SOME FURTHER NUMERICAL STUDIES OF LASER INDUCED MELTING AND VAPORIZATION

Peter J. Torvik

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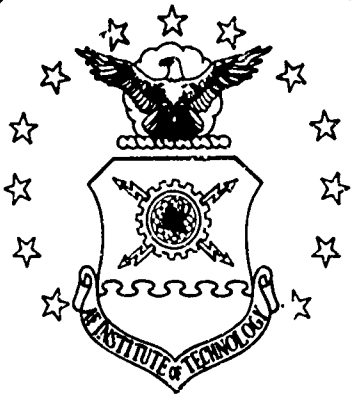

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TECHNICAL REPORT

AFIT TR 73-1 PETER J. TORVIK

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13. ABSTRACT

A method for predicting the melt through time for a metal sheet subjected to a continuous laser beam of Gaussian distribution of arbitrary size and peak intensity is given. The effect of completely retained and instantaneously removed melt layers on the melt through time are considered. It is shown that melt through time and the influence of retained melt are dependent on the value of a dimensionless variable representing power per unit thickness. The partial melting and vaporization of sheets by a large single pulse is also discussed and compared with experimental results.

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SOME FURTHER NUMERICAL STUDIES OF LASER INDUCED MELTING AND VAPORIZATION

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I. INTRODUCTION

In earlier reports (1, 2), a method for calculating the thermal response due to intensive heating was described. The present report describes extensions and applications of the method to other laser heating problems. In the second chapter, a general criterion for one dimensional heating under the assumption that the melt is instantaneously removed will be developed. The predictions of a simple analytical solution for radial conduction are combined with these results in the third chapter to produce a method for estimating from a single curve the melt through time for any arbitrary combination of flux, spot size, thickness and material. In the fourth chapter, melt through time is computed for the case of complete retention of the melt until vaporization occurs. Some predictions for heating due to a large single pulse will be presented in the fifth chapter and compared with the results of experiments featuring large single pulses.

II. A CRITERION FOR ONE DIMENSIONAL MELTING OF SOLIDS

The question of the flux (or power level) required to generate one-dimensional (axial) melting is particularly significant in the design and interpretation of experiments on laser effects. Radial heat conduction has been found to make the interpretation of some experimental results difficult. The following analysis leads to a criterion for one-dimensionality which, it is hoped, will prove useful in the design of experiments.

We assume that a melting front moves at a constant speed V through a slab of material having thermal properties K, ρ and Cp, and a melting temperature T_m. Material at distances greater than some length δ will essentially be at an ambient temperature T_o, i.e.,

$$T(\delta)-T_o/(T_m-T_o) = \varepsilon \tag{1}$$

We assume that the heat flow can be regarded as one dimensional (axial) if the depth of heating is small compared to the width, or if

$$\delta/\sigma<N \tag{2}$$

where σ is some characteristic dimension of the heated region, and N is assumed to be independent of flux intensity and thermal properties. In essence, we assume one-dimensionality if temperature changes take place over distances much less than the dimension of the heated area. If the heat flow is one dimensional, the heat conduction equation reduces to

$$\kappa \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \tag{3}$$

where the diffusivity, κ is defined by

$$\kappa = \frac{K}{\rho C_p} \tag{4}$$

and x is taken to be normal to the surfaces of the slab. In terms of a

coordinate system moving with the front, i.e.,

$$z = x - Vt \tag{5}$$

Equation (2) becomes

$$\kappa \frac{\partial^2 T}{\partial z^2} = - V \frac{\partial T}{\partial z} \tag{6}$$

which has the solution

$$T = T_0 + (T_m - T_0) e^{-Vz/\kappa} \tag{7}$$

From 1,

$$\delta V/\kappa = \ell n\ (1/\epsilon) \tag{8}$$

One dimensional axial heat flow may also exist in a sheet so thin as to be of nearly uniform temperature, i.e., if the characteristic dimension, σ, of the heated region is large compared with ℓ, the thickness, or

$$\ell/\sigma < M \tag{9}$$

M is a dimensionless number, assumed to be independent of material properties. More generally then, we expect that the heat flow will be predominantly axial (or one dimensional), if either of inequalities 1 and 9 are satisfied, or if a new inequality combining the two is satisfied, i.e., if

$$\ell\delta/\sigma^2 < M^* \tag{10}$$

where M^* is assumed to be independent of flux intensity and thermal properties.

The steady speed, V, at which the liquid-solid interface moves may be readily determined from a heat balance. In any time Δt, a thickness Δz of material at a temperature arbitrarily close (ϵ) to the ambient temperature T_0 enters the heated layer. During that time increment, a like amount of material is melted. Thus, for any unit area, a thickness Δz is raised from the ambient to melting temperature, and undergoes phase transformation.

The heat required is supplied by the absorbed intensity I_{pa}, and a simple heat balance on a unit area subject to the peak intensity yields

$$I_{pa}\Delta t = [Cp(T_m - T_o)+L_m]\rho\Delta z \tag{11}$$

where L_m is the heat of fusion. Thus if I_{pa} (the peak absorbed energy per unit area per unit time) is constant, the speed at which the liquid-solid interface penetrates the solid portion at the point of maximum intensity is given by

$$V = \frac{\Delta z}{\Delta t} = \frac{Ipa}{\rho[Cp(T_m-T_o)+L_m]} \tag{12}$$

Substituting this result, and Equation 8 into inequality 10, we find

$$\frac{\ell\kappa\rho}{\sigma^2 I_{pa}}[C_p(T_m-T_o)+L_m] < \frac{M^*}{\ell n(1/\epsilon)} \tag{13}$$

If we assume the beam profile to be Gaussian and axi-symmetric, the distribution of absorbed intensity is

$$I(r) = I_{pa}e^{-r^2/(2\sigma^2)} \tag{14}$$

the total power absorbed is given by

$$P_a = 2\pi\sigma^2 I_{pa} \tag{15}$$

and inequality 13 may be written as

$$\frac{P_a}{\ell} > m^* \tag{16}$$

This furnishes a theoretical basis for the criterion for one dimensional axial heat flow deduced in earlier work [2] from the results of computer experiments on magnesium where m* was found to be 40 KW/cm.

Equation 13 suggests, however, the existence of a single constant, a dimensionless power per unit thickness, applicable to any material. Let

$$P_{\ell a} = \frac{Pa}{\ell\kappa\rho[L_m+C_p(T_m-T_o)]} \tag{17}$$

and

$$P_{\ell a}^* = \frac{2\pi\ell n(1/\epsilon)}{M^*} \tag{18}$$

Then the criterion for one dimensional melting becomes

$$P_{\ell a} > P_{\ell a}^* \tag{19}$$

The assumption of some profile other than Gaussian (e.g., a "flat-top"), could affect at most the numerical value of the dimensionless $P_{\ell a}^*$.

Equation 13 may be written in another form, through the use of the time t_1 required for complete melt through under the one-dimensional condition. The required time is obtained from a one-dimensional heat balance as

$$t_1 = \frac{[C_p(T_m - T_0) + L_m]\rho\ell}{I_{pa}} \tag{20}$$

Substituting into 13 leaves a dimensionless parameter known as a Fourier number.

$$\frac{\kappa t_1}{\sigma^2} < H^*/\ln(1/c) \tag{21}$$

We have seen that the conditions necessary for one-dimensional melting are met if a Fourier number, defined through

$$F = \kappa t_1/\sigma^2 \tag{22}$$

is less than some critical value, or if the dimensionless power per unit thickness, defined through Equation 17 and related to the Fourier number through

$$P_{\ell a} = \frac{2\pi}{F} \tag{23}$$

is greater than some critical value. It is at this point unproven that the critical value of these dimensionless parameters are independent of material properties, although the results given earlier for magnesium are a satisfactory indication that the critical value of the parameter $P_{\ell a}$ is independent of the flux intensity, dimensions of the heated area, and sample thickness.

In order to ascertain if the critical value of P_{2a}, the dimensionless absorbed power per unit thickness, is independent of material parameters, calculations using the previously described numerical method were undertaken. Material properties as given in Table I were assumed, with results as given in Table II. In all cases, a disk 2.5 cm diameter by .1 cm thick was divided into 20 annular rings and 10 layers. The parameter σ describing the gaussian beam was taken to be .26 cm. The results are plotted in dimensionless form as Figure 1, and suggest that for absorbed dimensionless power per unit thickness of 70 or greater, the melting time will be within 10% of the value calculated from the one-dimensional heat balance (Equation 20).

Table I

THERMAL PROPERTIES OF SEVERAL MATERIALS

Material	ρ gm	L_m Joule/gm	K Joule/(cm sec °K)	C_p $\frac{\text{Joule}}{\text{gm°K}}$	T_m °K
Al_2O_3	3.8	1070	.104	.855	2313
Aluminum	2.79	375	1.84	1.0	911
Magnesium	1.77	338	.96	1.04	905
Stainless Steel	7.9	290	.24	.42	1700
Titanium	4.42	390	.145	.77	1900

Table II

COMPUTED MELT THROUGH TIMES FOR VARIOUS LEVELS OF ABSORBED FLUX

Material	Flux (Watts/cm^2)	t_m-sec	t_1-sec
Al_2O_3	1000	1.4197	1.084
	2000	.6226	.5418
	3000	.3985	.3612
	5000	.232	.2167
	10000	.1140	.1084
	30000	.0377	.03612
Aluminum	2000	.3131	.1371
	3000	.1518	.09137
	5000	.0733	.05482
	10000	.0318	.02741
Magnesium	1000	.4128	.1712
	1500	.1962	.1141
	2000	.1266	.0856
	3000	.0737	.05706
	5000	.0402	.03424
	10000	.0188	.0171
	30000	.0061	.00571
Stainless Steel	1000	1.1068	.6936
	3000	.270	.2312
	5000	.1532	.1387
	10000	.0742	.06936
	30000	.0244	.02312
Titanium	300	9.098	2.395
	1000	1.09	.7185
	3000	.271	.2395
	5000	.157	.1437
	10000	.078	.07185
	30000	.0255	.02395

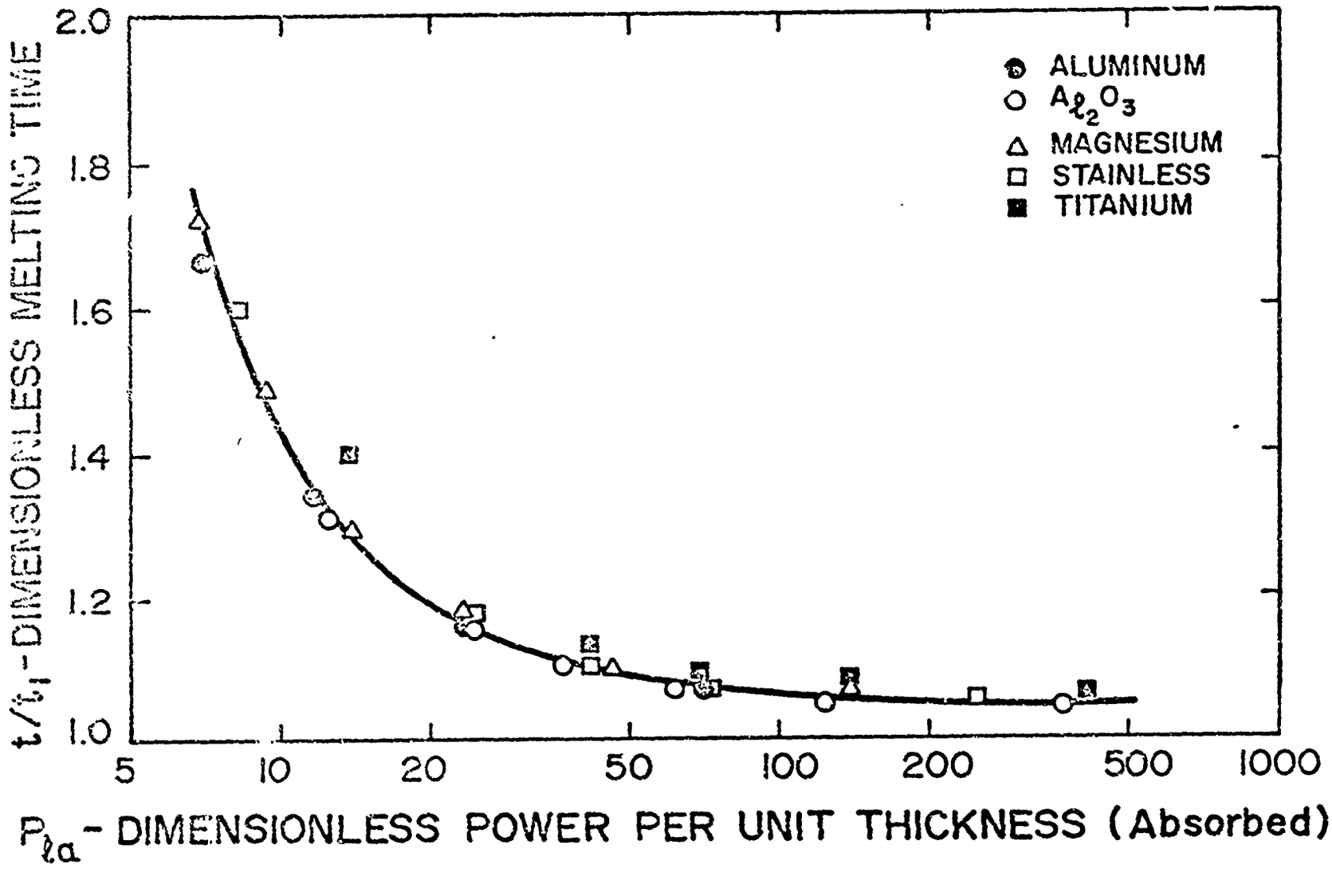

Fig. 1 Dimensionless Melting Time for Several Materials

III. MELTING TIMES AT LOW POWER

Up to this point, only the times necessary for complete melt through have been considered for cases where the incident flux is relatively high, i.e. the heat transfer primarily axial. Further insight may be gained through consideration of the other extreme, i.e. assuming that the heat flow is primarily radial.

Carslaw and Jaeger[3] have given the temperatures distribution in a solid due to the application of a flux Q per unit area applied over the interior of a cylindrical hole in a infinite domain. The temperature is

$$T(r,t)-T_o = -\frac{2Q}{\pi k}\int_0^\infty (1-e^{-\kappa u^2 t})\ \frac{J_0(ur)Y_1(uR)-Y_0(ur)J_1(uR)}{J_1^2(uR)+Y_1^2(uR)}\ \frac{du}{u^2} \qquad (24)$$

where R is the radius of the hole

κ is the diffusivity

k the conductivity

J and Y are bessel functions of first and second kind

and T_o is the initial temperature

For large values of time, an approximation[3] is

$$T-T_o = \frac{QR}{2k}\left\{\ln\frac{4\kappa t}{Cr^2}+\frac{R^2}{2\kappa t}\ln\frac{4\kappa t}{Cr^2}+\frac{R^2}{4\kappa t}\left[1+\frac{r^2}{R^2}-2\ln\frac{R}{r}\right]+\cdots\right\} \qquad (25)$$

where $C = e^\gamma = 1.781$. This approximation has been used to predict melting times for thin sheets exposed to small lasers.[4] Retaining only the first term leaves

$$T-T_o = \frac{QR}{2k}\ln\frac{4\kappa t}{Cr^2} \qquad (26)$$

as an approximate temperature in a thin sheet. In order to apply this solution to the problem of laser melting, we assume a uniform flux of intensely F_o to be applied over a radius R on a sheet of thickness ℓ. For a small ℓ, we assume the temperature under the beam (i.e. $r \leq R$) to be uniform and equal to the temperature at R. For moderate values of flux, we assume a quasi steady temperature under the beam, viz., that the transient term may be disregarded. Hence, a heat balance on the mass under the spot reads

$$F_o \cdot \pi R^2 = Q \cdot 2\pi R \cdot \ell \tag{27}$$

where Q is the radial flux, as in Equations 24 and 26. Using Equation 26 we then have as a temperature distribution

$$T(r,t) = T_o + \frac{R^2 F_o}{4k\ell} \ln \frac{4\kappa t}{CR^2} \quad \text{on } 0<r<R \tag{28a}$$

$$T(r,t) = T_o + \frac{F_o R^2}{4k\ell} \ln \frac{4\kappa t}{Cr^2} \quad \text{on } r \geq R \tag{28b}$$

It should be noted that certain assumptions made in the development of this result are not completely consistent. The assumptions of long time and low intensity are not compatible with the neglect of losses due to convection and radiation, hence the temperatures predicted by 28b are at best upper bounds, and melting times which might be predicted are lower bounds.

Such melting times may be estimated from Equation 28a as

$$t_m = \frac{CR^2}{4\kappa} \exp \frac{4k\ell}{F_o R^2} (T_m - T_o) \tag{29}$$

This melting time can be compared with that predicted for one dimensional axial flow with instantaneous removal,

$$t_1 = \frac{\rho \ell}{F_o} \left\{ L_m + C_p (T_m - T_o) \right\} \tag{30}$$

hence

$$\theta = \frac{t_m}{t_1} = P_\ell \frac{C}{4\pi} \exp \left\{ \frac{4\pi}{P_\ell \left[1 + \dfrac{L_m}{C_p (T_m - T_o)} \right]} \right\} \tag{31}$$

where

$$P_\ell = \frac{\pi R^2 F_o}{\ell \kappa \rho [L_m + C_p (T_m - T_o)]} \tag{32}$$

The dimensionless power per unit thickness is again seen to characterize the melting time ratio. For small P_ℓ, θ becomes large. For large P_ℓ,

$$\theta \to \frac{C P_\ell}{4\pi} \tag{33}$$

but the assumption of one dimensional radial flow is inapplicable in this regime. Equation 31 has a mimimum at

$$P_\ell = \frac{4\pi}{\left[1 + \dfrac{L_m}{C_p (T_m - T_o)} \right]} \tag{34}$$

when

$$\theta_{min} = \frac{C P_\ell e}{4\pi} = \frac{4Ce}{1 + \dfrac{L_m}{C_p (T_m - T_o)}} \tag{35}$$

and should not be used for values of P_ℓ above that given by Equation 34.

The previous results were deduced under the assumption of a

"flat-top" beam profile. For a Gaussian beam, the assumption that the material under the beam ($R = 2\sigma$) melts uniformly is somewhat questionable, but will enable comparison with the results obtained by the numerical method. Equation 27 then reads

$$\frac{1}{2} F_o \pi R^2 = Q 2\pi R \ell \tag{36}$$

and Equation 29 becomes

$$t_m = \frac{C\sigma^2}{\kappa} \exp \left\{ \frac{2k\ell}{F_o \sigma^2} (T_m - T_o) \right\} \tag{37}$$

normalizing by the one dimensional melt through time (Equation 20) and introducing the dimensionless power per unit thickness for a gaussian beam

$$P_\ell = \frac{2\pi\sigma^2 I_{pa}}{\ell \kappa \rho [L_m + C_p (T_m - T_o)]} \tag{38}$$

leaves

$$\theta = \frac{t_m}{t_1} = \frac{CP_\ell}{2\pi} \exp \left\{ \frac{4\pi}{P_\ell} \frac{1}{\left[1 + \frac{L_m}{C_p (T_m - T_o)} \right]} \right\} \tag{39}$$

A minimum again occurs at the value of P_ℓ which make the exponent of Equation 39 unity, and leaves

$$\theta_{min} = \frac{2Ce}{1 + \frac{L_m}{C_p (T_m - T_o)}} \quad \text{at } P_\ell = \frac{4\pi}{1 + \frac{L_m}{C_p (T_m - T_o)}} \tag{40}$$

The parameter $1 + L_m / C_p (T_m - T_o)$ ranges from 1.613 in the case of aluminum to 1.316 in the case of Titanium, with intermediate values of 1.537 for magnesium and 1.493 in the case of stainless steel. Taking 1.5 as a representative value introduces an approximation equivalent to 10% uncertainties in the dimensionless power per unit thickness and

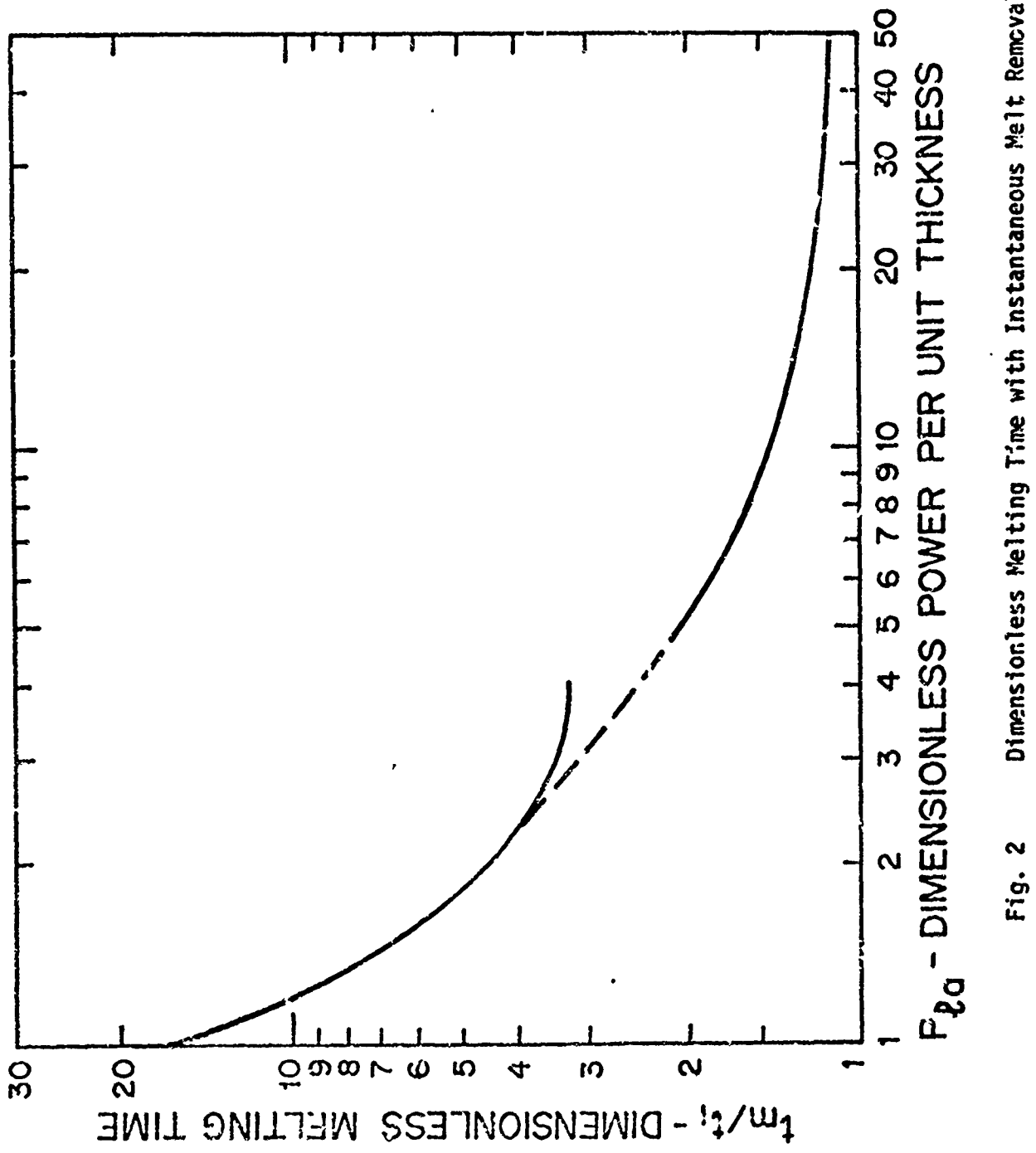

Fig. 2 Dimensionless Melting Time with Instantaneous Melt Removal

the upper solid curve of Figure 2 is found to result. The lower solid curve is a reproduction of the results given as Figure 1, and the dashed curve presents a smooth interpolation joining the two solid segments. The resulting composite curve is suggested as a means of rapidly estimating the time required for complete melt through, together with the definition of dimensionless power per unit thickness (Equation 38) and the one dimensional melt through time (Equation 20).

As noted earlier, these results were obtained under the assumption that no heat is lost to the surroundings. If heat is lost from each unit area by connection and radiation, then the temperature must satisfy the differential equation

$$k\nabla^2 T = \rho C_p \frac{\partial T}{\partial t} + \frac{2h(T-T_0)}{\ell} + \frac{2\varepsilon\sigma^2(T^4-T_0^4)}{\ell} \qquad (41)$$

where T_0 is the temperature of the surroundings, h is the surface heat taansfer coefficient, ε is the emissivity and σ is the Stephan-Boltzman constant. An approximate solution for the radially symmetric steady state temperature distribution can be obtained by setting

$$h^* = h + 4\varepsilon\sigma T_{ave}^4 \qquad (42)$$

which yields

$$T = T_0 + A I_0 \left(\sqrt{2\beta}\,\frac{r}{\ell}\right) + B K_0 \left(\sqrt{2\beta}\,\frac{r}{\ell}\right) \qquad (43)$$

where $\beta = \frac{h^*\ell}{k}$, is a Biot number and I_0 and K_0 are modified Bessel functions of the first and second kind. For small arguments the following approximations may be used:

$$K_0(z) \sim - \ln z \tag{44a}$$

$$K_1(z) \sim 1/z \tag{44b}$$

Assuming a flux F_0 is again uniformly distributed over a circle of radius R, and a quasi steady temperature to exist in the region under the spot, then a heat balance on material under the spot gives

$$F_0 \pi R^2 = 2\pi R \ell \left(-k \frac{\partial T}{\partial r}\right)_{r=R} \tag{45}$$

Solving for the constants A and B, we find A = 0 as the temperatures must remain finite at large r and

$$B = \frac{F_0 R}{2k\sqrt{2\beta}\, K_1(\sqrt{2\beta}\, R/\ell)} \tag{46}$$

The steady state temperature distribution, asymptotically approached at large time is

$$T = \frac{F_0 R\, K_0(\sqrt{2\beta}\, R/\ell)}{2k\sqrt{2\beta}\, K_1(\sqrt{2\beta}\, R/\ell)} + T_0 \text{ for } r \geq R \text{ and } T = T(R) \text{ for } r \leq R. \tag{47}$$

For $\sqrt{2\beta}\, R/\ell \ll 1$, the asymptotic values may be used to obtain

$$T(R) = \frac{F_0 R^2}{\ell} \ln \left\{ \frac{\ell}{R\sqrt{2\beta}} \right\} \quad \text{or} \tag{48}$$

$$T(R) = T_0 + \frac{P_\ell}{2\pi} \left[\bar{T}_m - T_0 + \frac{L_m}{C_p} \right] \ln \left(\frac{\ell}{R\sqrt{2\beta}} \right) \tag{49}$$

The critical power per unit thickness required to bring the area under a uniform beam to the melting temperature is therefore

$$P_{\ell\, crit} = \frac{2\pi}{1 + \dfrac{L_m}{C_p(\bar{T}_m - T_0)}} \frac{1}{\ln \ell/R\sqrt{2\beta}} \tag{50}$$

An identical result is obtained for a gaussian beam of radius $R = 2\sigma$. At lower values of power per unit thickness, melting will not be achieved. For values not substantially above the critical value, the times predicted by the method described in the previous section (or any other method which ignores losses) will be much increased.

IV. RETENTION AND VAPORIZATION OF MELT

In the preceding sections, the time required for complete melt-through under a Gaussian distribution was considered and compared with the times which would be required assuming one dimensional heat conduction. The assumption of complete removal of the melted material may be justified under conditions of strong air flow, but the alternative, i.e., molten material retained as a liquid until the vaporization temperature is reached, should also be considered.

For this study, the finite element computer program, as described earlier, was modified. The specific heat was taken to be constant below T_L, the temperature below which the material is solid, and above T_U, the temperature at which the material becomes liquid. The first phase change (melting) was accounted for by assuming an effective specific heat C_p over the melting range, i.e., between the lower and upper melting temperatures. The heat of fusion is assumed to be added uniformly over the melting range rather than at a fixed, single melting temperature. Let

$$C_p = C_{po} \quad \text{for} \quad T < T_L \text{ or } T > T_U \tag{51}$$

and

$$C_p = C_{po} + L_m/(T_U - T_L) \quad \text{for} \quad T_L < T < T_U \tag{52}$$

where L_m is the heat of fusion. The second phase change, vaporization at a temperature T_v, was handled as melting was treated in the original program, i.e., the finite increment of mass is assumed to remain at T_v until the entire heat of vaporization is absorbed. At this time, the entire cell is assumed to be instantaneously removed.

Table III

PHASE TRANSFORMATION PROPERTIES FOR SEVERAL MATERIALS

Material	T_L / ^0K	T_U / ^0K	T_V / ^0K	L_V / Joules/g_m	L_V/L_m
Aluminum	916	930	2873	10492	28.0
Magnesium	878	905	1380	5852	17.3
Stainless Steel	1673	1728	3273	6360	21.9
Titanium	1800	1900	3550	8820	22.6

Melt-through times were computed for the four materials Titanium, Aluminum, Stainless Steel and Magnesium. The Gaussian beam was assumed to have a diameter (4σ) of 1.64 cm and the sample thickness was assumed to be .08 cm. For Magnesium, Stainless Steel and Titanium, the density, specific heat, conductivity and heat of fusion were taken as given previously (Table I), while the properties of aluminum were taken to be $\rho = 2.7$ gm/cm^3 $K = 2.01$ Joule/cm sec °K $C_p = .9305$ Joule/gm°C $L_m = 395.4$ Joules/gm. Other thermal properties were taken as given in Table III. Computed melt-through times (t_m, the time at which the rear surface reaches the upper melt temperature T_m) were computed and are compared with t_1, the time which would be required for one-dimensional heating with complete removal at the same intensity. The results are tabulated in Table IV and shown graphically in Figure 3, where the ratio of melt-through times is given as a function of dimensionless power per unit thickness. The solid line is the average of the results for all materials given in Figure 1, and represents melting with instantaneous melt removal. From Figure 3 it can be seen that the retained melt causes only a slight increase in melt-through time until a critical value of dimensionless power per unit thickness is reached. It is interesting to note that this value is comparable to that required for one-dimensional heating. At larger values of P_{ta}, the time ratio increases, as more and more of the incident energy is "wasted" in evaporation of the melt. In this case, the results for all materials do not fall on a single curve. This can be attributed to the fact that the ratio of heat of vaporization to heat of fusion is not the same for all materials (Table III). The results in Figure 3 can be seen to be ordered as the ratio of heats.

Table IV

COMPUTED MELT THROUGH TIMES FOR SEVERAL MATERIALS

Material	I_{p_a} -Kw/cm^2	tm-sec	t_1-sec
Titanium	.5	2.122	1.151
	.8	1.069	.7192
	1	.807	.5754
	2	.3755	.2877
	5	.160	.1151
	10	.0965	.05754
	20	.0750	.02877
	30	.0640	.01918
	50	.0505	.01151
	100	.0318	.005754
Magnesium	1	.2833	.1373
	2	.0996	.06863
	5	.0355	.02745
	10	.0184	.01373
	20	.0112	.006863
	30	.0096	.004575
	50	.0081	.002745
	100	.0059	.00137
Stainless Steel	1	.8688	.5629
	2	.3736	.2814
	5	.149	.1126
	10	.0820	.05629
	20	.056	.02814
	30	.0496	.01876
	50	.0423	.01126
	100	.0302	.005629
	200	.0192	.002814
Aluminum	2	.228	.1062
	3	.1166	.07081
	5	.0591	.04249
	10	.0273	.02124
	20	.0140	.01062
	50	.00622	.004249
	100	.0035	.002124
	200	.0024	.001062
	500	.0019	.0004249
	800	.0017	.000266

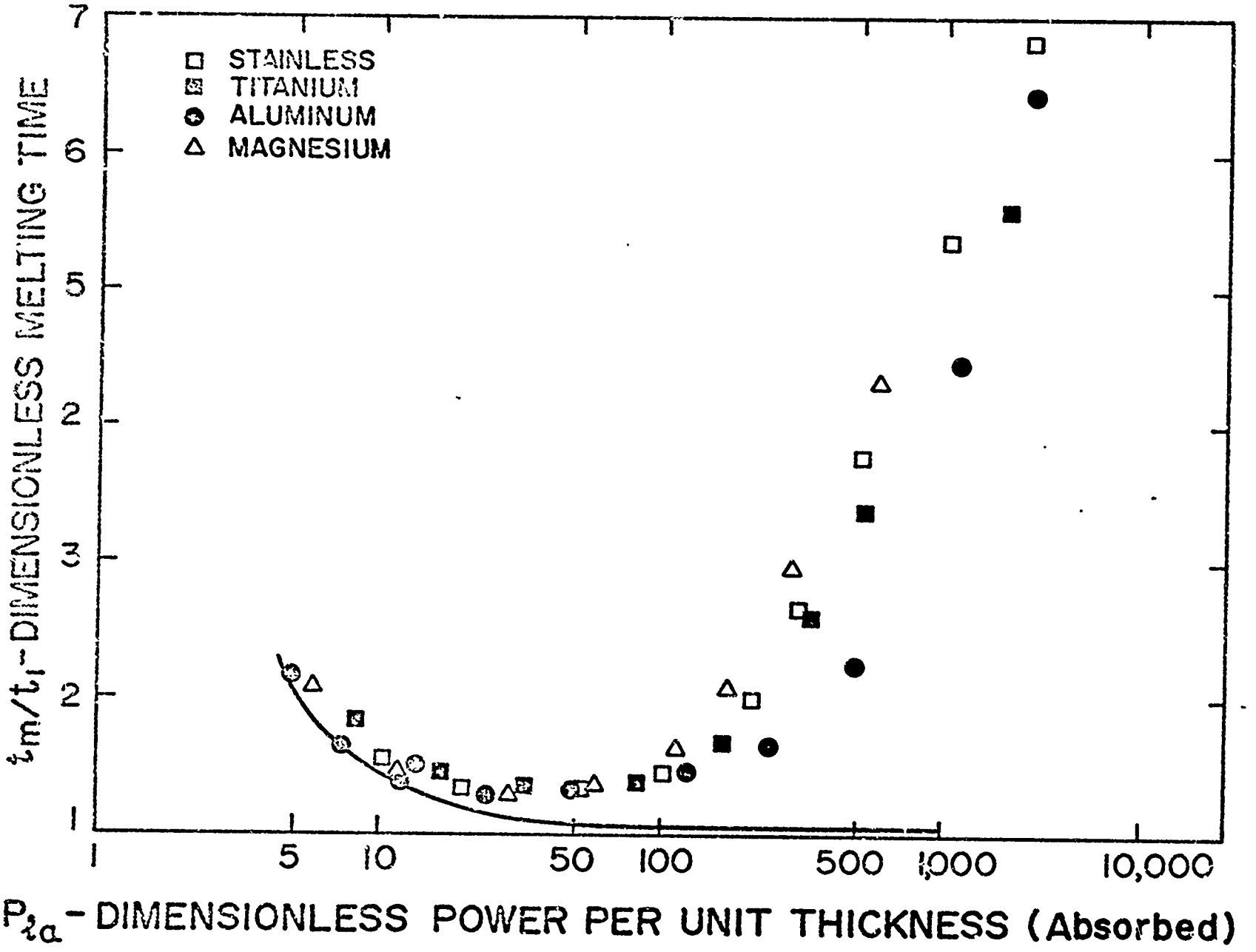

Fig. 3 Melt Through Times for Several Materials with Retained Melt

These results suggest that vaporization effects in a fully retained melt first become significant at a value of power per unit thickness which is comparable to that required to achieve one-dimensional axial heat flow. Consequently, it is to be expected that the heat conduction aspects of laser material interaction problems may be treated as one dimensional in the regime where vaporization is significant. Conversely, it is to be expected that the vaporization of retained melt will be negligible in cases where radial conduction is significant.

V. MELTING AND VAPORIZATION DUE TO A SINGLE PULSE

The computer program as modified to model two dimensional heating and melting with the melt being retained through to complete vaporization was further modified to treat single or repeated pulses. For pulses long compared to the time steps used in the finite element method, the method is applicable during the duration of a pulse which is constant or which increases in intensity with time. In order to analyze the heat transfer and melting which occurs after the incident energy is set to zero or reduced significantly (as at the end of a pulse), account must be taken of the fact that the temperature of certain cells will decrease as time progresses. A cell which, at the cessation of heat addition, was in the process of vaporizing must either be assumed to "give up" as heat flowing into other cells that portion of the heat of vaporization which had been supplied or else must be assumed to partially vaporize. The latter approach was found to lead to smoother and more consistent results in the cases considered, particularly when the thickness was divided into only ten layers. A pulse with a long, slowly diminishing "tail" would require special treatment, as the material under the beam may lose energy by conduction faster than supplied by the incident radiation. Repeated pulses present no difficulty once the program is modified to permit "cooling down" between pulses. In all cases considered here, the pulse was assumed to be of uniform intensity for the duration of the single pulse.

Limited calculations were performed

for .08 cm thick sheets of aluminum and titanium, assuming gaussian beam profiles of σ = .179 and .228 cm and material properties as given in the previous section. Various absorbed intensities were assumed in temporally uniform pulse of .005 seconds duration. Twenty divisions through the thickness were used in all cases.

The results obtained for aluminum showed that no vaporization is to be expected. The peak absorbed intensity required to produce melting to various depths is shown in Fig. 4. Little dependence on spot size was found, as is evidenced by the results from a one dimensional calculation shown for comparison.

In the case of Titanium (Figure 5) the sheet is found to be partially melted and partially vaporized by the beam. The portion melted is seen to increase rapidly with intensity until the threshold for vaporization occurs. Above this intensity, a fairly uniform melt thickness (about .008 cm in this case) is found in front of the vaporization depth. These limited results suggest that the spot size is not critical, particularly in regard to the prediction of the vaporization depth.

These calculations were compared with experimental data obtained from the Air Force Weapons Laboratory in order to determine an average value for the absorbtivity. If a gaussian profile is assumed, the comparison suggests an absorbtivity of 6.9% for the experiment on aluminum in vacuum and values of 13.7% and 9.2% respectively for the two experiments on Titanium in air. The beam shape in these experiments would, however, have a pronounced effect on the results of this comparison. Absorbtivities of twice there values would have been necessary if the beam was "flat," while

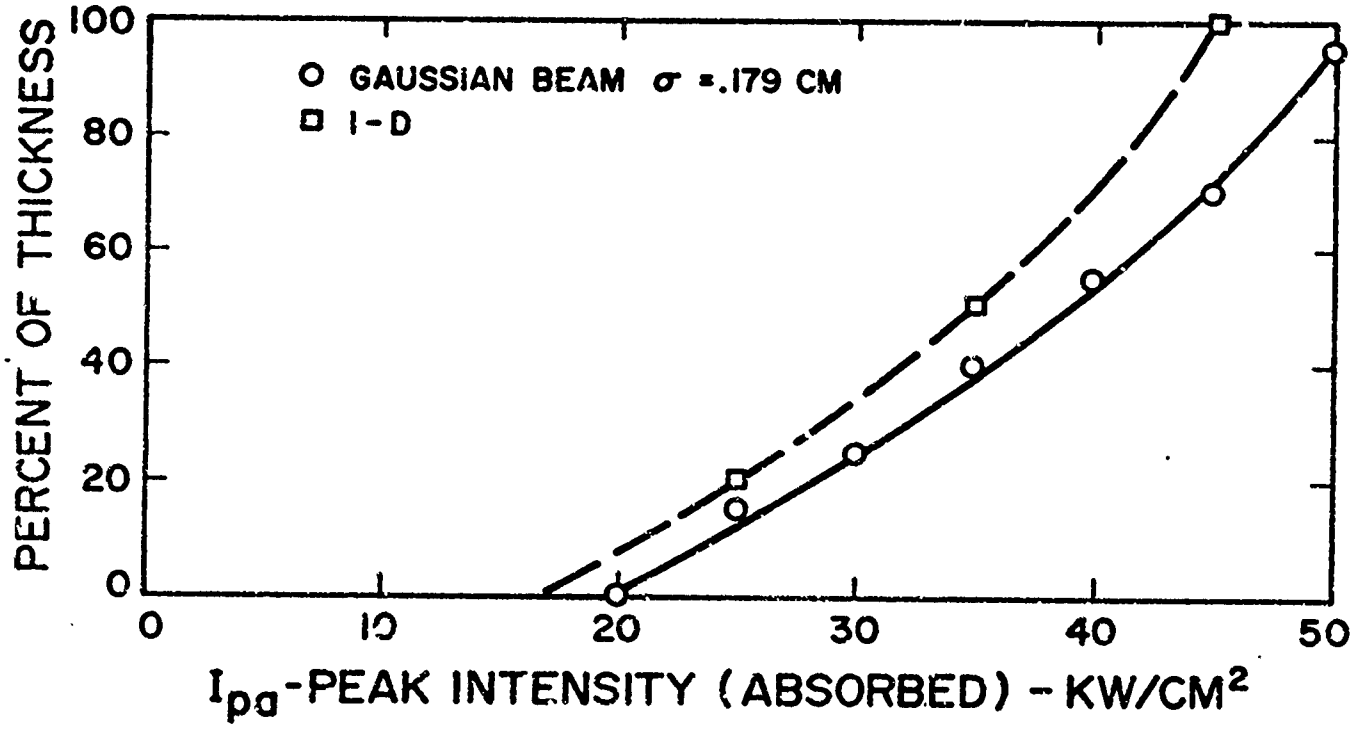

Fig. 4 Melting of .08 cm Aluminum Sheet due to .005 Second Pulse

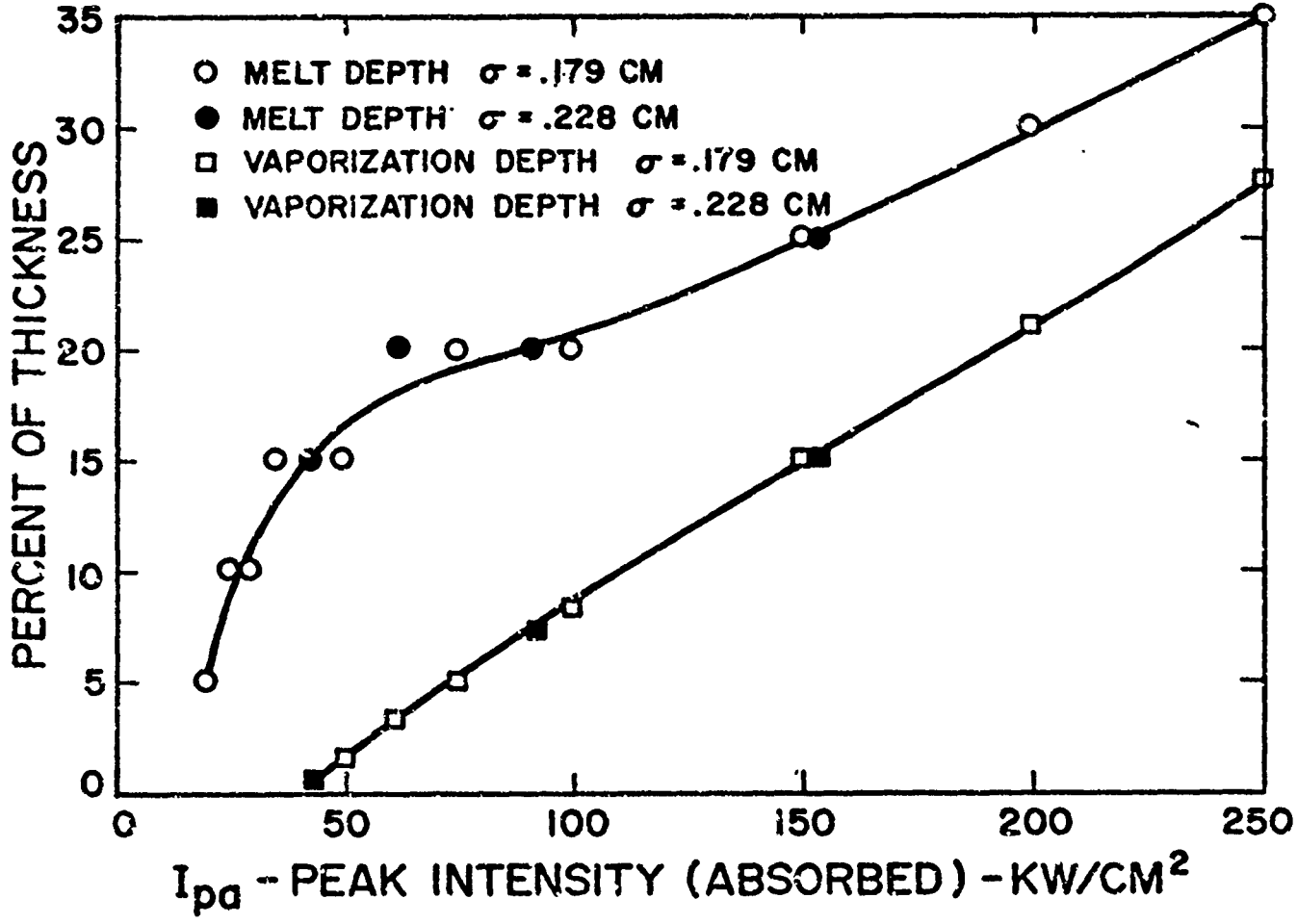

Fig. 5 Melting and Vaporization of .08 cm Titanium Sheet due to 5 Millisecond Pulses of Various Peak Intensities

the observed depths of melting and vaporization would be possible with a lower absorbtivity if the vaporization and melting occured at a local "hot-spot" in the beam. These limited results do suggest, however, a method for determining absorbtivity if the beam profile is well characterized.

VI. CONCLUSIONS

Further theoretical studies of laser induced melting and vaporization using the finite element numerical method previously described have shown the method to be adaptable to several new problems.

Principal new results are as follows:

(1) A single curve has been developed which permits the estimation of time required to produce melt-through by a continuous beam of gaussian profile even for cases where radial conduction is substantial. The melt-through time was shown to be a unique multiple of the time required for one dimensional melting at the same intensity, where the multiple is a function only of a certain dimensionless power per unit thickness, independent of intensity, spot size and material.

(2) A simple analytical result was developed which shows that for values of dimensionless power per unit thickness below a critical value (which does depend on material, spot size and thickness) melting of thin sheets cannot be produced.

(3)At high values of dimensionless power per unit thickness, all problems may be treated as one dimensional (axial) heat conduction. The critical value is about $P_{\ell a} = 70$.

(4) It is particularly significant that the geometrical condition necessary for one dimensionality is not a ratio of spot dimension to thickness, but rather a ratio of spot area (dimension squared) to thickness.

(5) The value of power per unit thickness necessary to produce one dimensional axial heat conduction is also approximately the threshold where vaporization effects first become significant. At higher values

of $P_{\ell a}$ the dimensionless power per unit thickness, large amounts of input energy serve only to produce vaporization of the melt.

(6) The method was also found applicable to the prediction of the degree of melting and vaporization due to single pulses of high intensity and provides a method for estimating absorbtivities if the melt depth due to a well characterized pulse is known.

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REFERENCES

1. Torvik, P. J. A Numerical Procedure for Two Dimensional Heating and Melting Calculations with Applications to Laser Effects, AFIT TR-72-2, March 1972.

2. Torvik, P. J. A Practical Approach to Laser Heating Calculations, Laser Division Digest, Spring 1972, LDR-72-1, June 1972.

3. Carslaw, H. S. and Jaeger, J. C. Conduction of Heat in Solids, Oxford University Press, 1959.

4. Gonsalves, J. N. and Duley, W. W. Interaction of CO_2 Laser Radiation with Solids, Canadian Journal of Physics, Vol. 49, pp. 1708-1713, 1971.

LIST OF FIGURES