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LANDING APPROACH AUTOMATIC FLIGHT CONTROL SYSTEM DESIGN VIA REDUCED ORDER OPTIMAL CONTROL LAW

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13 ABSTRACT

A set of optimal feedback gains is used as a basis for designing
a practical longitudinal automatic flight control system for the landing
approach task. A procedure is developed to give a good first-cut design.
The procedure is systematic and straightforward.

The procedure is used to design two control systems for a DC-8
aircraft. In one case it is assumed that pitch, pitch rate, normal
acceleration, and longitudinal airspeed are continuously measured on
board the aircraft. The second system is similar to the first with the
exception that longitudinal airspeed is deleted as one of the measured
variables. These systems are compared with a high-performance automatic
flight control system that was designed using classical control techniques.
The procedure is also used to design two control systems for a hypothetical
aircraft with direct lift control capability.

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LANDING APPROACH AUTOMATIC FLIGHT CONTROL
SYSTEM DESIGN VIA REDUCED ORDER
OPTIMAL CONTROL LAW

THESIS

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LANDING APPROACH AUTOMATIC FLIGHT CONTROL SYSTEM DESIGN

VIA REDUCED ORDER OPTIMAL CONTROL LAW

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

Jerry D. Pfleeger, B.S.E.E.

1st Lieutenant   USAF

Graduate Guidance and Control

June 1973

Preface

My initial motivation for this thesis was to develop and evaluate
a new design procedure. I would be less than honest if I failed to also
admit that I was further motivated to complete a thesis which would meet
the requirements for graduation. Somewhere along the way my motivation
shifted. I hope the many long hours of work summarized in this thesis
might be of use to others working on the landing problem. It is then,
to this end, that I have written this thesis. I have included the
equations and the numerical values which I used so others will be able
to reproduce my results. I have also tried to write this thesis in a
clear, understandable manner for the benefit of the prospective reader.

Those who have written a thesis can appreciate the many long hours
of work required and the fact that a thesis is not just an individual
effort. I would like to express my sincere appreciation to those who
helped make this thesis possible. I especially want to thank my advisor,
Major James D. Dillow, for his many hours of help and encouragement. His
cheerful "press-on" attitude turned a dreaded task into a learning exper-
ience. Major Dillow deserves a majority of the credit for this study.
I would also like to thank my sponsor, Robert Huber, and Ronald Anderson,
both of the Air Force Flight Dynamics Laboratory, for their help.

Finally, I wish to express my deep appreciation to my wife, Joyce,
for her patience, understanding, and encouragement, and for typing the
rough draft of this thesis. Her sacrifices during not only this thesis
but the last four years have helped immeasurably in my education. With-
out her encouragement, I would not have come this far.

<div align="right">Jerry D. Pfleeger</div>

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Table of Notation

A   A matrix of coefficients used in the first-order representation of the system differential equations

AFCS   Automatic Flight Control System

B   A matrix of coefficients used in the first-order representation of the system differential equations

d   Glide slope deviation (ft)

$E\{\ \}$ Expected value operator

F   Feedback matrix used in control law

g   Constant of gravity, 32.2 ft/sec^2

H   Observation or measurement matrix

h   Altitude (ft)

J   Quadratic cost functional

J_1 A quadratic cost functional with weighting on σ_d and rms control activity

K   Gain matrix used in discrete Kalman filter

k_1, k_2 Weighting coefficients used in quadratic cost functional J_1

M   Pitching moment divided by pitching moment of inertia

M_q $\partial M/\partial q$ (1/sec)

M_u $\partial M/\partial u$ (1/ft-sec)

M_w $\partial M/\partial w$ (1/ft-sec)

M_{δ_d} $\partial M/\partial \delta_d$ (1/sec^2)

M_{δ_e} $\partial M/\partial \delta_e$ (ft/sec^2)

$M_{\delta_{th}}$ $\partial M/\partial \delta_{th}$ (1/percent rpm-sec^2)

n   Denotes the nth sample data measurement and is used interchangeably with nT_0

P   The solution to the nonlinear matrix equations used to solve for the optimal feedback matrix F and estimator gains K

PMA	Probability of Missed Approach
Q	Matrix used to weight the state in the quadratic cost functional J
q	Pitch rate (rad/sec)
R	Matrix used to weight the control in the quadratic cost functional J
s_h	Wind shear velocity gradient (ft/sec/ft)
T_e	Lag time constant for elevator response to a commanded input (sec)
T_d	Lag time constant for direct lift control response to a commanded input (sec)
T_0	Time interval between sample data measurements (sec)
T_{th}	Lag time constant for thrust response to commanded input (sec)
t	Time (sec)
U_e	Nominal longitudinal velocity (ft/sec)
u	Perturbed longitudinal velocity (ft/sec), or control input vector
u_{as}	Perturbed longitudinal airspeed (ft/sec)
u_g	Longitudinal gust intensity (ft/sec)
u_w	Longitudinal component of wind intensity due to headwind and wind shear (ft/sec)
$\tilde{V}$	Covariance of "measurement" noise, $E(n(n)n(m)') = V\delta_{nm}$
w	Perturbed normal velocity (ft/sec); or a vector for which the non-zero entries are the states which are not continuously and perfectly measured
w_g	Normal gust intensity (ft/sec)
w_{sh}	Wind intensity due to wind shear (ft/sec)
X	Longitudinal force divided by aircraft mass
X_w	$\partial X/\partial w$ (1/sec)
X_{δ_d}	$\partial X/\partial \delta_d$ (ft/sec^2)
X_{δ_e}	$\partial X/\partial \delta_e$ (ft/sec^2)

$X_{\delta_{th}}$	$\partial X/\partial \delta_{th}$ (ft/percent rpm-sec^2)
x	State vector for the system differential equation
$\hat{x}$	Estimate of x
x^o	Optimal estimate of x
y	Sample data measurement
y_1	State representing measurement errors due to fluctuation noise (ft)
y_2	State representing integral of glideslope deviation d (ft-sec)
Z	Normal force divided by aircraft mass for defining stability derivatives, or steady-state covariance matrix for z
Z_u	$\partial Z/\partial u$ (1/sec)
Z_w	$\partial Z/\partial w$ (1/sec)
Z_{δ_d}	$\partial Z/\partial \delta_d$ (ft/sec^2)
Z_{δ_e}	$\partial Z/\partial \delta_e$ (ft/sec^2)
$Z_{\delta_{th}}$	$\partial Z/\partial \delta_{th}$ (ft/sec^2)
Δd	Glideslope deviation tolerance used to define the 100 ft decision altitude window (ft)
Δy	Lateral centerline deviation tolerance used to define the 100 ft decision altitude window (ft)
δ_d	Direct lift control deflection (rad)
δ_{d_c}	Commanded direct lift control deflection (rad)
δ_e	Elevator deflection (rad)
δ_{e_c}	Commanded elevator deflection (rad)
δ_{th}	Perturbed thrust (percent rpm)
δ_{th_c}	Commanded perturbed thrust (percent rpm)
$\delta(t-s)$	Impulse function
η	Gaussian amplitude white noise measurement (ft)
θ	Perturbed pitch altitude (rad)
γ_o	Nominal glide path angle

ζ	Gaussian amplitude discrete white noise process used as an input to the system differential equations due to finite bandwidth measurement noise and gust disturbances
ζ^*	Gaussian amplitude white noise process used as an input to the system differential equations due to finite bandwidth measurement noise and gust disturbances
ξ_{fn}	Gaussian amplitude white noise process used in the model for the fluctuation noise errors y_2
ξ_{u_g}	Gaussian amplitude white noise process used in the model for the longitudinal gust disturbances u_g
ξ_{w_g}	Gaussian amplitude white noise process used in the model for the normal gust disturbances w_g
σ_d	The standard deviation of d
σ_{fn}	The standard deviation of the fluctuation noise y_2
σ_{u_g}	The standard deviation of u_g
σ_{w_g}	The standard deviation of w_g
$\sigma_{\delta c_i}$	The standard deviation of the ith control activity
σ_{δ_e}	The standard deviation of δ_e
$\sigma_{\dot\delta_e}$	The standard deviation of $\dot\delta_e$
$\sigma_{\delta th}$	The standard deviation of δ_{th}
σ_θ	The standard deviation of θ
ω_{fn}	Break frequency of the fluctuation noise y_2
ω_{u_g}	Break frequency of the longitudinal gust intensity
ω_{w_g}	Break frequency of the normal gust intensity

Superscripts

$\cdot$	Derivative with respect to time, d/dt
$'$	Matrix transpose

LANDING APPROACH AUTOMATIC FLIGHT CONTROL SYSTEM DESIGN
VIA REDUCED ORDER OPTIMAL CONTROL LAW

I. Introduction

Background

A new National Landing System is being developed to provide improved landing guidance information to aircraft. One approach which has shown promise involves the use of a microwave scanning beam to provide landing aircraft with glide path deviation and azimuth information. James D. Dillow conducted a study (Ref 1) to analyze the pitch plane data rate requirements for the proposed microwave scanning beam system. The data rate study used an optimal model for the aircraft flight control system.

As a part of the data rate analysis, a digital computer program was developed to implement the landing approach model and to automatically compute the probability of a missed approach as a function of the data rate of the glide slope deviation information. The computer program takes as inputs the data rate; the aircraft stability derivatives; nominal longitudinal airspeed; glide path angle; atmospheric disturbances (gust, headwind, windshear); guidance noise parameters; constraints on the control activity; and the tolerances on the aircraft variables that define a missed approach (i.e., the "window dimensions"). The program is structured o that changes in the system equations can be easily implemented in the program. Changes in the system equations result from

1. Changes in the aircraft equations of motion (for example, inclusion of flexure modes and effect of sensor location).

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2. Addition or deletion of control points (i.e., autothrottle, direct lift control, etc.).

3. Changes in the measurement model or sensor complement (for example, consideration of sensed normal acceleration, sensed glideslope deviation rate, sensed longitudinal airspeed, and noise in the continuous measurements).

The results obtained by Dillow (Ref 1) using the optimal model were compared with results obtained using conventional analysis techniques (Ref 2). This comparison validated the optimal model.

One of the "by-products" of the digital computer program used in the data rate analysis is a "full-blown" optimal control law with feedback gains and Kalman Filter gains. This control law is optimal in the sense of approach performance and accounts for control authorities that would be imposed on the automatic flight control system. The purpose or object of this study was to determine if the optimal model and the optimal control law from the data rate analysis could be used as a design tool. The study was prompted by the success of the data rate analysis and the fact that the "optimal" was optimal with respect to a meaningful measure of performance while accounting for limitations in control activity. The digital computer program used in the data rate analysis offers several advantages over classical techniques in control design. The program structure makes changes in the system equations a matter of changing data cards rather than redrawing root locus or Bode plots as is done with classical techniques. There is no need to go through tedious loop closure procedures. The time required to evaluate the effect of one or more changes is much less than that needed when using classical techniques.

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Multi-input, multi-output control systems are also handled much easier using the data rate analysis computer program.

Hypothesis

The optimal feedback matrix F*, from the data rate analysis, can be used in a systematic procedure to develop a practical control law. The systematic procedure is subject to one constraint. When the data rate analysis program develops the optimal control law, certain variables such as pitch, pitch rate, normal acceleration, and nominal longitudinal airspeed are considered to be measured "on board" the aircraft. The data rate analysis program further assumes that glide path deviation is measured on the ground and transmitted to the aircraft. Therefore, flight path deviation information is in sampled data form for the data rate analysis. The constraint is that the continuously measured variables and sampled data measurements used in the data rate analysis must also be used as feedbacks in the practical control law. Any control law not employing all these feedback gains will be unstable or display poor flying or poor ride qualities. These poor qualities show up as large pitch attitude deviations, high control actuator rates, high probability of missed approach or slow settling time when the aircraft is perturbed.

Investigation

The object of this investigation is to determine a design procedure starting with the "optimal" feedback gains of the data rate analysis and ending with a practical (and realizable) automatic flight control system for an aircraft performing the landing approach task and to compare the resulting suboptimal automatic flight control system against some "standard."

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The measures of "goodness" for the evaluation are the probability of missed approach, PMA (minimized); the rms control activity (within bounds); and rms pitch attitude σ_θ (ride quality). System response to a pitch rate impulse will also be checked for low damping. If the damping is too low, the system will be slow in damping out oscillations. These oscillations are not evident in the rms performance measures.

The "standard" used as a basis for comparison was designed by Systems Technology, Inc. (Ref 2). It represents an advanced automatic flight control system designed with classical control techniques. The "standard" is not currently in use, but rather represents a system of the quality deemed necessary to meet the Category II landing requirements.

In order to compare results with the "standard" a common basis must be established. The following ground rules have been established to arrive at that basis:

1. The glideslope deviation d is assumed to be measured continuously in the "standard" and sampled at a rate of 6 samples/sec in the suboptimal control system.

2. The sample data glideslope deviation measurement is assumed to contain noise where the noise, which is superimposed on the true glideslope deviation, accounts for the effects of fluctuation noise (due to sampled data measurement) and white noise. This accounts for using a scanning beam landing guidance system. The noisy measurement is filtered with a low pass filter which is described in detail later. When airspeed is used, it is assumed that any noise which might be on the measurement is filtered out by the measurement system. The measurement system for airspeed is approximated by a first-order lag.

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3. The atmospheric environment is one of severe turbulence.

4. The microwave scanning beam has low beam noise.

5. The height Δd of the landing "window" is ± 12 ft.

6. Steady winds and wind shear are both assumed to be zero.

7. The measured variables are pitch, pitch rate, normal acceleration, and, in special cases, longitudinal airspeed.

The individual ground rules are explained in greater detail as they are encountered later in this report. The procedure is explained here in general and in detail in Chapters V and VI.

A basic requirement for the procedure is that it be a "clean" and logical process. The starting point is the full optimal control law using the optimal feedback matrix F* from the data rate analysis. The goal is a reduced order control law employing feedback of the measured variables (states) only. Feedbacks on the measured states are referred to in this report as desired feedbacks. Since the optimal feedback F* includes some feedback gains for variables not measured (i.e., wind gust, elevator position, etc.), the procedure must systematically remove the feedbacks on the unmeasured states. The unmeasured state feedbacks are referred to as undesired feedbacks. Once the undesired feedbacks have been removed and only the desired feedbacks remain, the logical approach would be to try to further simplify the control law by reducing the number of remaining feedbacks. In the hypothesis, it was stated that removal of any desired feedback would produce unacceptable results. The next step in the procedure is to validate this constraint. The final step involves placing noise on those measurements (glideslope deviation) assumed to have additive noise present. The measurement and the noise are then filtered

through a low pass filter and the filtered measurement used for the practical control law. The practical control law is developed without "fiddling" with feedback gains or use of compensators.

II. Mathematical Model

In this section the mathematical model of the landing approach process is described. This model is used to develop the automatic flight control system. The assumptions used in developing the model are described and the equations used in this study are given. The development of the equations is found in Ref 1. The system of equations is a set of stochastic differential equations and accounts for the aircraft dynamics, the atmospheric environment, the landing guidance system data rate and errors, the aircraft onboard sensors, and the flight control system capabilities. The approach performance is determined by the probability of missed approach.

The Aircraft Equations of Motion

The aircraft is assumed to be adequately represented by a set of perturbed, linear differential equations of motion. This is because the landing approach task basically requires tracking a fixed rectilinear path in space with disturbances induced by the atmospheric environment (gusts) and measurement errors induced by sensor noise and noisy or erroneous guidance measurements. It is further assumed that the longitudinal and lateral equations of motion are uncoupled. This assumption is justified for trimmed flight with small perturbations in Ref 3. It is assumed that the longitudinal motion variables dominate the probability of missed approach (Ref 1), thus only these equations are considered. The variables considered in the longitudinal equations of motion are pitch attitude θ, pitch rate q, longitudinal perturbed

velocity u; normal perturbed velocity w; and glideslope deviation d.
Additional states are introduced as needed to account for lags between
commanded control input and the resulting force or motion generated,
additive finite bandwidth noise, or sensor lags.

The specific equations presented here represent the system model
for the basic DC-8 aircraft and are adopted from Ref 1. They are*

$$\dot{u} = X_u u + X_w w - g\theta \cos \gamma_o - X_u u_g - X_w w_g + X_{\delta_e} \delta_e + X_{\delta_{th}} \delta_{th}$$

$$\dot{w} = Z_u u + Z_w w + U_o q - g\theta \sin \gamma_o - Z_u u_g - Z_w w_g + Z_{\delta_e} \delta_e + Z_{\delta_{th}} \delta_{th}$$

$$\dot{q} = M_u u + M_w w + M_q q - M_u u_g - M_w w_g + M_{\delta_e} \delta_e + M_{\delta_{th}} \delta_{th}$$

The elevator and thrust responses are modeled by first-order lags.
The respective differential equations are

$$\dot{\delta}_e = -\frac{1}{T_e} \delta_e + \frac{1}{T_e} \delta_{e_c}$$

$$\dot{\delta}_{th} = -\frac{1}{T_t} \delta_{th} + \frac{1}{T_t} \delta_{th_c}$$

where δ_{e_c} and δ_{th_c} are commanded inputs, δ_e and δ_{th} are the resulting
elevator position and throttle setting, respectively, and the values for
T_e and T_t are taken to be .06666 sec and 1.0 sec, respectively.

Gusts

The gust disturbance, taken from Ref 1, is assumed to have two
independent components. These components are the longitudinal gust

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*$Z_{\dot{w}}$ and $M_{\dot{w}}$ were taken to be zero in these equations.

velocity u_g and the normal gust velocity w_g. These are described by the following first-order stochastic differential equations:

$$\dot{u}_g = -\omega_{u_g} u_g + \xi_{u_g}$$

$$\dot{w}_g = -\omega_{w_g} w_g + \xi_{w_g}$$

where ω_{u_g} and ω_{w_g} are the half power or break frequencies for the longitudinal and normal gusts, respectively; ξ_{u_g} and ξ_{w_g} are zero mean, Gaussian amplitude, white noise processes. The statistics of ξ_{u_g} and ξ_{w_g} are given by

$$E\{\xi_{u_g}(t)\xi_{u_g}(s)\} = 2\omega_{u_g}\sigma_{u_g}^2 \delta(t-s)$$

$$E\{\xi_{w_g}(t)\xi_{w_g}(s)\} = 2\omega_{w_g}\sigma_{w_g}^2 \delta(t-s)$$

$$E\{\xi_{u_g}(t)\xi_{w_g}(s)\} = 0$$

where σ_{u_g} is the rms longitudinal gust intensity and σ_{w_g} is the rms normal gust velocity; $\delta(t-s)$ is an impulse function.

The parameters used to describe the gust disturbance are given in Table I. These values were taken from Refs 1 and 2. The rms gust intensities used here represent severe turbulence. Severe turbulence was considered because previous studies (Refs 1 and 2) indicated the landing approach task required landing guidance and flight control systems to suppress the effects of gusts.

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TABLE I
Atmospheric Disturbance Parameters--DC-8

$$\sigma_{u_g} = 10.000 \text{ ft/sec}$$

$$\omega_{u_g} = 0.340 \text{ rad/sec}$$

$$\sigma_{w_g} = 6.500 \text{ ft/sec}$$

$$\omega_{w_g} = 3.950 \text{ rad/sec}$$

Measurements

The measurement model was developed in Ref 1 and accounts for
the two different types of measurements associated with a low visibility
landing approach with a scanning beam guidance system. The first part of
the measurement model accounts for the glideslope deviation measurements
derived from the scanning beam system, and as such, are considered to be
sample data measurements. These measurements are usually associated with
guidance measurements or guidance inputs to the flight control system.
The other measurements considered in the measurement model are those
usually associated directly as a part of the flight control system and
are made continuously on board the aircraft. These measurements may
include sensed pitch attitude, pitch rate, airspeed, normal acceleration, etc.

The model for the glideslope deviation measurement as derived from
the scanning beam guidance system is taken from Ref 1. This model for
measured glideslope deviation accounts for three error components in the
difference between the measured value of glide slope and the true reference
landing glide slope. These components include a fixed bias, fluctuation

noise associated with the sampled data feature of the measurement, and white noise representing the very broad band thermal noise in the circuits and random electromagnetic noise.

The fixed bias is taken as a zero mean, Gaussian random variable. It is not directly included as a part of the measurement since it is assumed that this error cannot be detected without some other external reference. Thus the true glideslope plus the fixed bias is taken as the reference glideslope track for the aircraft. Its effect is accounted for by computing a root-sum-squared glideslope deviation using the fixed bias and the rms glideslope deviation due to all other disturbances and measured errors.

The fluctuation noise y_2 is modeled by a first-order, Gaussian, shaped noise of the form

$$y_2 = - \omega_{fn} y_2 + \xi_{fn}$$

where ω_{fn} is the half-power frequency of the fluctuation noise and ξ_{fn} is a zero mean, Gaussian amplitude, white noise process. Furthermore,

$$E\{\xi_{fn}(t)\xi_{fn}(s)\} = 2\omega_{fn}\sigma_{fn}^2\delta(t-s)$$

where σ_{fn} is the rms fluctuation noise. The break frequency is taken to be

$$\omega_{fn} = \frac{2.8}{T_0} \text{ rad}$$

where T_0 is the information update interval (or $1/T_0$ is the data rate).

Thermal noise and random electromagnetic noise are modeled by zero mean, Gaussian amplitude, white noise η. It is assumed that η is statistically independent of y_2.

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The combined measured glideslope deviation from the true reference glideslope is represented pictorially in Fig. 1. The measured deviation from the reference glideslope track y is given by

$$y = d + y_2 + n$$

The measured glideslope deviation y is sampled at the information update intervals and $y(nT_0)$ is the measured sample data glideslope deviation on the time interval $nT_0 \leq t < (n+1)T_0$, where T_0 is the sampling interval and $1/T_0$ is the sampled data rate.

Fig. 1. Glideslope Deviation Measurement Model

In addition to the sampled data measurement of glideslope deviation, it is assumed that certain aircraft variables are continuously measured. The mathematical model of Ref 1 was developed to include consideration for continuously sensed motion variables; however, the following

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restriction was imposed on the continuous measurements considered in Ref 1:

- Only those variables appearing explicitly as states in a first-order state-variable representation of the system equations, $\dot{x} = Ax + Bu + \zeta^*$, can be measured continuously. Furthermore, those measurements are perfect, i.e., not noisy.

The reason for this restriction is discussed in Ref 1. This restriction does not generally limit the type of continuous measurements that can be considered but in some cases it requires "fiddling around" with the state equations to get them in a suitable form so that the restriction is satisfied; for example, when it was desired to use airspeed feedback in the control law. In order to consider longitudinal airspeed (longitudinal velocity with respect to air mass) as a sensed variable, it is convenient to introduce the longitudinal airspeed u_{as} as a state in the aircraft equations. Let u_g denote the inertial velocity of the air mass. Then

$$u_{as} = u - u_g$$

and the time derivative of the longitudinal airspeed is

$$\dot{u}_{as} = \dot{u} - \dot{u}_g$$

The aircraft equations of motion, after replacing longitudinal velocity with longitudinal airspeed, are

$$\dot{u}_{as} = X_u u_{as} + X_w w - g\theta \cos \gamma_0 + \omega_{u_g} u_g - X_w w_g + X_{\delta_e}\delta_e + X_{\delta_{th}}\delta_{th}$$

$$w = Z_u u_{as} + Z_w w + U_0 q - g\theta \sin \gamma_0 - Z_w w_g + Z_{\delta_e}\delta_e + Z_{\delta_{th}}\delta_{th}$$

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$$\dot{q} = M_u u_{as} + M_w w + M_q q - M_w w_g + M_{\delta_e} \delta_e + M_{\delta_{th}} \delta_{th}$$

In a similar manner, noisy measurements of aircraft motion variables can be considered. This is done by introducing the appropriate linear stochastic differential equations describing the measurement noise and introducing a new state equation representing the sum of the sensed motion variable and the measurement noise.

System Differential Equation

The equations of motion for the aircraft along with the control lags, gust equations, and measurement equations make up a set of differential equations which describe the system. The system differential equation is

$$\dot{x}(t) = Ax(t) + Bu(t) + \zeta^*(t) \tag{1}$$

where

$$x = \begin{bmatrix} u_{as} \\ w \\ \theta \\ q \\ u_g \\ w_g \\ y_2 \\ d \\ \delta_e \\ \int d \\ \delta_{th} \end{bmatrix}$$

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$$
A = \begin{bmatrix}
X_{\delta_{th}} & 0 & X_{\delta_e} & 0 & 0 & -X_w & X_u & 0 & -g\cos\gamma_0 & X_w & X_u \\
Z_{\delta_{th}} & 0 & Z_{\delta_e} & 0 & 0 & -Z_w & 0 & U_0 & -g\sin\gamma_0 & Z_w & -Z_u \\
M_{\delta_{th}} & 0 & M_{\delta_e} & 0 & 0 & M_w & M_w & M_w & 0 & M_w & M_u \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \tfrac{g}{U_0} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \tfrac{g}{U_0} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & U_0 & -1 & 0 \\
0 & 0 & -\tfrac{1}{\tau_e} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\tfrac{1}{\tau_t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

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$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/T_e & 0 \\ 0 & 0 \\ 0 & 1/T_t \end{bmatrix}, \quad u = \begin{bmatrix} \delta_{e_c} \\ \delta_{th_c} \end{bmatrix}, \quad \xi' = \begin{bmatrix} -\xi_{u_g} \\ 0 \\ 0 \\ 0 \\ \xi_{u_g} \\ \xi_{w_g} \\ \xi_{fn} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The stability derivatives and aircraft parameters are taken from Ref 1 and are included here in Table II. The system block diagram is shown in Fig. 2.

Measures of "Goodness"

Three measures of "goodness" were used to evaluate a possible flight control system. The first is the probability of missed approach which is used as the primary measure of approach performance. The second is the rms control activity, and the third is the rms pitch angle.

The measure of approach performance used in this study is based on the assumption that if certain prespecified aircraft variables are within a given tolerance at the decision altitude, then the landing

TABLE II
DC-8 Stability Derivatives and Aircraft Parameters--Landing Approach

$$X_u = -0.03730 \ 1/\text{sec}$$

$$X_w = 0.13600 \ 1/\text{sec}$$

$$Z_u = -0.28300 \ 1/\text{sec}$$

$$Z_w = -0.75000 \ 1/\text{sec}$$

$$M_u = 0.0 \quad 1/\text{sec-ft}$$

$$M_w = -0.00461 \ 1/\text{sec-ft}$$

$$M_q = -0.59400 \ 1/\text{sec}$$

$$X_{\delta_e} = 0.0 \quad \text{ft/rad-sec}^2$$

$$X_{\delta_{th}} = 0.10600 \ \text{ft/percent rpm-sec}^2$$

$$Z_{\delta_e} = -9.25000 \ \text{ft/rad-sec}^2$$

$$Z_{\delta_{th}} = -0.00097 \ \text{ft/percent rpm-sec}^2$$

$$M_{\delta_e} = -0.92300 \ 1/\text{rad-sec}^2$$

$$M_{\delta_{th}} = 0.00007 \ 1/\text{percent rpm-sec}^2$$

$$U_0 = 228 \quad \text{ft/sec}$$

$$\gamma_0 = -3 \quad \text{deg}$$

$$T_e = 0.06666 \ \text{sec}$$

$$T_t = 1.0 \quad \text{sec}$$

Fig. 2. Block Diagram of the Landing Approach Model

proceeds to touchdown. Otherwise a missed approach has occurred and a "go around" is executed. The tolerances used to specify a successful approach are called the window. With this definition of a missed approach, the probability of a missed approach (PMA) is a quantitative and computable measure of landing performance. The definition, a complete description, and a method for computing PMA are found in Ref 4.

The current FAA Category II landing accuracy criteria of ±72 ft lateral deviation and ±12 ft vertical deviation are used to define the window. The window is shown graphically in Fig. 3. Since the results of Ref 2 indicate the vertical errors dominate the probability of missed approach (with this window definition), only the vertical window dimension was used in this study to define a missed approach.

Fig. 3. The Recommended Category II Window

After minimizing the probability of missed approach, the rms elevator actuator rate and the rms pitch angle are checked to assure

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that both fall within acceptable limits. The rms elevator actuator rate serves as a measure of the control authority required by the automatic flight control system. Limits are generally set on the control authority of an automatic flight control system with a margin of safety to allow sufficient manual authority to override a hard-over failure of the automatic system. The control constraint used as an upper bound on activity was .15 rad/sec for the rms elevator actuator rate. This value was never exceeded by any of the cases considered. By relaxing or tightening the rms actuator rate constraints, possible trade-offs can be evaluated in terms of the change in the probability of missed approach for a given change in the rms actuator rate.

The rms pitch angle was considered as a measure of the "ride" qualities associated with the automatic flight control system and hence was used to evaluate each automatic flight control system configuration. The upper limit was set at 6° (Ref 1). This figure was primarily used to detect an automatic flight control system design which results in a high-frequency oscillation in glideslope deviation. There are cases where an automatic flight control system could have a low probability of missed approach and reasonable actuator rate, but the oscillatory tendencies would show up by a marked increase in the rms pitch angle. The probability of missed approach and rms elevator actuator rate appears reasonable in these cases because oscillations around the mean tend to cancel out in the rms measures.

III. Optimal Feedbacks

The technique used to develop the optimal feedback gains is presented in this chapter for completeness and reader convenience. This coverage is not intended to be mathematically rigorous. A more in-depth treatment is found in Refs 1 and 4.

In developing the model for the automatic flight control system, steady winds and wind shear were taken to be zero. The steady wind is a time invariant wind assumed to be blowing horizontally with the ground and may be either a head wind or tail wind. With respect to the aircraft, it has a steady longitudinal and normal component depending on the aircraft attitude with respect to the horizon. The wind shears account for a gradient in the wind intensity as a function of altitude. The wind shear w_{sh} is represented by the equation

$$w_{sh} = \begin{cases} 0 & , \quad h \geq 200 \text{ ft} \\ s_h(200-h), & 100 \text{ ft} \leq h \leq 200 \text{ ft} \end{cases}$$

where s_h denotes the linear rate at which the wind velocity changes as a function of altitude; s_n may be positive or negative. With respect to the aircraft, the wind shear has a longitudinal and normal component depending upon the aircraft attitude with respect to the horizon.

The control law representing the automatic flight control system is developed for the linear Gaussian system of equations,

$$\dot{x}(t) = Ax(t) + Bu(t) + \xi^*(t) \tag{1}$$

where $u(t)$ represents the control input generated by the control law.

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Assuming that the glideslope (and of course localizer) has been captured, the landing approach (not considering flare, touchdown and rollout) becomes a tracking problem with respect to the deviation from the reference glideslope track and with respect to the nominal or trim values of the other aircraft motion variables. For a fixed reference glideslope track angle, this is a regulator problem from a control point of view. Thus the model for the flight control system is developed from the control theory relating to the optimal regulator problem.

As a brief review of the optimal control regulator problem, consider the linear system

$$\dot{x}(t) = Ax(t) + Bu(t) + \zeta^*(t) \tag{1}$$

where the observations are of the form

$$y(t) = Hx(t) + n(t) \tag{2}$$

where n is a zero mean, Gaussian amplitude, white noise process. Let J be a quadratic functional in state and control defined by

$$J(u) = \lim_{T \to \infty} E\left\{ x(T)'Qx(T) + u(T)'Ru(T) \right\} \tag{3}$$

where Q and R are symmetric, Q is non-negative definite, and R is positive definite. It is well known that the control law defined on the observation y, which minimize the functional (3), subject to the differential constraint (1), is of the form (Ref 5)

$$u(t) = Fx^*(t) \tag{4}$$

where F is the optimal feedback for the deterministic problem (i.e., $\zeta^* = 0$) and is given by the equation

$$F = -R^{-1}B'P \tag{5}$$

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and P is the solution to the nonlinear matrix Ricatti equation

$$AP + PA' + Q - PBR^{-1}B'P = 0 \tag{6}$$

In Eq (4), x^* is the "best estimate" of x and is the output of a system
of differential equations (called a Kalman filter) which have as their
input the observations y.

Based on this familiar optimal control result for the so-called
linear, quadratic, Gaussian problem, the flight control system is modeled
by the control law

$$u(t) = F\hat{x}(t) \tag{7}$$

where F is the optimal feedback matrix for a quadratic cost function of
the form given by Eq (3), and $\hat{x}(t)$ is a "sub-optimal" estimate of the
state x, and is described in Ref 1.

The quadratic cost function used to define F is of the form

$$J_1(u) = \sigma_d^2 + k_1 \sigma_{\delta_{c_1}}^2 + k_2 \sigma_{\delta_{c_2}} \tag{8}$$

where σ_d is the rms glideslope deviations from the reference glideslope
track$\sigma_{\delta_{c_1}}$, $i = 1,2$, are the rms control inputs. For the DC-8 these
control inputs are commanded elevator position and commanded engine rpm.
In the case where direct lift control is considered, these control inputs
are commanded elevator position and commanded direct lift. The weighting
coefficients, k_1 and k_2, were selected to minimize the probability of a
missed approach (PMA) subject to prespecified limits on the rms control
activity. In this manner, the structure of the solution to the linear,
quadratic, Gaussian optimal control problem was used to define the feed-
back gains, and the cost functional (8) was determined so as to optimize
the landing approach performance while accounting for the control

authorities that are imposed on the flight control system. The value of the structure of the solution to the linear, quadratic, Gaussian problem is that Eqs (5) and (6) for the optimal feedback matrix F can be easily and rapidly solved via modern digital computer techniques. Thus the problem is reduced to solving for k_1 and k_2 of Eq (3) which minimizes PMA subject to specified rms constraints on the control activity. This was done numerically by a direct search minimization technique called "pattern search." The details of this minimization technique and the digital computer implementation are given in Ref 4.

As previously mentioned, a "sub-optimal" state estimate $\hat{x}$ is used in the control law formulation given by Eq (7). The form of the "sub-optimal" state estimator and the reasons for using the "sub-optimal" state estimate instead of an optimal estimate of the state in the control definition are given in Refs 1 and 4. This aspect of the optimal control law is not particularly pertinent to this study since it is the optimal feedback gains which we really want. The interested reader can find complete coverage in Refs 1 and 4.

Since the control law given by Eq (7) was developed for the case where disturbance and measurement noise are zero mean and Gaussian, it does not directly deal with cases where steady winds and wind shears are considered. This approach was taken to avoid the intractability of dealing with a probabilistic description of these types of disturbance. It would also be unrealistic to prescribe a flight control law which took into account a given deterministic wind directly because they are in fact random from day to day and from one geographic location to another. It is possible, however, to develop a control law which suppresses the effect of steady winds and wind shear on the deviation from glide slope. This

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is done by including a state in the system equations, Eq (1), which is the integral of glideslope deviation. In this way, the control law given by Eq (7) incorporates an integral control which suppresses steady errors in the glideslope deviation due to steady winds and compensates for the wind shears. This approach was taken for the DC-8 using integral feedback in glideslope tracking. Recall, however, that steady winds and wind shear are taken as zero in this study.

The resulting automatic flight control system model used in the data rate analysis is shown schematically in Fig. 4. The discrete Kalman filter and the intersample extrapolator shown in the figure are used to derive the state estimate $\hat{x}$ and are described in Ref 1. The portion of the control system model that is of particular interest in this study is the optimal feedback gains.

Fig. 4. Block Diagram of the Landing Approach Model with the Optimal AFCS

IV. Standard

In this chapter a classically designed, advanced automatic flight control system is described. This AFCS is taken from Ref 2 and it is used as a standard in this study. This standard served to validate the computational techniques of the digital computer program and was used to compute baseline values for such figures of merit as the probability of missed approach (PMA), rms glideslope deviation (σ_d), rms pitch angle deviation (σ_θ), rms elevator angle deviation (σ_{δ_e}) and the rms elevator actuator rate ($\sigma_{\dot{\delta}_e}$). These baseline figures are used as a measure of "goodness" with which to compare the sub-optimal AFCS design. Included in this chapter are the feedback control law for the standard, the state equation formulation for the standard, and a comparison of numerical results from Refs 2, 6, and this study for the various figures of merit.

The standard automatic flight control system was designed using "classical" multi-loop control techniques, and the design philosophy as well as the control system is described in Ref 2. A block diagram of the standard system is shown in Fig. 5. The standard does not represent an existing system, but rather an advanced, high performance automatic pilot and approach coupler of the type that would be required for successful Category II landing approach operations in a moderate-to-severe turbulence environment.

The inner loop of Fig. 5 is used to feed back pitch and pitch rate. Pitch rate $\dot{\theta}$ is fed back for short-period damping and to extend the path-following bandwidth. A conventional feedback of pitch attitude θ to

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Fig. 5. Block Diagram of Automatic Glidepath Displacement Control System of the Standard

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achieve short-period stiffness together with path damping is modified by a washout with a comparatively small time constant T_{wo}. This has the effect of retaining the short-period attitude stiffness, but it trades path damping for a reduced glide path displacement due to normal gusts. The gains and time constants used in the inner loop closure are given in Table III.

TABLE III

Selected Gains and Time Constants for the AFCS of Ref 2

Pitch Rate and Attitude Stability Augmentation

$$1/T_a = 15.0 \text{ rad/sec}$$

$$1/T_{wo} = 0.7 \text{ rad/sec}$$

$$K_{\dot{\theta}} = -2.0 \text{ sec}$$

$$K_{\theta} = -2.0$$

Path-Following Regulation and Control

$$1/T_f = 2.0 \text{ rad/sec}$$

$$K_d = -0.00867 \text{ rad/ft}$$

$$K_{\ddot{d}} = -0.0013 \text{ 1/ft-sec}$$

$$K_{\dot{d}} = -0.0256 \text{ sec/ft}$$

The outer loop provides the feedback and filtering of glideslope displacement information and is referred to as a path-following loop. Since the pitch attitude θ feedback is modified by a washout, the aircraft has no absolute attitude reference. This dictates the need for a high quality path damping signal. In practice, this could be the derived beam rate $\dot{d}$, the incremental altitude rate $\dot{h}$, or the result of a complementary filtering which could include output of a normal accelerometer.

The analysis presented in Ref 2 assumed that in a high-performance system, and on a relatively shallow glidepath, such as -3.0 deg, the complementary filtering can be performed in such a way that, over the bandwidth of the path-following loop, a pure or noise-free d signal can be provided.

A beam displacement signal is required for path acquisition and stiffness. To this is added the integral of the beam displacement to keep the aircraft on the reference glidepath in the presence of a headwind or a long wavelength updraft. The gain on the integral term is limited by considerations of path-following stability, so that its effectiveness is only felt in regulating against, at most, slowly changing winds. Both the beam displacement and integral of beam displacement are shown, in Fig. 5, to be filtered by a low-pass filter with a time constant T_f. This filter is representative of the combined impedance of the filter capacitor and the receiver conventionally used in the VHF-UHF ILS. Alternatively, T_f can be taken to closely approximate the combined characteristics of a receiver boxcar hold and ripple filter such as might be used in connection with a microwave scanning beam system. The numerical values for the gains and time constants used in the outer loop closure are given in Table III.

The feedback control law for the system of Fig. 5 is arrived at by closing both feedback loops and assuming $d_c = 0$. The control law δ_{e_c} is given by

$$\delta_{e_c} = -Y_\theta \theta + \theta_c$$

The first term Y_θ is given by

$$Y_\theta = \frac{K_\theta s}{s + 1/T_{wo}} + K_{\dot\theta}s = \frac{K_{\dot\theta}s(s + 1/T_E)}{s + 1/T_{wo}}$$

where

$$\frac{1}{T_E} = \frac{K_\theta}{K_{\dot\theta}} + \frac{1}{T_{wo}}$$

The second term θ_c is given by

$$\theta_c = -Y_d d$$

where

$$Y_d = \frac{K_{\bar d} + K_d s}{s(s + 1/T_f)} + K_{\dot d} s$$

$$= \frac{K_{\dot d}(s + 1/T_{d_1})(s + 1/T_{d_2})(s + 1/T_{d_3})}{s(s + 1/T_f)}$$

and

$$\frac{1}{T_{d_1}} = \frac{K_{\bar d}}{K_d} , \qquad \frac{1}{T_{d_2}} = \frac{K_d}{K_{\dot d}} , \qquad \frac{1}{T_{d_3}} = \frac{1}{T_f}$$

Expressing the control law equations in state-variable form requires a little "fiddling around" and the establishment of at least three dummy states. For the benefit of the inquisitive reader the equations used are

$$\dot u_{a_s} = X_u u_{a_s} + X_w w - g\theta \cos \gamma_\bullet + \omega_{u_g} u_g - X_w w_g + X_{\delta_e}\delta_e + X_{\delta_{th}}\delta_{th}$$

$$\dot w = Z_u u_{a_s} + Z_w w + U_\bullet q - g\theta \sin \gamma_\bullet - Z_w w_g + Z_{\delta_e}\delta_e + Z_{\delta_{th}}\delta_{th}$$

$$\dot q = M_u u_{a_s} + M_w w + M_q q - M_w w_g + M_{\delta_e}\delta_e + M_{\delta_{th}}\delta_{th}$$

$$\dot\theta = q$$

$$\dot d = U_\bullet \theta - w$$

$$\dot{\delta}_e = -\frac{1}{T_E}\delta_e + \frac{1}{T_E}\delta_{e_c}$$

$$\dot{\theta}_f = -\frac{1}{T_{wo}}\theta_f + K_\theta q$$

$$\dot{d}_f = -\frac{1}{T_f}d_f - d - y_2$$

$$y_2 = d_f$$

where θ_f, d_f, and y_2 are dummy states representing the output of the washout, the output of the glideslope receiver filter, and the integral of the filtered glideslope deviation. The state y_2 represents the additive noise placed on the glideslope deviation d. The gains, time constants, and stability derivatives are given in Tables II and III.

Comparison of Results

The numerical results of Ref 2 were validated by two methods: one, using the digital computer program that was used in this study to compute the numerical results, and two, an analog simulation technique reported in Ref 6. The comparison is shown in Table IV.

The figures in Table IV show that exact agreement was not achieved between the data of Ref 2 and the digital solution. Also note the results of the analog simulation of Ref 6 do not agree exactly with those of Ref 2. The digital computer results and those of Ref 6 tend to vary in the same direction when compared to the results of Ref 2. The root sum squared disturbance correlated error figures show fairly good agreement between all three studies, and the values for σ_d and σ_θ of Ref 2 are used as a basis for comparison in this study. A value for the rms elevator rate

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TABLE IV

Standard Deviations of System Response to Stochastic Disturbances

Contribution To	Source of Data*	Disturbance Source			Root Sum Squared Disturbance Correlated Error (Standard Deviation)
		u_g σ_{u_g} = 10 ft/sec	u_g σ_{u_g} = 6.5 ft/sec	Microwave Scanning Glide Slope Fluctuation σ_g = 1.32 ft T_c = 0.2 sec	
σ_{dc_g} (ft)	1	4.41	3.68	0.314	5.75
	2	5.17	3.2	0.316	5.98
	3	4.70	2.7	0.203	5.43
σ_θ (rad)	1	0.0183	0.0215	0.00519	0.0287
	2	not available	not available	not available	-
	3	0.0158	0.0178	0.0005	0.0223
σ_{δ_e} (rad)	1	0.0269	0.0315	0.00827	0.0422
	2	0.022	0.0110	0.0	0.0240
	3	0.022	0.0156	0.0015	0.0264
$\sigma_{\dot{\delta}_e}$ (rad/sec)	1	not available	not available	not available	-
	2	not available	not available	not available	-
	3	0.08165	0.147	0.0229	0.1696

*Row 1 data are from digital computer study performed by STI (Ref 2); Row 2 data are from analog simulation (Ref 6); Row 3 data are digital computer results of this study.

is not given in Ref 2; therefore, a value of .1692 rad/sec, as computed by the digital program, will be used for comparing control activities. The response of the "standard" system to a pitch rate impulse is shown in Fig. 12 of Chapter VII.

V. Developing the Practical System

The main objective of this study is to determine if a practical control system could be developed from a full optimal system through some logical procedure. The step by step procedure is presented for two cases using the equations of motion for the DC-8 aircraft as given in Chapter II. In the first case, airspeed was assumed to be measured and available for the control law, while the second case did not use airspeed. Before starting, a brief review of the complete procedure is given.

As explained in Chapter II, the optimal control law is developed to minimize a given cost functional subject to certain constraints. In order to arrive at this control law, it is necessary to specify the measurements which are assumed to be available. As previously mentioned, some of these measurements are associated with the flight control system and are available continuously, free of noise. Those measurements considered here include sensed pitch attitude, pitch rate, and normal acceleration. These are common to both cases presented in this chapter. The remaining measurements are the glideslope deviation, the associated errors as mentioned in Chapter II, and for the first case, airspeed with noise and measurement errors. The integral of glideslope deviation is derived from the glideslope measurement.

Once given the desired measurements, the optimal control law can be solved to obtain the optimal feedback gains. The feedback gains include a set of gains for an autothrottle. Since the approach flight condition is at a trim speed above the speed for trim response reversal, both glidepath tracking and speed regulation can be achieved with elevator control

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alone (Ref 1). Since autothrottle is not needed, the first step is to remove those feedbacks (i.e., zero the appropriate gains). The procedure continues, removing feedbacks, one at a time, until only those desired feedbacks are left. Of those final feedbacks, glideslope deviation and airspeed must have measurement noise added and then be filtered. The filtered measurements are used in the final control law since they represent the information as it would really be available for use in the AFCS.

If the feedback control law is simplified further by removing feedbacks on measurements assumed available in the optimization process, the resulting control law will be either unstable, seriously degrade approach performance, result in high actuator rates, or result in poor pitch attitude characteristics. The results of deleting the prespecified measurements are given in Table V and reinforce the original hypothesis (i.e., the measurements specified for optimization must be retained by the practical system).

All data presented in this paper assumed a scanning beam system as described in Ref 1. A sampled data rate of 6 samples/sec was used for all cases. This figure was found to be an acceptable data rate for an AFCS incorporating normal acceleration information in the control law.

Control Law with Airspeed Feedback

In the first case, airspeed information is assumed to be available in formulating the control law. The procedure is described a step at a time and the results are explained at each step. The numerical data for each step are given in Table V.

> Step 1. The equations are solved using the full state feedbacks from the optimization routine (data rate analysis program). This gives the performance attainable with full state feedback.

Step 2. The optimization procedure develops two control laws: the elevator control law and a throttle control law. Auto-throttle was not used in Ref 2 for the standard and is not needed, so the autothrottle feedbacks are deleted.

Step 3. Elevator position is fed back in the full optimal system. The standard does not use elevator position so this feedback is deleted.

Step 4. The optimal feedback gains include feedback of the wind gust intensities. Wind gusts cannot be measured easily so feeding them back is not practical. These gains are set to zero.

At this point, the control law consists of only the desired feedbacks: airspeed u_{as}, normal velocity w, pitch attitude θ, pitch rate q, glideslope deviation d, glideslope deviation rate $\dot{d}$, and the integral of glideslope deviation $\int d$. To verify the constraint on the hypothesis of Chapter I, the prespecified feedbacks are now individually zeroed.

Step 5. Zero the feedback on airspeed u_{as}, and the PMA increases.

Step 6. Zero the feedback on normal velocity w, and the PMA increases.

Step 7. Zero the feedback on pitch attitude θ, and the system is unstable.

Step 8. Zero the feedback on pitch rate q. This step deserves special comment. The previous steps showed a definite detrimental effect on the performance while this step has improved the approach performance PMA. Notice, however, that the rms pitch attitude deviation has increased as has the rms actuator rate. Removing pitch rate feedback decreases the response time but causes the system to have poor short-period damping. The light damping is more evident when the system response to a pitch rate impulse is plotted as in Fig. 6. Inasmuch as PMS is computed using the rms glideslope deviation, these high-frequency oscillations, having a low rms component, contribute little to the rms glideslope deviation and PMA. While the PMA is acceptable, the high-frequency oscillations produce unacceptable ride qualities and unacceptable pitch attitude characteristics.

Step 9. Zero the feedback on glideslope deviation d, and the system is unstable.

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Fig. 6. System Response Without Pitch Rate Feedback

Step 10. Zero the feedback on the integral of glideslope deviation $\int d$ and the control system will no longer compensate for steady winds.

As hypothesized in Chapter I, the removal of any prespecified feedback results in an unsuitable control system. Now returning to the control system as it was configured in Step 4, the glideslope deviation and airspeed feedbacks must be made to resemble the "real world" measurements.

Step 11. The glideslope deviation measurement noise y_2 and errors such as explained in Chapter II are considered first. The noise is added to the glideslope deviation and a new state is introduced to represent the filtered glideslope deviation measurement d_f. The filter is shown in Fig. 7. The time constant T_f was chosen to minimize the PWA. The value selected was 0.1 sec. It was assumed the d_f is also used to derive the integral of d feedback.

Fig. 7. Glideslope Deviation Measurement Model

Step 12. Including airspeed in the control law requires that the problems associated with measuring airspeed be considered. There is noise on the measurement. But more important, there exists significant time lag in the measurement caused by the mechanism used to make the measurement. Airspeed measurement devices currently in use incur a time delay in the order of 0.5 sec. It is reasonable to assume a lag in sensed airspeed of 0.5 sec.* The measurement device was approximated by a first-order lag. It was

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*Suggested by Ronald Anderson, Air Force Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio.

TABLE V

Numerical Results of Sub-optimal Control System with Airspeed Feedback

Step	PMA	3σ Value of Elevator Actuator Rate (rad/sec)	1σ Value of Pitch Angle (rad)
1	.02084	.16572	.02119
2	.02090	.16572	.02153
3	.01719	.16974	.02178
4	.01074	.23367	.02436
5	.19926	.04219	.02097
6	.24631	.24463	.01781
7	unstable		
8	.00187	.27216	.03177
9	unstable		
10	.00468	.23137	.02374
11	.01235	.23932	.02422
12	.03980	.10700	.02518
Standard	.0366	.168	.0287

assumed that most of the sensor noise present would be filtered out by the device itself. The measurement model is shown in Fig. 8, where T_L=0.5 sec. The measured airspeed is called u_m. The system performance is extremely sensitive to the airspeed measurement. A delay of 1.0 sec was found unacceptable as the system performance deteriorated greatly (PMA=.07304).

Fig. 8. Airspeed Measurement Model

The results of Step 12 represent the final system. The numerical results are compared with those for the standard in Chapter VII. The system response to a pitch rate impulse of 1 deg/sec is shown in Chapter VII, Fig. 13.

Control Law Without Airspeed Feedback

This case is developed much as the previous case except airspeed is not considered as one of the measurements available in the development of the optimal feedback control law. Developing the control law with the same measurements as those used in Ref 2 permits the comparison of a classically designed advanced system with the realizable sub-optimal system where the same feedbacks are assumed to be available. The numerical results for this case are found in Table VI. The step-by-step procedure is as follows:

Step 1. As in the first case, the initial step is to use the full state feedback.

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Step 2. Delete all feedbacks to the throttle.

Step 3. Zero the elevator position feedback.

Step 4. Zero the wind gust intensity feedbacks.

Step 5. Zero the airspeed feedback. It should be pointed out here that even though airspeed was not a prespecified feedback, airspeed feedback does have a significant effect on approach performance and control activity.

The control law now consists of just the desired (prespecified) feedback gains, and measurement noise or filtering is not yet considered. As in the previous case, these desired feedback gains are now individually set to zero, one at a time, to validate the constraint on the hypothesis in Chapter I and to show the resultant effect.

Step 6. Zero the normal velocity feedback.

Step 7. Zero the pitch attitude feedback.

Step 8. Zero the pitch rate feedback.

Step 9. Zero glideslope deviation feedback.

Step 10. Zero the integral of glideslope deviation feedback. The improvement in this step is expected since integral feed-backs have a destabilizing effect. The cost of this improvement is the loss of compensation for steady winds.

With fewer measurements available for the control law, the gains are higher and the result of removing one of these feedbacks is much more pronounced. This is apparent when comparing the effects of Steps 6 through 10 of Table VI with the corresponding steps of Table V. Now returning to the control system as it was configured in Step 5, the glideslope deviation must be made to resemble the actual measurements.

Step 11. The glideslope deviation is corrupted with noise y_2, and filtered as in the previous case. Originally the filter in Fig. 7 was used; however, the effect of filtering glideslope deviation was a reduced elevator actuator rate and consequently a poor PMA. The filter of Fig. 7 was modified by including a gain G in the numerator to

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compensate for the effect of filtering. This gain was
varied to achieve the minimum PMA. This filter is shown
in Fig. 9, where T_f is 0.5 sec and G is 1.27.

Fig. 9. Glideslope Deviation Measurement Model with Increased Gain

Step 12. The control system of Step 11 has a much poorer per-
formance than the standard. The standard achieved a
PMA of .0366 while the sub-optimal control system, at
this point, results in a PMA of .07316. A closer look
at the data reveals that one cause for the poorer per-
formance is a much lower rms elevator actuator rate:
.08058 rad/sec for the sub-optimal as opposed to .16779
rad/sec for the standard. The logical "fix" is to
increase the rms elevator actuator rate in the sub-
optimal system. To get the higher actuator rate, the
optimization routine was rerun with relaxed constraints
on actuator activity. This resulted in higher feedback
gains and in the final system, higher rms actuator rates.
It is important to point out the ease and quickness with
which this improvement was achieved. Because the equa-
tions are mechanized for computer solution, only one
card needed to be changed and the entire revision was
completed in less than a day. The final system response
to a pitch rate impulse is shown in Chapter VII, Fig. 14.

The final sub-optimal control law results in an oscillation when

the system is perturbed by a pitch rate impulse as shown by Fig. 13 of

Chapter VII. A classically designed system (Ref 7) similar to the

standard but without the washout also experienced those oscillations.

If these oscillations are considered undesirable, the pitch attitude

feedback can be modified by a washout. This has the effect of retaining

the short-period attitude stiffness, but it trades path damping for a

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reduced response or glide path displacement due to normal gusts or pitch rate impulses.

The realizable systems described in this chapter will be compared to the advanced, classically designed standard in Chapter VII. The numerical values for the feedback gains are given in Appendix A. The directness and simplicity of this technique should be quite evident at this point.

TABLE VI

Numerical Results of Sub-optimal Control System Without Airspeed Feedback
(6 samples/sec)

Step	PMA	3σ Value of Elevator Actuator Rate (rad/sec)	1σ Value of Pitch Angle (rad)
1	.00131	.26750	.02443
2	.00131	.26750	.02443
3	.00100	.27791	.02469
4	.00136	.34038	.02633
5	.05172	.07137	.02250
6	.31705	.14866	.01940
7	unstable		
8	unstable		
9	.79171	.06014	.03216
10	.05173	.12369	.01895
11	.07316	.08058	.02510
12	.02154	.15095	.02680
Standard	.0366	.168	.0287

VI. Application to Direct Lift Control

In this chapter, the technique described in Chapter V is used to develop realizable control laws for a hypothetical aircraft which has the capability of using direct lift control. Direct lift control has attracted considerable attention, particularly for possible use on STOL aircraft. The primary purpose for considering DLC is to conceptually show the significant improvement which can be achieved in glideslope tracking and to demonstrate the versatility of this design technique. Direct lift control represents a multiple input, multiple output system which, for classical control techniques, is at best extremely difficult. The optimal approach handles this case with comparative ease.

This chapter describes how DLC was conceptually considered for the DC-8. The modified equations of motion and actuator equations are given. Two cases are presented in detail: the first case includes airspeed measurements in the control law, while the second case does not. The procedure follows closely that described in the previous chapter. The numerical results for each step of the procedure are in Table VIII. The system response to a pitch rate impulse is found in Chapter VII, Figs. 15 and 16. The numerical values of the optimal feedback gains are presented in Appendix A.

Aircraft Model with Direct Lift Control

The investigation of DLC was performed on a hypothetical DC-8 incorporating idealized direct lift control. The model is taken from Ref 1, where it was assumed that ideal DLC would generate normal forces

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on the aircraft without significant changes in pitch attitude. The basic aircraft equations were adapted to include direct lift control by the introduction of a new control input vector

$$ u = \begin{bmatrix} \delta_{e_c} \\ \delta_{d_c} \end{bmatrix} $$

where δ_{d_c} is the commanded input to the direct lift control surface. Thus the throttle control was replaced by a hypothetical direct lift control.

The equations of motion in Chapter II were modified by replacing the thrust stability derivatives $(X_{\delta_{th}}, Z_{\delta_{th}}, M_{\delta_{th}})$ with a hypothetical set of direct lift stability derivatives $(X_{\delta_d}, Z_{\delta_d}, M_{\delta_d})$. The equations for the hypothetical "direct lift" DC-8 are

$$ \dot{u}_{as} = X_u u_{as} + X_w w - g\theta \cos \gamma_0 + \omega_{u_g} u_g - X_w w_g + X_{\delta_e} \delta_e + X_{\delta_d} \delta_d $$

$$ \dot{w} = Z_u u_{as} + Z_w w - g\theta \sin \gamma_0 + U_0 q - Z_w w_g + Z_{\delta_e} \delta_e + Z_{\delta_d} \delta_d $$

$$ \dot{q} = M_u u_{as} + M_w w + M_q q - M_w w_g + M_{\delta_e} \delta_e + M_{\delta_d} \delta_d $$

$$ \dot{\theta} = q $$

$$ \dot{d} = U_0 \theta - w $$

The values for the direct lift stability derivatives are given in Table VII. The remaining stability derivatives are left the same and are found in Table II. The disturbances are the same as described in Chapter II. The differential equations for the elevator actuator and direct lift control surface are

$$\dot{\delta}_e = -\frac{1}{T_e}\delta_e + \frac{1}{T_e}\delta_{e_c}$$

$$\dot{\delta}_d = -\frac{1}{T_d}\delta_d + \frac{1}{T_d}\delta_{d_c}$$

where δ_d is the direct lift control surface deflection, T_d and T_e are the control lags on the direct lift surface and the elevator; T_d and T_e were both taken to be .06666 sec.

TABLE VII

Direct Lift Stability Derivatives (Ref 1)

$$X_{\delta_d} = 0.0 \text{ ft/rad-sec}^2$$

$$Z_{\delta_d} = -50.0 \text{ ft/rad-sec}^2$$

$$M_{\delta_d} = 0.0 \text{ 1/rad-sec}^2$$

The numerical value for Z_{δ_d} corresponds to a control effectiveness of roughly 0.1 g normal acceleration for a 4° surface deflection. The choice of $X_{\delta_d} = M_{\delta_d} = 0$ causes the direct lift control to be relatively uncoupled from the θ, q, and u_{as} equations. This is especially true since $M_{\dot{w}}$ is zero.

Control Law with Airspeed

The first case considered for the hypothetical aircraft with DLC included airspeed feedback in the control law. The other measurements used in the optimization process were normal velocity, pitch attitude, pitch rate, and sampled glideslope deviation. The procedure follows

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closely that used in the previous chapter. There are two control laws of interest when considering DLC: the elevator control law and the direct lift control law. Therefore, each variable has two feedback gains. The numerical results are given in Table VIII. The step-by-step procedure is as follows:

Step 1. The equations are solved using full state feedback to evaluate the system capability using the full set of optimal feedbacks.

Step 2. The wind gust intensity feedbacks are deleted because wind gusts are not measured in a realizable flight control system.

Step 3. Feedback gains on the position of both the elevator and the direct lift control surface are set to zero. This case represents the prespecified feedbacks without considering measurement noise or filtering of the measured signals.

The following series of steps were used to verify the original hypothesis of Chapter I which stated that removing a measurement from the control law which was assumed present for the original optimization results in a poor control system. In Table VIII, case (a) represents the result when the particular feedback was deleted from the elevator control law, while case (b) is the result when the same feedback was deleted from the direct lift control law.

Step 4. Zero the airspeed feedback gain.

Step 5. Zero the normal velocity feedback gain.

Step 6. Zero the pitch attitude feedback gain.

Step 7. Zero the pitch rate feedback gain. Deleting pitch rate feedback results in lower response times but higher settling times. Just as in similar cases in Chapter V, the PMA is satisfactory because the high frequency oscillations contribute little to the rms glideslope deviation. The oscillations cause a marked increase in the rms pitch attitude. Removing pitch rate feedback results in poor ride quality.

TABLE VIII

Numerical Results of Sub-optimal Direct Lift Control System
With Airspeed Feedback (Data rate, 6 samples/sec)

Step	PMA	3σ Value of Elevator Actuator Rate (rad/sec)	3σ Value of Direct Lift Actuator Rate (rad/sec)	1σ Value of Pitch Angle
1	.02428	.02900	.19461	.01163
2	.00050	.04361	.27452	.01187
3	.00035	.04423	.27898	.01199
4a	.00596	.00254	.27924	.01114
b	.16504	.04454	.03987	.01250
5a	.00162	.04546	.27907	.00997
b	.08778	.04443	.29341	.01163
6a	.00146	.04554	.27968	.01561
b	.12760	.04463	.29146	.01615
7a	.00028	.04622	.27889	.01268
b	.00000	.04423	.29391	.01247
8a	.00053	.04425	.27889	.01149
b	.08345	.04420	.27892	.01181
9	.00047	.04420	.27862	.01189
10	.00280	.01709	.11152	.01208
Standard	.0366	.168	n/a	.0287

NOTE: Case a represents the result when the particular feedback was
deleted from the elevator control law, while Case b is the
result when the same feedback was deleted from the direct lift
control law.

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Step 8. Zero the glideslope deviation feedback gain.

The results of the last five steps validate the original hypothesis of Chapter I. Now going back to the control system in Step 3, the sampled glideslope deviation and airspeed will be handled as in the first case in the previous chapter.

Step 9. The glideslope deviation was assumed to be sampled at a rate of 6 samples/sec and to have the same errors as described in Chapter II (i.e., broadband additive noise). The noisy measurement information was fed into a filter as shown in Fig. 7. The filter time constant was picked to minimize the probability of missed approach. In this case, the value selected was T_f=0.2 sec. The filtered glideslope deviation was then fed back. The filtered glideslope deviation was also used to derive integral of glideslope deviation.

Step 10. The airspeed was handled in the same manner as in the first case in Chapter V. As before, it was assumed that any noise on the measured airspeed would effectively be filtered by the mechanism of the measurement device. It was also assumed that the measurement would incur a time lag due to the mechanical nature of the measurement device. The measurement model is shown in Fig. 8 of Chapter V. The time lag T_L was again taken to be 0.5 sec.

The realizable control system gives a very low PMA (.00280). This demonstrates the benefit of using DLC. The rms direct lift control surface rate dropped more than half when the lag was placed on the airspeed measurement. This points out the strong reliance on good airspeed measurement for any AFCS which considers airspeed as a prespecified measurement in developing the optimal feedbacks.

Control Law Without Airspeed

Prespecifying airspeed in the control law creates several problems. As the previous case demonstrates, the control system with airspeed feedback tends to be very dependent on this feedback. This dependency results in the need for high quality sensors which will create a minimum time lag in the measurement and also filter out most of the noise. These disadvantages

provide motivation for developing an optimal control law without airspeed
feedback. The procedure parallels that of the second case in Chapter V.
The numerical results are listed in Table IX.

Step 1. The system of equations is solved using the full state
feedbacks from the optimization procedure. The system
response to a pitch rate impulse of 1 deg/sec is shown
in Fig. 10.

Step 2. All feedbacks to the direct lift control surface are set
to zero to determine if the direct lift control law can be
eliminated. The resulting AFCS is unstable. Since DLC
is a prespecified control and is weighted heavily in the
data rate analysis, the practical system will have to
include DLC.

Step 3. The elevator control gains were deleted to determine if
the elevator could be eliminated. The numerical results
in Table IX indicate that elevator control is not needed;
however, these results are based on rms values and can be
misleading. The control system without the elevator is
actually slightly underdamped. The system response to a
pitch rate impulse of 1 deg/sec is shown in Fig. 11. The
effect of elevator control is quite evident when Figs. 10
and 11 are compared. The elevator control law is left in
the system.

Step 4. Zero the airspeed feedback gain.

Step 5. Zero the position feedback gains on both controls.

Step 6. Zero the wind gust intensity feedback gains. This system
represents the final system, before adding noise y_2 to the
glideslope deviation.

Step 7. As in previous cases, the glideslope deviation is now com-
bined with the fluctuation noise and then filtered. The
filter is shown in Fig. 6. The same time constant is used
as in the past case where $T_f = .2$ sec.

The final system gave very good results. No attempt was made to
improve performance through adding gains to the glideslope filter or by
relaxing rms actuator constraints in the optimization routine. In an
actual application, the actuator rates would be fixed and the design would
then be made to meet the rates while here a hypothetical case was considered

TABLE IX

Numerical Results of Sub-optimal Direct Lift Control System
Without Airspeed Feedback (Data rate, 6 samples/sec)

Step	PMA	3σ Value of Elevator Actuator Rate (rad/sec)	3σ Value of Direct Lift Actuator Rate (rad/sec)	1σ Value of Pitch Angle
1	.00000	.00000	.28877	.01377
2	unstable			
3	.00000	.00000	.28877	.01377
4	.00835	.00000	.24163	.01585
5	.00593	.00000	.24658	.01572
6	.00440	.00000	.14584	.01482
7	.00668	.00000	.13543	.01518
Standard	.0366	.168	n/a	.0287

Fig. 10. Pitch Rate Impulse Response of DLC System
With Full State Feedback

Fig. 11. Pitch Rate Impulse Response of DLC System
Without Elevator Control

to point out the ease with which the optimal approach could handle this multi-input, multi-output situation--not to achieve the best possible performance.

The actual numerical values for the feedback gains are listed in Table XIV for the case with airspeed feedback, and in Table XV for the case without airspeed feedback. Both tables are located in Appendix A.

VII. Summary of Results, Conclusions, and Recommendations

The summary of results, conclusions, and recommendations reached in determining a design procedure using the "optimal" feedback gains of the data rate analysis are enumerated in this chapter.

Summary of Results

The results of this study are summarized in the form of a table and five time histories showing aircraft responses. In Table X, the PMA, rms elevator actuator rates, rms pitch attitude, and rms direct lift control rates are listed for the "standard" and the final form of each of the four cases considered. The time histories (Figs. 12, 13, 14, 15, and 16) show the system response to an initial pitch rate impulse of 1 deg/sec for the "standard" and for each case considered. The "standard" and the two cases with airspeed have approximately the same closed-loop frequency. The two cases without airspeed seem to have a higher closed-loop frequency.

Conclusions

1. It has been shown that a set of optimal feedback gains can be the basis for a practical control system, provided the following considerations are properly accounted for in the optimization:

    a. The cost functional reflects system performance.

    b. Control activity is realistically accounted for.

    c. The measurements available for feedback are taken into account in the optimal design.

TABLE X

Comparison of the Final System for Each Case and the Standard

	PMA	RMS Elevator Actuator Rate (rad/sec)	RMS Pitch Attitude (rad)	RMS Direct Lift Control Rate (rad/sec)
			Measures of "Goodness"	
Standard	.03660	.16800*	.02870	
DC-8 with Airspeed Measurements	.05090	.10582	.02680	
DC-8 without Airspeed Measurements	.02154	.15095	.02680	
DLC with Airspeed Measurements	.00280	.01709	.01208	.11152
DLC without Airspeed Measurements	.02395	.00000	.01351	.10052

*This value was computed by the digital computer program and was not given in Ref 2.

Fig. 12. Pitch Rate Impulse Response of the Standard System

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Fig. 13. Pitch Rate Impulse Response of the DC-8
With Airspeed Feedback

Fig. 14. Pitch Rate Impulse Response of the DC-8
Without Airspeed Feedback

Fig. 15.   Pitch Rate Impulse Response of the Direct Lift Control
DC-8 with Airspeed Feedback

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Fig. 16. Pitch Rate Impulse Response of the Direct Lift Control
DC-8 without Airspeed Feedback

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2. A systematic procedure was developed which gives a good "first cut" at a practical control system.

3. The procedure is "clean" and requires no "games" or "fiddling" with the feedback gains.

4. The AFCS achieved in the case of the DC-8 without airspeed feedback compared quite well with the standard as shown in Table X and by comparing Figs. 12 and 14. If the high-frequency oscillations in pitch attitude, shown in Fig. 14, are considered undesirable, a washout on pitch attitude feedback can be used to eliminate them (Ref 7).

5. The incorporation of airspeed feedback in the control law results in improved approach performance (i.e., lower PMA) and a reduced sensitivity to changes in the other feedbacks. When interpreting the results in Table X, the reader must consider the value of the rms actuator rate as well as PMA to avoid being misled.

6. Direct lift control was handled easily. The results for the two DLC cases are presented in Table X and Figs. 15 and 16. The merit of DLC in the landing approach task is evident from the results in Table X.

7. The rms measures of "goodness" are good for a "first cut" design but are not sufficient to assure a desirable, safe, flyable AFCS. Further analysis (i.e., pitch rate response plots, stability tests, phase and gain margin checks, etc.) is required to assure a safe system.

Recommendations

The following recommendations are suggested:

1. A better or improved measure of landing approach performance is needed. PMA is not adequate even when rms pitch attitude and rms control activity are considered. What is needed is a measure of ride

qualities or pilot rating. Ride quality can be evaluated by looking at the acceleration spectrum at the cockpit in the 3 to 5 cps region.

2. Adopt the approach to tasks other than landing approach, for instance, weapon delivery.

3. Apply this procedure to design an automatic landing control system for remotely piloted vehicles.

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APPENDIX A

Numerical Values of Feedback Gains

The information in this appendix is included for the benefit of anyone who might want to duplicate this study or do further research in this area. All values presented here are identical to the values used in this study. The feedbacks retained in the practical system are marked with an asterisk (*).

The feedbacks on glideslope deviation and airspeed are moved from the pure states $(d, \int d, u_{as})$ in the full state feedback system to the filtered or sensed states $(d_f, \int d_f, u_m)$ in the practical system.

TABLE XI

Feedback Gains for the Standard System
(data rate, continuous)

State	Feedback to Elevator
u_{as}	0.0
w	-2.5576×10^{-2}
θ	5.8313
q	2.0000
u_g	0.0
w_g	0.0
θ_f	-1.0000
y_2	0.0
d	0.0
δ_e	0.0
$\int d$	7.6803×10^{-4}
d_f	8.6700×10^{-3}

TABLE XII

Feedback Gains for the Sub-optimal System with Airspeed Feedback
(data rate, 6 samples/sec)

State	Feedback to Elevator	Feedback to Throttle
u_{as}	3.3678×10^{-3}*	$-.22531$
w	-6.4191×10^{-3}*	9.5716×10^{-2}
θ	2.2657*	-32.134
q	$.85460$*	-13.897
u_g	1.0442×10^{-3}	$-.18910$
w_g	-3.2762×10^{-4}	9.5802×10^{-3}
y_2	0.0	0.0
d	3.0483×10^{-3}*	-5.0314×10^{-2}
δ_e	-4.7493×10^{-2}	$.78816$
$\int d$	2.6690×10^{-4}*	-2.3104×10^{-2}
δ_{th}	2.1731×10^{-4}	-2.3412×10^{-3}

*This feedback is retained in the practical system.

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TABLE XIII

Feedback Gains for the Sub-optimal System Without Airspeed Feedback
(data rate, 6 samples/sec)

State	Feedback to Elevator	Feedback to Throttle
u_{as}	4.8803×10^{-3}	-1.3055×10^{-4}
w	$-1.1965 \times 10^{-2}*$	6.2504×10^{-5}
θ	$4.3265*$	-2.1616×10^{-2}
q	$1.4094*$	-7.9522×10^{-3}
u_g	1.2629×10^{-3}	-1.1141×10^{-4}
w_g	-7.8283×10^{-4}	6.9985×10^{-6}
y_2	0.0	0.0
d	$7.4510 \times 10^{-3}*$	-4.3170×10^{-4}
δ_e	-7.6428×10^{-2}	4.4071×10^{-4}
$\int d$	$3.4793 \times 10^{-4}*$	-1.3305×10^{-5}
δ_{th}	5.7354×10^{-4}	-2.4500×10^{-6}

*This feedback is retained in the practical system.

TABLE XIV

Feedback Gains for Sub-optimal System Without Airspeed Feedback
Relaxed Control Activity Constraints
(data rate, 6 samples/sec)

State	Feedback to Elevator (Gains for Practical System Only)
u_{as}	0.0
w	-2.1154×10^{-2}
θ	7.7203
q	2.1266
u_g	0.0
w_g	0.0
y_2	0.0
d	1.6108×10^{-2}
δ_e	0.0
$\int d$	4.6272×10^{-4}
δ_{th}	0.0

TABLE XV
Feedback Gains for Direct Lift Control System with Airspeed Feedback

State	Feedback to Elevator	Feedback to DLC Surface
u_{as}	6.4943×10^{-4}*	-4.0792×10^{-3}*
w	-9.8753×10^{-4}*	7.1767×10^{-3}*
θ	$.34236$*	-2.5057*
q	$.14456$*	$-.98193$*
u_g	2.3002×10^{-4}	-1.3089×10^{-3}
w_g	-4.0206×10^{-5}	3.4499×10^{-4}
y_2	0.0	0.0
d	3.3161×10^{-4}*	-3.4505×10^{-3}*
δ_e	-8.1636×10^{-3}	5.4893×10^{-2}
δ_{dlc}	3.2540×10^{-3}	-2.3556×10^{-2}

*This feedback is retained in the practical system.

TABLE XVI

Feedback Gains for Direct Lift Control System Without Airspeed Feedback

State	Feedback to Elevator	Feedback to DLC Surface
u_{as}	1.8430×10^{-4}	-5.4850×10^{-3}
w	-3.5975×10^{-4}*	1.4798×10^{-2}*
θ	$-.12807$*	-5.2815*
q	5.2266×10^{-2}*	-1.7123*
u_g	4.9343×10^{-5}	$-.10923$
w_g	-1.3738×10^{-5}	9.5572×10^{-4}
y_2	0.0	0.0
d	1.3172×10^{-4}*	-9.3767×10^{-4}*
δ_e	-2.9351×10^{-3}	9.2860×10^{-2}
δ_{dlc}	1.1819×10^{-3}	-4.8113×10^{-2}

*This feedback is retained in the practical system.

VITA

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