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DYNAMIC COMPATIBILITY OF ROTARY-WING AIRCRAFT PROPULSION COMPONENTS

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Fort Eustis, Virginia

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It is recommended that studies be made to justify an intelligent standardization of engine vibration limit specifications for helicopters, that impedance-mobility methods be developed for optimizing engine/airframe interface design, that research and development of helicopter power transmission shafts and couplings be carried out to solve whirling problems, and that new methods and hardware be developed to eliminate torsional instabilities in helicopter drive systems with automatic fuel control.

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A study of problems related to vibration and dynamic loads in helicopter propulsion systems has been made. It has been found that engine vibration, shaft whirling, and dynamic instabilities seriously limit helicopter performance and reliability.

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FOREWORD

In this report, the dynamic compatibility of helicopter propulsion components is considered from the standpoint of assessing what can be done in this area to improve helicopter reliability and performance.

The subject is divided into four major subareas: (1) engine vibration limits, (2) engine/airframe vibratory interface design, (3) drive train dynamics, and (4) torsional stability. A section on each subarea briefly describes the state of the art, states the major problems encountered, provides some motivation for the recommendations to be made, and recommends research and development in specific directions.

The importance of the references listed at the end of the report should not be underestimated. It is strongly suggested that anyone intending to pursue the recommendations contained in this report should become familiar with these references. They in turn will suggest further study.

Additional background material and pertinent information obtained from published references are given in the appendixes.

The work reported herein was authorized by DA Task 1G162207AA7104, House Task 71-26.

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LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

a	displacement amplitude of a lumped mass
A_n	vibratory displacement amplitude in the n^{th} mode, in.
c	damping constant, $\frac{\text{lb-sec}}{\text{in.}}$
C	cross-sectional dimension from neutral axis to outer surface of a beam, in.
D	determinant of matrix
e	the base of Napierian logarithm, 2.718
E	Young's modulus, lb/in.^{2}
F	force in a force generator, lb
F_b	blocked force, lb
F_e	force in added element, lb
F_i	internal force in Norton's equivalent system, lb
F_1, F_2 F_3, F_4	force at stations 1, 2, 3, and 4, lb
i, j	$\sqrt{-1}$
I	area moment of inertia, in.^{4}
k, k_R, k_B $k_{RB}, k_c,$ k_m	spring constant, lb/in.
ℓ	length of a beam, in.
$m, m_1,$ m_2	mass, $\frac{\text{lb-sec}^2}{\text{in.}}$
M	moment, in.-lb

M_e, M_i, M_1, M_2, $\overline{M}_2$	mobility, in./lb-sec
n	an integer defining a vibratory mode
N	an integer
t	time, sec
V_n	vibratory velocity in the n^{th} mode, in.
V_o	free velocity, in./sec
V_1, V_2, V_3, V_4	velocity of stations 1, 2, 3, and 4, in./sec
x	position along a flexible member, in.
X_n	vibratory displacement of a lumped mass in the n^{th} mode, in.
y_n	vibratory displacement along a flexible member, in the n^{th} mode, in.
Z	impedance, $\frac{\text{lb-sec}}{\text{in.}}$
ϵ_x	strain in the x direction, in./in.
ρ	mass per unit length, $\frac{\text{lb-sec}^2}{\text{in.}^2}$
σ_x	stress in the x direction, lb/in.^{2}
ω	frequency, rad/sec
ω_n	frequency of the n^{th} mode, rad/sec

SYMBOLS IN APPENDIX I

f	force in basic physical elements, lb
F	amplitude of f, lb

m	mass, $\frac{\text{lb-sec}^2}{\text{in.}}$
M_{21}	transfer mobility, in./sec-lb
v	vibratory velocity of basic element, in./sec
V	amplitude of v, in./sec

INTRODUCTION

Vibration, shaft whirling, and dynamic instabilities have imposed major limitations on the performance and reliability of helicopters since their inception. In addition to the limitations that have been directly attributed to vibration, there is little doubt that vibration contributes to problems attributed to other causes. For example, turbine blade failures in helicopter engines, which are normally associated with thermal effects, might occur less frequently in a less severe dynamic environment.

Even if indirect effects are ignored, however, vibration-related failures of helicopter propulsion components in the field make mandatory the establishment of better dynamic compatibility of these components.

Vibratory excitation in a helicopter propulsion system can be either from mechanical sources, such as the whirling of shafts, or from aerodynamic sources, such as the N/rev excitation at the hub of the rotary wing, generated by the air loads on the blades as they alternately advance and recede in the direction of flight. Although neither of these sources of excitation can be completely eliminated, efforts can be and are being made to reduce the magnitude of both.

Mechanical excitations can be reduced by improved balancing of rotating components, by the avoidance of near-resonant conditions, and by the elimination of self-excited whirl and vibration. These techniques will be discussed further in subsequent sections.

The subject of aerodynamic excitation is so broad and complex, and is being researched so thoroughly by other investigators, that it will not be discussed in detail here. Rather, it is assumed that a certain amount of aerodynamic excitation will always be present in helicopters, and the question of optimum propulsion system design to survive in this environment will be considered.

Devices to dynamically isolate the rotary-wing assembly (mast, hub, and blades) from the rest of the helicopter are under development. Some of these devices are "active", requiring external energy to provide counterbalancing excitations over a wide frequency range; others are "passive", similar to the Frahm dynamic absorber. Reference 1 summarizes the most promising concepts.

It should be recognized that the aerodynamic sources of excitation are governed to a large extent by the basic design of the helicopter, such as the number of blades on the rotor. If additional flight test data is obtained, it may become possible to quantify these effects to the extent that vibration can have proper consideration in early design decisions. This report, however, is aimed at dealing directly with the current problems of dynamic compatibility in helicopters of contemporary design.

The adoption of turbine engines originally designed for fixed-wing applications has emphasized the need to consider the dynamic interactions of engine, drive train (shafting, transmissions, and rotors), and helicopter airframes. In addition to being originally intended for a dynamic environment less severe than that provided by the typical helicopter, the turboshaft engine has also required development of power transmission systems with very high overall speed ratios and high horsepower capabilities. Since speed reduction must often be accomplished in several stages that may be physically remote from one another, it is difficult to prevent whirling and vibration generated by the many rotational frequencies inherent in the shafting and gearboxes.

The advent of automatic speed governors on large helicopters has created a special problem of maintaining torsional stability of the engine and drive train while at the same time providing a sufficiently rapid response to demand for power and speed changes.

Thus the overall problem of achieving dynamic compatibility in helicopter propulsion systems can be conveniently divided into four major areas: (1) engine vibration limits, (2) design of the engine/airframe interface, (3) whirling and vibration of power transmission shafting, gearboxes, and bearings, and (4) torsional stability of the engine and drive train with closed-loop fuel controls. These four areas are treated in detail in the sections that follow, each with a review of the state of the art and recommendations for future research and development.

ENGINE VIBRATION LIMITS

The effects of vibration on turboshaft engine reliability (and consequently on helicopter performance limits) are recognized by the engine manufacturers' recommendations for engine vibration limits. One of the chief characteristics of these limits is that each manufacturer expresses them in a different form. Reference 2 discusses the desirability of standardizing the methods of establishing and applying these limits.

Engine vibration limits are usually expressed as maximum allowable displacement, velocity, acceleration, or bending angle, versus discrete frequency of the passing filter or harmonic analysis. Sometimes limits on overall amplitudes, summed over all frequencies present, are also given.

Both the location of the measurement transducers and their orientation are specified differently for each engine.

Some of these limits were established with fixed-wing aircraft as the expected application, and there is a question to be answered about their applicability to helicopters because of the lower frequencies and higher vibratory magnitudes encountered. This question really boils down to a more basic question: What are the relative contributions to engine vibration from (1) the helicopter rotor and airframe and (2) the engine itself (such as turborotor unbalance). In fact, there are several such questions which must be answered by experimental and analytical research before engine vibration limits can be intelligently standardized. Some of these questions are:

1. What is the typical vibratory environment encountered by helicopter engines? Can this environment be improved by airframe designers with a reasonable amount of effort? Can it be improved by proper engine mount design? How does the vibratory environment vary with mission profiles? With the type of helicopter (i.e., number of blades, number of rotors, etc.)?

2. What is the best parameter (displacement, velocity, acceleration) to use in measuring and specifying engine vibration? Which is the easiest to measure?* The most convenient for analysis? The most directly related to potential structural damage?

3. What are the best locations and directional orientations for engine vibration transducers to measure severity of environment? To measure potential of structural damage? To measure response of internal parts (such as engine rotor whirl and flexure)?

*The parameter easiest to measure may not be best for other purposes.

4. What is the most useful and realistic method of data analysis (i.e., discrete frequency magnitude, power spectral density, 1/3 octave averaging, etc.)? Is more than one method of data analysis required in order to establish meaningful vibration limits?

Of course, all of these questions relate to the problem of establishing engine vibration criteria that are both realistic and at the same time will improve helicopter reliability and performance.

With respect to question 1, the only way to accurately define the vibrational environment in contemporary helicopters is through acquisition and analysis of flight test data. The present situation is best summarized by a quotation from Reference 1:

> "...the most important finding of this study is as follows: An appallingly small amount of directly applicable experimental data exists on the vibration environment in operational helicopters. Moreover, almost no data have been collected under controlled conditions in an operational setting to determine the effects of this helicopter vibration regime on flight crew performance and physiology."

The same statement applies here if the last five words are changed to "engine performance and reliability." The key to this problem is that vibration data must be acquired at the proper locations and using the techniques which are designed for the use to which the data are to be put. There are some data available, but they were obtained mostly with the objective of defining aerodynamic loads and consequently are of little use in defining the engine environment. Ideally, of course, engine data and aerodynamic data should be obtained simultaneously so that the effect of the latter on the former can be determined. In practice, such a test would tax instrumentation capabilities to the limit and beyond.

It is recommended that existing flight test data be analyzed to determine:

- The spectrum of engine vibration amplitudes for typical military missions.
- The relative contributions of engine and airframe to engine vibration.
- Amplitude versus discrete frequency over the range from 10 to 10,000 Hz for flight conditions dominating the mission profiles.
- Predominant responding mode shapes for the engine (both rigid body and flexural, if possible).

It is further recommended that additional flight test data be acquired from a representative group of the different types of helicopters for analysis as described above.

On first consideration of the question as to whether displacement, velocity, or acceleration is best for vibration measurement and specification, many vibration engineers assert that it makes no difference since they are directly related (for discrete frequencies) by the frequency ω. However, there are some whose experience in measuring engine vibration has led them to strong opinions in favor of one parameter over the others.

For example, White[3] suggests that the magnitude of vibratory velocity is the best measure of the destructive potential of vibration. He bases this hypothesis on (1) observations of the trends of test results and (2) consideration of the velocity-strain relationship for simple mathematical models in which strain is proportional to curvature (e.g., beams). To illustrate this concept, consider a simply supported uniform beam executing undamped free vibration in each of its natural modes (Figure 1a). The displacement and velocity of the nth mode are given by

$$Y_n = A_n \cos \omega_n t \sin \frac{n\pi x}{\ell}$$

$$\dot{Y}_n = -A_n \omega_n \sin \omega_n t \sin \frac{n\pi x}{\ell}$$

where $\omega_n = n^2\pi^2 \sqrt{\frac{EI}{\rho \ell^4}}$

Thus the peak velocity is

$$V_{max} = -A_n n^2\pi^2 \sqrt{\frac{EI}{\rho \ell^4}}$$

for the n^{th} mode.

The bending strain ϵ_x is given in terms of curvature by

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{Mc}{EI} = c \frac{d^2y}{dx^2}$$

Thus for the n^{th} mode, the strain is

$$\epsilon_{x_n} = A_n \frac{c}{\ell^2} n^2\pi^2 \cos \omega_n t \sin \frac{n\pi x}{\ell}$$

and the maximum strain is

$$\epsilon_{x_{max}} = A_n \, n^2 \pi^2 \, \frac{c}{\ell^2}$$

The ratio of maximum vibratory velocity to maximum vibratory strain is then (for each mode)

$$\frac{V_{max}}{\epsilon_{x_{max}}} = \sqrt{\frac{EI}{\rho c^2}}$$

which is seen to be independent of frequency.

Vibration does not always involve bending flexure, however. Especially at lower frequencies, a measured velocity may indicate rigid-body modes, in which the strain in springlike elements is proportional to displacement rather than velocity. For example, consider the lumped mass system of Figure 1b, in which the springs can represent any element which deflects in direct tension, compression, shear, or torsion. Displacement and velocity for the two modes are given by

$$\left.\begin{aligned} X_1 &= a \sin \omega_1 t \\ X_2 &= a \sin \omega_1 t \\ \dot{X}_1 &= a\, \omega_1 \cos \omega_1 t \\ \dot{X}_2 &= a\, \omega_1 \cos \omega_1 t \end{aligned}\right\} \quad \omega_1 = \sqrt{\frac{k}{m}}$$

and

$$\left.\begin{aligned} X_1 &= a \sin \omega_2 t \\ X_2 &= -a \sin \omega_2 t \\ \dot{X}_1 &= a\, \omega_2 \cos \omega_2 t \\ \dot{X}_2 &= -a\, \omega_2 \cos \omega_2 t \end{aligned}\right\} \quad \omega_2 = \sqrt{\frac{k + 2k}{m}}$$

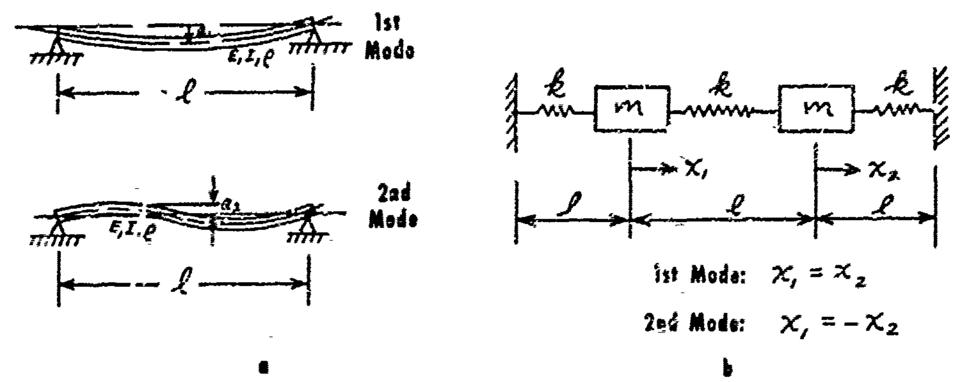


Figure 1. Two Systems in Which the Ratio of Vibratory Strain to Vibratory Velocity Is (a) and Is Not (b) Independent of Frequency.

Consider a measurement to be made of the peak velocity of X_1 which is to be related to the strain in the spring to the left of X_1. The strain is $\epsilon = \frac{a}{\ell}$. The peak velocity in either mode is $V = a\,\omega$. The ratio of measured velocity to peak strain is $\frac{V}{\epsilon} = \omega \ell$, which is dependent on the frequency. Clearly, then, in this case vibratory velocity alone would not be a sufficient measure of the destructive potential of vibration.*

It is recommended that a study be made to determine the extent to which vibration-related engine component failures can be reliably related to vibratory velocity alone. This study should consist of two phases:

1. A survey of vibratory engine failure histories made in conjunction with vibration surveys (flight test data analyses) of the same engine installations.

2. Analysis to relate vibratory strain in complex engine structural components to vibratory velocity measured at the locations recommended by engine manufacturers.

Another reason given by White[3] for preference of vibratory velocity as a measurement parameter is that the measured vibratory response tends to remain constant with frequency. It is recommended that a study be made to rationalize these observations on a sound theoretical basis, thus improving our understanding of engine vibratory response to both internal and external excitation.

The question of vibration transducer location is treated in Article 3.17.3 of MIL-E-8593 by the statement: "The points of attachment for the vibration detectors shall

*Based on the maximum strain theory.

be shown on the engine installation drawing, and shall provide for determination of vibration in three mutually perpendicular planes. In particular, the vibration pickup mounting points shall be located in close proximity of at least two main bearings."

The evident attempt in this statement to require transducers to be located such that bearing whirl (and thus, presumably, rotor whirl) would be detected is laudable, but it can be shown that the possibility exists for severe rotor whirl or vibration to occur without significant vibratory response of the external bearing housings or engine case.

Consider a simplified engine case/rotor assembly (Figure 2) in which the engine case is represented by a beam of uniformly distributed mass and stiffness, the engine rotor is represented by a massless beam with a centrally located massive disc, and the engine mounts and rotor bearings are represented by springs. Since the system is assumed to be linear, response to external (airframe) excitation and internal (engine unbalance) excitation can be considered independently and later added together to give the total response, if desired. The effect of damping is neglected.

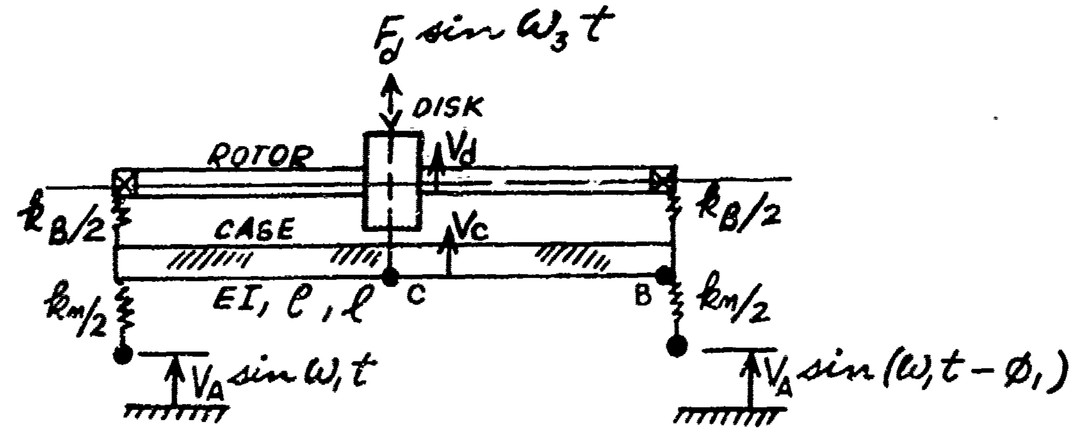


Figure 2. An Engine Case/Rotor System With Disc Unbalance and Vibrating Mount.

To demonstrate the relationship which a measured steady-state vibratory velocity at "B" or "C" on the engine case has to the velocity of the rotor disc, the mobility method of analysis will be used.

Impedance-mobility methods for vibration analysis are described clearly in Reference 4, and a condensed description is given in Appendix I of this report. These methods are especially well suited for problems encountered in establishing vibratory compatibility of helicopter propulsion components. Figure 3 shows a mobility model of the case/rotor assembly, with a velocity generator representing airframe excitation at the engine mounts.

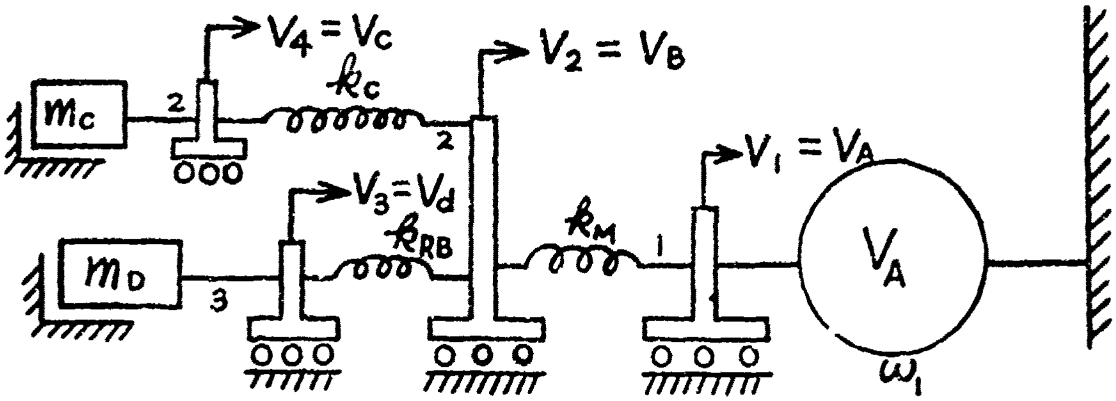


Figure 3. Mobility Diagram for Case/Rotor Assembly With Airframe Excitation.

The combined flexibility of the rotor and bearing supports is

$$k_{RB} = \frac{k_R k_B}{k_R + k_B}$$

The mobility equations are

$$\frac{V_1 - V_2}{F_1} = \frac{i\omega_1}{k_M} \quad , \quad \frac{V_2 - V_3}{F_3} = \frac{i\omega_1}{k_{RB}}$$

$$\frac{V_2 - V_4}{F_2} = \frac{i\omega_1}{k_C} \quad , \quad \frac{V_3}{F_3} = \frac{1}{i\omega_1 m_d}$$

$$\frac{V_4}{F_2} = \frac{1}{i\omega_1 m_c} \quad , \quad F_1 = F_2 + F_3$$

which can be reduced by substitution to the matrix equation

$$\begin{bmatrix} (k_c + k_m) - \omega_1^2 m_d & - k_c \\ k_{RB} & (\omega_1^2 m_d - k_{RB}) \quad 0 \\ k_c & 0 \quad (\omega_1^2 m_c - k_c) \end{bmatrix} \begin{Bmatrix} V_2 \\ V_3 \\ V_4 \end{Bmatrix} = \begin{Bmatrix} k_m V_1 \\ 0 \\ 0 \end{Bmatrix}$$

with solution

$$V_2 = V_B = \frac{k_m \, (\omega_1^2 m_d - k_{RB}) \, (\omega_1^2 m_c - k_c)}{D} V_1$$

$$V_3 = V_d = \frac{-k_{RB}k_m\ (\omega_1{}^2 m_c - k_c)}{D}\ V_1$$

$$V_4 = V_c = \frac{-k_c k_m\ (\omega_1{}^2 m_d - k_{RB})}{D}\ V_1$$

where

D = determinant of the matrix

$$= (k_c + k_m)\ (\omega_1{}^2 m_d - k_{RB})\ (\omega_1{}^2 m_c - k_c) + k_c{}^2\ (\omega_1{}^2 m_d - k_{RB})$$

$$+ k_{RB}\omega_1{}^2 m_d\ (\omega_1{}^2 m_c - k_c)$$

Of special interest are the ratios of vibratory velocity at the rotor disc to vibratory velocity at points "B" and "C" on the engine case:

$$\frac{V_3}{V_2} = \frac{V_d}{V_B} = \frac{-k_{RB}}{\omega_1{}^2 m_d - k_{RB}}$$

$$\frac{V_3}{V_4} = \frac{V_d}{V_c} = \frac{k_{RB}\ (\omega_1{}^2 m_c - k_c)}{k_c\ (\omega_1{}^2\ m_d - k_{RB})}$$

The conclusions that can be drawn from this model of engine respone to airframe excitation are as follows:

1. In general, rotor disc response does not follow the same trend as response at engine case location "B" or "C". In particular, $V_d/V_b \neq 1$, $V_d/V_c \neq 1$.

2. When $V_c = 0$ at antiresonance, $V_d = \frac{-k_m}{k_{RB}} V_1$. At other frequencies and when the airframe excitation is fixed, the best way of reducing V_c to "acceptable" levels is through a reduction in mount stiffness k_m, since this also reduces the rotor response V_d.

3. When $\frac{k_c}{m_c} < \omega_1{}^2 < \frac{k_{RB}}{m_d}$, that is, when the airframe excitation frequency falls between the first engine bending mode and the first rotor mode,

V_c and V_d are 180 degrees out of phase, a condition that maximizes the possibility for blade rub to occur. Practical values of damping will modify this conclusion only slightly.

4. The engine case response near bearing locations (point "B") could be zero under two conditions:

   (a) $k_{RB} = \omega_1^2 \, m_d$, or

   (b) $k_c = \omega_1^2 \, m_c$

For case (a), $V_d = \dfrac{-k_m}{k_{RB}} V_1$.

For case (b), $V_d = 0$.

Therefore, from the standpoint of engine rotor response, a low measured level of vibration at "B" can be more significant than a low level at "C", especially if the system is closer to case (b) than case (a) (which holds true, for example, in the CH-53/T64-6 installation[5]), or if the engine mounts are of low stiffness.

A similar analysis of the response to engine rotor unbalance (Figure 4) is given in Appendix I. The results lead to the following conclusions:

1. In general, rotor disc response does not follow the same trend as response at engine case locations "B" or "C". In particular, $V_d/V_b \neq 1$, $V_d/V_c \neq 1$.

2. If point "B" is at antiresonance ($V_B = 0$), then the rotor response to unbalance, $V_d = \dfrac{i\omega_2 F_1}{(\omega_2^2 \, m_d - k_{RB})}$, can be quite large (i.e., a low measured level of vibration near bearing supports may not indicate a low level of rotor response).

3. If $\dfrac{k_c}{m_c} < \omega_2^2 < \dfrac{k_{RB}}{m_d}$, and if the engine rotor unbalance is fixed, a reduction in mount stiffness k_m will lessen the rotor response to unbalance.

Of course, the mathematical models considered above are greatly oversimplified, and the conclusions may not be applicable to some real engine installations. However, they do illustrate how measurements of engine vibration at external locations may not be representative of the vibratory response of internal engine parts (specifically rotors).

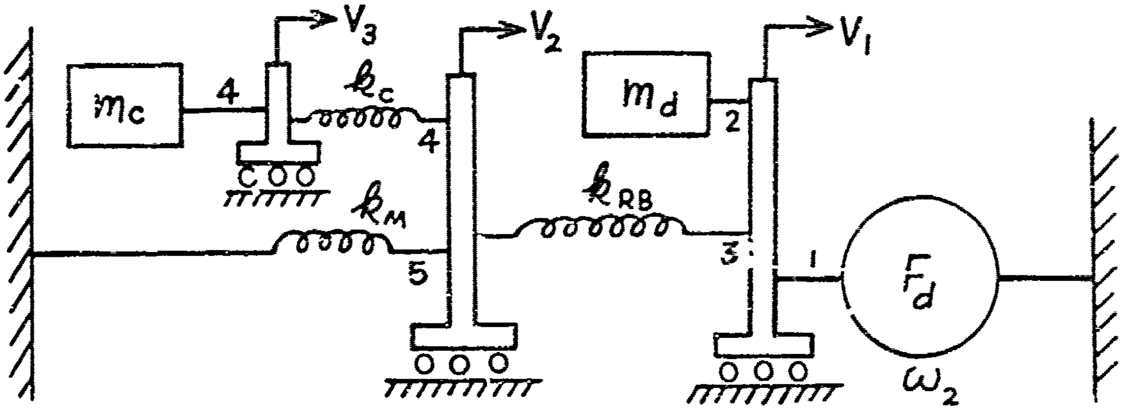


Figure 4. Mobility Diagram for Case/Rotor Assembly With Unbalance Excitation.

If engine case vibration is to be used as a measure of acceptability for engine vibratory limits, it is recommended that thorough investigations be conducted to determine the relationship of vibration at the points of measurement to response at other critical locations.

With regard to engine rotor response, it is recommended that special attention be given to the effect of squeeze film bearing dampers being used in the most recent engine designs. These dampers may render the conclusions reached from the mathematical models considered above invalid.

If external engine vibration measurements are to be related to engine rotor whirl and vibration, better methods of measuring rotor whirl will have to be developed. This is especially true of flexural whirl, which cannot be measured at bearing journal locations. It is therefore recommended that an investigation be made to determine the best methods for measuring engine rotor flexure under operating conditions, and that the most promising methods be developed to a practical state.

The final question to be answered before engine vibration limits can be standardized, with regard to the best method for data analysis, may have a multiple answer. The discrete frequency method, which is in fairly common use for expressing engine limits, has the following disadvantages:

- Amplitudes at several different frequencies may be individually within limits but may sum to unacceptable values, especially if one frequency is a multiple integer of another.

- It is difficult to perform discrete frequency measurements. They require sophisticated equipment and extensive processing.

On the other hand, measurement of overall vibratory amplitude, summed over all frequencies, may not be sufficient since there is no way of determining mode shapes or the relative contributions from airframe and engine excitation.

This question of data analysis is closely tied in with the previous question of transducer location. For example, an ambiguous aspect of some specifications presently furnished by engine manufacturers is caused by all measurements' being taken in one plane, especially a transverse plane. This type of specification makes a measurement of both amplitude and frequency almost mandatory.

It is recommended that vibration data acquisition and analysis on future programs be conducted with sufficient detail to identify both amplitudes and modes of response until enough is known about engine limits to allow a simplification of methods without sacrificing predictability of failures. The identification of modes will usual' require phase data in addition to frequency data, and will also require intelligent placement of transducers at more locations than is presently customary.

ENGINE/AIRFRAME INTERFACE DESIGN

An investigation to determine optimum engine/airframe interface characteristics of a medium transport helicopter was supported by the Navy BuWeps as an addition to the helicopter development program. Except for stiffness in the roll mode to resist engine torque, the engine mounts were made "soft" in order to isolate the engine from the airframe at frequencies above about 15 cps.

Results from the mobility analyses of the simplified engine case/rotor models considered in the previous section also indicate that, for several cases of interest, a reduction of engine mount stiffness can reduce vibratory response to both airframe excitation and engine rotor unbalance.*

The impedance-mobility methods of analysis have characteristics which make them uniquely well suited to the problem of determining optimum engine mount properties. This is especially true since the engine vibration response is to be measured only at a few select points of interest. The mobility methods to be illustrated below are formed in terms of the ratios of the vibratory velocities of these points of interest to the driving force. If the point of interest is the same point at which the driving force is applied, the ratio is called "driving point mobility"; otherwise, the ratio is "transfer mobility". The response of complex structures, or several combined structures, to steady-state excitation can be analyzed in terms of "mobility boxes", which greatly simplify the system conceptually and which eliminate the unnecessary consideration of internal parameters (i.e., parameters not associated with the points of interest).

Another factor which makes mobility methods especially well suited to engine/airframe interface design analysis is the usual early availability of the designated engine in hardware form while the airframe is still on the drawing board. Since the engine mobilities can be determined by testing, a very accurate and condensed model of the engine structure can be included in the overall dynamic model in the form of these mobilities.

Finally, there are theorems in mobility analysis, to be illustrated below, which greatly facilitate determination of the effects of inserting a single connecting element into a structure, such as an engine mount.**

Consider first the problem of determining the effect of replacing a very stiff engine mount by a softer one in an already existing engine/airframe design. Figure 5 shows the airframe and engine represented schematically by mobilities M_1 and M_2, with

*A significant factor not considered, however, is engine-transmission shaft misalignment, which can become a serious problem when soft engine mounts are used

**Engine mount is used in the singular sense here because the models to be used for illustration will be simple and one dimensional, requiring combination of the effects of all mounts, say, in the vertical direction, into a single effective "mount".

points 1 and 2 to be connected rigidly. The airframe mobility M_1 is a driving-point mobility looking back from engine mount point 1. The engine mobility M_2 is a driving-point mobility looking into the point of attachment 2. A source of airframe excitation is designated by $Fe^{j\omega t}$, representing a vector component (say, vertical) of the sum of aerodynamic blade forces at the hub for frequency ω .

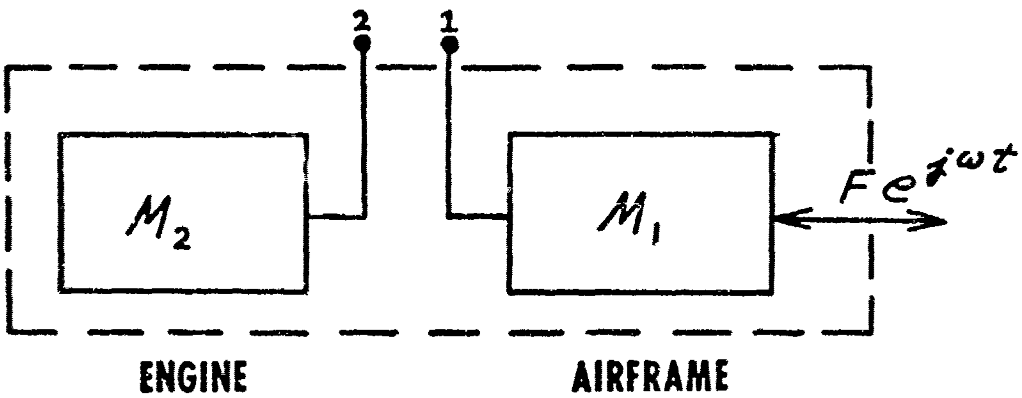


Figure 5. Mobility Schematic of Engine and Air' ..ne To Be Connected by Mounting Element M_e Between Points 1 and 2.

Figure 6 shows an equivalent system obtained by application of Norton's theorem,[4] which can be used to determine the force and relative velocity across the soft mount.

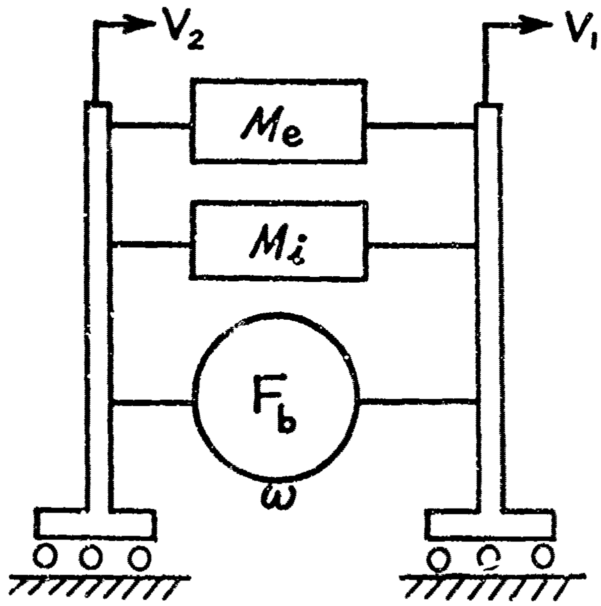
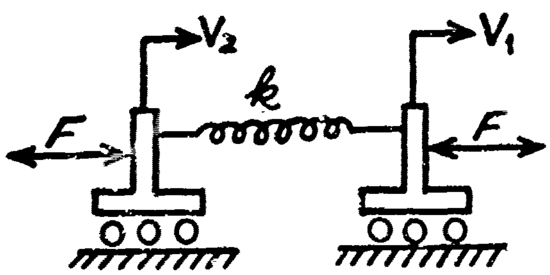


Figure 6. Equivalent System (for M_e) Obtained by Application of Norton's Theorem.

M_e represents the mobility of the soft mount, M_i is the "internal mobility" of the engine/airframe system without M_e, and F_b is the steady-state force that would exist in a rigid link connecting points 1 and 2 (i.e., in the rigid mount). The internal mobility M_i is simply the sum of driving point mobilities M_1 and M_2 .

An analysis of the equivalent system yields the steady-state force F_e in the soft mounts and the relative velocity between engine and airframe at the points of attachment. The force F_e can then be used in conjunction with engine transfer mobilities (perhaps experimentally measured) to predict vibratory response at points of interest on the engine.

To illustrate this method, let the airframe and engine be simply represented by single rigid masses m_1 and m_2 respectively, and let the soft engine mount consist of a spring of stiffness k . From Figure 7, the engine mount mobility is given by $M_e = i\omega/k$ (see Appendix II).



$$M_e = \frac{V_1 - V_2}{F} = \frac{i\omega}{k}$$

Figure 7. Mobility M_e of Soft Engine Mount.

From Figure 8, the internal mobility is

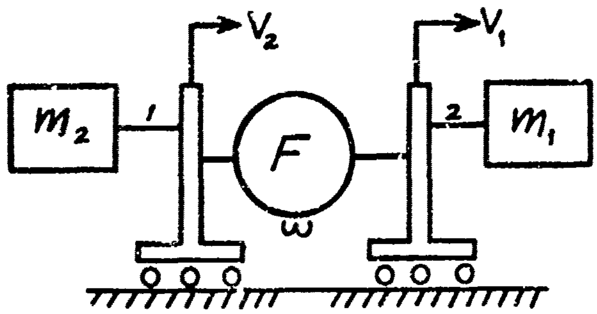
$$M_i = \frac{m_2 + m_1}{i\omega m_1 m_2} = M_1 + M_2$$

From Figure 9, the "blocked force" is $F_b = \frac{m_2}{m_1 + m_2} F$. An analysis of the equivalent system shown in Figure 6 yields

$$F_b = F_e + F_i = \frac{V_1 - V_2}{M_e} + \frac{V_1 - V_2}{M_i}$$

$$V_1 - V_2 = F_b \frac{M_e M_i}{M_e + M_i} = \frac{\omega^2 m_2 \; F}{i\omega\,[\,\omega^2 m_1 m_2 - k\,(m_1 + m_2)\,]}$$

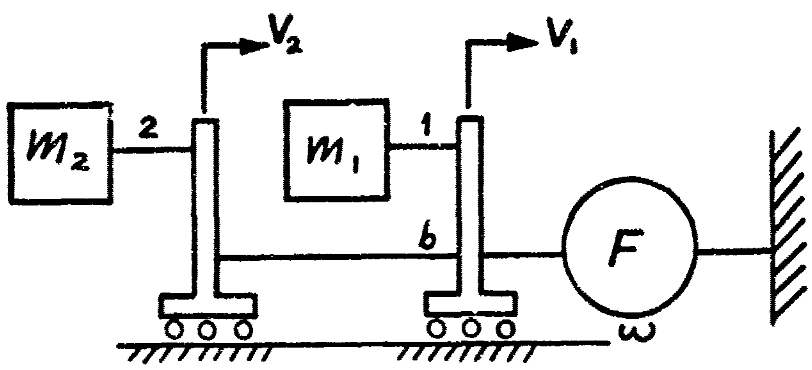
$$F_e = \frac{V_1 - V_2}{M_e} = \frac{-\,k m_2 \; F}{\omega^2 m_1 m_2 - k\,(m_1 + m_2)}$$



$$F = F_1 = F_2$$

$$M_i = M_{21} = \frac{V_1}{F} - \frac{V_2}{-F} = \frac{V_1 + V_2}{F} = \frac{m_2 + m_1}{i\omega\, m_1 m_2}$$

Figure 8. Internal Mobility of Engine/Airframe Assembly.



$$V_1 = V_2 \;; \quad F = F_1 + F_2 \;; \quad F_b = F_2$$

$$\frac{V_1}{F_1} = \frac{1}{i\omega m_1} \;; \quad \frac{V_1}{F_2} = \frac{1}{i\omega m_2}$$

$$F_b = F_2 = i\omega m_2 V_1 = \frac{m_2}{m_1 + m_2} F$$

Figure 9. Blocked Force F_b in Rigid Mount.

Then, to find V_2 ,

$$\overline{M}_2 = \frac{V_2}{F_e} = \frac{1}{i\omega m_2} , \quad V_2 = \frac{-kF}{i\omega [\omega^2 m_1 m_2 - k (m_1 + m_2)]}$$

Of course, for such an oversimplified model, a less complicated method of analysis will yield the same result. However, the example illustrates the following possibilities for real systems:

1. The blocked force F_b can be directly measured in an existing helicopter with a rigid engine mount.

2. The internal mobility M_i is made up of driving point mobilities M_1 and M_2, either one of which can be either calculated using existing computer analyses or measured in a shake test.

3. For a more realistic model, the mobility $\overline{M}_2$ used in the final step would be a transfer mobility for the response of some particular point of interest on the engine; this mobility could be accurately determined from engine shake tests.

A similar scheme for applying Norton's theorem in terms of electrical analogies is given in Reference 6 for a missile-payload problem.

Another theorem that can be used to advantage in dynamic interface problems is Thévenin's theorem.[4] Figure 10 shows the airframe represented by mobility M_1, excitation at the rotor hub represented by a harmonic forcing function, and with an engine and mount assembly to be represented by mobility M_e and installed between points 1 and 3, where 3 is ground. Figure 11 shows the equivalent system obtained by Thévenin's theorem. M_i is the driving-point mobility looking back into the air-frame from the engine mount point 1, and V_o is the free vibratory velocity of point 1 without M_e.

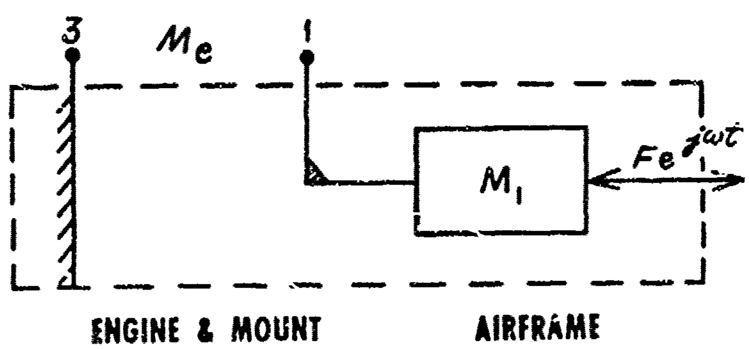


Figure 10. Mobility Schematic of Airframe With Engine and Mount To Be Installed Between Points 1 and 3.

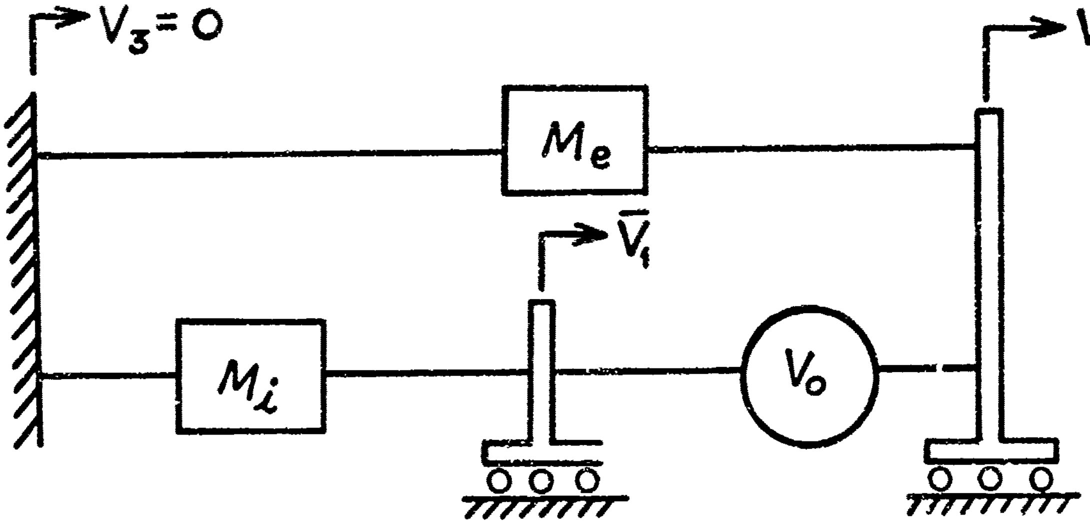


Figure 11. Equivalent System (for M_e) Obtained by Application of Thévenin's Theorem.

As a simple illustration of application, consider the same system as for the previous example, with the airframe and engine each represented by a rigid mass and the engine mount represented by a spring.

From Figure 12, the mobility of the engine/mount assembly is

$$M_e = \frac{k - \omega^2 m_2}{i\omega k m_2}$$

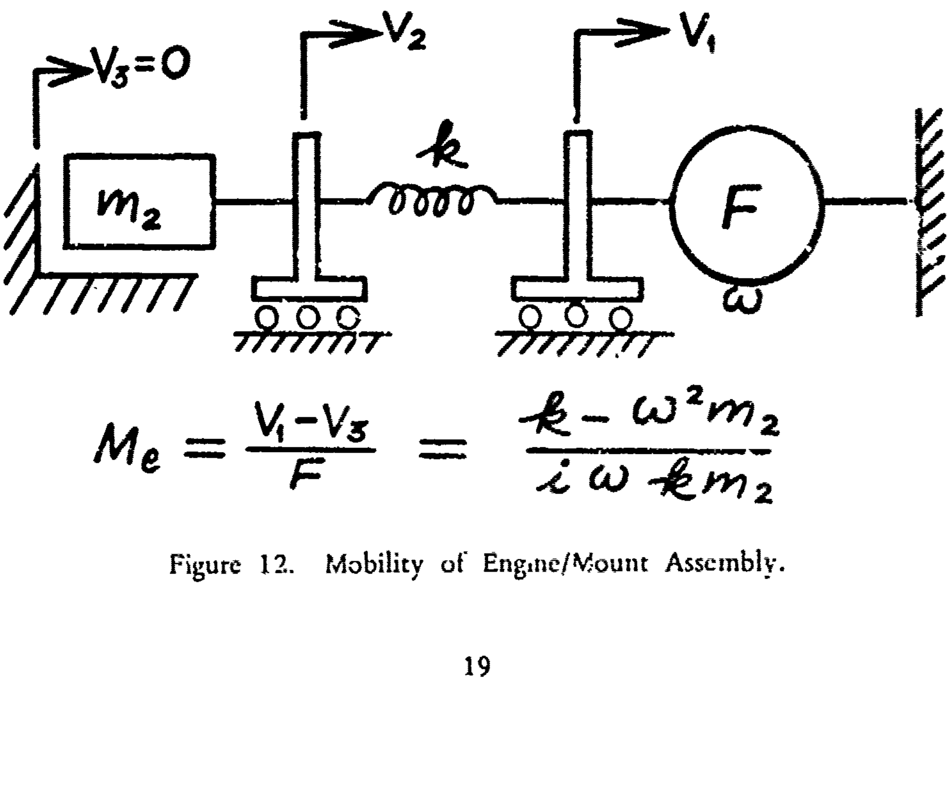
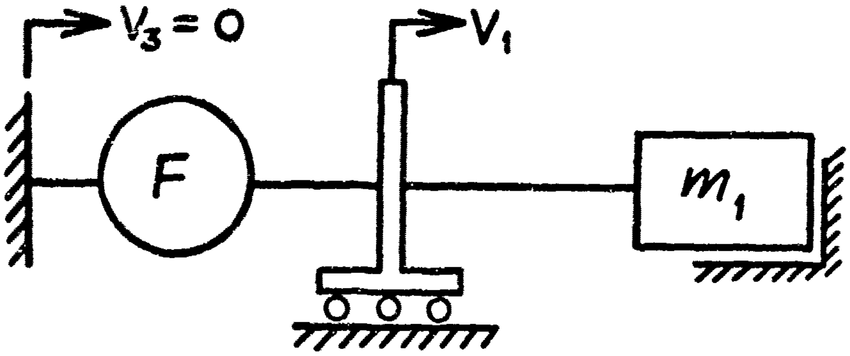


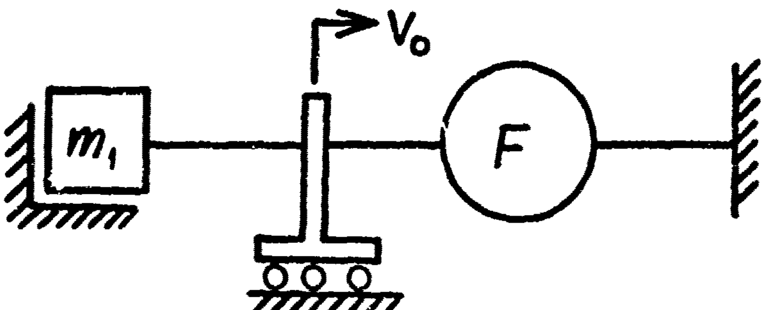
Figure 12. Mobility of Engine/Mount Assembly.

From Figures 13 and 14, the internal mobility of the airframe is $M_i = \frac{1}{i\omega m_1}$, and the free velocity of the engine mount point is $V_o = \frac{F}{i\omega m_1}$.



$$M_i = \frac{V_1 - V_3}{F} = \frac{V_1}{F} = \frac{1}{i\omega m_1}$$

Figure 13. Internal Mobility of Airframe.



$$\frac{V_o}{F} = \frac{1}{i\omega m_1} \quad , \quad V_o = \frac{F}{i\omega m_1}$$

Figure 14. Free Velocity of Mount Point.

Analysis of the equivalent system shown in Figure 11 yields the force in the engine as

$$F_e = \frac{V_o}{M_e + M_i} = \frac{-km_2 \ F}{\omega^2 m_1 m_2 - k(m_1 + m_2)}$$

Once again, the oversimplification of the system does not justify the method of analysis. However, for realistic models, it can be seen how the mobility method allows experimental measurements or computer results to be integrated into the analysis for maximum advantage. This might be done in the analysis just described as follows:

- The mobility M_e can be determined from an engine shake test.
- The internal mobility M_i can be calculated with existing computer programs for structural dynamics, such as NASTRAN (see Appendix II).
- The free velocity V_o can be determined from a program such as NASTRAN and later verified by an airframe shake test.
- For a realistic model, the final step would require a transfer mobility V_N/F_e, where V_N is the velocity of a point of interest on the engine. This mobility can be obtained from an engine shake test.

More extensive application of mobility methods to the problems of engine/airframe vibration should make the relatively independent analytical amd experimental studies of engine contractors and airframe contractors easier to combine, resulting in more compatible systems.

Therefore, it is recommended that studies be made to develop schemes similar to those illustrated on the simple models above for applying mobility methods to realistic multidimensional analyses of helicopter/engine installations. References 6 through 9 may be helpful as a starting point.

In developing these methods, special efforts should be made to use the already existing computer analyses and vibration test facilities that have been developed by the engine and airframe contractors for vibration studies. A list of the major computer programs* that are applicable to the subject program is given in Appendix II.

*The list is restricted to those programs that the author became aware of during this study.

DRIVE TRAIN DYNAMICS

Whirling and vibration of shafts, couplings, and gearboxes are significant sources of dynamic loads in helicopter propulsion systems. The recent vibration-related engine failures encountered in helicopters illustrate how engine/airframe interface characteristics, drive shaft whirl, and fuel control response can all interact in complex ways to cause problems.

In particular, cross shaft problems point out an area in which design analysis can be profitably improved. Figure 15 is a schematic of a typical shaft assembly, which is typical of helicopter drive shafts. Figure 16 shows the shape of the first flexural whirl mode, which is generally assumed to be the speed-limiting factor for subcritical shaft design. Figures 17 and 18 show "rigid-body" modes that can occur at speeds less than the first flexural critical speed under the following design conditions:

1. Couplings capable of allowing angular misalignment used in conjunction with splines requiring radial clearance. This includes the typical cross shaft design as well as designs using Hooke's joints.

2. Couplings capable of allowing angular misalignment used in conjunction with flexible bearing supports or housings on the outboard ends.

3. Couplings capable of allowing both angular and radial misalignment, even if used without splines and with rigid bearing supports.

The last case is considered and discussed in Reference 10, which is one of the few published analyses of this type of problem.*

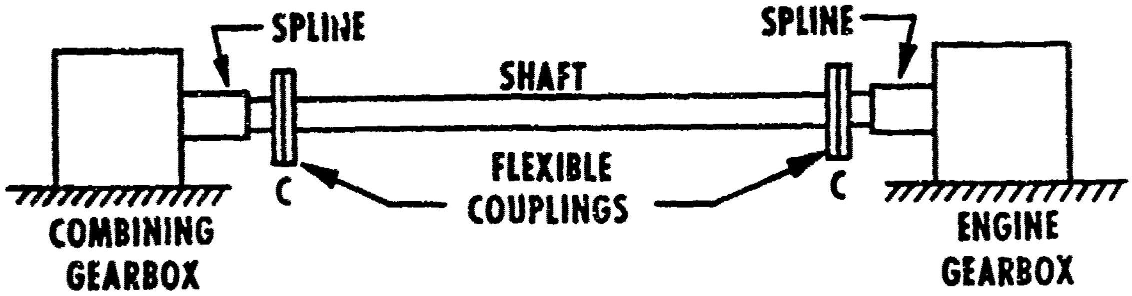


Figure 15. Schematic of Typical Cross Shaft Assembly.

*Russian literature excepted.

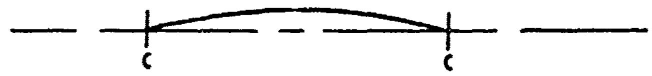


Figure 16. First Flexural Mode Shape for Drive Shaft.

Figure 17. Rigid-Body Mode Shape, Cylindrical Whirl.

Figure 18. Rigid-Body Mode Shape, Conical Whirl.

Rigid-body modes, which can occur in many drive shaft configurations using flexible couplings, are often ignored in the preliminary design analysis of a helicopter propulsion system. If whirling problems occur at a later stage of development, these modes may then be investigated, but the analytical tools generally used for this purpose have some or all of the following limitations:

1. The whirling problem is represented by a lateral vibration model.

2. As a result of 1, gyroscopic moments and the coupling of whirl with lateral vibration of housings are often neglected.

3. The effect of rotational acceleration on whirl is neglected.

4. The effect of internal friction in couplings, splines, sleeves, or U-joints is neglected.

These problems have been generally overlooked or underestimated in the past because unbalance response in synchronous rigid-body whirl modes :, usually limited in amplitude by nonlinearities or discontinuities in system parameters, thus allowing passage through these critical speeds without dire consequences. However, the last-mentioned effect of internal friction can cause a drive shaft to become unstable and self-destructive at speeds above the rigid-body criticals. Cases of severely violent whirl in shaft-coupling systems which have been observed in past development programs have sometimes been attributed to this effect, but the solution has usually been to change the design*, thus preventing verification of the hypothesis or analysis of the cause.

*Such as a change to a different type of shaft coupling, perhaps sacrificing desirable characteristics of the original coupling that was the basis of its choice.

A quote from Reference 10 describes the situation for a particular type of coupling, but it applies to many other configurations as well:

> "A significant feature of the results for this example is the wide gap which exists between the second and third critical speeds. This gap may provide a suitable region for high speed, super-critical applications. Such operations could permit the Bossler couplings to be designed with more misalignment capability and with less weight. Supercritical operation also opens the possibility for very smoothly operating designs which employ dynamic self-balancing. However, it is important to note that a potential problem area also exists. The built-up nature of the Bossler coupling creates a possibility for a whirling instability involving non-synchronous precession (Reference 11) when operated above the first critical speed. The dynamic stability characteristics of supercritical systems employing Bossler couplings are unknown at present."

It is important to note that the critical speeds referred to above are rigid-body criticals, not the flexural criticals which are often used as criteria for shaft design and which the research on supercritical shaft design by Battelle Memorial Institute[11] was related to.

A report of whirling induced by internal friction in a pinion shaft test rig is given in Reference 12. Reference 13 reports a similar occurrence and the "fix" for a turbine engine rotor shaft.

The source of internal friction inducing the whirl reported in Reference 13 was a spline coupling. In analyzing the problem, the spline friction force had to be calculated from assumed values of the friction factor, since no appropriate data on spline friction was available.

The tendency for whirl speed to remain constant regardless of shaft speed, a characteristic of friction-induced whirl, is illustrated by an experimental observation from Reference 12:

> "Rotating single-mass systems with large spans continued to whirl at the natural frequency as the rotating speed increased to twice that frequency."

This could be a partial explanation of the wide discrepancies sometimes noted between vibration measurements made on the same helicopter propulsion system by different investigators, since filters are sometimes used which admit only the frequencies expected at speeds synchronous with the various rotational speeds.

Even if the rigid-body modes in a drive train do not result in friction-induced whirl instability, the synchronous response to unbalance can cause rapid wear of couplings, splines, and bearings, especially during start-up, when grease may be cold and clearances may be large.

It is therefore recommended that a study be made to identify all of the specific rigid-body shaft whirl modes that can and do occur in contemporary helicopter propulsion systems, and that analyses be conducted to determine ways to reduce the magnitude of response to these modes and to ensure stability under all operating conditions.

In order to effectively apply the results of this study, it will be necessary to have detailed information on the dynamic properties (such as stiffness, damping, etc.) of the different types of couplings now in use or being considered for use. It is therefore recommended that experimental studies of these properties be made and methods developed for predicting these properties under various operating conditions.

One of the oldest and simplest types of shaft coupling is the Hooke's joint, or universal joint. This coupling has proved to be efficient and reliable in many applications for transmitting torque while accommodating misalignments, but recent high-speed applications have proved troublesome. As a result, other more expensive types of couplings have been developed for high speeds which sacrifice some of the desirable properties of the U-joint associated with its simplicity.

At high speeds, the dynamics of a shaft with U-joint couplings are quite complex. This is due in part to the oscillating speed characteristic of the coupling and in part to its bearing friction which plays the part of the internal friction discussed above. As a result, when past experience with U-joint couplings in low-speed applications has been applied to high-speed power transmission shafts, problems of whirl and vibration have been encountered.

It is possible that a more complete understanding of the dynamics of shafts with U-joint couplings can make possible an advantageous use of these couplings in high-speed applications

It is therefore recommended that a study be made, preferably both analytical and experimental, to determine optimum design criteria for smooth and efficient operation of shafts with U-joints at high speeds for helicopter applications. Reference 14, which treats torsional dynamic effects and bending moments induced by misalignment, may be found helpful as a starting point.

Finally, shaft couplings should be recognized as potential transmitters of vibration from one component to another in a drive train, as well as sources of excitation.

For example, the engine-to-transmission drive shaft on the UH-1 helicopter uses a spherical spline coupling to accommodate misalignment between the rigidly mounted engine and the compliantly mounted transmission. The transverse vibratory shears and moments transmitted by spline friction across the interface are unknown quantities. It can be surmised that these shears and moments vary with spline clearance, lubrication, misalignment, transmitted torque, vibratory velocity or displacement, and vibratory frequency.

It is recommended that measurements be made on existing helicopters to determine the magnitude of the transverse vibratory shears and moments transmitted across spline shaft couplings used in power transmission shafts.

If these measurements indicate that the transmitted shears and moments are significant, it is recommended that experimental and analytical research be carried out to determine how they vary with the factors listed above or with any other factors found to be significant.

It should be possible to predict analytically the transverse shears and moments transmitted across other types of couplings by using information obtained from the studies of coupling dynamic properties which were recommended earlier for shaft whirl analysis.

TORSIONAL STABILITY OF HELICOPTER DRIVES WITH AUTOMATIC FUEL CONTROL

Although this topic properly belongs in the preceding section on drive train dynamics, it has been treated independently by SAE-Aerospace Recommended Practice 704 because of its connection with automatic speed governors, and accordingly it is treated as a separate topic here.

In the past, the approach to designing helicopter drive systems with automatic speed control has been to define a set of fuel control characteristics which, when combined with a predetermined mechanical drive, would result in acceptable control response without objectionable oscillations or instabilities. This approach has not always led to optimum speed control characteristics, since the desired high gain of the governor must sometimes be sacrificed to obtain stability.

The methods and tools presently available for stability analysis are not always successful in predicting conditions for torsional stability in helicopter propulsion systems. This is due in part to the linear restrictions on analytical tests for stability and in part to the designer's incomplete knowledge of system parameters.

For example, Reference 15 reports on stability problems encountered in a medium transport helicopter which were not predicted by the initial design analysis. A Holzer torsional analysis initially predicted a natural frequency involving rotor blade lag motion which turned out to be about 25 percent too low. A study of system parameters eventually revealed that the lag damper characteristics used in the analysis were in error. It is significant that the final solution was to reduce the fuel control gain, since a proposed modification to the lag damper was reported to produce unacceptable ground resonance characteristics. It can also be observed that a fuel control modification is usually less expensive to an airframe manufacturer than a design change in the helicopter drive train or rotor.

Another point of significance in this example is that analytical predictions of drive train response were ultimately made from digital simulations, marching out step-by-step solutions to the equations of motion for each test case. That is, linear servo analysis was not adequate for the problem due to the inherent nonlinearities and complexity of the system. Computational expense could have been greatly reduced or eliminated if a satisfactory method had been available for finding regions of stability in concise form for complex nonlinear systems.

One possibility is to extend the present methods for predicting stability of linear systems to include "equivalent linear" systems. That is, the significant nonlinearities in a drive train and governor might be represented by predictable linear characteristics to produce nearly the same response.

Another possibility is to apply the second method of Lyapunov, which is the most general approach currently available for the study of stability in dynamic systems.[16] The generality of the method is the basis of considerable difficulty encountered in developing systematic techniques for its application to specific classes of problems. Significant work has been done, however, toward developing methods which are well suited to high-speed digital computation of the "Lyapunov functions".[17]

A second-rate substitute or first-class companion to the development of better stability analyses would be the development of faster and more efficient marching solutions for digital simulation. For example, a method for direct application of Hamilton's principle to the derivation of first-order difference equations for dynamic systems has been derived by the author of this report.[18] Since this method eliminates the need to take second derivatives in writing equations of motion and thereby eliminates one of the usual steps in machine computation,* it should be possible to reduce man and/or machine effort in obtaining solutions to test cases.

It is therefore recommended that studies be made to develop improved methods for stability analysis of complex nonlinear drive systems with closed-loop fuel control, and/or methods for more rapid digital simulation of drive train dynamic response.

As pointed out in the introductory paragraphs to this section, the usual solution to drive train torsional instabilities is to modify the fuel control characteristics. This is because modifications to the mechanical components of already existing shafts, transmissions, and rotors are costly and sometimes result in less-than-optimum system performance by some other criteria. On the other hand, the modifications usually necessary to the fuel control to obtain stability often result in poor speed control response characteristics.

It is possible that new mechanical components for helicopter drive trains can be developed which will allow improvements in speed control response without sacrificing torsional stability.

For example, Reference 19 describes the design and application of a shaft coupling which uses centrifugal force to provide a zero torsional stiffness characteristic at a particular point on the torque-speed curve. The zero stiffness property of the coupling effectively decouples the torsional inertias on opposite sides of the coupling and has been found to eliminate the first resonant mode completely and to reduce the second mode significantly in a two-degree-of-freedom shaft-rotor system. The coupling is now in service in a tugboat drive, with one of the previously resonant torsional frequencies completely eliminated. A similar coupling developed in Russia is described in Reference 20.

It is recommended that the application of zero-torsional-stiffness couplings to helicopter drive shafts be investigated with the objective of eliminating or favorably modifying resonant modes of torsional oscillation.

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*For example, in the Runge-Kutta integration algorithm, the exact first integrals of many of the accelerations are obtained without numerical integration.

It is also recommended that favorable consideration be given to the development of any other mechanical devices which appear capable of enhancing the torsional stability of helicopter drive trains without compromising speed control characteristics.

CONCLUSION

The basic conclusion of this report is that the reliability and performance of U. S. Army helicopters can be improved by specific research and development work in the areas of engine/airframe vibratory compatibility, power transmission shaft dynamics, and drive system/governor stability. The programs which appear to offer the greatest potential for practical results at the present time are described in the Summary of Recommendations.

SUMMARY OF RECOMMENDATIONS

1. It is recommended that existing flight test data on engine vibration in helicopters be analyzed to determine:

   - The spectrum of engine vibration amplitudes for typical military missions.
   - The relative contributions of engine and airframe to engine vibration.
   - Amplitude versus discrete frequency over the range of 10 to 10,000 Hz for flight conditions dominating the mission profiles.
   - Predominant responding mode shapes for the engine, both rigid-body and flexural.

   A list of some of the presently available flight test data is given in Appendix I.

2. It is recommended that additional flight test data be acquired from a representative group of the different types of helicopters (i.e., single rotor, tandem rotor, two blades, four blades, etc.) for analysis as described in recommendation 1.

3. It is recommended that a study be made to determine the extent to which vibration-related engine component failures can be reliably related to vibratory velocity alone (with no frequency dependence). This study should consist of two phases:

   - A survey of vibratory engine failure histories made in conjunction with vibration surveys (flight test data analyses) of the same engine installations.
   - Analysis to relate vibratory strain in engine structural components to vibratory velocity measured at the locations recommended by engine manufacturers.

4. It has been observed that measured engine vibratory velocity tends to remain constant over a wide frequency range. It is recommended that a study be made to rationalize this observation on a sound theoretical basis, thus improving our understanding of engine vibratory response to both internal and external excitation.

5. It is recommended that investigations be conducted to determine the relationship of vibration at recommended points of measurement to vibratory response of critical engine components, especially rotors. With regard to engine rotor response, it is recommended that special attention be given to the effect of the squeeze film bearing dampers being used in many recent engine designs.

6. It is recommended that an investigation be made to determine the best methods for measuring engine rotor flexure under operating conditions, and that the most promising methods be developed to a state that will encourage their use.

7. It is recommended that vibration data on future helicopter development programs be acquired and analyzed in sufficient detail to identify both amplitudes and modes of responses until enough is known about engine vibratory limits to allow a simplification of methods without sacrificing predictability of failures. The identification of modes will usually require phase data in addition to frequency data, and will also require intelligent placement of transducers at more locations than is presently customary.

8. It is recommended that impedance-mobility methods be developed for engine/airframe vibratory interface analysis. A primary objective should be to achieve optimum use of mobilities obtained from shake tests or from existing computer programs in the overall analysis. A list of some existing computer programs is given in Appendix II.

9. It is believed that problems associated with rigid-body whirl modes* in helicopter drive shafts with flexible couplings have been treated too lightly in design analysis. It is therefore recommended that a study be made to identify all of the specific rigid-body shaft whirl modes that can and do occur in contemporary helicopter drive trains, and that analyses be conducted to determine methods for reducing the magnitude of response to these modes and ensuring stability under all operating conditions.

10. It is recommended that experimental studies be made to determine the dynamic properties, such as stiffness, damping, and inertia, of the different types of shaft couplings now in use or being contemplated for use. Although some of these properties are known and available, they are incomplete for an adequate dynamic analysis. In making these studies, shaft couplings should be recognized as potential transmitters of vibration from one component to another in a drive train, as well as sources of excitation.

*Modes that cannot be determined simply from a consideration of lateral bending vibration of shafts, modeled as beams.

11. It is recommended that an analytical and experimental study be made to determine optimum design criteria for smooth and efficient operation of shafts with Hooke's joints at high speeds for helicopter applications. Reference 14, which treats torsional dynamic effects and bending moments induced by misalignment, may be helpful as a starting point. The effects of flexible bearing mounts or support housings should not be neglected unless they are proved to be insignificant for a particular configuration.

12. It is recommended that the magnitude of the transverse vibratory shears and moments transmitted across spline shaft couplings used in power transmission shafts be measured on existing helicopters. If these measurements indicate that the transmitted shears and moments are significant, it is recommended that experimental and analytical studies be made to determine how they vary with the pertinent parameters, such as spline clearance, misalignment, and transmitted torque.

13. It is believed that the methods presently used for torsional stability analysis of governor-controlled rotor drive systems are not adequate for the problems being encountered. It is therefore recommended that studies be made to develop improved methods for stability analysis of complex nonlinear drive systems with closed-loop fuel control, and/or methods for more rapid digital simulation of drive train torsional response.

14. It is recommended that the application of zero torsional stiffness couplings to helicopter drive shafts be investigated with the objective of eliminating or favorably modifying resonant modes of torsional oscillation. It is also recommended that favorable consideration be given to the development of any other mechanical devices that appear to be capable of enhancing the torsional stability of helicopter drive trains without compromising speed control characteristics.

LITERATURE CITED

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APPENDIX I
IMPEDANCE – MOBILITY METHODS FOR VIBRATION ANALYSIS

Impedance – mobility methods are ideally suited for steady-state vibration analysis of complex structures when the following conditions prevail:

- Vibration is to be measured at only a few selected points in the structure, and the response of these points is to be predicted by analysis.
- It is desired to combine experimental measurements with the analytical model so as to improve the accuracy of simulation.
- It is desired to predict the effect of changing specific elements in isolated parts of the structure or at an interface
- It is desired to predict the vibratory response of an assemblage of component structures, when the vibratory response of each component is known.

Impedance – mobility methods were first developed by electrical engineers for circuit design analysis, and they were later adopted for vibration analysis by some mechanical engineers. However, acceptance of these methods has not been widespread in the mechanical engineering field, mainly because they are so often presented in terms of electrical analogies. An excellent elementary position of these methods is given in purely mechanical terms in Reference 4. The remainder of this appendix will be devoted to summarizing a few of the basic features of mobility analysis and motivating their application to helicopters.

In mobility analysis it is convenient to express vibratory force and velocity as complex quantities:

$$f = Fe^{i\omega t} = F\ (\cos\ \omega t + i\ \sin\ \omega t) \tag{1}$$

$$v = Ve^{i\omega t} = V\ (\cos\ \omega t + i\ \sin\ \omega t) \tag{2}$$

where F and V in general are complex.

In linear vibration analysis of lumped parameter systems, the three basic physical elements are the mass, damper, and spring. The equations of equilibrium for these components are

$$\text{mass:} \quad m\,\frac{dv}{dt} = f \tag{3}$$

$$\text{damper:} \quad cv = f \tag{4}$$

$$\text{spring:} \quad k\int_{t_o}^{t} v dt = f \tag{5}$$

Substitution of Equations (1) and (2) into (3), (4), and (5) yields

$$\text{mass:} \quad i\omega mV = F \tag{6}$$

$$\text{damper:} \quad cV = F \tag{7}$$

$$\text{spring:} \quad \frac{k}{i\omega} V = F \tag{8}$$

It should be recalled that the imaginary component of a complex vector corresponds to the sine term of harmonic motion while the real component corresponds to the cosine term. Thus the complex notation expresses both the amplitude of a vibratory vector (given by the modulus) and the phase angle.

Mechanical impedance is defined as the ratio of vibratory force to vibratory velocity,

$$Z = \frac{F}{V}$$

Mobility is the inverse of impedance and is therefore defined as the ratio of vibratory velocity to vibratory force.

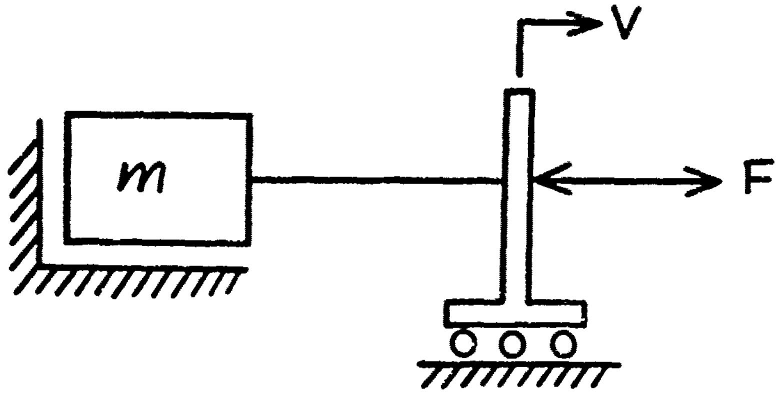
$$M = \frac{V}{F} = \frac{1}{Z}$$

The choice of either impedance or mobility as a working quantity is basically arbitrary, but may be influenced by special characteristics of the problem of interest. Mobility will be used in this report, because velocity is the quantity to be measured most often, while the excitation will often be expressed in terms of force.

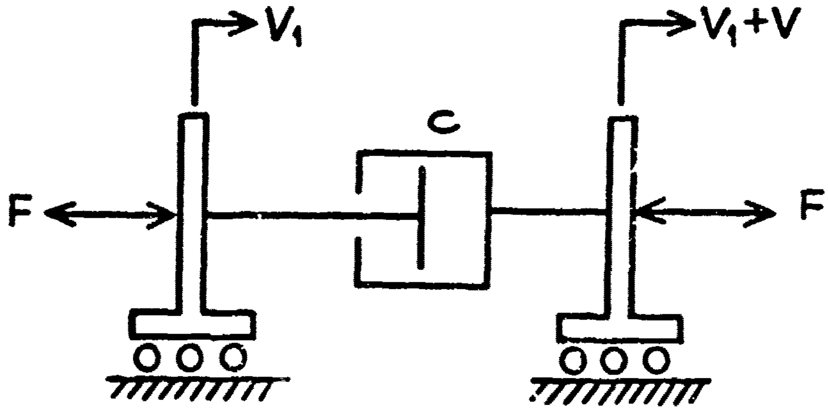
If the velocity and force are taken at the same point in a structure, the resulting mobility is called "driving point mobility". If the velocity and force are taken at different locations, the resulting mobility is called "transfer mobility".

Every element or combination of elements in a structure has a characteristic mobility. The mobilities of the three basic physical elements are obtained from Equations (6), (7), and (8), as shown in Figure 19, by dividing the relative velocity across each element by the exciting force. Special notice should be taken of the fact that the velocity of one side of a mass element must always be taken as zero, since Newton's Second Law is valid only in an inertial reference frame. That is, the velocity used to obtain mass mobility must be absolute.

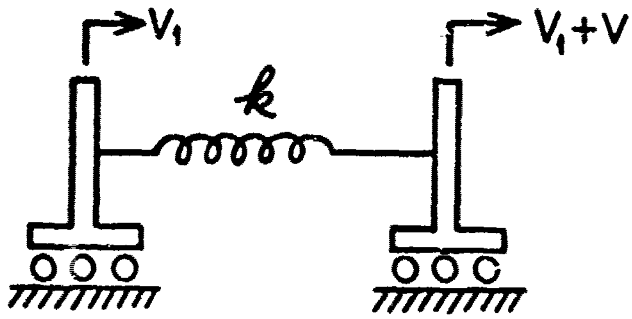
Mobilities of combinations of elements are obtained in exactly the same way, and the analysis of any structure is reduced to the solution of a set of algebraic equations by the requirements of velocity compatibility and force equilibrium.



Mass Mobility $= M_m = \frac{V}{F} = \frac{1}{i\omega m}$ **from Equation (6)**



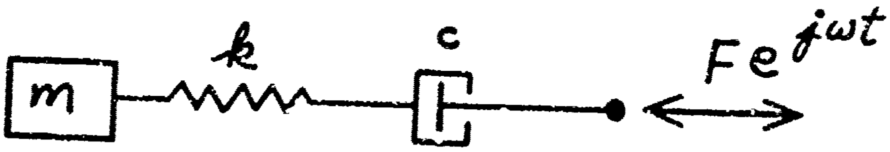
Damper Mobility $= M_c = \frac{V}{F} = \frac{1}{c}$ **from Equation (7)**



Spring Mobility $= M_k = \frac{V}{F} = \frac{i\omega}{k}$ **from Equation (8)**

Figure 19. Mobilities of the Basic Elements.

For example, consider a damper, spring, and mass connected in series and excited by a harmonic force generator. A mobility schematic is shown in Figure 20, with the links numbered to identify forces.



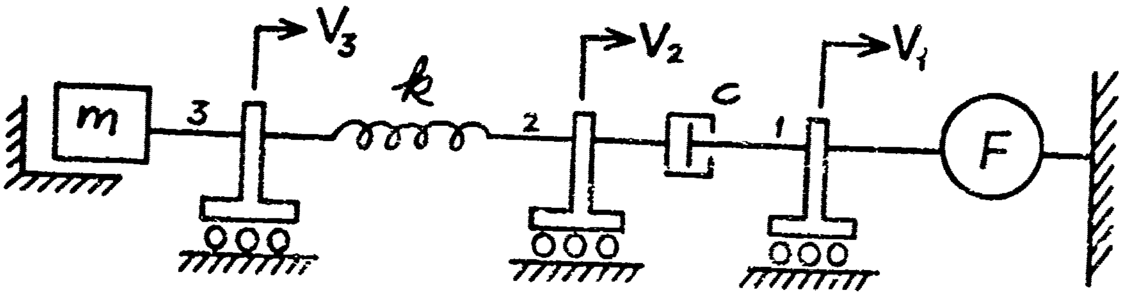


Figure 20. Mobility Schematic for Damper, Spring, and Mass in Series.

The mobility equations are

$$\frac{V_1 - V_2}{F_1} = \frac{1}{c} ; \frac{V_2 - V_3}{F_2} = \frac{i\omega}{k} ; \frac{V_3}{F_3} = \frac{1}{i\omega m}$$

and equilibrium of forces requires

$$F_1 = F_2 = F_3 = F$$

The driving point mobility which the force generator "looks in to" is calculated as

$$M_1 = \frac{V_1}{F} = \frac{V_1 - V_2}{F} + \frac{V_2 - V_3}{F} + \frac{V_3}{F}$$

$$= \frac{1}{c} + \frac{i\omega}{k} + \frac{1}{i\omega m}$$

$$= \frac{m\omega k + ic\ (\omega^2 m - k)}{m\omega kc}$$

which illustrates the basic rule that mobilities connected in series can be added to obtain the mobility of the resulting assembly. The transfer mobility for velocity V_2 is

$$M_{21} = \frac{V_2}{F} = \frac{V_1}{F} - \frac{V_1 - V_2}{F} = M_1 - \frac{1}{c}$$

$$= \frac{i\,(\omega^2 m - k)}{k\omega m}$$

Figure 21 shows the mobility schematic of the same elements connected in parallel. The mobility equations are

$$\frac{V_1}{F} = \frac{1}{i\omega m}\,;\ \frac{V_2}{F_3} = \frac{i\omega}{k}\,;\ \frac{V_2}{F_4} = \frac{1}{c}$$

The equilibrium of forces requires

$$F = F_1 + F_2\ ;\ F_2 = F_3 + F_4$$

and the compatibility of velocities requires $V_1 = V_2$.

The driving point mobility seen by the force generator is calculated as

$$\frac{V_1}{F} = \frac{M_m\ M_k\ M_c}{M_k\ M_c + M_m\ M_c + M_m\ M_k}$$

$$= \frac{i\omega}{(k - \omega^2 m) + i\omega c}$$

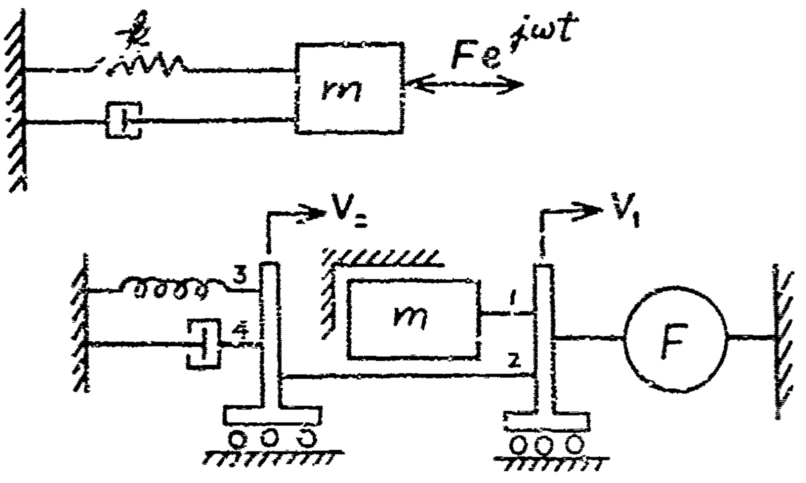


Figure 21. Mobility Schematic for Damper, Spring, and Mass in Parallel.

To illustrate applications of these techniques, consider the problem of determining the vibratory response of a turboshaft engine to rotor unbalance if the response is to be measured at an external location on the engine case. A greatly simplified model of the the rotor/case assembly is shown in Figure 22. The case is represented by a uniform beam of mass per unit length ρ and stiffness EI. The rotor is represented by a massless shaft carrying a centrally located disc of mass M_d. The bearing supports and engine mounts are represented by springs of stiffness $k_{B/2}$ and $k_{m/2}$ respectively.

Figure 23 shows the mobility model with rotor disc unbalance represented by a force generator. The engine case is reduced to a single-degree-of-freedom system in the mobility model, thus preserving only the fundamental mode. The case stiffness and rotor stiffness are represented by k_c and k_R respectively.

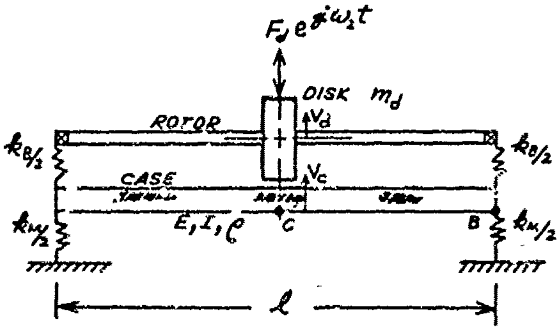
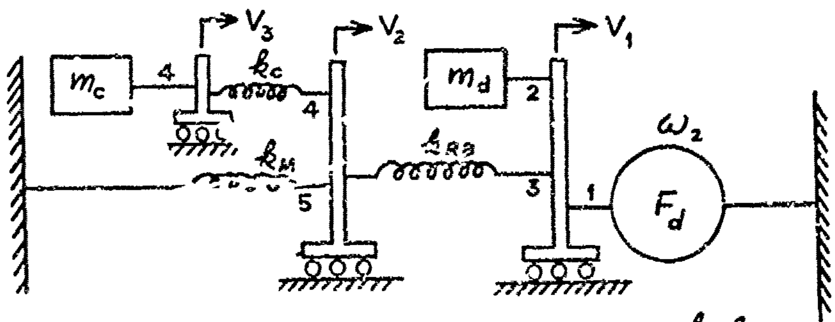


Figure 22. Simplified Model of Engine Case/Rotor Assembly.



$$m_c = \rho \ell/2 \;, \quad k_c = \frac{48EI}{\ell^3} \;, \quad k_{RB} = \frac{k_R k_B}{k_R + k_B} \;,$$

$$V_1 = V_d \;, \quad V_2 = V_B \;, \quad V_3 = V_c \;, \quad F_1 = F_d$$

Figure 23. Mobility Model for Response of Case to Rotor Unbalance.

It is desired to compare the vibratory velocity at locations B and C on the engine case to the response of the rotor disc. The mobility equations are

$$\frac{V_1}{F_2} = \frac{1}{i\omega_2 m_d}\ , \quad \frac{V_1 - V_2}{F_3} = \frac{i\omega_2}{k_{RB}}$$

$$\frac{V_2 - V_3}{F_4} = \frac{i\omega_2}{k_c}\ , \quad \frac{V_2}{F_5} = \frac{i\omega_2}{k_m}$$

$$\frac{V_3}{F_4} = \frac{1}{i\omega_2 m_c}$$

and the equations for equilibrium of forces are

$$F_1 = F_2 + F_3\ ;\ F_3 = F_4 + F_5$$

Taken together, these represent a system of seven algebraic equations in seven unknowns, which can easily be reduced by substitution to the following third-order matrix equation:

$$\begin{bmatrix} (\omega_2^2\ m_d - k_{RB}) & k_{RB} & \\ 0 & k_c & (\omega_2^2\ m_c - k_c) \\ k_{RB} & -(k_m + k_{RB}) & \omega_2^2\ m_c \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} i\omega F_1 \\ 0 \\ 0 \end{Bmatrix}$$

Cramer's rule yields the solution as

$$V_1 = V_d = \frac{k_c\omega_2^2 m_c + (k_m + k_{RB})\ (\omega_2^2 m_c - k_c)}{D} \quad (i\omega_2 F_1)$$

$$V_2 = V_b = \frac{k_{RB}\ (\omega_2^2\ m_c - k_c)}{D} \quad (i\omega_2 F_1)$$

$$V_3 = V_c = \frac{-k_{RB}k_c}{D} \quad (i\omega_2 F_1)$$

$$D = (k_c + k_m)\ (\omega_1^2 m_d - k_{RB})\ (\omega_1^2 m_c - k_c) + k_c^2\ (\omega_1^2 m_d - k_{RB})$$

$$+ k_{RB}\omega_1^2 m_d\ (\omega_1^2 m_c - k_c).$$

Of special interest are the ratios of rotor disc response to the response which would be measured at locations B and C:

$$\frac{V_1}{V_2} = \frac{V_d}{V_b} = \frac{k_m + k_{RB}}{k_{RB}} + \frac{k_c\omega_2^2 m_c}{k_{RB}\,(\omega_2^2 m_c - k_c)}$$

$$\frac{V_1}{V_3} = \frac{V_d}{V_c} = 1 - \frac{k_m\ (\omega_2^2 m_c - k_c) + k_{RB}\,\omega_2^2\, m_c}{k_{RB}k_c}$$

Inspection of the solutions shows that the rotor disc response can be very large even when the measured response on the case is zero. For example, if

$$V_b = 0 \quad (\text{antiresonance, } k_c = \omega_2^2 m_c)$$

$$V_d = \frac{i\omega_2 F_1}{(\omega_2^2 m_d - k_{RB})}$$

Mobility (or impedance) analysis of large and complex structural assemblies is facilitated by the application of special matrix techniques as described and/or used in References 4 and 8.

The mobility approach is made especially powerful for analysis of the effects of changing or inserting individual elements in a complex structure by the use of Thévenin's theorem and Norton's theorem.

Consider a system which is to have an element (or combination of elements) installed between two junction points, say, points 1 and 2. Figure 24 illustrates the concept, with the mobility of the added element represented by M_e.

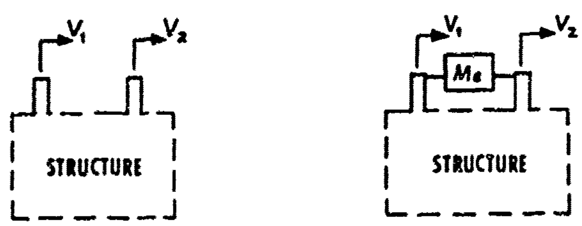


Figure 24. Mobility M_e Added Between Two Points in an Existing Structure.

The force and relative velocity across mobility M_e installed in the structure, with all force and velocity generators activated, can be determined from an analysis of a relatively simple equivalent system given by Thévenin's theorem. Thévenin's equivalent system is shown in Figure 25, where

M_e = mobility of the added element

M_i = driving point mobility of point 2 with respect to 1 with all generators deactivated (a deactivated velocity generator becomes a rigid link while a deactivated force generator disappears) and no M_e

V_o = free velocity $V_2 - V_1$ with all generators active and no M_e.

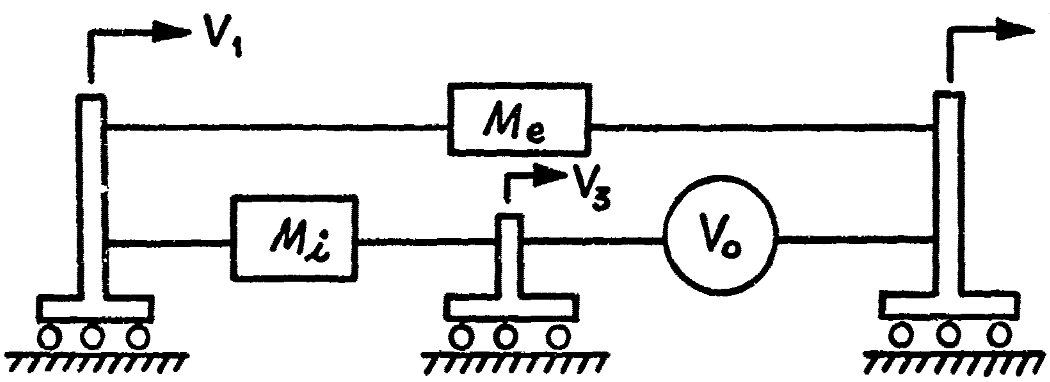


Figure 25. Thévenin's Equivalent System.

An alternate equivalent system which can be constructed from Norton's theorem is shown in Figure 26, where F_b is the "blocked force" that would be transmitted through a rigid link if it were inserted to prevent relative motion of points 1 and 2 when the generators are activated. The mobilities M_e and M_i are defined the same as for Thévenin's equivalent system. An example of direct application of Norton's equivalent system is the determination of vibratory force and velocity across a soft engine mount to be installed between an engine and airframe. The blocked force F_b would be the force measured or calculated in a rigid engine mount at the same location.

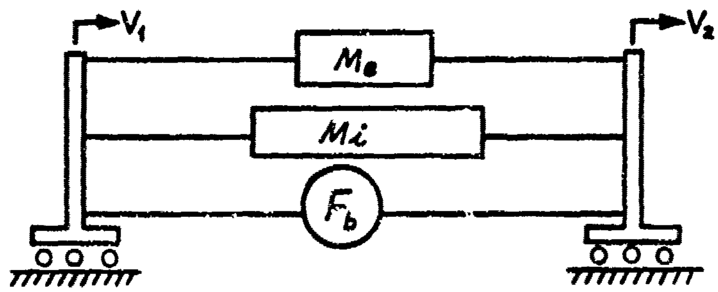


Figure 26. Norton's Equivalent System.

APPENDIX II
COMPUTER PROGRAMS CURRENTLY IN USE OR AVAILABLE FOR HELICOPTER ENGINE/AIRFRAME VIBRATION ANALYSIS

The major specialized computer programs for engine/airframe vibratory analysis with which the author of this report became familiar during the study preceding the report are listed below. The list is intended solely as a convenience for future researchers in this area and does not constitute in any way a commendation by inclusion or a criticism by omission of any contractor or other organization. All of the helicopter engine and airframe contractors have general analytical capabilities which may not be included in the list.

1. VAST, "System Vibration and Static Analysis"

   a. Source and User: General Electric, Aircraft Engine Group, Lynn, Massachusetts.

   b. Description: Based on Prohl's method for calculating critical speeds of rotors and expanded to include the engine frame and associated components modeled as beams. Effects of bearing clearances, squeeze film dampers, aircraft maneuver loads, etc., have been added. Calculates critical speeds and forced response of turboshaft engines.

   c. Pertinent references:

      (1) Sevcik, J. K., "System Vibration and Static Analysis", ASME Paper No. 63-AHGT-57, presented at the Aviation and Space, Hydraulic, and Gas Turbine Conference and Products Show, Los Angeles, California, March 3-7, 1963.

      (2) Prohl, M. A., "A General Method for Calculating Critical Speeds of Flexible Rotors", Journal of Applied Mechanics, September 1945, pp. A-142 through A-148.

2. Critical Speed Analysis of Turbo Rotor Systems (Lycoming)

   a. Source and User: Lycoming Division, Avco Corporation, Stratford, Connecticut.

b. Description: Based on a planar multiple beam model that allows reasonably accurate predictions for critical speeds but which is dependent on an accurate knowledge of damping for reliable predictions of forced response. The capability exists for determining the response to shock or impulse inputs at the engine mounts. The program calculates both critical speeds and forced response of turboshaft engines.

c. Pertinent reference:

Bohm, R. T., "Designing Complex Turbo Rotor Systems With Controlled Vibration Characteristics", SAE Paper No. 928B, presented at the National Transportation, Powerplant, and Fuels and Lubricants Meeting, Baltimore, Maryland, October 19-23, 1964.

3. COSMOS – USA, Dynamic Structural Analysis of Helicopter Airframes

a. Source and user: The Boeing Company, Vertol Division, Philadelphia, Pennsylvania.

b. Description: Based on the stiffness method for calculating eigenvalues of an elastic structure. Uses a "unified structural analysis (USA)" method which avoids breaking a structure into substructures. Eigenvalues are obtained by similarity transformations. Mass elements are strategically located at relatively few points in the model. The method is efficient but is not well adapted to studying the response of a small part of the structure in great detail (e.g., an engine or a transmission). Calculates critical frequencies and mode shapes of complicated structures and assemblages.

c. Pertinent references:

(1) Sciarra, J. J., "A Computer Method for Dynamic Structural Analysis Using Stiffness Matrices", Journal of Aircraft, Vol. 6, No. 1, Jan-Feb 1969, pp. 3-8.

(2) Kiersky, L. B., "COSMOS, A Computer Program for Structural Analysis", Doc. D2-4513, The Boeing Co., 1962.

4. NASTRAN, "NASA Structural Analysis Computer System"

a. Source: NASTRAN, Systems Management Office, NASA Langley Research Center, Hampton, Virginia.

b. User: Bell Helicopter Co., Hurst, Texas.

c. Description: This program performs both static and dynamic structural analysis. The dynamic analysis solves for natural frequencies, mode shapes, transient response in terms of modal coordinates, transient response in terms of grid-point coordinates, and forced response to a spectrum of harmonic inputs. Output is obtainable as curves against time or frequency, or as plots of structural deformation at specific time intervals.

d. Pertinent references:

(1) Butler, T. G., and Michel, D., "NASTRAN, A Summary of the Functions and Capabilities of the NASA Structural Analysis Computer System", NASA SP-260, 1971.

(2) McCormick, C. W., "NASTRAN Beginner's Guide", MS 139-1, prepared for NASA Langley Research Center, Hampton, Virginia, by the MacNeal-Schwendler Corporation.