AD-757 084

TROPICAL CYCLONES

Francis E. Fendell

TRW Systems Group

Prepared for:

Advanced Research Projects Agency

March 1973

DISTRIBUTED BY:

NTIS

National Technical Information Service

U. S. DEPARTMENT OF COMMERCE

5285 Port Royal Road, Springfield Va. 22151

AD757084

TRW
SYSTEMS GROUP
ONE SPACE PARK • REDONDO BEACH, CALIFORNIA

TRW Report 18524-6003-RU-00

FINAL REPORT

TROPICAL CYCLONES

by

Francis E. Fendell
Principal Investigator

Contract DAHC04-71-C-0025
19 April 1971 - 18 March 1973
ARPA Order 1786
Program Code 1030

TRW Systems Group
One Space Park
Redondo Beach, California 92078
(213) 535-2789

Sponsored by the Advanced Research Projects Agency of the Department of Defense under ARPA Order No. 1786. The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the Advanced Research Projects Agency or the U.S. Government.

I

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) TRW Systems Group One Space Park Recondo Beach, California 90278	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP N/A
3. REPORT TITLE Tropical Cyclones	
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Final Report (19 April 1971 - 18 March 1973)	
5. AUTHOR(S) (First name, middle initial, last name) Francis E. Fendell	

6. REPORT DATE March 1973	7a. TOTAL NO. OF PAGES ~~178~~ 179	7b. NO. OF REFS 161
8a. CONTRACT OR GRANT NO DAHC04-71-C-0025 b. PROJECT NO ARPA Order 1786 c. d. Program Code 1030	9a. ORIGINATOR'S REPORT NUMBER(S) 18524-6003-RU-00 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
10. DISTRIBUTION STATEMENT Approved for public release; distribution unlimited.		
11. SUPPLEMENTARY NOTES N/A Details of illustrations in this document may be better studied on microfiche.	12. SPONSORING MILITARY ACTIVITY Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, Virginia 22209	

13. ABSTRACT

After a brief summary of salient observational facts about cumulus convection in the tropical ambient and about tropical cyclones, models for the cyclone-scale structure and maintenance of the quasisteady mature stage of a hurricane are presented and evaluated. Principal attention is devoted to the model developed by Riehl and Malkus in the late 1950's, and to the model developed by Carrier in the early 1970's. In order to explain hurricane maintenance, Riehl and Malkus find it necessary to postulate the existence of a latent and sensible heat transfer (from the underlying sea to the air in the frictional inflow layer within about 100 km of the storm center) that is augmented by about an order of magnitude over the ambient-level enthalpy transfer in the autumnal tropics. Because Carrier permits some outward sloping of the eye-eyewall interface (in contrast with Riehl and Malkus), he finds that hurricane-strength winds can be explained without an oceanic heat source postulate; Carrier believes that oceanic enthalpy transfer is increased in a hurricane only modestly over ambient levels. Further, Carrier emphasizes the role of the large mass of warm moist air advected with the storm from its inception in understanding hurricane lifetimes. Analyses relating to maximum swirl speed estimates, solutions describing boundary-layer dynamics and energetics, and interpretations of temperature measurements are criticized to evaluate the relative merits of the two models and their implications for successful storm modification by current seeding practices. Attention is then called to unresolved questions concerning tropical-cyclone intensification; further progress on the deepening of tropical depressions will probably entail more quantitative description of the processes that maintain convective instability in the tropical ambient, along with better understanding of the spatial concentration of cumulus convection by a CISK-like mechanism.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

Unclassified
Security Classification

Unclassified
Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Conditional Instability of the Second Kind						
Eye						
Eyewall						
Hurricane						
Tropical Cyclogenesis						
Tropical Cyclone						
Tropical Depression						
Tropical Storm						
Typhoon						

Unclassified
Security Classification

CONTENTS

			Page
1.	INTRODUCTION		1
	1.1	The Dangers and Benefits of Hurricanes	1
	1.2	Some Observational Facts on Hurricanes	2
	1.3	Tropical Cyclone Generation	5
	1.4	Properties of Mature Hurricanes	8
	1.5	Path Prediction	15
	1.6	The Importance of Tropical Cyclones in the Global Circulation	17
2.	ASPECTS OF TROPICAL METEOROLOGY		23
	2.1	Stability in the Tropical Atmosphere	23
	2.2	Tropical Cumulonimbi	31
	2.3	CISK	36
3.	MODELS OF A MATURE TROPICAL CYCLONE		40
	3.1	Introduction	40
	3.2	The Carrier Model	41
	3.3	Critique of the Carrier Model	47
	3.4	Maximum Swirl Speed Estimate According to the Carrier Model	50
	3.5	The Swirl-Divergence Relation for the Frictional Boundary Layer	65
	3.6	The Energetics of the Frictional Boundary Layer and Throughput Supply	73
	3.7	The Riehl-Malkus Postulate of an Oceanic Heat Source	77

	Page
3.8 The Intensity of a Tropical Cyclone and the Underlying Sea-Surface Temperature	83
3.9 Critique of the Riehl-Malkus Model	88
3.10 Numerical Simulation of Hurricanes on Digital Computers	98
3.11 Implications of Hurricane Models on Seeding	103
4. THEORY OF TROPICAL CYCLONE INTENSIFICATION	109
4.1 Carrier's Outline of Intensification	110
4.2 Critique of the Carrier Model of Intensification	115
4.3 The Distribution of Cumulonimbi during Intensification	118
4.4 The Time-Dependent Flowfield during Intensification	120
5. CONCLUDING REMARKS	127
APPENDIX A - ESTIMATING THE KINETIC ENERGY AND WATER CONTENT OF HURRICANES	129
APPENDIX B - THE MOIST ADIABAT	132
PARTIAL LIST OF SYMBOLS	138
ACKNOWLEDGEMENT	141
FOOTNOTES	142
REFERENCES	148

1. INTRODUCTION

1.1 The Dangers and Benefits of Hurricanes

In an average year the Atlantic and Gulf coastal states of America suffer over $100 million damage and 50 - 100 fatalities owing to hurricanes; in a severe year damage will exceed $1 billion (Meyer, 1971a). Hurricane Agnes from June 19-26, 1972 caused damage from Florida to Maine that has been estimated at $3 billion (Anonymous, 1972), and Hurricane Camille in August 1969 cost 258 lives (White, 1972). The threat consists of high winds [over 200 mph is known (Riehl, 1972a)]; in rainfall [up to 27 inches in 24 hours (Schwarz, 1970) -- since 1886 hurricanes have caused over 60 floods in the United States (Alaka, 1968), and Hurricane Agnes is estimated to have rained in toto 28.1 trillion gallons (Anonymous, 1972)]; in storm surges, especially along concavely-curving coasts [coastal ocean levels can rise 15-20 feet, with Hurricane Camille setting the American record at 24 feet along the Gulf coast (White, 1972)];[1] and in ocean wave heights [wave heights of 45 feet are known (Riehl, 1954, p. 298)]. On the positive side, on a long-term basis the hurricane-associated rainfall over the Eastern seaboard states is about one-third of the total annual rainfall (Penner, 1972); without hurricanes, droughts seem inevitable on the west coast of Mexico, in southeast Asia, and elsewhere.

Globally, there are about eighty tropical storms annually, of which about fifty intensify into hurricanes; hurricanes occur in all oceans except the South Atlantic (Atkinson, 1971). Under the current state of the art for path predictions for either military or civilian use, three times the area actually hit by a hurricane is typically placed under hurricane warnings (Meyer, 1971a; Malone and Leimer, 1971). In the greater Miami area, current estimates suggest that at least $2 million are spent on preparations whenever hurricane warnings are issued (Simpson, 1971). Unnecessary preparation by the United States Department of Defense installations owing to false hurricane and typhoon warnings costs $8.3 million annually, exclusive of diversion-of-manpower costs. Thus, the mere threat of hurricanes incurs expenditure of prodigious sums.

1.2 Some Observational Facts on Hurricanes

"A tropical cyclone starts out as a tropical disturbance in which there is a slight surface circulation and perhaps one closed isobar. When the wind increases to about 20 knots and there is more than one closed isobar around the center, it is called a tropical depression. When the wind rises to more than 34 knots, and there are several closed isobars, it becomes known as a tropical storm. If the winds exceed 64 knots (74 miles/hour), it is classified as a hurricane or typhoon or cyclone (depending on location)" (Day, 1966, p. 187). Thus any low-latitude low-pressure circulation is technically a tropical cyclone; however, probably to emphasize the physical similarity of all very intense tropical lows despite the plethora of local appellations, in practice tropical cyclone often refers to the hurricane stage only. Following this practice, the name tropical cyclone will be mainly used to refer categorically to the hurricane stage; hurricane will also be used as a synonym for variation. The other local names will be used mainly when geographically appropriate. The local designations for tropical cyclones (Fig. 1) are hurricanes (North Atlantic Ocean), typhoons (western North Pacific Ocean), papagallos (eastern North

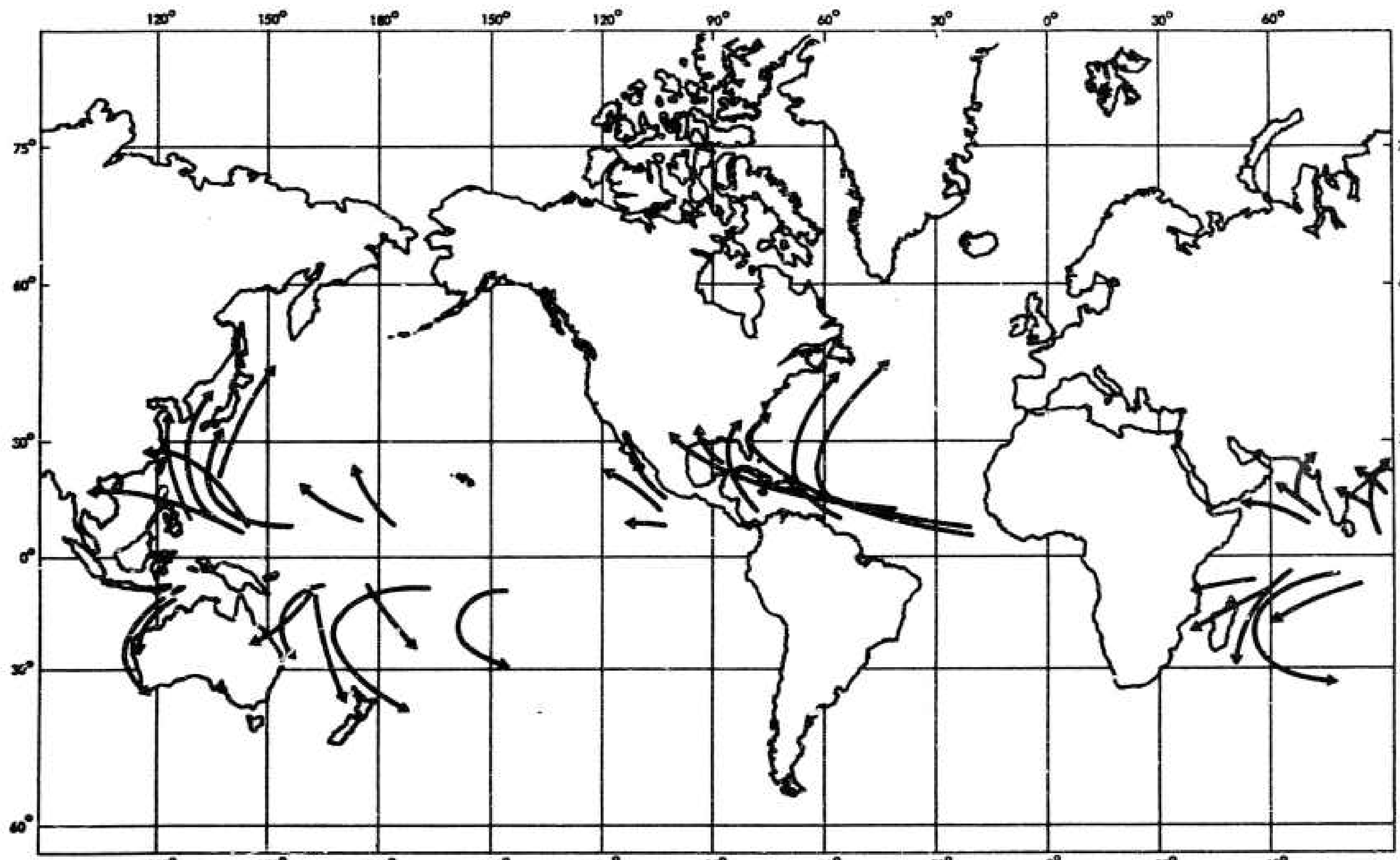

Fig. 1. Typical paths of tropical storms and cyclones.

Pacific Ocean), baguios (Philippine Islands), cyclones (North Indian Ocean), trovadoes (near Madagascar), and willy-willies [near Australia -- a term in use in the early twentieth century, although now it is reserved for dust devils (Spark, 1972; Vollsprecht, 1972)].

Annually, especially in the late summer and early fall, some disturbances over tropical oceans warmed by solar radiation (usually at least 26°C - 27°C, often 28°C and higher) intensify into tropical cyclones, which are typically at least a thousand miles in diameter and ten miles in height (i.e., they extend from sea-level to tropical tropopause). Not only do most typhoons form in the autumn, but also the most intense ones occur then (Brand, 1970a). These vortical storms are cyclonic in the Northern Hemisphere and anticyclonic (North Pole reference) in the Southern Hemisphere, and take many days to intensify -- indicating that the small Coriolis force is the source of angular momentum and explaining why intensification within 5° of the geographic equator is very rare. In fact, conservation of angular momentum in itself indicates that a fluid particle in the tropics drawn in about five hundred miles will swirl at several hundred miles per hour.

With satellite photography, inspection of broad ocean expanses has improved, and some former estimates about the frequency of tropical cyclones have had to be revised. About three-quarters of the annual global total of fifty tropical cyclones occur in the Northerm Hemisphere. In the North Atlantic about 60% of the nine tropical storms that typically occur annually intensify into tropical cyclones; in the eastern North Pacific, 33% of 14; and in the western North Pacific, 80% of 30 (Atkinson, 1971). Since there are daily disturbances in the tropics in autumn, a weak disturbance has a poor chance (∿10%) of becoming a tropical storm, but any disturbance that does manage to become a tropical storm has a good chance to become a hurricane (Palmén and Newton, 1969, p. 503). Since globally just over 60% of tropical storms become hurricanes, there appears to be no criterion such that once a developing depression exceeds it, the depression will definitely become a hurricane. No one today can infallibly predict which tropical disturbances will intensify into hurricanes.

1.3 Tropical Cyclone Generation

Hurricanes form where there is sustained local convective activity over warm tropical seas, with surface temperature 27°C or higher (so that low-level air lifted on a moist adiabat remains convectively unstable relative to the undisturbed ambient up to 12 km); where there is enhanced cyclonic shear (as occurs when the Intertropical Convergence Zone lies at a considerable distance from the geographical equator); and where there is weak vertical shear of the zonal wind. Each of these three criteria warrants elaboration.

The first criterion regarding stability tends to be fulfilled only in the fall, and is discussed in detail in the discussion of tropical-atmosphere stability (Section 2.1). With regard to the need for increased cyclonic shear as well as warm ocean temperatures, it is noteworthy that, globally, 65% of the disturbances that later become tropical storms were first detected between 10° and 20° of the equator, with only 13% poleward of this region and only 22% equatorward (Atkinson, 1971). The absence-of-vertical-wind-shear criterion may explain the anomalous cyclone season for the northern Indian Ocean (in the Arabian Sea, the Bay of Bengal, and even the South China Sea); in these areas, rather than a single autumnal peak, twin peaks in cyclone frequency occur in fall and spring, with a relatively uneventful summer season (Palmén and Newton, 1969). Gray (1968) suggests that unless cumulonimbi retain vertical integrity, the atmospheric lightening associated with them is dissipated by being advected in different directions at different altitudes ("ventilated"). Climatologically, regional differences in vertical wind shear within the tropics become most evident in the upper troposphere. Gray (1968) also asserts that the up- and down-drafts in cumulonimbi themselves help suppress enhancement of vertical wind shear as baroclinicity increases during intensification of tropical depressions. Vertical wind shear at upper tropospheric levels prevents the higher structure of some typhoons to get fully organized, although the structure may be well defined in the lower troposphere; such so-called shallow typhoons are invariably of minimal hurricane intensity (Varga, 1971).

Particular features of the North Atlantic hurricane season are now enumerated. As the peak of the hurricane season approaches, the region where tropical storms reach hurricane intensity moves eastward from the Gulf of Mexico and the Caribbean to the Cape Verde Islands; as the peak hurricane season passes, the spawning ground moves westward again to the Caribbean (Meyer, 1971a). Coincidentally, there is a latitudinal movement northward in the first half of the season, then a retreat equatorward (Riehl, 1954, p. 323). Cyclogenesis poleward of 20°N is a particular characteristic of the North Atlantic; in fact, for the past four years the tropical North Atlantic has been mostly free of hurricanes, which have been forming at 25°N and higher (Simpson and Frank, 1972).

At one time discussion of tropical cyclogenesis inevitably evoked discussion of waves in the easterlies (Riehl, 1954). But in recent years attention has focused on (1) the role of the ICTZ and (2) twice-weekly autumnal disturbances that begin as large cyclonic sandstorms of 1000 n mi extent over Africa and that drift westward at up to 10 kts. There may well be two different sources of hurricane seedlings, and if so, the relative roles of latent and sensible heat in each may differ (Garstang, 1972).

1. Tropical cyclones tend to form on the poleward side of the equatorial trough; in fact, 80% - 85% of synoptic-scale tropical disturbances form within 2° - 4° of the equatorial trough on the poleward side. Disturbances are rarer, smaller, and weaker when the trough is closest to the equator. The trough is maintained by the CISK process (Section 2.3), since low-level meridional moist inflow sustains the persistent cloudiness of this perennially low-pressure region. Further, there is low-level cyclonic shear from interaction of easterlies poleward of the trough and westerlies equatorward of the trough. On a zonally-averaged basis around the globe, the equatorial trough annually migrates from 15°N to 5°S, lagging the solar zenith by about two months as it does so (Riehl, 1972a). Byers (1944) attributes the absence of hurricanes in the South Atlantic largely to the failure of the ITCZ to become displaced south of the equator, even in February, at longitudes extending from the eastern Pacific to western Africa. Agee (1972) has presented an interesting sequence of satellite photographs documenting a case of tropical cyclogenesis in the vicinity of the ITCZ in late July, 1972.

2. If it is possible to correlate tropical North Atlantic disturbances with the ITCZ, it is also possible to correlate them with disturbances first formed over the mountainous east African bulge, that migrate westward (Carlson, 1969). Half the disturbances over the tropical North Atlantic can be so traced; further, half of these disturbances can then be traced across Central America to the eastern Pacific. Actually 75% of eastern

Pacific storms originate east of Central America. A significant percentage of midseason hurricanes (August and September) have African origins; analyses of dust samples taken on Caribbean isles after hurricane passage reportedly confirm the African origin of the storm systems (Jennings, 1970). An interesting case documented by Denny (1971) is an African seedling identified on September 7, 1971, which became a depression on September 11 and Atlantic Hurricane Irene on September 18. The system crossed southern Nicaragua on September 20 and regenerated to eastern Pacific Hurricane Olivia (948 mb central pressure) before dying on September 28 near Baja California.

There are many other correlations of tropical cyclone formation that may be attempted; for example Carpenter, Holle, and Fernandez-Partagas (1972) have recently suggested a major peak in hurricane formation near a new moon, a minor peak at full moon, and minima at last quarter and several days after first quarter. However, the correlations with the ITCZ and African seedlings just discussed have the advantage of inherent plausible physical mechanisms. Discussion of modeling of tropical cyclogenesis will be taken up in Section 4.

1.4 Properties of Mature Hurricanes

Tropical cyclones have a structure characterized in the mature stage by a relatively cloud-free calm eye (winds usually well below 15 mph) of about 10-20 mi radius. The eye is characterized by low pressure at sea level (often below 960 mb, infrequently below 910 mb) and high temperatures aloft (often 10°C above ambient at the same altitude). Such pressure drops are particularly spectacular in the tropics, where surface pressure usually varies by little more than 0.3% (Riehl, 1954, chapter 11). The eye is surrounded by an eyewall, an approximately ten-mile-wide annulus of intense convection, torrential rainfall, and deep thick couldiness (marked by large numbers of cumulonimbi). Outside the eyewall are convective rainbands that appear like pinwheels or logarithmic spirals in some satellite photographs and/or radar displays taken from above the storm (Figs. 2 and 3).

The principal velocity component in much of the storm is azimuthal (or tangential). Hurricane-force winds begin at the eyewall and extend outward 50 to 70 miles; winds usually fall to moderate-gale level (~35 mph) at distances of 100 to 150 miles from the center. The vertical component of velocity is largest in the eyewall. In the Northern Hemisphere, there is low-level cyclonic inflow (below three km) and high-level outflow (above eight km, with the maximum outflow near 12 km); toward the outer regions (between, say, 200 to 275 miles from the axis) the outflow turns from cyclonic to anticyclonic (relative to an observor rotating with the earth). If one subtracts off the symmetric flow from the total flow, the outflow layer possesses a horizontal anticyclonic eddy to the right of the path vector and a cyclonic eddy to the left; these are seemingly due to ambient streaming around the high-level outflow (Black and Anthes, 1971). In the midtroposphere (say, three to eight km above sea level) there is little radial flow. While there is much spray, the lowest few hundred feet (at least) of the inflow layer remain cloud-free in as far as the eyewall (Riehl, 1954).

It is often agreed that there is slow downward motion in the eye and in the outer regions of the storm to balance the strong updrafts in the eyewall; that radiational cooling of about 1.5°C/day attends sinking in the outer, less cloudy regions; that, either directly or indirectly, the

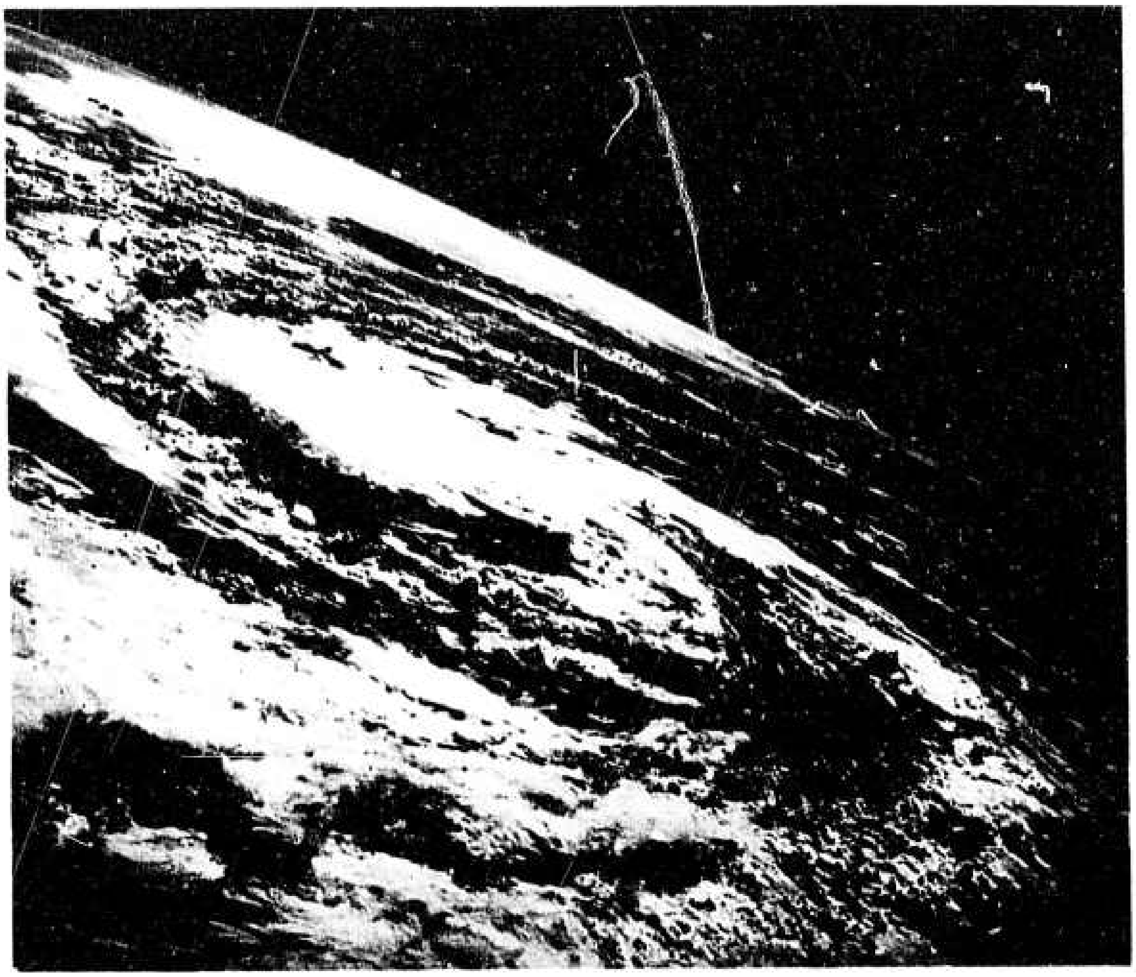

Fig. 2. NASA photograph of Hurricane Gladys west of Naples, Florida taken from Apollo 7 on October 7, 1968. The maximum wind speed was 65 kts at this time, and the tropopause was at 54,000 ft.

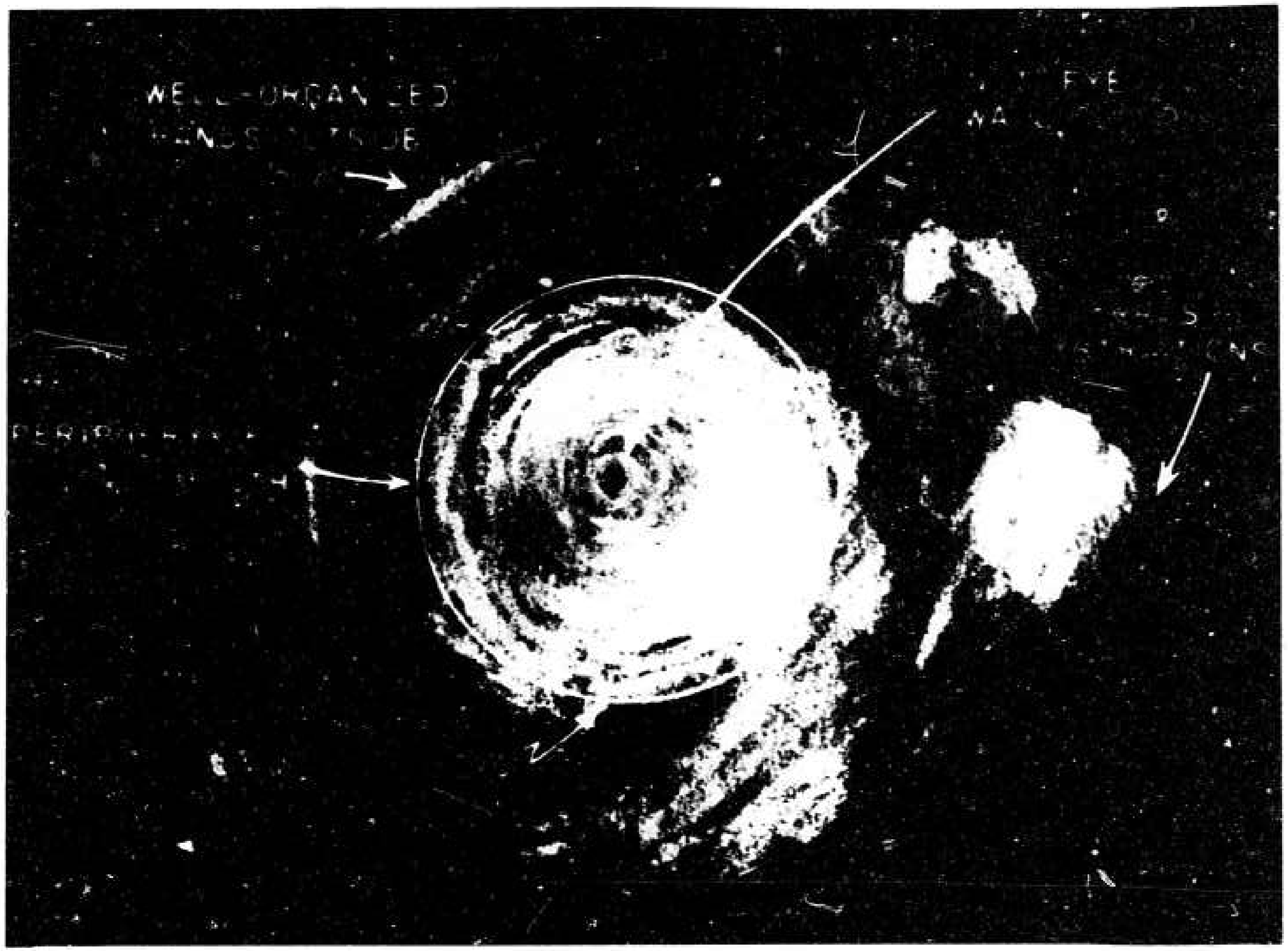

Fig. 3. NOAA radar photograph of mature hurricane revealing lower-level rainbands outside the region of overcast (O/C) and upper-level cirrus clouds of the outflow.

release of the latent heat of condensation reduces the density in the eyewall to establish a large radial pressure deficit relative to ambient conditions, from hydrostatic considerations; that a still further pressure deficit from ambient occurs in the eye owing to roughly dry adiabatic recompression of air that has arisen along the near-moist-adiabat vertical sequence of states that characterize an appreciable portion of the eyewall; and that a cyclostrophic balance (balance of radial pressure gradient and centrifugal force), together with an algebraic decay of the swirl with increasing radial distance from the axis, yields on the average a fairly good estimate of the maximum swirl speed.

Outside the eyewall, the precipitation rate falls off roughly linearly with distance from the center of the tropical cyclone. Cloud cells in the eyewall are typically 5 to 20 km thick and a few of the smaller ones can rain over 15 cm/hr. Spiral bands out to 150 km yield 1 cm/hr, and further out 0.25 cm/hr -- though individual convective cells of 1 km horizontal scale can give much heavier rainfall. In the low-rainfall area the precipitation is probably snow that turns to rain at the melting level (Meyer, 1971a, 1971b). Incidentally, clouds in a hurricane are probably strained by the rapid swirl into the spiral band pattern seen on radar screens or in satellite photographs. The spiral bands give visualization to parts of the strain pattern, rather than streamline pattern. A rainband persists typically for one-to-two hours (Gentry, 1964).

In addition to rainfall, storm surge, large waves, and high winds, tropical cyclones can spawn tornadoes and waterspouts of moderate intensity (Orton, 1970). These are usually reported for the region outside the domain of hurricane winds, but this may only mean that twisters are more easily discernible from the general vortical intensity in the outer portions.

One would expect tornadoes to be most closely associated with the intense convection and large swirl of the inner rainbands. Most tornadoes are reported for the right forward quadrant, with respect to an observor looking along the direction of translation. This seems plausible because for North Atlantic hurricanes that quadrant is also the most severe with respect to rainfall, rainbands, and winds (Hawkins, 1971). As previously noted, only near the center are tropical cyclones axisymmetric to good approximation; near the outer edges there is asymmetry. The additive translational contribution to the aximuthal winds has been cited as one plausible source of the asymmetry in wind speeds (Riehl, 1954, p. 290). But Riehl also notes that the largest radial inflow occurs in the right forward quadrant once translational effects are subtracted out. It may be remarked that the so-called beta-plane effect (variation of the Coriolis force with latitude for fixed velocity) is not negligible over a system as large as a hurricane, which is normally at least of diameter 1000 mi.

The longest recorded lifetime for an Atlantic tropical cyclone was Ginger (September 5 - October 5, 1972); of this 31-day lifespan, 20 were spent as a hurricane. Previously, Carrie (1957) had lasted 18 days as a hurricane and Faith (1966) had lasted 26 days as a system (Simpson and Frank, 1972).

Hurricanes tend to weaken moderately rapidly over land. The central pressure of Camille (1969) rose from 905 to 990 mb in about 13.5 hr after land fall (Bradbury, 1972).[2] Agnes (1972), a minimal hurricane with 986 mb central pressure and maximum winds of 75 mph with gusts to 95 mph over the Gulf of Mexico, was downgraded to a tropical storm eight hours after crossing the Florida panhandle (Anonymous, 1972). In an interesting report

Grossman and Rodenhuis (1955) cite Hurricane Able (1952) and Hurricane Diane (1955) as examples of hurricanes which only weakly interacted with their environment and still maintained appreciable energy, circulation, and precipitation rates after passing inland. For example, 24 hours after Diane passed inland over the Carolinas on August 17, 1955 the central pressure had risen from about 985 mb to 1000 mb, and the core rainfall had decreased from 9.1 cm/day to half as much. Convective instability in the core decreased, and maximum rainfall occurred 75 mi from the center. However, in the next 24 hours the central pressure began to fall a few millibars and the central rainfall at the center of the storm began to increase; the convective instability in the core increased again. Hurricane Diane, although inland, encountered no major orographic changes during this time while moving northeastward. Grossman and Rodenhuis emphasize the uncertainty in numerical modeling of the role of latent-heat release in complicated situations in which soundings indicate layers of stable and of convectively unstable air (Section 3.10). This lack of knowledge renders detailed analysis of how tropical cyclones die inland difficult.

Matano and Sekioka (1971), after examining several typhoons near Japan, seem to suggest that a tropical cyclone gradually decays without strong interaction with other atmospheric systems if it moves into extratropical regions where there is no marked midlatitude baroclinicity. If there is marked baroclinicity, the tropical cyclone will tend to interact with extratropical cyclones, either pre-existing or else induced on fronts passing close to the typhoon core.

1.5 Path Prediction

Tropical cyclones move westerly in the trades, usually at 15 to 25 mph, before turning poleward, often at greater translational speeds. Except when the westerlies are furthest north in midsummer, one frequently observes a recurvature eastward in the midlatitudes along the western side of high-pressure cells (Riehl, 1972a); usually a decrease in intensity follows recurvature (Riehl, 1972b). However, there are so many special circumstances that many exceptions could be cited to virtually every generalization about path. For example, coexistent tropical cyclones of comparable size and intensity in the same hemisphere rotate about one another (Fujiwhara effect) (Brand, 1970b), while binary systems in different hemispheres tend to move parallel (Cox and Jager, 1969) -- as suggested by classical hydrodynamic potential-flow theory for line vortices. Prior passage, within a month, of a previous cyclone can also have an effect (Brand, 1971). Tropical cyclones can interact with extratropical cyclones; some typhoons are large enough to alter anticyclonic highs, the result on path being similar to that just cited for coexistent typhoons in different hemispheres (Palmén and Newton, 1969). Riehl (1954, p. 289) notes that hurricanes produce long waves that travel three times as fast as the storm and provide early warning. When hurricane swell arrives at a coastline, the normal wave frequency of 10 to 15 per minute, is reduced to two to four; the wave direction can be interpreted to yield the path.

Tropical-cyclone path prediction is in an imperfect state. Since forecast errors involving the intensity, rainfall, and movement of hurricanes can have serious consequence, the subject is pursued here briefly. For 1971, short term (12-36 hours) path forecasts by the National Hurricane Center were best made by the use of past climatological and analogue data (what did previous hurricanes in a similar situation do?). Methods based on historical data are referred to as "objective." For 1971, for long term (48-72 hours) NHC found methods based on dynamical principles superior (R. Simpson, private communication). Today the average error for 24-hour predictions of hurricane movement is 129 n mi; the average landfall error for a 24-hour prediction is about 100 n mi. The reduction is due to the closer monitoring of tropical storms as they approach the Atlantic and Gulf coasts (Simpson, 1971). The less accurate forecasts often entail faster moving storms, and storms at latitudes poleward of the trades, where recurvature may occur.

Many dynamical techniques treat the tropical cyclone as a point vortex steered in a current, which has been smoothed to remove the influence of the circulation of the storm itself. The steering current may be the flow at a specific level, usually in the mid- or upper troposphere [Byers (1944, p. 447) cited 10,000 feet]. More recent work emphasizes the use of a steering layer; Riehl (1954, p. 345) advocated the use of a pressure weighted mean flow from the surface to 300 mb, over a band 8° latitude in width centered on the storm, to predict hurricane direction and speed. Today prognostic flow is sometimes used to predict path. Further, the barotropic model of Sanders and Burpee (1968) averages over the depth of the troposphere from 100 to 1000 mb, and does <u>not</u> involve reduction of the tropical cyclone to a point vortex. Despite such improvements, the outlook for reducing path forecast errors by more sophisticated dynamical prediction models is not favorable:

> Numerical models for predicting the movement and development of hurricanes remain a frail source of guidance to the hurricane forecaster for three reasons. First, an error in

direction of movement as small as 8 to 10 degrees -- nominally an acceptable one in a 24-hour forecast for extratropical storm centers -- can yield disastrous results in hurricane warnings if followed literally. Second, the performance of most hurricane prediction models depends significantly upon the initial direction of movement of the center, which in turn depends upon an exact knowledge of the current center position and the position 6 and 12 hours earlier. The average positioning error is more than 20 nm, and often leads to initial direction errors of 15-20 degrees. Finally the forecaster remains hard put to identify and diagnose the frailties of numerical prediction models for individual forecasts. All too often this has led to near abandonment of the guidance materials and the application of empirical and individual experience factors in decision making. (Simpson, 1971, p. 1.)

1.6 The Importance of Tropical Cyclones in the Global Circulation

Lorenz (1966, p. 409, 418) suggested that hurricanes were of secondary, rather than primary, importance in the global circulation of the atmosphere (Fig. 4). However, he noted that it was unclear how great a role hurricanes play in maintaining the currents of larger scale. He also noted that hurricanes often do not appear in numerical simulations of the general circulation on advanced digital computers. Of course, appearance is hardly to be expected when restraints on computing time severely limit resolution; for example, the Mintz-Arakawa model is currently treated for a five-degree longitudinal and a four-degree latitudinal spatial grid with a six-minute time step, such that about twenty minutes

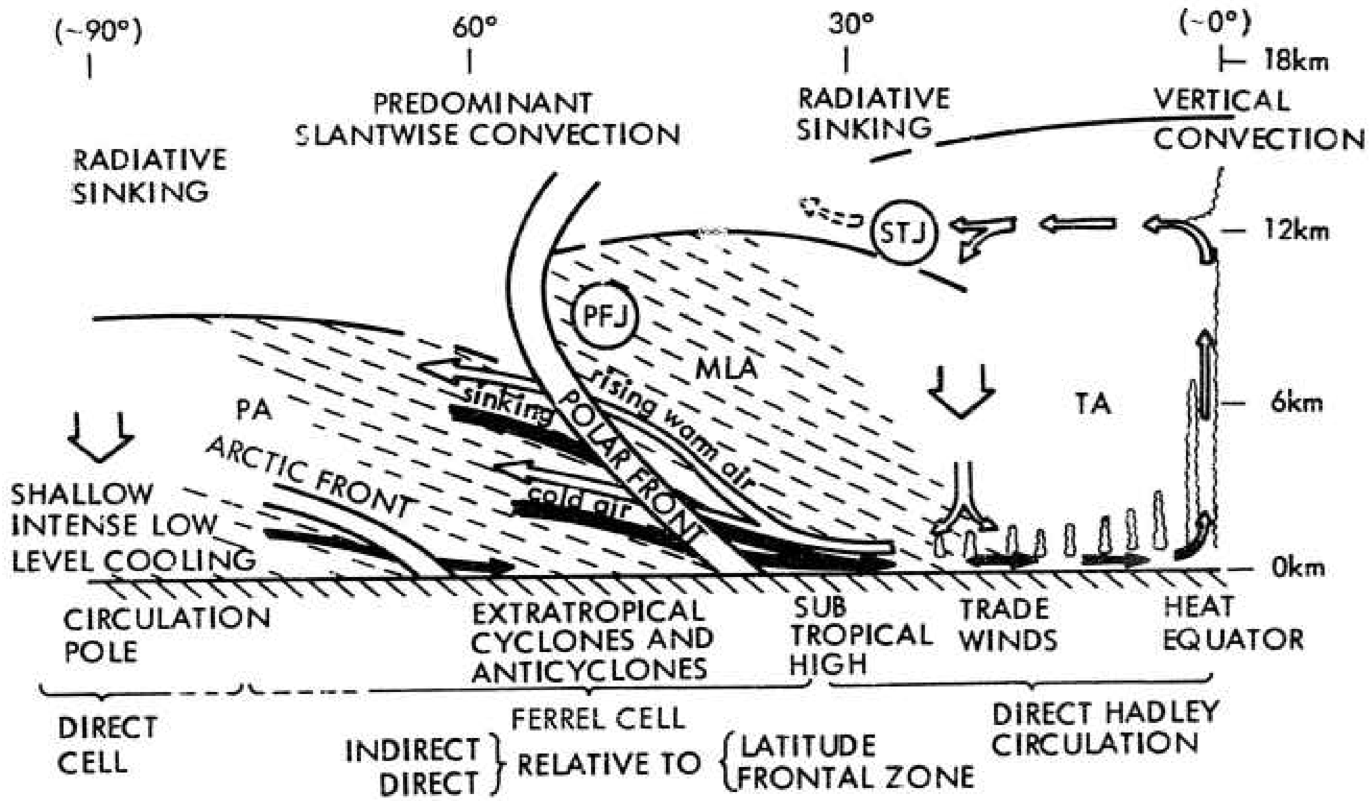

Fig. 4. Schematic features of the atmosphere in winter. Latent heat evaporated from tropical oceans to the lower-level trades is carried toward the ITCZ by the sketched meridional flow; the air rises as the ITCZ is approached, and the latent heat ultimately is released as sensible heat and gravitational potential energy. Height of the local tropopause is marked. Portions of the general circulation are less well defined in summer. TA denotes tropical air; MLA, midlatitude air; PA, polar air; STJ, subtropical jet stream; and PFJ, polar front jet stream. (By permission from E. Palmén and C. W. Newton, Atmospheric Circulation Systems, Academic Press, p. 569.)

of computer time is needed to simulate a day of climate (Rapp, 1970; Gates, Batten, Kahle, and Nelson, 1971). The resolution for disturbances of dimension less than 1000 n mi is poor, but two weeks of computer time per day of simulated climate would be required on most computers if satisfactory resolution on a 100 n mi scale were sought. Meteorologists at the Rand Corporation in Santa Monica, California planned to program the two-level Mintz-Arakawa model of the general atmospheric circulation for the advanced ILLIAC IV computer. Even if such plans are realized, a grid no finer than 1° x 1° seems practical; thus, if and when hurricanes are added to the global circulation model, their role may have to be introduced by parameterization guided by more fundamental studies (such as a subroutine with finer grid activated on appropriate occasions). After all, cumulus convection, radiational cooling, and turbulent mixing must currently be parameterized into computer simulations of tropical cyclones themselves.

Manabe, Holloway, Jr., and Stone (1970), in their computer simulation of the global circulation, do observe the two-week intensification of tropical disturbances of scale 2000 km - 3000 km near the ITCZ in regions where tropical cyclones are observed; further, there is low-level convergence and upper-level divergence. However, the resolution is not great, the pressure fell only to about 980 mb, and the disturbance appears more like subtropical cyclones (warm core above 800 mb, especially above 400 mb, and cold core below 800 mb). Appreciable doubt remains whether these disturbances are to be identified as hurricanes; Bates (1972) discusses possible inadequacies in the parameterization of cumulus convection adopted.

Some observational evidence has been presented that in certain regions during certain seasons hurricanes do play a role in the global circulation. From interpretation of satellite photographs of tropical storms and typhoons for 1967 - 1969, Erickson and Winston (1972) have suggested that substantial heat and moisture transfer from tropical cyclones to midlatitude westerlies plays a role in the autumnal buildup of the planetary-scale circulation. A tropical storm is believed to transport energy from the lower to upper tropical atmosphere; the energy is then carried to fronts in the midlatitudes in broad extensive cloud bands extending northeastward over 20° latitude. The warming so conveyed to the extratropics intensifies the midlatitude zonal westerlies around the 300 mb level. When the Hadley-cell circulation is weak in late summer, perhaps hurricanes do locally serve as a substitute mechanism for conveying energy from the tropics to the extratropics.

That hurricanes could locally and seasonally play such a role seems at least possible. First, some models of hurricane structure require for self-consistency an export of heat from the storm to the rest of the atmosphere. For example, various models require an export of roughly 10^{26} ergs/day (Palmén and Riehl, 1957; Erickson and Winston, 1972); unless this energy (in the form of sensible heat), produced by transformations within the hurricane, is removed by other members of the general circulation via high-level advection on the periphery of the storm, the models break down (Riehl, 1954, p. 338; Palmén and Riehl, 1957, pp. 158-159). Anthes and Johnson (1968), by applying the theory of available potential energy, estimate that about 4×10^{24} ergs/day are contributed by a hurricane to the global scale. Second, a hurricane has been estimated to possess a kinetic energy comparable to that of a hydrogen bomb, or 4×10^{23} ergs (Battan, 1961; see also Appendix A), and one has rained over 95 inches on one location in four days (Silver Hill, Jamaica in November, 1909) (Alaka, 1968). The daily production of condensed water for precipitation in a mature hurricane has been estimated at 1.6×10^{16} g (Ooyama, 1969, p. 29). A weather system with such energy and water content, that extends radially hundreds of miles and vertically from sea level to the tropical tropopause, and that persists for weeks, would seem no local accident. Third, hurricanes occur annually, mainly in the autumn after the long heating of the tropical oceans by solar radiation, and hurricanes do turn poleward after drifting westward in the trades. These facts invite the previously stated speculation that hurricanes are seasonally part of a substitute mechanism for relaxing energy poleward when the Hadley-cell mechanism is not sufficient. If so, then the paradox discussed by Bates (1972, pp. 2, 14) is perhaps resolved: "The Hadley cell of the summer hemisphere is weak or non-existent. . .. There seems to be a paradox in the fact that while the Hadley cell is most intense in winter, the frequency of oceanic tropical disturbances, which one would expect to be an important contributor to its rising branch, is greatest in summer." Even if temporally and longitudinally varying eddy transfer associated with pressure troughs, rather than the zonally symmetric mean meridional (Hadley) cell, is the primary mechanism by which moisture and angular momentum are transported from the tropics to the midlatitudes (Riehl, 1954, chapter 12; Riehl, 1969a), still, a significant portion of the eddy transfer occurs in the upper troposphere. The mode by which quantities to be transferred are convected to appreciable height in the tropics is cumulonimbus clouds, and, as discussed later, these occur in abundance in hurricanes. The basic suggestion remains: hurricanes may play some role in the fall in the export of energy from the tropics to the midlatitudes by what they export themselves, and by what they convect to the upper tropospheric levels in the tropics for other mechanisms to export.

However, on an annual and global basis, vertical transfer in the tropics is carried out by smaller-scale, but more numerous convective systems (Palmén and Newton, 1969, p. 572). In fact, it is now suggested that only about one percent of the total energy annually exported from the tropics to the midlatitudes is conveyed by hurricanes. For convenience, suppose in a year there are 100 hurricanes, each lasting 18.25 days and covering 10^6 mi^2. The surface area of the tropics (with 30° serving as the limit) is about 10^8 mi^2. The fraction of the total annual total static enthalpy H transferred by hurricanes is

$$R = \frac{(100)(18.25)10^6\ (\partial H/\partial Z)_{o,h}}{(365)10^8\ (\partial H/\partial Z)_{o,a}} \tag{1}$$

where the numerically unassigned quantity in the denominator is the ambient sea-level transfer rate, and that in the numerator is the <u>rate above ambient</u> associated with hurricanes (the transfer coefficients have been cancelled in numerator and denominator). According to Carrier, Hammond, and George (1971), the ratio of the two unassigned quantities is about 0.2, so $R \approx 0.01$. The value of R would be larger if the model of Malkus and Riehl (1960) were used. These authors assert that at the sea surface in a moderate-intensity hurricane (966 mb central pressure) the Bowen ratio (ratio of sensible to latent heat transfer) is about 0.2, and that the latent heat transfer is augmented by a factor of 10 - 12 over the ambient level in the trades.

2. ASPECTS OF TROPICAL METEOROLOGY

2.1 Stability in the Tropical Atmosphere

Stability in the tropical ambient atmosphere introduces consideration of circumstances rare in the extratropics. For convenience these properties of the tropical atmosphere curcial to tropical cyclogenesis are introduced at this point.

Stability is conveniently described in terms of the total static enthalpy H where

$$H \equiv c_pT + gz + LY; \tag{2}$$

here c_p is the heat capacity of the atmosphere (effectively that of dry air, and independent of temperature for present purposes); T is static temperature; g the magnitude of the gravitational acceleration; z height about sea level; L the relevant latent heat of phase transition for water substance; and Y the mass fraction of water vapor. Throughout this paper, whenever H is (loosely) described as a temperature, reference is to (H/c_p). Actually the total enthalpy of a fluid particle is the total stagnation enthalpy

$$H_t \equiv c_pT + gz + LY + q^2/2 \tag{3}$$

where q is the magnitude of the velocity vector (the wind speed).

The magnitude of the various contributions in the tropics to H and H_t are worth mention. As shown in Fig. 5, the static enthalpy decreases

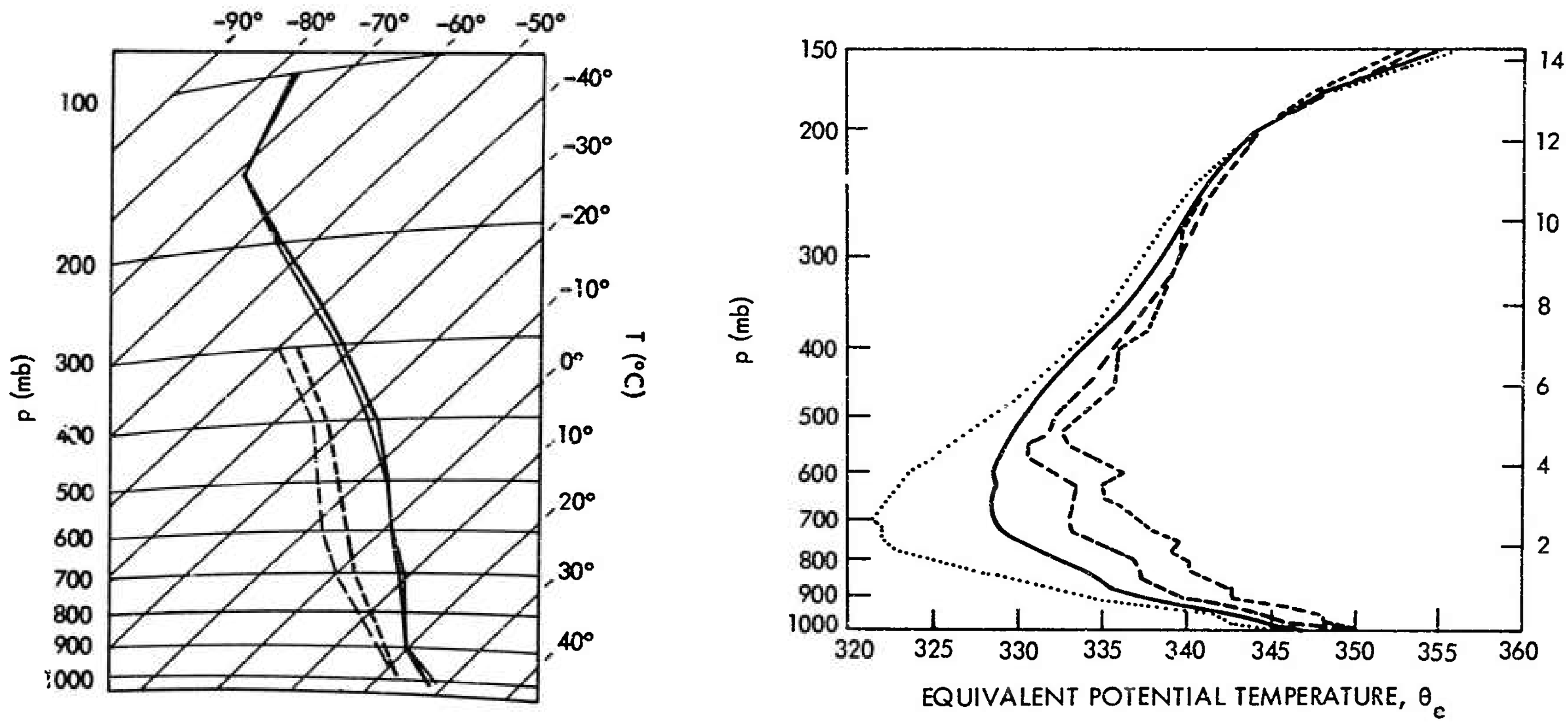

Fig. 5. Left, the equivalent potential temperature θ_e as a function of height z and pressure p, from measurements near Barbados in the Lesser Antilles in July-August, 1968. Four characteristic weather types are represented: average (solid), suppressed convection (dotted), moderately enhanced convection (long dashes), and strongly enhanced convection (short dashes). (From Warsh, Echternacht, and Garstang, 1971, p. 127; with permission of the American Royal Meteorological Society.) Right, soundings taken at Gan Island in the Indian Ocean (00°41'S; 73°09'E). The heavier curves (solid for static temperature, dashed for dew point) are the average for 42 wet days; the lighter curves are the average for 113 dry days. Measurements were made in Julys 1960-1964. (From Johnson, 1969, p. 122; with permission of the Royal Meteorological Society.)

from a sea level maximum of about 350°K (L = heat of condensation plus fusion) to a midtropospheric minimum of about 330° at about 650 mb (around 13,000 ft), before recovering the sea level value of 350° just below the tropical troposphere (that is, at almost 100 mb or 50,000 ft). This midtropospheric minimum is observed with monotonous regularity throughout the tropics in all seasons, although the local sea-level and near-tropopause maxima are reduced from the 350° level in the equatorial trough to 340° near the subtropics. Over the lowest few tens of millibars there may be a layer of well-mixed air in which H is virtually constant, but this is of no major consequence here. The dynamic contribution of $(q^2/2)$ is always small, and usually negligible; for example, even in a 200-mph hurricane, $(q^2/2c_p) \doteq 4°K$, or just over a 1% contribution to (H/c_p) near sea level. For stability one can virtually always consider H, rather than H_t. At sea level, the static temperature contribution T is about 300°K near the trough, the latent heat contribution (LY/c_p) is about 50°K, and, of course, the gravitational-potential-energy contribution (gz/c_p) is zero. The moisture content of the tropical ambient falls off roughly exponentially with increasing height; the e-folding distance is crudely 650 mb. Whatever modest variation in H normally occurs from day to day over a locale in the tropics is more likely due to changes in water vapor content than to changes in temperature (Johnson, 1969, pp. 122-123); the lapse rate alters very little as one moves from cloudy areas to clear areas, but there is a drop in the dew point (Gray, 1972b, p. 14). In any case, certainly by 400 mb (or 25,000 ft) for current purposes $H \doteq \theta$, where the potential-temperature-like quantity (actually an enthalpy) θ is defined by

$$\theta \equiv c_p T + gz. \tag{4}$$

A quantity describing the stagnation potential-like enthalpy may also be defined:

$$\theta_t \equiv c_p T + gz + q^2/2. \tag{5}$$

At 50,000 ft the gravitational contribution (gz/c_p) is about 150°K, or over 40% of θ (or H), and hence no longer negligible.

Clearly H plays a role much like the equivalent potential temperature used by most meteorologists, but H is preferred here. For one thing, what a dry-bulb and a wet-bulb thermometer measure (of relevance below) is readily understood in terms of H_t and θ_t. However, unless L is constant, H is not rigorously a thermodynamic state variable. However, L varies slowly with temperature and H remains a very useful concept.

The condition that convection is absent (mechanical equilibrium) is that the entropy increases with height. If air is approximated as a perfect gas, the unsaturated atmosphere is stable if $(\partial H/\partial z) > 0$, convectively unstable if $(\partial H/\partial z) < 0$. If the atmosphere is saturated. then the atmosphere is stable if $(\partial H/\partial z) > 0$, statically unstable if $(\partial H/\partial z) < 0$. Convectively unstable means a perturbed particle will resume its former position unless lifted enough for the onset of condensation, in which case it will rise until its density discrepancy relative to ambient is reduced to zero (i.e., rise to a new equilibrium position). Statically unstable means unstable without the requirement of a sufficiently large displacement. If the atmosphere is dry, or if for some reason one wants to consider stability excluding the role played by condensation (dry ascent as opposed to moist ascent), then the atmosphere is stable if $(\partial\theta/\partial z) > 0$, unstable if $(\partial\theta/\partial z) < 0$.

In the extratropics, H and θ both increase monotonically with altitude usually, so the atmosphere is stable to both dry and moist ascent (Fig. 6). Only in exceptional circumstances, as thunderstorms, does will penetrative convection, with its vertical (as opposed to slantwise) ascent, occur. In the tropics θ increases monotonically with height, but H (as previously noted) has a midtropospheric minimum (Fig. 7); thus air is stable to dry ascent, but air in the lower atmosphere is unstable to moist ascent (convectively unstable). How turbulent mixing, cumulus convection, radiational cooling, cumulonimbi (see below), and large-scale circulation maintain this condition of convective instability deals with tropical meteorology in general, and lies largely outside the scope of this review of tropical cyclones (although such distinction will probably soon prove unsound with incipient progress in tropical cyclogenesis). Actually full understanding of how the tropical ambient is maintained does not exist; the complexity is indicated by the fact that, as will now be explained, the H profile generates different types of clouds, which in turn sustain the H profile.

In the convectively unstable tropical atmosphere, the larger the lapse rate, the more suppressed is the cumulus activity. Only as the ambient lapse rate approaches the moist adiabat is there penetrative convection. For example, Malkus (1960) notes that $-(\partial T/\partial z) \sim 6.6°C/km$ from 900 to 200 mb in the tropics normally, but $-(\partial T/\partial z) \sim 6.0°C/km$ in the inner rain area of Hurricane Daisy (1958). There have been many confirmations since, that the less pronounced the midtropospheric minimum of the total static enthalpy in the tropics, the more convective activity is likely to be present (Garstang, LaSeur, Warsh, Hadlock, Peterson, 1970; Aspliden, 1971).

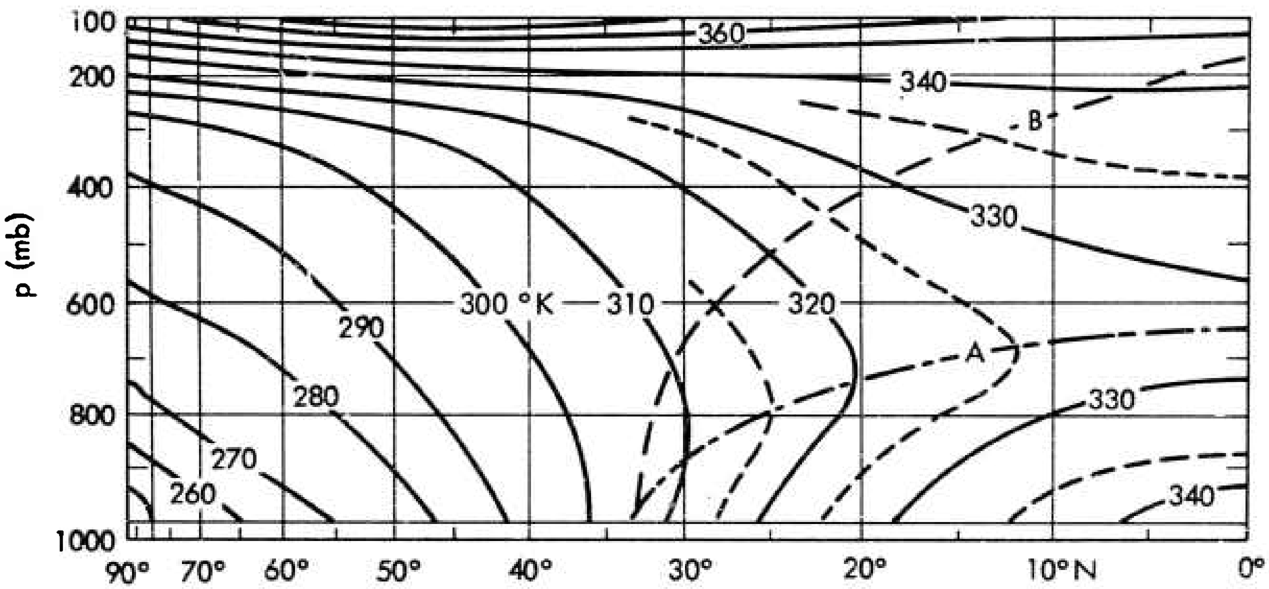

Fig. 6. The average total static temperature (H/c_p) for the Northern Hemisphere in winter. The curve marked A denotes the locus of the minimum of (H/c_p); free convection can most readily occur below level A, and only undilute ascent can continue much above A; undilute ascent continues to level B, where sea-level values are recovered. (By permission from E. Palmén and C. W. Newton, <u>Atmospheric Circulation Systems</u>, Academic Press, p. 574.)

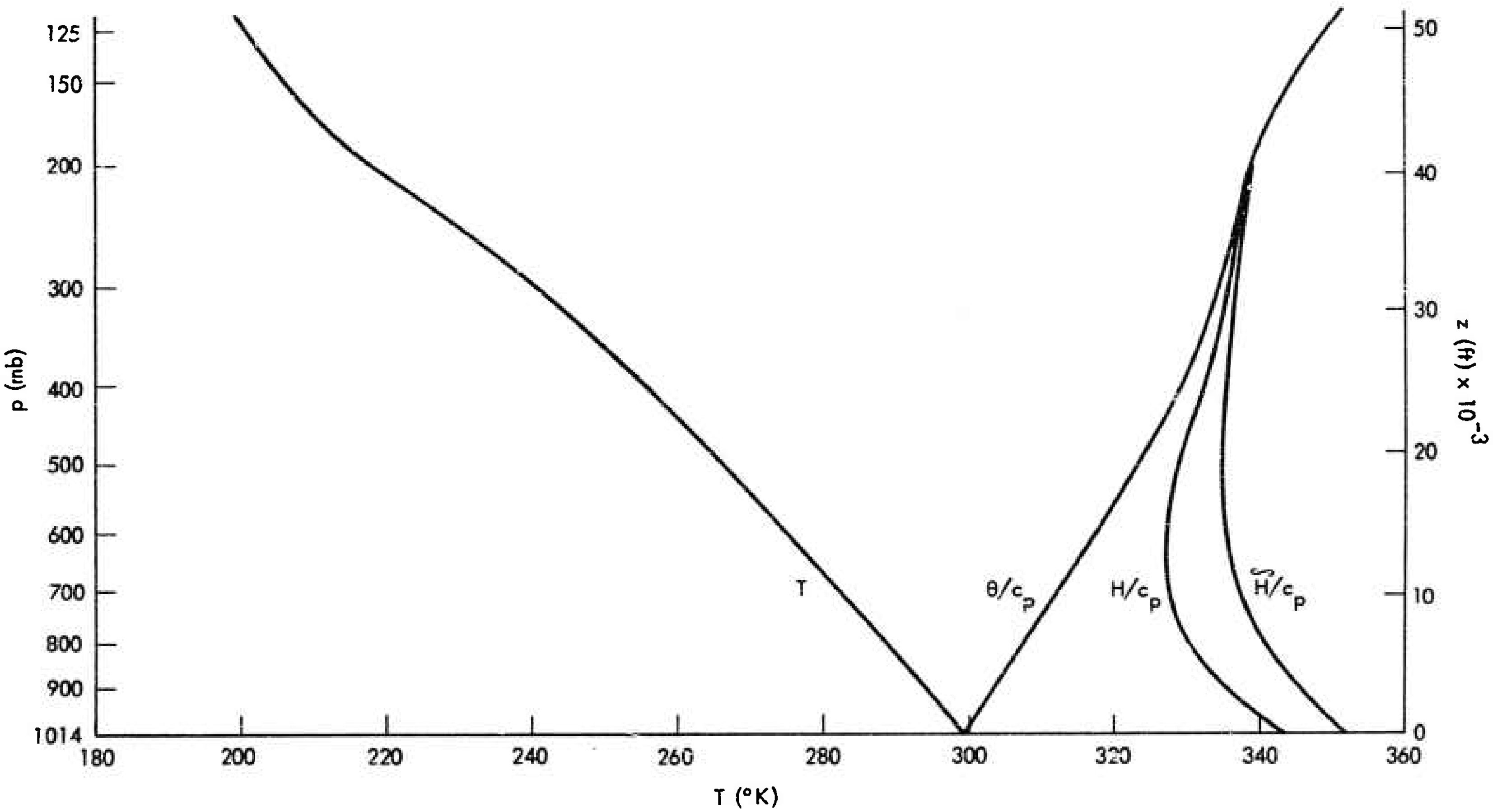

Fig. 7. Static temperature T, potential-temperature-like measure (θ/c_p), total static temperature (H/c_p), and the total static temperature for a hypothetical atmosphere saturated at the actual local temperature and pressure $(\tilde{H}/c_p)$ vs. height and pressure for the West Indies ambient for September. Based on data given by Jordan (1957). A parcel at height z_1 will become buoyant at z_0 where (H/c_p) at z_1 exceeds $(\tilde{H}/c_p)$ at $z_0 (>z_1)$; the inequality is satisfied for air in the planetary boundary layer, and little else.

If a glob of very low-level tropical air (buoyant element) is displaced vertically enough for condensation to occur, ascent will continue many kilometers virtually to the tropical tropopause, under the stability criteria just presented, if the ascent is rapid enough for no mixing or radiational cooling to occur during ascent. Near-sea-level air would rise dry adiabatically until saturated (roughly, 50 mb); thereafter, enough precipitation would fall out to leave the air just saturated at the local pressure and temperature, the air retaining the latent heat of phase transition. Such a locus of thermodynamic states is conventionally referred to as the moist adiabat; for rapid ascent in which ambient processes are too slow to act, $H \doteq$ const.[3] (Air that has risen to its level of neutral buoyancy will not be unstable to descent, because any compressional heat from work done on the fluid by gravitational forces cannot be absorbed by the condensed water substance, which has precipitated out. However, descending flow will accompany ascent, and this will be addressed in Section 2.3.)

The core of a towering cumulonimbus is described by a moist adiabat. The tropical ambient must be close to moist adiabatic for cumulonimbi to be significant; this is in fact the case (Riehl, 1969a, 1969b). If the ambient were moist adiabatic, cumulonimbi would be unnecessary because the ambient would be too unstable (there would be gross convective lifting). If the ambient were far removed from moist adiabatic, cumulonimbi would be nonexistent because the atmosphere would be too stable (there would be no lifting, just heating up of the air).

2.2 Tropical Cumulonimbi

About one percent of the area within the tropics is receiving precipitation on average; this area is likely to be in the equatorial trough, but is not uniformly distributed (Riehl, 1969b). The precipitation is concentrated in large-scale disturbances consisting of a few cumulonimbi immersed in a larger number of cumuli (Figs. 8 and 9); Gray (1972b) estimates that 10% - 20% of the rain areas are covered by active cumulonimbi. Agglomerations of cumulonimbi are sometimes described as cloud clusters. Cumulonimbi tend to align themselves in east-west rows, or bands; the bands can be tens-to-hundreds of kilometers long (Kuettner, 1971), and are typically 20 km to 50 km wide (Charney, 1972). This east-west alignment is consistent with the low-level wind field of large-scale disturbances in the tropics. The ITCZ, itself composed of one-or-more narrow bands of vigorous cumulonimbus convection, is typically of width 300 km (Charney. 1971). It is emphasized that the ITCZ, while a region of variable winds, appreciable cloudiness, large precipitation, and low pressure (Godshall, 1968), does not consist of a long unbroken band of heavy cloudiness. Rather, it consists of intermittent cloud clusters (with strong low-level convergence, appreciable vertical movement with precipitation, cyclonic vorticity, and upper-level outflow with possibly anticyclonic relative voriticity) with interstitial clear areas (with divergence, subsidence, and anticyclonic vorticity) (Holton, Wallace, and Young, 1971).[4] Gray (1972b) also emphasizes this patchwork nature of the entire tropical belt, which consists of 20% cloud area (which contains the cumulonimbi, that can rain at the rate of 2.5 gm/cm^2-day), and 40% variable cloud area and 40% clear area (this 80% of the area yields negligible precipitation). There tends to be moist ascent in the cloudy areas, and

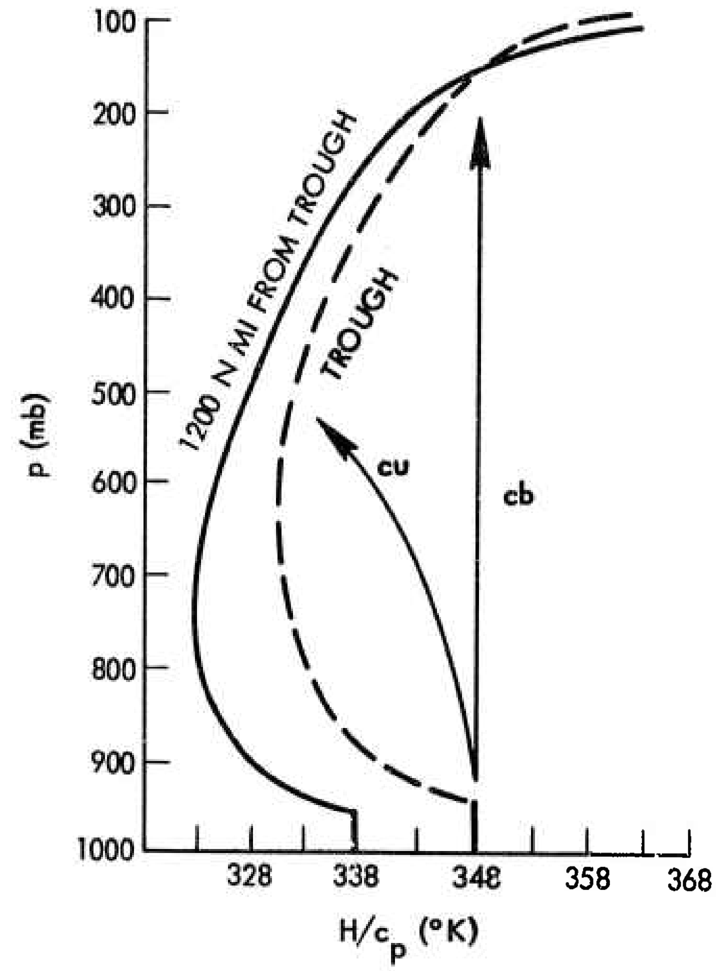

Fig. 8. The total static temperature (H/c_p) over the ocean in the equatorial trough and in the tropics near the subtropics, as a function of pressure. Both typical cumulus loci (cu) and also cumulonimbus loci (cb) are noted. (Based on, with permission, E. Palmén and C. W. Newton, Atmospheric Circulation Systems, Academic Press, p. 575.)

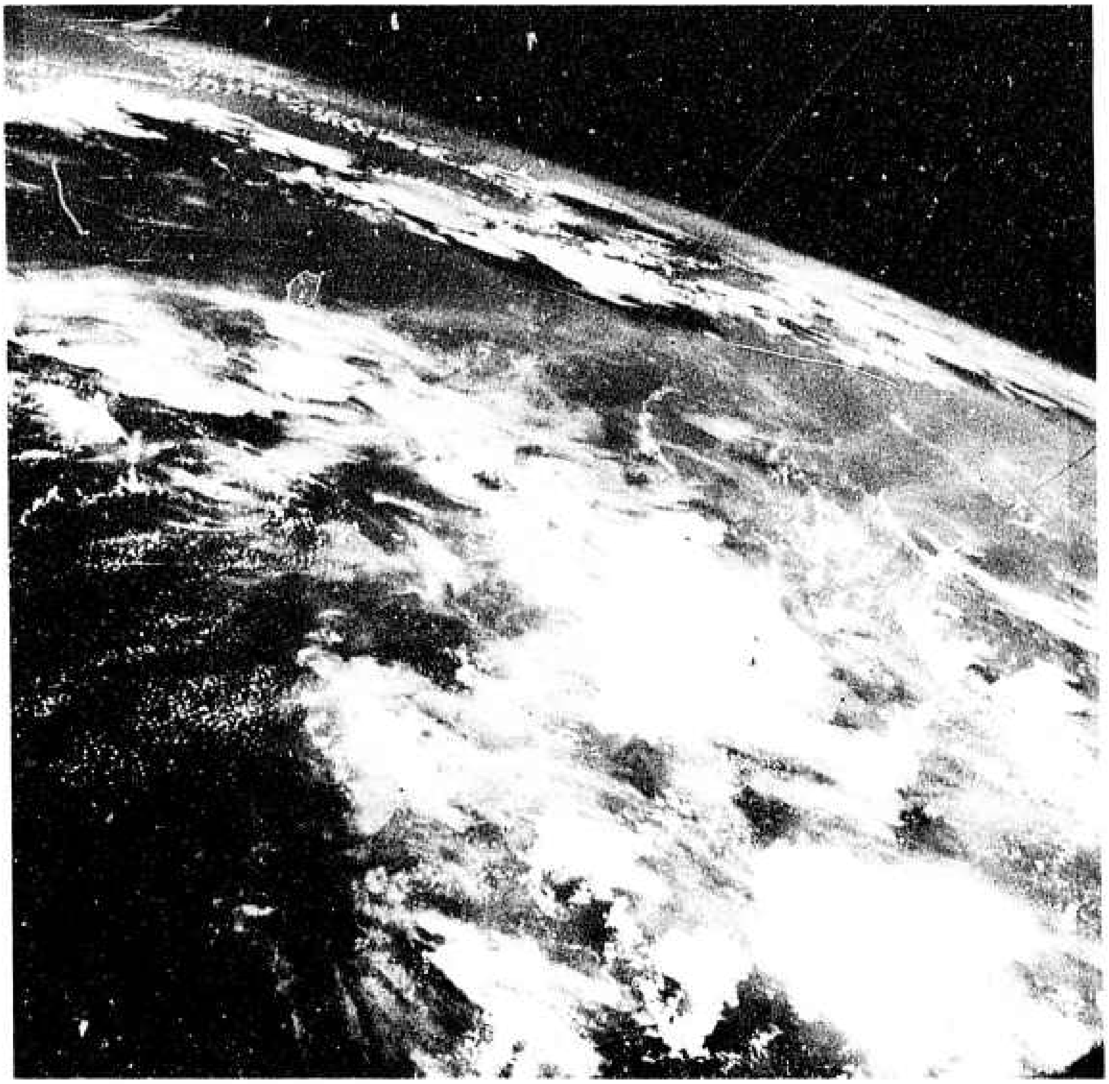

Fig. 9. NASA photograph taken in June, 1965 from Gemini IV of cumulus, cumulonimbus, and cirrus clouds over the Pacific Ocean off the western coast of Central America; the view is northeast toward Mexico.

dry descent in the less cloudy areas, though by evaporation of liquid water and by diffusion of water vapor, water substance is transferred from the cloudy to less cloudy regions.

These cloud systems are described here because in some still incompletely understood manner, they play a role in tropical cyclogenesis (Palmén and Newton, 1969, p. 585). Cumulonimbi are given particular attention because although they cover only one-thousandth (Palmén and Newton, 1969, p. 440) to possibly four-thousandths (Gray, 1972, p. 33) of the area of an equatorial belt extending 10° latitude on each side of the equatorial trough, they cover one-tenth of the area of the trough itself (Bates, 1972, p. 2). Tropical cyclogenesis often occurs near the equatorial trough, and cumulonimbi occur in concentration in the hurricane eyewall (Section 4.3). In fact, the absence of cumulonimbi in <u>subtropical</u> cyclones occurring off western India, over the Pacific, and over the Altantic serves as one means of distinguishing these storms (in which maximum intensity is reached in midtroposphere, and in which a warm core occurs at high levels but a cold core at low levels) from <u>tropical</u> cyclones (Walker, 1972). [Simpson and Pelissier (1971) discuss various so-called hybrid storms in and near the tropics, that may have hurricane-force winds at least transiently, but do not have conventional hurricane structure.][5]

A cumulonimbus (cb) is typified by a radius of 2 km, a lifespan of 30 to 40 min, a height of 14 km, a peak water substance content of 4 g/cm^3, maximum updraft speed of 25 m/sec, a thermal anomaly of 6°C, and a total rainfall of 2.4 cm. In contrast, a cumulus (cu) is typified by a radius of 1 km or less, a lifetime of 10 to 25 min, a height of 6 km, a peak water content of 1 g/cm^3, maximum updraft speed of 6 m/sec, a thermal anomaly of 4°C, and a total rainfall of 0.3 cm (Lopez, 1972a). Cumulonimbi are observed to overshoot their neutral-stability level and then oscillate about it; further, only the core of a cloud rises undilute because only the larger clouds reach upper-tropospheric levels (Malkus, 1960). Riehl and Malkus (1961) estimate that a cumulonimbus conveys about 6×10^{10} g/sec and that a fluid particle rises the full vertical extent in about 30 min.

In the summer tropics there is far more local ascent and descent than might be suggested by the mean synoptic-scale motion (Johnson, 1969; Gray, 1972b); in fact, if air in the equatorial trough rose only with the mean vertical flow, the ascent would be so slow that radiational cooling alone would stop it in midtroposphere (Palmén and Newton, 1969) and cumulonimbi would not exist. Gray (1972b) suggests that there is ten-to-twenty times the mean synoptic convergence in the lower summertime tropical atmosphere, and this is the source of the initial forced vertical displacement that leads to condensation and permits free convection to continue undiluted ascent in the unsaturated conditionally unstable ambient conditions. Without one or two orders of magnitude more up-and-down recirculatory local motion than suggested by the mean synoptic-scale motion, the water substance distribution in the summertime tropics (in which individual cumulonimbi can rain at 2.5 cm/day even though the evaporation rate at the sea surface is a uniform 0.5 cm/day) is inexplicable. This large local recirculation may have implications for the momentum balance because it would seem to inhibit vertical-wind-shear development.

Malkus and Riehl (1961) described the tropical cumulonimbus as a hot tower because sensible heat from condensation was supposedly available to warm the ambient surrounding air. Lopez (1972b), however, asserts that cooling tower would be more apropos because the condensational heat is primarily used to raise the air in the column to its level of neutral stability (i.e., the heat goes into potential energy). The air that is detrained from the cloud serves to cool the environment because the heat extracted from the ambient to re-evaporated condensed moisture in the detrained air exceeds any sensible heat transferred to the surrounding environment. Measurements around cumulus activity indicate that the static temperature often falls (e.g., Riehl, 1969b, p. 588). Of course, the ascent in a cloud cluster does indirectly engender drying and warming through the subsidence of surrounding air that inevitably accompanies ascent in the cloud; this heating counteracts heat loss to detrainment and radiation. Thus on a large scale cumulonimbus activity generates a net warming, but the effect on the locally surrounding air is cooling (Gray, 1972b). Most models parameterizing the role of cumulus convection in large-scale disturbances have the direct sensible-heat-transfer model, rather than the indirect warming-by-subsidence mechanism, in mind (see Section 3.10). The Riehl-Malkus hot-tower model is inconsistent with the existence of large local recirculatory motion (Gray, 1972b).

2.3 CISK

Among the most important and fruitful concepts introduced in tropical meteorology in several decades is an idea due to Charney (Charney and Eliassen, 1964; Charney, 1971); the concept of CISK (conditional instability of the second kind), in addition to whatever contribution it may make to extratropical meteorology, seems of great utility in describing tropical phenomena of widely different scales, ranging from a single cumulonimbus to hurricane rainbands to cloud clusters to tropical cyclones to the ITCZ (the intertropical convergence zone is, for present purposes, taken as identical with the equatorial trough, the doldrums, and the trade confluence). In all these phenomena there is low-level convergence with large moisture content in the presence of appreciable Coriolis force. Phillips (1970) notes that while the physical processes involved in CISK can be qualitatively described, much remains to be done with respect to detailed quantitative description (Geissler, 1972).

The CISK process is the feeding of convective activity in a swirling flow by frictionally-induced inflow in a surface boundary layer. The idea is that rotating flows like that in a local low over the ocean tend to suppress convergence except in frictional boundary layers (about a kilometer or two thick and continguous to the ocean). In this frictional layer there is radial inflow because the Coriolis and centrifugal forces (which balance the radial pressure gradient above the boundary layer) are diminished by friction; in the frictional layer radial advective transport responds to the "partially unbalanced" pressure gradient. The radial inflow in the boundary layer is furnished by slight sinking in the swirling flow above the boundary layer, in those regions where the swirl speed decreases with radial distance from the center of the low. In the tropics the converging boundary layer flow is warm and moist, because of transfer from the warm ocean during inflow and because the boundary layer air is warm and moist at the outset. The boundary layer flow erupts near the center of the disturbance in the region where the swirl speed of the fluid above the boundary layer increases with radial distance. In the conditionally unstable tropical atmosphere the core of the erupting, rotating column can rise vertically without dilution to the tropical tropopause. [Buoyant elements from higher in the boundary layer may have somewhat lower total static enthalpy, and may rise to a level of neutral buoyancy somewhat lower than that of sea-level air. In fact, a buoyant element may contain air from a wide range of the boundary layer thickness; the core of the element, which does not mix with outside air, may not mix internally either, so there may well be shedding off of fractional pieces at various upper-tropospheric levels. This refinement need not be pursued here. But it should be noted that there is good laboratory evidence that entrainment into a buoyant plume is reduced appreciably with increasing rotation rate of the environment, for all but very small rotation rates (Emmons and Ying, 1967).] The hydrostatically computed weight of the convective column, which thermodynamically is described by a moist-adiabatic locus of states, is taken to be less than the weight of a column of nearby air. The difference in weight of the two columns implies a radial pressure gradient, which (by the conservation of radial momentum) in turn implies swirling. The swirling leads to further downflux into the surface frictional layer, further inflow, further moist adiabatic ascent, and hence maintenance (or even augmentation) of the radial pressure gradient. A crucial additional point about the CISK process is that it furnishes the basis of explaining how, instead of competing, cumulus-scale activity can cooperatively interact with cyclone-scale activity to their mutual enhancement.

The emphasis in CISK on both Coriolis force and conditional instability suggests that both the ITCZ and tropical cyclogenesis occur near, but not at, the equator. The critical point of the CISK process is that the weight of the convective column is lighter than that of the ambient gas. From a facile view, the release of the appreciable heat of condensation during moist adiabatic ascent can only lighten the column, and the existence of a radial pressure gradient is obvious. However, Gray (1972b) has recently viewed this typical summary of the CISK mechanism as an incomplete description of certain tropical phenomena, because it fails to emphasize the actual indirect heating mechanism associated with cumulus activity and consequentially fails to emphasize the large local up-and-down recirculation above the mean synoptic motion (Section 2.2).

Another problem with quantitative analysis of the CISK process is that the surface frictional layer over the oceans may not be adequately described even in time average by a linear quasi-Ekman-layer theory, and particularly not at low latitudes. Gray (1972a) notes that dissipation of kinetic energy in the tropical frictional layer so exceeds production that indeed there must be mesoscale and synoptic-scale sinking of new momentum-carrying fluid to sustain the boundary layer. However, Gray claims that, contrary to Ekman's theory, measured planetary boundary-layer thickness does not increase with decreasing latitude; he suggests vertical stability and turbulence scales, rather than rotation, determine the boundary-layer thickness. Gray's general remarks, it may be noted, are not addressed to the exceptional circumstances of a tropical cyclone. Mahrt (1972a, 1972b) has attempted theoretically to demonstrate the sensitivity of tropical-disturbance development to the details of the sub-cloud-level moisture convergence; especially equatorward of the ITCZ where the Coriolis parameter is small and its latitudinal variation is large, nonlinear advective acceleration may so alter the boundary-layer flow that spiraling may be in the opposite sense to that predicted by Ekman's theory. The thickness, latitudinal variation, and strength of cross-isobar flow may deviate from what would be predicted by linear analysis.

3. MODELS OF A MATURE TROPICAL CYCLONE

3.1 Introduction

In sorting out the thermohydrodynamics of a tropical cyclone, one is faced with understanding the interaction of two scales of phenomena: the larger cyclone scale ($\sim$500 to 1000 mi), and the smaller cumulus scale ($\sim$1 mi). The cyclone must feed the cumulus scale, which in turn sustains the cyclone scale, in a cooperative interdependence. Such ideas suggest that the CISK process is not only relevant to the maintenance of an individual cumulonimbus cloud, but also (properly scaled) to the maintenance of the tropical cyclone in the large (Section 2.3).

The cyclone-scale thermohydrodynamics for the <u>mature stage</u>, taken to be axisymmetric and quasisteady for most purposes to a lowest order of approximation, will be set forth in terms of a model introduced in the last three years by Carrier and his co-workers (Carrier, 1970; Dergarabedian and Fendell, 1970; Carrier, Hammond, and George, 1971; Carrier, 1971a; Carrier, 1971b; McWilliams, 1971; Dergarabedian and Fendell, 1972b). These articles delineate <u>overall</u> dynamics and thermodynamics, and imply (in contrast to other models to be introduced later) no major augmentation of ambient sensible and latent heat transfer is needed to explain hurricanes. These articles emphasize that many details and refinements in structure still remain to be quantitatively treated.

3.2 The Carrier Model

Subsequently (Section 4) transient analysis will be devoted to how a severe vortical storm may be generated in the tropics. Here, the quasi-steady mature hurricane is studied.

The Carrier model, on the basis of subdividing the tropical cyclone into segments where different processes and scales predominate, is a four-part analysis. The four regions, indicated in Fig. 10 are the throughput supply I, the frictional boundary layer II, the eyewall and efflux region III, and the eye IV. Some of this subdivision is conventional, some not. Besides clarifying locally dominant physical processes, subdividing permits retention of the minimal number of terms in locally valid quantitative formulation; this procedure simplifies the mathematical solution in a manner unavailable to any direct finite-differencing of uniformly valid equations.

The Carrier model is closed for convenience -- there is no very significant amount of mass convected across any boundary, although mass may be diffused across any boundary. The cylindrical-like volume encompassing the entire storm has the sea surface for its bottom; its sides lie far enough from the center (about 500 to 1000 mi) so that the winds are virtually reduced to ambient, and the swirl relative to the earth is taken as zero for compatibility with the Ekman condition for no radial inflow across the outer boundary. The top of the storm is taken to be that height ($\sim$150 mb) at which sea-level air in the outer part of the storm, if lifted rapidly so that the total enthalpy of a fluid particle remained constant because relatively slow ambient-maintaining processes would not have

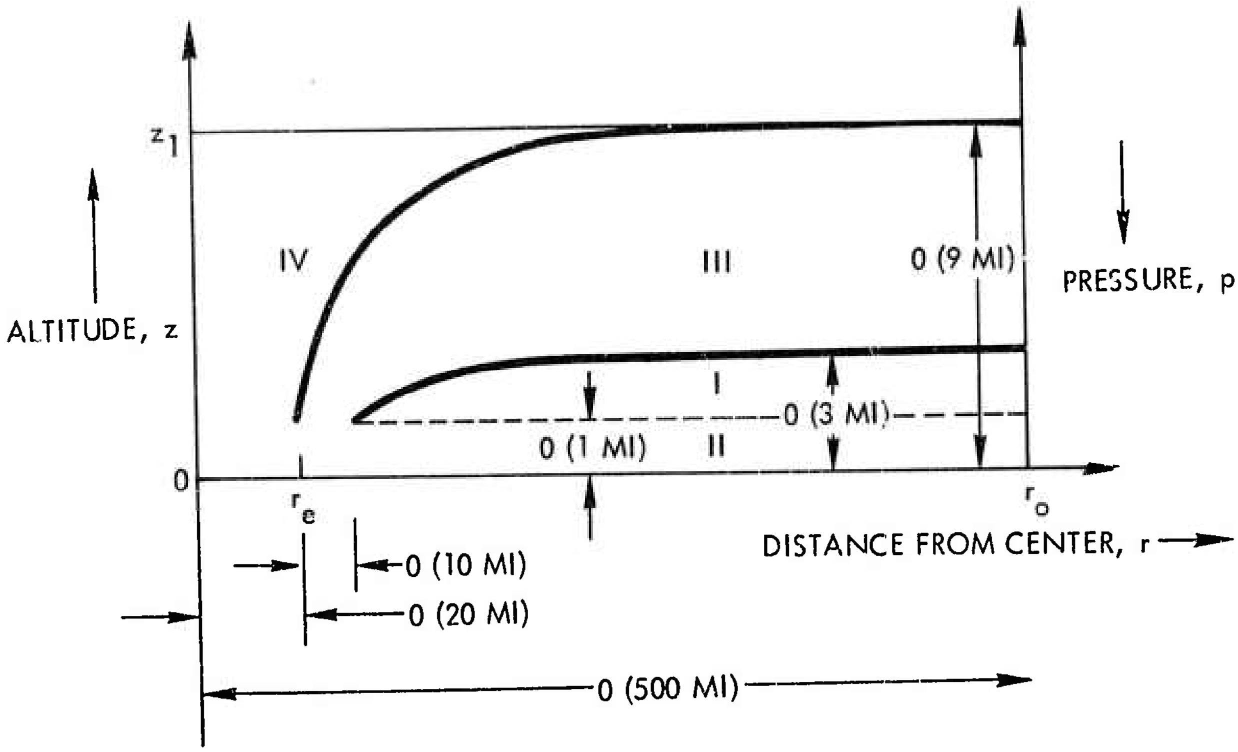

Fig. 10. This conjectured configuration of a mature hurricane with rough order-of-magnitude dimensions is not drawn to scale. The sub-domains are: I, throughput supply, a region of rapid swirl and very slow downdraft; II, frictional boundary layer; III, eyewall; and IV, eye. Across the boundary layer there is about a 100 mb drop, and across I, a further drop of about 200 mb; the pressure at the top of the hurricane is about 150 mb, i.e., the top is near the tropopause. The eye-eyewall interface is taken to slope outward, though the effect may not be so pronounced as sketched.

time to act, would no longer be unstable relative to the local ambient air. Such rapid lifting is, of course, the moist adiabatic ascent discussed in Section 2.1. This "instability lid" lies at so great a height that there is virtually negligible swirl, as explained below; the ambient pressure and temperature at this height is taken to describe all radial positions at this height, from the center to the outer edge. Thus, the top of the storm is an isothermal, isobaric, constant-altitude lid with no water vapor content for current purposes.

Discussion now turns to describing each of the four regions comprising the tropical cyclone in some detail.

In region I there is warm moist air typical in stratification of the ambient atmosphere in which the hurricane was generated. This air spun up under conservation of angular momentum as it moved in toward the axis of symmetry during the formative stage. As the mature stage was approached, a gradient-wind balance of pressure, Coriolis, and centrifugal forces chcked off any further inflow; the inflow is only enough to prevent the eyewall III from diffusing outward, and that requires only an exceedingly small radial flow. The air in I, then, is rapidly swirling, the azimuthal velocity component greatly increasing and the pressure greatly decreasing from the edge to the center. Under such a radial profile for the swirl, there is a small downflux from the throughput supply I into the frictional boundary layer II. The small downflux leads to a large net mass flux into the frictional layer because of the large area involved. Furthermore, the downdraft is only a gross temporal and azimuthal average because locally and transiently there is intense convective activity by which clouds form and rain falls.

In the frictional boundary layer II, the only region in which angular momentum is not conserved but is partially lost to the sea, there is appreciable influx. In fact, the azimuthal and radial velocity components are of comparable magnitude; typically, for fixed radial position, the maximum inflow speed at any axial position in the boundary layer is about one-third the maximum azimuthal speed. [This fraction is about the one reported by Hughes (1952) from flight penetration of hurricanes at altitudes of 1000 feet or less.] The vertical velocity component is <u>much</u> smaller. The reason for the inflow is, as previously discussed, that the no-slip boundary condition reduces the centrifugal acceleration, and a relatively uncompensated pressure gradient drives the fluid toward the axis of symmetry (so-called "tea cup effect"). Far from the axis in II the classical balance of the linear Ekman layer (friction, pressure, and Coriolis forces) suffices; since the downdraft from I to II is probably fairly independent of radial position (especially far from the eyewall) for swirl distributions of practical interest, three-quarters of the flux inward through II comes from downflux across the interface between II and I where $(r_0/2) < r < r_0$, where r_0 is the radial extent of the storm. Closer in to the axis the nonlinear accelerations, especially the radial acceleration of radial and tangential momentum and also centrifugal acceleration, must enter.

As the pressure gradient in I lets more air sink into II, the influx into II drives boundary layer air moderately rapidly up a cloudy eyewall III. In the eyewall hydrostatic and cyclostrophic approximations hold; the locus of thermodynamic states is the moist adiabat based on sea-level conditions in the eyewall (this particular point of the Carrier model will be examined in the critique to follow). The swirl is reduced in III owing to boundary layer friction. Near the top of III the flow moves further away from the axis of symmetry such that, in the outflow, the air seems to an observor on earth to be rotating opposite in sense to the rotation in I and II (which is cyclonic). The air in III slips over the air in I with no interaction; there is no large radial pressure gradient in much of III, unlike I.

At low altitudes the eyewall flushes moist air out of IV, and at high altitude rained-out air is entrained into IV. In time the eye becomes drier and better defined; it is the central core in which relatively dry air sinks, is warmed by compression, and is entrained out into the eyewall or recirculated within the eye (stagnation at the base of the eye would permit transport processes to cool the eye). The relatively light eye permits much greater pressure deficits from ambient and hence supports much higher swirling speeds. At a fixed altitude, the density in the eye is less than that in the eyewall, which in turn is less than that of the ambient gas at the storm edge. Since (except in cumulonimbi) the hydrostatic approximation is uniformly valid, spatially and temporally, in a hurricane, the sea-surface eye pressure is less than the sea-surface eyewall pressure, which is less than the sea-surface ambient pressure. While Carrier has not explicitly written so, it would seem that as a general trend, the smaller the eye radius, the more spin-up under conservation of angular momentum. Hence the more intense storm might generally have a relatively small eye, the eye broadening as the storm weakens.[6]

Carrier's model thus pictures the tropical cyclone as a once-through process in which a "fuel supply" -- the warm moist air in I, which is part of the tropical cyclone at its inception and is convected with the storm -- is slowly exhausted. The storm weakens because the air drifting down into the frictional layer toward the end is typical of the higher tropical environment and hence of lower total static enthalpy. Eventually the fuel supply is exhausted, and the boundary between III and I sinks toward II. Some models (e.g., Eliassen and Kleinschmidt, 1957) picture a recirculation through the storm of outflow air; the storm does not survive long enough for this, nor could such recycled air maintain the storm.

There is one important omission to the foregoing description that has been intentionally deferred: the energetics of the surface frictional layer and associated questions of air/sea transfer of latent and sensible heat. The relevant quantity to consider is the total stagnation enthalpy (the sum of static enthalpy, the heat associated with condensible moisture, gravitational potential energy, and kinetic energy contributions). This quantity is conserved at roughly its ambient stratification throughout regions I and II; therefore, it is described by a profile that decreases with height from 1000 mb to 650 mb, and then increases with height, as mentioned above. The implication is that the heat and mass transfer from the ocean to the atmosphere is <u>about the same</u> within the hurricane as in the ambient. This transfer helps compensate for the rain-out in the spiral bands and helps maintain the warm, moist nature of the air in I. [Occasionally the Carrier model is still grossly misrepresented as proposing adiabatic conditions (constant total stagnation enthalpy in II so the net heat and mass transfer from sea to air is zero); such a solution cannot possibly satisfy the parabolic boundary-value problem describing the energetics of the frictional boundary layer because it obviously violates the initial condition at $r = r_0$, the outer edge. In fact, if the supplemental flux from the ocean is entirely eliminated, as from passage over land, the spin-down time is $O(a/\nu^{1/2}\Omega^{1/2})$ where the eddy viscosity $\nu \doteq 10^{-2}$ mi^2 hr^{-1}, the normal component of the rotation of the earth $\Omega \doteq 2 \times 10^{-1}$ hr^{-1}, and the height of the throughput supply a $\doteq$ 1 mi -- so the spin-down time is half a day to two days.] The model of total stagnation enthalpy fixed at its ambient stratification breaks down in the eyewall III; there the vertical velocity component is at least one, probably two orders of magnitude larger than the relatively small downdraft into the boundary layer; the result is that convection dominates the slow ambient-sustaining processes, so for at least portions of the eyewall the total stagnation enthalpy is virtually constant at its local sea-level value, which (as just explained) is roughly its ambient sea-level value.

3.3 Critique of the Carrier Model

A closed system seems naturally definable since a single hurricane probably does not interact significantly with the entire planetary atmosphere, especially when structure rather than path is under consideration. However, the closed system is considered optional by Carrier because the same conclusions would also hold for an open system, although the arguments would be far more difficult to construct. Clearly treatment of asymmetric effects should be a future goal of the Carrier model since complete elucidation of the spiral-band phenomena and other properties are probably unattainable otherwise. Initial emphasis on axisymmetry has been the traditional path of development for tropical-cyclone models.

The sketch in Fig. 10 is schematic, but it will appear below that in lieu of a greatly augmented oceanic sea-to-air latent-and-sensible heat transfer, the Carrier model relies upon the pressure deficit achievable by dry-adiabatic recompression in the eye to be available to maintain dynamic equilibrium in a system with wind speeds known to reach 200 mph. Thus, some small but finite outward sloping of the eyewall at least down to midtropospheric levels, if not lower, is required for internal consistency in the Carrier model. Such ideas were present in the work of Haurwitz (1935) and in the early papers of Palmén (1948).

The particular emphasis placed on careful scrutiny of the dynamics and energetics of the frictional inflow layer is a major contribution of the Carrier model. [Incidentally, the thickness of this layer is too great to let increased frictional drag explain the accelerated decay after landfall, unless the coastline is mountainous. Even then, the orographic effects are more likely to be felt in deleterious premature rain-out owing to lifting of I, which would result in relatively dry air descending into the boundary layer and running inward toward the eyewall, so that moist-adiabatic ascent would no longer be possible. The rate of decay of intensity owing to sea-to-air enthalpy-transfer reduction after landfall, according to the Carrier model, was discussed earlier. The fact that friction effects the low-level moisture convergence suggests small increases in friction may even cause intensification.] However, the most novel contribution of the Carrier model is identification of the "fuel supply" in I, which requires only ambient-level sea-to-air total-stagnation enthalpy transfer for maintenance. That the fluid in I does sink down into II is suggested by the observation by Gentry (1964, p. 64) that at lower altitudes the temperatures are lower in the outer rainbands than in the surrounding air. There is little undilute ascent from the inflow to the outflow layer. Rather, even the air which ascends in the outer rainbands also descends again in the storm area, and is <u>not</u> immediately carried away through the outflow layer in the upper troposphere.

The grossest feature of the Carrier model is the absence of any refinement to the eyewall structure. While for decades modelers have presented mean soundings in the eyewall that suggest moist adiabatic ascent from the surface layers (Palmén and Newton, 1969, pp. 477-482), Shea (1972) emphasizes that the ascent is limited to cumulonimbi that cover only 10 to 20% of the eyewall area. Subsidence occurs in the regions of the eyewall outside cumulonimbi. Shea, from flight data that include intensifying and decaying as well as mature tropical cyclones, asserts that the relative humidity probably is not 100% throughout the eyewall, and there is a small midtropospheric minimum in mean vertical profiles of the equivalent potential temperature (far less pronounced than the ambient minimum). In fact, wet-bulb effects may have led to spuriously low temperature measurements at midtropospheric heights (Gentry, 1964). In any case, the failure to delineate this structure does not have significant repercussions for Carrier's work on the mature hurricane, but does have important implications on his work on intensification, to be discussed later (Section 4). Shea also suggests that the gradient-wind balance does not hold to good approximation in the eyewall; if so, this is probably due to a contribution from the transient partial derivative of the radial velocity component, and is not taken to modify analyses of a mature hurricane appreciably.

In the following sections three specific analyses among the many carried out by Carrier and his co-workers to help substantiate the model will be briefly reviewed. The three analyses involve maximum swirl speed estimation, the dynamics of a nonlinear Ekman layer, and the energetics of the surface frictional layer. The quantitative results achieved to date concerning the Carrier model have been attained without large-scale digital computation. The emphasis on subdivisional investigation of the four regions of the storm, with interfacial compatibility, permits a substantially analytic approach. It should be noted that although certain linearizations are sometimes adopted for tractability by Carrier and his co-workers in the course of their analysis, Carrier's model for the mature hurricane is definitely nonlinear, and solutions derived by such linearizations are acceptable only if, a posteriori, they can be demonstrated to satsify the original nonlinear boundary-value problem with acceptably small error. Without prior proof of internal self-consistency by subdivisional analyses, a full numerical treatment of the basic boundary-value problem would seem premature.

3.4 Maximum Swirl Speed Estimate According to the Carrier Model

An upper and lower bound on the central pressure deficit achievable in a known spawning atmosphere will now be set forth by use of the Carrier hurricane model, of hydrostatics, and of the thermodynamics of moist and dry air. Specifically, the weights of various columns of air in the storm will be determined in light of different moisture content and thermodynamic processes involved. The bounds on the central pressure deficit can then be translated into an estimate of bounds on the maximum swirl speed through dynamics (the radial momentum equation). Fletcher (1955) had suggested use of the cyclostrophic balance once pressure deficits were known, and Malkus (1958) had suggested that pressure deficits could be calculated from moist adiabatic considerations for the eyewall and dry adiabatic considerations for the eye. Here the concepts are combined to achieve quantitative bounds, but just as important, to demonstrate that hurricane speeds could be achieved without requiring any enthalpy transfer from the ocean greatly in excess of the ambient transfer, provided the eyewall is not perfectly vertical. Miller (1958) performed somewhat similar calculations to those to be described and also noted that the Riehl-Malkus postulate of large oceanic latent and sensible heat transfer to air flowing in through the frictional layer was not required to explain low central pressure.

The first step is to neglect the frictional boundary layer II, which is relatively thin and across which, except for hydrostatic variations, the pressure does not change according to lowest-order boundary layer theory.

The variation of pressure p, density ρ, and temperature T with height above the ocean z, for any ambient tropical atmosphere in which a hurricane forms, may be computed from

$$P_a = \rho_a R_a T \qquad (a = \text{dry air}); \tag{6}$$

$$P_v = \rho_v R_a T/\sigma \qquad (v = \text{water vapor}; \sigma = 0.622); \tag{7}$$

$$p = p_a + p_v, \ \rho = \rho_a + \rho_v, \ p_v = P(T)\ (RH); \tag{8}$$

$$\frac{dp}{dz} = -\rho g; \tag{9}$$

$$T = f(p), \ RH = g(p), \tag{10}$$

where the temperature profile f(p) and the relative humidity (RH) profile g(p) are taken as known from measurement. The saturation pressure P(T) is well-tabulated for vapor and liquid phases above freezing, and vapor and solid below freezing (Keenan and Keyes, 1936); a convenient and accurate expression for P(T) in mb was given by Tetens (Murray, 1967):

$$P(T) = 6.1078 \exp\left[\frac{a(T - 273.16)}{(T - b)}\right], \tag{11a}$$

$$a = 21.8745584,\ b = 7.66 \quad \text{over ice;} \qquad a = 17.2693882,\ b = 35.86 \quad \text{over water,} \tag{11b}$$

where T is in °K. The integration proceeds from the sea level upward in altitude z; data typically extend from about 1000 mb to 150 mb (Fig. 11). The top of the storm is normally taken as the height at which the ambient total stagnation enthalpy (for which the kinetic energy contribution is negligible) recovers its sea-level value, as noted earlier; here, however, a slightly different procedure explained below will be used. The sea-level ambient state is henceforth denoted by subscript s.

In a fully developed storm the air rising up the eyewall ascends in cumulonimbus clouds, and thus follows moist adiabats. The initial states to be used for the moist adiabats are not necessarily known a priori. <u>If</u> one believes the total stagnation enthalpy is constant along a streamtube in the surface frictinal layer from the ambient to the eyewall, then the sea-level tropical ambient state may characterize the total stagnation enthalpy of the streamtube rising in the eyewall closest to the storm center (procedure A). [Actually, mixing inevitably occurs so perhaps a lower total stagnation enthalpy characteristic of some height above sea level should be used, but the small distinction is not worth the effort (Carrier, 1971b, p. 158)]. While such a choice for the initial state of an eyewall moist adiabat is sometimes made, and while such results will be presented here, another <u>preferable</u> procedure B will also be developed. In this alternative procedure, the temperature and relative humidity of the sea-level state of the moist adiabat will be taken as known, but the initial pressure will be taken as unknown (to be determined by iteration for self-consistency to be explained below).

There is also a comment worth noting concerning the equation that describes the moist adiabat. If one simply takes $dH_t = 0$ with L held constant (Charney, 1971, pp. 357-358), where again H_t is the total stagnation enthalpy, then after manipulation with (6) - (9):

$$\frac{dT}{dp} = \frac{\frac{1}{\rho} + \frac{L\sigma}{p^2x^2}P - \frac{d(q^2/2)}{dp}}{c_p + \frac{L\sigma}{px^2}\frac{dP}{dT}}, \quad x = 1 - \frac{(1-\sigma)P}{p}. \tag{12}$$

The $[d(q^2/2)/dp]$ contribution is negligible and is henceforth dropped. A more careful derivation from basic principles (Appendix B) yields a slightly different expression

$$\frac{dT}{dp} = \frac{1 + \dfrac{\sigma LP}{R_a\, Tp(1 - \frac{P}{p})}}{\frac{\gamma}{\gamma-1}\left(\frac{p}{T}\right)\left[1 + \dfrac{\sigma L}{c_p\, p(1 - \frac{P}{p})}\dfrac{dP}{dT}\right]} . \tag{13}$$

However, if one takes $p \doteq p_a$ [so $x = 1$ in (12)] and $\rho \doteq \rho_a$, which incurs an error of only about three percent at most at any point in the flowfield, then the two expressions are equivalent.

The procedure for computing the moist adiabat is to use the dry adiabatic relation $T \sim p^{\gamma/(\gamma-1)}$, $\gamma = 1.4$, from the sea-level eyewall conditions to that pressure at which RH = 1, at which point one switches to the moist adiabat and continues the calculation. The integration is terminated at that pressure p_1 for which the temperature calculated from the moist adiabat and from the ambient are equal; this temperature is denoted T_1, and the height above sea level at which T_1 occurs is denoted

z_1 (the "lid" on the cyclone). One then integrates (6) - (9) and the differential equation for the moist adiabat from $z = z_1$ (where $T = T_1$ and $p = p_1$) to $z = 0$; RH = 1 initially during this integration, but one switches to the dry adiabat where appropriate. Under procedure A the sea-level pressure for the eyewall $p(z = 0) \equiv p_e$ is that computed. Under the superior procedure B, an estimate of the sea-level eyewall pressure p_e had to be adopted to compute the moist adiabat, and this value must be recovered for convergence. If no eye existed in the vortex -- as seems to be the case for some tornadoes and waterspouts -- then the just-calculated $p(z = 0) \equiv p_e$, $\rho(z = 0) \equiv \rho_e$ would characterize conditions at the center of the vortex.

The lid on the storm as calculated here will be higher in procedure B than in procedure A, for a given ambient. The taller the storm, the more intense it is according the calculations developed here. The same relation holds observationally (Riehl, 1972, p. 248).

In a mature hurricane a pressure deficit in excess of $(p_s - p_e)$ is achieved by having rained-out air entrained from the eyewall sink in a relatively dry eye under adiabatic recompression. Thus in a hurricane $(p_s - p_e)$ is a lower bound on the central pressure deficit. For an upper bound on the deficit that may be achieved, one may adopt the idealized model that the eye is completely dry (so no compressional heat is lost to re-evaporation) and that the air entrained into the eye is drawn from the top of the eyewall (or, in any case, has $T = T_1$, $p = p_1$, $Y = 0$ at $z = z_1$). The relevant equations are (6) - (9), RH = 0, and $T \sim p^{(\gamma-1)/\gamma}$; integration in the direction of decreasing z yields $p(z = 0) \equiv p_c (< p_e < p_s)$ and $\rho(z = 0) \equiv \rho_c (< \rho_e < \rho_s)$ -- the density discrepancies so calculated are at most 25% and Fletcher (1955) estimates the density does not vary by even

15%, so the density may be held constant at its ambient value throughout the dynamical calculations now discussed.

If one adopts the cyclostrophic balance, holds ρ constant at (say) ρ_s, and (since the core is observed not to rotate) lets

$$v(r) = \begin{cases} 0 & 0 \leq r \leq R \\ (v)_{max}\,(R/r)^n & R \leq r \leq \infty \end{cases}, \tag{14}$$

then

$$(v)_{max} = \left[2n\left(\frac{p_s - p_c}{\rho_s}\right)\right]^{1/2} . \tag{15}$$

First, for a one-cell vortex (when there is no eye so the moist adiabat calculation is appropriate all the way to the axis), a rigid-body-like rotation lies near the core so then

$$v(r) = \begin{cases} (v)_{max}\,(r/R) & 0 \leq r \leq R \\ (v)_{max}\,(R/r)^n & R \leq r \leq \infty \end{cases}, \tag{16}$$

and from the cyclostrophic balance

$$(v)_{max} = \left[\frac{2n}{n+1}\left(\frac{p_s - p_e}{\rho_s}\right)\right]^{1/2} . \tag{17}$$

Clearly, (15) holds for the mature hurricane; the maximum swirl is larger in (15) than in (17) by the factor $(n + 1)^{1/2}$ because none of the pressure deficit from ambient needs to be expended to maintain rotation of the central column. Next, although power-law decays of swirl with radial distance are frequently adopted and suffice for current purposes, it will become evident that other forms are at least as plausible, and more convenient, for $r \geq R$. In any case, Miller (1967) suggested from limited data that $0.5 < n < 0.65$ usually suffices, and Riehl (1963) had chosen $n = 0.5$; earlier, Byers (1944, p. 435) had also recommended $n \doteq 0.5$ and Hughes (1952) considered $n \doteq 0.6 - 0.7$ suitable for an average hurricane. The upshot is that estimates will be made on the limits $n = 0.5$ and $n = 1.0$, although $n \doteq 0.5$ seems the more realistic. Finally, the gradient wind equation would probably be more appropriate than the cyclostrophic equation, but the more complicated formula would give maximum speeds reduced by only five percent from those obtained from the simple forms (15) and (17).

First, results using procedure A (i.e., taking the total static enthalpy H constant throughout the radial inflow as well as eyewall region) will be given. If H is held constant at sea-level ambient conditions [p_s = 1014 mb, T_s = 299.4°K, $(RH)_s$ = 0.84], then use of (12) for the moist adiabat gives condensation at 972 mb ($T \doteq$ 295°K, $z \doteq$ 180 ft). One finds $z_1 \doteq$ 48,150 ft, p_1 = 138 mb, T_1 = 203°K. The eyewall pressure at sea level p_e = 978 mb [with $T_e \doteq$ 296°K, $(RH)_e \doteq$ 0.975]. The eye pressure at sea level p_c = 894 mb (with T_c = 346°K); this upper bound reflects, it is reiterated, the idealization of a completely moisture-free eye. As a variant, if sea level ambient conditions were revised slightly for computing the moist adiabat only, [p_s = 1014 mb, T_s = 299.4°K, $(RH)_s$ = 1.0)], then use of (12) gives z_1 = 52,500 ft, p_1 = 110 mb, T_1 = 196.6°K. Also, $p_e \doteq$ 945 mb, T_e = 297°K; p_c = 850 mb, T_c = 352.7°K.

As an illustration of the change attendant upon using (13) in place of (12), one finds for H referenced to the unsaturated sea-level ambient [p_s = 1014 mb, T_s = 299.4°K, $(RH)_s$ = 0.84], saturation again occurs at 972 mb, $z \doteq$ 450 ft; however, z_1 = 46,500 ft, p_1 = 150 mb, T_1 = 205.4°K; further, p_e = 988 mb, T_e = 297.2°K, $(RH)_e$ = 0.94 (Figs. 11-13). If the

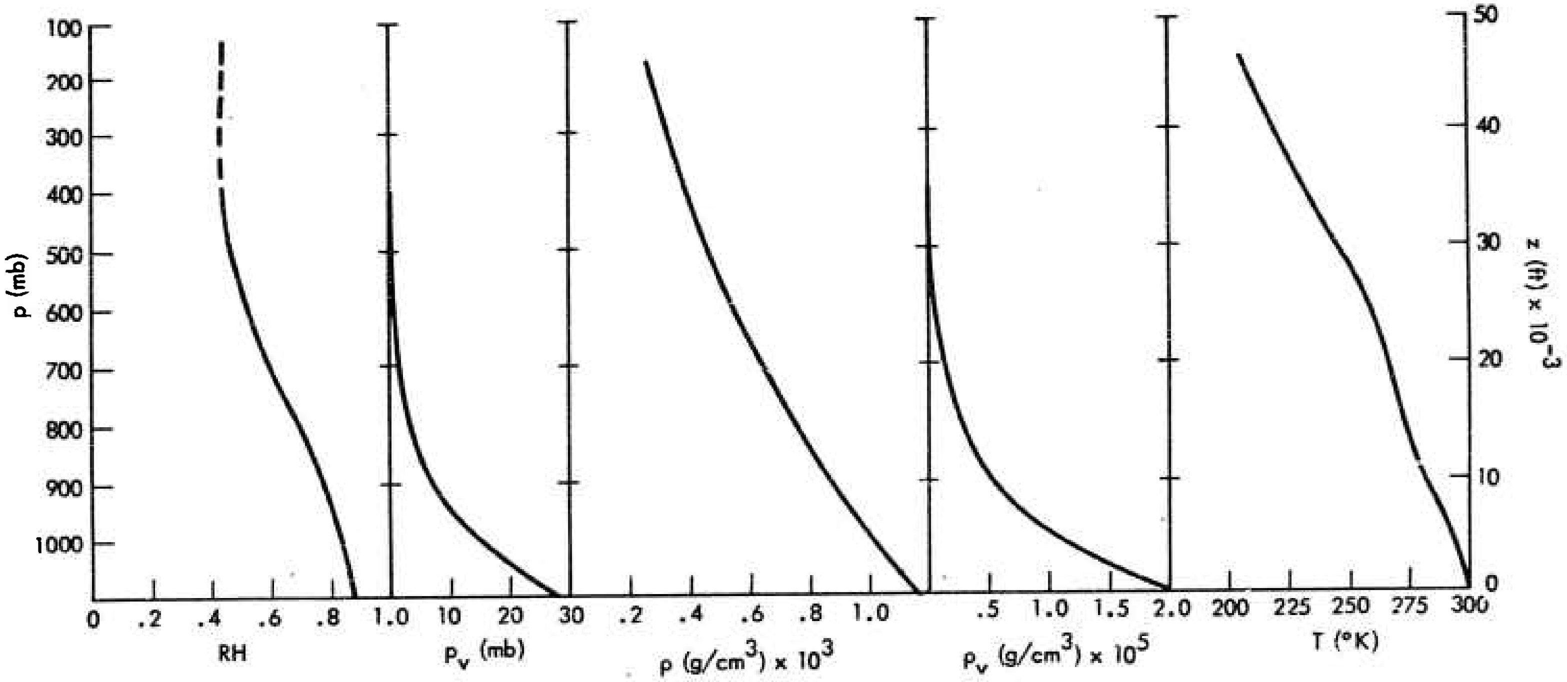

Fig. 11. Typical vertical profiles of thermodynamic state variables for the West Indies in September, based on data by Jordan (1957). The relative humidity RH, the water vapor pressure p_v, the total gas density ρ, the water vapor density ρ_v, and temperature T are plotted. Jordan's data for the relative humidity stop at 400 mb.

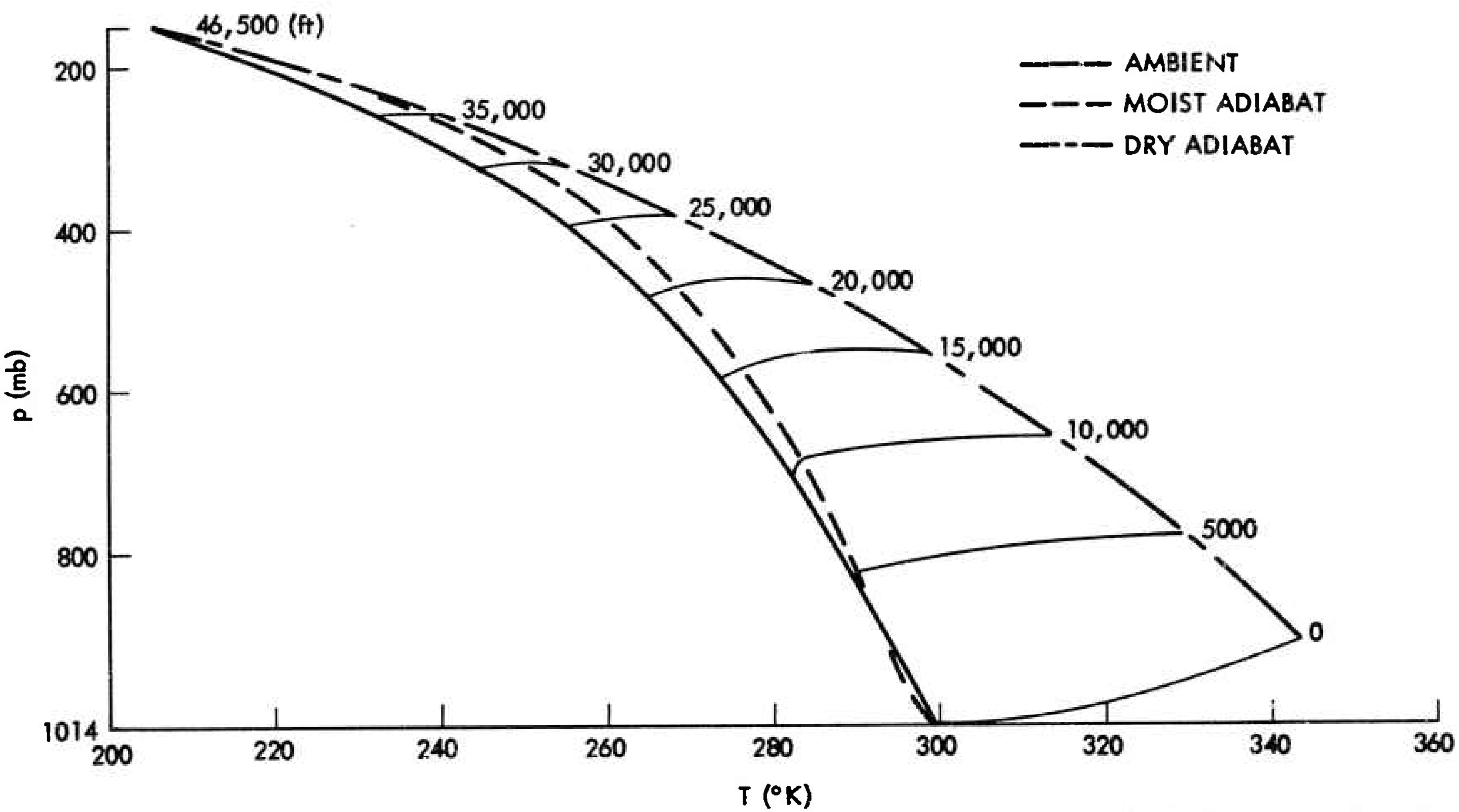

Fig. 12. The ambient pressure-temperature curve is based on data for the Caribbean in September given by Jordan (1957). The moist adiabat is based on having sea-level ambient air undergo dry-adiabatic expansion until saturation, then moist adiabatic ascent; the resulting sea-level pressure deficit from ambient gives a lower bound on the central pressure deficit under the adopted model. An upper bound on the deficit is furnished by having the air that rose on the so-called moist adiabat, recompressed dry adiabatically back down to sea level. Altitudes are associated with the thermodynamic states by use of hydrostatics and the equations of state for dry air and water vapor.

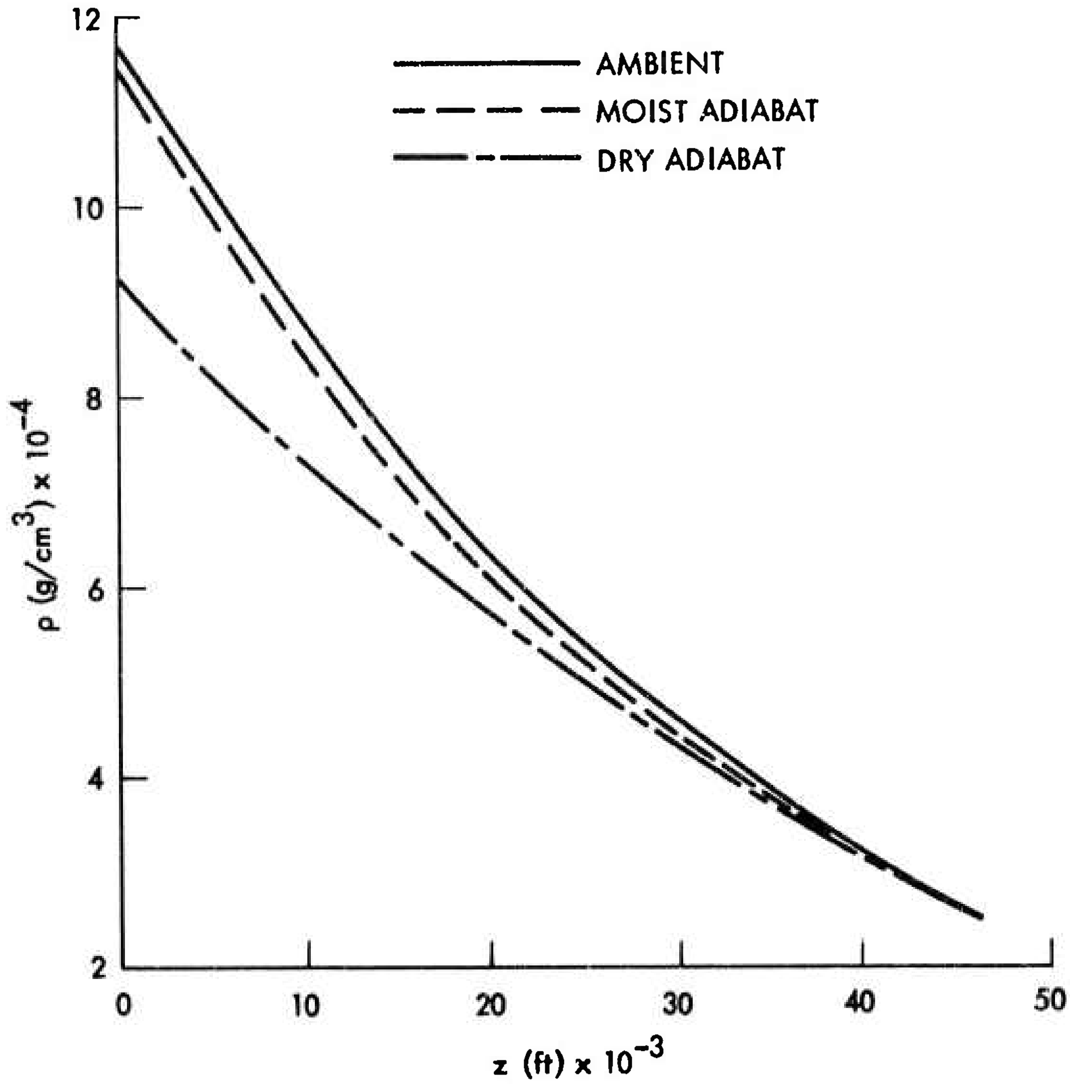

Fig. 13. The density vs. altitude relation for each of the three thermodynamic loci of Fig. 12 are presented. The density variations from ambient within the storm are less than 30%, and speeds are far below sonic everywhere.

sea-level ambient state is taken as saturated for computing the moist adiabat only, z_1 = 50,530 ft, p_1 = 122 mb, T_1 = 199.6°K; whence, p_e = 958 mb, T_e = 297.5°K.

These results, to reiterate, have followed procedure A (moist adiabat based on sea-level ambient, as given by Jordan (1957) or modified to be saturated at the same temperature and pressure). The results have been presented here because such calcualtions have been performed in the past, and because the size of variances owing to use of (13) in place of (12) seemed worth investigating.

For the <u>superior</u> procedure B (iteration to determine the consistent pressure at the base of the eyewall moist adiabat), only (13) will be used. If one adopts T_e = 299.4°K, $(RH)_e$ = 0.84, one finds convergence for p_e = 958 mb (Figs. 14 and 15); incidentally, z_1 = 50,100 ft, $p_1 \doteq$ 125 mb, $T_1 \doteq$ 200°K; further, saturation occurs at 918 mb, $z \doteq$ 1300 ft. If L is given the value of the heat of condensation only, $(H/c_p)_s$ = 343.4°K (sea-level ambient), and $(H/c_p)_e \doteq$ 346.4°K (sea level under the moist adiabat). [Under (13), (H/c_p) is not precisely constant on a moist adiabat.] Also, p_c = 875 mb, T_c = 349°K. Thus it would seem that the sea-level total static temperature needs only to increase about 3°K, with relative humidity and sea surface temperature held fixed, for an appreciably reduced pressure under eyewall cumulonimbi cores. According to (15), for the deficit $(p_s - p_e)$ = 56 mb, $(v)_{max}$ = 155 mph for n = 0.5 and $(v)_{max} \doteq$ 219 mph for n = 1.0. Any eyewall bending would permit appreciably higher speeds; in fact, since $(p_s - p_c)$ = 139 mb, theoretically sufficient bending could explain any speed up to 244 mph for n = 0.5 and 345 mph for n = 1.0. If one takes again T_e = 299.4°K, but now lets RH = 1.0, one finds convergence for p_e = 890 mb. Here z_1 = 57,500 ft, p_1 = 84 mb, T_1 = 191.8°K. In addition, $(H/c_p)_e$ = 359.4°K;

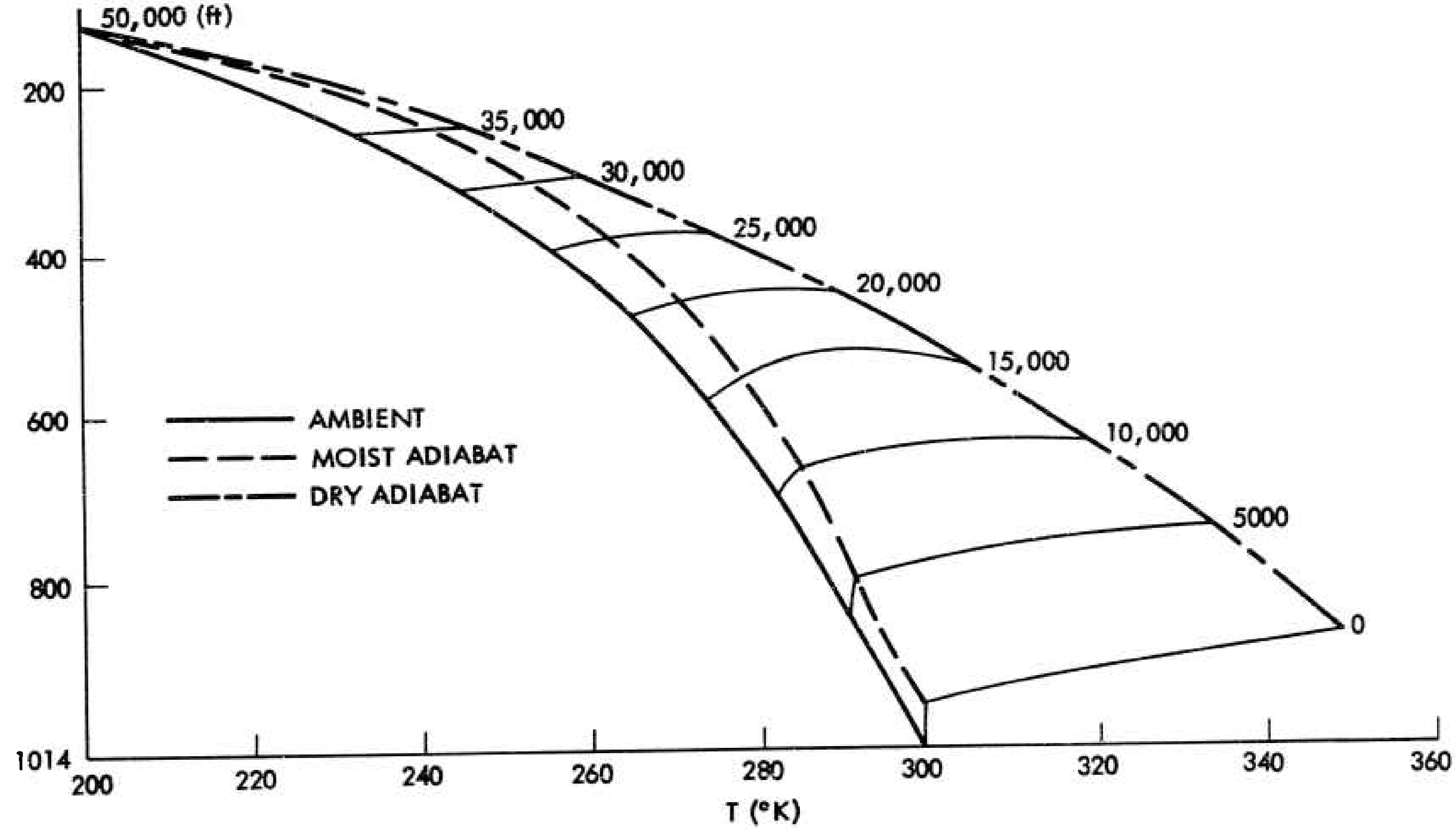

Fig. 14. The three curves are analogous to those of Fig. 12, except that the moist adiabat, while still taken to be based on ambient sea-level values for relative humidity and temperature, is now computed from an iteratively-determined sea-level pressure consistent with the definition of the top of the storm as an isobaric, isothermal surface of constant altitude.

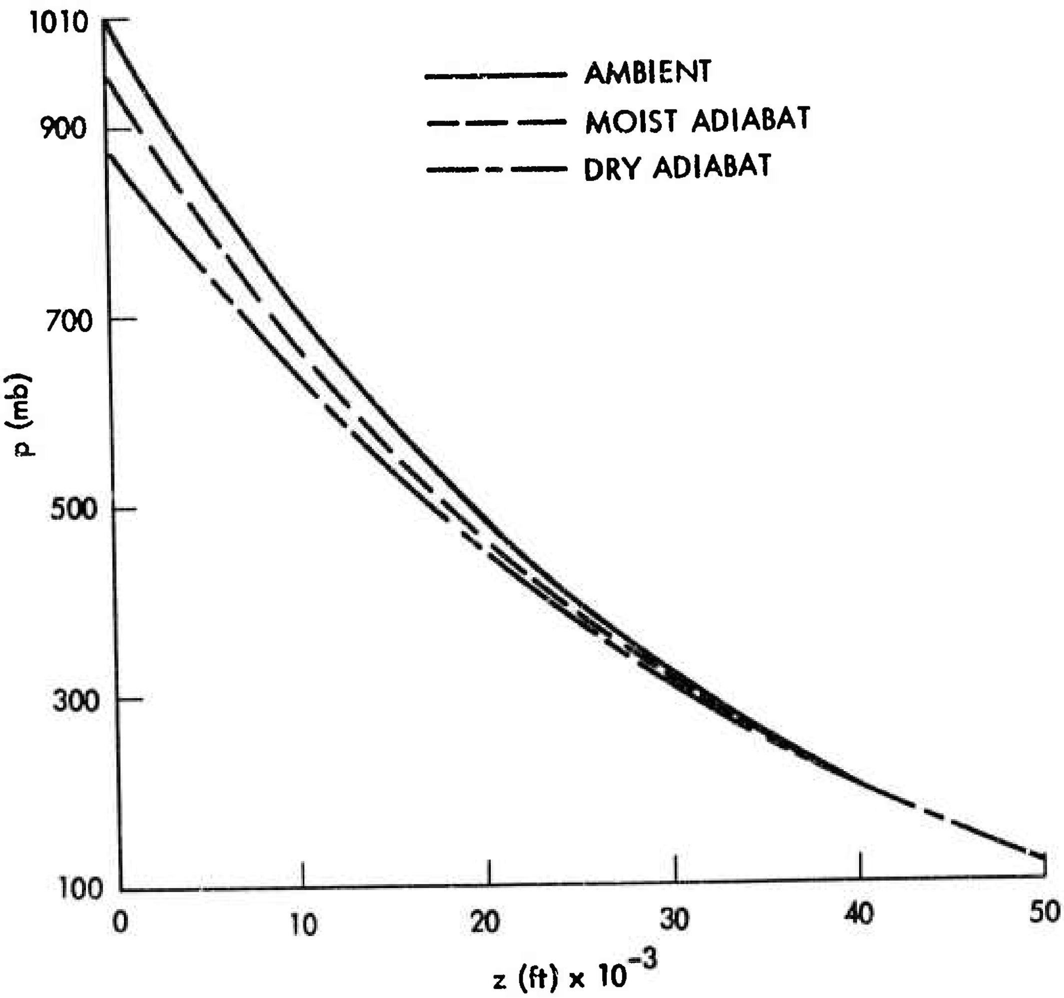

Fig. 15. In this replotting of Fig. 14, explicit pressure vs. altitude curves for the ambient, the moist-adiabat taken to characterize the eyewall, and the dry-adiabat recompression taken to characterize the eye are presented. The moist adiabat is computed by the iterative procedure described in the caption to Fig. 14.

so an increase of about 16°K in the total static temperature could explain any recorded hurricane wind intensity without any eyewall bending <u>if</u> the sea-level eyewall air were saturated, <u>and if</u> a moist adiabat were taken to be an adequate approximation to the eyewall sounding. In fact, increases above 16°K appear impossible under these conditions as long as the sea surface is taken as an isothermal surface. Of course, use of the moist adiabat for the entire eyewall and the dry adiabat for the eye involves idealizations yielding upper estimates, but the large magnitude of the winds so calculated suggests the ideas behind the estimates are correct.

At this point a brief summary of the calculated results and their implication seems apropos. Even though cumulonimbi occupy only 10 to 20% of the eyewall (Charney, 1971, pp. 358-359), results have been presented for moist adiabatic ascent and hydrostatics in the eyewall which suggest that significant pressure deficits relative to ambient may be achieved by moist-adiabatic ascent of air whose total static temperature (H/c_p) is increased by only one percent over ambient, and <u>very</u> intense storms may be achieved by ascent of air with a five-percent increase over ambient. The five-percent increase involves arbitrarily increasing the eyewall relative humidity at shipboard height to 100% from an ambient level of 84%, and it would seem more likely that the means by which the winds of very intense hurricanes are achieved is that the eye-eyewall interface, as well as it can be defined, slopes at least slightly outward from the top of the inflow layer to the top of the storm. The calculation of wind speed for a storm with an eye implicitly assumes such sloping. The additional dry-adiabatic-compression-associated pressure deficit available to sustain swirling is reduced if the eye-eyewall interface is vertical to a certain height, and only then slopes outward; the additional pressure deficit is unavailable if the eye-eyewall interface is perfectly vertical virtually to the top of the storm. (Whether or not the eye-eyewall interface slopes outward does not alter the conceptual thermodynamic stratification of the eye nor the sea-level eye pressure deficit from ambient.) Dissenting publications will now be discussed.

Malkus and Riehl (1960) find that moist-adiabatic ascent of ambient sea-level air to the level of neutral buoyancy produces a sea-level pressure of 1000 mb; insufficient detail is presented to ascertain what precise calculation was conducted. In any case, such a pressure drop is insufficient to sustain even a moderate hurricane, and since these authors believe the eye-eyewall to be virtually perfectly vertical, another means of achieving and sustaining winds known to arise in hurricanes was required. The means ultimately postulated is latent and sensible heat transfer from sea to air greatly in excess of ambient transfer rates; the air ascending in the eyewall would then possess large enough equivalent potential temperature to achieve the pressure deficits necessary to sustain even the highest known hurricane winds. The Riehl-Malkus extra-oceanic-heat-source postulate will be examined below (Sections 3.7 - 3.9). Here it will be noted that even extrapolation of low-speed transfer coefficients to extreme hurricane conditions could not justify the equivalent potential temperature increases required by Riehl and Malkus; thus for very intense hurricanes (only) an outward sloping of the eyewall was conceded by these modelers. Such a procedure directly contradicts the conclusions of Shea (1972), based on consideration of flight radar data; Shea finds that the eye-eyewall interface slopes modestly outward for moderate hurricanes, but questions whether it does so for intense hurricanes. However, radar data are based on returns from precipitation and err on the side of verticality because the strongest returns will come from the torrential rains of cumulonimbi, rather than from weaker or decaying cumuli (Palmén and Newton, 1969, pp. 487-491). Direct observational distinction between an interfacial slope of a few degrees and no slope at all, especially when some cloudiness may occur in the eye, is difficult, and observational evidence can be cited to support either position. For instance, Palmén (1948) sketches an eyewall with appreciable outward slope down to nearly inflow-layer heights, based on observations of a September 1947 hurricane off Tampa, Florida. Palmén and Newton (1969) construct a schematic diagram of Hurricane Helene, based on observational data of September 26, 1958, that indicates outward eyewall slope commencing a few kilometers above the sea surface. On the other hand, Riehl (1954, pp. 312-313) cites reports published in 1945 that described an observed eye-eyewall interface that did not have appreciable outward slope until 30,000 ft. Definitive proof concerning whether or not dry-adiabatic recompression contributes to the pressure difference available to sustain hurricane winds appears nonexistent.

This section is concluded by noting that maximum wind speeds in hurricanes are estimated, in the absence of aircraft reconnaissance, by means of previously correlated formulae involving the area of the overcast "circle" on satellite photographs (Hubert and Timchalk, 1969)(Fig. 3).

3.5 The Swirl-Divergence Relation for the Frictional Boundary Layer

Because the maximum speed achieved in a tropical cyclone is rarely much over 200 mph, the Mach number rarely reaches even 0.3. Hence, when examining the dynamics (as opposed to the energetics), an incompressible constant-property model suffices.

A steady axisymmetric flow of an incompressible fluid is now studied to confirm the crucial point that, under rapid swirling, there is downflux from region I to region II, and sufficient downflux enters the surface frictional layer to account for the mass flux up the eyewall. The analysis will be carried out in a noninertial coordinate system rotating at the constant speed of that compoennt of the rotation of the earth which is normal to the local tangent plane. Because the boundary-layer divergence under an impressed swirl (the major constraint furnished by the boundary layer on the inviscid flow above it) is relatively small in magnitude, careful formulation and solution of the coupled quasilinear parabolic partial differential equations and boundary conditions describing the layer are required.

The relevant equations are

$$\nabla \cdot \vec{q} = 0, \tag{18}$$

$$\nabla(q^2/2) + (\nabla \times \vec{q}) \times \vec{q} + 2\vec{\Omega}_e \times \vec{q} = -\nabla\tilde{p} - \nu\, \nabla\times(\nabla \times \vec{q}), \tag{19}$$

where $\tilde{p} = (p/\rho) + (\vec{\Omega}_e \times \vec{r})^2/2 + gz$, the gravitational acceleration $\vec{g} = -g\hat{z}$, the velocity in noninertial coordinates $\vec{v} = \vec{\Omega}_e \times \vec{r} + \vec{q}$, the component of the rotation of the earth normal to the local tangent plane $\vec{\Omega}_e = \Omega\hat{z}$, and the kinematic viscosity (later given eddy-diffusivity values) is ν.

Nondimensionalization is effected by letting $\vec{q}\,' = \vec{q}/(\Psi_0\Omega)^{1/2}$, $p' = \tilde{p}/(\Psi_0\Omega)$, $\vec{r}\,' = \vec{r}/(\Psi_0/\Omega)^{1/2}$, and $E = \nu/\Psi_0$ where the Ekman number $E << 1$ and Ψ_0 characterizes the circulation away from the boundary (such as the maximum swirl speed times the radius at which it occurs). Dropping primes, one has

$$\nabla \cdot \vec{q}, \tag{20}$$

$$\nabla(q^2/2) + (\nabla \times \vec{q}) \times \vec{q} + 2\hat{z} \times \vec{q} = -\nabla p - E\, \nabla\times(\nabla \times \vec{q}). \tag{21}$$

These equations are studies in axisymmetric cyclindrical polar coordinates

$$\vec{q} = u\,\hat{r} + v\,\hat{\theta} + w\,\hat{z}, \quad \vec{r} = r\,\hat{r} + z\,\hat{z}. \tag{22}$$

Away from the boundary (i.e., in region I) the following expansions are adopted:

$$p = \pi(r,z) + \ldots, \; v = V(r,z) + \ldots, \; w = E^{1/2}W(r,z) + \ldots, \; u = o(E^{1/2}). \tag{23}$$

Substitution of (23) in (20) and (21) gives the gradient-wind equation:

$$\pi_z = 0, \quad W_z = 0, \quad \pi_r = 2V + V^2/r. \tag{24}$$

Subscripts r and z here (and x and ζ below) denote partial differentiation. The axially invariant solution is complete when $\pi(r)$ or $V(r)$ is specified [here $V(r)$ will be given]; $W(r)$ is found by matching the solution to (19) to the solution for the frictional layer II, and in this sense $W(r)$ is determined by the boundary-layer dynamics.

If $\zeta = z\,E^{-1/2}$ [which implies that the frictional layer is $O(E^{1/2})$ in thickness] and if near the boundary

$$u = u_b(r,\zeta) + \ldots, \; v = v_b(r,\zeta) + \ldots, \; w = E^{1/2}w_b(r,\zeta) + \ldots, \; p = p_b(r,\zeta) + \ldots, \tag{25}$$

then the axial component of the momentum conservation equation degenerates to $(\partial p_b/\partial\zeta) = 0$ in conventional fashion, so the pressure field in the boundary layer is known from (24). If

$$\psi = rv_b, \quad \Psi = r\,V, \quad \phi = ru_b, \quad x = r^2, \quad \tilde{w} = 2^{-1/2}w_b, \quad \tilde{\zeta} = 2^{1/2}\,\zeta, \tag{26}$$

then in terms of dimensional quantities $\phi = ru/\Psi_0$, $\psi = rv/\Psi_0$, $\tilde{w} = w/(2\Omega\nu)^{1/2}$, $\tilde{\zeta} = z/(\nu/2\Omega)^{1/2}$, $x = \Omega r^2/\Psi_0$, the boundary layer thickness is $O(\nu/\Omega)^{1/2}$. In terms of quantities introduced in (26), one has from (20) and (21), upon dropping the tildes,

$$\phi_x + w_\zeta = 0, \tag{27}$$

$$\phi\phi_x + w\phi_\zeta + (\Psi^2 - \psi^2 - \phi^2)/2x - (\psi - \Psi) - \phi_{\zeta\zeta} = 0, \tag{28}$$

$$\phi\psi_x + w\psi_\zeta + \phi - \psi_{\zeta\zeta} = 0. \tag{29}$$

Matching of expansions gives

$$\zeta \to \infty: \quad \phi \to 0, \ \psi \to \Psi(x) \text{ given}, \tag{30}$$

and at $\zeta = 0$ no-slip conditions are adopted:

$$\zeta = 0: \quad \phi = w = \psi = 0. \tag{31}$$

Initial conditions are conveniently given by noting that at $x = x_0$, for x_0 large enough, solution is given by discarding all the nonlinear terms and retaining the linear equations treated by Ekman, in which x enters parametrically only. The solution to the balance of Coriolis, pressure, and friction forces is well known:

$$\phi = -[\Psi(x)] \sin (2^{-1/2}\zeta) \exp (-2^{-1/2}\zeta), \tag{32}$$

$$\psi = [\Psi(x)] [1 - \cos (2^{-1/2}\zeta) \exp (-2^{-1/2}\zeta)), \tag{33}$$

$$w = 2^{-1/2} [\Psi_x(x)] \left\{1 - [\sin (2^{-1/2}\zeta) + \cos (2^{-1/2}\zeta)] [\exp (-2^{-1/2}\zeta)]\right\}. \tag{34}$$

Specifically what is sought is $w(r,\zeta\to\infty) = W(r)$ for $\Psi(r)$ of interest. For r large, from (34)

$$w(r,\zeta\to\infty) = W(r) = 2^{-1/2}\,\Psi_x(x). \tag{35}$$

Numerical integration by finite-difference methods is formidable because the flow component in the time-like direction, u, is, in successively thinner strips lying parallel to the boundary, alternately in the direction of integration (numerically stable) and opposite to it (unstable). Though the radial flow is, on net, in the direction of integration, the integration is marginally stable. The only finite-difference results applicable to the hurricane problem known to the author were given by Anthes (1971) and are discussed later (Section 3.10).

George (Carrier, Hammond, and George, 1971) and Dergarabedian and Fendell (1972b) independently but simultaneously applied the method of weighted residuals to the boundary value problem, and found that, except near the eyewall where the method was inadequate, the linear result given in (35) sufficed for the nonlinear problem as well (Fig. 16). Thus, for a form like $\Psi = A(1 - x/x_0)$ [for which $\phi(r_0,\zeta) = 0$ according to (32) -- as required by the closed-system model] $w(r,\zeta\to\infty) \to -(A/x_0)$, a small negative constant quantity (equivalent to about 0.005 mph downdraft for physically interesting values).

Incidentally, the form just mentioned for the rapid swirl in the inviscid flow above the frictional inflow layer is stable for parametric values of practical interest, according to Rayleigh's criterion (Greenspan, 1968, pp. 271-272).

The reason weighted-residual calculations fail near the axis, as discovered by Carrier (1971a) and McWilliams (1971) by modified Oseen

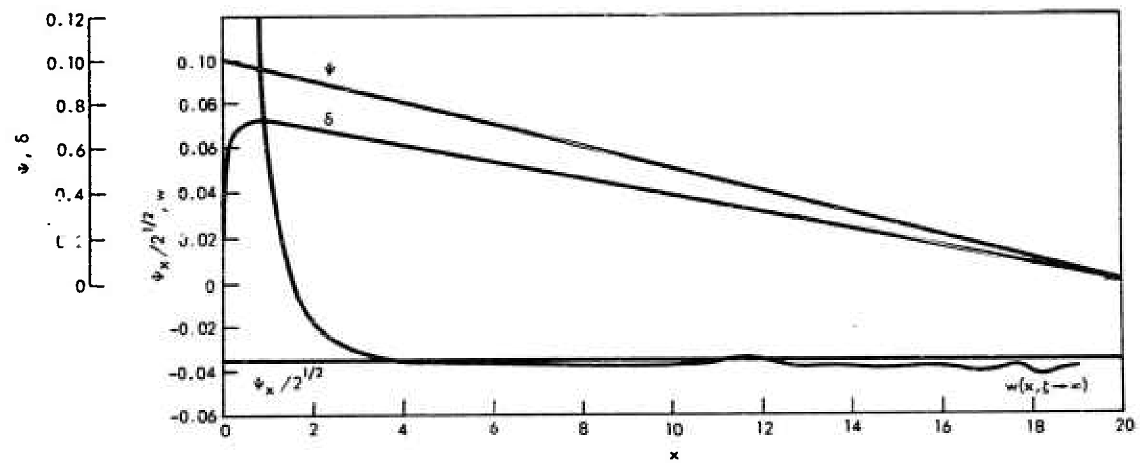

Fig. 16. Nondimensional results for the frictional boundary layer obtained by the method of weighted residuals, from Dergarabedian and Fendell (1972). The divergence $w(x,\zeta \to \infty)$ and the volumetric flux $\delta = -\int_0^\infty \phi(x,\zeta)\, d\zeta$ are presented for the impressed swirl $\psi = 1 - x/x_1$, $x_1 \doteq 20$, believed pertinent to a hurricane outside the eyewall. Except near the axis where nonlinear inertial effects dominate, the linear Ekman layer result, $w(x,\zeta \to \infty) \doteq \psi_x/2^{1/2}$, is an excellent approximation to the numerical results. The volumetric flux $\delta(x)$ is thus linearly proportional to $(x_1 - x)$ to good approximation. Normalized residuals indicate large errors for $x < 3$, and discount the premature eruption as an artifact of the method. The solution by Carrier (1971a) for small x indicates the adequacy of the linear Ekman result for the divergence to within a factor of two. Since $\nu \doteq (1/75)\ mi^2/hr$ and $\Omega \doteq (1/16)$ rad/hr, dimensionally the results imply the boundary layer is of thickness $O(\nu/\Omega)^{1/2} = O(1\ mi)$, the downflux into the boundary layer is $(2\nu\Omega)^{1/2}\, w(x,\zeta \to \infty) = O(5 \times 10^{-3}\ mi/hr)$, and the volumetric flux erupting up the eyewall is $(2\pi^2\ \Psi_0^2\ \nu/\Omega)^{1/2}\, \delta(x) = O(7 \times 10^3\ mi^3/hr)$ where Ψ_0 characterizes the eyewall relative angular momentum per unit mass. The implication is that the fluid initially in the boundary layer sustains the hurricane for about a week, and the fluid lying above the boundary layer (with supplementary replenishment of moisture from the ocean) can readily sustain the hurricane for more than a week more.

linearization and by Burggraf, Stewartson, and Belcher (1971) by semi-numeri⁻¹ analysis, is that the structure of the boundary layer so varies with radial distance that adequate representation in terms of one set of orthonormal polynomials is difficult. Far from the axis of symmetry, friction is important across the entire layer of thickness $O(\nu/\Omega)^{1/2}$; near the axis, friction is significant only in a small sublayer near the wall of thickness $O(r^2\nu/\Psi)^{1/2}$, and the remainder of the inflow layer of $O(\nu/\Omega)^{1/2}$ thickness is inviscidly controlled. Still, for conditions of interest in hurricanes, (35) is everywhere correct to within a factor of two, and often far better.

Still further confirmation of the adequacy of the classical linear Ekman swirl-divergence relation for the nonlinear frictional inflow layer under a hurricane vortex has recently been given, in as yet unpublished work, by Prof. K. K. Tam of McGill University. Prof. Tam has applied the theory of differential inequalities to the solution proposed by the Carrier group to the boundary-value problem (27) - (31) and indicated the satisfactory accuracy of the result.

But while the solution may satisfy the boundary-value problem, one may ask how well the boundary-value problem reflects the actual situation, since a constant eddy viscosity has been adopted to model the turbulent transfer. It would seem premature to adopt a second-order (or field-closure) model that reduces the Reynolds time-averaged formulation to a determinate set by means of a differential relationship between the rate of change of the Reynolds stress and diffusion, production, and dissipation of turbulence; still, an eddy viscosity invariant with distance from the air-sea interface seems outdated, and further, some small slip might better model the boundary condition at the air-sea interface, taken planar.

It should be noted, however, that only one quantitative result from the nonlinear Ekman layer plays a role in the Carrier model, and that is the swirl-divergence relation appropriate at the extreme outer edge of the frictional inflow layer. Furthermore, the eddy viscosity is almost always taken as independent of distance from the "wall" over at least the outer ninety percent of the turbulent boundary layer thickness, aside from intermittency effects, which play very little role in altering the profiles of time-averaged macroscopic variables and are very often entirely neglected in calculations. In fact, one does not really know how to model slip and effective roughness effects in the hurricane viscous sublayer, so restoration of eddy-viscosity dependence on the normal coordinate would be a very speculative undertaking at best.

Other workers introduce a constant or wind-dependent drag coefficient to model frictional effects near the air-sea interface for computer solutions. There seems to be no inherent reason why either of these empirical, phenomenological devices (drag coefficients, eddy viscosity coefficients) is superior to the other; either could be made to succeed by proper adjustment (curve-fitting). Both face the problem that empiricism developed for relatively small wind speeds and relatively smooth interfaces must be extrapolated to higher wind speeds and very rough seas (Ooyama, 1969, p. 19). There exist attempts to study the frictional inflow layer entailing both simpler and more detailed modeling than that just presented; for example, Riehl (1963) invokes conservation of potential vorticity for the frictional inflow layer, and Riehl and Malkus (1961), who assert that dissipation of kinetic energy in the surface layer is independent of distance from the center, find that they must adopt exceptionally large eddy transfer coefficients to resolve problems that arise in computing a refined mechanical energy budget.

3.6 The Energetics of the Frictional Boundary Layer and Throughput Supply

For the frictional boundary layer II and throughput I, Carrier, Hammond, and George (1971) take the following approximations as adequate for the quasisteady mature phase:

1. the Prandtl and Schmidt numbers are unity;
2. the hydrostatic approximation holds;
3. the boundary layer approximation holds (derivatives normal to the boundary exceed those tangential to the boundary, but velocity components parallel to the boundary exceed those normal to the boundary);
4. the eddy transfer is adequately modeled by the laminar flux-gradient relations for diffusion of mass, momentum, and heat (as given by Fick, Newton, and Fourier, respectively), except that the augmented kinematic (eddy) viscosity may vary with radial position (but not with axial position); and
5. the mixture of dry air and water vapor may be taken as a perfect gas with constant heat capacity over the range of temperatures of interest here.

In view of the limited understanding of quantitative formulation of cumulus convection and turbulent transfer, on the cyclone scale of interest here these five approximations seem reasonable. Clearly the familiar Reynolds analogy, together with the constancy of the eddy viscosity over most of the boundary layer, is being adopted. It can then be shown that for L held constant, the following equation, a generalization of ones given by Crocco in fluid dynamics and Shvab and Zel'dovich in combustion, holds in the meteorological context of interest here:

$$u \frac{\partial H}{\partial r} + w \frac{\partial H}{\partial z} = \nu \frac{\partial^2 H}{\partial z^2} + D, \tag{36}$$

where D denotes radiational loss, taken by Carrier, Hammond and George (1971) to be representable as

$$D = -f(z)\, H,$$

with f(z) (to be discussed below) known. The H used throughout this section is H_t as defined in (3); the subscript t is dropped to avoid double subscripts below. The boundary-initial conditions are taken to be

$$z = 0: \quad H = H_s; \; z = z_1: \quad H = H_1; \tag{37}$$

$$r = r_o: \quad H = H_a(z), \tag{38}$$

where, again, r_o is the outer edge of the storm and z_1 is the top of the storm. The ambient profile $H_a(z)$ is given; $H_s = H_1 = H_a(z = z_1) = H_a(z = 0)$. Taking $H_s = H_a(z = 0)$ implies that for r > R (where R is the radius of the eyewall), the ocean surface temperature is a constant and the water vapor mass fraction (which takes on its saturation value at the nominally plane sea surface z = 0) is taken independent of pressure (see below). Since the thermal conductivity of water greatly exceeds that of air, uniform sea-surface temperature appears to be a good approximation. That $H_1 = H_a(z = 0)$ follows from the definition of the lid on the storm.

The terms, from left to right, in (36) represent radial advection; axial convection; turbulent diffusion and cumulus convection (both parameterized in ν, which will henceforth be treated as constant, though this is not necessary); and radiation loss. Throughout I, and in II at $r = r_0$ (36) may be approximated as

$$w \frac{\partial H}{\partial z} - \nu \frac{\partial^2 H}{\partial z^2} = -f(z)\ H; \tag{39}$$

this follows because $\phi(x_0) = 0$ [cf. (29)] and $u(\partial H/\partial r)$ is negligible in I [cf. (23)]. In view of the boundary conditions $f(z)$ must be chosen to permit $H(z) = H_a(z)$, where $H_a(z)$ is given (Jordan, 1957); Carrier, Hammond, and George (1971) take $w(r_0,z) \equiv w_a(z) = 0$, but there appears to be no need to require this. Thus,

$$w_a(z) \frac{\partial H_a}{\partial z} - \nu \frac{\partial^2 H_a}{\partial z^2} = -f(z)\ H_a; \tag{40}$$

in fact, taking $H = H_a(z)$ throughout I and II satisfies the initial and boundary conditions, and renders $u(\partial H/\partial r) = 0$. Further, the function $w(r,z)$ is available from the nonlinear boundary layer dynamics just discussed: the dynamics reveals $(w)_{max} = w(r,\zeta\to\infty) = W(r)$, the value at the outer edge of the boundary layer -- and W is independent of r over a wide expanse. Only if W is appreciably different from w_a can H depart from H_a, and such a variance occurs only in the eyewall, where vertical convection dominates all other terms in (36) because (by simple continuity considerations) w is increased two orders of magnitude over its maximum magnitude in I or II. Hence for at least portions of the eyewall,

$$w \frac{\partial H}{\partial z} = 0, \tag{41}$$

and the relevant boundary data for this hyperbolic suboperator is that given at $z = 0$ (37). All this discussion, more carefully argued in Carrier, Hammond, and George (1971), leads to two significant results:

1. throughout the frictional boundary layer and the throughput supply, outside the eyewall the total stagnation enthalpy is approximately fixed at its ambient stratification. Furthermore, the small correction owing to inertial effects in a hurricane is readily seen to be a decrease of H with z such that the enthalpy gradient at $z = 0$ is increased slightly (about ten percent or so);
2. in portions of the eyewall, the total stagnation enthalpy is, to good approximation, constant at its sea-level value, i.e., columns of air are very nearly rising on a moist adiabat.

Numerical values of interest are the ambient net sea/air enthalpy transfer and the eddy viscosity (Carrier, Hammond, and George, 1971):

$$-\rho\nu[\partial H(r,z = 0)/\partial z] \doteq -\rho\nu[\partial H_a(r,z = 0)/\partial z] = 1.8 \times 10^5 \text{ ergs cm}^{-2} \text{ sec}^{-1}; \tag{42}$$

$$\nu = 2.7 \times 10^5 \text{ cm}^2 \text{ sec}^{-1}.$$

Because this result is not compatible with many existing hurricane models, it becomes necessary to identify the reasons for disagreement. Hence in the following sections the Riehl-Malkus theory is scrutinized.

It way be worth emphasizing that (36) yields a steady solution for the frictional inflow layer with all inertial terms considered. It is <u>not</u> based on obviously inadequate models of the inflow layer, such as frictionless or purely horizontal flow.

However, this result is really not quite so accurate as suggested by Carrier, Hammond, and George, owing to the omission of compressibility effects. A decrease in gas density attends the decrease in gas pressure (of up to 12%) as one moves radially inward in the frictional inflow layer from the outer edge toward the eyewall. This decrease in air density is negligible for the dynamics, but not the energetics. If the gas density decreases as one goes radially inward, but the total-stagnation-enthalpy sea-to-air transfer remains approximately at the ambient rate (Carrier, 1971b, p. 158 footnote), then the vapor mass fraction (ratio of vapor density to gas density) increases because there is less mass to accept the same transfer. Carrier, Hammond, and George (1971, p. 162) acknowledge the effect, but fail to account for it in their calculations. It seems more appropriate to study the vapor density, which at sea level depends only on sea-surface temperature, rather than the vapor mass fraction. A rough first estimate is that perhaps a 20% increase over tropical ambient levels in latent and sensible heat transfer from the ocean to the atmosphere occurs near the eyewall of hurricanes owing to this density-reduction effect.

3.7 The Riehl-Malkus Postulate of an Oceanic Heat Source

While the exposition of the mature tropical cyclone has been given here in terms of the Carrier model, most published works adopt the framework of the Riehl-Malkus theory alluded to earlier (Riehl, 1954; Palmén and Riehl, 1957; Malkus, 1958; Malkus and Riehl, 1960; Riehl and Malkus, 1961; Malkus, 1962; Riehl, 1963). The cornerstones of this theory have been summarized by Malkus (1962, p. 232):

> The new model relates core maintenance to mechanism, namely, cumulonimbus convection and sea-air exchange. <u>To</u>

sustain the required pressure gradients, two coupled processes are necessary: first, a greatly magnified oceanic input of sensible and latent heat, and secondly, the undilute release of the latter in concentrated hot tower ascent, so that air of high heat content is pumped rapidly into the upper troposphere. The quantitative establishment of these crucial relationships was carried out in a joint analytic and observational framework (Malkus and Riehl, 1960).

The important role of large numbers of cumulonimbi in the eyewall has been confirmed, although the Riehl-Malkus concept of a direct hot-tower heating mechanism has since been challenged by Lopez (1972b), as discussed in Section 2.2. Here the other major contention [". . . it is postulated that lowering of surface pressures in hurricanes arises mainly through an 'extra' oceanic heat source in the storm's interior" (Malkus and Riehl, 1960, p. 12)] is questioned. There is undoubtedly some augmentation of the ambient-level sensible and latent heat transfer from sea to atmosphere over a wide ocean expanse under a hurricane, and the warm autumnal tropical seas are in a major way responsible for the atmospheric total static enthalpy profile to begin with; however, an augmentation of over an order of magnitude in oceanic heat transfer concentrated according to Malkus and Riehl largely in the region 30 to 90 km from the cyclone center seems unnecessary and unlikely (Section 3.8).

The Riehl-Malkus theory, as noted earlier in Section 3.4, takes the eye-eyewall interface to be effectively perfectly vertical so some other mechanism than dry adiabatic recompression must be found to sustain hurricane-force winds. Why this mechanism is stated to be an "extra" oceanic heat source is understood by reconstructing their logic. It may be worth noting at this point that Riehl and Malkus permit the relative vorticity (rather than the relative velocity) to vanish at the outer edge of the storm; workers using their concepts tend to permit convective, but not diffusive, transport across the outer cylindrical boundary, while workers on the closed Carrier model permit diffusive, but not convective, transport across boundaries.

Malkus and Riehl (1960, pp. 3, 7) acknowledge the existence of a low-level frictional inflow layer, although in a sample calculation there is efflux from the boundary layer from the outer edge of the eye out to 500 km at an average ascent rate of 0.7 mph. Thus air flows in through the boundary layer from the outer edge of an open system, rather than sinking down into the frictional layer as in Carrier's model. The boundary layer air, according to Malkus and Riehl, undergoes adiabatic expansion as it spirals inward toward the center, yet it remains isothermal. This requires a vast, rather localized increase in sea-to-air transfer between the ocean and the contiguous atmosphere. That the gradient normal to the air/sea interface of temperature and of water vapor mass fraction is large enough to be consistent with vastly increased air/sea transfer is accepted as possible:

> In the outskirts of a hurricane the temperature of the inflowing air drops slowly due to adiabatic expansion during (horizontal) motion toward lower pressure. It is one of the remarkable observations in hurricanes that this drop ceases at pressures of 990 - 1000 mb and that thereafter isothermal expansion takes place. Presumably, the temperature difference between sea and air attains a value large enough for the oceanic heat supply to take place at a sufficient rate to

keep the temperature difference constant. (Malkus and Riehl, 1960, p. 9).

The actual transports [between sea and air], of course, are very large in the hurricane compared to the trades. Sensible heat pickup is 720 cal/cm^2/day, and increase by a factor of 50 over the trades. . .; latent heat pickup is 2420 cal/cm^2/day, higher by a factor of 12-13. (Malkus and Riehl, 1960, p. 12).

Similar arguments are made elsewhere by Riehl (1954, pp. 286-287):

Many published records, notably those by Deppermann. . ., have proved that the surface temperature outside the eye is constant or decreases very slightly toward the center. The implications of this remarkable fact passed without notice until Byers. . . drew attention to it. The temperature of the surface air spiraling toward a center should decrease if adiabatic expansion occurred during pressure reduction. For instance, air entering the circulation with the average properties of the mean tropical atmosphere should reach the 930 mb isobar with a temperature of 20.5°C and specific humidity of 17 g/kg. Because of condensation, a dense fog should prevail at the ground inward from the 970 mb isobar. But this is never observed. It follows that <u>the potential temperature of the surface air increases along the inward trajectories</u>. We also know that the specific humidity increases and that the cloud bases remain between a few hundred and 1000 feet.

The surface air thus <u>acquires both latent and sensible heat during its travel toward lower pressure</u> . . ..

A source for the heat and moisture increment is obvious. The ocean is greatly agitated, and large amounts of water are thrown into the air in the form of spray. It is hard to say where the ocean ends and where the atmosphere begins! As the air moves toward lower pressure and begins to expand adiabatically, the temperature difference between ocean and air suddenly increases. Since the surface of contact between air and water increases to many times the horizontal area of the storm, rapid transfer of sensible and latent heat from ocean to air is made possible. In the outskirts, say beyond the 990 mb isobar, the turmoil is less and the process of heat transfer is not operative.[7]

The Riehl-Malkus theory that greatly augmented heat and mass transfer sustains the tropical cyclone has, in fact, been parameterized into all existing computer simulations (see Section 3.10). For example, Rosenthal (1971b) closely reflects the Riehl-Malkus theory and notes similar logic in the work of another computer modeler of tropical cyclones (Ooyama, 1969):

> Air-sea exchanges of sensible and latent heat have long been considered important ingredients in the development and maintenance of tropical storms. Palmén (1948) showed, on a climatological basis, that tropical storms form primarily over warm ocean waters ($T_{sea} > 26°C$). Malkus and Riehl (1960) showed that the deep central pressures associated with hurricanes could not be explained hydrostatically unless the equivalent potential temperature, θ_e, in the boundary layer was 10° to 15°K greater than that of the mean tropical atmosphere. Byers (1944) pointed

out that the observed near-isothermal conditions for inward spiraling air in the hurricane boundary layer required a source of sensible heat to compensate for the cooling due to adiabatic expansion. . ..

Ooyama (1969) found drastic reductions in the strength of his model storm when the air-sea exchanges of sensible and latent heat were suppressed. He pointed out that at sufficiently large radii, the boundary layer is divergent (the so-called Ekman layer "sucking"). . .. This subsidence tends to decrease the boundary layer θ_e since $\partial\theta_e/\partial z < 0$ in the lower troposphere. Ooyama argued that unless the energy supply from the ocean can again raise the θ_e of the boundary layer air to sufficiently large values before the inflowing air reaches the inner region, the convective activity will diminish in those regions and, hence, the storm will begin to weaken.

Ooyama's line of reasoning can be extended to show that evaporation is far more important than sensible heat flux. The air sucked into the boundary layer has a higher potential temperature than the original boundary layer air. The subsiding air has a smaller θ_e only because it is relatively dry. (Rosenthal, 1971b, p. 772).

3.8 The Intensity of a Tropical Cyclone and the Underlying Sea Surface Temperature

Palmén and Riehl (1957, p. 156) state: "Evidently, a cyclone will decay rapidly if it encounters thermally unfavorable conditions. The generation term in [the equation for conservation of kinetic energy] will then decrease rapidly; it will become negative if the air ascending in the core is cooler than the surroundings in spite of release of latent heat. This would happen when a storm moves over a relatively cold ocean surface, over a continent, or when colder air masses invade the cyclone near the surface."

In discussing the Riehl-Malkus theory, one should distinguish between what these authors themselves assert, and what others citing their work have added. For example, Shuleykin (1970; 1972) goes beyond Riehl and Malkus and asserts that the ". . . power of the hurricane. . . increases sharply with the temperature of the underlying water surface" (Shuleykin, 1972, p. 1). However, attempts to correlate central pressure deficit and sea surface temperature, without sufficient account of other factors, seems simplistic. This particular point will be now discussed.

First, ocean temperatures are difficult to determine accurately from currently available records and by currently available methods (Perlroth, 1967; Hidy, 1972, p. 1091). Next, Gentry (1969a, p. 406) presents data relating ". . . the maximum intensity of several tropical cyclones to the temperatures of the sea beneath them and shows that both severe and weak tropical cyclones occur when ocean temperatures are relatively high. This suggests that variations in parameters other than the transfer of heat from the ocean to the atmosphere also influence the storm's intensity . . . . " Actually Gentry's data (Fig. 17) indicate that some intense tropical cyclones lie over relatively cold water (<28.0°C). While Brand (1971) cites a supposed correlation of central pressure deficit with sea-surface temperature throughout the lifetime of Hurricane Ester of September 9-26, 1961, it may be noted that during one twelve-hour period of constant sea temperature, the central pressure rose 15 mb; for three days while the sea temperature hovered about 84°F, the central pressure

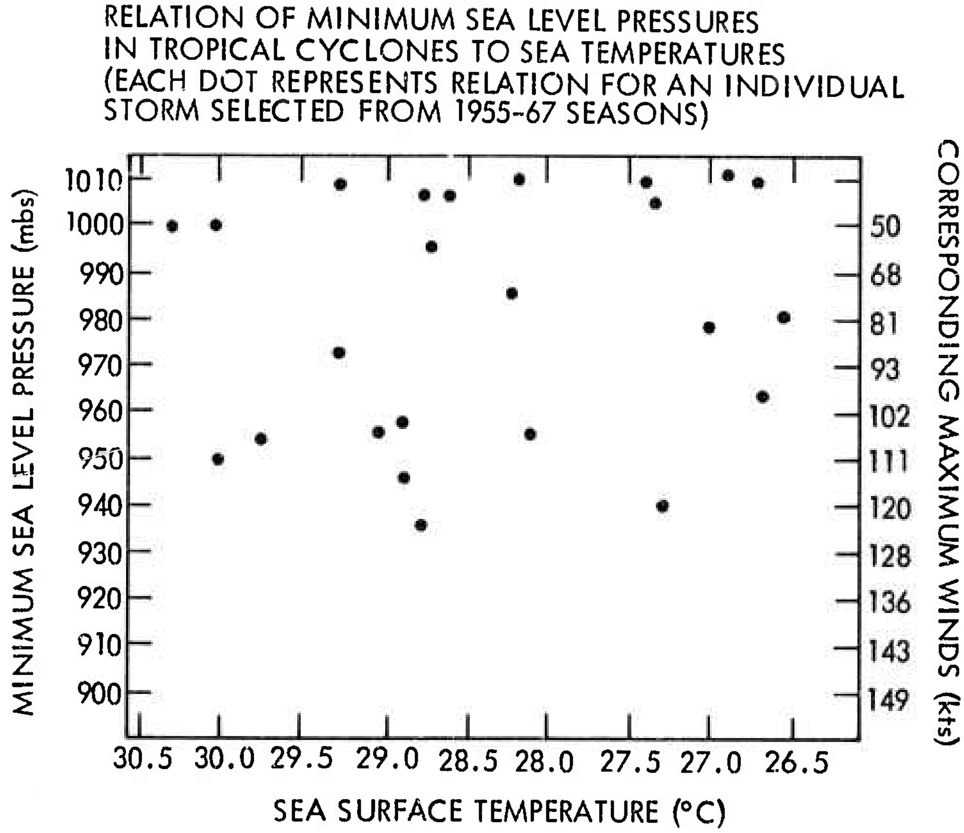

Fig. 17. Measurements of minimum sea-level pressure vs. local sea-surface temperature for several tropical cyclones (from Gentry, 1969a, p. 406; reproduced with permission of the American Meteorological Society).

nonmonotonically rose from 930 mb to 955 mb; and at various times when the sea temperature was at 86°F, the central pressure was at values as low as 927 mb and as high as 953 mb. Perlroth (1967) shows that for Hurricane Ginny of October 21-28, 1963, for five days while the sea temperature hovered near 80°F, the central pressure fell from 995 mb to 970 mb (Fig. 18). Perlroth (1969) cites the importance in tropical storm generation of considering not only high sea-surface temperature (above 26°C) but also small oceanic temperature variation with depth (less than 7°C within 200 ft of the surface). Perlroth emphasizes that the oceanic environment is only one factor in tropical storm intensity, and the sea-surface temperature is only one factor in the oceanic environment.

Finally, the fact that hurricanes leave cold wakes in the ocean might suggest that they do drain the upper sea layers of heat. However, the reason for the sea-surface temperature dropping about two degrees Centigrade after hurricane passage is the upwelling of lower, colder water owing to convection currents induced in the ocean by the pressure deficits of the hurricane (Perlroth, 1967). Slow-moving intense tropical cyclones can cause upwelling, from depths of 40 to 65 m, within the radius of hurricane-force winds; compensatory downwelling of as much as 80 to 100 m occurs 45 to 110 n mi from the path of the storm (Revesz, Jr., 1971; Leipper, 1967). The warm ocean water near the center of the hurricane flows radially outward, and is replaced by colder water from lower depths; outside 65 n mi from the center, below the outflow, there is compensatory inflow toward the center of the hurricane to complete the convection cell (Leipper, 1967; Wright, 1969).

Leipper finds the temperature readings at depths in the outer portion of the hurricane path to be higher than the temperatures before the storm.

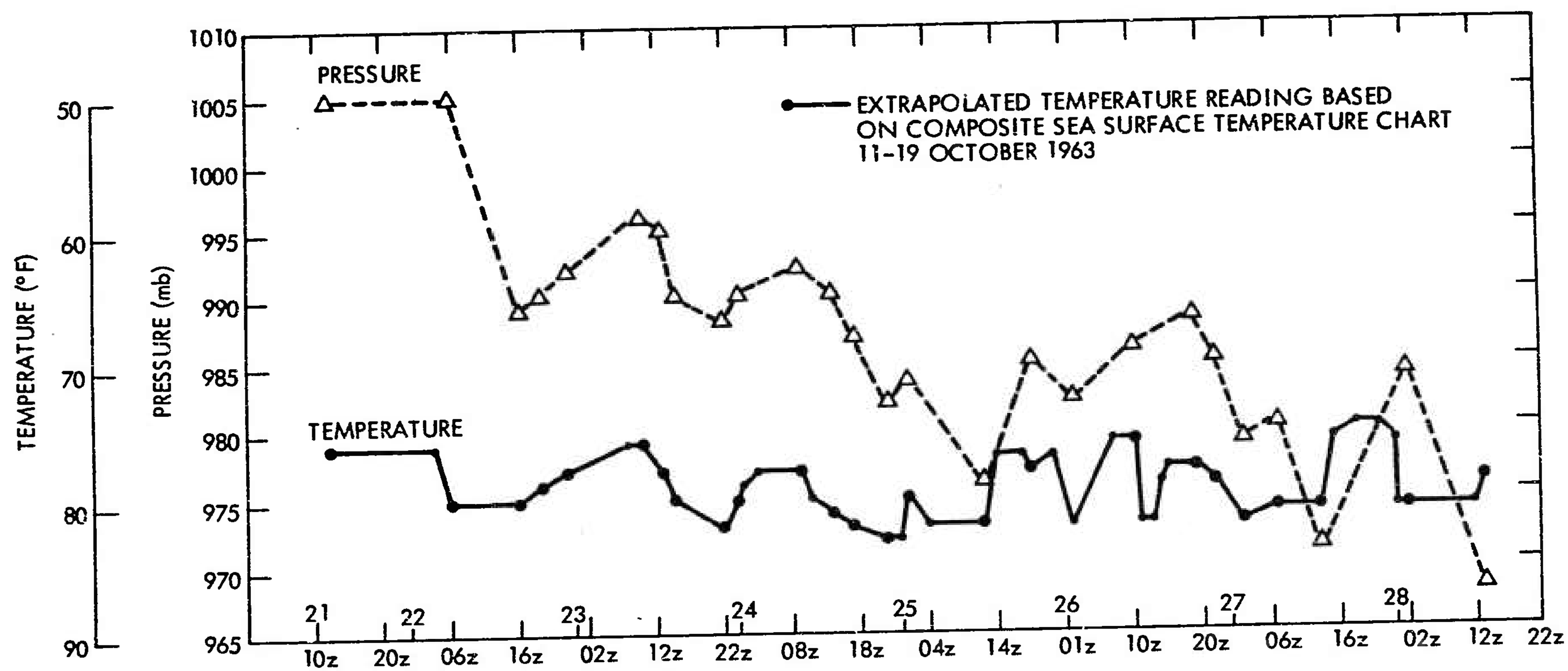

Fig. 18. The central sea-level pressure is plotted as a function of the local sea-surface temperature for Hurricane Ginny, 21-28 October 1963, which passed over the Gulf Stream on 24 October (from Perlroth, 1967, p. 266).

Thus a significant portion of the heat of the upper layers would seem to be transported radially outward, vertically downward, and then radially inward. Leipper and Wright, however, postulate that all the heat of the relatively warm upper oceanic level in excess of that of the colder upwelling is transferred to the atmosphere as an augmentation of the ambient rate;[8] Leipper suggests a transfer rate of 4500 cal/cm^2-day under Hurricane Hilda (1964), and Wright suggested 7200 cal/cm^2-day under Typhoon Shirley (1965). The previously cited figure of Malkus and Riehl (1960) for a moderate hurricane was 2420 cal/cm^2-day. Leipper acknowledges that the transfer so predicted is so much larger to the right of the path that even observed asymmetries in intensity are inadequate for his estimation. Further reasons for doubt concerning Leipper's calculation for Hurricane Hilda will be given in Section 3.9.1.

Leipper and Volgenau (1972) later introduced the concept of a hurricane heat potential, which they applied to oceanographic data for the Gulf of Mexico. The idea is that all the heat content of the upper oceanic layer (top 500 cm) implied by water temperature in excess of 26°C is available to sustain a hurricane. The authors acknowledge that their theory implies that in a hurricane, the air controls the sea temperature, although in almost every other physical situation, the sea is regarded as characterizing the air temperature. If all this heat were really available for transfer, why would a hurricane be required (rather than just the normally operative turbulent transfer mechanisms) to effect the transfer? One wonders how such a large potential could be developed, and why hurricanes are not always formed over waters with surface-layer temperatures above 26°C.

3.9 Critique of the Riehl-Malkus Model

A critique of the Riehl-Malkus reasoning that an extra oceanic source of latent and sensible heat maintains the tropical cyclone might begin by noting that use of the adiabatic-expansion relation $p \sim T^{\gamma/(\gamma-1)}$ is inappropriate for a (turbulent) frictional layer. Such a relation implies that the entropy is constant for each fluid element. Riehl (1954, p. 26) is correct that neglect of diffusion gives almost a 6°C drop (with condensation) in passing from 1014 mb (autumnal sea-level ambient) to 930 mb. In fact, the inflow layer is largely cloud-free, and the same adiabatic expansion taken as dry produces a 7.3°C drop. Such a large temperature drop does not occur (though whether the static temperature is really constant as Riehl believes is uncertain), and should never have been expected. But Riehl (1954, p. 286) was surprised at measurements he interpreted to imply constant static temperature during inflow in the frictional layer, and passed immediately from the (clearly inappropriate) isentropic expansion to the postulate of latent and sensible heat transfer from the ocean greatly in excess of ambient transfer levels.[9]

3.9.1 Evaluating the Arguments for Greatly Augmented Enthalpy Transfer

There are no indisputable semi-empirical theories or direct measurements of the heat and mass transfer from the ocean to the atmosphere in a hurricane: "unfortunately there is little information on C_E ('the nondimensional coefficient for the air-sea energy interchange') under hurricane conditions, other than the semispeculative guess that the exchange coefficients of latent heat, sensible heat and momentum are probably of the same magnitude" (Ooyama, 1969, p. 15). Hidy (1972, pp. 1086, 1097) notes the lack of quantitative progress on air-sea transfer, cites the paucity of data on the drag coefficient at sea level for winds over 20 mph, and mentions the difficulties in extrapolating data accurately; in fact, one of few known results casts doubt on the validity of the Reynolds analogy relating turbulent transfer of heat and momentum at the air-sea interface. There have been attempts to extrapolate to hurricane conditions empirical laws relating ocean evaporation rates to wind speed. The actual measurements refer to low wind speed and involve relatively dry air, far from saturation; extrapolation to hurricane winds and nearly saturated low-level tropical air is unjustified and misleading. Garstang (1967) emphasizes that a critical assumption in the theory of Riehl and Malkus is that the sea-to-air transfer coefficients increase linearly with wind speed indefinitely into the hurricane regime. Zipser (1969, p. 813) found that using such linearly-extrapolated bulk turbulent transfer relations, which imply an order-of-magnitude increase in latent heat transfer under moderately strong tropical disturbances over the ambient rate of transfer, would have produced clouds in an observed tropical disturbance with unsaturated convective downdrafts on a time scale an order-of-magnitude too short. Zipser also notes that Riehl and Malkus expected downdrafts around cumulonimbi to be saturated, but the observed downdrafts were unsaturated and so details of the Riehl-Malkus predictions on sea-to-air transfers near cumulonimbi must be modified. In the absence of more quantitative evidence Riehl (1954, p. 287) asserts that much sea spray is tossed into the air in a hurricane. This does not in itself assure augmented net enthalpy transfer from sea to air, since heat must be drawn from the air to evaporate the drops for later condensation of the water vapor in the eyewall and inner rainbands. Evaporation of spray leads to a temperature decrease as well as a dew-point increase, such that the total stagnation enthalpy remains unchanged.

Possibly the closest approximation to a direct measurement of the Riehl-Malkus postulate of an augmented oceanic heat source in a hurricane are the tritium-tracing measurements by Östlund in Hurricane Faith (1966). The tritium deposited initially in the stratosphere following American and Russian fusion bomb tests in 1961 and 1962 temporarily created trace amounts in the troposphere that exceeded the amounts in the ocean; determining tritium concentrations could theoretically determine the source of water substance in a hurricane. While the experiment cannot be repeated without the resumption of atmospheric nuclear testing, and while the present author does not regard the experiment as highly reliable, Hidy (1972) reported the result without reservations. Östlund found that the ambient rate of latent heat transfer from sea to atmosphere in the tropics (which is equivalent to evaporation of about 0.5 cm/day of water) was increased by a factor of about three within the inner 100 km radius of the hurricane, as opposed to the order-of-magnitude increase predicted by the theory of Riehl and Malkus.

There seems to be no indisputable direct evidence for the internal ocean heat source postulate of the Riehl-Malkus theory; the question does remain whether or not Carrier and his co-workers have obviated the indirect justification, specifically, the assertion by Riehl and Malkus that the oceanic heat source is necessary to satisfy the laws describing conservation of energy and momentum in the storm.

In this respect the earlier quotation from Rosenthal (1971b, p. 772), citing Ooyama (1969, pp. 14-15), should be discussed. Rosenthal asserts that higher-level air sinking down into the surface frictional layer will be drier and of lower equivalent potential temperature than the original boundary-layer air. If the quasisteady approximation of the Carrier model is justified, then in the mature stage the total stagnation enthalpy is a function mainly of position. According to Carrier and his co-workers, the plume and radiative transfer mechanisms that carry heat and moisture up into the ambient tropical atmosphere persist, with neither augmentation nor diminution to any appreciable degree, in the presence of the storm. These transfer mechanisms help compensate for rain-out in the outer spiral bands such that by the time a particle has slowly sunk into the boundary layer at 0.005 mph, it has been enriched in total stagnation enthalpy to the ambient value at its current height. In other words, much of the sea-to-air transfer of latent and sensible heat continues to pass across the boundary layer with little diminution, just as in the ambient, such that the enthalpy of air originally in the 700-900 mb strata increases to that of air originally in the boundary layer owing to enrichment of its total static enthalpy as it slowly descends. Only as the hurricane leaves the tropical oceans is this ambient transfer reduced; the air entering the boundary layer eventually is of lower total static enthalpy, since it comes from air which is originally higher in the tropical ambient (hence colder and drier), and since that air is not appreciably enriched as it descends. In this way traverse over ocean patches of varying temperature can help cause the well-known nonmonotonic perturbations in hurricane intensity within the general level of strength computed from the spawning ambient as discussed earlier (see also Section 4.4).

In this critique of the Riehl-Malkus model, one should note a later modification of earlier statements. Riehl and Malkus (1960, p. 17) write: "The total added heat energy from the ocean is on the order (for the moderate storm) of 2.5 cal/gm, while the average normal heat content (latent plus sensible) of tropical air is about 80 cal/gm." Thus the enthalpy contribution added by the ocean to ambient-air enthalpy is about three percent for a moderate storm. "The heat. . . gained by the air is only a minute fraction of that carried inward through the cylinder at the distance of the 1000-mb isobar. . .. Nevertheless, it is these small increments that produce the strong inward warming. . .. They are thus of utmost importance for generation and maintenance of tropical storms, even though they may be wholly neglected in a general energy balance. . . " (Palmén and Riehl, 1957, p. 156). That such a marginal amount of heating is critical to both generation and maintenance seems intuitively surprising. Later Malkus (1962, p. 247-249) limited the role of greatly augmented enthalpy transfer strictly to the maintenance of the hurricane, since during development the high winds that supposedly permit the increased turbulent transfer do not exist.

Hawkins and Rubsam (1968) have computed the structure and budgets for Hurricane Hilda for October 1, 1964, when it was an intense storm over the Gulf of Mexico. An unprecedented five-level collection of data was available for exhaustive analysis, and the total static enthalpy budget was calculated very similarly to the approach used by Malkus and Riehl (1960). Hawkins and Rubsam found, however, that (upon inclusion of the postulated latent and sensible enthalpy flux from the sea) the amount of energy estimated to be radiated away from the top of the storm was twenty times what seems physically possible. The kinetic energy budget computed for Hilda was also at significant variance from the one computed by Riehl and Malkus (1961) for Hurricane Daisy (1958). In view of this, Leipper's (1967) suggestion that the oceanic heat transfer to Hilda would be <u>underestimated</u> using Malkus's formulae, by 50-90% seems incomprehensible.

3.9.2 Temperature Measurements in Hurricanes

If numerous accurate temperature measurements in rapidly swirling, heavily raining portions of the hurricane could be readily carried out, resolution of doubt about the existence of a large ocean heat source within hurricanes could be quickly accomplished. However, such measurements are still subject to controversy and inaccuracy. Aircraft measurements tend to be taken above the inflow layer for safety reasons: ". . . the preferred altitude for reconnaissance traverses of a hurricane is from 10,000 to 14,000 feet" (Meyer, 1971a, p. 58). Currently used instrumentation may suffer from slow response time, from wet-bulb effects, and from the need to deduce the static temperature from a measurement that includes the dynamic contribution (Meyer, 1971b, pp. 19-37). Riehl and Malkus (1961, p. 188) discuss possible unresolved errors in midtropospheric temperature measurements of a few degrees Centigrade in magnitude.

First, to a reasonable degree of accuracy a dry-bulb thermometer measures $[T + (q^2/2\ c_p)]$, and a wet-bulb thermometer (with knowledge of the pressure) yields $[T + (q^2/2\ c_p) + (LY/c_p)]$ (Hess, 1959, p. 61). At very low altitudes, the dry-bulb temperature is the total potential-like temperature (θ_t/c_p), and the wet-bulb temperature is then related to the total stagnation temperature (H_t/c_p), because the missing gravitational contribution (gz/c_p) is negligible. Here the recovery factor (usually denoted r) for the thermometers is taken as unity; for the relevant Mach and Reynolds numbers this seems satisfactory, since for a well-designed instrument $r > 0.9$ for turbulent flow in air. The belaboring of the dynamic contribution would normally be unjustified in meteorology, but a 200-mph wind is equivalent to a contribution of about 4°C; this amount is in excess of one percent of both the static temperature (~300°K) and also the total stagnation temperature (~350°K) for the sea-level autumnal tropical ambient. Whether temperature measurements made several decades ago, when hurricanes generally were not believed to be capable of such speeds, are properly corrected for this dynamic contribution is not always clear. It should also be recalled that, while the theory needs some improvement, the still-quite-useful result of Carrier, Hammond, and George (1971) implies that the wet-bulb temperature would remain approximately radially invariant in a traverse of a hurricane along a ray and at constant height up to about 700 mb altitude, from the outer eyewall to the outer edge.

Deppermann (1937, p. 5), whose work is frequently cited by Riehl, writes: ". . . remarkably uniform diurnal temperature oscillations are maintained both before and after a typhoon; but when the station is under typhoon influence, a reduction of the maximum temperature by about 3°C occurs, while the minimum temperature remains the same." If by maximum and minimum temperatures Deppermann means dry-bulb and wet-bulb temperatures, respectively, and if one assumes Deppermann correctly accounted for the dynamic contribution ($q^2/2\,c_p$), then Deppermann finds that the static temperature T decreases by about 3°K in a typical typhoon, while the total stagnation temperature increases by a few degrees Centigrade (5°C-6°C at very most). In any case, there is no evidence of large increases in (H/c_p), as an oceanic heat source would purportedly cause. Also, if (H/c_p) is roughly radially constant in the inflow layer, then a decrease in T of about 3.7°C from sea-level autumnal tropical ambient is necessary for condensation, and this is reportedly not achieved in a typical typhoon, so a cloud-free frictional inflow layer is no surprise in Carrier's model.

Arakawa (1954, p. 119) reported that the air temperature dropped 1°C from 28°C to 27°C, and the wet-bulb temperature remained about constant radially at approximately 26.5°C, during the passage of an 898-mb typhoon over a Japanese weather ship in October 1944; only in the eye did the two temperatures rise sharply. Winds rose to about 120 mph in the eyewall. Again, this seems compatible with the Carrier-Hammond-George predictions, which include no oceanic heat transfer within hurricanes much above ambient level. Palmén and Newton (1969, p. 478) assert with no elaboration that the measurement implies an increase of the equivalent potential temperature from 360°K to 385°K, and thus is a confirmation of the internal heat source postulate; the author does not understand how this result was achieved.

To avoid difficulties in boundary layer measurements, Riehl (1963, p. 277) studied equivalent potential temperatures deduced from aircraft data taken at the 245-250 mb level at the inner edge of the eyewall. Little complication from moisture is expected at such heights, and the variation of equivalent potential temperature with height is not expected to be large. Riehl reports equivalent potential temperature of 370°K for a hurricane of central pressure of 960 mb, but provides no further details on the instrumentation or data reduction whatever. If the result is correct, very significant evidence exists for an oceanic heat source. However, Dr. R. H. Simpson of the National Hurricane Center in Miami, Florida has informed the author that a vortex thermometer was used (private correspondence). If one recalls that 250 mb corresponds very roughly to approximately 35,000 ft, then the remarks of Gentry (1964, p. 12) are illuminating: "The wind tunnel tests of the vortex probe under icing conditions (Ruskin and Schecter. .) indicated that the recorded temperature might be too high due to the change of state on impact at probe entry. . .. No evidence is readily available as to whether much ice accumulated on the vortex probe of the aircraft flying at about 35,000 ft or higher. Ordinarily, little liquid water would be encountered at the higher elevations and the ice accumulation should be minor. Some data collected by the research aircraft in 1961 and 1962 indicated, however, that at times the ice accumulation might be significant. There is the possibility, therefore, that the temperatures recorded for the upper troposphere are too high by an unknown amount. There will probably always remain a question as to the accuracy of the temperature measurements until some absolute standard of comparison is developed."

3.9.3 On the Magnitude of Possible Increases in the Wet-Bulb Temperature at Sea Level

Some insight into possible increases in the total stagnation temperature $(H/c_p) = T + (gz/c_p) + (q^2/2\ c_p) + (LY/c_p)$ at sea level within a hurricane will now be sought. Very near $z = 0$, $q \doteq 0$. Also, the air temperature assumes the temperature of the sea; since the sea temperature is held spatially constant in all hurricane analyses known to the author [with one experimental exception (Anthes, 1972, pp. 473-474)], and since the sea temperature for an intense slowly translating hurricane decreases toward the center of the storm if it changes at all, the conservative approximation for current purposes is to hold T everywhere constant at the ambient sea-level value for the tropical autumn. Then the only changes in (H/c_p) are due to changes in (LY/c_p), or more specifically, to changes in Y since L and c_p are taken constant for current purposes. Further, with only one-percent error,

$$Y \equiv \frac{\rho_v}{\rho} \equiv \frac{\sigma(RH)\ P(T)}{p_a + \sigma(RH)\ P(T)} \doteq \frac{\sigma(RH)\ P(T)}{p}, \tag{44}$$

so the vapor mass fraction Y increases as the relative humidity RH increases or as the total pressure p decreases, for temperature T fixed. In the autumnal tropical ambient, if L represents latent heat of condensation plus glaciation, at sea-level $(LY/c_p) \doteq 50°K$. The ambient sea-level relative humidity is 0.84 and the pressure is 1014 mb. [The relative humidity is unity at the air-sea interface but sea-level measurement generally means readings at shipboard height, or more phenomenologically, it means measurement in the postulated, roughly thirty-foot-thick constant-stress layer near the air-sea interface (Hidy, 1972, p. 1084).] For a moderate hurricane with central pressure of 966 mb, the increase in (H/c_p) is 2.5°K, and for an intense hurricane with central pressure of 910 mb, the increase in (H/c_p) is 5.7°K -- provided RH is held at 0.84. If RH increases from 0.84 to unity under the eyewall, then (H/c_p) increases by 12.5°K for the moderate hurricane and 16.3°K for an intense hurricane -- although the justification for adopting such an increase in RH appears unestablished, and Anthes and Johnson (1968, p. 297) cite evidence that the relative humidity of the boundary layer tends to remain between 80% and 90%. Malkus and Riehl (1960, p. 16) claim the equivalent potential temperature, which is clearly closely related to (H/c_p), increases by 12.5°K from ambient to eyewall in the moderate hurricane [although Riehl (1963, p. 277) seems to augment this to 16°K], and by 35°K in the intense hurricane. By the reasoning adopted here the physical basis of the 12.5°K increase becomes evident, but the 35°K increase for an intense hurricane represents a physically impossible relative humidity of over 100%. Succinctly, while Carrier and his co-workers discuss increases in the first normal derivative of H at sea level in hurricanes over ambient on the order of 10%, Riehl and Malkus discuss occasional increases in H itself on the order of 10%. Carrier and his co-workers neglected the full effect of decreased air density as the eyewall is approached, and found H to be radially constant; the conjecture here is that the error in H so incurred is about one percent or so.

3.10 Numerical Simulation of Hurricanes on Digital Computers

In recent years there has been a proliferation of computer models of the tropical cyclones (Yamasaki, 1968; Ooyama, 1969; Rosenthal, 1970; Sundqvist, 1970; Kurihara, 1971; Anthes, 1972). These models warrant a review in themselves, and only a limited discussion is undertaken here.

First, it must be understood that these models are by no means exact solutions of the full conservation equations, subject to appropriate boundary and initial conditions. In fact, to paraphrase Lorenz (1967), one might remark that detailed reproduction of the hurricane lifespan by numerical methods might not appreciably increase physical insight, because the total behavior is so complex that the relative importance of various features might be no more evident from examination of the computer solutions than from direct observations of the real atmosphere. In any case, computer solutions are today far from exact solutions; in fact, today no computer solution even attempts to treat observed initial data (Ooyama, 1969, pp. 35-37; Rosenthal, 1971b, p. 35), but rather describes how a general hurricane grows and dies -- in fact, works in computer simulation emphasize results for the <u>later</u> stages of intensification and the peak-intensity portion of the lifetime. The adopted initial conditions almost invariably have pronounced circulations, and only major alteration of parameterizations prevent inevitable hurricane formation.

Mention of parameterizations introduces another area of compromise with exact solution. Because many phenomena on the cumulus scale and smaller are crucial to the hurricane-scale flow, computer solution today is feasible only if the roles of turbulent diffusion, radiative transfer, and (especially) cumulus convection are simulated by parameterization. No one knows with certainty how to describe the role of these phenomena in terms of variables occurring in the hurricane-scale solution, or even if adequate description of this type is possible. Reviewing the parameterizations of cumulus convection currently used in numerical simulations of hurricanes, Ooyama (1971, p. 744) acknowledges that ". . . it is generally agreed that these methods are essentially stopgap measures which are used for want of better alternatives." Gray (1972, p. 57,60) writes: ". . . the authors (Gray and Lopez) feel that it is very difficult (or impossible) at the present time for any numerical model of tropical motion to come to grips with the real problems of incorporating the cumulus convection in terms of the broader-scale flow. . .. In that the cumulus cloud is such a distinctive physical unit, it would appear that it should be independently treated. . .. The physics of the cumulus-broadscale interaction may not necessarily be overcome by applying the primitive equations to even smaller grids and time steps without additional insight into the character of the individual convective elements." While there are some physical concepts that guide currently employed parameterizations (such as weak integral constraints on energy and water substance), the parameterizations remain rather arbitrary and experimental (for a summary of them, see Haltiner, 1971; Bates, 1972). Often the parameterizations are freely adjusted until the cyclone-scale results are consistent with measurements and/or intuition.

The computer results are very sensitive to the assumptions made concerning the formulation of cumulus convection and eddy transfer (Ramage, 1971, p. 40; Haltiner, 1971, p. 262; Garstang, 1972, p. 619). Without any disparagement, such adjustment of parameterizations may be termed curve-fitting. What is learned from current computer experiments is mute. Of course, no model can today entirely escape such curve-fitting. The question to be pondered is how refined a solution is currently warranted in view of the limited physical understanding of certain critical phenomena?

A further complication in evaluating numerical models is that sometimes the finite differencing, intentionally or inadvertantly, adds to the stated formulation in the sense that effects not in the differential equations are present in the difference equations. The most accurate differencing of the differential equations is not necessarily preferred, but rather numerical techniques are rated ". . . on intuitive meteorological inspection of results" (Anthes, Rosenthal, and Trout, 1971, p. 747). For instance, errors introduced by less accurate upstream-difference schemes were retained to simulate lateral mixing when more accurate central-difference schemes gave less realistic results (Rosenthal, 1970, pp. 657-658). Later the differential formulation was modified so that an accurate differencing of the modified expression for lateral diffusion gave the numerical contribution desired. However, the modified expression ". . . is not very satisfying from a physical point of view" (Anthes, Rosenthal, and Trout, 1971, p. 747). What can be learned from such procedures again seems mute.

Even if the parameterizations for cumulus convection, turbulent mixing, and radiative transfer were adequately known, and the finite-differencing were adequately accurate, the question of feasibility in seeking a uniformly valid solution for the entire cyclone would remain.

Large gradients occur over relatively small scales in important subregions of the storm (e.g., the frictional boundary layer and the eyewall), while small gradients occur over relatively large scales in the bulk of the storm (the rapidly swirling regions and the outflow layer aloft). On current computers usually a more-or-less fixed grid of 10 km or 20 km radial resolution and at most thirteen layers of vertical resolution is adopted by practical considerations; the overwhelming bulk of the grid points then lie outside the eye, the eyewall, and the frictional boundary layer -- where important processes are occurring. The domain sizes are quite limited (typically 440 km) so relatively weak boundary conditions (requiring only that purely advective influx occur at the side boundaries) are employed and much of the storm lies outside the domain of computation. Garstang (1972, p. 619) states numerical results are often very sensitive to the treatment of the outer limits. An alternative to direct numerical solution of the uniformly valid equations is to subdivide the storm into natural portions where different gradients and phenomena are operative. This is the approach, with provision for interfacial compatibility, used by Carrier and his co-workers.

Some evidence for uncertainty concerning current numerical models for the hurricane is provided by results for the frictional inflow layer. First, with his model, Rosenthal (1971b) performs numerical experiments in which the latent heat transfer from the sea is greatly augmented over ambient but the sensible heat transfer is set to zero. The goal is to show that the latent heat transfer is the important portion of the total enthalpy transfer. Of course, Rosenthal is well aware that the sensible heat transfer is required to permit the latent heat transfer (Riehl, 1954, p. 336). The computation is cited as evidence of how incompletely the full conservation equations are represented in current models. Thus the current computer programs can not prove the physical validity of the Riehl-Malkus oceanic-heat-source postulate; the solutions which the current computer programs generate merely reflect the incorporation of the Riehl-Malkus postulate in their formulation.[10] Next, the study by Anthes (1971) of the incompressible axisymmetric flow in a frictional layer under an intense Rankine-like vortex furnishes an opportunity to test how well the finite-difference techniques used in computer simulations of hurricanes succeed in a problem of reasonable complication to which the answer is apparently known. Interestingly, Anthes introduces an initial-value problem as a convenient device for achieving the desired steady solution, without claiming validity for the transient phase necessarily computed. There are three points about the solution that will be noted. (1) The multilayer solution reveals that the one-layer treatments of Ooyama, Rosenthal, and Yamasaki underestimate the tangential and (particularly) the radial velocity components of the frictional layer, and give a premature eruption from the Ekman layer at incorrectly large radial distances from the center. (2) While both diffusion coefficients are held constant, Anthes takes the radial diffusion coefficient four orders of magnitude larger than the axial. Either by this means, or by large truncation errors through forward-differencing of advective terms, for plausible solution radial diffusion plays a role in regions of the flow where boundary-layer theory suggests radial diffusion should be negligible. (3) The Ekman layer divergence changes from a weak downdraft to a large updraft about 125 km from the axis of symmetry. The spuriousness of this result, and the actual two-part structure of the frictional layer at distances from the axis at which nonlinear terms are significant [as revealed by analytic and more sophisticated numerical techniques (Carrier, Hammond, and George, 1971; Carrier, 1971a; Burggraf, Stewartson, and Belcher, 1971; Dergarabedian and Fendell, 1972b)] remain obscured (Section 3.5).

Nevertheless, more elaborate computer simulations are continually developed. Anthes (1972) has attempted to account for departures from azimuthal symmetry. The axis of the simulated hurricane vortex undergoes a curious anticyclonic rotation at approximately eight mph about the center of the storm. Anthes suggests the path of a cyclone is not influenced by horizontal sea-surface thermal gradients, although Brand (1971) claims observational evidence to the contrary.

3.11 Implications of Hurricane Models on Seeding

The current mode of attempting artificial modification of tropical cyclones is to introduce silver iodide crystals in the supercooled water believed to exist in appreciable, naturally unfrozen quantities high in the eyewall, and perhaps also in rainbands and cumulus clouds both close to and far from the eyewall. In a typical field experiment, over two hundred rocket cannisters, each containing about 150 g of silver iodide, are released at about 33,000 feet over a 30 km distance (entirely outside the eye), along a ray 90° clockwise to the direction of storm translation; such an experiment takes two-to-three minutes to execute.[11] In laboratory studies, a gram of silver iodide produces 10^{12} to 10^{14} crystals, which serve as ice nuclei and which grow to three-micron diameter. The goal is that there will be 10^3 nuclei/liter-of-space over a depth of several kilometers a few minutes later; this concentration is reduced to negligible levels in a couple of hours (Gentry, 1971a; Penner, 1972). The silver iodide will hopefully cause the supercooled water droplets to freeze, and thus to release the heat of fusion (Battan, 1969).

The proposed mechanism by which this heat release (and supposed attendant temperature rise of a couple of degrees Celsius, and density decrease) is efficacious has been altered on several occasions (Simpson and Malkus, 1964; Rosenthal, 1971a; Gentry and Hawkins, 1971), and at times several alternative possible mechanisms have been set forth (Gentry, 1971a). A concise history of the various suggestions has been given by Riehl (1972a, pp. 267-270). One accepted point is that seeding in the nascent eye of a developing tropical storm should be avoided since this procedure would probably abet intensification (Rosenthal, 1971a). The most recent proposal has been to seed cumuli <u>outside</u> the eyewall and above the 0°C isotherm (roughly, 600 mb or 15,000 ft) with silver iodide. The aim is not to capitalize directly upon the heat of fusion released by supercooled water droplet freezing, but rather to cause these cumuli outside the eyewall to grow several thousand feet in height from midtropospheric to upper tropospheric levels. This would hopefully initiate a chain of events from which an order-of-magnitude more heat than that associated with freezing will indirectly be realized. This added heat would come from some low-level inflowing warm moist air erupting not up the eyewall, but prematurely up the growing cumulus, with associated condensation and additional glaciation. If entirely successful, proponents have suggested that perhaps a substitute eyewall further from the center, and hence (from conservation of angular momentum) with reduced swirling speeds, could be realized from this premature eruption of boundary-layer air (Gentry and Hawkins, 1971; Rosenthal, 1971c).

Fukuta (1972) suggests that the field experiments conducted to date have employed overseeding of supercooled water droplets to form small ice particles; the energy release would be small and the effect over in less than half an hour. Instead, Fukuta suggests reducing the seeding material to 1% or even 0.1% of that used in past field experiments, to form large precipitable ice particles. The latent heat of fusion these falling particles could extract from the inflow-updraft stream would supposedly raise the central pressure for hours, reduce swirling, and induce spreading out of the storm. Detailed analysis fully substantiating either the concepts of Gentry and Hawkins or the concepts of Fukuta has not appeared.

Clearly, like numerical simulation of hurricanes, the seeding of hurricanes warrants an entire review unto itself. If fact, with the proposed extension of the U.S. seeding effort [Project Stormfury, a joint effort of the Department of Commerce (NOAA) and the Department of Defense (Navy)] from the western North Atlantic to the western North Pacific Oceans (Mallinger, 1971), this might be an apt time for such a review.

Attention here will be confined to a few observations, especially what the hurricane models suggest about seeding procedure.

Even proponents of seeding anticipate only a 10 to 15% decrease in maximum winds. In the one case in which larger decreases were noted, the anomaly is now attributed to synoptic peculiarities relating to upper-level outflow (Hawkins, 1971). This is not to disparage such decreases; statistical treatment of a model suggests multimillion dollar annual savings in damage from hurricane seedings that would reduce peak winds by fifteen percent (Boyd, Howard, Matheson, and North, 1971). The problem is that the storm is naturally oscillating in intensity by the same order of magnitude, so it is very difficult to distinguish natural and artificially induced changes. In fact, the gradual increase of the maximum winds of Hurricane Ginger by 15% during the day after initial seeding on September

26, 1971 was ascribed to natural forces by Project Stormfury personnel, as was the 11% decrease after the second seeding on September 28 (Lieb, 1972). Fujita (1972) could find no evidence that seeding altered Ginger; since the central pressure of this weak hurricane was only 980 mb at the times of seeding, he suggests that perhaps the cloud-top heights were too low for the silver iodide to be effective. The threat of litigation has constrained the number of seeding experiments severely.

The National Hurricane Research Laboratory (Gentry, 1969a; Gentry, 1969b; Gentry, 1970) has discussed a six-to-twelve hour cycle of amelioration after seeding; physical basis for this time scale has yet to come forth (see below). Further, if the central pressure deficit is reduced as reported, eyewall seeding must alter the eye in a still unidentified manner, although Black, Senn, and Courtright (1972) suggest that one hour after seeding outside the eye, the eye may expand more than is consistent with natural variability. More complete post-seeding probing of the tropical cyclone would be helpful in evaluating these claims.

Rather similar computer models have produced different guidance with regard to current seeding practice. The reasons for the discrepancy are not fully available because some details remain unpublished. Rosenthal (1971c) notes that both the magnitude and the duration of the heating taken to simulate seeding in his computer experiments seem excessively large. Nevertheless, in those numerical experiments believed to most closely simulate the field experiments performed on Hurricane Debby, August 18 and 20, 1969, Rosenthal (1971a) predicts an increase after seeding in the maximum wind, and over wide radial extents an augmentation in wind level, at 700 mb; the comparable measurements at 12,000 ft indicated a reduction in winds (Hawkins, 1971). Rosenthal predicts a decrease in the maximum wind at sea level after seeding (though in substantial portions of the hurricane, surface winds are predicted to increase); no post-seeding measurements were made at sea level. Quantitative detail is omitted here because Rosenthal (1971a, p. 415) notes: ". . . at best, the results should be considered qualitative guidance material." Sundqvist (1971) states flatly that seeding will intensify a hurricane. Estoque (1971, p. 4), noting the tentative nature of his conclusions, remarks:

> We have [studied] the properties of a hurricane model which is suitable for simulating the effects of artificial seeding. The main improvement of this model over previous ones is the explicit inclusion of cloud micro-physical processes, including the prediction of the liquid water distribution. Thus, the model is able to simulate more realistically the release of the latent heat of fusion due to artificial seeding. The model has been used to simulate artificial seeding over three different radial locations in the vicinity of the maximum in the surface wind. The expected reduction and outward displacement of the maximum radial surface pressure gradient did not materialize. Instead, the pressure gradient and also the wind increased in intensity. However, the seeded storm intensity decreased after the seeding stopped. The greatest effect occurs when seeding is done over a region just inside the surface wind maximum location.
>
> An interesting feature of the unseeded hurricane is the occurrence of periodicity in time of the hurricane intensity. The magnitude and the period of this periodicity is about 10 m sec^{-1} and 8 hours, respectively. This magnitude is about

the same order as that of changes induced artificially by seeding. If this periodicity is real, it should be taken into account in the interpretation of seeding experiments in the actual atmosphere.

The effectiveness of current seeding practices remains unresolved. If the Carrier model is valid, silver iodide seeding can only transiently upset the stable hurricane configuration. Under this model, warm-fog dispersal methods would have to be applied to the entire "throughput supply" layer of warm moist air to achieve the significant goal of premature rainout in outer spiral bands. The layer of warm moist air is so spatially extensive that such attempts seem somewhat impractical. Dergarabedian and Fendell (1971) have discussed use of warm-fog dispersal methods for lower levels in a hurricane for premature rainout of the throughput-flux; about simultaneously, so did Mathews (1971, p. H-8), albeit more optimistically:

> The feasibility of warm cloud modification prior to cold cloud modification should be examined because warm cloud modification may permit growth of small warm clouds to temperatures at which cold cloud modification will be effective. The combined use of warm cloud and cold cloud modification techniques would permit selective seeding in all regions without cloud top temperature restrictions.

Seeding to divert path seemingly holds little better promise than seeding to alleviate intensity, since there appears to be no way to discern what path alterations were due to human intervention under current understanding.[12]

4. THEORY OF TROPICAL CYCLONE INTENSIFICATION

There exists no satisfactory theory explaining how, annually, a small number of tropical disturbances intensify to become hurricanes, and why in contrast most disturbances do not. Charney (1971) notes that, unlike in the midlatitudes, tropical temperature gradients are weak, and tropical cyclones most often form over the tropical oceans where the surface temperatures are particularly uniform. No theory of how pronounced baroclinicity develops in the barotropic tropics is likely to emerge until the detailed local balances and large-scale circulations of the tropical ambient are better understood quantitatively, because early stages of cyclogenesis will probably involve perturbation about that ambient.

Carrier (1971b) has presented an intensification process by which his quasisteady mature model evolves in time from a tropical depression. Tracing back to even earlier evolution seems premature at the current state of understanding. Although Carrier's theory has been reproduced by Penner (1972) without objection, Carrier's theory of intensification is not currently entirely satisfactory. Thus, it is used here only as an interesting vehicle to raise some significant points about the later, briefer stage of intensification, as opposed to the longer, earlier stage. Ooyama (1969, pp. 35-37), among many others, has written about an initial very modest rate of pressure fall in intensifying tropical cyclones, followed by an appreciably accelerated rate of fall -- it is this latter stage which is principally addressed here.

4.1 Carrier's Outline of Intensification

Carrier begins with a schematic view of an axisymmetric tropical depression in which there is a weak Rankine-vortex-like swirling, the maximum azimuthal speed lying relatively far from the axis of rotation. There is radial inflow in region I during the transient phase (Fig. 19). The swirling would quickly establish a shear layer beneath it; there is weak upflux out of (downflux into) the surface layer where the swirling speed increases (decreases) with radial distance. Since linear theory correctly predicts the downflux in the mature stage with rapid swirl, linear theory probably suffices to predict the boundary-layer divergence during intensification. However, since equilibration of the boundary layer requires times of $O(\Omega^{-1})$, or roughly sixteen hours in the tropics, and since this is possibly an appreciable fraction of the later-stage rapid-intensification time-scale, a transient linear theory will be required. It must yield a radial influx through the boundary layer II in excess of the radial inflow speed in I. [Incidentally, the times characterizing significant change in the theomodynamic state of the lower tropical ambient are probably on the order of 100 hours, a time span partly related to the typical magnitude of eddy viscosity appropriate for the tropical atmosphere. Hence, unless the intensification time from depression to cyclone greatly exceeds 100 hours -- and this seems dubious -- the thermodynamic state of the ambient may possibly be justifiably taken as fixed throughout the later stages of intensification. This statement by no means implies that the ambient can be taken as fixed throughout the entire intensification process. The very fact that descent speeds in the outer portions of even a mature hurricane are roughly comparable to ambient tropical descent speeds suggests that the tropical ambient probably cannot be held fixed throughout intensification.]

The air erupting from the boundary layer begins to displace the air initially in the central core of the developing storm. The air initially present in the core is of slightly lower pressure than the ambient air at the edge of the storm, but not vastly different in vertical stratification. The air erupting from the boundary layer under the Rankine-vortex-like swirl displaces the air initially present in the core vertically upward; since there is a "lid" on top of the storm, the vertically displaced initial air is, near the top of the core, squeezed radially outward.

The following competition develops. The new air rising out of the boundary layer is drawn entirely from relatively warm moist air near the bottom of the atmosphere. Thus, displacing the air initially present in the core is relatively light air. On the other hand, the convective motion of new air is small, especially at early times, and the ambient processes (turbulent diffusion, radiational cooling, cumulus convection) try to maintain the original, near-ambient stratification in the core. If the convective displacement wins out, then the core becomes lighter and lighter, relative to a column of air at the outer edge of the storm. This paragraph contains those points of Carrier's intensification model that seem most unsettling, and will be subject to close scrutiny in the critique below.

However, Carrier's basic proposition is that the swirling in I has led to a downflux in II, a spiraling inward in the boundary layer and an upflux into the core, and a lightening of the core by hydrostatic considerations. For dynamic consistency, the centrifugal force (anticipated to be the dominant inertial effect) must increase to balance the augmented radial pressure gradient. Since angular momentum is conserved in I, where friction is negligible, the fluid particles must necessarily move in closer to the axis of symmetry (axis of rotation). The result is that in time, in the Rankine-vortex-like swirl distribution, the maximum azimuthal speed increases in magnitude and the position of the maximum lies closer to the axis (Figs. 19 and 20). Hence, the more the pressure falls in the core, the more fluid sinks into the boundary layer to spiral inward, erupt upward, and cause further pressure reduction in the core. If the crucial early competition is resolved in favor of the organized convection, ultimately the particles erupting out of the boundary layer rise so quickly that they lie on a moist adiabat, and the greatly lightened core is entirely flushed of its original fluid.

While most descriptions (Palmén and Newton, 1969) tend to picture the eye as formed gradually as the pressure deficit develops, in the Carrier model the central core may well be completely flushed of ambient-like air, so that the air in the core lies on a moist adiabat, before much trace of an eye is to be found. At first, a Rankine-vortex-like swirl holds everywhere. From this fully developed one-cell structure with at most only the rudiments of an eye, a well-defined two-cell structure with a calm center region emerges rapidly, probably in much less than an hour, owing to inertial oscillation, in the following way.

As the pressure falls in the core relative to ambient, the particles in I necessarily move in closer to the axis to permit a compensating centrifugal force to develop. Once the core is flushed and moist adiabatic ascent characterizes the full height of the core, no further pressure deficit can be generated. By inertia, the spinning particles continue to move in, a dynamic imbalance is created, and a radial acceleration develops

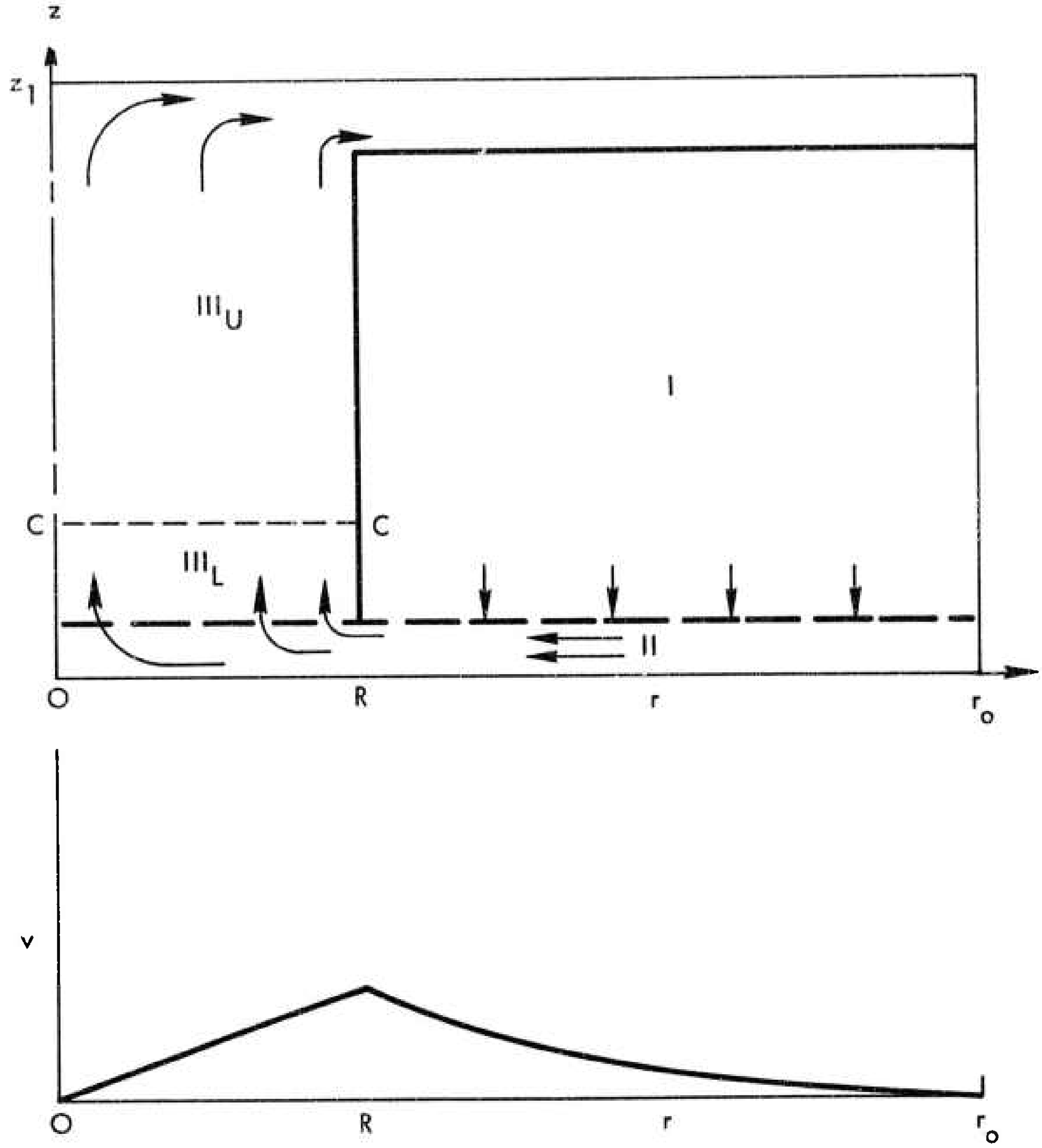

Fig. 19. In this schematic view by Carrier of the flow configuration and circumferential velocity distribution in an intensifying tropical depression at some early time $t = t_1$ (say), the interface C-C between the new and initial air in the core is idealized as horizontal for convenience. However, it is easy to show that for all but very modest circulations (i.e., except for angular velocities less than three times ambient), a rigidly rotating core (which implies uniform updraft velocity at the top of the nonlinear frictional layer) could not accept all the fluid pumped through the Ekman boundary layer. Thus a uniform ascent in the core is dynamically impossible, and in fact ascent in the eyewall is concentrated in cumulonimbi. For the gross balances being discussed, many points can be made without accounting for such refinements in eyewall structure, though ultimately details of the structure are crucial. (From Carrier, 1971b, p. 146).

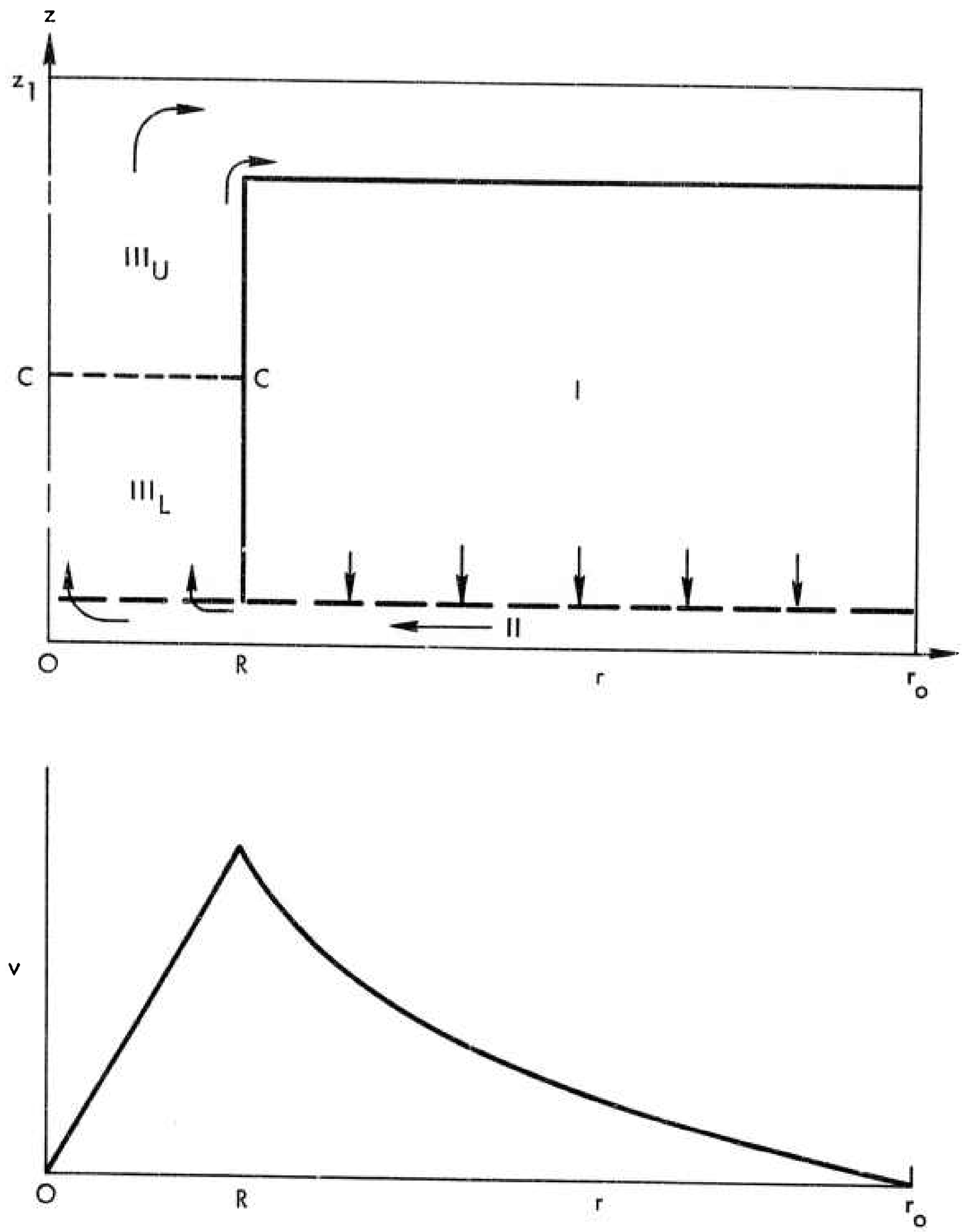

Fig. 20. Intensification from tropical depression to hurricane has progressed to a more advanced stage in this schematic diagram, holding at $t = t_2 > t_1$, according to Carrier's model. The magnitude of the maximum swirl is increased and its position lies closer to the axis of symmetry. (From Carrier, 1971b, p. 146.)

to force the particles away from the axis of symmetry. This reverse motion creates a rarefaction at the center, and relatively dry warm motionless air sinks down the axial column to form an eye. This air may be air from above the storm or rained-out, slowly swirling air entrained out of the top of the moist adiabatic column (Fig. 21). Because there is no appreciable swirl (hence no associated pressure gradient) in the eye, there is no frictional boundary layer under the eye. The moist adiabatic column becomes an annulus displaced from the axis, i.e., the eyewall; the inertial oscillations of the eyewall eventually damp in time.

4.2 Critique of the Carrier Model of Intensification

Carrier has offered more than the conventional linearized stability analysis to determine what scales of motion grow in what time interval (Yanai, 1964; Ogura, 1964; Yamasaki, 1969). [Bates (1970) did study the growth to finite amplitude in the ITCZ of nonzonally symmetric equatorial wave disturbances of 2000-km longitudinal scale, which are observed to move westward at 13 kts. The time scale for early growth owing to barotropic instability was about two days; frictional dissipation eventually curtails further growth effected by conversion of condensational heat to eddy available potential energy. Because Charney's concept that condensational heating in the tropics is dependent on convergence of moisture in the planetary boundary layer was incorporated, the disturbance had a warm core with maximum intensity in the lower atmosphere. Holton (1972) furnishes further lucid insight into equatorial wave disturbances, and in fact concisely discusses many other topics arising in tropical meteorology.] Carrier has also emphasized the role of lightening of the core rather than (say) large-

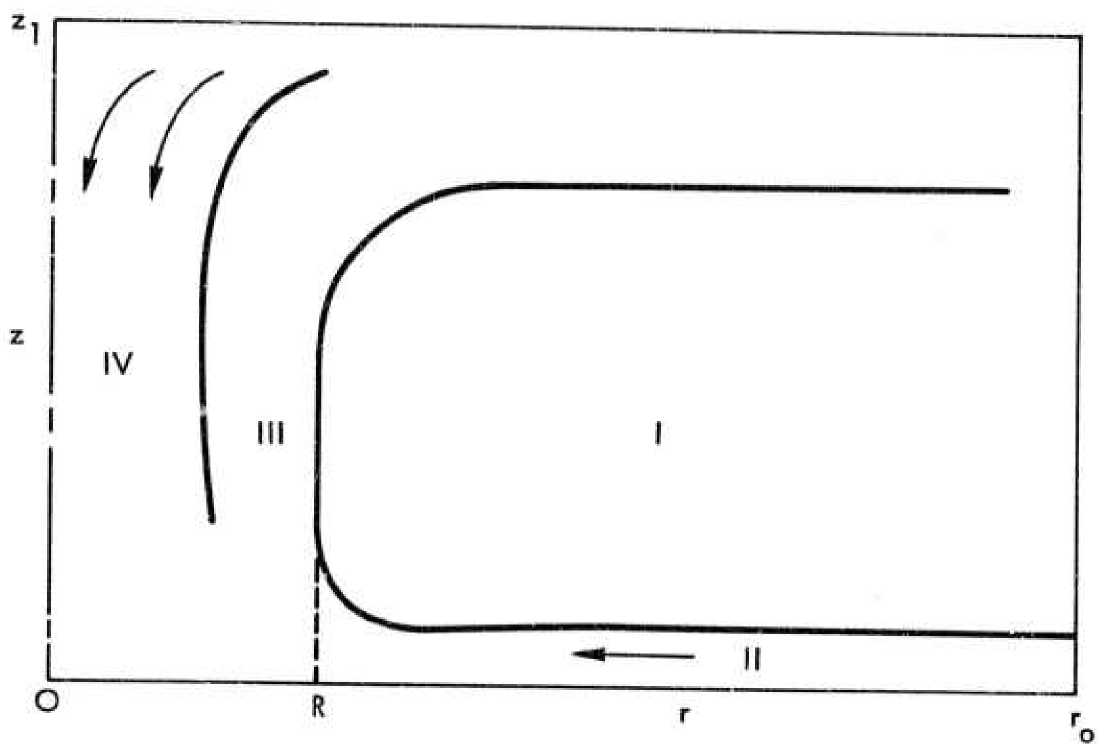

Fig. 21. Schematic picture by Carrier of the flow configuration which prevails when the radius of maximum swirl R is increasing and an incompletely formed eye is being filled with relatively dry and motionless air, which sinks down from the top of the storm under dry-adiabatic compression. With the formation of an eye lighter in weight than the eyewall, the terminal stages of intensification and the beginnings of quasisteady mature-stage structure are realized. (From Carrier, 1971b, p. 150.)

scale horizontal turbulent diffusion. However, there are problems concerning the nature of the convective lifting in the core, as now discussed.

The ambient distribution of total stagnation enthalpy is such that when a column stratified like the ambient is lifted as a whole adiabatically, warm air is squeezed radially outward at the top, even if warm air is added at the bottom (Fig. 7). The net effect for small convective motion, according to a method-of-characteristics numerical integration of the resulting hyperbolic boundary-value problem, is not a lightening of the developing core (Dergarabedian and Fendell, 1972a, pp. 62-67).[13]

A closely related observation is to note Carrier (1971b), in his discussion of intensification, seemed more concerned with how a hurricane is formed than whether it is formed. Hence, he adopted effectively the following proportionality:

$$\frac{\text{pressure fall from ambient}}{\text{total pressure fall for a moist adiabatic core}} = \frac{\text{new air into core}}{\text{total air required to flush core}}$$

This statement is true at the outset, and, if a hurricane forms, at the end, of flushing the core. In fact, however, such a simplification lets a hurricane always be formed. In truth, the pressure fall for small times is less than that predicted by the fractional-volume proportionality. As the quasisteady mature state approaches, the initially slow pressure fall probably accelerates to a rapid decrease, and the thermodynamic stratification of the emerging eyewall asymptotically approaches a near-moist-adiabat state.

The problem cannot be satisfactorially buried in parameterizations of turbulent transfer, radiative cooling, and cumulus convection in the core. What appears needed is explicit recognition that a broad uniform ascent in the core does not account for the actual mechanism of convection, which would seem to entail ascent in a concentrated number of intensified cumulonimbi. The somewhat tentative nature of this remark stems from statements like that by Palmén and Newton (1969, p. 490): "It seems clear from. . . observations and from descriptions of turbulent encounters that, in some cyclones, ascent in the inner region is dominated by cumulonimbus, . . . while in others a broad-scale and more uniform ascent takes place."

If cumulonimbi do play an essential role in the core ascent during intensification, some semblance of an eye would probably form at an earlier stage of hurricane development than Carrier suggests.[14] The eye would probably not be entirely created on a time scale of 20 min at the end of intensification, as Carrier suggests, although inertial oscillations at the end of intensification might rapidly delineate a previously ill-defined eye.

The tritium-tracing analyses by Östlund (1968) of Hurricane Hilda (1964) and Hurricane Betsy (1965) suggest that while the eye may have partially consisted of air entrained from the eyewall, there had been substantial subsidence of stratospheric air down into the eye. Thus, Carrier seems correct in citing two possible sources for the air in the eye.

4.3 The Distribution of Cumulonimbi during Intensification

Riehl (1972, p. 249) notes: "Very fine weather, with hardly a cloud in the sky, often precedes the arrival of a hurricane by 1 day, when the storm is moving westward in the tropics. In the past this deceptively beautiful weather has been the cause of many disasters with heavy loss of human life in the Caribbean and elsewhere."[15] In a possibly related observation, Lopez (1972b) calls attention to the work of Oliver and Anderson (1969), in which satellite photographs are used to indicate that the majority of hurricanes form around a circulation center in the clear area ahead of (rather than directly under) a convective region. The concentration of cloudy convective regions amid clear calm regions in tropical cyclogenesis may be related to characteristics of subsidence that, by continuity, must accompany convection.

However, perhaps also there is a concentrating of pre-existing dispersed cumulonimbi that is a concomitant of large-scale tropical cyclogenesis. This admittedly speculative subject is included because it may be worth pursuing. Furthermore, Malkus, Ronne, and Chafee (1961) observed that for the inner rain area of radius 200 n mi in Hurricane Daisy (1958), in the pre-hurricane stage there were 60 cumulonimbi that covered one percent of the area; during intensification two and one-half percent of this area was covered by cumulonimbi. In the mature stage there was a persistence of cumulonimbus patterns relative to the center, and there were 200 cumulonimbi that covered four percent of the area. By studying measurements in Hurricanes Carrie (1957) and Cleo (1957), as well as Daisy, Gentry (1964) estimated five-percent coverage of the inner eighty-nautical-mile-radius area by cumulonimbi (total cloud coverage in this area was under 25%). Gentry adds that both aircraft penetration and radar returns suggest that the area of strong convective activity is larger in more intense hurricanes and is located closer to the center.

It might also be speculated [in contrast to conjectures by Gray (1972, p. 64) on conditions for tropical cyclogenesis] that for a synoptic-scale pressure difference to develop not all the air ascending in the cumulonimbi should locally subside, but some of the descent should be deferred to a larger horizontal scale.

4.4 The Time-Dependent Flowfield during Intensification

Although the energetics of the core may need revision, Carrier's modeling of the dynamics in the outer portions during the transient phase should remain substantially unchanged. Thus, this portion is developed here in some detail.

For axisymmetric intensification, a set of uniformly valid equations for the dynamics is now given in cylindrical polar coordinates. If subscripts r, z, and t denote partial differentiation, for a noninertial coordinate system rotating at the speed of the normal component of the rotation of the earth (i.e., normal to a plane locally tangent to the earth at about 15° latitude so $\Omega \doteq 0.06$ rad hr^{-1}):

$$u_t + u\,u_r + w\,u_z - 2\Omega v - (v^2/r) + \rho^{-1}\,p_r = \nu\,u_{zz}, \qquad (44)$$

$$(rv)_t + u(rv)_r + w(rv)_z + 2\Omega ru = \nu\,(rv)_{zz}, \qquad (45)$$

$$p_z + \rho g = 0, \qquad (46)$$

and

$$(ru)_r + rw_z = 0. \qquad (47)$$

These equations hold in $0 \leq r \leq r_0$, $0 \leq z \leq z_1$ where, again, z_1 is the top of the storm (a slippery lid) and r_0 is the outer edge. The boundary-initial conditions of relevance here are

$$r = r_0:\; u = v = 0;\; z = 0:\; u = v = w = 0; \qquad (48)$$

$$t = t_0:\; u, v \text{ small.} \qquad (49)$$

There is angular momentum in the initial flowfield in an inertial frame of reference, and this is conserved with the anticipated radial inflow toward the axis of symmetry; the picture of the flow is that given in Fig. 19.

For convenience the following notation is adopted. The radial, azimuthal, and axial velocity components will be denoted by capitals (U, V, W) in I and by lower-case letters, u, v, w in the frictional boundary layer II.

From results given earlier for the mature stage boundary-layer dynamics, and from other considerations developed below for the transient boundary layer, it is anticipated that $\Psi = r\ V$ is a linear function of r^2, and independent of z; then there is a downflux into the frictional boundary layer II from the supply region I which is independent of r to lowest order. It follows from (45) and (47), together with (48) and (49):

$$\Psi = r\ V(r,t) = \Omega \left[R_o^{\ 2} - R^2(t) \right] \frac{r_o^{\ 2} - r^2}{r_o^{\ 2} - R^2} , \tag{50}$$

$$\Phi = r\ U(r,t) = R\ \dot{R}\ \frac{r_o^{\ 2} - r^2}{r_o^{\ 2} - R^2} , \tag{51}$$

and

$$W(r,z,t) = \frac{2\ R\ \dot{R}}{r_o^{\ 2} - R^2(t)}\ z , \tag{52}$$

where $R(t_o) = R_o$. The associated pressure field from (44) and (46) is discussed later. This solution holds in I, i.e., in $R \leq r \leq r_o$. At $t = t_o$, in I, i.e., in $R_o \leq r \leq r_o$, $\Psi = 0$. The particles at $r = R_o$ at $t = t_o$, at a later time $t > t_o$ lie at $r = R < R_o$. Typically, $r_o =$

$O(1000\ \text{mi})$, $R_o = O(200\ \text{mi})$, and $(R_o/12) \leq R(t) \leq R_o$.

The next step is to develop the boundary layer in II that lies under the swirling flow in I given by (50) - (52). The equations for the viscous layer are taken to be the linear transient parabolic equations appropriate for low-Rossby-number flow. While the inequality $(\phi^2 + \psi^2) << \Omega^2 R^4$ may not formally hold everywhere in the flowfield, it is widely valid and, in addition, from experience with the mature quasisteady storm, the key output, the boundary-layer divergence, is adequately given by linearized theory. Hence,

$$\nu\, \phi_{zz} - \phi_t = -2\Omega\psi, \tag{53}$$

$$\nu\, \psi_{zz} - \psi_t = 2\Omega\phi, \tag{54}$$

where

$$\left.\begin{aligned} \phi = r[u(r,z,t) - U(r,t)],\ \psi = r[v(r,z,t) - V(r,t)], \\ w = W(r) + w'(r,z,t)\ . \end{aligned}\right\} \tag{55}$$

If $\chi = \phi + i\psi$,

$$\nu\, \chi_{zz} - \chi_t = 2\Omega i \chi, \tag{56}$$

$$\phi_r + \phi_r + r\, w'_z = 0\ , \tag{57}$$

subject to

$$\chi(r,z=0,t) \rightarrow -(\phi + i\psi),\ \chi(r,z\rightarrow\infty,t) \rightarrow 0,\ \chi(r,z,0) = 0\ . \tag{58}$$

If $(r_o^2 - R^2) \rightarrow r_o^2$, an excellent approximation, then it is convenient to define

$$F(r,t) \equiv F_1(t) \left(1 - \frac{r^2}{r_o^2}\right) \equiv [R_o^2 - R^2(t)] \left(1 - \frac{r^2}{r_o^2}\right). \tag{59}$$

Carrier (1971a) studied the special form $F_1(t) = R_o^2 \exp(\alpha t)$. For the more general form (59), by Laplace transformation (Dergarabedian and Fendell, 1972, pp. 56-57), the relevant boundary-layer divergence is

$$w(r,z\to\infty,t) = \frac{\nu^{1/2}}{r_o^2} \operatorname{Re} \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F_1(s) \frac{s - 2\Omega i}{(s + 2\Omega i)^{1/2}} [\exp(st)]\, ds$$

$$\doteq \left(\frac{\nu}{\Omega}\right)^{1/2} \frac{1}{r_o^2} \left(\frac{6}{7} \frac{\partial}{\partial t} - \Omega\right) (R_o^2 - R^2) \tag{60}$$

for the R(t) of practical interest. The divergence, a function of time only, is largest in magnitude in the steady state; if $R \to (R_o/12)$ typically,

$$w(r,z\to\infty) \doteq - \left(\frac{R_o}{r_o}\right)^2 (\nu\Omega)^{1/2} = 0(10^{-3} \text{ mph}) .$$

The net volumetric flux down into II, and hence (in an incompressible model) up into the core, Q(t) is given by

$$Q = Q_o \left(\frac{6}{7\Omega} \frac{\partial}{\partial t} - 1\right) \left[1 - \left(\frac{R}{R_o}\right)^2\right] \tag{61}$$

where the quasisteady discharge into the base of the core III is given by

$$Q_o = \pi(r_o^2 - R^2) (R_o/r_o)^2 (\nu\Omega)^{1/2} . \tag{62}$$

The previously discussed requirement for compatibility between (1) the nonlinear spin-up of the inviscid flow in I under conservation of angular momentum and (2) the pressure deficit generated between a column in the core and a column in the ambient owing to partial flushing of the core with boundary layer air driven radially inward by the outer inviscid flow, is now made quantitative. It is necessary to model the radial and azimuthal velocity components in the core; kinematic and continuity considerations suggest that, at least in the lower regions of III, for this purpose it is adequate to take

$$r\,u = \phi = r^2\,\dot{R}/R\ , \tag{63}$$

$$r\,v = \psi = \Omega(R_o^{\,2} - R^2)\ r^2/R^2\ . \tag{64}$$

Accordingly (44) is integrated from $r = 0$ to $r = r_o$ at some fixed z above the boundary layer; the profiles for u and v are given by (63) and (64) for $0 \le r \le R$ and by (50) and (51) for $R \le r \le r_o$. The term $u\,u_r$ in (44) is a perfect differential and yields precisely zero; $w\,u_z$ yields zero because u is independent of z and w is small anyway. It follows that the time-dependent, Coriolis, centrifugal, and pressure gradient terms yield, for $\rho \doteq$ const. and neglecting terms of $O(R^2/r_o^{\,2})$ relative to those of order unity,

$$p(r_o,\ z{=}0,t) - p(r{=}0,\ z{=}0,t) = \frac{\rho\,\Omega^2\,R_o^{\,2}\,(R_o^{\,2} - R^2)}{R^2}\left\{1 + \left[2\,\ln\left(\frac{r_o}{R}\right) - 1\right]\frac{R^2}{R_o^{\,2}}\right\}$$

$$+\ \rho\,\frac{d}{dt}\left[R\,\dot{R}\,\ln\left(\frac{R}{r_o}\right)\right]. \tag{65}$$

The last term on the right-hand side is characterized by the inverse square of the time for intensification; this time probably is much larger than the time for equilibration of the boundary layer. Thus normally the time-derivative term can be neglected.

However, when inertial oscillations associated with the continued radial influx and rebound of air in I occur, a shorter time-scale enters and the time-derivative term must be retained. Carrier (1971b, p. 157) suggests that the time-scale of the oscillation is $O(4R_f^2/\Omega R_o^2)$ where R_f is the final equilibrium position of the eyewall $R(t)$. For typical values, with $R_o \doteq 250$ mi, this time scale is 20 min. In fact, holding the convection-core pressure deficit from ambient fixed at about 35 mb, holding ρ at its ambient sea-level value, letting $R_o \doteq 250$ mi and $r_o \doteq 1000$ mi, one finds from (70) -- by dropping the temporal derivative -- that $R = R_f = O(35$ mi$)$. Restoring the temporal term, adopting as initial conditions $R = O(35$ mi$)$ and $\dot{R} \doteq -\Omega(R_o^2 - R^2)/2R$ (Carrier, 1971b, pp. 152 and 155), and holding the core pressure deficit relative to ambient fixed at 35 mb (near-quasisteady conditions), one finds from (70) that R decreases to about 16 mi in about a half-hour before $\dot{R} = 0$. Thus, numerical values confirm the inertial oscillation period suggested by Carrier. Such oscillations probably reinforce the eye, rather than create it, as noted earlier (Section 4.2). The oscillation of the eyewall is dwelled upon because it seems to be the first proposal of a mechanism <u>internal</u> to the hurricane that might explain a crudely periodic variability in intensity of 10 to 20% in peak winds, on the time-scale of a few hours; such variability appears to be observed sometimes, and attribution to changes in the ambient atmospheric and underlying oceanographic states may not be always satisfactory.

The increased low-level radial inflow toward the center of the developing cyclone seems consistent with the previously discussed reorganization of the statistically steady cumulonimbus activity which helps maintain the ambient enthalpy transport process. Such radial inflow implies a stronger circulation (or vorticity), and this implies a decreased mixing of updraft air with surrounding air; the result would be a decreased density in the updraft region and hence a decreased pressure. As cumulonimbus activity increases in the central region of the incipient cyclone, there is probably diminution of such activity in the surrounding region. However, the downdraft in the surrounding region should differ little from ambient since the total low-level inflow into the whole region will not have changed appreciably. Thus, the anticipation is that the flowfield presented in this section probably remains substantially correct, within the axisymmetric approximation.

5. CONCLUDING REMARKS

Several viewpoints not universally accented have been adopted in this review. The existence of, and necessity for, a greatly augmented oceanic heat source in the inner regions of a hurricane as the basic sustaining mechanism has been questioned. The validity of the hot-tower thesis of heating from the tropical cumulonimbus has been scrutinized. The incomplete understanding of the processes and time scales of tropical cyclogenesis and intensification has been emphasized. The conviction that improved quantitative understanding of the tropical ambient is prerequisite to further progress on the deepening of tropical disturbances has been stated. Reasons for reservations concerning the probable success of current hurricane seeding techniques have been offerred.

This unconventionality hopefully conveys two major points to the reader. First, any serious worker in hurricanes would be very remiss not to consult the referenced works directly and decide for himself whether or not the viewpoints adopted here are in fact valid. Second, this review on tropical cyclones describes a vital subject in rapid flux to which many basic physical and mathematical contributions are still to be made. This is not a field in which only exquisite refinements to elaborate computer programs remain. In fact, many valuable theoretical contributions in recent years have been achieved by Carrier and Gray via approximate solution of simplified models with only modest computing, as opposed to attempting direct numerical simulation of the full conservation laws. The meteorologist with insight will hopefully agree that the tropical cyclone is neither a closed subject, nor a subject to which only those with access to very sophisticated computing facilities can aspire to contribute.

APPENDIX A

ESTIMATING THE KINETIC ENERGY AND WATER CONTENT OF HURRICANES

To estimate very quickly and roughly the total kinetic energy content of a mature hurricane of moderate intensity, one must evaluate I where

$$I = \frac{1}{2} \int_V \rho\, v^2\, dV = \pi \int_{z=0}^{z=z_1/2} dz \int_{r=r_e}^{r=r_o} \rho [v(r)]^2\, r\, dr\ . \qquad \text{(A.1)}$$

The speed is virtually all swirl, and about half the nine-mile-high, axisymmetric storm is rotating like (say) a potential vortex. The swirl is (suppose) 100 mph at the edge of the (nonrotating) eye r_e = 30 mi, and the storm extends out to r_o = 500 mi. The density may be set to a constant average value of 5 x 10^{-4} g cm^{-3}. Then

$$I \doteq \frac{\pi \rho z_1}{2} (v_{max})^2\, r_e^{\,2} \int_{r_e}^{r_o} \frac{dr}{r} \doteq 10^{24} \text{ ergs}\ . \qquad \text{(A.2)}$$

Battan (1961, p. 21) characterizes the kinetic energy of a hurricane as 10^{10} kilowatt-hours, or 3.6 x 10^{23} ergs.

An <u>upper bound</u> on the rainfall per day for a hurricane hovering over a spot is now calculated according to the Carrier model. The vertical velocity down into the boundary layer in the mature stage is

$$w(r,\ z \to \infty,\ t \to \infty) \doteq -\frac{R_o^{\,2}}{r_o^{\,2}} (\nu\Omega)^{1/2}\ . \qquad \text{(A.3)}$$

The volumetric flux down into the boundary layer is, for $r_o^2 >> R^2$,

$$Q_o = \pi r_o^2 w(r, z \to \infty, t \to \infty) \doteq - \pi R_o^2 (\nu\Omega)^{1/2} ; \qquad (A.4)$$

this is also the flux up into the eyewall. The mass of air per unit time is given by ρQ_o where ρ is fairly constant at a fixed height. The water content of this mass of air, which falls out as precipitation, is given in mass per time by

$$J = \pi\rho R_o^2 (\nu\Omega)^{1/2} Y(z = z_t) , \qquad (A.5)$$

where z_t is a typical ambient-profile height for characterizing the mass fraction of water vapor for the flux into the eyewall. For $\rho \doteq 1.1 \times 10^{-3}$ g cm^{-3}, $\nu \doteq 2 \times 10^5$ cm^2 sec^{-1}, $\Omega \doteq 6.25 \times 10^{-2}$ hr^{-1}, and [from the data of Jordan (1957)] $Y(z_t = 1000 \text{ ft}) \doteq 1.65 \times 10^{-2}$, $J \doteq 1.8 \times 10^{11}$ g sec^{-1}. If this falls entirely in an annulus from 20 to 30 miles, one gets 4.3×10^{-3} g cm^{-2} sec^{-1}, or since 1 g of liquid water occupies about 1 cm^3, about 370 cm day^{-1}. The record rainfall over a spot in one day is associated with a hurricane: 117 cm (Baguio, Phillippines in July 1911). Of course, for a hurricane translating at 20 mph, no point is likely to lie under the eyewall for more than an hour, and $(370/24) \doteq 15$ cm hr^{-1}.

Because of the flood-spawning torrential rains over Virginia associated with Hurricane Camille, it is interesting to consider whether the water-vapor content of this hurricane at landfall is consistent with the large total rainfall over the Southeast. From isohyetal maps published by Schwarz (1970) it may be crudely estimated that in eastern Kentucky, Virginia, and West Virginia an area of 150 mi x 180 mi received an average of five inches of rain and an area of 100 mi x 360 mi received three inches.

In addition 5×10^4 mi^2 of Mississippi, Alabama, Louisiana and Tennessee, received an average of three inches. This totals 2.58×10^{15} g, which is probably a conservative estimate. From Jordan's data one can estimate that the Caribbean hurricane contained

$$Q = \pi r_o^2 \int_0^{z_1} \rho Y \, dz = \pi r_o^2 I_1 \qquad (A.6)$$

where $\rho Y = \rho_v$ = density of water vapor. For an August-September ambient, $I_1 = 4.71 \text{ g cm}^{-2}$. Equating Q to the total rainfall, one finds $r_o \doteq 260$ mi. This is an interesting value in that Camille was a small intense hurricane with hurricane winds extending outward from the center about 60 mi and gale winds, 110 mi (Meyer, 1970, p. 5). However, the results here indicate merely that $r_o = O(260 \text{ mi})$ satisfies a conservative estimate of the total precipitation requirement; it is <u>not</u> intended to imply r_o could necessarily be taken so small, though if one fits Carrier's proposed swirl profile [given below (30)] to the data, $r_o = O(225 \text{ mi})$.

APPENDIX B

THE MOIST ADIABAT

Because of the significant role it plays in understanding tropical cyclones, a derivation of the relation describing the moist adiabatic process seems worth including. The following is based on unpublished notes of G. F. Carrier of Harvard University.

When moist air expands adiabatically, the temperature eventually decreases to a value at which the water vapor density reaches its saturation level. For all further decreases in temperature, condensation occurs. Here it is taken that the condensed water has zero volume (a good approximation), and that the liquid precipitates out as it is formed.

An air-vapor mixture is at temperature T, the mass of air in volume V being m_a, and the saturated vapor mass in V being m_v. The corresponding densities are $\rho_a \equiv (m_a/V)$ and $\rho_v \equiv (m_v/V)$; the pressure $p = p_a + p_v$. A relationship between T and p (say) as T changes is obtained by studying the moist adiabatic process in several steps (Fig. 22). The mixture is separated into two fluids (dry air and vapor), to be later recombined; this step is taken to require no expenditure of work.

Initially, the mass m_v of vapor occupies the volume V as in the top box of Fig. 22. That volume is decreased by a process in which dm_v of the vapor is condensed while it and the remaining vapor are at temperature T and pressure p_v. Thus, the new volume (that of the second box) is $V[1 - (dm_v/m_v)]$. The work done by external forces during this process is $(p_v \, V \, dm_v/m_v)$, and is indicated by the arrow to the left of the boxes.

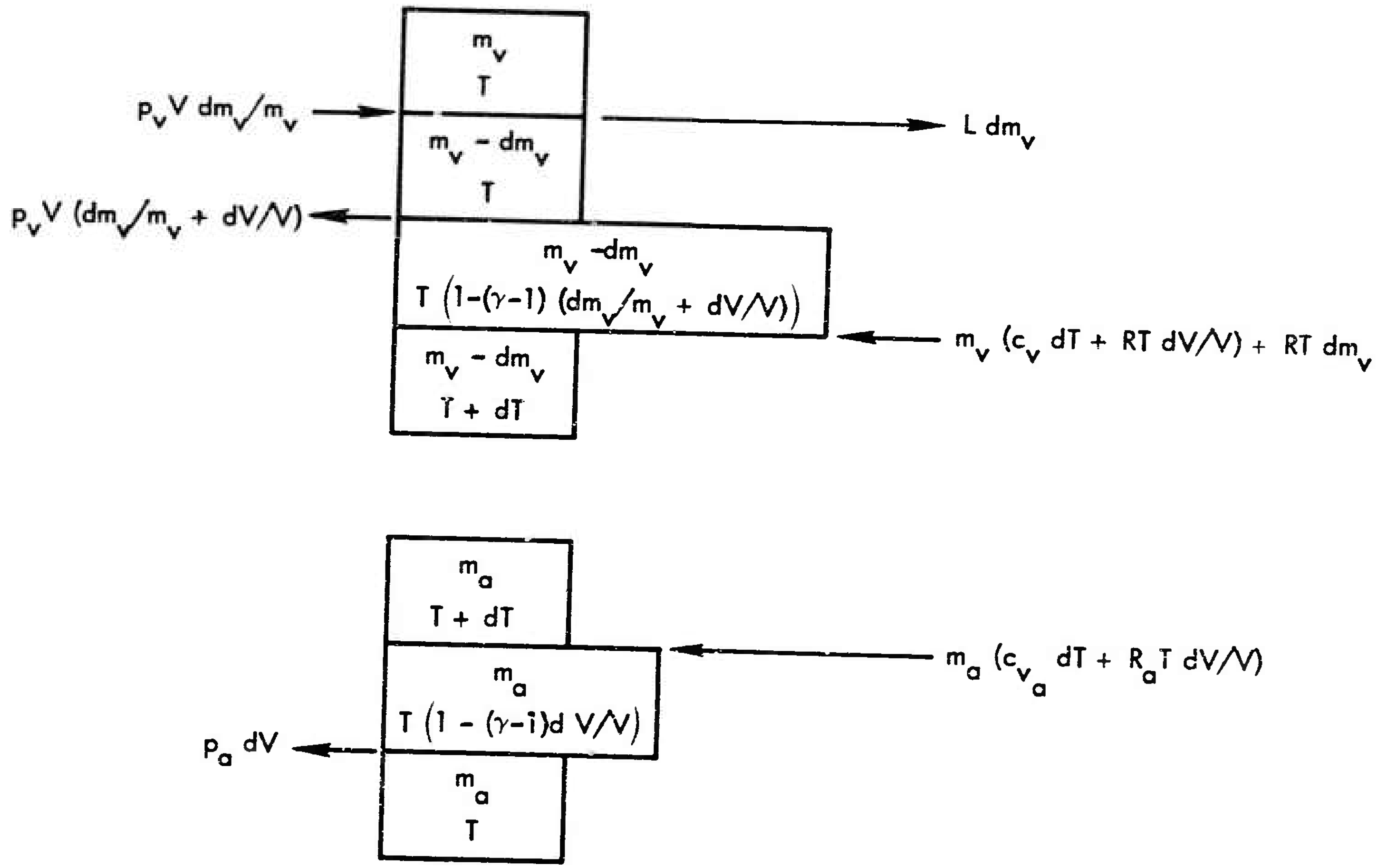

Fig. 22. Schematic diagrams of the sequence of thermodynamic steps by which the equation describing the moist-adiabatic process is derived. The upper diagram sketches operations on the water vapor component, and the lower diagram sketches operations on the dry air component; the diagrams read from top to bottom. Arrows on the left of the diagrams denote work done on the system (arrow in) and work done by the system (arrow out). Arrows on the right of the diagrams indicate heat removed from the system (arrow out) and heat added to the system (arrow in).

The heat gained $L\ dm_v$ is shown at the right. The vapor now expands adiabatically and isentropically to volume $(V + dV)$ as shown in box 3; the new temperature T' is given by

$$\frac{T'}{T} = \left\{\frac{V[1 - (dm_v/m_v)]}{V + dV}\right\}^{\gamma_v - 1} = 1 - (\gamma_v - 1)\left(\frac{dm_v}{m_v} + \frac{dV}{V}\right) + \cdot\cdot\cdot . \quad \text{(B.1)}$$

In the third step, the gas in box 3 is heated to temperature $(T + dT)$, as indicated in box 4. The heat that must be added during this process is

$$(m_v - dm_v)(c_v)_v \left[dT + (\gamma_v - 1)\left(\frac{dm_v}{m_v} + \frac{dV}{V}\right)T\right]$$

$$= m_v\left[(c_v)_v\ dT + R_v T\ \frac{dV}{V}\right] + R_v T\ dm_v + \cdot\cdot\cdot \quad , \quad \text{(B.2)}$$

where $(c_v)_v$ is the heat capacity at constant volume of the vapor, $(c_p)_v$ the heat capacity at constant pressure of the vapor, $R_v = (c_p)_v - (c_v)_v$, and $\gamma_v = (c_p)_v/(c_v)_v$.

The dry air, on the other hand, starts off in the bottom box of volume V with mass m_a at temperature T. It expands isentropically to volume $(V + dV)$ so that its temperature becomes $T[1 - (\gamma_a - 1)(dV/V)]$. Henceforth $\gamma_v = \gamma_a = \gamma$. The dry air is then heated at constant volume to temperature $(T + dT)$ by the addition of heat in the amount $m_a[(c_v)_a\ dT + R_a T(dV/V)]$.

Thus at the end of the process, a vapor mass $(m_v - dm_v)$ and a dry air mass m_a occupies a volume $(V + dV)$ at temperature $(T + dT)$. By definition, the process undergone will have been the moist adiabatic one if the net heat added is zero, and if the final $(\rho_v + d\rho_v)$ is the saturation value for the final temperature $(T + dT)$:

$$m_a\left[(c_v)_a\, dT + R_a T\,\frac{dV}{V}\right] + m_v\left[(c_v)_v\, dT + R_v T\,\frac{dV}{V}\right] + R_v T\, dm_v = -L\, dm_v\ ;$$

(B.3)

$$\rho_v = \frac{m_v}{V} = \rho_v(T) \qquad \text{(B.4)}$$

where $\rho_v(T)$ is the function that describes saturation density.

By the equation of state for dry air,

$$\frac{dV}{V} = -\frac{d\rho_a}{\rho_a} = \frac{dT}{T} - \frac{dp_a}{p_a}\ . \qquad \text{(B.5)}$$

But $(d\rho_v/\rho_v)$ is comprised of two contributions: that due to change of volume, and that due to the loss of mass via condensation. Thus

$$\frac{\rho_v + d\rho_v}{\rho_v} = \frac{m_v - dm_v}{m_v}\,\frac{V}{V + dV} = 1 - \frac{dm_v}{m_v} - \frac{dV}{V} + \cdot\cdot\cdot\ . \qquad \text{(B.6)}$$

Hence, after division by V, upon use of the equations of state,

$$\left\{\rho_a[(c_v)_a + R_a]\, dT - dp_a\right\} + \left\{\rho_v[(c_v)_v + R_v]\, dT - dP\right\} = -\frac{L}{V}\, d(\rho_v\ V)\ , \qquad \text{(B.7)}$$

or

$$\left[\frac{\gamma}{\gamma - 1}(p_a + P) + \left(\frac{L}{R_v} - T\right)\frac{dP}{dT}\right]\frac{dT}{T} = \left(1 + \frac{LP}{R_v T\, p_a}\right) dp_a\ , \qquad \text{(B.8)}$$

where P(T) is the saturation vapor pressure.

Alternatively, the moist adiabat may be derived as a special case of the equation for conservation of energy, written as the sum of internal plus kinetic energy for a multicomponent mixture. Here Cartesian tensor notation will be used instead of vector notation, and a comma denotes partial differentiation. Furthermore, the symbolism previously adopted in the main text for describing the dynamics of an effectively one-component gas will not be used here. The air velocity is denoted u_i, with $u_i u_i = q^2$ and $u_i \delta_{i3} = w$, where the gravitational acceleration $g_i = -g\, \delta_{i3}$. The water vapor velocity is $(u_i + v_i)$. The vapor quantities ρ_v, $(c_v)_v$, $(c_p)_v$, R_v and the air quantities ρ_a, $(c_v)_a$, $(c_p)_a$, R_a have the same designations as above. The specific internal energy of the air is e_a, and of the vapor e_v. The specific heat of the liquid water substance is c_ℓ. The heat lost by radiation and all other processes is Q, with units (mass/length-time^3). The conservation of energy for current purposes may be written (upon neglect of the kinetic energy contribution relative to internal energy contributions)

$$[\rho_a e_a + \rho_v e_v]_{,t} + [\rho_a e_a u_i + \rho_v e_v(u_i + v_i)]_{,i}$$
$$= -(\rho_a + \rho_v)\, gw + (\sigma_{ij} u_i)_{,j} + (kT_{,i})_{,i} - c_\ell T\omega - Q \qquad \text{(B.9)}$$

where σ_{ij} is the total stress tensor, k is the thermal conductivity, and

$$\rho_{v,t} + \rho_v(u_i + v_i)_{,i} = -\omega . \qquad \text{(B.10)}$$

In general, ω (the mass of water vapor condensed per unit time) must be specified explicitly or through a prescribed mechanism; when the fluid is saturated and $(dT/dt) < 0$, ω is known.

For steady flow with densities, pressures, and temperature dependent on the vertical coordinate z only, with negligible diffusion so $v_i = 0$ and $\sigma_{ij} = -p\, \delta_{ij}$, in the hydrostatic approximation $p_{,z} + \rho g = 0$, it follows from continuity that

$$(\rho_a w)_{,z} = 0 \ , \quad (\rho_v w)_{,z} = -\omega \ ; \tag{B.11}$$

hence,

$$\rho_a w e_{a,z} + \rho_v w e_{v,z} - \omega e_v + p w_{,z} + c_\ell T\omega = 0 \ . \tag{B.12}$$

With the aid of the continuity equations and the equations of state,

$$\rho_a w \left[(c_p)_a T\right]_{,z} + \rho_v w \left[(c_p)_v T\right]_{,z} - w p_{,z} = \omega(e_v + R_v T - c_\ell T) \ . \tag{B.13}$$

Since

$$e_v = c_\ell T + L(T) - R_v T \ , \tag{B.14}$$

and

$$\omega = -w \rho_a \left(\frac{\rho_v}{\rho_a}\right)_{,z} = -\frac{1}{V} d(\rho_v V) \ , \tag{B.15}$$

upon substitution and cancellation of the common factor w, one recovers (B.7).

PARTIAL LIST OF SYMBOLS

English Symbols

c_ℓ	heat capacity of liquid water
c_p	heat capacity at constant pressure
c_v	heat capacity at constant volume
e	internal energy
E	Ekman number, ν/Ψ_0
g	magnitude of gravitational acceleration, $\lvert g_i \rvert$
g_i	acceleration of gravity
H	total static enthalpy, $c_pT + gz + LY$
H_t	total stagnation enthalpy, $H + q^2/2$
L	specific latent heat of phase transition
m_i	molecular weight of species i
n	power of algebraic decay of swirl with radial distance
p	pressure
P	saturation vapor pressure
q	wind speed, $\lvert q_i \rvert$
q_i	velocity vector for a fluid in noninertial frame rotating with earth
Q	net volumetric flux per time; heat loss per volume-time
Q_0	steady-state net volumetric flux per time
r	radial coordinate in cylindrical polar coordinates
R	radial position of maximum azimuthal velocity v
R_i	gas constant for species i, $\overline{R}/m_i$

$\bar{R}$	universal gas constant
RH	relative humidity
t	time
T	temperature
u	radial velocity component of q_i
u_i	velocity vector for dry air
U	radial velocity component of q_i in the inviscid flow exterior to the frictional boundary layer
v	azimuthal velocity component of q_i
v_i	velocity vector for water vapor relative to dry air; velocity vector for a fluid in a noninertial system
V	azimuthal velocity component of q_i in the inviscid flow exterior to the frictional boundary layer; volume
w	vertical velocity component of q_i
W	vertical velocity component of q_i in the inviscid flow exterior to the frictional boundary layer
x	$\Omega r^2/\psi_0$; a factor in the moist adiabat based on $dH = 0$
Y	water vapor mass fraction, ρ_v/ρ
z	altitude above sea level

Greek Symbols

γ	c_p/c_v
ρ	density
ϕ	ru [sometimes, $r(u - U)$; sometimes, nondimensional]
Φ	rU
ψ	rv [sometimes, $r(v - V)$; sometimes, nondimensional]

Ψ	rV
σ	ratio of molecular weight of water vapor to dry air
σ_{ij}	total stress tensor
ν	kinematic viscosity (usually given turbulent transfer values)
ω	mass of water vapor condensed per volume-time
Ω	Coriolis parameter (component of the angular velocity of the earth perpendicular to a plane locally tangential to the sea surface in the tropics)
θ	azimuthal coordinate in cylindrical polar coordinates; potential-like enthalpy, $c_pT + gz$
θ_t	stagnation potential-like enthalpy, $\theta + q^2/2$
ζ	$z/E^{1/2}$

Superscript

$\cdot$	total derivative with respect to time
$\rightarrow$	vector (Gibbs notation)

Subscripts

a	dry air; ambient
c	at the axis of summetry at sea level
e	eyewall at sea level
i	vector (Cartesian tensor notation)
o	outer edge of hurricane; initial value; typical value
s	ambient at sea level
t	total
v	vapor

ACKNOWLEDGEMENT

The author is very deeply indebted to Prof. George F. Carrier of Harvard University for generous and indispensable guidance and assistance on every aspect of the work described here. He also wishes to thank Dr. Paul Dergarabedian of Aerospace Corporation for many stimulating discussions. However, full responsibility for all errors lies entirely with the author. All computations were programmed by Mr. Phillip S. Feldman, of TRW Systems. This study was initiated owing to the interest of Capt. Hugh Albers of the Interdepartmental Committee on Atmospheric Sciences (Department of Commerce); the encouragement and helpful suggestions of Drs. John Perry and Lawrence G. Roberts of ARPA Information Processing Techniques (Department of Defense), Drs. Robert Rapp and W. L. Gates of Rand Corporation, and Mr. James Murray of the Army Research Office (Durham) are gratefully acknowledged. Finally, the author wishes to thank Dr. H. E. Landsberg for the opportunity to present this paper. This work was carried out under contracts DAHC04-71-C-0025 and DAHC04-67-C-0015 with the U.S. Army Research Office (Durham, North Carolina). Although this work was sponsored in part by the Advanced Research Projects Agency of the Department of Defense under ARPA Order 1786, the views and conclusions contained in this paper are those of the author and should not be interpreted as necessarily representing the official policies, either expressed or implied, of ARPA or the U.S. Government.

FOOTNOTES

[1]For the East Coast of the United States, the hurricane-generated storm surge depends on storm direction, but can be roughly estimated from $s = 2.90 - 9.70 \times 10^{-2} (v)_{max} + 1.33 \times 10^{-3} (v)^2_{max}$, $74 < (v)_{max} < 149$, where $(v)_{max}$ is the maximum wind speed in mph and s is the surge above normal in ft [based on Saffir, H. S. (1973). Military Engineer 65, 4]. In addition to onshore wind-driven waves, with the arrival of a hurricane and persisting often for about a day after its passing along a coast with a sloping bottom (say, of inclination angle β), arise so-called edge waves. These waves travel parallel to the coast; their crests are normal to the coast. If the component of the hurricane translational velocity parallel to the coast is denoted U, the period of the waves is $(2\pi U/g \sin \beta)$ and the wavelength is $(2\pi U^2/g \sin \beta)$; typically for the East Coast of the United States the period is five-to-seven hours, and the wavelength is 400 km. The amplitude vanishes rapidly from the shore seaward, and is negligible at the distance of one wavelength; the amplitude is roughly given by the inverse barometer rule (one centimeter water rise per millibar of atmospheric pressure drop) so heights of two-to-three feet are typical [Munk, W., Snodgrass, F., and Carrier, G. (1956). Science 123, 127].

[2]According to data collected on 30 typhoons for 1960-1970, tropical storms approaching the Philippines with maximum winds in excess of 90 kts leave with winds reduced by 40-50%, while a storm with peak winds less than 90 kts experience only a 10-15% reduction recovered within a day after leaving

(and storms with peak winds under 60 kts typically undergo an intensification in crossing the islands). The average crossing time is 14.5 hr, with the weaker storms crossing more quickly [Brand, S., and Blellock, J. W. (1972). Changes in the characteristics of typhoons crossing the Philippines. Environmental Prediction Research Facility Technical Paper 6-72, Naval Postgraduate School, Monterey, California].

[3]The convective instability of the tropical ambient explains how cumulonimbi can occur, but cumulonimbi explain how high-static-enthalpy levels of the upper tropical troposphere can arise. This interdependence exemplifies the earlier remark on the link between the presence of clouds and the circumstances regarding stability.

[4]These authors emphasize that the ITCZ consists of westward propagating cloud clusters that pass at four-to-five-day intervals.

[5]Further discussion of hybrid (or semi-tropical) storms, which rely for energy source not only on latent heat release but also on baroclinicity associated with positioning of warm and cold air masses, is given by Spiegler (1972).

[6]Data on this matter appears contradictory. Using all recorded data for Atlantic hurricanes for 1961-1968, Sheets (1972) found no correlation between the size of the radar eye and storm intensity indicators (such as maximum wind speed or minimum sea level pressure); there was reportedly a slight tendency toward smaller eye diameters at lower latitudes. Questions concerning the use of radar by itself to determine the eye radius will be discussed in Section 3.4. In contrast, the eye diameter increased

typically from 20 n mi to 33 n mi, while the intensity typically decreased by 33%, according to data for typhoons crossing the Philippines during 1960-1970; furthermore, for typhoons near the Philippines the eye diameters for intense typhoons (maximum wind speed over 90 kts) are typically 13% smaller than for weaker typhoons (maximum wind speeds under 90 kts). Incidentally, if the size is determined by the average diameter of the closed surface isobar, the size of a storm decreases by 17% in area in crossing the Philippines, and near the Philippines the intense typhoon has a mean outer circulation diameter 60 n mi - 150 n mi greater than that of a weaker typhoon [Brand, S., and Blellock, J. W. (1972). Changes in the characteristics of typhoons crossing the Philippines. Environmental Prediction Research Facility Technical Paper 6-72, Naval Postgraduate School, Monterey, California].

[7]From "Tropical Meteorology" by Herbert Riehl. Copyright 1954 by McGraw-Hill Book Company. Used with permission.

[8]Most discussions emphasize the cold wake left in the upper oceanic layers by intense, slowly moving hurricanes; however, the reduction of surface-layer temperature by upwelling and mixing may be occurring directly under the hurricane. Black and Mallinger (1972, p. 74) find evidence of radial outflow and downwelling of warm upper-layer water for Hurricane Ginger (1971); the upwelling of colder water was found to reduce the sea-surface temperature by 4°C on one day, and by 2.5°C on another day when the hurricane was translating faster. Reduced latent and sensible enthalpy transfer from sea to atmosphere ensued, with a reduction in intensity. Thus, the interaction of a hurricane with the ocean may be such that

reduced transfer from the sea, not augmentation of transfer over ambient level, may occur (cf. Section 3.9.1).

[9]Kraus emphasizes that the Riehl-Malkus theory depends on an isothermal inflow layer and discusses whether frictional effects are fully accounted for in that theory: "The importance of [an] additional energy supply to air in the hurricane, before it ascends in the warm core, was stressed particularly by Malkus and Riehl (1960). The argument would be weakened somewhat if a (verbal) suggestion by Carrier was found to be true. Carrier maintains that surface friction must cause a deceleration and compression of a surface air parcel along its trajectory; this compression compensates to some extent for the expansion that would be associated otherwise with the externally imposed pressure reduction" [Kraus, E. B. (1972). "Atmospheric-Ocean Interaction." Clarendon Press, Oxford, England, p. 208].

[10]Black and Mallinger (1972, p. 64) characterize most models as giving a 50% reduction in maximum wind speed for a 2°C drop in sea-surface temperature. Doubt concerning such sensitive dependence of hurricane intensity on sea-surface temperature was discussed in Section 3.8. Recently the existence of a closed oceanic convection cell induced by hurricane winds, already discussed, has been further confirmed by additional measurements on Typhoons Trix (which was situated over the central South China Sea on October 23-24, 1952) and Wilma (which was similarly situated on October 28-29, 1952), and on other storms [Ramage, C. S. (1972). Weather 27, 484]. Ramage (1972, p. 491) notes: "When they struck the Philippines Trix and Wilma were greatly, equally intense typhoons. Trix reintensified over the South China Sea and Wilma probably did. Paired aircraft reconnaissances on the 23rd and 28th and the 24th and 29th. . . show little

intensity difference, in contrast to Ooyama's numerical hurricane model . . . in which a smaller surface temperature difference is associated with a significantly large intensity difference." Furthermore, the path for Wilma was much less influenced by oceanic cooling owing to the prior passage of Trix than the work of Brand (1971) (previously cited in Section 1.5) would suggest, even though Brand considered typhoons generally more separated in time. This is one more item of evidence suggesting that ocean temperatures are not solely determining concerning where hurricanes occur. Ramage (1972, p. 493) concludes: "The history of typhoon Wilma suggests that surface temperature may not always control storm intensity nor significantly affect storm movement, for Wilma moved along the cold water wake of Trix and remained intense until reaching Indochina."

[11]Typically five runs of the kind described, at two-hour intervals, starting from the radius of maximum winds and proceeding radially away from the center, characterize the so-called eyemod experiment. Most of the discussion is oriented toward this type of experiment. However, it may be noted that the seeding of Hurricane Ginger (1971) discussed below was a so-called rainsector experiment: all the water-containing clouds in a 45° sector from 50 n mi to 100 n mi from the center are seeded at about 22,000 ft to divert low-level inflow to premature eruption and high-level outflow outside the eyewall, with resultant dispersal of latent-heat energy over a wider region of the storm (Hawkins, Bergman, and Gentry, 1972). In concept, a rainsector experiment consists of four fifty-minute seedings, at fifty-minute intervals. The reason a rainsector, rather than eyemod, experiment was attempted on Ginger was that it was a large diffuse

storm (moderate gale winds out to 250 n mi, circulation out to 400 n mi) with only an ill-defined eyewall (maximum winds of about 60-70 kts at 40 n mi to 60 n mi from the center).

[12]Dr. R. Cecil Gentry, director of the National Hurricane Research Laboratory and of Project Stormfury, stated recently that budgetary reductions had grounded the four aircraft used in cloud-seeding experiments, and probably no full-scale attempts at hurricane modification would be undertaken in the next three years ("U.S. Hurricane Control Test Program Cut," Los Angeles Times, 4 February 1973, part 8, p. 5).

[13]Calculation that undilute lifting of the autumnal Caribbean ambient produces reduced temperatures at all tropospheric levels was also carried out by Riehl [Riehl, H. (1963). Science 141, 1001].

[14]Consistent with his concept that it is the compressional heating of the compensatory downdraft that warms the air, because heat directly released by cumulus convection is almost entirely expended in raising the buoyant column, Gray (Shea, 1972, pp. 118-122) suggests that turbulent diffusion and mixing conveys heat from the eye to warm the eyewall.

[15]From "Introduction to the Atmosphere" by Herbert Riehl. Copyright 1972 by McGraw-Hill Book Company. Used with permission.

REFERENCES

Agee, E. M. (1972). Monthly Weather Rev. 100, 733.

Alaka, M. A. (1968). Climatology of Atlantic tropical storms and hurricanes. ESSA Technical Report WB-6, U.S. Govt. Printing Office, Washington, D.C.

Anonymous (1972). Weatherwise 25, 174.

Anthes, R. A. (1971). Monthly Weather Review 99, 261.

Anthes, R. A. (1972). Monthly Weather Rev. 100, 461.

Anthes, R. A., and Johnson, D. R. (1968). Monthly Weather Rev. 96, 291.

Anthes, R. A., Rosenthal, S. L., and Trout, J. W. (1971). Monthly Weather Rev. 99, 744.

Arakawa, H. (1954). Papers Meteorol. Geophys. (Tokyo) 5, 114.

Aspliden, C. I. (1971). On energy distribution in the tropical troposphere. Department of Meteorology Report, Florida State University, Tallassee, Florida.

Atkinson, G. D. (1971). Forecasters guide to tropical meteorology. Technical Report 240, USAF Air Weather Service, Scott AFB, Illinois.

Bates, J. R. (1970). Quart. J. Roy. Meteorol. Soc. 96, 677.

Bates, J. R. (1972). Quart. J. Roy. Meteorol. Soc. 98, 1.

Battan, L. J. (1961). "The Nature of Violent Storms." Doubleday, Garden City, New York.

Battan, L. J. (1969). "Harvesting the Clouds -- Advances in Weather Modification." Doubleday, Garden City, New York.

Black, P. C. and Anthes, R. A. (1971). J. Atmos. Sci. 28, 1348.

Black, P. G. and Mallinger, W. D. (1972). The mutual interaction of Hurricane Ginger and the upper mixed layer of the ocean. Appendix D, Project Stormfury Annual Report 1971, National Hurricane Research Laboratory, Coral Gables, Florida.

Black, P. G., Senn, H. V., and Courtright, C. L. (1972). Monthly Weather Rev. 100, 208.

Boyd, D. W., Howard, R. A., Matheson, J. E., and North, D. W. (1971). Decision analysis of hurricane modification. Project 8503 Final Report, Stanford Research Institute, Menlo Park, California.

Bradbury, D. L. (1971). The filling over land of Hurricane Camille, August 17-18, 1969. Satellite and Mesometeorology Research Project

Research Paper 96, Department of the Geophysical Sciences, University of Chicago, Chicago, Illinois.

Brand, S. (1970a). Geographic and monthly variation of very large and very small typhoons of the western North Pacific Ocean. Technical Paper 13-70, Naval Weather Research Facility, Norfolk, Virginia.

Brand, S. (1970b). J. Appl. Meteorol. 9, 433.

Brand, S. (1971). J. Appl. Meteorol. 10, 865.

Burggraf, O. R., Stewartson, K., and Belcher, R. (1971). Phys. Fluids 14, 1821.

Byers, H. R. (1944). "General Meteorology," 2nd ed. McGraw-Hill, New York.

Carlson, T. N. (1969). Monthly Weather Rev. 97, 716.

Carpenter, T. H., Holle, R. L., and Fernandez-Partagas, J. J. (1972). Monthly Weather Rev. 100, 451.

Carrier, G. F. (1970). SIAM Rev. 12, 175.

Carrier, G. F. (1971a). J. Fluid Mech. 49, 133.

Carrier, G. F. (1971b). J. Fluid Mech. 49, 145.

Carrier, G. F., Hammond, A. L., and George, O. D. (1971). J. Fluid Mech. 47, 145.

Charney, J. (1971). _In_ "Mathematical Problems in the Geophysical Sciences. I. Geophysical Fluid Dynamics (Lectures in Applied Mathematics, Vol. 13)" (W. H. Reid, ed.), Vol. 1, pp. 355-368. American Mathematical Society, Providence, Rhode Island.

Charney, J. G. and Eliassen, A. (1964). _J. Atmos. Sci. 21_, 68.

Cox, J. L. and Jager, G. (1969). A satellite analysis of twin tropical cyclones in the western Pacific. Technical Memorandum WBTM SOS 5, ESSA Space Operations Support Division, Silver Spring, Maryland.

Day, J. A. (1966). "The Science of Weather." Addison-Wesley, Reading, Massachusetts.

De Angelis, R. M. (1969). _Weatherwise 22_, 173.

Denny, W. J. (1972). _Monthly Weather Rev. 100_, 276.

Deppermann, C. E. (1937). Wind and rainfall in selected Philippine typhoons. Manila Weather Bureau Central Observatory Report, Bureau of Printing, Manila, Philippines.

Dergarabedian, P. and Fendell, F. (1970). _J. Astronaut. Sci. 17_, 218.

Dergarabedian, P. and Fendell, F. (1971). Tornado and hurricane thermo-hydrodynamics. Technical Report, TRW Systems, Redondo Beach, California.

Dergarabedian, P. and Fendell, F. (1972a). Tropical cyclones. Report 18524-6000-RO-01, TRW Systems, Redondo Beach, California.

Dergarabedian, P. and Fendell, F. (1972b). J. Astronaut. Sci. 20, 9.

Eliassen, A. and Kleinschmidt, E. (1957). In "Encyclopedia of Physics (Geophysics II)" (S. Flügge and J. Bartels, eds.), Vol. 48, pp. 1-54. Springer, Berlin, Germany.

Emmons, H. W., and Ying, S.-J. (1967). In "Eleventh Symposium (International) on Combustion," pp. 475-486. Combustion Institute, Pittsburgh, Pennsylvania.

Erickson, C. O., and Winston, J. S. (1972). J. Appl. Meteorol. 11, 23.

Estoque, M. A. (1971). Hurricane modification by cloud seeding. Rosentiel School of Marine and Atmospheric Science Report, University of Miami, Coral Gables, Florida.

Fletcher, R. D. (1955). Bull. Am. Meteorol. Soc. 36, 247.

Fujita, T. T. (1972). Use of ATS pictures in hurricane modification. Satellite and Mesometeorology Research Project Research Paper 106, Department of the Geophysical Sciences, University of Chicago, Chicago, Illinois.

Fukuta, N. (1972). Modification of hurricanes by cloud seeding. Report, Denver Research Institute, Denver, Colorado.

Garstang, M. (1967). Tellus 19, 492.

Garstang, M. (1972). Bull. Am. Meteorol. Soc. 53, 612.

Garstang, M., La Seur, N. E., Warsh, K. L., Hadlock, R., and Peterson, J. R. (1970). Am. Scientist 58, 482.

Gates, W. L., Batten, E. S., Kahle, A. B., and Nelson, A. B. (1971). A documentation of the Mintz-Arakawa two-level atmospheric general circulation model. Report R-877-ARPA, Rand Corp., Santa Monica, California.

Geissler, J. E. (1972). J. Atmos. Sci. 29, 240.

Gentry, R. C. (1964). A study of hurricane rainbands. National Research Project Report 69, U.S. Weather Bureau, Washington, D. C.

Gentry, R. C. (1969a). Bull. Am. Meteorol. Soc. 50, 404.

Gentry, R. C. (1969b). Project Stormfury, 1969. Paper presented at Sixth Technical Conference on Hurricanes, Miami, Florida.

Gentry, R. C. (1970). Science 168, 473.

Gentry, R. C. (1971a). Science J. 7, 49.

Gentry, R. C. (1971b). Bull. Am. Meteorol. Soc. 52, 775.

Gentry, R. C., and Hawkins, H. F. (1971). A hypothesis for modification of hurricanes. Appendix B, Project Stormfury Annual Report 1970, National Hurricane Research Laboratory, Coral Gables, Florida.

Godshall, F. A. (1968). Monthly Weather Rev. 96, 172.

Gray, W. M. (1968). Monthly Weather Rev. 96, 669.

Gray, W. M. (1972a). A diagnostic study of the planetary boundary layer over the oceans. Department of Atmospheric Science Paper 179, Colorado State University, Fort Collins, Colorado.

Gray, W. M. (1972b). Cumulus convection and larger-scale circulations. Part 3. Broadscale and mesoscale circulations. Department of Atmospheric Science Paper 190, Colorado State University, Fort Collins, Colorado.

Greenspan, H. P. (1968). "The Theory of Rotating Fluids." University Press, Cambridge, England.

Grossman, G., and Rodenhuis, D. (1972). The effects of release of latent heat on the vorticity of a tropical storm over land. Technical

Note BN-722, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland.

Haltiner, G. J. (1971). "Numerical Weather Prediction." John Wiley, New York.

Haurwitz, B. (1935). Monthly Weather Rev. 63, 45.

Hawkins, H. F. (1971). Monthly Weather Rev. 99, 427.

Hawkins, H. F., Bergman, K. H., and Gentry, R. C. (1972). Report on seeding of Hurricane Ginger. Appendix B, Project Stormfury Annual Report 1971, National Hurricane Research Laboratory, Coral Gables, Florida.

Hawkins, H. and Rubsam, D. T. (1968). Monthly Weather Rev. 96, 617.

Hess, S. L. (1959). "Introudction to Theoretical Meteorology." Henry Holt, New York.

Hidy, G. M. (1972). Bull. Am. Meteorol. Soc. 53, 1083.

Holton, J. R., Wallace, J. M., and Young, J. A. (1971). J. Atmos. Soc. 28, 275.

Holton, J. R. (1972). "An Introduction to Dynamic Meteorology." Academic, New York.

Hubert, L. and Timchalk, A. (1969). Monthly Weather Rev. 97, 382.

Hughes, L. A. (1952). J. Meteorol. 9, 422.

Jennings, G. (1970). "The Killer Storms -- Hurricanes, Typhoons, and Tornadoes." J. B. Lippincott, Philadelphia, Pennsylvania.

Johnson, D. H. (1969). In "The Global Circulation of the Atmosphere" (G. A. Corby, ed.), pp. 113-136. Royal Meteorological Society, London, England.

Jordan, C. L. (1957). A mean atmosphere for the West Indies area. National Hurricane Research Project Report 6, U.S. Department of Commerce, Washington, D. C.

Keenan, J. H. and Keyes, F. G. (1936). "Thermodynamic Properties of Steam Including Data for Liquid and Solid Phases." John Wiley, New York.

Kuettner, J. P. (1971). Tellus 23, 404.

Kurihara, Y. (1971). Bull. Am. Meteorol. Soc. 52, 769.

Leipper, D. F. (1967). J. Atmos. Sci. 24, 182.

Leipper, D. F., and Volgenau, D. (1972). J. Physical Oceanog. 2, 218.

Lieb, H. S. (1972). Project Stormfury director reveals results of Hurricane Ginger seeding experiments. News Release NOAA 72-4, U.S. Department of Commerce, Washington, D. C.

Lopez, R. E. (1972a). Cumulus convection and larger-scale circulations. Part 1. A parametric model of cumulus convection. Department of Atmospheric Science Paper 188, Colorado State University, Fort Collins, Colorado.

Lopez, R. E. (1972b). Cumulus convection and larger-scale circulations. Part 2. Cumulus and meso-scale circulations. Department of the Atmospheric Science Paper 189, Colorado State University, Fort Collins, Colorado.

Lorenz, E. N. (1966). Am. Scientist 54, 402.

Lorenz, E. N. (1967). "The Nature and Theory of the General Circulation of the Atmosphere." World Meteorological Organization, Geneva.

Mahrt, L. J. (1972a). J. Atmos. Sci. 29, 1477.

Mahrt, L. J. (1972b). J. Atmos. Sci. 29, 1485.

Malkus, J. S. (1958). J. Meteorol. 15, 337.

Malkus, J. S. (1960). In "Cumulus Dynamics (Proceedings of the First Conference on Cumulus Convection" (C. E. Anderson, ed.), pp. 65-84. Pergamon Press, New York.

Malkus, J. S. (1962). In "The Sea" (M. N. Hill, ed.), Vol. 1, pp. 88-294. Interscience, New York.

Malkus, J. S. and Riehl, H. (1960). Tellus 12, 1.

Malkus, J. S., Ronne, C., and Chafee, M. (1961). Tellus 13, 8.

Mallinger, W. D. (1971). Project Stormfury experimental eligibility in the western North Pacific. Appendix L, Project Stormfury Annual Report 1970, National Hurricane Research Laboratory, Coral Gables, Florida.

Malone, M. J. and Leimer, D. R. (1971). Estimation of the economic benefits to DOD from improved tropical cyclone forecasting. Report, Headquarters Military Airlift Command, Scott AFB, Illinois.

Manabe, S., Holloway, Jr., J. L., and Stone, H. M. (1970). J. Atmos. Sci. 27, 580.

Matano, H. and Sekioka, M. (1971). J. Meteorol. Soc. Japan 49, 736.

Matthews, D. A. (1971). Ice-phase modification potential of cumulus clouds in hurricanes. Appendix H, Project Stormfury Annual Report 1970, National Hurricane Research Laboratory, Coral Gables, Florida.

McWilliams, J. C. (1971). The boundary layer dynamics of symmetric vortices. Ph.D. thesis, Harvard University, Cambridge, Massachusetts.

Meyer, J. W., ed. (1970). Airborne severe storm surveillance. Vol. 1. Summary and recommendations. Technical Note 1970-43, Lincoln Laboratory, Lexington, Massachusetts.

Meyer, J. W. (1971a). <u>(MIT) Technology Rev. 74</u>, 53.

Meyer, J. W., ed. (1971b). Airborne severe storm surveillance. Vol. 2. Reports of working panels. Technical Note 1970-43, Lincoln Laboratory, Lexington, Massachusetts.

Miller, B. I. (1958). <u>J. Meteorol. 15</u>, 184.

Miller, B. I. (1967). <u>Science 157</u>, 1389.

Murray, F. W. (1967). <u>J. Appl. Meteorol. 6,</u> 203.

Ogura, Y. (1964). <u>J. Atmos. Sci. 21</u>, 610.

Oliver, V. J. and Anderson, R. K. (1969). <u>Bull. Am. Meteorol. Soc. 50</u>, 702.

Ooyama, K. (1969). J. Atmos. Sci. 26, 3.

Ooyama, K. (1971). J. Meteorol. Soc. Japan 49, 744.

Orton, R. (1970). Monthly Weather Rev. 98, 541.

Östlund, H. G. (1968). Tellus 20, 577.

Östlund, H. G. (1970). J. Geophys. Res. 75, 2303.

Palmén, E. (1948). Geophysica 3, 26.

Palmén, E., and Riehl, H. (1957). J. Meteorol. 14, 150.

Palmén, E. and Newton, C. W. (1969). "Atmospheric Circulation Systems." Academic, New York.

Penner, S. S. (1972). Astronautica Acta 17, 351.

Perlroth, I. (1967). Tellus 19, 258.

Perlroth, I. (1969). Tellus 21, 230.

Phillips, N. A. (1970). In "Annual Review of Fluid Mechanics" (M. Van Dyke, W. G. Vincenti, and J. V. Wehausen, eds.), Vol. 2, pp. 251-292. Annual Reviews, Palo Alto, California.

Ramage, C. S. (1971). "Monsoon Meteorology." Academic, New York.

Rapp, R. (1970). Climate modification and national security. Paper presented at 1970 Meteorological Technical Exchange Conference, Rand Corp., Santa Monica, California.

Revesz, Jr., W. (1971). Comparison of effects of various tropical storms on the vertical temperature of the ocean using pictorial representation. Master's thesis, Naval Postgraduate School, Monterey, California.

Riehl, H. (1954). "Tropical Meteorology." McGraw-Hill, New York.

Riehl, H. (1963). J. Atmos. Sci. 20, 276.

Riehl, H. (1969a). Weather 24, 288.

Riehl, H. (1969b). Bull. Am. Meteorol. Soc. 50, 587.

Riehl, H. (1972a). "Introduction to the Atmosphere," 2nd ed. McGraw-Hill, New York.

Riehl, H. (1972b). J. Appl. Meteorol. 11, 613.

Riehl, H. and Malkus, J. (1961). Tellus 13, 181.

Rosenthal, S. L. (1970). Monthly Weather Rev. 98, 643.

Rosenthal, S. L. (1971a). Monthly Weather Rev. 99, 414.

Rosenthal, S. L. (1971b). Monthly Weather Rev. 99, 767.

Rosenthal, S. L. (1971c). Hurricane modeling at the National Hurricane Research Laboratory (1970). Appendix C, Project Stormfury Annual

Report 1970, National Hurricane Research Laboratory, Coral Gables, Florida.

Sanders, F. and Burpee, R. W. (1968). J. Appl. Meteorol. 7, 313.

Schwarz, K. (1970). Monthly Weather Rev. 98, 851.

Shea, D. J. (1972). The structure and dynamics of the hurricane's inner core region. Department of Atmospheric Science Paper 182, Colorado State University, Fort Collins, Colorado.

Sheets, R. C. (1972). Some statistical characteristics of the hurricane eye and minimum sea-level pressure. Appendix I, Project Stormfury Annual Report 1971, National Hurricane Research Laboratory, Coral Gables, Florida.

Shuleykin, V. V. (1970). Izv. Acad. Sci. USSR Atmospheric and Oceanic Physics Ser. (English Transl.) 6, 729.

Shuleykin, V. V. (1972). Izv. Acad. Sci. USSR Atmospheric and Oceanic Physics Ser. (English Transl.) 8, 1.

Simpson, R. H. (1971). The decision process in hurricane forecasting. NOAA Technical Memorandum NWS SR-53, National Weather Service Southern Region Headquarters, Fort Worth, Texas.

Simpson, R. H. and Frank, J. R. (1972). Monthly Weather Rev. 100, 256.

Simpson, R. H. and Malkus, J. S. (1964). Scientific Am. 211, 27.

Simpson, R. H. and Pelissier, J. M. (1971). Monthly Weather Rev. 99, 269.

Spark, E. H. (1971). Bull. Am. Meteorol. Soc. 52, 574.

Speigler, D. B. (1972). Bull. Am. Meteorol. Soc. 53, 1174.

Sundqvist, H. (1970). Tellus 22, 359.

Sundqvist, H. (1971). Bull. Am. Meteorol. Soc. 52, 574.

Varga, G. (1971). Some observations on the structure of shallow typhoons. Master's thesis, Naval Postgraduate School, Monterey, California.

Vollsprecht, R. (1972). Bull. Am. Meteorol. Soc. 53, 888.

Walker, J. M. (1972). Weather 27, 178.

Warsh, K. L., Echternacht, K. L., and Garstang, M. (1971). J. Physical Oceanog. 1, 123.

White, R. M. (1972). Bull. Am. Meteorol. Soc. 53, 631.

Wright, R. (1969). Tellus 21, 409.

Yamasaki, M. (1968). Papers Meteorol. Geophys. (Tokyo) 19, 559.

Yamasaki, M. (1969). Papers Meteorol. Geophys. (Tokyo) 20, 289.

Yanai, M. (1964). Rev. Geophys. 2, 367.

Zipser, E. J. (1969). J. Appl. Meteorol. 8, 799.

CAPTIONS TO FIGURES

Fig. 1. Typical paths of tropical storms and cyclones.

Fig. 2. NASA photograph of Hurricane Gladys west of Naples, Florida taken from Apollo 7 on October 7, 1968. The maximum wind speed was 65 kts at this time, and the tropopause was at 54,000 ft.

Fig. 3. NOAA radar photograph of mature hurricane revealing lower-level rainbands outside the region of overcast (O/C) and upper-level cirrus clouds of the outflow.

Fig. 4. Schematic features of the atmosphere in winter. Latent heat evaporated from tropical oceans to the lower-level trades is carried toward the ITCZ by the sketched meridional flow; the air rises as the ITCZ is approached, and the latent heat ultimately is released as sensible heat and gravitational potential energy. Height of the local tropopause is marked. Portions of the general circulation are less well defined in summer. TA denotes tropical air; MLA, midlatitude air; PA, polar air; STJ, subtropical jet stream; and PFJ, polar front jet stream. (By permission from E. Palmén and C. W. Newton, Atmospheric Circulation Systems, Academic Press, p. 569.)

Fig. 5. Left, the equivalent potential temperature θ_e as a function of height z and pressure p, from measurements near Barbados in the Lesser Antilles in July-August, 1968. Four characteristic weather types are represented: average (solid), suppressed convection

(dotted), moderately enhanced convection (long dashes), and strongly enhanced convection (short dashes). (From Warsh, Echternacht, and Garstang, 1971, p. 127; with permission of the Americal Meteorological Society.) Right, soundings taken at Gan Island in the Indian Ocean (00°41'S; 73°09'E). The heavier curves (solid for static temperature, dashed for dew point) are the average for 42 wet days; the lighter curves are the average for 113 dry days. Measurements were made in Julys 1960-1964. (From Johnson, 1969, p. 122; with permission of the Royal Meteorological Society.)

Fig. 6. The average total static temperature (H/c_p) for the Northern Hemisphere in winter. The curve marked A denotes the locus of the minimum of (H/c_p); free convection can most readily occur below level A, and only undilute ascent can continue much above A; undilute ascent continues to level B, where sea-level values are recovered. (By permission from E. Palmén and C. W. Newton, Atmospheric Circulation Systems, Academic Press, p. 574.)

Fig. 7. Static temperature T, potential-temperature-like measure (θ/c_p), total static temperature (H/c_p), and the total static temperature for a hypothetical atmosphere saturated at the actual local temperature and pressure ($\tilde{H}/c_p$) vs. height and pressure for the West Indies ambient for September. Based on data given by Jordan (1957). A parcel at height z_1 will become buoyant at z_0 where (H/c_p) at z_1 exceeds ($\tilde{H}/c_p$) at $z_0 (>z_1)$; the inequality is satisfied for air in the planetary boundary layer, and little else.

Fig. 8. The total static temperature (H/c_p) over the ocean in the equatorial trough and in the tropics near the subtropics, as a function of pressure. Both typical cumulus loci (cu) and also cumulonimbus loci (cb) are noted. (Based on, with permission, E. Palmén and C. W. Newton, Atmospheric Circulation Systems, Academic Press, p. 575.)

Fig. 9. NASA photograph taken in June, 1965 from Gemini IV of cumulus, cumulonimbus, and cirrus clouds over the Pacific Ocean off the western coast of Central America; the view is northeast toward Mexico.

Fig. 10. This conjectured configuration of a mature hurricane with rough order-of-magnitude dimensions is not drawn to scale. The sub-domains are: I, throughput supply, a region of rapid swirl and very slow downdraft; II, frictional boundary layer; III, eyewall; and IV, eye. Across the boundary layer there is about a 100 mb drop, and across I, a further drop of about 200 mb; the pressure at the top of the hurricane is about 150 mb, i.e., the top is near the tropopause. The eye-eyewall interface is taken to slope outward, though the effect may not be so pronounced as sketched.

Fig. 11. Typical vertical profiles of thermodynamic state variables for the West Indies in September, based on data by Jordan (1957). The relative humidity RH, the water vapor pressure p_v, the total gas density ρ, the water vapor density ρ_v, and temperature T are plotted. Jordan's data for the relative humidity stop at 400 mb.

Fig. 12. The ambient pressure-temperature curve is based on data for the Caribbean in September given by Jordan (1957). The moist adiabat is based on having sea-level ambient air undergo dry-adiabatic expansion until saturation, then moist adiabatic ascent; the resulting sea-level pressure deficit from ambient gives a lower bound on the central pressure deficit under the adopted model. An upper bound on the deficit is furnished by having the air that rose on the so-called moist adiabat, recompressed dry adiabatically back down to sea level. Altitudes are associated with the thermodynamic states by use of hydrostatics and the equations of state for dry air and water vapor.

Fig. 13. The density vs. altitude relation for each of the three thermodynamic loci of Fig. 12 are presented. The density variations from ambient within the storm are less than 30%, and speeds are far below sonic everywhere.

Fig. 14. The three curves are analogous to those of Fig. 12, except that the moist adiabat, while still taken to be based on ambient sea-level values for relative humidity and temperature, is now computed from an iteratively-determined sea-level pressure consistent with the definition of the top of the storm as an isobaric, isothermal surface of constant altitude.

Fig. 15. In this replotting of Fig. 14, explicit pressure vs. altitude curves for the ambient, the moist-adiabat taken to characterize the eyewall, and the dry-adiabat recompression taken to characterize the eye are presented. The moist adiabat is computed by

the iterative procedure described in the caption to Fig. 14.

Fig. 16. Nondimensional results for the frictional boundary layer obtained by the method of weighted residuals, from Dergarabedian and Fendell (1972). The divergence $w(x,\zeta \to \infty)$ and the volumetric flux $\delta = -\int_0^\infty \phi(x,\zeta)\, d\zeta$ are presented for the impressed swirl $\Psi = 1 - x/x_1$, $x_1 \doteq 20$, believed pertinent to a hurricane outside the eyewall. Except near the axis where nonlinear inertial effects dominate, the linear Ekman layer result, $w(x,\zeta \to \infty) \doteq \Psi_x/2^{1/2}$, is an excellent approximation to the numerical results. The volumetric flux $\delta(x)$ is thus linearly proportional to $(x_1 - x)$ to good approximation. Normalized residuals indicate large errors for $x < 3$, and discount the premature eruption as an artifact of the method. The solution by Carrier (1971a) for small x indicates the adequacy of the linear Ekman result for the divergence to within a factor of two. Since $\nu \doteq (1/75)$ mi^2/hr and $\Omega \doteq (1/16)$ rad/hr, dimensionally the results imply the boundary layer is of thickness $O(\nu/\Omega)^{1/2} = O(1$ mi$)$, the downflux into the boundary layer is $(2\nu\Omega)^{1/2}\, w(x,\zeta \to \infty) = O(5 \times 10^{-3}$ mi/hr$)$, and the volumetric flux erupting up the eyewall is $(2\pi^2\, \Psi_0^{\,2}\, \nu/\Omega)^{1/2}\, \delta(x) = O(7 \times 10^3$ mi^3/hr$)$ where Ψ_0 characterizes the eyewall relative angular momentum per unit mass. The implication is that the fluid initially in the boundary layer sustains the hurricane for about a week, and the fluid lying above the boundary layer (with supplementary replenishment of moisture from the ocean) can readily sustain the hurricane for more than a week more.

Fig. 17. Measurements of minimum sea-level pressure vs. local sea-surface temperature for several tropical cyclones (from Gentry, 1969a, p. 406; reproduced with permission of the American Meteorological Society.)

Fig. 18. The central sea-level pressure is plotted as a function of the local sea-surface temperature for Hurricane Ginny, 21-28 October 1963, which passed over the Gulf Stream on 24 October (from Perlroth, 1967, p. 266).

Fig. 19. In this schematic view by Carrier of the flow configuration and circumferential velocity distribution in an intensifying tropical depression at some early time $t = t_1$ (say), the interface C-C between the new and initial air in the core is idealized as horizontal for convenience. However, it is easy to show that for all but very modest circulations (i.e., except for angular velocities less than three times ambient), a rigidly rotating core (which implies uniform updraft velocity at the top of the nonlinear frictional layer) could not accept all the fluid pumped through the Ekman boundary layer. Thus a uniform ascent in the core is dynamically impossible, and in fact ascent in the eyewall is concentrated in cumulonimbi. For the gross balances being discussed, many points can be made without accounting for such refinements in eyewall structure, though ultimately details of the structure are crucial. (From Carrier, 1971b, p. 146.)

Fig. 20. Intensification from tropical depression to hurricane has progressed to a more advanced stage in this schematic diagram,

holding at $t = t_2 > t_1$, according to Carrier's model. The magnitude of the maximum swirl is increased and its position lies closer to the axis of symmetry. (From Carrier, 1971b, p. 146.)

Fig. 21. Schematic picture by Carrier of the flow configuration which prevails when the radius of maximum swirl R is increasing and an incompletely formed eye is being filled with relatively dry and motionless air, which sinks down from the top of the storm under dry-adiabatic compression. With the formation of an eye lighter in weight than the eyewall, the terminal stages of intensification and the beginnings of quasisteady mature-stage structure are realized. (From Carrier, 1971b, p. 150.)

Fig. 22. Schematic diagrams of the sequence of thermodynamic steps by which the equation describing the moist-adiabatic process is derived. The upper diagram sketches operations on the water vapor component, and the lower diagram sketches operations on the dry air component; the diagrams read from top to bottom. Arrows on the left of the diagrams denote work done on the system (arrow in) and work done by the system (arrow out). Arrows on the right of the diagrams indicate heat removed from the system (arrow out) and heat added to the system (arrow in).