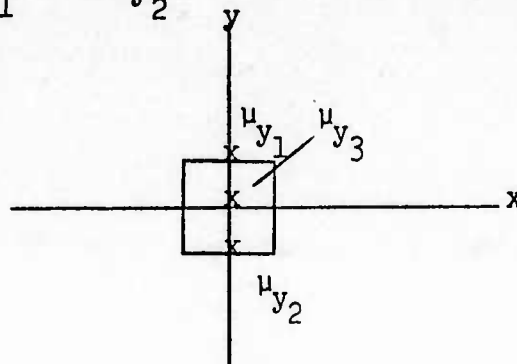


Figure 7

The probability of killing the target is then

$$\begin{aligned}
 P_K(\mu, 0, -\mu) = & \frac{W^2}{W^2 + \sigma_a^2 + \sigma_l^2 + \sigma_b^2} + \frac{2W^2}{W^2 + \sigma_l^2 + \sigma_a^2 + \sigma_b^2} \exp^{-\frac{1}{2} \left\{ \frac{\mu^2}{W^2 + \sigma_a^2 + \sigma_l^2} \right\}} \\
 & - \frac{W^3 \exp^{-\left\{ \frac{\mu^2}{W^2 + \sigma_b^2} \right\}}}{(W^2 + \sigma_b^2)[2(\sigma_l^2 + \sigma_a^2) + W^2 + \sigma_b^2]} - \frac{2W^3 \exp^{-\frac{1}{2} \left\{ \frac{(\sigma_l^2 + \sigma_a^2 + \sigma_b^2) \mu^2}{(W^2 + \sigma_b^2)(2\sigma_a^2 + 2\sigma_l^2 + \sigma_b^2 + W^2)} \right\}}}{(W^2 + \sigma_b^2)[2(\sigma_a^2 + \sigma_l^2) + W^2 + \sigma_b^2]} \\
 & + \frac{W^4 \exp^{-\left\{ \frac{W^2}{W^2 + \sigma_b^2} \right\}}}{(W^2 + \sigma_b^2)[2(\sigma_a^2 + \sigma_l^2) + W^2 + \sigma_b^2]} .
 \end{aligned} \tag{28}$$

Under the above assumptions, the case when N rounds are to be fired at μ_{y_1} and N rounds at μ_{y_2} , where $N \geq 2$, was not pursued due to the problems of integrating the product of two sums over a double integral. The equation to be solved is

$$P_K(\mu_{y_1}, -\mu_{y_2}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{x_L, y_L} \left(\sum_{K=0}^N \binom{N}{K} \left[\frac{-W^2}{W^2 + \sigma_L^2 + \sigma_a^2} \right]^K \exp \frac{-K}{2} \frac{(\mu_{y_1} - y_L)^2 + x_L^2}{W^2 + \sigma_L^2 + \sigma_a^2} \right. \\ \left. \times \sum_{K=0}^N \binom{N}{K} \left[\frac{-W^2}{W^2 + \sigma_L^2 + \sigma_a^2} \right]^K \exp \frac{-K}{2} \frac{(-\mu_{y_2} - y_L)^2 + x_L^2}{W^2 + \sigma_L^2 + \sigma_a^2} \right) dx_L dy_L \quad (29)$$

Thus, when the true target location is unknown, in attempting to maximize the probability of hitting the target and thus of incapacitating it, the rounds should be delivered so as to cover both the suspected target center and the area immediately surrounding the target. This can be accomplished at short ranges by using grenade, shotgun or flechette type ammunition, while at longer ranges ball ammunition fired from a light machine gun or similar rapid fire system would be appropriate.

V. CONCLUSIONS

When the target's true location is known, i.e., no target location error, the optimal aim point is one which places the expected impact point of the round on the target center (see Maximizing P_{SSK}). Therefore, as is obvious, a desirable weapon system is one which places a round on the target center with a probability near one. A high degree of accuracy and a slow rate of fire are two characteristics of such a weapon system.

However, except in very rare instances, targets do not remain motionless and in clear view. More often than not, the target appears as a fleeting target whose true location is unknown. Under these more realistic conditions dispersion about the suspected target location is desirable. Depending on the range from the firer to the target this can be accomplished by:

(a) Spraying the target area with ball ammunition from a fairly inaccurate but rapid firing system.

(b) Firing fragmentation ammunition, whether it be shotgun, grenade, or flechette, at the suspected target location.

APPENDIX A
DIFFUSE TARGET APPROXIMATION

To obtain a solution in closed-form for a sharply defined target the diffuse target approximation is employed, i.e. rounds impacting near the target produce a nearly full effect on the target, while those falling far from the target produce a very small effect on the target. This approach was originally suggested by J. von Neumann [4].

The negative exponential function is used for this approximation. Thus,

$$f_{X_d, Y_d}(x_b, y_b) = \exp^{-\frac{1}{2} \frac{(x_L - x_b)^2 + (y_L - y_b)^2}{W^2}}$$

The negative exponential is used since (1) the negative exponential drops off sharply outside the target area and (2) it greatly simplifies the calculations.



Figure A-1 shows the original situation while Figure A-2 shows the approximation.

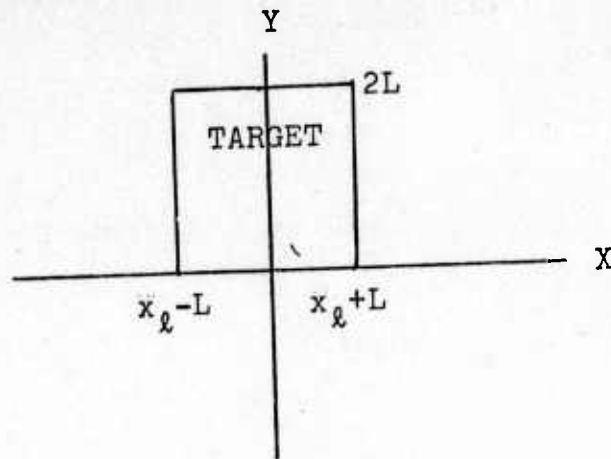


Figure A-1

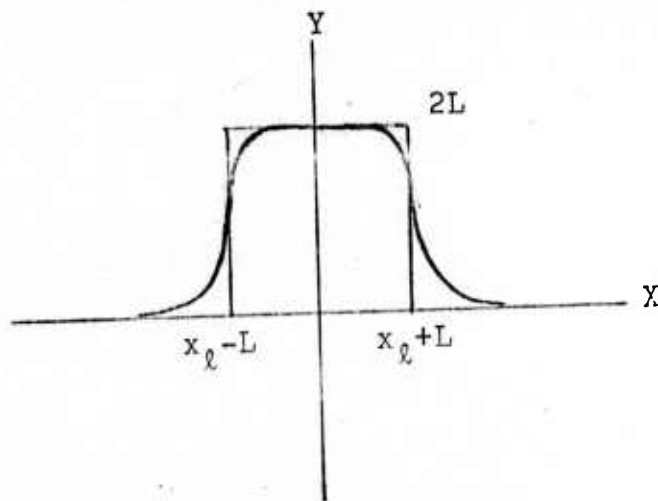


Figure A-2

The diffuse target approximation of Figure A-2 allows the integration to be carried out from $-\infty$ to $+\infty$ with a minimum amount of probability in the tails, i.e. beyond $x_l + L$ and $x_l - L$. To further minimize the discrepancies between the original situation and the approximation the parameter W must be adjusted.

Adjusting W so that the areas of Figure A-1 and A-2 are equal implies

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp^{-\frac{1}{2} \frac{(x_{\ell} - x_b)^2 + (y_{\ell} - y_b)^2}{W^2}} dx dy = 4L^2$$

Performing the above integration implies that:

$$W^2 2\pi = 4L^2$$

$$W^2 = \frac{4L^2}{2\pi}$$

$$W^2 = \frac{\text{Target Area}}{2\pi}$$

APPENDIX B

COMPUTER OUTPUT FOR A SALVO OF 5 ROUNDS

This table compares the results of the diffuse target approximation for a salvo of $N = 5$ rounds.

The inputs are;

$N = \text{NRDS} =$ Number of rounds in the salvo

$$\text{RATA} = \frac{(2)^{\frac{1}{2}} \sigma_b}{L}$$

$$\text{RATB} = \frac{\left[2(\sigma_a^2 + \sigma_\ell^2) \right]^{\frac{1}{2}}}{L}$$

$\text{BINOMC}(N) =$ Binomial coefficients

See Appendix D for the computer program.

N	RATB	RATA	$P_K(N)$	
			DIFFUSE	EXACT
5	1.	1.	0.872415	0.893900
5	1.	2.	0.672554	0.709200
5	1.	4.	0.302942	0.311400
5	1.	7.	0.118124	0.119300
5	1.	11.	0.050578	0.050700
5	1.	16.	0.024403	0.024300
5	2.	1.	0.544349	0.525200
5	2.	2.	0.493774	0.510400
5	2.	4.	0.264370	0.270800
5	2.	7.	0.111875	0.113000
5	2.	11.	0.049400	0.049500
5	2.	16.	0.024125	0.024100
5	4.	1.	0.204383	0.184500
5	4.	2.	0.229438	0.229500
5	4.	4.	0.173284	0.175800
5	4.	7.	0.092206	0.092900
5	4.	11.	0.045181	0.045300
5	4.	16.	0.023075	0.023000
5	7.	1.	0.074655	0.065800
5	7.	2.	0.091964	0.090400
5	7.	4.	0.088312	0.038800
5	7.	7.	0.061983	0.062300
5	7.	11.	0.036566	0.036600
5	7.	16.	0.020606	0.020600
5	11.	1.	0.031285	0.027300
5	11.	2.	0.039817	0.038900
5	11.	4.	0.042590	0.042600
5	11.	7.	0.036080	0.036100
5	11.	11.	0.025803	0.025800
5	11.	16.	0.016701	0.016700
5	16.	1.	0.014973	0.013000
5	16.	2.	0.019296	0.018800
5	16.	4.	0.021605	0.021500
5	16.	7.	0.020219	0.020200
5	16.	11.	0.016619	0.016600
5	16.	16.	0.012319	0.012300

APPENDIX C

COMPUTER OUTPUT FOR A SALVO OF 10 ROUNDS

This table compares the results of the diffuse target approximation for a salvo of 10 rounds with the tabulated values of Reference 7.

The inputs are:

$N = NRDS$ = Number of rounds in the salvo

$$RATA = \frac{(2)^{\frac{1}{2}} \sigma_b}{L}$$

$$RATB = \frac{\left[2(\sigma_a^2 + \sigma_b^2) \right]^{\frac{1}{2}}}{L}$$

$BINOMC(N)$ = Binomial coefficients

See Appendix D for the computer program.

N	RATB	RATA	$P_K(N)$	
			DIFFUSE	EXACT
10	1.	1.	0.963104	0.961100
10	1.	2.	0.887269	0.908400
10	1.	4.	0.513899	0.525700
10	1.	7.	0.222290	0.224500
10	1.	11.	0.098597	0.098900
10	1.	16.	0.048210	0.048100
10	2.	1.	0.679618	0.633700
10	2.	2.	0.705957	0.713900
10	2.	4.	0.456730	0.465900
10	2.	7.	0.211172	0.213100
10	2.	11.	0.096356	0.096600
10	2.	16.	0.047668	0.047600
10	4.	1.	0.273233	0.235900
10	4.	2.	0.351053	0.344300
10	4.	4.	0.308397	0.312000
10	4.	7.	0.175413	0.176700
10	4.	11.	0.088294	0.088500
10	4.	16.	0.045616	0.045600
10	7.	1.	0.101787	0.085400
10	7.	2.	0.144124	0.138900
10	7.	4.	0.160042	0.160600
10	7.	7.	0.118929	0.119500
10	7.	11.	0.071679	0.071800
10	7.	16.	0.040776	0.040700
10	11.	1.	0.042912	0.035700
10	11.	2.	0.062891	0.060200
10	11.	4.	0.077764	0.077600
10	11.	7.	0.059592	0.069700
10	11.	11.	0.050724	0.050800
10	11.	16.	0.033091	0.033000
10	16.	1.	0.020583	0.017000
10	16.	2.	0.030568	0.029200
10	16.	4.	0.029568	0.039400
10	16.	7.	0.039100	0.039100
10	16.	11.	0.032731	0.032700
10	16.	16.	0.024437	0.024400

COMPUTER PROGRAM FOR TWO DIMENSION MODEL
WITH CIRCULAR NORMAL DISTRIBUTIONS

```

C      THIS PROGRAM COMPUTES SALVO KILL PROBABILITIES FOR
C      THE TWO DIMENSION SALVO MODEL WITH CIRCULAR NORMAL
C      DISTRIBUTIONS
C      IMPLICIT REAL*8(A-H,O-Z)
C      THE FOLLOWING MUST BE DIMENSIONED TO NRDS, THE
C      NUMBER OF ROUNDS FIRED IN THE SALVO
      DIMENSION TERM(5), RATA(6), RATB(6,6), BINOMC(5), TAB(6,6)
      READ(5,1000) ((RATA(J), J=1,6)
      READ(5,1001) ((RATB(M,L), L=1,6), M=1,6)
      READ(5,1002) (BINOMC(K), K=1,5)
      READ(5,1003) ((TAB(M,L), L=1,6), M=1,6)
1000  FORMAT (18F4.0)
1001  FORMAT (15F6.0)
1002  FORMAT (8F9.6)
      NRDS=5
      N=NRDS
      DO 99 I=1,6
      DO 103 J=1,6
      DO 100 K=1,N
      TERM2A=1.273+(RATB(I,J))*2
      TERM2=(-1.273/TERM2A)**(K-1)
      TERM3A=TERM2A+(K*(RATA(I))*2)
      TERM3=1.273/TERM3A
      TERM(K)=BINOMC(K)*TERM2*TERM3
100  CONTINUE
      SUM=0.0
      DO 101 K=1,N
      SUM=SUM+TERM(K)
101  CONTINUE
      PKILL=SUM
      GO TO 104
104  WRITE(06,1001) NRDS,RATA(I),RATB(I,J),PKILL,TAB(I,J)
1001  FORMAT(21X,13,5X,F4.0,5X,F4.0,5X,F9.6,5X,F9.6)
103  CONTINUE
99   CONTINUE
      STOP

```

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Salvo Diffuse Target						