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A Survey of Mathematical Techniques for Solid Waste Management

Air Force Weapons Laboratory

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A SURVEY OF MATHEMATICAL TECHNIQUES FOR SOLID WASTE MANAGEMENT

Dennis E. Lundquist
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13 ABSTRACT

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The major portion of monies expended for solid waste management by civilian authorities is for collection activities. Similarly, Air Force solid waste managers find that collection exhausts a major portion of their solid waste management budget. Therefore, improvement in the efficiency of Air Force solid waste collection operations could result in significant cost reductions. One means of improving collection efficiency is through the use of mathematical modeling and other analytic techniques. This research effort briefly summarizes some of the more significant efforts in quantifying solid waste collection and transportation systems. These techniques are described and evaluated for potential Air Force use. The intent is to educate Air Force solid waste managers to the availability of analytic techniques for their use and not to recommend any one in particular. The hope is that some Air Force solid waste managers will acknowledge their deficient systems and endeavor to apply analytic techniques to them. Furthermore, it is hoped that successful implementation will motivate other Air Force solid waste managers to investigate the utility of these techniques.

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FOREWORD

The research was performed under Program Element 63723F, Project 683M3S04.

Inclusive dates of research were January 1972 through July 1972. The report was submitted 12 December 1972 by the Air Force Weapons Laboratory Project Officer, Lt Dennis E. Lundquist (DEE).

This technical report has been reviewed and is approved.

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SECTION I

INTRODUCTION

Solid waste studies in civilian communities have shown that collection costs are often four to five times disposal costs (Ref. 1). There is little doubt that a similar situation presently exists on Air Force installations.

While private contractor pickup of Air Force solid waste is significant, the majority of Air Force solid waste is collected by Air Force personnel (Ref. 2). Therefore, a logical place for significant economic progress would be in developing more efficient solid waste collection practices for Air Force bases.

This survey will delineate a few of the many mathematical techniques and models developed for solid waste collection. The techniques presented are the state of the art in this subject and are the most pertinent to Air Force operations. Quantitative analysis toward an "optimum," however, is only a partial answer toward provision of a "best" system because of the limitations in attempting to abstract the real world into mathematical terms. Factors such as frequency of pickup, location of containers, type and number of containers, collection equipment, segregation of refuse, crew size, crew motivation, etc., are not always included in optimum analyses, and yet bear close examination because of their vital effect on total system efficiency.

Political and social constraints usually enter into the problem, but they are rarely quantified into a mathematical analysis or model. Therefore, the decision-making process involves many facets, both objective and subjective. Quantitative management techniques should be thought of as tools to aid the decision maker and not as a panacea to his problems.

Mathematical programming or operations research as some would have it, has been around for some time and has contributed quite significant cost reductions to properly analyzed operations. However, the use of mathematical programming to develop solid waste policies has not been widespread. But in limited application to solid waste decision making, this technique has shown excellent promise.

The objectives of this report are: (1) to inform Air Force base solid waste managers of mathematical programming as an aid in formulating more economic solid waste policies, (2) to discuss some of the programming techniques available and their possible Air Force applicability, and (3) to stimulate interest in this area and generate applications at Air Force bases whereby mathematical techniques are used to aid in optimizing base solid waste management systems.

Mathematical programming, for our purposes, can be defined as consisting of an objective function which describes what you want to achieve, and constraining relations that limit the range or size of the parameters to be maximized or minimized in the objective function. The procedural analysis for solving the mathematical problem usually involves repetition of an operation which is called an algorithm. The complexity of the objective function and the constraining relations determine the degree of difficulty for the algorithm in obtaining the optimal solution, i.e., the best solution possible. Heuristic algorithms are also found in the literature which use a set of rules that ensure a "good" solution is found although it may not be the best or "optimal" solution.

The literature has been reviewed and can be categorized into three areas: (1) location models; (2) simulation models; and (3) vehicle selection, scheduling, and routing models. These analyses will be discussed qualitatively with most of the quantitative relationships enumerated.

SECTION II

LOCATION MODELS

Three locational models will be discussed: (1) transfer facilities, (2) disposal sites, and (3) garaging facilities.

1. TRANSFER FACILITIES

The fundamental question lies in the desirability of having transfer facilities to transfer wastes to specialized vehicles for long-haul transportation. Marks and Liebman (Ref. 3) have looked at the problem in depth and present an algorithm that guarantees an optimal solution and can handle fairly large problems within a moderate computational time. Once the desirability of transfer facilities has been substantiated, questions such as number, location, capacity, and specific use arise. Essentially, these decisions may be looked upon as a trade-off between building of the facilities and the cost of transportation (Ref. 3).

The brief statement of the problem by Marks and Liebman is as follows.

There is a set, K, of sources of waste with an amount S_k generated at each source. Also, there is a set of sinks, J, for the waste, each with an upper and lower bound on demand of D_j^u and D_j^l. A set of possible facility sites I has been suggested as trans-shipment points between the sources and the sinks. Each proposed trans-shipment site has a fixed charge, F_i, a variable unit cost linearly associated with the amount shipped through the facility, V_i, and a capacity Q_i. Simply stated, the problem is to find which facilities should be built and which sources and sinks each facility serves so that the total cost of the operation is minimized (Ref. 3).

In mathematical form Marks and Liebman (Ref. 3) present the problem as follows.

$$\text{Minimize:} \quad \sum_{i=1}^{m} F_i Y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^{*} X_{ij}^{*} + \sum_{i=1}^{m} \sum_{k=1}^{p} C_{ki}^{**} X_{ki}^{**} \qquad (1a)$$

subject to the constraints:

$$\sum_{i=1}^{m} X_{ki}^{**} \geq S_k \qquad k=1, 2, \ldots, p \qquad (1b)$$

$$\sum_{j=1}^{n} X_{ij}^{**} = \sum_{k=1}^{p} X_{ki}^{**} \qquad i=1, 2, \ldots, m \qquad (1c)$$

$$\sum_{k=1}^{p} X_{ki}^{**} \leq Q_i Y_i \qquad i=1, 2, \ldots, m \qquad (1d)$$

$$D_j^u \geq \sum_{i=1}^{m} X_{ij}^{*} \geq D_j^l \qquad j=1, 2, \ldots, n \qquad (1e)$$

X_{ij}^{*}, X_{ki}^{**} are nonnegative integers (1f)

$Y_i = (0,1)$ (1g)

where

Y_i = 1 if the ith facility is built

Y_i = 0 otherwise

X_{ij}^{*} = flow of material from facility i to sink j

X_{ki}^{**} = flow of material from source k to intermediate point i

$C_{ij}^{*} = C_{ij} + r_j$ = unit cost associated with a transfer of material from facility i to sink j (dollars per unit)

C_{ij} = unit shipping cost from facility i to sink j (dollars per unit)

r_j = unit variable cost associated with using sink j (dollars per unit)

$C_{ki}^{**} = C_{ki}' + t_k + V_i$ = unit cost associated with transfer of material from source k to facility i (dollars per unit)

C_{ki}' = unit shipping cost from source k to facility i (dollars per unit)

t_k = unit variable cost associated with using source k (dollars per unit)

V_i = unit variable cost associated with using facility i (dollars per unit)

F_i = fixed charge for establishing facility i (dollars)

S_k = amount supplied at source k

D_j^u = upper bound on amount demanded at sink j

D_j^l = lower bound on amount demanded at sink j

Q_i = capacity of the ith facility

m = number of proposed facility sites

n = number of demand areas

p = number of supply points

Inequality (1b) requires that flow from the source cannot exceed the supply of material. Equation (1c) states that the flow entering the ith facility must be equal to the flow leaving it. Inequality (1d) expresses the fixed charge nature of facility i. If the ith facility does not exist, $Y_i = 0$ and no flow is allowed to pass through it. If $Y_i = 1$, the ith facility exists and flow up to Q_i may pass through it. Inequality (1e) maintains that the flow into the sink must be within its upper and lower bounds (Ref. 3).

Marks and Liebman discuss various problem solutions and present their algorithm in great detail. This algorithm guarantees an optimal solution and can handle fairly large problems within a moderate computational time.

Applying this technique to an Air Force installation might simplify the mathematical formulation. Proposed transfer facilities might not have an initial cost depending on the availability and suitability of any vacant buildings on base. Utilization of unused buildings would cause the initial cost of the ith facility to be zero, and thus simplify the objective function, equation (1a). In any case, Air Force applicability would be limited to those bases with large populations and/or long-haul distances to disposal sites.

2. DISPOSAL SITES

Helms and Clark (Ref. 4) present an interesting article using mathematical techniques to aid in determining which of various candidate disposal sites (including incineration) should be used and at what level of capacity. Buffalo,

New York, is used as an example with the objective of minimizing the total yearly haul plus disposal cost for providing service to 27 collection districts. The authors use an operations research technique known as the "fixed charge solution" in solving the problem. The authors present the mathematical formulation as follows.

$$\text{Minimize:} \quad \sum_{j=1}^{7} \delta_j f_j + \sum_{ij}\sum C_{ij} X_{ij} \tag{2a}$$

where i = 1, 2,..., 27 denotes the collection district and j = 1,..., 7 denotes the disposal facility. The amount of solid waste in tons going from collection district i to disposal facility j is X_{ij}. In equation (2a), f_j is the fixed cost for facility j and δ_j takes on only the values of zero and one. The coefficient C_{ij} is the variable cost associated with allocating X_{ij} tons of waste from district i to facility j. If Z_j is the tons processed at each facility, $\delta_j = 0$ if $Z_j = 0$ and $\delta_j = 1$ if $Z_j > 0$ (Ref. 4).

The objective function (2a) to be minimized is subject to the following set of constraints.

$$\sum_{i=1}^{27} \sum_{j=1}^{7} X_{ij} = T_i \tag{2b}$$

which represent the requirement that all refuse produced in a collection district be disposed of, and T_n, n=1,..., 27, is the annual tons of solid waste produced in each collection area (Ref. 4).

These constraining relations

$$\sum_{j=1}^{7} \sum_{i=1}^{27} X_{ij} - Z_j = 0 \tag{2c}$$

represent the upper limit of the number of tons capable of being processed at facility j (Ref. 4).

The mathematical solution envisions a system that saves about 7 percent over the next "best" hypothetical system. The obvious limitation of the approach is its inability to consider time variations of refuse production. However, even

with this limitation the technique's use can be a valuable aid for planning purposes in sorting out the various alternatives for disposal available. Future situations could be evaluated by having tonnage figures appropriately modified.

Discussion of the Helms and Clark article by Liebman points out that while the Walker algorithm is an excellent heuristic technique, it does not always find the optimal solution (Ref. 5). Liebman suggests that an exact algorithm developed by Marks (Ref. 6), and applied to solid waste facilities by Marks and Liebman (Ref. 7) might provide a better solution to the problem.

Air Force applicability would be at bases where various alternative disposal sites exist with their associated costs known. This tool can be a valuable aid in planning for Air Force base solid waste management if accurate waste production data can be obtained.

3. DECENTRALIZED GARAGE FACILITIES

In another paper by Clark and Helms (Ref. 1) the "fixed charge approach" is used in determining the most efficient location for garaging facilities for solid waste vehicles. Buffalo, New York, is used as the case study to determine whether employment of one, two, or three decentralized facilities should be used in place of, or in addition to, the present facility.

The objective of the analysis is to minimize the average daily cost for providing service to the 27 collection districts. The objective function is as follows.

$$\text{Minimize total cost} = \sum_{j=1}^{4} f_j + a_j z_j + \sum_{i=1}^{27} \sum_{j=1}^{4} C_{ij} X_{ij} \qquad (3a)$$

where i = number of collection districts; j = number of facilities; f_j = fixed cost for the facility; z_j = the level at which facility j is being used; C_{ij} = variable cost associated with allocating X_{ij} trucks to district i from facility j; X_{ij} = number of trucks operating from facility j to collection district i; and a_j = average cost per truck of operating facility j (Ref. 1).

The objective function is subject to the following set of constraints.

$$\sum_{i=1}^{27} \sum_{j=1}^{4} a_{ij} X_{ij} \geq T_i \qquad i=1,\ldots, 27 \qquad (3b)$$

where a_{ij} = number of truckloads of refuse collected in district i from facility j in one day; and T_n is the average number of truckloads generated daily in each district. Further, the constraints

$$\sum_{j=1}^{4} Z_j = 100 \tag{3c}$$

$$Z_1 \leq 100,\ Z_2 \leq 50,\ Z_3 \leq 50,\ Z_4 \leq 50$$

set the total fleet size and the capacity limitations on each facility (Ref. 1).

The authors use the Walker algorithm (Ref. 8) to solve the problem and forecast a 19-percent savings in costs by using two of the three decentralized garage facilities. While the authors note the Walker algorithm will not guarantee the optimal solution, they feel it is computationally efficient and yields the optimal solution enough of the time to be effectively used.

Helms and Clark also discuss the question of reduction in fleet size as a possible extension of the problem. Reformulation of the problem to include the above extension required only that equation (3c) (Ref. 1) be modified to

$$\sum_{j=1}^{4} Z_j \leq 100 \tag{3d}$$

Solving the revised formulation suggested a six-vehicle reduction in fleet size and an increase in the hypothetical cost savings over the next best system from 19 percent to 23 percent.

Discussion of this article by Heaney (Ref. 9) suggests an alteration of the mathematical formulation which may strengthen the validity of the problem solution.

Techniques of this kind could be especially useful on larger Air Force installations where vacant buildings could be used as potential sites for garaging facilities.

While this method fails to explicitly consider time variations, future situations could be examined by adjusting data. However, the situation on an Air Force installation should not be as critical as a municipality that is growing. Because of an easier forecast data base, bases with stable populations should be much easier to plan for future operations.

SECTION III

SIMULATION

Another useful application of mathematical techniques to solid waste management is modeling to simulate the collection process. The more accurate a representation the simulation provides of the collection system the more useful it becomes. Then, by using data from the existing system, simulation can be used to forecast the result of various parametric changes on system effectiveness. In this way a manager can examine changes in the collection system without costly field experimentation.

Bodner, Cassell, and Andros (Ref. 10) developed a model which simulates the operations of a refuse collection system for the purpose of designing and optimizing collection routes for individual vehicles which are responsible for servicing some defined collection area. The route determination through a given street grid is by having a simulated vehicle "randomly walking" through the network, making decisions at each intersection until all streets have been serviced. The authors give a flow chart symbolizing their procedure as shown in figure 1 (Ref. 10). The procedural analysis generates various feasible alternative solutions and picks the one with minimum mileage, but not necessarily the optimal route. It is possible by programming the procedure to simulate many weeks of route operations under variable refuse production enabling the engineer to examine the hypothetical operational characteristics of the chosen route before being placed in service.

The model was applied to collection routing in Potsdam, New York, for various sets of parameters. For each series of parameters, 100 routes were generated by the model. The "optimum" route chosen was the one that exhibited the minimum mileage (Ref. 10).

Once an optimum route was chosen, further investigation was undertaken to see what effect variations in refuse production had on route mileage. As expected, variant refuse production generally increased average route mileage, whereas invariant refuse production did not (Ref. 10).

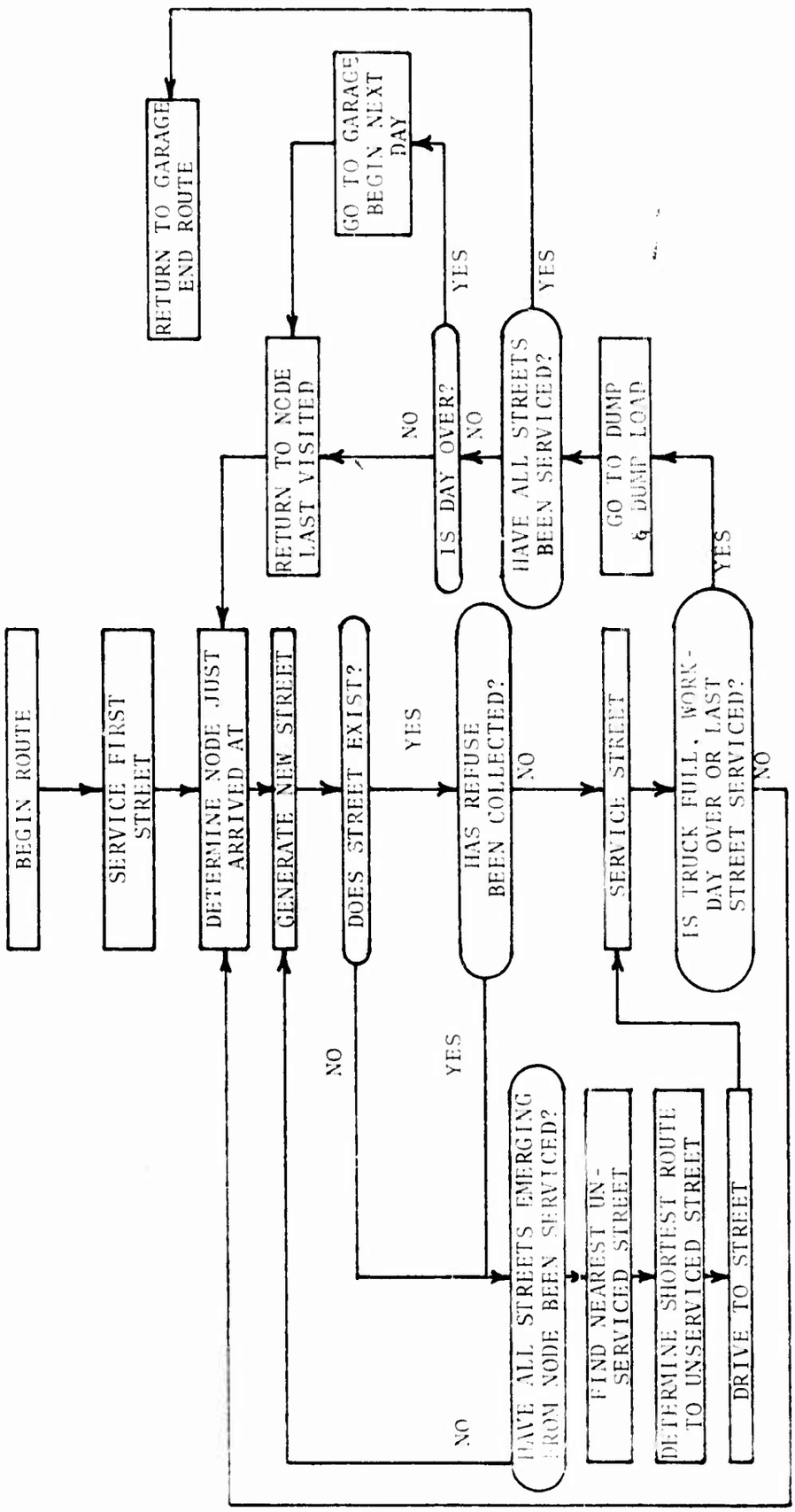

Figure 1. Flow of Control

Discussion by Marks and Liebman (Ref. 11) warns against use of the above method for large problems and the use of the term "optimal" for the solution. They give Stricker (Ref. 12) as a reference for an algorithm that guarantees the optimal solution.

Bodner, Cassell, and Andros (Ref. 13) qualify that while their method could not handle a city the size of Baltimore, Maryland, which they tried to simulate, it did handle sections of the city which required eight to ten vehicles to service.

Another article using simulation for investigation of solid waste collection systems is one by Truitt, Liebman, and Kruse (Ref. 14). They investigated system changes (measured as cost/ton of collected refuse) to alteration of various parameters in three different models.

Models 1 and 2 dealt with cost responses of a synthetic system to variations in season, haul distance, truck capacity, trash collection frequency, and collection density (Ref. 14).

Model 3 was designed to simulate the complete collection system under three different policies. Model 3A collected refuse and transported it directly to the disposal site. Model 3B is similar to 3A except it included a transfer station with sufficient loading docks to unload the trucks immediately. Model 3C is similar to 3B except there are only two docks at which to unload.

Model 3 was used for investigation of the solid waste system in Baltimore, Maryland, for the purpose of (1) proving the model, (2) evaluating system cost changes resulting from changing collection frequency from two to three pickups per week, (3) investigating the desirability of one or more transfer stations in the area, and (4) determining the effects of different operational policies at a transfer station (Ref. 14).

The model proved to be satisfactory in representing the real-life system in cost per ton of refuse collected, tons refuse collected per day, and number of collection trucks needed per day.

The model predicted an increase of $1 per ton of refuse collected when frequency of collection was increased from two to three times per week. A transfer station became economical when the haul distance became 8 or more miles,

and more than one transfer station was found to increase costs slightly. Finally, compaction equipment at the transfer station resulted in cost reductions for the simulated system when the haul distance was large (Ref. 14).

Thus, the model proved quite versatile in examining costs of the system proposals for the Baltimore, Maryland, study area.

The simulation of Quon, Tanaka, and Wersan (Ref. 15) evaluates operational characteristics of collection crews employed on a constant length work day. The model is evaluated under several different operating policies. One result was the preference of assessing system cost as dollars per service per week instead of dollars per ton of refuse collected. Again this model provides a basis for forecasting hypothetical service provided by various operational systems.

Simulation of Air Force solid waste collection systems could help forecast system cost under various operational policies. However, the utility of simulation modeling is not well known. Additional research needs to be done to determine if benefits from this technique warrant its application to Air Force solid waste management systems.

SECTION IV

VEHICLE SCHEDULING, SELECTION, AND ROUTING

1. VEHICLE SCHEDULING IN A GIVEN NETWORK

Marks and Liebman (Ref. 3) approach a flow of commodities (different types of solid waste) from sources (of refuse production) through intermediate points (transfer facilities) to sinks (disposal sites) in given networks. They discuss two phases of the problem, one with single commodity routing and the other multi-commodity routing. They propose to solve the multi-commodity problem by use of the "out-of-kilter" algorithm and present the procedural analysis. Their formulation of the multi-commodity truck assignment problem is as follows (Ref. 3).

$$\text{Minimize:} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} b_{ijk}\, x_{ijk} + \sum_{i=1}^{m}\sum_{a=1}^{f}\sum_{k=1}^{p} b^{*}_{iak}\, x^{*}_{iak} + \sum_{a=1}^{f}\sum_{j=1}^{n}\sum_{k=1}^{p} b^{**}_{ajk}\, x^{**}_{ajk} + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ji}\, x_{ji} \qquad (4a)$$

Subject to

$$\sum_{j=1}^{m} x_{ijk} + \sum_{a=1}^{f} x^{*}_{iak} \le S_{ik} \qquad \begin{matrix} i=1, 2, \ldots, m \\ k=1, 2, \ldots, p \end{matrix} \qquad (4b)$$

$$\sum_{j=1}^{n} x^{*}_{ajk} \le V_{ak} \qquad \begin{matrix} a=1, 2, \ldots, f \\ k=1, 2, \ldots, p \end{matrix} \qquad (4c)$$

$$D_{jk} \le \sum_{a=1}^{f} x^{*}_{ajk} + \sum_{i=1}^{m} x_{ijk} \qquad \begin{matrix} j=1, 2, \ldots, m \\ k=1, 2, \ldots, p \end{matrix} \qquad (4d)$$

$$\sum_{i=1}^{m} x^{**}_{iak} = \sum_{j=1}^{n} x^{*}_{ajk} \qquad \begin{matrix} a=1, 2, \ldots, f \\ k=1, 2, \ldots, p \end{matrix} \qquad (4e)$$

$$\sum_{k=1}^{p}\sum_{i=1}^{m} x_{ijk} + \sum_{k=1}^{p}\sum_{a=1}^{f} x^{*}_{ajk} = \sum_{i=1}^{m} x_{ji} \qquad j=1, 2,\ldots, n \quad (4f)$$

$$\sum_{k=1}^{p}\sum_{j=1}^{n} x_{ijk} + \sum_{k=1}^{p}\sum_{a=1}^{f} x^{**}_{iak} = \sum_{j=1}^{n} x_{ji} \qquad i=1, 2,\ldots, m \quad (4g)$$

$$x_{ijk} \le Q_{ijk} \qquad \begin{matrix} i=1, 2,\ldots, m \\ j=1, 2,\ldots, n \\ k=1, 2,\ldots, p \end{matrix} \quad (4h)$$

$$x^{**}_{iak} \le Q^{**}_{iak} \qquad \begin{matrix} i=1, 2,\ldots, m \\ a=1, 2,\ldots, f \\ k=1, 2,\ldots, p \end{matrix} \quad (4i)$$

$$x^{*}_{ajk} \le Q^{*}_{ajk} \qquad \begin{matrix} a=1, 2,\ldots, f \\ j=1, 2,\ldots, n \\ k=1, 2,\ldots, p \end{matrix} \quad (4j)$$

$$x_{ji} \le Q_{ji} \qquad \begin{matrix} j=1, 2,\ldots, n \\ i=1, 2,\ldots, m \end{matrix} \quad (4k)$$

x_{ijk}, x^{*}_{ajk}, x^{**}_{iak}, x_{ji} are nonnegative integers (4m)

where

a = index relating to intermediate nodes

i = index relating to supply points

j = index relating to demand points

k = index relating to commodities

x_{ijk} = number of truckloads of commodity k sent directly from source i to demand j

x^{**}_{iak} = number of truckloads of commodity k sent from source i to intermediate point a

x^{*}_{ajk} = number of truckloads of commodity k sent from intermediate point a to demand j

x_{ji} = number of empty trucks returned from demand j directly to source i

b_{ijk} = unit cost of supplying demand for commodity k at sink j directly from source i = $c_{ijk} + t_{ik} + r_{jk}$

b^{*}_{iak} = unit cost of shipping to a as an intermediate point for commodity k from source k = $c^{*}_{iak} + t_{ik}$

b^{**}_{ajk} = unit cost of using a as an intermediate point for supplying demand for k at j = $c^{**}_{ajk} + u_{ak} + r_{jk}$

c_{ijk} = cost per truckload of commodity k from i directly to j

c^{**}_{iak} = cost per truckload of commodity k from i to a

c^{*}_{ajk} = cost per truckload of commodity k from a to j

c_{ji} = cost per empty truck from j to i

t_{ik} = cost per truck load of shipping commodity k from source i

r_{jk} = cost per truckload of receiving commodity k at demand j

u_{ak} = cost per truckload of trans-shipping commodity k at intermediate point a

S_{ik} = supply in truckloads of commodity k at source i

D_{jk} = demand in truckloads for commodity k at demand j

Q_{ijk} = upper bound on flow of k from i to j

Q^{**}_{iak} = upper bound on flow of k from i to a

Q^{*}_{ajk} = upper bound on flow of k from a to j

Q_{ji} = upper bound on flow of k from j to i

V_{ak} = upper bound on trans-shipment of commodity k at intermediate point a

Constraining relations (4b), (4c), and (4d) express, respectively, the amount of each commodity shipped from i, trans-shipped through a, and received by j. Equations (4e), (4f), and (4g) express the constraint that the number of trucks leaving a point must be the same as the number entering it. The inequality relations (4h), (4i), (4j), and (4k) express the requirement that the flow between points can be no more than any bound. The requirements of nonnegativity and integers (4m) is satisfied by the solution of the problem as an out-of-the-kilter graph (Ref. 3).

Computational time on an IBM 7094 computer using randomly generated data for 15 sources, five intermediate nodes, two sinks and two commodities was less than 5 seconds (Ref. 3).

This technique would be applied to Air Force installations where transfer facilities exist, are presently being examined, or are feasible in the future. One conceivable case would be where the Air Force is using civilian disposal facilities but is collecting and transporting the waste with Air Force personnel. If the haul distance dictates transfer facilities to be economic, then analytic techniques to determine the flow would be needed to aid in the planning. This technique seems to be best suited for the larger installations with its immediate Air Force applicability not well known.

Marks and Liebman (Ref. 3) also approach the problem of routing of vehicles from given locations. Specifically, they approach the problem of routing of vehicles from transfer stations through a set of demand areas (collection areas). It is requisite that the vehicles do not have the capacity to service all the demand areas before returning to the transfer station. This problem is called the m-salesman traveling salesman problem and the authors give a detailed algorithm to solve a special case. The problem formulation is (Ref. 3)

$$\text{Minimize: } \sum_{i=1}^{N} \sum_{j=1}^{N} f_{ij}\, x_{ij} + \sum_{t=1}^{s} \sum_{c=1}^{N} \left(d_{ct}\, x^{*}_{ct} + d^{*}_{tc}\, x^{**}_{tc} \right) \qquad (5a)$$

subject to

$$\sum_{c=1}^{N} x^{*}_{ct} = \sum_{c=1}^{N} x^{**}_{tc} \qquad t=1, 2, \ldots, s \qquad (5b)$$

$$\sum_{\substack{i=1 \\ i\neq j}}^{N} x_{ij} = \sum_{\substack{i=1 \\ i\neq j}}^{N} x_{ji} \qquad j=1, 2,\ldots, N \qquad (5c)$$

$$\sum_{\substack{i=1 \\ i\neq j}}^{N} x_{ij} = 1 \qquad j=1, 2,\ldots, N \qquad (5d)$$

$$\sum_{c=1}^{N} x^{**}_{tc} = m \qquad c=1, 2,\ldots, s \qquad (5e)$$

x^{*}_{ct}, x^{**}_{tc}, x_{ij} are nonnegative integers (5f)

where

x_{ij} = the number of salesmen who travel from city i to city j

x^{*}_{ct} = the number of salesmen who travel from city c to terminal t

x^{**}_{tc} = the number of salesmen who travel from terminal t to city c

f_{ij} = distance from city i to city j ($d_{ii} = \infty$)

d_{ct} = distance from city c to terminal t

d^{*}_{tc} = distance from terminal t to city c

m = number of salesmen dispatched from each terminal

N = number of cities

s = number of terminals

In the context of solid waste collection the salesmen are the solid waste collection vehicles, the terminals are the transfer stations, and the cities are small collection areas, each of which generates the same amount of solid waste in each time period (Ref. 3).

The objective function (equation (5a)) minimizes the distance traveled by the salesmen (trucks) while equations (5b) and (5c) express the requirement that the number of salesmen (trucks) entering a city (collection area) or

terminal (transfer station) must equal the number leaving it. Equation (5d) requires that exactly one salesman (truck) must visit each city (collection area) and (5e) requires that m salesmen (trucks) must visit each terminal (transfer station) (Ref. 3).

Computational time using randomly generated data for two vehicles, two transfer stations and 12 collection areas was about 1/2 minute.

2. FLEET SELECTION FOR SOLID WASTE COLLECTION SYSTEMS

Clark and Helms (Ref. 16) present a linear programming formulation for replacement of solid waste collection vehicles with the most cost-effective new ones. The problem was to minimize the average daily cost associated with 16, 20, and 25 cubic yard replacement vehicles, the original fleet being composed entirely of 16 yarders. The mathematical model the authors developed was

$$\text{Minimize: } \sum_k d_k t_k + \sum_i \sum_k C_k X_{ik} \tag{6a}$$

Subject to

$$\sum_k a_{ik} X_{ik} \geq T_i \qquad i=1,\ldots, I \tag{6b}$$

$$\sum_i X_{i_1} = t_1 + W_1 \tag{6c}$$

$$\sum_i X_{ik} = t_k \quad (k=2, 3) \tag{6d}$$

$$W_1 = C_1 \tag{6e}$$

where

d_k = average daily crew and amortization cost of truck type k

t_k = number of each type replacement

C_k = average daily operating cost of truck type k

X_{ik} = number of vehicles of type k assigned to collection district i

a_{ik} = average number of residences that can be serviced daily by truck type k in collection district i

T_i = average number of residences to be serviced daily in collection district i

X_{i_1} = number of 16-cubic-yard vehicles assigned to district i

T_1 = number of 16-cubic-yard trucks to be added to fleet

W_1 = number of 16-cubic-yard vehicles that will be retained in the fleet

The objective function (6a) minimizes the average daily cost of the existing fleet as well as the average daily costs of the replacement alternatives. Inequality (6b) requires that enough trucks are assigned to a collection district to pick up the solid waste generated there daily. Equation (6c) specifies that the total number of trucks serving a collection district i be equal to the number of 16-cubic-yard trucks not replaced and the number of new 16-cubic-yard trucks that will be added to the fleet. In equation (6d),

$$\sum X_{ik}$$

represents all of the 20-yard or 25-yard replacement vehicles, or both, assigned to the collection districts, and t_k = the number of replacement vehicles of both types 2 and 3 that will be purchased. Finally, equation (6e) gives the number of trucks in the existing fleet that will not be replaced (Ref. 16).

The linear programming solution resulted in replacing some of the 16-yard trucks with 25-yard vehicles and elimination of other 16-yard trucks altogether. No 20-yard trucks were programmed as replacement vehicles. The solution forecast a daily savings of $270.00, which amounted to 14 percent of the total daily cost (Ref. 16).

The most obvious Air Force applicability would be at bases with large vehicle fleets. However, even though associated economics would be greater with large vehicle fleets, smaller fleets may also benefit. This technique needs to be applied and results implemented to determine its relative value as a cost-reduction technique.

3. ROUTING FOR PUBLIC SERVICE VEHICLES

Marks and Stricker (Ref. 17) approach the problem of finding the shortest route for a vehicle of insufficient capacity to service an entire collection area

in one trip. Stricker develops a heuristic decomposition algorithm for this problem and uses it to determine a "good" route through a section of Cambridge, Massachusetts. The algorithm was executed by hand in less than 3 hours and was judged by the author to be "optimal" or very close to optimal. It seems that the algorithm will continue to be feasibly executed by hand because computer technology does not enable pattern recognition.

The solution found represented a saving of 46 percent of unnecessarily duplicated streets and 13 percent of total distance traveled. To the city of Cambridge, which spends $1,500,000.00 per year on solid waste collection and disposal, any significant increase in efficiency results in considerable cash savings (Ref. 17).

This procedural analysis seems to be suited to almost any Air Force base and involves little data gathering and computational time. Further, computational results provide an almost immediate answer to whether or not collection routing can be improved.

SECTION V

CONCLUSION

The foregoing discussion has shown that considerable interest exists to quantify solid waste operations to improve management decisions in this area. Hopefully, interest will be generated in the form of Air Force research to quantify solid waste operations to achieve a more efficient economic operation. However, the question of applying the foregoing techniques to Air Force solid waste operations is unknown. Virtually no pertinent data exist and, therefore, most of the techniques could not be applied at present. Even with necessary data, improvements in Air Force solid waste operations could not be accurately forecase due to the limited knowledge gained in civilian use.

Generally, Stricker's routing technique mentioned in the survey offers the best initial opportunity for applying analytic techniques to Air Force solid waste operations successfully. This technique requires the least amount of data accumulation, and thus, potentially should offer the greatest benefit for least initial cost.

Simulation techniques may aid in optimizing present solid waste operational procedures and can aid in future planning by using forecasted data. This technique's usefulness to the Air Force is unknown.

The other mathematical techniques discussed range in complexity and some might have to be amended before successful use by the Air Force. Again, the benefits derived in these areas are unknown.

The hazy, uncertain picture presented above is by no means an indictment against using analytic techniques as an aid in Air Force solid waste management. Indeed, mathematical techniques have been applied very successfully to many operations with resulting cost reduction and should prove amenable to Air Force operations. It is anticipated that the best course of action at this time would be to select one or more of these techniques and fully investigate their potential application for Air Force use. The success of one of these techniques, even if only a small cost reduction occurs, should prove to offer substantial cost reductions Air Force wide.

In conclusion, the Environics Branch at the Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico, has completed a state-of-the-art survey in this area and is presently evaluating the utility of applying these techniques to Air Force base solid waste systems.

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