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Research Contribution 222

A Phased-Procurement Model for Application to F-14 Spare Parts Provisioning

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When a new aircraft, such as the F-14, is being bought, decisions on the quantities of spare parts to be bought are made even though information on expected demands, operating programs, and the final configuration of the aircraft is limited. For high-value, low-usage parts, which are those we consider, the minimum-cost strategy might be to defer procurement until demands occur. During the period of deferral, the Navy would buy the needed spare parts from a stock carried by the manufacturer or from the production line if no stock is held. This paper describes an algorithm for determining in what cases this would be the best policy.

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1. Enclosure (1) is forwarded as a matter of possible interest. This is the third in a series of Research Contributions on F-14 PHOENIX provisioning and completes CNA's work on this project. References (a) and (b), the other two Research Contributions, as well as several memoranda on the subject, were distributed previously. A summarization of the research methodology and results on the general topic of inventory management will be forwarded shortly.

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CENTER FOR NAVAL ANALYSES

Institute of Naval Studies

Research Contribution 222

A PHASED-PROCUREMENT MODEL FOR APPLICATION TO F-14 SPARE PARTS PROVISIONING

November 1972

Bernard L. Perlman, U.S.N.
Arnold N. Schwartz

Work conducted under contract N00014—68—A—0091

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ABSTRACT

When a new aircraft, such as the F-14, is being bought, decisions on the quantities of spare parts to be bought are made even though information on expected demands, operating programs, and the final configuration of the aircraft is limited. For high-value, low-usage parts, which are those we consider, the minimum-cost strategy might be to defer procurement until demands occur. During the period of deferral, the Navy would buy the needed spare parts from a stock carried by the manufacturer or from the production line if no stock is held. This paper describes an algorithm for determining in what cases this would be the best policy.

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THE MODEL

To determine the optimal procurement policy, a function relating the costs associated with different procurement policies is minimized over a single period identified with an initial phase of the life cycle of the system. At some point in the life cycle, a final procurement decision will be made by the Navy.

We consider five pure policies:

(1) An initial purchase of some quantity of the spare part.

(2) The purchase of spare parts from the manufacturer when demands occur, paying him to keep a buffer of completed parts.

(3) The purchase of spare parts from the manufacturer when demands occur, paying him to keep a buffer of semi-finished parts.

(4) The purchase of spare parts from the manufacturer when demands occur, paying him to keep a buffer of raw materials.

(5) The purchase of spare parts from the manufacturer when demands occur, paying a premium for the disruption of his production line.

The model allows for mixed policies. Specifically, a total inventory of $N \geq 0$ units is allocated among the first four policies. We assume that the stock procured under policy i, N_i, is exhausted before the stock of the $(i+1)$ policy is tapped. All demands above N are satisfied under the fifth policy.

There are four distinct costs the Navy incurs by procuring a quantity, N_1, of the part early in the life cycle of the weapon system:*

$$N_1 C_o - \sum_{j=o}^{N_1} (N_1 - j) P(j) \frac{C_o}{1+r} : \text{ the cost of procurement.} \qquad (1)$$

Since end-of-period inventory, $N_1 - j$, can be used to satisfy future demands, we subtract from the gross procurement cost the present value of end-of-period stock.

$$\sum_{j=o}^{N_1} (N_1 - j) P(j) C_o h: \text{ the expected holding cost.} \qquad (2)$$

$$\sum_{j=o}^{N_1} (N_1 - j) P(j) C_o \alpha \beta: \text{ the expected cost of design changes.} \qquad (3)$$

*See table 1 for definitions of the parameters.

$$\sum_{j=1}^{N_i} KT_1 \sum_{\ell=j}^{\infty} P(\ell): \text{ the imputed cost of downtime.} \qquad (4)$$

The costs of keeping a buffer stock, $N_i \geq 0$, and filling demands from it are:

$N_i C_i \mu$: the manufacturer's charge for holding the buffer. (5)

$$\sum_{j=1}^{N_i} (C_o + C^H) \sum_{\ell=j+N_1+\ldots+N_{i-1}}^{\infty} P(\ell)$$

: the cost of procurement plus additional shipping and handling costs generated by accelerated shipment from the manufacturer. (6)

$$\sum_{j=1}^{N_i} KT_i \sum_{\ell=j+N_1+\ldots+N_{i-1}}^{\infty} P(\ell)$$

: the imputed cost of downtime. (7)

All demands above $N = N_1+N_2+N_3+N_4$ must be satisfied from the manufactuer's production line, generating two costs:

$$\sum_{j=N+1}^{\infty} (C_o + \Delta) \sum_{\ell=j}^{\infty} P(j):$$

the cost of procurement plus a charge for disrupting the flow of work and for expediting shipping and handling. (8)

$$\sum_{j=N+1}^{\infty} KT_5 \sum_{\ell=j}^{\infty} P(j): \text{ the imputed cost of downtime.} \qquad (9)$$

The total cost, as a function of the policy (N_1, N_2, N_3, N_4), is thus given as:

$$C(N_1, N_2, N_3, N_4) = N_1 C_o + \mu(N_2 C_2 + N_3 C_3 + N_4 C_4) +$$

$$+ \sum_{j=o}^{N_1} (h + \alpha\beta - \frac{1}{1+r})(N_1 - j) P(j) + \sum_{j=1}^{N_1} KT_1 \sum_{\ell=j}^{\infty} P(\ell) +$$

$$+ \sum_{i=2}^{4} (KT_i + C_o + C^H) \sum_{j=1}^{N_i} \sum_{\ell=j+N_i+\ldots+N_{i-1}}^{\infty} P(j) + (KT_5 + C_o + \Delta) \sum_{j=N+1}^{\infty} \sum_{\ell=j}^{\infty} P(j) .$$

The optimal values of N_1, N_2, N_3, and N_4 are found by an iterative search through all quadruples (N_1, N_2, N_3, N_4) such that $N_1 + N_2 + N_3 + N_4$ is not greater than some predetermined upper limit. This method has been quite satisfactory in the analysis done to date. However, should the demand rate or time period become large, which would necessitate an increase in this upper limit, or should the analyst be constrained by computer usage time, other search methods are available.

For example, this model can be expressed in the "optimal allocation" format for dynamic programming. Denote by M the upper limit on $N = N_1 + N_2 + N_3 + N_4$ defined above. Let C_i, for i=1, 2, 3, 4, or 5, represent the costs attributable to policy i; thus C_i is a function of N_1 and $N_1 + \ldots + N_{i-1} = M_{i-1}$ and C_5 is a function of N. Define the recursive function F_i, i=2, 3, 4, as follows:

$$F_i(M_{i-1}) = \begin{cases} \min\limits_{0 \le N_4 \le M-M_3} \left[C_4(M_3, N_4) + C_5(M_3+N_4) \right] & i=4 \\ \min\limits_{0 \le N_i \le M-M_{i-1}} \left[C_i(M_{i-1}, N_i) + F_{i+1}(M_{i-1}+N_i) \right] & i=2, 3 \end{cases}$$

The optimal value of the cost function is $\min\limits_{0 \le N_1 \le M} \left[C_1(N_1) + F_2(N_1) \right]$, defining also the optimal value of N_1. The optimal values of N_2, N_3, and N_4 are found from the above equation.

TABLE 1

DEFINITIONS OF PARAMETERS OF COST FUNCTION

C_o	procurement cost per unit
$P(j)$	probability of j demands, with distribution P
r	discount rate
h	Navy's holding cost per dollar value
α	probability of obsolescense
β	cost of a design change per dollar value
K	penalty cost
C_i	value of buffer stock per unit, under policy $i, i=2,3,4$
μ	manufacturer's holding cost per dollar value of buffer
C^H	additional shipping and handling costs per unit
Δ	charge for disrupting flow of work and expediting shipping and handling
T_i	expected resupply time under policy $i, i=1,2,3,4,5$.

APPLICATIONS

ESTIMATES OF PARAMETERS

At present, the Navy supports new aircraft being introduced into the fleet by purchasing spare parts early in the phase-in period. Since early provisioning is current policy, we bias all cost estimates in favor of it.

C_o, λ (demand rate). A list of F-14 parts that have been provisioned is given in table 2. For each part, the probability distribution of demands is assumed to be a Poisson distribution whose mean, λ, is the demand rate listed in table 2. The unit costs and demand rates were obtained from the manufacturer's provisioning documents. The demand rates are rough engineering estimates of the numbers of these parts required for 68 aircraft over a 2-year period.

TABLE 2

UNIT COSTS AND DEMAND RATES OF TEN F-14 PARTS

Part	C_o	λ
Nose landing gear	$20,000	20.56
Collar assembly	2,940	20.56
Cylinder assembly	4,765	20.56
Piston assembly	5,000	1.96
Servoactivator, series input DLC	1,784	19.60
Servocylinder, pitch parallel	2,300	34.76
Servocylinder, outboard spoiler	2,561	19.60
Servocylinder, mid spoiler (RH & LH)	2,946	19.60
Servocylinder, inboard spoiler	2,473	19.60
Servoactivator, series input	4,035	19.60

r. The discount rate we use is 10 percent. This is the value currently used by the Navy Supply System Command.

α, β. Owing to the uncertainty of the final configuration of a new aircraft, the probability of a design change early in the program is high. Therefore, we set $\alpha = 1$. The cost of the design change per dollar value of the component was set at 10 percent. We feel that this is an underestimate of the cost, but it is a bias in favor of the first policy.

K. The Navy does not explicitly consider the penalty cost, K, since it forms no part of any appropriation. For nonprofit-making organizations, penalty cost is a dummy variable closely related to the available budget. If the projected cost of the program exceeds its budget, the penalty cost should be lowered, sacrificing output (in this case, lowering readiness), and vice versa. Such an iterative procedure is conducted until the projected cost equals the budget. As a first approximation, K is calculated directly from the life-cycle costs. For the F-14, with an estimated life of 11.8 years, this is approximately $7,000 per day. (For an analysis of the sensitivity of our model to variations in the penalty cost, see table 9.)

$\underline{C_i}$. Reference (2) estimates that a high-value spare part accumulates approximately 16 percent of its completion value through procurement of raw materials and 72 percent of its final value through component fabrication and sub-assembly. For the procurement policy in which the manufacturer maintains an excess of raw materials, we set the unit value of the buffer stock $C_4 = .16C_o$; of semi-finished units, $C_3 = .72C_o$; of completed units, $C_2 = C_o$.

$\underline{\mu, h}$. After discussion with various manufacturers, the Aviation Supply Office (ASO) informed us that manufacturers' estimates of holding costs per dollar value of buffer stock range from 1 percent to a maximum of 10 percent per year. This factor includes interest, overhead, and depreciation on additional fixtures needed for holding the extra finished spares. We chose the 10 percent estimate, since we wanted to be liberal in estimating the costs of the phased procurement alternatives. ASO also estimated the Navy's holding cost for the F-14 to be approximately 23 percent per dollar value per year.

$\underline{C^H, \Delta}$. Additional shipping and handling costs to expedite shipment of the unit from the manufacturer to Norfolk or Oakland were estimated by ASO to be about 0.4 percent of the value of the part. Whenever a demand is satisfied by the fifth policy, the Navy will incur an incremental charge for disrupting the flow of work. We estimated this charge at 100 percent of the value of the unit. ASO stated that the cost of disruption would be high, and a charge of 100 percent therefore seems reasonable.

$\underline{T_i}$. Estimates of resupply times were obtained from the manufacturers, ASO, and reference (1).

Policy	T_i (days)
1. Navy buy	22
2. Manufacturer keeps an excess of completed parts	25
3. Manufacturer keeps an excess of semi-finished parts	30*
4. Manufacturer keeps an excess of raw materials	35*
5. Navy buys parts from production line	25

*In these cases the excess will not be the supply used to satisfy demands. The additional semi-finished parts or raw materials are carried by the manufacturer in the production line to maintain the production schedule.

The estimates of parameters other than unit cost and demand rate are compiled in table 3.

TABLE 3

ESTIMATES OF PARAMETERS

Parameter	Estimate
r	.21
h	.46
α	1.0
β	.10
K	$7000
C_2	$$C_o$
C_3	$$.72C_o$
C_4	$$16C_o$
μ	.20
C^H	$$.004C_o$
Δ	$$C_o$
T_1	22 days
T_2	25 days
T_3	30 days
T_4	35 days
T_5	25 days

APPLICATION TO F-14 PARTS

The model was run for the 10 parts listed in table 2, with the results listed in table 4.

The optimal procurement policy is, of course, dependent upon the length of the planning horizon used in the analysis, since the model considers only a single decision made at the beginning of the planning horizon. The optimal length of this time period - the initial phase of the system life-cycle - depends on the quality of the parameter estimates and the rapidity with which they undergo change. The tradeoff between the value of postponing final procurement until parameter estimates stabilize and the costs of doing so is not exploited in this model, but might be if estimates of the time required for such stabilization were available.

TABLE 4

RESULTING NAVY INVENTORY AND MANUFACTURER'S BUFFER STOCK OF F-14 PARTS

Part	N_1	N_2
Nose landing gear	23	1
Collar assembly	27	0
Cylinder assembly	26	0
Piston assembly	4	0
Servoactuator, series input DLC	27	0
Servocylinder, pitch parallel	44	0
Servocylinder, outboard spoiler	26	0
Servocylinder, mid spoiler (RH & LH)	26	0
Servocylinder, inboard spoiler	26	0
Servoactuator, series input	25	0

Note: Time period = 2 years; number of aircraft = 68.

A second deficiency exists with the assumption of a single period planning horizon apart from its length. The procurement plan derived from the model is based upon minimizing expected total costs. However, once the plan is implemented, the random occurrences of demand might suggest that a re-evaluation of the plan is necessary, for example, if demands were large enough to exhaust the stock in the first week of the planning horizon. One possibility, discussed further in sensitivity analysis 4, would be to re-apply the model, possibly with newly updated parameter values, at that point. A more complete treatment of the general phased procurement problem, however, would take into account this possibility of future revisions in choosing an initial plan and formulate the problem dynamically rather than in a static context as done here. Introducing these two aspects of time dependence into the model should be a fruitful and important area for further research.

SENSITIVITY ANALYSES

Sensitivity analyses were performed, varying the parameter(s) of interest over a specified range while holding all other parameters constant. Table 5 shows the parameters and ranges chosen.

The usefulness of a sensitivity analysis such as performed here is somewhat limited as a result of the large number of parameters in the model. At best, only a subset of them can be varied simultaneously, and the results obtained are conditional on the remaining parameters taking on the assumed values. Thus generalizations about the effects of changes in any one parameter must be viewed cautiously: the tables and graphs displayed here are valid only for the assumed values of the nonvarying parameters. Figure 1 illustrates instances in which the effects of changes in two of the parameters differ considerably when a third parameter is varied. For any subsequent applications of this model, it is unlikely that the numerical results here can be directly applied.

Analysis 1 examines the relative importance of the parameters μ, Δ, and T_5. The results of this analysis, for representative values of the above three parameters, are shown

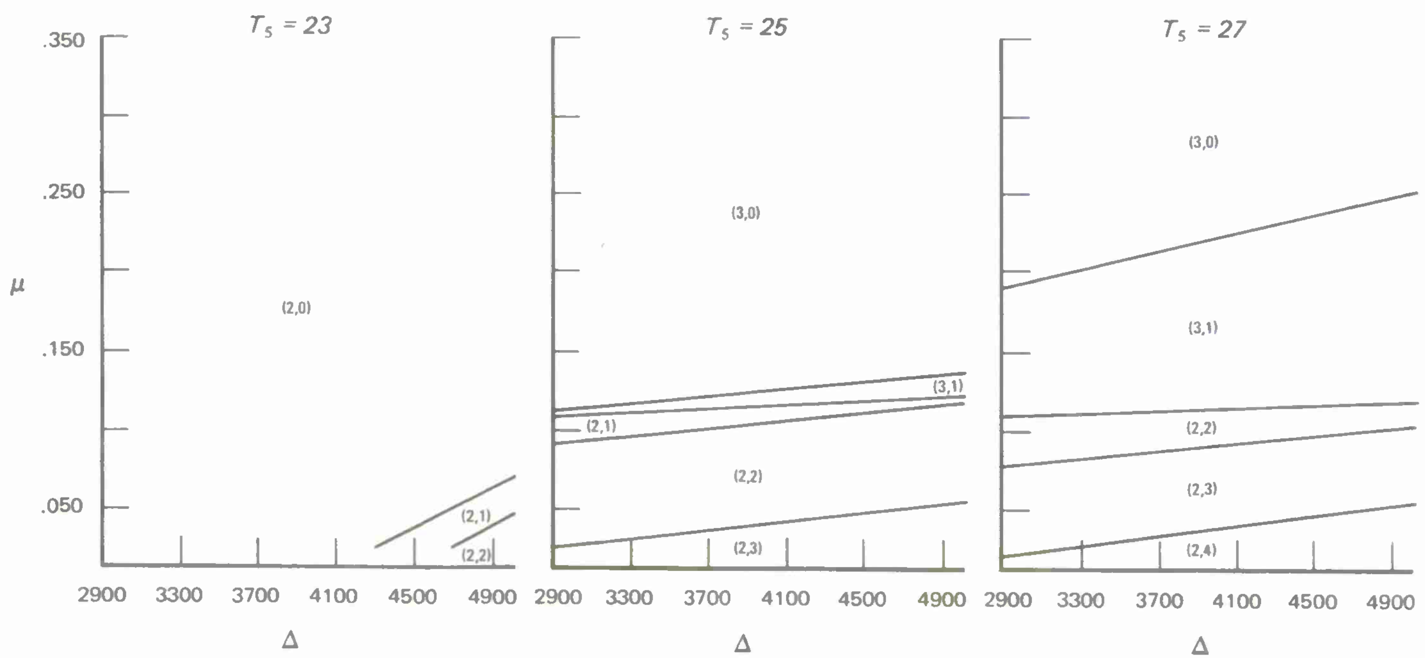

FIG. 1-A: SENSITIVITY ANALYSIS OF μ, T_5, Δ

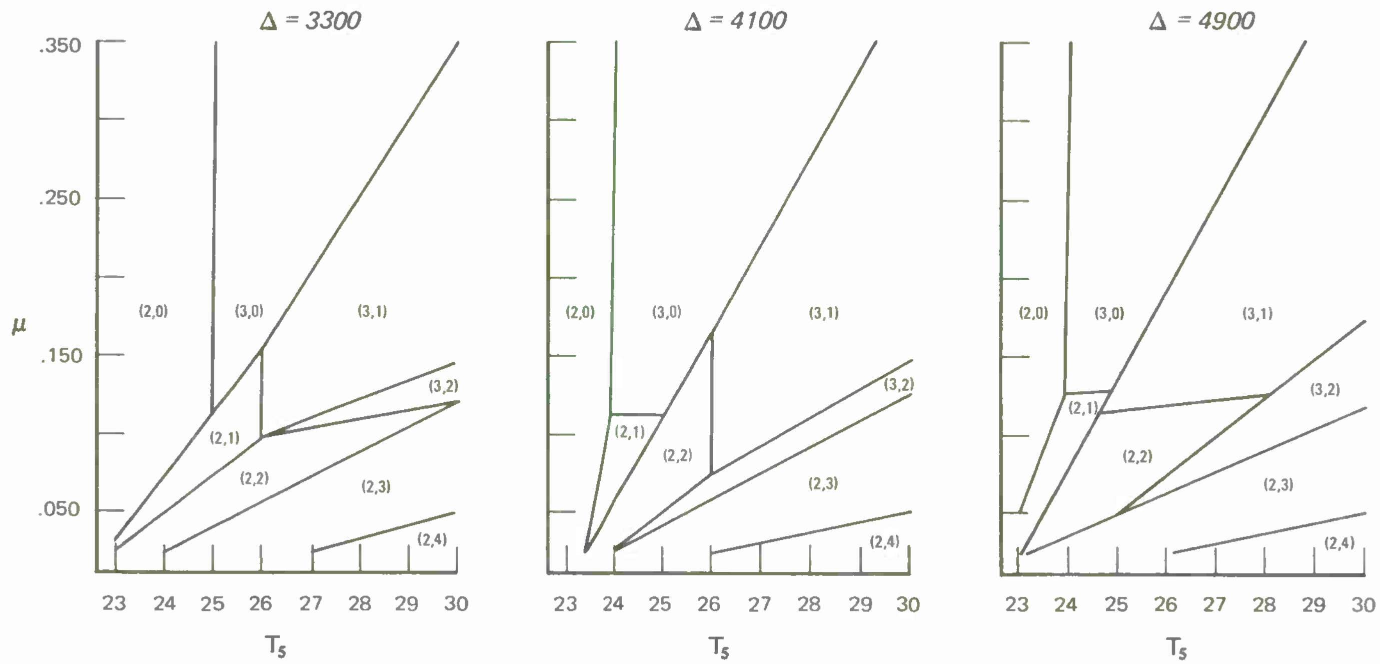

FIG. 1-B: SENSITIVITY ANALYSIS OF μ, T_5, Δ

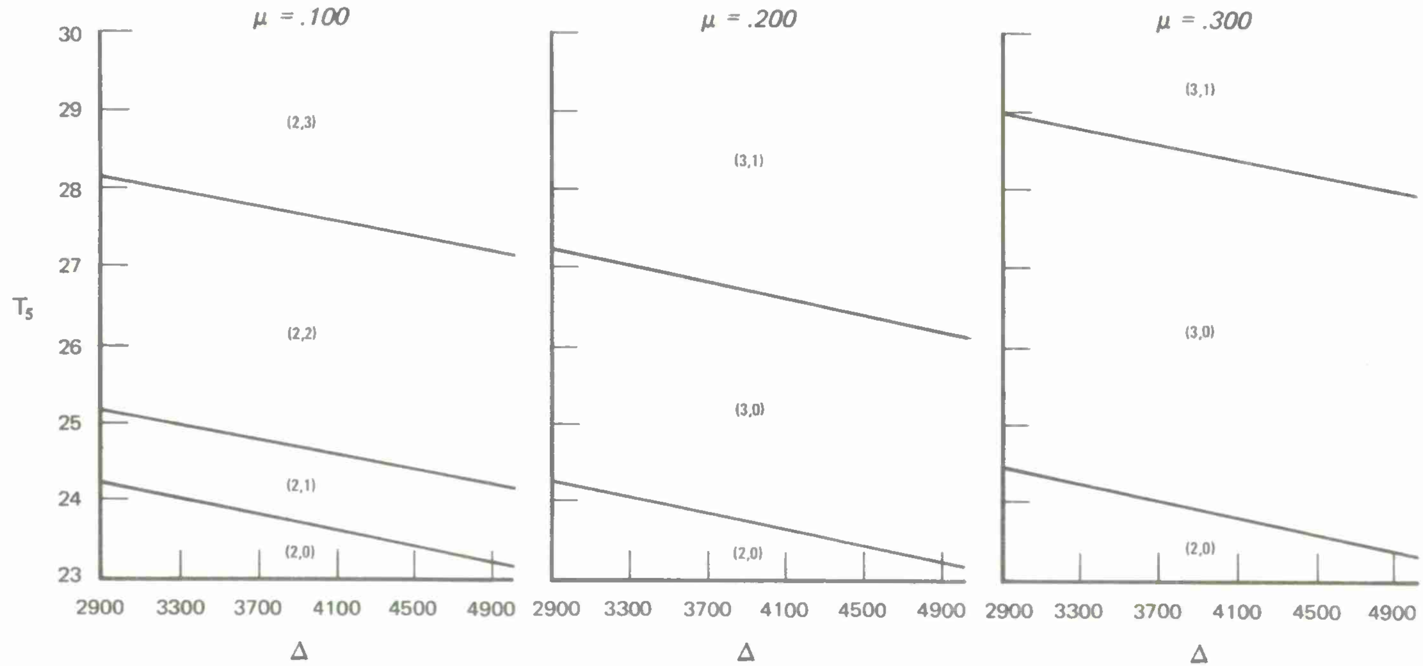

FIG. 1-C: SENSITIVITY ANALYSIS OF μ, T_5, Δ

in figure 1. The graphs in figure 1 are interpreted as follows. Each figure is based on assumed (and unchanging) values of all of the model's parameters other than the pair displayed on the axes. For these two parameters, any values may be chosen, and the optimal policy is determined by the decision variables (N_1, N_2) associated with the point determined by the values chosen for the two parameters. For example (using the graph in the upper right-hand corner of figure 1), if $\mu = .3$ and $\Delta = 3000$, the optimal policy is $N_1 = 3$, $N_2 = 0$, while if $\mu = 0$, $\Delta = 4100$, the optimal policy is $N_1 = 2$, $N_2 = 4$. We again note these policy results are conditional on the values of all remaining parameters.

TABLE 5

Parameter		Range of parameter
	Sensitivity analysis 1	
μ		.033 - .25
T_5		23 - 30
Δ		\$2900 - 5000
	Sensitivity analysis 2	
$\Pi = h + \alpha\beta - \frac{1}{1+r}$		(-.90) - (+.50)
T_2		21 - 29
T_5		21 - 29 ($T_5 = T_2$)
μ		.025 - .350
	Sensitivity analysis 3	
T_1		1 - 30
	Sensitivity analysis 4	
C_o		\$1000 - 25,000
λ		1.0 - 49.0
	Sensitivity analysis 5	
K		\$200 - 10,000

Note: C_o = \$5000, λ = 2.00, K = \$2000. The remaining parameters have the values, when constant, listed in table 3.

Observe that variations in Δ have only slight effect upon policy variations (figure 1, A and C). Note also that there are sudden policy changes for $T_5 = 24, 25, 26$, i.e., $T_5 \approx T_2$, and that N_2 depends upon the ratio $\frac{T_5}{\mu}$ (figure 1, B). The cost function was regressed against μ, Δ, and T_5; the regression equation is cost = 94,925 + 3936μ + 313 T_5 + .20Δ with R_2 = .81 .*.

Analysis 2 examines the behavior of the cost function with respect to changes in h, α, β, r, T_2, T_5, and μ, with the restriction that $T_2 = T_5$. It is evident from functions (1) through (3) that h, α, β and r combine to form a single cost parameter, $\Pi = h + \alpha\beta - \frac{1}{1+r}$. For example, the functional relationship between Π and the chosen time period, for the values of h, α, β, and r as given in the previous section, is displayed below.

Time period (in years)	h	$\alpha\beta$	$\frac{1}{1+r}$	Π
0.5	0.12	0.03	0.95	-0.80
1.0	0.23	0.10	0.91	-0.58
1.5	0.35	0.14	0.86	-0.37
2.0	0.46	0.21	0.83	-0.16
3.0	0.69	0.33	0.75	0.27

Note: the values for $\alpha\beta$ and $\frac{1}{1+r}$ are compounded while those for h are not.

The results of Analysis 2 are shown in figure 2. Notice that the policy is determined by the ratio $\frac{\Pi}{T_2}$ and $N_1 + N_2$ in Case 2, and N_1 is determined by T_2 and $N_1 + N_2$ by μ in Case 3. It appears that T_2 is the most important parameter of the three. This is clear also since the product $(T_2 - T_1)K$ is large compared to the other costs for T_2 even slightly greater than T_1. (See also Analysis 5, which examines the behavior of the model for variations in K.) The regression equation is:

$$C(\Pi, \mu, T_2 = T_5) = 78,713 + 383\ T\Pi + 5877\ \mu + 979\ T_2 \text{ with } R^2 = .77 .$$

Analyses 1 and 2 examine the tradeoff between expenditures for policies 1, 2, and 5. These analyses could be used to determine the tradeoff between the investment in the resupply system and the inventory system. Analysis 3 (see table 6) shows how inventory costs vary directly with Navy resupply time. This function can be summed with a resupply system cost function, which varies inversely with resupply time, to determine the optimal combination of inventory system and resupply system investments. This tradeoff has been exploited by others; for example, see reference (3).

*Regressions are used in this paper in a curve-fitting sense and should not be taken to indicate the presence of any random element.

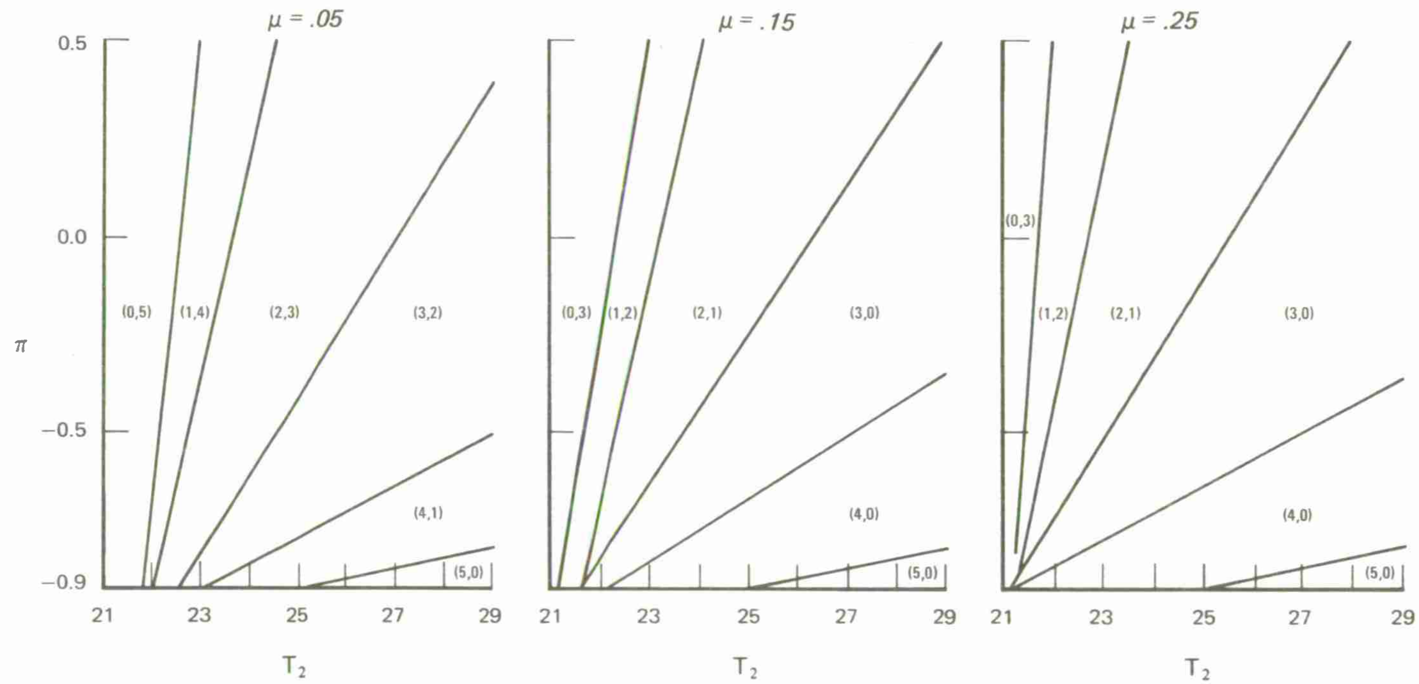

FIG. 2-A: SENSITIVITY ANALYSIS OF π, T_2, μ

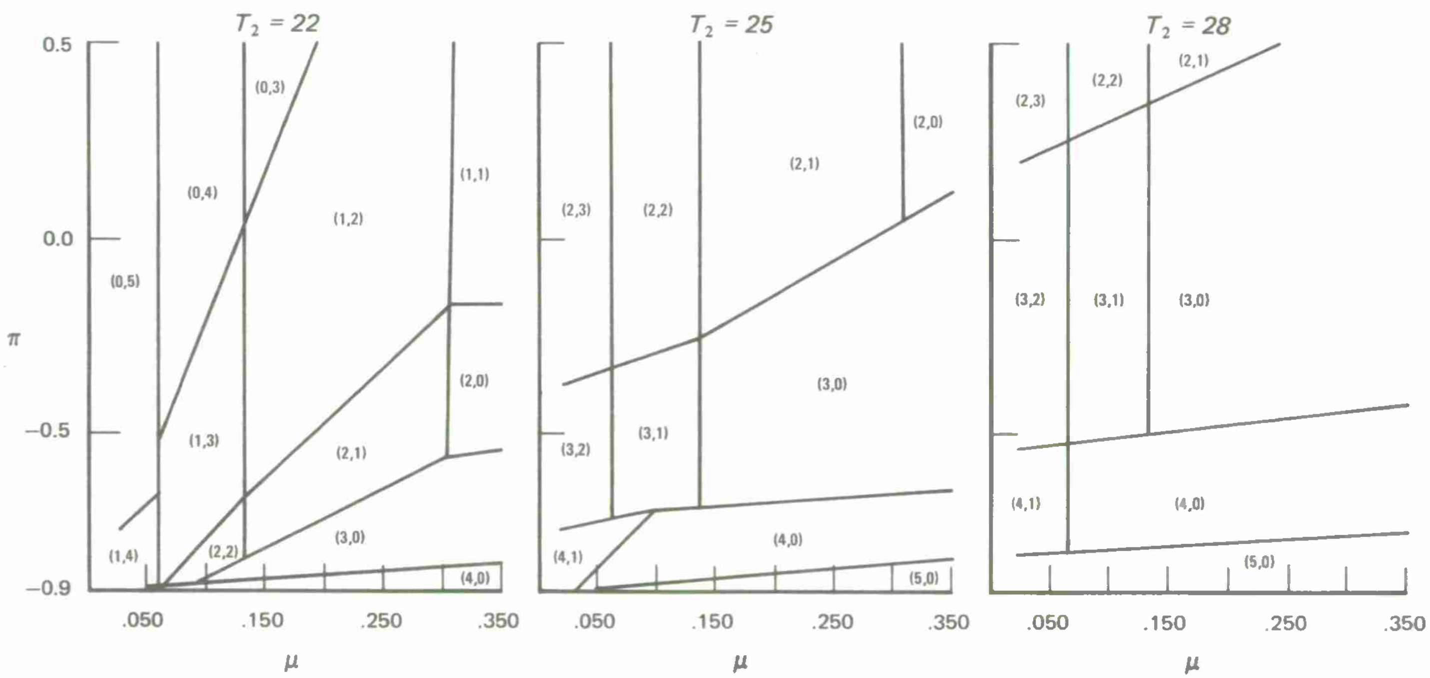

FIG. 2-B: SENSITIVITY ANALYSIS OF π, T_2, μ

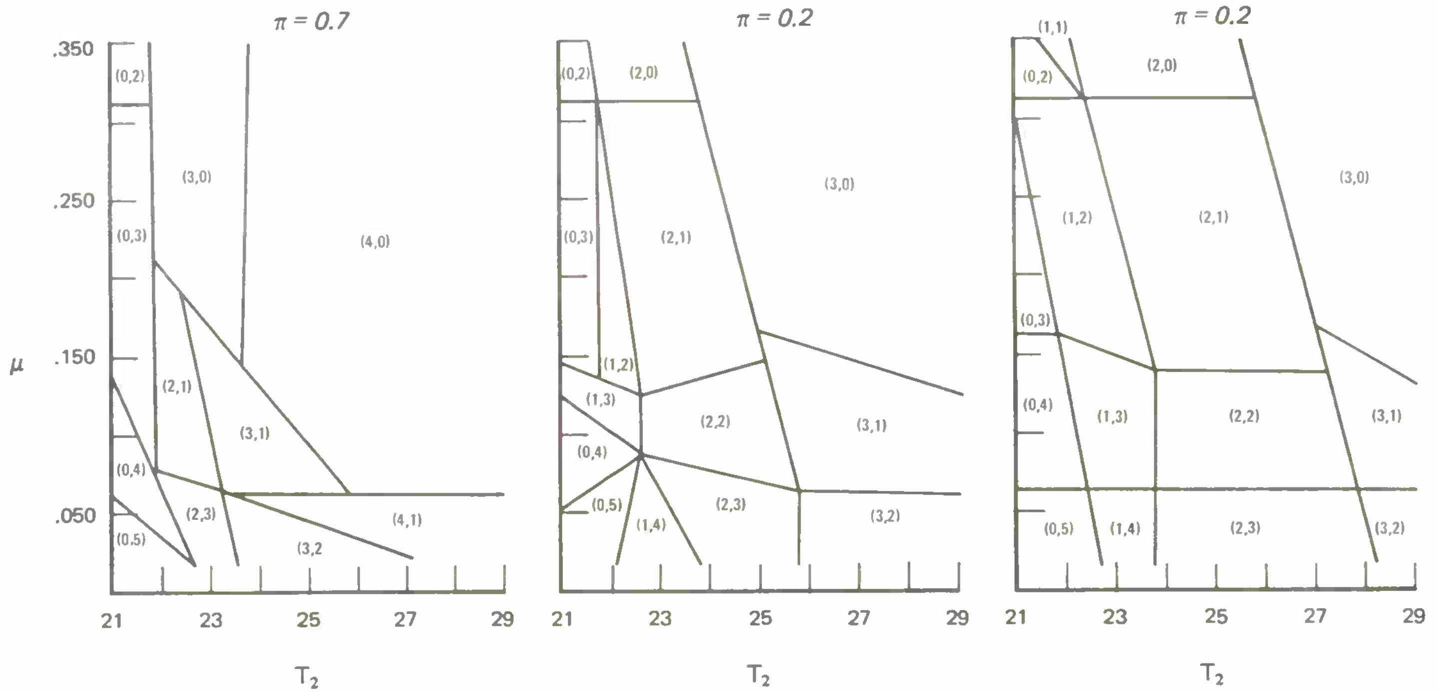

FIG. 2-C: SENSITIVITY ANALYSIS OF π, T_2, μ

Analysis 4 is potentially the most useful of these sensitivity analyses. It describes the relative influence of item cost, C_o, and demand rate, λ, upon policy and system cost. Table 7 exhibits these results for representative ranges of λ and C_o. For a given time, with a given cost, the relationships between N_1 and λ, N_2 and λ, and cost and λ have good linear fits, as shown in table 8.

TABLE 6

SENSITIVITY ANALYSIS OF T_1

T_1	N_1	N_2	Cost
1	4	0	25,593
4	4	0	37,142
7	4	0	48,691
10	4	0	60,241
13	4	0	71,790
16	4	0	83,339
19	3	0	94,173
22	3	0	104,865
23	2	1	108,247
24	2	1	111,164
25	1	2	113,604
26	0	3	114,125

Note: The regression equation is cost = 23345 + 3646 T_1 with $R^2 = .99$.

Time can be made a parameter of the demand rate. Thus, the length of the relevant initial phase of the system life-cycle need not be known at the time this analysis takes place; the proper value of λ should be substituted into the equations of table 8, for the relevant value of C_o. As an example, suppose that the time period is 1 year and the demand is λ. Also suppose that there is a demand during the first week. Do we place a reorder, if the demand is satisfied under the first policy, or do we increase the size of the manufacturer's buffer stock, if the demand is satisfied under policy 2? The answer is immediately available by entering the appropriate line of table 8, using a demand rate of $\frac{51}{52}\lambda$.

TABLE 7

SENSITIVITY ANALYSIS OF C_o AND λ

λ \ C_o($)	1,000	7,000	13,000	19,000	25,000
1	(2,0) $46,535	(1,1) 57,229	(1,1) 66,676	(1,1) 76,123	(1,1) 85,571
7	(11,0) 318,729	(8,1) 373,283	(7,2) 425,260	(7,2) 476,491	(7,2) 527,723
13	(18,0) 589,958	(14,2) 685,197	(13,3) 776,789	(13,3) 867,349	(12,4) 957,691
19	(25,0) 860,948	(21,2) 995,744	(19,4) 1,126,315	(19,4) 1,255,512	(18,5) 1,384,336
25	(32,0) 1,131,798	(27,2) 1,305,554	(26,3) 1,474,566	(25,4) 1,641,985	(24,5) 1,808,902
31	(38,0) 1,402,546	(33,3) 1,615,027	(32,4) 1,822,135	(31,5) 2,026,617	(30,6) 2,232,448
37	(45,0) 1,673,203	(39,3) 1,924,161	(38,4) 2,169,113	(37,5) 2,412,414	(36,6) 2,654,952
43	(52,0) 1,943,832	(46,2) 2,233,019	(44,4) 2,515,804	(43,5) 2,796,815	(42,6) 3,076,976
49	(58,0) 2,214,410	(52,3) 2,541,638	(50,5) 2,862,089	(49,6) 3,180,647	(48,7) 3,498,282

TABLE 8

REGRESSION ANALYSIS

C_o	N_1	R^2	N_2	R^2	Cost	R^2
$ 1,000	$2.5+1.2\lambda$	.99	0.0	1.00	$2516+45152\lambda$	1.00
7,000	$.39+1.1\lambda$	.99	$1.1+.04\lambda$	.67	$10785+51706\lambda$	1.00
13,000	$-.14+1.03\lambda$	.99	$1.7+0.7\lambda$	.80	$17239+58149\lambda$	1.00
19,000	λ	1.00	$1.5+.09\lambda$	.92	$23196+64556\lambda$	1.00
25,000	$-.29+.98\lambda$	.99	$1.8+.11\lambda$	.88	$29290+70948\lambda$	1.00

Analysis 5 (see table 9) shows the sensitivity of our model to variations in K . As might be expected, an increase in K forces the inventory levels, N_1 and N_2 , and system cost to increase. The regression equation of system cost against K provides the correspondence between penalty cost and budget. This equation might be used instead of the iterative procedure described in the previous section on parameter estimation.

TABLE 9

SENSITIVITY ANALYSIS OF K

K	N_1	N_2	Cost
$ 200	1	2	$ 23,086
400	2	1	32,331
1,400	2	1	77,955
1,600	3	0	87,003
5,600	3	0	265,620
5,800	4	0	274,494
10,000	4	0	460,241

Note: The regression equation is cost = 15,753 + 45K with $R^2 = 1.00$.

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