

AD-758 254

A STUDY OF NARROW BAND NOISE GENERATION
BY FLOW OVER VENTILATED WALLS IN TRAN-
SONIC WIND TUNNELS

James P. Woolley, et al

Nielsen Engineering and Research, Incorporated

Prepared for:

Air Force Office of Scientific Research

February 1973

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

AD 758254



DDC
RECEIVED
APR 11 1973
RECEIVED
B

Approved for public release;
distribution unlimited.

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
U S Department of Commerce
Springfield VA 22151

EAR

**NIELSEN ENGINEERING
AND RESEARCH, INC.**

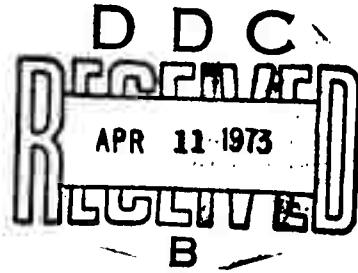
A STUDY OF NARROW BAND NOISE GENERATION
BY FLOW OVER VENTILATED WALLS IN
TRANSONIC WIND TUNNELS

By

James P. Woolley
Krishnamurty Karamcheti*

Approved for public release;
distribution unlimited.

NEAR TR 50
February 1973



This research was supported by the AFOSR
under Contract F44620-72-C-0010.

NIELSEN ENGINEERING & RESEARCH, INC.
Mountain View, Calif. 94040

*Professor, Department of Aeronautics and Astronautics, Stanford
University, Stanford, California. Consultant to NEAR.

ia

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be carried when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) NIELSEN ENGINEERING & RESEARCH, INC. 850 MAUDE AVENUE MOUNTAIN VIEW, CA 94040		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	
		2b. GROUP	
3. REPORT TITLE A STUDY OF NARROW BAND NOISE GENERATION BY FLOW OVER VENTILATED WALLS IN TRANSONIC WIND TUNNELS			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Scientific Interim			
5. AUTHOR(S) (First name, middle initial, last name) JAMES P WOOLLEY KRISHNAMURTY KARAMCHETI			
6. REPORT DATE February 1973	7a. TOTAL NO. OF PAGES 82	7b. NO. OF REFS 33	
8a. CONTRACT OR GRANT NO. F44620-72-C-0010	9a. ORIGINATOR'S REPORT NUMBER(S) NEAR TR-50		
b. PROJECT NO. 9781-02			
c. 61102F	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) AFOSR - TR - 78 - 0503		
d. 681307			
10. DISTRIBUTION STATEMENT Approved for public release; distribution unlimited.			
11. SUPPLEMENTARY NOTES TECH, OTHER		12. SPONSORING MILITARY ACTIVITY AF Office of Scientific Research (NAM) 1400 Wilson Blvd. Arlington, Va. 22209	

13. ABSTRACT

This report is concerned with the problem of environmental noise in transonic wind tunnels with ventilated walls. Specifically, it describes an analytical study aimed at understanding the generation of discrete sounds by the perforated walls of a transonic tunnel. A brief critical review of past experimental investigations of the such sounds is first given. It is indicated that no systematic studies, either experimental or analytical, have yet been directed towards understanding the main features of this sound and the physical factors underlying its generation. On the basis of the existing knowledge, it is asserted that the instability of the separated shear layer over the cavities in a perforated wall should be the main agency for the generation of this sound. With this in mind, it is first shown that stability analysis on the basis of a parallel shear flow does not describe the phenomenon adequately. Thus, a stability analysis of a nonparallel shear flow, which is more representative of the flows in question, is undertaken. It is then shown that the results of such an analysis for an almost-parallel flow leads to a satisfactory explanation of the generation of the tones and their main features. Furthermore, relations for quantities, such as the Strouhal numbers and minimum breadths, are given in terms of wind-tunnel aerodynamic parameters such as local Reynolds number, Mach number, shear layer thickness, and mean velocity profile.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
TRANSONIC WIND TUNNELS						
AERODYNAMIC NOISE, DISCRETE FREQUENCY						
EDGETONES						
FLUID DYNAMIC STABILITY						
NONPARALLEL FLOW STABILITY						
COMPRESSIBLE FLOW STABILITY						
TURBULENT FLOW STABILITY						

PREFACE

The problem of high-intensity noise generation by high-speed flow past the ventilated walls of a transonic wind tunnel has assumed considerable importance recently. There are a number of engineers and research workers dealing with transonic velocities who desire an understanding of the mechanism of such noise generation. The main factors underlying the generation and characteristics of such noise are not usually fully appreciated. This being the case and bearing in mind that interested engineers and researches may not be sufficiently familiar with some aspects of this report; such as the stability of free shear layers which plays a central role in the present problem, this report is written in various contexts in a style that is more explanatory than may seem warranted when viewed by a sophisticated reader. We ask for the indulgence of such a reader, particularly with respect to the section 3.1.

James P. Woolley
Krishnamurty Karamcheti

TABLE OF CONTENTS

	<u>Page No.</u>
1. INTRODUCTION	1
2. NATURE OF PROBLEM AND BACKGROUND	4
2.1 Noise Generated by Cavities in High Speed Flow	5
2.2 Edgetones	10
2.3 Parameters in the Problem of Sound Generation by Ventilated Walls	14
2.4 Some Investigations of Sound from Ventilated Walls of Transonic Tunnels	16
2.5 Discussion	19
3. FLOW STABILITY CONSIDERATIONS	23
3.1 Stability of Incompressible Two-Dimensional, Parallel Shear Flows	24
3.2 Application of Parallel Flow Stability to the Cavity Tone Problem	29
3.3 Formulation of Stability Analysis of Incompressible, Two-Dimensional, Nonparallel Shear Flows	32
3.4 Stability of "Almost-Parallel Flows"	36
3.5 General Features of a Disturbance in an Almost- Parallel Flow	42
3.6 Main Features of Cavity Tones Implied by the Stability of Almost-Parallel Shear Flow	51
3.7 Effects of Flow Compressibility and Turbulence	55
3.7.1 Flow Compressibility Effects	55
3.7.2 Stability Considerations for a Turbulent Mean Velocity Profile	63
4 CONCLUDING REMARKS	68
REFERENCES	69
FIGURES 1-1 THROUGH 3-6	72
DD FORM 1473	

A STUDY OF NARROW BAND NOISE GENERATION
BY FLOW OVER VENTILATED WALLS IN
TRANSONIC WIND TUNNELS

By James P. Woolley and Krishnamurty Karamcheti*
Nielsen Engineering & Research, Inc.

1. INTRODUCTION

It is well recognized that satisfactory wind-tunnel simulation of actual transonic flows associated with flight vehicles is beset with problems in evaluating the effects of the tunnel itself on the test results. Questions regarding Reynolds number effects, tunnel-wall effects, and the effects of environmental noise and turbulence (that are independent of the presence of a test model) in the tunnel have been raised. The environmental noise in a transonic tunnel is usually, as presently noted, of a high level (of the order of 160 dB or more) and consists of both broad band and narrow band components. Such noise levels, as should be expected, will seriously affect both steady-state and dynamic investigations in a transonic wind tunnel and thereby impair its utility. In order to improve this situation and achieve satisfactory utilization of a transonic tunnel, research studies concerning environmental noise in such a tunnel are needed, specifically basic studies are needed to understand the generation of the noise, to predict its characteristics in terms of the geometrical and flow parameters associated with a given wind-tunnel configuration, and to bring about its abatement. With such a broad intent, a program of basic study on the environmental noise in the transonic wind tunnels was initiated at NEAR, Inc. during July 1971 under the sponsorship of AFOSR through the Contract No. F44620-72-C-0010 and is continuing.

*Professor, Department of Aeronautics and Astronautics, Stanford University, Stanford, California. Consultant to NEAR.

The principal sources of environmental noise in a transonic wind tunnel are the flow-creating mechanisms (such as compressors, guide vanes, screens, and honeycombs), the wall boundary layers, and the ventilated test-section walls. These are depicted in figure 1-1. Our concern here is only with the noise generated by the boundary layers and the ventilated walls. Noise radiated by wall boundary layers is expected to be significant in transonic wind tunnels as in other high speed wind tunnels. However, in transonic tunnels, unlike in other tunnels, the ventilated test-section walls are known to have a significant influence on environmental noise contributing large noise levels at discrete frequencies. Figure 1-2, taken from Dods and Hanly (1972), compares pressure fluctuations on a model surface in flight in the atmosphere, in a ventilated wall transonic wind tunnel, and in a smooth wall supersonic tunnel and illustrates the statement.

Hardly any efforts have yet been made to understand the features of noise generated by the ventilated walls, such as the mechanism of its generation and its characteristics. Although in principle an attempt could be made to analyze the noise generated by the wall boundary layers, no such attempt appears to have been directed towards the problem of transonic wind-tunnel noise. Studies of this type are the objectives of the present program.

During the past year, effort was directed principally toward the problem of noise generation by a ventilated wall, particularly towards the problem of relating the noise generation and its main characteristics to the stability characteristics of the shear layers over the holes or cavities in the wall; and thus to the parameters, such as Reynolds number, Mach number, and boundary-layer thickness, characterizing the main flow. Attention was also given to the problem of the noise generated by the wall boundary layers and to that of the noise through the main stream, taking into account reflections at the walls of the tunnel. These latter items will be covered in a subsequent report.

This report is concerned only with the problem of sound radiation from a vented wall of a transonic tunnel. It begins with a review of the investigations concerning sound generation by high speed flow past cavities and ventilated walls and of the nature of the problem characterizing such sound generation. It is indicated that the free shear layer over a cavity is the crucial agency for initiating and controlling the main features of the sound radiated by a cavity. Therefore, the rest of the report is devoted to an analysis of the stability of nonparallel shear flows and of the generation of sound by cavities (and thus ventilated walls) and to a discussion of its features on the basis of the stability characteristics of the free shear layers in question.

2. NATURE OF PROBLEM AND BACKGROUND

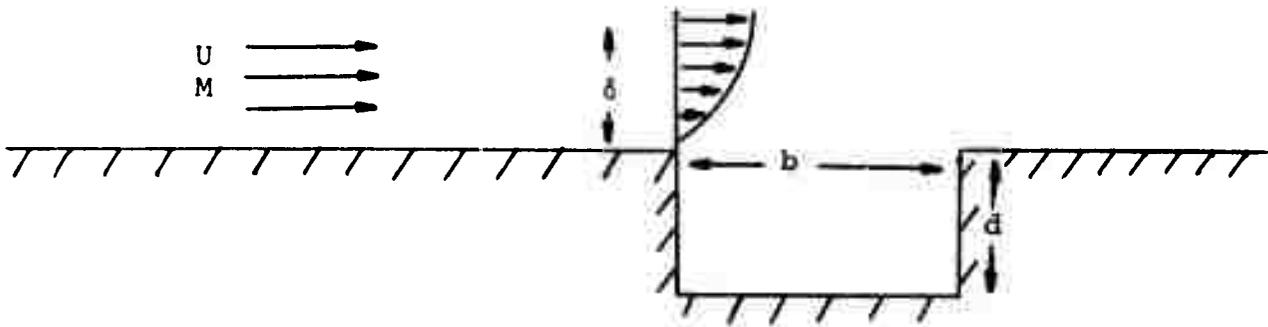
In this chapter we shall review briefly the general nature of the problem of sound generation by flow past the ventilated walls of a transonic tunnel, and the investigations concerning the generation and features of such sound. The generation of such sound is similar to the generation of sound by flow past any hole or cavity in a solid surface. Following the work of Krishnamurty Karamcheti (1955, 1956) there have been a number of other investigations relating to sound generation from two-dimensional cavities exposed to subsonic, transonic and supersonic flow. See for instance Rossiter (1966), Spee (1966), East (1966), and McGregor and White (1970).

Freestone and Cox (1971) investigated some aspects of sound generated by high speed flow past isolated (normal and inclined) holes, and a row of holes typical of those in a perforated wall of a transonic wind tunnel. They observed that the mechanism for production of the sound waves is similar to that observed from two-dimensional cavities.

Sound generation by flow past perforated or slotted walls of a transonic tunnel, although involving more complex geometrical and flow conditions, should be similar in its essentials to that by flow past isolated (normal or inclined) holes or a row of holes and, thus, by flow past two-dimensional cavities. With this in mind, and noting that no systematic investigations have yet been made of the problem of sound generation from ventilated transonic wind-tunnel walls, it is worthwhile to recall the main features of sound generation by two-dimensional cavities so as to realize the various parameters that are likely to play a part in this problem. Thus, we shall begin our brief review with a presentation of the features of sound generation by cavities in high speed flow. We draw attention to only those studies pertinent to the present discussion. Reference to other studies will be found in those cited.

2.1 Noise Generated by Cavities in High Speed Flow

Krishnamurty Karamcheti (1955, 1956) was the first to study extensively the problem of generation of sound by high speed flow past two-dimensional cavities. His studies were conducted on cavities of breadth (b) to depth (d) ratios of 1 to 5, over a Mach number range of 0.4 to 1.5, with laminar and turbulent boundary layers ahead of the cavity. See sketch below.



The general features of the sound radiation are the following:

At a fixed velocity, the frequency of the sound decreases linearly as the breadth increases.

At a fixed breadth, the frequency increases with velocity.

The observed frequencies are in the range of about -100 KHz.

The radiation exhibits characteristic direction properties.

The intensity of the radiation is high, about 160 decibels or more as measured by a Mach Zehnder interferometer.

For very shallow cavities the frequency and minimum breadths are affected by changes in the cavity depth but for $b/d \gg 1$ or greater there is little effect.

The downstream edge of the cavity is an important factor in generating the noise.

Sound emission occurs from cavities of different shapes and size, and from a row of cavities. The frequency of emission is generally controlled by the breadth of the cavity and the Mach number. The variables influencing the phenomenon are the geometry

of the gap given by its shape and typical dimensions (such as the breadth and depth), the free-stream speed, the local speed of sound, the density, the viscosity, and the boundary-layer thickness just ahead of the cavity. Frequency, intensity, and directionality characterize the sound field itself. The physical dependence of the frequency f on these variables may be expressed by a relation of the form

$$S = \frac{fb}{U} = S(M, Re_{\delta}, \frac{b}{\delta}, \frac{d}{\delta}, \text{shape of cavity, laminar or turbulent nature of the boundary layer})$$

Herein S is a nondimensional frequency or Strouhal number, M the local free-stream Mach number, Re_{δ} the local Reynolds number based on the boundary-layer thickness δ ahead of the cavity, and b and d are respectively the breadth and depth of the cavity. Similar functional relations exist for the intensity field.

Karamcheti discussed the possible mechanisms for generating noise having these characteristics. He considered as possible mechanisms:

- (a) Random excitation of acoustic modes of the cavity.
- (b) Helmholtz resonance of the cavity due to influx and efflux of the flowing fluid.
- (c) Formation of an unstable vortex system within the cavity.
- (d) The inherent instability of the separated shear layer over the cavity.

Each of the first three mechanisms were rejected on the grounds that they would place some untenable physical requirements on the fluid flow situation or otherwise predict features of the radiated sound which varied in a manner contrary to that observed. He concludes that the instability of the free shear layer plays the dominant role in the generation of the sound and in the control of some of its main features. The instability of the shear layer

makes available a certain range of frequencies of disturbances to which the layer is susceptible. In the case of sound emission with a laminar boundary layer ahead of the cavity, the Strouhal number at the onset of emission is estimated assuming that the minimum breadth corresponds to the smallest wavelength of a disturbance to which the shear layer is unstable.

Rossiter (1966) studied certain aspects of subsonic and transonic flow over rectangular cavities set in the roof of a wind tunnel. The ratio b/d was varied over the range 1-10, the Mach number over the range 0.4-1.2. The boundary layer was turbulent and the ratio δ^*/d was less than 0.13. He measured the pressure fluctuations in the cavity and on the tunnel wall downstream of the cavity, and observed by shadowgraph methods radiation from the cavities. The main conclusions are the following:

"The unsteady pressures acting in and around a rectangular cavity in a subsonic or transonic airflow may contain both random and periodic components. In general the random component predominates in shallow cavities ($b/d > 4$) and the periodic component predominates in deep cavities ($b/d < 4$)."

"The pressure fluctuations may contain a number of periodic components whose frequencies lie (at any given Mach number) in a sequence $(m - \beta)$ where $m = 1, 2, 3 \dots$ and β (a constant) < 1 ."

"The frequency of any component is inversely proportional to b and increases with tunnel Mach number or speed."

"The shadowgraphs show that the pressure fluctuations are accompanied by the periodic shedding of vortices from the front lip of cavity and by acoustic radiation from the cavity, the principal source being close to the rear lip of the cavity."

"It is found that the frequencies measured could be represented by the empirical equation

$$S = \frac{fb}{U_\infty} = \frac{m - \beta}{(1/K + M)} \quad (2.1)$$

where K is a constant." Note that in general β and K depend on b/d . For cavities with $b/d > 4$, the relation (2.1) represents closely the experimental results.

As a possible physical model of the flow that leads to such a formula, Rossiter envisages a feedback phenomenon, similar in principle to that producing so-called edgetones, see for instance, Curle (1953) and Powell (1961). He assumes a periodic shedding of vortices from the front lip of the cavity and acoustic radiations from the rear lip. Furthermore, it is assumed that the acoustic radiation within the cavity (not discerned in the observations) initiates the vortex shedding and that the passage of the vortices over the rear lip of the cavity is responsible for the sound radiation. Assuming that an identified phase of the acoustic radiation leaves the rear lip when a vortex is at a downstream distance of $\beta(u_c/f)$, where u_c is the average speed with which the vortices are transported over the cavity and f is the frequency of the vortex shedding (which is also the frequency of the sound radiation) and that a vortex is shed from the front lip just when that particular phase of acoustic radiation reaches it. Rossiter readily obtains, by putting

$$u_c = KU_\infty$$

the relation

$$S = \frac{fb}{U_\infty} = \frac{m - \beta}{(Ma_\infty/c + 1/K)} \quad (2.2)$$

where m is the sum of the number of the participating complete wavelengths of the vortex motion and those of the acoustic radiation, and c is the mean speed with which sound waves travel upstream in the cavity. If c is set equal to the speed of sound, a_∞ , in the free stream, relation (2.2) becomes identical with (2.1).

McGregor and White (1970) in experimental studies of the drag of rectangular cavities in transonic and supersonic flow found that the measured frequencies of the pressure oscillations within the

cavity, which are the same as those of the associated acoustic radiation, could be represented reasonably by the relation (2.2) if $\beta = 0.25$, $c = a_\infty$, and $K = u_j/U_\infty$ where u_j is the speed along the free shear layer streamline. It was not stated what value of K was used! The ratio b/d was in the range 0.5 to 3, and the Mach number, M_∞ , in the range 0.3 to 3.0. The boundary layer was turbulent.

Plumlee, Gibson and Lassiter (1962) attributed narrow band acoustic radiation from rectangular cavities in a cylindrical body entirely to resonant modes of the cavity excited by boundary-layer turbulence. They derived relations for the impedance of a rectangular cavity with five rigid walls and uniform flow over the sixth. It was determined that shallow cavities resonate primarily in longitudinal modes while deep cavities ($b/d \leq 1$) respond primarily to depth modes. Width modes, while predicted, seemed to be of little consequence. Extensive charts and tables are given for calculations of resonant frequencies and cavity amplification factors for the various modes. However, the basic hypothesis that the excitation of these modes is merely from the boundary-layer turbulence appears to be too restrictive since Karamcheti obtained tones with a laminar boundary layer.

Freestone and Cox (1971), as mentioned before, investigated sound radiation from perforated tunnel walls, normal and inclined single holes, and a row of normal holes. The holes in the perforated walls have a value of b/d equal to about unity. For the single holes, b/d was in the range 0.3 to 1.5. For the holes in the row, b/d is about unity and they are placed at 3.8 cm. pitch. The tests with the perforated walls were made over Mach number range of 0.6 to 1.3, those with the single holes and the row of holes were made at a single Mach number of 1.2. The boundary layer was turbulent. The ratio of the displacement thickness to the hole diameter was less than 0.1 for most tests. Some of the conclusions are as follows:

In the tests with the walls the frequency varied from 50 to 65 KHz over the Mach number range of 0.6 to 1.3. In these tests, as in the tests on a row of holes, a coupling between the disturbances from the different holes was indicated. It is inferred that the coupling is provided by a disturbance propagating along (or near) the wall surface (that is the streamside surface of the holes).

Strong sound waves can be radiated from both normal and inclined holes.

Strouhal numbers for single holes, apart from occasional mode changes, do not vary significantly with hole depth or if the bottom of the holes is fully or partly closed. This indicates that the frequency depends on disturbances having their origin in the shear layer passing over the cavity.

Strouhal numbers obtained in the tests can be represented by an empirical relation of the form given by Rossiter.

A general summary of the investigations of Strouhal numbers from single cavities is shown in figure 2-1. Shown for comparison are Strouhal numbers calculated from Rossiter's empirical formula, equation (2.2), with $\beta = 0.25$, $K = 0.67$ and c set equal to the speed of sound in the free stream, i.e. $c = a_{\infty}$. Also shown for comparison are Strouhal numbers calculated from an empirical formula proposed by McCanless for correlation of tones produced by transonic wind-tunnel walls. This formula will be described in Section 2.4.

2.2 Edgetones

Sound generation by high speed flow past cavities is similar to the phenomenon of edgetones, where a pure tone of sound is produced by allowing a thin jet of fluid (such as air or water)

coming from a slit to impinge on a wedge-shaped edge placed a short distance from the slit. See for instance Brown (1937), Curle (1953), Nyborg et al (1952), Powell (1961), and Karamcheti et al (1969).

We will now briefly review the edgetone phenomenon, which has been studied in great detail, to provide additional insight into the nature of cavity tones.

Experimental observations show that the main features of edgetone operation, as illustrated in Sketches 1 and 2 below, are the following:

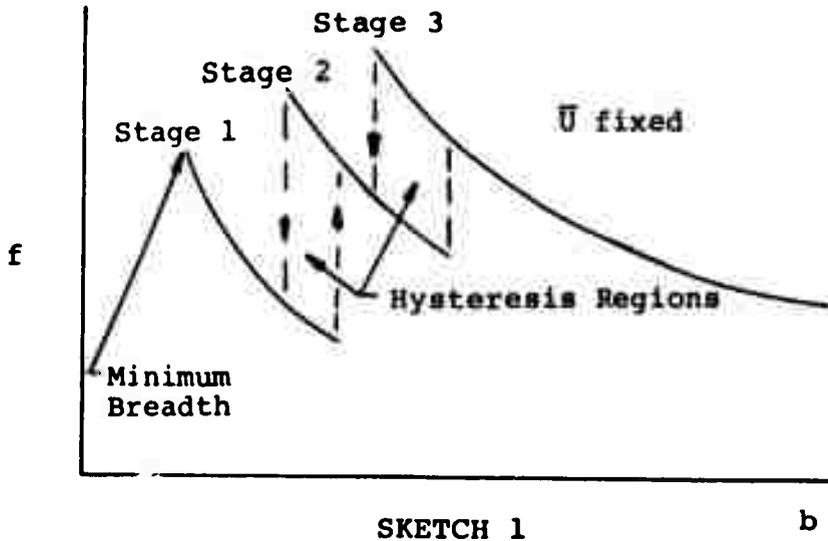
(a) When the mean speed \bar{U} of the jet at the slit is maintained constant, there is a minimum edge distance b_{\min} (i.e., from edge to slit) at which sound production commences. This distance is known as the minimum breadth.

(b) At a fixed mean speed, the frequency of the edgetone decreases gradually as the edge distance is increased over the minimum breadth. This frequency decreases until a certain edge distance is reached where the edgetone will suddenly jump to a new higher frequency or mode of operation. The new mode of operation is known as Stage 2; the first mode, as Stage 1. As the edge distance is further increased, the frequency of Stage 2 again decreases with distance until another jump in frequency takes place, whence the edgetone is said to operate in Stage 3.

The appearance of the higher stages depends on the experimental conditions. At large edge distances, the edgetone is lost in the appearance of irregular or turbulent flow.

When the edgetone is operating in the second or higher stage, and the edge distance is decreased, the frequency will increase until a certain edge distance is reached where the tone will jump down to the next lower stage. The distance at which this occurs does not coincide with the distance where the jump in frequency from one stage of operation to a higher one takes place as the edge distance is increased. Thus, the edgetone exhibits hysteresis regions of operation.

(c) In a given stage, the frequency is almost inversely proportional to the edge distance.

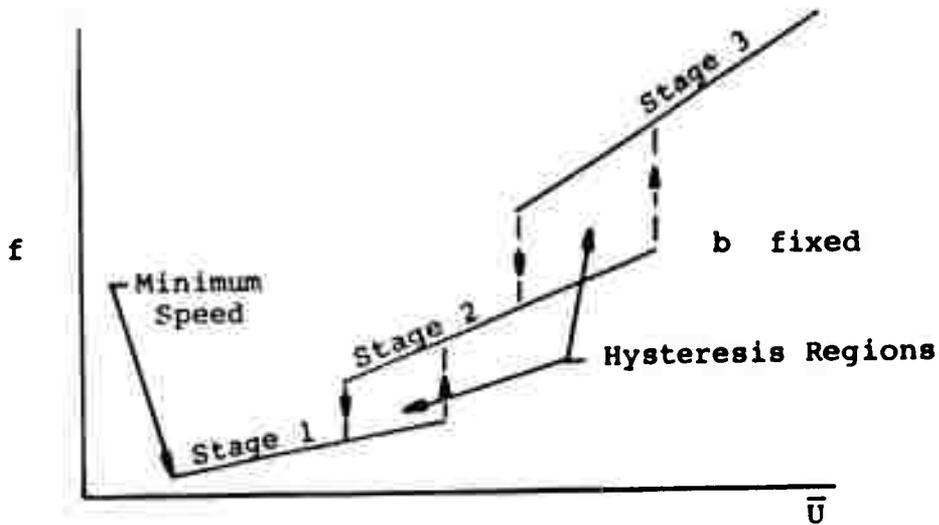


(d) At a fixed edge distance, there is a minimum mean speed below which no edgetone appears.

(e) If, at a fixed edge distance, the mean speed is increased gradually, the frequency of the tone increases until a certain value of the mean speed is reached where a jump to a higher frequency and stage of operation will take place. As the speed is further increased, still higher frequencies and stages of operation will appear until the phenomenon is again limited by irregular or turbulent flow.

When the mean speed is decreased, the frequency decreases, and jumps to lower stages occur. Hysteresis regions are also associated with the frequency jumps between stages as the mean speed is changed.

(f) In any given stage, the tone frequency is approximately proportional to the mean speed.



SKETCH 2

Also, detailed measurements by Nyborg et al (1952) and by Powell and Unfried (1964) reveal that the sound field associated with the edgetone exhibits directional properties characteristic of a dipole field.

Various theories have been proposed to explain the generation of edgetones, to derive a formula for the frequency of the tone, and to explain the reasons for the jumps in the frequency. For the recent theories of edgetones reference may be made to Curle (1953), Nyborg (1954), and Powell (1961), where a review of the earlier ideas may be found. These theories of edgetones consider the mechanism of their generation as a feedback mechanism. Curle and Powell emphasize the role of the stability characteristics of the jet and of the interaction of the jet with the edge. Powell's work is most comprehensive, detailing the role of the various physical factors that underlie the mechanism of edgetones. Nyborg's work does not involve detailed fluid mechanical considerations and takes no account of the stability characteristics of the jet.

Central to the theories of Curle and Powell is the assumption of the so-called phase criterion:

$$\frac{b}{\lambda} = n + \frac{1}{4}, \quad n \text{ being an integer}$$

which is the basis of their formula for the edgetone frequency

$$f = \frac{n + 1/4}{b} u_c$$

where b is the jet slot to edge distance, u_c is an average convection velocity of a disturbance through the jet, λ is the disturbance wavelength in the jet, and n denotes the stage of the edgetone operation. Rossiter (1966), as mentioned before, utilizes ideas similar to those of Curle and Powell in his analysis of the cavity tone.

2.3 Parameters in the Problem of Sound Generation by Ventilated Walls

With the background of the sound generated by two-dimensional cavities and by holes such as investigated by Freestone and Cox (1971), and noting that the mechanism of sound generation by the ventilated walls of a transonic tunnel is similar to that by cavities and holes, we can now enumerate the various parameters that play a part in the problem. For the present discussions, we shall concern ourselves with a perforated wall. The typical geometry of such a wall is shown in figure 2-2. The holes (perforations) are open on one side to the test section and on the other to a plenum chamber, where the pressure is generally lower than in the test section. There is flow through the holes, directed from the plenum, although there may be a few holes where, locally the flow is into the test section, e.g. where an expansion wave from a model intersects the wall. Regardless of such details, we may generally say that over each of the perforations, there is a shear layer (separated boundary layer) which impinges on the downstream edge or side of the perforation.

The parameters influencing the generation of sound by a perforated wall of a transonic tunnel may be listed as follows:

- (1) The free-stream unit Reynolds number, $Re = \rho_\infty U_\infty / \mu_\infty$ or the free-stream Reynolds number $Re = \rho_\infty U_\infty \delta / \mu_\infty$ based on a characteristic length, δ .

(2) The free-stream Mach number M .

(3) Nature of the boundary layer, laminar or turbulent.

Generally it is turbulent.

(4) Mean velocity profile in the boundary layer ahead of the perforations or more specifically the profiles of the free shear layers over the perforations.

(5) A characteristic length, δ , of the shear layer such as the displacement or momentum thickness ahead of the perforations.

(6) The maximum streamwise dimension, b , of the hole.

(7) The depth of the hole.

(8) Geometry of the upstream and downstream edges of the hole.

(9) Inclination, ψ , of a hole.

(10) Spacing, s , between the holes.

(11) Manner of disposition of the holes in the array forming the perforations.

(12) Porosity, τ , of the ventilated wall.

(13) Parameters characterizing the flow through the holes into or out of the plenum chamber.

(14) Plenum chamber geometry and its acoustical characteristics.

(15) Tunnel geometry and its acoustical characteristics.

The sound field may be described by the frequency, f , and the directional distribution of its intensity. Experimental and theoretical investigations should aim at determining these quantities as functions of the parameters described above. For instance, introducing the nondimensional frequency, the so-called Strouhal number, $S = fL/U$, where L is a characteristic length, such as b , and U a characteristic speed such as the free-stream speed, we wish to determine S as a function of the nondimensional parameters such as R , M , b/δ , τ , ψ , and other parameters characterizing the velocity profiles and the geometries of the holes, the plenum

chamber and the tunnel. In the same manner one wishes to determine in a nondimensional form the dependence of the sound intensity distribution on these parameters.

2.4 Some Investigations of Sound from Ventilated Walls of Transonic Tunnels

A number of experimental investigations have been made of the overall noise environment in existing transonic wind tunnels throughout the world. These have been summarized by Mabey (1970), and more comprehensively by Boone and McCanless (1968) and McCanless (1971). For a report of recent studies of noise in the AEDC transonic wind tunnels (designated AEDC-PWT Tunnels 16T and 4T), reference may be made to Credle (1971) where additional references to earlier work in these tunnels may also be found. Similar studies conducted by NASA/Ames Research Center were discussed by Dods and Hanly (1972). We shall not enter here into a detailed report of these various studies. Only a few salient features will be given.

Measurements indicate that the narrow band noise, attributable to generation by the perforations or slots in the test section walls in the various tunnels, may consist of discrete frequencies in the range of 350 Hz to 27 KHz. The tones could be very intense. In terms of the rms pressure coefficient defined by

$$\Delta C_p = \frac{\sqrt{\overline{(p')^2}}}{1/2 \rho_\infty U_\infty^2}$$

where $\overline{(p')^2}$ denotes the mean of the square of the fluctuating pressure p' . The overall level of the narrow band noise at the centerline of the test section can reach a value as high as about $\Delta C_p = 3$ to 4 percent. It thus controls the maximum noise levels in the test section as is evidenced by the appearance of pronounced spikes in the power spectra obtained by Dods and Hanly (1972), see figure 1-2.

At a given Mach number the measured discrete frequencies, as in the case of the tones generated by rectangular cavities (see

for instance Rossiter, 1966), lie in different groups or stages designated by n , where $n = 1, 2, 3 \dots$. Figure 2-3 shows in a nondimensional form the various discrete frequencies of tones generated by the ventilated walls of different wind tunnels.

A nondimensional frequency, S , is introduced such that

$$S = \frac{fb}{U_\infty} \quad (2.3)$$

where

- f is the measured frequency
- b is the maximum streamwise dimension of the hole
- U_∞ is the free-stream speed

Figure 2-3 is a plot of measured values of S versus M , the free-stream Mach number. It is seen that at any given M the measured values of S lie roughly in different stages, and that in any given stage, S varies with M .

It is thus found that the measured frequencies for a given wind tunnel could be represented by an empirical relation of the form

$$S = S(M, n) \quad (2.4)$$

Freestone and Cox (1971) suggest that Rossiter's equation (2.2 above) may be used to express the functional relation (2.4).

McCanless (1971), based on his analysis of the measured values of S at different M , suggests the following empirical relation:

$$S = 0.15 \frac{n^{1.60}}{1 + M} \quad (2.5)$$

The variation of S with M for the different stages n , as given by equations (2.2) and (2.5) is also presented in figure 2-3. It is seen that, as to be expected, there is some general agreement.

Credle (1971) in order to account for the observed effects of variations in Mach number, test-section wall angle and wall porosity,

on the frequency spectra measured in the AEDC tunnels, concluded that McCanless' relation is inadequate and used instead the following:

$$s = \frac{fb}{\bar{U}_{\text{wall}}} = 0.16 \frac{n^{1.55}}{0.7 + \bar{M}_{\text{wall}}} \quad (2.6)$$

where

\bar{U}_{wall} = the spatial average of the longitudinal wall velocity measured by boundary-layer pitot pressure probes that were closest to each of the four walls

and

$$\bar{M}_{\text{wall}} = \bar{U}_{\text{wall}}/a_{\infty}$$

This relation, however, proved unsatisfactory in accounting for the effects of porosity, τ , and a universal definition of U_{wall} will be difficult to arrive at for different probes and wind-tunnel wall boundary layers.

Anderson, Anderson and Credle (1970) also made studies of the effect of the geometrical and acoustical conditions of the plenum chamber on the discrete frequency noise field. They found that changes of the ratio of the plenum-to-test-section volumes between 8.3 and 0.8 had no significant effect on tunnel acoustics, although some changes in the steady flow field were observed for Mach numbers above 0.95. Thus, resonances with the plenum volume do not appear to be significant sources of noise in the tunnel.

Credle (1971) performed an experimental study in which the plenum of the AEDC PWT-4T wind tunnel was lined with acoustically absorbent material. This insulation reduced the level of random noise in the wind tunnel as might be expected, but had little effect on the narrow band noise.

This concludes the brief presentation of some of the main features of the experimental investigations of the noise produced by the perforated walls of transonic wind tunnels.

2.5 Discussion

It is clear that the studies of noise generated by flow past ventilated walls are limited in many respects. They are mainly experimental investigations of noise levels in existing transonic tunnels or attempts to eliminate specific problems associated with such noise. Systematic and comprehensive studies have yet to be undertaken to assess the role of even some of the important parameters mentioned in section 2.3 (such as the Reynolds number, Mach number, the hole dimensions relative to the boundary-layer thickness, the parameters characterizing the interaction between the holes) and to obtain a satisfactory understanding of the factors underlying the sound generation.

Analysis of existing experimental results so as to seek the effects of the various parameters and the mechanics of the sound generation is rather difficult because, in test involving existing transonic wind tunnels, variation of one parameter independently of the others is not always possible.

The main result of the investigations so far, is the presentation of the variation of the measured frequencies in the various tunnels with the Mach number and the representation of this variation by empirical relations such as proposed by Rossiter or McCanless. However satisfactory these relations appear to be, they throw no light on the factors influencing the occurrence of the sound generation nor is there any hope of developing expressions for the sound intensity from this approach. Furthermore, although the relations may be made to correlate the frequency data from a given wind tunnel quite well, it is clear that they cannot account for the differences observed between wind tunnels. The reason for this inability is obviously the fact that of all the parameters of the problem, only the Mach number and the cavity breadth are accounted for in these relations.

It appears, that although the generation of sound by ventilated walls is recognized to be similar to that by cavities and by jet-edge

systems, no serious account is taken of these studies, [such as those of Curle (1953), Krishnamurty Karamcheti (1956), Powell (1961), Karamcheti and Bauer (1963), Stegen and Karamcheti (1967), Shields and Karamcheti (1967)] directed towards understanding the details of the mechanics of sound generation by cavities and jet-edge systems.

Consider for instance the problem of the minimum or critical distance, b_{\min} , for initiation of sound in any particular stage of operation. From observations relating to the tones from cavities and to the (jet) edgetones, one would expect b_{\min}/δ (where δ is a characteristic dimension of the shear layer in question) to be a function of such important parameters as the Reynolds number, the Mach number, and of the mean velocity profile of the shear layer. That is,

$$\frac{b_{\min}}{\delta} = \text{fn. } (R, M, \text{ velocity profile})$$

Sound emission begins when

$$\frac{b}{\delta} > \frac{b_{\min}}{\delta}$$

and thus, the initiation of sound depends on Reynolds number, Mach number, and the mean velocity profile. Similarly, there is a value of b at which sound radiation of a particular stage ceases. If we denote this b by b_{\max} (which also will be a function of Reynolds number, etc.) we may state that each stage of sound radiation occurs such that

$$\frac{b_{\min}}{\delta} < \frac{b}{\delta} < \frac{b_{\max}}{\delta}$$

or

$$\frac{\delta}{b_{\max}} < \frac{\delta}{b} < \frac{\delta}{b_{\min}}$$

where the values of b_{\min} and b_{\max} are different for the different

stages as well as being dependent on the designated flow parameters. Such criteria need to be investigated systematically for the case of sound radiation from cavities and ventilated walls.

McCanless (1971) recognized that such a criterion exists for the sound radiation from a ventilated wall of a transonic tunnel. He gives, in analogy to the jet edgetones, values of the parameter δ^*/b that correspond to the four stages of operation noted. It is, however, not clear how these values were arrived at.

Mabey (1969, 1971), based on his measurements, states that tones could be avoided if

$$\frac{\delta^*}{b} < 0.5$$

or

$$\frac{b}{\delta^*} > 2$$

where δ^* is the displacement thickness. But Freestone and Cox point out with emphasis that their tests show that strong sound waves can be radiated from both normal and inclined holes as used in perforated lines for transonic tunnels for

$$\frac{\delta^*}{b} < 0.5$$

or

$$\frac{b}{\delta^*} > 2$$

It is possible that, among other influencing factors, Mabey's observations may refer to δ^*/b_{\max} for one stage while those of Freestone and Cox may refer to values of δ^*/b for another stage. However, it is readily appreciated that such confusing inferences relating to even important criteria for sound generation from ventilated walls exist at present and that they arise from lack of knowledge of the mechanism of sound generation.

Existence of criteria such as mentioned above, and the various studies directed at understanding the details of the factors underlying sound generation by jet-edge systems and cavities suggest strongly that the stability of the free shear layer in these systems plays an important part in the initiation of the sound radiation and in the control of some of its main features. Several investigations were carried out at Stanford by Karamcheti et al (see the references cited before) to seek the role of the stability of the jet in the generation of jet edgetones. These indicate that indeed the stability characteristics of the jet play an important role in the generation of the edgetones and that many of the features of generation can be predicted on the basis of theoretical results concerning the stability of nonparallel shear flows (see Woolley 1973). In view of this, it is to be expected that the main features of the sound radiation from cavities and ventilated walls of transonic tunnels are principally governed by similar results concerning the stability of the free shear layers involved. It is possible that other factors such as the acoustic resonant characteristics of the cavities may also come into the picture, but, as observations seem to indicate, the main agency for initiation of the sound radiation is the free shear layer.

In light of these considerations, the main objective of the present study, as pointed out before, has been to investigate the role of the stability of the shear layer in the generation of noise by flow past a ventilated wall.

The next chapter presents the analysis and results of such a study.

3. FLOW STABILITY CONSIDERATIONS

The generation of tones by flow past ventilated transonic wind-tunnel walls is, as described before, in many respects similar to the generation of the so-called jet and cavity edgetones. The principal agency in initiating such tones and, perhaps controlling their main features should be, as pointed out, the stability (or instability) of the free shear layer past a cavity, such as the one past a transonic wind-tunnel wall, or of the jet, in a jet-edgetone system. The tone itself results from the interaction of the oscillating shear layer with the rigid reattachment surface, the edge. In order to examine the role of the stability characteristics of such shear layers in the generation of tones, it is first appropriate to investigate the extent the stability characteristics of free shear layers (i.e. without the rigid surface) with different mean (or basic) velocity fields would explain qualitatively and quantitatively the initiation of the tones and their features. It is known from observations that the mean velocity field is not like that of a parallel shear flow but is, generally, a nonparallel shear flow. Departure from parallel flow appears to be greatest in the vicinity of the reattachment surface. The mean velocity fields finally applied to the cavity tone problem, therefore, should reflect the effect of the rigid surface.

With this in mind we shall investigate the problem of determining the stability characteristics of a nonparallel shear flow, and then show how such characteristics for an "almost-parallel flow" can be determined from analyses of corresponding parallel flows. Then we shall describe how the stability characteristics of an almost-parallel flow will already explain some of the most important features of the cavity tones, such as occur in a transonic wind tunnel. We shall begin with the discussion of the stability of a parallel shear flow and the inadequacy of its role in understanding the cavity tones.

3.1 Stability of Incompressible Two-Dimensional, Parallel Shear Flows

We consider an incompressible, two-dimensional, steady shear flow subjected to small, time-dependent disturbances. Choose a Cartesian coordinate system x, y with the x -axis in the direction of the basic steady flow (see fig. 3-1). The velocity of this flow is denoted by

$$\vec{U} = U(y)\vec{i}$$

and the disturbance velocity field by

$$u(x, y, t)\vec{i} + v(x, y, t)\vec{j}$$

The velocity components of the combined flow are given by

$$\hat{U} = U(y) + u(x, y, t) \quad (3.1)$$

$$\hat{V} = v(x, y, t) \quad (3.2)$$

The basic flow satisfies the steady Navier-Stokes equations while the combined flow satisfies the unsteady equations, which we express as the equations of continuity and vorticity.

$$\frac{\partial \hat{U}}{\partial x} + \frac{\partial \hat{V}}{\partial y} = 0 \quad (3.3)$$

$$\left(\frac{\partial}{\partial t} + \hat{U} \frac{\partial}{\partial x} + \hat{V} \frac{\partial}{\partial y} \right) \hat{\Omega} = \nu \nabla^2 \hat{\Omega} \quad (3.4)$$

where $\hat{\Omega}$ is the vorticity given by

$$\hat{\Omega} = \frac{\partial \hat{V}}{\partial x} - \frac{\partial \hat{U}}{\partial y} \quad (3.5)$$

with

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

and ν is the kinematic viscosity. Substituting in (3.3) and (3.4) for \hat{U} , \hat{V} , and $\hat{\Omega}$ from equations (3.1), (3.2), and (3.5), noting that the velocity component $U(y)$ of the basic flow also satisfies equations of the form (3.3) and (3.4), and neglecting all nonlinear terms in the disturbance quantities, we obtain the following linear equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3.6)$$

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \zeta - \nu \frac{d^2 U}{dy^2} = \nu \nabla^2 \zeta \quad (3.7)$$

where ζ is the disturbance vorticity given by

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (3.8)$$

Equation (3.6) is automatically satisfied by introducing a stream function

$$\psi = \psi(x, y, t)$$

for the disturbance field by means of the relations

$$u = \frac{\partial \psi}{\partial y} \quad (3.9)$$

$$v = - \frac{\partial \psi}{\partial x}$$

The disturbance vorticity is then given by

$$\zeta = - \nabla^2 \psi \quad (3.10)$$

Equation (3.7) now takes the form

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \nabla^2 \psi - \frac{d^2 U}{dy^2} \frac{\partial \psi}{\partial x} = \nu \nabla^4 \psi \quad (3.11)$$

This is the equation governing the disturbance field.

We now discuss solutions of equation (3.11) that represent traveling harmonic waves propagating in the streamwise direction. Thus, we assume a solution of the form

$$\psi(x,y,t) = \phi(y)e^{i(\alpha x - \omega t)} \quad (3.12)$$

where ϕ and the constants α and, ω , in general, may be complex. We shall assume that ω is real, thus representing a frequency. We write

$$\alpha = \alpha_r + i\alpha_i$$

Then equation (3.12) may also be expressed as

$$\psi(x,y,t) = \phi(y)e^{-\alpha_i x} e^{i(\alpha_r x - \omega t)} = \phi(y)e^{-\alpha_i x} e^{i\alpha_r(x-ct)} \quad (3.13)$$

where

$$c = \frac{\omega}{\alpha_r} \quad (3.14)$$

Thus, equation (3.12) or (3.13) represents a harmonic wave that travels in the x-direction with a constant speed, c , given by equation (3.14), and one that receives an exponential amplification in space, the amplification factor being α_i . If α_i is zero, the disturbance travels unattenuated, if α_i is positive the disturbance is damped, and if it is negative, the disturbance is amplified.

Substituting equation (3.12) into equation (3.11), we obtain

$$\left[\left(U - \frac{\omega}{\alpha} \right) (D_y^2 - \alpha^2) - D_y^2 U + \frac{i\nu}{\alpha} (D_y^2 - \alpha^2)^2 \right] \phi = 0 \quad (3.15)$$

where

$$D_y \equiv \frac{d}{dy}$$

This equation may be expressed in nondimensional form by introducing U_m , the maximum velocity, and δ , a representative length of the shear layer, respectively as characteristic velocity and length (see fig. 3-1). Adopting now the following notation

$$v = y/\delta$$

$$\bar{U} = U/U_m$$

$$\bar{\phi} = \phi/\delta U_m$$

$$\bar{\alpha} = \alpha \delta$$

$$\bar{\omega} = \omega \delta / U_m$$

$$D_\eta \equiv d/d\eta$$

we express equation (3.15) in the nondimensional form

$$\left[(\bar{U} - \bar{\omega}/\bar{\alpha}) (D_\eta^2 - \bar{\alpha}^2) - D_\eta^2 \bar{U} + \frac{1}{i \text{Re}_\delta \bar{\alpha}} (D_\eta^2 - \bar{\alpha}^2)^2 \right] \bar{\phi} = 0 \quad (3.16)$$

This equation is to be solved with the appropriate boundary conditions on $\bar{\phi}$. There results an eigenvalue problem in the quantities $\bar{\omega}$, $\bar{\alpha}$, and Re_δ , the solution of which depends on the mean velocity profile $\bar{U}(\eta)$. One may specify, for a given mean velocity profile $\bar{U}(\eta)$, any two of the other parameters and determine the third from the solution of the specific eigenvalue problem. For instance, one may determine either experimentally or analytically, as is usually convenient, $\bar{\alpha}$ (that is $\bar{\alpha}_r$ and $\bar{\alpha}_i$) for given $\bar{\omega}$, Re_δ and $\bar{U}(\eta)$, thus obtaining

$$\bar{\alpha} \equiv \bar{\alpha}_r + i\bar{\alpha}_i = \bar{\alpha}[\bar{\omega}, \text{Re}_\delta, \bar{U}(\eta)] \quad (3.17)$$

With $\bar{\alpha}_r$ known, we also obtain the nondimensional phase velocity \bar{c} (which is equal to the actual phase velocity divided by U_m) as a function of $\bar{\omega}$, Re_δ for given $\bar{U}(\eta)$ from the relation

$$\bar{c}[\bar{\omega}, Re_{\delta}; \bar{U}(\eta)] = \frac{\bar{\omega}}{\bar{\alpha}_r} \quad (3.18)$$

Typical solutions of this nature are illustrated in figures 3-2a and b for the different types of shear layers which are also shown in the figures. In these figures f denotes the dimensional frequency, in cycles per second.

Of primary interest is the nature of $\bar{\alpha}_i$, the disturbance amplification rate. It is noted that only a limited range of frequencies are amplified by the flow field, that is, has associated eigenvalues of $\bar{\alpha}_i$ which are less than zero. The flow field is said to be unstable to these disturbances. Disturbance frequencies with $\bar{\alpha}_i$ greater than zero are damped and the flow field is said to be stable to disturbances of this type. A neutral disturbance is one whose frequency is associated with an $\bar{\alpha}_i$ equal to zero.

The range of frequencies which receive amplification is also seen to be a function of Reynolds number for the flow. Below a certain Reynolds number, called $Re_{crit.}$, no frequency is amplified. As Reynolds number becomes very large, two distinct patterns, which are dependent on certain details of the velocity profile, may develop. These different patterns are depicted in the a and b parts of figure 3-2. If the velocity profile contains an inflection point (i.e., $d^2U/dy^2 = 0$ at any point of the flow field), as in figure 3-6b, some frequencies will receive amplification at infinite Reynolds number. If no inflection point exists, as in figure 3-6a, the region of amplified disturbances will vanish at high Reynolds number. The latter is illustrated by boundary layers on solid surfaces with zero or negative (favorable) pressure gradients, while the former is illustrated by such boundary layers with positive pressure gradients and by the entire class of free shear flows; such as jets, wakes, and separated shear layers.

From the solutions such as those illustrated in the figure, one may obtain, for given $\bar{U}(\eta)$ and Re_{δ} , the variation of $\bar{\alpha}_i$ and \bar{c} with $\bar{\omega}$. For instance, the variation of $\bar{\alpha}_i$ with $\bar{\omega}$ is as

shown in figure 3-3. Different frequencies have different amplification rates with one nondimensional frequency $\bar{\omega}_{\max}$ receiving the maximum (nondimensional) amplification rate, $\bar{\alpha}_i \max$.

While the above represents the attenuation of a sinusoidal disturbance of small amplitude and single frequency by the mean flow, it gives only part of the picture for a general disturbance which may contain many Fourier components. It has been noted that this attenuation is frequency dependent; therefore, the various components will receive different rates of amplification and will propagate at different speeds, since c is a function of frequency also. The initial disturbance wave form, thus, may be considerably distorted when observed at a distance from its injection into the flow field even though the attenuation is accomplished in a "linear" manner.

3.2 Application of Parallel Flow Stability to the Cavity Tone Problem

Krishnamurty Karamcheti (1956) examined the role of stability of a free shear layer in the generation of sound by high speed flow past a rectangular cavity in an aerodynamic surface. He utilized the then available results of the stability analysis for a laminar free shear layer between two parallel streams, one of which is at rest. By assuming that at large Reynolds numbers, the shortest wavelength disturbance which receives neutral amplification is the one which will be maintained in a sustained oscillation when sound generation begins (for given free-stream parameters), and further, assuming that the minimum breadth is equal to this shortest wavelength, he was able to estimate the minimum breadth and the corresponding nondimensional frequency and show that they were of the same order of magnitude as observed experimentally.

The role of parallel-flow stability characteristics of a thin jet in the related phenomenon of jet edgetones was considered by Brown (1937), Curle (1955), Powell (1961), Karamcheti et al (1969), Karamcheti and Bauer (1963), Stegen and Karamcheti (1967), Shields

and Karamcheti (1967), and Woolley and Karamcheti (1973). Brown noted that experimentally observed frequencies of jet edgetones were all within the range of frequencies which receive amplification in the jet flow field. Curle and Powell suggested that the edgetone oscillations occur at frequencies of disturbances that receive maximum amplification through the jet. Based on this notion and on the consideration that the edgetone operation is a feedback circuit in which the amplification of a disturbance through the jet is governed by parallel-flow stability characteristics, Powell, obtained the phase and frequency formulas (also previously given by Brown and Curle)

$$\frac{b}{\lambda} = \left(n + \frac{1}{4} \right)$$

$$f = \left(n + \frac{1}{4} \right) \frac{c}{b}$$

where

b = the distance from the slot, from which the jet issues, to the edge

λ = the wavelength of the disturbance through the jet

n = integer denoting the stage of operation of the edgetone

f = frequency of the disturbance in cycles per second

and

c = speed of propagation of the disturbance

and further showed how the minimum breadth, the minimum jet speed, the different stages of operation, and the hysteresis loops might be accounted for.

Investigations by Karamcheti et al at Stanford were undertaken to examine the details of these stability considerations. Velocity fluctuations along a jet in an edgetone system were measured with hot wire anemometers. Measurements of frequency, amplitude and phase were recorded under different operating conditions. Measurements were obtained for stage I, ($n = 1$), stage II and simultaneous

stage I and II operation. It was found that the amplification of the disturbance was not a pure exponential function of distance, as suggested by parallel flow stability. Neither was the phase velocity c constant along the jet, as would be expected from this theory. And, finally, as regards the phase criterion used by both Curle and Powell, the phase change between the jet exit and the edge was not $n + 1/4$ cycles, as supposed in the theories. It was found to be more like $n/2$ cycles. Furthermore, the phase, as would be expected according to the theoretical notions, does not change linearly with distance along the jet. Thus, there exists no fixed phase relation to determine the oscillation frequency as a function of free jet length, b .

Without a separate criterion, parallel flow stability cannot relate the frequency of a cavity or jet edgetone to the length of the free shear layer or the jet. The only frequency selection criterion available on the basis of flow stability is that of dominance of the most amplified frequency. For a parallel flow there is only one frequency which receives the greatest amplification and this frequency is independent of the length of the free shear layer or jet.

In order to explain the observed features of edgetones on the basis of flow stability, a theory for which the most amplified frequency is dependent on distance must be developed. Since the free shear layers present in edgetone systems are almost, but not totally, parallel flow fields, it would appear that investigation of the stability of flow fields which are nonparallel might be rewarding. In the following sections such an investigation is outlined and some of its implications are indicated.

3.3 Formulation of Stability Analysis of Incompressible, Two-Dimensional, Nonparallel Shear Flows

We now consider an incompressible, two-dimensional, steady nonparallel shear flow subjected to small, time-dependent disturbances. Introduce, as before, a Cartesian coordinate system x, y in the plane of motion such that the basic steady velocity field is described by

$$\vec{V}(x, y) = U(x, y)\vec{i} + V(x, y)\vec{j} \quad (3.19)$$

and the disturbance velocity field, as before, by

$$u(x, y, t)\vec{i} + v(x, y, t)\vec{j} \quad (3.20)$$

The velocity components of the combined flow field are then given by

$$\hat{U} = U(x, y) + u(x, y, t) \quad (3.21)$$

$$\hat{V} = V(x, y) + v(x, y, t) \quad (3.22)$$

The linearized equations for the disturbance field, namely for u and v , can be obtained, as in the parallel flow case, by substituting equations (3.21) and (3.22) into the unsteady Navier-Stokes equations (continuity and vorticity equations), by noting that the steady flow components (U and V) themselves satisfy the steady Navier-Stokes equations, and by neglecting all nonlinear terms in the disturbance quantities. Thus if the vorticity $\hat{\Omega}$ of the combined flow is given by

$$\hat{\Omega} = \Omega(x, y) + \zeta(x, y, t) \quad (3.23)$$

where

$$\Omega(x, y) = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \quad (3.24)$$

is the vorticity of the basic flow, and

$$\zeta(x, y, t) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (3.25)$$

is the vorticity of the disturbance flow, we obtain the following linearized equations for u and v :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3.26)$$

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \zeta + \frac{\partial \Omega}{\partial x} u + \frac{\partial \Omega}{\partial y} v = \nu \nabla^2 \zeta \quad (3.27)$$

where ζ is given by equation (3.25). Compare these with the corresponding equations (3.6) and (3.7) for the parallel flow case.

The continuity equation (3.26) is automatically satisfied by introducing a stream function

$$\psi = \psi(x, y, t)$$

for the disturbance field by means of the relations

$$u = \frac{\partial \psi}{\partial y} \quad ; \quad v = - \frac{\partial \psi}{\partial x} \quad (3.28)$$

The disturbance vorticity is then given by

$$\zeta = - \nabla^2 \psi \quad (3.29)$$

Equation (3.27) now takes the form

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \nabla^2 \psi - \frac{\partial \Omega}{\partial x} \frac{\partial \psi}{\partial y} + \frac{\partial \Omega}{\partial y} \frac{\partial \psi}{\partial x} = \nu \nabla^4 \psi \quad (3.30)$$

This is the basic equation governing the stability characteristics of a nonparallel, incompressible shear flow. Equation (3.30) reduces to the corresponding equation (3.11) for the parallel shear flow; for the latter $v = 0$, $U = U(y)$, and $\Omega = - dU/dy$.

We now discuss solutions of equation (3-30) that represent traveling harmonic waves propagating in the x-direction but with varying spatial attenuation rate and phase speed. Thus, we assume a solution of the form

$$\psi(x,y,t) = \phi(x,y)e^{i[\theta(x)-\omega t]} \quad (3.31)$$

where ω denotes frequency and is independent of x and y . Both ϕ and θ may in general be complex. Thus, let us set

$$\theta(x) = \theta_r(x) + i\theta_i(x) \quad (3.32)$$

Furthermore, expressing

$$\theta(x) = \int_0^x \alpha(\xi) d\xi = \int_0^x [\alpha_r(\xi) + i\alpha_i(\xi)] d\xi \quad (3.33)$$

We have

$$\theta_r(x) = \int_0^x \alpha_r(\xi) d\xi \quad (3.34)$$

$$\theta_i(x) = \int_0^x \alpha_i(\xi) d\xi \quad (3.35)$$

Thus equation (3.31) may be rewritten as

$$\psi(x,y,t) = \phi(x,y)e^{-\theta_i(x)} e^{i[\theta_r(x)-\omega t]} \quad (3.36)$$

where θ_r and θ_i are given by equations (3.34) and (3.35).

We note that θ_i accounts for attenuation while θ_r accounts for propagation. With this in mind we may refer to $\alpha_i(x)$, in light of equation (3.35), as the local spatial amplification rate.

The phase of ψ is given by $\theta_r(x) - \omega t$. Thus, a surface of constant phase (a wave front) is described by

$$F(x,t) = \theta_r(x) - \omega t = \text{a const.} \quad (3.37)$$

If the wave front travels with a (wave or phase) speed

$$c = c(x)$$

we know that, since there is no change in the function $F(x,t)$ as we follow the motion of a phase surface, we have

$$\frac{\partial F}{\partial t} + c \frac{\partial F}{\partial x} = 0$$

or, equivalently

$$c = - \frac{\partial F / \partial t}{\partial F / \partial x} = \frac{\omega}{d\theta_r / dx} \quad (3.38)$$

In light of equation (3.34) we obtain

$$c(x) = \frac{\omega}{\alpha_r(x)} \quad (3.39)$$

as the relation for the local phase speed. We may refer to α_r as the local wave number. If $\lambda = \lambda(x)$ is the local wavelength we further have

$$\alpha_r(x) = \frac{2\pi}{\lambda(x)}$$

Substitution of equation (3.36) into equation (3.30) with the use of equations (3.34) and (3.35) yields

$$\left\{ -i\omega [D_y^2 + (i\alpha + D_x)^2] + U[i\alpha + D_x] [D_y^2 + (i\alpha + D_x)^2] \right. \\ \left. + v [D_y^2 + (i\alpha + D_x)^2] D_y + (D_x(D_y U) - \Sigma_x^2 V) D_y \right.$$

(Continued on next page)

$$- \left(D_Y^2 U - D_Y D_X V \right) \left[i\alpha + D_X \right] - v \left[D_Y^2 + (i\alpha + D_X)^2 \right]^2 \phi = 0 \quad (3.40)$$

where the operator notation

$$D_X = \frac{\partial}{\partial x}$$

$$D_Y = \frac{\partial}{\partial y}$$

$$D_Y^2 = \frac{\partial^2}{\partial y^2}$$

and

$$\left[i\alpha + D_X \right] \phi = i\alpha \phi + \frac{\partial \phi}{\partial x}$$

$$\left[i\alpha + D_X \right]^2 \phi = -\alpha^2 \phi + 2i\alpha \frac{\partial \phi}{\partial x} + i\phi \frac{d\alpha}{dx} + \frac{\partial^2 \phi}{\partial x^2}$$

is employed.

3.4 Stability of "Almost-Parallel Flows"

In order to examine the stability of an "almost-parallel flow" it is first necessary to define what is meant by the term. It refers first to the nature of the mean flow and is intended to imply a slight relaxation of parallel flow restrictions as would be encountered in a free or wall shear layer. An important feature of these types of flows is that gradients of field variables are much greater in the direction transverse to than in the direction of the main flow. In a parallel flow the streamwise gradients are identically zero, not just small. Thus, if a measure of the shear layer thickness is denoted by ϵ , it may be implied that streamwise changes over a unit distance are the same order of magnitude as transverse changes over a distance of the order ϵ or, alternatively,

the scale of x is unity and the scale of y is ϵ . If ϵ is small compared to unity, the flow is "almost parallel" in the present sense.

It is well known that such conditions as are imposed here imply certain relations between the flow field components. In particular, from the continuity equation it is found that the transverse velocity, V , must be of order ϵ compared to the stream-wise velocity, U . This is obtained formally by considering ϵ to be a non-dimensional parameter and by instituting the following transformation.

$$\begin{aligned}x^* &= \epsilon x \\y^* &= y\end{aligned}\tag{3.41}$$

Such that the scales of x^* and y^* are of the same order. Then,

$$D_x = \epsilon D_{x^*}, D_y = D_{y^*}$$

The continuity equation for the mean flow now becomes

$$\epsilon D_{x^*} U + D_{y^*} V = 0$$

The consistent order of this equation under the present conditions requires that

$$\frac{V}{U} = O(\epsilon)$$

Therefore, let

$$\begin{aligned}U &= U^* \\V &= \epsilon V^*\end{aligned}\tag{3.42}$$

If the transformations (3.41) and (3.42) are applied to equation (3.40), one obtains¹

$$\begin{aligned} & \left\{ -i\omega \left[D_{y^*}^2 + (i\alpha + D_{x^*})^2 \right] + U \left[i\alpha + \epsilon D_{x^*} \right] \left[D_{y^*}^2 + (i\alpha + \epsilon D_{x^*})^2 \right] \right. \\ & + \epsilon V^* \left[D_{y^*}^2 + (i\alpha + \epsilon D_{x^*})^2 \right] D_{y^*} + \left(\epsilon D_{x^*} D_{y^*} U - \epsilon^3 D_{x^*}^2 V^* \right) D_{y^*} \\ & \left. - \left(D_{y^*}^2 U - \epsilon^2 D_{y^*} D_{x^*} V^* \right) \left[i\alpha + \epsilon D_{x^*} \right] - v \left[D_{y^*}^2 + (i\alpha + \epsilon D_{x^*})^2 \right]^2 \right\} \phi = 0 \end{aligned} \quad (3.43)$$

This equation may be rearranged and grouped into like orders of ϵ yielding

$$\left\{ \left(U - \frac{\omega}{\alpha} \right) \left[D_Y^2 - \alpha^2 \right] - \left(D_Y^2 U \right) + \frac{1}{\alpha} \left[D_Y^2 - \alpha^2 \right]^2 \right\} \phi = 0 \quad (3.44)$$

to order ϵ^0 , where the $()^*$ notation has been suppressed.

Higher order corrections to the stability characteristics may be obtained by expansion of ϕ and α such that $\phi = \phi_1 + \epsilon \phi_2 + \dots$ and $\alpha = \alpha_1 + \epsilon \alpha_2 + \dots$, etc., and substitution of these relations into equation (3.43). The differential equation for ϕ_1 and α_1 will be equation (3.44). Quantities ϕ_2 and α_2 and higher corrections may be found from the solution of equations obtained by the usual methods applied in small parameter expansions. Such refinements do not appear to be warranted at the present time since the first-order solutions have not yet been quantitatively assessed.

¹Note that $\int_0^x \alpha(\xi) d\xi = \frac{1}{\epsilon} \int_0^{x^*} \alpha^*(\xi^*) d\xi^*$

$$\text{thus } D_x \int_0^x \alpha(\xi) d\xi = D_{x^*} \int_0^{x^*} \alpha^*(\xi^*) d\xi^* = \alpha(x) = \alpha^*(x^*)$$

Equation (3.44) is of the same form as equation (3.15), but now U and α are functions of x . This shows that for an "almost-parallel flow" the local stability characteristics, at any given x location, are the same as those of a parallel flow having a mean velocity profile with the same y dependence as the local nonparallel flow. This immediately enables us, as shall be discussed, to determine the overall stability characteristics of the nonparallel shear flow on the basis of the stability analysis of appropriate parallel flows.

We shall now express equation (3.44) in nondimensional form. The scaling quantities for nondimensionalization will be chosen to be the local maximum velocity, $U_m(x)$, and a local characteristic shear layer dimension, $\delta(x)$ (e.g., the boundary-layer thickness), as was done for the parallel flow. The following nondimensional quantities may now be defined as they were for the parallel flow.

$$\bar{U}(x, \eta) = U(x, \eta) / U_m(x)$$

$$Re_\delta(x) = \frac{U_m(x) \delta(x)}{\nu}, \text{ local Reynolds number}$$

$$\bar{\alpha} = \alpha \delta$$

(3.45)

$$\bar{\alpha}(x) = \alpha(x) \delta(x)$$

$$\bar{\phi}(x, \eta) = \phi / U_m \delta$$

$$\eta = y / \delta$$

$$\bar{\omega} = \omega \delta / U_m$$

Upon substitution of equations (3.45), equation (3.44) becomes

$$\left[(\bar{U} - \bar{\omega}/\bar{\alpha}) (D_{\eta}^2 - \bar{\alpha}^2) - D_{\eta}^2 \bar{U} + \frac{i}{\text{Re}_{\delta} \bar{\alpha}} (D_{\eta}^2 - \bar{\alpha}^2)^2 \right] \bar{\phi} = 0 \quad (3.46)$$

Eigenvalues for $\bar{\alpha}$ and $\bar{\omega}$ which are valid for any $U_m(x)$ and $\delta(x)$ having the same \bar{U} are found from equation (3.46) and the appropriate boundary conditions, e.g.,

$$\bar{\phi} = D_{\eta} \bar{\phi} = 0 \quad \text{at} \quad \eta = \pm \infty$$

for free shear layers, jets, wakes, etc.

It is seen that at each streamwise position, x , the solution to equation (3.46) is the solution of the Orr-Sommerfeld equation (eq. (3.16) of section 3.2) for a mean streamwise velocity field identical to the local mean streamwise velocity. The local eigenfunction $\bar{\phi}$, and eigenvalues $\bar{\alpha}$, $\bar{\omega}$ and Re_{δ} for the developing flow are just those for a "parallel flow" having a mean streamwise velocity field whose dependence on η is identical to that of $\bar{U}(x, \eta)$. Note, however, that \bar{U} does not have to correspond to any real parallel flow. It is only necessary that U satisfy the steady Navier-Stokes equation. In this manner the map of stability characteristics, similar to those shown in figures 3-2 for parallel flows, may be computed for an almost-parallel flow field. Following is a description of the method of calculation.

Given $U(x, y)$ one may determine $U_m(x)$ and $\delta(x)$ and thereby compute $\text{Re}_{\delta}(x)$ and $\bar{U}(x, \eta)$. One then proceeds to solve equation (3.46) with the paired functions, $\bar{U}(x, y)$ and $\text{Re}_{\delta}(x)$, at a sufficient number of stations ($\text{Re}_{\delta}(x)$'s) to represent the flow field. These solutions will yield $\bar{\alpha}(\bar{\omega}, \text{Re}_{\delta}(x))$, as in the parallel case

$$\bar{\alpha}(x) = \bar{\alpha}_r(x) + i \bar{\alpha}_i(x) = \bar{\alpha}[\bar{\omega}, \text{Re}_\delta(x)]$$

with $\bar{U}(x, \eta)$ implied by $\text{Re}_\delta(x)$. The stability characteristic map for the given flow field may then be constructed and will be similar in appearance to one of those shown in figure 3-2.

The determination of the eigenfunction, ϕ , and eigenvalues, α and ω , in general requires tedious numerical computations even for the simplest parallel flow fields. The flow fields of present interest are two dimensional and hence a sufficient number of separate solutions must be obtained to represent the changes in characteristics as the flow field develops. The development of the flow field, and hence, the changing of its stability characteristics (the eigenvalues) may be separated into changes in scale and distortions of the fundamental velocity profile. It will be found that a large number of flow fields differ only in scale, e.g., flows of the Falkner-Skan classification. For these flow fields (called self-similar flow fields) each set of appropriate boundary conditions will yield a unique, nondimensional velocity profile $\bar{U}(\eta)$ which is a function of only the nondimensional variable η and normalizing parameters, δ and U_m . The scale of the flow is determined from specific values of these parameters for an individual case. If the solution of the stability problem for these flows is carried out in nondimensional terms, as for equation (3.46), a single set of nondimensional eigenvalues; $\bar{\alpha}$, $\bar{\omega}$, and Re_δ will enable the determination of the actual eigenvalues at any point in the self-similar field by supplying the local values of the parameters, U_m , δ and Re_δ , at each x . It will then be necessary to solve the eigenvalue problem anew only when distortions occur in the velocity profile.

The fate of a disturbance in an almost-parallel flow field may now be determined by integration of the local stability characteristics of the flow field along the path corresponding to that disturbance. Such integrations will necessarily be numerical since

closed form solutions to equation (3.46) exist only for extremely simple \bar{U} .

Before embarking on the task of constructing the computer programs necessary for computation of the stability characteristics for almost-parallel flow fields and the additional computations necessary to apply the results to the problem at hand, the cavity tone, it will be well to discuss the general behavior of disturbances under the new theory and to assess the implications of this behavior with regard to the cavity tone.

3.5 General Features of a Disturbance in an Almost-Parallel Flow

Let us first recall the features of propagation of a disturbance in a given parallel flow. In a parallel flow with given $U(y)$, the quantities U_m , δ , and Re_δ are all constants with the downstream distance x . The nondimensional frequency characterizing a disturbance of a fixed frequency f , which is given by $\bar{\omega} = 2\pi f\delta/U_m$ is also a constant with x . This being the case, in a parallel flow, a disturbance of given frequency propagates in the x -direction with a constant speed c and a constant amplification rate α_1 , each of which is completely specified by a point in the $\bar{\omega} - Re_\delta$ plot. Thus, we may say that in a parallel flow the features of propagation in the direction of x of a disturbance of fixed f are completely given by a point in the $\bar{\omega} - Re_\delta$ plot.

In a nonparallel flow the situation is different. Now, since $U = U(x,y)$, the quantities U_m , δ , and Re_δ all vary with x . Thus, as a disturbance of a given frequency f propagates in the direction of x , the nondimensional frequency $\bar{\omega} = 2\pi f\delta/U_m$ also varies with x . At any given station x , the amplification rate (or, similarly the phase speed) is given by a point in the $\bar{\omega}, Re_\delta$ plot, with those values of $\bar{\omega}$ and Re_δ corresponding to the x -station in question. As the disturbance travels in the direction of x , its local spatial amplification rate (or its phase speed) varies with x and this variation (that is of α_1 with x) is given by a curve in the $\bar{\omega} - Re_\delta$ plane. We note that there is a one-to-one correspondence between Re_δ and x .

In order to determine the features (namely, $\alpha_i(x)$ and $c(x)$) of propagation of a disturbance of given frequency f , in a non-parallel flow, we need to determine its path in the $\bar{\omega} - Re_\delta$ plane, that is the curve along which f is a constant. We may then determine the local propagation features from the $\bar{\alpha}_i(\bar{\omega}, Re_\delta)$ and $\bar{c}(\bar{\omega}, Re_\delta)$ evaluated along this path in $\bar{\omega} - Re_\delta$ plane. This is readily done as follows. Since,

$$\bar{\omega} = \frac{2\pi f \delta(x)}{U_m(x)} = \frac{\omega \delta(x)}{U_m(x)}$$

and

$$Re_\delta = \frac{U_m(x) \delta(x)}{v}$$

we have

$$\bar{\omega} = \frac{2\pi f v}{U_m^2(x)} Re_\delta = \frac{\omega v}{U_m^2(x)} Re_\delta$$

It follows that a curve describing $f = \text{a constant}$ is given by the relation

$$\bar{\omega} = \beta_1 \frac{Re_\delta}{U_m^2(x)} \tag{3.47}$$

where

$$\beta_1 = 2\pi v f = \omega v \text{ a constant for given } f,$$

since $U_m(x)$ is known and values of x can be expressed in terms of those of Re_δ , the disturbance path in the $\bar{\omega} - Re_\delta$ plane may be computed from equation (3.47). Different values of f , that is of β_1 lead to curves showing the paths of disturbances with different frequencies.

We shall concern ourselves with flow over a cavity and thus with a free shear layer only. In such a case, $U_m = U_\infty$, a constant. This being the case, the path of a disturbance in the $\bar{\omega} - Re_\delta$ plane is given by

$$\bar{\omega} = \beta Re_\delta$$

when

$$\beta = \frac{2\pi\nu}{U_m^2} f = \text{constant for given } f$$

The path is a straight line as depicted by the lines A to F in figure 3-4. As f increases, β increases, and the slope of the lines increases. Thus, the lines from A to F denote paths of disturbances with progressively decreasing frequencies.

We shall now discuss the features of propagation of disturbances in a nonparallel (especially an almost-parallel) free shear layer.

In order to examine the growth of disturbances, let us now consider the ratio of the stream function at two different points in space and time.

$$\begin{aligned} \frac{\psi(x, y, t)}{\psi(x_0, y_0, t_0)} &= \frac{\phi(x, y) \exp i \left[\int_0^x \alpha(\xi) d\xi - \omega t \right]}{\phi(x_0, y_0) \exp i \left[\int_0^{x_0} \alpha(\xi) d\xi - \omega t_0 \right]} \\ &= \frac{\phi(x, y)}{\phi(x_0, y_0)} \exp i \left[\int_{x_0}^x \alpha_r(\xi) d\xi - \omega(t - t_0) \right] \exp - \int_{x_0}^x \alpha_i(\xi) d\xi \end{aligned}$$

The ratio $\phi(x, y)/\phi(x_0, y_0)$ evaluated at $y = y_0$ merely represents the distortion of the disturbance by changes in the mean flow between x_0 and x . It is recalled that for a parallel

flow, ϕ is independent of x and there is no distortion, i.e., $\phi(x, y_0)/\phi(x_0, y_0) = 1$ for a parallel flow. For an almost-parallel flow we may assume that the distortion is small, i.e., $\phi(x, y_0)/\phi(x_0, y_0) \approx 1$ for almost-parallel flows.

The first exponential term in the above expressions represents the phase differences between the two points. It is of interest only for instantaneous comparisons. By choosing t such that the bracketed function is equal to $2n\pi$, with n an integer, this term becomes unity.

The remaining term represents the spatial amplification of a disturbance by the almost-parallel mean flow field. It is this term which primarily determines the fate of a disturbance in a given flow field. We shall designate it as

$$\frac{A^*(\omega, x)}{A^*(\omega, x_0)} = A(\omega, x, x_0) = \exp - \int_{x_0}^x \alpha_i(\xi) d\xi$$

It is implied that the α_i in this equation is computed from local solutions to the stability equation for the given ω . This dependence may be made more explicit by writing α_i as $\alpha_i(\omega, x)$. This slight change in notation should not cause any appreciable confusion. Thus, the amplification of a particular disturbance between the points x_0 and x may be expressed as

$$A(\omega, x, x_0) = \exp - \int_{x_0}^x \alpha_i(\omega, \xi) d\xi \quad (3.48)$$

Since it is planned to solve equation (3.46) for the non-dimensional stability characteristics of the flow field, let us now express the above in those terms. Recall that we have previously defined the nondimensional terms such that

$$\alpha_i(\omega, x) = \frac{\bar{\alpha}_i(\bar{\omega}(x), \text{Re}_\delta(x))}{\delta(x)} \quad (3.49)$$

and

$$\bar{\omega}(x) = \frac{\omega v}{U_m^2(x)} \text{Re}_\delta(x) = \beta \text{Re}_\delta(x)$$

and

$$\delta(x) = v/U_m(x) \text{Re}_\delta(x) \quad (3.50)$$

By using these relations in equation (3.48), we may obtain the following

$$A(\omega, x, x_0) = \exp - \int_{x_0}^x \frac{\bar{\alpha}_i(\beta(\xi) \text{Re}_\delta(\xi), \text{Re}_\delta(\xi)) U_m(\xi)}{v \text{Re}_\delta(\xi)} d\xi$$

If $U_m(x)$ is a constant, as it is for the cavity flow, simplification of this expression is possible. Under such conditions β is independent of x and depends only on the disturbance frequency, $\beta(\omega) = \omega v/U_m^2$. We may now change the dependent variable of integration from x to Re_δ

$$\frac{d\text{Re}_\delta}{dx} = \frac{U_m}{v} \frac{d\delta}{dx}$$

which may be expressed as a function of $\text{Re}_\delta(x)$ through equation (3.50), we obtain

$$A(\omega, x, x_0) = \exp - \int_{\text{Re}_\delta(x_0)}^{\text{Re}_\delta(x)} \frac{\bar{\alpha}_i(\beta(\omega) \text{Re}_\delta, \text{Re}_\delta)}{\text{Re}_\delta (d\delta/d\xi)} d \text{Re}_\delta \quad (3.51)$$

We shall use this expression to examine the behavior of disturbances of different frequency in an almost-parallel flow field.

The characteristics map and disturbance paths ($\beta = \text{const}$) depicted in figure 3-4 may be used for this purpose.

Let us assume that disturbances with frequencies corresponding to β_A through β_F (i.e., ω_A through ω_F) are introduced into the flow field at $x = x_0$. For simplicity, let us assume further that $x_0 = 0$ so that equation (3.51) becomes

$$A(\omega, x, 0) = \exp - \int_0^{\text{Re}_\delta(x)} \frac{\bar{\alpha}_i \beta(\omega) \text{Re}_\delta, \text{Re}_\delta}{\text{Re}_\delta (d\delta/dx)} d \text{Re}_\delta \quad (3.52)$$

Amplitude histories for disturbances introduced at $x_0 > 0$ may be obtained by evaluating equation (3.51) using the appropriate $\text{Re}_\delta(x_0)$ beginning at the intersection of the appropriate $\beta = \text{constant}$ path with the line $\text{Re}_\delta = \text{Re}_\delta(x_0)$.

If we may further assume that $d\delta/dx$ is positive throughout, we see from figure 3-4 and equation (3.52) that all disturbances are initially damped. Frequencies greater than ω_B (e.g., ω_A) are never amplified (regardless of x or x_0) because their paths lie completely in the stable region of the $\bar{\omega} - \text{Re}_\delta$ plot. For $\omega < \omega_B$, the path will cross the neutral curve, $\bar{\alpha}_i = 0$, at some $\text{Re}_\delta(x)$ and begin to be amplified.

The first frequency to begin receiving amplification if $x_0 \leq x$ is ω_C . The path of ω_C intersects the neutral curve at a lower Re_δ than any other frequency. This Reynolds number, $\text{Re}_\delta(x_1)$ corresponds to the Re_{crit} for a parallel flow and may be thought of as representing Re_{crit} for the local, almost-parallel flow.

If $x_0 > x$ all frequencies whose paths are in the amplified region, $\bar{\alpha}_i < 0$, will receive initial amplification; e.g., if

²Note that if $x_0 > x_1$ some frequency (e.g., at least ω_C) will be initially amplified.

$Re_\delta(x_0)$ corresponds to point 3 in figure 3-4, $Re_\delta(x_0) = Re_\delta(x_3)$, all $\omega \leq \omega_D$ will be amplified at the outset.

At some $Re_\delta(x)$, e.g., $Re_\delta(x_3)$ for $\omega = \omega_D$, the disturbance path will leave the unstable region and $A(\omega_D, x, x_0)$ will again begin to decrease.

The amplitude histories, $A(\omega, x, 0)$, described above are depicted in figure 3-5. It is noted that for $\beta \geq \beta_D$ the disturbance amplitude, $A(\omega, x, 0)$, is maximum at its point of introduction. For $\beta \leq \beta_D$, however, there is some region of the flow field, e.g., between $Re_\delta(x_4)$ and $Re_\delta(x_5)$ for $\beta = \beta_E$, where the amplitude exceeds the initial amplitude. This represents the condition for which there has been a net transfer of energy from the mean flow to the disturbance. Point 3 represents the beginning of such regions since it is at this point where $A(\omega, x, 0)$ first regains its input magnitude, $A^*(\omega, 0)$. The Reynolds number at point 3, $Re_\delta(x_3)$, is the nonparallel flow equivalent of the critical Reynolds number for a parallel flow since this is the least $Re_\delta(x)$ at which $A(\omega, x, x_0) \geq 1$ for any ω . It shall be designated $Re_{cr}(x_0)$. We note that $Re_{cr}(x_0)$ must be greater than or equal to the critical Reynolds number for the local parallel flow Re_{crit} (i.e., $Re_\delta(x)$) in this case, regardless of x_0 , but that it is a function of x_0 as well as the flow field.

Given the information contained in figure 3-5 for all frequencies, a map can be constructed in the $\omega - Re_\delta$ plane showing amplitude ratio, A , as a parameter similar to $\bar{\alpha}_1$ in the $\bar{\omega} - Re_\delta$ plane. The features of such a map are depicted in figure 3-6 for the present case. It is observed that this figure is strikingly similar in features to that of a parallel flow with uninflected mean velocity profile even though the local amplification rates are those of an inflected velocity profile (i.e., $d^2U/dy^2 = 0$ somewhere in the shear layer).

It is observed that the greatest amplitude disturbance at each x station is associated with values of β , and therefore ω , which decrease as x increases. Thus, as one moves downstream from the point of introduction of a general disturbance, the dominant frequency observed should decrease. This is a general relation, independent of the choice of x_0 (i.e., $Re_\delta(0) \neq 0$). A general expression may be derived from which this frequency may be computed, given the functional form of $\bar{\alpha}_1(\bar{\omega}, Re_\delta)$. This relation is obtained from the familiar condition for the extremum of a function.

$$\left[\frac{\partial A(\omega, x, x_0)}{\partial \omega} \right]_{\omega=\omega_m} = \frac{\partial A(\omega_m, x, x_0)}{\partial \omega_m} = 0$$

if ω_m = the extremum frequency at station x .

The extremum expression may be stated more explicitly as

$$\begin{aligned} \frac{\partial}{\partial \omega_m} \int_{Re_\delta(x_0)}^{Re_\delta(x)} \frac{\bar{\alpha}_1(\omega_m \gamma Re_\delta, Re_\delta)}{Re_\delta(d\delta/d\xi)} d Re_\delta \\ = \int_{Re_\delta(x_0)}^{Re_\delta(x)} \frac{\partial \bar{\alpha}_1(\omega_m \gamma Re_\delta, Re_\delta)}{\partial \omega_m} \frac{d Re_\delta}{Re_\delta(d\delta/d\xi)} \\ = \int_{Re_\delta(x_0)}^{Re_\delta(x)} \gamma \frac{\partial \bar{\alpha}_1(\bar{\omega}_m, Re_\delta)}{\partial \bar{\omega}_m} \frac{d Re_\delta}{(d\delta(\xi)/d\xi)} = 0 \end{aligned} \quad (3.53)$$

where $\bar{\omega}_m(x) = \omega_m \gamma Re_\delta(x)$

$$\gamma = v/U_m^2.$$

A second derivative could be computed to ascertain whether the extremum is a maximum or a minimum, but this information is probably more easily obtained by inspection. If it is known that

$-\bar{\alpha}_i$ passes through a maximum within the interval, the extremum is a maximum. If $-\bar{\alpha}_i$ passes through a minimum, then the extremum is a minimum. This conclusion arises from the observation that in order for the integral in equation (3.53) to vanish non-trivially, $\partial\bar{\alpha}_i/\partial\bar{\omega}$ must be positive for part of the interval of integration and negative for a balancing portion of the interval. Therefore, $\partial\bar{\alpha}_i/\partial\bar{\omega} = 0$ at some point in the interval (x_0, x) . If this point represents a maximum of $-\bar{\alpha}_i$, the above mentioned second derivative of the integral with respect to ω_m has the proper sign for $A(\omega, x, x_0)$ to be a maximum at ω_m , i.e.,

$$\int_{\text{Re}_\delta(x_0)}^{\text{Re}_\delta(x)} \gamma \frac{\partial^2 \bar{\alpha}_i(\bar{\omega}, \text{Re}_\delta)}{\partial \bar{\omega}^2} \frac{d \text{Re}_\delta}{d\delta(\xi)/d\xi} > 0$$

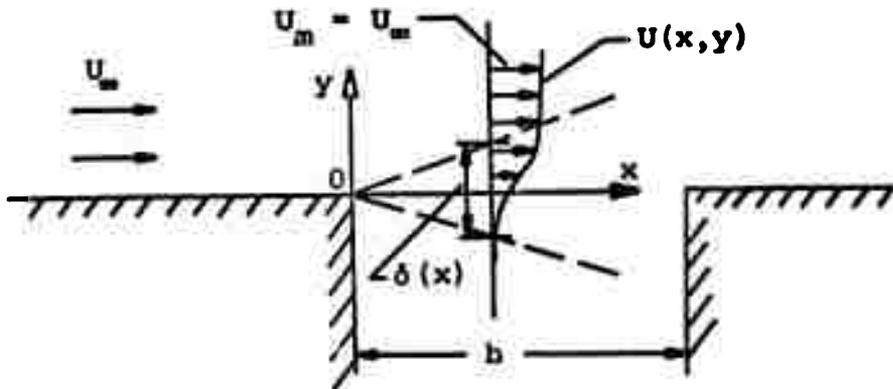
We have in equation (3.53) an integral expression for obtaining ω_m at any position x for a given point of introduction of disturbances. It can be shown that the conditions leading to this expression are the general conditions for determining the envelope of a set of curves, $A(\omega, x, x_0)$, having the parameter ω . Thus the function $A(\omega_m(x), x, x_0)$ is the envelope of the curves for $x_0 = 0$ shown in figure 3-5 and represents the expected amplitude. It is easily observed from figure 3-5 that when the amplitude histories become tangent to the envelope, the disturbance is still being amplified, i.e., $\alpha_i < 0$ at that point. Of course, this is only true if the amplification of successive ω_m 's is increasing locally with x (i.e., $dA/dx > 0$). If $dA(\omega_m(x), x, x_0)/dx < 0$, i.e., the amplitude is decreasing with x , however, the disturbance will be in the stable region of the characteristics map when it is the most amplified disturbance and α_i will be greater than zero at that point.

The above general description of the behaviour of disturbances in an almost-parallel flow field, while incomplete, illustrates that some strikingly new features appear in nonparallel flow stability in contrast to those in parallel-flow stability. A more complete

discussion of the nonparallel flow stability theory will be presented in a forthcoming report by the present authors. We now proceed to discuss the application of this theory to the edgetone and the problem of narrow band-noise generation in wind tunnels.

3.6 Main Features of Cavity Tones Implied by the Stability of Almost-Parallel Shear Flow

We shall now seek to express the main features of cavity tones in light of the stability of an almost-parallel free shear layer. Let us consider the generation of edgetones by means of flow past a cavity as shown by the sketch below.



The breadth of the cavity, which is the dimension in the direction of the main stream, is denoted, as before by b .

For the present discussions we shall assume that the free shear layer over the cavity begins at its upstream edge, that is $\delta(0) = 0$. The mean velocity profile $U(x,y)$ of the free shear layer is that which actually exists with the downstream edge in place.

With respect to the generation of a tone in such a situation we assert the following:

(a) If a tone is generated the frequency of the tone is the frequency of the disturbance which receives the maximum total amplification over the distance b , for the given $U(x,y)$; $\omega = \omega_m(b)$.

(b) A tone is generated only if the total amplification over the distance b for the disturbance with the frequency which

receives maximum amplification over that distance is equal or greater than unity:

$$A(\omega_m(b), b, 0) \geq 1$$

On the basis of these two criteria, which can be determined purely on the basis of the stability of the nonparallel free shear layer with the given $U(x,y)$ we can conclude the following with regard to the main features of the cavity tones (reference may be made to fig. 3-5):

(1) No tone will be generated for $Re_\delta(x)$ less than a certain value which we shall denote as $(Re_\delta)_{min}$. For

$$Re_\delta < (Re_\delta)_{min}$$

the amplitudes of all disturbances are less than their initial excitation, that is

$$A < 1$$

(2) Since

$$Re_\delta = \frac{U_m \delta(x)}{\nu} ,$$

where U_m is a constant in this case when a tone is first generated, we have the condition

$$U_m \delta(b) = \nu (Re_\delta)_{min}$$

where the right-hand member is known as discussed in (1) above.

(a) If U_m is given, the value of $\delta(b)$ or equivalently b (since there is a one-to-one correspondence between $\delta(b)$ and b for a given profile $U(x,y)$) corresponding to $(Re_\delta)_{min}$ is obtained. We shall refer to this value of b as the minimum

breadth denoted by b_{\min} . Thus, we have

$$\delta(b_{\min}) = \frac{v(Re_{\delta})_{\min}}{U_m} \quad \text{for given } U_m \text{ and } \bar{U}.$$

For tone generation with a given profile, \bar{U} , and U_m , we must have

$$\delta(b_{\min}) \geq \frac{v(Re_{\delta})_{\min}}{U_m}$$

or equivalently, since $\delta(b)$ increases with b , we must have

$$b \geq b_{\min}$$

(b) If b is given, $\delta(b)$ is known, and the value of U_m corresponding to $(Re_{\delta})_{\min}$ is obtained. We may refer to this value as the minimum speed, and denote it by $(U_m)_{\min}$. We have

$$(U_m)_{\min} = \frac{v(Re_{\delta})_{\min}}{\delta(b)} \quad \text{for given } b \text{ and } \bar{U}.$$

Thus, we see that in the generation of cavity tones, with a given mean profile, $\bar{U}(x, \eta)$, of the free shear layer, there is a minimum breadth b_{\min} for a fixed value of the characteristic speed U_m , and that there is a minimum speed $(U_m)_{\min}$ for a fixed value of the breadth b . For $b < b_{\min}$ at fixed U_m , or for $U_m < (U_m)_{\min}$ at fixed b there is no sound radiation.

(3) Consider now the case for which the downstream edge of the cavity is at a distance greater than b_{\min} . Since δ increases with increase in b , we have

$$Re_{\delta}(b) > (Re_{\delta})_{\min}$$

From figure 3-5, as we have discussed before, we see that (a) there are disturbances for which the maximum total amplification over the distance b is greater than unity and (b) the frequency of such disturbances decreases with increasing $Re_{\delta}(b)$.

Since this is the case,

$$Re_{\delta}(b) = \frac{U_m \delta(b)}{\nu}$$

where U_m is a constant and $\delta(b)$ increases with b , we may state that with a given profile, the frequency of the cavity tone at a fixed speed U_m decreases with increasing breadth b , and at a fixed b , it increases with the speed, U_m .

(4) The actual frequencies of the tones generated at $(Re_{\delta})_{\min}$ and $(Re_{\delta}) > (Re_{\delta})_{\min}$ can be readily inferred from the stability characteristics such as depicted in figure 3-5.

(5) Further features such as the growth and propagation speed over the distance b of the disturbance during a cavity tone operation can also be deduced from the stability characteristics. The above conclusions (1) to (3) are well supported by experimental observations. We have thus seen that some of the main features of the phenomenon of cavity tones can be well understood on the basis of the stability characteristics of nonparallel free shear layers. Furthermore, quantitative analysis of such features and their dependence on the given parameters of the problem can also be undertaken on this basis.

In applying these notions to the problem of sound generation by high speed flow past ventilated transonic wind-tunnel walls, it will be necessary first to extend the stability considerations to include compressibility and turbulent shear layers, and then to take into account the actual geometrical and flow configurations characterizing the ventilated walls. Preliminary attempts with regard to the first aspect are briefly described in the following sections.

3.7 Effects of Flow Compressibility and Turbulence

To this point we have discussed the stability of incompressible, steady, laminar flow. The flow fields on the walls of transonic wind tunnels will rarely if ever be laminar and compressibility effects are generally present in the mean flow, if not in the disturbance field. In order to apply our analysis to the wind-tunnel environment, we consider extensions of the analysis to compressible and turbulent flows. Through such extensions the nature and extent of the effects may be evaluated. Such effects as arise from these considerations are expected to represent modifications to the basic theory and not major revisions. It is with this in mind that we have constructed the analysis.

3.7.1 Flow Compressibility Effects

Some analytical treatment of compressible flows has been undertaken previously; for example, Lees (1946), Lees and Lin (1947), Dunn and Lin (1955), and Lees and Reshotko (1962). Experimental studies have also been carried out by Demetriades (1958 and 1960) and by Laufer and Vrebalovich (1960).

These analytical studies indicate two major effects of compressibility considerations. First, the dependence of stability on the mean velocity profile is altered by gradients in the mean temperature and density through the shear flow. Second, fluctuations in temperature and density, which may occur for compressible flows, introduce new sources of instability besides the velocity fluctuations experienced by incompressible flows. The additional terms introduced by compressibility into the equation governing the stability of the flow field could result in fundamental changes in the nature of its solution (e.g., inclusion of all these effects changes the governing differential equation from fourth order for incompressible flow to sixth order for compressible flow). This is particularly true of the temperature and density fluctuations. These analyses also indicate the possibility of disturbances which travel at supersonic speeds with respect to parts of the flow field,

as well as the subsonic disturbances, which are correspondent to disturbances associated with incompressible flows. The reader is referred to the referenced studies for a detailed account of these effects.

The experimental studies performed suggest that the important features of compressible flow stability in the range of present interest have a character very similar to those of geometrically similar incompressible flow fields. The study by Laufer and Vrebalovich in particular suggests that a simple correlation with incompressible stability characteristics may exist for compressible flows up to a Mach number of about 2.2 for flat plate boundary layers. Such results indicate that it is primarily the alterations to the mean flow field which affect the flow stability in the compressible flows of present interest.

In order to express these experimental findings in analytical form we have developed an analysis for the simplified case of "almost incompressible flow", in the same spirit as the other extensions to stability theory presented herein. This analysis is presently restricted to "inviscid stability" considerations (i.e., large Reynolds numbers) and will treat the mean flow as parallel.

It is noted at the outset that the existence of strong shock waves intersecting the shear layer may invalidate the "almost-parallel flow" assumption for compressible flows, but the treatment of such complications must await firmer understanding of the more simple situations.

In the following derivation the mean flow field will be assumed to be sufficiently parallel that transverse components may be neglected to the order of concern here. It is noted that under such conditions, the mean static pressure, P , may be a function of the streamwise variable, x , only. In the present case it will be assumed constant, P_0 . Although viscous effects are neglected

for the unsteady components of flow, they may be active in the mean flow. Indeed, the viscous effects and the boundary conditions will determine the mean velocity profile. It is assumed, however, that the steady flow problem may be solved separately from the unsteady problem and that the mean velocity profile, $U(y)$, is an input to the unsteady problem.

The unsteady field variables will be assumed to be two dimensional and sinusoidal

$$\hat{q}(x,y,t) = q(y) \exp i (\alpha x - \omega t)$$

where

- $\hat{q}(x,y,t)$ is any unsteady field variable
- $q(y)$ is the amplitude function of q and may be complex
- α is the complex wave number of the disturbance q
- ω is the circular frequency of the disturbance (radian/sec) and is assumed to be real

The linearized governing equations for the unsteady amplitudes, $q(y)$ are:

Mass

$$i\alpha(U - \omega/\alpha)r + \rho'/\rho v + v' + i\alpha u = 0 \quad (3.54)$$

x-Momentum

$$i\alpha(U - \omega/\alpha)u + U'v + i\alpha \pi/\rho = 0 \quad (3.55)$$

y-Momentum

$$i\alpha(U - \omega/\alpha)v + \pi'/\rho = 0 \quad (3.56)$$

Entropy

$$i\alpha(U - \omega/\alpha)s + S'v = 0 \quad (3.57)$$

State

$$\rho'/\rho + S'/c_p = 0 \quad (3.58)$$

and

$$\rho'/\rho + T'/T = 0$$

$$\text{(perfect gas at constant pressure)} \quad (3.59)$$

where

$$\bar{\rho} = \text{fluid density} = \rho(y) [1 + \hat{r}(x,y,t)]$$

$$\bar{P} = \text{fluid static pressure} = P_0 + \hat{\pi}(x,y,t)$$

$$\bar{S} = \text{entropy} = S(y) + \hat{s}(x,y,t)$$

$$\bar{T} = \text{temperature} = T(y) + \hat{t}(x,y,t)$$

$$\vec{V} = \text{velocity} = [U(y) + \hat{u}(x,y,t)] \vec{e}_x + \hat{v}(x,y,t) \vec{e}_y$$

and

$$()' = d/dy()$$

In view of the small magnitude of the unsteady variables the following relations may also be obtained from thermodynamic considerations.

$$\pi = \rho a^2 (r + s/c_p) \quad (3.60)$$

where

$$a^2 = (\text{sound speed})^2 = (\partial P / \partial \rho)_S = \gamma P / \rho \quad (\text{perfect gas}) \quad (3.61)$$

$$\gamma = \text{ratio of specific heats, } c_p / c_v$$

Equation (3.60) may be rearranged and, with the aid of equation (3.57), put in the form

$$r = \frac{\pi}{\rho a^2} - \frac{s}{c_p} = \frac{\pi}{\rho a^2} + \frac{S'}{c_p} \frac{v}{i\alpha(U - \omega/\alpha)}$$

By employing equation (3.55), π may be eliminated from the above.

$$r = - \frac{(U - \omega/\alpha)}{a^2} u - \frac{U'}{i\alpha a^2} v + \frac{S'}{c_p} \frac{v}{i\alpha(U - \omega/\alpha)}$$

With this equation r may be eliminated from equation (3.54), which, after some rearrangement and use of equation (3.58), becomes:

$$i\alpha u \left[1 - \frac{(U - \omega/\alpha)^2}{a^2} \right] - \left[\frac{U'(U - \omega/\alpha)}{a^2} \right] v + v' = 0 \quad (3.62)$$

Next, equations (3.55) and (3.56) are combined to eliminate π .

$$i\alpha \left[\left(U - \frac{\omega}{\alpha} \right) \rho u \right]' + (U' \rho v)' = - \alpha^2 \left(U - \frac{\omega}{\alpha} \right) \rho v \quad (3.63)$$

u is now eliminated between equations (3.62) and (3.63), yielding

$$\alpha^2 (U - \omega/\alpha) \rho v = - \left[(U - \omega/\alpha) \rho \frac{U'(U - \omega/\alpha)v/a^2 - v'}{1 - (U - \omega/\alpha)^2/a^2} + U' \rho v \right]'$$

(Continued on next page)

$$= - \left[\frac{\rho}{1 - (U - \omega/\alpha)^2/a^2} \times \right. \\ \left. [U'v (U - \omega/\alpha)^2/a^2 + 1 - (U - \omega/\alpha)^2/a^2 - U - \omega/\alpha)v'] \right]'$$

or

$$\alpha^2 (U - \omega/\alpha) \rho v = - \left[\frac{\rho [U'v - (U - \omega/\alpha)v']}{1 - (U - \omega/\alpha)^2/a^2} \right]' \quad (3.64)$$

Equation (3.64) is the form of the inviscid stability equations obtained by Lees and Lin (1946) with the substitution of density for temperature. Equation (3.64) may be greatly simplified in form by changing velocities to Mach numbers defined as follows:

$\hat{M} = (U - \omega/\alpha)/a =$ relative Mach number of the local fluid with respect to the disturbance

$m = v/a =$ Mach number of transverse velocity fluctuation

Noting that

$$a^2 \left[\frac{U'v - (U - \omega/\alpha)v'}{a^2} \right] = a^2 [\hat{M}'m - \hat{M}v(1/a)'] - \hat{M}m' + \hat{M}v(1/a)'] \\ = a^2 (\hat{M}'m - \hat{M}m')$$

since

$$U' = (U - \omega/\alpha)'$$

and that from equation (3.61) and $P = P_0$

$$a^2 = \frac{\gamma P_0}{\rho}$$

It is found that equation (3.64) simplifies to

$$\alpha^2 \hat{M} m = \left(\frac{\hat{M} m' - \hat{M}' m}{1 - \hat{M}^2} \right)' \quad (3.65)$$

Expressing the inviscid stability of a parallel compressible flow entirely in terms of Mach numbers instead of velocities, as is the case for incompressible flow.

The inviscid Orr-Sommerfeld equation for incompressible flow is

$$(U - \omega/\alpha)(v'' - \alpha^2 v) - (U - \omega/\alpha)''v = 0 \quad (3.66)$$

Now let

$$\hat{M} = \hat{M}_\infty \bar{M} \quad (3.67)$$

and

$$m = \hat{M}_\infty \bar{m}$$

where

$$\hat{M}_\infty = (U_\infty - \omega/\alpha)/a_\infty$$

U_∞ = the maximum or free-stream mean velocity

a_∞ = speed of sound at the position of $U = U_\infty$

Equation (3.65) with the substitution of equation (3.67), is now expanded to a form similar to equation (3.66).

$$\hat{M}_\infty^2 \bar{M} \left[\bar{m}'' - \alpha^2 (1 - \hat{M}_\infty^2 \bar{M}^2) \bar{m} \right] + \hat{M}_\infty^3 \frac{2 \bar{M} \bar{M}'}{1 - \hat{M}_\infty^2 \bar{M}^2} \bar{m}' - \hat{M}_\infty^2 \left[\frac{2 \hat{M}_\infty^2 \bar{M} (\bar{M}')^2}{1 - \hat{M}_\infty^2 \bar{M}^2} + \bar{M}'' \right] \bar{m} = 0 \quad (3.68)$$

Let it be assumed that the phase velocity of the disturbance having wave number α is everywhere subsonic with respect to the fluid. Then $1 \geq \hat{M}_\infty \geq |M|$ and $|\bar{M}| \leq 1$.

Solutions for \bar{m} may be sought which are of the form

$$\bar{m} = \bar{m}_0 + \hat{M}_\infty \bar{m}_1 + \hat{M}_\infty^2 \bar{m}_2 + \dots \quad (3.69)$$

Substitution of equation (3.69) into equation (3.68) yields the following equations for \bar{m}_0 and \bar{m}_1 , the first and second-order solutions for \bar{m} .

$$\bar{M}(\bar{m}_0'' - \alpha^2 \bar{m}_0) - \bar{M}'' \bar{m}_0 = 0 \quad (3.70)$$

$$\bar{M}(\bar{m}_1'' - \alpha^2 \bar{m}_1) - \bar{M}'' \bar{m}_1 = -2 \bar{M} \bar{M}' \bar{m}_0' \quad (3.71)$$

Equation (3.70) is the identical form of equation (3.66), the inviscid Orr-Sommerfeld equation. The velocities have been replaced by Mach numbers in the compressible flow case, but the first-order form of the equations are identical to the incompressible case. The mean velocity profile of the incompressible case has been replaced essentially by the mean Mach number profile.

It is seen, however, that the appropriate scaling parameter for velocity is not the speed of the free stream, but its speed relative to the disturbance phase velocity, ω/α . In order that the expansion converges it is sufficient that $|\hat{M}|$ and \hat{M}_∞ both be less than one; that is, all velocities are subsonic with respect to the phase velocity at local conditions.

On the basis of the present analysis one can gain some insight into the results of Laufer and Vrebalovich (1961) showing the correspondence between compressible and incompressible stability characteristics. If the major features of the Mach number profile and the velocity profile are similar, the solution of equation (3.70)

for $\alpha_i = 0$, neutral stability, will be little different from that of equation (3.66). The phase velocity, ω/α , however, is dependent on \bar{M} instead of U . Thus, a Mach number effect should be observed in the eigenvalues for the phase velocity due to the speed of sound gradients in the shear layer, as is noted by Laufer and Vrebalovich. A complete understanding of these effects will require further detailed study to extend the analysis into the viscous domain.

The analytical results indicate that a simple extension to the incompressible theory would account for the major compressibility effects if the disturbances are considered to propagate subsonically with respect to the mean flow. This extension merely requires that the mean and fluctuating flow velocities be replaced by Mach numbers in the inviscid Orr-Sommerfeld equation. Such results appear to substantiate the observations of Laufer and Vrebalovich in their experiments with the flat-plate boundary layer. On the basis of the analysis, however, their observation that the major compressibility effects on stability arise from alterations to the mean velocity profile appears to apply generally for subsonic disturbances. Further work is necessary, however, to carry these results over to low Reynolds numbers where viscous effects may be important.

Since the flow fields of interest in the present study are generally in the high Reynolds number regime, it is believed that the compressibility modifications indicated by the simplified inviscid analysis will give satisfactory results in the present application.

3.7.2 Stability Considerations for a Turbulent Mean Velocity Profile

The analysis for the stability characteristics of a flow field was originated as a means to predict the transition of a laminar flow into a turbulent flow. In the foregoing sections we have used the characteristics derived from such an analysis to

describe the generation of edgetones, cavity tones in particular. Such tones, however, are known to exist in turbulent as well as laminar flows. It is the present task to show how the analysis can be carried over to turbulent flows. The previously described theory for generation of cavity tones will thereby be shown to apply to turbulent as well as laminar flow fields.

This problem was approached by Malkus (1956) for turbulent channel flow. Reynolds and Tiederman (1967) studied this particular problem further. Along the lines employed by Malkus, the present analysis shows that the stability characteristics of a turbulent flow field may be computed in an appropriate sense in the same manner as for a laminar flow. The velocity profile to be employed for the turbulent flow analysis may usually be the velocity profile normally defined for "steady" turbulent flows.

For an incompressible fluid the motion is governed by the equations of momentum and continuity

$$\frac{D\hat{\vec{V}}}{dt} = -\nabla\hat{P} + \nu\nabla^2\hat{\vec{V}} \quad (3.72)$$

and

$$\nabla \cdot \hat{\vec{V}} = 0 \quad (3.73)$$

If $\hat{\vec{V}}$ is made up of a spatial (or ensemble) average flow and small amplitude random fluctuations

$$\hat{\vec{V}}(x,y,t) = \vec{V}(x,y,t) + \vec{V}'(x,y,t) \quad (3.74)$$

$$\hat{P}(x,y,t) = \bar{P}(x,y,t) + P'(x,y,t)$$

Equations (3.72 and (3.73) may be expanded to yield

$$\frac{\partial\vec{V}}{\partial t} + \frac{\partial\vec{V}'}{\partial t} + \vec{V} \cdot \nabla\vec{V} + \vec{V}' \cdot \nabla\vec{V}' + \vec{V}' \cdot \nabla\vec{V} + \vec{V} \cdot \nabla\vec{V}' - \nabla\bar{P} - \nabla P' + \nu\nabla^2\vec{V} + \nu\nabla^2\vec{V}' \quad (3.75)$$

and

$$\nabla \cdot \vec{V} + \nabla \cdot \vec{V}' = 0 \quad (3.76)$$

If there is no spatial correlation between the time dependence of \vec{V} and \vec{V}' , taking a spatial average of equations (3.75) and (3.76) tranverse to the main direction of flow yields

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} + \overline{V' \cdot \nabla V'} = -\nabla \bar{P} + \nu \nabla^2 \vec{V} \quad (3.77)$$

$$\nabla \cdot \vec{V} = 0 \quad (3.78)$$

Note that the assumption of no correlation between the mean and the random quantities eliminated such terms as $\vec{V} \cdot \nabla V'$ from the time average equation. Should a correlation exist these terms would survive this operation and proper treatment of the term $\overline{\Delta(V' \cdot \nabla V')}$ introduced by Malkus to account for this will be required.

Now, the mean flow is expanded in the form

$$\vec{V} = U(y)\vec{e}_x + \epsilon \vec{V}_1(x, y, t) + \epsilon^2 \vec{V}_2(x, y, t) \quad (3.79)$$

$$\bar{P} = P(x, y) + \epsilon \bar{P}_1(x, y, t) + \epsilon^2 (P_2(x, y, t))$$

where ϵ is an arbitrary parameter denoting the order of magnitude of the time dependent components of the mean flow.

Substitution of equation (3.79) into equation (3.77) results in

$$\begin{aligned} \epsilon \frac{\partial \vec{V}_1}{\partial t} + \dots + \epsilon U(y) \frac{\partial \vec{V}_1}{\partial x} \dots + \epsilon \vec{V}_1 \cdot \frac{dU}{dy} \dots + \overline{V' \cdot \nabla V'} \\ = -\nabla P - \epsilon \nabla P_1 \dots + \nu \frac{d^2 U}{dy^2} + \epsilon \nu \nabla^2 \vec{V}_1 \dots \end{aligned}$$

Where the noncorrelation between the mean and random flows has been employed. Collecting terms of like order in the parameter ϵ yields

$$\overline{\vec{v}' \cdot \nabla \vec{v}'} = -\nabla \bar{P} + \nu \frac{d^2 U}{dy^2} \quad (3.80)$$

for ϵ^0 , and

$$\frac{\partial \vec{v}_1}{\partial t} + U \frac{\partial \vec{v}_1}{\partial x} + \vec{v}_1 \cdot \vec{e}_y \frac{dU}{dy} = -\nabla_2 P_1 + \nu \nabla_2^2 \vec{v}_1 \quad (3.81)$$

for ϵ^1 , where

$$\nabla_2 = \vec{e}_x \frac{\partial}{\partial x} + \vec{e}_y \frac{\partial}{\partial y}$$

Equation (3.80) governs the motion of the time independent mean flow. Equation (3.81) may be used to investigate the stability of the time mean flow with respect to organized disturbances. Taking the curl of equation (3.81) and setting \vec{v}_1 into the functional form³

$$\vec{v}_1 \cdot \vec{e}_x = \frac{\partial \psi(x, y, t)}{\partial y} = \frac{d\phi(y)}{dy} \exp i(\alpha x - \omega t)$$

and

$$\vec{v}_1 \cdot \vec{e}_y = -\frac{\partial \psi(x, y, t)}{\partial x} = -i\alpha \phi(y) \exp i(\alpha x - \omega t)$$

yields the familiar equation

$$\left(U - \frac{\omega}{\alpha} \right) \left(\frac{d^2 \phi}{dy^2} - \alpha^2 \phi \right) - \frac{d^2 U}{dy^2} \phi = \frac{\nu}{1\alpha} \left(\frac{\partial^4 \phi}{\partial y^4} - 2\alpha^2 \frac{\partial^2 \phi}{\partial y^2} + \alpha^4 \phi \right)$$

³The pressure is of similar form but is eliminated from the governing equation by the curl operation and need not be considered further.

This equation is of identical form to the Orr-Sommerfeld equation used for the examination of the stability of laminar flows. Thus, with the appropriate interpretations, the sensitivity of the mean turbulent flow to organized disturbances may be examined by substitution of the mean velocity profile, $U(x,y)$, into the Orr-Sommerfeld equation and obtaining the resultant characteristics in the same manner as if one were examining a laminar flow. Additional complications can arise of course, if there is significant correlation between the "random" and the organized disturbance fields. These are expected to result in only minor modifications to the present theory and not alter the central ideas. As such, their investigation can be delayed to a future effort.

Thus, the central idea of the analysis of the cavity tone also remains essentially unchanged for the turbulent shear flow. The flow field stability characteristics will change, however, because the mean velocity profile for a turbulent shear layer is not the same as for a laminar one.

4. CONCLUDING REMARKS

This study has concentrated on the development of the theory of discrete tone generation by flow over cavities. Such tones are shown to be generated by the flow over the perforated walls of transonic wind tunnels. It is seen that the instability of the shear layer over these perforations is the main agency for the generation of such discrete tones of sound. The results of the analysis of the stability of an almost-parallel flow not only offer a satisfactory explanation of the sound generation and its main features, but in their quantitative form lead also to relations expressing quantities such as the Strouhal number and the minimum breadth in terms of tunnel aerodynamic parameters such as the Reynolds number, Mach number, shear layer thickness and the mean velocity profile.

In order to obtain quantitative results, it is necessary to carry out the stability analysis for typical shear layers that are representative of the the flow past the cavities in such perforated walls. The calculations require the input of mean velocity profiles for the shear layers at several streamwise positions in order to ascertain the changes in the stability characteristics due to flow development. These characteristics are then integrated along disturbance paths in the $\omega - Re_\delta$ plane to determine the features of disturbance propagation in the nonparallel flow field. From these features the frequency of the most amplified disturbance and the amplification it receives may be determined. The latter may be used to indicate the intensity of the generated noise, but actual calculation of the intensity requires further considerations.

REFERENCES

- Boone, J. R. and McCanless, G. F.: Evaluation of Acoustic Sources of Background Noise in Wind-Tunnel Facilities. NASA CR 98-155, June 1968.
- Brown, G. B.: The Vortex Motion Causing Edgetones. The Proceedings of the Physical Society, Vol. 49, Pt. 5, pp. 493-507, Sept. 1937.
- Credle, O. P.: Perforated Wall Noise in the AEDC-PWT 16-Foot and 4-Foot Transonic Tunnels. AEDC-TR-71-161, Aug. 1971.
- Curle, N.: The Mechanics of Edgetones, Proc. Roy. Soc., (A) Vol. 216, p. 416, 1953.
- Demetriades, A.: An Experimental Investigation of the Stability of the Hypersonic Laminar Boundary Layer. GALCIT Hypersonic Res. Project Memo No. 43, 1958.
- Demetriades, A.: An Experiment on the Stability of the Hypersonic Laminar Boundary Layers. JFM Vol. 7, p. 385, 1960.
- Dods, J. B. and Hanly, R. D.: Evaluation of Transonic and Supersonic Wind Tunnel Background Noise and Effects of Surface Pressure Fluctuation Measurements. AIAA Paper No. 72-1004, AIAA 7th Aerodynamic Testing Conference, Palo Alto, CA, Sept. 13-15, 1972.
- Dunn, D. W. and Lin, C. C.: On the Stability of the Laminar Boundary Layer in a Compressible Fluid. Jour. of the Aero. Scis., Vol. 22, pp. 455-477, July 1955.
- East, L. F.: Aerodynamically Induced Resonance in Rectangular Cavities. Jour. Sound & Vibration, Vol. 3, No. 3, pp. 227-287, 1966.
- Freestone, M. M. and Cox, R. N.: Sound Fields Generated by Transonic Flows over Surfaces Having Circular Perforations. Reference 24 in AGARD CP NO. 83-71 on Facilities and Techniques for Aerodynamic Testing at Transonic Speeds and High Reynolds Number. AD 731 150, Aug. 1971.
- Karamcheti, Krishnamurty: Acoustic Radiation from Two-Dimensional Rectangular Cutouts in Aerodynamic Surfaces. NACA TN 3487, Aug. 1955.
- Karamcheti, K.: Sound Radiation from Surface Cutouts in High Speed Flow. Ph.D. Thesis, California Institute of Technology, 1956.

- Karamcheti, K. and Bauer, A. B.: Edgetone Generation. Stanford University Dept. Aeronautics and Astronautics Report SUDAER 162, July 1963.
- Karamcheti, K., Bauer, A. B., Shields, W. L., Stegen, G. R., and Woolley, J. P.: Some Features of an Edgetone Flow Field. Basic Aerodynamic Noise Research Conference, NASA SP-207, July 1969.
- Laufer, J. and Vrebalovich, T.: Stability and Transition of a Supersonic Laminar Boundary Layer on an Insulated Flat Plate. JFM Vol, 9 p. 257, 1960.
- Lees, L. and Lin, C. C.: Investigation of the Stability of the Laminar Boundary Layer in a Compressible Fluid. NACA TN 1115, 1946.
- Lees, L.: The Stability of a Laminar Boundary Layer in a Compressible Fluid, NACA Rept. 876, 1947.
- Lees, L. and Reshotko, E.: Stability of the Compressible Laminar Boundary Layer. JFM Vol. 12, p. 555, 1962.
- Mabey, D. C.: Elimination of Edge-Tones in the Perforated Working Section of the 3 ft x 3 ft Tunnel at R.A.E. Bedford, 32nd Semi-annual Meeting of Supersonic Tunnel Association, Sept. 1969.
- Mabey, D. G.: Flow Unsteadiness & Model Vibration in Wind Tunnels at Subsonic and Transonic Speeds, R.A.E. Tech. Rept. 70184, 1970.
- Malkus, W. V. R.: Outline of a Theory of Turbulent Shear Flow. JFM Vol. 1, p. 521, 1956.
- McCanless, G. F.: Additional Correction of 4% Saturn V Protuberance Test Data. NASA CR-103064, Jan. 1971.
- McGregor, O. W. and White, R. A.: Drag of Rectangular Cavities in Supersonic and Transonic Flow Including the Effects of Cavity Resonance. AIAA Journal, Nov. 1970, pp. 1959-1964.
- Nyborg, W. L., Burkhard, M. D. and Schilling, H. K.: Acoustical Characteristics of Jet-Edge and Jet-Edge-Resonator Systems. JASA, Vol. 24, p. 293, 1952.
- Nyborg, W. L.: Self-Maintained Oscillations of the Jet in a Jet-Edge System. JASA Vol. 26, No. 2, pp. 174-182, Mar. 1954.
- Plumblee, H. E., Gibson, J. S., and Lassiter, L. W.: A Theoretical and Experimental Investigation of the Acoustic Response of Cavities in an Aerodynamic Flow. WADD-TR-61-75, Mar. 1962.
- Powell, A.: On the Edgetone. JASA, Vol. 33, No. 4, 1961.

- Powell, A. and Unfried, H. H.: An Experimental Study of Low Speed Edgetones, U.C.L.A. Dept. of Engr., Rept. No. 64-49, Nov. 1964.
- Reynolds, W. C. and Tiederman, W. G.: Stability of Turbulent Channel Flow, with Application to Malkus' Theory. JFM, Vol. 27, p. 253, 1967.
- Rossiter, J. E.: Wind Tunnel Experiments on the Flow Over Rectangular Cavities at Subsonic and Transonic Speeds. RAE R & M No. 3438, 1966.
- Shields, W. L., and Karamcheti, K.: An Experimental Investigation of the Edgetone Flow Field. Stanford University, Dept. Aero. and Astro., Rept. SUDAAR 304, Feb. 1967.
- Spee, B. M.: Wind Tunnel Experiments on Unsteady Cavity Flow at High Subsonic Speeds. AGARD C.P. No. 4, 1966.
- Stegen, G. R. and Karamcheti, K.: On the Structure of an Edgetone Flow Field. Stanford University Dept. of Aero. and Astro. Rept. SUDAAR 303, Feb. 1967.
- Woolley, J. P.: An Investigation of Edgetones and the Role of Flow Field Instability in their Generation, Ph.D. Thesis, Department of Aeronautics and Astronautics, 1973, Stanford University.
- Woolley, J. P. and Karamcheti, K.: An Experimental and Analytical Investigation of Jet and Vortex-Wake Edgetones. Stanford Univ., Dept. of Aero. and Astro. Rept. (To be published) 1973.

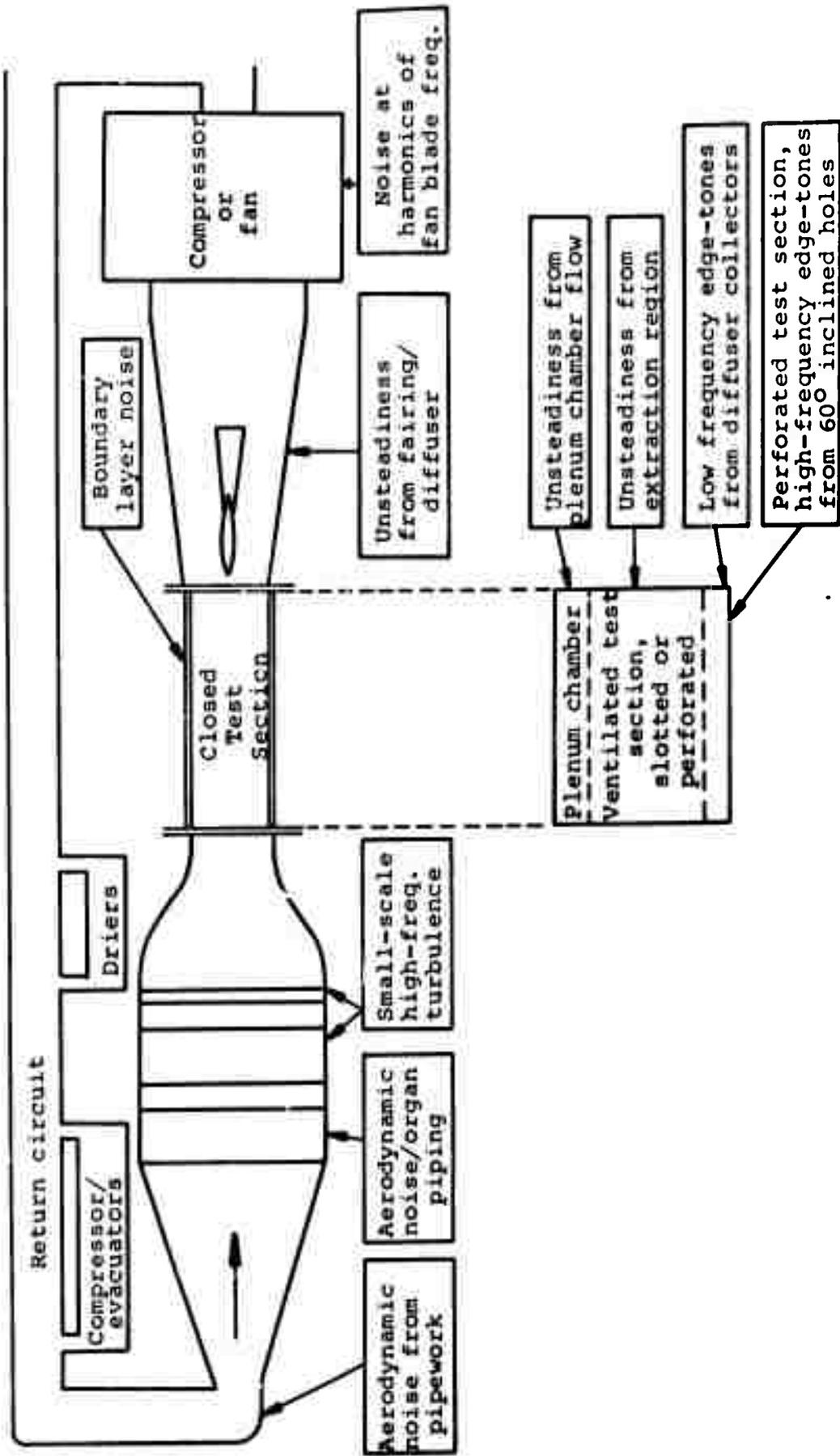


Figure 1-1.- Typical Noise Sources in a Transonic Wind Tunnel

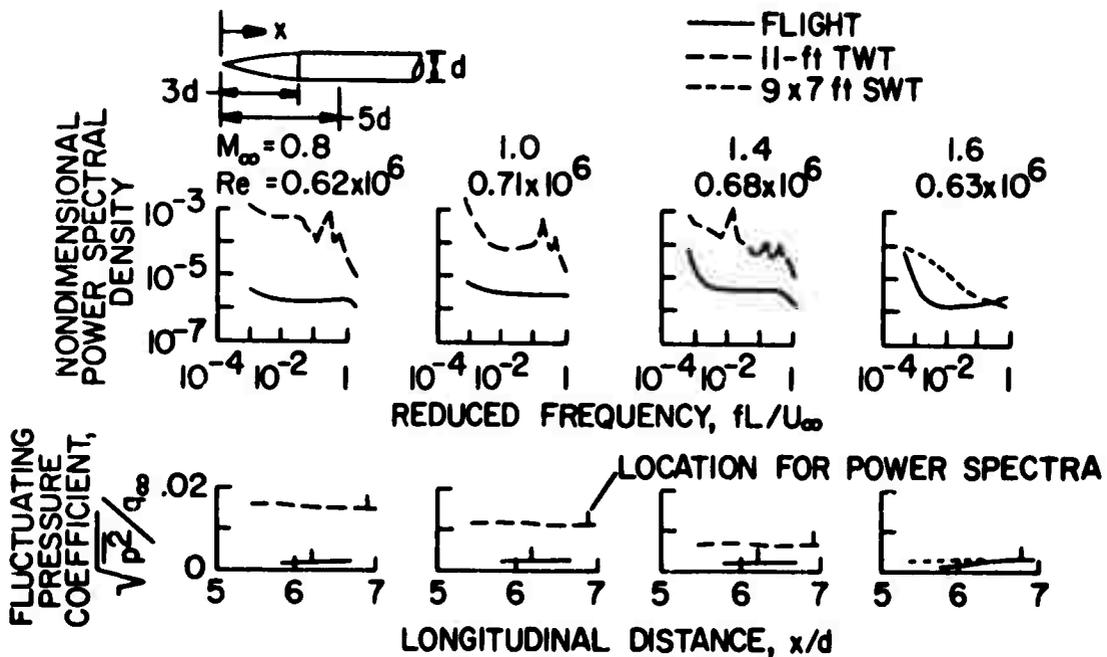


Figure 1-2.- Broadband pressure fluctuations and typical power spectra measured in flight and in wind tunnels at Ames Research Center (reproduced from Dods and Hanly, 1972).

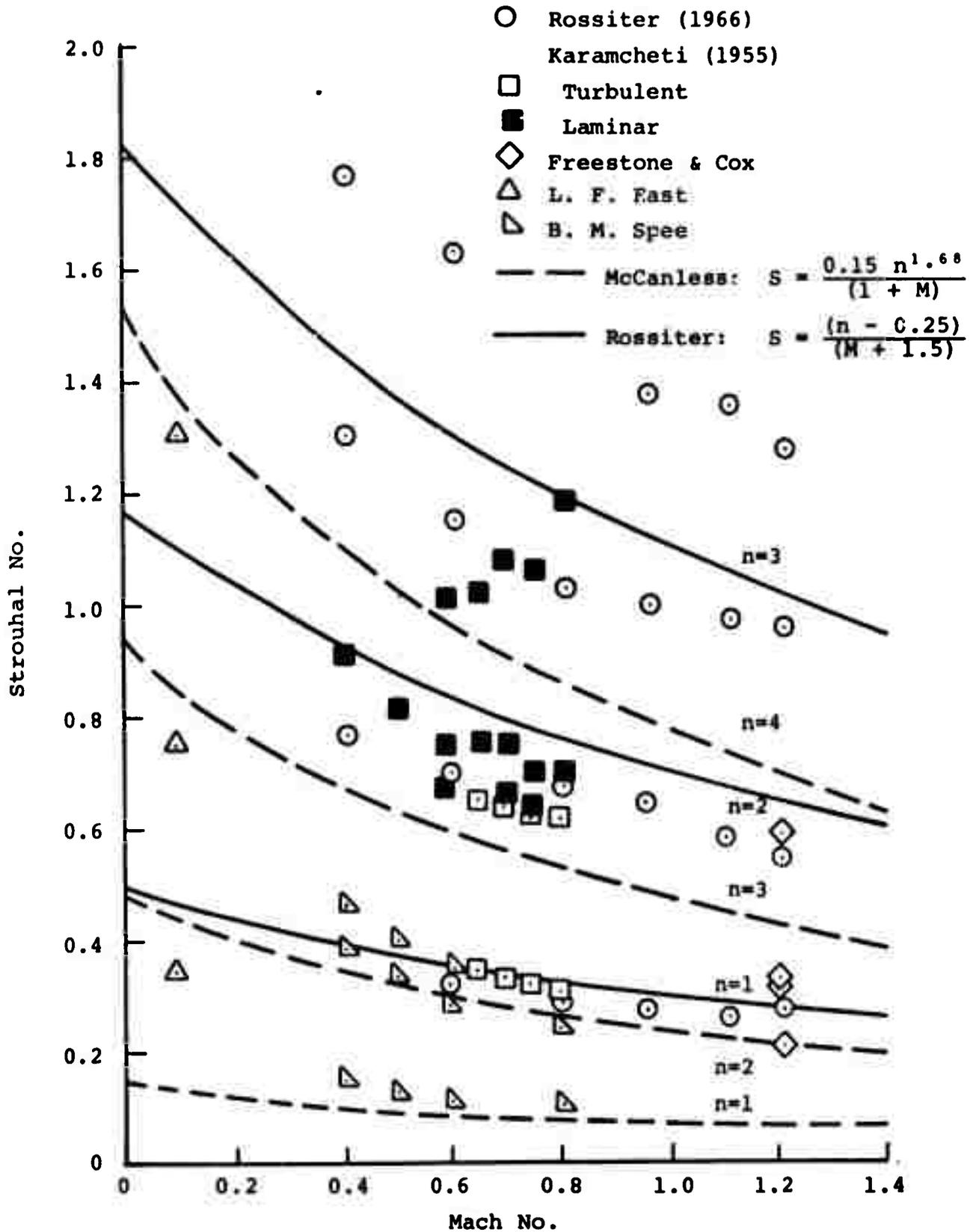


Figure 2-1.- Frequency of narrow band noise from single cavities.

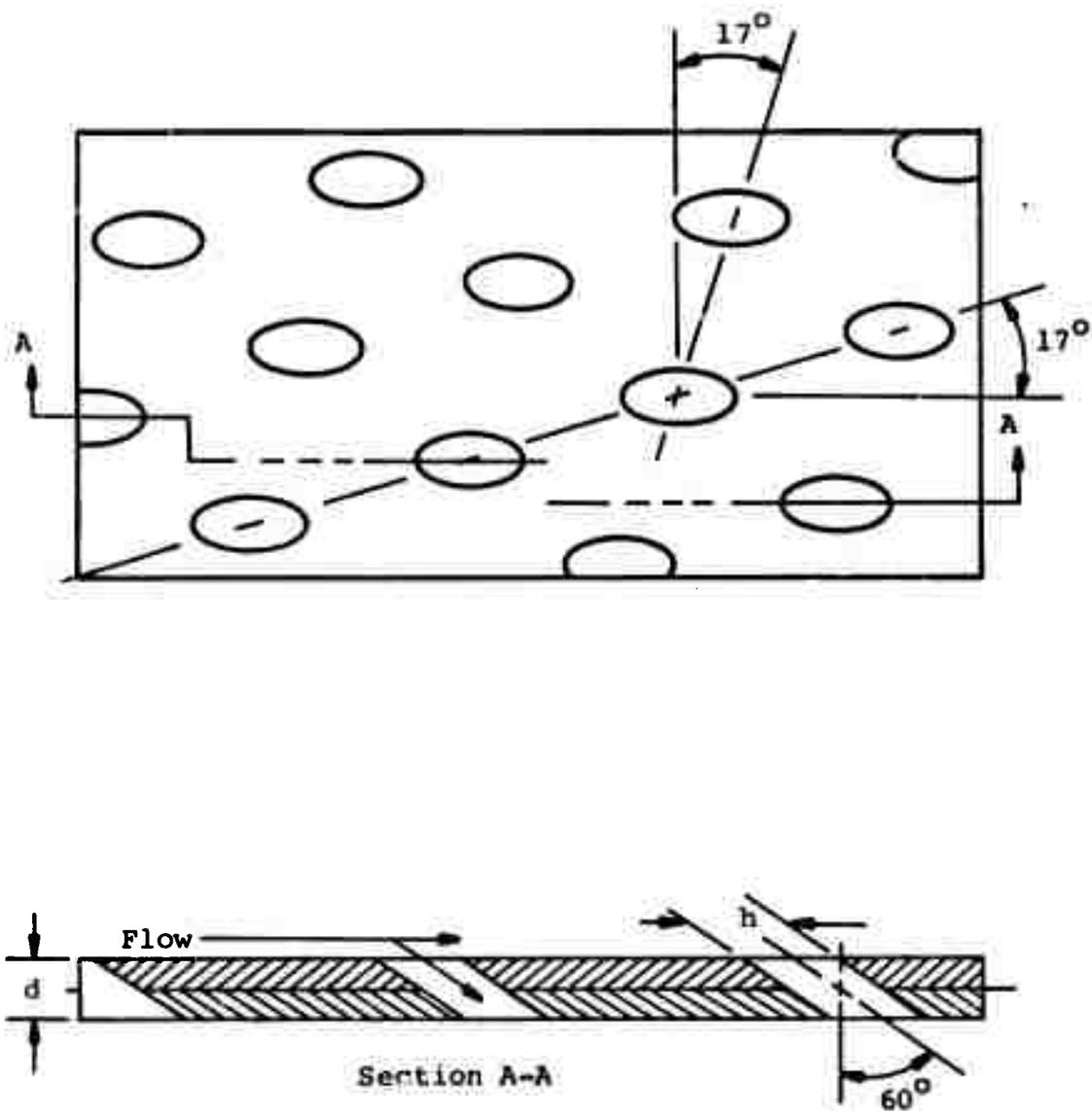


Figure 2-2.- Typical Transonic Wind Tunnel Perforated Wall

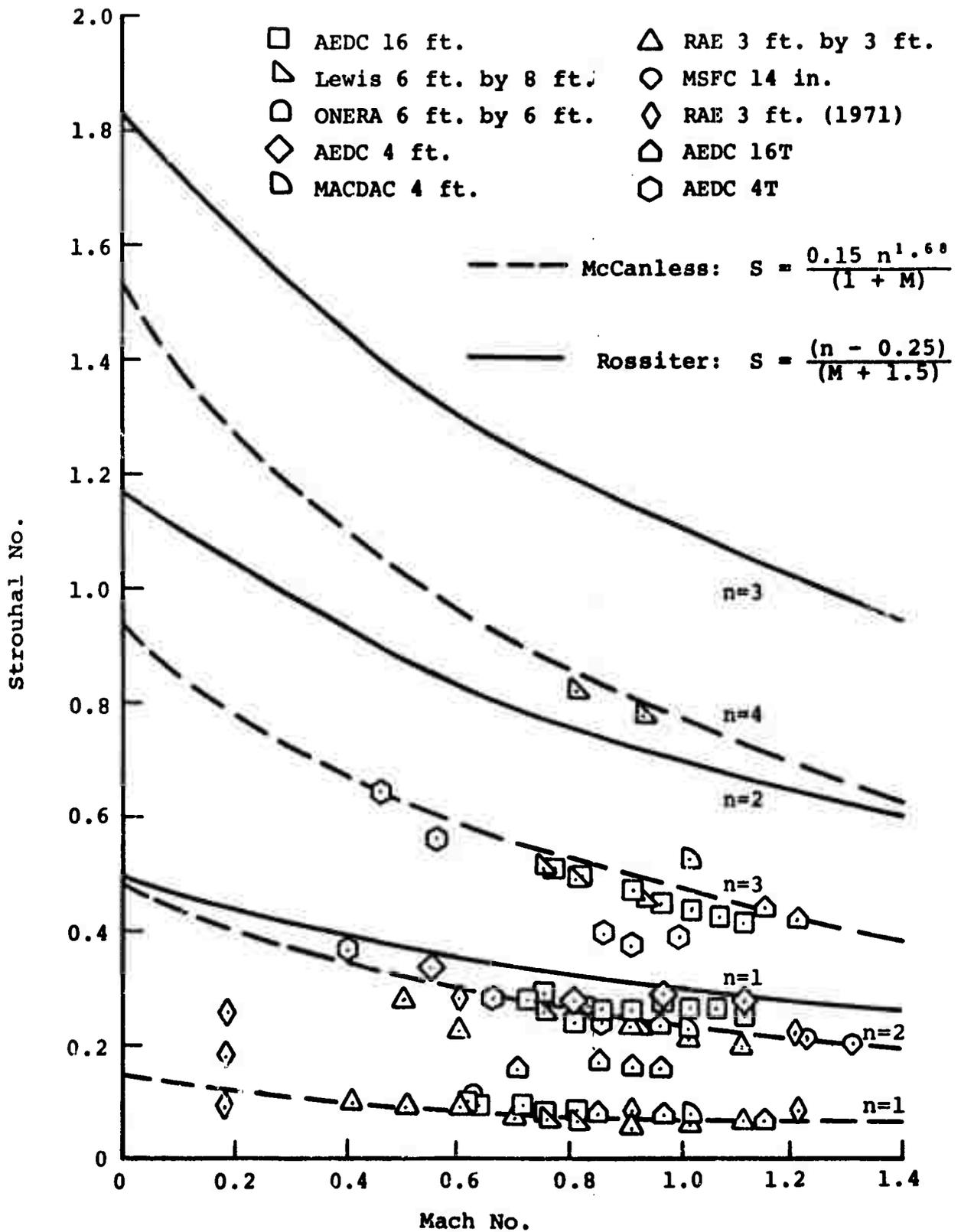


Figure 2-3.- Frequency of narrow band noise from transonic wind tunnels.

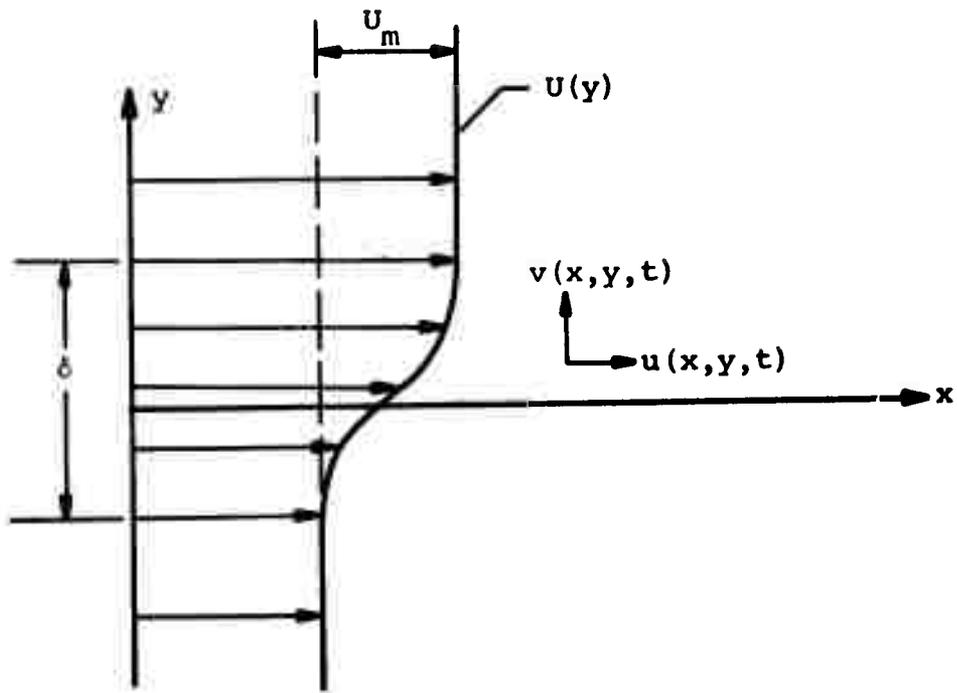
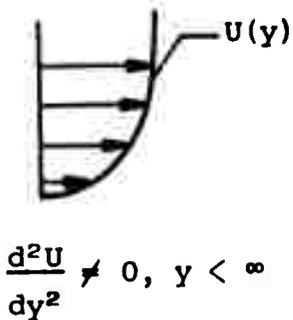
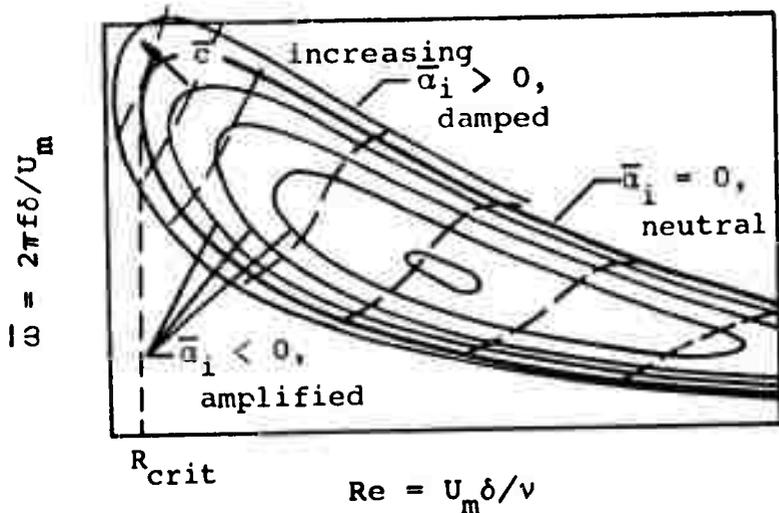


Figure 3-1.- A parallel flow field with two-dimensional time-dependent disturbances.

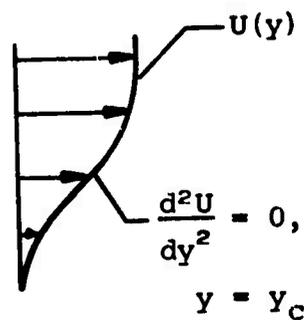
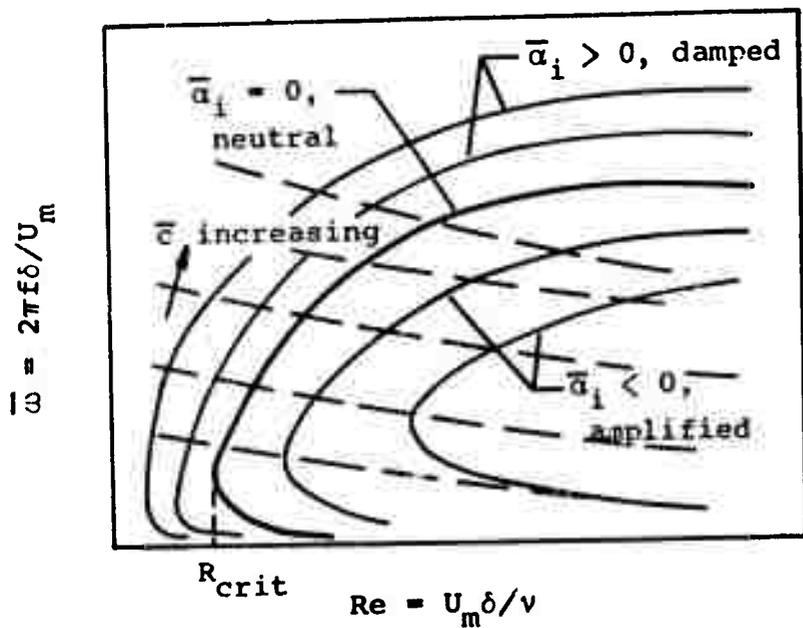
Stability Characteristics

Velocity Profiles



(a) Parallel flow velocity profiles without inflection point.

— $\bar{\alpha}_i = \alpha_i\delta = \text{constant}$
 - - $\bar{c} = \frac{c}{U_m} = \text{constant}$



(b) Parallel flow velocity profiles with an inflection point.

Figure 3-2.- Typical stability characteristics maps.

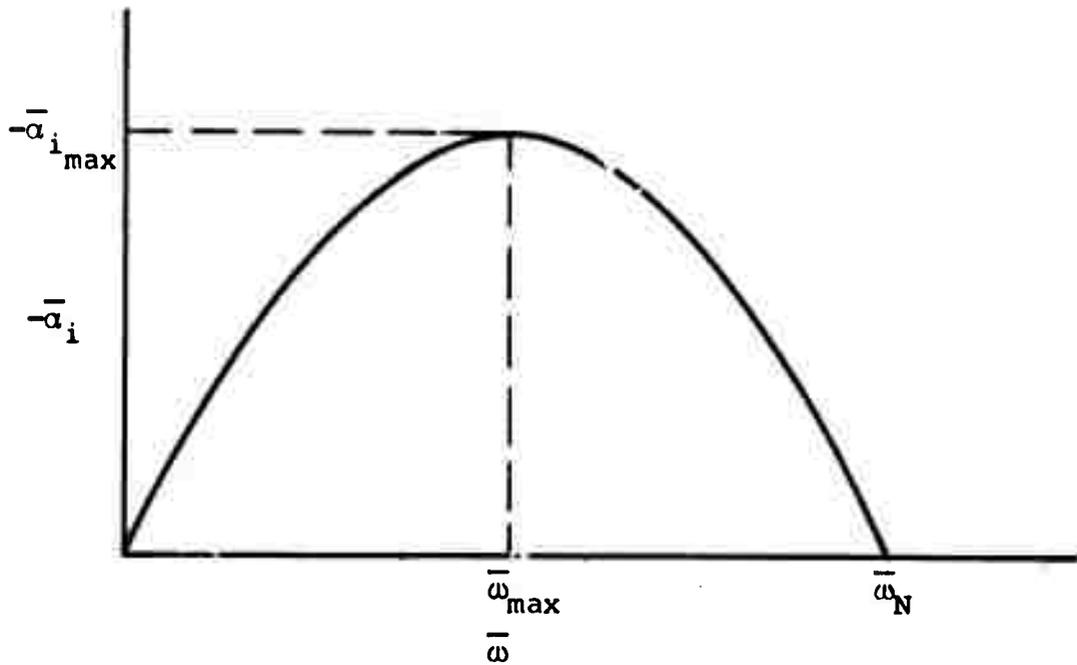


Figure 3-3.- Typical dependence of amplification factor on frequency (nondimensional) at a given Reynolds number.

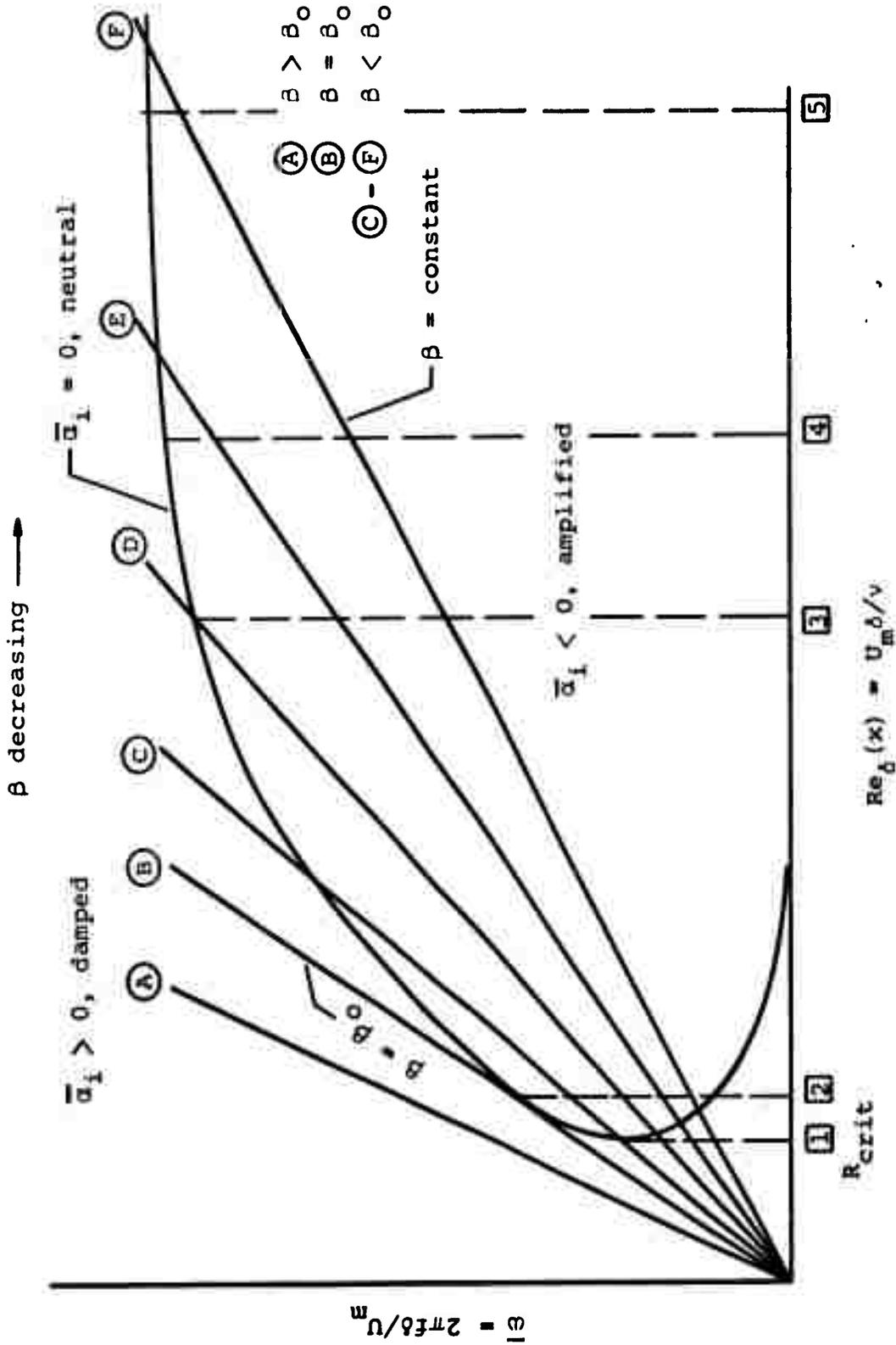


Figure 3-4.- Stability characteristics of a developing flow field showing the paths of disturbances in the characteristic's domain.

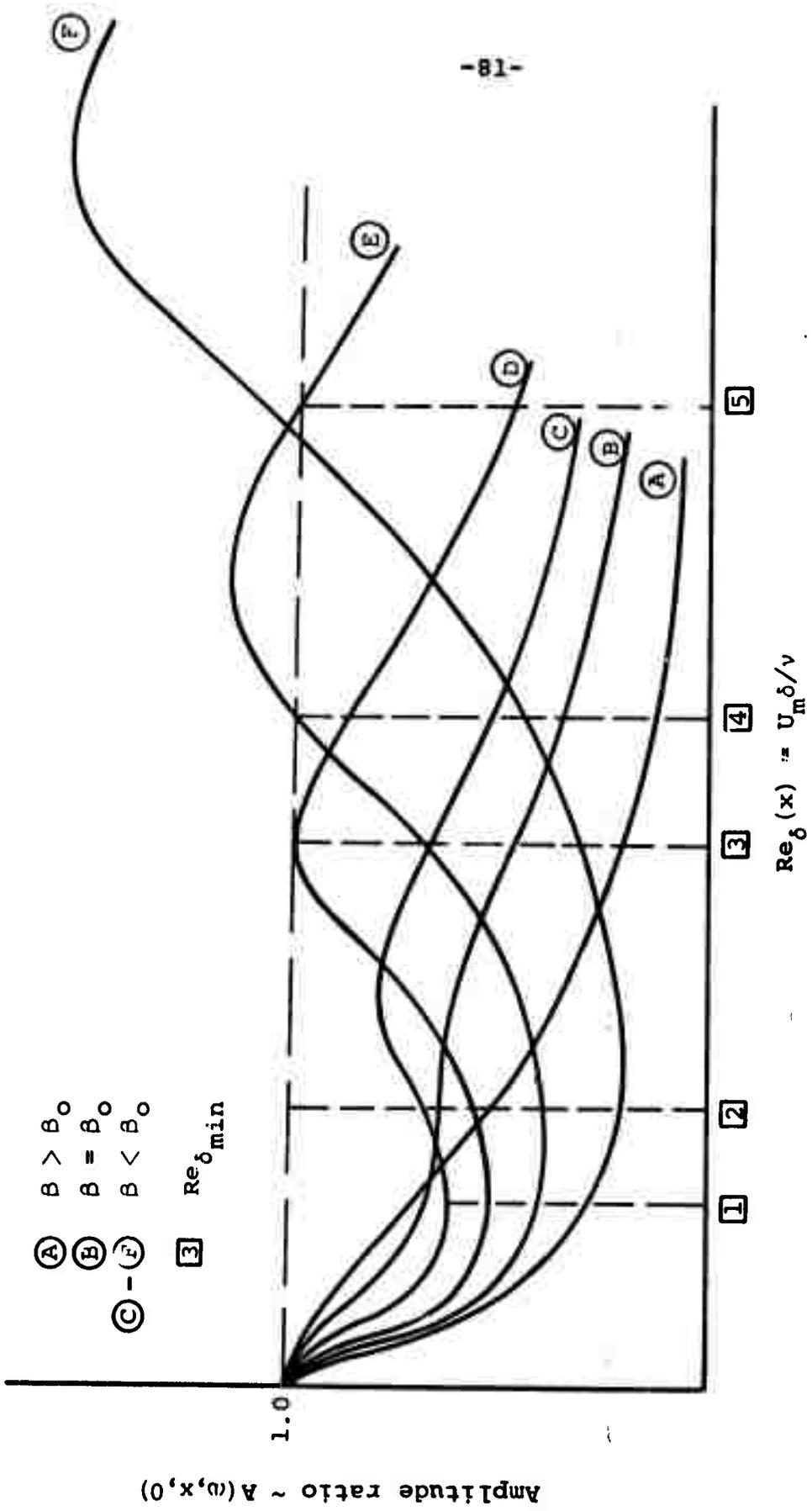


Figure 3-5.- Amplitude histories of disturbances in a developing flow field.

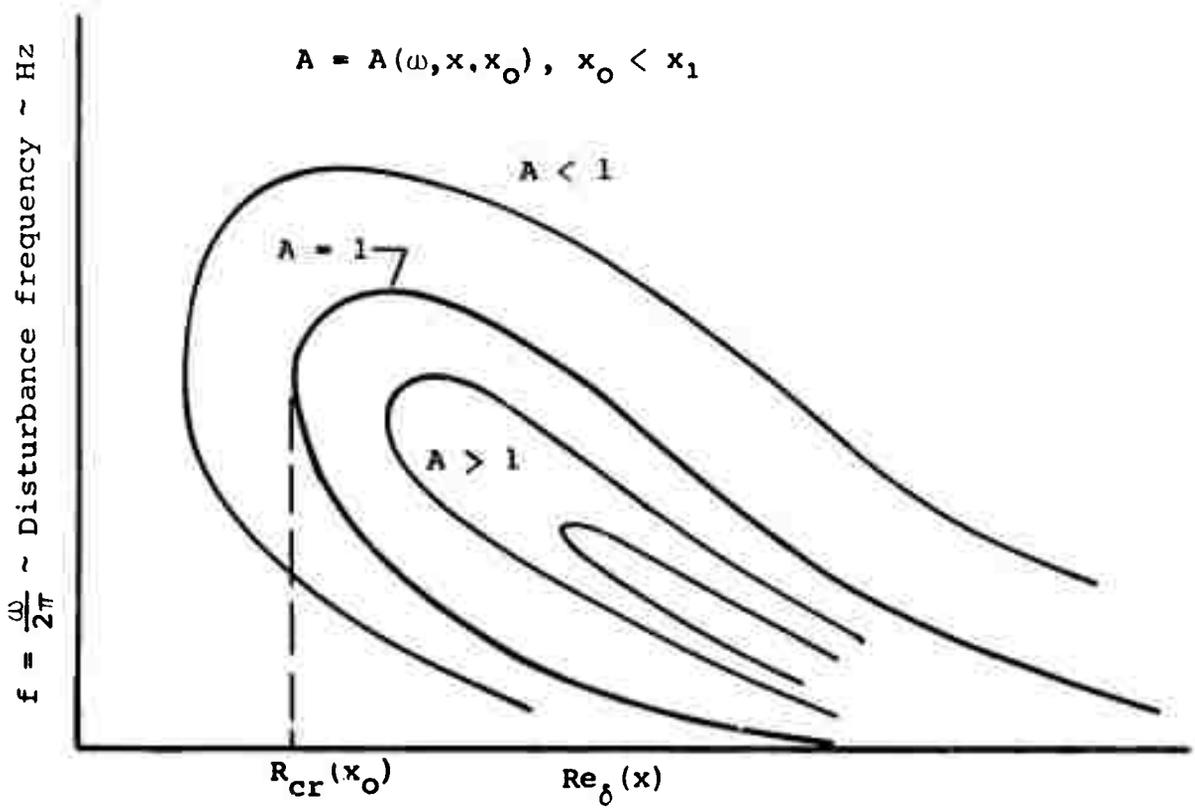


Figure 3-6. - Amplitude ratio map of disturbances in a non-parallel flow field.