AD-758 059

PICOSECOND LASER PULSES

William H. Glenn

United Aircraft Research Laboratories

Prepared for:

Advanced Research Projects Agency Office of Naval Research

29 March 1973

DISTRIBUTED BY:

National Technical Information Service U. S. DEPARTMENT OF COMMERCE 5285 Port Reyal Road, Springfield Va. 22151 Report Number: M-920479-39 Semi-Annual Techical Report for the period 1 August 1972 to 28 February 1973

PICOSECOND LASER PULSES

ARPA Order No. Program Code: Contractor: Effective Date of Contract: Contract Expiration Date: Amount of Contract: Contract Number: Principal Investigator Scientific Officer: Short Title: Reported By:

58059

9 most

> 1806 AMEND 6/07-18-72 3E90 United Aircraft Research Laboratories 1 August 1966 31 August 1973 \$533,133.00 NOOO14-66-C-0344 Dr. William H. Glenn (203) 565-5411 Dr. Robert E. Behringer Picosecond Laser Pulses W. H. Glenn

The views and conclusions contained in this document are those of the author and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the Advanced Research Projects Agency or the U. S. Government.

> Sponsored By Advanced Research Projects Agency ARPA Order No. 1806

DISTRIBUTION STATEMENT A

Approved for public release; Distribution Unlimited Reproduced by NATIONAL TECHNICAL INFORMATION SERVICE U S Department of Commerce Springfield VA 22151



Remains classifiers at the	DOCUMENT CONTRO	otation must be an	U Iterad when the o	varall raport (a classified)	
(Security classification of titls, body of shatract and indexing and CRIGINATING ACTIVITY (Corporate author)		28. REPORT SECURITY CLASSIFICATI		SECURITY CLASSIFICATION	
United Aircraft Research Laboratories			Unclassified		
Silver Lane/ East Hart	ford, Connecticut	eticut 06108		25. GROUP	
EPORT TITLE PICOSECOND LASER PULSE	NS				
DESCRIPTIVE NOTES (Type of repa					
Technical, Semi-Annual	L 1 August 1972 - 2	8 February	1973		
AUTHOR(5) (First name, middle initi					
Glenn, W. H.					
REPORT DATE		78. TOTAL NO. 43	OF PAGES	76. NO. OF REFS	
March 29, 1973			R'S REPORT NU		
N00014-66-00344		M-9204			
PROJECT NO.					
				athes numbers that may be and	
		9b. OTHER REPORT ND(5) (Any other numbers that mey be essinities report)			
1.					
DISTRIBUTION STATEMENT					
SUPPLEMENTARY NOTES		12 SPONSORIN	IG MILITARY AC	TIVITY	
1. SUPPLEMENTARY NOTES		12. SPONSORIN	NG MILITARY AC	TIVITY	
This report discuss resolution imaging the successful demo	es the application of radar systems. The nstration of a labor novel ultrafast dat	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The nstration of a labor	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The nstration of a labor	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The nstration of a labor	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The nstration of a labor	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The nstration of a labor	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The nstration of a labor	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The nstration of a labor	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The nstration of a labor	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The nstration of a labor novel ultrafast dat	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The nstration of a labor	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The nstration of a labor novel ultrafast dat	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	
This report discuss resolution imaging the successful demo	radar systems. The nstration of a labor novel ultrafast dat	of ultrafas principal : ratory scal	t laser pu results re e range Do	lses to high ported include . ppler radar,	

United Aircraft Research Laboratories Semi-Annual Report M-920479-39 For The Period 1 Aug. 1972 - 28 Feb. 1973

TABLE OF CONTENTS

I.	TECHNICAL REPORT SUMMARY	Page
II.	COHERENT IMAGING RADARS	
	Discussion of Principles Preliminary Imaging Experiment	II-l II-4
REFERENCI	S	
FIGURES		·
III.	ADDITIONAL TECHNIQUES APPLICABLE TO IMAGING	
	Frequency Domain Sampling Fabry Perot Modulator	III-l III-7

REFERENCES

FIGURES

DISTRIBUTION LIST

T

Section I

Technical Report Summary

This report discusses several aspects of the application of ultrashort optical pulses to high resolution imaging optical radar. In the work reported herein, attention has been concentrated on the extension of the principles involved in microwave synthetic aperture radar (SAR) to the optical region. Significant acco plishments during this period include the successful demonstration of a laboratory scale synthetic aperture optical radar with two dimensional resolution, and the development of a novel scheme for ultrafast optical data processing involving frequency domain sampling.

SAR systems make use of range-Doppler processing to achieve twodimensional resolution of targets in situations where the target cannot be resolved by conventional means, i.e., where the target is smaller than the diffraction limited resolution of the transmitting and receiving aperture. SAR processing can be used whenever there is relative motion of the radar and the target. This motion can be due to motion of the radar as in airborne side looking radars, or to motion of the target as in radar astronomy of planetary surfaces where the motion is due to the planetary rotation. Most of the discussions here will be concerned with rotating targets. The resolution of a SAR system in the down range direction is determined by the reciprocal of the bandwidth of the transmitted signal. In the case of a rotating target, the cross range resolution is determined by the wavelength of the transmitted radiation and the coherent processing time.

A simple and useful form of transmitted signal for a synthetic aperture radar system consists of a periodic train of coherent short pulses. The signal processing involved in treating the received data involves the sampling of different time segments of the received signal to give the range resolution elements, and a spectral analysis of the received signal from each range element to determine the distribution of Doppler shifts within each range resolution element. More elaborate forms of the transmitted signal may be useful in some applications. For any form of signal, however, the range resolution will be determined by the reciprocal of the signal bandwidth and the Doppler resolution by the reciprocal of the total signal duration (assuming that can all be coherently processed).

The choice of a laser for a practical synthetic aperture optical radar would be governed by many considerations including the available power, the range and Doppler resolution desired, the nature of the transmission medium, and the coherence and stability of the source Under the present contract, a laboratory scale demonstration of a synthetic aperture optical radar has been set up; it has successfully demonstrated two dimensional resolution of

I-1

a laboratory target and has produced a range-Doppler image of the target. In this experiment a continuously mode-locked Nd-YAG laser was used. This laser produced a periodic train of \sim 100 psec duration pulses and allowed a range resolution of 1-2 cm Further experiments are planned to investigate the ultimate resolution capabilities of this system.

In the laboratory scale synthetic aperture radar, the required time sampling was accomplished by heterodyning the received signal against a local oscillator that consisted of a time-shifted and frequency offset version of the transmitted pulse train. The distribution in range was determined by changing the relative arrival times of the signal pulses and the local oscillator pulses at the photo surface of the detector. In a more efficient system, parallel detectors, each with its appropriately delayed local oscillator, could be used to sample the range elements simultaneously rather than sequentially as was done in the demonstration. This would be a satisfactory system to obtain an image but it does not make optimum use of the received signal energy. An inefficiency results from the way that sampling is carried out. Each of the detectors receives a fraction of the total received signal, together with the local oscillator pulse. Only that portion of the received signal that is coincident in time with the local oscillator pulse is actually used. The remaining portion is absorbed in the detector (which is necessarily absorptive if it is to have a high quantum efficiency), and is wasted. To achieve N range resolution elements in such a system the received signal would have to be divided into N channels with consequent reduction in the signal level of each channel. The problem could be eliminated if a very fast and lossless optical modulator or beam deflector were available. A fast beam deflector, for example, could divert different time segments of the received signal to different detectors and no inefficiency would result. Such modulators, however, are beyond the present state-of-the-art. Lossless sampling can in principle be achieved by using nonlinear optical effects such as sum frequency generation but such schemes are impractical at low signal levels. A considerable amount of attention was paid to this problem during the present contract period, and a scheme has been devised to overcome it. The solution to the problem lies in the observation that while it is impossible in principle to sample a signal in time and to return the remainder for further processing using a linear electromagnetic device, it is easy to sample a portion of the frequency spectrum of the signal and to return the rest without loss. An interference filter or a Fabry-Perot etalon for example does precisely this. This observation leads to a scheme that should be useful not only for the processing of synthetic aperture radar data but for other ultrafast optical data processing problems as well. In the simplest form of the system the received signal, having a large bandwidth product, and a local oscillator signal consisting of a short duration pulse are combined with parallel wavefronts on a beam splitter. The composite signal is then analyzed by a bank of narrow band filters. The output of each filter is square law detected and integrated over a time equal to the total duration of the received signal. A number of parallel cutputs are

thus obtained. Under the proper conditions, it may be shown that these outputs constitute a sampled version of the Fourier Transform of the original received signal. If a sufficient number of samples are taken, these completely characterize the received signal, and the original amplitude and phase of signal as a function of time can be reconstructed. The time resolution of such a system is limited only by the duration of the local oscillator pulse, assuming the proper choice of filters. This scheme is quite general and can be extended to cases where the local oscillator signal is more complicated than a simple short pulse. The scheme discussed at greater length in the body of this report. It is expected that a substantial portion of the future work on this contract would involve further analysis of this system, investigation of means for experimentally implementing the system, experimental demonstration and application of the system to a laboratory synthetic aperture optical radar.

As mentioned above, the range resolution of an SAR system is determined by the reciprocal of the bandwidth of the signal (or the pulse duration for a train of transform limited pulses). In 1963, E. I. Gorden and J. D. Rigden proposed and analyzed a modulator consisting of a Fabry-Perot interferometer with an internal phase modulator that is driven at a frequency equal to the c/2L axial mode spacing frequency. Such a modulator can act as an extremely fast, repetitive optical shutter. In effect, the very narrow passband of the Fabry-Perot is swept back and forth at the modulation frequency. If a cw signal is incident upon the interferometer a train of very short duration pulses will be transmitted. The seemingly contradictory result that the effective transmission can be varied in a time short compared to the energy storage time of the interferometer can be readily confirmed by a simple analysis. It is a result of the periodicity of the modulating signal. Recently T. Kobayashi, T. Sueta, Y. Cho and Y. Matsuo used such an interferometer as the output reflector for a helium neon laser. The modulator was driven at a frequency of 1.34 GHz and produced a train of pulses of 21 psec duration and at a rate of 2.68×10^9 pps. This is rather remarkable since the pulse duration is more than an order of magnitude shorter than the reciprocal of the available laser line width. It is also possible, by adjusting the modulator parameters, to achieve optimum coupling out of the laser. Such a technique would be ideal to generate a signal for a synthetic aperture radar. This technique appears capable of generating pulses that are much shorter than can be achieved by modelocking, and since the resolution of an SAR improves with decreasing pulse duration, such pulses would be very useful A modulator has been designed and fabricated for use with a YAG laser and initial tests are planned for the near future. If this type of modulator is successful, it will be used to demonstrate improved range resoulution.

SECTION II

COHERENT IMAGING PROBLEMS

Discussion of Principles

The section briefly reviews some of the basic principles involved in a coherent imaging radar (Refs. II-1 and 2).

In a conventional radar imaging system, the resolution is determined by the diffraction limit of the system, which, with an aperture D and a wavelength λ , is,

$$\Delta \theta = \lambda / D$$

So that the resolution at a distance R is

 $\Delta X = \lambda R/D$

The maximum size of the aperture is determined by scintillations and angle of arrival fluctuations in the visible and infrared regions of the spectrum and in addition, in the microwave region, by fabrication problems. In the visible region, the largest useable aperture is of the order of 10 cm. At 10.6 μ it is approximately one meter. (The coherence area actually increases as $\lambda^{6/5}$ and depends on the state of turbulence of the atmosphere. The value used here are quite optimistic. See, for example, Ref. II-3, pp 128-144.) Microwave apertures of tens to hundreds of meters or larger have been successfully operated. In the visible and IR regions then, the minimum beam width is approximately 10⁻⁵ radians. Obtainable

R (km)	$\Delta X \text{ (meters)}$			
1	.01			
10	.1			
100	1			
200	2			
1000	10			
3×10^{4}	300			
5 x 10 ⁵	5×10^3 (moon)			

Setting aside considerations of detectability for the moment, obtaining an image of an object of a few meters in size at a distance of 100-500 km is on the far edge of the capability of a conventional imaging system. Simple range gating

United Aircraft

using short pulses of duration τ can provide down-range resolution of the order of $\tau c/2$, independent of range. Nanosecond pulses can thus provide resolution of the order of 15 cm and shorter pulses correspondingly better resolution. If periodic trains of pulses can be processed coherently, then comparable or better cross range resolution on moving targets can be obtained from Doppler information.

Coherent synthetic aperture techniques can be used whenever there is relative motion between the radar and the target. Two cases are usually considered, the rotating body range-doppler imaging used in radar astronomy and the side-looking radar used for terrain mapping. Fig. II-la shows the geometry for a rotating sphere and Fig. II-lb for a rotating cylinder. Range gating provides resolution in the down range direction, and Doppler processing provides the resolution in the cross range direction. In the simple case of the rotating sphere, the contours of constant velocity are parallel to the axis of rotation as shown and together with the contours of constant range, provide a coordinate system on the target. The doppler shift is given by

$$\omega_d = 2 \frac{\omega v}{c} = 2 \frac{\omega \Omega x}{c}$$

where Ω is the rotation rate of the target, and x is distance from the axis of rotation.

The resolution in x will depend on how well we can resolve the doppler frequency, i.e.,

$$\Delta \mathbf{x} = \frac{\mathbf{c}}{2\omega\Omega} \quad \Delta \omega_{\mathrm{dr}}$$

where $\Delta \omega_{dr}$ is the doppler frequency resolution limit. If we observe the target for a time T, then $\Delta \omega_{dr} \approx 2\pi/T$ and

$$\Delta \mathbf{x} = \frac{2\pi C}{2\omega \Omega T} = \frac{\lambda}{2\Omega T} = \frac{\lambda}{2\Delta \theta}$$

where $\Delta\theta$ is the angle through which the target has turned during the observation time T. The cross range resolution can be very good at optical wavelengths. There is, of course, the problem of movement of features through the resolution elements during the observation time. There are techniques to handle this. We can estimate the magnitude of this effect. The x or cross range position of a given feature will change by $\delta x = R\Delta\theta$ during the observation time, and if it is not to move out of a range resolution cell

$$\mathbf{\hat{e}x} = \mathbf{R} \ \Delta \ \mathbf{\theta} < \Delta \mathbf{x} = \frac{\lambda}{2\Delta \mathbf{\theta}}$$

United Aircraft United Aircraft Research Laboratories

II-2

or

$$R \Delta \theta \Delta x < \Delta x^{2}$$
$$\frac{R \lambda}{2} < \Delta x^{2}$$

So that if $\lambda = 10 \mu$ R = 10 meters

$$\Delta x^{2} > \frac{10^{-5} \cdot 10}{2}$$

$$\Delta x > \frac{1}{\sqrt{2}} \quad 10^{-2} \text{ meters}$$

i.e., as long as we do not try to resolve better than about 1 cm., the motion of features through doppler resolution lines will not be a great problem.

If $\lambda = 1 \mu$, R = 1 meter then

$$\Delta x^{2} > \frac{10^{-6}}{2}$$

 $\Delta x^{2} > \frac{1}{\sqrt{2}}$ 10⁻³ meters

and the resolution is better by a factor of 10.

Application of synthetic aperture processing to side looking radars has been discussed in UARL Report L-920479-36, Ref. II-4. This report discusses the power and energy requirements, the frequency resolution, signal to noise ratio and detection sensitivity for a synthetic aperture optical radar system.

The basic problem involved in the processing of optical synthetic aperture radar data is indicated schematically in Fig. II-2. This shows one possible type of transmitted signal although many other types could be used. The target is illuminated by a periodic sequence of transform limited short duration optical pulses as shown in Fig. II-2a. The return signal consists of a series of longer pulses as shown in Fig. II-2b. In principle, this series of pulses could be heterodyned against a stable local oscillator having the same center frequency as the transmitted pulse train to give a signal as shown in Fig. II-2c. To produce an image, it is necessary to sample each range element in the signal and to spectrally analyze the history of the optical phase within the element to determine the distribution of doppler shifts within the range element. The range and doppler

distribution of the received signal can then be used to provide a twordimensional image of the target since the range and doppler contains constitute a coordinate system on the target.

The simple scheme discussed above cannot be used in practice. To achieve resolution of 1.5 cm, for example, would require an optical heterodyne detector with a time resolution of 100 psec together with electronics of comparable response time. Electronic sampling circuits with this response time are not available.

A possible way of processing the signal is illustrated in Fig. II-3. The incoming signal is split into a number of channels corresponding to the desired number of range resolution elements. A local oscillator consisting of a train of short pulses (areplica of the transmitted signal with an offset frequency) is provided. This signal is appropriately delayed for each range channel so that it arrives at the mixer coincident in time with the range element that it is desired to sample. Since heterodyning will occur only when the local oscillator is present, this performs the necessary sampling. The signal is then amplified at the IF frequency. The resulting signal is subject to a spectral analysis to determine the distribution of doppler shifts in the range element. This is repeated for each range element in the received signal. The output of this system thus contains all the information necessary to synthesize a range-doppler image of the target. The detection sensitivity of such a system after the splitting of the original beam is excellent. It is a heterodyne system and can be operated in the quantum noise limited region. A fundamental inefficiency arises, however, due to the beam division. A scheme to avoid this has been devised and is discussed in Section III of this report. A modified version of the scheme illustrated in Fig II-3 is being used in imaging experiments.

Preliminary Imaging Experiments

The ideal laser for the imaging system discussed above would be one that emitted a train of ultrashort pulses with highly reproducible characteristics. The pulses should have the shortest possible duration, possess uniform phase fronts too and be phase coherent from pulse to pulse.

In an operational system the coherence of the local oscillator used for heterodyning must be maintained over the round trip time of the signal from the target. Provisions must be made to compensate for the potentially large doppler shift due to translation of a spinning target. Finally, sufficient power must be available to overcome the large propagation loss from a distant target, to ensure an adequate signal to noise ratio at the receiver. In a laboratory simulation, the coherence requirement may be easily met since propagation times are very small.

United Aircraft U Research Laboratories Q Imaging of a purely rotating target eliminates the need for extremely large local oscillator offsets. The power requirements, however, are not reduced by the reduction in target range. This arises because of the spatial coherence requirements imposed by the optical heterodyne detector. If a distant target intercepts all of the power in the beam from the transmitter, and **if** it scatters the power uniformly in angle, then the power received by the receiver is just the solid angle subtended by the receiver aperture as viewed from the target. In the case where the target is just resolved, this will be of the order of

$$Pr = Po \quad \frac{1}{4\pi} \left(\frac{\lambda}{d} \right) \stackrel{?}{=} \frac{Po}{Po} \frac{1}{4\pi} \qquad \frac{D^2}{R^2}$$

where d is the target diameter, D is the receiver aperture and R is the distance. In the case of a target at very close range, essentially all of the power scattered back toward the receiver can be intercepted by the receiving aperture. Only a small fraction of this however is useful for the heterodyne detection. The angular coherence interval for power scattered from different point scatters on the target is approximately $\Delta \theta = \frac{\lambda}{d}$. The field of view of the receiver must be limited to this to avoid loss of signal. Again assuming isotropic scattering, the total useful received power is again of the order of Po $(\lambda/d)^2/4\pi$.

The choice of wavelength for an imaging radar is governed by a number of considerations, some of which are discussed in the previously referenced UARL Report L-920479-36 (Ref. II-4). It is generally advantageous to go to a long wavelength and the detection sensitivity increases approximately as λ^3 . The larger usable aperture discussed in Section II accounts for a λ^2 dependence and the reduced quantum noise at lower frequencies accounts for a λ dependence. In an operating system, this would indicate a choice of the 10.6 μ CO₂ laser over the 1.06 μ Nd:YAG laser. Bandwidths available from CO₂ lasers are capable of giving range resolutions of only a foot or so while a Nd:YAG laser can give resolution of about a centimeter. A mode-locked YAG laser is better suited to conduct a laboratory investigation of synthetic aperture imaging techniques.

A closed loop mode-locked laser has been fabricated for use in these experiments. The laser head is illustrated in Fig. II-4 and the associated control rack is sketched in Fig. II-5. In this laser a detector is used to sample the cutput from the high reflectivity end of the Nd:YAG laser. The component at the fundamental mode spacing frequency of the laser is filtered and amplified. This signal is then properly phase shifted and is used to drive an intracavity Lithium Nibate phase modulator. When the phase is properly adjusted, this results in mode-locking of the laser and a continuous train of \sim 100 psec pulses are produced.

United Aircraft **Research Laboratories**

II-5

This laser was used in the initial laboratory demonstration of an imaging optical radar. The system that was used is identical in concept to that shown in Fig. II-3 except that both the range and Doppler distribution of the received signal were measured sequentially rather than simultaneously as shown in the figure. The basic idea is illustrated schematically in Fig. II-6. The beam emerges from the laser with horizontal polarzation. The initial $\lambda/2$ plate is used to produce a small amount of vertical polarzation which is subsequently split off by a Glan-Thompson prison and serves as the local oscillator. This is then sent to an adjustable delay. The main beam is sent through an expanding telescope to illuminate the rotating target. The return beam is deflected by the Glan-Thompson prison, is frequency shifted by 40 MHZ by an acoustic modulator and is combined on a beam splitter with the local oscillator. If we assume that the return beam is completely polarized, then the $\lambda/4$ plate serves to render the returning beam vertically polarized, and it is completely reflected at the prism interface. If there is significant depolarization, the polarized component can be measured with the $\lambda/4$ plate in place and the depolarized component with it removed. The heterodyne signal, containing the complete Doppler distribution for a given range element, can then be sent to a spectrum analyzer and the output of the analyzer used to modulate the intensity of a storage oscillascope. The horizontal sweep is driven in synchronism with the spectrum analyzer and provides a direct display of the Doppler distribution at a given range. If the local oscillator delay and the vertical position of the trace are then scanned slowly and in synchronism, a complete range-Doupler map of the target is produced.

A slightly modified version of this system is being used in the laboratory experiments, and is illustrated in Fig. II-7. The primary difference is that the target range is scanned rather than the delay of the local oscillator. The two arrangements are entirely equivalent. A synchronizing signal is derived from the rotation of the target and is used to initiate the scan of the spectrum at the same point in each rotation of the target. In an actual radar system it is not necessary to have a priori knowledge of the phase or rate of the target rotation. The reason why it is necessary here is due to the fact that the spectrum analyzer and the range sampling operates sequentially. If the range sampling were done in parallel as indicated in Fig. II-3, then the entire image could be obtained in a small fraction of one rotation and the changing range-Doppler map could be observed as the object rotated.

The telescope used for the initial experiment consisted of a 2 cm positive lens and a 5" diameter, 1 meter radius concave mirror that was operated slightly off axis. The target that was used consisted of an aluminum bar holding two retroreflectors(glass corner cubes) of 1 cm diameter separated by a vertical distance of 3.5 cm. The target was rotated about a horizontal axis at a rate

M=920479-39

of 0.5 rps. Typical images that could be obtained are shown in Fig. II-7. The spectrum was taken when the bar was at an angle of 45° with the upper half receding from the transmitter. The horizontal axis of the display is the Doppler axis and the vertical axis is the range axis. The upper image of the figure is obtained where the laser is operated in a single axial mode (or only a few modes). The display shows the upper reflector (receding) with a negative Doppler shift and the lower reflector (approaching) with a positive Doppler shift. The vertical line between the images corresponds to the axis of rotation of the target (zero Doppler shift), and is actually due to a small amount of feedthrough from the acoustic modulator into the spectrum analyzer. As would be expected, the image is independent of range, since the laser is running in a single mode. The lower part of Figure II-8 shows the result obtained where the laser is mode-locked. The targets are very clearly resolved in the range direction. The upper portion corresponds to longer range and the lower portion to nearer range. The top (receding) target is seen with down Doppler shift and at a greater range while the bottom target (approaching) is seen with up Doppler and at a closer range.

The preliminary experiment clearly demonstrates the excellent spatial resolution that can be achieved with a synthetic aperture optical radar system. The next experiments to be undertaken with this system will involve imaging of more complex targets. This will involve some modifications of the optical train to achieve signal detection sensitivity that is close to the theoretical limit. The present system is limited by some aberrations and moudulator inefficiencies that limit its detection sensitivity. Beyond these experiments, the alternate signal processing techniques and signal generating schemes that are discussed in the next section will be investigated.

M=920479-39

of 0.5 rps. Typical images that could be obtained are shown in Fig. II-7. The spectrum was taken when the bar was at an angle of 45° with the upper half receding from the transmitter. The horizontal axis of the display is the Doppler axis and the vertical axis is the range axis. The upper image of the figure is obtained where the laser is operated in a single axial mode (or only a few modes). The display shows the upper reflector (receding) with a negative Doppler shift and the lower reflector (approaching) with a positive Doppler shift. The vertical line between the images corresponds to the axis of rotation of the target (zero Doppler shift), and is actually due to a small amount of feedthrough from the acoustic modulator into the spectrum analyzer. As would be expected, the image is independent of range, since the laser is running in a single mode. The lower part of Figure II-8 shows the result obtained where the laser is mode-locked. The targets are very clearly resolved in the range direction. The upper portion corresponds to longer range and the lower portion to nearer range. The top (receding) target is seen with down Doppler shift and at a greater range while the bottom target (approaching) is seen with up Doppler and at a closer range.

The preliminary experiment clearly demonstrates the excellent spatial resolution that can be achieved with a synthetic aperture optical radar system. The next experiments to be undertaken with this system will involve imaging of more complex targets. This will involve some modifications of the optical train to achieve signal detection sensitivity that is close to the theoretical limit. The present system is limited by some aberrations and moudulator inefficiencies that limit its detection sensitivity. Beyond these experiments, the alternate signal processing techniques and signal generating schemes that are discussed in the next section will be investigated.

SECTION II

REFERENCES

- II-1. Harger, R. O.: Synthetic Aperture Radar Systems. Academic Press, New York.
- II-2. Notes Entitled "Principles or Imaging Radars", University of Michigan Engineering Summer Conferences, July 20-31, 1970, University of Michigan, Ann Arbor, Micigan.
- II-3. Pratt, W. K.: Laser Comminication Systems. John Wiley and Sons, Inc., New York, 1969.
- II-4. Glenn, W. H., and A. R. Clobes: UARL Report L920479-36, Annual Report under Contract NO0014-66-C0344 for the period 1 August 1971 to 31 July 1972.

I-4

ı

ROTATING BODY RADAR



FROM

RADAR







a) SPHERE





SIDE



b) CYLINDER

1

SYNTHETIC APERTURE SIGNAL PROCESSING





ł

FIG. 11 - 3





.



MODEL 2001/2010 MODE-LOCKED LASER

٩

INSTRUMENTATION RACK UARL MODEL 2001 - MODE-LOCKED LASER

FIG. II - 5







FIG. II-<u>6</u>





.



RANGE







LASER MODE-LOCKED

TARGET AXIS DOPPLER

II-16



Section III

Additional Techniques Applicable To Imaging Frequency Domain Sampling

As has been noted in Section II, the time-shifted heterodyne system is capable of performing the sampling required to process the SAR signal, but it is far from optimum in terms of its detection sensitivity. A fast optical switch could perform the sampling without loss of efficiency if such a switch could be built. The sampling times of 100 picoseconds or less that are required are beyond the present state of the art in optical modulators. During the course of the present contract, a considerable amount of attention was directed to this problem and a possible solution to the problem has been devised. The solution involves sampling in the frequency domain rather than in the time domain. This is advantageous because it is possible to spatially or temporally separate the various frequency components of an optical signal using a linear, passive optical device. e.g., a spectrometer separates the frequency components spatially and a dispersive propagation path separates them temporally. It is not possible to separate and measure the various time segments of a signal without the use of a nonlinear effect such as an electrooptic switch. An extremely fast detector could be used in a heterodyne system but such detectors are unava lable.

The proposed scheme measures a sampled version of the Fourier Transform of a complex optical signal. If a sufficient number of samples are taken, this will completely characterize the optical signal. The system depends for its operation on the availability of an extremely short duration optical pulse having the same approximate optical center frequency as the pulse to be measured. This pulse serves as a local oscillator in a frequency domain multiplexed detector. The ultimate effective time resolution is determined by the duration of the pulse.

The discussion below shows in a somewhat formal way how the system operates. This discussion shows the ability of the system to fully characterize an optical signal on an extremely fast time scale, with high detection sensitivity and without the need for fast detectors. Possible practical implementations will then be discussed.

Let us assume that we want to measure a received optical signal s(t). This signal will be assumed to be of the type that would be obtained by illuminating an extended, moving target by an extremely short duration optical pulse. The signal will be of finite total duration, in the range of a fraction of a nanosecond to a few nanoseconds. It will have a wide bandwidth in terms of electronic detectors (in the range of 10^{11} Hz). This bandwidth is due primarily to the bandwidth of the original optical pulse that was transmitted, but also contains a small contribution from the doppler spread of the target. Although the bandwidth is wide in terms of electronic circuit bandwidths, the relative bandwidth referred to the optical carrier will be small, of the order of 0.1%. The Fourier Transform of this signal will be denoted by

$$S(w) = \frac{1}{(2\pi)^2} \int_{e}^{-iwt} s(t) dt$$

We shall also assume that there is a reference the available, and that this pulse is a very short duration, transform limited pulse with the same optical center frequency as the received pulse (it could be a replica of the transmitted pulse). This pulse will be denoted by r(t) or its transform $R(\omega)$. These two signals are combined with parallel wavefronts on a beam splitter. The spectrum of the composite signal is then

$$F(\omega) = R(\omega) \pm S(\omega)$$

where the plus or minus sign is determined by the phase shift upon reflection from the beam splitter; both signals can be obtained from opposite sides of the beamsplitter. We now assume that we have a bank of N narrowband optical filters covering the spectral extent of F(w). The transmission of each of these filters will be denoted by $T_n(w)$. At the output of each filter, the spectrum is given by

$$G_{n}(\omega) = [R(\omega) \pm S(\omega)] [T_{n}(\omega)]$$

At the output of each filter, the signal is square law detected and integrated over time, producing an output

$$U_{n} = \int_{-\infty}^{+\infty} |g_{n}(t)|^{2} dt$$
$$= \int_{-\infty}^{\infty} |R(w)|^{2} |T_{n}(w)|^{2} dw$$

where the last equation follows from Parseval's Formula. Expanding, we find

$$U_{n} = \int \{ |R(\omega)|^{2} \pm [R(\omega)S^{*}(\omega) + R^{*}(\omega)S(\omega)] + |S(\omega)|^{2} \}$$
$$|T_{n}(\omega)|^{2} d_{\omega}$$

We now assume that the spectrum of R(w) is real and essentially constant over the spectral range of S(w). This will be true if r(t) is a transform limited pulse as assumed and is shorter than or comparable to the duration of the original transmitted pulse. Let us denote this value by R, So

$$U_{n} = \int_{-\infty}^{\infty} \left[R^{2} \pm R \left[S(\omega) + S^{*}(\omega) \right] + \left| S(\omega) \right|^{2} \right] \left| T_{n}(\omega) \right|^{2} d_{\omega}$$

In this expression, the third term will generally be negligible compared to the first and second; it can always be made so by a sufficiently large local oscillator signal.

The function $T_n(w)$ is a narrow band transmission function. We choose its width to be sufficiently narrow so that S(w) does not vary appreciably over the range in which $|T_n(w)|^2$ is large. If we denote the center frequency of $|T_n(w)|^2$ by w_n , then we have

$$U_{n} = K_{n} \left[R^{2} \pm R \left(S \left(\omega \right) + S^{*}(\omega) \right) \right]$$
$$= K_{n} \left[R^{2} \pm 2R \operatorname{Re} \left(S \left(\omega \right) \right) \right].$$

where

$$K_{n} = \int_{-\infty}^{\infty} |T_{n}(\omega)|^{2} d\omega$$

Since signals can be obtained corresponding to either sign in the above equations, we can take the difference of the two to obtain

$$V_n = U_n^+ - U_n^- = 2K_nR Re(S(\omega))$$

It is straightforward to show that if an identical scheme is used but with S $(_{\omega})$ initially shifted by one quarter wavelength to yield S' $(_{\omega}) = i S (_{\omega})$, the corresponding result is

$$V_n^* = 2K_n R \text{ Im } (S(\omega))$$

We, thus, have a scheme for sampling both the real and the imaginary parts of the spectrum S (w). If a sufficiently large number of samples are taken, the original signal S(t) can be reconstructed. To summarize the important points involved in this system:

- (1) The sampling is done in the frequency domain rather than in the time domain. This eliminates the problem of inefficient sampling in the time domain due to the necessity of splitting the signal into separate detectors.
- (2) The detection system in each channel is a heterodyne system and by making the local oscillator signal large enough so that its shot noise is the dominant source of noise, quantum noise limited operation can be achieved.
- (3) The system does not require fast detectors. For detection of a single signal, they may be arbitrarily slow. To detect a series

of pulsed signals, they must have a time response faster than the time between the pulses.

(4) The basic operation of the system can be described simply. What it does is to split the incoming signal into a number of frequency bins, spaced across the total spectrum of the signal. If a local oscillator were available at each frequency bin, the amplitude and phase of the signal in each bin could be measured. The system provides just such a set of local oscillators by frequency filtering of a short duration pulse. The system, thus, divides the original wide time-bandwidth product signal into a number of parallel channels having a unity time-bandwidth product.

The operation of this type of system might be understood more clearly by a more concrete example. Consider a signal consisting of a repetitive train of complex pulses

$$s(t) = \sum_{n = -N}^{n = N} a_n \left[\cos (\omega_0 + n\Omega) t + \phi_n \right]$$

where the repetition period is $2\pi/\Omega$. Consider also a local oscillator signal consisting of a train of short pulses

$$r(t) = \sum_{n = -N}^{n = N} b_n \cos (\omega_0 + \omega_1 + n_0) t$$

Here the center frequency of the pulse train has been offset by a frequency w_1 for convenience. We shall assume that $w_1 << \Omega$. The introduction of this offset will simplify the discussion immensely. The composite signal is

$$f(t) = \sum \left(a_n \cos \left[(w_0 + m) t + \phi_n \right] + b_n \cos \left[(w_0 + w_1 + m) t \right] \right)$$

This signal is then subjected to the filtering operation. In its most idealized form, this simply selects out a range of n values; (since $u_1 << \Omega$). We obtain

$$g_{N'}(t) = \sum_{n=N_{1}}^{n=N_{2}} \left\{ a_{n} \cos \left[(w_{0} + n_{0})t + \phi_{n} \right] + b_{n} \cos \left[(w_{0} + w_{1} + n_{0})t \right] \right\}$$

м-920479-39

After square law detection, this signal becomes

$$(g_{N'}(t))^{2} = \sum_{\substack{n = N_{1} \\ n = N_{1}}} \sum_{\substack{m = N_{1} \\ m = N_{1}}} \left\{ a_{n} \cos[(\omega_{0} + m_{1})t + \phi_{n}] + b_{n} \cos[(\omega_{0} + \omega_{1} + m_{1})t] \right\}$$

$$\left\{ a_{m} \cos[(\omega_{0} + m_{1})t + \phi_{m}] + b_{m} \cos[(\omega_{0} + \omega_{1} + m_{1})t] \right\}$$

Since the detector does not respond at optical frequencies, we may ignore all the sum frequency terms in this expression and obtain

$$(g_{N}, (t))^{2} = \sum_{n m} \sum_{m} \frac{a_{n m}}{2} \cos \left((m-n) \Omega t + \phi_{m} - \phi_{n} \right) + \frac{b_{n m}}{2} \cos \left((m-n) \Omega t \right)$$

$$+ \frac{a_{n m}}{2} \cos \left((m-n) \Omega t - \omega_{1} t + \phi_{m} \right)$$

$$+ \frac{a_{n m}}{2} \cos \left((n-m) \Omega t - \omega_{1} t + \phi_{n} \right)$$

$$(g_{N},(t))^{2} = \sum_{n} \sum_{m} \left\{ \frac{a_{n}a_{m}}{2} + \frac{b_{n}b_{m}}{2} + \frac{a_{m}b_{n}}{2} \cos \left((m-n)_{\Omega}t - \omega_{1}t + \phi_{m}\right) + \frac{a_{n}b_{m}}{2} \cos \left((n-m)_{\Omega}t - \omega_{1}t + \phi_{n}\right) \right\}$$

We now suppose that we perform an electrical filtering operation on the signal and look only at frequencies near the offset frequency w_1 . This eliminates all terms except those for m = n and we obtain

$$(g_{N'}(t))_{\text{filtered}}^{2} = \sum_{n=N_{1}}^{n=N_{2}} a_{n}b_{n} \cos(\omega_{1}t - \phi_{n})$$

And since by assumption the optical filters are narrow enough so that a_n , b_n and ϕ_n are essentially constant, we have

$$(\mathbf{g}_{\mathbf{N}},(\mathbf{t}))_{\mathbf{i}}^{2} = (\mathbf{N}_{2} - \mathbf{N}_{1}) \mathbf{a}_{\mathbf{N}}, \mathbf{b}_{\mathbf{N}}, \cos(\mathbf{w}_{1}\mathbf{t} - \mathbf{\phi}_{\mathbf{N}})$$

where a_N , b_N , and ϕ_N , represent the averager values of these quantities over the interval $N_1 \le n \le N_2$. Thus, the output of the system is a sinusoid whose amplitude and phase are simply related to the amplitude and phase of the Fourier components of the original signal. Although this discussion has been concerned only with a truly periodic train of pulses, it is clear that if the characteristics of the pulses change slowly compared to ω_1 , it will be true that

$$(e_{v_1}(t))_{f}^2 = (N_2 - N_1) a_{v_1}(t) b_{v_1} \cos(\omega_1 t - \phi_{v_1}(t))$$

The relevant information, a_N , and ϕ_N , can be extracted by means of a pair of quadrature detectors. The system is really quite a bit more general than the simplified description. Note that we have not used the fact that the local oscillator is a short pulse, only the fact that its Fourier components are constant over the width of the filter and that they are known quantities.

A summary of the signal processing involved in this second example is shown in Fig. III-1.

Some remarks now should be made about the number of filters required and the bandwidth of the filters. Let us assume that the time duration of the received signal from a single transmitted pulse is T. To reconstruct a strictly periodic version of this signal, we would have to construct a Fourier series consisting of frequency components separated by f = 1/T. With such a series, we can reconstruct an arbitrary function with this periodicity. Since each component has a real and a quadrature component, we are actually specifying 2 FT frequency components in an interval F. This should be compared with the familiar result from time sampling theory where it is known that it is necessary to sample a signal at time intervals of t = 1/2F where F is the highest frequency component in the signal and are, thus, also specifying 2 FT components in a time interval T. Thus, for a duration T of 1 nanosecond, the filters should be spaced by 10^9 Hz, for 10 nanoseconds by 10^8 Hz.

The bandwidth of the individual filters determines the response time of the system. Clearly, the bandwidth should not exceed the spacing of the filters as this will simply lead to "crosstalk" between them, with the same information appearing on more than one filter. The bandwidth should not be less than the heterodyne offset signal or all information will be lost. Between these limits, the bandwidth determines how fast the system can respond to changing pulse characteristics. There are a number of possible ways to realize the bank of narrowband optical filters required for the frequency domain sampling system. The optimum choice is, as yet, undetermined and the evaluation of possible realizations would be part of the proposed amendment. The narrow bandwidths required may readily be achieved by commercially available Fabry-Perot interferometers. A simple means for realizing the filter bank is shown in Fig. III-². Here beam splitting prisms and waveplates are used to distribute the signal to a bank of Fabry-Perot filters spread across the spectrum of the incoming signal. While simple in principle, this system would be quite expensive to implement experimentally. Another means is shown in Fig. III-3. Here the incident beam is

caused to reflect back and forth between two inclined etalons. At each reflection, the angle of the beam changes so that upon the next pass, the transmission of the etalon is shifted in frequency. The resulting frequency filtered components of the signal emerge from both sides of the device and are dispersed in space and angle. This is far simpler than the system of Fig. III-2 but for the parameters of the SAR imaging system it suffers from a walkoff problem. The required frequency filtering could also be obtained by a diffraction grating as shown in Fig. III-4. In the configuration illustrated, a double pass is used to double the dispersion. This is probably the most practical system, although it requires the use of a fairly large and good quality grating to achieve the desired bandwidth of the filters. Other schemes are possible and would be investigated under the proposed amendment.

For a feasibility demonstration of the frequency domain sampling technique, the system shown in Fig. III-1 will be used. The target will be chosen to produce a known repetitive return signal. One channel will be used and the filter would be a tunable Fabry-Perot filter. The center frequency will be gradually scanned to produce the output from the various frequency elements sequentially in time rather than in parallel. These will be recorded and used to synthesize the return signal. At the present time, the appropriate filters, amplifiers and balanced mixers that are necessary to realize this system are being fabricated.

FABRY PEROT MODULATOR

In 1963, E. J. Gorden and J. D. Rigdon (Ref. III-1) proposed a modulator consisting of a Fabry-Perot etalon with an internal phase modulator that was driven at the c/21 frequency of the etalon. Their analysis showed that this device could act as an extremely fast optical switch. Recently, Kobayashi, Sueta, Cho and Matsuo (Ref. III-2) demonstrated the use of such a modulator to generate trains of 21 picosecond optical pulses from a helium neon laser. The system that was used is illustrated in Fig. III-5. It may be shown by a simple derivation that the effective transmission of a synchronously modulated Fabry-Perot interferometer is given by

$$T = T_{max} \left\{ 1 + F \sin^2 \left[\left(v - v_0 \right) \pi / \Delta f + \Delta \theta \sin(wmt) \right] \right\}^{-1}$$

where

$$T_{\text{max}} = T_2 T_3 \exp(-2\delta) \left[1 - v_2 v_3 \exp(-2\delta)\right]^{-2}$$

F = $4r_2 r_3 \exp(-2\delta) \left[1 - r_2 r_3 \exp(-2\delta)\right]^{-2}$
 Δf = $c/2\ell$

 $\Delta \theta$ = depth of phase modulation

The quantities appearing in these expressions are:

 $r_2, r_3 = magnitudes of the amplitude reflectances of mirrors 2 and 3$ $<math>T_2, T_3 = magnitudes of the power transmittancies of mirrors 2 and 3$ $exp (-\delta) = one way transmission through the phase modulator$ f = mode spacing of the interferometer $<math>w_m = modulation \ frequency = 2\pi \ \Delta f$ $v_0 = n\Delta f = resonant \ frequency \ of \ interferometer$ $v = applied \ optical \ frequency$

The transmission of this Fabry-Perot filter is exactly the same as that of a conventional filter without the modulator except that the transmission is scanned back and forth in frequency at the modulation frequency ω_{m} . For a high finesse, the transmission of the filter is sharp and, because of the high scanning frequency, the filter acts as an extremely fast, repetitive shutter. The seemingly anomalous result that the effective transmission can be varied in a time short compared to the energy storage time of the filter is a result of the synchronous driving of the modulator. The modulator can thus produce an extremely short train of pulses. The time required to change the shape or amplitude of the transmitted pulses is, of course, related to the energy storage time of the interferometer which, for a high finesse, is long compared to the pulse repetition time. The device is not really a fast modulator, but rather a device to produce a repetitive series of very short pulses. It is shown in Ref. III-2 that if the Fabry-Perot interferometer is driven by a cw signal at $v = v_{0}$, the width of the pulses transmitted is given by

 $\Delta \tau = (\omega_m/2 \sqrt{F} \Delta \theta)^{-1}$

The significant point about this expression is that the pulse width is independent . of the laser line width. Mode locking of a homogeneously broadened laser produces a pulse duration that is proportional to $(w_m \ w_a)^{-1/2}$ where w_m is the modulation frequency and w_a the laser line width.

When operated with a homogeneously broadened gas laser such as the helium neon laser, this modulator has another very useful property, that of mode selection. The time averaged reflectivity is a function of the depth of modulation $\Delta \vartheta$. For $0 \leq \Delta \vartheta \leq \pi/2$, the reflectivity has a maximum at $\upsilon = (2n + 1)/2 \upsilon_0$, i.e., halfway between the resonances of the Fabry-Perot. For $\pi/2 < \Delta \vartheta < \pi$, the reflectivity has a maximum at $\upsilon = n \upsilon_0$, i.e., coincident with the resonances of the Fabry-Perot. This is the desired frequency of operation to produce a uniform, periodic train of pulses. For the helium neon laser, this frequency dependent reflectivity was sufficient to cause the laser to operate at a single frequency. The authors of Ref. III-2 used a value $\Delta \vartheta = \pi/2$ which

caused the laser to operate at $v = v_0$ and obtained a periodic train of pulses. The behavior of the modulator for $v \neq v_0$ is illustrated in Fig. III-6. For $v > v_0$, the output consists of a train of double pulses occurring in the negative part of the modulation waveform and for $v < v_0$ a train of double pulses occurring in the positive portion of the modulation waveform. This type of output will be produced as long as $|v - v_0| < \Delta \theta$. If $|v - v_0| > \Delta \theta$, a pulsed output is not produced since the transmission function does not have sharp maxima as a function of time.

In addition to the fact that the pulse width produced by this type of modulator is independent of the laser line width, the modulator has an additional attractive feature. The output coupling can be optimized by varying the parameters of the Fabry-Perot so that the output power in the pulse train can be made very close to the average power that would be obtained from the cw laser without modulation.

It should be possible to apply this modulation scheme to the output of a Nd:YAG laser to produce trains of short pulses at 1.06μ . The Nd:YAG laser, although it is homogeneously broadened, is subject to spatial hole burning problems. It is highly probable that the frequency dependent reflectivity of the Fabry-Perot modulator will be insufficient to constrain the laser to operate at a single frequency. A number of techniques are available to cause the laser to operate at a single frequency. These include the use of etalons in the cavity or the use of a unidirectional ring laser as described in Ref. II-4, this latter being a technique that was investigated under the present contract. If one of these schemes to cause single frequency operation is used, the single frequency may be tuned to the proper value by means of the system shown in Fig. III-7. This makes use of the property illustrated in Fig. III-6 to provide a discriminant to tune the laser frequency to the resonant frequency of the Fabry-Perot.

As mentioned above, an electro-optic phase modulator has been designed and fabricated to investigate the Fabry Perot modulator technique. The modulator consists of a Lithium Niobate crystal of dimensions 2 mm x 2 m x 10 mm, with Brewster angle end faces. This crystal is mounted in a re-entrant microwave cavity that is tunable in resonant frequency over the range of about 0.5 to 1.0 6 GHZ. Intial tests have shown that this modulator is capable of producing appreciable phase modulator with a few watts of RF impact power. When sufficient power is applied to produce a phase retardation of $\pi/2$, thermal distortion appears due to RF heating of the crystal. This problem may be simply solved by using a crystal of smaller cross sectional area, and such a crystal will be procured and tested. The present modulator, however, will be carried out.

SECTION III

REFERENCES

- III-1. Gorden, E. I., and J. D. Rigden: Bell System Technical Journal, <u>42</u>, 155 (1963).
- III-2. Kobayashi, T, T. Sueta, Y. Cho, and Y. Matsuo: Appl. Phys. Letters, <u>21</u>, 341, (1972).

11.10





SAMPLING DETECTOR

M920479-39



SCHEMATIC FREQUENCY DOMAIN SAMPLER

FREQUENCY DOMAIN SAMPLER USING MULTIPLE REFLECTIONS FROM FABRY FEROT ETALONS



亚-13

M920479-39



11-14

FREQUENCY DOMAIN SAMPLER USING A DIFF PACTION GRATING



FABRY-PEROT FAST OPTICAL SHUTTER (KOBAYA SHI, et al)

M920479-39

FIG. III - 5



BEHAVIOR OF FABRY PEROT TRANSMISSION OFF RESONANCE

 $T = T_{max} \left[1 + F_{sin}^2 \left\{ \{\mu\nu_0\}/\pi\Delta t + \Delta\theta_{sin}\omega_m t \} \right]^{-1} \right]$





11-17